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DEVELOPMENT OF A NEW HEURISTIC

METHOD FOR ASSEMBLY LINE BALANCING

A Thesis  
Submitted to the Faculty of Graduate Studies  
through the Department of Industrial Engineering  
in partial fulfillment of the requirements for the  
Degree of Masters of Applied Science at the  
University of Windsor

by

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Windsor, Ontario  
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1975


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## ABSTRACT

This research is directed towards the development of a new method of assembly line balancing which incorporates either constant or variable work element time values. This method is relatively simple and provides a good balance and has been designed for both manual and computer modes of solution. In this research the element time values are considered "Distribution Free". The coefficient of variation of the Station Time Distribution is taken into consideration while assigning the work elements to work stations. The main technique of this method is assigning of priorities to the work elements. During the assigning of elements to the work stations, the priority elements are preferred over the non-priority elements. A computer program has been written, which incorporates all the above features. This method was tested by solving nearly all the well known line balancing problems available in the literature.



## ACKNOWLEDGEMENT

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## CHAPTER 1

### INTRODUCTION:

The assembly line is a production system in which work stations are sequentially arranged. Assembly operations are performed as the product moves from one work station to the next. The entire assembly task is divided into small work elements, which are distributed among the stations, which comprises the assembly line. The most common criteria is to minimize the delay time at each station and to allocate the assembly work equally to all operators.

A production rate set by the management determines the minimum time (cycle time) to be assigned to any work station. In the line balancing problem, according to Gutjahr and Nemhauser (8) one is given a finite set of work elements, each having a processing time and a set of precedence requirements which specify the permissible ordering of elements. The problem is to assign the elements to an ordered sequence of stations such that the precedence relations are satisfied and that some measure of effectiveness is optimized. To be explicit the assembly line balancing problem so defined concerns a finite set of elemental tasks such that:

1. Each elemental task requires a known operation time per unit, independent of when performed.

2. A partial ordering exists between tasks.

Optimal solution of the problem consists of an assignment of elemental tasks to work stations such that:

1. Each elemental task is assigned to one, and only one work station.
2. The sum of the times of all elemental tasks assigned to any one station does not exceed cycle time.
3. The stations thus formed can be ordered such that the partial orderings among elemental tasks are not violated.
4. The number of work stations thus formed is minimum.

Considerable research has been devoted to the problem of achieving better balance. Since the assembly line is generally paced by the slowest station (heaviest work load), it follows that a certain amount of idle time is built into the faster stations (lighter work loads). It is towards the reduction of this idle time that almost all the research has been devoted.

By and large the line balancing methods developed so far can be classified into two categories as follows:

1. The methods assuming deterministic work element times.
2. The methods assuming variable work element times.

For methods using deterministic work element times, the results obtained cannot be implemented in practice because

of variations in performance time. To compensate for these variations in work element times, Tonge (27) suggested that a worker cannot be loaded over 80%. For methods using variable work element times, it is assumed that the element times are normally distributed. There is ample evidence in the literature that the work element time values are not necessarily normally distributed.

In this study a new method is developed which minimizes the variations in station time and which will not impose any restriction on the type of distribution of variable work element times. That is the variable work element times will be considered as distribution free. The method is developed so as to have a manual as well as computerized approach.

## CHAPTER 2

### LITERATURE SURVEY

Numerous methods have been made to develop analytical solutions for the line balancing problems. It has been formulated for solution by integer linear programming and by dynamic programming, but because of computational limitations, they are not practical. Hence heuristic approach is applied to find solution, within reasonable computational effort.

Till 1965 all the heuristic methods developed for assembly line balancing considered deterministic work element times. From 1965 onwards, the methods developed considered variable work element times as well. A review of the better known methods of assembly line balancing, giving their advantages, disadvantages and their limitations, is appended below.

Salveson (25) suggested to enumerate all possible work stations and then use linear programming to select the best combination that satisfies the constraints. The use of linear programming would require the inversion of large matrix and for realistic problems this would be computationally infeasible. Even for small problems it gave unacceptable set of stations, because integer programming was not known at that time. The work of Gomory (7) and others on integer programming has made solutions of this theoretically possible, although as yet it is computationally infeasible.

Bryton (5) in his unpublished thesis, improved Salveson's method by suggesting a "Convergence" procedure. He minimizes the idle time by varying the cycle time for a given number of work stations and interchanging the pair of elements between those largest and smallest stations, whose time difference is nearest to one half of the stations. This is repeated until no more improvement is obtained.

Some of the major deficiencies in this method as a direct approach to a large problem are:

1. It considers only transfer of single elements from each station rather than several elements.
2. In large problems, the simple concept of each transfer causing improvement is insufficient.
3. Production rate is not assumed constant, which generally is fixed and pre-determined by management.

Jackson's (13) method minimizes the number of work stations for a given cycle time. It enumerates all feasible work stations; then for each such work station, it constructs all feasible second work stations; for each first-second combination, construct all feasible third stations and so forth. At some point, say, after  $K^{\text{th}}$  station are constructed, it is likely to be found that one or more of the balances have assigned all the tasks. Jackson's dominance arguments, which concludes that when sequences are generated and if in one sequence 'A' it is possible to add an extra task in the  $K^{\text{th}}$  station, then this sequence

must be at least as good as others because the only difference between them is that in sequence 'A' there is one less task to assign. Therefore, if the other sequences

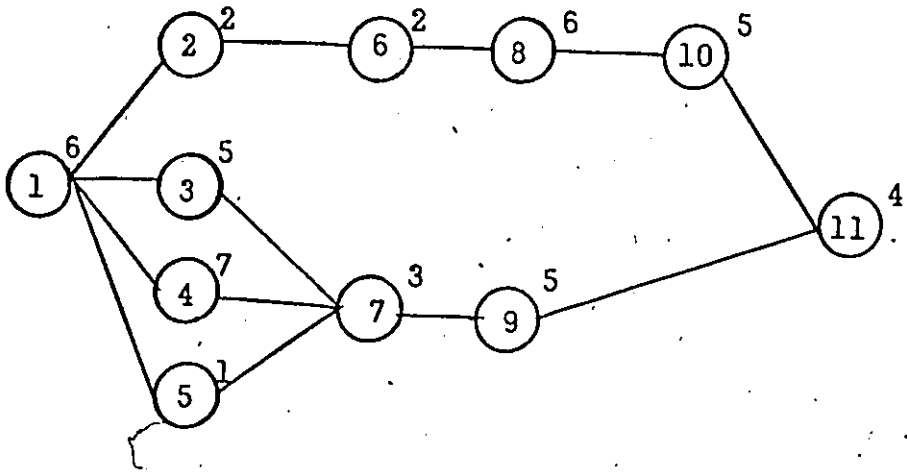


Fig. (1)

yield the minimum number of work stations, sequence 'A' must do so also, and hence the other sequences need not be considered. Thus many balances are eliminated, with the assurance that the method will yield at least one of the balances with the minimum number of work stations. The entire computation (making use of dominance) needed to solve the problem given in Fig. (1) for cycle 10 is shown in Fig. (2).

FIRST	STATIONS					BALANCE NUMBER
	SECOND	THIRD	FOURTH	FIFTH		
	3	8	3 10	4 7	9 11	1
1 2 6		4	3 7	3 10		2
	4	5	3 7	8		3
		8				4
1 2 5	6		Equivalent to Balance 1			5
	4	6	Equivalent to Balance 4			6
						7

Fig. (2)



Jackson has given proof that his method will find the optimal solution. In conclusion, Jackson's method is best suited for hand calculations, because for computer calculation it requires great amount of storage and running time.

Held, Karp and Shareshian (9) offer a method which yields a minimum work station balance for a given cycle time.

Tonge (26) has imbedded Jackson's method in his procedure and developed a heuristic line balancing procedure. His method consists of three phases.

Phase 1) Repeated simplification of the initial problem by grouping adjacent elemental tasks into compound tasks.

Phase 2) Obtaining the solution of the simpler problems thus created by assigning tasks to work stations.

Phase 3) Smoothing the resulting balance by transferring tasks among work stations until the distribution of assigned time is as even as possible.

The essential idea in Arcus (1) technique is the random generation of a feasible sequence. Arcus proceeds by assigning, at each stage, equal probability to all the tasks that could come next. Then, judging on the basis of the yield of good balances, he explores another method for weighting the tasks. This method of generating permits the same sequence to be generated more than once which is redundant.

Kilbridge and Wester's (14) procedure uses the precedence

diagram which is drawn so that assembly progresses from left to right. In column 1 of the diagram, all the work elements are listed which need not follow any work element. Then in the  $K^{\text{th}}$  column are entered those elements which must follow elements already on the diagram. Arrows are then drawn from work elements in columns  $K-1$  to work elements in Column  $K$  which must follow them as shown in Fig. (3). This procedure is repeated replacing  $K-1$  by columns  $K-2$  ----- 1, successively. Diagram thus drawn show that the elements in each vertical column are mutually independant and therefore can be permuted among themselves

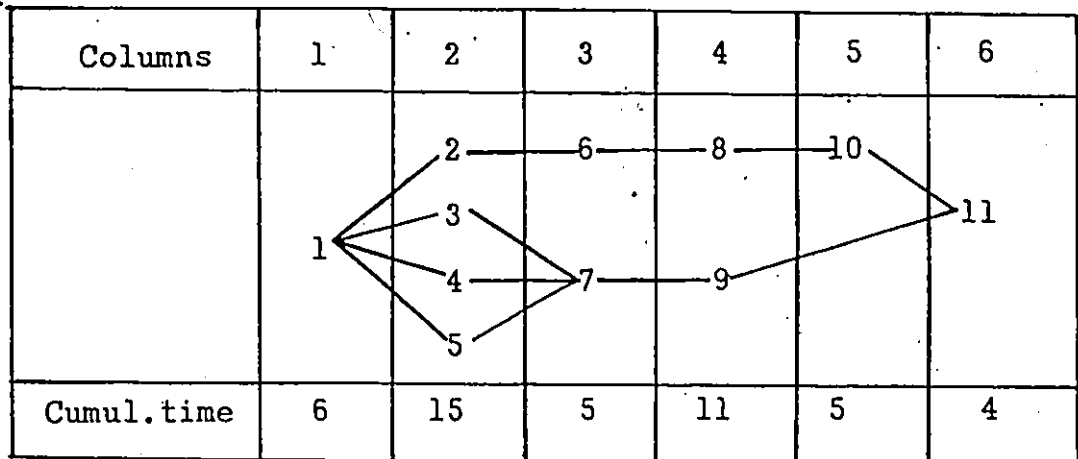


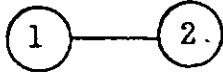
Fig. (3)

in any work sequence without violating the precedence restriction. Furthermore the elements can be moved laterally from their columns to positions on the right without violating the precedence restrictions. These two properties are used to achieve optimal balance.

This method illustrates that this technique is simple and powerful for large cycle times, when one station crosses

several columns. However for low cycle times, where one column may require two or more stations, much adjustment is necessary, with no guarantee of good results.

Hoffman's (11) work begins with the precedence matrix, whose element gives the same information as the arrows on Fig. (1). The precedence matrix is shown in Fig. (3b).

