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Study of energy levels and proton holes by the gallium-69 (helium-3,d)germanium-70 reaction.

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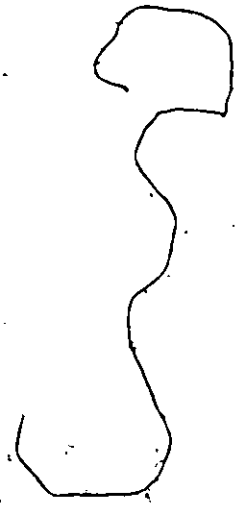
STUDY OF ENERGY LEVELS AND PROTON HOLES BY THE $^{69}Ga(^{3}He,d)^{70}Ge$ REACTION



by

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ABSTRACT

Excited levels of ^{70}Ge and proton holes in ^{69}Ga have been investigated by means of the (^{3}He, d) reaction at an incident beam energy of 22.5 MeV. Angular distributions were measured and are compared with the prediction of the Distorted-Wave-Born-Approximation theory in order to obtain the spectroscopic strengths of each level.

The number of proton holes in ^{69}Ga were obtained from the sum rule of the spectroscopic strengths. The vacancy probability U_j^2 and the center-of-gravity energy E_j for the $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ subshells are

Single-particle state	U_j^2	E_j(MeV)
$2p_{3/2}$	0.342	0.926
$1f_{5/2}$	0.957	3.008
$2p_{1/2}$	1.058	3.566

These are compared with the pairing model calculations.

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CHAPTER I

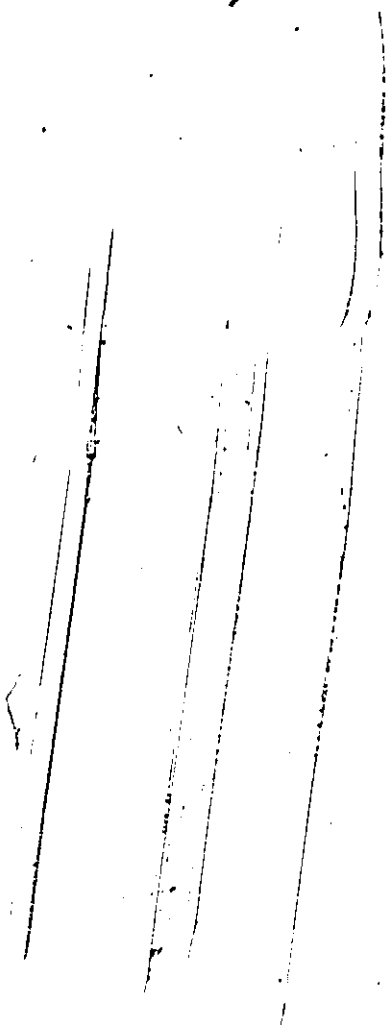
THEORY

1-1 Introduction

The ^{69}Ga (^{3}He, d) ^{70}Ge reaction was treated as a direct reaction process which involves only a few internal degrees of freedom of the system[1]. The incident nucleus is regarded as consisting of two nuclei held together in a bound state by their mutual interaction. The target nucleus is allowed to interact directly with one of these two subunits and absorb it in order to form the residual nucleus in its ground state or in an excited state.

The elastic scattering between two nuclei can be described by an optical potential well. In a direct reaction process, we add to this optical potential an additional interaction which causes the transition from the initial to the final state. We treated this additional interaction as a perturbation and this can lead to the Distorted-Wave - Born-Approximation (DWBA) expression for the transition amplitude which is given by the matrix element of the direct interaction with respect to the initial and final state wave functions. The relative

motion of the nuclei before and after the process is described by distorted waves fitting the elastic scattering cross-section in the two channels.



1-2 Distorted-Wave-Born-Approximation for a (^{3}He, d) Reaction

1-2-a) Introduction

Consider the A(h, d) B reaction, where h stands for the ^{3}He (helion) nucleus. This reaction can be represented in the following scheme

channel α ⟶ channel β

The total Hamiltonian of the system can be written as

$$H = H_h + H_A + T_{hA} + V_{hA} = H_\alpha + V_\alpha \ , \qquad (1\text{-}2\text{-}1)$$
$$= H_d + H_B + T_{dB} + V_{dB} = H_\beta + V_\beta \ .$$

Where $H_\alpha(H_\beta)$ is the free particle Hamiltonian of the channel $\alpha(\beta)$, consisting of the Hamiltonians $H_h(H_d)$ and $H_A(H_B)$ for the internal motion of the respective nuclei and of the Hamiltonian T_{hA} (T_{dB}) for the relative motion of the centers of mass of h(d) and A(B). $V_\alpha(V_\beta)$ is the interaction potential.

In the channel α, the Hamiltonian H reduces to H_α since the pair h, A are well separated from each other. Therefore we can define a complete set of eigenfunctions in

this channel which have the form

$$\phi_\alpha = \varphi_\alpha \psi_h \psi_A = \varphi_\alpha \Psi_\alpha \quad . \tag{1-2-2}$$

Such that

$$\begin{aligned} T_\alpha \varphi_\alpha &= E\ \varphi_\alpha \quad ; \\ H_h \psi_h &= E_h \psi_h \quad , \\ H_A \psi_A &= E_A \psi_A \quad . \end{aligned} \tag{1-2-3}$$

Where ψ_h and ψ_A are the internal wave functions of h and A with the energy eigenvalues E_h and E_A respectively. The ψ's are assumed to be normalized to unity and to be eigenfunctions of the magnitude and z-component of the total angular momenta (j_h, m_h) and (J_A, M_A) respectively. φ_α is the wave function of the relative motion of the pair. It is written as a plane wave.

1-2-b) Leading to the Distorted-Waves

We must now generate the distorted waves from the Schrödinger equation

$$\left[\frac{-\hbar^2}{2\mu} \nabla_{\underline{r}} + U(\underline{r}) + V_s(\underline{r})\underline{L}\cdot\underline{S} + V_{Coul}(\underline{r})\right] \chi(\underline{k},\underline{r}) = E\chi(\underline{k},\underline{r}) \quad . \tag{1-2-4}$$

Where $U(\underline{r})$ is the central optical-model potential, $V_{Coul}(\underline{r})$ is the Coulomb potential and $V_s(\underline{r})\underline{L}\cdot\underline{S}$ is the spin-orbit coupl-

ing potential for the LJ wave. The criterion for the choice of $U(\underline{r})$ is that it reproduces well the inelastic scattering reaction A(h,h')A.

If we write the Schrödinger equation (1-2-4)

$$(E - H_\alpha)\, \chi_\alpha(\underline{k},\underline{r}) = U_\alpha(\underline{r})\, \chi_\alpha(\underline{k},\underline{r}) \ , \qquad (1\text{-}2\text{-}5)$$

with

$$U_\alpha(\underline{r}) = U(\underline{r}) + V_s(\underline{r})\underline{L}\cdot\underline{S} + V_{Coul}(\underline{r}) \ , \qquad (1\text{-}2\text{-}6)$$

the corresponding homogeneous differential equation is

$$(E - H_\alpha)\, \phi_\alpha(\underline{k},\underline{r}) = 0 \ , \qquad (1\text{-}2\text{-}7)$$

where $\phi_\alpha(\underline{k},\underline{r})$ is the plane wave wave function defined in the equation (1-2-2). It also satisfies the equation

$$(E - H_\alpha)\, \phi_\alpha(\underline{k}',\underline{r}) = (E - E')\, \phi_\alpha(\underline{k}',\underline{r}) \ , \qquad (1\text{-}2\text{-}8)$$

the orthogonality condition

$$\int \phi_\alpha^*(\underline{k}',\underline{r})\, \phi_\alpha(\underline{k},\underline{r})\, d\underline{r} = (2\pi)^3\, \delta(\underline{k}'-\underline{k}) \ , \qquad (1\text{-}2\text{-}9)$$

and the closure relation

$$\int \phi_\alpha^*(\underline{k},\underline{r}')\, \phi_\alpha(\underline{k},\underline{r})\, d\underline{k} = (2\pi)^3\, \delta(\underline{r}'-\underline{r}) \ . \qquad (1\text{-}2\text{-}10)$$

If we multiply both sides of the equation (1-2-8) by the operator $(E-H_\alpha)^{-1}$ we get

$$(E-H_\alpha)^{-1}(E-H_\alpha)\, \phi_\alpha(\underline{k}',\underline{r}) = (E-E')(E-H_\alpha)^{-1}\, \phi_\alpha(\underline{k}',\underline{r}) \ . \qquad (1\text{-}2\text{-}11)$$

Rearranging the terms, we have

$$(E-H_\alpha)^{-1}\,\phi_\alpha(\underline{k}',\underline{r}) = \frac{1}{(E-E')}\,\phi_\alpha(\underline{k}',\underline{r}) \quad . \qquad (1\text{-}2\text{-}12)$$

Thus the $\phi_\alpha(\underline{k}',\underline{r})$ are eigenfunctions of the operator $(E-H_\alpha)^{-1}$ with the corresponding eigenvalues $(E-E')^{-1}$; where $E' > 0$ and $E \neq E'$. For $E = E'$, the operator $(E-H_\alpha)^{-1}$ is not defined.

If we operate the same way on the equation (1-2-5), we obtain

$$\chi_\alpha^{\pm}(\underline{k},\underline{r}) = (E-H_\alpha \pm i\epsilon)^{-1} U_\alpha(\underline{r})\,\chi_\alpha^{\pm}(\underline{k},\underline{r}) \quad . \qquad (1\text{-}2\text{-}13)$$

We have removed the divergence by inserting a small quantity ϵ, and we take the $\lim_{\epsilon \to 0}$ after calculating (1-2-13). We have two solutions depending on the way we remove the divergence by $+i\epsilon$ or $-i\epsilon$. The plus (minus) superscript on χ denotes the outgoing- (ingoing-) wave.

The equation (1-2-13) is a particular solution of the differential equation (1-2-5), the total solution is then

$$\chi_\alpha^{\pm}(\underline{k},\underline{r}) = \phi_\alpha(\underline{k},\underline{r}) + (E-H_\alpha \pm i\epsilon)^{-1} U_\alpha(\underline{r})\,\chi_\alpha^{\pm}(\underline{k},\underline{r}) \quad . \qquad (1\text{-}2\text{-}14)$$

We have that $(E-H_\alpha \pm i\epsilon)^{-1}$ is the free particle Green's operator.

