2004

# Space-time coding with imperfect channel estimates.

Bashir Iqbal Morshed
*University of Windsor*

# SPACE-TIME CODING WITH IMPERFECT CHANNEL ESTIMATES

by

Bashir Iqbal Morshed

A Thesis
Submitted to the Faculty of Graduate Studies and Research
through Electrical Engineering
in Partial Fulfillment of the Requirements for
the Degree of Masters of Applied Science at the
University of Windsor

Windsor, Ontario, Canada

2004

*Your file Votre référence*
*ISBN: 0-612-96399-3*
*Our file Notre référence*
*ISBN: 0-612-96399-3*

I hereby declare that I am the sole author of this thesis.
I authorize the University of Windsor to lend this thesis to other institutions or individuals for the purpose of scholarly research.

Bashir Iqbal Morshed

I authorize the University of Windsor to reproduce this thesis by photocopying or other means, in total or in part, at the request of other institutions or individuals for the purpose of scholarly research.

Bashir Iqbal Morshed

# Space-Time Coding with Imperfect Channel Estimates

by

Bashir Iqbal Morshed

Submitted to the Department of Electrical and Computer Engineering
on September 14th, 2004 in partial fulfillment of the requirements
for the Degree of Masters of Applied Science in Electrical and Computer Engineering

**Abstract** This thesis proposes a new decision rule for the space-time block (STB) coded wireless communication system in Rayleigh faded channel with partial knowledge of the channel state information (CSI). Also proposed is the frame-based iterative channel estimation algorithm for the same system with no knowledge of CSI.

Antenna diversity can provide high channel spectral efficiency necessary for the next generation wireless communication system. Receiver diversity is applicable for uplink from the mobile station (MS) to the base station (BS), while providing transmitter diversity for downlink from BS to MS. One of the transmitter diversity techniques is the STB code, proposed by Alamouti [1]. The STB code is highly efficient considering the decoding complexity. One major disadvantage of the STB coded system is that the error rate performance relies heavily on the estimated channel fading parameters, which are used in the decision rule. As the estimated fading parameters become unreliable with decreasing numbers of overhead pilot symbols, the performance degrades. If the receiver has partial knowledge of CSI, then this performance can be improved by including this information to the decision rule of the decoder. The state-of-the-art technique uses the modified decision rule proposed by Tarokh [28]. This thesis proposes a simpler modified decision rule which performs better in terms of bit error rate (BER) than the existing state-of-the-art technique using gray coded 16-QAM (Quadrature Amplitude Modulation) scheme with 2 transmitter antennas and 1 receiver antenna. Moreover, the proposed decision rule requires much less complexity from the implementation point of view compared to the state-of-the-art counterpart.

The thesis also proposes the frame-based iterative channel estimator when no knowledge of CSI is available at the receiver. The algorithm exploits the inherent orthogonal property of the STB code. The BER performance reaches within 1 dB of the perfect knowledge of CSI for the simplest case with BPSK (Binary Phase Shift Keying) modulation having 2 transmitter antennas and 1 receiver antenna. The proposed algorithm outperforms the state-of-the-art iterative decision-directed channel tracking algorithm [12] at the expense of increased receiver complexity.

**Thesis Supervisor:** Behnam Shahrrava
**Title:** Assistant Professor, Electrical and Computer Engineering

# Acknowledgements

I would like to thank all who made this work possible: my advisor Dr. Behnam Shahrrava, for thousands of suggestions and guidelines to maintain overall high standards; my friend Mr. Mehdi Hedjazi Moghari for his helpful hints; my wife Mrs. Sohely Perven, for giving me inspiration; my parents Dr. Monjur Morshed Mahmud and Mrs. Suraiya Begum, for fostering me up to this point without realizing the hardship of this world.

# Contents

# List of Tables

# List of Figures

# Chapter 1

# Introduction

The third generation (3G) mobile communication standards are expected to provide a wide range of bearer services, spanning from voice to high-rate data transmission. The requirements lead to a demand for increased capacity with limited radio spectrum available. Communication technology is being rapidly changed to meet this demand and new communication techniques are being developed for modern communication systems. In an effort to support such high rates, the bit/symbol capacity of band-limited wireless channels can be increased by employing multiple antennas. The traditional approach is to use multiple antennas at the receiver and use maximal-ratio receiver combining (MRRC) of the received signals for improved performance [11]. This receiver diversity scheme is attractive for the Base Stations (BS); however, applying this scheme at the Mobile Stations (MS) increases their complexity and size. Hence receiver diversity techniques typically have been applied for uplink only.

To achieve similar diversity gain for communication systems from the BS to the MS, different transmit diversity techniques have been introduced. Space-time trellis codes were proposed to meet the demand, but the receiver complexity becomes very high [27]. In addressing the issue of decoding complexity, Alamouti discovered a remarkable scheme for transmit diversity using two transmitter antennas [1]. Later this scheme was extended for three and more transmitter antennas by Tarokh *et al* [29]. This coding technique, known as the space-time block (STB) code, has a simple decoder structure. It can provide the best theoretical tradeoff between diversity gain, transmission rate, constellation size, signal space dimension and trellis complexity [28]. However, the decoding of STB code heavily depends on the accuracy of the estimate of the channel fading parameters [3]. Estimation of channel fading parameters requires overhead pilot symbols. To increase the data rate and decrease energy loss due to overhead, reduced pilot symbols are to be used. This causes the estimation to become unreliable, leading to a poor error rate performance of the system. Tarokh proposed a modified decision rule when partial knowledge of the channel state information (CSI) is available at the receiver [28]. The complexity of this decision rule is very high and the decision rule is valid only for high signal-to-noise ratio (SNR) [30]. On the contrary, when no knowledge of CSI is available, iterative channel estimation is proposed to improve the performance [4,12]. Practical importance of these two cases demand more research to resolve the complexity and performance gain issues before implementing the system efficiently in reality.

The goals of this thesis are as follows:

- Investigate the state-of-the-art of the STB coded system with imperfect channel estimates.
- Derive a modified decision rule with imperfect channel estimates when partial knowledge of CSI is available.
- Investigate iterative channel estimation algorithm for improved performance when no CSI is available.

These goals are realized in the following methodology:

- Partial CSI - a simple modified decision rule is derived for this case based on Alamouti's first model, which supersedes performance of Tarokh's model.

- No CSI - a novel frame-based iterative channel estimation algorithm is proposed for this case exploiting the orthogonal property of STB coded scheme.
- Comparison - performances are compared through software simulations with the corresponding state-of-the-art techniques.

The remainder of this introduction specifies the basic problems of decoding with imperfect channel estimates, and outlines the proposed solutions.

## 1.1 Challenges due to imperfect channel estimates

The decision rule for the STB code was derived assuming ideal case when the receiver has the perfect knowledge of CSI [1, 29]. In reality, the receiver never has the perfect knowledge of CSI as the channel parameters are random variables. There are two possible cases in practice:

- Case I - The receiver might have partial knowledge of CSI. This partial knowledge can be the variances of the estimation error of the channel fading parameters, which can be found easily and reliably.
- Case II - The receiver might have no knowledge of CSI.

In both cases, the decision rule derived assuming the perfect knowledge of CSI causes mismatch in the receiver. Performance degradation due to this type of mismatch in the channel fading parameters in the decision rule has been addressed in the standard literature [33]. It is shown in [3] that the STB code is more sensitive to the channel estimation error than the straightforward two branch diversity scheme because of its dependency on the removal of the cross-terms in the decision rule. This dependency on the channel estimation error increases as the number of transmitter and receiver antennas increases to achieve higher error rate performance [9]. Moreover, if the initial channel estimation is done with very few pilot symbols to reduce overhead loss, the estimation error becomes significant and the coding performance degrades further.

The case of partial knowledge of CSI was discussed and modification of the decision rule was proposed in [26,28]. Later, however, it was found that the scheme is only applicable when the SNR approaches infinity [30] and is approximate for the practical range of the SNR. Moreover, this modified decision rule requires much higher processing time as the complexity is very high. Still, this is the state-of-the-art method to combat imperfect channel estimates for the first case.

Techniques to overcome performance degradation due to the channel estimation error for the second case are also being extensively studied. In [14], a cyclic approach is considered to compensate the channel estimation error. Different iterative algorithms are being proposed to improve initial channel estimation using data symbols. Decision-directed channel estimation is proposed in [4,16,35]. An improved method of decision-directed channel tracking algorithm (which is in fact a modified version of the decision-directed algorithm, hence can be called as modified decision-directed method) is proposed in [12]. This is the state-of-the-art algorithm in this case for its high performance and very low complexity. However, there is still a significant performance gap between the ideal case and this algorithm.

## 1.2 Description of solutions

To resolve the issue of channel fading parameter estimation error, two different techniques for the two cases are proposed in this work. For the first case, a simple modified decision rule is derived. For the second case, a frame-based iterative channel estimator is proposed which exploits the inherent orthogonal property of the STB coding. The solutions are briefly discussed below.

### 1.2.1 Modified decision rule for the decoder

A low complexity modified decision rule has been proposed for the maximum likelihood (ML) decoder of the STB coded system proposed by Alamouti when variance of the estimation error is known or can be estimated reliably. Similar steps to Frenger's methodology for Turbo coded system with imperfect channel estimates has been adopted [8]. The basic approach is to include the known estimation error model while deriving the probability density function (pdf) of the received signal conditioned on the estimated channel parameters. The modified decision rule given in this work provides a similar or improved performance compared to the state-of-the-art method. Moreover, the proposed decision rule has much less complexity for practical implementation. The decision rule can easily be generalized or extended for any STB coded system and other similar systems, too.

### 1.2.2 Frame-based iterative channel estimation

The structure of the STB data blocks and of the orthogonal pilot sequences are similar. This inherent orthogonal property of the STB coding scheme is exploited in this algorithm. The effect of wrong detection of data blocks is minimized by iterating on a frame-basis instead of a block-basis. The initial channel fading parameters are estimated from a few known data or pilot sequences. These estimated channel parameters are used to decode the whole frame of the received signal. Then the whole data frame is used to find averaged channel fading parameters, which will obviously have higher reliability. This method of frame-based iteration is repeated for a number of times to have the desired performance of the error rate. The performance gain is very significant as it approaches ideal case performance after only a few iterations. However, the complexity is increased and higher processing time is expected.

## 1.3 Organization of the thesis

The organization of this thesis is as follows:

Chapter 2 discusses different types of channels and channel coding techniques.

Chapter 3 describes the STB coded system in detail with emphasis on the Alamouti's first model.

Chapter 4 describes the estimation technique of the channel fading parameters and explains the effect of imperfect channel estimates on the performance of the system.

Chapter 5 develops the proposed modified decision rule for the STB coded system in the Rayleigh faded channel with imperfect channel estimates having partial knowledge of CSI.

Chapter 6 develops the proposed model of the frame-based iterative channel estimation for the STB coded system having no knowledge of CSI.

Chapter 7 presents the simulation results showing performance comparison with the corresponding state-of-the-art methods.

Chapter 8 summarizes major accomplishments and identifies future research directions.

The Appendix A gives the steps of the derivation of the exact conditional pdf of the received signal in detail, which is needed for deriving the modified decision rule.

# Chapter 2

# Channel and channel coding

This chapter describes the basic concepts required for the modern communication system. As the thesis is based on a channel coding technique for wireless communication system, different types of channels involving this kind of system are explained. A brief discussion on different channel coding techniques are also given.

## 2.1 Modern communication system

Ever since the first transmission for radio communication by Guglielmo Marconi, the radio spectrum has become the fundamental resource on which every wireless communication system depends. Following those early pioneering times, the use of the radio spectrum has increased dramatically in recent days with the advent of mobile technology. Now-a-days, the available radio spectrum is heavily utilized by a variety of services based on land and sea, in air and space, for a vast array of different purposes. These services bring an enormous amount of benefits to human society. However, they also require appropriate radio spectrum management and regulation mechanisms to ensure economically viable use of this limited resource. Indeed, different wireless systems offering the services need to maximize their own spectrum efficiency to ensure that support can be given to as many users as possible.

The endeavor is to increase both user satisfaction and the revenues of the operator.

In the third generation of wireless communications, the demand for wide-band high data rate communication services will grow with the integration of internet and multimedia applications. To meet this demand, basic requirements of the modern communication systems can be categorized as:

- High data transmission rate.
- Limited bandwidth operation.
- Reliable communication.

Reliable communication, consisting of transmission and reception processes, has an acute impact on data transmission rate and capacity. A communication system with high data rate but low reliability suffers from high volume of requests for re-transmission of the same data packets. Similarly a reliable communication scheme with low capacity can outperform a unreliable communication scheme with higher capacity. Hence, emphasis is given on reliable communication to optimize the channel spectral efficiency. Increased reliability can be achieved by different coding techniques in different levels of transmission and reception. One of the coding techniques used to meet the uncertainty of the channel parameters is channel coding. Before discussing channel coding, a better understanding of a simple communication system and wireless channel is necessary.

## 2.2 A simple communication system

A simplified diagram of generalized digital communication system is shown in Figure 2-1 [32]. The digital signal sequences ($c(t)$) produced by the source are transmitted through the transmitter. Depending on the coding scheme, the number of transmitter antennas can be one or more. This transmitted symbol sequences ($s(t)$) pass through

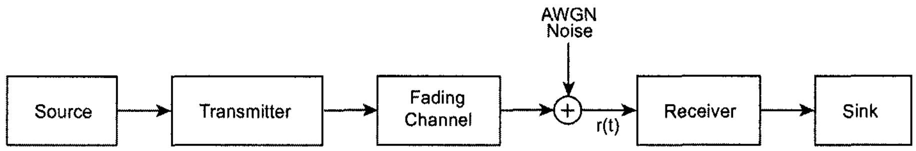


Figure 2-1: A simplified digital communication system.

the channel, which might be an acoustical channel for wireless communication systems. The channel introduces channel fading parameter ($h(t)$) to the transmitted symbols. This random fading parameter has probability distribution depending on the line-of-sight from the transmitter to the receiver. The worst case of fading is the Rayleigh fading, when there is no direct path from the transmitter to the receiver. In addition, at the receiver antenna, additive white Gaussian noise (AWGN) is generated ($n(t)$) and added to the faded symbols to constitute the received signal $r(t)$. Again, the number of receiver antennas can be one or more depending on the communication model. For the simplest case given in Figure 2-1, the received signal model can be expressed as,

$$r(t) = h(t)s(t) + n(t). \tag{2.1}$$

This received signal is sampled by the matched filter, the correlation detector or some other method. These samples constitute the received signal samples, which are then passed through the detector for detection of transmitted symbol sequences ($\hat{s}(t)$). A simple mapping can produce the detected data ($\hat{c}(t)$) sequences and are consumed at the sink. For estimation of channel fading parameter, some known pilot symbols are sent along with a data frame. A channel estimator (not shown in the figure) isolates the received signal samples due to pilot symbols and estimates the channel, which is fed to the receiver before the detection process starts. In the communication model, the source is a system having no input but producing an output; a sink has an input and no output.

One of the major challenges in wireless communication system is to overcome the channel fading caused by multipath and movement in the radio link. For a benign wireless channel, the receiver would simply be inverse of the transmitter and there would be no error in the detection process. But in practice, the wireless channel is not benign. Rather the channel introduces random destruction of the transmitted signal. This requires powerful channel coding to combat the malign effects of the channel. A brief discussion of wireless channels follows in the next section.

## 2.3 Types of wireless channel

The surrounding objects of a wireless environment act as reflectors or absorbers of the radio waves. Some obstacles produce reflected waves with attenuated amplitudes and phases. If a modulated signal is to be transmitted, multiple reflected waves of the transmitted signal will arrive at the receiving antenna (or antennas) from different directions with different amplitudes, propagation delays and phases. These reflected waves are called multipath waves [33]. The multipath waves at the receiver site have a combined effect due to the different arrival times, phases and amplitudes. When they are collected by the receiver antenna at any point, they may combine either in a constructive or a destructive way, depending on the random phases. The sum of these multipath components forms a spatially varying standing wave field. The mobile unit moving through the multipath field will receive a signal which can vary widely in amplitude and phase. When the mobile unit is stationary, the amplitude variations in the received signals are due to the movements of the surrounding objects in the radio channel. The amplitude fluctuation of the received signal is called channel fading. It is caused by the time-variant multipath characteristics of the channel. The effect of channel fading in the received signal samples is purely random.

The wireless channel suffers from time varying channel fading parameters due to the multipath propagation and destructive superposition of the signals received over different paths. From the quantitative point of view, channel fading parameters can be classified in two categories:

- Large-scale fading,
- Small-scale fading.

Large-scale fading is primarily caused by reflection, diffraction and scattering. This type of fading is usually deterministic. On the other hand, small-scale fading, or simply fading, is used to describe the rapid fluctuation of the amplitude of a radio signal over a short period of time or travel distance. This type of fading is described by stochastic process. The wireless channel considered in this work is the latter type.

Small-scale fading can again be classified into different categories. Depending on the Doppler shift, fading can be classified as fast or slow. In fast fading, the symbol duration is larger than coherence time, i.e. high Doppler rate. In slow fading, the symbol duration is smaller than the coherence time, i.e. low Doppler rate. Coherence time is a statistical measure of the time duration over which the channel impulse response is essentially invariant [22]. It quantifies the similarity of the channel response at different times. In other words, coherence time is the time duration over which any two received signals (at different times) have a strong correlation.