The matrix is constructed as follows. For each arrow on the Fig. (1), 1 is entered in the cell corresponding to the row of the task that precedes and the Column of the task that follows. The 1 in row 1, Column 2 means . This matrix is called X and S is defined arbitrarily as  $S = X + X^2 + X^3 + \dots + X^n$ . The elements of S are the number of paths from the task in the row to the task in the Column. Hoffman in (11) shows how operation on S yield all feasible sequences. He then suggests the "successive maximum elemental time" method in which the first station is selected as that feasible sequence that leaves the least idle time in the station; then select from the remaining tasks the subset that leaves the least idle time in the second station and so on.

Helegeson and Birnie (10) suggest the use of "ranked positional weights" (RPW) in choosing which elements to consider next for grouping in work stations. The positional weight of the element is the sum of its time plus the times of those elements which must follow it. After calculating the positional weights, the tasks are ranked such that task with the largest weight comes first as shown in Fig. (4).

	1	2	3	4	5	6	7	8	9	10	11
1		1	1	1	1						
2						1					
3							1				
4							1				
5							1				
6								1			
7									1		
8										1	
9											1
10											1
11											

Fig. (3b)

TASKS	1	2	4	3	6	8	5	7	9	10	11
POSITIONAL WT.	46	19	19	17	17	15	13	12	9	9	4
RANK	1	2	3	4	5	6	7	8	9	10	11

Fig.(4)

This method does not guarantee the best solution but requires relatively little computing effort. In evaluating this method, it must be realized that examination and improvement by an experienced engineer or technician is an integral part of the method.

Mansoor (17) has improved the ranked positional weight (RPW) method. He suggests that as the RPW method is applied the idle time in each work station should be added. Once the total idle in the stations assigned so far exceeds

$$(\text{Cycle time} \times \text{desired no. of stations}) - \sum t_i$$

where  $\sum t_i$  is the sum of element times in the work stations he backtracks, removing from the current stations some of the tasks that have already been assigned. He suggests first removing the last task assigned and trying RPW from this point. He recommends continuing the backtracking until either a balance with the desired number of stations is found or, after all possibilities have been examined and no such balance is found. If no balance is found then at least one more work station or a higher cycle time is needed. Mansoor claims that his method gives optimal balance, but the amount of work required may be quite large. Everytime a task with low positional weight must be ahead of a large number of tasks with higher weights in an optimal balance, it may take a lot of backtracking to get it assigned ahead of them. Hence it is not clear that Mansoor's method is practical for large lines.

Klein (16) presents a procedure for problems when the feasible sequences are given and the  $t_i$  are integers. For each feasible sequence, a minimum idle time balance is obtained and the best of these balances is elected. Since this method considers all the feasible sequences, it merits

consideration only for small lines.

Gutjahr and Nemhauser (8) developed an algorithm, based on finding a shortest route in a finite directed network for the assembly line balancing problem. Arc lengths are such that it is sufficient to find any path from the origin to destination node containing a minimal number of arcs. This method is an improvement over Klein (16) and is closely related to the dynamic programming approaches of Jackson (13) and Held et al (9), since dynamic programming is one of possible algorithm for finding shortest routes. This program is similar to that of Held et al (9), but was developed from a different view point. The major difference is that Gutjahr and Nemhauser (8) find an arc into a node, no other arcs into that node are considered but held et al (9) check all possible arcs into the node and then apply an unnecessary minimizing procedure to select one.

Apart from the advantages and disadvantages of various methods reviewed, one significant problem associated with them is the assumption of deterministic performance time. Performance times are usually variable, and this has an effect on the lines operation. A start on this problem has been made by Moodie and Young (20).

Moodie and Young developed a heuristic method for assembly line balancing which could be used for either constant or variable work element time values. This method is suitable for both manual and computer mode of solution.

It consists of two phases: Phase I attempts to assign work elements, so as to attain a minimum number of stations for a given amount of work and cycle time. It obtains a preliminary balance by using the "Largest Candidate Rule": Construct work stations sequentially by, at each stage, selecting from those tasks that are feasible and will fit in the current station. In phase II, heuristics are used to shift tasks between stations in an attempt to reduce idle time and allocate the tasks to work stations as uniformly as possible. The heuristics prescribe a series of transfers and trades of single elements between stations.

This method can be utilized with either a constant or variable work element values. Other researchers assume that the elemental tasks are independent and normally distributed random variable with known mean and variance. This is, probably, the first study conducted which has taken variability of elemental task times into consideration.

Mansoor and Tuvia (18), Brennecke (4), Ramsingh and Dowling (24) have also considered element time variations in their assembly line models. Brennecke (4) presents the result of a study which employed a two parameter assembly line balancing model. Most of the approaches to line balancing use single parameters - mean or expected values - for elemental time values.

Reviews of assembly line balancing methods have been made by Ingall (12) and Cauley (1968). Ingall (12) in his article presents the standard formulation of the assembly

line balancing problem and then reviews the methods available for actually balancing lines. The desirability of modifying the usual assumption of known tasks and constraints, constant performance times, a single product, and worker of equal ability are discussed. Mastor (1970) has made a comparative study of various methods.

Mansoor (17) in his algorithm for variable operator performance levels suggests that four steps:

- (1) Determine the minimum number of work stations,  $N$ , and corresponding minimum operating cycle time, that will satisfy the production requirement of  $P$  products per hour.

- (2) Selecting a suitable group of  $N$  operators from the labour pool containing  $M$  operators.

- (3) Calculating the amount of work each operator can perform during the operating cycle time.

- (4) Assigning work units to operators to meet both the precedence restriction and the appropriate operator work capacity, so that the resulting operating cycle time is a minimum.

This method is suitable only if the number of work elements is small, the computer can cope with the backtracking process satisfactorily, even to the extent of raising cycle time. With the sizeable assembly task, the backtracking process can prove to be a formidable task even for a faster computer.

From the review of the existing assembly line balancing methods we see that the element time values are considered



normally distributed. The literature on human performance tells us that element time values are not necessarily normally distributed. Also none of the existing methods seem to consider minimizing the variations in the station times, as the elements are assigned to the work stations.

In this study the element time values are considered to be "distribution free" (23), and a new heuristic method for assembly line balancing is developed. This method cites rules for assigning the elements to work stations, which result in the following:

- (1) Minimizes the variations in station time.
- (2) Minimizes the probability of station time exceeding the cycle time.
- (3) Leads to a minimum station balance.

CHAPTER 3ANALYSIS OF VARIABLE WORK ELEMENT TIME3.1 MINIMIZATION OF STATION TIME VARIATIONS:

The coefficient of variation of distribution is defined as the ratio of the mean to standard deviation. Brady and Drury (3) showed that "if a new element is added to a group of elements, the coefficient of variation (CV) of the new group will be less than that of the old group provided that the  $CV^2$  of the new element is less than  $(1+2r)$  times  $CV^2$  of the old group, where 'r' is the ratio of the mean of the old group to the mean of the new element".

This concept if incorporated in developing an assembly line method is likely to reduce the variations of station time. The proposed method will use this concept in the selection of elements to be allocated to the work station. Before allocating the element to the work station it will be checked to see whether the addition of the element increases the coefficient of variation of the work station. If it does, the element is not selected and the next available element is tried. If none of the variable elements fit in the work station, this condition is by passed for that particular case. Thus by utilizing the above concept, it is insured to the maximum possible extent that the addition of an element to the work station will decrease the variation in the station time exceeding

the cycle time and is also likely to lead to an increase in the overall performance of the system.

### 3. II DISTRIBUTION FREE ANALYSIS:

The element time values are usually assumed normally distributed and therefore the station time values are also considered normally distributed. In this study the element time values are not assumed normally distributed and analysis is carried out for any probability distribution. The two parameters used in this line balancing method are the mean  $\mu_{EL}$  and standard deviation  $\sigma_{EL}$  of the elements time distribution and the corresponding parameters  $\mu_{ST}$  and  $\sigma_{ST}$  of the station time distribution.

Tchebycheff's inequality states that for any probability distribution whatsoever, there is a simple relationship expressing the probability that the given variable will differ from its mean by some multiple of its standard deviation. Expressed specifically in terms of station time distribution the inequality states that

$$P(ST \geq CT) \leq \frac{1}{K^2 + 1}$$

where ST = Station time  
 CT = Cycle time  
 K = Number of Standard deviations

$$= P(ST \geq \mu_{ST} + K\sigma_{ST}) \quad \frac{1}{K^2 + 1}$$

$$= P(|ST - \mu_{ST}| \geq K \sigma_{ST}) \quad \frac{1}{K^2 + 1}$$

where  $|ST - \mu_{ST}|$  designates the absolute value of the

difference. In other words, positive and negative values of  $(ST - \mu_{ST})$  are both included in the statement. But since in this method, we need to find the probability where  $(ST - \mu_{ST})$  is positive, the inequality is written as,

$$P(ST \geq CT) \leq \frac{1}{2} \left( \frac{1}{K^2 + 1} \right)$$

Translated, this inequality states that the probability the station will exceed the cycle time is always less than or equal to  $\frac{1}{2} \left( \frac{1}{K^2 + 1} \right)$ .

Utilizing this concept, if the mean and standard deviation of the distribution of the station time of all work stations are known, the probability of the station time exceeding the cycle time for each work station can be determined. That is for a given cycle time, it is possible to determine the probability that each of the operators will be able to perform the assigned tasks in the time allotted.

This is accomplished by computing the number of standard deviations between the cycle time and the mean station time and obtaining the corresponding probability figure from Tchebycheff's inequality. The above procedure may be summarized as follows:

- (1) For each work station, compute the value of K from the expression

$$K = \frac{CT - \mu_{ST}}{\sigma_{ST}}$$

- (2) Compute the probability that station time exceeds cycle time, from the inequality.

$$P(ST \geq CT) \leq \frac{1}{2} \left( \frac{1}{K^2 + 1} \right)$$

CHAPTER 4A HEURISTIC METHOD FOR ASSEMBLY LINE BALANCING4.1 HEURISTICS FOR ASSEMBLY LINE BALANCING METHOD:

The technique employed in this method is as follows:

- (1) Obtain the critical path of the given problem for each end product that requires at least two tasks before its completion.
- (2) Assign priority to each of the elements that fall in the critical path. These are called critical elements.
- (3) Assign priorities to those elements which have immediate followers whose cumulative sum is greater than or equal to cycle time.
- (4) Assign priorities to those elements which are direct predecessors and immediate followers of the critical elements and whose cumulative sum is greater than cycle time.
- (5) Assign priority to those elements which are direct predecessors of critical element and have the lowest elemental time value.

The priorities assigned to the elements in the above steps are based on the following reasons. If the cumulative sum of the immediate followers of an element is greater than cycle time, then priority is given to that element, because if all of them are selected one after the other,

they can form one work station and this in fact is helping in making a good balance. The same reasoning goes for giving priorities to all those elements which are direct predecessors and immediate followers of the critical elements. Priority is also assigned to those prerequisites of critical element that has the lowest elemental time value, because if priority is not assigned, these elements will never be selected except by chance before all the other elements have been tested. This is because of the nature of the proposed method and is causing a restriction in the selection of tasks along the critical path. Hence to overcome this restriction a priority is given.

#### 4.2 THE ASSEMBLY LINE BALANCING METHOD:

The proposed method assigns elements to consecutive work stations on the assembly line by the "Largest Candidate Priority Rule". This entails, considering the priority elements first (in declining order of elemental time values) and then the non-priority elements (in declining order of elemental time values) for assigning them to the work station.