We want to solve the Schrodinger equation (1-2-5) in terms of the total Green's operator[2]. We multiply the equation (1-2-14) by $(E-H_\alpha \pm i\epsilon)$ and by adding and subtracting

$-U_\alpha(\underline{r})\ \phi_\alpha(\underline{k},\underline{r})$ on the right-hand side of the equation, we obtain

$$\chi_\alpha^{\pm}(\underline{k},\underline{r}) = \phi_\alpha(\underline{k},\underline{r}) + \frac{1}{(E-\hat{H}_\alpha \pm i\epsilon)}\, U_\alpha(\underline{r})\ \phi_\alpha(\underline{k},\underline{r}) \ , \tag{1-2-15}$$

$$\chi_\alpha^{\pm}(\underline{k},\underline{r}) = \hat{\Omega}^{\pm}\ \phi_\alpha(\underline{k},\underline{r}) \ . \tag{1-2-16}$$

Where $\hat{H}_\alpha$ is the total Hamiltonian of the Schrödinger equation (1-2-5)

$$\hat{H}_\alpha = H_\alpha + U_\alpha \ , \tag{1-2-17}$$

and $\hat{\Omega}_\alpha^{\pm}$ is the Møller wave operator

$$\hat{\Omega}_\alpha^{\pm} = 1 + \frac{1}{(E-\hat{H}_\alpha \pm i\epsilon)}\, U_\alpha(\underline{r}) \ . \tag{1-2-18}$$

So we see that our distorted waves $\chi_\alpha^{\pm}(\underline{k},\underline{r})$ are generated from the plane waves $\phi_\alpha(\underline{k},\underline{r})$ through the Møller wave operator $\hat{\Omega}_\alpha^{\pm}$.

1-2-c) Effect of the Interaction Potential on the Distorted-Waves

If we now include the perturbation potential V_α which causes the transition from the channel α to the channel β. The total Hamiltonian is given by

$$H = \hat{H}_\alpha + V_\alpha - U_\alpha \ . \tag{1-2-19}$$

We must now solve the Schrödinger equation of the system

$$H\Psi = E\Psi \quad . \tag{1-2-20}$$

We use again the Green's operator method. The solution of (1-2-20) will have the form

$$\Psi_\alpha^{\pm}(\underline{k},\underline{r}) = \Omega_\alpha^{\pm}\, \chi_\alpha^{\pm}(\underline{k},\underline{r}) \quad , \tag{1-2-21}$$

when we put the incident wave as being a distorted wave calculated from the equation (1-2-16).

Comparing (1-2-21) with (1-2-16) we immediately write

$$\Omega_\alpha^{\pm} = 1 + \frac{1}{(E-H\pm i\epsilon)}\, V_\alpha \quad . \tag{1-2-22}$$

1-2-d) Transition Matrix Elements

We shall now follow the method discussed by K. Kikuchi and M. Kawai[3] in order to derive the transition matrix elements.

Let us define the energy operators

$$e = E - H + i\epsilon \quad , \tag{1-2-23a}$$

and

$$\hat{e}_\alpha = E - \hat{H}_\alpha + i\epsilon \quad . \tag{1-2-23b}$$

From the equations (1-2-1 and 17), we have

$$e = \hat{e}_\alpha - (V_\alpha - U_\alpha) = \hat{e}_\beta - (V_\beta - U_\beta) \quad . \qquad (1\text{-}2\text{-}24)$$

The energy operator $\hat{e}_\beta$ is defined the same way as $\hat{e}_\alpha$ in (1-2-23b) by substituting β for α. We can write the equations (1-2-18 and 22) as

$$\hat{\mathcal{Q}}_\alpha^+ = 1 + \frac{1}{\hat{e}_\alpha} U_\alpha \quad , \qquad (1\text{-}2\text{-}25)$$

and

$$\mathcal{Q}_\alpha^+ = 1 + \frac{1}{e} V_\alpha \quad . \qquad (1\text{-}2\text{-}26)$$

We use the Gell-Mann-Goldberger identity which holds for two arbitrary operators a and b

$$\frac{1}{a-b} = \frac{1}{a}\left[1 + b \frac{1}{a-b} \right] \quad ,$$

$$= \left[1 + \frac{1}{a-b} b \right] \frac{1}{a} \quad , \qquad (1\text{-}2\text{-}27)$$

which can be proved by multiplying both sides by a from the left and by (a-b) from the right for the first identity and by multiplying both sides by a from the right and by (a-b) from the left for the second equation.

Applying this identity to the equation (1-2-24), we get

$$\frac{1}{(\hat{e}_\alpha - (V_\alpha - U_\alpha))} = \left[1 + \frac{1}{(\hat{e}_\alpha - (V_\alpha - U_\alpha))} (V_\alpha - U_\alpha) \right] \frac{1}{\hat{e}_\alpha} \quad ,$$

i.e. $$\frac{1}{e} = \left[1 + \frac{1}{e} (V_\alpha - U_\alpha) \right] \frac{1}{\hat{e}_\alpha} \quad . \qquad (1\text{-}2\text{-}28)$$

From the equation (1-2-26) we have

$$\Omega^{+}_{\alpha} = 1 + \frac{1}{e}(V_{\alpha} - U_{\alpha}) + \frac{1}{e} U_{\alpha} \quad ,$$

substituting the value of $\frac{1}{e}$ in the last term on the right-hand side, we get

$$\Omega^{+}_{\alpha} = \left[1 + \frac{1}{e}(V_{\alpha} - U_{\alpha})\right] + \left[1 + \frac{1}{e}(V_{\alpha} - U_{\alpha})\right] \frac{1}{\hat{e}_{\alpha}} U_{\alpha}$$

$$= \left[1 + \frac{1}{e}(V_{\alpha} - U_{\alpha})\right]\left[1 + \frac{1}{\hat{e}_{\alpha}} U_{\alpha}\right]$$

and from the equation (1-2-25), we get

$$\Omega^{+}_{\alpha} = \left[1 + \frac{1}{e}(V_{\alpha} - U_{\alpha})\right]\hat{\Omega}^{+}_{\alpha} \quad . \qquad (1\text{-}2\text{-}29)$$

If we multiply both sides of this equation from the left by the energy operator e, we get

$$e\,\Omega^{+}_{\alpha} = (e + (V_{\alpha} - U_{\alpha}))\,\hat{\Omega}^{+}_{\alpha} \quad .$$

Substituting the expression for e given by the equation (1-2-24), we get

$$\left[\hat{e}_{\beta} - (V_{\beta} - U_{\beta})\right]\Omega^{+}_{\alpha} = \hat{e}_{\alpha}\hat{\Omega}^{+}_{\alpha} \quad .$$

We then multiply from the left by $\hat{e}^{-1}_{\beta}$ and rearranging the expression, one gets

$$\Omega^{+}_{\alpha} = \frac{1}{\hat{e}_{\beta}} \hat{e}_{\alpha} \hat{\Omega}^{+}_{\alpha} + \frac{1}{\hat{e}_{\beta}}(V_{\beta} - U_{\beta})\,\Omega^{+}_{\alpha} \quad . \qquad (1\text{-}2\text{-}30)$$

Since $V_\beta \Omega_\alpha^+ - U_\beta \Omega_\alpha^+ = (V_\beta - U_\beta)\Omega_\alpha^+$,

then $V_\beta \Omega_\alpha^+ = U_\beta \Omega_\alpha^+ + (V_\beta - U_\beta)\Omega_\alpha^+$.

Substituting the expression for Ω_α^+ given in the equation (1-2-30) in the first term of the right-hand side, we obtain

$$V_\beta \Omega_\alpha^+ = U_\beta \left[\frac{1}{\hat{e}_\beta} \hat{e}_\alpha \hat{\Omega}_\alpha^+ + \frac{1}{\hat{e}_\beta}(V_\beta - U_\beta)\Omega_\alpha^+ \right] + (V_\beta - U_\beta)\Omega_\alpha^+$$

$$= U_\beta \frac{1}{\hat{e}_\beta} \hat{e}_\alpha \hat{\Omega}_\alpha^+ + \left[1 + U_\beta \frac{1}{\hat{e}_\beta} \right](V_\beta - U_\beta)\Omega_\alpha^+ ,$$

$$= U_\beta \frac{1}{\hat{e}_\beta} \hat{e}_\alpha \hat{\Omega}_\alpha^+ + \left[1 + \frac{1}{\hat{e}_\beta^*} U_\beta^* \right]^\dagger (V_\beta - U_\beta)\Omega_\alpha^+ ,$$

$$= U_\beta \frac{1}{\hat{e}_\beta} \hat{e}_\alpha \hat{\Omega}_\alpha^+ + \hat{\Omega}_\beta^{-\dagger} (V_\beta - U_\beta)\Omega_\alpha^+ .$$

(1-2-31)

where * and † indicate the complex and hermitian conjugate respectively and

$$\hat{\Omega}_\beta^- = \left[1 + \frac{1}{\hat{e}_\beta^*} U_\beta^* \right] . \qquad (1\text{-}2\text{-}32)$$

By comparing with the equations (1-2-18 and 16), we may write

$$\chi_\beta^- = \hat{\Omega}_\beta^- \phi_\beta . \qquad (1\text{-}2\text{-}33)$$

Where χ_β^- can be considered as a solution of a Schrödinger equation similar to the one given in (1-2-5) but with the

potential U_β^* instead. This type of function has been discussed by L. I. Schiff[4]. It is the time reversed eigenstate of χ_α^+, but it does not represent a physically possible behavior of the system since the Hamiltonian is not time reversal invariant.

The transition matrix element for the reaction from channel α to channel β is given by (See Appendix A and Ref.4 p.313)

$$T_{\beta\alpha} = \langle \phi_\beta | V_\beta | \Psi_\alpha^+ \rangle \quad . \tag{1-2-34}$$

Substituing the expression for Ψ_α^+ given by the equation (1-2-21), one obtains

$$T_{\beta\alpha} = \langle \phi_\beta | V_\beta | \Omega_\alpha^+ \chi_\alpha^+ \rangle \quad . \tag{1-2-35}$$

From the equation (1-2-31), it is clear that

$$T_{\beta\alpha} = \langle \phi_\beta | U_\beta \frac{1}{\hat{e}_\beta} \hat{e}_\alpha | \chi_\alpha^+ \rangle + \langle \phi_\beta | \hat{\Omega}_\beta^{-\dagger} (V_\beta - U_\beta) \Omega_\alpha^+ | \chi_\alpha^+ \rangle \quad . \tag{1-2-36}$$

The first term on the right vanishes since we have $\hat{e}_\beta \neq \hat{e}_\alpha$ and that $\hat{e}_\alpha \chi_\alpha^+ = i\epsilon \chi_\alpha^+$ which goes to zero when we take $\lim_{\epsilon \to 0}$. Then from the equations (1-2-33 and 21)

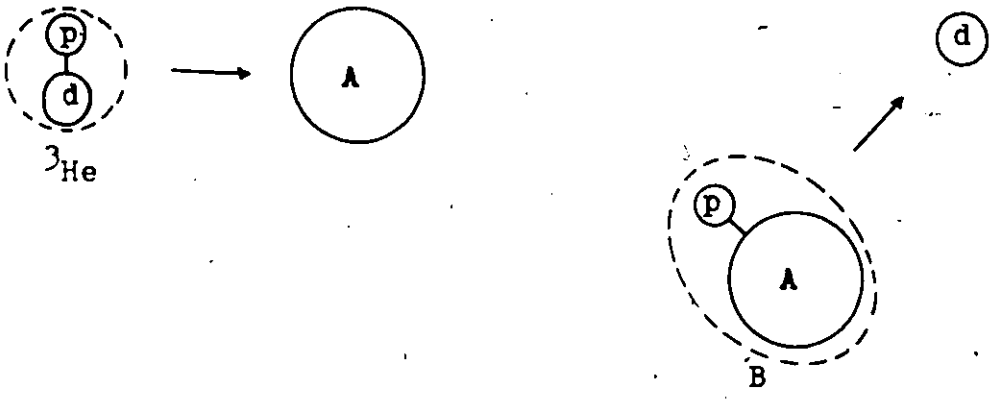
$$T_{\beta\alpha} = \langle \chi_\beta^- | V_\beta - U_\beta | \Psi_\alpha^+ \rangle \quad . \tag{1-2-37}$$

The distorted-waves-Born approximation consists of taking $\Psi_\alpha^+ \simeq \chi_\alpha^+$ in the equation (1-2-21). Then the DWBA expression for the transition matrix element is given by

$$T^{DW}_{\beta\alpha} = \langle \chi^-_\beta | V_\beta - U_\beta | \chi^+_\alpha \rangle \quad . \qquad (1\text{-}2\text{-}38)$$

Let us consider schematically the A(^{3}He,d)B reaction[5].

