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THE EFFECTS OF PROBABILITY LEVEL AND SAMPLE SIZE ON SUBJECTIVE PROBABILITY DISTRIBUTIONS

by

Stephen H. Kleiman

B. Sc. (Psychology), University of Michigan, 1980

A Thesis
Submitted to the Faculty of Graduate Studies
through the Department of Psychology
in Partial Fulfillment of the
Requirements for the Degree
of Master of Arts at the
University of Windsor

Windsor, Ontario, Canada

ABSTRACT

This study investigated how individuals assess the probability of an uncertain event by examining the effects of probability level (P) and sample size (N) on subjective probability estimates. Subjects were asked to produce a subjective probability distribution for one of nine different binomial distributions. Three different binomial populations were used (binomial p = .30, .50, and .75). Each population was tested for three different sample sizes (N = 10, 100, and 1000). Therefore, there were two independent variables, binomial probability and sample size. The dependent variable was the subjective probability estimate. Thus, a 3 X 3 between subjects experimental design was used. The number of subjects tested was 201.

Various analyses were performed in order to determine how individuals assess the probability of an uncertain event. Essentially, the subjects in this study were homogenous. No systematic relationship between subject characteristics and the subjects' probability estimates was found (i.e., neither age nor sex had an effect on the subjective probability estimates). These results may have been due to the complexity of the task.

Sample size did not significantly affect the subjective probability estimates of the various binomial distributions. In contrast, the probability level of the binomial distribution heavily influenced the subjects' subjective probability estimates. It was also found that subjects varied their subjective probability estimates on the basis of category. Overall, subjects underestimated the middle categories (i.e., categories 4 - 8) and overestimated the categories at the tails (i.e., categories 1 - 3 and categories 8 - 11). These results may have been due to the under/over bias effect (see Kahneman & Tversky (1972)).

Finally, the goodness of fit between the subjective probability estimates and the calculated objective probability values was tested. The subjects' subjective probability estimates differed significantly from the objective probability values across all problems and across all categories. However, subjects estimated the binomial distributions with 10 cases much better than binomial distributions with 100 or 1000 cases. The magnitude of the departure from the objective probability values was similar for binomial distributions of 100 or 1000 cases. This finding again illustrated that subjects ignored the effects of sample size. The relationship of the findings to decision theory was discussed, and suggestions for future research studies were presented.

ACKNOWLEDGEMENTS

I would like to express my thanks and my gratitude to Dr. Meyer Starr, my committee chairman. His ideas and his suggestions formed an integral part of this study.

I would also like to acknowledge the contributions of Dr. A.A. Smith and Dr. S.R. Paul, the other members of my thesis committee.

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CHAPTER I

INTRODUCTION

Individuals must often make decisions about people and about events on the basis of little or no information (e.g., success in a new job, future value of IBM stock, outcome of a sports event). These situations occur regularly because there is no time to obtain the necessary information or because the necessary information is unavailable. This study will investigate how individuals assess the probability of an uncertain event. In addition, it will investigate how sample size affects these probability estimates. Finally, it will examine how individual probability estimates differ from the calculated probability values.

Traditionally, the research on decision making has been divided into two separate but related areas; normative decision theory and descriptive decision theory. Normative decision theory seeks to determine the ideal or optimal principles of decison making. It assumes that men are rational and that they attempt to maximize their gains and minimize their losses. This area has been extensively studied by mathematicians (cf. Savage (1954)) and by economists (cf. Von Neumann & Morgenstern (1953)).

In contrast, descriptive (or behavioral) decision theory is concerned with the choices that men actually make. In so doing, it describes the decision making behavior of real people. Consequently, behavioral decision theory is of interest to applied psychologists (see Cascio (1978)). Research in this area has examined the various personal and situational factors involved in decision making (e.g., ability to process information, risk taking, gambling behavior, and game theory).

Literature Review

Subjective probability. Subjective probability is an important research area in behavioral decision theory (cf. Edwards (1961)). Subjective probabilities represent an individual's estimate of the likelihood of a given event. This estimate may or may not correspond to the objective probability. A number of studies have examined how individuals determine the subjective probabilities of events in a variety of contexts. For example, psychologists have studied decision making in the contexts of probability learning, intuitive staistics, and risk (see Cohen, Dearnaley, & Hansel (1957) and Peterson & Beach (1967)).

Frequently, decisions must be made under conditions of uncertainty (e.g., success of a business venture, outcome of an election, success of English majors in law school). These conditions arise when there is no time to obtain the necessary information or when the necessary information is unavailable. Under such conditions, a person bases his decision on his beliefs about the underlying objective probability distribution (see Luce & Raiffa (1957)). In many cases, however, an individual's beliefs about the underlying probability distribution are erroneous. Consequently, the subjective probability distribution does not match the objective probability distribution (see Tversky & Kahneman (1974)).

This study will examine how individuals make subjective probability estimates. Specifically, it will assess the effects of sample size (N) and binomial probability (P) on subjective probability estimates. Moreover, it will compare the subjective probability estimates and the objective probability values. Much research has been devoted to the question of how individuals evaluate the probabilities of uncertain events. Some relevant research studies are now examined and analyzed.

Savage (1954) looked at the formal properties of subjective probabilities. Given the assumption that individuals can rank order all events, he defined subjective probability as that number which represents an individual's beliefs about the likelihood of a given event. Furthermore, he postulated that subjective probabilites have the same mathematical properties as objective probabilities. Finally, he suggested that changes in subjective probability for a series of events are governed by Bayes's Theorem (i.e., $EV_i = \sum_{i}^{n}(P_iV_i)$). Subsequent research, however, has revealed that individuals do not always behave in accordance with Bayes's Theorem (see Edwards (1968)).

Cohen, Hansel, and their associates have also investigated subjective probabilities (John Cohen, Dearnaley, & Hansel (1956) and John Cohen & Hansel (1955)). Their research explored various facets of the relationship between subjective probability and objective probability. For example, they studied the addition and multiplication of subjective probabilities, the effects of task size on subjective probability, and the properties of subjective binomial distributions.

Based on the results of their research, Cohen and Hansel (1955) concluded that the relationship between subjective probability and objective probability is complex. In certain situations, subjective and objective probabilities tend to coincide; in other situations, they tend to differ, and these differences are systematic. In addition, they discovered that age and prior experience can greatly influence subjective probabilities. Finally, they determined that the number and the value of the alternatives offered can greatly affect subjective probabilities (see John Cohen, Dearnaley, & Hansel (1957)).

Tune (1964) studied how individuals perceive random events like coin tosses, which are binomial experiments with probability .50. The subjects were asked to generate a random sequence of hypothetical coin tosses. In response, they produced sequences of coin tosses in which the proportion of heads in any short segment stayed far closer to p = .50 than the laws of probability would predict. In a sequence of ten coin tosses, for example, subjects usually generated five heads and five tails.

The results of Tune's (1964) study indicated that individuals believe that a sample of any size must reflect the characteristics of the parent population. If a sample proportion differs from a population proportion, individuals expect a corrective bias in the opposite direction. For example, if seven consecutive heads occur in a series of coin tosses, most individuals expect that a tail will appear on the next coin toss. This error is known as the "gambler's fallacy". The "gambler's fallacy" demonstrates that many individuals do not understand the laws of chance (i.e., probability theory).

A number of studies have examined the properties of subjective binomial probability distributions (e.g., Peterson, DuCharme, & Edwards (1968) , Wheeler & Beach (1968) , and Kahneman & Tversky (1972)). These studies employed similar methodologies and reported similar findings. Upon close examination of these studies, a number of methodological shortcomings can be observed, which involve: 1) sample sizes of the binomial distributions were too small 2) different numbers of response categories were used to estimate different sample sizes 3) the number of subjects used in each experimental condition was inadequate 4) lack of variation in the probability level of the binomial distribution studied 5) each subject estimated several subjective binomial probability distributions.

Both the Peterson, DuCharme, & Edwards (1968) study and the Wheeler & Beach (1968) study suffered from some of the methodological flaws mentioned above. The sample sizes of the binomial populations estimated were too small (N = 3, 5, or 8). When the sample sizes estimated are too small, subjects may be able to enumerate the different possibilities. Moreover, different numbers of response categories were used for different sample sizes. For a sample of size N, subjects evaluated N+1 outcomes. When different numbers of response categories are used, between group comparisons are difficult to make. They employed less than 10 subjects in each experimental condition. Furthermore, subjects estimated several binomial probability distributions. It is likely that their subjective probability estimates were not independent. Finally, they only looked at binomial distributions with probability levels of .60, .70, or .80 . The probability levels used by Peterson, DuCharme, & Edwards (1968) are insufficient because they only involve one side of the binomial distribution and do not include the probability level of .50 .

Kahneman and Tversky (1972) corrected some of the methodological shortcomings of the studies previously described. First, subjects produced subjective probability distributions for sample sizes of 10, 100, and 1000 (i.e., N = 10, 100, and 1000). More importantly, subjects evaluated the same number of response categories for all sample sizes. Finally, the Kahneman and Tversky (1972) study used at least 45 subjects in each experimental condition.

However, the Kahneman & Tversky (1972) study did not correct all of the methodological weaknesses of the previous studies. The subjects in their study only evaluated binomial distributions with probability levels of .50 and .80 . They did not look at binomial distributions with probability levels of less than .50 . Additionally, Kahneman and Tversky used a repeated measures design. Each subject produced at least three subjective probability distributions. Consequently, the experimental results may have been contaminated by practice and fatigue effects. In 1982, Kahneman and Tversky commented that "within-subject designs are associated with significant problems of interpretation in many areas of psychological research. In studies of intuitions, they are liable to induce the effect which they are intended to test."

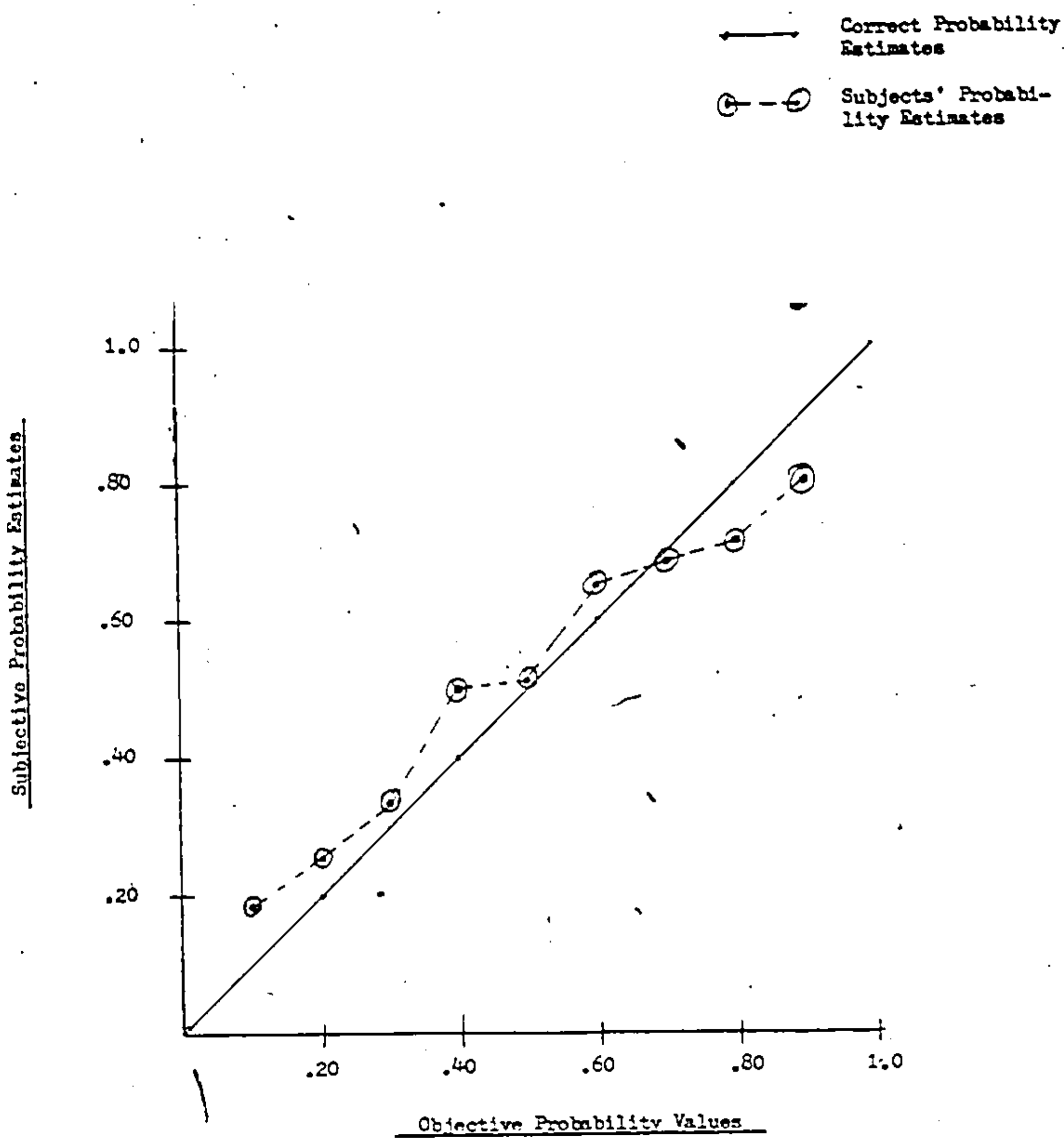
The present study attempts to correct the methodological shortcomings described above by : 1) using large sample sizes for the binomial distributions studied. Subjects will estimate sample sizes of 10, 100, and 1000 for all binomial distributions studied. 2) using the same number of response categories to estimate the different sample sizes. Subjects will evaluate 11 response categories for all sample sizes. 3) using at least 20 subjects in each experimental condition 4) using a variety of probability levels for the binomial distributions studied. In the present study, subjects will evaluate binomial distributions with probability levels of .30, .50, and .75 . 5) Subjects will produce only one subjective probability distribution. Consequently, the subjects' subjective probability estimates will be independent.

