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Optimal acquisition policy for a supply network with discount schemes and uncertain demands.

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OPTIMAL ACQUISITION POLICY FOR A SUPPLY NETWORK WITH DISCOUNT SCHEMES AND UNCERTAIN DEMANDS

by

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Abstract

Supplier selection – also referred to as procurement function – is a vital component of Supply Chain Management. The supplier selection decision affects a company's ability to compete in the market place, because purchasing-related costs frequently account for a large percentage of a product's costs, and the purchasing decision may involve a long-term contract as well.

The uncertainty in production caused by customer demands, and complex discount offers from a variety of potential suppliers/vendors all tend to complicate the procurement decision. While these two factors have been addressed individually in previous research, they have not commonly been considered simultaneously.

This study uses a mathematical programming approach in which a series of Mixed Integer Non-Linear Programming (MINLP) models are developed to represent a supply network for a manufacturer dealing with various quantity or volume discount schemes from suppliers, as well as incorporating uncertain product demands that follow Normal distributions. Furthermore, the manufacturer's optimal acquisition policy and production level are obtained simultaneously by solving the models with an objective of maximizing the expected value of the manufacturer's profit.

Although complicated by the employment of an integration function, the mathematical models are solved by a GAMS program with integrated SBB, CONOPT, MINOS, and SNOPT solvers working in collaboration. This research is one of the few studies in this field to use commercial optimization software for

solving such complex mathematical models. The MINLP models and the GAMS solution program are applied in two real-world cases, and the preliminary results justify the capabilities of both the mathematical models and the GAMS solution program. Numerical analysis supports the managerial implications regarding the acquisition policy, and the comparison between the quantity discount and the volume discount.

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Table of Contents

List of Tables

List of Figures

Chapter 1: Introduction

The first section of this chapter provides a brief overview of Supply Chain Management (SCM) topics that include SCM activities and the development, implementation, and industrial impact of SCM. The second section is an introduction to this thesis research project.

1.1 General Overview

Supply Chain Management (SCM) has been actively researched in both academic and practical domains for the past several decades. The topic has received enough attention from researchers and practitioners to prove that the SCM field deserves our attention considering a successful SCM system can deliver high efficiency and competition to businesses and organizations. With a global economy, and an increasingly volatile and dynamic market, the competition has become fierce. It follows that the success of Supply Chain Management is critical for business survival. From a broader point of view, with a strategic SCM system scarce resources can be more effectively applied to the development of our common world.

SCM serves as an umbrella process for the overall improvement of a business [Poirier, 2004]. It covers all activities from the acquisition of raw materials, to the delivery of finished goods or services. A supply chain affects every aspect of how companies organize and operate – a fact that supports Taylor's (2004) statement, "the way you manage the supply chain can make or break your business." Supply chain optimization guides the execution of all management activities to assure maximum customer satisfaction with a minimum investment or cost. That is why SCM is currently the most popular business performance improvement strategy and methodology [Taylor, 2004].

Many researchers and practitioners have tried to define SCM in a more accurate and comprehensible way. The technical definition can be easily found in any book that covers Supply Chain Management. Generally speaking, Supply Chain Management is a series of activities whose purpose is to plan and control the flow of material, information, and finance from the acquisition of raw materials to the delivery of the finished goods – even including the return of products at the expiration of their life cycles in some industrial areas. These activities are executed – in an integrated and coordinated manner – by various business entities that get involved in the supply chain, such as suppliers of raw materials, manufacturers, wholesale distributors, and retailers. The objective of Supply Chain Management, as stated in *From Mind to Market,* "...is all about having the right product in the right place, at the right price, at the right time and in the right condition" [Blackwell, 1997].

In a broad sense, Supply Chain Management is the coordination of several legally separated organizational units that might be economically dependent, working together to make their products more competitive. In the present business world, the competition between two products does not stem from two individual companies, but from two supply chain systems that complete a series of tasks in order to deliver products or services to their customers. Under these circumstances, SCM becomes more complicated when you consider that a successful SCM system means that profits should be created for every entity in the chain, simultaneously. However, some of said entities have conflicting objectives.

In a narrow sense, Supply Chain Management consists of intra-organizational activities for a multi-national company with several plant sites, its own raw material provider, and its own international distributor. Compared with the former situation, it is relatively easy to execute a Supply Chain Management system within a single organization, since there is only one top-level management committee [Taylor, 2004].

Consider the history of SCM. Prior to the 1980s, most organizations worked independently of their suppliers. The birth of SCM as an initiative that integrated external partners sprang from the grocery industry's efficient consumer response program. The most publicized SCM efforts, however, were at Wal-Mart. In the late 1980s, Wal-Mart installed the bar code scanning system in all of its stores and began updating its inventory data at points of sale. As a result, a large number of suppliers were pushed into this supply chain integration in order to compete with Wal-Mart's efficiency until, by the mid to late 1990s, the importance of SCM was widely recognized [Sherer, 2005].

Information plays a critical role in both the planning and execution of all SCM functions. Advancements in information technology have promoted the development of Supply Chain Management to some degree. Without it, the synchronized communication between two entities of a SCM system — as well as the sharing of information between them — would be impossible, even for an intra-organizational supply chain system. The deployment of Electronic Data

Interchange (EDI) as a convenient mechanism in particular has allowed companies to change information in a timely manner.

In order to efficiently implement SCM functions, numerous Information Technology systems have been developed to support the implementation of SCM with vendors like *i2 technologies*, *Manugistics Group*, and *PeopleSoft*. So far, SCM has evolved as an integral part of business management systems, mingling with Material Requirement Planning (MRP), Enterprise Resource Planning (ERP), and other Business Planning and Control Systems (BPCS). More information about SCM systems is available at the website – http://supplychain.ittoolbox.com.

1.2 Proposed Research

1.2.1 Research Topic

There are two factors that contribute to the complexity of Supply Chain Management issues [Simchi-Levi et al., 2004]. The first factor is the broadness of a supply chain system. In some supply chains, even the supplier's supplier or the customer's customer needs to be taken into account, since they have an impact on SCM's common goal. The implementation of SCM engages different business units with conflicting objectives, which makes an SCM problem a multi-objective optimization problem in terms of satisfying everyone involved in the chain.

The second factor is the uncertainty that exists throughout a supply chain. While the uncertainty of market demand is the root cause of many other uncertainties upstream, trucks also break down during transportation, and machines crash during the production process — making production capacity and transportation time essentially random in nature. Uncertainty is actually an internal property of the real world, and forecasting cannot eliminate it. While forecasting might be as close to reality as is possible, it will never equal it. However, SCM is about planning for the future, which makes uncertainty the most difficult element to handle in all SCM projects.

Supply Chain Management encompasses major issues such as strategic planning, demand forecasting, supplier selection and raw material procurement, production planning and scheduling, inventory control, distribution, and reverse logistics. Each activity – based on its property — can be classified into two categories: strategic, and operational SCM [Sabri and Beamon, 1999]. The strategic category includes plant site location and other long-term business planning movements. The operational category refers to mid-term or short-term planning activities such as MRP, daily production planning, monthly production planning, and sales planning.

One may ask why the procurement function plays such a large role in a manufacturer's SCM activities. In a typical manufacturing firm, the procurement and supply chain costs make up close to 50% of the cost of goods sold (COGS), while the manufacturing contributes 30% [Wincel, 2003]. It stands to reason that any improvement in these two areas would provide a significant opportunity for profit improvement. For example, a 20% gross profit business requires $5 of increased sales to equal the profit effect of $1 of procurement savings [Wincel, 2003]. In addition, all in-bound supply processes are executed by the procurement function.

The procurement activity occurs at the early stage of a forward supply chain, therefore it is important to deal with the uncertainty in the procurement function appropriately so that the activities downstream will benefit as a result. In recent supply chain optimization research, many quantitative models utilize statistical knowledge to handle the uncertainty problem. However, some papers only use the average value and other statistical parameters to simplify the problem — an unsatisfying approach in areas where high accuracy is required.

This thesis explores procurement issues in a supply network where a manufacturer is faced with uncertain demands for multiple products and different discount schemes from multiple suppliers for the provision of raw materials. The project seeks an optimal acquisition policy, and an optimal production level for the manufacturer in order to maximize its profit. The acquisition policy includes which suppliers to choose for the purchase of raw materials and how to allocate order quantities among those selected suppliers.

1.2.2 Research Methodology and Solution

There is a surplus of research on the procurement function in the Supply Chain Management domain [Kraljic, 1983; Ganeshan, 1999; Weber, 2000; etc.]. In some papers, the procurement function is analyzed as a decision-making process with all factors considered by decision-making tools like Data Envelopment Analysis (DEA). More frequently, quantitative models are developed for the final stage of the decision-making process.

This research project is an extension of the work done by Kim et al. (2002) and Zhang (2004). A series of Mixed Integer Non-Linear Programming models are developed for dealing with the different supply network situations described in the last section: different supplier discount schemes (quantity vs. volume) and varied manufacturer/buyer purchasing rules (split, no split, and conditional split allowed for the purchase of each raw material). The optimization models are solved by using commercial GAMS (General Algebraic Modeling System) software with its integrated solvers.

The mathematical models and the solution programs are applied in two real-world cases and fundamental results are obtained to verify the capability of the GAMS program, and justify the models. In addition, further numerical results support practical implications regarding procurement policies.

1.2.3 Organization of Thesis

Chapter 2's literature review of previous research covers important SCM topics, such as supplier selection and procurement, inventory management, discount models, and more. Chapter 3 provides mathematical models paired with detailed explanations. Chapter 4 presents the GAMS program solution approach and the numerical analysis. Finally, Chapter 5 draws conclusions from this research, and suggests directions for future work.

Chapter 2: Literature Review

This chapter is organized by theme into five sections: Supply Chain Management, supplier selection and procurement, inventory management, miscellaneous SCM topics, and discount model reviews. These topics are included in this chapter because of their conceptual ties with this thesis.

2.1 General Topics in SCM

Many of the studies that discuss Supply Chain Management attempt to cover the whole picture – methodology, analysis, process, movements, and development.

Beamon (1998) concluded that increasing attention is placed on the performance, design and analysis of the supply chain as a whole. The supply chain is an integrated manufacturing process, and encompasses two basic courses of action: the production planning and inventory control processes, and the distribution and logistics process. In the interests of specifics, the sub-processes of each of these courses of action are also listed in the paper. All processes interact with each other. The supply chain performance measurement is used to evaluate the efficiency and effectiveness of a supply chain system. There are qualitative measurements such as customer satisfaction, flexibility, information and material flow integration, and effective risk management; and quantitative measurements such as cost-based, customer response-based, fill rate maximization, product lateness minimization, customer response time minimization, and lead-time minimization.

Beamon (1998) sorts supply chain models into four categories based on modeling approach, which is, in turn, driven by the nature of the problem's input and objectives.

- Deterministic analytical models: the objective is to minimize the total cost, maximize the profit, or even maximize the supply chain performance;
- Stochastic analytical models: at least one of the variables is unknown, and is assumed to follow a particular probability distribution (Most models in the SCM field belong to these two categories);
- Economic models: built based on game theory knowledge (Few papers address this kind of model);
- Simulation models: these models evaluate the effect of supply chain strategy performances on demand amplification. The objective of a simulation model is to find the most effective strategy for a varying demand pattern. Simulation models are commonly developed to verify developed mathematical models.

Beamon (1998) also suggested that different methods applied in the SCM field should be evaluated comprehensively in terms of the accuracy of a problem representation, the time needed to collect and prepare data for the analysis, the solution time, the transparency of methodology and results, and the flexibility of the solution.

Sabri and Beamon (1999) pointed out that almost all strategic level models in the SCM field are deterministic. On the operational level, most of the models are not integrated with a consideration of all entities' benefits, even when the uncertainty is handled properly.

There is an absence of SCM research focusing on the integrated SCM considering the total benefit of all entities to all entities of the supply chain. Sabri and Beamon (1999) developed a comprehensive model that integrates both strategic and operational level SCMs. Four echelons are considered at the strategic level: suppliers, plants, distribution centers, and customer zones. One sub-model aims to minimize the total cost of these four echelons, while the other sub-model tries to maximize the flexibility of the supply chain – specifically plant volume flexibility and distribution volume flexibility. There are three sub-models at the operational level that correspond to the supplier control echelon, the plant echelon, and the distribution echelon – each of which minimizes the total cost of one echelon.

The analytical method and the non-linear programming technique are used for solving sub-models. The whole integrated model is solved iteratively until the optimal SCM performance is obtained. The output from one level, strategic or operational, is used to improve the other level as input while the e-constraint method is applied to deal with the multiple-objective problem. This complex model covers and integrates almost all SCM aspects on both strategic and operational levels. However, a fixed variable is employed in this model to represent an uncertain demand, so the uncertainty factor in SCM is not managed properly in this study.

Yusuf et al. (2004) explored the relationship between Supply Chain Management practices and manufacturing or business objectives. Seven dimensions of supply chain practice are set as research factors in the conceptual model, based on three supply chain practice patterns – traditional, lean, and agile. A multiple regression analysis is conducted with the data collected from questionnaires spanning 600 large companies in the UK. The study concludes that a long-term commercial relationship with customers and suppliers is desired, and meanwhile the data integration is widely popular since these two factors have strongest relationships with business objectives. The author suggests that lean and agile supply chain practices should be integrated in order to generate a greater synergy in their impact on manufacturing or business.

However more literature in the SCM field tends to be pertinent in individual aspects of the supply chain, rather than addressing SCM in a holistic fashion. In other words, most of the research only considers the benefits of one entity. For example, in many inventory models a manufacturer's profit is the only objective. Literatures regarding supplier selection and inventory control are reviewed in the following sections.

2.2 Supplier Selection and Procurement

Kraljic (1983) states, "Purchasing (an operating function) has evolved into supply management (a strategic one)." From this point of view, it is clear that the procurement function gains increasing importance in the Supply Chain Management system. Meanwhile, with outsourcing activities becoming more and more popular in the modern industrial world, companies find themselves dependent on their outsourcing partners and the purchasing function becomes more important to business success. The purchasing issue has been studied widely in both the academic and practical worlds. Many researchers have attempted to develop a systematic approach to automate the purchasing decision-making process.

Ganeshan (1999) concludes that the research done in the vendor selection policy field supports two trends. With the Just-In-Time (JIT) manufacturing philosophy, maintaining a stable and long-term relationship with a few suppliers has gained a positive reputation because it can lead to continuous improvement in the quality of the suppliers' product, due to the learning curve effect and a lower relationship management cost. On the other side, with the development of Electronic Data Interchange (EDI), the supplier relationship management cost becomes relatively minor. Maintaining multiple suppliers enables the buyer/manufacturer to split the purchase among those selected vendors. This reduces the buyer's inventory management cost, especially when the manufacturer is faced with uncertain demands. Moreover, it also decreases the effective lead-time when the lead-times from suppliers become uncertain. All this results in a trade-off for a manufacturer to determine the optimal number of suppliers to employ.

Kraljic (1983) summarized that the supplier selection problem is a multi-criteria issue. The four most important criteria are: price, product quality, vendor's delivery reliability, and capacity (Vendor and supplier are interchangeable in this report). Most mathematical models in this area use a linear weighted multi-objective programming technique when dealing with the four criteria. Quality is represented by a number, which is obtained from historical data as a percentage of good-quality products out of all products. The delivery reliability criterion is handled in the same way. Kraljic (1983) criticized that only one compromised solution is usually obtained with this method. However, it has been proven that "bad" weights might occasionally produce good solutions and "good" weights might produce bad solutions. In addition, the purchasing team in a real company may want a group of good solutions first, then more evaluation would be made among those solutions with consideration for other non-quantitative decision variables.

Karpak et al. (1999) first introduced the application of Visual Interactive Goal (VIG) programming in vendor selection problems. VIG considers constraints as indirective/flexible goals in a multi-goal problem. A range for a flexible goal is entered in Pareto Race – a multi-criteria analysis tool. In Pareto Race, bar graphs visually represent the performances of candidates with the ranges entered by users defining the bar lengths. The values of Pareto bars are optimized to obtain optimal solutions. The method is available in a personal computer as a menu-driven modeling program. With this method, a set of near-optimal solutions can be generated.

Sedarage et al. (1999) proposed a Mixed Integer Non-Linear Programming model for a multi-supplier, single-item inventory system, with stochastic supplier lead-times and demand arrival, and legitimate backorder. The objective of the model is to determine the following: the optimal purchasing plan, the reorder level and the order quantity for each supplier with minimizing the expected cost over one production period. Both dual sourcing and multiple sourcing systems are formulated and the uncertain demand – which can follow any probability distribution – is also addressed in this paper. All models can be solved by using the optimization software MINOS.

Weber et al. (2000) developed an approach that uses two optimization techniques – multi-objective programming, and Data Envelopment Analysis (DEA) – for a procurement situation with multiple suppliers and a single product. A multi-objective programming model is used to select a group of qualified vendors based on price, delivery, and quality criteria. The DEA is then used to evaluate the efficiency of those selected suppliers in order to find the best supplier and a case study is conducted with a manufacturer. The limitation of this paper lies in the demand, which is assumed as a fixed parameter.

Liu et al. (2000) discussed in detail how to use DEA to evaluate suppliers' performances in a supplier selection problem. A simplified DEA model was first presented in this paper to measure suppliers' overall performance strategically so that the buyer can determine how many and which suppliers to choose for the purchase. How to use the DEA method to help improve suppliers' performances after vendor selection is also discussed in this paper.

Boer et al. (2001) suggested that the purchasing problem should be studied in a systematic manner with consideration for the specific procurement situation, the classification of the problem, and the purchasing process. According to Boer, the purchasing items can be categorized into routine items, bottleneck items, leverage items, and strategic items based on two important factors: profit impact and risk management (as illustrated in Table 2.1). Some of the methodologies and techniques used in the problem solving process are also discussed in this paper.

	Low supply risk	High supply risk
Low profit impact	*Routine items*	*Bottleneck items*
	Many suppliers, Low value items, No frequent search for supplier selection, Rationalized systematic purchasing procedures	Limit of monopolistic suppliers, Unique specification items, Decision models are used for supplier selection, Long term contracts
High profit impact	*Leverage items*	*Strategic items*
	Many suppliers, Frequent supplier selection search justified by high value items, Short term contracts, Active sourcing	Few suppliers, Limit supplier selection, Medium contracts, Decision model used

Table 2.1 Classification of purchasing items (Boer et al., 2001)

Kim et al. (2002) considered a supply network consisting of a manufacturer with multiple products and multiple suppliers. A Non-Linear Programming model was developed, and an iterative analytical algorithm was applied to solve the model with numerical analysis conducted on two examples. Only one cycle of production period is considered for the supply network in this paper.

Zhang (2004) further studied the problem discussed in Kim et al. (2002) and extended the model to be a Mixed-Integer Non-Linear Programming model that considers quantity discount offers from suppliers. Some solution approaches were suggested in his presentation.

Minner (2003) proposed a two-supply-mode inventory system. In such as system, a regular supplier might offer a lower price due to the contracted delivery quantity, in turn reducing the supplier's demand uncertainty and proposing a long-term business relationship between the supplier and the manufacturer. A second supplier is used in order to increase the inventory position in case of emergency. This policy combines a constant supply push and flexible pull inventory system. In a global situation, the basic supplier can reside outside of the country and offer low prices, but with the drawback of a long distance whereas a second supplier can reside within the home country, but with the disadvantage of greater expense. This supplier selection strategy is a compromise that combines the push and pull philosophies.

Dahel (2003) presented a multi-objective mixed integer programming approach that simultaneously determines the number of vendors to employ, and the order quantities to allocate to these vendors in a multi-product, multi-supplier sourcing environment. Suppliers offer a volume discount based on the total value of what the buyer purchases. The linear-weighted method is used to handle the delivery reliability, quality, and price factors when suppliers are evaluated.

Crama et al. (2004) analyzed a complicated supplier selection situation in which a multi-plant company's suppliers offer complex discount schedules based on the total quantity purchased by the whole company, as well as by a single plant. The purchasing decision is made more complex by the existence of alternative production recipes for each product. A mixed 0-1 non-linear programming model is formulated with a cost-minimization objective. The non-linear part is linearized in order to solve the model. However, the uncertainty of the demand is not taken into account.

Cakravastia and Takahashi (2004) presented a multi-objective non-linear model to support the integration of the negotiation process and supplier selection within a make-to-order manufacturing environment. The model generates a series of effective decision-making alternatives in each negotiation period and a combination of Interactive Weighted Tchebycheff (IWT) and decomposition method is used to solve the model with IWT being used to generate a set of non-dominated solutions to a multi-objective problem [Steuer and Choo, 1983].

Electronic support for the direct procurement of finished goods has already matured since 1980s with the adoption of ERP and although the indirect raw material procurement via electronics method has been discussed broadly, the implementation is just emerging. Puschmann and Alt (2005) qualitatively explored the critical factors that companies must consider in order to implement e-procurement successfully, as well as how companies use the e-procurement using the benchmarking method.

With the implementation of JIT philosophy, manufacturers now try to maintain a long-term relationship with a few selective suppliers in a supply alliance, with the goal of building a supply chain. A new concept referred to as *supplier management* or *supplier development* has recently been brought to researchers' attention. Nelson et al. (2005) stated that the supplier selection and management strategy is used by the manufacturer/buyer to improve their supplier's performance and capabilities, in order to meet the manufacturer's short-term and/or long-term supply needs. The strategy is applied in an effort to guide supplier management activities. Nelson et al. (2005) also argued that getting suppliers involved early in product development activities and continuous product improvement will help suppliers meet the manufacturer's requirements more efficiently. Nelson et al. (2005) concludes that the supplier selection activities, based on technology, are critical to manufacturers who focus on product flexibility. Manufacturers who focus on volume flexibility should incorporate quality-based supplier selection activities.

According to Krause (1999), supplier development refers to "any effort by a buying firm to improve a supplier's performance and/or capabilities to meet the buying firm's short- and/or long-term supply needs" (Krause, 1999, p. 206). It is clear that the concepts of supplier development and supplier management are essentially the same.