Channel α Channel β



It is convenient to transform the equation (1-2-38) to the coordinate system ($\underline{r}_\alpha$, $\underline{r}_\beta$, ξ); where ξ stands for an aggregate of the internal coordinates orthogonal to $\underline{r}_\alpha$ and $\underline{r}_\beta$. The position-vector diagram is shown in fig 1[3,6].

From fig 1 we have

$$\xi_h = (\xi_d, \xi_p, \underline{r}_{pd}) \quad , \qquad (1\text{-}2\text{-}39a)$$

$$\xi_\alpha = (\xi_h, \xi_A) = (\xi_d, \xi_p, \underline{r}_{pd}, \xi_A) \quad , \qquad (1\text{-}2\text{-}39b)$$

and

$$\xi_\beta = (\xi_d, \xi_B) = (\xi_d, \xi_A, \xi_p, \underline{r}_{pA}) \qquad (1\text{-}2\text{-}39c)$$

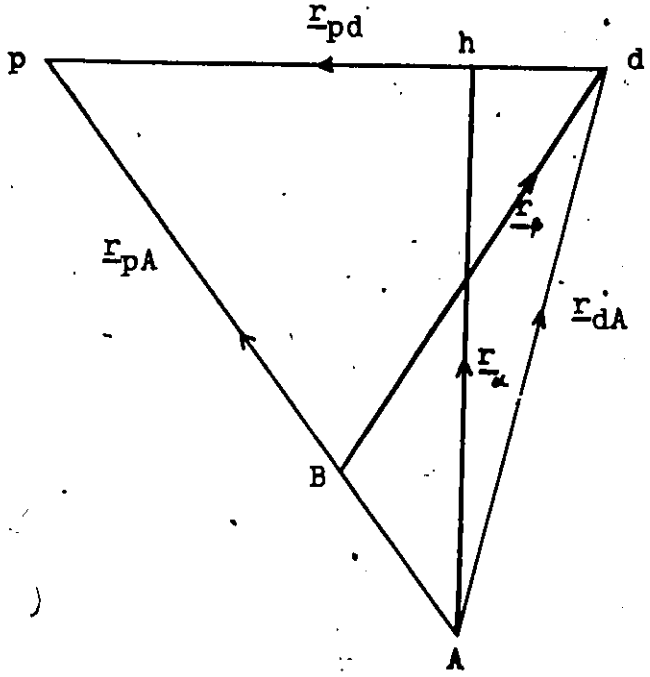


fig 1.

where ξ_i represents the internal coordinates of particle i.

We can express $\underline{r}_{pd}$ and $\underline{r}_{pA}$ in terms of $\underline{r}_\alpha$ and $\underline{r}_\beta$. We shall write p, d, h, A and B as the mass of the corresponding particles. From the vector-position diagram, we have that

$$\underline{r}_\alpha = \frac{p}{h}\,\underline{r}_{pA} + \frac{d}{h}\,\underline{r}_{dA} \qquad (1\text{-}2\text{-}40a)$$

and

$$\underline{r}_\beta = -\frac{p}{B}\,\underline{r}_{pd} + \frac{A}{B}\,\underline{r}_{dA} \qquad (1\text{-}2\text{-}40b)$$

Combining these two equations together, one gets

$$\frac{h}{d}\,\underline{r}_{\alpha} - \frac{p}{d}\,\underline{r}_{pA} = \frac{B}{A}\,\underline{r}_{\beta} + \frac{p}{A}\,\underline{r}_{pd} \quad . \tag{1-2-41}$$

Since

$$\underline{r}_{pd} = \underline{r}_{ph} + \underline{r}_{hd} \quad ,$$

and

$$p\,\underline{r}_{ph} = d\,\underline{r}_{hd} \quad ,$$

therefore

$$\underline{r}_{pd} = \underline{r}_{ph}\left[1 + \frac{p}{d}\right] = \frac{h}{d}\,\underline{r}_{ph} \quad . \tag{1-2-42}$$

We also have

$$\underline{r}_{pA} = \underline{r}_{\alpha} + \underline{r}_{ph} \quad . \tag{1-2-43}$$

Substituting (1-2-42) into (1-2-43), one gets

$$\underline{r}_{pA} = \underline{r}_{\alpha} + \frac{d}{h}\,\underline{r}_{pd} \quad . \tag{1-2-44}$$

Then from (1-2-41)

$$\frac{h}{d}\,\underline{r}_{\alpha} - \frac{p}{d}\,\underline{r}_{\alpha} - \frac{p}{h}\,\underline{r}_{pd} = \frac{B}{A}\,\underline{r}_{\beta} + \frac{p}{A}\,\underline{r}_{pd} \quad .$$

Rearranging this equation we obtain

$$\underline{r}_{pd} = \frac{hB}{p(B + d)}\left[\underline{r}_{\beta} - \frac{A}{B}\,\underline{r}_{\alpha}\right] . \tag{1-2-45}$$

Substituting (1-2-45) in (1-2-41) and rearranging the terms

$$\underline{r}_{pA} = \frac{dB}{p(B+d)}\ \underline{r}_{\beta} - \frac{h}{d}\ \underline{r}_{\alpha} \quad . \tag{1-2-46}$$

From the equations (1-2-39b,c) we have that

$$\xi = (\xi_d, \xi_p, \xi_A) \quad , \tag{1-2-47}$$

and the Jacobian of the transformation from the natural variables $\underline{r}_{pd}$ and $\underline{r}_{pA}$ to $\underline{r}_{\alpha}$ and $\underline{r}_{\beta}$ is simply [3,4,5,6,7]

$$J = \frac{\partial(\underline{r}_{pd}, \underline{r}_{pA})}{\partial(\underline{r}_{\alpha}, \underline{r}_{\beta})} = \left[\frac{hB}{p(B+d)}\right]^3 \quad . \tag{1-2-48}$$

The distorting potentials are not diagonal with respect to the channels. Therefore χ_{α}^{+} and χ_{β}^{-} are given as a linear combination of the channel wave functions (Appendix B)

$$\chi_{\alpha}^{+} = \sum_{m_{\alpha}'} \Phi_{\alpha\, m_{\alpha}'}^{+}\ \psi_{\alpha\, m_{\alpha}'} \quad , \tag{1-2-49}$$

and

$$\chi_{\beta}^{-} = \sum_{m_{\beta}'} \Phi_{\beta\, m_{\beta}'}^{-}\ \psi_{\beta\, m_{\beta}'} \quad , \tag{1-2-50}$$

where m_{α}' and m_{β}' represent the different values of the z-component of the spin of the ^{3}He and d, the $\psi_{m'}$'s are the internal wave functions for the m'-th state of the nuclei and the Φ's are the corresponding distorted wave functions of the relative motion.

The transition matrix element (1-2-38) then becomes

$$T^{DW}_{\beta\alpha} = J \sum_{m'_\alpha\, m'_\beta} \langle \Phi^{-}_{\beta\, m'_\beta}(\underline{r}_\beta) \,|\, F_{\beta\, m'_\beta \alpha\, m'_\alpha}(\underline{r}_\beta, \underline{r}_\alpha, \xi)$$

$$\times \,|\, \Phi^{+}_{\alpha\, m'_\alpha}(\underline{r}_\alpha) \rangle_{\underline{r}_\alpha,\, \underline{r}_\beta, \xi} \qquad (1\text{-}2\text{-}51)$$

1-2-e) Form Factor

From the equation (1-2-51), the form factor $F_{\beta\alpha}$ is given by

$$F_{\beta\alpha} = \int \psi^{*}_{J_B M_B}(\underline{r}_{pA}, \xi_p, \xi_A) \psi^{*}_{J_d M_d}(\xi_d)\, V$$

$$\times \psi_{J_A M_A}(\xi_A) \psi_{J_h M_h}(\underline{r}_{pd}, \xi_d, \xi_p)\, d\xi_A d\xi_d d\xi_p \qquad (1\text{-}2\text{-}52)$$

Where $V = V_\beta - U_\beta = V_{dB} - U_{dB}$. Since the nucleus B is formed from the target A by stripping a proton from ^{3}He, we may write

$$V_{dB} = V_{pd} + V_{dA},$$

so that $V = V_{pd} + (V_{dA} - U_{dB})$.

It is usually argued that the term V_{pd} dominates[8,9]. There is considerable cancellation between V_{dA} and U_{dB} but this can never be complete for finite nuclei. V_{dA} is considered

as a true, and therefore real interaction, whereas U_{dB} is an optical model potential with an important imaginary term. Thus U_{dB} can only reproduce the elastic scattering while V_{dA} also excites the core A. Even if V_{dA} was represented by an optical potential U_{dA}, the cancellation would still not be complete since the potential refer to slightly different nuclei. The only argument in favor of our assumption is that it gives satisfying results[8].

Then the form factor (1-2-52) is written

$$F_{\beta\alpha} = \int \psi^*_{J_B M_B}(\underline{r}_{pA}, \xi_p, \xi_A)\psi^*_{J_d M_d}(\xi_d)V_{pd}(\underline{r}_{pd}, \xi_d, \xi_p)$$

$$\times \psi_{J_A M_A}(\xi_A)\psi_{J_h M_h}(\underline{r}_{pd}, \xi_d, \xi_p)d\xi_A d\xi_d d\xi_p. \qquad (1\text{-}2\text{-}53)$$

It is assumed that V_{pd} is central, that is, scalar in the separation $\underline{r}_{pd}$, so that d, p are in an s state of relative motion within h[7]. It is also assumed that the projectile h and the outgoing particle d can be represented by s wave functions. Then $\underline{J}_d = \underline{s}_d$ and the z-component $M_d = m_d$, also $\underline{J}_h = \underline{s}_h$ and the z-component $M_h = m_h$.