Cognitive decision making. Coombs, Dawes, & Tversky (1970) in summarizing the results of the research on subjective probability distributions showed that subjective probability scales approximated linear functions of objective probability for most subjects. However, there were systematic discrepancies from linearity. Subjects tended to overestimate low probabilities (i.e., unlikely events) and underestimate high probabilities (i.e., likely events). This result, the under/over bettor bias, has been found in many different studies (e.g., Edwards (1968)). Figure 1 shows a typical subjective probability function.

Coombs, Dawes, & Tversky (1970) also discussed the role that cognition plays in decision making. Decision models based upon normative decision theory assume that the decision maker can evaluate all available alternatives and can examine all possible states and outcomes. In actuality, however, these conditions are rarely met. In many situations, the information available to the decision maker is vague and incomplete. Moreover, the number of available alternatives is so large that the decision maker can not properly evaluate all of them. Consequently, the individual must redefine the decision problem in some way. For these reasons, cognitive factors are important in the decision making process. They determine how a decision problem is formulated and what methods are used to find a solution. If a decision problem is too complex, an individual will attempt to simplify it in order to make it more manageable. Heuristics are often used to accomplish this task.

Figure Caption

Figure 1 Subjective probability estimates as a function of objective probability values (Taken from Coobs, Dawes, & Tversky (1970)).


Correct Probability Estimates
Subjects' Probability Estimates
1.0
.80
.60
.40
.20
Subjective Probability Estimates
.20
.40
.60
.80
1.0
Objective Probability Values

Tversky and Kahneman (1971, 1972, & 1974) have examined how heuristics are used in decision making. Heuristics can be defined as "rules of thumb" that enable a person to simplify a complex task. Tversky and Kahneman demonstrated that individuals often base their subjective probability estimates on three informal heuristics. These are: (1) the representativeness heuristic (2) the availability heuristic and (3) the anchoring-adjustment heuristic.

The representativeness heuristic is employed when individuals are asked to judge the probability that event A belongs to a certain class or process B. In such situations, the individual infers that a limited sample is representative of (i.e., similar to) the whole population. Unfortunately, similarity judgments are insensitive to factors such as base-rate information and sample size (cf., Tversky & Kahneman (1971)). Consequently, the use of the representativeness heuristic can lead to serious errors (see Tversky & Kahneman (1974)).

The availability heuristic is used when individuals are asked to assess the frequency of a class or the likelihood of a particular event. Under such conditions, the individual looks at the ease with which instances or events can be retrieved from memory. However, availability judgments can be biased. Errors can occur due to a variety of factors such as ease of retrievability, the effectiveness of a search set, and the lack of imaginability (see Tversky & Kahneman (1974)).

Finally, the anchoring-adjustment heuristic is used in situations involving numerical prediction from some initial value. In such situations, individuals make estimates by adjusting an initial value to yield a final answer. Errors are likely if the final answer is unduly biased by the initial value (i.e., individuals do not adequately adjust the initial value) (see Tversky & Kahneman (1974)).

The Kahneman & Tversky (1972) study previously discussed used the binomial distribution to study subjective probability distributions. In so doing, it demonstrated that the misuse of the representativeness heuristic can lead to incorrect subjective probability estimates. The experimental results indicated that the subjective sampling distributions produced were independent of sample size, N. That is, the subjective sampling distributions produced for samples of 10, 100, and 1000 were indistinguishable. Apparently, the subjects based their subjective probability estimates only on the sample proportion or on the sample mean. Kahneman and Tversky suggest that the subjects may have used the representativeness heuristic to aid them in their decisions of subjective probability estimates.

Of the three cognitive heuristics described by Kahneman & Tversky, the representativeness heuristic is most relevant to studies of subjective probability. Smaller populations do in fact resemble their larger parent populations. In particular, populations with sample sizes of 10 are similar to populations with sample sizes of 100 or 1000. Subjects can accurately estimate the sample mean but they overlook the effects of sampling error. Consequently, they systematically and reliably neglect differences in populations with different sampling errors. Specifically, they underestimate response categories with high probabilities and overestimate response categories with low probabilities. It may be that the subjects' inability to appreciate the concept of sampling error accounts for the under/over bettor bias so commonly reported in previous studies on subjective probability.

Evaluative Summary of Previous Research

A number of approaches have been used to study subjective probability. Some researchers (e.g., Savage (1954) have focused on normative decision theory while other researchers (e.g., Edwards (1968)) have focused on behavioral decision theory. Kahneman & Tversky (1972, 1974) looked at the cognitive processes involved in making subjective probability estimates. Unfortunately, little effort has been made to integrate the various theoretical perspectives. As a result, no attempt has been made to systematically study the differences betwea subjective and objective probability distributions.

The research reviewed suggested that the probability level of a distribution heavily influences subjects' subjective probability estimates. Various studies (e.g., Tversky & Kahneman (1972)) found that subjects could accurately estimate the mean of a subjective probability distribution. However, sample size did not influence the subjects' subjective probability estimates. Because subjects overlook the effects of sampling error, they tend to underestimate response categories with high probabilities and overestimate response categories with low probabilities. These errors of estimation become severe as sample size increases. Finally, a number of methodological deficiencies were noted in the previous research on subjective probability. These deficiencies made it difficult to adequately evaluate the findings reported.

Statement of Problem and Hypotheses

The purpose of this experiment is to study decision making under uncertainty. It will partially replicate and extend the work of previous researchers (e.g., Tversky & Kahneman (1972, 1974, & 1982) and Cohen, Dearnaley, & Hansel (1955, 1956, & 1957) . The probability level (P) and the sample size (N) of the binomial distributions presented to the subjects will be manipulated. As suggested by Tversky & Kahneman (1982), the subjects will produce only one subjective probability distribution. Because the subjects' subjective probability distributions are independent, practice and fatigue effects will be avoided. Finally, all binomial distributions will be evaluated with the same number of response categories. Previous research on subjective probability (e.g., Peterson, DuCharme, & Edwards (1968)) used different numbers of response categories for different binomial distributions.

In Tversky & Kahneman's (1972) study, each subject produced an entire subjective probability distribution that was appropriate to the binomial distribution presented. However, no comparisons between the subjects' subjective probability distributions and the objective probability distributions were made. Previous researchers have apparently ignored this aspect of probability estimation. An extensive review of the research literature did not produce any studies which examined the goodness of fit between an empirical and a theoretical probability distribution. In addition to examining methodological and theoretical issues, the major contribution of this study will be to systematically compare the goodness of fit between subjective and objective probability distributions. Estimates of the goodness of fit will be produced for all nine problems. The results obtained will specify the pattern of subject estimation errors and will quantify the effects of the under/over bettor bias.

In order to accomplish these objectives, nine different binomial distributions will be used in the experiment. These distributions will be constructed by using different combinations of probability level (P) and sample size (N). Three different probability levels will be used (P = .30, .50, and .75) . Each binomial distribution will be tested for three different sample sizes (N = 10, 100, and 1000) . Each subject will evaluate one of the nine binomial distributions. In doing so, he will produce a theoretical binomial distribution . Thus, different combinations of probability level and sample size will be used in a 3 X 3 between subjects experimental design with the subjective probability estimate as the dependent variable.

Statement of Hypotheses

This study will examine the effects of probability level and sample size on subjective probability estimates. It is predicted that probability level (P) will affect the subjective probability estimates. Therefore, the subjects will be able to accurately estimate the mean of the subjective probability distributions. In contrast, it is expected that sample size (N) will not affect the subjective probability estimates. Moreover, it is predicted that the interaction of probability level (P) and sample size (N) will not significantly affect the subjects' responses. If these hypotheses are correct, then the subjects will produce similar subjective probability distributions for sample sizes of 10, 100, and 1000.

One of the main purposes of the study is to investigate the relationship between subjective and objective probability distributions. It is predicted that the subjective probability distributions will approximate linear functions of the objective probability distributions (cf., Coombs, Dawes, & Tversky (1970). Specifically, it is expected that the goodness of fit between subjective and objective probability distributions will be better for binomial distributions with P = .50 than for binomial distributions with P = .30 or P =.75 . That is, symmetrical binomial distributions will be more accurately estimated than skewed binomial distributions. Additionally, it is hypothesized that the goodness of fit between subjective and objective probability distributions will become increasingly poor as sample size (N) increases. Finally, it is predicted that subjects will underestimate response categories with high probabilities and overestimate response categories with low probabilities. This pattern of estimation errors will also become increasingly severe as sample size (N) increases.

CHAPTER II

Method

Subjects

The subjects were undergraduate students enrolled in the introductory psychology courses at the University of Windsor. A total of 201 subjects were tested. They had a mean age of 23.9 years and a sex distribution of 88 males and 113 females. On the average, subjects had 4.26 years of high school mathematics and had taken 2.5 mathematics or statistics courses as an undergraduate student. Each subject received one course credit point for participating in the study.

Research Instrument

Nine versions of a two page questionnaire were constructed. The first page contained a short description of the study, directions to the subject, and requested demographic information about the subject. The second page contained one problem for the subjects to solve. The problem required the subject to estimate the probability of a given event. In so doing, each subject created a subjective probability distribution for the event. Different subjects were required to solve different problems. Altogether, there were nine different problems. These problems break down into a 3 X 3 experimental design. Table 1 shows the factorial design of the experiment.

Three different binomial probabilities ($P = .30$, $.50$, and $.75$) and three different sample sizes ($N = 10, 100,$ and 1000) were used in the formulation of the different problems. Probability estimates for each category were given in terms of percentages. Large sample sizes were used so that the subject could not calculate the objective probability values of the different categories. A typical

Table 1

Factorial Design for the Probability Level (P) by Event Size (N) Experiment

	Probability Level		
Event Size	P = .30	P = .50	P = .75
N = 10	P = .30 N = 10	P = .50 N = 10	P = .75 N = 10
N = 100	P = .30 N = 100	P = .50 N = 100	P = .75 N = 100
N = 1000	P = .30 N = 1000	P = .50 N = 1000	P = .75 N = 1000

problem is given in Table 2. All problems dealt with the distribution of physical traits (e.g., eye color) in a given sample. Appendix A gives all the different problems.

Procedure

With the permission of each instructor, I arrived at the classroom at a pre-arranged time. After I was introduced to the students, I said the following:

> I am here today to ask you to participate in a research study. This study is concerned with probability estimation. The experimental questionnaire is two pages long and takes only 5 - 10 minutes to complete. Additionally, the questionnaire is anonymous. The only personal information that you will supply is your age and your sex.
>
> Although participation in this study is voluntary, it will be to your advantage to participate. If you fill out the questionnaire, you will receive one point of extra credit toward your course grade.
>
> A short description of the study and complete directions are contained on the first page of the questionnaire. Be sure to read the problem carefully. I would also appreciate any comments that you might have about the questionnaire. These comments may be written directly on the questionnaire.
>
> Now please raise your hand if you would like to fill out the questionnaire. Once you receive the questionnaire, you may begin.

Subjects were then given a copy of the questionnaire. All nine versions of the questionnaire were distributed to each class tested. Prior to testing, I arranged the questionnaires into repeated sets of nine. Consequently, there

Table 2
Sample Problem

DISTRIBUTION OF EYE COLOR

A group of geneticists at the University of Toronto decided to study the inheritance of eye color. Specifically, they wanted to determine the percentage of individuals in Ontario that had brown eyes. Their research results indicated that the probability that an individual will have brown eyes is .75 (that is, 75%).

Assume that you are given the task of analyzing the data for 100 individuals that are randomly selected from the population studied. Based on the information given above, please determine the probability that each of the following categories will occur (in %) :

	(State probability in percent)
Up to 5 individuals have brown eyes	____________
5 to 15 individuals have brown eyes	____________
15 to 25 individuals have brown eyes	____________
25 to 35 individuals have brown eyes	____________
35 to 45 individuals have brown eyes	____________
45 to 55 individuals have brown eyes	____________
55 to 65 individuals have brown eyes	____________
65 to 75 individuals have brown eyes	____________
75 to 85 individuals have brown eyes	____________
85 to 95 individuals have brown eyes	____________
More than 95 individuals have brown eyes	____________
	Total = 100%

-Note that the categories include all possibilities, so your answers should add up to about 100%.

were many random replications of the task in each class tested. The subjects were allowed five to ten minutes to complete the questionnaire. I answered any questions that arose.

The completed questionnaires were returned to me. Subjects signed an experimental credit sheet that indicated that they had participated in the study. They were then told that the experimental results would be available in July 1983. Copies of the results were posted on the classroom walls. The correct answers were given for all nine probability estimation problems. Additionally, the subjects' pooled responses were reported for all nine groups. No individual results were reported because the questionnaires were anonymous.

Analytic Procedures

For each given binomial distribution, the subjects made 11 separate subjective probability estimates. The sum of each subject's probability estimates was 1.0 or 100% . The mean subjective probability estimate (M) was calculated for each subject and it served as the main dependent variable for the study.

A two-way ANOVA (probability level (P) by sample size (N)) was performed. It tested whether probability level, sample size, or their interaction affected the subjects' mean subjective probability estimates (M) . In addition, a three-way ANOVA (probability level (P) by sample size (N) by response category (C)) was performed. It used the subjective probability estimates in a given reponse category in order to determine if the distribution of subjective probability estimates differed across category as a function of probability level, sample size, or their interaction.

The goodness of fit between the subjective probability estimates and the objective probability distribution for each of the nine problems was examined. The objective probability values were calculated directly from the binomial theorem when the sample size estimated was 10 (i.e., N = 10) . A normal approximation to the binomial distribution was used to calculate the objective probability values when the sample size estimated was 100 or 1000 (i.e., N = 100 or 1000). For each subject in each response category, a variable called DP (i.e., discrepancy in probability) was constructed by subtracting the subjective probability estimate (SP) from the objective probability value (OP) . Then, for each of the nine problems, a repeated measures ANOVA was performed on the DP values in order to assess the goodness of fit between the empirical and theoretical probability distributions.