Depending on delay, small-scale fading can be classified into two categories: flat fading and frequency-selective fading. In a flat fading channel, the signal bandwidth is smaller than the coherence bandwidth. Meanwhile, in a frequency-selective fading channel, the signal bandwidth is larger than the coherence bandwidth. Here, coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered to be flat, i.e. having approximately equal gain and linear phase [22]. In other words, coherence bandwidth is the range of frequencies over which any two frequency components have a strong correlation. Flat fading channel is also called a narrowband system, as all spectral components of the transmitted signal are subject to the same fading attenuation. On the other hand, frequency-selective fading channel is also called wideband system, as the received signal spectrum becomes distorted due to the fact that the relationships between various spectral components are not the same as in the transmitted signal. The system model in this thesis is based on small-scale slow flat fading channel.

The channel fading parameter of interest can be described using one of the following two popular models depending on the existence of a line-of-sight from the transmitter antenna to the receiver antenna:

- Rayleigh fading model: no line-of-sight from the transmitter to the receiver.
- Rician fading model: line-of-sight exist from the transmitter to the receiver.

Among these models, Rayleigh fading model is the worst case where there is no line-of-sight from the transmitter antenna to the receiver antenna. This is the typical model for cellular wireless channel where the mobile unit receives only reflected waves. Rayleigh fading model is described in detail next.

When the number of reflected waves is large, according to the central limit theorem, two quadrature components of the received signal are uncorrelated Gaussian random processes with a zero mean and variance $\sigma^2$. Thus the envelope of the received signal at any time instant undergoes a Rayleigh probability distribution and its phase obeys a uniform distribution between $-\pi$ to $\pi$. The probability distribution function (pdf) of the Rayleigh fading model is given by [33]

$$p(a) = \begin{cases} \frac{a}{\sigma^2} e^{-a^2/2\sigma^2} & a \geq 0 \\ 0 & a < 0. \end{cases} \tag{2.2}$$

The mean and variance of the distribution is $1.2533\sigma$ and $0.4292\sigma^2$ respectively. If the pdf in (2.2) is normalized so that the average signal power ($E[a^2]$) is unity, then

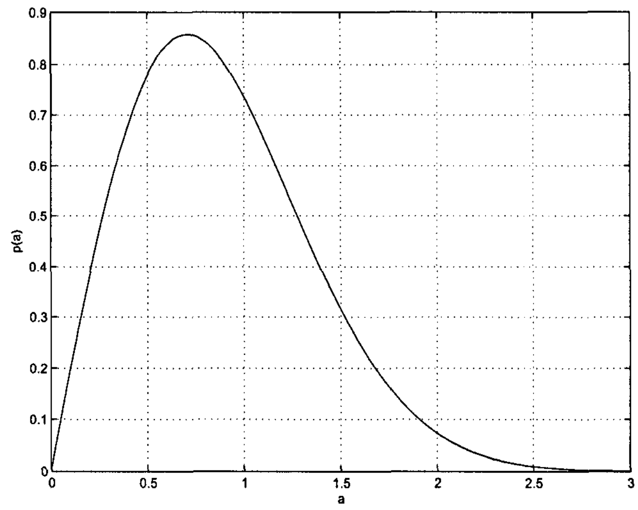


Figure 2-2: The pdf of normalized Rayleigh fading distribution.

the normalized Rayleigh distribution becomes

$$p(a) = \begin{cases} 2ae^{-a^2} & a \geq 0 \\ 0 & a < 0. \end{cases} \tag{2.3}$$

In this case, the variance in each quadrature component becomes 0.5. The pdf of normalized Rayleigh distributions is shown in Figure 2-2.

Again, depending on the number of transmitter and receiver antennas, a channel can be classified into four different categories:

- Single-Input Single-Output (SISO): having one transmitter antenna and one receiver antenna.

- Single-Input Multiple-Output (SIMO): having one transmitter antenna and two or more receiver antennas.

- Multiple-Input Single-Output (MISO): having multiple transmitter antennas with only one receiver antenna.

- Multiple-Input Multiple-Output (MIMO): having multiple numbers of trans-

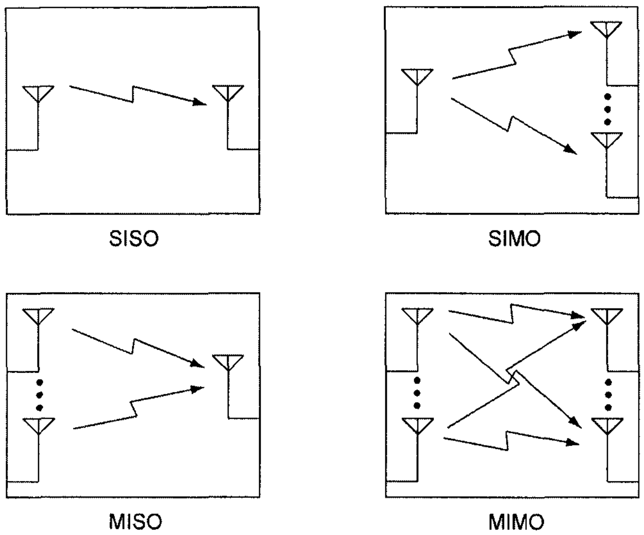


Figure 2-3: Different types of channels based on number of antennas.

mitter and receiver antennas.

A schematic diagram of the different cases are shown in Figure 2-3.

## 2.4 Channel coding

Channel coding is a state-of-the-art technique used to combat the detrimental effects of the channel fading parameters for improved reliability of communication. The basic concept of the technique is to introduce redundancy to the data streams in the transmitter to enable the receiver to detect or even correct transmission errors due to randomness of the channel, and hence to improve reliability of the transmitted data at the expense of lower data rate. The task of the channel coding is to represent the source information in such a systematic manner that minimizes the error probability in the decoding process.

There exist a vast number of channel coding schemes in the communication system. Some prominent techniques are discussed below.

### 2.4.1 Hamming codes

One primitive type of channel coding is the Hamming codes. In this type of coding, the codes maintain a minimum Hamming distance in the constellation diagram. The receiver decides in favor of the nearest code from the received signal sample. The coding gain provided through this technique is very low, hence this coding is no longer used in the practical field.

### 2.4.2 Parity-check codes

In this type of the channel coding technique, some parity-bits are added with the data symbols which contain some information of data bits. Using this parity-bits as a constraint, the decoder can decide whether the received data bits are correct or not. The receiver can even correct a certain degree of error using this technique. The scheme is still used in some simple data communication system where the random effect of the channel is very trivial.

### 2.4.3 Linear block codes

This was one of the popular channel coding technique for a long time. In this scheme, a low density generator matrix is used to linearly encode a block of data before transmission of the signal. The receiver uses a message passing algorithm to decide in favor of the correct codeword. The high complexity of the algorithm is a drawback of this type of encoding technique.

### 2.4.4 Convolution codes

Convolutional encoders are commonly specified by three parameters: the number of output bits, the number of input bits and the number of memory registers. The ratio of the number of input bits over the number of outputs bits is called the code rate, which is a measure of coding efficiency of this type of encoder. Viterbi decoding is used to decode the received data and decide in favor of lowest path metric. This coding scheme provides high gain and is the foundation of modern channel coding schemes.

### 2.4.5 Turbo codes

Turbo Codes have been adopted by both the third generation partnership program (3GPP) and 3GPP2 standards to enable higher network data capacity in CDMA systems, according to Association of Radio Industries and Business. Turbo encoder consists of two recursive systemic convolution (RSC) encoder and one interleaver. The data from the source is directly given to the output of the channel encoder, which is called systematic bit. Data is also fed directly to one RSC encoder and through the interleaver to the other RSC encoder. The output of the RSC encoders might be punctured to increase the data rate at the cost of lower error rate performance. Turbo decoder consists of two A Posteriori Probability (APP) decoders, an interleaver and a de-interleaver. The data is iterated several times within the APP decoders before deciding in favor of a codeword. The decoders use both extrinsic and intrinsic information to provide soft decisions. The error rate performance of this technique achieves near Shannon's lower limit of channel capacity in AWGN channel.

### 2.4.6 Space-time trellis codes

Space-time trellis codes (STTC) were first proposed by Tarokh, Seshadri and Calderbank [27]. They offer a substantial coding gain, spectral efficiency, and diversity improvement on flat fading channels. The encoder maps binary data to modulation symbols, where the mapping function is described by a trellis diagram. The decoder employs a Viterbi algorithm to perform maximum likelihood (ML) decoding and decides in favor of the minimum path metric. One disadvantage of this type of channel coding is its complex decoding structure.

### 2.4.7 Space-time block code

Space-time block (STB) coding was first proposed by Alamouti [1] and then later analyzed and extended by Tarokh *et. al.* [29]. The key feature of the scheme is that it achieves a full diversity gain with a simple ML decoding algorithm. A generator matrix is used to encode the data, while a combiner and a ML decoder is used to decode the received data. The advantage of the scheme is its ability to provide improved performance when concatenated with other channel coding schemes, especially Turbo codes. This combination provides extremely high performance in even the Rayleigh fading channel.

The thesis will focus on STB channel coding technique. Low complexity of implementation and feasibility of small physical size of the receiver, makes the STB code very suitable for the next generation mobile communication. The scheme is explained in detail in the next chapter.

# Chapter 3

# Space-time block coding

Multi-path propagation and destructive superposition of signals received over different paths are the principle reasons of performance loss in the wireless channel with time-varying fading. Special techniques have been applied to enable bandwidth efficient transmission. One of the widely applied approaches is to reduce the detrimental effects of multi-path fading using antenna diversity. Recently this type of diversity technique has been studied extensively because of its relative simplicity of implementation and feasibility of having multiple antennas at the base station. But due to the difficulty of efficiently using receive antenna diversity at the remote units since they should remain relatively simple, inexpensive and small, receive diversity has been nearly exclusively used at the base station [2]. In this chapter, the general structure of STB code is described with emphasis on Alamouti's proposed model.

## 3.1 Historical background

Space-Time Coding is a coding technique designed to use with multiple transmit antennas, where coding is performed in both spatial and temporal domains to introduce correlation between signals transmitted from various antennas at various time periods [33]. Historically, STB coding scheme was first proposed by Alamouti [1] for two

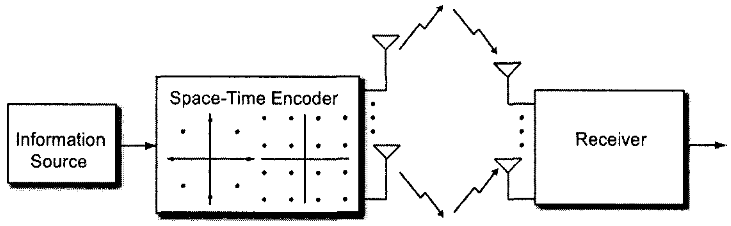


Figure 3-1: The block diagram of the transmitter and the receiver for space-time block Codes.

transmitter antennas and multiple receiver antennas. Later, the scheme was generalized for any number of transmitter and receiver antennas [27–29, 31, 34]. A general block diagram of STB coded system is given in Figure 3-1.

STB codes, can provide the best theoretical tradeoff between diversity gain, transmission rate, constellation size, signal space dimension and trellis complexity [28]. While receive diversity like maximal-ratio receiver combining (MRRC) uses multiple receive antennas, space-time block (STB) code uses multiple transit antennas to achieve performance gain. The advantage of STB code is that it allows us to achieve diversity gain while maintaining small physical size of the receiver.

## 3.2 Alamouti's first model

The Alamouti scheme is the first STB code to provide full diversity for systems with two transmit antennas. The encoder takes a block of two modulated symbols $s_0$ and $s_1$ in each encoding operation and maps them to the transmit antennas according to a code matrix given by

$$\mathbf{G} = \begin{pmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{pmatrix}. \tag{3.1}$$

Table 3.1 shows the encoding and transmission sequence for the STB coded system for Alamouti's first model given in Figure 3-2. Here $T$ is the time period for one

| | tx antenna 0 | tx antenna 1 |
|---|---|---|
| time $t$ | $s_0$ | $s_1$ |
| time $t+T$ | $-s_1^*$ | $s_0^*$ |

Table 3.1: The encoding and transmission sequence for the STB coded scheme of Alamouti's first model.

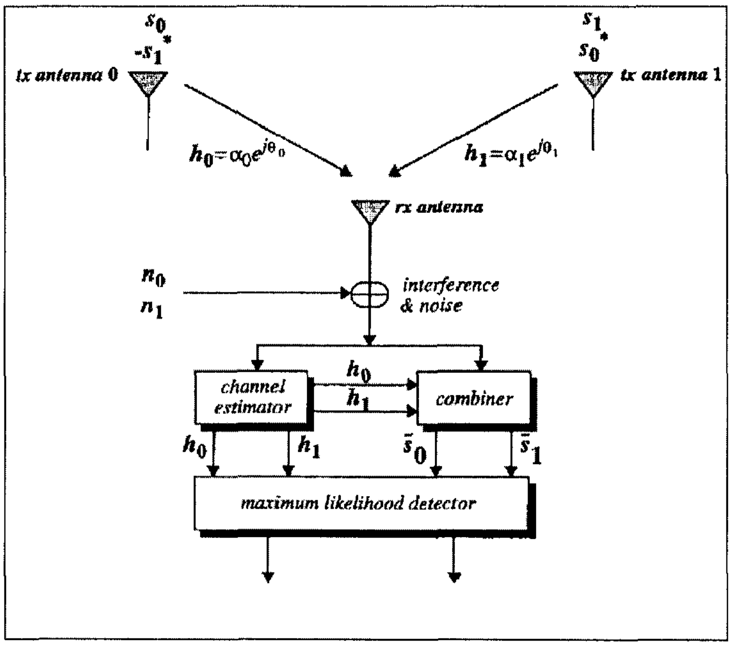


Figure 3-2: Alamouti's first model of STB coded system [1].

symbol. At time slot $t$, $s_0$ is transmitted from the transmitter antenna 0 and $s_1$ is transmitted from the transmitter antenna 1. At the next time slot, $-s_1^*$ is transmitted from the transmitter antenna 0 and $s_0^*$ is transmitted from transmitter antenna 1, where $(\cdot)^*$ denotes the complex conjugate. The key feature of Alamouti scheme is that the transmitted sequences from the two transmitter antennas are orthogonal.

For the Alamouti scheme, the codeword distance matrix has two identical eigenvalues. The minimum eigenvalue is equal to the minimum squared Euclidean distance in the signal constellation. This means for the Alamouti scheme, the minimum distance between any two transmitted code sequences remains the same as in the uncoded system. Therefore, the Alamouti scheme does not provide any coding gain relative to the uncoded modulation scheme, except diversity gain [33].

## 3.3 Other STB code models

Alamouti's simple scheme of STB coding was later generalized by Tarokh *et al.* All STB coded models are similar except the change of generator matrix and combining scheme due to different number of transmitter and receiver antennas. In every model, data is encoded to match the number of transmitter antenna using corresponding generator matrix. The rate of coding is defined as the number of data encoded in one block over the number of transmitter antennas. A full rate code has code rate of 1, and a partial rate code has a code rate less than 1. The criteria of encoding is that the transmitted symbol sequences must be orthogonal to each other. This is valid for both the full rate and partial rate encoders. The decoder with multiple receive antenna uses some sort of received signal combining scheme before delivering the received signal to the decoder. A generalized system model is given next, but emphasis will be on the Alamouti's first model.

## 3.4 A generalized system model

A wireless communication system is considered with $n$ transmit antennas at the base and $m$ receive antennas at the remote [Figure 3-1]. The STB encoder takes $p$ symbols in one block of data from the information source and uses the generator matrix to produce $q$ symbols for each transmitter antenna [1, 29]. Hence the generator matrix has dimension of $q \times n$. One frame of data symbols contains L blocks. If $p = q$, then the encoder is called full rate and if $p < q$, the encoder is partial rate. At each time slot $t$, one symbol $s_{i,t}$, $i = 1,2,\ldots,n$ is transmitted simultaneously from the $n$ transmit antennas. The channel is assumed to be flat fading and quasi-static, i.e., the path gains are constant over a frame and vary from one frame to another. The path gain from the $i$th transmitter antenna to the receiver antenna is denoted by $h_i$. The Rayleigh fading channel is modelled as samples of independent complex Gaussian random variables with variance of 0.5 per real dimension.

A sample of the received signal of the $j$th receiver antenna at time $t$ is the superposition of all signals sent from different transmitter antennas and is given by [29]

$$r_{j,t} = \sum_{i=1}^{n} h_{i,j} \cdot s_{i,t} + n_{j,t} \tag{3.2}$$

where, $n_{j,t}$ is the AWGN noise sample of a zero-mean complex Gaussian random variable at the $j$th receiver at time $t$ with variance $N_0/2$ per real dimension, where $N_0$ is the average noise energy. The average energy of the symbols transmitted from each antenna is normalized to be one.

Now for the case of Alamouti's first scheme shown in Figure 3-2 and the transmitted signal given in Table 3.1, the received signal at time slot 0 and 1 can be written respectively as

$$\begin{aligned} r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= h_0(-s_1^*) + h_1 s_0^* + n_1 \end{aligned} \tag{3.3}$$

where $n_0$ and $n_1$ are the AWGN noise samples at time slot 0 and 1, respectively.