Sanchez-Rodriguez et al. (2005) described supplier management activities as basic, moderate, and advanced. For example, basic supplier management activities include sourcing from a limited number of suppliers for a purchased item, supplier performance evaluation, parts standardization, and supplier qualification. Several descriptive models are developed to depict the relationship among supplier management activities and the relationship between those activities and purchasing performances. A large sample of data was analyzed with statistical techniques and it was determined that basic, moderate, and advanced supplier management activities correlate to one another, and have a positive impact on purchasing performances. Purchasing performances, however, benefit largely from basic supplier management activities. This paper built a bridge between the supplier management activities and purchasing performances, and provides practical suggestions about which supplier management activities should be pursued according to different purchasing performances and the characteristics of the industry.

2.3 Inventory Control

At the operational level, inventory management is closely connected to short-term procurement function because the inventory level is directly affected by order quantity and reorder frequency. That is why the purchasing issue is always accompanied by inventory management topics in most literature.

The inventory management cost per year consists of anywhere from 20% to 40% of the inventory itself, so it makes economic sense that there is an abundance of research in this field seeking a scientific way to maintain the inventory management cost at a minimal level [Ganeshan, 1999].

There are currently two contrasting inventory policy philosophies: the pull system and the push system. In a pull system the order is generated from retailers at the bottom of the supply chain, as opposed to a push system where the order comes from the central warehouse manager who decides how much to order, and when to reorder [Ganeshan, 1999].

Ramasesh et al. (1991) developed mathematical inventory models with stochastic lead-times for sole and dual sourcing, demonstrating how dual sourcing can reduce the uncertainty of the stochastic lead-time. Ouyang et al. (1996) presented a statistic model for an inventory system that allows both backorders and lost sales with an analytical algorithm for the solution that was explained in detail.

Ganeshan (1999) demonstrates a statistical model for a near optimal (s, Q) inventory policy (when the inventory level is under s and an order of Q is placed), minimizing the inventory management cost while subject to certain customer service levels. The system consists of multiple identical suppliers, one central warehouse, and multiple identical retailers. The mathematical model is verified by a simulation program in SLAM-II language and shows that the model is an accurate representation of the system – subject to some assumptions. The model suffered, however, from a limitation of identical suppliers and retailers. In reality, the carrying cost of inventory is often different in different locations. Another limitation of this model is that only a single product is discussed.

Matheus and Gelders (2000) proposed a solution for the (R, Q) inventory policy with subject to a target service level and a probabilistic compound Poisson demand pattern (the demand arrivals constitute a Poisson process, and the individual demand size follows some unspecified discrete distribution). The paper presented two models based on Markov chain theory. One is for the exact calculation of the reorder point R that requires, the vast historical data for the demand size. Another algorithm is used to give an approximate solution, in case the substantial volume of data for the demand size is not available. The limitation of the study is the assumption that there is constant lead-time from suppliers.

Robison (2001) developed a new inventory management technique called Inventory Profile Analysis (IPA). IPA is an improvement of the Months On Hand (MOH) methodology. MOH is formulated as the average monthly inventory divided by the monthly demand, both in units. IPA, on the other hand, compares the target demand and the inventory in terms of each Stock Keeping Unit (SKU), and summarizes the data into categories of "shortages" and "excess". Robison (2001) states that, "IPA does not net excesses in one SKU against shortage in another". A parallel study was conducted with a lens company between IPA and MOH over six months, and the results demonstrated that IPA has a higher correlation to the customer service level than MOH. IPA analysis is easy to implement using MS Excel or similar spreadsheet programs, and the data input can be readily obtained from common inventory management software.

Minner's (2003) study provides an overview of multiple-supplier inventory models. The paper reviews the functions of single stage inventory models with deterministic and statistic lead-times, as well as multi-stage inventory models. The latter is slightly different from multi-echelon inventory models. The multi-stage inventory system allows for trans-shipments – not only from warehouses to retail stores, but also between different parallel retail stores when practical. For example, Store A has one order from a customer, but is out of stock. It happens that Store B – located near Store A – has extra in stock. In such a case, the trans-shipment from Store B to Store A would clearly be beneficial. This kind of dynamic makes the inventory models much more complicated. Which situations these models are applied to, and advantages and disadvantages of using these models on supply chain performance are also addressed in this paper.

Giannoccaro et al. (2003) is one of the few studies where fuzzy set theory is applied to inventory management. The paper defines a supply chain inventory management policy based on echelon stock and fuzzy set theory concepts. The employment of echelon stock aims to manage the inventory system in an integrated manner, while the fuzzy set theory is applied in an effort to better handle the uncertainty of the demand and inventory costs. A simulation program proves that using this methodology is superior to the policy generated from a local stock inventory management system.

Ghalebsaz-Jeddi et al. (2004) proposed a non-linear programming model that determines an optimal, continuous review inventory policy that can be applied to a budget-constrained, multi-item, stochastic inventory system with marginal shortage costs. The uniqueness of the paper lies in its assumption that the purchasing cost is paid upon the delivery of orders – which arrive randomly – rather than when orders are placed (the frequent assumption in most research). An iterative algorithm based on the Lagrange multiplier technique is developed to solve the problem, and a benchmarking work and numerical analyses are also demonstrated.

Agrawal et al. (2004) discussed dynamic inventory allocation from a central warehouse to a group of retailers, and how to keep inventories balanced between all retailers – based on their stochastic demands. The paper uses a dynamic programming method to determine the scheduling of inventory shipments from a central warehouse to retailers, as well as how to rebalance the stock level among involved retailers.

2.4 Various SCM Topics

Besides the two primary SCM issues mentioned above, there are other worthy topics to address, such as the bullwhip effect, reverse logistics, and risk management. These might be supportive SCM activities, but they are important to some extent for the success of SCM.

Fleischmann et al. (1997) reviews quantitative models regarding reverse logistics. The study discusses the implication of reverse logistics in three domains: production planning, inventory control, and distribution planning. The paper proposes that more research should be conducted on the overall frame structure that surrounds reverse logistics.

Owen and Daskin (1998) reviews literature about facility location problems. The paper summarizes that the majority of the research in this area discusses simplified static and deterministic models, due to the complexity of the facility location problem which demands consideration for uncertain future events. Focused research on stochastic and dynamic aspects of the facility location problem had not been conducted until the year the paper was published. The dynamic approach focuses on the timing issues of facility location, while the stochastic approach focuses on the uncertainty of future events. The paper also addresses the strategic nature of facility location problems.

Chandra and Kumar (2001) proposes that the pipeline analysis is driven by the increasing industrial competition to reduce product throughput time and stock outs, removing non-value added activities along the supply chain and increasing profit margins. Pipeline analysis is used to improve interactions and linkages between supply-chain members through the application of various analytical methods and other systematic tools.

Chen et al. (2000) identified that the two factors commonly assumed to cause the bullwhip effect are demand forecasting and an order's lead-time. One of the most frequent suggestions for reducing the bullwhip effect is to centralize the demand information by supplying complete information regarding demand from the lower stages to the upper stages of a supply chain. However, centralizing customer demand information will not eliminate the bullwhip effect entirely. A quantitative model is given, outlining how the demand varies from the lower to upper stages of a supply chain. It is one of few papers that quantify the bullwhip of a supply chain.

Ballou (2001) addresses some unresolved issues in the supply chain network design. The paper reviews previous research and analyzes the network design problems, then focuses its discussion on all aspects of the facility location issue. The methodologies applied in this area are also evaluated.

Meade and Sarkis (2002) present a decision-making model based on Analytical Network Process (ANP) to support the selection of third-party reverse logistic providers. The reasons a company might use a third party logistic provider for its reverse logistics are discussed in detail. ANP technique is a general form of Analytical Hierarchy Process (AHP) (Satty, 1996). With an ANP, a model – considering all quantitative, and qualitative, tangible, and intangible factors – is developed to determine whether or not a third party logistic provider should be employed, and which one should be selected. The limitation of this method is that the weight placed on each factor is dependent on the decision maker, making the pair comparison cumbersome in certain situations.

Finch (2004) analyzes the risks that companies are exposed to in Supply Chain Management and concludes that large companies increase their exposure to risks by having small to medium-sized enterprises as supply chain partners.

The importance of undertaking risk assessment and management is also discussed in this paper.

Lodree et al. (2004) presents a stochastic production/inventory model that minimizes the customer response time for a two-stage supply chain system with variable lead-time and stochastic demand. The production quantity and the lead-time are both treated as decision variables in this model, which is solved by an analytical method. The authors assert that customer response time is one of the most important business performance indices in a fast-changing market that includes a short product life cycle. The limitation of the paper lies in the model, which is established based on the consumption of products at a linear rate.

Inderfurth (2004) discusses the optimal manufacturing/remanufacturing policy for a situation with a stochastic used product return and independent stochastic demands for both manufactured and remanufactured products. A single period with deterministic lead-times for manufacturing and remanufacturing is considered. Two models with optimal production quantities are formulated as decision variables for both product types, and the managerial meaning of the optimal policy is also explored.

2.5 Discount Models

Traditionally, vendors offer price discounts based on the quantity of an item purchased – prompting the buyer to order in large quantities despite the incurred inventory holding costs. A volume discount offers a discounted sum based on the total dollar value of all items purchased from one supplier. It has been proven that the joint business volume discount provides advantages for both vendors and buyers. For vendors, the balanced sales on all items can be achieved, while buyers can save the setup ordering cost and reduce the inventory with a large order for multiple items but small quantities of each item [Xu et al., 2000]. The quantity discount model has been extensively researched in the past, and there are some studies on volume discount models. In this section, the mathematical models for both kinds of discounts are reviewed.

Nahmias (2001) explores inventory models to find Economic Order Quantity (EOQ) values with an all-unit quantity discount schedule, and an incremental quantity discount schedule.

Sadrian and Yoon (1992) compare the practical impact of the business volume discount with that of the quantity discount and conclude that the joint business volume discount is more realistic for both vendors and buyers. Sadrian and Yoon (1994) extended their work with a mixed integer (0-1) programming model for a procurement problem with multi-suppliers, multi-buyers, multi-products, and two volume-discount schedules. A decision support system application was then developed to minimize the buyer's purchasing costs.

Parlar and Wang (1994) present a mathematical model that analyzes how to set quantity discounts so that the buyer and seller (vendor) simultaneously gain maximum profitability. The model is developed based on the assumptions of EOQ model where the lead-time is known and there are no lost sales or backorders. This paper concludes that the incentive for discounts is twofold: to reduce the inventory related costs, and attract more demands from customers. The model is solved by an analytical method.

Chiang et al. (1994) proposes a game theoretic approach to find the optimal quantity discount policy for both buyer and supplier in two situations – namely with and without co-operation between buyers and suppliers. Numerical examples are discussed as well.

Xu et al. (2000) develops a Mixed Integer Programming model for a multi-item lot size problem with a volume discount schedule. Rather than using the mathematical programming software, a heuristic approach based on dynamic programming is developed to solve the model more quickly. The model is still deterministic, however, and no back order is considered, but the author comments that the extended application of the model could be easily derived.

Wang and Wu (2000) proposed another discount policy based on the buyer's increase percentage of the order quantity before discount. The discount schedule – different from the traditional quantity discount – is defined as a discrete, all-unit quantity discount schedule with break points. The discount policy encourages buyers to purchase larger quantities at lower prices. An explicit vendor's optimal discount schedule is obtained by solving a mathematical model, but only one product is considered in the model.

Chapter 3: Mathematical Programming Models for the Optimal Acquisition Policy

The industrial procurement problem discussed in this thesis and the relevant previous research is introduced in the first section of this chapter. The problem is then formulated as a series of mathematical models with regard to different specific situations, namely different discount schemes and split or no-split purchase rule.

3.1 Background

As aforementioned in Chapter 2's literature review, Kraljic (1983) contended that the four most researched criteria for the supplier selection problem are price, product quality, delivery reliability, and supplier capacity. Some studies, such as Dahel (2003) and Crama et al. (2004), use linear weighted multi-objective programming techniques when dealing with the four criteria. Other studies use decision-making methodologies like DEA [Liu et al., 2000] to handle the quality and delivery reliability factors. Once the quality and the delivery reliability of suppliers have been screened, a group of qualified suppliers are selected with considerations for price. This is usually the last stage in the supplier selection process.

Supplier performance evaluation is always one of the essential procedures in terms of the supplier management activity – also discussed in Chapter 2 – for obtaining a group of qualified suppliers regardless of which strategy is used by a manufacturer, be it technology-based or quality-based. Supplier selection, or sourcing among limited suppliers based on price and/or other quantitative factors are also considered to be basic supplier management activities.

It follows that sourcing in a group of qualified suppliers based on price and/or quantitative factors is always essential, either for the supplier selection problem in the open market, or for the supplier management activity in a manufacturer's supplier alliance. Figure 3.1 illustrates a supply network with one manufacturer, multiple products, and multiple suppliers. In some cases, the problem becomes more complex with the introduction of a large number of qualified suppliers offering different discount schemes. Such a situation makes it difficult for the decision-makers to determine the best acquisition policy for which suppliers to employ and how to allocate order quantities among those chosen suppliers.

This thesis is closely related to and based on the research done by Kim et al. (2002) and extended by Zhang (2004). The procurement issue explored in these two papers is how to configure a supply network that consists of a manufacturer making multiple products with uncertain demands from the customers, with multiple suppliers providing the raw materials/components required for said products' fabrication (Raw materials and components are interchangeable in this report). Both studies solve the problem by finding the optimal production level and raw material acquisition policy for the manufacturer, based on uncertain demand and production requirement information.

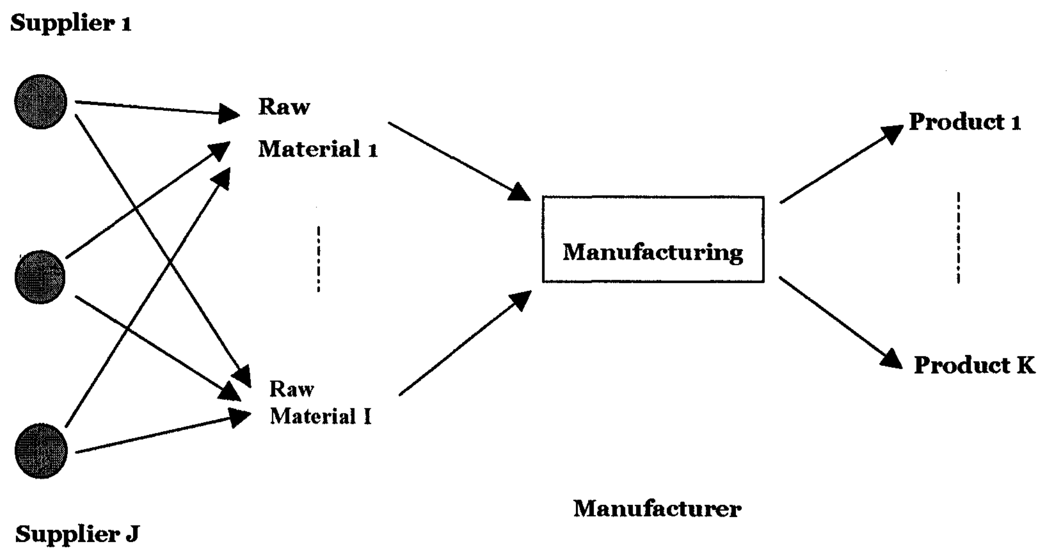


Figure 3.1 *Structure of the supply network (Zhang, 2004)*

Kim et al. (2002) developed a Non-Linear Programming model for this supply network that attempts to find the optimal acquisition policy and the production levels for the manufacturer – maximizing the profit. In order to express the expected value of the manufacturer's profit, with the uncertain demand following Normal distribution, the employment of an integration function in the objective function of the NLP model makes it difficult to solve. Kim et al. (2002) developed an iterative analytical algorithm to solve the model with the complicated optimality analysis and Zhang (2004) extended this research by considering quantity discount offers from suppliers and presenting a Mixed Integer Non-Linear Programming model. Some solution methodologies are also suggested by Zhang (2004).

This thesis project extends the quantity discount model (originally developed by Zhang (2004)) into a group of models with split, no-split, and conditional split purchasing rules. A solution approach is then developed by using a commercial mathematical programming package to solve the models. The volume discount models are developed and solved by similar solution programs. The work done in this thesis and the related previous work are illustrated in Table 3.1.

	Kim et al. (2002)	**Zhang (2004)**	**This thesis**
SCM problem	No discount; No supplier management cost	Only quantity discount	Quantity & volume discounts; Split, no split or conditional split
Mathematical model	NLP model	MINLP model	MINLP models
Solution	Analytical algorithm	Solution suggestions	GAMS solution program

Table 3.1 The work in this thesis and the related previous work

3.2 Model Development

The supply network shown in Figure 3.1 suggests a manufacturer with multiple products having uncertain customer demands and multiple suppliers providing different combinations of raw materials with quantity or volume discount offers. The problem is to find both a long-term strategic acquisition policy, and the optimal production level for the manufacturer in order to maximize its expected profit over the assumed production cycle.

The above problem is formulated as a series of Mixed Integer Non-Linear Programming models with the following assumptions:

Assumptions

- It is a two-tier supply chain problem;

- One cycle of the manufacturer's long-term production period is considered. This project concerns a long-term planning problem that explores which suppliers should be selected for the purchase of raw materials and how many raw materials should be ordered over the entire production cycle from each selected supplier, rather than a short-term planning problem that questions how often an order should be placed with each supplier.
- Because this is a long-term strategic production level and purchasing scheme configured for this project, some time-related management factors are not considered despite the fact that they might incur management costs. Management costs, such as holding cost, ordering cost, and lead-time of suppliers, are related to the short-term planning policy and should be considered in a subsequent stage, however which is not discussed in this project.
- Shortages in the inventory of finished products at the manufacturer's site is allowed, and an estimate of overall economic loss on each shortage unit is provided as the underage cost;
- An estimate of salvage value on each overstock unit is offered as the overage cost;
- It is assumed that the uncertain product demands of products follow normal distributions, with each product having different parameters for the normal distribution function;
- Suppliers offer either all-unit quantity discounts or volume discounts for any purchase.

The quantity discount is offered as an all-unit quantity discount, rather than an incremental quantity discount. The difference between these two types of quantity discounts is discussed in Nahmias (2001). The quantity discount varies with the size of the order on one single product from one supplier. The volume discount is based on the magnitude of the total dollar value of the items purchased from a supplier.

The notations for the mathematical model are as follows:

Notation:

i : index for a raw material/component, $i = 1,2,...I$;
k : index for a product, $k = 1,2,...K$;
j : index for a supplier, $j = 1,2,...J$;
l : index for a discount segment, $l = 1,2,...L$;

3.2.1 Quantity Discount Model

In both quantity and volume discount models, the objective function is to maximize the manufacturer's profit – represented by the sales revenue minus the purchasing cost, and losses from overstocking and understocking.

Parameters for the quantity discount model

g_{ik} : the number of units of raw material i required to produce one unit of product k;
c_{ijl} : the unit price of raw material i after discount from supplier j on discount segment l;
du_{ijl} : the upper bound of the quantity of raw material i from supplier j on discount segment l;
dl_{ijl} : the lower bound of the quantity of raw material i from supplier j on discount segment l;

m_j : the management cost to maintain supplier j in the manufacturer's supplier base;
n_{ij} : the resource consumed by supplier j if one unit of raw material i is produced;
t_k : the resource consumed by the manufacturer if one unit of product k is produced;
e_k : the production cost for one unit of product k;

z_k : the random variable of the demand for product k;
$f(z_k)$: the probability density function followed by the demand of product k;
μ_k : the mean of the Normal distribution followed by the demand of product k;
σ_k: the standard deviation of the Normal distribution followed by the demand of product k;
r_k : the unit sale price of product k;
a_k : the estimated underage cost of one unit of product k;
b_k : the estimated overage cost of one unit of product k;
q_j : the resource capacity of supplier j;
Q : the resource capacity of the manufacturer;

m_j: The management cost to maintain supplier j in the manufacturer's supplier base. The management cost refers to the cost created by any supplier management/development activity, such as supplier performance evaluation, supplier quality improvement, or any other efforts to keep the supplier qualified for the raw material supply.

n_{ij}, t_k: The resource can be any productive resource or capacity usage as long as the units for these two variables match the ones for parameters q_j,Q respectively. It can actually refer to any physical resource that contributes to the production and is crucial for limiting suppliers or the manufacturer in their production capacities. For example, if the reserved capacity by a supplier for the production of a raw material implicitly means its capacity in one specific physical resource, such as labor, then the resource consumption by one unit of raw material production n_{ij} must be expressed in terms of the unit of labor.

It is also assumed that the market demands for the final products are independent of each other. This assumption is not always valid but it applies to most situations. For example, the correlation between the demands for different products may be low, or the products might be sold in distinct regional markets [Kim et al., 2002].

Continuous decision variables:

x_{ijl} : the number of units of raw material i purchased from supplier j on quantity discount segment l;

y_k : the number of units of product k to be produced in one production cycle;

Binary decision variables:

u_{ijl} : $=1$, if the quantity discount segment l is taken for the purchase of raw material i from supplier j;
$=0$, otherwise;

v_{ij} : $=1$, if supplier j is chosen to supply raw material i;
$=0$, otherwise;

w_j : $=1$, if supplier j is chosen for any purchase, then the supplier management cost will be incurred;
$=0$, otherwise;

If the price c_{ijl} is applied (i.e. a discounted price is taken based on the setting in discount segment l), then the quantity of material i purchased from supplier j must fall within the quantity range of the corresponding discount segment – namely from dl_{ijl} to du_{ijl}. The discounted price c_{ijl} can be applied to all the raw material i purchased from supplier j because it is an all-unit quantity discount.

