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Design of 1-D and 2-D recursive digital filters based on a new stability test.

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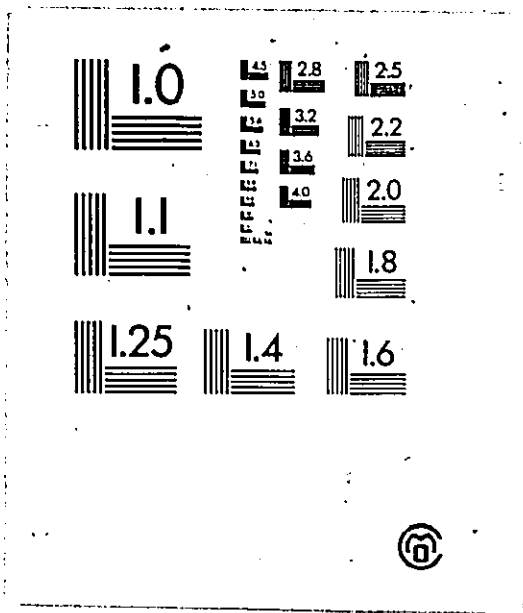
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Design of 1-D and 2-D Recursive Digital Filters
based on a new stability test

by

Mohamed T. Boraie

A thesis
presented to the University of Windsor
in partial fulfillment of the
requirements for the degree of
Master of Applied Science
in
Department of Electrical Engineering

Windsor , Ontario, 1984

ABSTRACT

Digital signal processing of 1-D and 2-D signals is becoming increasingly important, and is finding applications covering various scientific disciplines. One of the important branches in digital signal processing is digital filtering.

Among the number or structure of digital filters, the recursive (IIR) is known for its computational efficiency.

In this thesis an alternative approach to the direct design of 1-D recursive digital filters satisfying prescribed magnitude specifications with or without constant group delay is presented. The method uses an iterative technique to minimize the mean square error between the desired and the designed filter response (responses) in order to calculate the coefficients of the filter's transfer function.

Using a new stability test (9), the stability of the designed filter is guaranteed. Through the process of optimization the variable substitution technique is used to ensure that the parameter or the denominator satisfies the necessary and sufficient conditions for stability.

Based on the same stability test a method is presented for the design of a class of 2-dimensional recursive digital

filters with or without constant group delay characteristics and separable denominator transfer function.

Two 1-D polynomials in z_1 and z_2 having all their zeros inside the unit circle is generated and assigned to the denominator of the 2-D transfer function while the numerator is left to a general case of 2-D non-separable polynomial in z_1 and z_2.

The variable substitution technique is again used through the process of optimization to transfer the problem from a constrained optimization to a simple unconstrained optimization problem.

ACKNOWLEDGEMENTS

I would like to express my sincere appreciation to my
supervisor, Dr. M. Ahmadi, for his invaluable advice and
help during the course of this research. The help of all
the faculty members in the Electrical Engineering Department
is gratefully acknowledged.. In addition, the help of many
graduate student during the course of studying is sincerely
appreciated.

To me parents and wife, I extend my sincerest thanks.
Without their moral support and help, this work would not
have started.

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Chapter I

INTRODUCTION

1.1 DIGITAL SIGNAL PROCESSING IN (1-D) AND (2-D)

Signals arise in almost every field of science and engineering, e.g., in acoustics, biomedical engineering, communications, control systems, radar, seismology and telemetry. Two general classes of signals can be identified, namely continuous-time and discrete-time signals.

A continuous-time signal is one that is defined at each and every instant of time. Typical examples are a voltage waveform and the velocity of a space vehicle as a function of time. A discrete-time signal, on the other hand, is one that is defined at discrete instants of time, perhaps every millisecond, second, or day. Examples of this type of signal are the closing price of a particular commodity on the stock exchange and the daily precipitation as a function of time.

In signal processing a dimension can mean any physical domain in which a signal is defined. Time, space and frequency are examples of such domains. In some cases, the signal is defined in one or two dimensions. Acoustics sig-

nals are such examples of one dimensional signals.   Seismic
and geophysical signals are exapmles of two dimensional sig-
nals.

1.2   DIGITAL FILTERING

The last decade has witnessed some tremendous advanc-
es in the technology of digital systems, with a dramatic
drop in the cost of basic hardware elements used in imple-
menting such systems. As a result, many applications of di-
gital signal processing have become feasible, and this in
turn has stimulated the development of further theory. One
of the important branches in digital signal processing is
digital filtering which is the main theme of this thesis.

A digital filter, like an analog filter, can be
represented by a network which comprises a collection of in-
terconnected elements. The problem of designing filters is
one of finding filter coefficients such that some aspect of
the filter's response (e.g.,time response, frequency res-
ponse) approximate a desired behavior in a specified manner
(e.g., minimum mean square or minimax error).

As such, the "filter design problem" is basically a
mathematical approximation problem. The domain in which the
approximation problem is solved determines how and where the
resulting filter can be used.   Thus if the approximation
problem is solved in the z-plane, the resulting filter is a

digital filter, if it is solved in the s-plane, the result-
ing is an analog filter.   The common ground among all these
filters is the mathematics of functions with filter-like
properties.

1.2.1   Characteristics of 1-D and 2-D digital filters

Digital filters fall into two classes.   Filters whose
spatial response contains a finite number of non-zero sam-
ples, are called Finite Impulse Response (FIR) filters, and
those whose spatial response contains an infinite number of
non-zero samples, are called Infinite Impulse Response (IIR)
or recursive filters.

In the one dimensional case,   the output $y(m)$, of a
FIR filter, assuming an input sequence $x(m)$, is given by

$$y(m) = \sum_{k=0}^{N-1} h(k) \, x(m - k) \qquad (1.1)$$

where $h(k)$ is the impulse response defined over the interval
$0 \leq k \leq N-1$.   Similarly, in the two dimensional case, the
output array $y(m,n)$ can be written as

$$y(m, n) = \sum_{k=0}^{K-1} \sum_{\ell=0}^{L-1} h(k, \ell) \, x(m - k, n - \ell) \qquad (1.2)$$

where x(m,n) is the input array and h(k,l) is the impulse response defined over the interval $0 \leq k \leq K-1$, $0 \leq 1 \leq L-1$.

A one dimensional recursive filter, on the other hand, is characterized by the difference equation:

$$y(m) = \sum_{k=0}^{N} a(k) \; x(m-k) - \sum_{\ell=1}^{M} b(\ell) \; y(m-\ell) \quad \cdots\cdots\cdots (1.3)$$

where y(m) is the output sequence.
x(m) is the input sequence.

The impulse response, h(m), corresponding to (1.3) is causal, i.e., h(m)=0 for m < 0 and it extends up to infinity for positive values of m.

In the two dimensional case, on the other hand, a difference equation that characterizes a two dimensional IIR filter can be written as:

$$y(m,n) = \sum_{k=0}^{K_1} \sum_{\ell=0}^{L_1} a(k,l) \; x(m-k, n-\ell) - \sum_{i=0}^{I_1} \sum_{j=0}^{J_1} {}_{i+j\neq0} b(i,j) \; y(m-i,n-j) \quad (1.4)$$

where x,y are the input and the output arrays respectively,

K1,L1,I1 and J1 are all integers.

A two dimensional recursive digital filter is said to be causal if the impulse response h(m,n) is zero for m and n less than zero(11).   It can be seen that the impulse response of this filter is spread over only the upper quadrant or the right half plane in the spatial domain. This type of filter is also referred to as a quarter plane recursive fil-

ter, since its impulse response is confined to one quarter
of the spatial domain.

1.2.2 z-transform and filter transfer function

The z-transform of a sequence $x(n)$, for $-\infty \leq n \leq \infty$
is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^n \quad \dots\dots\dots\dots\dots\dots\dots (1.5)$$

where $\qquad Z = \exp(sT)$

where s and z are complex variables. The sequence $x(n)$ in
(1.3) is obtained be sampling a continuous signal $x(t)$ once
every T units of time (T is the sampling period which is
chosen based on the sampling theorem). The complex variable
s and the sampling period is defined as :

$$s = jw \qquad \text{and} \qquad T = 2\pi / w_s$$

where w is the continuous frequency variable and ws is the
sampling frequency (both expressed in radians). The complex
variable Z can be written as :

$$z = \exp(sT)$$

From the above definition, the transfer functions for nonrecursive and recursive digital filters could be deriven. Thus for a causal FIR filter whose impulse response $h(n)$ is zero for values of n outside of the range $0 < n < N-1$, the transfer function can be written as :

$$H(z) = \sum_{n=0}^{N-1} h(n) \ z^n.$$

(1.6)

Similarly, the transfer function corresponding to the causal recursive (IIR) filter (i.e., the impulse response $h(n)$ is defined for $0 \leq n \leq \infty$) is obtained as :

$$H(z) \sum_{n=0}^{\infty} h(n) \ z^n = \frac{\sum_{i=0}^{N} a_i \ z^i}{\sum_{j=0}^{M} b_j \ z^j}$$

(1.7)

with the assumption that no root of the denominator polynomial is cancelled by any root of the numerator polynomial.

A two dimensional z transform can be defined in exactly the same manner as the one dimensional z transform . Hence , for a two dimensional sequence $x(m,n)$, defined for all m and n, the z-transform is defined as :

$$X(z_1, z_2) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(m,n) \ z_1^m \ z_2^n$$

(1.8)

where $z_1 = \exp(s_1 T_1)$ and $z_2 = \exp(s_2 T_2)$

where s_1, s_2, z_1 and z_2 are complex variables. $X(m,n)$ is a sequence obtained by sampling a continuous two spatial directions x and y. With complex variables s_1, s_2 and the sampling periods T_1 and T_2, so that

$$s_1 = jw_1 \qquad s_2 = jw_2$$
$$T_1 = 2\pi / w_{s1} \qquad T_2 = 2\pi / w_{s2}$$

where w_1 and w_2 are continuous spatial frequency variables and ws_1 and ws_2 are the frequencies at which the signal $x(t_1, t_2)$ is sampled in x and y spatial directions. The complex variables z_1 and z_2 can be written as :

$$z_1 = \exp(s_1 T_1) \quad , \quad z_2 = \exp(s_2 T_2)$$

It follows from the above discussion, that given a FIR filter whose impulse response $h(m,n)$ is zero outside the region $0 \le m \le M$ and $0 \le n \le N$, the two dimensional transfer function can be obtained as :

$$H(z_1, z_2) = \sum_{m=0}^{M} \sum_{n=0}^{N} h(m, n) \ z_1^m \ z_2^m \qquad (1.9)$$

For a two dimensional recursive digital filter the transfer function can be obtained from the difference equation (1.2) as:

$$H(z_1, z_2) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} h(m, n) \; z_1^m \; z_2^m$$

$$= \frac{\sum\limits_{k=0}^{K_1} \sum\limits_{\ell=0}^{L_1} a(k, \ell) \; z_1^m \; z_2^{\ell}}{\sum\limits_{i=0}^{I_1} \sum\limits_{j=0}^{J_1} b(i, j) \; z_1^i \; z_2^j} \qquad (1.10)$$

1.2.3 Recursive and Nonrecursive Filters

Before a solution is sought for the approximation problem, a choice must be made between a recursive and a nonrecursive design. In recursive filters the poles of the transfer function can be placed anywhere inside the unit circle. A consequence of this degree of freedom is that high selectivity can easily be achieved with low-order transfer functions. In nonrecursive filters, on the other hand, with the poles fixed at the origin, high selectivity can be achieved only by using a relatively high order for the transfer function. For the same specification the requirement in a nonrecursive design can be as high as 5 to 10 times that in the recursive design. For example, the band-pass filter with the following specification (2) :

```
Minimum attenuation for      0   <  w  <   200  :  45  dB
Passband ripple       for   400   <  w  <   600  :  0.2 dB
Minimum attenuation for    700   <  w  <  1000  :  45  dB
```

sampling frequency 2000 rad/sec.

can be met using a nonrecursive filter of order 52 or a recursive filter of order 8.

In practice, the cost of a digital filter tends to increase and its speed tends to decrease as the order of the transfer function is increased. The nonrecursive filters have the advantage of constant group delay and stability while the recursive filters suffers from stability problems and the group delay can only be approximate.

1.2.4
Magnitude and Group Delay of a Filter

The transfer function $H(z)$ of a one dimensional filter can be written in the form:

$$H(z) = \left| H(z) \right| \exp(j\beta(z))$$

where $\left| H(z) \right|$ is the magnitude and $\beta(z)$ is the phase response of $H(z)$. The magnitude response of the filter is defined as:

$$\left| H(\omega) \right| = \left| H(z) \right|_{z=e^{j\omega T}} \sqrt{\operatorname{Re}(H(z))^2 + \operatorname{Im}(H(z))^2} \; \Big|_{z=e^{j\omega T}}$$

(1.11)

and the phase response is defined as:

$$\beta(\omega) = \beta(z)\Big|_{z=e^{j\omega T}} = \tan^{-1}\left\{ \frac{\operatorname{Im} H(z)}{\operatorname{Re} H(z)} \right\} \Big|_{z=e^{j\omega T}} \quad (1.12)$$

The group delay of a filter is a measure of average spatial or time delay as a function of frequency. The group delay is now defined as:

$$\tau(\omega) = -\frac{d\ \beta(\omega)}{d\ \omega} \tag{1.13}$$

$\tau(\omega)$ can be expressed in terms of $H(z)$ as:

$$\tau(\omega) = -\text{Re}\left[\frac{z}{H(z)} \cdot \frac{d\ H(z)}{d\ z}\right]\Bigg|_{z=e^{j\omega T}} \tag{1.14}$$

, The magnitude response and the group delay for two dimensional filters can be defined in a manner similar to the one dimensional case. For a two dimensional filter, there exist group delays in each of the spatial directions and are functions of both the spatial frequencies.

Consider a two dimensional filter transfer function expressed in a form similar to (1.7) as:

$$H(z_1, z_2) = |H(z_1, z_2)|\ \exp(j\ \beta(z_1, z_2)) \cdots \cdots \cdots \cdots \cdots \tag{1.11}$$

As before, the magnitude response is defined as:

$$|H(\omega_1, \omega_2)| = |H(z_1, z_2)|\Bigg|_{\substack{z_1=e^{j\omega_1 T}\\ z_2=e^{j\omega_2 T}}}$$

$$= \sqrt{\text{Re}\ \{H(z_1, z_2)\}^2 + \text{Im}\ \{H(z_1, z_2)\}^2}\Bigg|_{\substack{z_1=e^{j\omega_1 T}\\ z_2=e^{j\omega_2 T}}} \tag{1.15}$$

and the phase response defined as:

$$H(\omega_1,\omega_2) = H(z_1,z_2)\Big|_{\substack{z_1=e^{j\omega_1 T} \\ z_2=e^{j\omega_2 T}}} = \tan^{-1}\ \frac{Im\ H(z_1,z_2)}{Re\ H(z_1,z_2)}\Bigg|_{\substack{z_1=e^{j\omega\ T} \\ z_2=e^{j\omega\ T}}}$$

$$(1.16)$$

The group delays can now be defined as:

$$\tau_1(\omega_1,\omega_2) = -\frac{\partial\beta(\omega_1,\omega_2)}{\partial\omega_1} \qquad (1.17a)$$

$$\tau_2(\omega_1,\omega_2) = -\frac{\partial\beta(\omega_1,\omega_2)}{\partial\omega_2} \qquad (1.17b)$$

$\tau_1(\omega_1,\omega_2)$ and $\tau_2(\omega_1,\omega_2)$ can be expressed in terms of

the two dimensional filter transfer function $H(z_1,z_2)$ as:

$$\tau_1(\omega_1,\omega_2) = -Re\left\{\frac{z_1}{H(z_1,z_2)}\cdot\frac{\partial H(z_1,z_2)}{\partial z_1}\right\}\Bigg|_{\substack{z_1=e^{j\omega_1 T} \\ z_2=e^{j\omega_2 T}}}$$

$$(1.18a)$$

$$\tau_2(\omega_1,\omega_2) = -Re\left\{\frac{z_2}{H(z_1,z_2)}\cdot\frac{\partial H(z_1,z_2)}{\partial z_2}\right\}\Bigg|_{\substack{z_1=e^{j\omega_1 T} \\ z_2=e^{j\omega_2 T}}}$$

$$(1.18b)$$

1.3 IMPORTANCE OF PHASE IN 1-D AND 2-D SIGNAL PROCESSING

It is proven that for many images, the phase of the Fourier transform is more important than the magnitude (37,38). Specifically if,

$$F(u, v) = | F(u, v) | \cdot \exp(\beta(u, v))$$

denotes the two-dimensional Fourier transform of an image $f(x,y)$ then the inverse Fourier transform of $\beta(u,v)$ has many recognizable features in common with the original image, whereas the inverse Fourier transform of $|F(u,v)|$ generally bears no resemblance to the original image. Huang (38) illustrated this fact by modifying an image such that the phase is preserved but the magnitude of all the spectral components is set to unity and the inverse Fourier transform was computed to obtain the phase-only equivalent image. The phase-only image clearly retains many of the features of the original. But contrast to the magnitude-only image, i.e., the inverse Fourier or the $|F(u,v)|$ would have a small bright region near the origin in a dark background with no resemblance to the original image. It was evident that the phase-only image often has the general appearance of a high-pass filtered version of the original with additive broadband noise.

Oppenheim et al.(39), carried out a similar experiment with speech with similar results. Specifically, the magnitude of the Fourier transform of an entire sentence was set to unity and the inverse transform was computed to obtain the phase-only equivalent speech. The spectrogram of the original sentence and the spectrogram of the phase-only equivalent shows that the basic formant structure of the original sentence has been preserved. It was also reported (39) that by listening to the processed sentence a total intelligibility is retained although the speech has the general quality associated with high-pass filtering and the introduction of additive uncorrelated noise.

The previous discussion, shows that the design of recursive digital filters that approximates both magnitude and group delay responses is of greate importance in some applications in both 1-D and 2-D signal processing.

In this thesis the desqin of 1-D and 2-D recursive digital filters with or without linear phase characteristics based on a new stability test is introduced.

1.4 SUMMARY OF PREVIOUS WORK

Within the past decade vast amount of papers have been published in the area of filter design and stability of one-dimensional and two-dimensional recursive digital filters.

Since the concern is on the design of 1-D and 2-D recursive digital filters based on a new stability test, an overview of filter design methods in both 1-D and 2-D is given.

1.4.1
Design of 1-D recursive digital filters

Generally, there are two approaches to the design of 1-D recursive digital filters, namely indirect and direct methods. In indirect methods, one of the classical analog-filter approximations (e.g., Butterworth, Bessel, Chebyshev, Elliptic) is used to generate an analog transfer function which is subsequently discretized by the use of any transformation technique (e.g., bilinear transformation, the invariant impulse response, matched z-transform) (2). Among the techniques that fall into the category of direct digital IIR design are magnitude-squared function design and time domain methods (14). In magnitude-squared function design the magnitude-squared of the filter transfer function is expressed as the ratio of two trignometric functions of w and by suitable choice of the functions, various types of digital filters can be designed to match prescribed characteristics. This technique is readily extendable to several other classes of filters directily(i.e, without the use of frequency transformation technique) and need not be restricted to lowpass filters (14). The difficulties with this techni-

que are twofold.   First,   a suitable rational trignometric polynomial must be found to provide the desired filtering. Secondly, the magnitude-squared function must be factored to find the poles and zeros.   This factorization is generally nontrivial and therefore makes this a complex filter design method.

Burrus and Parks(15),   Brophy and Salozar (16),   have shown that it is possible to· design an IIR filter whose im-pulse response approximates a desired impulse response. Un-fortunately,   since this method involves only time domain considerations, the resulting frequency domain approximation to the filter stopband responses (where a 40-dB stopband loss is desired) is generally unacceptable.   The filter coefficients obtianed by this technique, however, may often be suitable as an initial guess to more sophisticated optim-ization algorithms for designing IIR filters in terms of frequency response specifications.

·Another approach to the design of IIR can be classi-fied as optimization method.   In this method the desired discrete-time transfer function is obtained directly from the given specifications through the use of an iterative method based on linear or non-linear programming (17-20) to minimize some error criterion subject to the appropriate de-·sign equations.   In the case of existing recursive digital filter design techniques, the linear phase is realized via

group delay equalization(14,26), where a non-linear optimization procedure is employed for the approximation. The overall design procedur(14,26) involves two steps, the approximation of magnitued followed by group delay equalization which compensates for the non-linearities in the phase response of the magnitude only filters. This method requiers two phases of optimization which is proven to be time consuming.

1.4.2 Design of 2-D recursive digital (IIR) filters

The area of 2-D recursive digital filters design have been of great interest to many researchers over the last decade. The first design method for 2-D (IIR) filter is due to E.L. HALL (21). He introduced the separable product technique which enables the design of any Two-dimensional rectangular cut-off boundary type filter by the use of two one-dimensional recursive filters in cascade.