In summary, various satistical analyses were performed to: (1) assess the effects of probability level (P), sample size (N), and their interaction (P X N) on the subjects' mean subjective probability estimates (M). (2) determine the distribution of subjective probability estimates across judgment categories (C) as a function of probability level (P) and sample size (N). (3) look at the goodness of fit between the subjective and objective probability distributions for each combination of probability level (P) and sample size (N).

CHAPTER III

Results

The results for this study are reported in two sections: (1) Analysis of individual differences in subjective probability estimates across problems and across categories. This section reports the effects of probability level (P), event size (N), and their interaction on the mean subjective probability estimates and on the subjective probability estimates within categories. (2) Test of the goodness of fit between the subjective and objective probability distributions. The goodness of fit is reported for each binomial distribution (i.e., each of the combinations of probability level and event size). The fits of the various binomial distributions are compared.

Contrary to expectations, a correlational analysis revealed no systematic relationship between subject characteristics and subject probability estimates with the possible exception of sex. Thus, only the effects of sex on subjective probability estimates are reported in the results.

The results are presented in two ways: (1) Mean subjective probability estimate (M) . The M value is the subject's average subjective probability estimate and is claculated from the grouped frequency distributions. (2) Individual subjective probability estimates within a particular response category. To clarify possible confusion, event size estimated is distinguished from sample size. Event size estimated refers to the size of the binomial distribution that the subjects estimated while sample size refers to the number of subjects in a particular experimental condition.

Tables 3 - 5 and Figures 2 - 11 provide all data obtained in the study. Table 3 gives SP, OP, and DP values for the three event sizes (10, 100, and 1000). Table 4 provides SP, OP, and DP values for the three probability levels (.30, .50, and .75).

Table 3

Subjective Probability Estimates (SP), Objective Probability Distributions (OP), And Differences Between Subjective and Objective Probabilities (DP) for the Three Event Sizes (10, 100, 1000)

	Number of Subjects	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	M
						Event Size = 10 (N = 1)							
	67												
SP		.044	.079	.081	.106	.117	.183	.121	.119	.070	.042	.038	.482
OP		.010	.047	.099	.135	.144	.136	.127	.122	.103	.062	.018	
DP		.033	.032	-.018	-.029	-.027	.047	-.005	-.003	-.033	-.020	.020	
						Event Size = 100 (N = 2)							
	70												
SP		.072	.065	.085	.128	.153	.160	.120	.084	.053	.042	.038	.453
OP		.000	.000	.045	.238	.099	.235	.057	.161	.161	.003	.000	
DP		.072	.065	.040	-.110	.055	-.075	.063	-.077	-.108	.039	.039	
						Event Size = 100 (N = 3)							
	64												
SP		.066	.072	.088	.129	.110	.150	.101	.109	.090	.044	.042	.473
OP		.000	.000	.000	.375	.001	.312	.000	.156	.156	.000	.000	
DP		.066	.072	.087	-.246	.109	-.161	.100	-.047	-.066	.044	.042	

Note. C = Category and M = Mean of the Subjective Probability Distribution

Table 4

Subjective Probability Estimates (SP), Objective Probability Distributions (OP), And Differences Between Subjective and Objective Probabilities (DP) for the Three Probability Levels (.30, .50, and .75)

	Number of Subjects	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	M
						Probability Level = .30 (P = 1)							
	71												
SP		.095	.110	.120	.195	.145	.105	.078	.063	.044	.028	.017	.374
OP		.009	.041	.124	.662	.112	.035	.013	.003	.001	.000	.000	
DP		.085	.069	-.005	-.467	.033	.070	.066	.060	.043	.028	.017	
						Probability Level = .50 (P = 2)							
	66												
SP		.043	.048	.068	.089	.142	.279	.133	.082	.049	.034	.031	.482
OP		.000	.003	.015	.040	.126	.633	.126	.040	.015	.003	.000	
DP		.043	.044	.053	.050	.016	-.354	.007	.043	.035	.031	.031	
						Probability Level = .75 (P = 3)							
	64												
SP		.041	.055	.062	.071	.093	.112	.135	.170	.122	.067	.071	.562
OP		.000	.000	.000	.001	.005	.019	.052	.416	.425	.065	.018	
DP		.041	.055	.062	.070	.088	.093	.084	-.246	-.303	.002	.053	

Note. C = Category and M = Mean of the Subjective Probability Distribution

Table 5

Means of Subjective Probability Estimates (SP), Objective Probability Distributions (OP), And Differences Between Subjective and Objective Probabilities (DP) for the Nine Problems

	Number of Subjects	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	M
		Problem 1 Probability Level = .30 (P = 1) Event Size = 10 (N = 1)											
	24												
SP		.090	.133	.119	.161	.134	.112	.094	.078	.043	.022	.013	.374
OP		.028	.121	.235	.266	.198	.102	.037	.009	.002	.000	.000	
DP		.062	.012	-.116	-.105	-.064	.010	.057	.069	.041	.022	.013	
		Problem 2 Probability Level = .50 (P = 2) Event Size = 10 (N = 1)											
	22												
SP		.016	.037	.063	.090	.129	.293	.138	.099	.056	.038	.042	.513
OP		.001	.010	.044	.118	.206	.247	.206	.118	.044	.010	.001	
DP		.015	.027	.019	-.028	-.077	.046	-.068	-.019	.012	.028	.041	
		Problem 3 Probability Level = .75 (P = 3) Event Size = 10 (N = 1)											
	21												
SP		.020	.061	.055	.061	.084	.148	.134	.187	.117	.069	.064	.574
OP		.000	.000	.000	.003	.016	.059	.146	.255	.281	.188	.056	
DP		.020	.061	.055	.058	.068	.089	-.012	-.068	-.164	-.119	.008	

Table 5 Continued

	Number of Subjects	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	M
		Problem 4 Probability Level = .30 (P = 1) Event Size = 100 (N = 2)											
	23												
SP		.124	.111	.122	.205	.177	.097	.068	.045	.028	.018	.006	.335
OP		.000	.001	.137	.724	.137	.001	.000	.000	.000	.000	.000	
DP		.124	.110	-.015	-.519	.040	.096	.068	.045	.028	.018	.006	
		Problem 5 Probability Level = .50 (P = 2) Event Size = 100 (N = 2)											
	24												
SP		.034	.041	.071	.097	.172	.269	.151	.078	.044	.027	.016	.471
OP		.000	.000	.000	.001	.157	.683	.157	.001	.000	.000	.000	
DP		.034	.041	.071	.096	.015	-.414	-.006	.077	.044	.027	.016	
		Problem 6 Probability Level = .75 (P = 3) Event Size = 100 (N = 2)											
	23												
SP		.060	.043	.062	.083	.110	.109	.142	.130	.086	.082	.093	.551
OP		.000	.000	.000	.000	.000	.000	.010	.490	.490	.010	.000	
DP		.060	.043	.062	.083	.110	.109	.132	-.360	-.404	.072	.093	

Table 5 Continued

	Number of Subjects	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	M
				Problem 7 Probability Level = .30 (P = 1) Event Size = 1000 (N = 3)									
	24												
SP		.071	.086	.118	.220	.124	.105	.073	.067	.061	.043	.033	.412
OP		.000	.000	.001	.999	.001	.000	.000	.000	.000	.000	.000	
DP		.071	.086	.117	-.779	.123	.105	.073	.067	.061	.043	.033	
				Problem 8 Probability Level = .50 (P = 2) Event Size = 1000 (N = 3)									
	20												
SP		.083	.067	.070	.079	.120	.277	.107	.070	.049	.040	.040	.459
OP		.000	.000	.000	.000	.001	.998	.001	.000	.000	.000	.000	
DP		.083	.067	.070	.079	.119	-.721	.106	.070	.049	.040	.040	
				Problem 9 Probability Level = .75 (P = 3) Event Size = 1000 (N = 3)									
	20												
SP		.042	.062	.069	.068	.082	.079	.128	.200	.168	.049	.054	.561
OP		.000	.000	.000	.000	.000	.000	.000	.500	.500	.000	.000	
DP		.042	.062	.069	.068	.082	.079	.128	-.301	-.333	.049	.054	

Note. C = Category and M = Mean of the Subjective Probability Distribution

Figure Caption

Figure 2 Subjective probability estimates for the three probability levels (.30, .50, and .75).

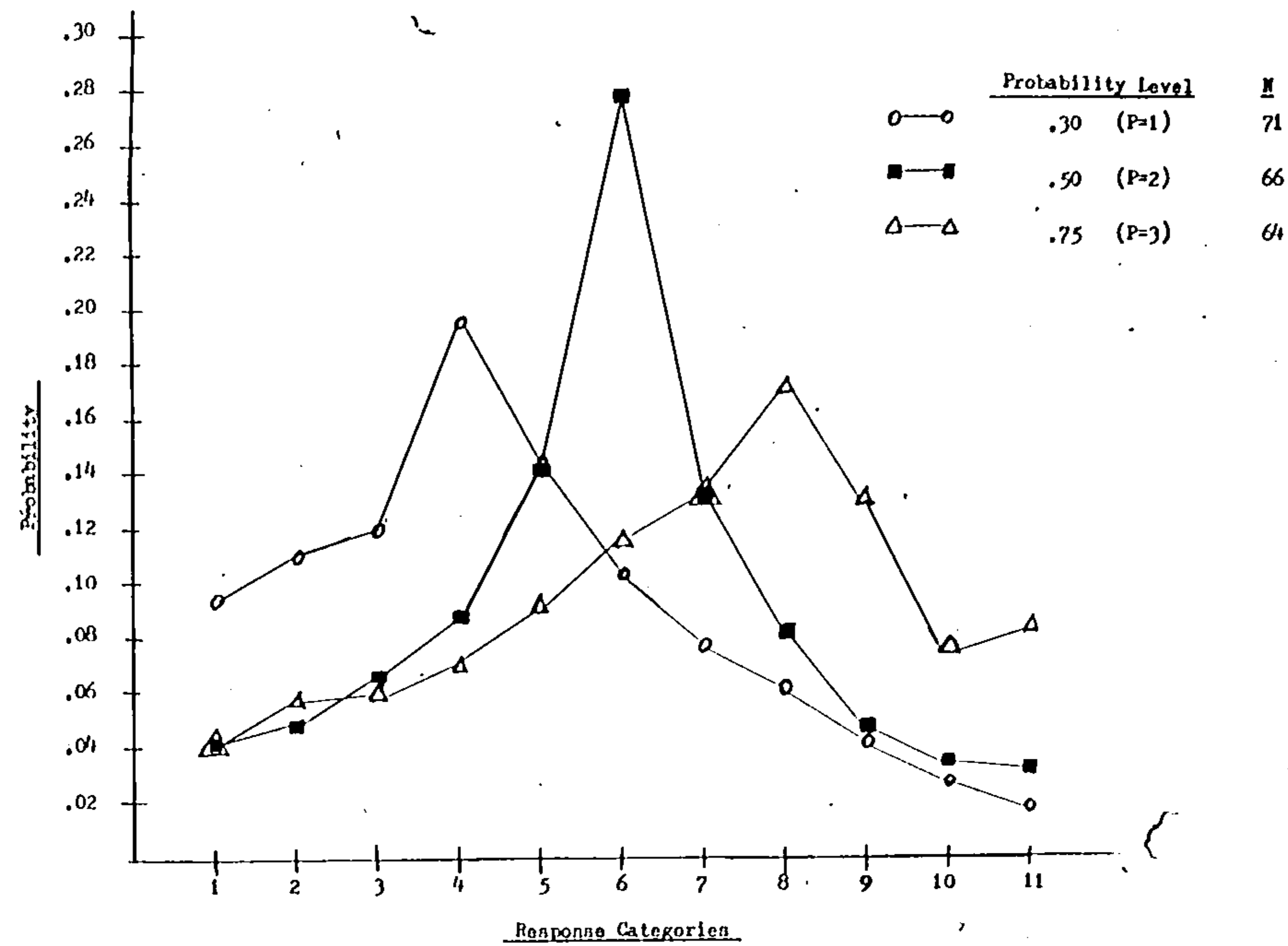

Probability
.30
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
1
2
3
4
5
6
7
8
9
10
11
Response Categories
Probability Level
N
.30 (P=1)
71
.50 (P=2)
66
.75 (P=3)
64

Figure Caption

Figure 3 Subjective probability estimates and objective probability distribution for probability level of .30 and sample size of 10 (N = 24 subjects).