The objective function:

Maximize

$$\sum_{k=1}^{K}\left\{\int_{0}^{y_k}\left[r_k z_k - b_k\left(y_k - z_k\right)\right]f\left(z_k\right)dz_k + \int_{y_k}^{\infty}\left[r_k y_k - a_k\left(z_k - y_k\right)\right]f\left(z_k\right)dz_k\right\}$$
$$-\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} x_{ijl}c_{ijl} - \sum_{k=1}^{K} e_k y_k - \sum_{j=1}^{J} m_j w_j$$

- $\int_{0}^{y_k}\left[r_k z_k - b_k\left(y_k - z_k\right)\right]f\left(z_k\right)dz_k$ is the manufacturer's expected revenue minus the overstock cost on product k when the production amount is above the actual demand level, and there is overstock cost incurred;
- $\int_{y_k}^{\infty}\left[r_k y_k - a_k\left(z_k - y_k\right)\right]f\left(z_k\right)dz_k$ is the manufacturer's expected revenue minus the shortage cost on product k when the production amount is lower than the actual market demand, and the manufacturer has a shortage cost incurred;
- $\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L} x_{ijl}c_{ijl}$ is the total purchasing cost for all raw materials;
- $\sum_{k=1}^{K} e_k y_k$ is the total production cost;
- $\sum_{j=1}^{J} m_j w_j$ is the total management cost to maintain the selected suppliers in the manufacturer's supplier alliance.

Constraints:

1. Raw material requirement constraint: For each raw material, the purchased quantity must be equal to or greater than the quantity required for the manufacturer's production.

$$\sum_{k=1}^{K} g_{ik} y_k \leq \sum_{j=1}^{J} \sum_{l=1}^{L} x_{ijl}; \quad \forall i;$$

2. Restrictions on the quantity range of a discount segment: If the quantity discount segment l is taken, then $u_{ijl} = 1$. It means the price c_{ijl} is applied with all raw material i purchased from supplier j. Meanwhile, the following constraint is applied to enforce that the purchased quantity x_{ijl} falls within the quantity range of the corresponding discount segment, which is from dl_{ijl} to du_{ijl}.

$$u_{ijl} dl_{ijl} \leq x_{ijl} \leq u_{ijl} du_{ijl}; \quad \forall j;$$

3. Resource capacity constraint:

$$\sum_{i=1}^{I} n_{ij} \left(\sum_{l=1}^{L} x_{ijl} \right) \leq q_j; \qquad \forall j;$$

This inequality is to ensure that the resource required for supplier j to produce a certain number of raw materials is within the supplier's resource capacity.

$$\sum_{k=1}^{K} y_k t_k \leq Q;$$

Likewise, the resource requirement for generating the total production amount must be satisfied by the manufacturer's capacity.

4. One discounted price constraint: for each raw material purchased from one supplier, only one discounted price can be applied. In another word, it is impossible to purchase the same material from one supplier at two prices. This can be realized by the constraint below.

$$\sum_{l=1}^{L} u_{ijl} = v_{ij}; \qquad \forall j, i;$$

5. Management constraint: The supplier management cost will be generated in order to maintain a supplier with supplier development activities if the supplier is selected to provide any raw material.

$$w_j \geq v_{ij}; \qquad \forall j, i;$$

Model I – Split Model: If the split for the supply of each raw material is allowed among as many suppliers as possible, the aforementioned objective function and 5 constraints form the entire model. Actually this situation is not common in the business world.

Model II – No Split Model: If there is no split allowed among suppliers for the provision of each raw material (i.e. only one supplier is allowed to be chosen for the purchase of one raw material), the following *No split constraint* is necessary for the model to meet this requirement.

* No split constraint:

$$\sum_{j=1}^{J} v_{ij} = 1; \qquad \forall i;$$

Model III – Conditional Split Model: If the split among N ($N \geq 2$) suppliers for the provision of one raw material is allowed, the following *Conditional split constraint* must be added to the first 5-constraint group.

* Conditional split constraint:

$$\sum_{j=1}^{J} v_{ij} \leq N; \qquad \forall i;$$

3.2.2 Volume Discount Model

Parameters for volume discount model

g_{ik} : the number of units of raw material i required to produce one unit of product k;
c_{ij} : the unit price of raw material i from supplier j before any volume discount is applied;
du_{jl} : the upper bound of the total dollar value for the purchase from supplier j on the volume discount segment l;
dl_{ijl} : the lower bound of the total dollar value for the purchase from supplier j on the volume discount segment l;
dr_{jl} : the volume discount rate on discount segment l from supplier j;
m_j : the management cost to maintain supplier j in the manufacturer's supplier base;
n_{ij} : the resource consumed by supplier j if one unit of raw material i is produced;
t_k : the resource consumed by manufacturer if one unit of product k is produced;
e_k : the production cost for one unit of product k;
z_k : the random variable of the demand for product k;
$f(z_k)$: the probability density function followed by the demand of product k;
μ_k : the average value of the Normal distribution followed by the demand of product k;
σ_k: the standard deviation of teh Normal distribution followed by the demand of product k;

r_k : unit sale revenue of product k;
a_k : the estimated underage cost of one unit of product k;
b_k : the estimated overage cost of one unit of product k;
q_j : the resource capacity of supplier j;
Q : the resource capacity of the manufacturer;

The meanings of m_j, n_{ij}, t_k are the same as those explained for the aforementioned quantitative discount model.

Continuous Decision Variables

x_{ij} : the number of units of raw material i purchased from supplier j;
y_k : the number of units of product k to be produced in one production cycle time;

Binary Decision Variables

u_{jl} : = 1, if the business volume discount rate dr_{jl} is applied;
= 0, otherwise;

v_{ij} : = 1, if supplier j is chosen to supply raw material i;
= 0, otherwise;

w_j : = 1, if supplier j is chosen for any purchase, then supplier management cost will be incurred;
= 0, otherwise;

The objective function

Maximize

$$\sum_{k=1}^{K}\left\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k y_k - a_k(z_k - y_k)]f(z_k)dz_k\right\}$$
$$-\sum_{i=1}^{I}\sum_{j=1}^{J}\left[x_{ij}c_{ij}\sum_{l=1}^{L}(1 - dr_{jl})u_{jl}\right] - \sum_{k=1}^{K}e_k y_k - \sum_{j=1}^{J}m_j w_j$$

The objective function is almost the same as in the quantity discount model, excluding the purchasing cost. In the quantity discount model, the price c_{ijl} is already a discounted price and it varies in relation to discount segment l. However, in the volume discount model, the price c_{ij} is an original price. Different prices $c_{ij}\sum_{l=1}^{L}(1-dr_{jl})$ will be applied corresponding to different discount segment l. Two kinds of discount price representations are used to illustrate different specific situations in the business world.

Constraints:

1. Raw material requirement constraint: It is the same as the one in the quantity discount model.

$$\sum_{k=1}^{K} g_{ik} y_k \le \sum_{j=1}^{J} x_{ij}; \qquad \forall i;$$

2. Restrictions on the dollar value range of a discount segment: It is similar to the one in the quantity discount model, but here it is the range for dollar value, instead of the purchased quantity.

$$\sum_{l=1}^{L} u_{jl} dl_{jl} \le \sum_{i=1}^{I} x_{ij} c_{ij} \le \sum_{l=1}^{L} u_{jl} du_{jl}; \qquad \forall j;$$

3. Resource capacity constraint: It is the same as those in the quantity discount model.

$$\sum_{i=1}^{I} x_{ij} n_{ij} \le q_j; \qquad \forall j;$$

$$\sum_{k=1}^{K} y_k t_k \le Q;$$

4. One discounted price constraint: It is similar to the one in the quantity discount model, except that one discount segment can be applied with the purchase volume from one supplier.

$\sum_{l=1}^{L} u_{jl} = w_j;$ $\forall j;$

5. Management constraint: It is the same as the one in the quantity discount model.

$w_j \geq v_{ij};$ $\forall j, i;$

6. Purchase constraint: This constraint is used to enforce the values of two variables to match each other. For example, if $x_{ij} > 0$, it means the manufacturer buys something from supplier j. Consequently, the value of the variable v_{ij} must be 1. Otherwise, if $x_{ij} = 0$, then $v_{ij} = 0$.

$x_{ij} \leq v_{ij} M;$ $\forall j, i;$

Models for three specific cases – split, no split, and conditional split — are given below:

Model I – Split Model:

From the above 6-constraint group for the volume discount situation,

Removing binary variable v_{ij};

Removing constraint 5 – Management constraint and constraint 6 – Purchase constraint;

Adding the following *Management2 constraint.*

* Management2 constraint: it is used to ensure that the values of two variables match each other. To put it in greater detail, the value of w_j equals 0 only when all u_{jl} equal 0 with all l for a specific j; on the other side, the value of w_j equals 1 as long as one u_{jl} value is 1 (actually at most only one $u_{jl} = 1$ for a specific j).

$$w_j \geq u_{jl}; \qquad \forall j, l;$$

Model II – No Split Model:

Adding the following *No split constraint* to the original 6-constraint group.

* No split constraint: It is the same as the one in the quantity discount model.

$$\sum_{j=1}^{J} v_{ij} = 1; \qquad \forall i;$$

Model III – Conditional Split Model:

Adding the following *Purchase2 constraint* and *Conditional split constraint* to the original 6-constraint group.

* Purchase2 constraint: It is similar to the *Purchase constraint*, but used to ensure that the values of two variables match each other in the opposing direction.

$$x_{ij} \geq v_{ij}\varepsilon; \qquad \forall j, i;$$

* Conditional split constraint: Split is allowed among N suppliers $N \geq 2$.

$$\sum_{j=1}^{J} v_{ij} \le N; \qquad \forall i;$$

Chapter 4: Solution and Numerical Results

This chapter presents a solution approach that uses commercial optimization software – namely GAMS and its integrated solvers – to solve the mathematical models developed in the last chapter. This chapter also provides a detailed interpretation of the GAMS solution program structure, the algorithms used in the program for solving the Mixed Integer Non-Linear Programming (MINLP) models, and the numerical results.

The mathematical models and the solution approach are applied to two real-world cases, and then the preliminary numerical results are presented to justify the capabilities of the models and the solution program. The managerial significance of the acquisition policy and the production level are explored by way of the numerical analysis of the two cases located in the final section of this chapter.

4.1 Solution Approach

In Kim et al. (2002), the in-depth mathematical analysis was conducted on the NLP model. The KKT first order optimality condition was derived, and then an iterative analytical algorithm was obtained to solve the NLP model. However, the introduction of $0-1$ variables in the MINLP model developed in this project makes it difficult to derive an analytical method for solution. Therefore, developing a new solution approach that takes advantage of commercial mathematical programming application packages is one of the main missions of this research.

In the stage of seeking qualified software, several popular commercial modeling systems and optimization software (both functions are integrated into one package for some software), such as Xpress-SLP, AMPL, GAMS and LINGO, were investigated for their abilities to express and solve the MINLP model. In fact, all constraints in the MINLP model in this project are linear, and only the objective function – which has continuous first order derivative function – is non-linear. Therefore an approach combining the Branch-and-Bound algorithm and a NLP solver will be competent for solving this MINLP model. The critical issue, however, is finding an appropriate modeling language capable of expressing the integration function – which can't be expressed in closed form.

In terms of the integration function, only a few mathematical application packages (Mathematica and Matlab) are able to evaluate numerical integration to some extent. For example, Matlab can do a numerical integration computation in a limited manner. The above-mentioned software was studied intensely and a simple prototypical integration function was developed to test the software. GAMS, with its integrated multiple solvers for integer models and NLP models, was ultimately chosen as the most suitable package for tackling this problem – thanks in part to its unique facility called External Function.

4.2 GAMS Introduction

"GAMS, standing for General Algebraic Modeling System, is a high-level modeling system for mathematical programming problems. It consists of a language compiler, which converts the GAMS modeling language to machine language, and a number of integrated high-performance solvers" (GAMS website). GAMS is tailored for complex, large-scale models.

Like most popular optimization modeling systems in the current market, GAMS supports most basic model types such as Linear Programming, Mixed-Integer Programming, Constrained Nonlinear Systems, Non-Linear Programming with Discontinuous Derivatives, Mixed Integer Non-Linear Programming and Quadratically Constrained Programming.

In order to use GAMS to solve an optimization problem, the mathematical model is expressed in GAMS modeling language following the structure of a GAMS model as inputs as below:

INPUTS (namely a GAMS model):

- *Sets*: Declaration, assignment of indexes;
- *Data* (parameters, tables, scalars): Declaration and assignment of parameters;
- *Variables*: Declaration, assignment of type, assignment of bounds or initial values for variables of the model;
- *Equations*: Declaration, definition of all constraints, and the equation, into which the objective function is converted by adding one objective function variable;
- *Model and solve statements*: Statements indicating the components of a model and the solver used to solve a model (http://www.gams.com);

The above elements have their own specific formats in GAMS, which are very close to the way we make them in natural language however. This makes it easy to read and write a GAMS program. Figure 4.1 is a screenshot of a GAMS model under GAMS IDE (GAMS Interactive Development Environment) and gives a glimpse of all the input elements.

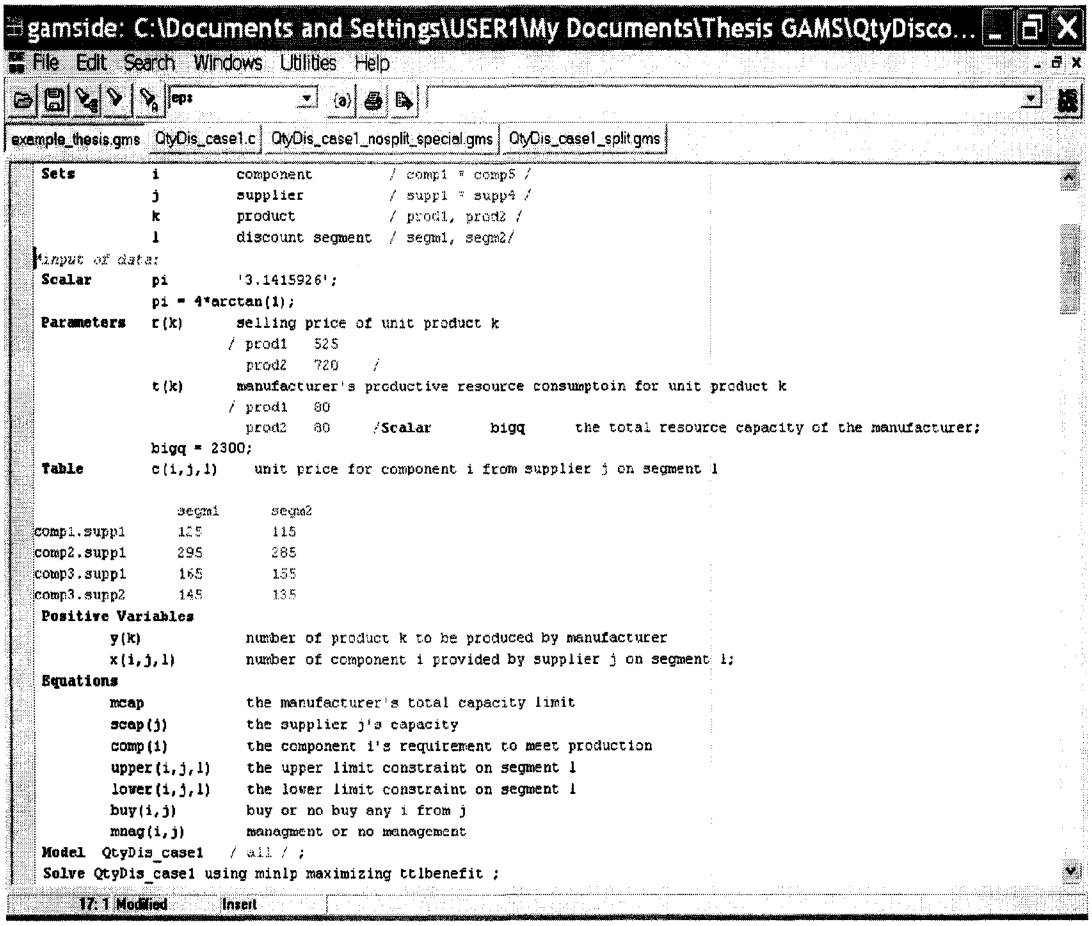


Figure 4.1 GAMS inputs elements and modeling environment

The results of a GAMS program as outputs are mainly solutions of a mathematical model. Meanwhile some other information, which is included in outputs as supportive material, is beneficial to the numerical analysis of an optimization model. Specifically the outputs contain several portions as the following:

OUTPUTS:

- *Echo print*: A copy of the input file including important error messages also, which is helpful for the debug;
- *Reference maps*: Summaries and analyses of the input file for the purposes of debugging and documentation. A list of model entities is also included in Reference Maps;
- *Equation listings*: The first-order Taylor approximations of nonlinear equations in nonlinear models;
- *Status reports and results*: The last section of outputs before GAMS invokes the solver. It is a group of statistics about the model's size such as how many variables and equations there are in the model (http://www.gams.com).

The last portion in GAMS outputs is the optimal objective function value, the variables value, and whatever is requested to be displayed by display statements.

All the outputs are included in two files, which are generated after solving the GAMS model. One is the log file with the same file name as your model file and *.log* as the file extension name. The other is the list file with *.lst* as the file extension name. As the name suggests, the log file records the iterative solving process, which includes the solution of each iteration and the statistical summary of the process. For example, information can be found such as how many NLP models have been solved; how many times the solver failed to solve a NLP model; The model's final status, the solver's final status, the program running time and so on. Appendix I provides a sample of a GAMS log file and a GAMS list file for reference.

There are 22 integrated solvers in GAMS. A solver itself is a software program, which is using one or several specific algorithms to solve one or some special types of optimization models. The 22 solvers are BARON, BDMLP, BENCH, COIN, CONOPT, CONVERT, CPLEX, DECIS, DICOPT, GAMS/AMPL, GAMS/LINGO, MINOS, SNOPT, MSNLP, SBB, XPRESS and so on. All of these solvers are superior on different optimization models respectively. Manuals for each solver can be accessed in the GAMS official website – http://www.gams.com. Some solvers, which are closely related to this project, are introduced subsequently.

CONOPT, developed by ARKI Consulting and Development in Denmark, is the solver by default for nonlinear optimization problems in GAMS. CONOPT3, a higher version of CONOPT, is a multi-method solver containing some sub-methods such as Steepest Descend, Quasi-Newton, Sequential Linear Programming, and Conjugate Gradients etc. Specific sub-method is selected dynamically based on the scale and features of a specific NLP model.

SNOPT is another often used solver for nonlinear optimization problems in GAMS. According to the solver manual, SNOPT uses Sequential Quadratic Programming (SQP) algorithm that obtains search directions from a sequence of quadratic programming (QP) sub-problems. First, SNOPT solver converts all constraints into equality constraints by adding slack variables for linear constraints and using Taylor series to linearize nonlinear constraints. In each iteration to find the optimal solution, a sequence of iterates is generated and they satisfy the constraints and converge to a point that satisfies the first-order conditions for optimality. At each iterate a QP sub-problem is used to generate a search direction towards the next iterate. Solving the QP sub-problem is itself an iterative procedure.

MINOS is another NLP solver in GAMS, which is also employed to solve the model in this project. Both SNOPT and MINOS are developed by Stanford University. MINOS uses Reduced Gradient algorithm, combined with Quasi-Newton algorithm, to solve a NLP model. For more information, the GAMS official website – http://www.gams.com – may be consulted.

SBB (Simple Branch-and-Bound) is the solver used to solve the Mixed Integer Programming model in this thesis project. Like CONOPT, SBB is also developed by ARKI Consulting and Development in Denmark. It uses the standard Branch-and-Bound algorithm for the integer programming problem. Different options for the SBB solver can be set up in GAMS to help optimize the solving process of the Branch-and-Bound algorithm regarding a specific integer-programming model. For example, different NLP sub-solvers can be designated in the option file for the root node and other sub-nodes. Different node selection methods for the Branch-and-Bound algorithm, such as depth first search, best estimate, and/or best bound, can be selected. These options greatly improve the efficiency of the solver.

All the NLP solvers in GAMS somehow complement each other. For a specific model, if one solver fails to solve it, it is highly possible that one of the others is able to handle it. This implies the improvement of the capability of GAMS, especially for Non-Linear optimization problems, which there is no one definite algorithm being able to solve all NLP models so far. On the other aspect, the result-oriented benchmarking can be done to compare solutions from different solvers for one problem in order to select an efficient solver as the first choice. Hence, the function of multi-solver makes GAMS more powerful.

Finally, the reason that GAMS is selected as the solution software for the MINLP model in this project lies in the modeling function of GAMS with External Function. Like any other common mathematical programming modeling language, GAMS modeling language itself, is actually not so advanced for expressing any mathematical function, especially with some complex functions. That is why the facility of External Function exists in GAMS. Using External Function can define any part of a model in another common programming language, such as C, Fortran, or Delphi, as an external module, which is connected into GAMS program by a GAMS interface. The GAMS interface creates a mapping between all variables in an external module and a GAMS program so values of variables can be exchanged smoothly. The part of a model corresponding to an External Function has to be written in a special format in GAMS (more information about External Function is available at the GAMS official website).

4.3 The GAMS Solution Program

The structure of the GAMS solution program for solving the MINLP model in this project is illustrated in Figure 4.2.