Shanks et al. (10) introduced the first transformation technique which takes a stable (1-D) analog filter and uses a transformation to rotate the amplitude response in (2-D). A bi-linear transformation is then used on each variable to produce a (2-D) digital transfer function. However, there is no guarantee of stability and the approach suffers from the warping effects of the bi-linear transformation on the frequency response.

Costa and Venetsanopculos (22) devised a method of using this approach to produce circularly symmetric filters while guaranteeing the stability. However, their technique produces filters of high order.

The transformation technique due to McClellen followed by a decomposition technique due to Pistor (23) was used by Bernabo et al. (24) in order to obtain four one-quadrant recursive digital filters. Obviously the filter is inefficient from the point of view of the number of multiplications per sample.

Another approach or the design of 2-D recursive digital filter is the direct design. Maria and Fahmy (25,26) developed lp design technique to approximate magnitude and group delay responses. The technique is an extinsion of iterative technique for the design of recursive filters due to Deczky (27).

Aly et al. (28) and chottera et al. (29) introduced linear-programming approach for the design of 2-D (IIR) filters that approximate the magnitude response as well as the group delay. The advantage of this technique is its speed and that the global mininum is obtained through the optimization process. However, it requires minimization of a weighted error function (untrue error) rather than the real one. The memory requirements are large due to the increased number of parameters, the stability constraints used in (29)

were only sufficient conditions, hence the designed filters belong to a subclass of stable filters.

Multi-variable network theory has found application in the design of two- and multi-dimensional recursive digital filters. Ramamoorthy and Bruton (30) have shown how a 2-D recursive digital filter can be designed by using the properties of the immittance function of a lossless frequency independent (n1+n2+1)-port network having n1-ports terminated by s1-type capacitors, the next n2-ports terminated by s2-type capacitors and the remaining port by a resistor. This technique, however, has failed to deal with the possibility of existence of 2-D transfer functions having poles with non-essential singularities of the second kind which could result in unstable 2-D digital filters.

Ahmadi and Ramachandran (31) modified the above technique by imposing the constraints for a 2-D SHP (strict hurwitz polynomial) to become a VHSP (very strict hurwitz polynomial) as penalty function (32) in the process of optimization thereby guaranteeing the stability. However, this method bears heavy computational costs. The generation of VSHP was further investigated (33,34) to ensure that the denominator of the filter is always a VHSP, to avoid the uncertainty of the method (30) and also does not required any constrained optimization method (32) which results in considerable savings in computational time.

Charalambous (35) showed that the design prcblem of 2-D circularly-symmetric digital filters can be simplifyed if separable denominator transfer function is used and that many usefull filters can still be designed. At the end of the optimization process the location of the poles is checked and the pole inversion technique due to Steiqlitz (36) is used to replace any pole outside the unit circle with its mirror-image with respect to the unit circle. This technique reduces the stability problem from 2-D case to 1-D case and reduces the number of variables be (n^2-1) where n is the order of the filter hence, it requeires less computaional time. Due to the use of pole inversion technique the linear-phase can not be obtained.

1.5
MOTIVATION AND THESIS ORGANIZATION

In the recent years the design of stable recursive filters has been and is still being carried cut due to the facts that recursive filters offer greater speed of filtering, smaller memory requirements and easier implementation compared to nonrecursive filters. In most of the design techniques of 1-D recursive digital filters using the optimization methods the stability of the designed filter can be quaranteed by replacing all possible poles which is placed outside the unit circle by its mirror image with respect to the unit circle, nence, stabilizing the designed unstable

filter. The main drawback with this method is that the phase response of the filter in not invariant under this method. In this thesis a method for the design of 1-D recursive digital filters with or without linear phase characteristics where the stability problem is avoided through the use of a new stability criterion is presented. . The method was extended to the design or a class of 2-D filters with separable denominator transfer function. Both the denominator polynomials satisfy the necessary and sufficient conditions for a polynomial to have all its zeros inside the unit circle. Hence, the stability of the designed filter is ensured and testing it is not necessary which reduces the compution time (also due to the fact that the number of the variables is reduced by n^2+3 where n is the order of the filter). Also approximation of magnitude as well as group delay response is now possible since the pole inversion method is not used. A primary result of the proposed method is reported in (42-44).

Chapter 2 discusses the stability of 1-D and 2-D recursive digital filters , useful definitions and theorems are given. Chapter 3 discusses the design of 1-D recursive digital filters, then, a direct design approach is presented. In order to illustrate the usefulness of the method several examples are given. Chapter 4 discusses the design of 2-D recursive digital filter. A design method of 2-D filter using Separable transfer function is introduced and several

examples are shown in order to illustrate the usfullness of the method. Chapter 5, the final chapter, is the conclusion of the thesis.

Chapter II

STABILITY

Generally the term stability is used to indicate that convolving a filter with any bounded input sequence of numbers always yields a bounded output sequence of numbers. Since, as it can be seen from Eqn.(1.3) and Eqn.(1.4), in recursive filters the past output values are used in calculating the present one, such value can become arbitrarily large irrespective of the size of the input values. Therefore, recursive filters may by unstable. In this chapter, definitions for the stability of 1-D and 2-D systems are given. Methods for testing the stability of recursive digital filters are discussed. Then a method for genarating a polynomial having all its zeros inside the unit circle is given.

2.0.1 stability of 1-D discrete system

In order to understand the various conditions and tests for stability some definitions must first be defined. These definitions will be first made for the 1-D case then the same can be extended to the 2-D case.

Definition 1 : A function i(n) in one variable n is said to be absolutely bounded if and only if :

$$|i(n)| \leq M < \infty$$

Definition 2 : A function i(n) in one variable n is said to be absolutely summable if and only if :

$$\sum_n |i(n)| \leq N < \infty$$

for all integer values of n where M and N are positive real numbers.

One kind of stability to consider is bounded input bounded output stability. A system is said to be BIBO stable if and only if for any bounded input, the output is always bounded.

The restriction to be placed on the filter's impulse response h(n) to ensure that the output of the system o(n) is also absolutely bounded can be derived by expressing the output o(n) as the convolution of the input i(n) and the impulse response h(n).

$$o(n) = \sum_p h(p) \ i(n-p+1) \quad \ldots\ldots\ldots\ldots\ldots (2.1)$$

and by applying Schwarz's inequality, gives :

$$|o(n)| < \sum_p |h(p)| \ |i(n-p+1)| \quad \ldots\ldots\ldots\ldots (2.2)$$

but since $|i(n)| < M$ for all integer values of its argument, the inequality reduces to

$$|o(n)| < M \sum |h(p)|$$

Which restricts the impulse response to an absolutely summable function. So that, if the impulse response h(p) is ab-

solutely summable, and the input i(n) is absolutely bounded, then the output o(n) is also absolutely bounded.

Another kind of stability is summable input, summable output (SISO). A system is said to be SISO if and only if for any summable input i(n) the output o(n) is also summable.

Here again the restrictions on the filter's impulse response h(n) can be derived through applying Schwarz's inequality to the convolution sum of equation (2.1).

$$o(n) = \sum_{p} h(p) \ i(n - p + 1)$$

$$\sum_{n} |o(n)| = \sum_{n} \sum_{p} |h(p) \ i(n - p + 1)|$$

$$\leq \sum_{n} \sum_{p} |h(p)| \ |i(n - p + 1)|$$

$$\leq N \sum_{p} |h(p)|$$

Therefore, a necessary and sufficient condition for stability is:

$$\sum_{p=0}^{\infty} |h(p)| < \infty \tag{2.4}$$

where h(n) is the impulse response of the given filter.

FIR filters satisfy the above conditions, since their impulse response is defined only over a bounded limite, i.e., h(n) is defined for $N < n < M$. However, in the case of recursive filters, the impulse response is unbounded and

in order to meet the stability criterion, it is required
that poles of the transfer function be inside the unit cir-
cle.

Thus given an IIR filter:

$$H(z) = \frac{A(z)}{B(z)} = \frac{\sum_{n=0}^{N} a(n)\, z^n}{\sum_{m=0}^{M} b(m)\, z^m} \quad , \quad z = e^{j\omega T} \tag{2.5}$$

in order for this to be stable, the singularities (often
referred to as zeros) of B(z) should be inside the unit cir-
cle in the z-plane. Therefore, in designing recursive fil-
ters of the form given in (2.5),proper stability constraints
have to be imposed on the coefficients b(n), so that the ze-
ros of B(z) lie inside the unit circle in the z-plane.

Methods are available in the literature regarding the
location of the roots of a polynomial in z within the unit
circle such as Schur-Cohn (1), Jury test (2), the inners
test (3), the use of inverse bilinear transformation (4).
Also, stability tests have been formulated based on the z-
domain continued fraction expansion (6,7) relative to the
bilinear function z-1/z+1 or in terms of (z-1) and (1-z^{-1})
factors (8). Based on Schussler's theorem (5) Ramachandran
et al. (9) proposed a new stability test and some necessary
conditions that have to be satisfied by the denominator po-
lynomial of a stable 1-D discrete system.

Recently, Schussler(5) formulated a set of conditions to determine the stability of a discrete system, it can be stated as folows :

Schussler's Theorem:

Let $D(z)$, a polynomial of degree m having real coefficients, be described as:

$$D(z) = a_m z^m + a_{m-1} z^{m-1} + \ldots + a1 \, z + a0$$

$$= \sum_{i=0}^{m} a_i \, z^i \quad \ldots\ldots\ldots\ldots\ldots (2.6)$$

where a_i is always positive.

$D(z)$ can be decomposed as the sum of the mirror-image polynomial,

$$F_1(z) = 1/2 \, [\; D(z) + z^m D(z^{-1}) \,]$$

and the anti-mirror image polynomial,

$$F_2(z) = 1/2 \, [\; D(z) - z^m D(z^{-1}) \,]$$

For $D(z)$ to have all zeros inside the unit circle, the necessary and sufficient conditions are :

(i)   The zeros of $F_1(z)$ and $F_2(z)$ are located on the unit circle,

(ii)   They are simple,

(iii)· They separate each other, and

(iv) $|a0/am| < 1$

The properties of $F_1(z)$ and $F_2(z)$ were studied by Ramachandran and et al (9) in order to generate a 1-D polynomial of order m which has all its zeros inside the unit circle, it has been shown that such a polynomial can be obtained using the following relationship.

case (a): (m even)

$$D(z) = K_e \prod_{i=1}^{n} (z^2 + 2\alpha_i z + 1) + (z^2 - 1) \prod_{i=1}^{n-1} (z^2 + 2\beta_i z + 1) \qquad (2.7)$$

$$n = m/2, \quad K_e > 1 \quad \text{and} \quad 1 > \alpha_1 > \beta_1 > \alpha_2 > \beta_2 \cdots > \beta_{n-1}$$
$$> \alpha_n > -1 \qquad (2.8)$$

case (b): (m odd)

$$D(z) = K_o (z + 1) \prod_{i=1}^{n} (z^2 + 2\alpha_i z + 1) + (z-1) \prod_{i=1}^{n} (z^2 + 2\beta_i z + 1) \qquad (2.9)$$

$$K_o > 1, \quad 1 > \alpha_1 > \beta_1 > \alpha_2 > \beta_2 \cdots > \alpha_n > \beta_n > -1 \qquad (2.10)$$

$$n = \frac{m - 1}{2}$$

This method of generating m order polynomial will be used later in the proposed design method.

2.1

STABILITY OF 2-D RECURSIVE DIGITAL FILTERS

The stability of 2-D recursive digital filter is determined by the coefficients of the denominator polynomial $B(z_1, z_2)$ of the z-transfer function of the filter.

The same definition for absolutely bounded and absolutely summable defined for the 1-D case can easily extend to the 2=D case in two variables, and the restriction on the impulse response can also be derived by applying Schwarz's inequality to the convolution sum as follows.

$$|o(m,n)| \leq \sum_{p_1} \sum_{p_2} |h(p_1,p_2)| |i(m-p_1+1, n-p_2+1)| \qquad (2.11)$$

and if the input is absolutely bounded, i.e.,

$$|i(m_1, m_2)| \leq M < \infty$$

for all integer pairs of its arguments; then

$$|o(m_1, m_2)| \leq M \sum_{p_1} \sum_{p_0} |h(p_1, p_2)|$$

which again restricts the impulse response to an absolutely summable function. The same procedure can be followed to derive the restriction for summable input summable output stability.

However, testing the stability then is difficult due to the fact that the fundamental theorems of algebra is not applicable to two-variable functions; namely, the denominator factorization is not always possible and hence testing the stability by finding the poles of the z-transfer function as in the case of 1-D, is not possible. The first stability test introduced by Shank (10), is a direct extension of the conditions for stability in the 1-D case to the 2-D case, which can be stated as follows:

Shank's Theorem:

Given that $B(z_1, z_2)$ is a polynomial in (z_1, z_2), for the coefficients of the expansion of $1/B(z_1, z_2)$ in a positive powers of z_1 and z_2, to converge absolutely, it is necessary and sufficient that $B(z_1, z_2)$ not be zero for $|z_1|$ and $|z_2|$ simultaneously less than or equal to one.

i.e., $B(z_1, z_2) \neq 0$ for $\bigcap_{i=1}^{2} |z_i| \geq 1$

stated another way, the theorem says that if there are any values (real or complex) of z_1, z_2 for which $B(z_1, z_2)$ is zero, and for which z_1 and z_2 are simultaneously less than or equal to one in magnitude, then the filter $1/B(z_1, z_2)$ will be unstable, if there are not the $1/B(z_1, z_2)$ will be stable. unfortunately, testing the stability using Shank's theorem is very tedious to apply since, for each particular value of

$z_1 = z_1^{0} \in d_1$ equation $B(z_1^{0}, z_2) = 0$ has to be solved

for z_2 and that has to be repeated for all (in practice a large number of points in d_1).

This theory was further simplified by Huang(11) in order to overcome the difficulties associated with it. This simplification leads to the following two conditions :

The system is stable if and only if,

1. The maps of $d_1 \equiv \left(z_1; |z_1| = 1\right)$ in the z_2 plane, according to $B(z_1, z_2) = 0$ lies outside $d_2 \equiv \left(z_2; |z_2| \leq 1\right)$;

2. No point in $d_1 \equiv \left(z_1; |z_1| < 1\right)$ maps into the point $z_2 = 0$ by the relation $B(z_1, z_2) = 0$.

Even though, Huang's Theorem could modify Shank's theorem to be easier but involves an infinite number of steps, it is still not simple to apply.

stability of 2-D filters. Anderson and Jury (3,12)
and also Maria and Fahmy (13) simplified these results by
putting the stability in terms of the root locations or po-
sitivity of a set of polynomials in one variable. While
this represented a considerable simplification, the proce-
dure is still computationally difficult except for low-order
cases(3).

As it could be seen from the previous discussion that
stability of 2-D filter is not as simple as the case of 1-D
and stabilizing the unstable filter is also more complicated
than that or 1-D. This problem and difficulty can be avoid-
ed if the following assumption is made, that is, the denomi-
nator polynomial can be expressed in the separable product
form of two polynomials on in each variable z_1 and z_2.

Chapter III

DESIGN OF 1-D RECURSIVE DIGITAL FILTERS

3.1 INTRODUCTION

Contrary to some popular beliefs, a great deal of 1-D recursive digital filter design does not depend intrinsically on continuous- time filter design but instead makes use of the wide body of knowledge that is available in the literature on the design of such filters. Instead of redeveloping the theory(i.e., restructuring the mathematics to the case of digital filters) for digital filters, simple mapping procedures can be used to transform filters in one domain to filters in the other domain.

A second method for designing recursive digital filters is direct closed form design in the z-plane. Beginning with the desired response of the filter, one can often decide where to place poles and zeros to approximate this response directly(14). A third way in which 1-D recursive digital filters are often designed is by using optimization procedures to place poles and zeros at appropriate positions in the z-plane to approximate in some sense the desired response.

3.2 1-D RECURSIVE DIGTAL FILTER DESIGN FROM CONTINUOUS-TIME FILTER

The most popular technique for designing 1-D filters is to digitize an analog filter that satisfies the design specifications. There are many techniques for designing analog prototype filters when the specifications are of the form of a lowpass, bandpass, highpass or bandreject filter. Among the well-known analog filter classes are the Butterworth, Bessel, Chebyshev and Elliptic filters.

An analog filter transfer function can be written as:

$$H_A(s) = \frac{A(s)}{B(s)} = \frac{\prod\limits_{i=1}^{N} (s - s_i)}{\prod\limits_{i=1}^{M} (s - p_i)} \qquad (3.1)$$

The most widely used procedures for digitizing the transfer function of Eq. (3.1) include,

1. The invariant impulse transformation.

2. The modified invariant impulse transformation.

3. The matched z-transform technique.

4. The bilinear transformation.

In the next section each of these techniques will be briefly discussed.

3.2.1

<u>Invariant Impulse Transformation</u>

The characteristic property of this transformation is that the impulse response of the resulting digital filter is a sampled version of the analog filter's impulse response. In order to understand this method consider an analog filter H(s), a corresponding digital filter, represented by H(z), can be derived by using the following procedure.

1. Deduce h(t), the impuse response of the analog filter.

2. Replace t by nT in h(t).

3. Form the z-Transform of h(nT).

Advantages of this method are that a stable analog filter yields to a stable digital filter, also the denominator's degree in H(z) can not exceed the numerator degree and H(z) is therefore realizable. This method preserves the phase as well as the loss characteristic of the analog filter. However, aliasing errors tend to restrict the invariant-impulse response method to the design of allpole filters. The higher sampling frequency gives better results because aliasing errors are less pronounced.

3.2.2

<u>Modified Invariant-Impulse response Method</u>

A modified version of the invariant-impulse response can be applied to filters with finite transmission zeros. Consider the transfer function

$$H_A(s) = \frac{H_o A(s)}{B(s)} = \frac{H_o \prod_{i=1}^{N} (s - s_i)}{\prod_{i=1}^{M} (s - p_i)} \qquad (3.2)$$

$$H_A(s) = \frac{H_o H_{A_1}(s)}{H_{A_2}(s)} ------ \qquad (3.3)$$

where,

$$H_{A_1}(s) = \frac{1}{B(s)} \qquad\qquad H_{A_2}(s) = \frac{1}{B_2(s)}$$

The design can be accomplished by using the following procedure.

1. Find the poles and residues of $H_{A_1}(s)$ and $H_{A_2}(s)$.

2. Form $H_{D_1}(z)$ and $H_{D_2}(z)$ using invariant-impulse response method.

$$H_D(z) = \frac{H_o A_1(z) B_2(z)}{A_2(z) B_1(z)} \qquad (3.4)$$

3.2.3

Matched z-Transform method

An alternative approximation method for the design of recursive digital filters is the so-called matched z-transformation.

In this method, given a continuous-time transfer function like that in Eqn. (3.1), a corresponding discrete-time transfer function can be formed as

$$H_D(z) = (z + 1)^L \; \frac{H \prod_{i=1}^{M} (z - e^{s_i T})}{\prod_{i=1}^{N} (z - e^{p_i T})}$$

where L is integer.

3.2.4.

Bilinear-Transformation Method

The bilinear transformation is a simple conformal mapping from the s-plane to the z-plane which preserves the desired algebraic form, it is defined by

$$S = \frac{2}{T} \frac{z - 1}{z + 1}$$

The nature of this mapping is that the entire jw axis in the s-plane is mapped onto the unit circle; the left-hand s-plane is mapped inside the unit circle in the z-plane and the right-half s-plane is mapped outside the z-plane unit circle. It also has the property that realizable, stable continuous systems are mapped to realizable, stable digital system. One disadvantage of this technige is that the frequancy response of the continuous system must be piece-wise constant to compensate for the effects of the nonlinear relation between analog and digital frequencies. Also neither the impulse response nor the phase response of the analog filter is preserved in a digital filter obtained by bilinear transformation.

3.3 ## DESIGN OF 1-D RECURSIVE DIGITAL FILTERS BASED ON A NEW STABILITY TEST.

Linear and non-linear programming procedures are frequently used for the design of recursive digital filters. The stability of the designed filter can be guaranteed by replacing all possible poles which are placed outside the unit circle by its mirror image with respect to the unit circle, hence, stabilizing the designed unstable filter. This stabilization procedure, however has a drawback, which is, the phase response of the filter is not invariant under this stabilization process. In the method to be presented in this thesis this problem has been avoided through imposing necessary and sufficient conditions on the coefficients of a 1-D polynomial of any order to have all its zeros inside the unit circle, based on a new stability test (9) throughout the optimization process.

3.3.1 ### Generation Of 1-D Stable Polynomial

Based on Schussler's theorem , Ramachandran and Gargour showed in chapter 2 that a 1-D polynomial of order m which has all its zeros inside the unit circle can be generated using the following relationship.