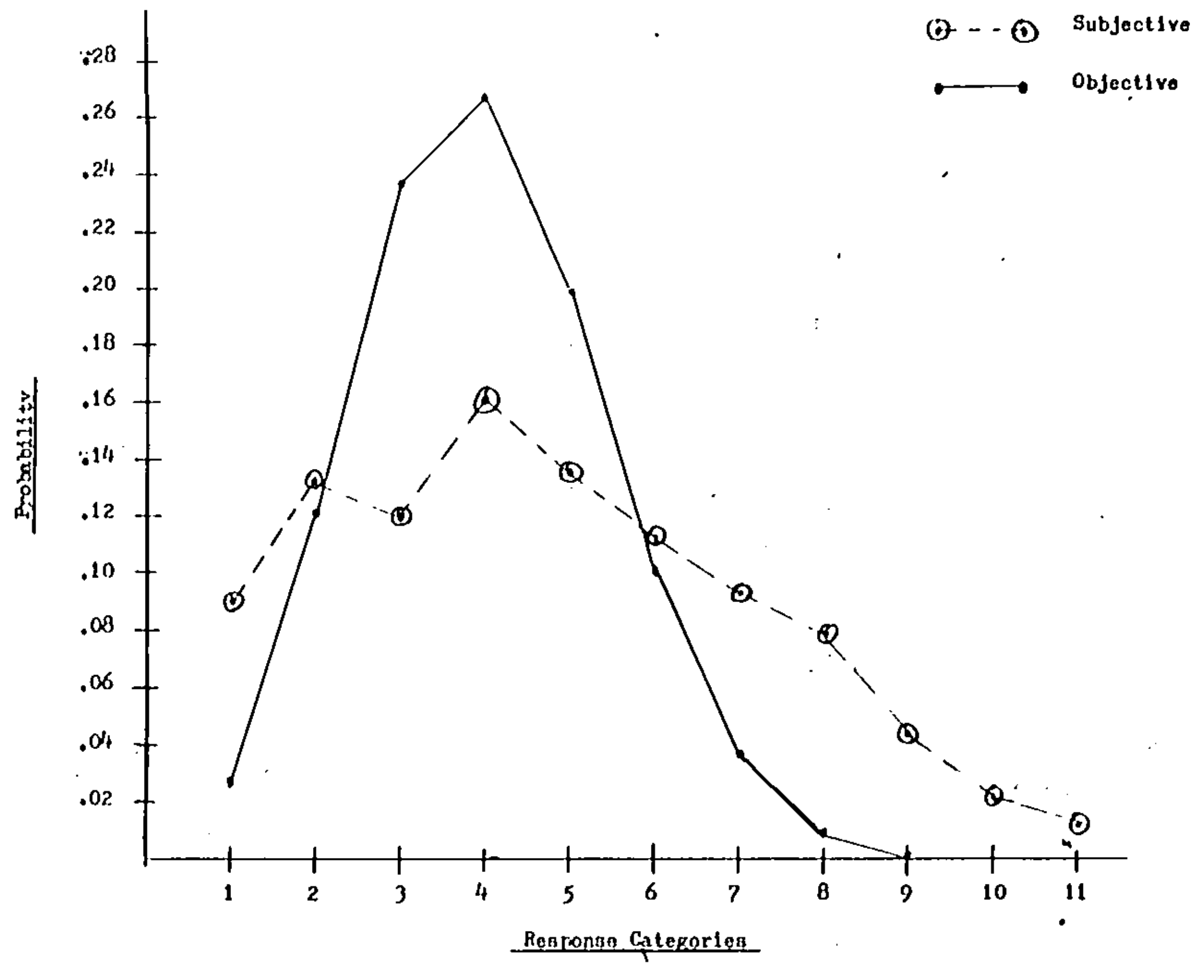
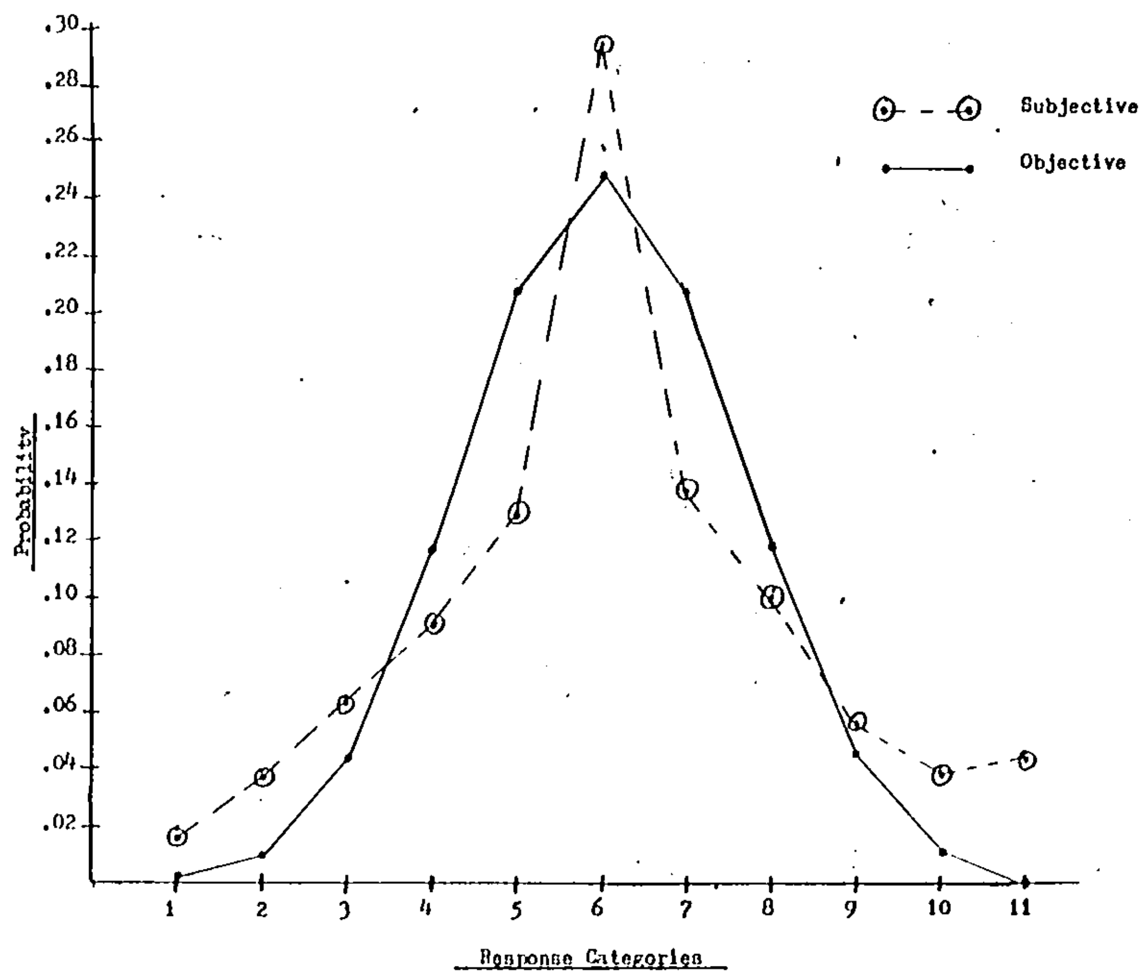

Subjective
Objective
Probability
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
1
2
3
4
5
6
7
8
9
10
11
Response Categories

Figure Caption

<u>Figure 4</u> Subjective probability estimates and objective probability distribution for probability level of .50 and sample size of 10 (N = 22 subjects).



Subjective
Objective
Probability
.30
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
1
2
3
4
5
6
7
8
9
10
11
Response Categories

Figure Caption

Figure 5 Subjective probability estimates and objective probability distribution for probability level of .75 and sample size of 10 (N = 21 subjects).

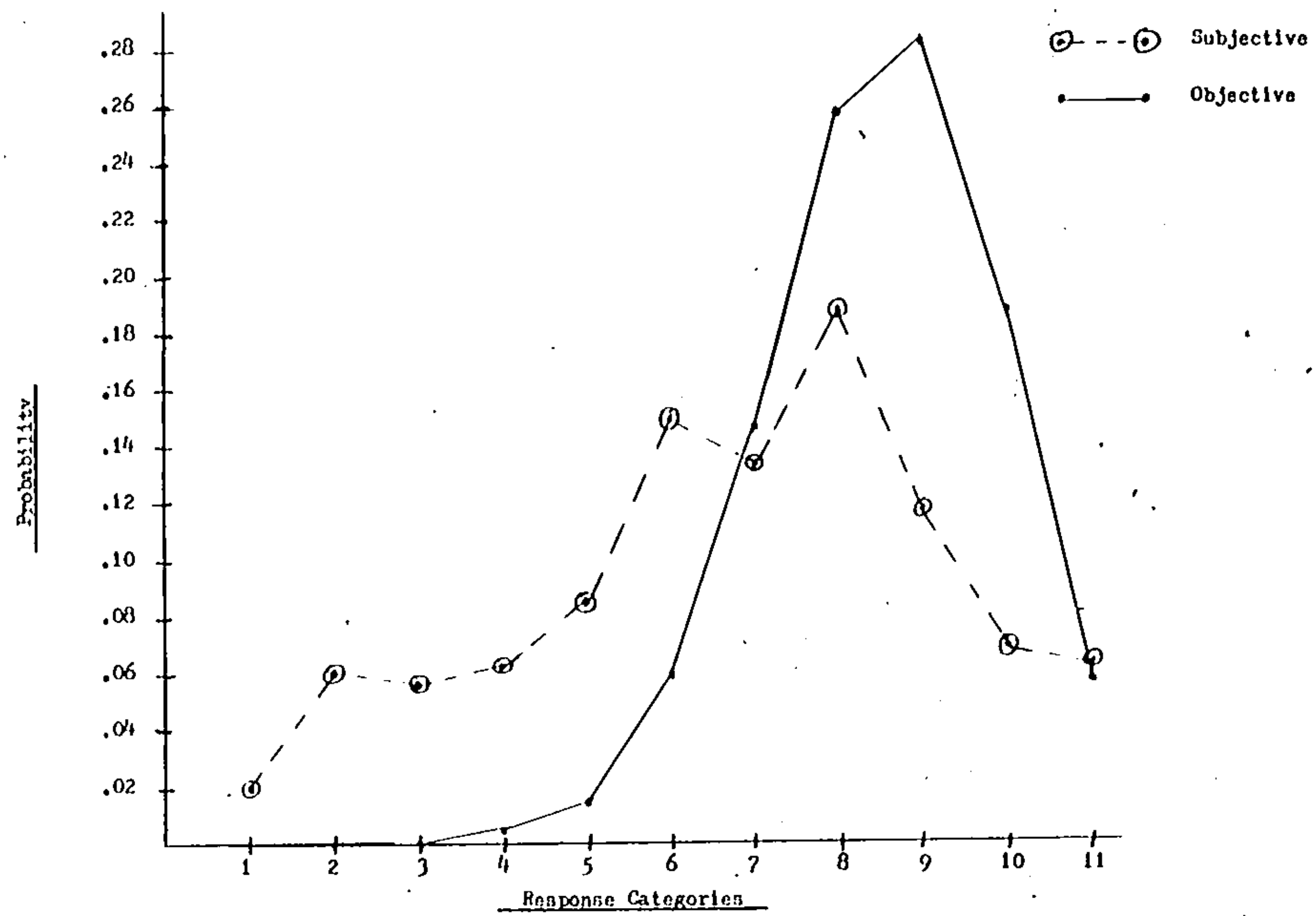

Subjective
Objective
Probability
.28
.26
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
1
2
3
4
5
6
7
8
9
10
11
Response Categories

Figure Caption

Figure 6 Subjective probability estimates and objective probability distribution for probability level of .30 and sample size of 100 (N = 23 subjects).

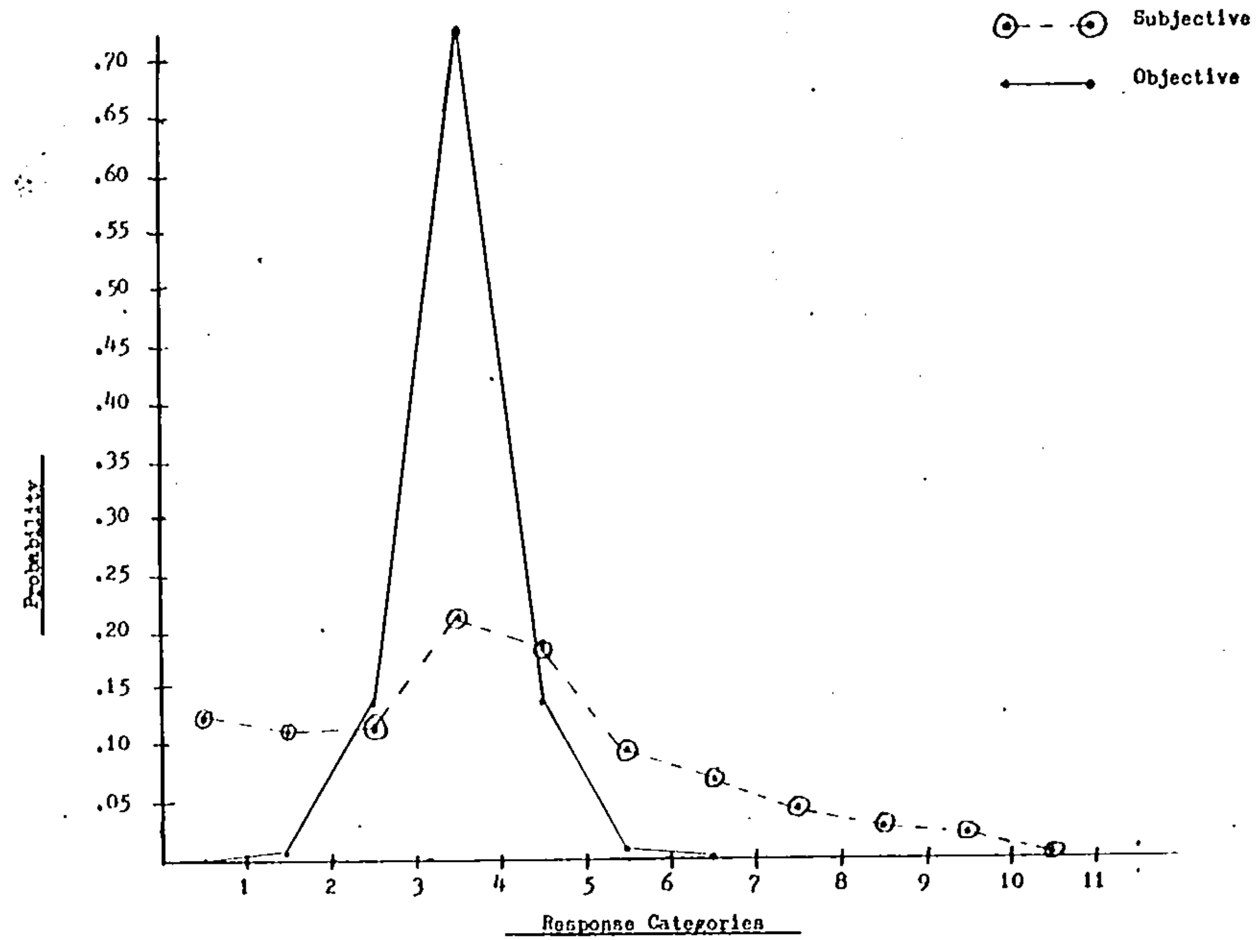

Subjective
Objective
Probability
.70
.65
.60
.55
.50
.45
.40
.35
.30
.25
.20
.15
.10
.05
1
2
3
4
5
6
7
8
9
10
11
Response Categories

Figure Caption

<u>Figure 7</u> Subjective probability estimates and objective probability distribution for probability level of .50 and sample size of 100 (N = 24 subjects).