In the external module, an advanced C program routine was developed as an External Function to define the integration function part, which is extracted from the original objective function because the integral of this function can not be expressed as a closed form. This piece of C code does numerical

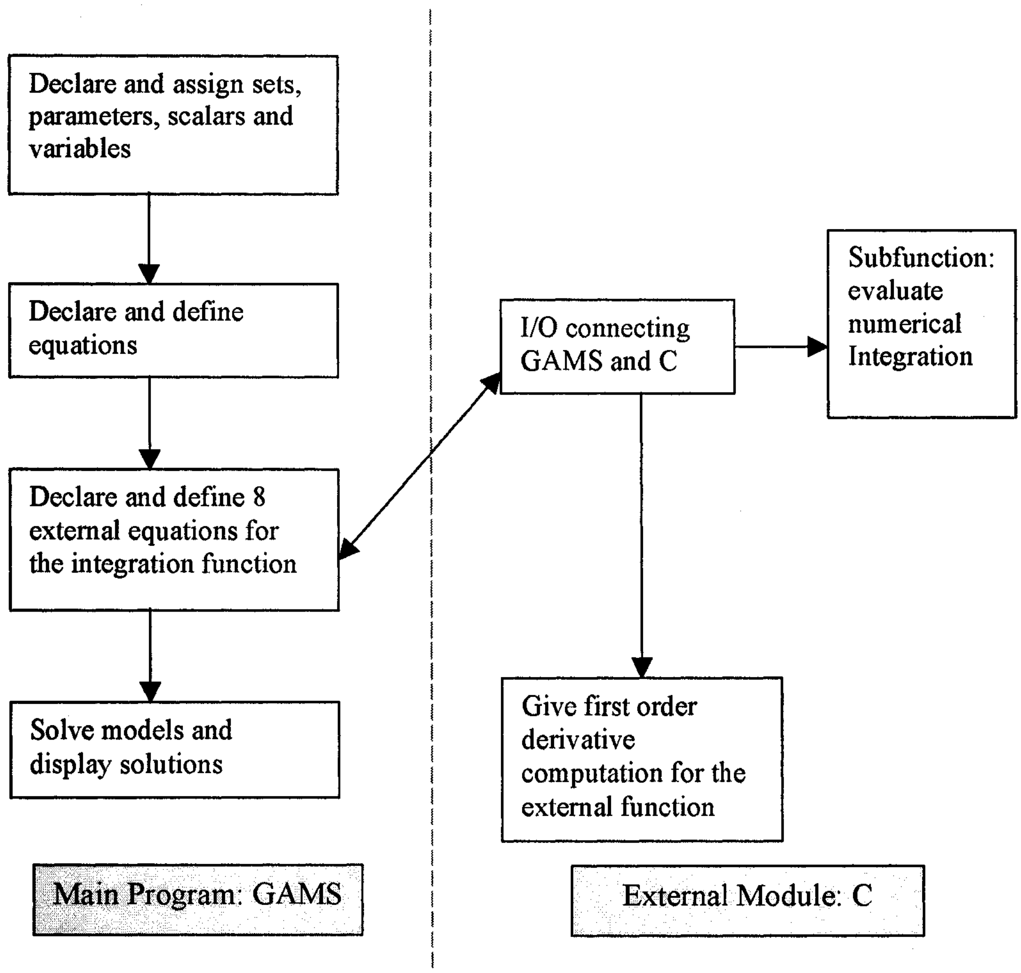


Figure 4.2 The structure of the solution program

integration evaluation by using the Romberg integration algorithm – originally coded by Press et al. (1992), and modified slightly for employment in this project. Romberg is a sophisticated numerical integration technique that uses the extended trapezoidal method (more details about this algorithm is available at the Mathworld website, http://mathworld.wolfram.com/RombergIntegration.htm). The external module also provides the first order derivative of the integration function (a single variable function) for the GAMS main program since it is required by the NLP solver. The external module is then integrated into the GAMS main program by a GAMS interface.

In the GAMS main program, the SBB solver is set as the main solver for the MINLP model. At each node with the Branch-and-Bound algorithm in the SBB solver, the relaxed problem is an NLP model. Different NLP solvers were tried with the program at the early stage of this project, then the results on the programming running time and the optimal solution were compared in order to find the most competent NLP solver for the MINLP model. Table 4.1 displays some results of the benchmarking among 3 NLP solvers - CONOPT, SNOPT and MINOS.

Table 4.1 illustrates Cases 1-5 from the Volume Discount model, and Cases 6-10 from the Quantity Discount model. Under the column of "Solver setting", *C->S* means the CONOPT solver is applied to each NLP model first, and the SNOPT solver will be called for solving the model only when the CONOPT solver fails. Similarly, *S->C* means the SNOPT solver will be applied to all NLP models first and, if it fails, the CONOPT solver will be called to solve the model. *M->C* means the MINOS solver will be the first choice for solving NLP models and the CONOPT solver serves as a backup solver. The word *stuck* implies the program running time is more than 3 minutes, at which point it is interrupted intentionally. In this situation, the solver is judged as being incapable of solving the model.

Case no	Solver setting	Total # of nodes solved	# of nodes solved by CONOPT	# of nodes solved by SNOPT	# of nodes solved by MINOS	Running time (seconds)	Optimal O.F. value
1	S->C	46	3	43	#	1.968	827
	M->C	49	1	#	48	9.218	827
	C->S	54	54	0	#	3.000	827
2	S->C	28	2	26	#	7.656	1301
	M->C	stuck on node 3 (more than3 minutes)					
	C->S	56	56	0	#	3.562	1301
3	S->C	58	5	53	#	1.734	1127
	M->C	stuck					
	C->S	56	54	0	#	7.093	1127
4	S->C	48	1	47	#	0.938	1229
	M->C	stuck					
	C->S	52	52	0	#	1.796	1229
5	S->C	52	0	52	#	0.828	1010
	M->C	stuck					
	C->S	54	54	0	#	1.921	1010
6	S->C	20	0	20	#	3.343	58572
	M->C	24	2	#	22	57.25	58572
	C->S	53	52	1	#	16.625	58572
7	S->C	31	0	31	#	2.062	71386
	M->C	29	0	#	29	19.265	71386
	C->S	12	12	0	#	9.468	71386
8	S->C	31	0	31	#	2.422	72952
	C->S	12	12	0	#	7.734	72952
9	S->C	67	6	61	#	25.968	71450
	C->S	12	12	0	#	30.250	73310
10	S->C	20	6	14	#	24.546	73240
	C->S	12	12	0	#	35.060	73240

Table 4.1 Compare different NLP solvers (from different models)

For example, with the *Solver setting* as *S->C*, 46 NLP models are solved totally in Case 1 and among those 46 NLP models, 43 are solved by the SNOPT solver – which is the first choice – and 3 are solved by CONOPT due to the failure of the SNOPT solver. The sign of "#" under the column of "# of nodes solved by MINOS" means the MINOS solver is not employed at all because it is not indicated in the setting of solvers. The program running time is 1.968 seconds for Case 1.

The performances of the NLP solvers can be evaluated from Table 4.1 consequently. The MINOS solver is incapable of solving the NLP models in this project, and the SNOPT solver does fail a small number of NLP models although SNOPT behaves slightly better than CONOPT in terms of the program running time. Apparently the CONOPT solver performs very well with all NLP models. The program running time is not a critical issue in this project due to the size of the model, so the CONOPT solver is selected as the first choice for the NLP solver. The SNOPT and MINOS solvers are employed as the second and third backup solvers, sequentially, if the preceding solver fails. It has turned out that the NLP models at all nodes in this project have been solved successfully by the collective work of these 3 NLP solvers.

4.4 Numerical Results

4.4.1 Two Real-World Cases

The MINLP model and the GAMS solution program are applied to two real-world cases, namely the numerical examples originally introduced in Kim et al. (2002).

- Case A: A computer company assembles and sells two models. The manufacturer has 2 products and 5 components from 4 suppliers for production. Due to the rapid depreciation in computers as high-tech products, the company has a strategic goal of "zero" inventory (Kim et al., 2002). This is why the overage cost is very high as indicated in Table A3 (Appendix II).

- Case B: The manufacturer has 5 products and needs to purchase 5 components from 5 suppliers for production.

All of the input data requirements for the two cases on the parameters of the suppliers' capacities, product demand distributions, and material requirements for production are listed in Appendix II [Kim et al., 2002].

For the Quantity Discount model-No Split with Case A, the GAMS List file, as the output of the GAMS program, tells the size of the model as below:

MODEL STATISTICS (from the GAMS List file)

BLOCKS OF EQUATIONS	13	SINGLE EQUATIONS	144
BLOCKS OF VARIABLES	10	SINGLE VARIABLES	115
NON ZERO ELEMENTS	361	NON LINEAR N-Z	6
DERIVATIVE POOL	13	CONSTANT POOL	20
CODE LENGTH	59	DISCRETE VARIABLES	64

From the GAMS List file of the Quantity Discount model-Split with Case B, the size of the model can be known as:

MODEL STATISTICS (from the GAMS List file)

BLOCKS OF EQUATIONS	12	SINGLE EQUATIONS	182
BLOCKS OF VARIABLES	10	SINGLE VARIABLES	156
NON ZERO ELEMENTS	576	NON LINEAR N-Z	15
DERIVATIVE POOL	22	CONSTANT POOL	28
CODE LENGTH	143	DISCRETE VARIABLES	80

Therefore Case A has 115 variables – as "Single variables" in the GAMS List files – for the Quantity Discount model with 64 of them identified as binary variables. Case B has 156 single variables for the Quantity Discount model, with 80 of them identified as binary variables. The Volume Discount models have almost the same number of variables as the Quantity Discount models. The GAMS program running time is less than 1 minute for the Quantity Discount models with both numerical examples. The Volume Discount model with Case A takes somewhere from 30 to 300 seconds, and the Volume Discount model with Case B takes longer (somewhere between 150 to 800 seconds) to run the programs. This is because the solving process of a NLP model does not only depend on the size of the model, but on the feature of the data also (the condition number of a matrix). Therefore, even for the same model, the solution time varies with different input data. With a large-scale problem, it is possible that the GAMS solution program runs out of time without finding an optimal solution, but the GAMS solution program solved all MINLP models in this project by the time this report was written.

4.4.2 Preliminary Results

First, a GAMS program was coded to realize the Non-Linear Programming model developed by Kim et al. (2002). Then, the optimal solutions from the GAMS program for the NLP model were compared with those originally obtained by Kim et al. (2002) using the analytical algorithm. The comparison of the objective function values, which stand for the manufacturer's profits, is displayed in Figure 4.3. It is obvious that the two sets of results for the manufacturer's profit are equal. Even in the numerical results displayed in Appendix III, the insignificant difference between two results – caused mainly by the numerical integration computation – can be disregarded when drawing conclusions. The numerical results for the purchasing quantities and production level of two solutions reveal the same thing.

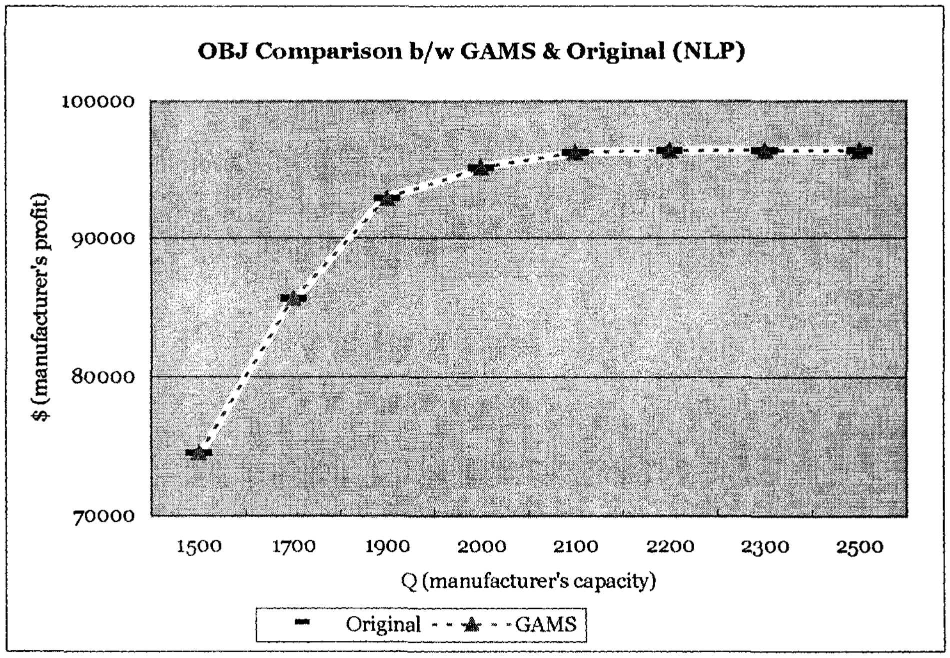


Figure 4.3 *Comparison of Objective function value (manufacturer's profit) between GAMS program and original results for the NLP model on Case B*

The results from the GAMS program for the MINLP models developed in this project, along with the special input data, are compared with the results from the corresponding NLP models. The purpose of special input data is to enforce that all original prices are applied to any purchase in the MINLP models so that the optimal solution from the MINLP model (the original prices are applied by using special input data) should be exactly the same as those from the NLP model (without discount setting). This special input data can be realized in many ways, such as adjusting the discount value or assigning special values to the lower/upper bound of the discount segments in the program. Some comparisons between the results from the MINLP models and those from the NLP models are displayed subsequently.

➢ The comparison between the Quantity Discount model-Split and the NLP model on Case A is displayed in Figure 4.4.

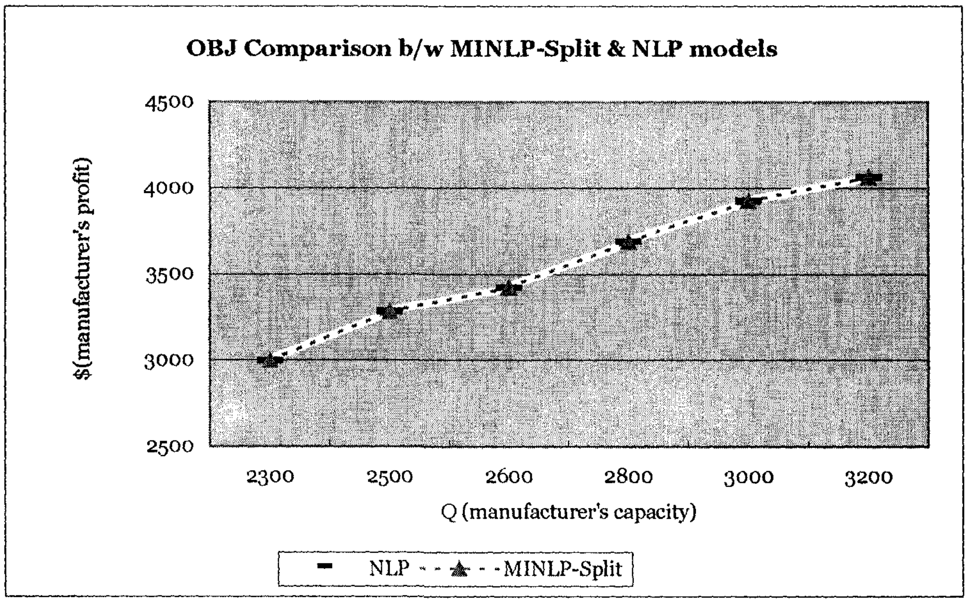


Figure 4.4 *Comparison between the Quantity Discount model- Split and NLP model on Case A*

➢ The comparison between the NLP model, the Quantity Discount Model-No Split, and the Volume Discount Model-No Split on Case B is illustrated in Figure 4.5.

One is prompted to ask why there is a difference between the NLP model results and the discount model results (the manufacturer's profit from the NLP model is 0.46% higher than that from the discount models) when Q (manufacturer's capacity) is bigger. When Q>=2100, the manufacturer's profit from the NLP model is higher than that from the Quantity or Volume Discount model-No Split. It is because in the optimal solution of the NLP model, there is a split on the purchase of Component 4. As a result, a sacrifice of the manufacturer's profit in the discount models occurs due to the no-split constraint. This point is well illustrated in the data results, which support the above Figures 4.3, 4.4 and 4.5 (in Appendix III).

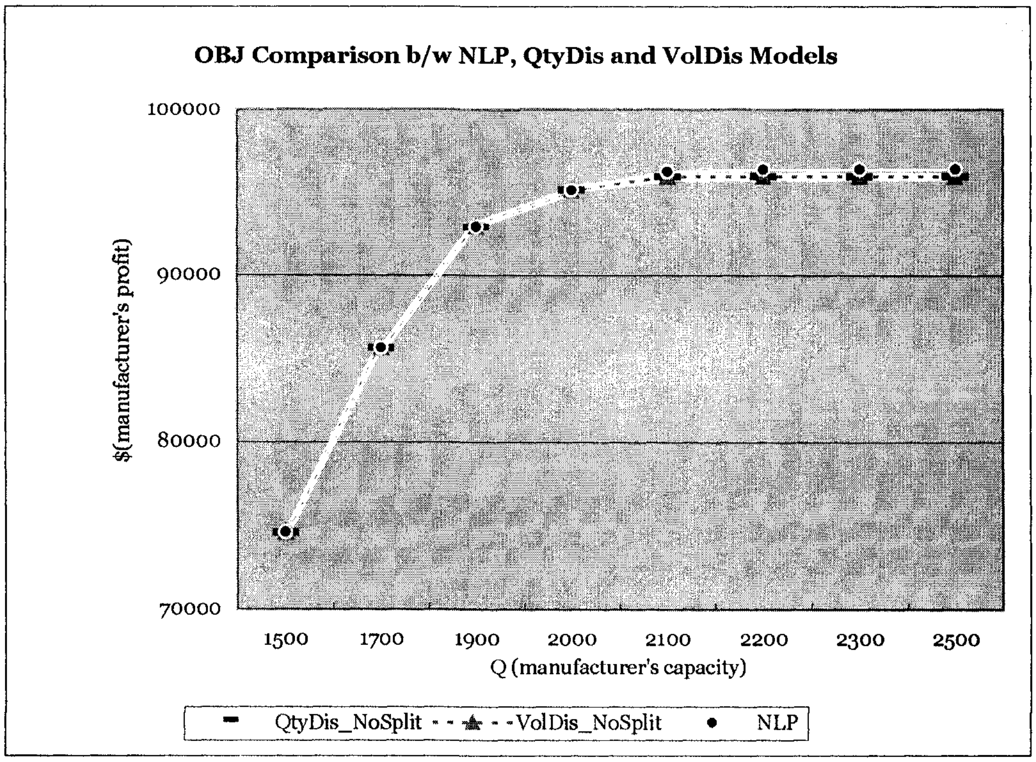


Figure 4.5 *Comparison between NLP, Quantity Discount Model-No Split and Volume Discount Model-No Split on Case B*

Pair comparisons are conducted between the results from all the MINLP models, namely the Quantity Discount models, Volume Discount models (with 3 cases for each), and the results from the corresponding NLP models. All comparisons support the conclusion that both sets of results (from the MINLP models with special input data and the NLP models) are equal, barring negligible error. This verifies the "correctness" of both the MINLP models, and justifies the capabilities of the MINLP models and the GAMS programs.

4.4.3 Numerical Analysis

Based on the preliminary results in the last section, the MINLP models and the GAMS solution program can be trusted to generate more numerical results. Therefore, a series of experiments were designed for the numerical analysis on the two practical cases in order to explore managerial implications.

Compared with the original input data in Kim et al. (2002), the supplier management cost is a new item in the input data requirement for the MINLP model. The management costs for maintaining a supplier in a manufacturer's supplier alliance are almost at the same level for all suppliers, who produce and provide the same products under comparable economic levels. In addition, it is difficult to contact the original source of the data in Kim et al. (2002). Therefore, with moderate research regarding the management cost and consideration of the specific circumstances that surround the two real-world cases, the values for the supplier management costs are assigned as recorded in Appendix II.

The numerical analysis in this project investigates the following:

- A complete solution;
- How the optimal objective function value (the manufacturer's profit) and the production amounts change as the manufacturer's capacity (Q) changes;
- What managerial implications the GAMS program output provides;
- What factors determine the optimal acquisition policy;
- How the manufacturer's profit and the purchased quantity from the supplier change when a supplier's capacity changes;

- How the manufacturer's profit and the purchased quantity from the supplier change when a supplier management cost changes;
- How the change in a product's demand uncertainty affects the other products' production amounts and the manufacturer's profit;
- Comparisons of the Split, No Split, and Conditional Split situations;
- Comparisons of the quantity discount and volume discount schemes;
- And analyses of the node selection modes in SBB solver.

A Complete Solution

To demonstrate the solution and results for the numerical analysis, a complete solution for a Quantity Discount model-No split on Case A from the GAMS solution program is illustrated below. Since the original prices (without discounts) are used, most input data for this example can be found in Appendix II, and the manufacturer's capacity is set as Q=3500. The complete solution of the problem is displayed in Table 4.2.

INPUT DATA							
Manufacturer's capacity	Product demand average		Suppliers' capacities				
	product 1	product 2	supp1	supp2	supp3	supp4	
3500	**25.10**	**25.15**	**39.4**	**40.0**	**36.7**	**41.8**	
OUTPUT SOLUTION							
Manufacturer's profit (OBJ)	Production amounts		purchased quantity				
	product 1	product 2	comp1	comp2	comp3	comp4	comp5
			supp1	supp1	supp2	supp4	supp4
2895.842	**19.641**	**19.759**	**19.641**	**19.759**	**39.400**	**19.641**	**19.759**

Table 4.2 A complete solution for a case of the quantity discount model-no split on Case A

- The optimal purchasing policy for the manufacturer from the above solution is, for example, purchasing 19.641 units of Component 1 from Supplier 1. The optimal production level suggests the production amount of Product 1 over the one production cycle is 19.641, which is lower than the average value of Product 1's demand. This is because the optimal solution is obtained by evaluating all factors that affect the manufacturer's production decision including the manufacturer's capacity, overage and underage costs, the supplier's capacity, and so on. This solution also suggests that it is not always wise to take the average value as the optimal production level for a Normal distributed demand when an accurate analysis of the manufacturer's entire situation is not available.
- With the optimal purchasing policy and production level, the manufacturer can achieve a profit of $2895.842.

How does the manufacturer's profit and the production amounts change as the manufacturer's capacity changes?

The results recorded in Table 4.3 are obtained by taking the Quantity Discount model—No Split on Case A as an example. It shows how the manufacturer's profit and the production amounts in the optimal solutions change when the manufacturer's capacity (Q) changes.

Figure 4.6 takes the data for the manufacturer's profit in Table 4.3 and illustrates that the manufacturer's profit increases linearly when its capacity grows and this trend continues until it comes to a point – Q=3200 in this case. Beyond said point, the manufacturer's profit maintains its level when the capacity continues to increase. The production amounts in the optimal production level exhibit the same behavior as the manufacturer's profit does, as can be observed in the data results listed in Table 4.3.