The form factor can be expanded into terms which correspond to the transfer to the nucleus of a definite angular momentum $\underline{j} = \underline{J}_B - \underline{J}_A$ which is composed of a spin part $\underline{s} = \underline{s}_h - \underline{s}_d$ and of an orbital part $\underline{\ell} = \underline{j} - \underline{s}$. The

respective z-components are $\mu = M_B - M_A$, $\nu = m_h - m_d$ and $m = M_B - M_A + m_d - m_h$.

The function of the residual nucleus may be expanded in terms of the eigenstates of the target

$$\Psi_{J_B M_B}(\underline{r}_{pA}, \xi_p, \xi_A) = \sum_{j\mu J'_A M'_A} \Psi_{J'_A M'_A}(\xi_{A'}) \Phi^{A'}_{j\mu}(\underline{r}_{pA}, \xi_p) \times C^{M'_A\ \mu\ M_B}_{J'_A\ j\ J_B} , \qquad (1\text{-}2\text{-}54)$$

where $\Psi_{J'_A M'_A}$ is the wave function of the nucleus A in the state A' and $\Phi^{A'}_{j\mu}(\underline{r}_{pA}, \xi_p)$ is the wave function of the proton in the orbit (ℓ, j) around A which is in the state A'.

We can further decompose $\Phi^{A'}_{j\mu}$ as

$$\Phi^{A'}_{j\mu} = \sum_{\ell s m} \mathcal{J}^{A'}(\ell_j)\, i^{\ell}\, Y^m_\ell(\Omega_{pA})\, \phi^{A'}_{\ell j}(r_{pA})\, \psi_{s\nu}(\xi_p) \times C^{m\ \nu\ \mu}_{\ell\ s\ j} . \qquad (1\text{-}2\text{-}55)$$

Where $\mathcal{J}^{A'}(\ell_j)$ is a spectroscopic amplitude aside from a factor $n^{\frac{1}{2}}$ which arises from antisymmetry considerations if there are n equivalent nucleons in the orbit[10]. The ususal spectroscopic factor is then just (see section 1-4)

$$S(\ell_j) = n(\mathcal{J}^{A'}(\ell_j))^2 \qquad (1\text{-}2\text{-}56)$$

$\phi^{A'}_{\ell j}$ is the radial function for the shell model orbit (ℓj) and $\psi_{s\nu}$ is the nucleon spin function. The sum over ℓ and s is superfluous since $s=\frac{1}{2}$ and $\ell=j\pm\frac{1}{2}$ according to the parity change. Then the form factor is given by

$$F_{\beta\alpha} = \int \sum_{j\mu J_{A'}M_{A'}} \psi^{*}_{J_{A'},M_{A'}}(\xi_{A'}) \sum_{\ell sm} i^{-\ell} \mathcal{J}^{A}(\ell_j)\, Y^{m*}_{\ell}(\Omega_{pA})$$

$$\times \phi^{A'*}_{\ell j}(r_{pA}) \psi^{*}_{s\nu}(\xi_p)\, C^{m\ \nu\ \mu}_{\ell\ s\ j}\, C^{M_{A'}\ \mu\ M_B}_{J_{A'}\ j\ J_B}$$

$$\times \psi^{*}_{s_d m_d}(\xi_d)\, V_{pd}(\underline{r}_{pd}, \xi_d, \xi_p)\, \psi_{J_A M_A}(\xi_A)$$

$$\times \psi_{s_h m_h}(\underline{r}_{pd}, \xi_d, \xi_p)\, d\xi_A\, d\xi_d\, d\xi_p \ .$$

Performing the summation over $J_{A'}, M_{A'}$; integrating over ξ_A and rearranging the terms, we get

$$F_{\beta\alpha} = \sum_{\ell sjm} \mathcal{J}^{A}\ i^{-\ell} Y^{m*}_{\ell}(\Omega_{pA})\, \phi^{A*}_{\ell j}(r_{pA})\ C^{m\ \nu\ \mu}_{\ell\ s\ j}$$

$$\times C^{M_A\ \mu\ M_B}_{J_A\ j\ J_B} \int d\xi_d \int d\xi_p\, \psi^{*}_{s\nu}(\xi_p) \psi^{*}_{s_d m_d}(\xi_d)$$

$$\times V_{pd}(\underline{r}_{pd}, \xi_d, \xi_p) \psi_{s_h m_h}(\underline{r}_{pd}, \xi_d, \xi_p) \ . \qquad (1\text{-}2\text{-}57)$$

The integral in (1-2-57) gives us a scalar function $D(r_{pd})$ times a Clebsh-Gordan coefficient[7] $C^{\nu\, m_d\, m_h}_{s\; s_d\; s_h}$. We have that

$$C^{\nu\, m_d\, m_h}_{s\; s_d\; s_h} = (-)^{s + s_d - s_h}\, C^{m_d\, \nu\, m_h}_{s_d\; s\; s_h} ,$$

and since $\underline{s} = \underline{s}_h - \underline{s}_d$, we can write

$$F_{\beta\alpha} = \sum_{\ell s j} i^{-\ell} \mathcal{J}^A(\ell j)\, Y^{m*}_{\ell}(\Omega_{pA})\, \phi^{A*}_{\ell j}(r_{pA})\, C^{m\, \nu\, \mu}_{\ell\; s\; j}$$

$$\times C^{M_A\, \mu\, M_B}_{J_A\; j\; J_B}\, C^{\nu\, m_d\, m_h}_{s\; s_d\; s_h}\, D(r_{pd}) . \qquad (1\text{-}2\text{-}58)$$

As pointed out before, it is usual to take the wave function of the incident particle as being a s wave. In this case the internal wave function of ^{3}He can be separated into a radial part and an internal wave function of ξ_d and ξ_p.

$$\Psi_{s_h m_h}(\underline{r}_{pd}, \xi_d, \xi_p) = \Phi_h(r_{pd})\, \Psi_{s_h m_h}(\xi_d, \xi_p) . \qquad (1\text{-}2\text{-}59)$$

Taking $V^{s_h}_{pd}$ to be a central potential, the integral in (1-2-57) becomes

$$\int d\xi_d \int d\xi_p\, \Psi^*_{s\nu}(\xi_p)\, \Psi_{s_d m_d}(\xi_d)\, V^{s_h}_{pd}(r_{pd})\, \Phi_h(r_{pd})$$

$$\times \Psi_{s_h m_h}(\xi_d, \xi_p) .$$

Integrating over ξ_p and ξ_d we get

$$C^{\nu}_{s\, s_d\, s_h}\, {}^{m_d\, m_h} V^{s_h}_{p_d}(r_{pd})\,\Phi_h(r_{pd}), \qquad (1\text{-}2\text{-}60)$$

where $V^{s_h}_{pd}$ is the value of V in the spin state s_h. By comparing (1-2-60) with (1-2-58) the scalar function D becomes

$$D(r_{pd}) = V^{s_h}_{pd}(r_{pd})\,\Phi_h(r_{pd}) . \qquad (1\text{-}2\text{-}61)$$

We must notice that since there are two protons in ^{3}He, the cross section will be two times that of the equation which describes the transfer of one particular proton.

It is convenient to define a function $H_{\ell sj}(r_{pA}, r_{pd})$ such as

$$H_{\ell sj}(r_{pA}, r_{pd}) \doteq \mathcal{J}^{A}(\ell j)\, \phi^{A^*}_{\ell j}(r_{pA})\, D(r_{pd}). \qquad (1\text{-}2\text{-}62)$$

$H_{\ell sj}(r_{pA}, r_{pd})$ refers to the system (r_{pA}, r_{pd}). We must transform it to the system $(\underline{r}_\alpha, \underline{r}_\beta)$. To do this, we define a function

$$G_{\ell sj,m}(\underline{r}_\alpha, \underline{r}_\beta) = J \left[\frac{2s_h + 1}{2s + 1}\right]^{\frac{1}{2}} Y^{m^*}_{\ell}(\Omega_{pA}) H_{\ell sj}(r_{pA}, r_{pd}), \qquad (1\text{-}2\text{-}63)$$

where J is the Jacobian of the transformation defined in the equation (1-2-48).

If we substitute (1-2-63) in the equation for the form factor (1-2-58) and using the following relation between the Clebsch-Gordan coefficient

$$C^{m_d\ \nu\ m_h}_{s_d\ s\ s_h} = \left[\frac{2s_h + 1}{2s + 1}\right]^{\frac{1}{2}} (-)^{s_d - m_d}\ C^{m_h - m_d\ \nu}_{s_h\ s_d\ s}$$

we obtain an expression identical to the one given by Satchler[7]

$$F_{\beta\alpha} = J^{-1} \sum_{\ell s j} i^{-\ell}\ G_{\ell s j, m}(\underline{r}_\alpha, \underline{r}_\beta)\ (-)^{s_d + m_d}$$

$$\times C^{M_A\ M_B - M_A\ M_B}_{J_A\ j\ J_B}\ C^{m_h\ -m_d\ m_h - m_d}_{s_h\ s_d\ s}$$

$$\times C^{m\ m_h - m_d\ M_B - M_A}_{\ell\ s\ j} \qquad (1\text{-}2\text{-}64)$$

An expansion similar to equation (1-2-64) can be made for the isospin exchange, except there is no analogue to the orbital transfer ℓ. Each G is then a sum of terms weighted by

$$C = C^{M_{T_A}\ m_t\ M_{T_B}}_{T_A\ t\ T_B}\ C^{m_{t_d}\ m_t\ m_{t_h}}_{t_d\ t\ t_h}$$

We have $t_d = 0$, $m_d = 0$ and $t_h = \frac{1}{2}$, $m_h = -\frac{1}{2}$. Then Clebsch-Gordan coefficient

$$C^{0\ m_t\ -\frac{1}{2}}_{0\ t\ \frac{1}{2}} = C^{-m_t\ 0\ \frac{1}{2}}_{t\ 0\ \frac{1}{2}} = \delta_{t\frac{1}{2}}\ \delta_{-m_t\frac{1}{2}}$$

The weight factor C is then given by[9)]

$$C = C \begin{matrix} M_{T_A} & -\frac{1}{2} & M_{T_B} \\ T_A & \frac{1}{2} & T_B \end{matrix} = C \begin{matrix} M_I & -\frac{1}{2} & M-\frac{1}{2} \\ T_A & \frac{1}{2} & T_B \end{matrix}$$

where $M_I = T_A = \frac{1}{2}(N-Z)$ of the target nucleus A.