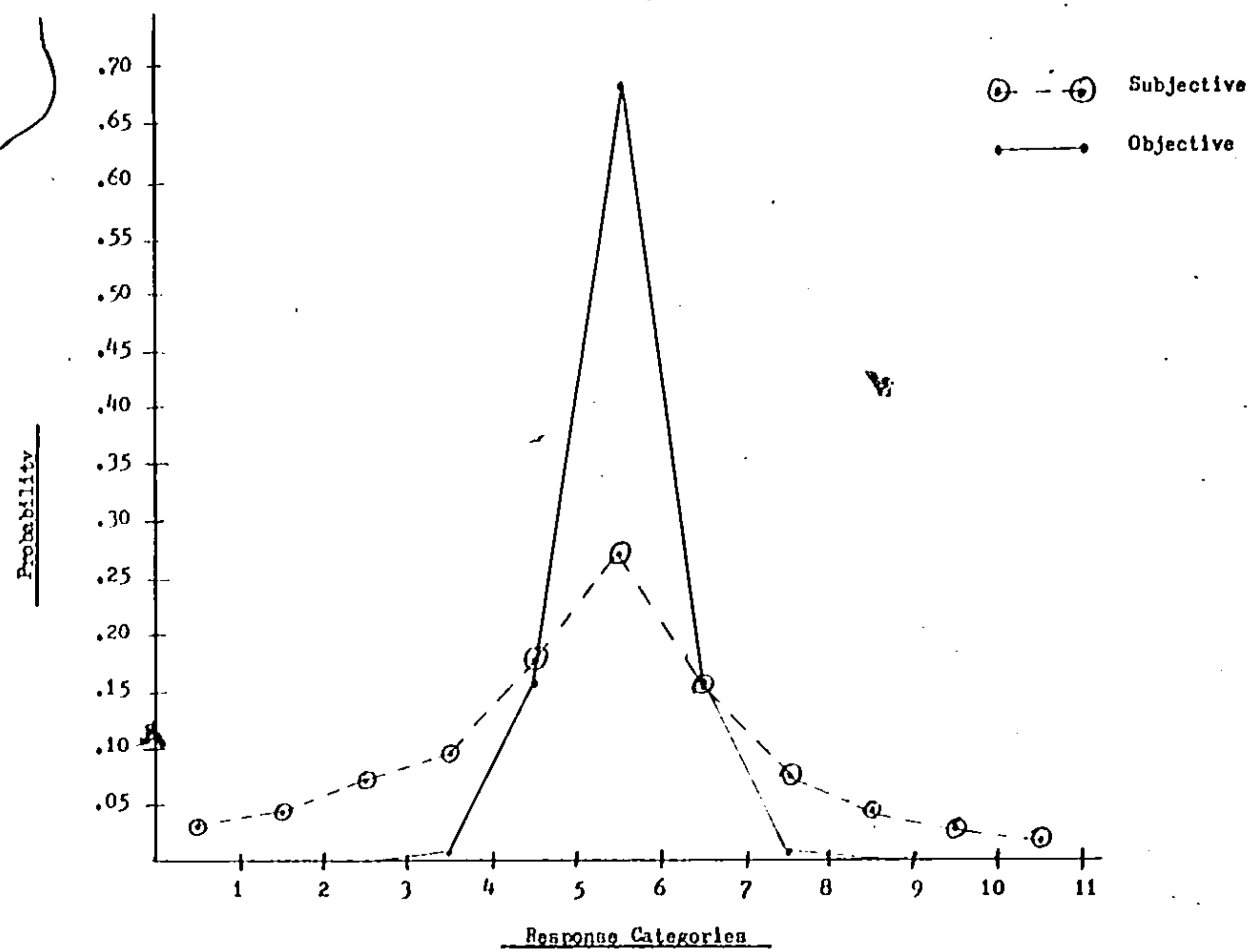

Subjective
Objective
Probability
.70
.65
.60
.55
.50
.45
.40
.35
.30
.25
.20
.15
.10
.05
1
2
3
4
5
6
7
8
9
10
11
Response Categories

Figure Caption

Figure 8 Subjective probability estimates and objective probability distribution for probability level of .75 and sample size of 100 (N =23 subjects).

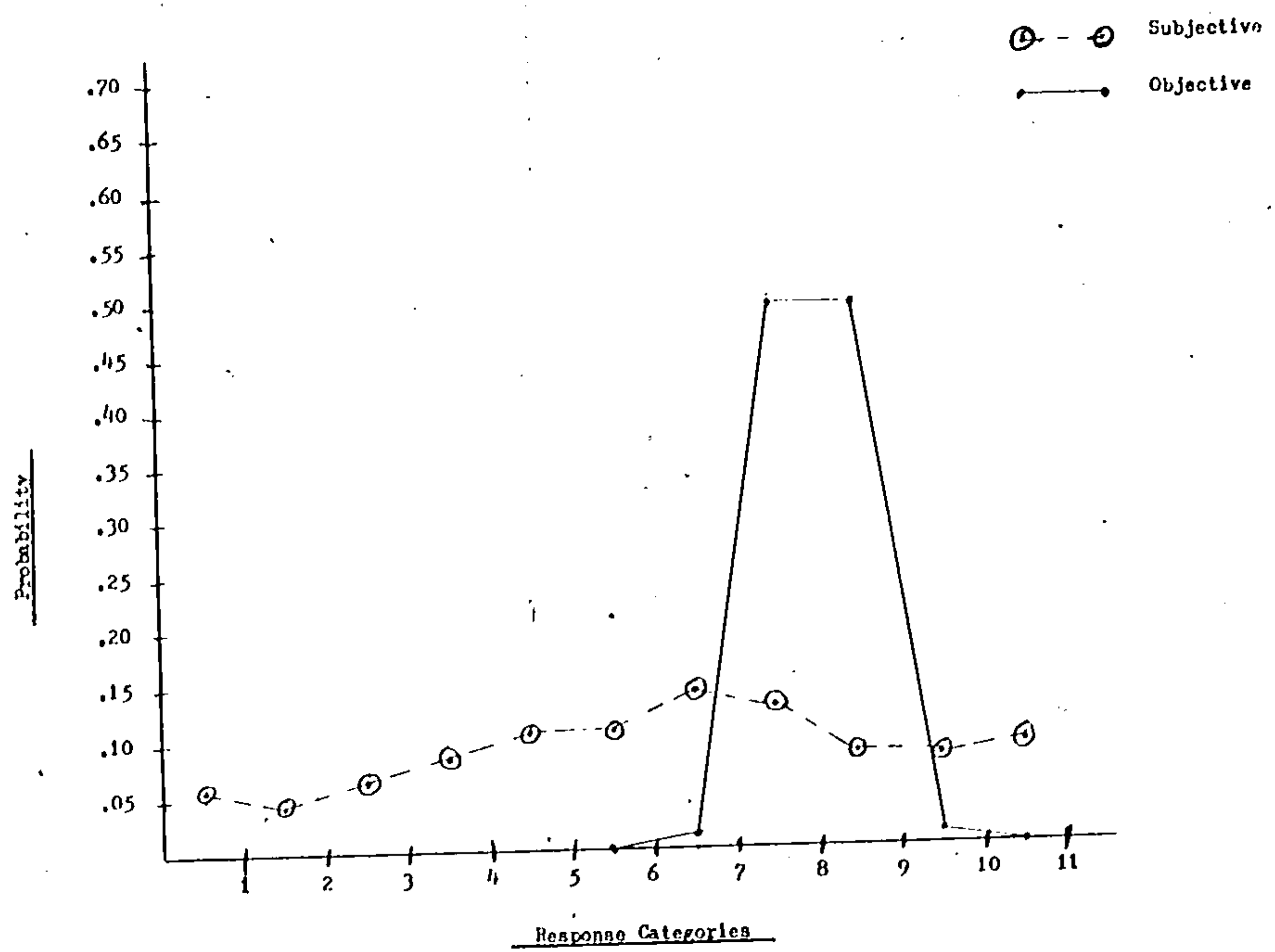
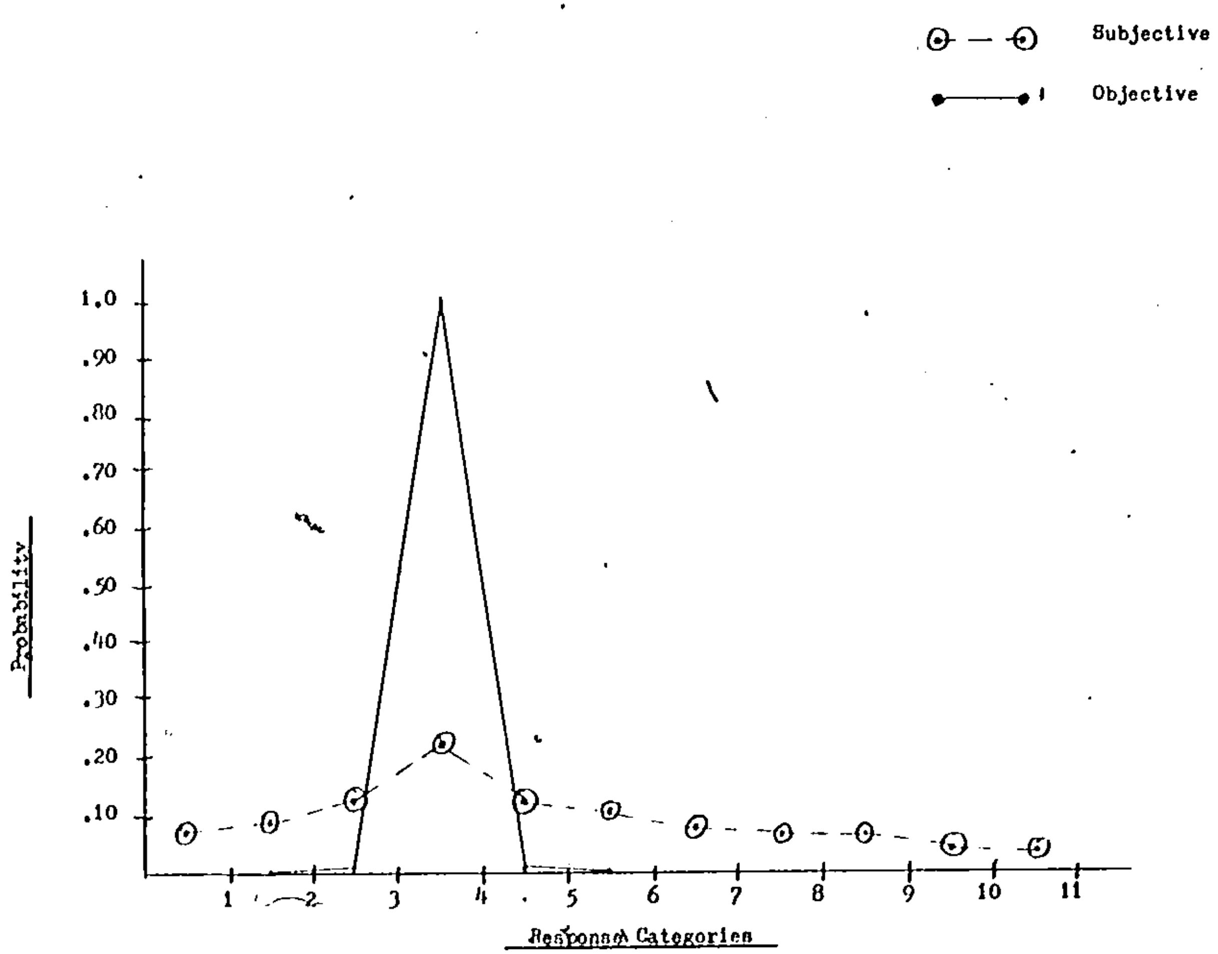

Subjective
Objective
.70
.65
.60
.55
.50
.45
.40
.35
.30
.25
.20
.15
.10
.05
Probability
1
2
3
4
5
6
7
8
9
10
11
Response Categories

Figure Caption

Figure 9 Subjective probability estimates and objective probability distribution for probability level of .30 and sample size of 1000 (N = 24 subjects).



Subjective
Objective
1.0
.90
.80
.70
.60
.50
.40
.30
.20
.10
Probability
1
2
3
4
5
6
7
8
9
10
11
Response Categories

Figure Caption

Figure 10 Subjective probability estimates and objective probability distribution for probability level of .50 and sample size of 1000 (N = 20 subjects).