Case (a): (m even)

$$D(z) = K_e \prod_{i=1}^{n} (z^2 + 2\alpha_i z + 1) + (z^2 - 1) \prod_{i=1}^{n-1} (z^2 + 2\beta_i z + 1)$$

(3.5)

with $K_e > 1$ and $n = \frac{m}{2}$

and condition, $1 > \alpha_1 > \beta_1 > \alpha_2 > \beta_2 \, \text{----} \, > \beta_{n-1} > \alpha_n > -1$ (3.6)

Case (b): (m odd)

$$D(z) = K_o (z + 1) \prod_{i=1}^{n} (z^2 + 2_i z + 1) + (z - 1) \prod_{i=1}^{n} (z^2 + 2_i z + 1)$$

(3.7)

with $K_o > 1$ and $n = \frac{m - 1}{2}$

3.3.2
Formulation of The Design Problem

3.3.2.1
Approximation of The Magnitued Response Only

Since 1-D polynomial on the form of Eqn. (3.5) or Eqn. (3.7) is shown to have all its zeros inside the unit circle if conditions of Eqn. (3.6) or Eqn. (3.8) respectively is satisfied. Eqn. (3.5) or Eqn. (3.7) is assigned to the denominator of a 1-D transfer function. The error between the ideal and the desigred magnitude response of the 1-D filter is calculated using the relationship.

$$E_M(j\omega_m, n, \alpha, \beta) = \left| H_I(e^{j\omega_m T}) \right| - \left| H_D(e^{j\omega_m T}) \right|$$

(3.9)

where E_M is the error of the magnitude response and $\left|H_I\right|$ and $\left|H_D\right|$ are magnitude response of the ideal and the designed filter respectively. The calculation of the cost function using Eqn. (3.9) could be obtained using any suitable error criterion, Among the most widely used error criterions,

1. The least mean squared error criterion

This error criterion calculates the cost function using the following relationship.

$$E_{\ell_2}(j\omega, \underline{n}, \underline{\alpha}, \underline{\beta}) = \sum_{m \in I_{ps}} E_M^2(j\omega_m) \tag{3.10}$$

2. The Minmax error criterion

In this case the cost function to be minimized is the maximum error in the passband and the stopband, which can be expressed in the following relationship,

$$E_{MinMax}(j\omega, \underline{n}, \underline{\alpha}, \underline{\beta}) = \underset{m \in I_{ps}}{Max} \left(\left| E_M(j\omega_m) \right| \right) \tag{3.11}$$

Where I_{ps} is the set of all discrete points in the passband and the stopband of the 1-D filter. Now any suitable non-linear optimization technique can be used to calculate the parameters of the 1-D filter's z-transfer function in such a way that the cost function is minimized subject to constraint of Eqn. (3.6) or Eqn. (3.8). This constraint optimiza-

tion problem can be transformed to an unconstraint optimization problem by the use of variable substitution method.

3.3.2.2   Variable substitution method (45)

Since, the polynomial of order m on the form of Eqn.(3.5) or Eqn.(3.7) assigned to the denominator of 1-D filter's transfer function, must satisfy the constraints,

Case (a): (m even)
$$1 > \alpha_1 > \beta_1 > \alpha_2 > \beta_2 ---- > \beta_{n-1} > \alpha_n > -1$$
Case (b): (m odd)
$$1 > \alpha_1 > \beta_1 > \alpha_2 > \beta_2 ---- > \alpha_n > \beta_n > -1$$

A variable substitution can be used to ensure that the constraints will always be satisfied through the optimization process, hence, the resulting filter is guaranteed to be stable. The following variable substitution is used (for the case of even order),

$$\alpha_n = \cos \pi \; e^{-\theta_1^2}$$

$$\beta_{n-1} = \cos \pi \; e^{-(\theta_1^2 + \theta_2^2)}$$

$$\alpha_{n-1} = \cos \pi \; e^{-(\theta_1^2 + \theta_2^2 + \theta_3^2)}$$

$$\vdots$$

$$\alpha_1 = \cos \pi \; e^{-\sum_{i=1}^{2n-1} \theta_i^2}$$

(3.12)

Now any unconstraint optimization method can be used for the calculation of the new filter's coefficients n and to minimize the cost function Eqn. (3.10) or Eqn. (3.11).

3.3.2.3 Approximation of the Magnitude and Group Delay Response

The error between the ideal and the designed group delay response of the 1-D filter is calculated in the same fashion the error of the magnitude response was calculated using the relationship,

$$E_{\tau_\omega} (j\omega_m) = \tau_I T - \tau_\omega (e^{j\omega_m T})$$

(3.13)

where τ_I is a constant representing the ideal group delay response of the filter and its value is chosen equal to the order of the filter (29) and τ_ω is the group delay response of the designed filter calculated as shown by Eqn. (1-14). The General cost function for both the magnitude and group delay can be calculated using either the least mean squared error criterion or the minmax error criterion,

The general mean squared error E_G is calculated using Eqn. (3.9) and Eqn. (3.13) in the following manner:

$$E_G(j\omega, \underline{\alpha}, \underline{\beta}, \underline{n}) = \sum_{m \,\epsilon\, I_{ps}} E_M^2 (j\omega_m) + \sum_{m \,\epsilon\, I_{ps}} E_{\tau_\omega} (j\omega_m)$$

(3.14)

The general minmax error E , on the other hand, can be calculated from the same two equations in the following manner:

$$E_G(j\omega, \underline{\alpha}, \underline{\beta}, \underline{n}) = \text{Max} \left[\underset{m \in I_{ps}}{\text{Max}} \left| E_m(j\omega_m) \right| , \underset{m \in I_p}{\text{Max}} \left| E_\tau(j\omega_m) \right| \right]$$

where I_{ps} is the set of all discrete points in the passband and stopband where I_p is the set of all points in the passband of the 1-D filter. Again to design a 1-D filter satisfying prescribed magnitude and constant group delay reponse $\underline{\alpha}$, $\underline{\beta}$, n should be calculated in such a way that E_G is minimized subject to constraint of Eqn. (3.6) or Eqn. (3.8) , E_G can be minimized either by utilization of any suitable nonlinear optimization routine with linear constraints or to transform the problem to an unconstraint one using the variable substitution method previously discussed.

3.3.3   Design Examples

In order to illustrate the usefulness of the proposed method, several examples of recursive digital filters satisfying a prescribed magnitude response with and without constant group delay characteristics were designed.
In the first example, design of a lowpass filter with the following amplitude specification is required.

The order of the filter is considered to be equal to four and the constant group delay characteristic is not required.

$$|H(jw)| = \begin{cases} 1 & \text{for} \quad 0 < |w| < 1 \\ 0 & \text{for } 2.5 < |w| < 5 \quad \text{rad/sec} \end{cases}$$

where the sampling frquency = 10 rad/sec.

Table (3.1) shows the values of n, α and β of the filter's transfer function obtained be minimizing the least mean square error Eqn. (3.10) while Fig. (3.1) shows the magnitude response of the designed filter. The stability of the filter can be seen from the locations of the ploes in Fig. (3.2). Table (3.2) shows the values of the zeros and ploes of the filter's transfer function.

The second example designs a bandpass filter with the following amplitude specification,

$$|H(jw)| = \begin{cases} 0 & \text{for} \quad 0 < |w| < 0.5 \text{ rad/sec} \\ 1 & \text{for} \quad 2 < |w| < 3 \quad \text{rad/sec} \\ 0 & \text{for } 4.5 < |w| < 5 \quad \text{rad/sec.} \end{cases}$$

where the sampling frequency = 10 rad/sec.

Again the order of the filter is considered to be equal to four and the constant group delay characteristic is not required. Table (3.3) shows the values of n, α and β of the filter's transfer function while Fig. (3.3) shows the magnitude response of the designed filter. The filter can be shown to be stable by the locations of the poles in Fig. (3.4), Table (3.4) shows the values of the zeros and ploes of the filter's transfer function.

In the last example a lowpass filter with following amplitude specification is designed,

$$I H \ (jw) \ I = \begin{cases} 1 & \text{for} \quad 0 < |w| < 1 \ \text{rad/sec} \\ 0 & \text{for} \ 2.5 < |w| < 5 \ \text{rad/sec}. \end{cases}$$

where the sampling frequency = 10 rad/sec.

and the constant group delay characteristic. The order of the filter in this example is considered to be equal to four so the in Eqn. (3.13) is set equal to four. Table (3.5) shows the value of the parameters of the designed filter while Fig. (3.5) and Fig. (3.6) show the magnitude and group delay response of the designed filter respectively. Fig. (3.7) and Table (3.6) show the location and values of the zeros and ploes of the designed filter's transfer function.

$\left|\, H_D(z)\,\right|$

.117E 01

.107E 01

.965E 00

.860E 00

.756E 00

.651E 00

.547E 00

.442E 00

.338E 00

233E 00

.129E 00

.246E-01

00E 00   .98E 00   .20E 01   .29E 01   .39E 01   .49E 01

ω rad/sec

XMIN =   .00000E 00   YMIN =   .24596E-01
XMAX =   .49000E 01   YMAX =   .10690E 01

Figure (3.1) Amplitude response of the lowpass filter with
the passband and stopband edges of 1. rad/sec
and 2.5 rad/sec respectively.

Numerator Coeff.	Denominator Coeff.	Typical Values of θ
$n_0 = 0.34060$	$\alpha_2 = -0.86360$	$\theta_1 = 0.42898$
$n_1 = 0.66972$	$\beta_1 = 0.01100$	$\theta_2 = 0.71845$
$n_2 = 0.56831$	$\alpha_1 = 0.92700$	$\theta_3 = 1.18537$
$n_3 = 0.30067$		
$n_4 = 0.09598$		

Table (3.1) The coefficients of lowpass filter

The Zeros of $H_D(z)$	The Poles of $H_D(z)$
$z(1) = -1.1278 + j0.3435$	$p(1) = -0.7227 + j0.2156$
$z(2) = -1.1278 - j0.3435$	$p(2) = -0.7227 - j0.2156$
$z(3) = -0.4385 + j1.5865$	$p(3) = 0.6767 + j0.3578$
$z(4) = -0.4385 - j1.5365$	$p(4) = 0.6767 - j0.3578$

Table (3.2) The values of the zeros and poles of the
designed filter's transfer function

Fig. (3.2) The location of the zeros and poles with
respect to the unit circle in the z-plane
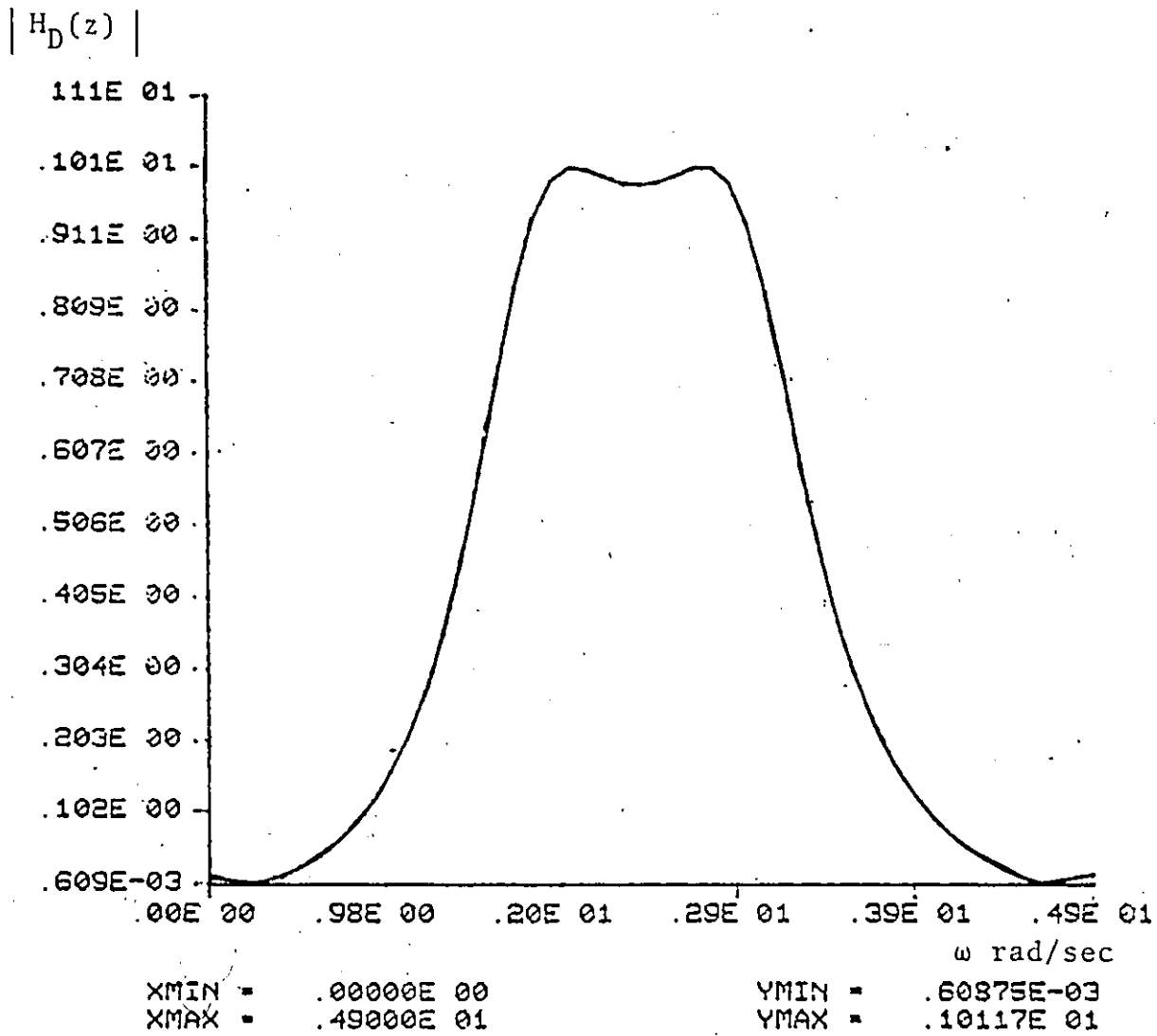
Figure (3.3) Amplitude response of the bandpass filter with the
              passband edges of w_{p1} = 2. rad/sec and w_{p2} = 3. rad/sec.

Numerator Coeff.	Denominator Coeff.	Typical Values of θ
$n_0 = 0.42153$		
$n_1 = -0.01460$	$\alpha_2 = -0.50610$	$\theta_1 = 0.725$
$n_2 = -0.76596$	$\beta_1 = -0.06845$	$\theta_2 = 0.480$
$n_3 = -0.01610$	$\alpha_1 = 0.39296$	$\theta_3 = 0.435$
$n_4 = 0.41503$		

Table (3.3) The coefficients of a bandpass
filter

The Zeros of $H_D(z)$	The Poles of $H_D(z)$
$z(1) = 0.9922 + j0.1558$	$p(1) = 0.3497 + j0.6796$
$z(2) = 0.9922 - j0.1558$	$p(2) = 0.3497 - j0.6796$
$z(3) = -0.9728 + j0.2457$	$p(3) = -0.2515 + j0.7122$
$z(4) = -0.9728 - j0.2457$	$p(4) = -0.2515 - j0.7122$

Table (3.4) The values of the zeros and poles of the
designed filter's transfer function