## 3.5 Receiver structure for perfect knowledge of CSI

The original receiver proposed by Alamouti consists of a combiner and a maximum likelihood (ML) decoder. The combiner combined the received signal over two time slots and gives the resultant data to the decoder. The following two combined signals are calculated at the combiner.

$$\tilde{s_0} = \hat{h}_0^* r_0 + \hat{h}_1 r_1^*$$
$$\tilde{s_1} = \hat{h}_1^* r_0 - \hat{h}_0 r_1^*. \tag{3.4}$$

Here $\hat{h}_0$ and $\hat{h}_1$ are the estimated channel parameters at the receiver. The ML decoder receives these data and computes the distance from the data given from the combiner ($\tilde{s_0}$ and $\tilde{s_1}$) with the signals of the constellation. It decides in favor of the symbols closest to the computed result from the combiner. However, later, it has been shown that the ML decoder can use a decision metric to decide in favor of a symbol, providing same performance [29]. Assuming perfect channel state information is available, the receiver computes the following decision metric

$$\sum_{t=1}^{l} \sum_{j=1}^{m} \left| r_{j,t} - \sum_{i=1}^{n} h_{i,j} s_{i,t} \right|^2 \tag{3.5}$$

over all possible combinations of the codeword and decides in favor of the codeword that minimizes the sum. Here $l$ is the length of one block of Space-Time codes.

Now for the case of Alamouti's first model, the decision metric given in (6.2) can be simplified to the following decision rules as given in [29]:

$$|(r_0 h_0^* + r_1^* h_1) - s_0|^2 + \left(-1 + \left(|h_0|^2 + |h_1|^2\right)\right) |s_0|^2 \tag{3.6}$$

for decoding of $s_0$ and

$$|(r_0 h_1^* - r_1^* h_0) - s_1|^2 + \left(-1 + \left(|h_0|^2 + |h_1|^2\right)\right) |s_1|^2 \tag{3.7}$$

for decoding of $s_1$.

Please note that the decision rule given here assumes perfect channel knowledge

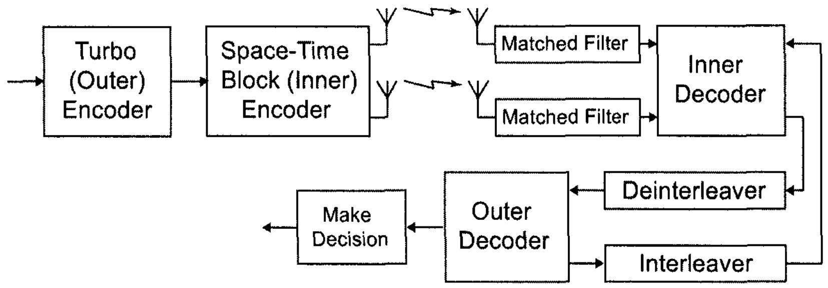


Figure 3-3: The block diagram of concatenated Turbo codes with STB codes (Transmitter and Decoder).

is available at the receiver. In practical cases, as we do not have access to the actual channel parameter $h$, we try to estimate it using channel estimation techniques. Let the estimated channel parameter be $\hat{h}$, which has a certain estimation error. Hence in (6.2), $h$ has to be replaced by estimated channel parameter $\hat{h}$, when no knowledge of the CSI is available. This mismatch causes degradation of performance compared to the perfect channel knowledge case [28]. This will be discussed in detail in the next chapter.

## 3.6 Advantages of STB coded systems

The STB code offers maximum diversity with simple transmitter and receiver structure. Another advantage of space-time code is that it can be used in concatenation with other channel codes. Two layer channel coding offers increased reliability of data transmission for highly noisy channels. An example is Turbo code as outer code and space-time code as inner code as shown in Figure 3-3 [2, 15, 25]. This kind of concatenation provides very high coding gain in both additive white Gaussian noise (AWGN) and Rayleigh faded channel. However, it has been shown that in the presence of one receiver antenna, little can be gained in terms of outage capacity by using more than four transmitter antennas. A similar argument shows that if there are two receiver antennas, almost all the capacity increase can be obtained using six transmitter antennas [6,27].

The STB coding scheme is very suitable to achieve diversity gain with relatively low complexity. However, the scheme performs poorly in the absence of the perfect channel knowledge of CSI. The effect of incorrect channel knowledge is discussed in the next chapter.

# Chapter 4

# Effect of imperfect channel estimation

The derivation of the decision rule for the decoding of the STB code, as given in [1,29], assumes that the receiver has perfect knowledge of the channel state information (CSI). To find the decision rule, the probability density function (pdf) of the received samples is derived. This system model is only valid when the receiver has the perfect knowledge of CSI. However, in practice, the receiver never has the perfect knowledge of the channel, as the channel parameters are random variables. The decoding of the STB code for practical implementation requires the knowledge of the MIMO channel parameters at the receiver end. In such situation, for practical implementation of the STB codes, the channel parameters are estimated using a channel estimation scheme. Simplicity and reliability of the orthogonal pilot sequence insertion (O-PSI) method makes it one of the state-of-the-art technique and is used in the present wireless devices [18]. However, all known techniques of the channel estimation suffer from certain estimation errors due to the noise at the receiver. From this point of view, all practical communication systems can be considered as systems with imperfect channel knowledge. This estimation error of the channel parameters cause performance degradation of the STB coded system in terms of error rates. In this chapter, different channel estimation techniques and the effect due to the imperfectness of the estimation of the channel fading parameters are discussed.

## 4.1 Channel estimation model

A wireless channel has random fading parameters, which distort both amplitude and phase of the signal. The commonly observed channel fading is Rayleigh fading and is given in (2.3). This fading can be modelled as samples of independent zero mean complex Gaussian random variables with variance of 0.5 per real dimension. Let the complex fading parameter be $h$. At the channel estimator, the estimated fading parameter $\hat{h}$ can be modelled as

$$\hat{h} = h + e \tag{4.1}$$

where $e$ is the complex estimation error samples due to the AWGN noise in the receiver. Hence the probability distribution of the error $e$ is also zero mean complex Gaussian random variables. Thus the probability distribution of the estimated channel parameter $\hat{h}$ is zero mean complex Gaussian distributed, too. The variance of the distribution of the estimated channel parameter is the summation of the variances of the actual fading parameter and that of the estimation error.

This generally accepted model of the estimated fading parameter is used in this work. Among different types of channel estimation technique, we used the O-PSI method and STB coded data aided channel estimation in this research work. The above mentioned model is valid for both methods as shown in the next section.

## 4.2 Channel estimation techniques

Among different types of channel estimation techniques for MIMO channel fading parameters estimation, one of the simplest and efficient method is to insert some orthogonal pilot sequences within a data frame. The method is called orthogonal pilot sequence insertion (O-PSI) method [18]. Another approach, special to the STB code, is to exploit inherent orthogonal property of the STB coded data [12]. These methods are discussed in this section and shown that the model given by (4.1) is valid for both of these models.

### 4.2.1 O-PSI method

O-PSI is a simple but powerful technique for channel estimation. In this method, some pilot sequences are inserted at the beginning (or middle) of a data frame. The receiver has perfect knowledge of the positions and magnitudes of the pilot sequences. For multiple transmit antennas, the pilot sequence of any transmitter must be orthogonal to other pilot sequences from other transmitter antennas to simplify channel estimator structure. For a system with $n$ transmit antennas, $n$ different pilot sequences $P_1, P_2, ..., P_n$ with the same length are needed. Let $k$ is the length of the pilot sequences, i.e., $P_i = [P_{i,1}\ P_{i,2}\ ...\ P_{i,k}]^T$ for the $i$th transmitter. To satisfy the orthogonality property, the pilot sequence of the $i$th transmitter has to satisfy the condition

$$P_i^T \cdot P_j^* = \begin{cases} 0 & \text{for } i \neq j \\ \|P_i\|^2 & \text{for } i = j \end{cases}$$

where $j$ is any other transmitter antenna. Here $(\cdot)^T$ denotes transpose and $(\cdot)^*$ denotes complex conjugate.

The receiver isolates the received signals due to the pilot symbols and sends those to the channel estimator for initial estimation of the channel before it decodes the received signals due to the data symbols. During the channel estimation, the received signal at the the receiver antenna at time $t$ can be represented by

$$r_t = \sum_{i=1}^{n} h_i \cdot P_{i,t} + n_t. \tag{4.2}$$

The received signal and noise sequence at the antenna can be represented as

$$r_p = [r_1 \, r_2 \, ... \, r_k]^T, \tag{4.3}$$

$$n_p = [n_1 \, n_2 \, ... \, n_k]^T. \tag{4.4}$$

The receiver estimates the channel fading parameter $h_i$ by using the observed sequences $r_p$. Since the pilot sequences $P_1, P_2, ..., P_n$ are orthogonal, the minimum mean square error (MMSE) estimate of $h_i$ is given by [28]

$$\begin{aligned}
\hat{h}_i &= r_p^T \cdot P_i^* / \|P_i\|^2 \\
&= (h_i P_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} h_j P_j + n_p)^T \cdot P_i^* / \, \|P_i\|^2 \\
&= [h_i (P_i^T \cdot P_i^*) + \sum_{\substack{j=1 \\ j \neq i}}^{n} h_j (P_j^T \cdot P_i^*) + (n_p^T \cdot P_i^*)] / \, \|P_i\|^2 \\
&= h_i + (n_p^T \cdot P_i^*) / \|P_i\|^2 \\
&= h_i + e_i
\end{aligned} \tag{4.5}$$

where $e_i$ is the estimation error due to the noise, given by

$$e_i = (n_p^T \cdot P_i^*) / \|P_i\|^2. \tag{4.6}$$

Since $n_p$ is a zero-mean complex Gaussian random variable with single-sided power spectral density $N_0$, the estimation error $e_i$ has a zero mean and single-sided power spectral density $N_0/k$ [28].

### 4.2.2 STB coded data

The generator matrices of all STB codes (either full rate or partial rate) are formed so that, for a block, the data symbol sequence of any transmitter antenna is orthogonal to the sequences of other transmitter antennas [1,29]. For example, with two transmitter antennas, the full rate generator matrix is given as

$$G = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \tag{4.7}$$

where each column, which represents the signals transmitted from an antenna at different time slots, is orthogonal to the other columns. Here $x$ denotes the data symbols. The transmitted signal sequences, in this case, are $s_1 = [s_{1,1}\ s_{1,2}]^T = [x_1\ -x_2^*]^T$ and $s_2 = [s_{2,1}\ s_{2,2}]^T = [x_2\ x_1^*]^T$. In general, if $s_i$ is the transmitted signal sequence of the $i$th transmitter for a block, then

$$s_i^T \cdot s_j^* = \begin{cases} 0 & \text{for } i \neq j \\ \|s_i\|^2 & \text{for } i = j \end{cases}$$

where $s_j$ is the transmitted signal sequence from any other transmitter antenna for the same block. This orthogonality of the transmitted symbol sequences is guaranteed due to the inherent orthogonal property of the generator matrices.

The inherent orthogonal property of the generator matrix makes the channel estimator simpler using MMSE criterion. When a block of the transmitted symbol sequence of the $i$th transmitter antenna is multiplied with the corresponding received signal, fading parameters of the other paths vanish. This property enables one to easily obtain the channel fading parameter of the corresponding transmission path.

Let the received signal vector for a block be $r_s = [r_1\ r_2\ ...\ r_q]^T$. As transmitted symbol sequence is not known to the receiver, a previous estimate of the channel fading parameter has to be used to decode the received signal of the block. The detected symbol sequence of a transmitter antenna is then utilized to estimate corresponding channel fading parameter. As for a quasi-static channel, the fading parameters of any block is the same during the whole frame, so this estimated fading parameters using data block can be used for the next iteration of the received signal of the same frame. Now, if the detected symbol sequence is correct, i. e. $\hat{s}_i = s_i$, then the estimate of desired channel parameter is the same as O-PSI method. The estimated channel fading parameter becomes

$$\begin{aligned}
\hat{h}_i &= r_s^T \cdot \hat{s}_i^* / \|\hat{s}_i\|^2 \\
&= (h_i s_i + \sum_{j=1, j\neq i}^{n} h_j s_j + n_s)^T \cdot s_i^* / \|s_i\|^2 \quad [\text{as } \hat{s}_i = s_i] \\
&= [h_i(s_i^T \cdot s_i^*) + \sum_{j=1, j\neq i}^{n} h_j(s_j^T \cdot s_i^*) + (n_s^T \cdot s_i^*)] / \|s_i\|^2 \\
&= h_i + (n_s^T \cdot s_i^*)/\|s_i\|^2 \\
&= h_i + e_i
\end{aligned} \tag{4.8}$$

where $n_s = [n_1\ n_2\ ...\ n_q]^T$ and $e_i$ is again the estimation error due to the AWGN noise given by

$$e_i = (n_s^T \cdot s_i^*)/\|s_i\|^2. \tag{4.9}$$

On the other hand, if an incorrect detection of a block of data occurs, i. e. $\hat{s}_i \neq s_i$, then the estimated channel fading parameter using that block also becomes incorrect. Let us assume that the channel fading parameter of wireless path from the $i$th transmitter antenna to the receiver antenna is to be found. Also assuming that due to incorrect detection, the detected block of data is the sequence sent from the $k$th transmitter antenna (i.e. $\hat{s}_i = s_k$). Then the obtained channel parameter becomes

$$\begin{aligned}
\hat{h}_i &= r_s^T \cdot \hat{s}_i^* / \|\hat{s}_i\|^2 \\
&= (h_i s_i + h_k s_k + \sum_{j=1, j \neq i,k}^{n} h_j s_j + n_s)^T \cdot s_k^* / \|s_k\|^2 \\
&= [h_i(s_i^T \cdot s_k^*) + h_k(s_k^T \cdot s_k^*) \\
&\quad + \sum_{j=1, j \neq i,k}^{n} h_j(s_j^T \cdot s_k^*) + (n_s^T \cdot s_k^*)] / \|s_k\|^2 \\
&= h_k + (n_s^T \cdot s_k^*) / \|s_k\|^2 \\
&= h_k + e_k \qquad (4.10)
\end{aligned}$$

where $e_k$ is the corresponding estimation error due to the AWGN noise given as

$$e_k = (n_s^T \cdot s_k^*) / \|s_k\|^2. \qquad (4.11)$$

If the $k$th sequence is not produced by any transmitter antenna, then the estimated channel parameter does not even exist.

It is worthwhile to mention here that the channel estimation technique using the data blocks of the STB coded system is only performed in the iterative channel estimator. This is because the method requires a previous estimate of the channel to decode the data sequences at the beginning, and only improves that previous estimation by providing another estimate of the channel.

## 4.3 Effects of imperfect channel estimates

The effect of the estimation error can be best shown by using the equation (3.4) used in the combiner. For a receiver designed without the combiner, however, the effect of the estimation error remains the same.

The cross terms in the received signals are removed at the combiner by using the combining scheme defined in (3.4). Assuming that the receiver has perfect knowledge of the channel, $\hat{h}_0 = h_0$ and $\hat{h}_1 = h_1$, we get

$$\begin{aligned}
\tilde{s_0} &= h_0^* r_0 + h_1 r_1^* \\
&= h_0^*(h_0 s_0 + h_1 s_1 + n_0) + h_1(h_0(-s_1^*) + h_1 s_0^* + n_1)^* \\
&= |h_0|^2 s_0 + h_0^* h_1 s_1 - h_1 h_0^* s_1 + |h_1|^2 s_0 + h_0^* n_0 + h_1 n_1^* \\
&= (|h_0|^2 + |h_1|^2) s_0 + h_0^* n_0 + h_1 n_1^* \qquad (4.12)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{s_1} &= h_1^* r_0 - h_0 r_1^* \\
&= h_1^*(h_0 s_0 + h_1 s_1 + n_0) - h_0(h_0(-s_1^*) + h_1 s_0^* + n_1)^* \\
&= h_1^* h_0 s_0 + |h_1|^2 s_1 + |h_0|^2 s_1 - h_0 h_1^* s_0 + h_1^* n_0 + h_0 n_1^* \\
&= (|h_0|^2 + |h_1|^2) s_1 + h_1^* n_0 + h_0 n_1^*. \qquad (4.13)
\end{aligned}$$

Thus the cross terms are perfectly removed for the ideal case of perfect channel knowledge. However, for the practical cases, where channel fading parameters are estimated using a channel estimation process having error model as given by (4.1), we have

$$\begin{aligned}
\tilde{s_0} &= \hat{h}_0^* r_0 + \hat{h}_1 r_1^* \\
&= (h_0^* + e_0^*)(h_0 s_0 + h_1 s_1 + n_0) - (h_1 + e_1)(h_0(-s_1^*) + h_1 s_0^* + n_1)^* \\
&= |h_0|^2 s_0 + h_0^* h_1 s_1 - h_1 h_0^* s_1 + |h_1|^2 s_0 + e_0^* h_0 s_0 + e_0^* h_1 s_1 - e_1 h_0^* s_1 + e_1 h_1^* s_0 \\
&\qquad + h_0^* n_0 + e_0^* n_0 + h_1 n_1^* + e_1 n_1^* \\
&= (|h_0|^2 + |h_1|^2) s_0 + (e_0^* h_0 + e_1 h_1^*) s_0 + (e_0^* h_1 - e_1 h_0^*) s_1 \\
&\qquad + h_0^* n_0 + h_1 n_1^* + e_0^* n_0 + e_1 n_1^* \qquad (4.14)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{s_1} &= \hat{h}_1^* r_0 - \hat{h}_0 r_1^* \\
&= (h_1^* + e_1^*)(h_0 s_0 + h_1 s_1 + n_0) + (h_0 + e_0)(h_0(-s_1^*) + h_1 s_0^* + n_1)^* \\
&= h_1^* h_0 s_0 + |h_1|^2 s_1 - |h_0|^2 s_1 + h_0 h_1^* s_0 + e_1^* h_0 s_0 + e_1^* h_1 s_1 - e_0 h_0^* s_1 + e_0 h_1^* s_0 \\
&\qquad + h_1^* n_0 + e_1^* n_0 + h_0 n_1^* + e_0 n_1^* \\
&= (|h_0|^2 + |h_1|^2) s_1 + (e_1^* h_0 + e_0 h_1^*) s_0 + (e_1^* h_1 - e_0 h_0^*) s_1 \\
&\qquad + h_1^* n_0 + h_0 n_1^* + e_1^* n_0 + e_0 n_1^*. \qquad (4.15)
\end{aligned}$$

Thus it is seen that the computed results have enhanced noise term and interference from the cross term symbol. Because of this reason, the effect of the estimation error of the fading parameter is very severe in the decoding of the STB coded system compared to the receiver diversity scheme [3]. As the STB code becomes attractive due to its simplicity of the decoding process, this issue has to be resolved ahead of efficient practical implementation of the scheme.