As an example consider the 21 element problem shown in Fig. (5) taken from Tonge (26). In Fig (6) the matrix on the left indicates the immediate predecessor of each element and on the right, the matrix indicates the immediate followers. The assignment of elements to obtain a minimum station balance proceeds in the method as follows:

- (1) From the P Matrix, the elements having all zeros in their rows are noted. From these elements, the priority and the non-priority elements are ranked in descending order of their elemental time values. First the priority elements are considered (in declining order of their elemental time values) for assigning them to work station and then the non-priority elements (in declining order of their elemental time values).
- (2) The elements in the row F Matrix of the assigned element are noted. In the corresponding rows in P Matrix indicated by the elements just noted, the assigned element number is replaced by a zero.
- (3) The elements are assigned according to step 1 and step 2 adhering to the following rules:
  - (a) The coefficient of variation should not increase when an element is added to the work station.
  - (b) The probability of the station time exceeding the cycle time does not exceed the assigned value.

When the P Matrix contains all zeros the problem is solved. However, if this is not possible for certain operating conditions increasing the cycle time will and in obtaining the minimum station balance.

Applying these rules to the 21 element problem for cycle time of 21, the balance obtained is given in Fig. (7). The smoothness index of the balance is zero. This problem is also solved manually as follows:



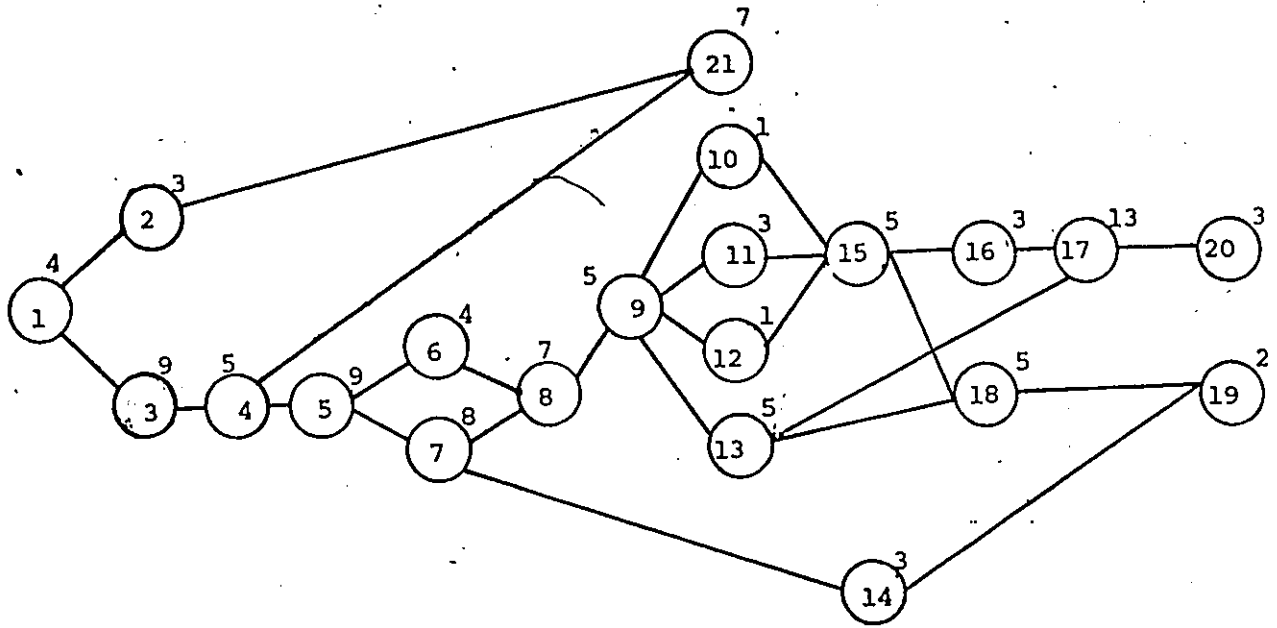


Figure (5). Precedence Diagram for Twenty-One-Element Problem Taken from Tonge (21).

1	0	0	0
2	1	0	0
3	1	0	0
4	3	0	0
5	4	0	0
6	5	0	0
7	5	0	0
8	6	7	0
9	8	0	0
10	9	0	0
11	9	0	0
12	9	0	0
13	9	0	0
14	7	0	0
15	10	11	12
16	15	0	0
17	16	13	0
18	13	15	0
19	14	18	0
20	17	0	0
21	2	4	0

P-Matrix

1	2	3	0	0
2	21	0	0	0
3	4	0	0	0
4	21	5	0	0
5	6	7	0	0
6	8	0	0	0
7	8	14	0	0
8	9	0	0	0
9	10	11	12	13
10	15	0	0	0
11	15	0	0	0
12	15	0	0	0
13	17	18	0	0
14	19	0	0	0
15	16	18	0	0
16	17	0	0	0
17	20	0	0	0
18	19	0	0	0
19	0	0	0	0
20	0	0	0	0
21	0	0	0	0

F-Matrix

Figure (6). Dual Precedence Matrices for Twenty-One-Element Problem.

WORK STATION	ELEMENT	ELEMENTAL TIME	STATION TIME
1	1	4	21
	3	9	
	4	5	
	2	3	
2	5	9	21
	7	8	
	6	4	
3	8	7	21
	21	7	
	9	5	
	10	1	
	12	1	
4	11	3	21
	15	5	
	16	5	
	13	3	
	18	5	
5	17	13	21
	20	3	
	14	3	
	19	2	

Smoothness index = 0

Fig. (7): Balance for twenty-one element problem for cycle time 21.

The critical path for each end product is obtained and the elements that fall on the critical path are blackened in Fig. (5). Priority is given to each of these elements and they are called critical elements.

To "assign priority to those elements which have immediate followers whose cumulative sum is greater than cycle time", the mean time of elements in each row of F Matrix are added. If the cumulative sum of any row is greater than cycle time, priority is given to the element representing that row.

It is seen from the Table (1) that no row has cumulative sum of mean time of the elements is greater than cycle time. Hence priority is not given to any element due to the said rule.

To "assign priority to those elements which are direct predecessors and immediate followers of the critical elements and whose cumulative sum is greater than cycle time", the mean time of all element in the rows of P Matrix, represented by critical elements are added. If the cumulative sum of any row is greater than cycle time, it is checked if the elements in that row are immediate followers of another critical element. If they are, then priority is given to each of the elements in that row.

It is seen from Table (2), that the cumulative sum of mean time of elements in the rows of P Matrix, represented by critical elements is not greater than cycle time. Hence priority is not assigned to any element due to the said rule.

ELEMENTS	CUMULATIVE SUM OF TIME OF ALL ELEMENTS IN ROW OF F-MATRIX	ELEMENTS	CUMULATIVE SUM OF MEAN TIME OF ALL ELEMENTS IN ROW OF F-MATRIX
1	12	12	5
2	7	13	18
3	5	14	2
4	16	15	8
5	12	16	13
6	7	17	3
7	10	18	2
8	5	19	0
9	10	20	0
10	5	21	0
11	5		

Table (1)

Critical Elements	Cumulative sum of Mean Time of Elements in the Rows of P-Matrix Represented by the Critical Elements
1	0
3	4
4	9
5	5
7	9
8	12
9	7
11	5
15	5
16	5
17	8
18	10
19	8
20	13
21	8

Table (2)

To "assign priority to those direct predecessors of critical elements, that have the lowest mean time", the mean time of the elements in the rows of P-Matrix represented by critical elements are noted. If any element in these rows, has lowest mean time value, priority is given to it. The lowest mean time for this problem is 1.

Noting the mean time of elements in the rows of P-Matrix, represented by critical elements, it is seen that elements 10 and 12 have the lowest mean time. Hence priority is assigned to these elements.

Now the elements are to be allocated to the work stations. Any row of P-Matrix having all zeros in its row signifies that the element representing that row has no prerequisites or in other words is free to be allocated to the work station. Every time an element is allocated to the work station, the element number of its immediate followers are noted. Then in the rows of P-Matrix indicated by these numbers; the assigned element number is replaced by zero. Thus implying that, to find the elements available for allocation, look up the rows of P-Matrix that have all zeros in their rows.

To begin with only element 1 has all zeros in its row in the P-Matrix. Hence it is available for allocation.

ELEMENT AVAILABLE	PRIORITY ELEMENT	NON PRIORITY ELEMENT
1	1	--

Table (3)

Element 1, being the only one available is assigned to the first work station. Station time is the cumulative sum of all elements assigned to the work station. Thus the station time of the first work station is 4.

Element numbers in the rows of F-Matrix, corresponding to element 1, are 2 and 3. Replace element number 1, with zeros, in the rows of P Matrix corresponding to elements 2 and 3. The modified P Matrix is shown in fig. (8)

1	0	0	0
2	0	0	0
3	0	0	0
4	3	0	0
5	4	0	0
6	5	0	0

Fig. (8)

Now the rows of elements 1, 2 and 3 have all zeros in their rows in P-Matrix. Element 1, being already assigned, elements 2 and 3 are available for allocation. These available elements are separated into priority and non-priority elements and are ranked in descending order of their mean time values as shown in table (4).

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
2	3	2
3		

Table (4)

Since preference is given to priority elements, element 3 is assigned to first station, thereby increasing the station time to 13.

Element numbers in the rows of F-Matrix corresponding to element 3 is 4. Replace element number 3 with a zero in

the row of P-Matrix corresponding to element 4 as shown in Fig (9).

1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	4	0	0
6	5	0	0
7	5	0	0

Fig. (9)

The rows in P-Matrix of element 1,2,3,and 4 have all zeros. Elements 1 and 3 being already assigned, elements 2 and 4 are available for allocation. They are separated into priority and non-priority elements and are ranked in descending order of their mean time values as shown in Table (5).

ELEMENTS	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
2 4	4	2

Table (5)

Element 4 being a priority element is assigned to the first station, increasing the station time to 18.

As before replacing element numbers in P-Matrix by zero and then rank the available priority and non-priority elements in descending order of mean time values as shown in table (6).



ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
2 5	5	2

Table (6)

As usual the priority element (Element 5) is assigned to first work station, increasing the station time to 27. Since station time exceeds cycle time, element 5 is rejected. No other priority element being available, element 2 is assigned to the first station, increasing the station time to 21. As the station time equals cycle time, no other element can be assigned to first station. Now elements are assigned to second work station .

Modifying the P-Matrix as before, the succeeding results are summarized in the tables below.