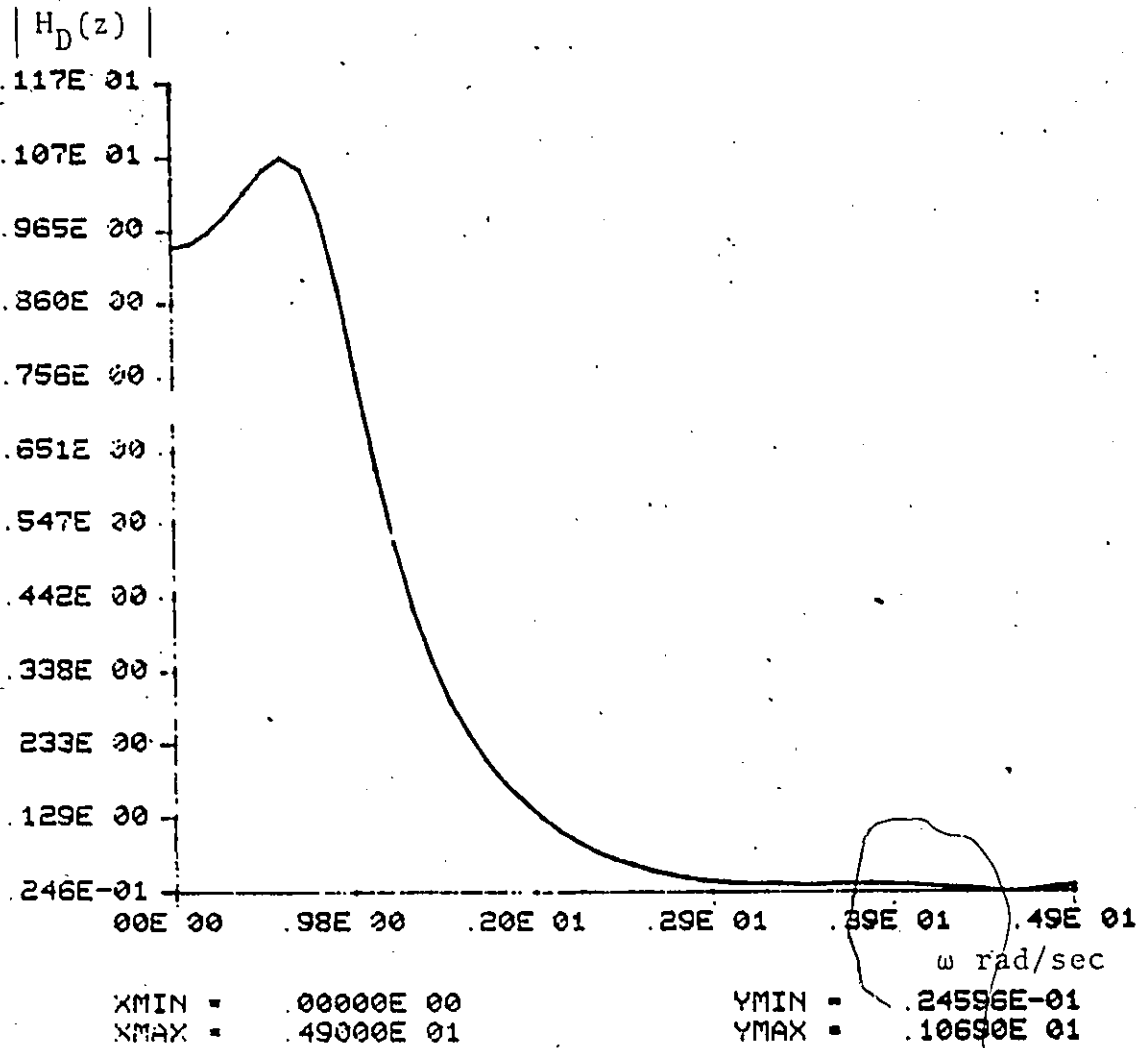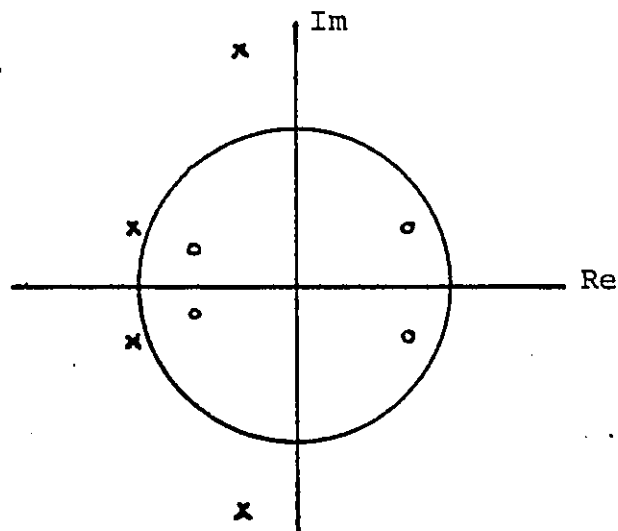Fig. (3.4)   The locations of the zeros and poles with
respect to the unit circle in the z-plane.
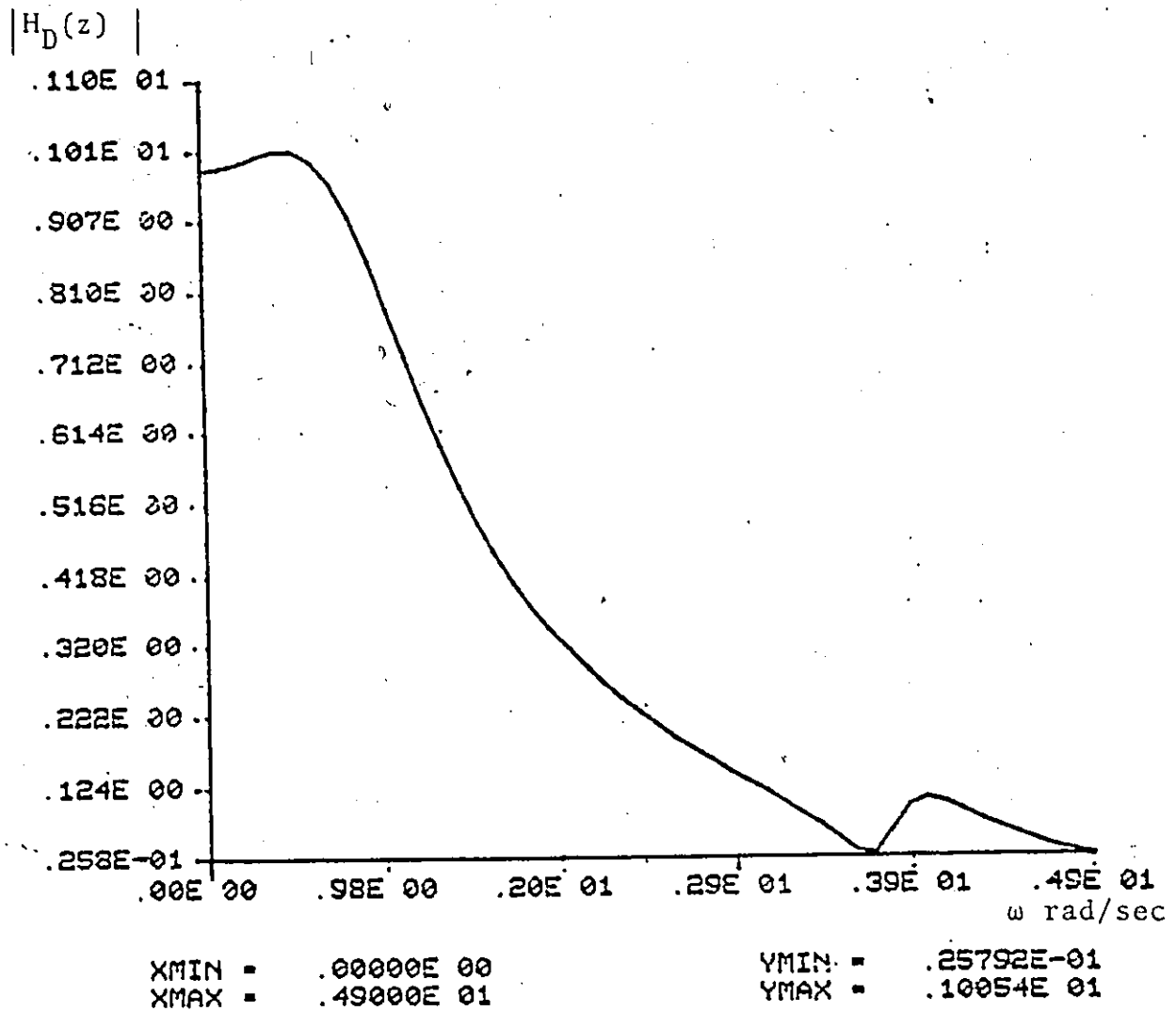
Figure(3.5) Amplitude response of the lowpass filter with the
passband and stopband edges of 1. rad/sec and
2.5 rad/sec respectively.
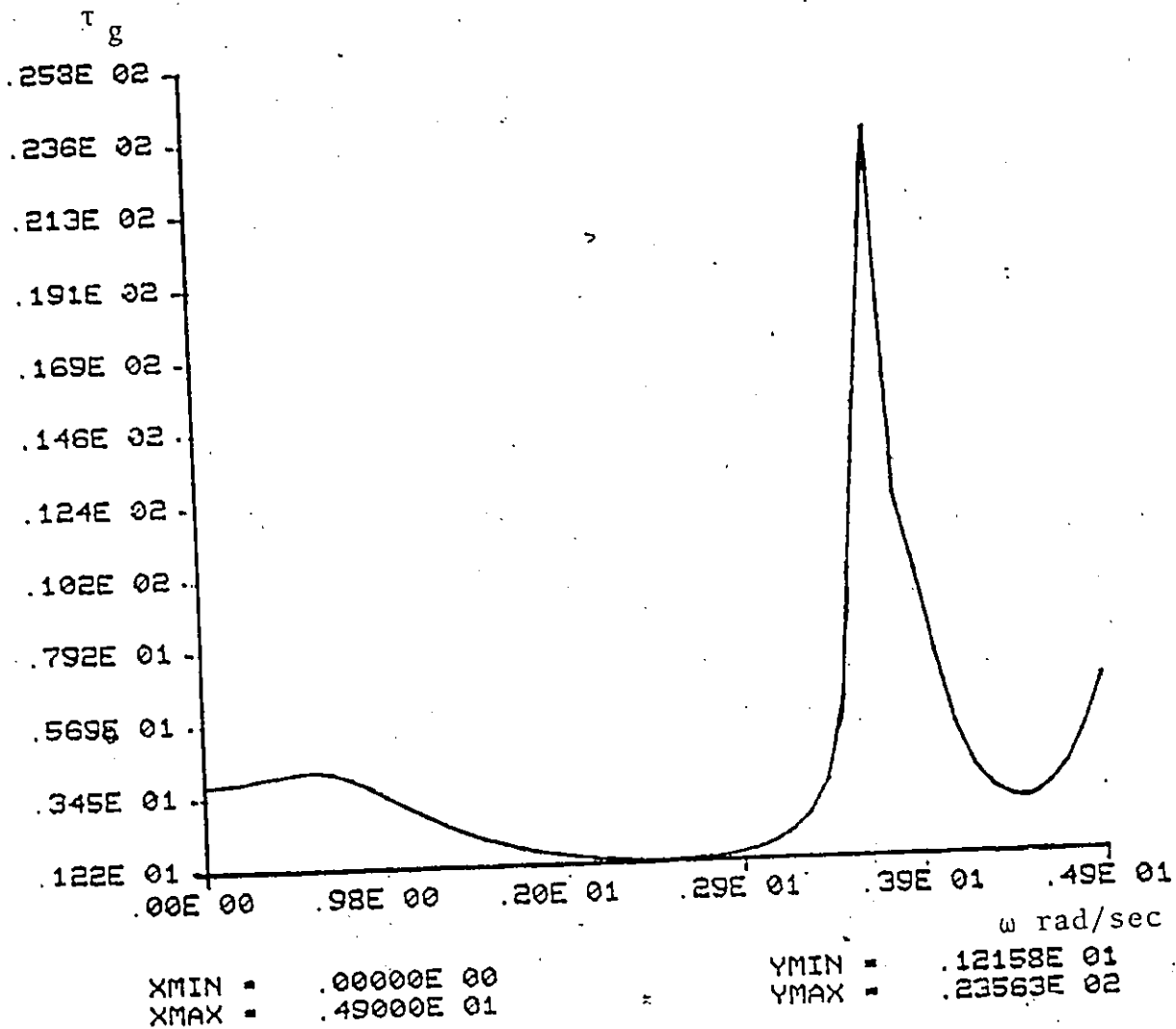
Figure (3.6) Group delay response of the lowpass filter
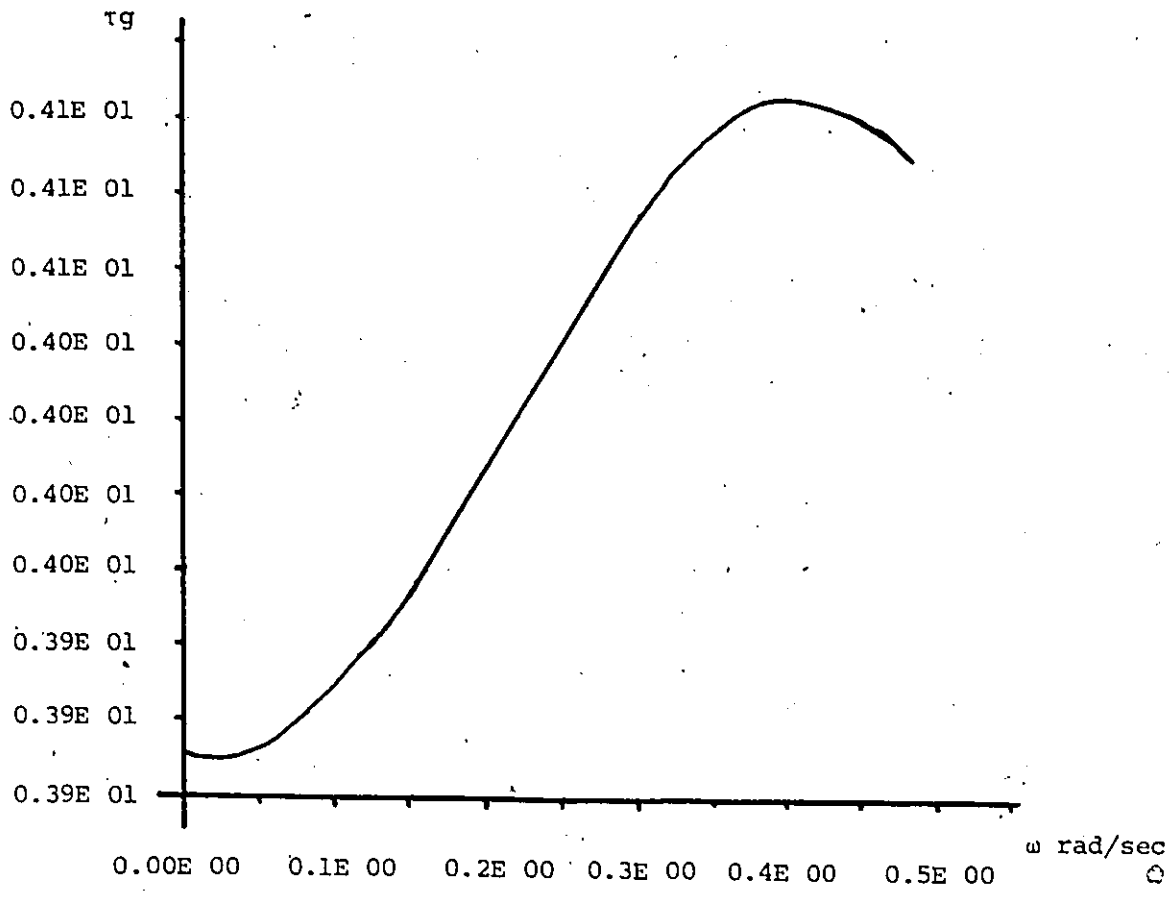
Fig. (3.6b) The group delay of the designed lowpass
            filter in the passband only

Numerator Coeff.	Denominator Coeff.	Typical Values of θ
$n_0 = 0.10940$		
$n_1 = -0.09563$	$\alpha_2 = -0.92450$	$\theta_1 = 0.36461$
$n_2 = 0.07519$	$\beta_1 = -0.76160$	$\theta_2 = 0.3841699$
$n_3 = 0.25029$	$\alpha_1 = -0.58240$	$\theta_3 = 0.32485$
$n_4 = -0.07875$		

Table (3.5) The coefficients of a lowpass
filter with constant group delay
characteristic

The Zeros of $H_D(z)$	The Poles of $H_D(z)$
$z(1) = -0.5242 + j0.4312$	$p(1) = 0.6448 + j0.2030$
$z(2) = -0.5242 - j0.4312$	$p(2) = 0.6448 - j0.2030$
$z(3) = 3.3177 + j0.0$	$p(3) = 0.6137 + j0.5940$
$z(4) = 0.9089 + j0.0$	$p(4) = 0.6137 - j0.5940$

Table (3.6) The values of the zeros and poles of the
designed filter's transfer function
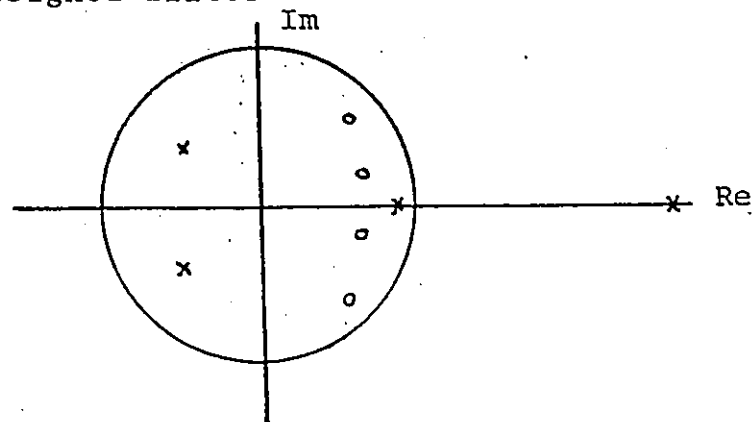
Fig. (3.7)   The locations of the zeros and poles with
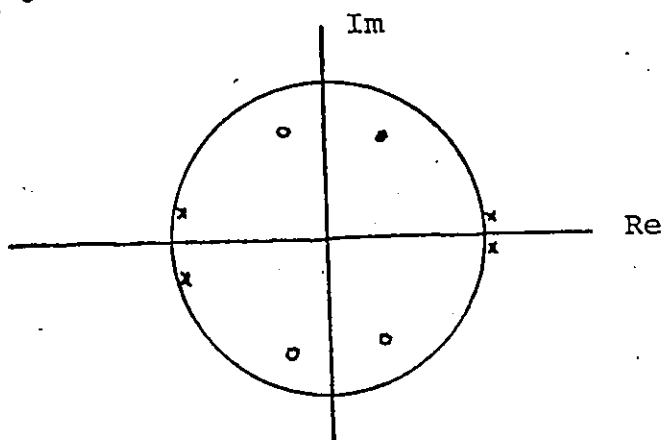respect to the unit circle in the z-plane.

In order to compare the results obtained using this technique with those of the direct design method the same examples were redesigned. The filter transfer function;

$$H(z) = \frac{\sum_{i=0}^{4} a_i \, z^i}{\sum_{j=0}^{4} b_j \, z^j}$$

was assigned to $H_D(z)$ in Eqn. (3.9) and the mean squared error was minimized. The location of the poles and zeros with respect to the unit circle in the z-plane was determined and the pole inversion technique due to Stiglitz(36) was used to stabilize the last example.

Fig. (3.8) shows the magnitude response of a 1-D low-pass filter with the same specifications of the first example. Table (3.7) shows the filter's coefficients and Fig. (3.9) shows the locations of the poles and zeros in the z-plane while Table (3.8) shows their values.

The magnitude response of 1-D bandpass filter is shown in Fig. (3.10) which approximates the specifications of the second example. Table (3.9) and Table (3.10) show the designed filter's coefficients and the values of the zeros and the poles respectively while Fig. (3.11) shows the ploes and zeros location in the z-plane.

A lowpass filter with a constant group delay characteristic was designed. Fig. (3.12) and Fig. (3.13) show the magnitude response and group delay response of the designed filter. Table (3.11) and Table (3.12) show the filter's coef-

ficients and the ploes and zeros of the the designed fil-
ter's transfer function.

As it can be seen from the poles locations, since no
restrictions on the poles locations were made, all the de-
signed filters were unstable.

In order to illustrate the effect of stabilizing an
unstable filter using the methode reported in (30) the low-
pass filter with the constant group delay specification was
stabilized. Fig. (3.13) and Fig. (3.14) show the magnitude and
group delay responses respectively.

From the resultes of the direct design we can see
that the stability or the designed filter is not guaranteed
and testing the stability is necessary. Stabilizing an un-
stable filter does not change the magnitude response (the
response is multiplied with a constant ractor) but the group
delay is no longer constant.

Figure (3.8) The magnitude response of a 1-D lowpass filter
designed using the direct approach

Numerator Coeff. a_i	Denominator Coeff. b_i
$a_0 = 0.2280899$	$b_0 = 2.9654980$
$a_1 = 0.5384600$	$b_1 = -0.4468799$
$a_2 = 0.6839299$	$b_2 = -1.2145990$
$a_3 = 0.5537899$	$b_3 = 0.4215700$
$a_4 = 0.2397199$	$b_4 = 0.4758399$

Table (3.7) The coefficients of lowpass
filter designed using the
direct approach

The Zeros of $H_D(z)$	The Poles of $H_D(z)$
$z(1) = -0.9062 + j0.3997$	$p(1) = 1.1813 + j0.7707$
$z(2) = -0.9062 - j0.3997$	$p(2) = 1.1813 - j0.7707$
$z(3) = -0.2489 + j0.9529$	$p(3) = -1.6242 + j0.7033$
$z(4) = -0.2489 - j0.9529$	$p(4) = -1.6242 - j0.7033$

Table (3.8) The values of zeros and poles of the
designed fourth order filters
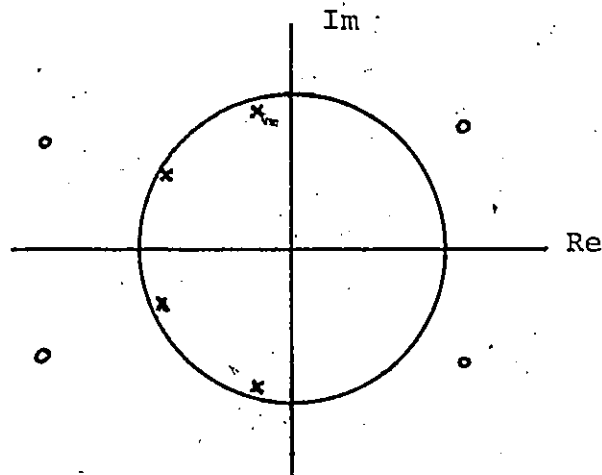
Fig. (3.9) The location of the poles and zeros with
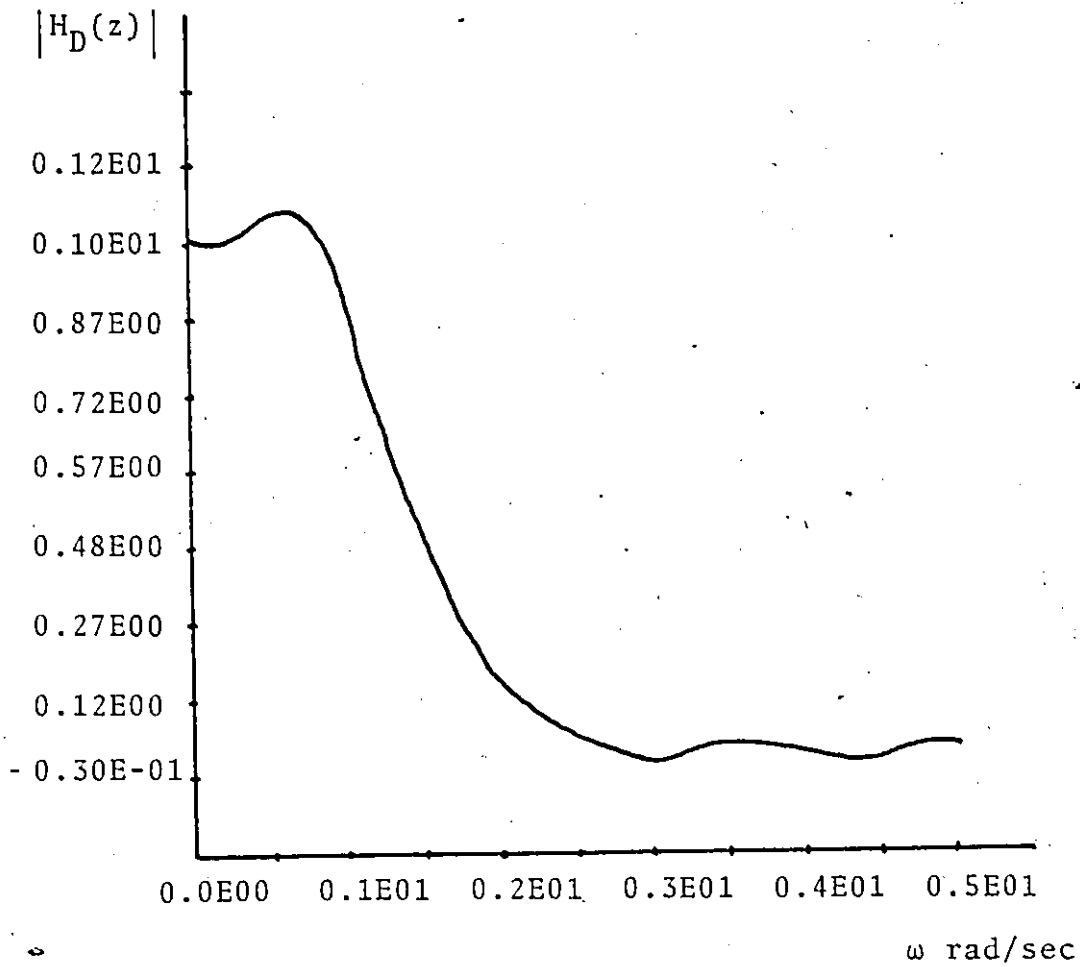respect to the unit circle in the z-plane.
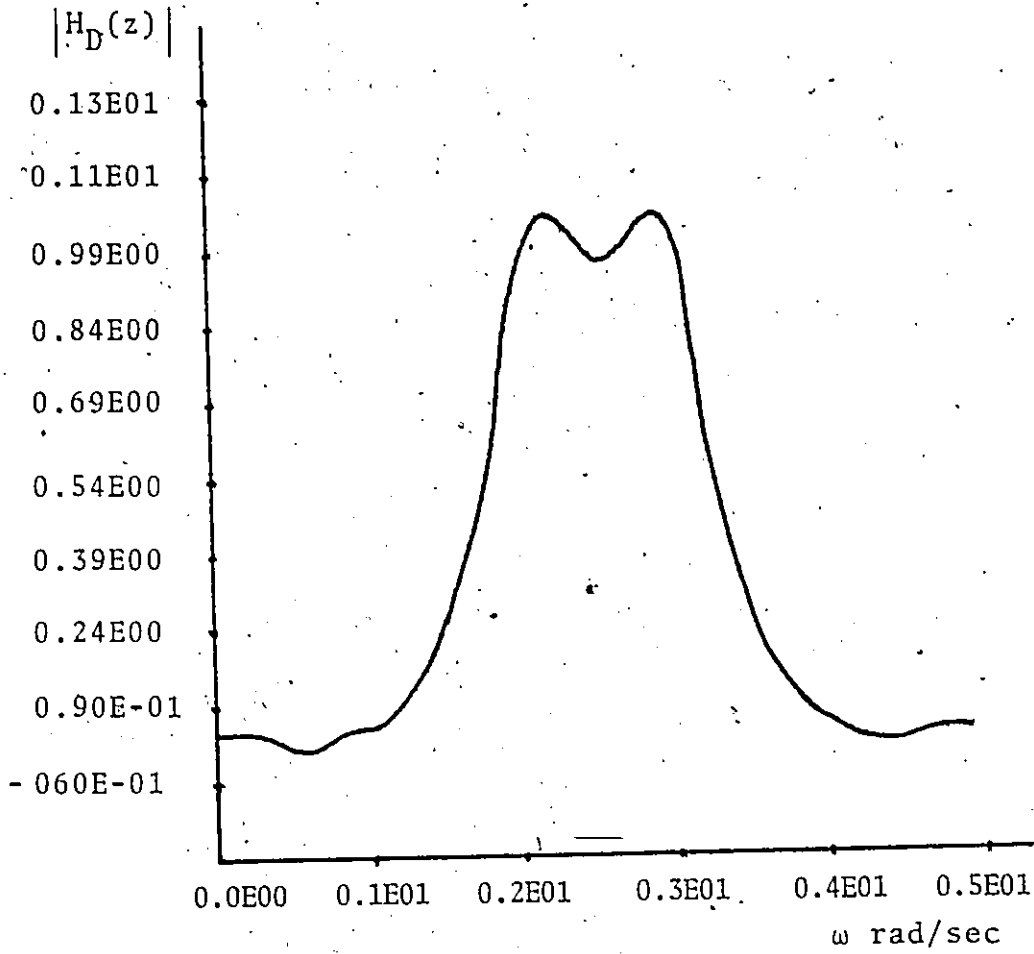
Figure (3.10) The magnitude response of 1-D bandpass filter designed using the direct approach