This thesis proposes modification of the state-of-the-art for both cases of partial knowledge of CSI and no knowledge of CSI. The next chapter deals with the case of partial knowledge of CSI and a new decision rule is proposed. The following chapter considers the case of no knowledge of CSI and an improved iterative channel estimator for STB coded system is proposed.

# Chapter 5

# Proposed decision rule for imperfect channel estimates

In this chapter, Alamouti's first model of the STB coded system with 2 transmitter antennas and 1 receiver antenna is considered. It is assumed that the receiver has partial knowledge of CSI. The estimation error model described in (4.1) has been used. The variance of the estimation error is assumed to be known. A simple decision rule is derived from the exact pdf of the received signal samples conditioned on the estimated channel parameters. The proposed decision rule is shown to be the same as the one for the ideal case when perfect knowledge of CSI is available. It is also shown that the proposed decision rule and Tarokh's decision rule becomes identical for very high SNR. The proposed decision rule shows at least same or even better performance in terms of the error rate than the one proposed by Tarokh. Moreover, the complexity of the proposed scheme compared to the Tarokh's scheme is much less from the implementation point of view, providing high gain in terms of the hardware requirements and prossing time.

## 5.1 Background

The derivation of the decision rule given in (6.2), (3.6) and (3.7) for the decoding of the STB code, assumes that the receiver has perfect knowledge of CSI is available at the receiver [1, 29]. In practice, however, the receiver never has the perfect knowledge of CSI, as the channel parameters are random variables. The parameters are estimated using a channel estimation technique as the decision rule requires the knowledge of these parameters. Thus all practical communication systems can be considered as systems with imperfect channel knowledge. For such STB coded systems with imperfect channel knowledge, if we employ the decision rule for perfect knowledge of CSI using imperfect channel parameter estimates in place of actual channel parameters, performance degradation of the whole system is observed due to mismatch. Performance degradation due to this type of mismatch in the channel parameters in the decision rule has been addressed in the standard literature [33]. It is shown in [3] that STB code is more sensitive to the channel estimation error than straightforward two branch diversity schemes, because of their dependency on the removal of the cross-terms in the decision rule. This dependency on the channel estimation error increases as the number of transmitter and receiver antenna increases to achieve high error performance [9].

To resolve this issue of mismatch due to the imperfect channel estimate, the case of partial knowledge of CSI was discussed and a modified decision rule was proposed by Tarokh [26, 28]. The partial knowledge of CSI utilized was the variance of the estimation error of the channel parameters, which can be easily and reliably obtained. Recently, however, it has been found that the scheme is only applicable when the SNR approaches infinity [30] and approximate for the practical range of SNR.

A systematic approach to include variance of the channel estimation error has been done by Frenger in [8] for Turbo coded systems. A similar approach is taken here to calculate a new metric for the STB coded system proposed by Alamouti.

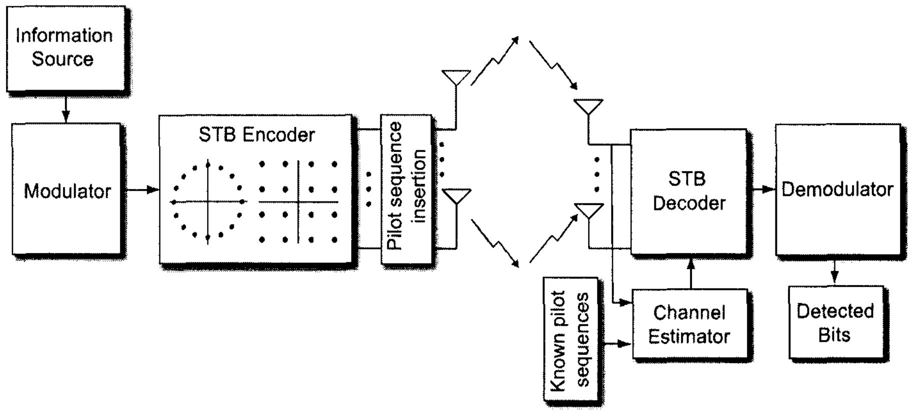


Figure 5-1: Simplified block diagram of the STB coded communication system.

This decision rule can easily be generalized for similar type of systems. The proposed modified decision rule performs equal or supersedes Tarokh's decision rule with much less complexity.

## 5.2 System model

A wireless communication system is considered with $n$ transmitter antennas at the base station and one receiver antenna at the remote station. A simplified block diagram is given in Figure 5-1. Extension of formulations for $m$ receiver antennas is straightforward. Data is modulated in the modulator before encoding it with the STB encoder. The encoder uses a generator matrix to encode the modulated data into different transmitter sequences, maintaining orthogonality of the sequences. Each of the $n$ transmitter antenna simultaneously transmit one symbol $s_{i,t}, i = 0, 1, 2, \cdots, n-1$ at any time slot $t$. The received signals at the receiver are combined and then decoded using the maximum likelihood (ML) decoding algorithm. These signals are then demodulated to obtain the data bits.

We assume a flat fading wireless channel with the path gain defined to be $h_i$ from the transmitter antenna $i$ to the receiver antenna. The path gains are modelled as samples of zero mean, independent complex Gaussian random variables with the variance defined as $E[|h_i|^2] = 2\sigma_h^2$, where $E[x]$ denotes the expected value of $x$. Furthermore, the wireless channel is assumed to be quasi-static so that the path gains are constant over a frame of length $L$ and vary from one frame to another. This incorporates the required assumption of the STB decoder to have constant fading for all the symbols of a block of transmitted symbol sequences having length of $l$. A frame consists of an integer number of blocks.

We consider the STB coded system considered in [1] with 2 transmitter antennas and one receiver antenna. Data are encoded with STB encoder using the generator matrix

$$\mathbf{G} = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix} \tag{5.1}$$

where each column represents the signals transmitted from a particular antenna at different time slots and each row represents the signal vector transmitted from all transmitter antennas at a particular time slot. Here $x^*$ denotes the complex conjugate of $x$.

After sampling of the received signal using the matched filter at the receiver, we have samples of the received signals [32]. Let the received signal at time slot 1 be $r_1$ and at time slot 2 be $r_2$, and the corresponding additive white Gaussian noise (AWGN) in the receiver be $n_1$ and $n_2$, respectively. So, the received signal model can be written in vector form as

$$\mathbf{r} = \mathbf{G}\mathbf{h} + \mathbf{n} \tag{5.2}$$

where $\mathbf{r} = [r_1 \; r_2]^T$, $\mathbf{h} = [h_1 \; h_2]^T$ and $\mathbf{n} = [n_1 \; n_2]^T$. Here $[\cdot]^T$ denotes transpose of the matrix or vector. Noise component $\mathbf{n}$ is a vector of complex valued Gaussian distributed elements with zero mean and the variance defined as $E[|n_1|^2] = E[|n_2|^2 =$

$2\sigma_n^2$. Actual fading parameter $\mathbf{h}$ is also a vector of complex valued Gaussian distributed elements with mean zero and the variance $E[|h_1|^2] = E[|h_2|^2 = 2\sigma_h^2$. All real and imaginary parts of $\mathbf{h}$ and $\mathbf{n}$ are assumed to be independent. It is straightforward to show that the variance of the sampled zero mean received signal becomes $E[|r_1|^2] = E[|r_2|^2] = 2\sigma_r^2 = 2\sigma_h^2(|s_1|^2 + |s_2|^2) + 2\sigma_n^2$.

Now, as shown in the previous chapter, the estimated channel parameter model can be expressed as

$$\hat{\mathbf{h}} = \mathbf{h} + \mathbf{e} \tag{5.3}$$

where the estimated channel vector $\hat{\mathbf{h}} = [\hat{h}_1\ \hat{h}_2]^T$ and the error vector $\mathbf{e} = [e_1\ e_2]^T$. Here $\mathbf{e}$ is complex valued Gaussian distributed estimation error with $E[|e_1|^2] = E[|e_2|^2] = 2\sigma_e^2$ and $E[\mathbf{e}] = 0$. Assuming $\mathbf{h}$ and $\mathbf{e}$ to be independent, we find that $\hat{\mathbf{h}}$ is also a vector of complex valued Gaussian distributed random variables with zero mean and variance defined as $E[|\hat{h}_1|^2] = E[|\hat{h}_2|^2] = 2\sigma_{\hat{h}}^2 = 2\sigma_h^2 + 2\sigma_e^2$. It is shown in [5] that this channel estimate model is valid for pilot-based channel estimation schemes. Furthermore, in [7], this model is shown to hold for decision-directed channel estimation schemes, assuming that the previous data symbols used for channel estimation were correctly detected. This model can be used in other channel estimation models as well.

Here, we use a pilot based channel estimation technique, where the channel fading coefficients are estimated by inserting orthogonal pilot sequences in the transmitted signals. In this method, some pilot sequences are inserted at the beginning (or middle) of a data frame. The receiver has perfect knowledge of the positions and magnitudes of the pilot sequences. For multiple transmit antennas, the pilot sequence of any transmitter antenna must be orthogonal to other pilot sequences from other transmitter antennas to simplify channel estimator structure. For a system with $n$

transmitter antennas, $n$ different pilot sequences $P_1, P_2, ..., P_n$ with the same length are needed. Let $k$ be the length of the pilot sequences, i.e., $P_i = [P_{i,1}\ P_{i,2}\ ...\ P_{i,k}]^T$ for the $i$th transmitter. To satisfy the orthogonality property, the pilot sequence of the $i$th transmitter has to satisfy the condition

$$P_i^T \cdot P_j^* = \begin{cases} 0 & \text{for } i \neq j \\ \|P_i\|^2 & \text{for } i = j \end{cases}$$

where $j$ is any other transmitter antenna.

The receiver isolates the received signals due to the pilot symbols and sends those to the channel estimator for initial estimation of channel before decoding of the received signals due to data symbols. During the channel estimation, the received signal at the the receiver antenna at time $t$ can be represented by

$$r_t = \sum_{i=1}^{n} h_i \cdot P_{i,t} + n_t. \tag{5.4}$$

The received signal and noise sequence at the antenna can be represented as

$$r_p = [r_1\ r_2\ ...\ r_k]^T, \tag{5.5}$$

$$n_p = [n_1\ n_2\ ...\ n_k]^T. \tag{5.6}$$

The receiver estimates the channel fading parameter $h_i$ by using the observed sequences $r_p$. Since the pilot sequences $P_1, P_2, ..., P_n$ are orthogonal, the minimum mean square error (MMSE) estimate of $h_i$ is given by [28]

$$\begin{aligned} \hat{h}_i &= r_p^T \cdot P_i^* / \|P_i\|^2 \\ &= (h_i P_i + \sum_{j=1, j\neq i}^{n} h_j P_j + n_p)^T \cdot P_i^* / \|P_i\|^2 \end{aligned}$$

$$= [h_i(P_i^T \cdot P_i^*) + \sum_{j=1, j\neq i}^{n} h_j(P_j^T \cdot P_i^*) + (n_p^T \cdot P_i^*)] / \|P_i\|^2$$
$$= h_i + (n_p^T \cdot P_i^*)/\|P_i\|^2$$
$$= h_i + e_i \tag{5.7}$$

where $e_i$ is the estimation error due to the noise, given by

$$e_i = (n_p^T \cdot P_i^*)/\|P_i\|^2. \tag{5.8}$$

Since $n_p$ is a zero-mean complex Gaussian random variable with single-sided power spectral density $N_0$, the estimation error $e_i$ has a zero mean and single-sided power spectral density $N_0/k$ [28].

In order to derive the modified decision metric, we need to know the exact pdf of the received signal conditioned on the estimated channel parameters and transmitted symbol sequences. To simplify our calculations, the following cross correlation coefficients are defined:

$$\mu_{11} = \frac{E[r_1 h_1^*]}{\sqrt{\text{var}(r_1)\text{var}(h_1)}}$$
$$\mu_{12} = \frac{E[r_1 h_2^*]}{\sqrt{\text{var}(r_1)\text{var}(h_2)}}$$
$$\mu_{21} = \frac{E[r_2 h_1^*]}{\sqrt{\text{var}(r_2)\text{var}(h_1)}}$$
$$\mu_{22} = \frac{E[r_2 h_2^*]}{\sqrt{\text{var}(r_2)\text{var}(h_2)}}.$$

It can be easily shown that $\mu_{11} = \mu_{22}^* = s_1\sigma_h^2/(\sigma_r\sigma_{\hat{h}})$ and $\mu_{12} = -\mu_{21}^* = s_2\sigma_h^2/(\sigma_r\sigma_{\hat{h}})$. We further define

$$|\mu|^2 = |\mu_{ij}|^2 + |\mu_{mn}|^2$$

$$= (|s_1|^2 + |s_2|^2)\frac{\sigma_h^4}{\sigma_r^2\sigma_{\hat{h}}^2} \quad (5.9)$$

where $i, j, m, n \in \{1, 2\}$ with the condition that if $i = m$, then $j \neq n$ or vice versa.

As shown in the Appendix, the required pdf can be expressed as

$$p_{r|\hat{h},s}(R|\hat{H}, S) = \frac{1}{(2\pi)^2\sigma_r^4\left(1 - |\mu|^2\right)^2} \exp\left[\frac{-1}{2\sigma_r^2\left(1 - |\mu|^2\right)} \left(\left|r_1 - \left(\mu_{11}\hat{h}_1 + \mu_{12}\hat{h}_2\right)\frac{\sigma_r}{\sigma_h}\right|^2 + \left|r_2 - \left(\mu_{11}^*\hat{h}_2 - \mu_{12}^*\hat{h}_1\right)\frac{\sigma_r}{\sigma_h}\right|^2\right)\right] \quad (5.10)$$

where $s$ is the vector of signals transmitted at a particular time slot.

## 5.3 Derivation of the decision metric

We can easily find that the pdf described in (5.10) can be expressed as multiplication of two pdf's, as follows

$$\begin{aligned} p_{r|\hat{h},s}(R|\hat{H}, S) =& \frac{1}{2\pi\sigma_r^2\left(1 - |\mu|^2\right)} \exp\left\{-\frac{1}{2\sigma_r^2\left(1 - |\mu|^2\right)}\left|r_1 - \left(\mu_{11}\hat{h}_1 + \mu_{12}\hat{h}_2\right)\frac{\sigma_r}{\sigma_{\hat{h}}}\right|^2\right\} \\ &\times \frac{1}{2\pi\sigma_r^2\left(1 - |\mu|^2\right)} \exp\left\{-\frac{1}{2\sigma_r^2\left(1 - |\mu|^2\right)}\left|r_2 - \left(\mu_{11}^*\hat{h}_2 - \mu_{12}^*\hat{h}_1\right)\frac{\sigma_r}{\sigma_{\hat{h}}}\right|^2\right\} \end{aligned} \quad (5.11)$$

which is in the form of

$$p_{r|\hat{h},s}(R|\hat{H}, S) = p_{r_1|\hat{h},s}(R_1|\hat{H}, S)p_{r_2|\hat{h},s}(R_2|\hat{H}, S). \quad (5.12)$$

It is obvious that the conditional distributions of $r_1$ and $r_2$ are independent, Gaussian distributed with conditional expected values of

$$E\left[\left|r_1|\hat{h}, s\right|\right] = (\mu_{11}\hat{h}_1 + \mu_{12}\hat{h}_2)\frac{\sigma_r}{\sigma_{\hat{h}}}, \quad (5.13)$$

$$E\left[\left|r_2|\hat{h},s\right|\right] = (\mu_{11}^*\hat{h}_2 - \mu_{12}^*\hat{h}_1)\frac{\sigma_r}{\sigma_{\hat{h}}}. \quad (5.14)$$

The conditional variances are equal and given as

$$E\left[\left|r_1|\hat{h},s\right|^2\right] = E\left[\left|r_2|\hat{h},s\right|^2\right] = 2\sigma_r^2(1-|\mu|^2). \quad (5.15)$$

Assuming that all the signals in the modulation constellation are equiprobable, a ML decoder chooses a pair of signals from the signal modulation constellation to minimize the distance metric

$$d^2(r_1, E[\left|r_1|\hat{h},\hat{s}\right|]) + d^2(r_2, E[\left|r_2|\hat{h},\hat{s}\right|]) \quad (5.16)$$

over all possible values of $\hat{s}$, the detected signal sequence vector. Here $d^2(x,y)$ is the squared Euclidean distance between signals $x$ and $y$ calculated using the expression $d^2(x,y) = (x-y)(x^*-y^*)$.

Putting values in (5.16) leads us to the minimization problem of the following distance metric

$$|r_1 - (\mu_{11}\hat{h}_1 + \mu_{12}\hat{h}_2)\frac{\sigma_r}{\sigma_{\hat{h}}}|^2 + |r_2 - (\mu_{11}^*\hat{h}_2 - \mu_{12}^*\hat{h}_1)\frac{\sigma_r}{\sigma_{\hat{h}}}|^2 \quad (5.17)$$

for all transmitted symbol sequences.