No	Q	Profit	Production amounts		Purchased quantity				
			Prod1	Prod2	Comp1	Comp2	Comp3	Comp4	Comp5
1	2300	2049.759	11.657	17.093	11.657	17.093	28.750	11.657	17.093
2	2400	2166.769	12.862	17.138	12.862	17.138	30.000	12.862	17.138
3	2500	2282.908	14.012	17.238	14.012	17.238	31.250	14.012	17.238
4	2600	2397.180	15.076	17.424	15.076	17.424	32.500	15.076	17.424
5	2700	2508.047	16.043	17.707	16.043	17.707	33.750	16.043	17.707
6	2800	2613.427	16.927	18.073	16.927	18.073	35.000	16.927	18.073
7	2900	2710.621	17.745	18.505	17.745	18.505	36.250	17.745	18.505
8	3000	2796.376	18.519	18.981	18.519	18.981	37.500	18.519	18.981
9	3100	2866.777	19.262	19.488	19.262	19.488	38.750	19.262	19.488
10	3200	2895.842	19.641	19.759	19.641	19.759	39.400	19.641	19.759
11	3300	2895.842	19.641	19.759	19.641	19.759	39.400	19.641	19.759
12	3400	2895.842	19.641	19.759	19.641	19.759	39.400	19.641	19.759
13	3500	2895.842	19.641	19.759	19.641	19.759	39.400	19.641	19.759
14	3600	2895.842	19.641	19.759	19.641	19.759	39.400	19.641	19.759

Table 4.3 Numerical results with the Quantity Discount model-No Split on Case A

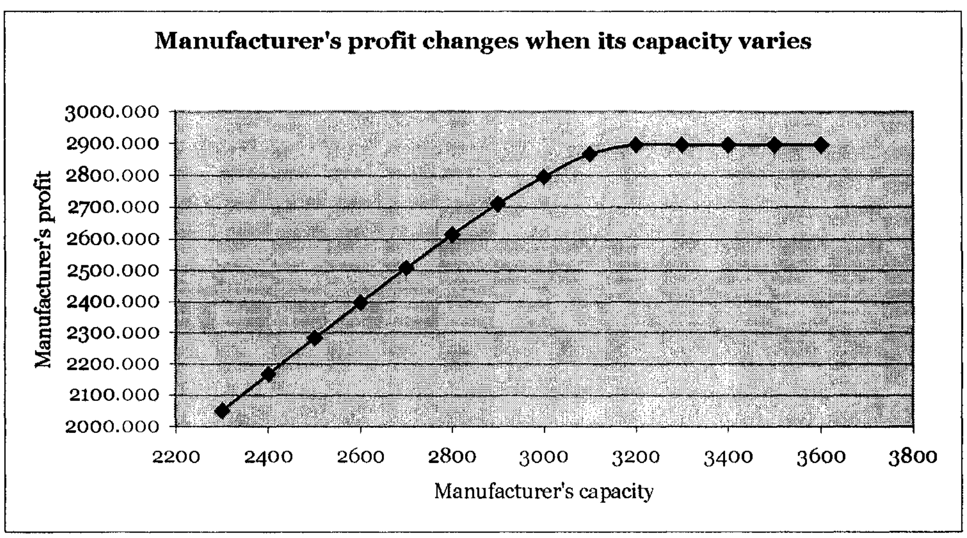


Figure 4.6 Manufacturer's profit changes as its capacity changes

Why does the manufacturer's profit stop increasing when it comes to a point and what is the significance of that point? These questions can be answered with the aid of the output of the GAMS program. In the GAMS List file, when Q<=3200, the manufacturer's capacity that is actually consumed for production – named the "level" value in the GAMS List file – is always equal to the actual Q value (called the "upper" value in the GAMS List file). That means that the manufacturer's capacity is always fully utilized. When Q>3200, the "level" value is lower than the "upper" value – i.e. the amount of the capacity that is actually utilized for the production is less than the one the company is able to offer. Therefore, when the manufacturer's capacity Q>3200 in Case A, it is considered underdeveloped. So at the point where Q=3200, the manufacturer's capacity stops being fully exploited. To explain further, when Q<=3200, the manufacturer's capacity acts like a bottleneck – preventing profit from improving. Accordingly, once the capacity increases, the profit improves. When Q>3200, the capacity no longer behaves like a bottleneck because there is extra capacity left. As a result, increasing the capacity no longer improves the profit of the manufacturer. Only when the bottleneck factor is increased can the profit increase continuously.

What other managerial implications does the GAMS program output provide?

Extending the last section's findings by using the GAMS output for analysis allows more information to be captured from the GAMS List file. Whether or not a constraint is binding is important for the sensitivity analysis in optimization problems. It is easy to find the answer to this question in the GAMS outputs. Meanwhile there are managerial implications (demonstrated in the following examples). Figure 4.7 is a screenshot of the GAMS List file when

Q=3300 (still with the Quantity Discount model-No Split on Case A). We already know that when Q=3300, the manufacturer's capacity is not the critical factor for improving the profit because there is extra capacity left unused. However, what is the cause that prevents the manufacturer from increasing its profit further?

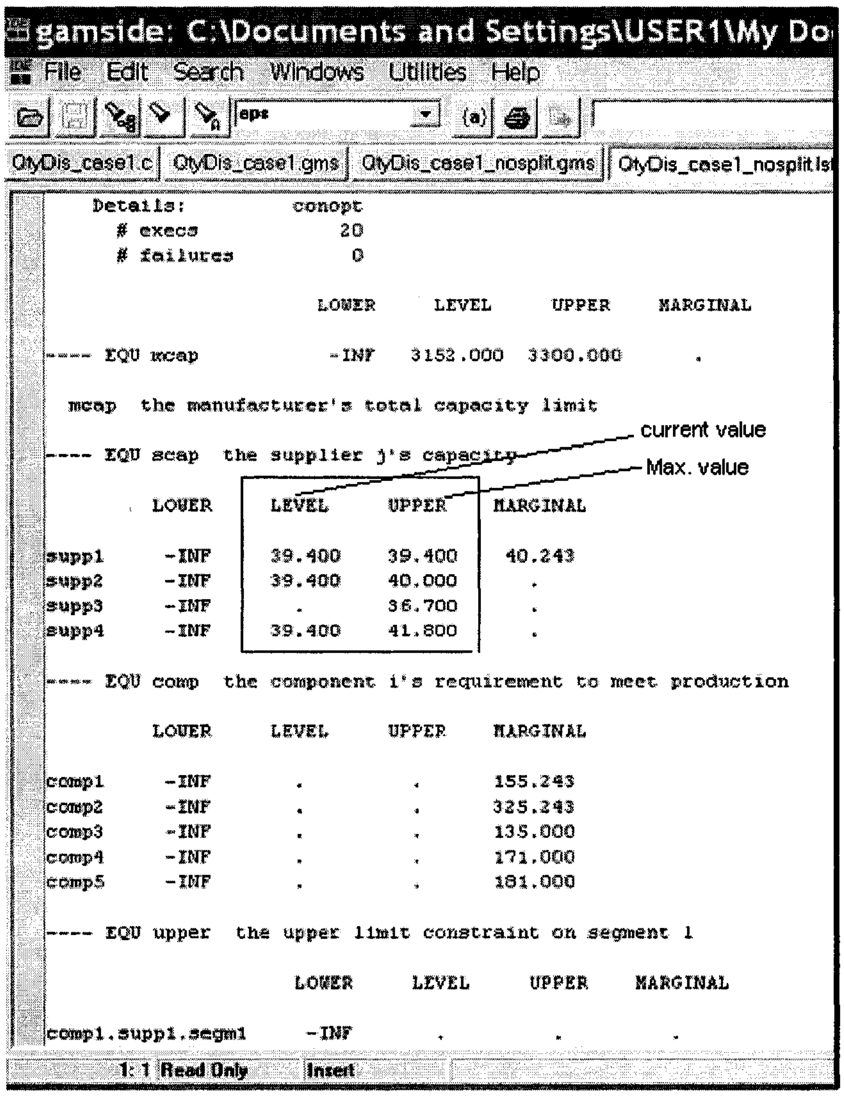


Figure 4.7 Screenshot of the GAMS List file: Demonstrate using the GAMS outputs for analysis

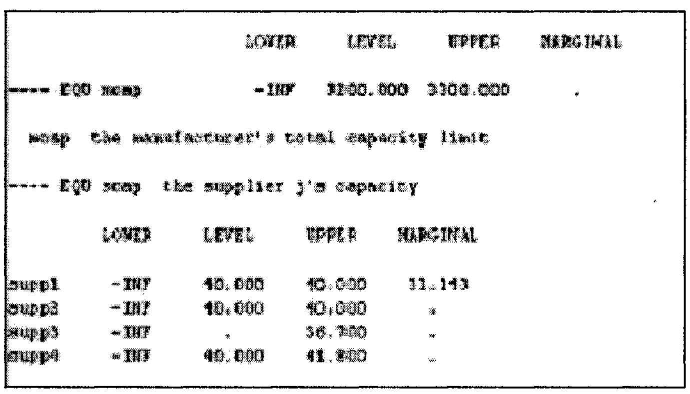
The GAMS List file discloses that when Q=3300, Supplier 1's capacity is fully utilized (the constraint is binding) as the *upper* value equals the *level* value.

What if Supplier 1's capacity increases? To find the outcomes, Supplier 1's capacity is increased from 39.4 to 40 in Case 2 in Table 4.4, and other input data is kept unchanged as in Case 1 in Table 4.4 – revealing that the manufacturer's profit improves, as in Table 4.4.

#	q1	q2	Q	Profit	Production amounts		Purchasing quantity				
					Y1	Y2	X11	X21	X32	X44	X53
1	39.4	40.0	3300	2895.842	19.641	19.759	19.641	19.759	39.400	19.641	19.759
2	40.0	40.0	3300	2917.331	19.987	20.013	19.987	20.013	40.000	19.987	20.013
3	42.0	42.0	3300	2942.897	20.699	20.551	20.699	20.551	41.250	20.699	20.551

Table 4.4 Case analysis from GAMS outputs

The screenshot of the GAMS List file for Case 2 is displayed in Figure 4.8, showing that the capacities of Supplier 1 and Supplier 2 are both used up. With further analysis, the capacities of both Supplier 1 and Supplier 2 are increased to 42 while the manufacturer's profit improves as displayed in Case 3 in Table 4.4.


```
                        LOWER      LEVEL      UPPER     MARGINAL

---- EQU mcap            -INF    3300.000  3300.000       .

  mcap  the manufacturer's total capacity limit

---- EQU scap  the supplier j's capacity

          LOWER      LEVEL      UPPER    MARGINAL

supp1      -INF     40.000     40.000     11.143
supp2      -INF     40.000     40.000       .
supp3      -INF       .        36.700       .
supp4      -INF     40.000     41.800       .
```

Figure 4.8 Screenshot of GAMS List file when q1=q2=40 (Case 2)

In this way, the manufacturer can technically identify the critical causes that prevent them from improving their profit. For example, if the bottleneck is a supplier's capacity, the purchasing team may negotiate with the supplier to see if the supplier's extra capacity can be purchased at an appropriate price. During the negotiation, the manufacturer already has the target information in mind. On the other side, if the bottleneck is the manufacturer's capacity, the manufacturer can try to resolve it using its own management team. If it happens that the supplier and the manufacturer are intra-organizational, the whole situation can be coordinated within one management group. This exactly illustrates the essence of Supply Chain Management – the coordination between two facilities in a supply chain.

Which factors determine the optimal acquisition policy?

The optimal acquisition policy – which suppliers are selected for the supply of raw materials and how many should be purchased from each selected supplier – is determined once the purchasing situation is carefully considered. The results from the Quantity Discount model-No Split on Case A illustrate how the procurement pattern changes when price, discount segment setting, supplier's capacity, and/or supplier management cost changes.

No	*Price*		*Profit*	*Production amount*		*Purchasing quantity*					
	c322	c332		y1	y2	x11	x21	x32	x33	x44	x54
1	135	147	2895.842	19.641	19.759	19.641	19.759	39.400	0	19.641	19.759
2	135	136	2895.842	19.641	19.759	19.641	19.759	39.400	0	19.641	19.759
3	135	135	2895.842	19.641	19.759	19.641	19.759	39.400	0	19.641	19.759
4	135	134	2895.842	19.641	19.759	19.641	19.759	39.400	0	19.641	19.759
5	135	134	2935.242	19.641	19.759	19.641	19.759	0	39.400	19.641	19.759
6	135	130	2926.500	18.028	18.672	18.028	18.672	0	36.700	18.028	18.672
7	135	131	2895.842	19.641	19.759	19.641	19.759	39.400	0	19.641	19.759

Table 4.5 Results for the Quantity Discount model-No Split on Case A when prices and suppliers' capacities change

In Case 1 in Table 4.5, the lowest price on Component 3, c_{322} is from Supplier 2, so Component 3 is purchased from Supplier 2 in the optimal acquisition policy originally. From Case 1 through Case 4, the price of Component 3 from Supplier 3 decreases gradually, approaching Supplier 2's price from 147 to 134. However, in Case 4, the price of component 3 from Supplier 3 is lower than that from Supplier 2, but Supplier 2 is still selected for the purchase of component 3 since Supplier 3's capacity is 36.7 – lower than 39.4, which is Supplier 2's capacity.

Finally, only in case 5 – Supplier 3's capacity is increased to 39.4 (the same as Supplier 2's) – Supplier 3 is chosen to supply Component 3 due to its lower price. In Case 6, Supplier 3's capacity is set back to 36.7 (its original value), and its price declines further. This experiment aims to see how low Supplier 3's price must drop in order to be chosen over Supplier 2 for the provision of Component 3 with its limited capacity. It turns out that Supplier 3 won't be chosen until its price is reduced to 130. That means Supplier 3 has to sacrifice $5 per unit on its price to compensate for its capacity limitation. Case 7 in Table 4.5 is illustrated in Figure 4.9 and demonstrates that a supplier's capacity contributes to the buyer's purchasing decision in addition to the price. The discount segment setting – the quantity boundary for a segment and the discount amount – directly affects the price of raw materials with specific quantities. Consequently, it ends up affecting the purchasing solution.

How do the manufacturer's profit and the purchased quantity from the supplier change when a supplier's capacity changes?

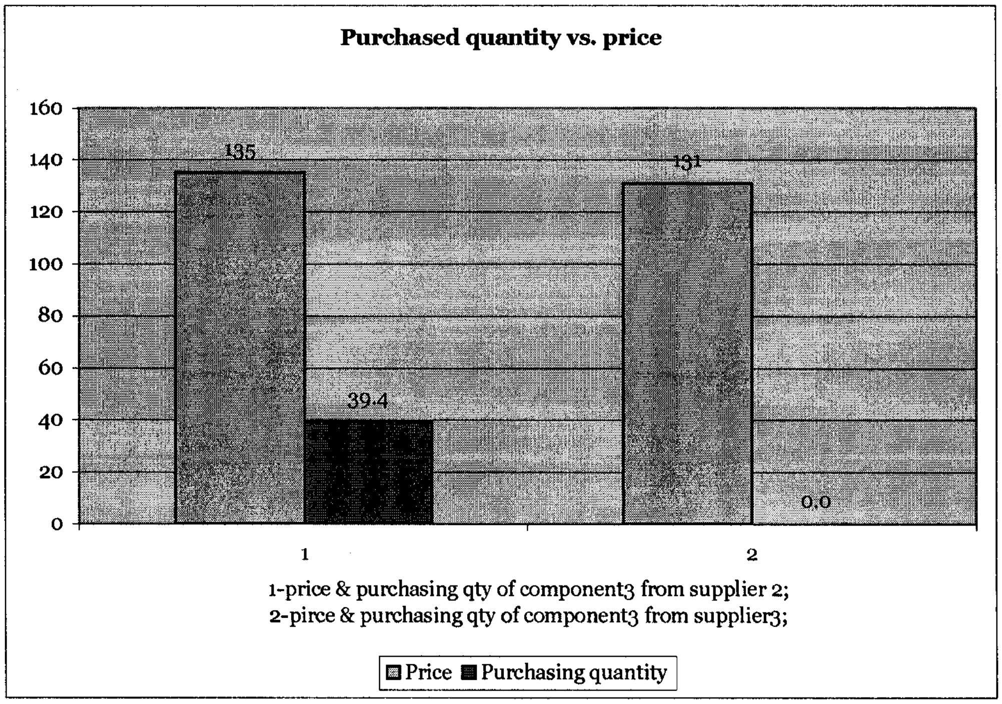


Figure 4.9 *Not only price affects the supplier selection issue, but the supplier's capacity also matters (Based on the data in Case 7 in Table 4.5)*

The Quantity Discount model-No Split on Case B is taken as example and the results are illustrated in Figures 4.10 and 4.11 – showing how a supplier's capacity affects the manufacturer's profit and the purchased quantity from this supplier.

- The above two figures reveal that when Supplier 2's capacity $q_2 < 5800$, the manufacturer purchases the raw material from another supplier, rather than from Supplier 2, although Supplier 2's price is lower. As a result, the manufacturer's profit is lower than that when Supplier 2 is selected to provide the specific raw material.

- When $6000 < q_2 < 7500$ roughly, Supplier 2 wins the opportunity to provide the material. The manufacturer's profit then increases as Supplier 2's capacity increases.

When $q_2 > 7500$, the manufacturer's profit and the purchased quantity from Supplier 2 maintain constant levels.

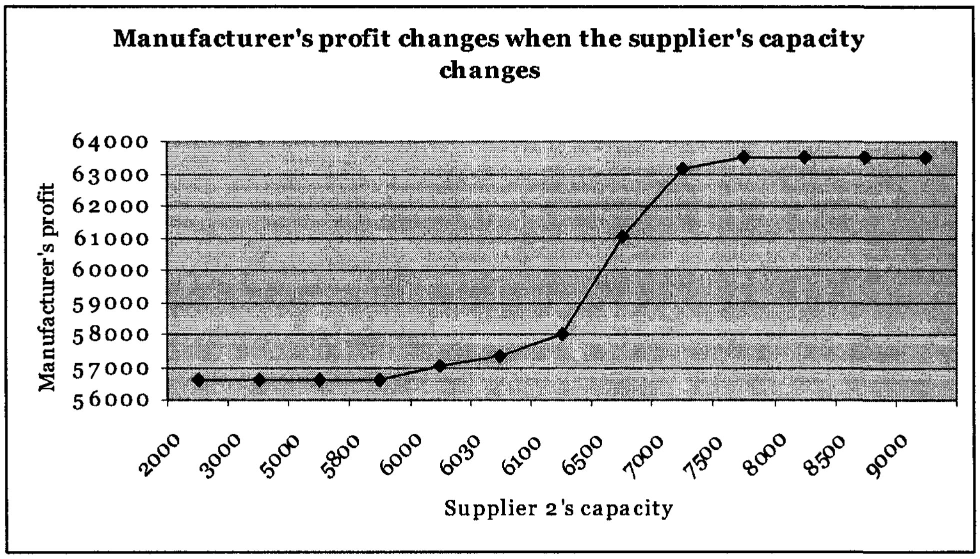


Figure 4.10 How the manufacturer's profit change when the supplier's capacity varies

The numerical results, when taking another supplier's capacity as an example, reveal the same pattern. This analysis suggests a practical explanation. If the supplier and the manufacturer are intra-organizational, the supplier's capacity – usually reserved for the production of the specific raw material – can be assigned optimally based on the information that the manufacturer has for the whole purchasing situation.

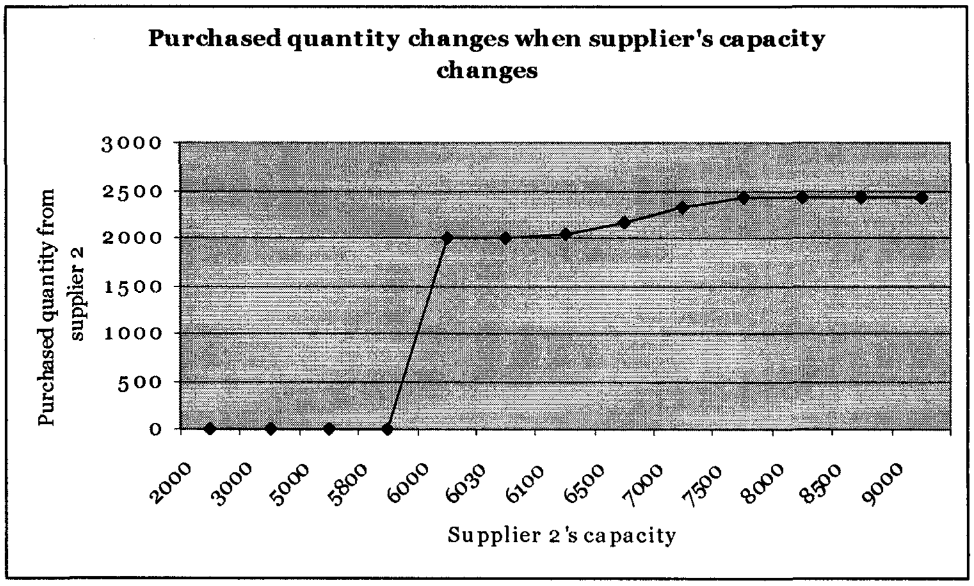


Figure 4.11 How the purchased quantity changes when the supplier's capacity varies

How do the manufacturer's profit and the purchased quantity change when a supplier management cost changes?

The Quantity Discount model-No Split on Case B is taken as an example in this analysis. Originally the management costs for 5 suppliers are identical because every supplier produces the same five components – suggesting that the manufacturing efforts of the production process for the five suppliers are at the same level.

Figure 4.12 and 4.13 originally reveal that the management cost for Supplier 3 is 350, and Supplier 3 is selected for the provision of Component 5 mainly due to its low price. This continues until the management cost for Supplier 3 increases to approximately 2200. The high management cost for Supplier 3 causes them to lose the opportunity to provide Component 5 (provided by Supplier 5). However, because of Supplier 5's high price, the purchase of Component 5 from Supplier 5 results in a lower manufacturer's profit.