Numerator Coeff. a_i	Denominator Coeff. b_i
$a_0 = -0.2917699$	$b_0 = 1.9727170$
$a_1 = 0.0093400$	$b_1 = 0.5448098$
$a_2 = 0.4181799$	$b_2 = 3.4707170$
$a_3 = 0.000550$	$b_3 = -0.3970497$
$a_4 = -0.3081398$	$b_4 = 1.9716070$

Table (3.9) The parameters of the designed bandpass filter using the direct approach

The Zeros of $H_D(z)$	The Poles of $H_D(z)$
$z(1) = 0.9092 + j0.3723$	$p(1) = -0.2649 + j0.7830$
$z(2) = 0.9092 - j0.3723$	$p(2) = -0.2649 - j0.7830$
$z(3) = -0.9083 + j0.3949$	$p(3) = 0.3656 + j1.1536$
$z(4) = -0.9083 - j0.3949$	$p(4) = 0.3656 - j1.1536$

Table (3.10) The values of poles and zeros of the designed filter's transfer function

Fig. (3.11)   The locations of zeros and poles with respect to the unit circle in the z-plane.
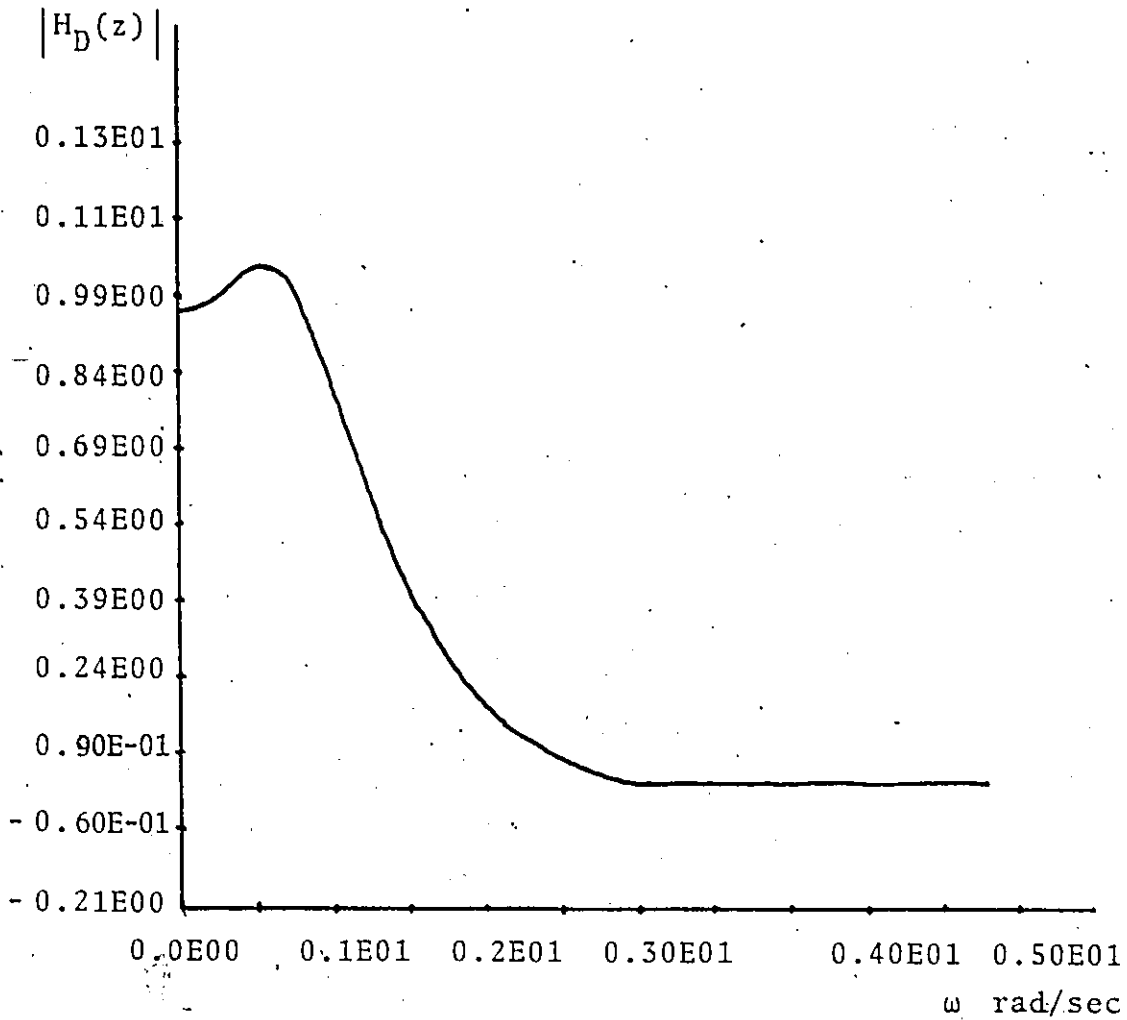
Fig.(3.12) The magnitude resposne of a 1-D lowpass filter with constant group delay characteristic
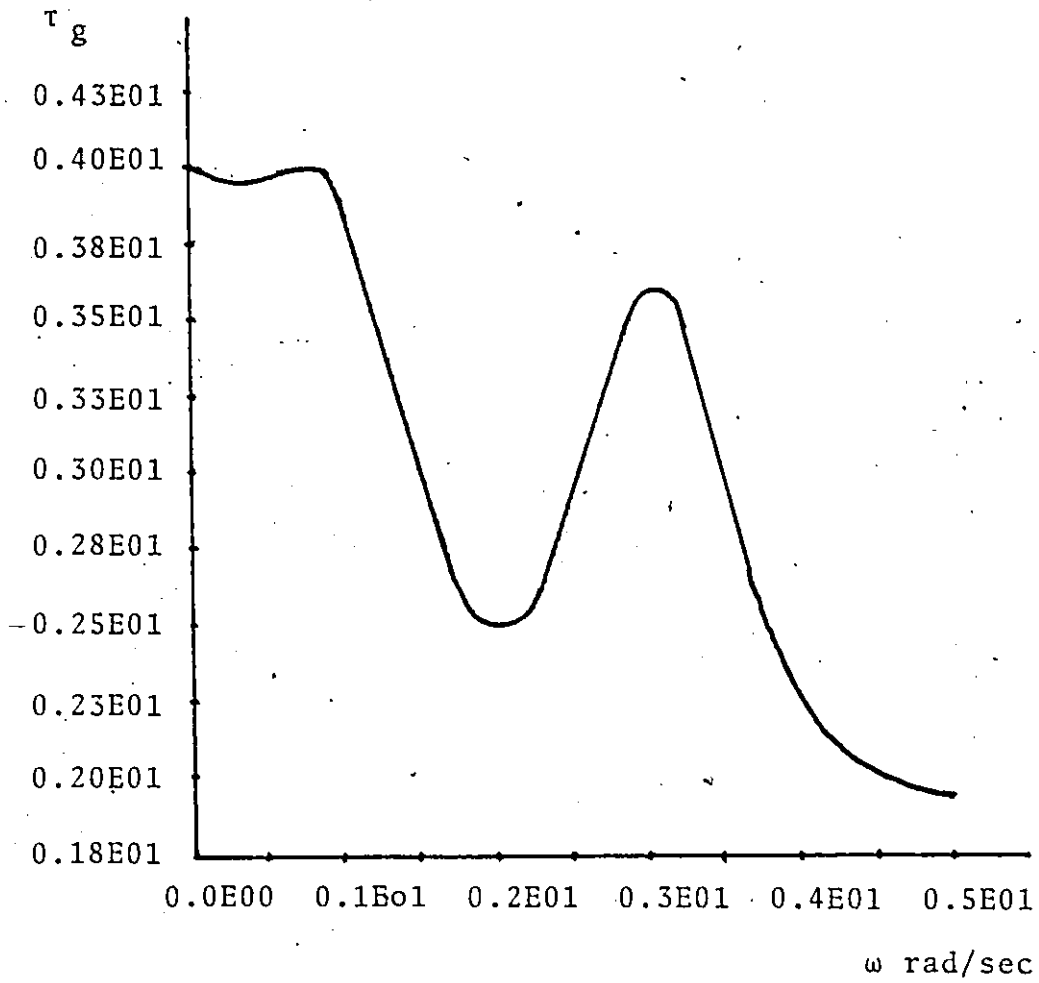
Figure (3.13) The group delay response of the designed
1-D lowpass filter

Numerator Coeff. a_i	Denominator Coeff. b_i
$a_0 = 0.29856530$	$b_0 = -0.17248460$
$a_1 = 0.00256598$	$b_1 = 1.45944200$
$a_2 = 0.02792882$	$b_2 = -3.58752300$
$a_3 = -0.06321770$	$b_3 = 3.71607300$
$a_4 = -0,02456379$	$b_4 = -1.16782500$

Table (3.11) The coefficients of a lowpass.
1-D filter with constant group
delay designed using the direct
approach

The Zeros of $H_D(z)$	The Poles of $H_D(z)$
$z(1) = 1.5395 + j0.0$	$p(1) = 0.1916 + j0.0$
$z(2) = -3.2622 + j0.0$	$p(2) = 1.8787 + j0.0$
$z(3) = -0.4255 + j1.4964$	$p(3) = 0.5559 + j0.3183$
$z(4) = -0.4255 - j1.4964$	$p(4) = 0.5559 - j0.3183$

Table (3.12) The values of zeros and poles of the
designed filter's transfer function
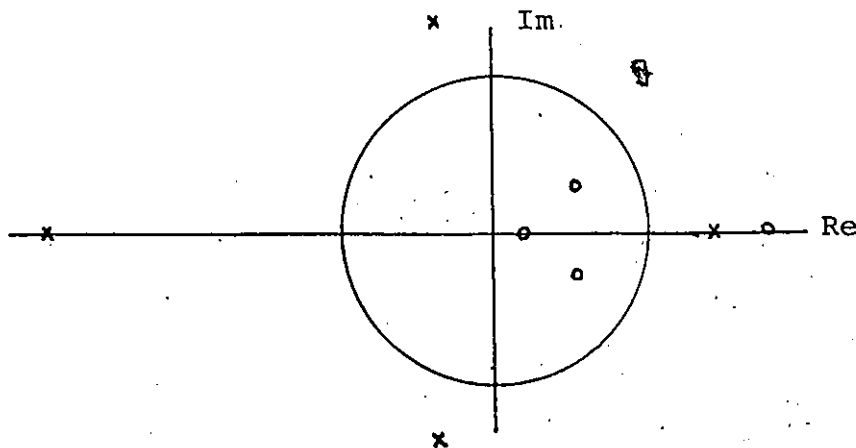
Fig. (3.16) The locations of the zeros and poles with
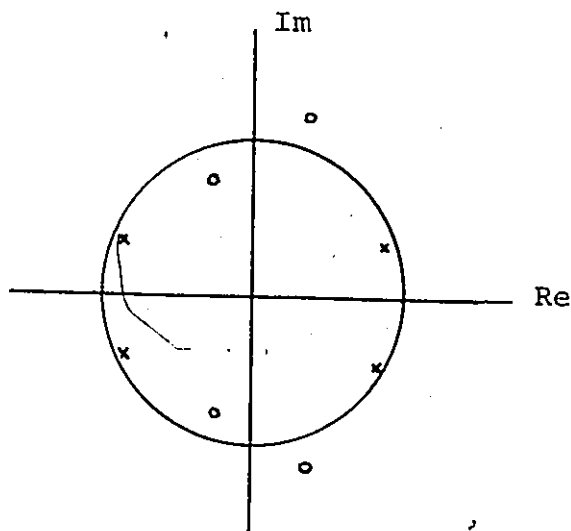respect to the unit circle in the z-plane.
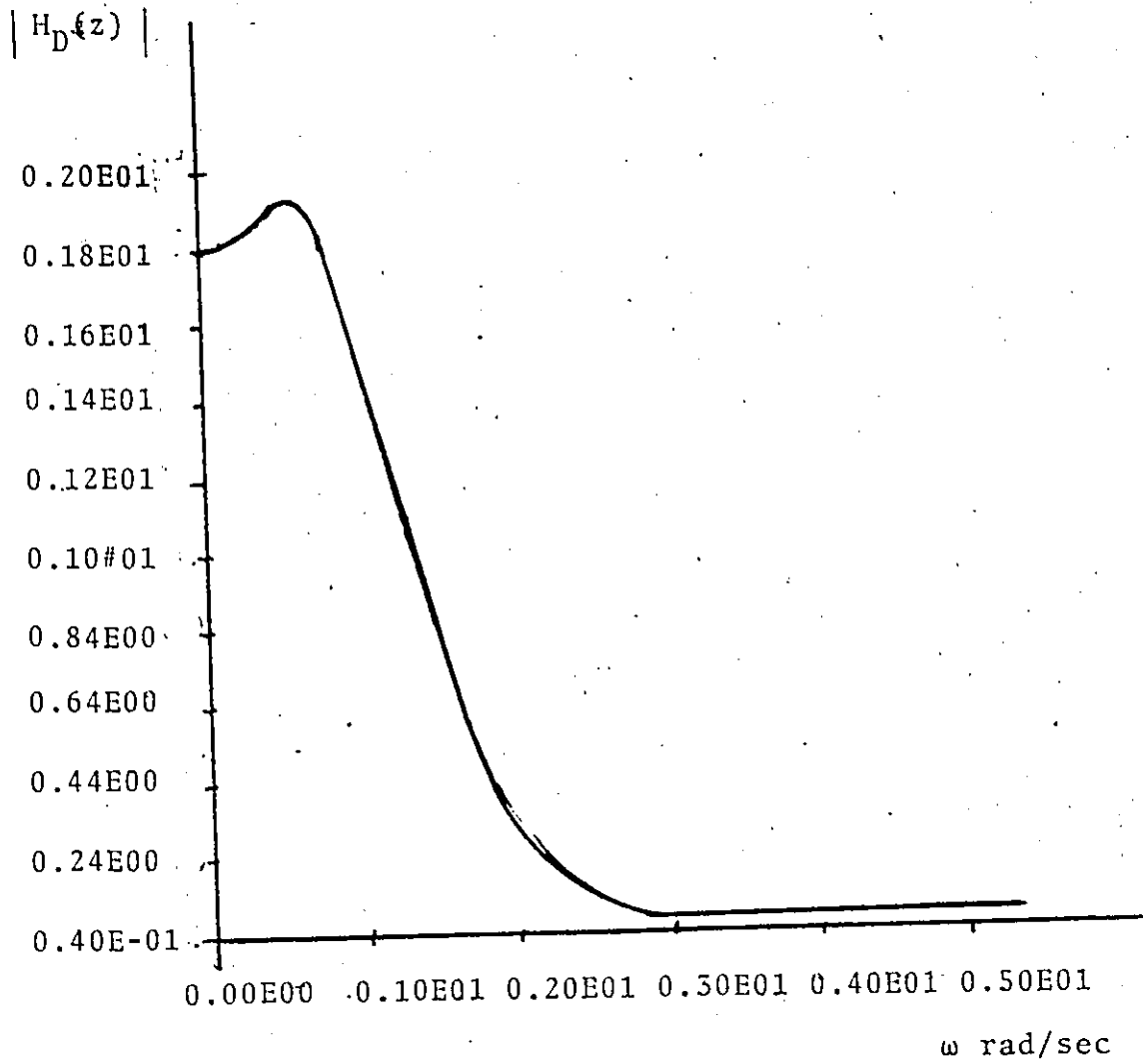
Figure (3.14) The magnitude response of the stabilized lowpass
filter
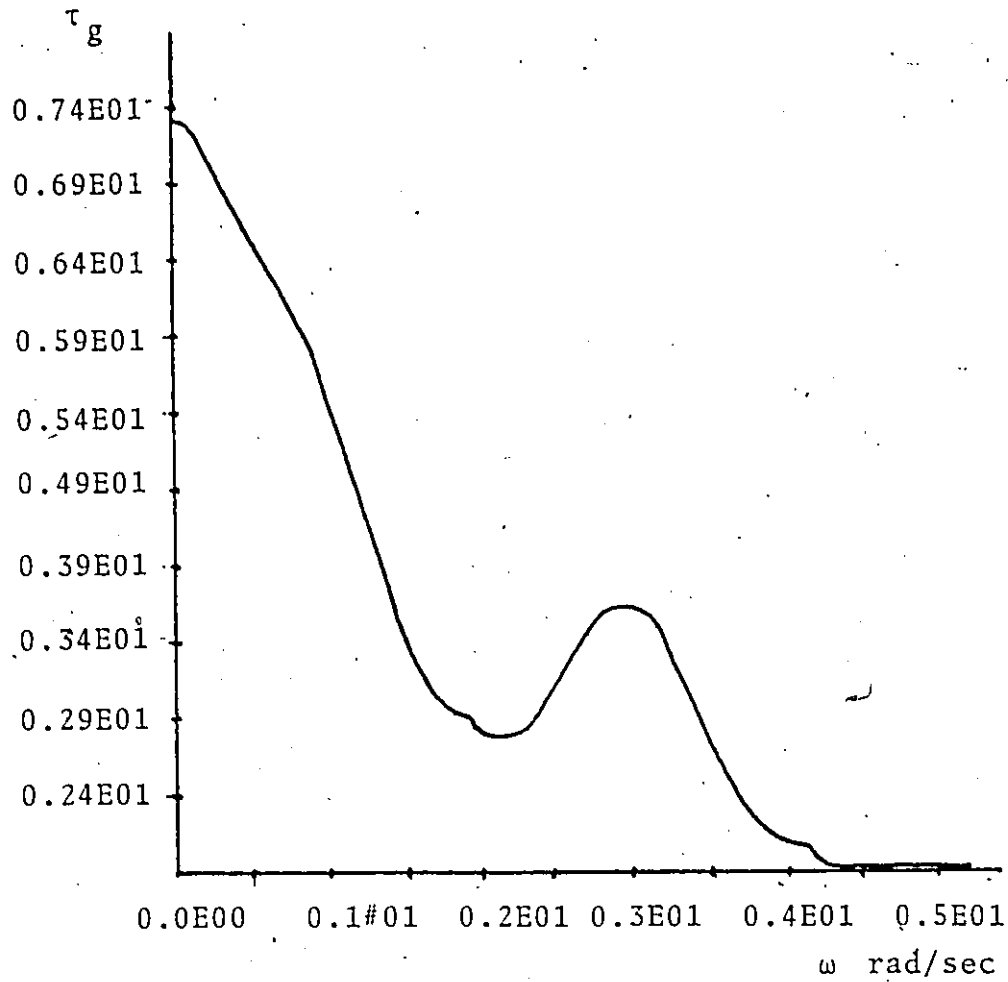
Figure (3.15) The group delay response of the stabilized lowpass filter

Chapter IV

DESIGN OF 2-D RECURSIVE DIGITAL FILTER

4.1

INTRODUCTION

The problems associated with the design of 2-d recursive digital filters which are highly non-linear with complex approximation problems are,

(i) Stability,

(ii) Extensive amount of computation time required even with large main frame computer.