ELEMENTS AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
5 21	5 21	--

Element Allocated to 2nd work station: Element 5

Station Time: 9

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
6 7 21.	7 21	6

Element allocated to 2nd work station : 7

Station Time : 17

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
6	21	6
21		14
14		

Element allocated to 2nd work station: 21

STATION TIME:  $24 \geq 21$  Element rejected

Element allocated to 2nd work station: 6

Station Time: 21

Station time = Cycle time : charge work station

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
8, 21	21	14
14	8	

Element allocated to 3rd work station: 21

Station time: 7

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
8	8	14
14		

Element allocated to 3rd work station: 8

Station time: 14

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
9	9	14
14		

Element allocated to 3rd station: 9

Station time: 19

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
10	11	13
11	10	14
12	12	
13		
14		

Element allocated to 3rd station : 11

Station time :  $22 \geq 21$  Element rejected

Element allocated to 3rd station : 10

STATION TIME : 20

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
11	11	13
12	12	14
13		
14		

Element allocated to 3rd Station : 11

Station time :  $23 \geq 21$  element rejected

Element allocated to 3rd Station : 12

STATION TIME : 21

Station time = Cycle time : Charge work station

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
11	11	13
13		14
14		

Element allocated to 4th work station : 11

Station time : 3

StationTime = Cycle Time : charge work station

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
13	15	13
14		14
15		

Element allocated to 4th Station: 15

Station time: 8

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
16	16	13
14		14
15		

Element allocated to 4th work station : 16

STATION TIME : 11

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
	—	13
14		14
15		

Element allocated to 4th work station: 13

Station time: 16

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
17	17	14
18	18	
14		

Element allocated to 4th work station: 17

Station time: 29  $\geq$  21 element rejected

Element allocated to 4th work station: 18

Station time: 21

Station time = Cycle time : Charge work station

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
17 14	17	14

Element allocated to 5th work station : 17

Station time : 13

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
20 14		

Element allocated to 5th work station: 20

Station time: 16

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
14		14

Element allocated to 5th work station : 14

Station time : 19

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
19	19	

Element allocated to 5th work station : 19

Station time : 21

ELEMENT AVAILABLE	PRIORITY ELEMENTS	NON PRIORITY ELEMENTS
ALL ELEMENTS ALLOCATED		

The summary of the results is given in Table (7).

WORK STATION	ELEMENTS	STATION TIME	IDLE TIME
1	1, 3, 4, 2	21	0
2	5, 7, 6	21	0
3	8, 21, 9, 10, 12	21	0
4	11, 15, 16, 13, 18	21	0
5	17, 20, 14, 19	21	0

Smoothness Index = 0

Table (7)

#### 4.3 ADDING A ZONING CONSTRAINT

The product to be assembled is characterized by time requirements for each final assembly task and by restrictions on the way these tasks may be grouped into work stations. These grouping restrictions are either ordering constraints, stating which tasks must be completed before other tasks can be started, or zoning constraints, specifying which tasks may not be grouped together because of production facility layout. The injunction against assigning tasks to be done from both the front and the back of the product to one worker, thus requiring him to cross the conveyor line, is a zoning constraint.

Many zoning constraints are added to the assembly line balancing problem to keep tasks at separate stations, not because it is impossible to do them together, but because they will take longer if done together. These zoning constraints are incorporated in this new method as follows:

One of the advantages sought from the heuristic approach to complex decision problems is the ability to redefine the problem, adding or deleting restrictions on a solution, with ease. As a specific example, the zoning restriction as given by Tonge (22) will be considered here. The zoning restriction for the elements is given in Table (8).

To incorporate this new restriction, the program can be slightly modified to produce only groupings within a

	Element Zones
Zone	Element Tasks
1	1, 2, 3, 4, 5, 7, 8, 21
3	3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18
4	13, 14, 15, 16, 17, 18, 19, 20

Table (8)

specified zone or zones, by rejecting the elements not belonging to the same zone. This is accomplished as follows:

- (1) Sort from the available elements, the element that can be placed only in that zone, and the elements that can be placed in that zone or some other zone.
- (2) Sort these groups of elements into priority and non-priority elements and arrange them in descending order of their elemental time value.
- (3) Preference (in descending order of the mean time values) is given to the elements as follows:
  - (a) Priority elements strictly to be assigned to that work station due to zoning constraint.
  - (b) Non priority elements strictly to be assigned to that work station due to zoning constraint.
  - (c) Priority elements that can be assigned to that work station.
  - (d) Non priority elements that can be assigned to that work station.



The above modifications can be made with relative ease without affecting other parts of the over-all program.

These modifications have been hand simulated for several problems. The twenty-one element problem has been hand simulated for cycle time 20.

## CHAPTER 5

### OPERATING RESULTS WITH THE LINE BALANCING PROGRAM

#### 5.1 Mechanization of the assembly line balancing procedure:

The assembly line balancing procedure described here is programmed in Fortran language to be run on the University's IBM 360/165 computer. A detailed description of the procedure is depicted in flow chart given in Appendix B. A completely documented computer program is also given in Appendix C.

#### 5.2 Operating Results:

Six sample problems were used in developing and testing this heuristic procedure: The 9 element problem taken from Moodie & Young (20), the 11-Element, 21-Element and 70-Element problem taken from Tonge (27), 35-Element problem taken from Bedworth (2), and 45-Element problem from Kilbridge and Wester (14). All the six problems are depicted in Appendix B. Although these few cases do not completely test the methods general validity, we can observe measure of performance.

The twenty-one element problem was solved with zoning constraint as given in Tonge (27). For the cycle time of 20, the results obtained are similar to those obtained by Tonge (27).

#### 5.3 Other Comparisons:

All the sample problems were taken from the assembly line balancing literature. The results obtained by the new

method described are compared with the results obtained by others. All the solutions obtained are either similar or better to the ones obtained by other researchers. For the seventy element problem the results of phase 3 of Tonge's method were not available and hence they were compared with phase 2 solution.

The summary of all the comparisons with other methods is given in Table (9). The problems solved and the results shown in Table (9) are based on the data which was assumed to be deterministic. To test the methodology for cases when mean and standard deviation of elemental time values are known, a twenty-one element problem shown in Fig. (5) was solved. The mean and variance of elemental time values are given in Table (8a). The computer program based on the flow chart in Appendix B was used to solve the problem. The results obtained are given in Table (9a).

NUMBER OF ELEMENTS	NUMBER OF WORK STATIONS	CYCLE TIME	METHOD	SI	$\eta$
9	3	14	Moodie & Young(20)	2	94.9
			PROPOSED	2	94.9
11	3	62	Mansoor(17)	1	99.5
			PROPOSED	1	99.5
21	5	25	Moodie & Young(20)	0	100
			PROPOSED	0	100
35	6	83	Helgison & Birnie(10)	4	98.6
			PROPOSED	4	98.6
45	3	184	Kilbridge & Wester(14)	0	100
			PROPOSED	0	100
70	22	176	Tonge(27) End of Phase sol. incom- plete sol.	80	92.6
			PROPOSED	51	95

Table (9)

Comparing proposed method with other methods.  
Results are compared for constant work element  
time and non-zoned conditions.

S.I. - Smoothness index \*

$\eta$  - Efficiency\*\*

- \* Smoothness Index consists of the square root of the sum of the squares of the time deviations for each of the stations in the balance from the maximum station time.
- \*\* Efficiency is defined as the ratio of total elemental time to the number of work stations into the maximum station time.

CYCLE TIME = 23

STATION NO.	ELEMENTS	STATION TIME	IDLE TIME
1	1 3 4 2 0 0 0 0 0 0	21	2
2	5 7 6 0 0 0 0 0 0 0	21	2
3	8 21 9 11 0 0 0 0 0 0	22	1
4	10 12 14 15 16 13 18 0 0 0	23	0
5	17 20 19 0 0 0 0 0 0 0	18	2

SMOOTHNESS INDEX = 2.24      OUTPUT EFFICIENCY = 91.3%

Table (9a)

POOR COPY

Element No.	Mean	Variance
1	4	1
2	3	1
3	9	2
4	5	2
5	9	3
6	4	1
7	8	3
8	7	3
9	5	2
10	1	0.5
11	3	1
12	1	0.5
13	5	1
14	3	1
15	5	2
16	3	1
17	13	5
18	5	1
19	2	1
20	3	1
21	7	2

Table (10)

CHAPTER 6C O N C L U S I O N S

The new method proposed in this paper is suitable for obtaining balances, either manually or by using a computer. A flow chart summarizing the new method is given in Appendix B and the computer program is given in Appendix C.

The new method has the following inherent characteristics:

- (1) It gives only one balance (and often the optimal) of many possible balances.
- (2) This method operates in two steps:
  - (i) Obtaining the critical path and assigning the priorities.
  - (ii) Assigning the elements to work stations.If it becomes apparent that a minimum station balance is unattainable with the given cycle time, the cycle time may be successively incremented by one time unit up to a specified time limit, as an aid in obtaining a minimum station balance.
- (3) The work element variance is incorporated in this method and hence, this method gives a better balance because the variance of the work element is a determining factor in assigning elements to work station.

This method has been tested for most of the assembly line balancing problems given in the literature.

The results obtained by the new method are similar or better than the results documented in literature. If in addition to the mean of element time value, variances are also known, this method is likely to yield more realistic balances for assembly lines than one gets by using only the means of element time values.



LIST OF VARIABLES

NCPM	Number of Columns of 'P'-Matrix.
NOE	Number of Elements
NCFM	Number of Columns of 'F'-Matrix
MINTIM	Minimum time (Elemental time)
CT	Cycle Time
NRPM	Number of rows of 'P'-Matrix
NRFM	Number of rows of 'F'-Matrix
REQPRO	Required probability
UNDEF	Indicator for probabilistic or deterministic study = 1.0 for deterministic study = 0.0 for probabilistic study
RPW	Earliest start
ISP(I)	Priority indicator of element I = 0 if non-priority element = 500 if priority element
P(I,J)	Element in 'P' Matrix
F(I,J)	Element in 'F' Matrix
GUM	Cumulative time of post requisites
TT	Station Time
IST(IJK)	Station time of station number IJK
L	Element under consideration for allocation
KIM	Elements position in the work station
TIME(L)	Mean time of element L
IJK	Station number
SIGMA(IJK)	Standard deviation of station IJK
SIG(L)	Standard deviation of element L
M	Number of elements in work station
WS(M)	Element number M in work station

ELE(IJK,K1M) Element allocated is stored in this variable

IDL(IJK) Idle time in station IJK

IJKK Total number of work stations

PW Expected earliest start

APPENDIX A

SAMPLE PROBLEMS

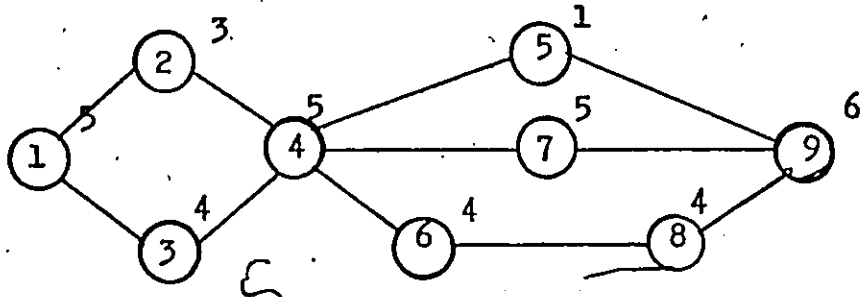


Fig. (10) Precedence Diagram for nine element problem taken from Moodie and Young (20)