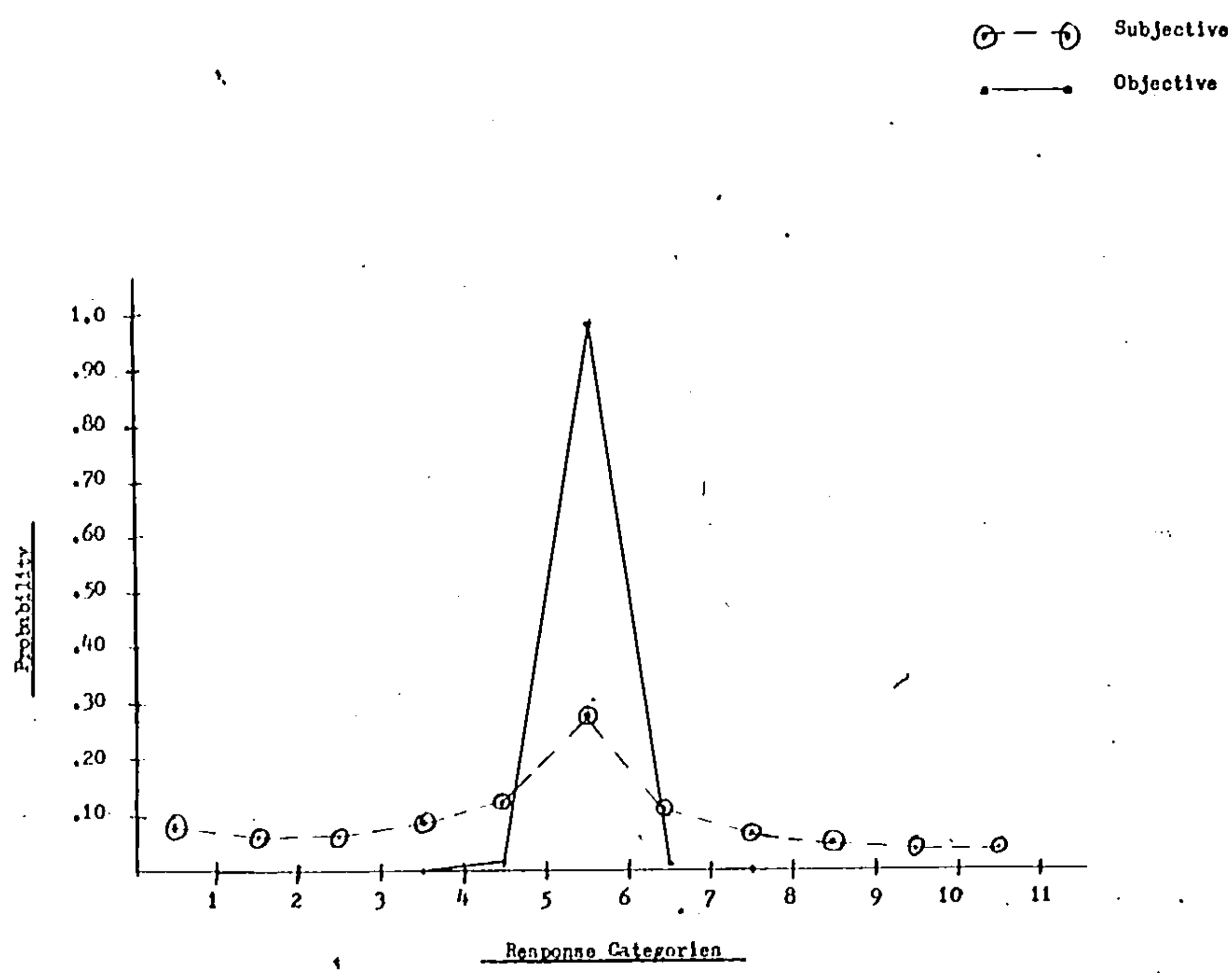

Subjective
Objective
Probability
1.0
.90
.80
.70
.60
.50
.40
.30
.20
.10
1
2
3
4
5
6
7
8
9
10
11
Response Categories

Figure Caption

Figure 11 Subjective probability estimates and objective probability distribution for probability level of .75 and sample size of 1000 (N = 20 subjects).

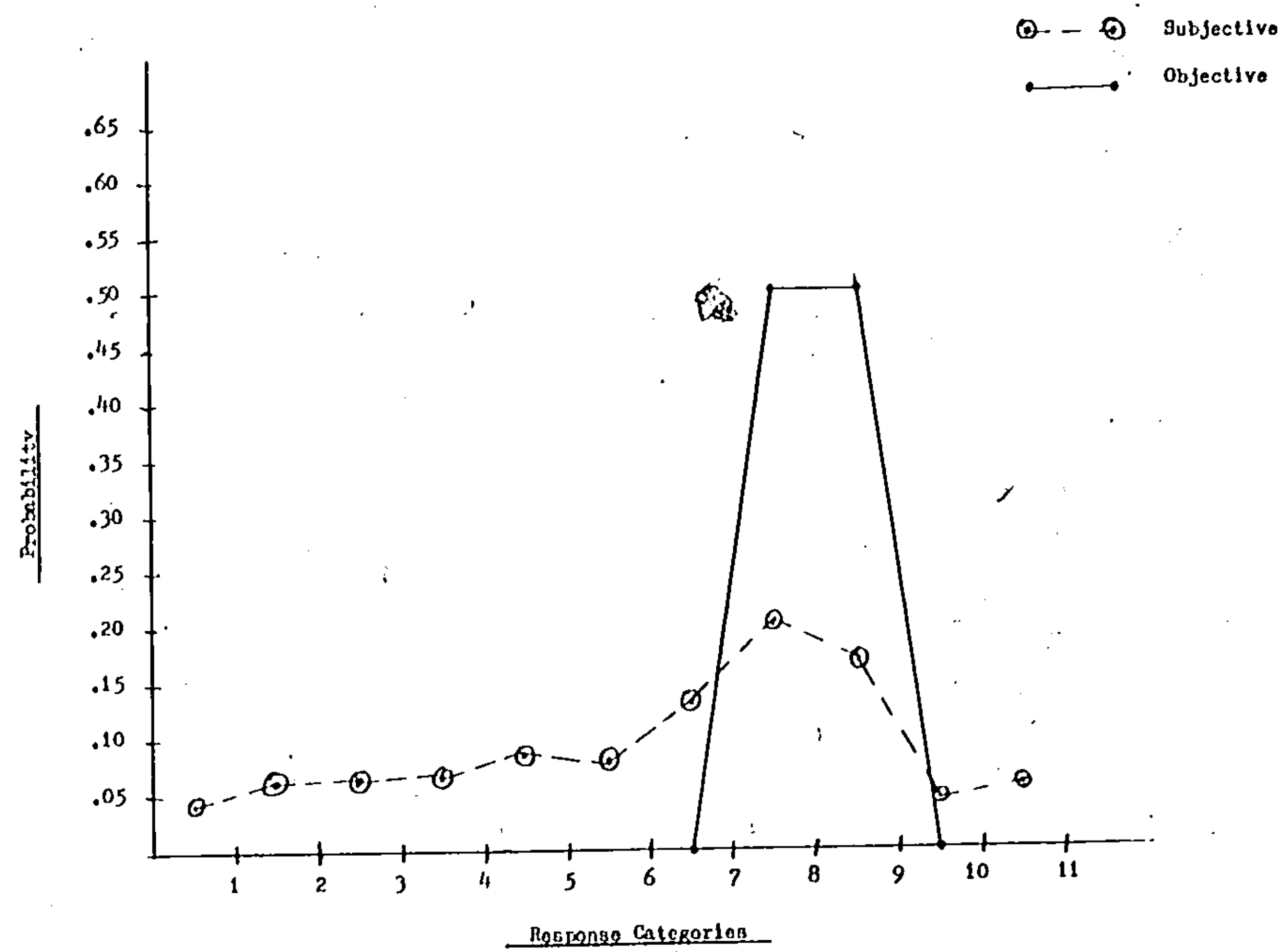

Subjective
Objective
Probability
.65
.60
.55
.50
.45
.40
.35
.30
.25
.20
.15
.10
.05
1
2
3
4
5
6
7
8
9
10
11
Response Categories

Table 5 provides SP, OP, and DP values for each binomial distribution estimated. Consequently, the data are reported for each probability level (P) and for each event size (N). Figure 2 - 11 are graphs of the data obtained. Figure 2 shows the subjective probability estimates for the three probability levels while Figures 3 - 11 present the subjective and objective probability distributions according to the binomial distribution estimated. Tables 3 - 5 and Figures 2 - 11 should be referred to when considering the results of the main analyses.

Before proceeding to the main analyses of the study, it is worthwhile to examine Figures 2 - 11 more closely. Figure 2 shows that subjects can estimate the mean of a given binomial distribution with accuracy. However, they can not estimate the tails of a binomial distribution as successfully. From Figures 3 - 11, it is evident that the fit between subjective and objective probability distributions is good for an event size of 10 cases. However, as the event size increases to 100 or 1000 cases, the fit between the subjective and objective probability distributions becomes progressively poorer. For the individual response categories, subjects underestimate the middle categories (i.e., Categories 4 - 8) while they overestimate the categories at the tails (i.e., Categories 1 - 3 and Categories 8 - 11).

Analysis of Individual Differences in Subjective Probability Estimates Across Problems and Across Categories

Table 6 presents an ANOVA table of the effects of sample size (i.e., event size) by probability level by sex on the mean subjective probability estimate (i.e., the M variable). A standard factorial ANOVA was performed on the data. The number of subjects was 201. Looking at the results presented in Table 6 , it can be seen that the three independent variables (sample size, probability level, and sex) accounted for a significant portion of the variance. The model F-Value was 4.85 and the R - Square value was .3107. A closer inspection of the ANOVA table reveals that sex and sample size did not exert significant effects on the mean subjective probability

Table 6

Analysis of the Means of the Individual Subjective Probability Distributions By Sample Size (N), Probability Level (P), and Sex (S)

Source:	df	SS	MS	F	R^2
Model	17	1.4289	.0841	4.85*	.3107
N	2	.0338	.0169	.97	
P	2	1.1859	.5930	34.23*	
S	1	.0044	.0044	.25	
N*P	4	.0531	.0133	.77	
N*S	2	.0061	.0031	.18	
P*S	2	.0050	.0025	.15	
N*P*S	4	.1064	.0266	1.54	
Error	183	3.1696	.0173		
Total	200	4.5986			

*$p < .01$

estimates. Additionally, none of the possible interactions was significant. The sum of squares values show that the probability level of the binomial distribution had the greatest effects on the subjects' mean subjective probability estimates. The F-Value for probability level was 34.23 which was significant at $p < .01$. The results shown in Table 6 indicate that the sex of the subject had no real effect on his/her subjective probability estimates. This finding was contrary to the findings of other researchers (cf., Cohen, Dearnaley, and Hansel (1955, 1956, & 1957). Finally, sample size (N) did not exert significant effects on the subjects' mean subjective probability estimates. As predicted, subjects ignored the size of the binomial distribution estimated.

Table 7 presents a repeated measures ANOVA table of the effects of sample size by probability level by category on subjects' subjective probability estimates within a particular category. Because there were 11 categories estimated for each binomial distribution, each subject made 11 subjective probability estimates. Consequently, the category variable (C) had 10 degrees of freedom. Since the study used 201 subjects, 2210 subjective probability estimates were made. Table 7 reveals that the model (sample size by probability level by category) had an F-Value of 7.73 which was significant at $p < .01$. The sum of squares values show that the sample size in a particular category did not affect the subjective probability estimates. In contrast, the probability level in a particular category (the P X C interaction) exerted tremendous effects on the subjective probability estimates. The obtained F- Value was 18.52 which was significant at $p < .01$. Therefore, subjects varied their subjective probability estimates on the basis of the category estimated and on the probability level in a particular category.

Table 7

Analysis of the Subjective Probability Estimates By Sample Size (N), Probability Level (P), and Response Category (C)

Source:	df	SS	MS	F	R^2
Model	98	7.0764	.0722	7.73*	.2640
C	10	2.9901	.2990	32.01*	
N*C	20	.2967	.0148	1.59	
P*C	20	3.4591	.1730	18.52*	
N*P*C	40	.3103	.0078	.83	
Error	2112	19.7283	.0093		
Total	2210	26.8048			

*$p < .01$

Test of the Goodness of Fit Between the Subjective and Objective Probability Distributions

Table 8 presents the results obtained from tests of the goodness of fit between the subjective probability estimates and the calculated objective probability values. The dependent variable was the DP values. The DP values were defined as the difference between the subjective probability estimates and the objective probability values in a particular category of a binomial distribution. ANOVA procedures were used to perform the goodness of fit tests. The rationale for the use of ANOVA procedures as opposed to chi-square techniques is that they permit the use of individual subjective probability estimates in each category rather than the mean subjective probability estimate in a category. Consequently, ANOVA procedures take advantage of sampling error .

Table 8 presents the results obtained according to the binomial distribution estimated. For problems 1, 2, and 3 , the event size estimated was 10 cases. Subjects estimated these three binomial distributions rather well. However, the subjects' subjective probability estimates differed significantly from the objective probability values across all categories estimated. The magnitude of the departure from the objective probability values was not large. The F-Values obtained were 5.07 to 13.72 .

For problems 4 - 6 , the event size estimated was 100 cases. Subjects did not estimate these binomial distributions well. In fact, the departure from the objective probability values accounted for 77% of the total variance. The R-Square values ranged from .7337 to .8040 . The F-Values obtained were 63.38 to 90.24. Apparently, subjects ignored the effects of increasing the event size estimated. In fact, the subjects' goodness of fit for problems 4 - 6 was on the average ten times worse than that obtained for problems 1 - 3.