After expanding the above metric and deleting the terms independent of the transmitted symbols, we reach the following equivalent metric to be minimized

$$\begin{aligned}
&\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}(-r_1\hat{h}_1^*s_1^* - r_1^*\hat{h}_1s_1 - r_1\hat{h}_2^*s_2^* - r_1^*\hat{h}_2s_2 - r_2\hat{h}_2^*s_1 \\
&- r_2^*\hat{h}_2s_1^* + r_2\hat{h}_1^*s_2 + r_2^*\hat{h}_1s_2^*) + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}(|s_1|^2|\hat{h}_1|^2 \\
&+ |s_2|^2|\hat{h}_2|^2 + |s_1|^2|\hat{h}_2|^2 + |s_2|^2|\hat{h}_1|^2).
\end{aligned}$$

We can decompose this term into two parts for the sake of the simplicity of the detection process, one of which

$$\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\left(-(r_1\hat{h}_1^* + r_2^*\hat{h}_2)s_1^* - (r_1^*\hat{h}_1 + r_2\hat{h}_2^*)s_1\right) + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(|\hat{h}_1|^2 + |\hat{h}_2|^2\right)|s_1|^2$$

is a function of $s_1$ only, and the other one

$$\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}\left(-(r_1\hat{h}_2^* - r_2^*\hat{h}_1)s_2^* - (r_1^*\hat{h}_2 - r_2\hat{h}_1^*)s_2\right) + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}\left(|\hat{h}_1|^2 + |\hat{h}_2|^2\right)|s_2|^2$$

is a function of $s_2$ only. Thus the minimization problem given in (5.17) is reduced to minimizing these two parts separately. This leads us to a faster decoding process with less complexity, especially with higher order modulation schemes. After some rearrangement and manipulation of the above two expressions, we reach the decision metric

$$|(r_1\hat{h}_1^* + r_2^*\hat{h}_2)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_1|^2 + (-1 + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}(|\hat{h}_1|^2 + |\hat{h}_2|^2))|s_1|^2 \tag{5.18}$$

for detecting $s_1$ and the decision metric

$$|(r_1\hat{h}_2^* - r_2^*\hat{h}_1)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_2|^2 + (-1 + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}(|\hat{h}_1|^2 + |\hat{h}_2|^2))|s_2|^2 \tag{5.19}$$

for detecting $s_2$. These are the desired modified decision rules and are used in our simulations for a STB coded system with the imperfect channel estimates.

For the ideal case in which we have perfect knowledge of CSI at the receiver, hence no estimation error, we have $\sigma_h^2 = \sigma_{\hat{h}}^2$ as $\sigma_e^2 = 0$ and $h_i = \hat{h}_i$, where $i \in \{1, 2\}$. Consequently (5.18) and (5.19) becomes the same as the decision rules for the perfect

| | for detection of $s_1$ | for detection of $s_2$ |
|---|---|---|
| known CSI | $\|(r_1 h_1^* + r_2^* h_2) - s_1\|^2 + (-1 + (\|h_1\|^2 + \|h_2\|^2))\|s_1\|^2$ | $\|(r_1 h_2^* - r_2^* h_1) - s_1\|^2 + (-1 + (\|h_1\|^2 + \|h_2\|^2))\|s_2\|^2$ |
| unknown CSI | $\|(r_1 \hat{h}_1^* + r_2^* \hat{h}_2)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_1\|^2 + (-1 + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}(\|\hat{h}_1\|^2 + \|\hat{h}_2\|^2))\|s_1\|^2$ | $\|(r_1 \hat{h}_2^* - r_2^* \hat{h}_1)\frac{\sigma_h^2}{\sigma_{\hat{h}}^2} - s_2\|^2 + (-1 + \frac{\sigma_h^4}{\sigma_{\hat{h}}^4}(\|\hat{h}_1\|^2 + \|\hat{h}_2\|^2))\|s_2\|^2$ |

Table 5.1: Decision rules for the ideal case of known CSI and the proposed scheme of unknown CSI.

channel knowledge, which are

$$|(r_1 h_1^* + r_2^* h_2) - s_1|^2 + (-1 + (|h_1|^2 + |h_2|^2))|s_1|^2$$

for detecting $s_1$ and

$$|(r_1 h_2^* - r_2^* h_1) - s_1|^2 + (-1 + (|h_1|^2 + |h_2|^2))|s_2|^2$$

for detection $s_2$ as given in (3.6) and (3.7). Table 5.1 is provided for comparison of the two decision metric, i.e. for the ideal case of known CSI and for the partial knowledge of CSI.

## 5.4 Comparison with Tarokh's decision metric

We now compare the proposed metric with the state-of-the-art metric given by Tarokh [26, 28] in the presence of the channel estimation error. Originally derived for the space-time trellis codes, the metric is however generally accepted for the STB codes. The mean and variance of the distribution function of the random variable $r_t$ conditioned on $\hat{h}_i$ as given by Tarokh are $\mu_{r'} = \mu'/(\sqrt{2}\sigma_{\hat{h}})\sqrt{E_s}\sum_{i=1}^{n} \hat{s}_{i,t}\hat{h}_i$ and $\sigma_{r'}^2 = N_0 + (1 - |\mu'|^2)E_s \sum_{i=1}^{n} |\hat{s}_{i,t}^2|$, respectively. Here $\mu' = 1/\sqrt{1 + 2\sigma_e^2}$ is the correlation coefficient, $N_0 = 2\sigma_n^2$ is the noise variance and $E_s$ is the energy per symbol, which is the factor by which the elements of the signal constellation are contracted to make the average energy of the constellation as 1. The decision metric proposed by Tarokh for $n$ transmitter antennas and one receiver antenna can be written as

$$\sum_{t=1}^{l}\left(\frac{\left|r_t-\frac{\mu'\sqrt{E_s}}{\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{n}\hat{s}_{i,t}\hat{h}_i\right|^2}{N_0+(1-|\mu'|^2)E_s\sum_{i=1}^{n}|\hat{s}_{i,t}|^2}+\ln(N_0+(1-|\mu'|^2)E_s\sum_{i=1}^{n}|\hat{s}_{i,t}|^2)\right). \tag{5.20}$$

For the case of PSK (phase shift keying) constellation, the metric given in (5.20) is reduced to the following:

$$\sum_{t=1}^{l}\left|r_t-\frac{\mu'\sqrt{E_s}}{\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{n}\hat{s}_{i,t}\hat{h}_i\right|^2. \tag{5.21}$$

According to a recent publication of Tarokh, these expressions are only valid for very high SNR [30]. For SNR of infinite, $N_0/E_s \to 0$, i.e. for a certain $E_s$, $N_0 \to 0$. Again, for the case of normalized Rayleigh fading channel, which is assumed by Tarokh, we have $\sigma_{\hat{h}}^2 = 0.5$ and $\sqrt{E_s} = 1$. Considering the case of 2 transmitter antennas and 1 receiver antenna, the mean of the pdf of the received data conditioned on the estimated channel parameters becomes

$$\begin{aligned}\mu_{r'} &= \frac{\mu'\sqrt{E_s}}{\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{2}\hat{s}_i\hat{h}_i\\ &= \frac{1}{\sqrt{1+2\sigma_e^2}\sqrt{2}\sigma_{\hat{h}}}\sum_{i=1}^{2}\hat{s}_i\hat{h}_i\\ &= \frac{1}{\sqrt{2\times 0.5+2\sigma_e^2}\sqrt{2\sigma_{\hat{h}^2}}}\sum_{i=1}^{2}\hat{s}_i\hat{h}_i\\ &= \frac{1}{\sqrt{2\sigma_{\hat{h}}^2}\sqrt{2\sigma_{\hat{h}}^2}}\sum_{i=1}^{2}\hat{s}_i\hat{h}_i\\ &= \frac{1}{2\sigma_{\hat{h}}^2}(\hat{s}_1\hat{h}_1+\hat{s}_2\hat{h}_2).\end{aligned} \tag{5.22}$$

Assuming correct detection, for $r_1$, the mean becomes

$$\begin{aligned}
\mu_{r_1'} &= \frac{1}{2\sigma_{\hat{h}}^2}(s_1\hat{h}_1 + s_2\hat{h}_2) \\
&= \frac{\sigma_h^2}{\sigma_{\hat{h}}^2}(\frac{\sigma_r}{\sigma_r}s_1\hat{h}_1 + \frac{\sigma_r}{\sigma_r}s_2\hat{h}_2) \\
&= \frac{\sigma_r}{\sigma_{\hat{h}}}(\frac{\sigma_h^2}{\sigma_{\hat{h}}\sigma_r}s_1\hat{h}_1 + \frac{\sigma_h^2}{\sigma_{\hat{h}}\sigma_r}s_2\hat{h}_2) \\
&= \frac{\sigma_r}{\sigma_{\hat{h}}}(\mu_{11}\hat{h}_1 + \mu_{12}\hat{h}_2)
\end{aligned} \tag{5.23}$$

which is equal to the mean given in (5.13). Similarly, for $r_2$, the mean becomes

$$\begin{aligned}
\mu_{r_2'} &= \frac{1}{2\sigma_{\hat{h}}^2}(-s_2^*\hat{h}_1 + s_1^*\hat{h}_2) \\
&= \frac{\sigma_h^2}{\sigma_{\hat{h}}^2}(\frac{\sigma_r}{\sigma_r}s_1^*\hat{h}_2 - \frac{\sigma_r}{\sigma_r}s_2^*\hat{h}_1) \\
&= \frac{\sigma_r}{\sigma_{\hat{h}}}(\frac{\sigma_h^2}{\sigma_{\hat{h}}\sigma_r}s_1^*\hat{h}_2 - \frac{\sigma_h^2}{\sigma_{\hat{h}}\sigma_r}s_2^*\hat{h}_1) \\
&= \frac{\sigma_r}{\sigma_{\hat{h}}}(\mu_{11}^*\hat{h}_2 - \mu_{12}^*\hat{h}_1)
\end{aligned} \tag{5.24}$$

which is equal to the mean given in (5.14). For the same conditions, the variance of the pdf of the received data conditioned on the estimated channel parameters becomes

$$\begin{aligned}
\sigma_{r'}^2 &= N_0 + \left(1 - |\mu'|^2\right) E_s \sum_{i=1}^{2} |\hat{s}_i|^2 \\
&= (1 - \frac{1}{1 + 2\sigma_e^2}) \sum_{i=1}^{2} |\hat{s}_i|^2 \\
&= \frac{2\sigma_e^2}{2\sigma_{\hat{h}}^2} \sum_{i=1}^{2} |\hat{s}_i|^2 \\
&= \frac{\sigma_e^2}{\sigma_{\hat{h}}^2}(|\hat{s}_1|^2 + |\hat{s}_2|^2)
\end{aligned} \tag{5.25}$$

for both $r_1$ and $r_2$. The variance of the received signal conditioned on the estimated channel parameter derived here is given in (5.15). Expanding this variance with the

| Modulation Scheme | Decision Metric | Complexity Index | Memory Requirements | Number of Comparisons |
|---|---|---|---|---|
| $p$-QAM | Proposed (5.18) (5.19) | $17\times p$ | 10 | $p(p-1)$ |
| $p$-QAM | Tarokh (5.20) | $20\times p^2$ | 13 | $p^2(p^2-1)/2$ |
| $p$-PSK | Proposed (5.18) (5.19) | $17\times p$ | 10 | $p(p-1)$ |
| $p$-PSK | Tarokh (5.21) | $11\times p^2$ | 11 | $p^2(p^2-1)/2$ |

Table 5.2: Comparison of the complexity issues of the proposed scheme with Tarokh's scheme.

conditions that $\sigma_n^2 \rightarrow 0$ as SNR goes to infinity i.e. $\sigma_r^2 = \sigma_h^2(|s_1|^2 + |s_2|^2) + \sigma_n^2 \rightarrow \sigma_h^2(|s_1|^2 + |s_2|^2)$ and $\sigma_h^2 = 0.5$, we have

$$\begin{aligned}
2\sigma_r^2(1-|\mu|^2) &= 2(\sigma_h^2(|s_1|^2+|s_2|^2))(1-(|s_1|^2+|s_2|^2)\frac{\sigma_h^4}{\sigma_r^2\sigma_{\hat{h}}^2}) \\
&= (|s_1|^2+|s_2|^2)(1-\frac{(|s_1|^2+|s_2|^2)\sigma_h^4}{\sigma_h^2(|s_1|^2+|s_2|^2)\sigma_{\hat{h}}^2}) \\
&= (|s_1|^2+|s_2|^2)(1-\frac{\sigma_h^2}{\sigma_{\hat{h}}^2}) \\
&= (|s_1|^2+|s_2|^2)\frac{\sigma_e^2}{\sigma_{\hat{h}}^2} \qquad (5.26)
\end{aligned}$$

which is exactly equal to (5.25). Hence it is found that the proposed scheme becomes the same as the scheme proposed by Tarokh for high SNR.

To address the complexity issues and memory requirements of the proposed scheme compared with that of Tarokh's scheme, some assumptions are made in order to simplify our comparative study. In this simplified approach, we are interested only in the complexity of the decision rule as the difference of the two schemes lies in the decision rules. Since multiplications and divisions are the most complex basic operations in designing the signal processors, we base our calculations on the requirements of these operations in the decision rules. We assume the complexity of a multiplier, a divisor, a squaring or a logarithm operation be the same. The complexity index is calculated by simply adding the required number of multiplications, divisions, squaring and logarithm operations. Table 5.2 gives the comparison of the proposed scheme with Tarokh's scheme for $p$-QAM and $p$-PSK modulation, where $p$ is the number of states of the modulation scheme. The metrics are to be computed for each possible combination of the signals transmitted for each state of the modulation scheme. Assume that factors are to be calculated once and then stored for reuse in the later computations, rather than calculating each time. This results in minimized processing time by reducing the number of multiplications with increasing storage size. The metrics proposed by Tarokh have to be computed $p^2$ times for different combinations of $\hat{s}_1$ and $\hat{s}_2$ and compared with each other. However, in the proposed scheme, each metric has to be computed only $p$ times because of the variable separation operation during the derivation of the metrics. This is why the complexity of the decision rule of the proposed scheme is much lower than that of Tarokh's scheme.

# Chapter 6

# Iterative channel estimation technique

For a STB coded system where the receiver has no knowledge of the CSI, there is no way to improve performance of the system by modifying the decision rule of the system. In such case, one solution to improve the performance is to use iterative channel estimator to have reliable estimate of the channel with a few overhead pilot symbols. In this chapter, the state-of-the-art iterative channel estimation technique is discussed and a novel approach of frame-based iterative channel estimation technique is proposed. The proposed iterative channel estimator shows much improved performance in terms of the error rate, but has higher complexity compared to the state-of-the-art technique.

## 6.1 Background

The decoding complexity of STB code for practical implementation is that it requires knowledge of the MIMO channel fading parameter at the receiver end. Performance degradation due to mismatch in the channel parameters has been addressed in standard literature [33]. It was shown in [3] that STB code is more sensitive to channel

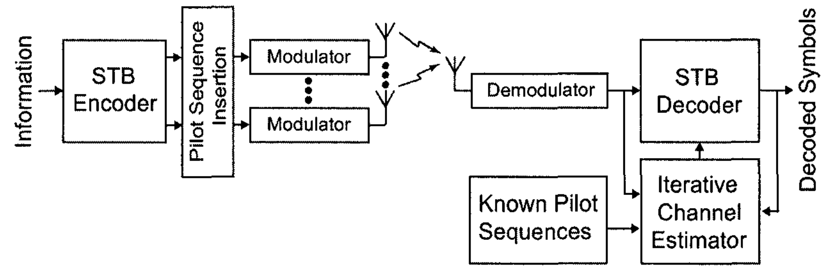


Figure 6-1: Block diagram for iterative channel estimation of STB coded system.

estimation error than straightforward two branch diversity scheme, because of its dependency on the removal of cross-terms in the decision rule. This dependency on the channel estimation error increases as the number of transmitter and receiver antennas increases to achieve same error performance [9].

Techniques to overcome performance degradation due to this type of channel estimation error in the absence of any CSI are being extensively studied. Iterative algorithms seem promising as they obviously outperform non-iterative approaches which use only initial estimates. In [14], a cyclic approach is considered to compensate the channel estimation error. The decision-directed iterative channel estimation method has been proposed in [4,16,35]. An improved method of the channel tracking, which is in fact a modified version of the decision-directed algorithm, which is in fact a modified version of the decision-directed algorithm, hence can be called as modified decision-directed method, has recently been proposed in [12]. Here, a frame-based iterative channel estimator is proposed that shows even better performance than the modified decision-directed approach.

## 6.2 System model

A wireless communication system is considered with $n$ transmitter antennas at the base station and one receiver antenna at the remote station. A simplified block diagram is given in Figure 6-1. Extension of formulations for $m$ receiver antennas is straightforward. The STB encoder takes $p$ symbols in one block of data from the information source and uses the generator matrix to produce $q$ symbols for each transmitter antenna [1, 29]. Hence the generator matrix has dimension of $q \times n$. One frame of data symbols contains L blocks. If $p = q$, then the encoder is called full rate and if $p < q$, the encoder is partial rate. At each time slot $t$, one symbol $s_{i,t}$, $i = 1,2,\ldots,n$ is transmitted simultaneously from the $n$ transmit antennas. The channel is assumed to be flat fading and quasi-static, i.e., the path gains are constant over a frame and vary from one frame to another. The path gain from the $i$th transmitter antenna to the receiver antenna is denoted by $h_i$. The Rayleigh fading channel is modelled as samples of independent complex Gaussian random variables with variance of 0.5 per real dimension.

A sample of the received signal at time $t$ is the superposition of all signals sent from different transmitter antennas and is given by [29]

$$r_t = \sum_{i=1}^{n} h_i \cdot s_{i,t} + n_t \tag{6.1}$$

where $n_t$ is a zero-mean complex Gaussian random variable with single-sided power spectral density $N_0$.