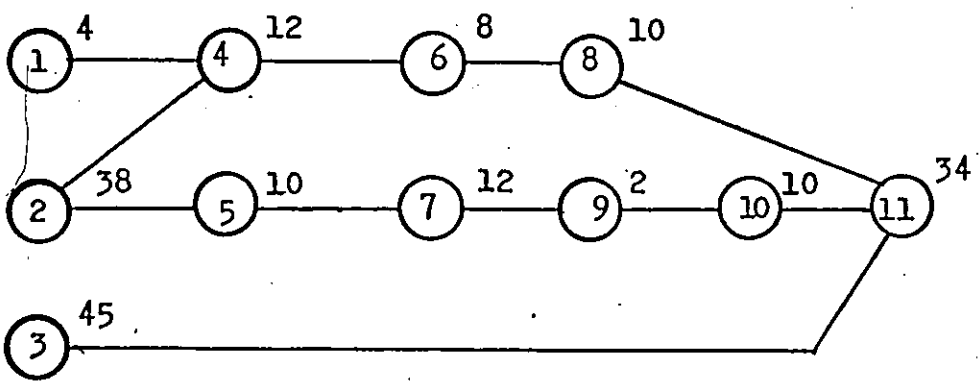


Fig. (11) Precedence Diagram for 11 Element problem taken from Tonge (26)

THE BALANCE FOR THE NINE ELEMENT PROBLEM TAKEN FROM MOODIE AND YOUNG (20)

CYCLE TIME = 13

STATION NO.	ELEMENTS	STATION TIME	IDLE TIME
1	1 3 2 0 0 0 0 0 0 0	12	1
2	4 6 8 0 0 0 0 0 0 0	13	0
3	5 7 9 0 0 0 0 0 0 0	12	1

SMOOTHNESS INDEX = 11.42

OUTPUT EFFICIENCY = 94.87

Table (10)

THE BALANCE FOR THE ELEVEN ELEMENT PROBLEM TAKEN FROM MANSOOR (17)

CYCLE TIME = 48

STATION NO.	ELEMENTS											STATION TIME	IDLE TIME
1	2	5	0	0	0	0	0	0	0	0	0	48	0
2	3	0	0	0	0	0	0	0	0	0	0	45	3
3	7	1	4	6	8	9	0	0	0	0	0	48	0
4	10	11	0	0	0	0	0	0	0	0	0	44	4

SMOOTHNESS INDEX = 5.00      OUTPUT EFFICIENCY = 96.35

Table (11)

THE BALANCE FOR THE ELEVEN ELEMENT PROBLEM TAKEN FROM MANSOOR (17)

CYCLE TIME = 62

STATION NO.	ELEMENTS	STATION TIME	IDLE TIME
1	2 5 7 9 0 0 0 0 0 0 0	62	0
2	3 1 4 0 0 0 0 0 0 0 0	61	1
3	6 8 10 11 0 0 0 0 0 0 0	62	0

SMOOTHNESS INDEX = 1.00

OUTPUT EFFICIENCY = 99.46

Table (11) Continued

THE BALANCE FOR THE ELEVEN ELEMENT PROBLEM TAKEN FROM MANSOOR (17)

CYCLE TIME = 97

STATION NO.	ELEMENTS	STATION TIME	IDLE TIME
1	2 3 5 1 0 0 0 0 0 0	97	0
2	4 6 8 7 9 10 11 0 0 0	88	9

SMOOTHNESS INDEX = 3.00

OUTPUT EFFICIENCY = 95.36

Table (11)



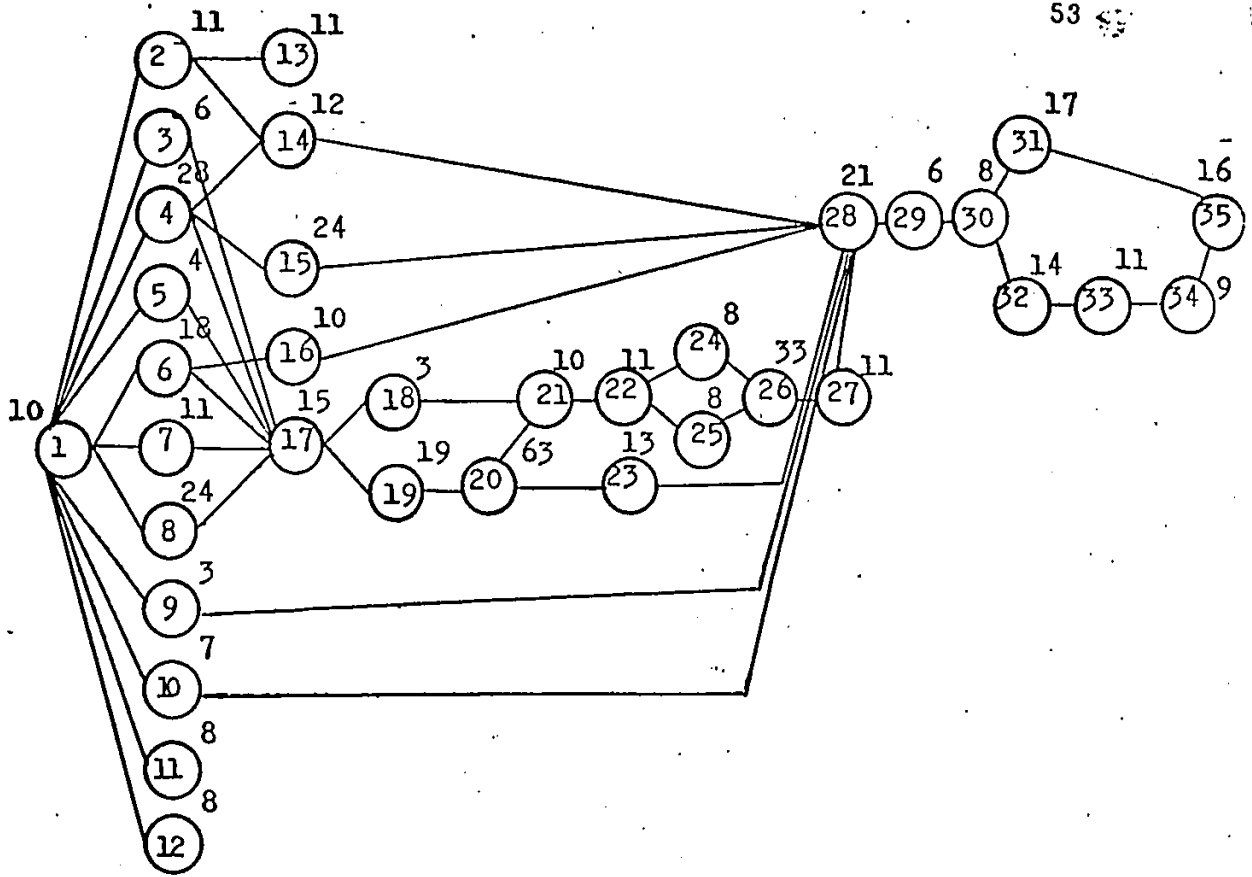


Fig. (12) Precedence diagram for Thirty Five Element problem taken from Helgeson and Birnie

THE BALANCE FOR THE THIRTY FIVE ELEMENT PROBLEM TAKEN FROM TONGE (26)

CYCLE TIME = 83

STATION NO.	ELEMENTS														STATION TIME	IDLE TIME	
1	1	4	8	6	9	0	0	0	0	0	0	0	0	0	0	83	0
2	2	13	7	3	5	17	19	18	0	0	0	0	0	0	0	80	3
3	20	21	16	0	0	0	0	0	0	0	0	0	0	0	0	83	0
4	22	24	25	26	27	14	0	0	0	0	0	0	0	0	0	83	0
5	15	23	11	12	10	28	0	0	0	0	0	0	0	0	0	81	2
6	29	30	32	33	34	31	35	0	0	0	0	0	0	0	0	81	2

SMOOTHNESS INDEX = 4.13      OUTPUT EFFICIENCY = 98.59

Table (12)

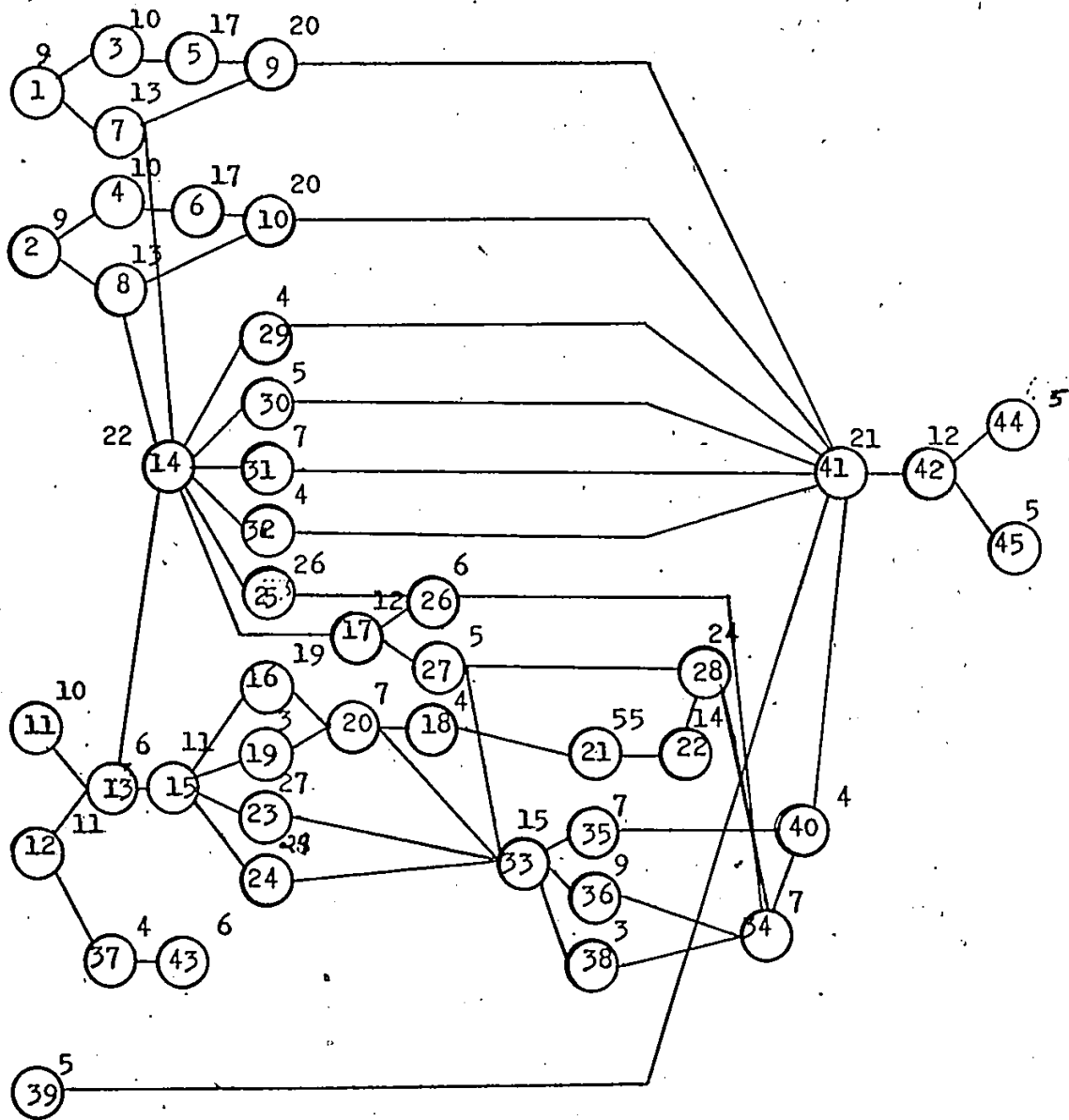


Fig. (13) Precedence Diagram For Forty Five element problem taken from Kilbridge and Wester (14)