It is often helpful to write $G_{\ell sj}$ as the product of two factors

$$G_{\ell sj,m}(\underline{r}_\alpha, \underline{r}_\beta) = A_{\ell sj} \; \zeta_{\ell sj,m}(\underline{r}_\alpha, \underline{r}_\beta) \quad . \qquad (1\text{-}2\text{-}65)$$

Where $A_{\ell sj}$ is a spectroscopic factor and $\zeta_{\ell sj}$ a form factor. From the equations (1-2-63) and (1-2-62) we have that

$$A_{\ell sj} \; \zeta_{\ell sj,m}(\underline{r}_\alpha, \underline{r}_\beta) = JC \left[\frac{2s_h + 1}{2s + 1}\right]^{\frac{1}{2}} \sqrt{2n} \; \mathcal{J}^A(\ell j) \, \phi^{A^*}_{\ell j}(r_{pA})$$

$$V^{s_h}_{pd}(r_{pd}) \; \Phi_h(r_{pd}) \; Y^{m^*}_\ell(\Omega_{pA}) \quad . \qquad (1\text{-}2\text{-}66)$$

Note that we have included in this expression the weight factor $\sqrt{2n}$ where the factor 2 comes from the fact that there are two protons in ^{3}He and n from the fact that there are n equivalent nucleons in the orbit (ℓj). J is the Jacobian and C is the isospin Clebsh-Gordan coefficient.

Let us recall the expression of the transition matrix element given in the equation (1-2-51) and include all the modifications used in computing the form factor.

$$T^{DW}_{\beta\alpha} = J \sum_{m'_h m'_d} \int d\underline{r}_\alpha \int d\underline{r}_\beta \Phi^{-*}_{m'_d m_d}(\underline{k}_\beta, \underline{r}_\beta)\, F_{\beta m'_d \alpha m'_h}(r_\beta, r_\alpha)$$

$$\times \Phi^{+}_{m'_h m_h}(\underline{k}_\alpha, \underline{r}_\alpha) \quad . \tag{1-2-67}$$

If we substitute in the expression we have calculated for the form factor equation (1-2-64), make use of (1-2-65) and rearrange the terms, we obtain

$$T^{DW}_{\beta\alpha} = \sum_{\ell sj} (2j+1)^{\frac{1}{2}} A_{\ell sj}\, C \begin{matrix} M_A & M_B - M_A & M_B \\ J_A & j & J_B \end{matrix} \beta^{\ell m m_h m_d}_{sj}(\underline{k}_\beta, \underline{k}_\alpha). \tag{1-2-68}$$

Where

$$(2j+1)^{\frac{1}{2}}\, i^{\ell}\, \beta^{\ell m m_h m_d}_{sj}(\underline{k}_\beta, \underline{k}_\alpha) = \sum_{m' m'_h m'_d} C \begin{matrix} m' & m'_h - m'_d & m' - m'_d + m'_h \\ \ell & s & j \end{matrix}$$

$$\times C \begin{matrix} m'_h & -m'_d & m'_h - m'_d \\ s_h & s_d & s \end{matrix} (-)^{s_d - m'_d}$$

$$\times \int d\underline{r}_\alpha \int d\underline{r}_\beta \Phi^{-*}_{m'_d m_d}(\underline{k}_\beta, \underline{r}_\beta)\, \mathcal{F}_{\ell sj, m'}(\underline{r}_\beta, \underline{r}_\alpha)$$

$$\times \Phi^{+}_{m'_h\ m_h}(\underline{k}_\alpha, \underline{r}_\alpha) \quad . \tag{1-2-69}$$

The distorted waves have the form[7] (Appendix B)

$$\Phi^{+}_{m_h' m_h}(\underline{k}_\alpha, \underline{r}_\alpha) = \frac{4\pi}{k_\alpha r_\alpha} \sum_{J_\alpha L_\alpha M_\alpha} C \begin{matrix} M_\alpha & m_h' & M_\alpha + m_h \\ L_\alpha & s_h & J_\alpha \end{matrix}$$

$$\times C \begin{matrix} M_\alpha + m_h - m_h' & m_h' & M_\alpha + m_h \\ L_\alpha & s_h & J_\alpha \end{matrix}$$

$$\times i^{L_\alpha} \Phi_{L_\alpha J_\alpha}(k_\alpha, r_\alpha)\, Y^{M_\alpha^*}_{L_\alpha}(\theta_{k_\alpha}, \varphi_{k_\alpha})\, Y_{L_\alpha}^{M + m_h - m_h'}(\theta_{r_\alpha}, \varphi_{r_\alpha}),$$

(1-2-70)

and

$$\Phi^{-*}_{m_d' m_d}(\underline{k}_\beta, \underline{r}_\beta) = (-)^{m_d - m_d'} \frac{4\pi}{k_\beta r_\beta} \sum_{J_\beta L_\beta M_\beta} C \begin{matrix} M_\beta & -m_d & M_\beta - m_d \\ L_\beta & s_d & J_\beta \end{matrix}$$

$$\times C \begin{matrix} M_\beta - m_d + m_d' & -m_d' & M_\beta - m_d \\ L_\beta & s_d & J_\beta \end{matrix} \; i^{-L_\beta}$$

$$\times \Phi_{L_\beta J_\beta}(k_\beta, r_\beta)\, Y^{M_\beta^*}_{L_\beta}(\theta_{k_\beta}, \varphi_{k_\beta})\, Y_{L_\beta}^{M_\beta - m_d + m_d'}(\theta_{r_\beta}, \varphi_{r_\beta})$$

(1-2-71)

One is referred to Appendix B for the definition of all the terms in the equations (1-2-70 and 71).

Since the form factor $f_{\ell sj,m}$ in the equation (1-2-66) transforms like Y_ℓ^{m*}, its expansion into a double series in spherical harmonics of Ω_{r_α} and Ω_{r_β} takes the form[11]

$$f_{\ell sj,m} = \sum_{L_1 L_2 M} P^{sj}_{\ell L_1 L_2}(r_\beta, r_\alpha)\, Y^{M^*}_{L_1}(\theta_{r_\beta}, \varphi_{r_\beta})\, Y^{m-M^*}_{L_2}(\theta_{r_\alpha}, \varphi_{r_\alpha})$$

$$\times C \begin{matrix} M & m-M & m \\ L_1 & L_2 & \ell \end{matrix} . \qquad (1\text{-}2\text{-}72)$$

Substituting the equations (1-2-70, 71, 72) in (1-2-69) and rearranging the terms, one gets

$$(2j+1)^{\frac{1}{2}}\, i^{\ell}\, \beta^{\ell m m_h m_d}_{sj}(\underline{k}_\beta, \underline{k}_\alpha) = \sum_{\substack{m' m_h' m_d' \\ J_\beta L_\beta M_\beta \\ L_1 L_2 M \\ J_\alpha L_\alpha M_\alpha}} (-)^{s_d - m_d}\, 4\pi\, i^{L_\alpha - L_\beta}$$

$$\times C \begin{matrix} m_h' & -m_d' & m_h'-m_d' \\ s_h & s_d & s \end{matrix} \quad C \begin{matrix} m' & m_h'-m_d' & m-m_d'+m_h' \\ \ell & s & j \end{matrix}$$

$$\times C \begin{matrix} M_\beta & -m_d & M_\beta - m_d \\ L_\beta & s_d & J_\beta \end{matrix} \quad C \begin{matrix} M_\beta - m_d + m_d' & -m_d' & M_\beta - m_d \\ L_\beta & s_d & J_\beta \end{matrix}$$

$$\times C \begin{matrix} M & m'-M & m' \\ L_1 & L_2 & \ell \end{matrix} \quad C \begin{matrix} M_\alpha & m_h & M_\alpha + m_h \\ L_\alpha & s_h & J_\alpha \end{matrix}$$

$$\times C \begin{matrix} M_\alpha + m_h - m_h' & m_h' & M_\alpha + m_h \\ L_\alpha & s_h & J_\alpha \end{matrix} \quad Y^{M_\alpha^*}_{L_\alpha}(\theta_{k_\alpha}, \varphi_{k_\alpha})\; Y^{M_\beta^*}_{L_\beta}(\theta_{k_\beta}, \varphi_{k_\beta})$$

$$\times \frac{4\pi}{k_\alpha k_\beta} \int r_\alpha dr_\alpha \int r_\beta dr_\beta\, \Phi_{L_\beta J_\beta}(k_\beta, r_\beta)\; P^{sj}_{\ell L_1 L_2}(r_\beta, r_\alpha)\, \Phi_{L_\alpha J_\alpha}(k_\alpha, r_\alpha$$

$$\times \int d\varphi_{r_\alpha} \int \sin\theta_{r_\alpha}\, d\theta_{r_\alpha}\; Y^{m'-M^*}_{L_2}(\theta_{r_\alpha}, \varphi_{r_\alpha})\; Y^{M_\alpha + m_h - m_h'}_{L_\alpha}(\theta_{r_\alpha}, \varphi_{r_\alpha})$$

$$\times \int d\varphi_{r_\beta} \int \sin\theta_{r_\beta}\, d\theta_{r_\beta}\; Y_{L_1}^{M^*}(\theta_{r_\beta}, \varphi_{r_\beta})\; Y_{L_\beta}^{M_\beta + m_d' - m_d}(\theta_{r_\beta}, \varphi_{r_\beta}) \,. \tag{1-2-73}$$

From the orthogonality relation of the spherical harmonics, we have that

$$\int d\varphi_i \int \sin\theta_i\, d\theta_i\; Y_{L_I}^{M_I}(\theta_i, \varphi_i)\; Y_{L_J}^{M_J}(\theta_i, \varphi_i) = \delta_{L_J L_I}\, \delta_{M_J M_I}$$

Then only the terms $L_1 = L_\beta$ and $L_2 = L_\alpha$ will contribute in the summation. We also have

$$m' - M = M_\alpha + m_h - m_h'$$

and

$$M = M_\beta - m_d + m_d' \,,$$

which imply that

$$m - M_\alpha = M_\beta \,. \tag{1-2-74}$$

Then one gets

$$Y_{L_\beta}^{M_\beta^*} = Y_{L_\beta}^{m - M_\alpha^*} = (-)^{m - M_\alpha}\, Y_{L_\beta}^{M_\alpha - m} \,.$$