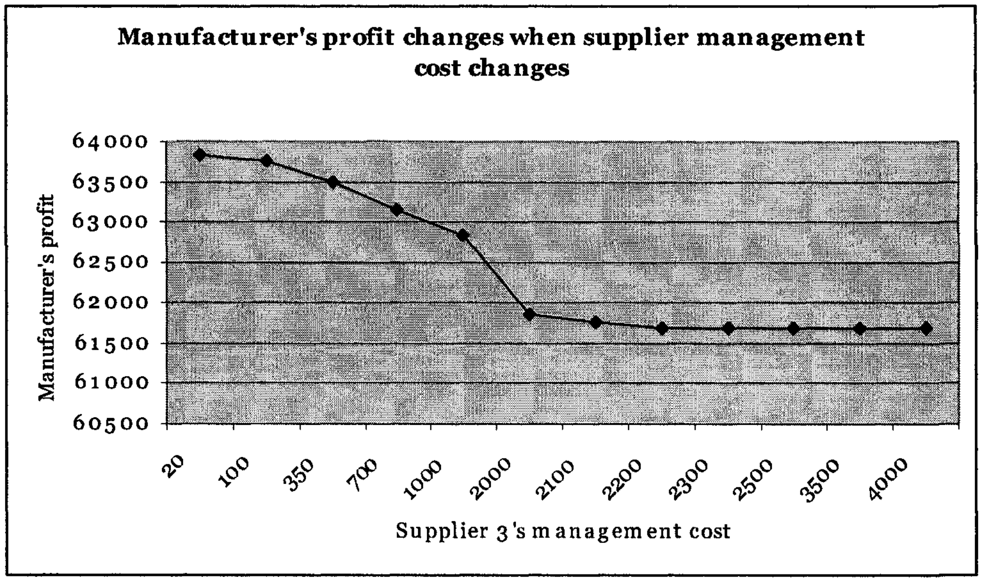


Figure 4.12 The manufacturer's profit changes when the management cost for Supplier 3 changes (Q=2400 on Case B)

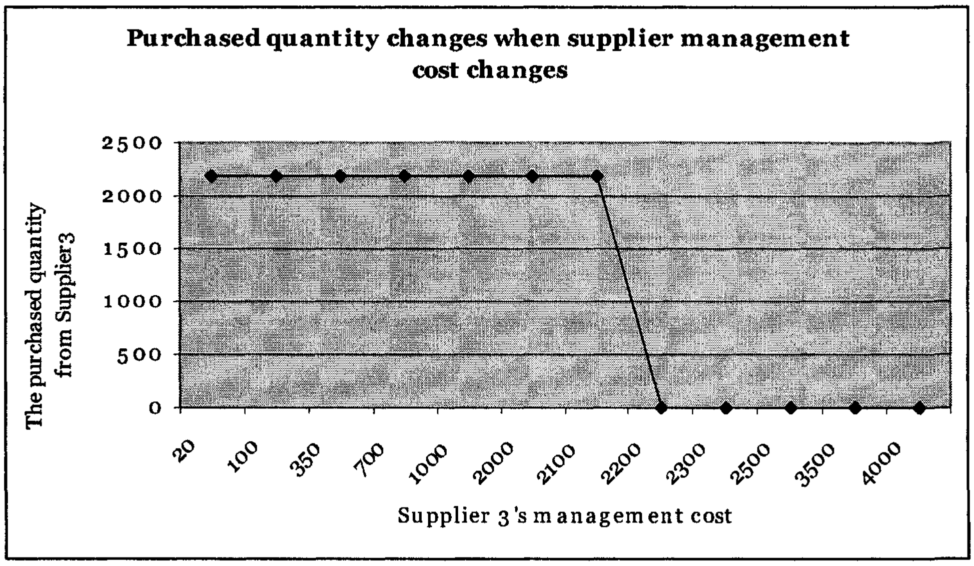


Figure 4.13 The purchased quantity from Supplier 3 changes when the management cost for Supplier 3 changes (Q=2400 on Case B)

Take another example with Case A. Originally Supplier 2 and Supplier 3 have the same management costs (80). Supplier 2 is selected for the provision of Component 3 due to its lower price. The numerical results reveal that only when the management cost for Supplier 2 increases to 700 – almost 10 times the original Supplier 2 loses its opportunity to supply Component 3 and Supplier 3 is selected instead to provide Component 3 for the manufacturer.

Usually there is not a significant difference between the management costs for two suppliers, especially when they produce and provide the same products. Therefore, the managerial implication is that the supplier management cost does not have an obvious impact on the optimal solution of whether the supplier is selected for a raw material's supply, although it does contribute to the decision. When there is significant difference between two suppliers' management costs — when one supplier is from the home country of the manufacturer, and another is from a foreign country, for example – then said costs require particular attention.

How does the demand uncertainty affect the production amounts and the manufacturer's profit?

The following figures illustrate how production amounts in optimal production levels and the manufacturer's profit change as the demand uncertainty of a product varies. The demand uncertainty is represented by the standard deviation of Normal distribution function in this thesis.

The Quantity Discount model-No Split on Case A is taken as an example in Figures 4.14 and 4.15 where the standard deviation of Product 2's demand changes from 1.000 to 19.000 (the original standard deviation of Product 2's

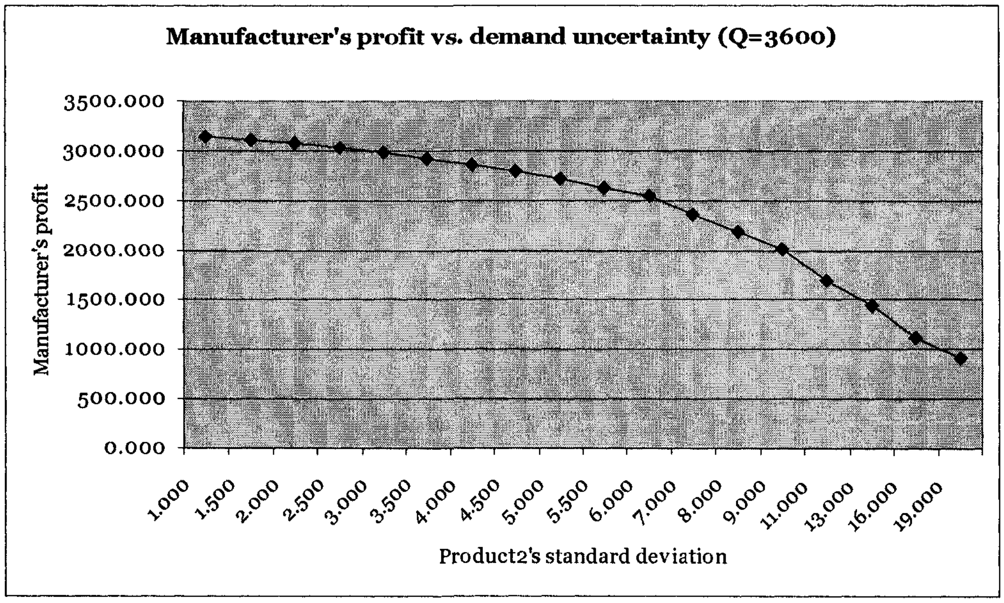


Figure 4.14 Manufacturer's profit changes as demand uncertainty varies (Q=3600 for Case A)

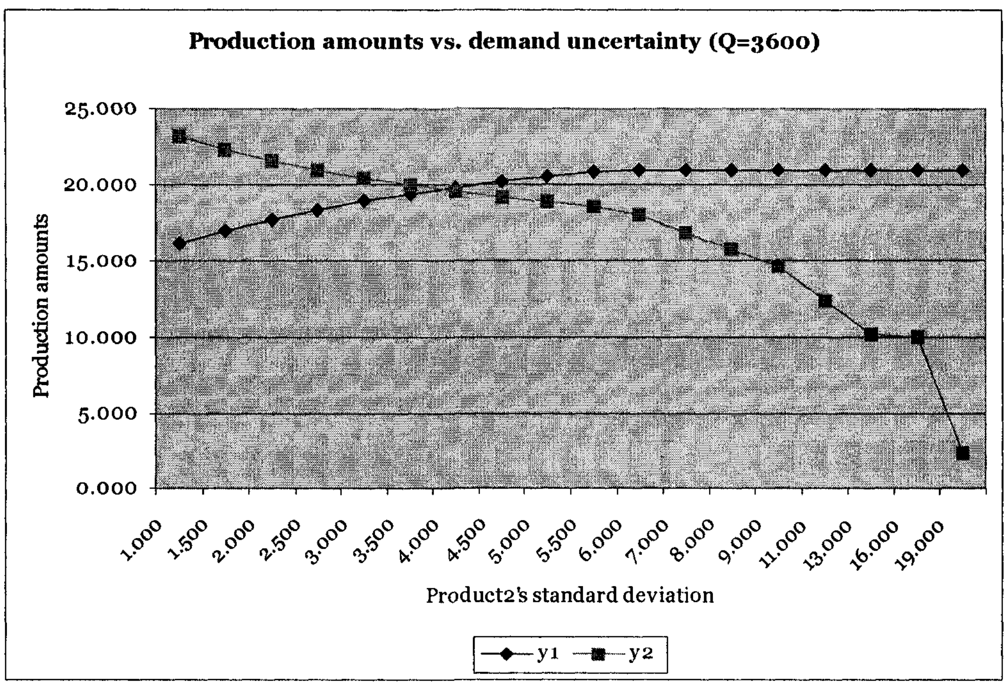


Figure 4.15 Production amounts change as demand uncertainty varies (Q=3600 for Case A)

demand is 3.742). The manufacturer's capacity is underdeveloped (Q=3600) for both figures. The data, which supports Figures 4.14 and 4.15, is displayed in Appendix IV.

A close look at Figure 4.14 reveals that the manufacturer's profit declines at a slower speed (a low sloping curve) when the standard deviation is relatively small. Once the standard deviation reaches approximately 6.000, the curve begins to slope high. The whole curve consists of two straight lines with two slopes. The smaller the standard deviation is, the more certain the demand of the product is. This suggests that the more random the demand is, the more difficult it is for the manufacturer to control its production level to meet the demand, and the less profit the manufacturer is able to make.

Figure 4.15 shows that the production volume of Product 1 increases, and the production volume of Product 2 decreases as the uncertainty of Product 2's demand grows. It follows that the manufacturer tries to shift its production capacity from Product 2 to Product 1 to avoid losses caused by the demand uncertainty of Product 2. When the production volume of Product 1 comes to a point that is close to the average value of Product 1's demand, the production amount of Product 1 stops increasing. The production amount of Product 2, however, is always under the average value of its demand in this case.

What if the manufacturer's capacity is fully utilized? Figures 4.16 and 4.17 display how the manufacturer's profit and production amounts change as the demand uncertainty changes when the manufacturer's capacity is 2700 in Case A.

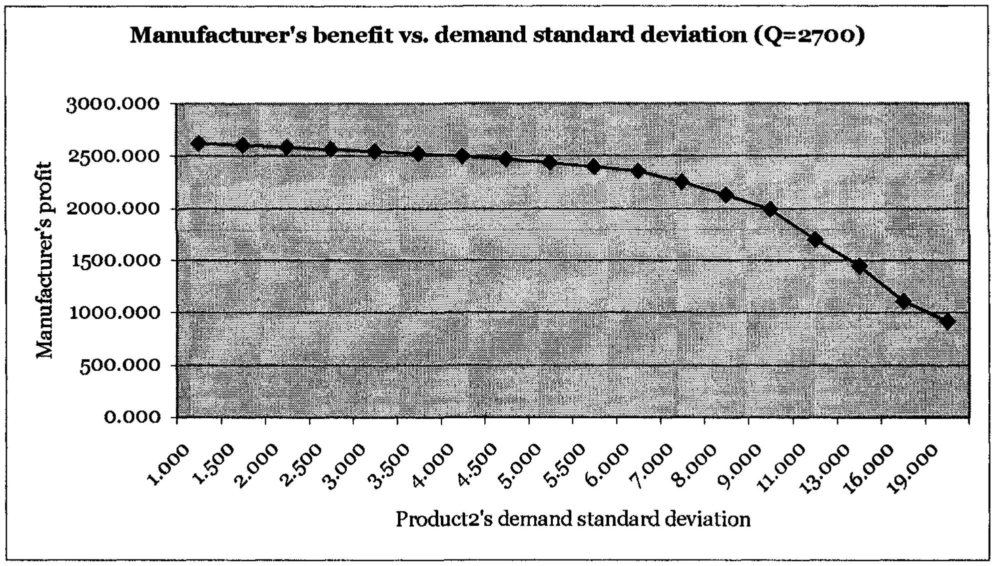


Figure 4.16 Manufacturer's profit changes as demand uncertainty varies (Q=2700 for Case A)

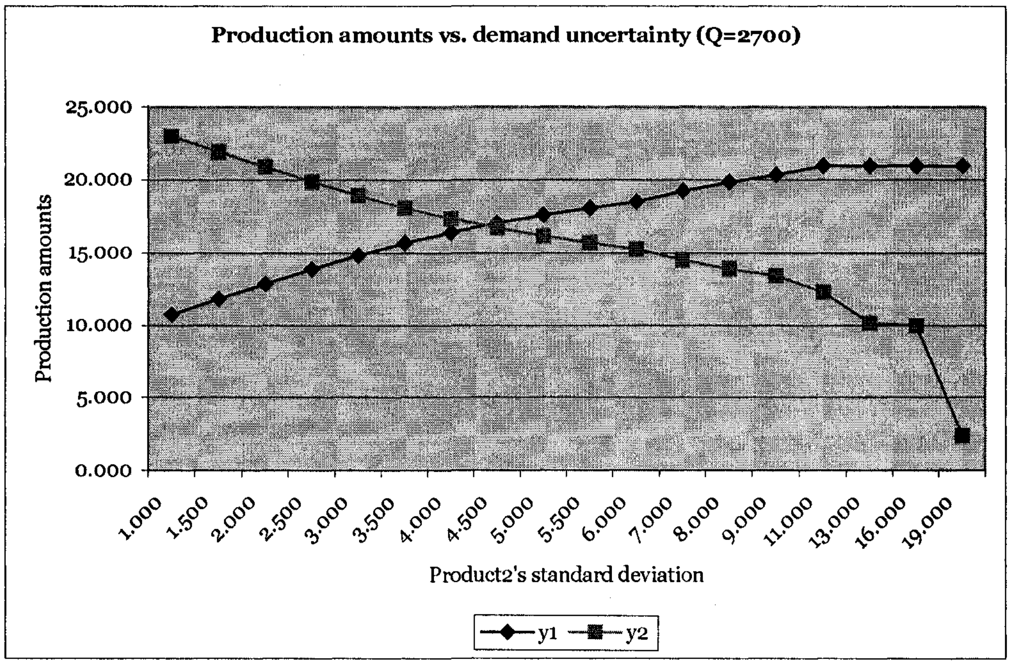


Figure 4.17 Production amounts change as demand uncertainty varies (Q=2700 for Case A)

Figures 4.16 & 4.17 show the same pattern as in Figures 4.14 and 4.15. Therefore, the manufacturer's profit and production amounts change in the same manner when Q=2700 and Q=3600. Another result of this observation suggests that the optimal production amounts of Product 2 and Product 1 are under their average values – 25.15 and 25.10. This is followed in both situations – whether the manufacturer's capacity is fully utilized or not.

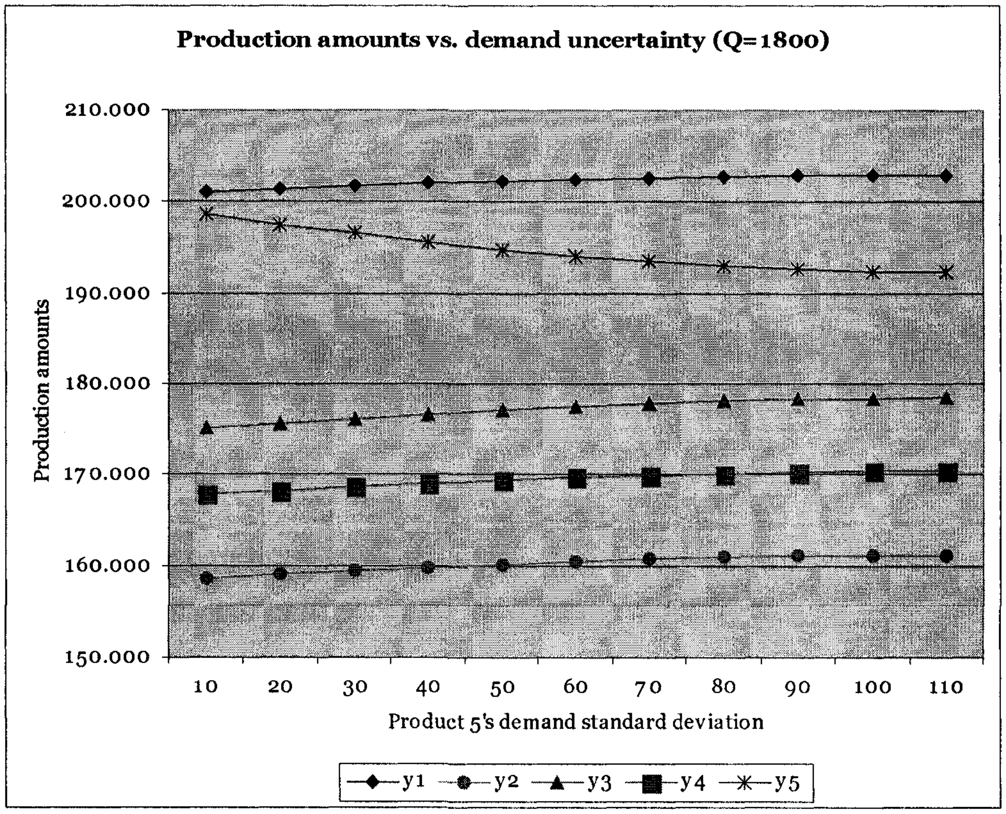


Figure 4.18 Production amounts change as demand uncertainty varies (Q=1800 for Case B)

To further prove the above conclusions, more numerical results are obtained from Case B. Figures 4.18 and 4.19 use the Quantity Discount model-Split on Case B as an example to illustrate how the production amounts change when the demand uncertainty changes.

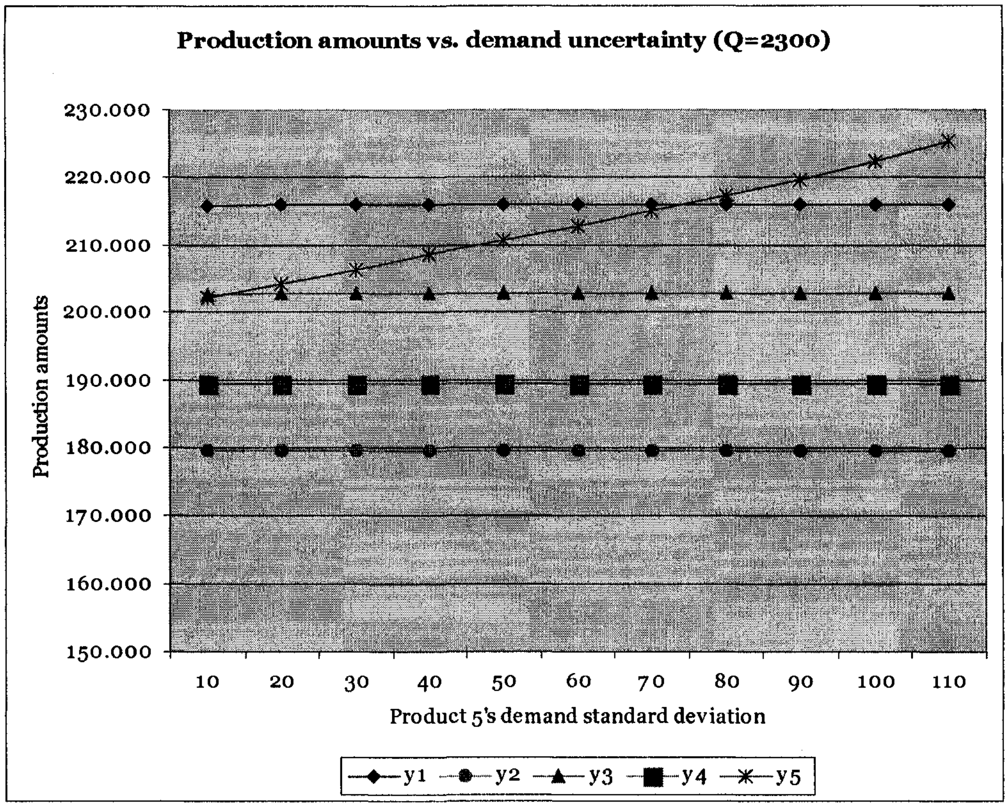


Figure 4.19 Production amounts change as demand uncertainty varies (Q=2300 for Case B)

In Figure 4.18, when Q=1800 the manufacturer's capacity is fully exploited the production amounts of product 5 decrease and the production amounts of other products increase slightly as the standard deviation of the demand on Product 5 increases. This implies that the manufacturer's capacity will shift from production of Product 5 to other products when the demand uncertainty of Product 5 increases.

In Figure 4.19, when Q=2300 and the manufacturer's capacity is under utilized, the production amounts of other products maintain a constant level and the production amount of Product 5 increases as the standard deviation of the demand of product 5 increases. This suggests that the production amounts of other products have already saturated the market, so the extra manufacturer's capacity will be used to improve the production amount of Product 5 despite its uncertainty.

Another point that deserves our attention is that the production amount of Product 5 is larger than 200 – the average value of the demand of Product 5 in Figure 4.19. The conclusion from Figures 4.18 and 4.19 with Case B differs from the one drawn from Figures 4.16 & 4.17 with Case A. The production amount of Product 2 always decreases as the demand of Product 2 increases. However, no matter which situation, the manufacturer's profit always decreases when the demand becomes more uncertain.