CYCLE TIME = 184

STATION NO.	ELEMENTS	STATION TIME	IDLE TIME
1	12 37 42 11 13 15 16 18 19 20	184	0
2	23 1 7 3 5 9 2 8 14 25	184	0
3	4 6 10 31 27 28 33 36 35 30	184	0

SMOOTHNESS INDEX = 0.00      OUTPUT EFFICIENCY = 100.00

Table (13)

THE BALANCE FOR THE SEVENTY ELEMENT PROBLEM TAKEN FROM TONGE (21)

CYCLE TIME = 176

STATION NO.	ELEMENTS						STATION TIME	IDLE TIME
	1	2	3	4	5	6		
1	15	1	70	69	5	0	167	9
2	16	2	41	0	0	0	172	4
3	18	0	0	0	0	0	176	0
4	3	68	17	0	0	0	176	0
5	19	20	21	4	0	0	175	1
6	6	9	24	0	0	0	168	8
7	10	11	7	0	0	0	176	0
8	8	12	57	0	0	0	171	5
9	14	22	0	0	0	0	175	1
10	13	30	58	54	0	0	168	8
11	23	33	0	0	0	0	175	1

Table (14) Continued

12	25	0	0	0	0	0	152	24
13	28	27	20	0	0	0	161	15
14	29	34	31	32	0	0	153	23
15	35	56	53	62	0	0	174	2
16	63	51	0	0	0	0	167	9
17	44	60	45	64	66	0	159	17
18	46	47	0	0	0	0	172	4
19	67	48	49	36	37	38	176	0
20	52	54	61	39	0	0	175	1
21	55	65	40	42	43	50	155	21

SMOOTHNESS INDEX = 53.22

OUTPUT EFFICIENCY = 95.36

Table (14)

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APPENDIX B

FLOW CHART

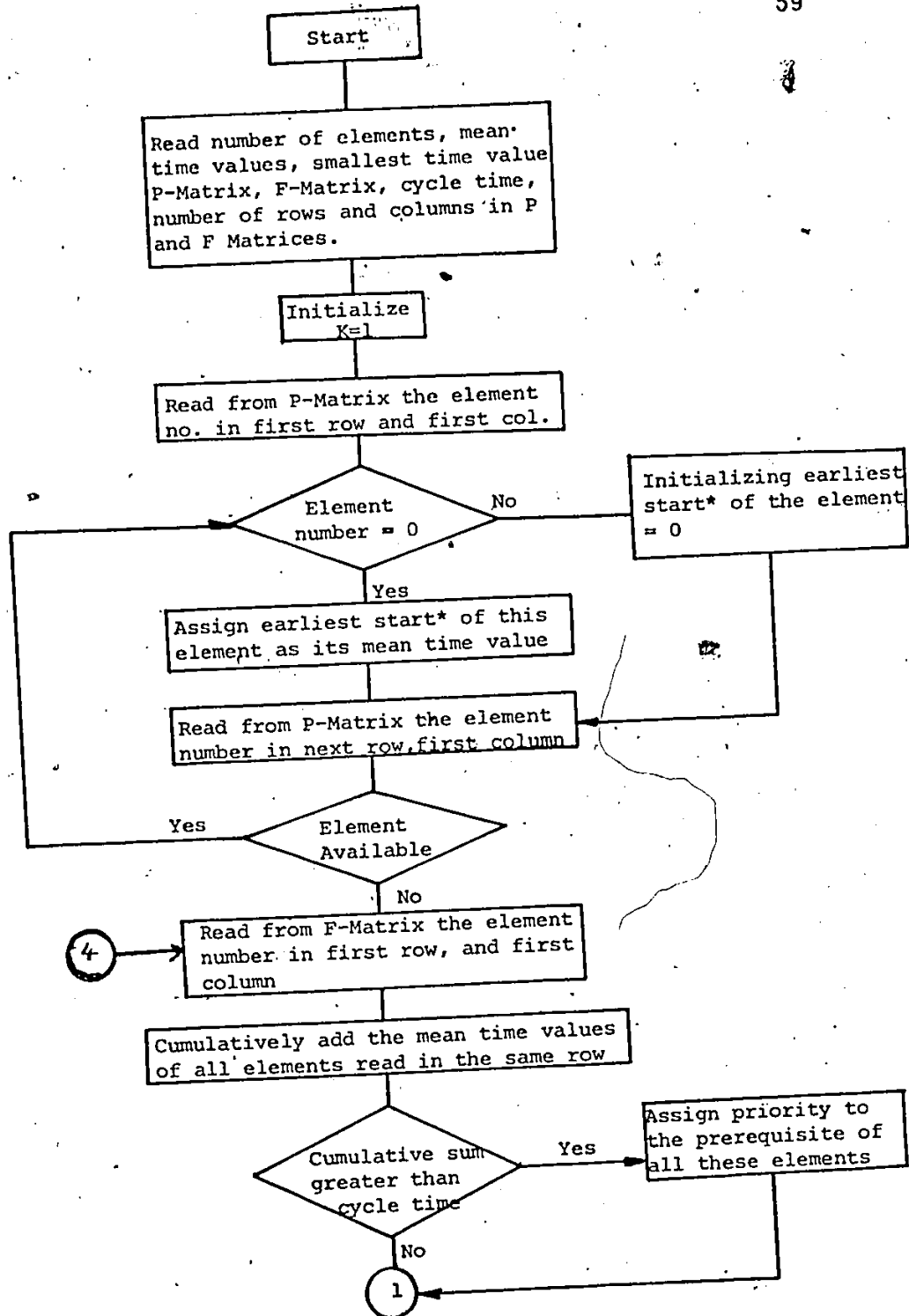


Figure (3). Flow Chart for the New Heuristic Method

\*Earliest start is the earliest time an element can begin when all preceding elements are completed, including itself. It is the highest of all the expected earliest starts of the element.



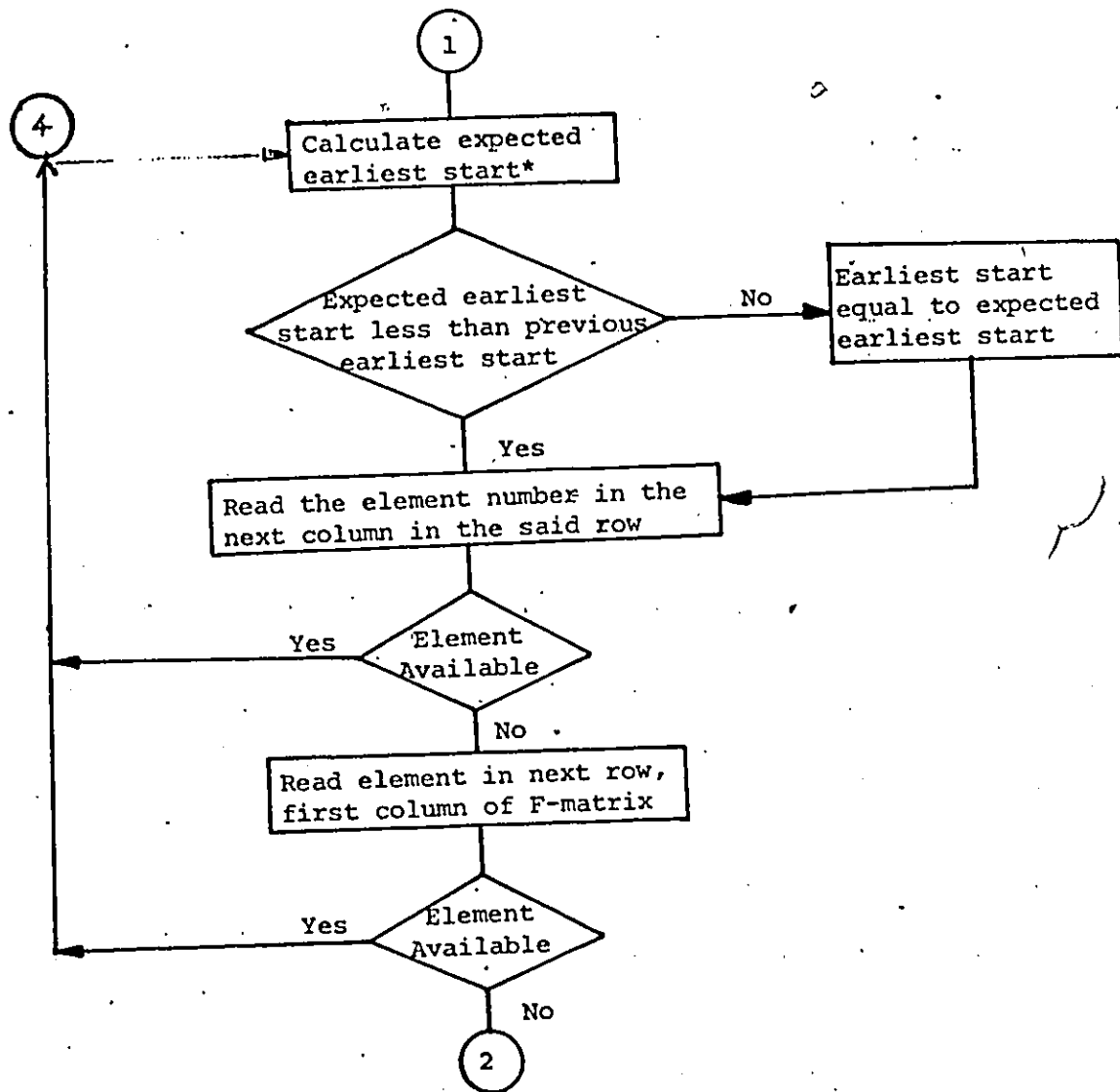


Figure (3) (Cont.)

\*Expected earliest start of the element is found by adding its mean time to the earliest start of its prerequisite. If the element has more than one prerequisite, expected earliest start is calculated considering each prerequisite and storing the highest value of expected earliest start as its earliest start.