Then the equation (1-2-73) reduces to

$$(2j+1)^{\frac{1}{2}}\, i^{\ell}\, \beta_{sj}^{\ell m m_h m_d}(\underline{k}_\beta, \underline{k}_\alpha) = 4\pi \sum_{\substack{m' \; m_h' \; m_d' \\ J_\beta \; L_\beta \; M_\beta \\ J_\alpha \; L_\alpha \; M_\alpha}} i^{L_\alpha - L_\beta}$$

$$\times (-)^{s_d-m_d+m-M_\alpha} C^{m_h' \; -m_d' \; m_h'-m_d'}_{s_h \quad s_d \quad s} \; C^{m' \quad m_h'-m_d' \quad m-m_d'+m_h'}_{\ell \quad s \quad j}$$

$$\times C^{M_\beta \; -m_d \; M_\beta-m_d}_{L_\beta \quad s_d \quad J_\beta} \; C^{M_\beta-m_d'+m_d' \; -m_d' \; M_\beta-m_d}_{L_\beta \quad s_d \quad J_\beta}$$

$$\times C^{M_\beta-m_d+m_d' \; M_\alpha+m_h-m_h' \; m'}_{L_\beta \quad L_\alpha \quad \ell} \; C^{M_\alpha \; m_h \; M_\alpha+m_h}_{L_\alpha \quad s_h \quad J_\alpha}$$

$$\times C^{M_\alpha+m_h-m_h' \; m_h' \; M_\alpha+m_h}_{L_\alpha \quad s_h \quad J_\alpha} \; Y^{M_\alpha^*}_{L_\alpha}(\theta_{k_\alpha}, \varphi_{k_\alpha}) \; Y^{M_\alpha-m}_{L_\beta}(\theta_{k_\beta}, \varphi_{k_\beta})$$

$$\times I^{\ell sj}_{L_\beta J_\beta L_\alpha J_\alpha} \; . \qquad (1\text{-}2\text{-}75)$$

Where $I^{\ell sj}_{L_\beta J_\beta L_\alpha J_\alpha}$ is the integral

$$I^{\ell sj}_{L_\beta J_\beta L_\alpha J_\alpha} = \frac{4\pi}{k_\alpha k_\beta} \int r_\alpha \, dr_\alpha \int r_\beta \, dr_\beta \, \Phi_{L_\beta J_\beta}(k_\beta, r_\beta)$$

$$\times P^{sj}_{\ell L_1 L_2}(r_\beta, r_\alpha) \, \Phi_{L_\alpha J_\alpha}(k_\alpha, r_\alpha) \; . \qquad (1\text{-}2\text{-}76)$$

By interchanging the terms in the Clebsh-Gordan coefficients the appropriate way and using their orthogonality relation, they contract to a Wigner 9-j symbol yielding[7,12,13,14]

$$\beta^{\ell m m_h m_d}_{sj}(\underline{k}_\beta, \underline{k}_\alpha) = 4\pi \sum_{\substack{L_\alpha \; J_\alpha \; M_\alpha \\ L_\beta \; J_\beta}} (2J_\beta+1)^{\frac{1}{2}}(2s+1)^{\frac{1}{2}}(2\ell+1)^{\frac{1}{2}} \, i^{L_\alpha-L_\beta-\ell}$$

$$\times C \begin{matrix} M_\alpha - m + m_d \\ J_\beta \end{matrix} \begin{matrix} m + m_h - m_d \\ j \end{matrix} \begin{matrix} M_\alpha + m_h \\ J_\alpha \end{matrix} C \begin{matrix} M_\alpha \\ L_\alpha \end{matrix} \begin{matrix} m_h \\ s_h \end{matrix} \begin{matrix} M_\alpha + m_h \\ J_\alpha \end{matrix} C \begin{matrix} M_\alpha - m \\ L_\beta \end{matrix} \begin{matrix} m_d \\ s_d \end{matrix} \begin{matrix} M_\alpha - m + m_d \\ J_\beta \end{matrix}$$

$$\times \begin{Bmatrix} j & \ell & s \\ J_\alpha & L_\alpha & s_h \\ J_\beta & L_\beta & s_d \end{Bmatrix} Y \begin{matrix} M_\alpha - m \\ L_\alpha \end{matrix} (\theta_{k_\alpha}, \varphi_{k_\alpha}) \; Y \begin{matrix} M_\alpha - m \\ L_\beta \end{matrix} (\theta_{k_\beta}, \varphi_{k_\beta})$$

$$\times I \begin{matrix} \ell s j \\ L_\beta J_\beta L_\alpha J_\alpha \end{matrix} . \qquad (1\text{-}2\text{-}77)$$

If we chose $\underline{k}_\alpha$ as our z-axis and $\underline{k}_\alpha \times \underline{k}_\beta$ as our y-axis as shown in fig. 2:

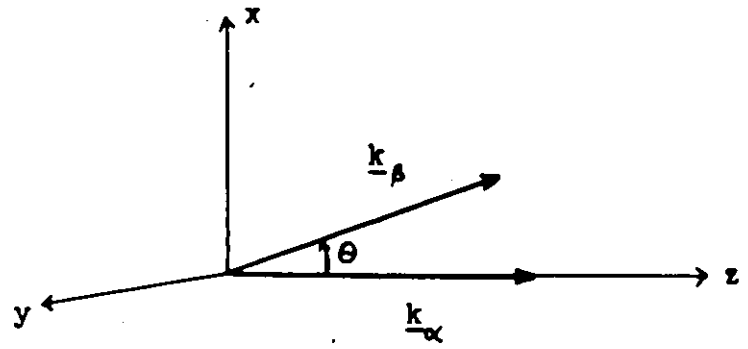


Fig. 2.

this choice of axes permits us to put

$$\theta_{k_\alpha} = \varphi_{k_\alpha} = \varphi_{k_\beta} = 0$$

and $$\theta_{k_\beta} = \theta .$$

Therefore[14)]

$$Y_{L_\alpha}^{M_\alpha^*}(0,\,0) = \delta_{M_\alpha,0}\left(\frac{2L_\alpha+1}{4\pi}\right)^{\frac{1}{2}} \qquad (1\text{-}2\text{-}78)$$

and[15]

$$Y_{L_\beta}^{M_\alpha -m}(\theta,\,0) = Y_{L_\beta}^{-m}(\theta,\,0) = (-)^m\, Y_{L_\beta}^{m^*}(\theta,\,0)$$

$$= \left[\frac{2L_\beta+1}{4\pi}\left[\frac{(L_\beta-m)!}{(L_\beta+m)!}\right]\right]^{\frac{1}{2}} P_{L_\beta}^{m}(\cos\theta) \qquad \text{for} \quad m>0 \qquad (1\text{-}2\text{-}79)$$

The integral $I_{L_\beta J_\beta L_\alpha J_\alpha}^{\ell s j}$ described by the equation (1-2-76) is taken over the two variables r_α and r_β. It is common to reduce it to an integration over only one variable by applying a zero-range approximation (ZR). The ZR approximation considers that the emitted particle is produced where the incident particle disappears. It essentially consists of making the outgoing deuteron and the centre of mass of ^{3}He (h) coincide as shown in fig. 3:

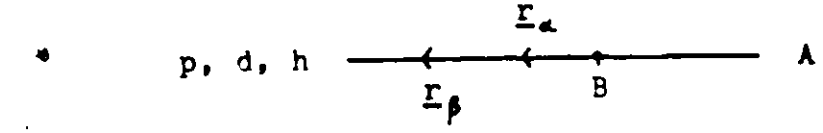


Fig. 3.

then $$\underline{r}_\beta = \frac{A}{B}\,\underline{r}_\alpha \,. \qquad (1\text{-}2\text{-}80)$$

Substituting the equations (1-2-78, 79 and 80) into (1-2-77)

$$\beta_{sj}^{\ell m m_h m_d ZR}(\theta) = \sum_{\substack{L_\alpha J_\alpha \\ L_\beta J_\beta}} i^{L_\alpha - L_\beta - \ell} (2J_\beta+1)^{\frac{1}{2}}(2s+1)^{\frac{1}{2}}(2\ell+1)^{\frac{1}{2}}(2L_\alpha+1)^{\frac{1}{2}}$$

$$\times(2L_\beta+1)^{\frac{1}{2}} \left[\frac{(L_\beta-m)!}{(L_\beta+m)!}\right]^{\frac{1}{2}} C\begin{matrix} m_d-m & m+m_h-m_d & m_h \\ J_\beta & j & J_\alpha \end{matrix} C\begin{matrix} 0 & m_h & m_h \\ L_\alpha & s_h & J_\alpha \end{matrix}$$

$$\times C\begin{matrix} -m & m_d & m_d-m \\ L_\beta & s_d & J_\beta \end{matrix} \begin{Bmatrix} j & \ell & s \\ J_\alpha & L_\alpha & s_h \\ J_\beta & L_\beta & s_d \end{Bmatrix} I^{\ell sj\ ZR}_{L_\beta J_\beta L_\alpha J_\alpha}, \qquad (1\text{-}2\text{-}81)$$

where

$$I^{\ell sj\ ZR}_{L_\beta J_\beta L_\alpha J_\alpha} = \frac{4\pi}{k_\alpha k_\beta} \int dr_\alpha \left(\frac{A}{B} r_\alpha\right)^{-2} \Phi_{L_\beta J_\beta}(k_\beta, \frac{A}{B} r_\alpha)$$

$$\times F^{sj}_{\ell L_\beta L_\alpha}(r_\alpha)\ \Phi_{L_\alpha J_\alpha}(k_\alpha, r_\alpha) . \qquad (1\text{-}2\text{-}82)$$

1-2-f) Differential Cross-Section

The differential cross-section for unpolarized projectile and an unpolarized target nucleus, averaged over the initial states and summed over the final states is given by[5,6,7] (See Appendix A)

$$\frac{d\sigma}{dw} = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} \frac{k_\beta}{k_\alpha} \frac{1}{(2J_A+1)(2s_h+1)} \sum_{\substack{M_A m_h \\ M_B m_d}} |T|^2 . \qquad (1\text{-}2\text{-}83)$$

From the equation (1-2-68), we have

$$\sum_{\substack{M_A\ m_h\\ M_B\ m_d}} |T|^2 = \sum_{\substack{M_A\ m_h\\ M_B\ m_d\\ \ell s j\ m}} (2j+1)\,|A_{\ell s j}|^2 \left|\beta_{sj}^{\ell m m_h m_d}\right|^2$$

$$\times C \begin{matrix} M_A & M_B-M_A & M_B \\ J_A & j & J_B \end{matrix} \cdot C \begin{matrix} M_A & M_B-M_A & M_B \\ J_A & j & J_B \end{matrix}$$

Transforming the Clebsh-Gordan coefficients

$$C \begin{matrix} M_A & M_B-M_A & M_B \\ J_A & j & J_B \end{matrix} = \left[\frac{2J_B+1}{2j+1}\right]^{\frac{1}{2}} (-)^{J_A-M_A} C \begin{matrix} M_A & M_B & M_B-M_A \\ J_A & J_B & j \end{matrix}$$

then summing over M_A and M_B and using their orthogonality relation:

$$\sum_{\substack{M_A\ m_h\\ M_B\ m_d}} |T|^2 = (2J_B+1) \sum_{\substack{j\ m\\ m_h\ m_d\\ \ell\ s}} |A_{\ell s j}|^2 \left|\beta_{sj}^{\ell m m_h m_d}\right|^2 \qquad (1\text{-}2\text{-}84)$$

Then

$$\frac{d\sigma}{dw} = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} \frac{k_\beta}{k_\alpha} \frac{(2J_B+1)}{(2J_A+1)(2s_h+1)} \sum_{\substack{j\ m\\ m_h\ m_d\\ \ell\ s}} \left|A_{\ell s j}\beta_{sj}^{\ell m m_h m_d}\right|^2 . \qquad (1\text{-}2\text{-}85)$$

In the ZR approximation, β^{ZR} is given by the equation (1-2-81). Usually in this approximation, the scalar

function $D(r_{pd})$ given by (1-2-61) is assumed to be of short range and may be replaced by a delta function:

$$D(r_{pd}) = V_{pd}^{s_h}(r_{pd})\,\Phi_h(r_{pd}) \simeq D_o\,\delta(\underline{r}_{pd}) \quad . \qquad (1\text{-}2\text{-}86)$$

From the equation (1-2-45),

$$\delta(\underline{r}_{pd}) = \delta\left(\underline{r}_\beta - \frac{A}{B}\,\underline{r}_\alpha\right) J^{-1} \quad , \qquad (1\text{-}2\text{-}87)$$

then the Jacobian J defined in (1-2-48) cancels out from the transition amplitude. The effect of this approximation is to consider $V_{pd}^{s_h}(r_{pd})$, which is the potential binding the proton to the deuteron to form the ^{3}He nucleus in the state s_h, as being of short range.