It has been shown in (35,40), that this problem for 2-D filters with circular symmetric characteristics can be simplified if separable denominator transfer function is used. In this case, the stability problem is of 1-D case which can be tested at the end of the design process and the poles outside the unit circles in z_1, z_2 plane can be replaced by its mirror image with respect to unit circles in z_1, z_2 plane, hence stability of the designed 2-D filters. Also, a reduction of (n^2-1) multipliers coefficients (where n is the order of the filter) can be achieved which leads to saving of considerable compution time. This stabilization techni-

que unfortunately changes the phase characteristic of the designed filter, i.e., if the designed filter has linear phase characteristics at the end of this process the phase response will no longer remain linear.

In the method presented in this chapter, Based on a new stability criterion(9), constraints are derived for the coefficients of a class of 1-D polynomial of order n to have all its zeros inside the unit circle. The derived 1-D polynomial is then assigned to the denominator of a 2-D separable product polynomial in z_1, z_2. Then using any suitable non-linear optimization technique to calculate the coefficients of the filter's z-transfer function in such a way so as to minimize a suitable error criterion (i.e., minmax, least mean square or Lp error criterion, etc.). between the ideal and the designed response of the 2-D filter subject to the stability constraints imposed on the coefficients of the two 1-D polynomials in z_1 and z_2 in the denominator. Therefore, a simultaneous design of amplitude and phase responses of a 2-D filter satisfying prescribed magnitude and phase responses can be achieved by the proposed method.

A variable substitution technique is used to transform the constraint optimization problem to an unconstraint optimization one. Although, this method designs only a subclass of 2-D filters, it is shown that many practical 2-D filters can be designed using the proposed method.

4.2

CHARACTERIZATION OF 2-D RECURSIVE DIGITAL FILTER WITH SEPARABLE DENOMINATOR.

While a 2-D recursive digital filter is characterized by its z-tranfer function

$$H_D(z_1, z_2) = \frac{N(z_1, z_2)}{D(z_1, z_2)} = \frac{\displaystyle\sum_{i=0}^{N} \sum_{j=0}^{N} n(i,j) \, z_1^i \, z_2^i}{\displaystyle\sum_{k=0}^{M} \sum_{\ell=0}^{M} d(k,\ell) \, z_1^k \, z_2^\ell} \qquad (4.1)$$

Where N and D are polynomials in ($z_i = \exp s_i T_i$, i=1,2) where s_i and T_i, i=1,2 are complex variables and sampling periods respectively. With the design problem of obtaining the polynomial's coefficients $n(i,j)$ and $d(k,\ell)$ such that the z-transfer function evaluated on the unit circles in z_1 and z_2 plan approximate to the desired response of the filter beside maintaining the stability of the filter. The later condition requires

$$D(z_1, z_2) \neq 0 \quad \text{for} \quad \bigcap_{i=1}^{2} |z_i| \geq 1 \qquad (4.2)$$

The approximation can be carried out in digital (z) domain by any suitable optimization method but the difficulty lies in maintaining the stability of the designed filter, and stabilizing the unstable filter.

A separable denominator filter, on the other hand, is one way to overcome this problem. Thus, if it is assumed that the denominator polynomial is of the form,

$$D(z_1, z_2) = D_1(z_1) D_2(z_2)$$

where,

$$D_i(z_i) = \sum_{j=0}^{M} d_{ij} z_i^{j}, \quad i = 1, 2,$$

(4.3)

In this case the 2-D filter is characterized by its z-transfer function of the form,

$$H_D(z_1, z_2) = \frac{N(z_1, z_2)}{D_1(z_1) D_2(z_2)} = \frac{\sum_{i=0}^{N} \sum_{j=0}^{N} n(i,j) j_1^{i} j_2^{j}}{\left[\sum_{i=0}^{M} d_{1i} z_1^{i}\right]\left[\sum_{j=0}^{M} d_{2j} z_2^{j}\right]} .$$

Now the stability problem is of 1-D case and testing the stability and stabilizing the unstable filter is much simpler task than before. It is shown later that simplification although excludes a large class of 2-D filter but does not effect the design of many useful 2-D filters which are widely used in the area of image processing and seismic data processing.

4.3 FORMULATION OF THE DESIGN PROBLEM

4.3.1 Approximation Of The Magnitued Response Only

Since, the denominator of the 2-D filter is on the form shown by Eqn.(4.3) one of the Eqn.(3.5) or Eqn.(3.7) is assigned to each of $D_1(z_1)$ and $D_2(z_2)$ in the following form,

$$D_i(z_i) = K_{e_i} \prod_{j=1}^{n} (z_1^2 + 2\alpha_{ji} z_i + 1)$$

$$+ (z_i^2 - 1) \prod_{j=1}^{n-1} (z_i^2 + 2\beta_{ji} z_i + 1) \qquad (4.5)$$

where $n = \frac{m}{2}$ for $i = 1, 2$

so that the z-transfer function of the filter will be on the form,

$$H_D(z_1, z_2) = \frac{\displaystyle\sum_{i=0}^{N} \sum_{j=0}^{N} n(i, j) z_1^{i} z_2^{j}}{\left[K_{e_1} \displaystyle\prod_{i=1}^{n} (z_1^2 + 2\alpha_{1i} z_1 + 1) + (z_1^2 - 1) \displaystyle\prod_{i=1}^{n-1} (z_2^2 + 2\beta_{1i} z_1 + 1) \right]} \text{ X}$$

$$\left[K_{e_2} \prod_{i=1}^{n} (z_2^2 + 2\alpha_{2i} z_2 + 1) + (z_2^2 - 1) \prod_{i=1}^{n-1} (z_2^2 + 2\beta_{2i} z_2 + 1) \right]$$

$$(4.6)$$

The error between the ideal and the designed response of the 2-D filter calculated using the following relationship,

$$E_M\left(j\omega_{1m}, j\omega_{2n}\right) = \left| H_I\left(e^{j\omega_{1m}T}, e^{j\omega_{2n}T}\right) \right| - \left| H_D\left(e^{j\omega_{1m}T}, e^{j\omega_{2n}T}\right) \right|$$

$$(4.7)$$

Where E_M is the error of the magnitude response and H_I and H_D are magnitude response of the ideal filter and the magnitude response of the designed filter from Eqn. (4.6) respectively. The norm used in this method is the least mean square error ($E_{\ell 2}$) criterion which is defined as

$$E_{\ell 2} = \sum_{m,\ m\epsilon I_{ps}} \sum E_M^2 \ (j\omega_{1m}, \ j\omega_{2n})$$

Where l_{ps} is the set of all discrete points in the passband and the stopband of the filter. Now any suitable non-linear optimization technique can be used to calculate the parameters of the 2-D filter's z-transfer function so as to minimize $E_{\ell 2}$ in Eqn. (4.8) subject to the constraints of Eqn. (3.6) or (3.8). This constraint optimization problem can be transformed to unconstraint problem by the use of variable substitution method similar to that used in the 1-D case with two sets of variables one for each of the polynomials D_1 (z_1) and $D_2(z_2)$ as described below.

Now any unconstraint optimization method can be utilized for calculation of new filter's coefficients n and to minimize Eqn. (4.8).

$$\alpha_{nij} = \cos \pi \, e^{-\theta_{ij}^2}$$

$$\beta_{n-1,ij} = \cos \pi \, e^{-\left(\theta_{1j}^2 + \theta_{2j}^2\right)}$$

$$\vdots$$

$$\alpha_{1,i,j} = \cos \pi \, e^{-\sum_{i=1}^{2n-1} \theta_{ij}^2} \tag{4.9}$$

for $j = 1, 2$

4.3.2 Approximation of the Magnitude and group delay responses.

It is often desired to design a 2-D filter that approximates magnitude response as well as linear phase characteristics(i.e., constant group delay responses) Since for many images, the phase of the Fourier transform is more important than the magnitude of response (37,38).

The group delay response with respect to w_1 and w_2 is calculated using Eqn.(1.18.a) and Eqn.(1.18.b) respectively. The error between the ideal and the designed group delay of 2-D filter is calculated using the following relationship,

$$E_{\tau_{\omega_i}}\left(\omega_{1m}, \omega_{2n}\right) = \tau_{I_i} T - \tau_{\omega_i}\left(e^{j\omega_{1m}T}, e^{j\omega_{2n}T}\right) \tag{4.10}$$

and $i = 1, 2$

where τ_{I_1} and τ_{I_2} are constants representing the ideal group delay with respect to w_1 and w_2 respectively and these values

are chosen equal to the order of the filter(29). The τ_{ω_1} and τ_{ω_2} are the group delay responses of the designed filter with respect to w_1 and w_2 respectively.

The general mean squared error E_G is calculated using Eqn. (4.8) and Eqn. (4.10) in the following manner:

$$E_G\left(\omega_{1m}, \omega_{2n}\right) = \sum_{m, m \in I_{ps}} \sum E_M^2 \left(\omega_{1m}, \omega_{2n}\right) + \sum_{m, m \in I_{ps}} \sum E_{\tau_{\omega_1}}^2 \left(\omega_{1m}, \omega_{2n}\right)$$

$$\sum_{m, m \in I_p} \sum \tau_{\omega_2} \left(\omega_{1m}, \omega_{2n}\right) . \qquad (4.11)$$

where I_{ps} is the set of all discrete points in the stopband and passband where I_p is the set of all discrete points in passband only.

4.4   DESIGN EXAMPLES

To illustrate the usefulness of the proposed technique, Several design examples of 2-D recursive digital filters satisfying magnitude response with and without constant group delay characteristics are designed.

In the first example, design of a lowpass filter with the following amplitude specifications is required:

$$\left| \hat{H}_I\left(e^{j\omega_{1m}T}, e^{j\omega_{2n}T}\right) \right| = \begin{cases} 1 \text{ for } 0 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 1 \text{ rad/sec} \\ 0 \text{ for } 2.5 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq \frac{\omega_s}{2} = 5 \text{ rad/sec} \end{cases}$$

$$(4.12)$$

The constant group delay characteristic is not required.
The order of the designed filter is chosen to be four.
Therefore, $N(z_1, z_2)$ in Eqn.(4.6) is of the form:

$$N(z_1, z_2) = \sum_{i=0}^{4} \sum_{j=0}^{4} n(i, j) z_1^i z_2^j \qquad (4.13)$$

and $D(z_i)$ for $i = 1, 2$ is also chosen to be fourth order as follows:

$$D(z_i) = K_{e_i} \prod_{j=1}^{2} (z_i^2 + 2\alpha_{ji} z_2 + 1)$$

$$+ (z_i^2 - 1)(z_i^2 + 2\beta_{1i} z_i + 1) \qquad (4.14)$$

for $i = 1, 2$

The filter's transfer function is formed by substituting
Eqn.(4.13) and Eqn.(4.14) in Eqn.(4.6) as follows:

$$H_D(z_1, z_2) = \frac{N(z_1, z_2)}{D(z_1) \, D(z_2)}$$

$$= \frac{\displaystyle\sum_{i=0}^{4} \sum_{j=0}^{4} n(i, j) z_1^i z_2^i}{\left[K_{e_i} \prod (z_1^2 + 2\alpha_{i1} z_1 + 1) + (z_1^2 - 1)(z_1^2 + 2\beta_{i1} z_1 + 1) \right] \times}$$

$$\left[K_{e_1} \prod_{i=1}^{2} (z_2^2 + 2\alpha_{i2} z_2 + 1) + (z_2^2 - 1)(z_2^2 + 2\beta_{i2} z_2 + 1) \right] \qquad (4.15)$$

The E is formed by substituting Eqn.(4.12) and Eqn.(4.15)
in Eqn.(4.7) In order to ensure that the stability const-
raints of Eqn.(4.6) (since, the polynomials $D(z_i)$ for $i=1,2$

is of even order) is satisfied the variable substitution method of Eqn.(4.9) is utilized as follows:

$$\alpha_{21} = \cos \pi \, e^{- \theta_{11}^2}$$

$$\beta_{11} = \cos \pi \, e^{- (\theta_{11}^2 + \theta_{21}^2)}$$

$$\alpha_{22} = \cos \pi \, e^{- (\theta_{11}^2 + \theta_{21}^2 + \theta_{31})}$$

$$\alpha_{12} = \cos \pi \, e^{- \theta_{12}^2}$$

$$\beta_{12} = \cos \pi \, e^{- (\theta_{12}^2 + \theta_{22}^2)}$$

$$\alpha_{12} = \cos \pi \, e^{- (\theta_{12}^2 + \theta_{22}^2 + \theta_{32}^2)}$$

Now any suitable non-linear optimization method can be used to minimize $E_M(\omega_{1m}, \omega_{2n}, \underline{n}, \underline{\theta})$ in Eqn.(4.7). The technique used in this example is due to Fletcher and Powell (41). The gradient of the E with respect to variables vector n, are calculated using the relationship;

$$\frac{\nabla E_{\ell 2}}{\partial \underline{n} \text{ or } \theta} = 2 \sum_{m_1} \sum_{m_2} \left[\left| H_D(j\omega_{1m}, j\omega_{2n}, \underline{n}, \underline{\theta}) \right| - \left| H_I(j\omega_{1m}, j\omega_{2n}) \right| \times \right.$$

$$\left. \frac{1}{\left| H_D(j\omega_{1m}, j\omega_{2n}, \underline{n}, \underline{\theta}) \right|} \times \text{Re} \; \overline{H_D \, j\omega_{1m}, \, j\omega_{2n}, \, \underline{n}, \, \underline{\theta}} \cdot \frac{\partial H_D \; j\omega_{1m}, j\omega_{2n} \, \underline{n}, \, \underline{\theta})}{\partial \underline{n} \text{ or } \underline{\theta}} \right]$$

Figs.(4.1) and (4.2) show the 3-D plot of the magnitude response and the contour plot of the designed 2-D filter respectively. Table (4.1) shows the values of the n, and of the designed filter. The poles of the transfer function are shown in Table (4.2) and their location with respect to the unit circles in z_1 and z_2 are shown in Fig.(4.3).

In the second example, design of a bandpass filter with the following specification is required:

$$\left| H_I\left(e^{j\omega_{1m}T}, e^{j\omega_{2n}T}\right) \right| = \begin{cases} 0 \text{ for } 0 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 1 \text{ rad/sec} \\ 1 \text{ for } 2 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 3 \text{ rad/sec} \\ 0 \text{ for } 4 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 5 \text{ rad/sec} \end{cases}$$

The order of the filter is again considered to be four. The transfer function will be the same as that of the first example. Fig. (4.4) and (4.5) show the 3-D plot of the designed filter and the contour plot respectively. Table (4.3) shows the values of n, $\underline{\alpha}$ and $\underline{\beta}$ of the designed filter. The poles and zeros of the transfer function are shown in Table (4.4). The location with respect to the unit circles in z_1 and z_2 is shown in fig. (4.6).

The last example, is for the design of 2-D lowpass filter with constant group delay characteristics with the following magnitude specifications:

$$\left| H_I\left(e^{j\omega_{1m}T}, e^{j\omega_{2n}T}\right) \right| = \begin{cases} 1 \text{ for } 0 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 1 \text{ rad/sec} \\ 0 \text{ for } 2.5 \leq \sqrt{\omega_1^2 + \omega_2^2} \leq 5 \text{ rad/sec} \end{cases}$$

The ideal group delay is chosen equal to the order of the filter which is four. The group delay response with respect to ω_1 and ω_2 is calculated using Eqn.(1.18a) and Eqn.(1.18.b), this is to be substituted into Eqn.(4.10) to formulate E_{τ_1} and E_{τ_2}. The general mean squared error is formulated by substituting Eqn.(4.8) and Eqn.(4.10) into Eqn.(4.11) and minimized to find the values of $\underline{n}$, $\underline{\theta}$ that approximates both the magnitude and the constant group delay.

Fig.(4.7), (4.8) and (4.9) show the 3-D plots of the magnitude response, group delay response with respect to ω_1 and the group delay response with respect to ω_2 of the designed filter respectively.

Table(4.5) is the values of the n, $\underline{\alpha}$ and $\underline{\beta}$ of the designed filter, where table(4.6) shows the poles of the two denominator ploynomials and fig.(4.10) shows the location of the poles with respect to the unit circles in both the z_1 and z_2 planes. Fig.(4.11) shows the contour plot of the magnitude response.
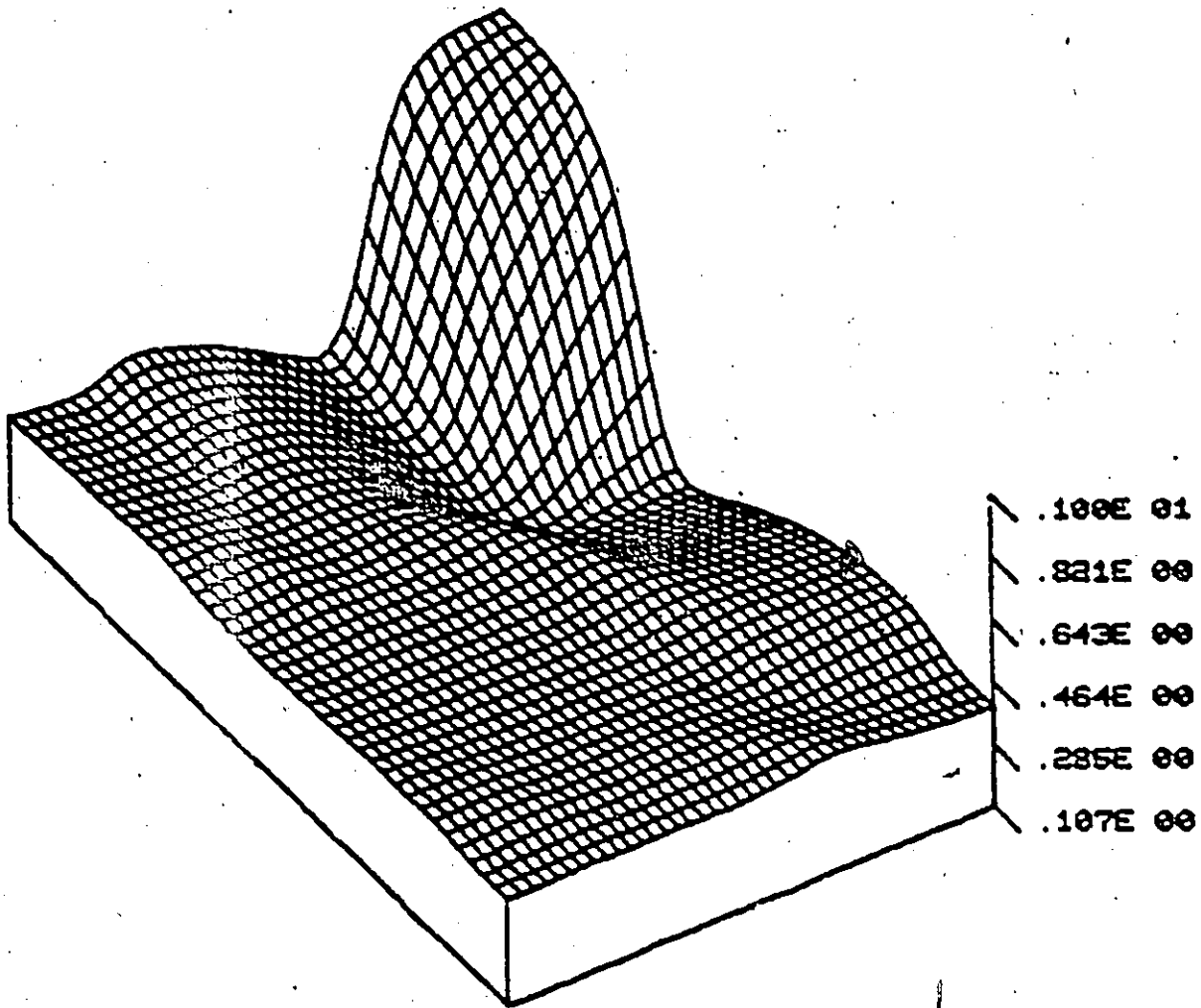
Figure (4.1) The 3-D plot of the normalized magnitude response
of the designed 2-D lowpass filter