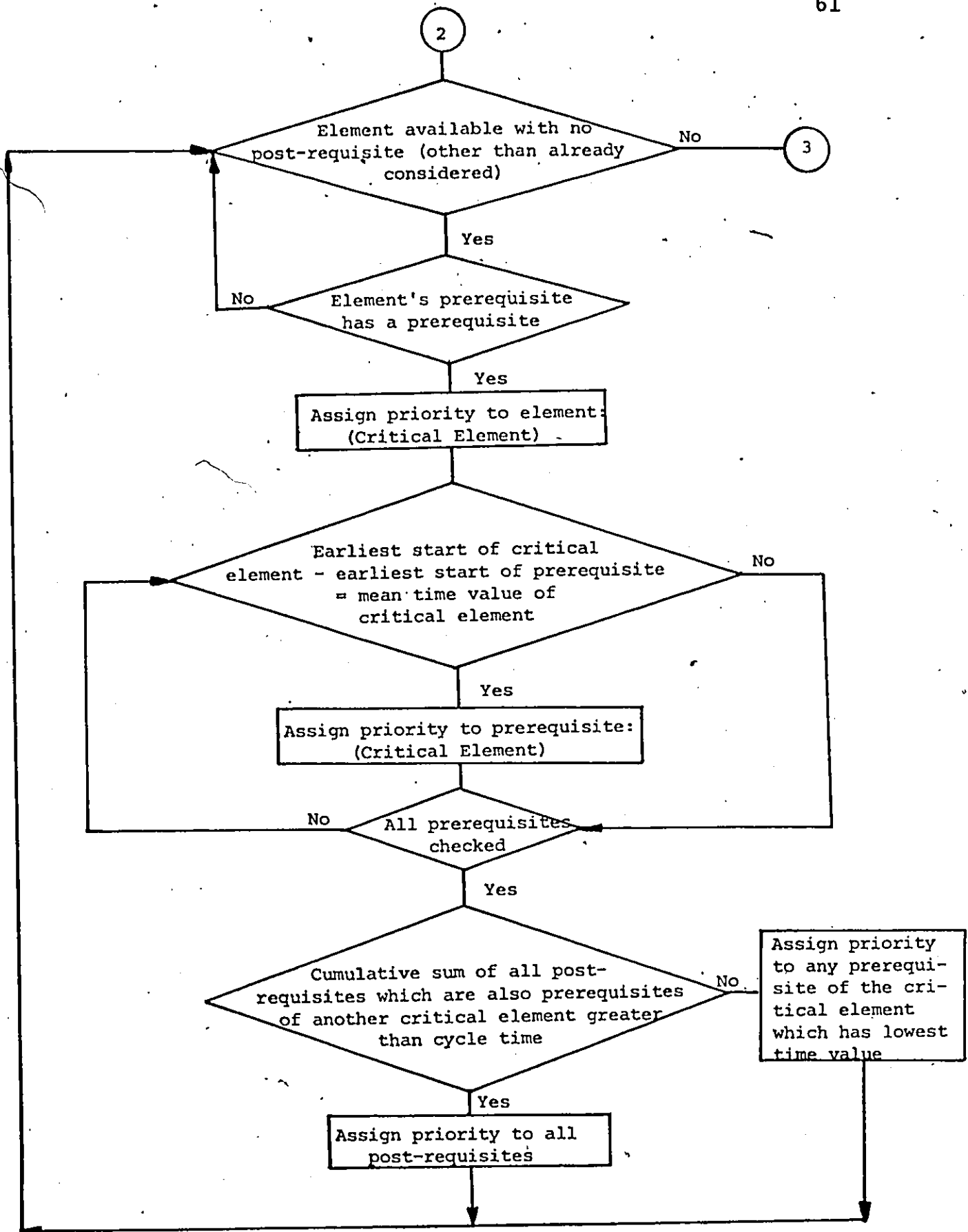


Figure (3) (Cont.)

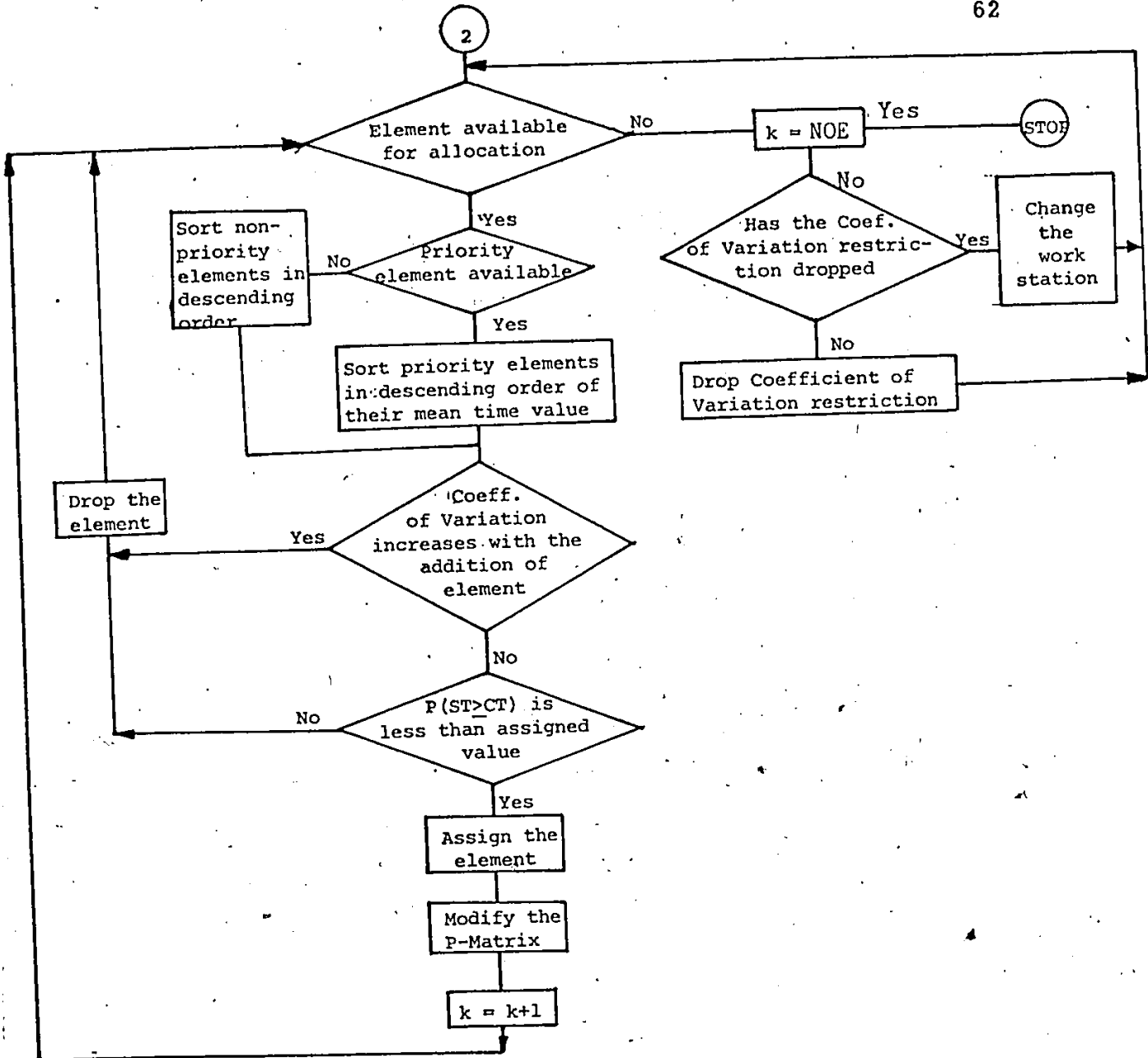


Figure (3) (Cont.)

APPENDIX C

Computer Program

5JOB WATFIV G12600C223,TIME=15

ASSEMBLY LINE BALANCING USING CRITICAL PATH APPROACH

THIS PROGRAM IS DIVIDED INTO TWO PARTS FOR SIMPLICITY
PART I ----- DETERMINES THE CRITICAL PATH AND ASSIGNS PRIORITIES
PART II ----- ALLOCATES THE ELEMENTS TO THE WORK STATION

P A R T I

THIS PROGRAM IS SET UP TO SOLVE ANY PROBLEM HAVING LESS THAN 100 ELEMENTS

THE FOLLOWING DATA IS TO BE GIVEN

- 1. THE FIRST READ CARD WILL HAVE THE VALUES OF NDE, NRPM, NCPM, NRPM, NCFM, CT, MINTIM, REQPRD, UNDEF. THE FORMAT OF FIRST SEVEN VALUES IS 7I3 AND LAST TWO VALUES IS F4.2
2. ON THE SECOND READ CARD THE TITLE OF THE PROBLEM IS GIVEN. THERE SHOULD BE TWO READS CARDS FOR THIS. IF THE TITLE IS WRITTEN ON ONE CARD, THEN A BLANK CARD MUST BE ATTACHED
3. CHANGE CARD NO'S. 15 - 17 ----- FORMAT CARDS ACCORDING TO THE DATA PUNCHED ON THE DATA CARDS

IF MORE THAN 100 ELEMENT PROBLEM HAS TO BE SOLVED THEN IN ADDITION TO THE ABOVE CHANGES, CARD NO. 2 ----- THE DIMENSION CARD HAS TO BE CHANGED ACCORDINGLY

DIMENSION P(100,15),F(100,15)
DIMENSION ISP(100),IFEJ(100),PEND(200),MIME(100),PW(100),RPW(100),
1WS(100),SI(100),TIME(100),TFT(100),MRA(100),IPPE(200)
DIMENSION ELE(30,25),NU(25),IST(25),IDL(25)
DIMENSION SSIG(50),SIG(30),SIGMA(50),ARC(33)
INTEGER P,F,CT,T,TT,TIME,A,RAN,PW,RPW,FLE

1
2
3
4
5

```

C
C THE VARIOUS PARAMETERS ARE DEFINED AS FOLLOWS
C
C CT.....CYCLE TIME
C NOE.....NUMBER OF ELEMENTS
C NCPM.....NUMBER OF COLUMNS OF 'P' MATRIX
C NCFM.....NUMBER OF COLUMNS OF 'F' MATRIX
C NRPM.....NUMBER OF ROWS OF 'P' MATRIX
C NRFM.....NUMBER OF ROWS OF 'F' MATRIX
C MINTIM.....MINIMUM ELEMENTAL TASK TIME
C
C*****
C*****
C
C
C READ(5,666) NOE, NRPM, NCPM, NRFM, NCFM, CT, MINTIM, REQPRO, UNDEF
6 6665 FORMAT(7I3,F4.2,F4.2)
7
8 READ(5,1333) ABC
9 1333 FORMAT(20A4,/,13A4)
10 WRITE(6,1334) ABC
11 1334 FORMAT(1H,33A4,////)
C
C*****
C*****
C
C READING THE INPUT DATA ---- P-MATRIX, F-MATRIX, AND ELEMENTAL
C TASK TIMES
C
C*****
C*****
C
C READ(5,1)((P(I,J),J=1,NCPM),I=1,NRPM)
12 READ(5,2)((F(I,J),J=1,NCFM),I=1,NRFM)
13 READ(5,3)(TIME(I),I=1,NOE)
14 1 FORMAT(3I2)
15 2 FORMAT(4I2)
16 3 FORMAT(2I12)
17 RESE0=0.5
18 KIM=0
19
C
C*****
C*****
C
C INITIALIZING ALL THE ELEMENTS
C READING FROM P-MATRIX THE ELEMENT NUMBERS. IF THE ELEMENT NUMBER
C IS ZFRO INITIALIZE THE ELEMENT TO ITS MEAN TIME VALUE
C*****
C*****
C
C DO 8200 N=1,NOE
20 RPW(N)=0
21 ISP(N)=0
22 IF(P(N,1).EQ.0) RPW(N)=TIME(N)
23 IF(P(N,1).EQ.0) PW(N)=TIME(N)
24 8200 CONTINUE
25 KK=0
26 MON=0
27 NON=0
28 MT=0
29 MBM=0
30
C
C*****
C*****
C
C THE EARLIEST START OF EACH ELEMENT IS DETERMINED
C*****
C*****
C
C DO 8001 I=1,NOE
31
C

```

\*\*\*\*\*  
\*\*\*\*\*

READING FROM F-MATRIX THE ELEMENT NUMBERS

\*\*\*\*\*  
\*\*\*\*\*

32  
33  
34  
35

```
DO 8001 J=1, NCFM
IF(F(I,J).EQ.0)GO TO 8001
K=F(I,J)
NUM=0
```

\*\*\*\*\*  
\*\*\*\*\*

CUMULATIVELY ADDING THE MEAN TIME VALUES OF ALL ELEMENTS READ  
IN THE SAME ROW.