The wave function $\Phi_h(r_{pd})$ was previously defined as the wave-function of the helion nucleus. It then satisfies the Schrödinger equation

$$(\nabla^2 - k^2)\,\Phi_h(r_{pd}) = \frac{2}{\hbar^2}\,\frac{pd}{h}\,V_{pd}^{s_h}(r_{pd})\,\Phi_h(r_{pd}) \quad . \qquad (1\text{-}2\text{-}88)$$

The potential used to generate the Φ_h is usually an optical-model potential with spin-orbit coupling chosen to describe the scattering of protons on deuterons. To estimate the magnitude of D_o, one may take[6)]

$$D_o = \int dr_{pd}\; V_{pd}^{s_h}(r_{pd})\,\Phi_h(r_{pd}) \quad . \qquad (1\text{-}2\text{-}89)$$

If we now apply the ZR approximation to the form factor $f_{\ell sj,m}$ in the equation (1-2-65), we get

$$f^{ZR}_{\ell sj,m}(\underline{r}_\beta, \underline{r}_\alpha) = \delta(\underline{r}_\beta - \frac{A}{B}\underline{r}_\alpha)\int f_{\ell sj,m}(\underline{s} + \frac{A}{B}\underline{r}_\alpha, \underline{r}_\alpha)d\underline{s},$$

$$= F_{\ell sj}(r_\alpha)\, Y^{m*}_{\ell}(\theta_{r_\alpha}, \varphi_{r_\alpha})\,\delta(\underline{r}_\beta - \frac{A}{B}\underline{r}_\alpha) \quad . \qquad (1\text{-}2\text{-}90)$$

Comparing the equations (1-2-66, 86 and 90) and noting that $\underline{r}_{pA} = \underline{r}_\alpha$ when $\underline{r}_{pd} = 0$, we can set $F_{\ell sj} = \phi^{A*}_{\ell j}$, which is the radial shell model wave function for the proton in the orbit (ℓj) around the target nucleus A. The coefficient $A_{\ell sj}$ then becomes

$$A_{\ell sj} = C\left[\frac{2s_h+1}{2s+1}\right]^{\frac{1}{2}} \sqrt{2n}\; \mathcal{J}^A(\ell j)\, D_o \quad , \qquad (1\text{-}2\text{-}91)$$

with (1-2-56)

$$A_{\ell sj} = C\left[\frac{2s_h+1}{2s+1}\right]^{\frac{1}{2}} \sqrt{2}\; S(\ell j)^{\frac{1}{2}}\, D_o \quad , \qquad (1\text{-}2\text{-}92)$$

where C is the isospin Clebsch-Gorden coefficient defined earlier.

Substituting (1-2-92) into the expression of the cross-section (1-2-85), one obtains

$$\frac{d\sigma}{dw} = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2}\,\frac{k_\beta}{k_\alpha}\,\frac{2J_B+1}{(2J_A+1)(2s_h+1)}$$

$$\times \sum_{\substack{m\, m_h\, m_d \\ \ell\; s\; j}} 2C^2\,\frac{(2s_h+1)}{(2s+1)}\, S(\ell j)\, D_o^2 \left|\beta^{\ell m m_d m_h\ ZR}_{sj}(\Theta)\right|^2 . \qquad (1\text{-}2\text{-}93)$$

For a particular transfer (ℓsj) of a proton, the differential cross-section becomes

$$\frac{d\sigma_{\ell sj}}{dw} = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} \frac{k_\beta}{k_\alpha} \frac{2J_B+1}{2J+1} C^2 D_o^2 \frac{2}{2s+1}$$

$$\times S(\ell j) \sum_{m\, m_h\, m_d} \left| \beta_{sj}^{\ell m\ m_d\ m_h}\, ZR(\Theta) \right|^2 , \qquad (1\text{-}2\text{-}94)$$

$$\frac{d\sigma_{\ell sj}}{dw} = N \left[\frac{2J_B+1}{2J_A+1}\right] C^2 S(\ell j) \left[\frac{2s+1}{2} \frac{\sigma_{DW}(\Theta)}{2j+1}\right] , \qquad (1\text{-}2\text{-}95)$$

where $\sigma_{DW}(\Theta)$ is the cross-section calculated by the DW program code DWUCK for a given form factor $F^{sj}_{\ell L_\beta L_\alpha}(r)$.

From the equations (1-2-81, 82, 94 and 95), we have that $\sigma_{DW}(\Theta)$ has the form

$$\sigma_{DW}(\Theta) = \frac{1}{N} \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} \frac{k_\beta}{k_\alpha} D_o^2 \frac{4}{(2s+1)^2} (2j+1) \frac{16\pi^2}{(k_\alpha k_\beta)^2}$$

$$\left(\frac{A}{B}\right)^4 \left[\sum_{\substack{m\, m_h m_d \\ L_\alpha J_\alpha L_\beta J_\beta}} \sqrt{(2J_\beta+1)(2s+1)(2\ell+1)(2L_\alpha+1)(2L_\beta+1)} \sqrt{\frac{(L_\beta-m)!}{(L_\beta+m)!}} \right.$$

$$\times C \begin{matrix} m_d-m & m+m_h-m_d & m_h \\ J_\beta & j & J_\alpha \end{matrix} \quad C \begin{matrix} 0 & m_h & m_h \\ L_\alpha & s_h & J_\alpha \end{matrix} \quad C \begin{matrix} m & m_d & m_d-m \\ L_\beta & s_d & J_\beta \end{matrix}$$

$$\left. \times \begin{Bmatrix} j & \ell & s \\ J_\alpha & L_\alpha & s_h \\ J_\beta & L_\beta & s_d \end{Bmatrix} \int dr_\alpha\, r_\alpha^2\, \Phi_{L_\beta J_\beta}\left(k_\beta, \frac{A}{B} r_\alpha\right) F^{sj}_{\ell L_\beta L_\alpha}(r_\alpha) \right.$$

$$\times \Phi_{L_\alpha J_\alpha}(k_\alpha, r_\alpha) \Big]^2 \quad . \qquad (1\text{-}2\text{-}96)$$

The constant N is taken to be[6] 4.42 for the (^{3}He, d) reaction.

The spectroscopic strength of a given level of the residual nucleus B, for a given set ℓj of the transferred proton is defined as[10, 24]

$$\frac{2J_B+1}{2J_A+1} \; C^2S(\ell j) \quad . \qquad (1\text{-}2\text{-}97)$$

Thus from (1-2-95), the spectroscopic strength of a given level is

$$\frac{2J_B+1}{2J_A+1} \; C^2S(\ell j) = \frac{(2j+1)}{4.42} \; \frac{2}{(2s+1)} \; \frac{\frac{d\sigma_{\ell sj}}{dw}}{\sigma_{DW}(\theta)} \quad . \qquad (1\text{-}2\text{-}98)$$

1-3 Non-Local Calculations and Finite-Range Effects

The zero-range approximation (ZR) in a ^{3}He stripping process corresponds to the assumption that the transition amplitude, equation (1-2-51), which is proportional to the matrix element of the deuteron-proton potential taken between the distorted waves in the incident and exit channels, receives contributions only from the region where the coordinates of the deuteron and the proton coincide.

The transition amplitude involves the combination (1-2-61)

$$D(r_{pd}) = V_{pd}^{s_h}(r_{pd})\, \Phi_h(r_{pd}) \quad , \tag{1-3-1}$$

and the ZR approximation amounts to assuming (1-2-86)

$$D(\underline{r}_{pd}) \simeq D_0\, \delta(\underline{r}_{pd}) \quad . \tag{1-3-2}$$

Where the constant D_0 was given in the equation (1-2-89) as

$$D_0 = \int dr_{pd}\, V_{pd}^{s_h}(r_{pd})\, \Phi_h(r_{pd}) \quad . \tag{1-3-3}$$

Since it is generally believed that the optical potential is non-local[19], we must include a finite range correction in our previous calculations. The local energy approximation[15] (LEA) constitutes a good approximation to the finite range correction. The distorted waves will be

taken to be spin independent in order to simplify the formalism. Then the equation (1-2-75) becomes with (1-2-72, 66 and 61)

$$(2j+1)^{\frac{1}{2}}\, i^{\ell}\, \beta^{\ell m}_{sj}(\underline{k}_\beta, \underline{k}_\alpha) = \frac{(4\pi)^2}{k_\alpha k_\beta} \int d\underline{r}_\alpha \int d\underline{r}_\beta\, \Phi^{-*}_\beta(\underline{k}_\beta, \underline{r}_\beta)$$

$$\times \phi^{A*}_{\ell j}(r_{pA})\, D(r_{dp})\, \Phi^{+}_\alpha(\underline{k}_\alpha, \underline{r}_\alpha). \qquad (1\text{-}3\text{-}4)$$

We recall from (1-2-44) that

$$\underline{r}_\alpha = \underline{r}_{pA} + \frac{d}{h}\, \underline{r}_{dp} \quad , \qquad (1\text{-}3\text{-}5)$$

and by using the same technique as for deriving (1-2-44), we have

$$\underline{r}_\beta = \frac{A}{B}\, \underline{r}_{pA} + \underline{r}_{dp} \quad . \qquad (1\text{-}3\text{-}6)$$