Product	**1**	**2**	**3**	**4**	**5**
a_k	100	90	50	90	150
b_k	60	40	20	10	100
r_k	1.67	2.25	2.50	9.00	1.50

Table 4.6 Parameters of products on Case B

Figure	Case	Manufacturer's capacity	Demand uncertainty	Demand	Production amount of current product	Production amounts of other products	Ratio of underage cost/overage cost
4.11	Case A	developed	increase	below average	decrease	increase	low
4.13	Case A	under developed	increase	below average	decrease	increase	low
4.14	Case B	developed	increase	below average	decrease	increase	low
4.15	Case B	under developed	increase	above average	increase	constant	low
Kim	Case B	developed	increase	below average	decrease	increase	high
Kim	Case B	under developed	increase	above average	increase	decrease	high

Table 4.7 Summary of the demand uncertainty analysis

When the above conclusions are compared with the findings by Kim et al. (2002), and interesting phenomenon arises. When the manufacturer's capacity is under developed, the production amounts of other products decrease (instead of keeping constant in Figure 4.19) as the demand of Product 4 becomes more uncertain. To further investigate the real reason behind this phenomenon, a parameter (r_k) – the ratio of the underage cost (a_k) over the overage cost (b_k) – is introduced. Among the five products in Case B, the ratio of Product 4 (9.00) is the highest, and the ratio of Product 5 (1.50) is the lowest as displayed in Table 4.6. A higher r_k means that if there is a shortage in the effort to meet the demand of Product 4, the loss incurred by the shortage for the manufacturer will be higher. On the contrary, if there is a shortage on Product 5, the loss will be lower. Therefore once the manufacturer has some extra capacities (under developed), the production amount of Product 5 will be improved to fulfill the uncertain demand on the premise that production amounts of other products are satisfied as a result of Product 5's low ratio. When it comes to Product 4, however, the manufacturer would like to reduce the production amounts of other products – the shortage on which will lead to lower loss – in order to meet the uncertain demand of Product 4 (the shortage of which will bring a higher loss).

Comparisons of split, no split, and conditional split

Models for split, no split, and conditional split cases are discussed in Chapter 3. The differences between these three cases is explored in this section by two examples with the Quantity Discount models on Case B recorded in Table 4.8. The manufacturer's capacity is set as 2400 (under developed) in both cases

		Production amounts					**Purchased quantity**					
	OBJ	**y1**	**y2**	**y3**	**y4**	**y5**	**comp1**	**comp2**	**comp3**	**comp4**	**comp5**	**comp4**
							supp4	**supp5**	**supp1**	**supp2**	**supp3**	**supp5**
Case1: Quantity Discount model on Case B (Q=2400, q2=7500)												
Nosplit	59778.694	215.869	179.552	202.7200	189.513	217.226	2060.64	1783.929	2184.711	2454.173	2190.668	0
Cond Split	59778.694	215.869	179.552	202.7200	189.513	217.226	2060.64	1783.929	2184.711	2454.173	2190.668	0
Split	59778.694	215.869	179.552	202.7200	189.513	217.226	2060.64	1783.929	2184.711	2454.173	2190.668	0
Case2: Quantity Discount model on Case B (Q=2400, q2=3500)												
Nosplit	52841.589	211.929	178.116	198.9250	184.965	212.329	2020.701	1753.749	2143.133	0	2151.044	2404.037
Cond Split	55991.589	211.929	178.116	198.9250	184.965	212.329	2020.701	1753.749	2143.133	1166.667	2151.044	1237.370
Split	55991.589	211.929	178.116	198.9250	184.965	212.329	2020.701	1753.749	2143.133	1166.667	2151.044	1237.370

Table 4.8 Comparisons of Split, No Split and Conditional Split

			Production amounts					**Purchased quantity**					**Supplier's total sale**
		OBJ	**y1**	**y2**	**y3**	**y4**	**y5**	**comp1**	**comp2**	**comp3**	**comp4**	**comp5**	
								supp4	**supp5**	**supp1**	**supp2**	**supp3**	
Case 1	qty	59778.694	215.869	179.552	202.720	189.513	217.226	2060.64	1783.929	2184.711	2454.173	2190.668	10923.555
								comp1	comp2	comp3	comp4	comp5	
								supp1	supp5	supp1	supp2	supp3	
	vol(23%)	62192.293	217.940	180.583	203.147	191.819	217.501	2070.227	1792.806	2202.912	2468.219	2202.126	21233.810
								comp1	comp2	comp3	comp4	comp5	
								supp4	supp5	supp1	supp2	supp3	
Case 2	qty	59778.694	215.869	179.552	202.720	189.513	217.226	2060.64	1783.929	2184.711	2454.173	2190.668	10923.555
	vol(25%)	62859.633	217.552	180.154	204.320	191.458	219.284	2077.469	1796.649	2202.31	2475.348	2207.391	8258.663
	vol(10%)	61007.894	216.530	179.792	203.358	190.287	218.048	2067.357	1789.006	2191.729	2462.621	2197.341	9862.781

Table 4.9 Comparisons of the quantity discount and the volume discount schemes

to avoid the manufacturer's capacity being the reason preventing more production. In Case 1, Supplier 2's capacity is set as 7500. The results from split, no split, and conditional split situations are exactly the same. This happens in a situation where there is no split purchase in the optimal solution, even when a split is allowed. As a result, the optimal solution with no split policy will be the same. In Case 2, Supplier 2's capacity is set as 3500. The split model and the conditional split model give the same result with the purchase on component 4 being split between Supplier 2 and Supplier 5. Due to the non-split restriction, the result from the no split model provides the worst results. The manufacturer has to purchase component 4 from Supplier 5 at a higher price because Supplier 2's capacity is small despite a lower price. Generally speaking, with no split or conditional split constraints it is possible for the manufacturer to get a lower profit with an inferior purchasing scheme if the no split or conditional split restrictions have a negative impact on the manufacturer's purchase.

Comparisons of the quantity discount and volume discount schemes

As discussed in Chapter 2, both the buyer and the vendor benefit from the volume discount. In this analysis, two examples are designed to perceive the effect the two types of discount schemes have on the optimal solutions. Two examples are taken from the Quantity Discount model-No Split and the Volume Discount model-No Split on Case B with the results illustrated in Table 4.9. For the same reason used in the last section – namely the comparison of the split, no split, and conditional split cases, the manufacturer's capacity is set at 2300 (under fully developed). Case 1 takes Supplier 1 for the analysis. Considering the prices from Supplier 1 for all components in the data requirement table (Table B3 in Appendix II), the quantity discount offered by Supplier 1 is set (with the intention to sell more by offering more discount when the price is high) as: 10% off for Component 1, 30% off for Component 2, no discount for Component 3, 30% off for Component 4 and 50% off for Component 5. It is assumed that Supplier 1 knows which price is high compared to the price from other suppliers in the open market. The rough average of the quantity discounts for all components is 24% and the volume discount that Supplier 1 offers is set as 23%. The results suggests that the manufacturer has a higher profit with the volume discount schemes. Supplier 1's sale volume is also provided in Table 4.9, and reveals that Supplier 1's sales increase with the volume discount, which increases the manufacturer's purchase of component 1.

Case 2 takes Supplier 2 for the analysis. Similarly the quantity discount offered by Supplier 2 is set as: 20% off for Component 1, 60% off for Component 2, 20% off for Component 3, no discount for Component 4, and 40% off for Component 5. The rough average of the quantity discounts for all components is 28% and the volume discount that Supplier 2 offers is set as 25% off. The results suggest that the manufacturer has a higher profit with the volume discount schemes. Even the results with 10% off as the volume discount provide a better solution for the manufacturer.

Analyses of the node selection mode in SBB solver

As a by-product of this project, some comments about the GAMS package – specifically regarding the SBB solver – are summarized here. When the SBB solver is called for dealing with the MINLP model in GAMS, as aforementioned when the SBB solver was introduced, different options can be made for the setting of the Branch-and-Bound algorithm to improve the efficiency of the algorithm. For instance, in the SBB solver, different node selection modes (listed in the following) can be assigned for the Branch-and-Bound algorithm in the option file to specify which mode is preferred:

0—automatic
1—Depth First Search (DFS)
2—Best Bound (BB)
3—Best Estimate (BE)
4—DFS/BB/ mix
5—DFS/BE mix
6—DFS/BB/BE mix

Based on the experience gained in the course of this project, the following comments about the node selection mode in SBB solver apply:

- The node selection mode that gives the optimal solution for a specific MINLP model is random. However, for a specific MINLP model, one node selection mode can be counted on for optimal solutions if it always gives the optimal solution with a certain number of tries. For this project, except node selection mode 2 (Best Bound), other node selection modes behaves at the same level in terms of program running time and finding optimal solutions.
- The program running time is related to the node selection mode for a specific MINLP model. This can be identified in Figure 4.20, which displays the program running time for the Volume Discount model-Split on Case B by using different node selection modes. Apparently it takes

longer to run the program when the node selection is 2 (Best Bound mode).

- For the NLP models, the GAMS program running time is not only dependent on the size of the model, but on the feature of the data as well. Therefore even for similar models with different data input, the program running time might change significantly. For example, with the same volume discount model the program running time on Case B is much longer than on Case A. However with the same quantity discount model, the program running time on Case A and Case B are almost the same although the size of Case B is slightly larger than that of Case A.

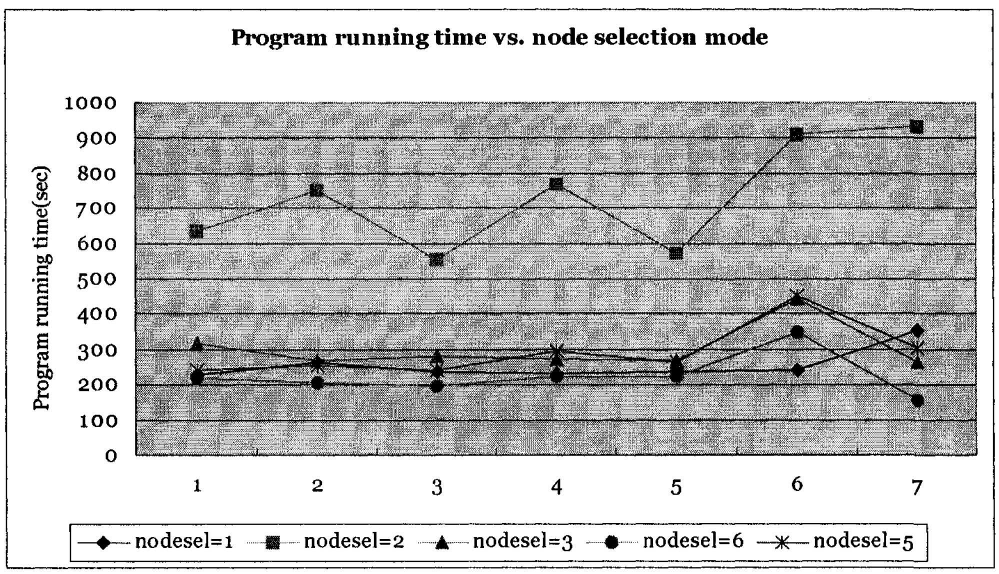


Figure 4.20 The GAMS program running time and the node selection mode

The above comments are limited by the number of times the GAMS program is run and the size of the models applied in this project. It is not supported by a series of scientific statistic experiments, which would demand a formidable amount of extra work that does not address the purpose of this project.

Chapter 5: Conclusions and Future Work

5.1 Conclusions

This thesis presents a series of Mixed Integer Non-Linear Programming models in order to obtain the optimal acquisition policy and simultaneously determine the production level for a supply network with different discount schemes from multiple suppliers and uncertain demands of multiple products. With the assumptions stated in Chapter 3, the mathematical model formulates the supply chain problem with the introduction of an integration function and a probability density function in the model to handle the uncertainty of the demand. The GAMS-based solution program is developed and verified to be capable of solving the MINLP models efficiently.

The supply chain problem discussed in this thesis is a strategic acquisition policy – a relatively long-term purchasing policy for a manufacturer. With the probability density function of the uncertain demand and other required production and supplier information as inputs, the solution program serves as a tool to integrate demand market information, resources, and supplier discount schemes and convert them into solution options – outputs for the management decision-makers regarding the production level and optimal purchasing strategy.

The GAMS solution program allows the product demands to follow any other probability density function (P.D.F) – especially when the result of the integration function, with the P.D.F as part of the integrand, can't be explicitly expressed in closed-form in addition to the Normal distribution that is cited in this report.

The research presented in this thesis extended the quantity discount model developed by Zhang (2004) to cover different split situations, and verified these models by comparing preliminary results from the GAMS program with those from the analytical algorithm. In addition, a group of volume discount models are developed and similarly verified. Based on the results from this work, the MINLP models for the quantity or volume discount are trusted with more numerical results, which support the numerical analysis and managerial implications. As one of the ultimate goals of this research work, the managerial implications below are derived from the numerical analysis in Chapter 4.

- ➢ The price, supplier management cost, discount segment setting, and the supplier's capacity are all decision factors for the supplier selection issue discussed in this thesis. How the manufacturer's profit and the purchased quantity from the supplier are impacted when a supplier's capacity or the management cost for a supplier changes is illustrated in Chapter 4.
- ➢ The optimal solution for the production level and the purchasing schemes are provided by the GAMS solution program, in addition to the analysis of the GAMS program output – specifically the List file, which reveals valuable information with managerial implications. Finding whether a constraint is binding or not identifies the bottleneck factor, which prevents the manufacturer from improving its profit.
- ➢ With the demand of one product being more uncertain, the manufacturer, who has multiple products, will try to shift its production capacity to other

products – namely increase the production amounts of other products by reducing the production amounts of said product if the manufacturer's capacity is fully exploited (no extra capacity left). If the manufacturer has extra capacity, however, the manufacturer will increase the production amount of this product to meet the uncertain demand. Whether the manufacturer increases or decreases production amounts of other products is related to a parameter – the ratio of the underage cost over the overage cost of the product. No matter which situation, the manufacturer is always losing profit when the demand is more uncertain.

- In most situations, a manufacturer can achieve a higher profit if a split is allowed in the purchase of raw material, but not always. Comparisons of split, no split, and conditional split cases reveal that the manufacturer probably suffers from the no split or conditional split constraints at the cost of lower profits.
- Comparisons of the quantity discount and volume discount schemes disclose that a volume discount offer is a better option for a manufacturer and, at times, for a supplier.
- In most optimal solutions, the optimal production amount is not the average value of the uncertain demand. This gives a better sense of the average value and suggests that – it is not always wise to use the average value to replace an uncertain demand (which follows Normal distribution) when the accurate analysis is not available.

- The CONOPT solver is the most competent NLP solver in terms of solving the NLP models at each node with Branch-and-Bound algorithm in the MINLP model in this project.
- Different node selection modes affect a MINLP model with SBB solver in terms of finding the optimal solution and the program running time. But the program running time of the MINLP model is more closely related to the data feature.

5.2 Contribution

The volume discount is gaining popularity in the business world, and has been proven beneficial for both buyers and vendors [Xu et al., 2000]. This study considers a supply network having quantity or volume discount offers from suppliers and multiple products with uncertain demands as one of few papers combing two factors together.

It is very difficult to solve the Mixed Integer Non-Linear Programming model with an integration function to calculate the expected value in the mathematical model. In previous research, an analytical algorithm solution approach was most often suggested and applied for this kind of complex mathematical model. However, the analytical algorithm tends to be problem dependent. This thesis is one of the few research projects using commercial optimization software to solve this type of MINLP model. The advantages of using software – speed and convenience – allow the solution to be easily put into practice. Consequently, this research contributes one option for developing the complement e-procurement in raw material/components because the programming-based solution approach make it more likely that the procurement process will be automated.

Another by-product of this research work regards the GAMS software. The analysis and comparisons in Chapter 4 regarding the node selection mode with the Branch-and-Bound algorithm in SBB solver, and the performances of different NLP solvers for the MINLP model in this project, are valuable for GAMS users.

The results of this research – as discussed in Chapter 3 – can be applied to a general supplier selection process, or the final stage of finding the best supplier among a limited number of qualified suppliers after a screening process. Both activities are common to a company's purchasing management personnel in the competitive business world.

The analysis regarding the impacts of uncertain demand and all the other factors from the manufacturer and the suppliers are gathered from the numerical results. This, in turn, provides useful information to the manufacturer and the supplier's decision makers for the supplier selection issue.

Besides the direct application of this research to a supply chain problem, the methodologies and the solution approach applied in this thesis provide some ideas for projects in other areas with similar circumstances – namely handling uncertainty, finding expected value, using a mathematical programming approach, and so on.

5.3 Future Work

Based on the mathematical models, the solution approach, and the results developed in this thesis, future work not covered in this study, due to the time limit is suggested in this section.

The mathematical model discussed in this report could be extended to deal with outsourcing activities, although in this report only raw material/components purchasing movements are covered. The MINLP models would fit in the outsourcing activity with some modifications.

A quantitative analysis of how the uncertain demand affects the production amounts could be developed. Such a study would be interesting to both practitioners and academic researchers in the Supply Chain Management field.

More mathematical models can be developed to explore topics in other tiers and other stages of the supply chain, based on the solution approach and the technique dealing with the uncertainty applied in this research. For example, a model could be developed considering the short-term purchasing plan, production plan, and the inventory management cost incurred accordingly. Suppliers' lead-time could be taken into account as a factor, which would definitely impact the purchasing plan. This would create a cost-ownership model, minimizing the manufacturer's total cost from the perspective of the operational level. Another mathematical model for the tier of suppliers could be built by considering the supplier's total benefit in terms of the production plan (corresponding to the buyer's purchasing plan) and the inventory management. A model using Game theory knowledge could be developed to analyze the relationship between the vendor and the manufacturer.

Finally, all the above models correlate with one another through some kind of linkage between different entities along the supply chain. This co-relationship could be used to integrate those models into one complete optimization problem. The entire model might be solved by multiple solution methodologies such as analytical method and other optimization tools. Commercial optimization applications could be used for solving each specific sub-model. The entire model would be one supply chain model for the integrated strategic and operational levels. Not much research has been done in an integrated manner in the Supply Chain Management field, although a lot of work has been done in each sub-area.

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GAMS software: http://www.gams.com

Romberg algorithm: http://mathworld.wolfram.com/RombergIntegration.htm

Appendix I: Screenshots of the log file and the list file in GAMS

1. A sample of the log file in GAMS

```
GAMS Rev 142  Copyright (C) 1987-2005 GAMS Development. All rights reserved
Licensee: Guoqing Zhang                     G050624:1833AP-WIN
          University of Windsor               DC5369
          License for teaching and research at degree granting institutions
--- Starting compilation
--- VolDis_case1_split.gms(214) 3 Mb
--- Starting execution
--- VolDis_case1_split.gms(207) 4 Mb
--- Generating model VolDis_case1
--- VolDis_case1_split.gms(208) 6 Mb
---   43 rows, 47 columns, and 159 non-zeroes.
--- VolDis_case1_split.gms(208) 4 Mb
--- Executing SBB

Simple B&B    Apr  2, 2005 WIN.SB.SB 21.7 016.031.041.VIS


Reading user supplied options file C:\Documents and Settings\USER1\My Documents\Thesis
GAMS\VolumeDiscount\sbb.opt
Processing...
> rootsolver   CONOPT
> failseq          SNOPT MINOS
> acceptnonopt  1
> nodesel          3
>

Sequence for solver failure:
   SNOPT option file: -
   MINOS option file: -
Root node solver: conopt.0
 47 columns (16 discrete), 43 rows, 159 nonzeros

Inside the External3.DLL.
Ext: n=10, m=8, nz=16
C O N O P T 3 Apr  2, 2005 WIN.CO.CO 21.7 016.054.041.VIS Library 314J


   C O N O P T 3   Intel /MS Window version 3.14J-016-054
   Copyright (C)   ARKI Consulting and Development A/S
                   Bagsvaerdvej 246 A
                   DK-2880 Bagsvaerd, Denmark

 Using default options.

 Reading Data

  Iter Phase Ninf  Infeasibility  RGmax   NSB   Step InItr MX OK
    0   0        5.2250028094E+01 (Input point)
                       Pre-triangular equations:     0
                       Post-triangular equations:    1
    1   0        5.2250028094E+01 (After pre-processing)
```

```
  2  0     5.2250028094E+01 (After scaling)

** Feasible solution. Value of objective =  -552.800311567

 Iter Phase Ninf    Objective     RGmax    NSB  Step InItr MX OK
   6  3      1.5303892606E+04 4.0E+03    5 2.1E-01   4 F  T
  11  3      1.5558639603E+04 8.3E+00    2 3.2E-02     F  T
  16  4      1.5558881137E+04 1.5E-08    2

** Optimal solution. Reduced gradient less than tolerance.

Root node solved locally optimal.
Pseudo Cost Initialization...
 Node  Act. Lev.  Objective   IInf  Best Int.   Best Bound   Gap  (0 secs)
   0    2   0  15558.8811   8        -  15558.8811      -
   1    1   1  infeasible   -        -  15558.8811      -
   2    2   1  15320.7372   8        -  15320.7372      -
   3    3   2   8189.5789   6        -  15320.7372      -
   4    4   2  10811.4973   8        -  10811.4973      -
*  5    3   3   -552.8003   0  -552.8003  10811.4973      -
   6    4   3   9105.7113   6  -552.8003   9105.7113      -
   7    3   4  infeasible   -  -552.8003   9105.7113      -
   8    4   4   8986.4885   6  -552.8003   8986.4885      -
   9    3   5  infeasible   -  -552.8003   8986.4885      -
  10    4   5   6237.2084   6  -552.8003   8189.5789      -
  11    5   3   8068.3264   6  -552.8003   8189.5789      -
  12    4   3  infeasible   -  -552.8003   8068.3264      -
  .........................Solving process
  49    5  12      pruned   -   654.0920   2323.8662  0.718533
  50    4  12      pruned   -   654.0920    860.0633  0.239484
  51    3  11  infeasible   -   654.0920    860.0633  0.239484
* 52    2  11    827.4271   0   827.4271    859.9553  0.037825
Solution satisfies optcr
Statistics:
  Iterations    :       498
  NLP Seconds   :       3.000000
  B&B nodes     :        52
  MIP solution  :    827.427085 found in node 52
  Best possible :    859.955273
  Absolute gap  :     32.528188    optca :  0.000000
  Relative gap  :     0.037825    optcr :  0.100000
  Model Status  :         8
  Solver Status :         1

NLP Solver Statistics
  Total Number of NLP solves  :       54
  Total Number of NLP failures:        0
  Details:      conopt
   # execs        54
   # failures      0
Terminating.
--- Restarting execution
--- VolDis_case1_split.gms(208) 0 Mb
--- Reading solution for model VolDis_case1
--- Executing after solve
--- VolDis_case1_split.gms(214) 3 Mb
*** Status: Normal completion
```