\*\*\*\*\*  
\*\*\*\*\*

36  
37  
38  
39

```
DO 8888 MN=1, NCFM
KIN=F(I, MN)
IF(KIN.EQ.0)GO TO 8889
NUM=NUM+TIME(KIN)
```

\*\*\*\*\*  
\*\*\*\*\*

CHECK IF THE CUMULATIVE SUM IS GREATER THAN CYCLE TIME

\*\*\*\*\*  
\*\*\*\*\*

40

```
IF(NUM-CT)8288,8288,8890
```

\*\*\*\*\*  
\*\*\*\*\*

ASSIGN PRIORITY

\*\*\*\*\*  
\*\*\*\*\*

41  
42

```
8890 ISP(I)=500
8888 CONTINUE
```

\*\*\*\*\*  
\*\*\*\*\*

IF EXPECTED EARLIEST START IS LESS THAN PREVIOUS EARLIEST START  
READ THE NEXT ELEMENT NUMBER. IF IT IS GREATER THAN EQUATE EARLIEST  
START EQUAL TO EXPECTED EARLIEST START.

\*\*\*\*\*  
\*\*\*\*\*

43  
44  
45  
46  
47  
48  
49

```
PW(K)=RPW(I)+TIME(K)
IF(PW(K).GE.RPW(K))GO TO 8002
GO TO 8001
8002 RPW(K)=PW(K)
8001 CONTINUE
DO 8100 MM=1, NOE
8100 CONTINUE
```

C  
C





```

C*****
C
69      DO 8033 MN=1, NCFM
70      MIN=F(KJ, MN)
71      IF(MIN.EQ.0)GO TO 8033
72      IF(MBM.LF.2)GO TO 8051
73      DO 8999 JA=1, NCFM
74      IF(F(MIN, JA)-IPPE(MBM-2))8999, 8044, 8999
75      8999 CONTINUE
76      GO TO 8051
77      8044 GUM=GUM+TIME(MIN)
78      IF(GUM-CT)8051, 8051, 8050
79      8050 DO 8052 LIN=1, NCFM
80      KL=F(KJ, LIN)
81      IF(KL.EQ.0)GO TO 8052
82      DO 7777 MM=1, NCFM
83      LAN=F(KL, MM)
84      IF(LAN-IPPE(MBM-2))7777, 7777, 7777
C      ASSIGN PRIORITY TO ALL POST REQUISITES
85      7778 ISP(KL)=500
86      7777 CONTINUE
87      8052 CONTINUE
88      8051 CONTINUE

```

```

C*****
C*****
C      IF THE POST-REQUISITE HAS THE MINIMUM TIME VALJE, PRIORITY IS
C      PLACED ON THAT ELEMENT
C*****
C*****
C

```

```

89      IF(TIME(MIN).EQ.MINTIM)GO TO 8034
90      GO TO 8033
91      8034 MEA(KK)=MIN
92      KK=KK+1
93      8033 CONTINUE
94      KK=KK-1
95      IF(KK.LE.0)GO TO 8004
96      DO 8037 IKK=1, KK
97      NIL=MRA(IKK)
98      ISP(NIL)=500
99      8037 CONTINUE
100     8004 CONTINUE
101     KI=SKI
102     IF(SKI.EQ.0)GO TO 8003
103     SKI=0
104     GO TO 8007
105     8003 CONTINUE
106     DO 8040 II=1, NOE
107     8040 CONTINUE

```

```

C*****
C*****

```

PART II

```

C*****
C*****
C      INITIALIZING
C*****
C*****

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```

C*****
C
108      M=0
109      C=0
110      TT=0.0
111      IJK=1
112      KIM=0
113      14 CONTINUE
114      ID=0
115      K=0
116      FB=0
117      DO 70 I=1,NDE
118      TET(I)=0.0
119      70 CONTINUE
120      DO 12 I=1,NDE
121      IF(M.EQ.0)GO TO 6
122      DO 5 N=1,M

```

```

C*****
C*****
C      ELEMENT IS PICKED UP FOR ALLOCATION
C      IF THE ELEMENT IS ALREADY ALLOCATED, THEN IT SELECTS THE NEXT
C      AVAILABLE ELEMENT.
C*****
C*****
C

```

```

123      IF(I.EQ.WS(N))GO TO 12
124      5 CONTINUE
125      6 CONTINUE
126      DO 30 J=1,NCPM
127      IF(P(I,J).NE.0)GO TO 12
128      30 CONTINUE
129      L=I
130      TET(L)=TIME(L)
131      K=K+1
132      ID=ID+1
133      SI(K)=I
134      PEND(ID)=I
135      IF(TET(L).GT.FB)IP=L
136      IF(TET(L).GT.FB)IQ=K
137      IF(TET(L).GT.FB)FB=TET(L)
138      L=IP
139      K=IQ
140      12 CONTINUE
141      RAN=0
      PRIORITY ELEMENT AVAILABLE

```

```

C*****
C*****
C
142      IF(ISP(L).EQ.500)GO TO 515
143      507 IMP=ID

```

```

C*****
C*****
C      SORT PRIORITY AND NON PRIORITY ELEMENTS.
C*****
C*****
C

```

```

144      DO 501 IF=1,IMP
145      IB=0
146      DO 505 IKC=1,IMP
147      IE=PEND(IKC)
148      IF(RAN.EQ.0)GO TO 508
149      DO 525 MAN=1,RAN

```

```

150     IF(IF.EQ.IREJ(MAN))GO TO 505
151     525 CONTINUE
152     508 CONTINUE
153     IF(TIME(IE).GT.IB)GO TO 506
154     GO TO 700
155     506 IR=TIME(IE)
156     IL=IF
157     L=IL
158     700 CONTINUE

```

```

C
C
C*****
C*****

```

```

C
C
C     IF AVAILABLE THEY ARE ASSIGNED IF POSSIBLE, OTHERWISE OTHER
C     ELEMENTS ARE ASSIGNED ACCORDING TO 'LARGEST CANDIDATE RULE'.
C
C

```

```

C*****
C*****

```

```

159     IF(ISP(IE)-500)505,701,701
160     701 L=IF
161     DO 702 IGG=1,IMP
162     IG=PEND(IGG)
163     IF(RAN.EQ.0)GO TO 707
164     DO 706 MAN=1,RAN
165     IF(IG.EQ.IREJ(MAN))GO TO 702
166     706 CONTINUE
167     707 CONTINUE
168     IF(IG.EQ.IE)GO TO 702
169     IF(ISP(IG)-500)702,704,704
170     704 IF(TIME(IG)-TIME(IE))702,702,703
171     703 L=IG
172     702 CONTINUE
173     GO TO 502
174     505 CONTINUE
175     502 CONTINUE
176     TT=TT+TIME(L)
177     IF(TT-CT)60,60,10
178     10 TT=TT-TIME(L)
179     RAN=RAN+1
180     IREJ(RAN)=L
181     501 CONTINUE
182     TT=0
183     KIM=0
184     IJK=IJK+1
185     GO TO 14
186     515 CONTINUE
187     TT=TT+TIME(L)
188     IF(TT-CT)60,60,520
189     520 TT=TT-TIME(L)
190     GO TO 507
191     60 M=M+1
192     WS(M)=L
193     KIM=KIM+1
194     ELE(IJK,KIM)=L
195     IF(UNDEF.EQ.1.0)GO TO 2222
196     IF(KIM.EQ.1)SSIG(KIM)=SIG(L)
197     IF(KIM.EQ.1)GO TO 2229
198     SSIG(KIM)=SIG(L)+SSIG(KIM-1)
199     2229 SIM=SSIG(KIM)
200     SIGMA(IJK)=SQRT(SIM)
201     2222 CONTINUE
202     NU(IJK)=KIM
203     IAC=NU(IJK)+1
204     IAD=10
205     DO 6161 IAB=IAC,IAD
206     ELE(IJK,IAB)=0
207     6161 CONTINUE
208     IST(IJK)=TT
209     IDL(IJK)=CT-TT

```

```

C
C
C*****
C*****

```

## MODIFYING THE P-MATRIX

C  
C  
C  
C  
CC\*\*\*\*\*  
C\*\*\*\*\*

```

210      DO 4 J=1,NCFM
211      II=F(L,J)
212      IF(II.EQ.0)GO TO 4
213      DO 80 A=1,NCPM
214      IF(P(II,A).EQ.0)GO TO 80
215      P(II,A)=0
216      GO TO 4
217      80 CONTINUE
218      4 CONTINUE
219      C=C+1
220      IF(C-NCF)15,16,16
221      15 GO TO 14
222      16 CONTINUE
223      WRITE(6,99999)CT
224      99999 FORMAT(' ','CYCLE TIME =',I3,////)
225      WRITE(6,7000)
226      7000 FORMAT(' ','STATION NO.',10X,'ELEMENTS',30X,'STATION TIME',10X,
227      1 'IDLE TIME',//)
228      IJK=IJK
229      DO 1338 KJK=1,IJK
230      IF(IDL(KJK).GE.MON)MIDL=IDL(KJK)
231      IF(IDL(KJK).GE.MON)MON=IDL(KJK)
232      IF(IST(KJK).GE.NON)MACT=IST(KJK)
233      IF(IST(KJK).GE.NON)NON=IST(KJK)
234      1338 CONTINUE
235      ISQ=0
236      MT=0
237      DO 1212 KJK=1,IJK
238      MT=MT+IST(KJK)
239      ISQ=(MACT-IST(IJK))*2
240      ISQ=ISQ+ISQ
241      WRITE(6,7001)KJK,(ELF(KJK,J),J=1,IAD),IST(KJK),IDL(KJK)
242      7001 FORMAT(' ',2X,I3,9X,I2,2X,I2,2X,I2,2X,I2,2X,I2,2X,I2,2X,I2,2X,I2,2X,I2,
243      12X,I2,2X,I2,10X,I4,15X,I4,////)
244      1212 CONTINUE
245      AISI=ISQ**0.5
246      AMT=MT
247      AIJK=IJK
248      AMACT=MACT
249      EFF=(AMT*100)/(AIJK*AMACT)
250      WRITE(6,6667)AISI,EFF
251      6667 FORMAT(' ','SMOOTHNESS INDEX =',F5.2,10X,'OUTRJT EFFICIENCY =',
252      1 F6.2,////)
253      IF(UNDF.EQ.1.0)GO TO 2223
254      2226 CONTINUE
255      IJA=IJK
256      CP=0
257      DO 2227 IJM=1,IJA
258      AK=(CT-IST(IJM))/SIGMA(IJM)
259      AP=1/((AK**2)+1)
260      AP=AP/2.0
261      CP=CP+AP
262      IF(CP.GT.1.0)CP=1.0
263      2227 CONTINUE
264      2224 WRITE(6,2225)CT,CP
265      2225 FORMAT(' ','THE PROBABILITY OF EXCEEDING THE CYCLE TIME OF',I4,4X,
266      1 ' =',F8.5)
267      IF(CP.LE.REOPRO)GO TO 2223
268      CT=CT+1
269      GO TO 2226
270      2223 CONTINUE
271      STOP
272      END

```

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