Assuming perfect channel state information (CSI) is available, the STB decoder computes the decision metric

$$\sum_{t=1}^{q} \left| r_t - \sum_{i=1}^{n} h_i \cdot s_{i,t} \right|^2 \tag{6.2}$$

over all possible combinations of transmitted symbol sequences ($s_i = [s_{i,1}\ s_{i,2}\ \ldots\ s_{i,q}]^T$), for $i = 1, \ldots, n$, and decides in favor of the symbol sequences that minimizes the sum. In practical cases, as the receiver does not have access to the actual channel fading parameter $h_i$, it tries to estimate the $h_i$'s using a channel estimation technique. Let the estimated channel parameter be $\hat{h}_i$, which has a certain estimation error. Hence in (6.2), $h_i$ is to be replaced by estimated channel parameter $\hat{h}_i$, which causes degradation of performance compared to perfect channel knowledge [28]. A better estimate of fading parameter improves performance but higher number of overhead symbols are required for this purpose. An efficient iterative channel estimator reduces the number of pilot symbols needed to achieve the same or even better performance.

## 6.3 Iterative channel estimation algorithm

For iterative channel estimation, an initial estimate of channel fading parameters is done using O-PSI pilot sequences, known data symbols or some other methods. The initial estimate is given to the detector for detecting the received signal. This initial estimate can be updated exploiting the orthogonal property of STB code using either the modified decision directed (tracking) mode or the frame-based approach proposed in this section.

The need for pilot symbols can be reduced if some known data from the transmitter can produce the same effect of transmitting the pilot sequence. For example, if 2 bits of a transmitted data are known, it is equivalent to 2 pilot symbols inserted within the frame when modulated using BPSK modulation scheme. Considering space-time Turbo codes as a specific example, the Turbo code is concatenated with the space-time code for higher performance gain [2]. With the coding rate of 1/3, two data from the two recursive systematic convolution (RSC) encoders are sent with one bit of systematic data. As both encoders are initially at all zero state, the first outputs from two RSC encoders are always zero irrespective of data. If these two data are sent before the systematic bit in the frame, then 2 known bits per frame are found. This is an example of reducing the need of pilot symbols on an ad hoc basis.

Whatever method is used, the obtained initial estimate during the training process has certain estimation error due to noise in the receiver. The initial estimate of the channel fading parameter of the $i$th path can be expressed as

$$\hat{h}_i^0 = h_i + e_i^0 \tag{6.3}$$

where $e_i^0$ is the initial estimation error. Here the superscript indicates the index number of iterative estimation process.

### 6.3.1 Modified decision-directed channel estimation

In this method, the initial estimation of the channel fading parameter is changed after the detection of every block of data throughout the entire frame as decoding progresses [12]. The estimated channel parameter for block $l$ with a received signal vector $r_s^l = [r_1^l \; r_2^l \; ... \; r_q^l]^T$ and a detected signal sequence $\hat{s}_i^l = [\hat{s}_{i,1}^l \; \hat{s}_{i,2}^l \; \cdots \; \hat{s}_{i,q}^l]^T$ is

$$\begin{aligned} \hat{h}_i^l &= (r_s^l)^T \cdot (\hat{s}_i^l)^* / \|\hat{s}_i^l\|^2 \\ &= h_i + (n_s^l)^T \cdot (s_i^l)^* / \|s_i^l\|^2 \\ &= h_i + e_i^l \end{aligned} \tag{6.4}$$

where $n_s^l$ is AWGN noise and $e_i^l$ is the corresponding estimation error. Note that (6.4) holds only when the detected signal sequence is correct ($\hat{s}_i^l = s_i^l$). This estimated fading parameter $\hat{h}_i$ is then time averaged over previous estimations to get the averaged estimate for the next iteration. Thus the averaged parameter for iteration $m = 1, 2, ..., M-1$ is

$$\begin{aligned} \hat{h}_i^1 &= (\hat{h}_i^0 + h_i + e_i^1)/2 = h_i + \frac{1}{2}(e_i^0 + e_i^1) \\ \hat{h}_i^2 &= (\hat{h}_i^1 + h_i + e_i^2)/3 = h_i + \frac{1}{3}(e_i^0 + e_i^1 + e_i^2) \\ &\cdots \quad \cdots \quad \cdots \end{aligned}$$

$$\hat{h}_i^{M-1} = h_i + \frac{1}{M}\sum_{m=0}^{M-1} e_i^m. \tag{6.5}$$

Now, let the first incorrect detection of block ($\hat{s}_i^l = s_k^l$) occurs at the $M$th iteration. Using (4.10) in (6.5), the averaged estimated channel fading parameter becomes

$$\hat{h}_i^M \approx \frac{M}{M+1}h_i + \frac{1}{M+1}h_j + \frac{1}{M+1}\sum_{m=0}^{M} e_i^m. \tag{6.6}$$

The approximation is used to simplify the estimation error term. Iteration continues until the end of the frame. The algorithm can be summarized as follows:

```
Find the initial estimate
For data block 1 to L
        Detect data block using present estimate
        Use detected data block to find new estimate
        Time average new estimate to get present estimate
End
```

### 6.3.2 Proposed frame-based iterative channel estimation

The proposed method is a simple extension of the state-of-the-art method, but the performance gain is substantial. The analytical reasoning is given in the next section. Here the mathematical modelling of the proposed scheme is described. The proposed scheme uses the property that, if a transmitted symbol sequence of a particular antenna is orthogonal to other transmitted symbol sequences for several blocks of data, then the combined transmitted symbol sequence for multiple blocks of data of that antenna will still be orthogonal to the combined transmitted symbol sequences of the other antennas. Let the transmitted signal sequence for a frame from the $i$th transmitter antenna be $S_i^T = [(s_i^1)^T \ (s_i^2)^T \ ... \ (s_i^L)^T]$, where one data frame contains $L$

blocks of data. According to this orthogonality property

$$S_i^T \cdot S_j^* = \begin{cases} 0 & for\ i \neq j \\ \|S_i\|^2 & for\ i = j \end{cases}$$

where $S_j$ is the transmitted symbol sequence from any other transmitter antenna for that frame.

In the frame-based iterative method, the initially estimated channel fading parameter is used to decode the whole frame similar to non-iterative methods. Then the whole decoded frame of data is used to find a new channel estimation parameter. For example, an estimate of $h_i$ using the received signal vector of the frame ($R_s^T = [(r_s^1)^T\ (r_s^2)^T\ ...\ (r_s^L)^T]$) and the detected data frame ($\hat{S}_i^T = [(\hat{s}_i^1)^T\ (\hat{s}_i^2)^T\ ...\ (\hat{s}_i^L)^T]$) can be obtained as follows:

$$\hat{h}_i^z = R_s^T \cdot (\hat{S}_i^{z-1}) * / \left\| \hat{S}_i^{z-1} \right\|^2 \tag{6.7}$$

where $z$ denotes number of iteration. Noting that $\|s_i^1\|^2 = \|s_i^2\|^2 = ... = \left\|s_i^L\right\|^2$ and assuming $a$ blocks are correctly detected and other $L - a$ blocks produce symbol sequences of the $k$th transmitter antenna in one frame of data during detection process of the last iteration, (6.7) can be rewritten as follows:

$$\begin{aligned} \hat{h}_i^z &= \left[a \cdot h_i(s_i^T \cdot s_i^*) + (L-a)h_k(s_k^T \cdot s_k^*) + N^T \cdot (\hat{S}_i^{z-1})^*\right] / [L \left\|(\hat{s}_i)^{z-1}\right\|^2] \\ &= \frac{a}{L}h_i + \frac{L-a}{L}h_k + \frac{1}{L}N^T \cdot (\hat{S}_i^{z-1})^* / \left\|\hat{s}_i^{z-1}\right\|^2. \end{aligned} \tag{6.8}$$

This new estimated channel fading parameter is used for the next iteration, and the process is repeated for a desired number of iterations. The proposed algorithm can be summarized as follows:

```
Find the initial estimate
For number of iterations 1 to Z
        Detect whole frame of data using present estimate
        Use all data in the frame to find new estimate
        Set new estimate to present estimate, discard previous
End
```

## 6.4 Analysis of performance

The difference of performance observed between the modified decision-directed estimation and the proposed frame-based iterative estimation is mainly based on the effect of incorrect detection within the frame. In the modified decision-directed estimation, one incorrect detection of a data block changes the channel estimation substantially, leading to a higher probability of incorrect detection of subsequent data blocks. However, in the frame-based estimation method, the effect of incorrect detection of a particular block is less. Because the effect of the incorrect fading parameter due to incorrect detection of a data block blends with other correct fading parameters of the same frame. From (6.8), as the value of $a$ reduces in each iteration, so $a \approx L$ and $L - a \approx 0$, leading to $\hat{h}_i^z \approx h_i$. This ensures better estimation of channel fading parameter in each iteration of the frame under consideration.

To have a better understanding of the difference in performance, let us study a particular case. With average BER of $7.7 \times 10^{-3}$, one block of data might be incorrect among 65 blocks of data. For the frame-based iterative method, from (6.8) with $a = 64$ and $L = 65$, the effect of the incorrect channel fading parameter is $1/64$ compared to the correct parameter. But for the modified decision-directed method, the effect depends on the number of blocks correctly detected before that error actually occurred. Suppose if the incorrect block happens before detecting half of the frame, from (6.6) with $M = L/2$, the effect of incorrect channel fading parameter is $2/65$ compared to the correct parameter. If an incorrect block is encountered within the first quarter of the frame ($M = L/4$), the effect is $4/65$ . The worst case is when

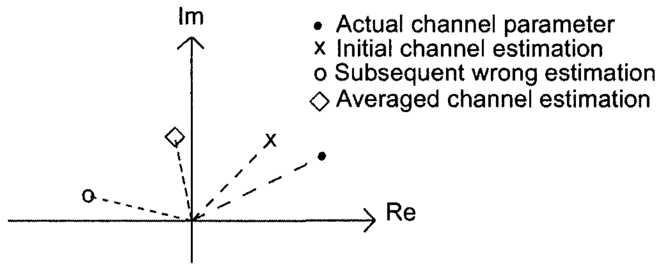


Figure 6-2: Effect of drastic channel estimation error if error occurs in the first block of data detection using modified decision directed method.

an incorrect detection occurs just after the initial estimation ($M = L/65$) and the effect is equal to the correct parameter. This case is shown in Figure 6-2, where the initial estimate of the channel fading parameter is close to the actual parameter, but the detection of the first data block is incorrect. Subsequently the channel parameter estimated using this data block might be away from the actual fading parameter of that channel, because this is a channel parameter of some other path as indicated in (4.10). As a result, the time averaged channel parameter diverts greatly from the actual channel parameter, increasing the probability of incorrect detection for subsequent data blocks. Further incorrect detection of data blocks will make the estimated channel parameter divert further away from the actual channel parameter.

From the analysis of performance for the case of incorrect detection of data block, it is clear that the proposed frame-based iterative algorithm is robust against this type of error compared to the modified decision-directed algorithm. The simulation results are also in agreement with this analysis.

# Chapter 7

# Simulation results

The proposed schemes are simulated extensively using software. Comparison is done for the applying the same conditions and criterions. All the simulations have been performed assuming Alamouti's first model of the STB codes with two transmitter antennas and one receiver antenna with the generator matrix given by (4.7). The channel is assumed to be quasi-static and flat faded. For pilot symbol insertion, the total transmit energy per frame is kept constant for a fair comparison. The simulation results are given in this chapter for both cases discussed in this thesis: the proposed decision metric and the frame-based iterative channel estimation.

## 7.1 Simulation result of the proposed modified decision rule

Normalized Rayleigh fading is assumed with the variance per complex dimension as $\sigma_h^2 = 0.5$. The transmitted signal sequences are modulated using 16-PSK and a gray coded 16-QAM modulation. The constellation diagram of the 16-PSK and gray coded 16-QAM modulation is given in Figure 7-1 and 7-2, respectively. In gray coding scheme, two adjacent symbols have only one bit change. Gray coding reduces

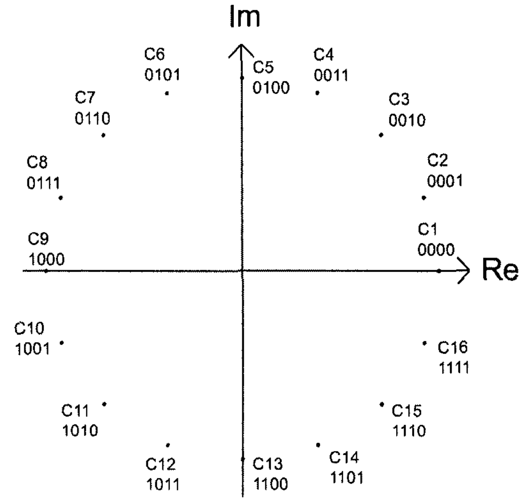


Figure 7-1: Constellation diagram for a 16-PSK modulation scheme.

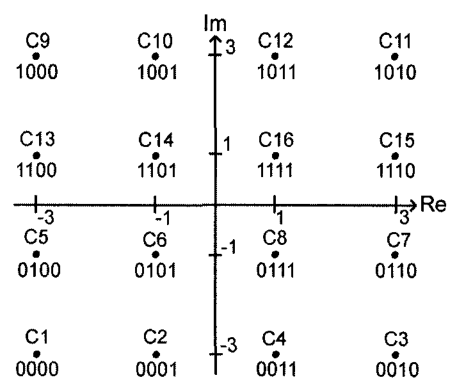


Figure 7-2: Constellation diagram for a 16-QAM gray coded modulation scheme.

the number of errors in the detection of bits even if the symbol detection is wrong. This is because most of the erroneous detection of symbols end up in one of the adjacent symbols in the constellation diagram, which will cause only one bit error in case of gray coding. We have used a higher order QAM modulation scheme to have better observability of the effect of performance degradation due to imperfect channel knowledge.

The channel estimation error variance $\sigma_e^2$ depends on the actual channel estimation scheme and can be computed as a function of bit energy-to-noise ratio and the

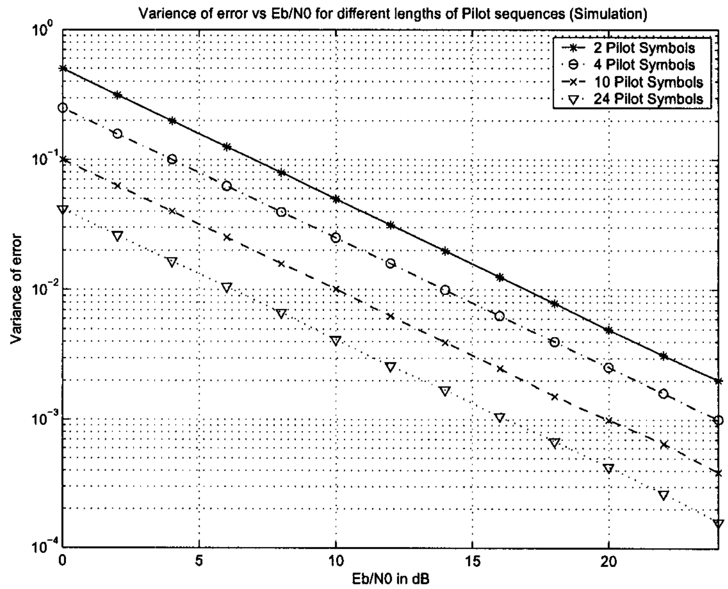


Figure 7-3: Variance of estimation error vs Eb/N0 for different lengths of pilot sequences in slow Rayleigh fading channel (Simulation results).

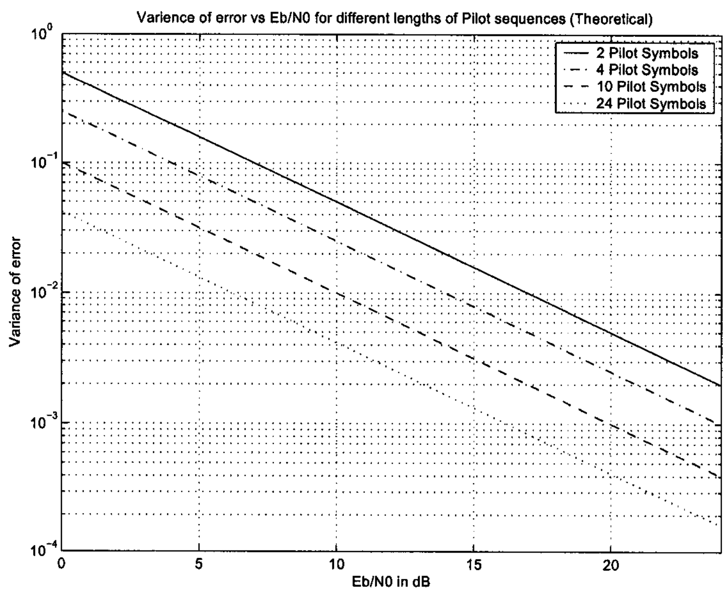


Figure 7-4: Variance of estimation error vs Eb/N0 for different lengths of pilot sequences in slow Rayleigh fading channel (Theoretical results).

number of pilot symbols [28]. To find these variances of estimation error for different lengths of pilot symbols, we have transmitted only pilot symbols from the transmitters and estimated the channel using (5.7) and plotted it against a range of $E_b/N_0$ as shown in Figure 7-3, where $E_b$ is the bit energy and $N_0$ is the noise variance. The estimation error is found to decrease with increasing signal power and increasing length of pilot symbols. Theoretically the estimation error variances can be found from the expression $N_0/2kE_s$ per dimension, where k is the length of the pilot sequence. The theoritical result is plotted in Figure 7-4. The theoretical results are found to be virtually identical to that of the experimental values. However, the experimental values of the estimation error variances are used in the later simulations which is obtained from a look-up table. This is done for better resemblance to the practical implementation issues.