2. A sample of the list file in GAMS

```
GAMS Rev 142  Intel /MS Window                12/27/05 13:54:57 Page 1
Volume Discount Model_Complete Split_case A
C o m p i l a t i o n

 12
 13  Sets      i        component        / comp1 * comp5 /
 14            j       supplier         / supp1 * supp4 /
 15            k        product          / prod1, prod2 /
 16            l       discount segment  / segm1, segm2,segm3/
 17
 18  *input of data:
 19  Scalar      pi       '3.1415926';
 20            pi = 4*arctan(1);
 21  Scalar      sqrt2    '1.414';
 22            sqrt2 = sqrt(2);
 23  Scalar      bigq     the total resource capacity of the manufacturer;
........................................
208  Solve VolDis_case1 using minlp maximizing ttlbenefit ;
209  Display ttlbenefit.l;
210  Display y.l;
211  Display x.l;
212  display u.l;
213  * display a.l;
214  display w.l;
--------------------------------------------ECHO PRINT-----------------------------------------------------------

COMPILATION TIME    =       0.000 SECONDS    3.2 Mb  WIN217-142 Apr 27, 2005
GAMS Rev 142  Intel /MS Window                12/27/05 13:54:57 Page 2
Volume Discount Model_Complete Split_case1
Equation Listing    SOLVE VolDis_case1 Using MINLP From line 208
---- mcap  =L=  the manufacturer's total capacity limit
mcap..  80*y(prod1) + 80*y(prod2) =L= 2200 ; (LHS = 0)
---- scap  =L=  the supplier j's capacity
scap(supp1)..  x(comp1,supp1) + x(comp2,supp1) + x(comp3,supp1) =L= 39.4 ;
   (LHS = 0)
scap(supp2)..  x(comp3,supp2) =L= 40 ; (LHS = 0)
scap(supp3)..  x(comp3,supp3) =L= 36.7 ; (LHS = 0)
REMAINING ENTRY SKIPPED

.......................................................


---- integ4  the value of the 4th integral item

integ4(prod1)
         (.LO, .L, .UP = -INF, 0, +INF)
    (-9)     integra4(prod1)
     10      obj

integ4(prod2)
         (.LO, .L, .UP = -INF, 0, +INF)
   (-10)     integra4(prod2)
     12      obj


---- ttlbenefit  total benefit of the supply network & the object funciton value

ttlbenefit
```

```
        (.LO, .L, .UP = -INF, 0, +INF)
   1    obj

----------------------------------------------------REFERENCE MAP---------------------------------------------

GAMS Rev 142  Intel /MS Window              12/27/05 13:54:57 Page 4
Volume Discount Model_Complete Split_case1
Model Statistics    SOLVE VolDis_case1 Using MINLP From line 208


MODEL STATISTICS

BLOCKS OF EQUATIONS      12    SINGLE EQUATIONS       43
BLOCKS OF VARIABLES       9    SINGLE VARIABLES       47
NON ZERO ELEMENTS       159    NON LINEAR N-Z         25
DERIVATIVE POOL          34    CONSTANT POOL          28
CODE LENGTH             227    DISCRETE VARIABLES     16

=X= PROCEDURE = C:\Documents and Settings\USER1\My Documents\Thesis GAMS\VolumeD
                            iscount\VolDis_case1.dll
=X= ROWS = 8  COLS = 10  NONZEROS = 16
GENERATION TIME    =     0.015 SECONDS   4.0 Mb  WIN217-142 Apr 27, 2005
EXECUTION TIME     =     0.015 SECONDS   4.0 Mb  WIN217-142 Apr 27, 2005
GAMS Rev 142  Intel /MS Window              12/27/05 13:54:57 Page 5
Volume Discount Model_Complete Split_case1
Solution Report    SOLVE VolDis_case1 Using MINLP From line 208


        S O L V E     S U M M A R Y
   MODEL  VolDis_case1      OBJECTIVE  ttlbenefit
   TYPE   MINLP             DIRECTION  MAXIMIZE
   SOLVER  SBB              FROM LINE  208

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      8 INTEGER SOLUTION
**** OBJECTIVE VALUE             827.4271

 RESOURCE USAGE, LIMIT          3.000      1000.000
 ITERATION COUNT, LIMIT       498        100000
 EVALUATION ERRORS              0             0

Simple B&B    Apr  2, 2005 WIN.SB.SB 21.7 016.031.041.VIS
Reading user supplied options file C:\Documents and Settings\USER1\My Documents\
                           Thesis GAMS\VolumeDiscount\sbb.opt
Processing...
> rootsolver       CONOPT
> failseq          SNOPT MINOS
> acceptnonopt     1
> nodesel                 3
>

Sequence for solver failure:
    SNOPT option file: -
    MINOS option file: -
................................

---- EQU comp  the component i's requirement to meet production

      LOWER    LEVEL    UPPER    MARGINAL

comp1    -INF      .        .      100.161
comp2    -INF      .        .      248.226
comp3    -INF      .        .      135.000
```

```
comp4   -INF    .     .   171.000
comp5   -INF    .     .   181.000

.....................

---------------------------------------------EQUATION LISTINGS----------------------------------------------

**** REPORT SUMMARY :       0     NONOPT
                    0 INFEASIBLE
                    0  UNBOUNDED
                    0     ERRORS
GAMS Rev 142  Intel /MS Window               12/27/05 13:54:57 Page 6
Volume Discount Model_Complete Split_case1
E x e c u t i o n


----   209 VARIABLE ttlbenefit.L        =     827.427  total benefit of the
                                             supply network & the
                                             object funciton value

----   210 VARIABLE y.L  number of product k to be produced by manufacturer

prod1  8.776,    prod2 18.724


----   211 VARIABLE x.L  number of component i provided by supplier j

        supp1      supp2      supp4

comp1      8.776
comp2     18.724
comp3     10.673     16.827
comp4                    8.776
comp5                   18.724

----   212 VARIABLE u.L  equals 1 if business volumen discount dr(jl) is taken
              and 0 otherwise
        segm1      segm2

supp1              1.000
supp2      1.000
supp4      1.000

----   214 VARIABLE w.L  equals 1 if buy anything from j and 0 otherwise

supp1 1.000,    supp2 1.000,    supp4 1.000

EXECUTION TIME       =       0.000 SECONDS    2.9 Mb   WIN217-142 Apr 27, 2005

USER: Guoqing Zhang                          G050624:1833AP-WIN
    University of Windsor                          DC5369
    License for teaching and research at degree granting institutions

----------------------------------------STATUS REPORTS & RESULTS--------------------------------------------
```

Appendix II: Input Data for Numerical Examples

1. Parameters for Case A (Kim et al. 2002)

Table A1. Product specification—Component requirement (g_{ik})

Product	Component (i)				
	Celeron (1)	Pentium (2)	Motherboard (3)	HDD4.3 (4)	HDD6.4 (5)
Model C ($k=1$)	1		1	1	
Model P ($k=2$)		1	1		1

Table A2. Original supply costs (c_{ij})

Supplier (j)	Management Cost (m_j)	Capacity (q_j)	Component (i)				
			Celeron (1)	Pentium II (2)	Motherboard (3)	HDD4.3 (4)	HDD6.4 (5)
IS (1)	120	39.4	115	285	155		
SOYO (2)	80	40.0			135		
LG (3)	80	36.7			147		
Samsung (4)	120	41.8				171	181

Table A3. Product information

Product	r_k	t_k	μ_k	σ_k	e_k	a_k	b_k
$k=1$	525	80	25.10	3.972	20	10	100
$k=2$	720	80	25.15	3.747	23	12	170

In Case A, $n_{ij}=1$ for all i, j.

2. Parameters for Case B (Kim et al. 2002)

Table B1. Product-related parameters

k	1	2	3	4	5
r_k	150	200	220	230	250
t_k	1	2	2	2	3
μ_k	200	160	180	160	200
σ_k	80	60	70	60	80
e_k	24	32	35.2	36.8	40
a_k	100	90	50	90	150
b_k	60	40	20	10	100

Table B2. g_{ik} (The amount of component *i* for per unit of product *k*)

i/k	1	2	3	4	5
1	2	1	3	1	3
2	1	3	2	1	2
3	3	2	1	4	1
4	2	1	2	3	4
5	1	3	2	2	3

Table B3. c_{ij} (The unit cost of material *i* from supplier *j*)

i/j	1	2	3	4	5
1	8	8	12	6	15
2	10	15	8	10	5
3	5	7	14	9	8
4	9	5	10	13	8
5	12	9	5	7	6

Table B4. n_{ij} **(The resource usage of supplier *j* per unit of material *i* produced)**

i/j	1	2	3	4	5
1	1.5	2	3	1	3
2	2	1	1	3	1
3	2	1.5	1	3	2.5
4	1.5	3	2.5	2	3
5	3	2	3	2	1.5

Table B5. q_j **(The capacity of supplier *j*)**

j	1	2	3	4	5
m_j	350	350	350	350	350
q_j	10,000	7,500	9,000	6,000	12,500

Appendix III: Preliminary Results

1. Original results from Kim and from GAMS on Case B

Q	Original results from Kim						Results from NLP					
	Obj	Y1	Y2	Y3	Y4	Y5	obj	y1	y2	y3	y4	y5
1500	74559.796	189.683	134.455	144.361	146.433	153.272	74560.010	189.683	134.455	144.361	146.433	153.272
1700	85615.126	201.539	151.848	167.909	163.216	177.505	85614.980	201.539	151.848	167.909	163.215	177.505
1900	92897.610	214.009	169.382	191.211	181.113	200.860	92896.920	214.009	169.382	191.211	181.113	200.860
2000	95108.445	220.221	178.171	202.921	190.503	212.197	95107.440	220.221	178.171	202.921	190.502	212.197
2100	96219.376	224.602	189.026	215.490	199.148	222.690	96218.090	224.602	189.026	215.490	199.148	222.690
2200	96361.126	227.969	194.192	222.461	204.895	228.942	96359.670	227.969	194.191	222.460	204.893	228.942
2300	96361.126	227.969	194.192	222.461	204.895	228.942	96359.670	227.969	194.191	222.460	204.893	228.942
2500	96361.126	227.969	194.192	222.461	204.895	228.942	96359.670	227.969	194.191	222.460	204.893	228.942

2. Results from Quantity Discount Model-Split & NLP Model on Case A

Q	**QtyDisMod_ComSplit**								**Results from NLP**		
Q	Y1	y2	obj	x11	x21	x32	x44	x54	y1	y2	obj
2300	11.371	17.379	2996.474	11.371	17.379	28.750	11.371	17.379	11.371	17.379	2996.481
2500	13.767	17.483	3279.996	13.767	17.483	31.250	13.767	17.483	13.766	17.484	3280.229
2600	14.871	17.629	3419.769	14.871	17.629	32.500	14.871	17.629	14.870	17.630	3419.752
2800	16.802	18.198	3687.833	16.802	18.198	35.000	16.802	18.198	16.802	18.198	3687.833
3000	18.446	19.054	3923.430	18.446	19.054	37.500	18.446	19.054	18.446	19.054	3923.430
3200	19.590	19.810	4063.198	19.590	19.810	39.400	19.590	19.810	19.590	19.810	4063.222

3. Results from Volume Discount Model-No Split on Case B

Q	**obj**	**y1**	**y2**	**y3**	**y4**	**y5**	**x1-**	**x2-**	**x3-**	**x4-**	**x5-**
1500	74559.796	189.683	134.455	144.361	146.433	153.272	x14=1553.156	x25=1334.750	x31=1721.327	x42=1854.933	x53=1634.455
1700	85615.126	201.539	151.848	167.909	163.216	177.505	x14=1754.385	x25=1511.127	x31=1906.591	x42=2090.413	x53=1851.848
1900	92897.61	214.009	169.382	191.211	181.113	200.86	x14=1954.726	x25=1687.409	x31=2097.314	x42=2326.600	x53=2069.382
2000	95108.445	220.221	178.171	202.921	190.503	212.197	x14=2054.469	x25=1775.471	x31=2194.133	x42=2444.750	x53=2178.171
2100	95923.51	218.442	190.473	212.596	192.974	217.133	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397
2200	95923.51	218.442	190.473	212.596	192.974	217.133	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397
2300	95923.51	218.442	190.473	212.596	192.974	217.133	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397
2500	95923.51	218.442	190.473	212.596	192.974	217.133	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397

4. Results from Quantity Discount Model-No Split on Case B

Q	obj	y1	y2	y3	y4	y5	x1-	x2-	x3-	x4-	x5-
1500	74559.796	189.683	134.455	144.361	146.433	153.272	x14=1553.156	x25=1334.750	x31=1721.327	x42=1854.933	x53=1634.455
1700	85615.126	201.539	151.848	167.909	163.216	177.505	x14=1754.385	x25=1511.127	x31=1906.591	x42=2090.413	x53=1851.848
1900	92897.610	214.009	169.382	191.211	181.113	200.860	x14=1954.726	x25=1687.409	x31=2097.314	x42=2326.600	x53=2069.382
2000	95108.445	220.221	178.171	202.921	190.503	212.197	x14=2054.469	x25=1775.471	x31=2194.133	x42=2444.750	x53=2178.171
2100	95923.510	218.442	190.473	212.596	192.974	217.133	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397
2200	95923.510	218.442	190.473	212.596	192.974	217.133	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397
2300	95923.510	218.442	190.473	212.596	192.974	217.133	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397
2500	95923.510	218.442	190.473	212.596	192.974	217.133	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397

5. Results from NLP Model on Case B

	Obj	Y1	Y2	Y3	Y4	Y5	x1-	x2-	x3-	x4-	x5-	
1500	74559.796	189.683	134.455	144.361	146.433	153.272	x14=1553.156	x25=1334.750	x31=1721.327	x42=1854.933	x53=1634.455	
1700	85615.126	201.539	151.848	167.909	163.216	177.505	x14=1754.385	x25=1511.127	x31=1906.591	x42=2090.413	x53=1851.848	
1900	92897.610	214.009	169.382	191.211	181.113	200.860	x14=1954.726	x25=1687.409	x31=2097.314	x42=2326.600	x53=2069.382	
2000	95108.445	220.221	178.171	202.921	190.503	212.197	x14=2054.469	x25=1775.471	x31=2194.133	x42=2444.750	x53=2178.171	
2100	96219.376	224.602	189.026	215.490	199.148	222.690	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397	x45=57.414
2200	96361.126	227.969	194.192	222.461	204.895	228.942	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397	x45=125.505
2300	96361.126	227.969	194.192	222.461	204.895	228.942	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397	x45=125.505
2500	96361.126	227.969	194.192	222.461	204.895	228.942	x14=2109.515	x25=1842.291	x31=2237.895	x42=2500	x53=2252.397	x45=125.505

Appendix IV: Numerical Results

1. Data results for Figure 4.14 and Figure 4.15

Numerical results from the quantity discount model-No split with case A when Q=3600

No	*Sigma2*	*Sigma1*	*Profit*	*Production amounts*		*Purchasing quantity*				
				y1	*y2*	*x11*	*x21*	*x32*	*x44*	*x54*
1	1.000	3.972	3135.883	16.219	23.181	16.219	23.181	39.400	16.219	23.181
2	1.500	3.972	3106.817	17.058	22.342	17.058	22.342	39.400	17.058	22.342
3	2.000	3.972	3073.074	17.785	21.615	17.785	21.615	39.400	17.785	21.615
4	2.500	3.972	3031.737	18.412	20.988	18.412	20.988	39.400	18.412	20.988
5	3.000	3.972	2982.948	18.955	20.445	18.955	20.445	39.400	18.955	20.445
6	3.500	3.972	2926.522	19.427	19.973	19.427	19.973	39.400	19.427	19.973
7	4.000	3.972	2862.558	19.845	19.555	19.845	19.555	39.400	19.845	19.555
8	4.500	3.972	2791.351	20.215	19.185	20.215	19.185	39.400	20.215	19.185
9	5.000	3.972	2713.226	20.545	18.855	20.545	18.855	39.400	20.545	18.855
10	5.500	3.972	2628.617	20.842	18.558	20.842	18.558	39.400	20.842	18.558
11	6.000	3.972	2539.042	20.950	18.092	20.950	18.092	39.042	20.950	18.092
12	7.000	3.972	2359.547	20.950	16.917	20.950	16.917	37.867	20.950	16.917
13	8.000	3.972	2182.541	20.950	15.746	20.950	15.746	36.696	20.950	15.746
14	9.000	3.972	2011.168	20.950	14.585	20.950	14.585	35.535	20.950	14.585
15	11.000	3.972	1698.697	20.950	12.325	20.950	12.325	33.275	20.950	12.325
16	13.000	3.972	1439.911	20.950	10.179	20.950	10.179	31.128	20.950	10.179
17	16.000	3.972	1106.772	20.950	10.000	20.950	10.000	30.950	20.950	10.000
18	19.000	3.972	918.012	20.950	2.348	20.950	2.348	23.298	20.950	2.348

2. Data results for Figure 4.16 and Figure 4.17

Numerical results from the quantity discount model-No split with case A when Q=2700

No	Sigma2	Sigma1	Profit	Production amounts		Purchasing quantity				
				y1	y2	x11	x21	x32	x44	x54
1	1.000	3.972	2616.500	10.754	22.996	16.219	23.181	33.750	16.219	23.181
2	1.500	3.972	2598.763	11.824	21.926	17.058	22.342	33.750	17.058	22.342
3	2.000	3.972	2580.702	12.876	20.874	17.785	21.615	33.750	17.785	21.615
4	2.500	3.972	2562.029	13.889	19.861	18.412	20.988	33.750	18.412	20.988
5	3.000	3.972	2542.069	14.828	18.922	18.955	20.445	33.750	18.955	20.445
6	3.500	3.972	2520.004	15.669	18.081	19.427	19.973	33.750	19.427	19.973
7	4.000	3.972	2494.926	16.401	17.349	19.845	19.555	33.750	19.845	19.555
8	4.500	3.972	2466.035	17.036	16.714	20.215	19.185	33.750	20.215	19.185
9	5.000	3.972	2432.671	17.587	16.163	20.545	18.855	33.750	20.545	18.855
10	5.500	3.972	2394.390	18.071	15.679	20.842	18.558	33.750	20.842	18.558
11	6.000	3.972	2350.948	18.499	15.251	20.950	18.092	33.750	20.950	18.092
12	7.000	3.972	2248.540	19.225	14.525	20.950	16.917	33.750	20.950	16.917
13	8.000	3.972	2127.297	19.821	13.929	20.950	15.746	33.750	20.950	15.746
14	9.000	3.972	1991.501	20.318	13.432	20.950	14.585	33.750	20.950	14.585
15	11.000	3.972	1698.697	20.950	12.325	20.950	12.325	33.275	20.950	12.325
16	13.000	3.972	1439.911	20.950	10.179	20.950	10.179	31.129	20.950	10.179
17	16.000	3.972	1106.772	20.950	10.000	20.950	10.000	30.950	20.950	10.000
18	19.000	3.972	918.012	20.950	2.348	20.950	2.348	23.298	20.950	2.348

3. Data results for Figure 4.18

Numerical results from the quantity discount model-No split with case B when Q=1800

No	Sigma5	Profit	Purchasing quantity				
			Y1	Y2	Y3	Y4	Y5
1	10	71385.817	201.147	158.611	174.974	167.787	198.703
2	20	69406.822	201.476	159.075	175.592	168.255	197.560
3	30	67425.964	201.768	159.487	176.139	168.671	196.546
4	40	65443.845	202.030	159.855	176.629	169.043	195.639
5	50	63461.298	202.264	160.185	177.068	169.379	194.824
6	60	61486.358	202.474	160.482	177.462	169.677	194.095
7	70	59546.998	202.655	160.737	177.800	169.936	193.466
8	80	57679.907	202.800	160.940	178.070	170.142	192.966
9	90	55914.244	202.901	161.083	178.260	170.287	192.613
10	100	54263.549	202.959	161.165	178.368	170.370	192.412
11	110	52727.999	202.977	161.190	178.402	170.396	192.349

4. Data results for Figure 4.19

Numerical results from the quantity discount model-No split with case B when Q=2300

No	Sigma5	Profit	Purchasing quantity				
			Y1	Y2	Y3	Y4	Y5
1	10	73310.150	215.769	179.450	202.563	189.409	202.093
2	20	71289.751	215.869	179.552	202.720	189.513	204.243
3	30	69339.454	215.869	179.552	202.720	189.513	206.364
4	40	67389.097	215.869	179.552	202.720	189.513	208.485
5	50	65439.620	215.869	179.552	202.720	189.513	210.608
6	60	63499.872	215.869	179.552	202.720	189.513	212.741
7	70	61599.359	215.869	179.552	202.720	189.513	214.926
8	80	59778.694	215.869	179.552	202.720	189.513	217.226
9	90	58070.082	215.869	179.552	202.720	189.513	219.699
10	100	56488.883	215.869	179.552	202.720	189.513	222.382
11	110	55036.162	215.869	179.552	202.720	189.513	225.286

VITA AUCTORIS

Liping Ma was born in 1972 in China. She earned her first Bachelor degree in Engineering from Beijing Jiaotong University in 1994. From there she worked in China until 2001. She obtained a B.Sc. in Industrial Engineering at the University of Windsor in 2004. She is currently a candidate for the Master's degree in Industrial Engineering at the University of Windsor and hopes to graduate in Spring 2006.