N U M E R A T O R C O E F F.		DENOMINATOR COEFF.
$n_{00} = 0.51000$		
$n_{01} = 0.45000$	$n_{24} = 0.89000$	
$n_{02} = 0.98990$	$n_{30} = 0.19000$	
$n_{03} = 0.45000$	$n_{31} = 1.05000$	
$n_{04} = 0.05000$	$n_{32} = 1.37000$	$\alpha_{12} = -0.60300$
$n_{10} = 0.75000$	$n_{33} = 1.22000$	$\beta_{11} = 0.07920$
$n_{11} = 1.01000$	$n_{34} = 0.82000$	$\alpha_{11} = 0.57890$
$n_{12} = 1.29000$	$n_{40} = 0.01000$	$\alpha_{22} = -0.64360$
$n_{13} = 1.55000$	$n_{41} = 0.16000$	$\beta_{21} = -0.01300$
$n_{14} = 0.42000$	$n_{42} = 0.59000$	$\alpha_{21} = 0.50790$
$n_{20} = 0.70000$	$n_{43} = 0.46000$	
$n_{21} = 1.55000$	$n_{44} = 0.57000$	
$n_{22} = 1.44990$		
$n_{23} = 1.51000$		

Table (4.1) The coefficients of the designed 2-D
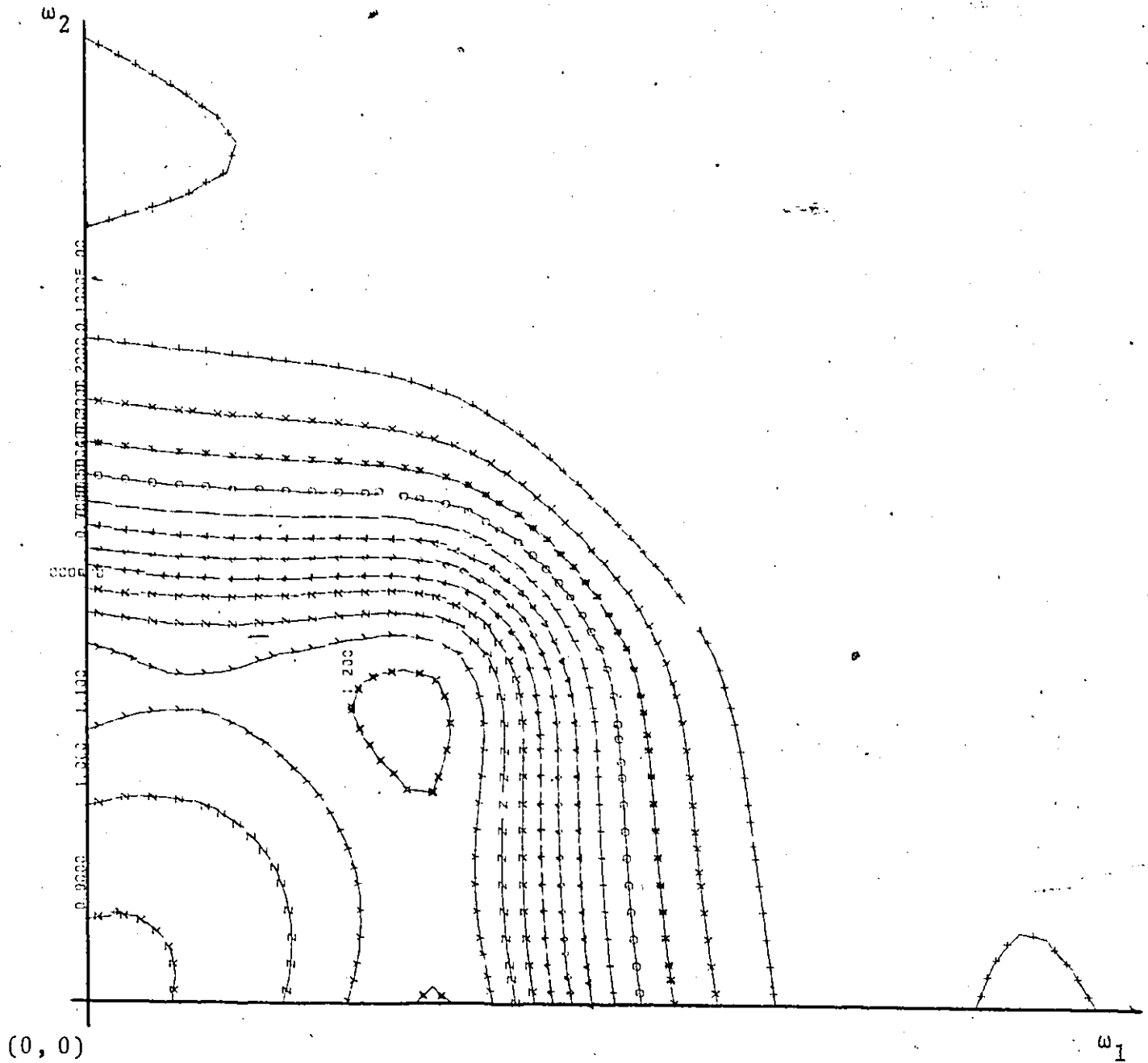lowpass filter

Figure (4.2) The contour plot of the designed 2-D lowpass filter

The Zeros of $D_1(z_1)$	The Zeros of $D_2(z_2)$
$z(1) = -0.4404 + j0.6646$	$z(1) = -0.3722 + j0.6908$
$z(2) = -0.4404 - j0.6646$	$z(2) = -0.3722 - j0.6908$
$z(3) = 0.4300 + j0.5826$	$z(3) = 0.4670 + j0.5684$
$z(4) = 0.4300 - j0.5826$	$z(4) = 0.4670 - j0.5684$

Table (4.2) The values of the zeros of $D_1(z_1)$ and
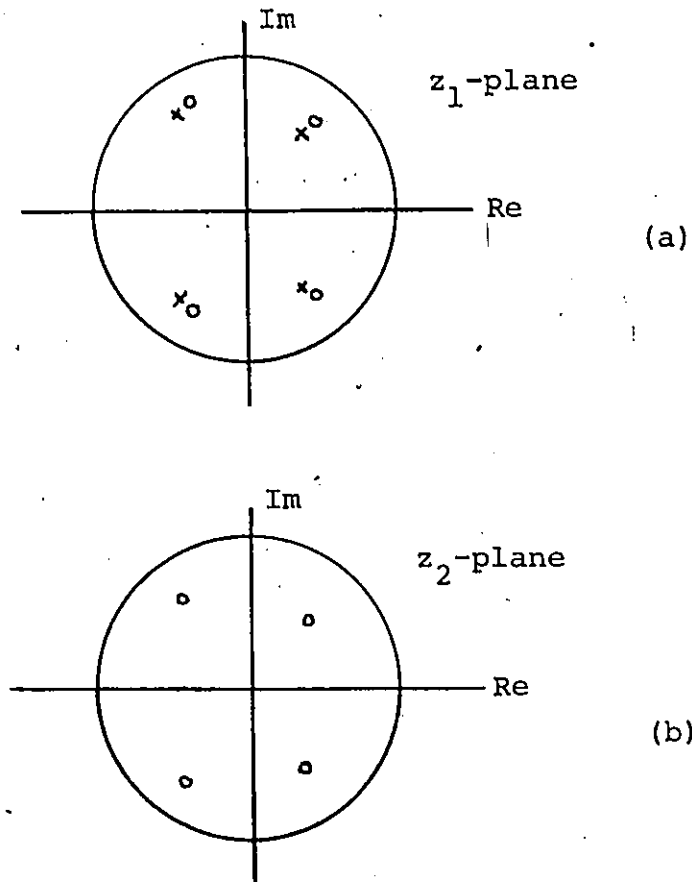$D_2(z_2)$ of the designtd 2-D lowpass filter

(a)

(b)

Fig. (4.3)　(a)　The locations of the zeros of $D_1(z_1)$
in the z_1-plane.
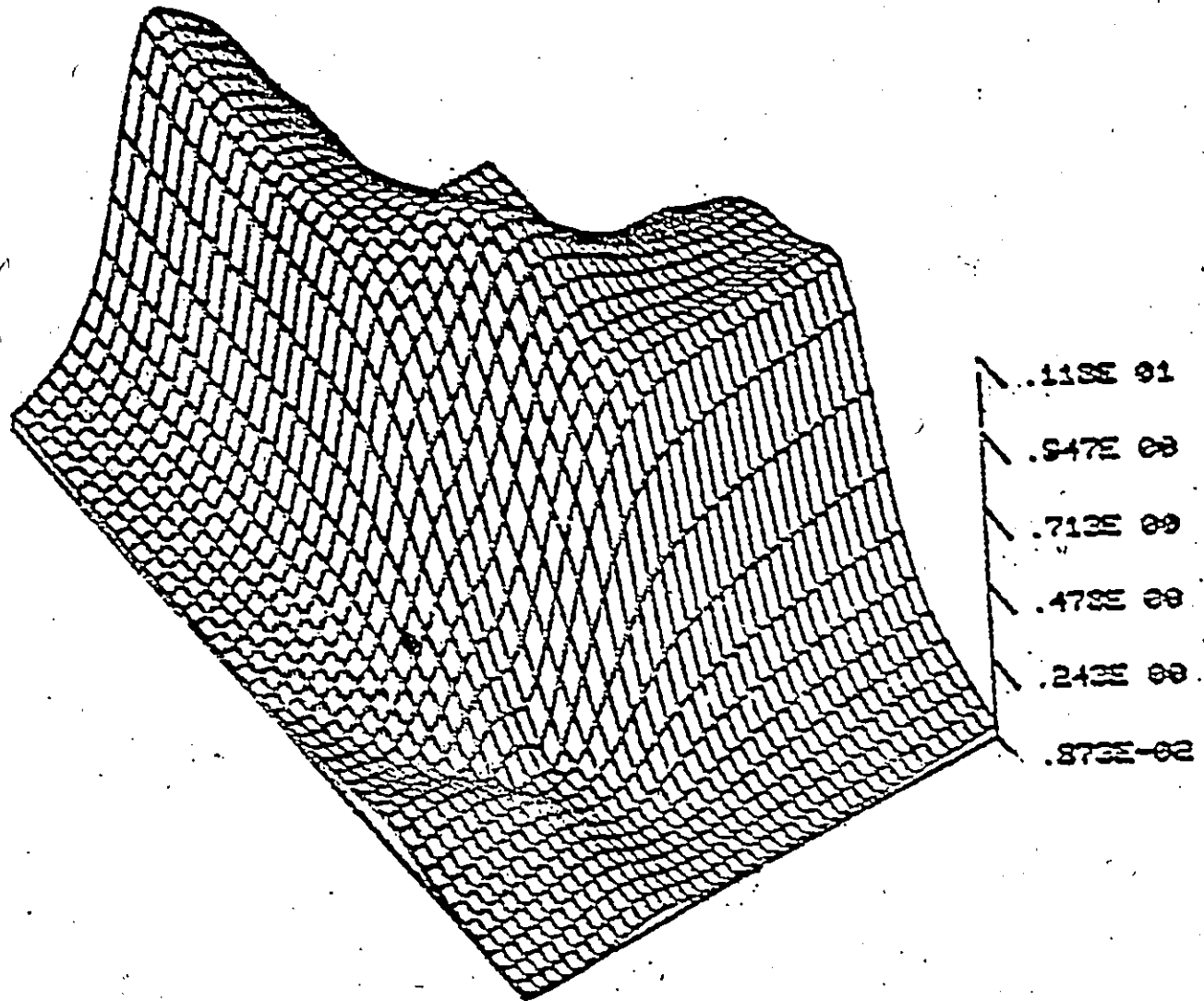(b)　The locations of the zeros of $D_2(z_2)$
in the z_2-plane.

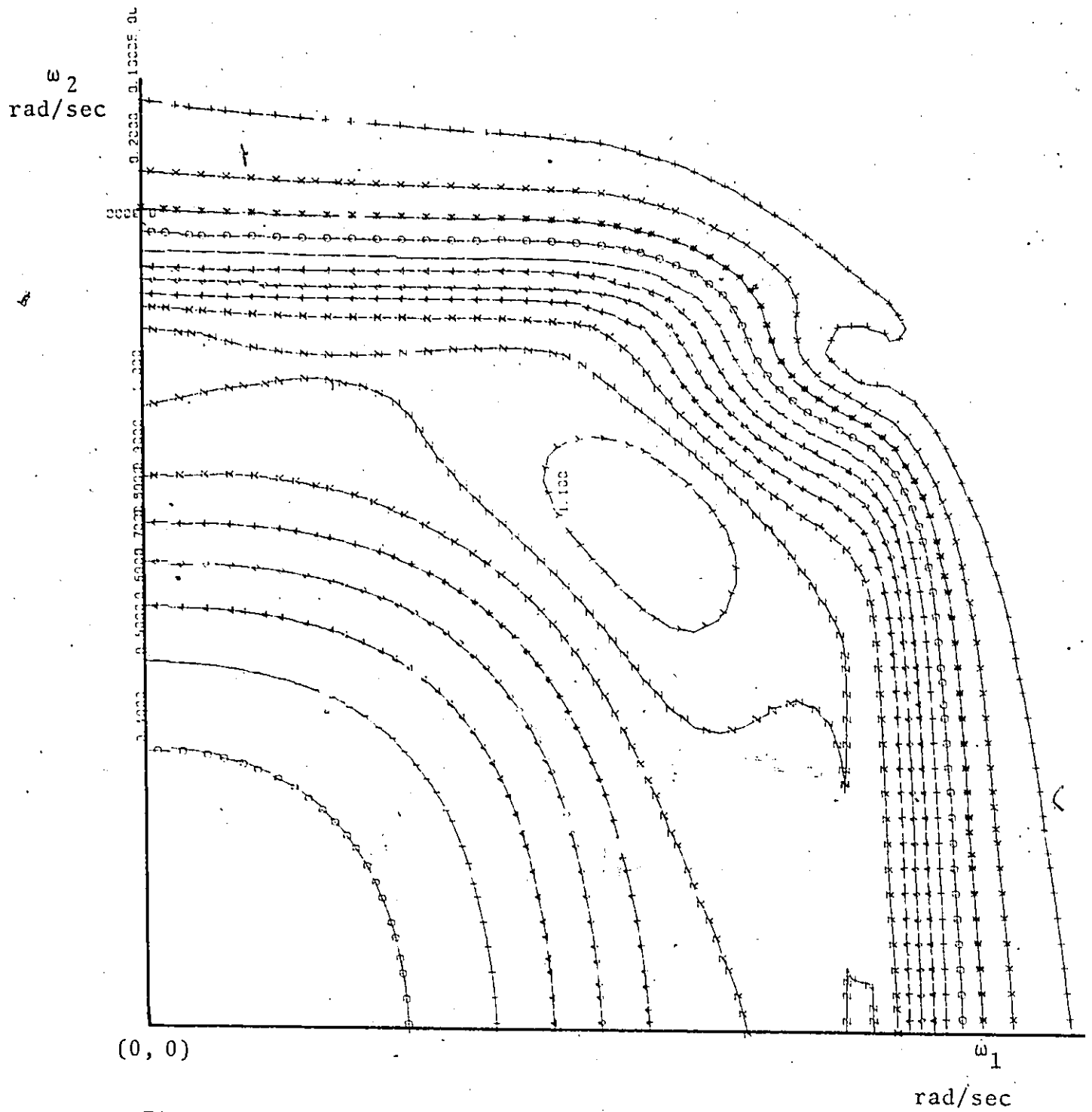Figure (4.4) 3-D plot of the magnitude response of the
2-D bandpass filter.

Fig.(4.5) The contour plot of the designed bandpass filter

NUMERATOR COEFF.		DENOMINATOR COEFF.
$n_{00} = -0.29500$		
$n_{01} = -0.02000$	$n_{24} = 1.21000$	
$n_{02} = 0.86500$	$n_{30} = 0.02000$	
$n_{03} = 0.57500$	$n_{31} = 0.46000$	
$n_{04} = 0.19500$	$n_{32} = 1.47898$	$\alpha_{12} = -0.29546$
$n_{10} = 0.44500$	$n_{33} = 1.21498$	$\beta_{11} = 0.15016$
$n_{11} = -0.07501$	$n_{34} = 0.73000$	$\alpha_{11} = 0.38061$
$n_{12} = 2.03499$	$n_{40} = -0.10000$	$\alpha_{22} = -0.28257$
$n_{13} = 2.24499$	$n_{41} = -0.32000$	$\beta_{21} = 0.09558$
$n_{14} = 0.59500$	$n_{42} = -0.09500$	$\alpha_{21} = 0.34292$
$n_{20} = 0.68500$	$n_{43} = 0.29500$	
$n_{21} = 1.38499$	$n_{44} \quad -0.25000$	
$n_{22} = 1.97500$		
$n_{23} = 3.84990$		

Table (4.3) The values of the coefficients. of the
designed 2-D bandpass filter

The Zeros of $D_1(z_1)$	The Zeros of $D_2(z_2)$
$z(1) = -0.2570 + j0.8168$	$z(1) = -0.1972 + j0.8082$
$z(2) = -0.2570 - j0.8168$	$z(2) = -0.1982 = j0.8082$
$z(3) = 0.1502 + j0.6573$	$z(3) = 0.1252 + j0.6826$
$z(4) = 0.1502 - j0.6573$	$z(4) = 0.1252 - j0.6826$

Table (4.4) The values of the zeros of the denominator
$D_1(z_1)$ and $D_2(z_2)$ of the designed 2-D
bandpass filter

(a)