Table 8

Goodness of Fit Between the Subjective Probability Estimates and the Objective Probability Distribution

Source:	df	SS	MS	F	R^2
Problem 1 Probability Level = .30 (P = 1) Event Size = 10 Cases (N = 1)					
Category	10	1.0308	.1031	6.78*	.2276
Error	230	3.4984	.0152		
Total	240	4.5292			
Problem 2 Probability Level = .50 (P = 2) Event Size = 10 Cases (N = 1)					
Category	10	.3905	.0391	5.07*	.1944
Error	210	1.6185	.0077		
Total	220	2.0090			
Problem 3 Probability Level = .75 (P = 3) Event Size = 10 Cases (N = 1)					
Category	10	1.4452	.1445	13.72*	.4069
Error	200	2.1063	.0105		
Total	210	3.5515			

Table 8 Continued

Source:	df	SS	MS	F	R^2
Problem 4 Probability Level = .30 (P = 1) Event Size = 100 Cases (N = 2)					
Category	10	7.2513	.7251	84.24*	.7929
Error	220	1.8937	.0086		
Total	230	9.1450			
Problem 5 Probability Level = .50 (P = 2) Event Size = 100 Cases (N = 2)					
Category	10	4.7435	.4744	63.38*	.7337
Error	230	1.7215	.0075		
Total	240	6.4650			
Problem 6 Probability Level = .75 (P = 3) Event Size = 100 Cases (N = 2)					
Category	10	8.3895	.8390	90.24*	.8040
Error	220	2.0454	.0093		
Total	230	10.4349			

Table 8 Continued

Source:	df	SS	MS	F	R^2
Problem 7 Probability Level = .30 (P = 1) Event Size = 1000 Cases (N = 3)					
Category	10	16.2027	1.6203	176.36*	.8846
Error	230	2.1130	.0092		
Total	240	18.3157			
Problem 8 Probability Level = .50 (P = 2) Event Size = 1000 Cases (N = 3)					
Category	10	11.5696	1.1570	97.09*	.8363
Error	190	2.2640	.0119		
Total	200	13.8336			
Problem 9 Probability Level = .75 (P = 3) Event Size = 1000 Cases (N = 3)					
Category	10	5.0113	.5011	38.59*	.6701
Error	190	2.4676	.0130		
Total	200	7.4789			

*$p < .01$

For problems 7 - 9, the event size estimated was 1000 cases. Once again, subjects estimated these binomial distributions poorly. The departure from the objective probability values accounted for 80% of the total variance. The R-Square values ranged from .6701 to .8846. The F-values obtained were 38.59 to 176.36 . Overall, the goodness of fit for problems 7 - 9 was only slightly worse than that obtained for problems 4 - 6. Curiously, subjects estimated problem 9 better than problem 6. It appears that the subjects could not distinguish between a binomial distribution with N = 100 and a binomial distribution with N = 1000. The results presented in Table 8 show that subjects estimate event sizes of 100 or 1000 cases very poorly. They forgot that the variability of a binomial distribution diminishes as sample size (N) increases. The data in Tables 3 - 5 and the graphs of the subjective and objective probability distributions (see Figures 3 - 11) support the arguments presented above.

Summary of Results

Figures 2 - 11 are graphs of the data obtained. An examination of these figures reveals that subjects can accurately estimate the mean of a binomial distribution. However, they underestimate the middle categories (i.e., Categories 4 - 8) and they overestimate the categories at the tails (i.e., Categories 1 - 3 and Categories 8 -11). Moreover, Figures 3 - 11 show that the fit between the subjective and objective probability distributions becomes progressively poor as the event size estimated increases from 10 cases to 1000 cases.

The results of a factorial ANOVA indicated that sex and sample size did not exert significant effects on the mean subjective probability estimates. In contrast, the probability level of the binomial distribution heavily influenced the mean subjective probability estimates. In addition, it was found that subjects varied their subjective probability estimates on the basis of category estimated and on the probability level in a particular category.

Table 8 presents the results obtained from tests of the goodness of fit between the subjective probability estimates and the calculated objective probability values. The results showed that the subjects' subjective probability estimates differed significantly from the objective probability values across all problems and across all categories. However, subjects estimated the binomial distributions with 10 cases much better than the binomial distributions with 100 or 1000 cases. Interestingly, the discrepancy between the subjective and objective probability distributions was similar for binomial distributions with 100 cases or 1000 cases. This finding demonstrates that subjects ignore the effects of increasing sample size on the standard error of a binomial distribution.

CHAPTER IV

Discussion

The discussion section of this study is divided into three sections : (1) Relationship of the results obtained to the stated hypotheses of the study (2) Theoretical implications of the results obtained (3) Suggestions for future research studies.

Relationship of the results obtained to the stated hypotheses of the study

Previous researchers (e.g., Cohen, Dearnaley, and Hansel ((1955, 1956)) found that subject characteristics such as age, sex, and mathematics/statistics background influenced subjects' subjective probability estimates. Interestingly, the results obtained in this study found no relationship between differences in subject characteristics and subject probability estimates. That is, age, sex, and prior experience with mathematics/statistics had little effect on subjects' subjective probability estimates. Essentially, the subjects in this study were homogenous. Because the probability estimation task was complex, the effects of individual differences in the subjects were negated.

Unlike previous research on subjective probability, this study systematically manipulated various probability levels and various sample sizes of the binomial distributions estimated. The results obtained indicated that probability level (P) affected the subjective probability estimates but that event size estimated (N) and the interaction of probability level and event size (P X N) did not affect the subjective probability estimates. In light of Kahneman and Tversky's (1972 and 1974) research, the results obtained are not surprising. Apparently, subjects ignored the effects of variability in a sampling distribution and focused solely on the central tendency (i.e., probability level) of the binomial distribution estimated.

Figure 2 shows that subjects recognize the importance of the probability level of a distribution. Subjects accurately estimated the means of the binomial distributions especially when the sample size estimated was 10 cases. Even though the effects of sampling variability are considerable as sample size increases, subjects ignore its effects. Kahneman and Tversky might suggest that the results obtained in this study are a reflection of the representativeness heuristic. That is, that individuals believe that a small sample resembles its parent population in all respects. The results of this study support Kahneman and Tversky's notion that subjects use the representativeness heuristic in complex estimation tasks. Unlike Kahneman and Tversky's work, this study also specifies the pattern and the size of subject estimation errors. Consequently, it indicates the particular errors resulting from the misuse of the representativeness heuristic.

Table 8 presents the results of the goodness of fit tests between the subjective probability estimates and the objective probability values. The subjects' subjective probability estimates differed significantly from the objective probability values across all binomial distributions estimated and across all categories estimated. However, subjects estimated symmetrical distributions better than skewed distributions. Additionally, subjects estimated binomial distributions with 10 cases more accurately than binomial distributions with 100 or 1000 cases. Moreover, the size of subject estimation errors was similar for binomial distributions with 100 cases or 1000 cases. These findings reveal that subjects focused on the probability level of the distribution but ignored its sample size. With increasing sample size, events in a binomial distribution tend to become less variable. However subject estimation errors revealed that subjects believe that events in a binomial distribution maintain a constant variability even though the sample size increases greatly.

Figures 3 - 11 further specify the pattern of subject estimation errors. Overall, subjects underestimated the middle categories which have high objective probability values (i.e., categories 4 - 8) . This finding occurred for all distributions estimated except for a probability level of .50 and a sample size estimated of 10 cases. For that distribution, subjects overestimated the mean of the binomial distribution. Again, this finding may be due to the subjects' lack of appreciation for sampling variability. In contrast, subjects overestimated the categories at the tails which have low objective probability values (i.e., categories 1 - 3 and categories 8 - 11). These findings can be explained by the under/over bettor bias effect (cf. Coombs, Dawes, & Tversky (1970)) .

The under/over bettor bias is a common error in decision making. Individuals have a tendency to underestimate likely events and to overestimate unlikely events. The results obtained in this study suggest that the under/over bias in subject estimation errors may be the result of misuse of the representativeness heuristic. The most representative feature of a population is its central tendency or its probability level. In making probability estimates about a sample, subject focus on this property of a population. They forget that factors such as sample size can affect the central tendency of a distribution. Therefore, samples of varying sample size have different means and different standard errors even though they are drawn from the same population. Estimation errors resulting from an ignorance of the effects of sample size increase in severity as sample size increases.

The methodology of the present study was a marked improvement over previous research studies on subjective probability. As suggested by Kahneman and Tversky

(1982), subjects produced only one subjective probability distribution. This procedure ensured that subjects' subjective probability estimates were independent and that practice and fatigue effects were avoided. Additionally, at least 20 subjects were used in each experimental condition. Previous researchers used only 7 - 8 subjects in each experimental condition. Finally, subjects evaluated 11 response categories for all binomial distributions estimated. By using the same number of response categories for all problems, the accuracy of subjects' subjective probability estimates could be compared across all problems and across all categories.

Theoretical implications of the results obtained

The results obtained indicate that different measures of central tendency may be appropriate for different types of distributions. In the present study, the subjects' mean subjective probability estimate was used because it was the expected value of the probability distribution and was the appropriate value for parametric statistical analysis. For symmetrical distributions (e.g., binomial distributions with a probability level of .50) , the mean, the median, and the mode are all equivalent. In skewed distributions, however, the mean, the median, and the mode are not equivalent.(see Horowitz (1974)). Instead, the mean is displaced towards the center of the distribution (i.e., towards a value of .50). Consequently, the mode may be a more appropriate measure of central tendency for skewed distributions. To illustrate this phenomenon, consider Figure 9. Figure 9 presents the subjective and objective probability distributions for a probability level of .30 and a sample size of 1000 cases. From Figure 9, it can be seen that the mode is approximately .25 . In contrast, the mean subjective probability estimate calculated for the subjective probability distribution is .412. Thus, the measure of central tendency selected for statistical analysis may not be the most descriptive.

Like previous research on subjective probability, this study utilized ipsative data. However, the use of ipsative data may create problems in the interpretation of the data obtained. Because the subjective probability estimates are ipsative, both skewness and kurtosis (i.e., peakedness) are constrained. Due to these imposed constraints, the skewness of the non-symmetrical distributions estimated is really indicative of their central tendency and the kurtosis of those distributions is really indicative of their variability. This phenomenon is known as "aliasing" in the statistical literature.

Unlike previous research on subjective probability, this study directly measured the goodness of fit between empirical and theoretical probability distributions. In doing so, the F test was used rather than the chi-square test. In the present study, the use of the F test had several advantages over the use of the chi-square test. First, the F test permitted full exploitation of the properties of the data base. Individual category estimates were used rather than the mean category estimates. Thus, the sampling error across subjects was able to be retained. The F test was also preferable because of considerations of statistical power incorporated into the anlysis. More research needs to be done to explore non-orthodox uses of the F test.

Suggestions for future research

A number of improvements could be made on the methodology of this study. Primarily, improvements would center on the design of the response categories. First, the order of presentation of the categories should be varied. In the present study, the categories estimated always went from low frequencies to high frequencies for a particular event. In order to counterbalance for the effects of order of category presentation, the categories estimated should be systematically varied so that high frequencies for an event are evaluated first as well as last.

Another issue in category design is that the category boundaries selected must be consistent for all distributions estimated. In the present study, binomial distributions with probability levels of .30, .50, and .75 were studied. Although the number of categories estimated was the same for all distributions, the category boundaries were not. Consequently, the goodness of fit for distributions with a probability level of .75 appeared to be better than it actually was (see Figure 11 and Table 8). In order to prevent such experimental artifacts, the probability levels studied should permit consistent category boundaries to be selected. In the present study, these difficulties could have been avoided by using binomial distributions with probability levels of .30, .50, and .70 .

This study did not investigate whether subjects could be taught to recognize the effects of sample size on the central tendency and the standard error of a distribution. This would be a fruitful area for further research. Perhaps, subjects could be taught to adjust their subjective probability estimates on the basis of prior information about the sample size of the distribution. Practice exercises could be designed to highlight certain features of probability estimation. The effects of sample size and sampling variability on subjective probability estimates could be stressed. Since Bayesian methods of statistical inference are still largely undeveloped, such a study would make a valuable contribution to the statistical literature.

It would also be interesting to study the subjective probability estimates made by different groups of subjects. In the present study, all subjects were introductory psychology students between 18 -24 years of age. Perhaps, different groups of subjects would perform differently on probability estimation tasks. For example, a researcher could study the differences in the subjective probability estimates of professional gamblers, graduate students/professors in mathematics, and professionals in other fields (e.g., doctors, lawyers, and engineers).

Summary of discussion

Contrary to previous research on subjective probability, individual differences in subject characteristics had little effect on subjects' subjective probability estimates. Perhaps the complexity of the probability estimation task negated the effects of individual differences in the subjects. The results obtained indicated that probability level affected the subjective probability estimates but event size estimated and the interaction of probability level and event size did not produce similar effects. Apparently, subjects ignored the effects of sampling variability on the binomial distributions estimated.

The results obtained in this study support Kahneman and Tversky's idea that subjects use the representativeness heuristic in complex estimation tasks. However, this study also specifies the pattern and the size of subject estimation errors. The results obtained from goodness of fit tests reveals that subjects estimated symmetrical distributions better than skewed distributions and binomial distributions with 10 cases better than binomial distributions with 100 cases or 1000 cases. Again, these findings reveal that subjects focused on the probability level of a distribution but ignored its sample size.

Overall, subjects underestimated the middle categories which have high objective probability values and overestimated categories at the tails which have low objective probability values. These findings suggest that the under/over bettor bias is a common error in decision making. They also suggest that the under/over bias in subject estimation errors may be the result of misuse of the representativeness heuristic. The probability level or central tendency of a population appears to be its most representative feature. Finally, methodological improvements made by this study were detailed.

The theoretical implications of the results were also discussed. The results obtained indicated that different measures of central tendency may be appropriate for different types of distributions and for different purposes. For example, the mean subjective probability estimate was used in this study because it was the appropriate value for parametric statistical analysis. However, it was not as descriptive as the mode for skewed distributions. Some problems created by the use of ipsative data were detailed. Due to the constraints imposed by ipsative data, care must be taken in the interpretation of skewness and kurtosis of non-symmetrical distributions. Finally, the rationale for using the F test as a goodness of fit test was explained. It was shown that the F test has several advantages over the chi-square test because it permits use of the sampling error across subjects. More research needs to be done on the non-orthodox uses of the F test.