Figure 7-5 shows the bit error rate (BER) and frame error rate (FER) performance curves of the system under consideration with the proposed decision metric and is compared with the conventional decision rule. Two pilot symbols are added to each frame containing information of 32 symbols for channel estimation. The perfect channel knowledge curve is also given as a lower bound of achievable performance. Significant improvement for BER is observed with little improvement in FER.

Experimental results show that the gain in error rate with respect to Tarokh's model is insignificant for PSK modulation. However, for QAM modulation, significant gain is observed. Here the results for the frame length of 128 bits/frame and 512 bits/frame with 16-QAM gray coded modulation are presented in Figure 7-6 and 7-7 respectively. Two pilot symbols are added to each frame for the channel estimation. As observed, the gain using the proposed method is higher with higher number of bits per frame.

The gain in error rate with the proposed metric over Tarokh's metric is plotted in Figure 7-8 for better understanding of the nature. It is seen that the gain decreases

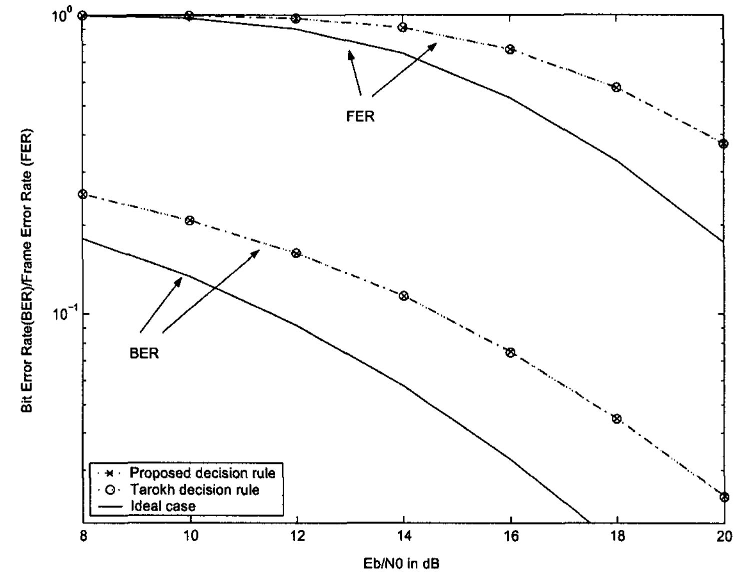


Figure 7-5: BER and FER performance for 16-PSK modulation (128 bits/frame) with a STB coded system using two pilot symbols per frame for the proposed decision rule, Tarokh's decision rule and the ideal case of known CSI.

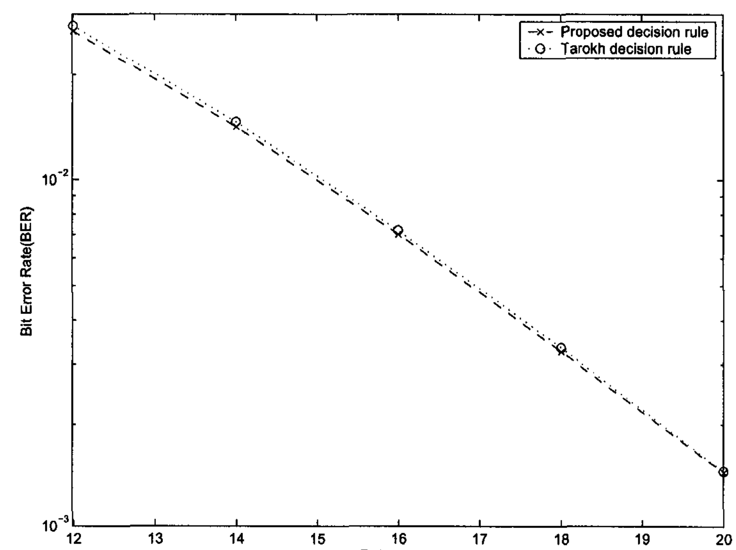


Figure 7-6: BER performance for 16-QAM gray coded modulation (128 bits/frame) with the STB coded system .

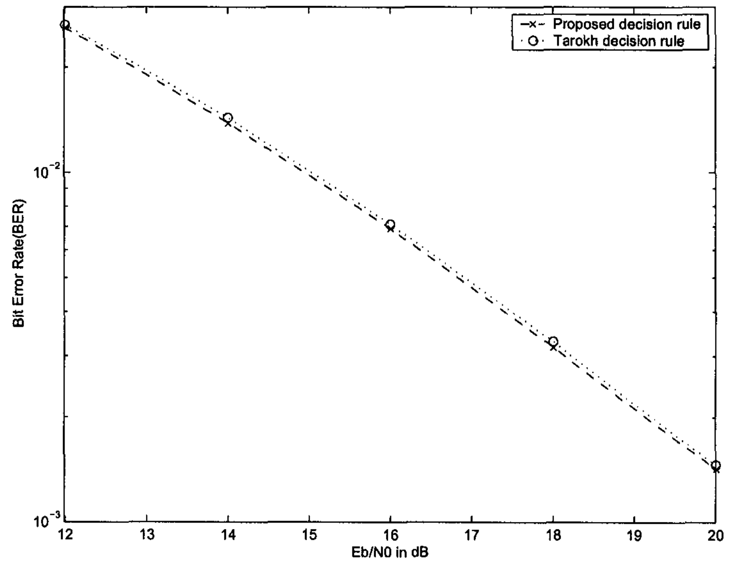


Figure 7-7: BER performance for 16-QAM gray coded modulation (512 bits/frame) with the STB coded system.

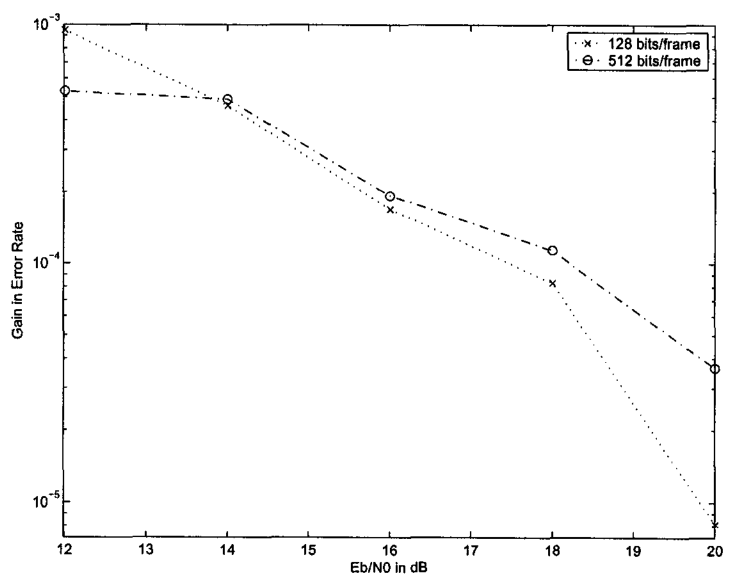


Figure 7-8: BER gain obtained using proposed metric over Tarokh's one for 16-QAM gray coded modulation.

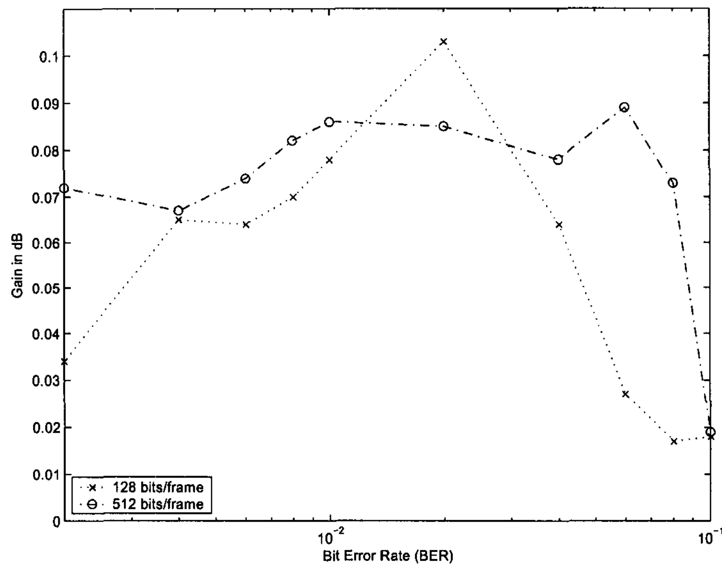


Figure 7-9: Gain in dB obtained using the proposed metric over Tarokh's one for 16-QAM gray coded modulation.

with increasing SNR and becomes identical for very high SNR as predicted in Chapter 5.

In Figure 7-9, the gain in dB is plotted against the BER. It shows that the gain remains high for low BER with larger frame length. So, it can be predicted that the gain will be substantial for practical range of frame size and error rate.

## 7.2 Simulation results for frame-based iterative channel estimation

The simulations have been done for the BPSK and QPSK modulation scheme with 1 receiver and 2 transmitter antennas with Alamouti's space-time block codes [1] for sufficient number of times to achieve statistical independence. The channel is assumed to be quasi-static, i. e. fading is constant over a frame and independent of other frames. The initial estimation is done assuming only 2 known symbols per frame, and hence found that 2 known symbols channel estimation performs similarly

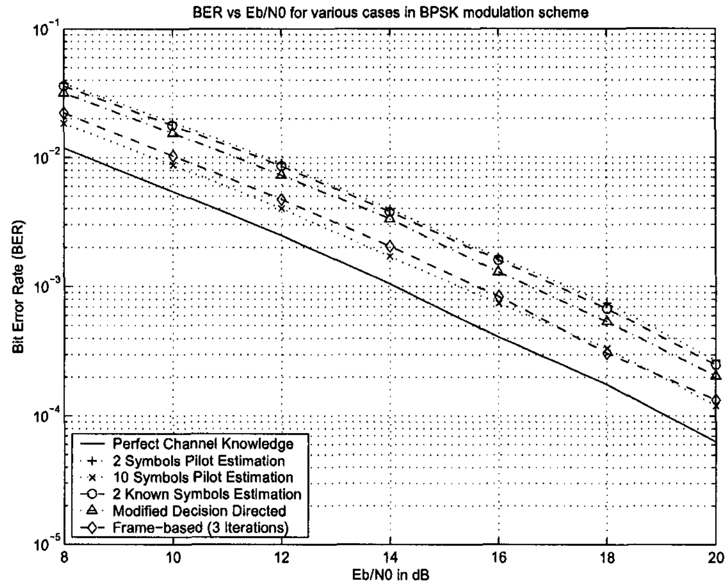


Figure 7-10: BER vs Eb/N0 for various cases (BPSK modulation scheme).

to 2 pilot symbols channel estimation (except minor performance improvement due to energy loss in pilot symbols).

Figure 7-10 shows the bit error rate (BER) for the modified decision-directed and frame-based iteration methods for BPSK modulation scheme with the frame length of 130 bits. The performance of the conventional pilot signal estimation (using 2 and 10 symbols) and also the performance of the system with perfect channel knowledge (as a lower bound) are given for comparison purposes. It is seen that a substantial gain is achieved using the proposed algorithm, which almost supersedes performance achieved even by 10 pilot symbols. For instance, to achieve the error rate of $3 \times 10^{-4}$, the gain is 1.2 dB with 3 iterations using the frame-based channel estimation method over the modified decision-directed method.

In Figure 7-11, the frame error rate (FER) of the corresponding cases are given. Both the modified decision-directed and frame-based iterative channel estimation methods outperform the conventional method using 10 pilots. The performance of the frame-based channel estimation (with 3 iterations) is near to the lower bound. For

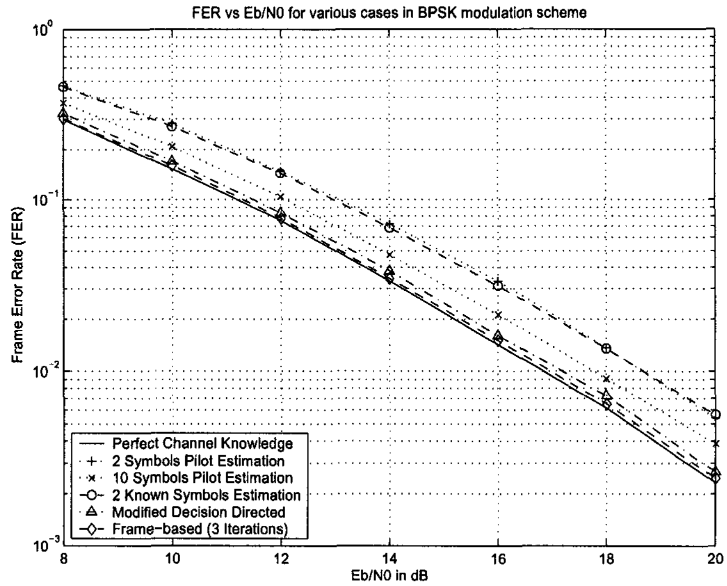


Figure 7-11: FER vs Eb/N0 for various cases (BPSK modulation scheme).

example, to achieve the frame error rate of $10^{-2}$, compared to the case with perfect channel knowledge, the proposed method requires 0.04 dB additional signal power, whereas the modified decision-directed method requires 0.17 dB. Further iterations of the frame-based iterative approach have marginal additional gain in terms of BER or FER.

In Figure 7-12 and 7-13, BER and FER for various cases are given using QPSK modulation and with the same frame length. BER of the proposed method is found to reach 10 symbols pilot estimation for high SNR, and even better in the case of FER performance. Substantial improvement of error rate is observed over the modified decision-directed method.

Finally, in Figure 7-14 and 7-15, a comparison is made for different frame lengths. It is seen that, BER is independent of the frame length in all the case. For FER, better performance is found with a shorter frame length. One interesting point to notice is that, to achieve the same FER using the proposed scheme compared to the modified decision-directed scheme, shorter frame length provides higher performance

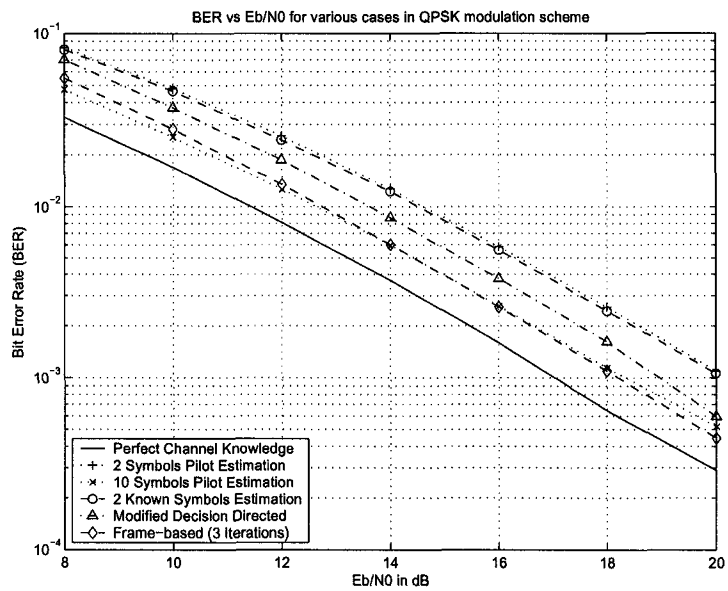


Figure 7-12: BER vs Eb/N0 for various cases (QPSK modulation scheme).

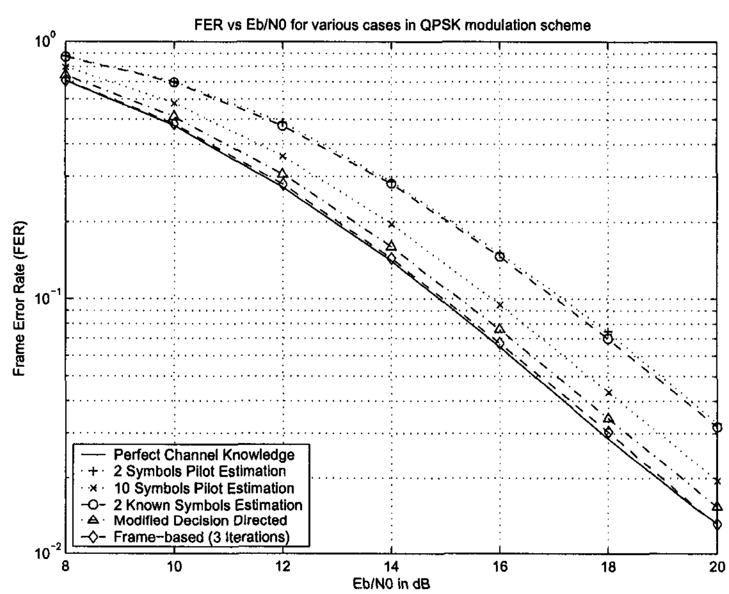


Figure 7-13: FER vs Eb/N0 for various cases (QPSK modulation scheme).