(b)
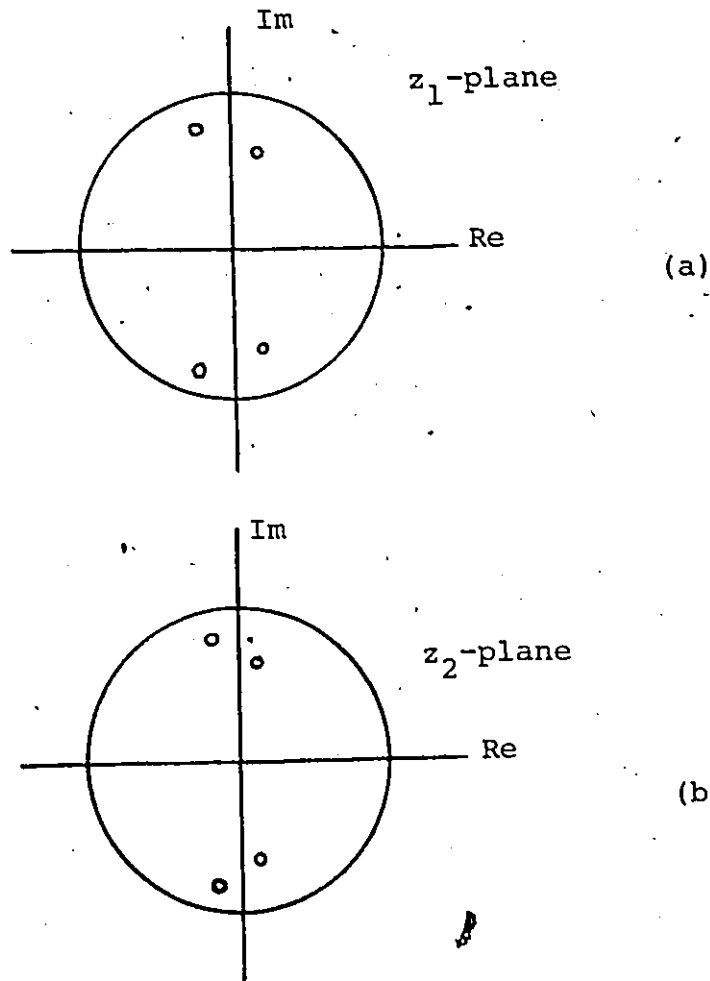
Fig.(4.6) (a) The location of the zeros of $D_1(z_1)$ in
the z_1-plane.
(b) The location of the zeros of $D_2(z_2)$ in
the z_2-plane.
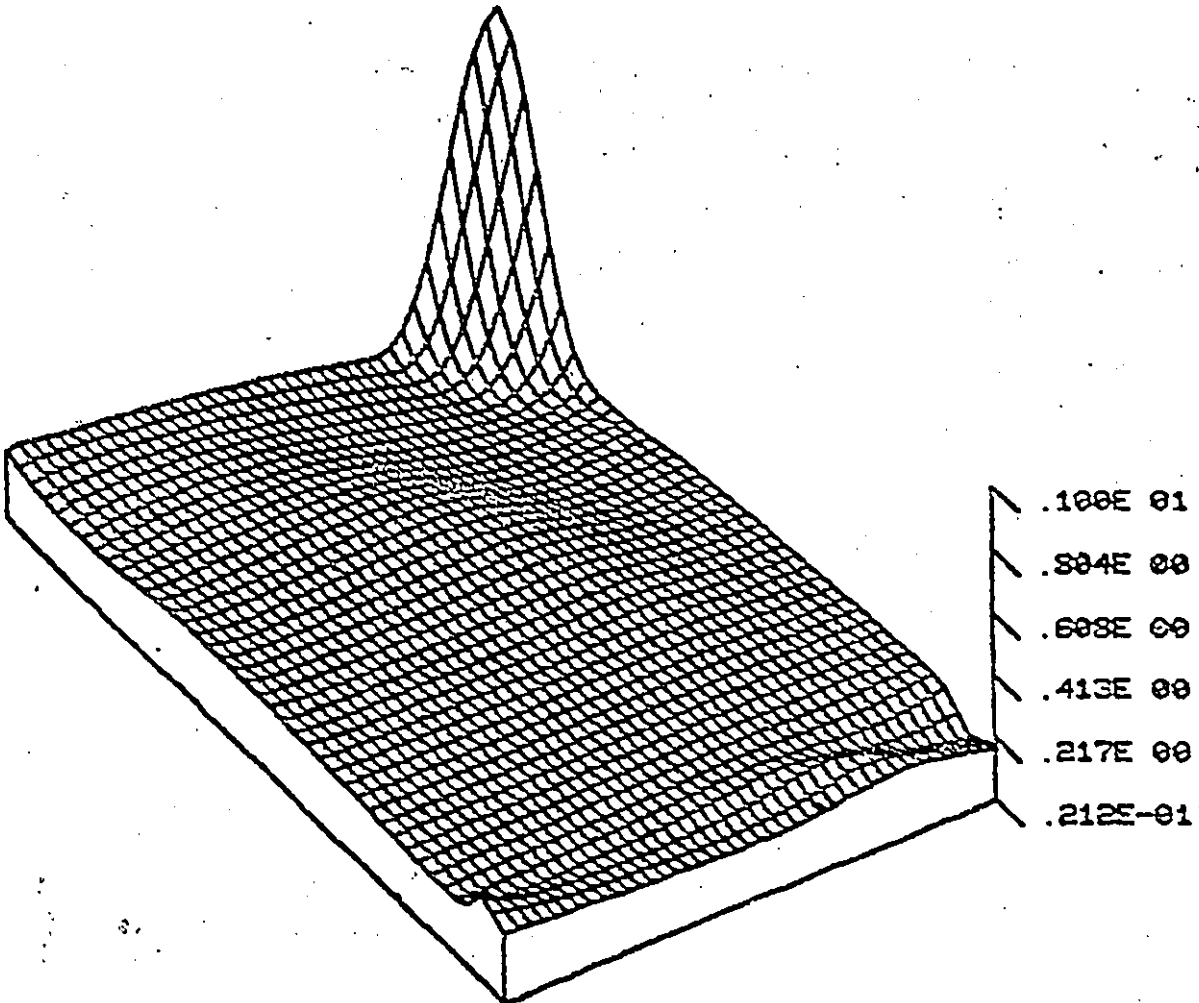
Figure (4.7) (3-D) Plot of the Magnitude Response of the
2-D Lowpass Filter
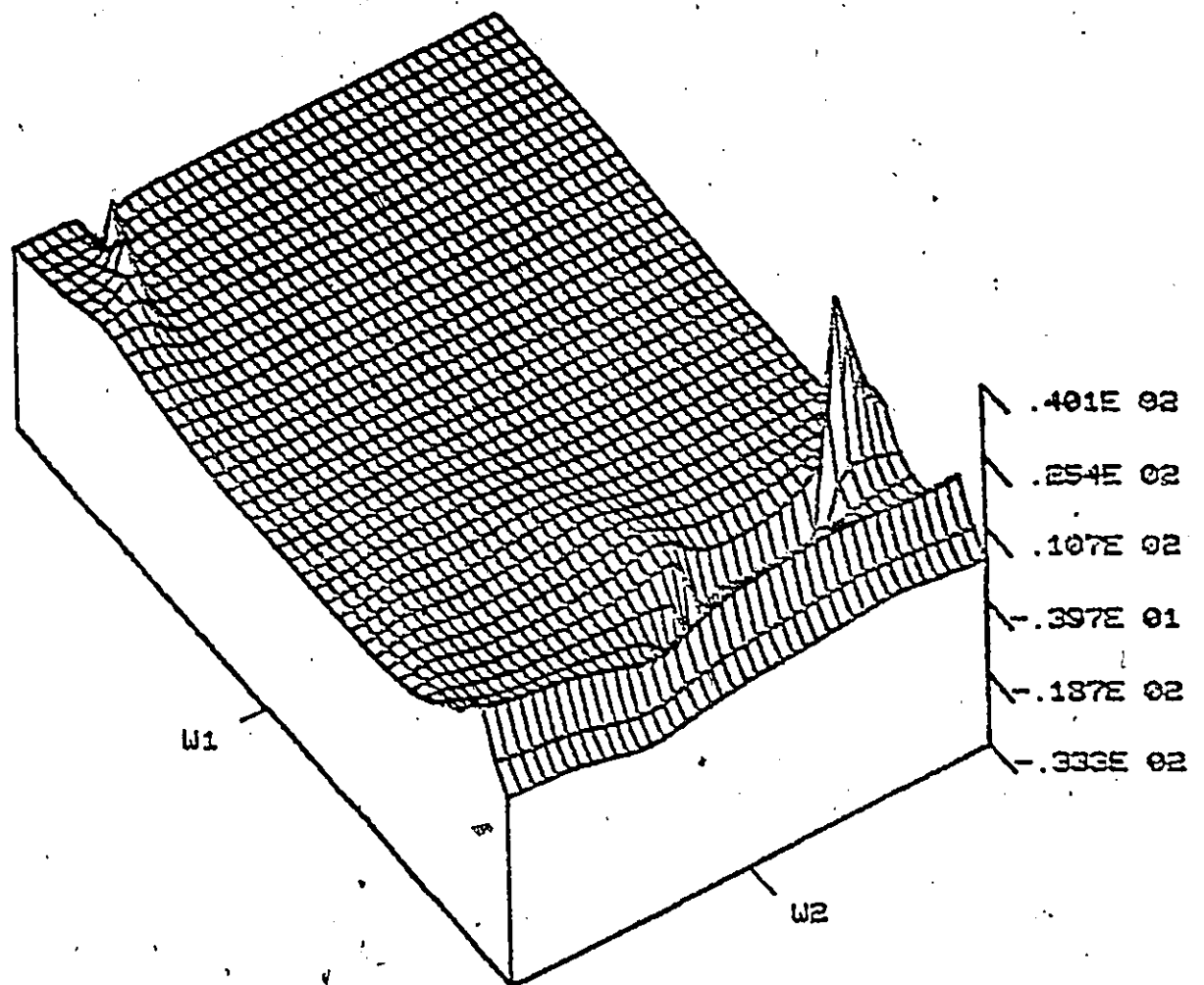
Figure (4.8) (3-D) Plot of the Group Delay Response With Respect to ω_1 of the 2-D Filter
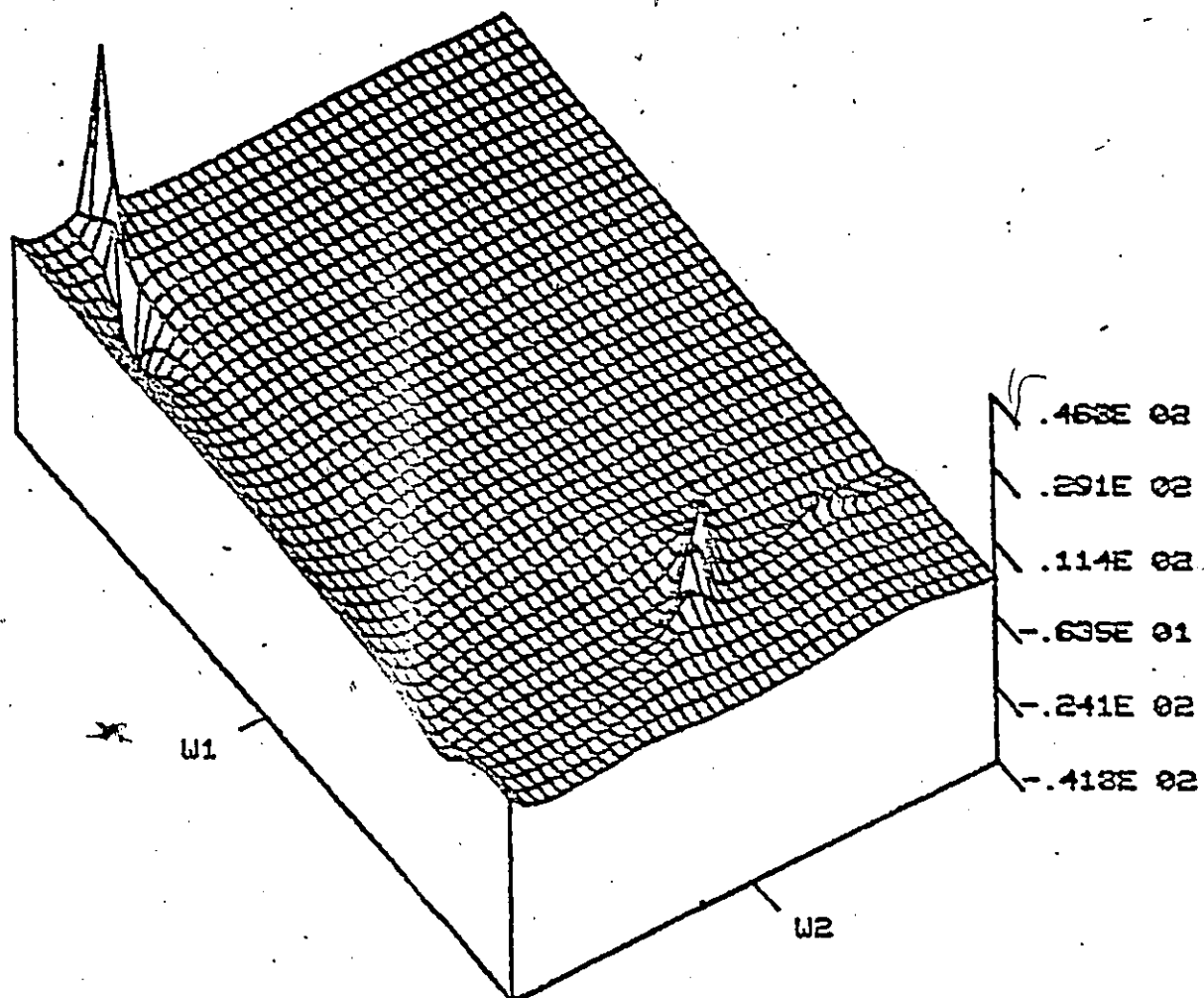
Figure (4.9) (3-D) Plot of the Group Delay Response with Respect
to ω_2 of the 2-D Filter
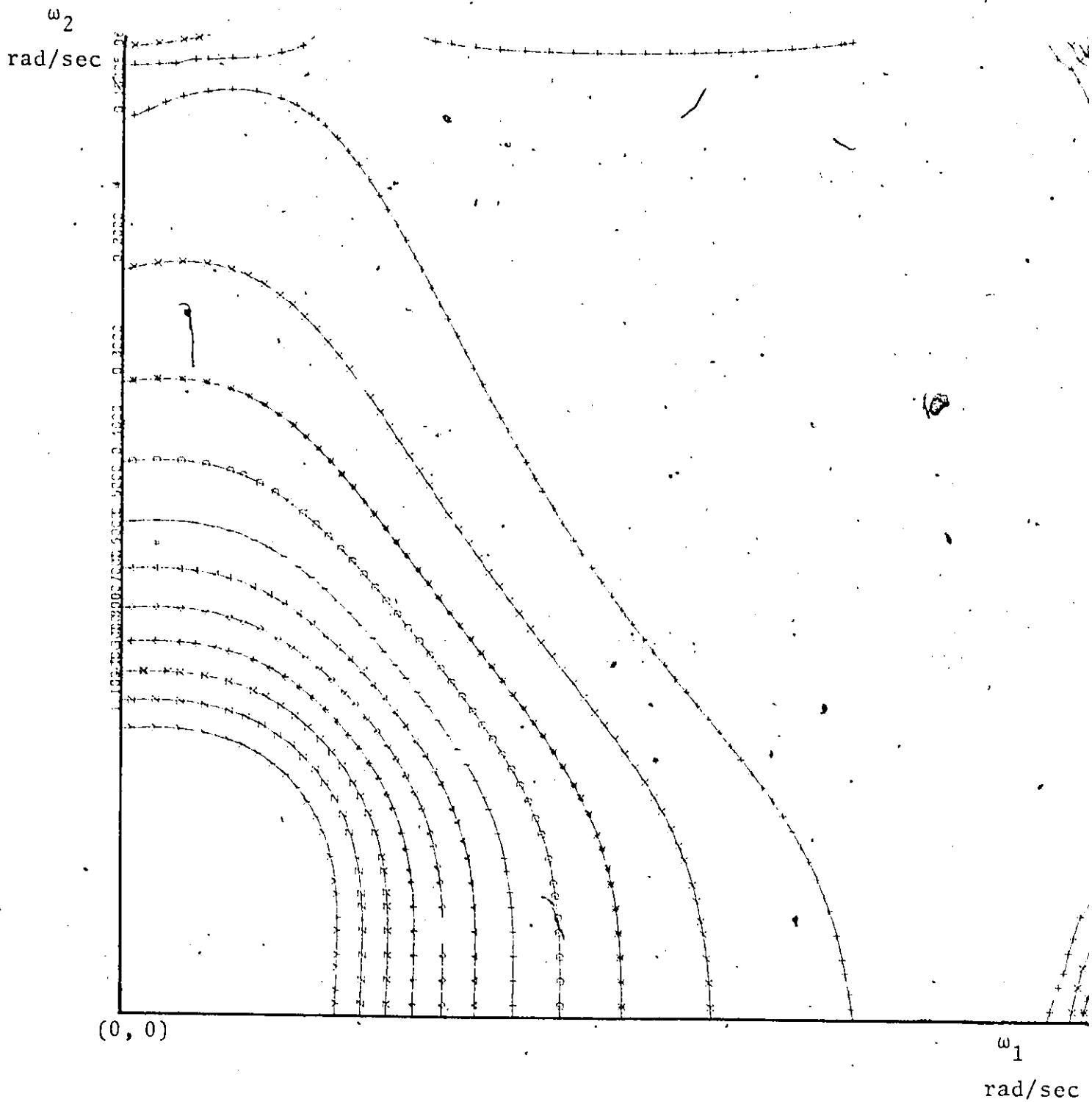
Fig.(4.10) The contour plot of the designtd 2-D lowpass filter
with constant group delay

N U M E R A T O R C O E F F.		DENOMINATOR COEFF.
$n_{00} = -0.49500$		
$n_{01} = 0.13000$	$n_{24} = 1.30000$	
$n_{02} = 0.96500$	$n_{30} = 0.12000$	
$n_{03} = 0.69500$	$n_{31} = 0.60000$	
$n_{04} = 0.24500$	$n_{32} = 1.60999$	$\alpha_{12} = -0.29546$
$n_{10} = 0.41500$	$n_{33} = 1.25498$	$\beta_{11} = 0.15016$
$n_{11} = 0.12499$	$n_{34} = 0.74000$	$\alpha_{11} = 0.37127$
$n_{12} = 2.10499$	$n_{40} = -0.05000$	$\alpha_{22} = -0.28257$
$n_{13} = 1.33499$	$n_{41} = -0.30000$	$\beta_{21} = 0.08156$
$n_{14} = 0.71500$	$n_{42} = 0.14500$	$\alpha_{21} = 0.32189$
$n_{20} = 0.85500$	$n_{43} = 0.32500$	
$n_{21} = 1.56499$	$n_{44} = -0.56000$	
$n_{22} = 1.98500$		
$n_{23} = 3.06599$		

Table (4.5) The values of the coefficients of the designed 2-D lowpass filter with constant group delay

The Zeros of $D_1(z_1)$	The Zeros of $D_2(z_2)$
$z(1) = -0.2518 + j0.8266$	$z(1) = -0.7126 + j0.00$
$z(2) = -0.2518 - j0.8266$	$z(2) = -0.8405 + j0.0$
$z(3) = 0.1512 + j0.6508$	$z(3) = 0.7231 + j0.1833$
$z(4) = 0.1512 - j0.6508$	$z(4) = 0.7231 - j0.1833$

Table (4.6) The valuesof the zeros of $D_1(z_1)$ and $D_2(z_2)$ of the designed lowpass filter with constant group delay characteristics
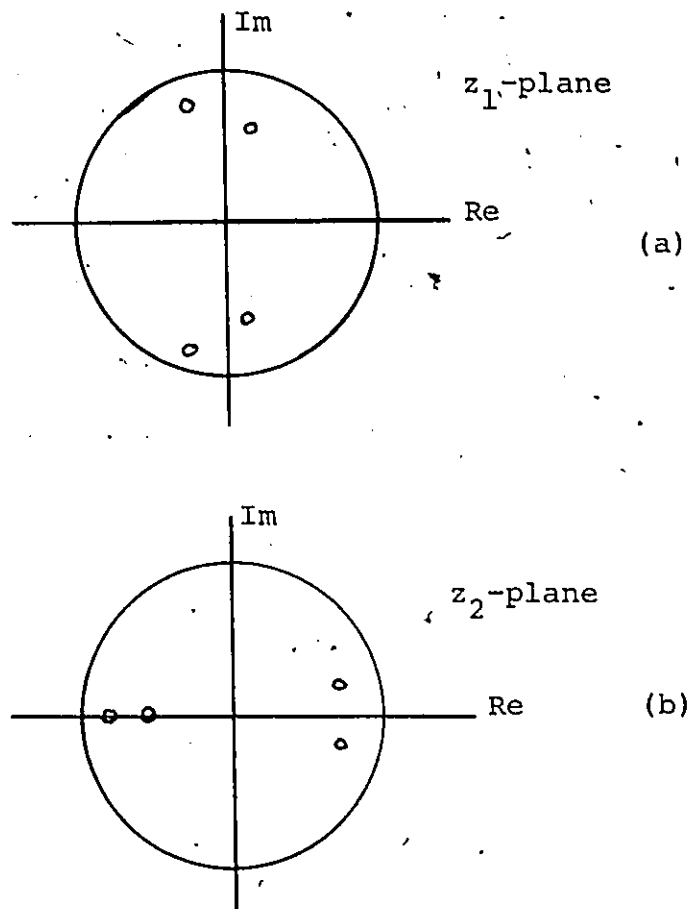

(a)

(b)

Fig. (4.10)   (a) The locations of the zeros of $D_1(z_1)$ in the z_1-plane.

(b) The locations of the zeros of $D_2(z_2)$ in the z_2-plane.

Chapter V

CONCLUSION

Based on a recently reported stability criterion(9) an iterative method for the design of 1-D recursive digital filter satisfying a given magnitude response with or without constant group delay characteristics is presented. Conditions are imposed on the parameters of the denominator polynomial of the filter's z-transfer function to ensure the stability of the designed filter.

The method was compared with those of the direct design and it has been shown that, while the proposed method always leads to stable filters the direct design approach does not. Also, the proposed method can approximate the magnitude response with constant group delay characteristics which can not be obtained by the direct approach if stabilization technique(30) is used.

A design method for a class of 2-D filters having a separable denominator transfer function is presented. Based on the same stability criterion the stability of the designed filter is guaranted. The method is extended to the design of linear-phase 2-D filters.

The method require $n^2 + 3$ less variables than that of the general case of 2-D and hence, the method requires a

modest amount of compution time. The stability problem is reduced to 1-D case which is more simple than that of general case of 2-D. Many useful filters can be designed using the proposed method as shown by the desing examples. The constraint optimization problem of calculating the parameters of the designed filter is transformed to an unconstraint optimization problem by the application of the variable substitution method.

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