Suggestions for future research studies were also made. A number of improvements could be made in this study, especially in the area of category design. The order of the presentation of the categories should be counterbalanced. In addition, the category boundaries selected should be consistent for all distributions studied. It would also be interesting to determine if subjects could be taught to adjust their subjective probability estimates on the basis of prior information about the sample size of the distribution. Finally, the performance of different groups of subjects could be compared. Differences in the subjective probability estimates of professional gamblers, mathematics professors, and other professionals (e.g., doctors, lawyers) could be studied.

References

Cascio, W. (1978). Applied psychology in personnel management. Reston, VA: Reston Publishing Company.

Cohen, Jacob. (1962). The statistical power of abnormal-social psychological research. Journal of Abnormal and Social Psychology, 65, 145-153.

Cohen, Jacob. (1969). Statistical power analysis in the behavioral sciences. New York: Academic Press.

Cohen, John., Dearnaley, E.J., & Hansel, C.E.M. (1956). The addition of subjective probabilities: the summation of estimates of success and failure. Acta Psychologica, 12, 371-380.

Cohen, John., Dearnaley, E.J., & Hansel, C.E.M. (1957). Measures of subjective probability: estimates of success in performance in relation to size of task. British Journal of Psychology, 48, 272-275.

Cohen, John., & Hansel, C.E.M. (1955). The idea of a distribution. British Journal of Psychology, 46, 111- 121.

Cohen, John., & Hansel, C.E.M. (1956). Risk and gambling. New York: Philosophical Library.

Coombs, C.H., Dawes, R.M., & Tversky, A. (1970). Mathematical psychology: An elementary introduction. Englewood Cliffs, NJ: Prentice-Hall.

Edwards, W. (1954). The theory of decision making. Psychological Bulletin, 51, 380-417.

Edwards, W. (1961). Behavioral decision theory. Annual Review of Psychology, 12, 473-498.

Edwards, W. (1968). Conservatism in human information processing. In B. Kleinmuntz (Ed.), Formal representation of human judgment (pp. 17-52) . New York: Wiley.

Horowitz, L. (1974). Elements of statistics for psychology and education. New York: McGraw-Hill.

Kahneman, D. & Tversky, A. (1972). Subjective probability: A judgment of representativeness. Cognitive Psychology, 3, 430-454.

Kahneman, D., & Tversky, A. (1982). On the study of statistical intuitions. Cognition, 11, 123-141.

Luce, R.D., & Raiffa, H. (1957). Games and decisions: Introduction and critical survey. New York: Wiley.

Peterson, C.R., & Beach, L.R. (1967). Man as intuitive statistician. Psychological Bulletin, 68, 29-46.

Peterson, C.R., & Ducharme, W.M., & Edwards, W. (1968). Sampling distributions and probability revisions. Journal of Experimental Psychology, 76, 236-243.

Savage L.J. (1954). The foundations of statistics. New York: Wiley.

Tune, G.S. (1964). Response preferences: A review of some relevant literature. Psychological Bulletin, 61, 286-302.

Tversky, A., & Kahneman, D. (1971). Belief in the law of small numbers. Psychological Bulletin, 76, 105-110.

Tversky, A., & Kahneman, D. (1973). Availability: A heuristic for judging frequency and probability. Cognitive Psychology, 5, 207-232.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. Science, 185, 1124-1131.

Tversky, A., & Kahneman, D. (1982). Variants of uncertainty. Cognition, 11, 143-157.

Von Neumann, J., & Morgenstern, O. (1953). Theory of games and economic behavior (3rd ed.). Princeton, NJ: Princeton University Press.

Wheeler, G. & Beach, L.R. (1968). Subjective sampling distributions and conservatism. Organizational Behavior and Human Performance, 3, 36-46.

Appendix A

All Versions of the Experimental Questionnaire

PROBABILITY ESTIMATION-TASK

(Do NOT write your name on this questionnaire)

Sex : ____________ Age, in years : ____________

Number of years of high school mathematics : ____________

Number of mathematics/statistics courses completed as an University of Windsor undergraduate : ______________

Individuals must often make decisions about people and about events on the basis of little or no information (e.g., success ~~in a~~ new job, future value of IBM stock, outcome of a ~~sports~~ event). These situations occur regularly because there is no time to obtain the necessary information or because the necessary information is unavailable. I believe that it would be valuable to study how individuals assess the probability of an uncertain event. On the following page, you are given a problem to solve. Please read the problem carefully and then indicate your response on the blanks provided. Answer the problem to the best of your ability. Thank you for your cooperation.

DISTRIBUTION OF HAIR COLOR

Researchers at Statistics Canada decided to study the distribution of hair color of individuals living in Canada. They determined that .30 (that is, 30%) of Canada's population had fair hair (that is, blond or red hair).

Assume that you are given the task of analyzing the data for 10 individuals that are randomly selected from the population studied. Based on the information given above, please determine the probability that each of the following categories will occur (in %) :

	(State probability in percent)
0 individuals have blond or red hair	__________
1 individual has blond or red hair	__________
2 individuals have blond or red hair	__________
3 individuals have blond or red hair	__________
4 individuals have blond or red hair	__________
5 individuals have blond or red hair	__________
6 individuals have blond or red hair	__________
7 individuals have blond or red hair	__________
8 individuals have blond or red hair	__________
9 individuals have blond or red hair	__________
10 individuals have blond or red hair	__________
	Total = 100%

- Note that the categories include all possibilities , so your answers should add up to about 100%.

DISTRIBUTION OF SEXES

In Peterborough, Ontario approximately 10 babies are born every day. As you know, about .50 (that is, 50%) of all babies are boys. However, the exact percentage of baby boys varies from day to day. Sometimes, it is higher than 50%, sometimes lower.

Please predict the percentage of days that the number of boys among 10 babies will be as follows :

(State probability in percent)

0 boys are born in Peterborough, Ontario	__________
1 boy is born in Peterborough, Ontario	__________
2 boys are born in Peterborough, Ontario	__________
3 boys are born in Peterborough, Ontario	__________
4 boys are born in Peterborough,Ontario	__________
5 boys are born in Peterborough, Ontario	__________
6 boys are born in Peterborough, Ontario	__________
7 boys are born in Peterborough,Ontario	__________
8 boys are born in Peterborough, Ontario	__________
9 boys are born in Peterborough,Ontario	__________
10 boys are born in Peterborough, Ontario	__________
	Total = 100%

-Note that the categories include all possibilities, so your answers should add up to about 100%.

DISTRIBUTION OF EYE COLOR

A group of geneticists at the University of Toronto decided to study the inheritance of eye color. Specifically, they wanted to determine the percentage of individuals in Ontario that had brown eyes. Their research results indicated that the probability that an individual will have brown eyes is .75 (that is, 75%).

Assume that you are given the task of analyzing the data for 10 individuals that are randomly selected from the population studied. Based on the information given above, please determine the probability that each of the following categories will occur (in %) :

(State probability in percent)

0 individuals have brown eyes	________
1 individual has brown eyes	________
2 individuals have brown eyes	________
3 individuals have brown eyes	________
4 individuals have brown eyes	________
5 individuals have brown eyes	________
6 individuals have brown eyes	________
7 individuals have brown eyes	________
8 individuals have brown eyes	________
9 individuals have brown eyes	________
10 individuals have brown eyes	________
	Total= 100%

-Note that the categories include all possibilities, so your answers should add up to about 100%.

Researchers at Statistics Canada decided to study the distribution of hair color of individuals living in Canada. They determined that .30 (that is, 30%) of Canada's population had fair hair (that is, blond or red hair).

Assume that you are given the task of analyzing the data for 100 individuals that are randomly selected from the population studied. Based on the information given above, please determine the probability that each of the following categories will occur (in %) :

	(State probability in percent)
Up to 5 individuals have blond or red hair	______
5 to 15 individuals have blond or red hair	______
15 to 25 individuals have blond or red hair	______
25 to 35 individuals have blond or red hair	______
35 to 45 individuals have blond or red hair	______
45 to 55 individuals have blond or red hair	______
55 to 65 individuals have blond or red hair	______
65 to 75 individuals have blond or red hair	______
75 to 85 individuals have blond or red hair	______
85 to 95 individuals have blond or red hair	______
More than 95 individuals have blond or red hair	______
	Total = 100%

-Note that the categories include all possibilities, so your answers should add up to about 100%.

In Toronto, Ontario approximately 100 babies are born every day. As you know, about .50 (that is, 50%) of all babies are boys. However, the exact percentage of baby boys varies from day to day. Sometimes, it is higher than 50%, sometimes lower.

Please predict the percentage of days that the number of boys among 100 babies will be as follows :

	(State probability in percent)
Up to 5 boys are born in Toronto, Ontario	______
5 to 15 boys are born in Toronto, Ontario	______
15 to 25 boys are born in Toronto, Ontario	______
25 to 35 boys are born in Toronto, Ontario	______
35 to 45 boys are born in Toronto, Ontario	______
45 to 55 boys are born in Toronto, Ontario	______
55 to 65 boys are born in Toronto, Ontario	______
65 to 75 boys are born in Toronto, Ontario	______
75 to 85 boys are born in Toronto, Ontario	______
85 to 95 boys are born in Toronto, Ontario	______
More than 95 boys are born in Toronto, Ontario	______
	Total = 100%

–Note that the categories include all possibilities, so your answers should add up to about 100%.

DISTRIBUTION OF EYE COLOR

A group of geneticists at the University of Toronto decided to study the inheritance of eye color. Specifically, they wanted to determine the percentage of individuals in Ontario that had brown eyes. Their research results indicated that the probability that an individual will have brown eyes is .75 (that is, 75%).

Assume that you are given the task of analyzing the data for 100 individuals that are randomly selected from the population studied. Based on the information given above, please determine the probability that each of the following categories will occur (in %) :

	(State probability in percent)
Up to 5 individuals have brown eyes	______
5 to 15 individuals have brown eyes	______
15 to 25 individuals have brown eyes	______
25 to 35 individuals have brown eyes	______
35 to 45 individuals have brown eyes	______
45 to 55 individuals have brown eyes	______
55 to 65 individuals have brown eyes	______
65 to 75 individuals have brown eyes	______
75 to 85 individuals have brown eyes	______
85 to 95 individuals have brown eyes	______
More tham 95 individuals have brown eyes	______
	Total = 100%

-Note that the categories include all possibilities, so your answers should add up to about 100%.

DISTRIBUTION OF HAIR COLOR

Researchers at Statistics Canada decided to study the distribution of hair color of individuals living in Canada. They determined that .30 (that is, 30%) of Canada's population had fair hair (that is blond or red hair).

Assume that you are given the task of analyzing the data for 1000 individuals that are randomly selected from the population studied. Based on the information given above, please determine the probability that each of the following categories will occur (in %) :

	(State probability in percent)
Up to 50 individuals have blond or red hair	
50 to 150 individuals have blond or red hair	
150 to 250 individuals have blond or red hair	
250 to 350 individuals have blond or red hair	
350 to 450 individuals have blond or red hair	
450 to 550 individuals have blond or red hair	
550 to 650 individuals have blond or red hair	
650 to 750 individuals have blond or red hair	
750 to 850 individuals have blond or red hair	
850 to 950 individuals have blond or red hair	
More than 950 individuals have blond or red hair	
	Total = 100%

-Note that the categories include all possibilities, so your answers should add up to about 100%.

In Canada, approximately 1000 babies are born every day. As you know, about .50 (that is, 50%) of all babies are boys. However, the exact percentage of baby boys varies from day to day. Sometimes, it is higher than 50%, sometimes lower.

Please predict the percentage of days that the number of boys among 1000 babies will be as follows :

	(State probability in percent)
Up to 50 boys are born in Canada	______
50 to 150 boys are born in Canada	______
150 to 250 boys are born in Canada	______
250 to 350 boys are born in Canada	______
350 to 450 boys are born in Canada	______
450 to 550 boys are born in Canada	______
550 to 650 boys are born in Canada	______
650 to 750 boys are born in Canada	______
750 to 850 boys are born in Canada	______
850 to 950 boys are born in Canada	______
More than 950 boys are born in Canada	______
	Total = 100%

-Note that the categories include all possibilities, so your answers should add up to about 100%.

DISTRIBUTION OF EYE COLOR

A group of geneticists at the University of Toronto decided to study the inheritance of eye color. Specifically, they wanted to determine the percentage of individuals in Ontario that had brown eyes. Their research results indicated that the probability that an individual will have brown eyes is .75 (that is, 75%).

Assume that you are given the task of analyzing the data for 1000 individuals that are randomly selected from the population studied. Based on the information given above, please determine the probability that each of the following categories will occur (in %) :

	(State probability in percent)
Up to 50 individuals have brown eyes	________
50 to 150 individuals have brown eyes	________
150 to 250 individuals have brown eyes	________
250 to 350 individuals have brown eyes	________
350 to 450 individuals have brown eyes	________
450 to 550 individuals have brown eyes	________
550 to 650 individuals have brown eyes	________
650 to 750 individuals have brown eyes	________
750 to 850 individuals have brown eyes	________
850 to 950 individuals have brown eyes	________
More than 950 individuals have brown eyes	________
	Total = 100%

-Note that the categories include all possibilities, so your answers should add up to about 100%.

Appendix B

List of Abbreviations

Abbreviations

C., Response category

DP., Difference between subjective and objective probabilities

M., Mean of the subjective probability distribution

N., Sample size or event size

OP., Objective probability values

P., Probability level

S., Sex of the subjects

SP., Subjective probability estimates

VITA AUCTORIS

1959	Born on January 18,1959 to Dr. and Mrs. Martin Kleiman in Chicago, Illinois
1976	Graduated from Lakeview High School, Battle Creek , Michigan
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1980	Enrolled in the doctoral program in Applied Social Psychology at the University of Windsor, Windsor, Ontario
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