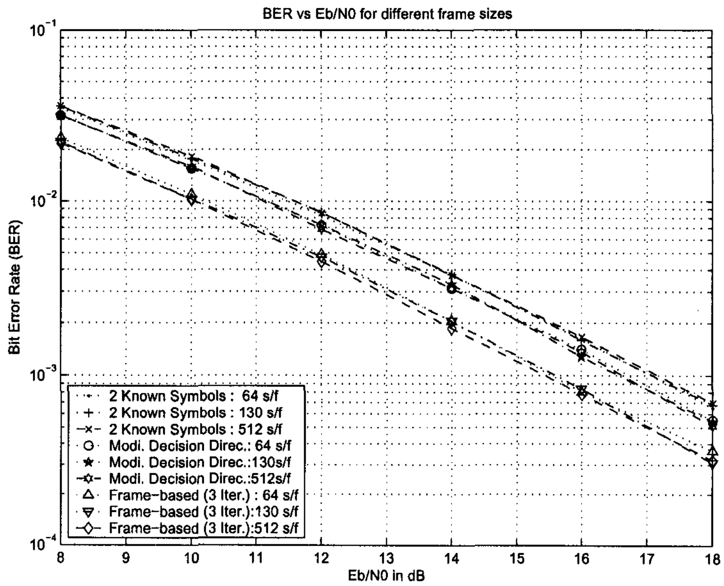


Figure 7-14: BER vs Eb/N0 for different frame sizes for the cases under consideration (BPSK modulation scheme).

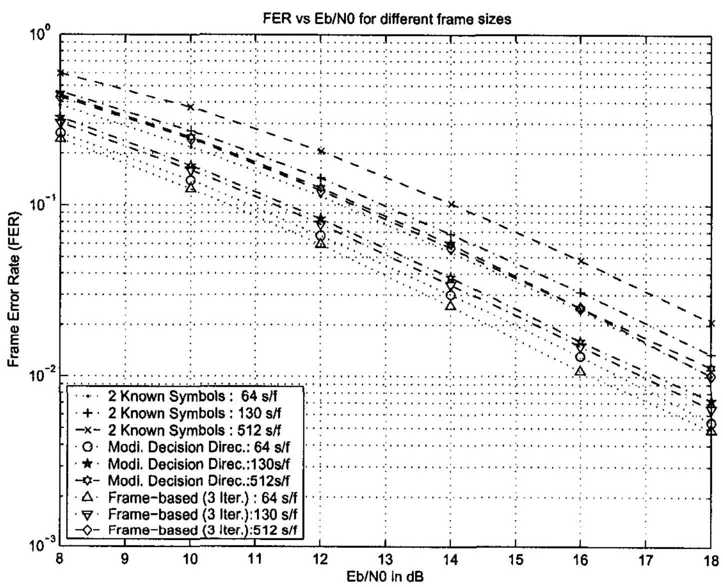


Figure 7-15: FER vs Eb/N0 for different frame sizes for the cases under consideration (BPSK modulation scheme).

gain over longer frame length.

The simulation results varify the theoretical conclusions drawn in the last two chapters. However, further simulation and research is expected ahead of practical implemantation of the proposed schemes.

# Chapter 8

# Conclusions and future directions

This thesis considers the practical implementation issue of the STB coded system when imperfect channel knowledge is available at the receiver. The cases of partial knowledge of CSI and no knowledge of CSI are considered separately. Efforts have been given aiming to push the limits of the state-of-the-art of the corresponding cases.

In the first case of the partial knowledge of CSI, an exact pdf of the received signal conditioned on the estimated channel parameters and the transmitted symbol sequences is derived. Using this pdf, a new modified decision rule has been derived for the decoding of the STB coded system with partial knowledge of CSI. Simulation results show that same or improved performance is obtained using the proposed method over the state-of-the-art method in terms of error rate. Moreover, there is a huge reduction of the system complexity. The proposed scheme performs especially well for QAM modulation, which is commonly found in the practical systems. Simulation results are obtained using parameters estimated from the channel, not from the theoretical values of the channel estimation error. Hence the performance observed incorporates the degradation due to the of the variance of the estimation error of the pilot symbol sequences.

The modified decision rule derived in this work is for a system with with 2 transmitter antennas and 1 receiver antenna with STB coded communication channel proposed by Alamouti. It is straightforward to extend it for a higher number of transmitter and receiver antennas using the same approach. Hence a generalized modified decision rule can be obtained easily to be used for any STB coded system, which is a future step to be taken. The extremely low complexity of the proposed scheme makes it an attractive solution for practical implementation.

For the second case of no knowledge of CSI, a frame-based iterative algorithm for channel fading parameter estimation of STB coded system is proposed in this work. It is found than the proposed scheme outperform state-of-the-art modified decision directed scheme. The BER and FER of the proposed scheme with 2 pilot symbol supersedes the performance of the conventional pilot signal estimation even with 10 pilot symbols in high SNR. The algorithm is applicable in space-time Turbo coded systems with very little modification. The improved CSI can be utilized in decoding of a Turbo decoder for even better performance gain. The simulations show that high performance gain can be achieved with fewer iterations. The proposed method significantly reduces the number of pilot symbols needed to achieve the same or even better performance, but requires higher processing complexity. Proposed modified decision rules can also be combined with the proposed iterative channel estimator and should produce even better performance in terms of error rate performance, but is an issue of future research.

# Appendix A

# Derivation of pdf for conditional received signal

In this appendix, we derive the exact pdf of the received signal conditioned on the estimated channel parameter and the transmitted symbol sequences.

The received signal samples are complex Gaussian distributed with $\mathbf{r} \sim N^c(\mu_r, C_r) \in \Im^2$, where $\Im^2$ denotes complex vector of dimension 2 and $N^c$ denotes complex normal distribution. Here the mean of the distribution is $\mu_r$, the covariance matrix is $C_r$. It is straightforward that $\mu_r = 0$, $C_r = 2\sigma_r^2 I_2$, where $I_2$ denotes $2 \times 2$ unit matrix. Hence the pdf of the complex received signal vector can be expressed as [19]

$$p_{r|s}(R|S) = \frac{1}{\pi^2 |C_r|} \exp\left(-r^H C_r^{-1} r\right). \tag{1}$$

Here $s$ is the transmitted signal vector at any time slot and $|X|$ denotes the determinant, while $X^H$ denotes Hermitian of the matrix $X$.

Similarly distribution of the estimated channel parameter is $\hat{h} \sim N^C(\mu_{\hat{h}}, C_{\hat{h}}) \in \Im^2$, where the mean is $\mu_{\hat{h}}$ and the covariance matrix is $C_{\hat{h}}$. Again, we have $\mu_{\hat{h}} = 0$ and

$C_{\hat{h}} = 2\sigma_{\hat{h}}^2 I_2$. Thus $\hat{\mathbf{h}}$ has the complex distribution function as follows

$$p_{\hat{h}}(\hat{H}) = \frac{1}{\pi^2 |C_{\hat{h}}|} \exp\left(-\hat{h}^H C_{\hat{h}}^{-1} \hat{h}\right). \tag{2}$$

Now, the joint distribution function of $\mathbf{r}$ and $\hat{\mathbf{h}}$ is [13]

$$p_{r,\hat{h}|s}(R, \hat{H}|S) = \frac{1}{\pi^4 \left|C_{r,\hat{h}}\right|} \exp\left(-\begin{bmatrix} r \\ \hat{h} \end{bmatrix}^H C_{r,\hat{h}}^{-1} \begin{bmatrix} r \\ \hat{h} \end{bmatrix}\right) \tag{3}$$

where the correlation matrix can be expressed as

$$C_{r,\hat{h}} = E\left[\begin{bmatrix} r \\ \hat{h} \end{bmatrix} \begin{bmatrix} r^H & \hat{h}^H \end{bmatrix}\right] = \begin{bmatrix} 2\sigma_r^2 I_2 & 2\sigma_h^2 G \\ 2\sigma_h^2 G^H & 2\sigma_{\hat{h}}^2 I_2 \end{bmatrix}.$$

We have found that $\left|C_{r,\hat{h}}\right| = 2^4 \sigma_r^4 \sigma_{\hat{h}}^4 \left(1 - |\mu|^2\right)^2$ and

$$C_{r,\hat{h}}^{-1} = \frac{1}{\left|C_{r,\hat{h}}\right|^{1/2}} \begin{bmatrix} 2\sigma_{\hat{h}}^2 I_2 & -2\sigma_r \sigma_{\hat{h}} C_\mu \\ -2\sigma_r \sigma_{\hat{h}} C_\mu^H & 2\sigma_r^2 I_2 \end{bmatrix} \tag{4}$$

where $C_\mu$ is defined as

$$C_\mu = \begin{bmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{bmatrix}.$$

Putting all these values in (3) and doing simple multiplications, we reach

$$p_{r,\hat{h}|s}(R, \hat{H}|S) = \frac{1}{\pi^4 |C_r| |C_{\hat{h}}| \left(1 - |\mu|^2\right)^2} \exp\left\{\frac{-1}{2^2 \sigma_r^2 \sigma_{\hat{h}}^2 \left(1 - |\mu|^2\right)} \left(2\sigma_{\hat{h}}^2 r^H r + 2\sigma_r^2 \hat{h}^H \hat{h} - 2^2 \sigma_r \sigma_{\hat{h}} \Re\left[r^H C_\mu \hat{h}\right]\right)\right\} \tag{5}$$

where $\Re[X]$ denotes the real axis portion of the complex scaler X, and the definition of $|\mu|^2$ is given in (5.9).

According to Bayes theorem, we can use the following expression to get the pdf of the received signal conditioned on the estimated channel parameters:

$$p_{r|\hat{h},s}(R|\hat{H},S) = \frac{p_{r,\hat{h}|s}(R,\hat{H}|S)}{p_{\hat{h}|s}(\hat{H}|S)} = \frac{p_{r,\hat{h}|s}(R,\hat{H}|S)}{p_{\hat{h}}(\hat{H})} \tag{32}$$

as $\hat{h}$ is independent of the transmitted symbols.

Putting values from (2) and (5) in (32), and doing some simple manipulations, we finally reach the desired conditional pdf, which is given in (5.10). This is the exact conditional pdf required for derivation of the decision rule in the presence of partial knowledge of CSI.

# Glossary

| | |
|---|---|
| APP | A posteriori probability |
| AWGN | Additive white Gaussian noise |
| BER | Bit error rate |
| BPSK | Binary phase shift keying |
| BS | Base station |
| CSI | Channel state information |
| FER | Frame error rate |
| MIMO | Multiple input multiple output |
| MISO | Multiple input single output |
| ML | Maximum likelihood |
| MMSE | Minimum mean square error |
| MRRC | Maximal ratio receiver combining |
| MS | Mobile station |
| O-PSI | Orthogonal pilot sequence insertion |
| pdf | Probability distribution function |
| PSK | Phase shift keying |
| QAM | Quadrature amplitude modulation |
| QPSK | Quadrature phase shift keying |
| RSC | Recursive systemic convolution |
| SIMO | Single input multiple output |
| SISO | Single input single output |

| | |
|---|---|
| SNR | Signal-to-noise ratio |
| STB | Space-time block |
| STTC | Space-time trellis code |

# Bibliography

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communications", *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[2] F. Bauch, "Concatenation of space-time block codes and Turbo-TCM", *IEEE Intl. Conf. Commun.*, vol. 2, pp. 1202-1206, June 1999.

[3] R. M. Buehrer, N. A. Kumar, "The impact of channel estimation error on Space-Time Block codes", *Vehicular Tech. Conf.*, vol. 3, pp. 1921-1925, Sept. 2002.

[4] X. Deng, A. M. Haimovich, J. Garcia-Frias, "Decision-directed iterative channel estimation for MIMO systems", *IEEE Intl. Conf. Commun.*, vol. 4, pp. 2326-2329, May 2003.

[5] O. Edfors *et al.*, "OFDM channel estimation by singular value decomposition", in *Proc. IEEE Veh. Tech. Conf.*, Atlanta, GA, pp. 924-927, May 1996.

[6] G. J. Foschini, M. J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas", *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311-335, March 1998.

[7] P. Frenger, A. Svensson, "Decision-directed coherent detection in multicarrier systems on Rayleigh fading channels", *IEEE Trans. Veh. Tech.*, vol. 48, pp. 490-498, March 1974.

[8] P. Frenger, "Turbo decoding for wireless systems with imperfect channel estimates", *IEEE Trans. Commun.*, vol. 48, no. 9, pp. 1437-1440, Sept. 2000.

[9] P. Garg, R. K. Mallik, H. M. Gupta, "Performance analysis of Space-Time coding with imperfect channel estimation", *IEEE Intl. Conf. Per. Wire. Commun.*, pp. 71-75, Dec. 2002.

[10] Y. Gong, K. B. Letaief, "Low complexity channel estimation for Space-Time coded wideband OFDM systems", *IEEE Trans. Wireless Commun.*, vol. 1, no. 5, pp. 876-882, Sept. 2003.

[11] L. Hanzo, T. H. Liew, B. L. Yeap, *Turbo Coding, Turbo Equalization and Space-Time Coding*, Southampton, Englad : Wiley, 2002.

[12] I. Harjula, A. Mammela, "Channel estimation algorithm for space-time block coded OFDM systems", *GLOBECOM '03 IEEE*, vol. 1, pp. 143-147, Dec. 2003.

[13] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Englewood Cliffs, NJ: Prentice-Hall, 1993.

[14] E. G. Larsson, P. Stoica, J. Li, "Space-time block codes: ML detection for unknown channels and unstructured interference", *Asilomar Conf. Signals, Systems, Comp.*, vol. 2, pp. 916-920, Nov. 2001.

[15] Y. Liu, M. P. Fitz, O. Y. Takeshita, "Full rate Space-Time Turbo codes", *IEEE J. Select. Areas Commun.*, vol. 19, no. 5, pp. 969-980, May 2001.

[16] R. Matsuo, T. Ohtsuki, I. Sasase, "Simple EM based channel estimation and data detection using estimation in the preceding frame in Space-Time coded signals", *ISIT 2001*, p. 222, June 2001.

[17] L. L. Mendes, G. G. R. Gomes, "An efficient educational approach for the study of 16 QAM and block codes", *VIII Intl. Conf. Engg. Tech. Edu. - INTERTECH 2002*, March 2002.

[18] A. F. Naguib, V. Tarokh, N. Seshadri, A. R. Calderbank, "A Space-Time coding modem for high-data-rate wireless communications", *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1459-1478, Oct. 1998.

[19] A. Papoulis, S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed., New York: McGraw-Hill, 2002.

[20] J. G. Proakis, "Probabilities of Error for Adaptive Reception of M-Phase Signals", *IEEE Trans. Commun. Tech.*, vol. COM-1, no. 1, pp. 71-81, Feb. 1968.

[21] J. G. Porakis, *Digital Communications*, 3rd ed. NY, USA: McGraw-Hill, 2000.

[22] T. S. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed. Upper Saddle River, NJ: Prentice Hall, 2002.

[23] B. Sklar, "A Primer on Turbo Code Concepts", *IEEE Commun. Magazine*, pp. 94-102, Dec. 1997.

[24] G. L. Stuber, *Principles of Mobile Communication*, Boston, MA: Kluwer Academic Press, 2000.

[25] J. Suh, M. K. Howlader, "Concatenation of Turbo codes to space-time block codes with no channel estimation", *MILCOM 2002*, vol. 1, pp. 726-731, Oct. 2002.

[26] V. Tarokh, A. F. Naguib, N. Seshadri, A. R. Calderbank, "Space-Time codes for high data rate wireless communication: mismatch analysis", *IEEE Intl. Conf. Commun.*, vol. 1, pp. 309-313, June 1997.

[27] V. Tarokh, N. Seshadri, A. R. Calderbank, "Space-Time codes for high data rate wireless communication: performance criterion and code construction", *IEEE Trans. Info. Theory*, vol. 44, no. 2, pp. 744-765, March 1998.

[28] V. Tarokh, A. Naguib, N. Seshadri, A. R. Calderbank, "Space-Time codes for high data rate wireless communication: performance criteria in the presence of channel estimation errors, mobility, and multiple paths", *IEEE Trans. Commun.*, vol. 47, no. 2, pp. 199-207, Feb. 1999.

[29] V. Tarokh, H. Jafarkhani, A. R. Calderbank, "Space-Time Block coding for wireless communications: performance results", *IEEE J. Select. Areas Commun.*, vol. 17, no. 3, pp. 451-460, March 1999.

[30] V. Tarokh, A. Naguib, N. Seshadri, A. R. Calderbank, "Errata to Space-Time codes for high data rate wireless communications: performance criteria in the presence of channel estimation errors, mobility, and multiple paths", *IEEE Trans. Commun.*, vol. 51, no. 12, p. 2141, Dec. 2003.

[31] O. Tirkkonen, A. Hottinen, "Tradeoffs between rate, puncturing and orthogonality in space-time block codes", *IEEE Intl. Conf. Commun.*, vol. 4, pp. 1117-1121, June 2001.

[32] H. L. V. Trees, *Detection, Estimation, and Modulation Theory: Part I*, New York: John Wiley and Sons, 2001.

[33] B. Vucetic, J. Yuan, *Space-Time Coding*, West Sussex, England: Wiley, 2003.

[34] H. Wang, X. Xia, "Upper bounds of rates of complex orthogonal space-time block codes", *IEEE Trans. Info. Theory*, vol. 49, no. 10, pp. 2788-2796, Oct. 2003.

[35] J. Yang, Y. Sun, J. M. Senior, N. Pem,"Channel estimation for wireless communications using space-Time Block coding techniques", *Proc. Intl. Circuits and Symbols*, vol. 2, pp. II-220-II-223, May 2003.

# VITA AUCTORIS

Bashir Iqbal Morshed was born in Bhola, Bangladesh, in 1976. He graduated from Chittagong College in 1994. He received B. Sc. in engineering with honors from the Electrical and Electronic Engineering department of Bangladesh University of Engineering and Technology (BUET) in 2001. He is currently a candidate for the Master of Applied Science in Electrical Engineering at the University of Windsor and hopes to graduate in Summer 2004.