Setting $A/B \simeq 1$,

$$\underline{r}_\beta = \underline{r}_{pA} + \underline{r}_{dp} \quad . \qquad (1\text{-}3\text{-}7)$$

the essential step in the LEA is to do a Taylor series expansion of the distorted wave functions Φ^{-*}_β and Φ^{+}_α; one then obtains[15, 20]

$$\Phi^{-*}_\beta(\underline{k}_\beta, \underline{r}_{pA} + \underline{r}_{dp}) = \Phi^{-*}_\beta(\underline{k}_\beta, \underline{r}_{pA}) + \underline{r}_{dp} \cdot \nabla_{\underline{r}_{pA}} \Phi^{-*}_\beta(\underline{k}_\beta, \underline{r}_{pA})$$

$$+ \tfrac{1}{2}(\underline{r}_{dp} \cdot \nabla_{\underline{r}_{pA}})^2\, \Phi^{-*}_\beta(\underline{k}_\beta, \underline{r}_{pA}) + \cdots \qquad (1\text{-}3\text{-}8a)$$

$$\Phi^{+}_{\alpha}(\underline{k}_{\alpha}, \underline{r}_{pA} + \frac{d}{h}\,\underline{r}_{dp}) = \Phi^{+}_{\alpha}(\underline{k}_{\alpha}, \underline{r}_{pA}) + \frac{d}{h}\,\underline{r}_{dp} \cdot \nabla_{\underline{r}_{pA}}$$

$$* \Phi^{+}_{\alpha}(\underline{k}_{\alpha}, \underline{r}_{pA}) + \tfrac{1}{2}\,(\frac{d}{h}\,\underline{r}_{dp} \cdot \nabla_{\underline{r}_{pA}})^2\ \Phi^{+}_{\alpha}(\underline{k}_{\alpha}, \underline{r}_{pA}) + \cdots$$

(1-3-8b)

These expansions can be written symbolically as

$$\Phi^{-*}_{\beta}(\underline{k}_{\beta}, \underline{r}_{\beta}) = \exp(\underline{r}_{dp} \cdot \nabla_{\underline{r}_{\beta}})\ \Phi^{-*}_{\beta}(\underline{k}_{\beta}, \underline{r}_{pA}) \quad ,$$

$$= \exp(i(\underline{r}_{dp} \cdot (\bar{k}_{\beta})_{op}))\ \Phi^{-*}_{\beta}(\underline{k}_{\beta}, \underline{r}_{pA}) \qquad (1\text{-}3\text{-}9a)$$

$$\Phi^{+}_{\alpha}(\underline{k}_{\alpha}, \underline{r}_{\alpha}) = \exp(\frac{d}{h}\,\underline{r}_{dp} \cdot \nabla_{\underline{r}_{\alpha}})\ \Phi^{+}_{\alpha}(\underline{k}_{\alpha}, \underline{r}_{pA}) \quad ,$$

$$= \exp(i\,\frac{d}{h}\,(\underline{r}_{dp} \cdot (\bar{k}_{\alpha})_{op}))\ \Phi^{+}_{\alpha}(\underline{k}_{\alpha}, \underline{r}_{pA}) \quad . \qquad (1\text{-}3\text{-}9b)$$

Where the subscripts on the operators $(\bar{k}_{\alpha})_{op}$ and $(\bar{k}_{\beta})_{op}$ serve as reminders that they operate on $\Phi^{+}_{\alpha}(\underline{k}_{\alpha}, \underline{r}_{pA})$ and $\Phi^{-*}_{\beta}(\underline{k}_{\beta}, \underline{r}_{pA})$ respectively.

Substituting the equations (1-3-9a and b) into (1-3-4), we get

$$(2j+1)^{\frac{1}{2}}\ i^{\ell}\ \beta^{\ell m}_{sj}(\underline{k}_{\beta}, \underline{k}_{\alpha}) = \frac{(4\pi)^2}{k_{\beta} k_{\alpha}} \int d\underline{r}_{\alpha} \int d\underline{r}_{\beta}\ D(r_{dp})$$

$$\times \exp(i\,\underline{r}_{dp}\cdot(\frac{d}{h}(\bar{k}_\alpha)_{op}+(\bar{k}_\beta)_{op})\;\Phi^{-*}_\beta(\underline{k}_\beta,\underline{r}_\beta)\;\phi^{A*}_{\ell j}(r_{pA})$$

$$\times\Phi^{+}_\alpha(\underline{k}_\alpha,\underline{r}_\alpha)\quad .$$

$$=\frac{(4\pi)^2}{k_\beta k_\alpha}J^{-1}\int d\underline{r}_{pA}\int d\underline{r}_{dp}\;D(\underline{r}_{dp})\exp(i\,\underline{r}_{dp}\cdot\bar{K}_{op})$$

$$\times\Phi^{-*}_\beta(\underline{k}_\beta,\underline{r}_\beta)\;\phi^{A*}_{\ell j}(r_{pA})\;\Phi^{+}_\alpha(\underline{k}_\alpha,\underline{r}_\alpha)\quad . \qquad (1\text{-}3\text{-}10)$$

Where J is the Jacobian of the transformation from the coordinate system $(\underline{r}_{pA},\underline{r}_{dp})$ to $(\underline{r}_\beta,\underline{r}_\alpha)$ given by the equation (1-2-48). We note that the Jacobian cancels out of the transition matrix element. The operator $\bar{K}_{op}$ is defined by

$$\bar{K}_{op}=\frac{d}{h}(\bar{k}_\alpha)_{op}+(\bar{k}_\beta)_{op}=-\,i(\frac{d}{h}\nabla_\alpha+\nabla_\beta)\,. \qquad (1\text{-}3\text{-}11)$$

The integration is carried out over $\underline{r}_{dp}$. We notice that the operator $\bar{K}_{op}$ is independent of $\underline{r}_{dp}$, thus

$$(2j+1)^{\frac{1}{2}}\,i^{\ell}\,\beta^{\ell\,m}_{sj}(\underline{k}_\beta,\underline{k}_\alpha)=\frac{(4\pi)^2}{k_\beta k_\alpha}J^{-1}\int d\underline{r}_{pA}\;D^{F}(\bar{K}_{op})$$

$$\times\Phi^{-*}_\beta(\underline{k}_\beta,\underline{r}_{pA})\;\phi^{A*}_{\ell j}(r_{pA})\;\Phi^{+}_\alpha(\underline{k}_\alpha,\underline{r}_{pA})\quad . \qquad (1\text{-}3\text{-}12)$$

Where

$$D^F(\bar{K}_{op}) = \int d\underline{r}_{pd}\, D(r_{pd}) \exp(i\, \underline{r}_{dp} \cdot \bar{K}_{op}) \quad . \qquad (1\text{-}3\text{-}13)$$

Clearly D^F is the operator corresponding to the Fourier transform of D. It's effects on the distorted waves Φ^{-*}_{β} and Φ^{+}_{α} and on the bound state wave function $\phi^{A*}_{\ell j}$ have been estimated by several autors[9, 15, 20, 21].

We will give here essentially the results obtained by Perey and Saxon[19]. The operator D^F modulates $\phi^{A*}_{\ell j}$ with a form factor

$$\Lambda(r) = 1 - \left[\frac{(U_h(r) - U_d(r) - U_p(r) - Be)}{\frac{\hbar^2}{MR^2}} \right] \quad . \qquad (1\text{-}3\text{-}14)$$

Here $U_i(r)$ is the optical-model potential for the particle i, M is an atomic mass unit, Be is the binding energy of the proton in the residual nucleus and R is a range as defined by Bassel[23].

D^F also modulates each distorted wave Φ with a form factor

$$N_i(r) = \left(1 - \frac{\mu_i \beta_i^2}{2\hbar^2} U_i(r)\right)^{\frac{1}{2}} \quad , \qquad (1\text{-}3\text{-}15)$$

where μ_i is the reduced mass in the channel considered, β_i is the non-locality range and U_i is the equivalent local potential.

1-4 Spectroscopic Strengths and Expectation Values for the Number of Proton Holes

We shall now give a summary of the theoretical analysis of the reduced widths by Macfarlane and French[10,24]. We will restrict ourselves to the jj representation and generalize without demonstration to the jjT representation.

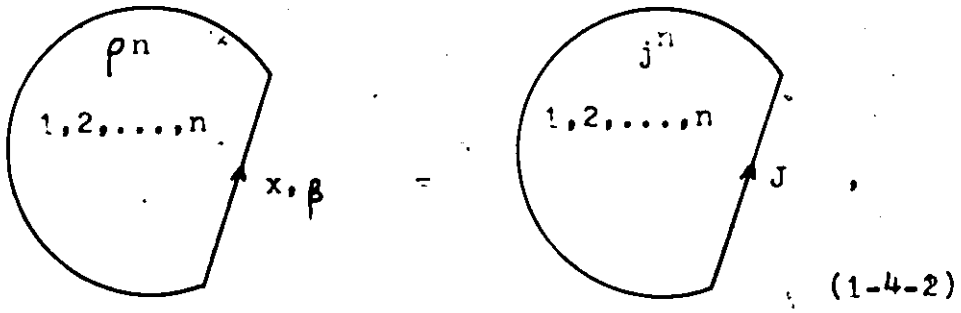
The nuclear shell model is based on two fundamental assumptions. The first one is the existence of single-particle orbits characterized by a radial quantum number n and an orbital quantum number ℓ. The second is the existence of a strong spin-orbit interaction which depresses each $j = \ell + \frac{1}{2}$ level relative to the corresponding $j = \ell - \frac{1}{2}$ level, with $\underline{j} = \underline{\ell} + \underline{s}$. These two basic postulates are summarized in writing the nuclear shell model Hamiltonian in the following form

$$H = \sum_{i=1}^{A} V(r_i) + \sum_{i<j=1}^{A} H_{ij} + a \sum_{i=1}^{A} \ell_i \cdot \underline{s}_i \quad , \qquad (1\text{-}4\text{-}1)$$

where $V(r_i)$ is the central shell model potential. This potential may conveniently be chosen to possess harmonic oscillator radial dependence; since in that case, the eigenfunctions are well known. H_{ij} is an effective two-body interaction operator and the last term in (1-4-1) is the one-body spin-orbit potential. These last two terms are usually adjusted so that the shell model Hamiltonian gives a good

approximation of the experimental properties of the considered nucleus. In a shell model calculation, it is necessary in practice to restrict the number of states which enter the calculation and include only the states of a few of the lowest configurations, regarding the A nucleons of the nucleus as filling the single-particle states of V(r) in ascending order and in accordance with Pauli's exclusion principle.

Let us consider n equivalent nucleons, i.e. nucleons in the same subshell. We wish to construct completely antisymmetric states in the jj representation

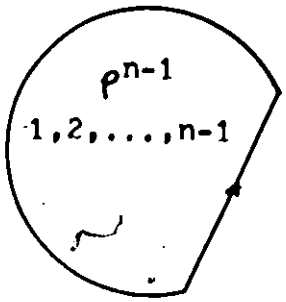


(1-4-2)

with $\rho = \{ n\ell j \}$, $\beta = \{ J \}$, where n is the principal quantum number of a nucleon, ℓ the orbital angular momentum of a nucleon, $\underline{j} = \underline{\ell} + \underline{s}$ the total angular momentum of a nucleon, s being the spin of the nucleon. J is the total angular momentum of the nucleus. The quantum numbers $\{n\ell j\}$ are absorbed for convenience into a single quantum number ρ and J into β. An additional quantum number x has been added to represent all the non-angular momentum quantum numbers.

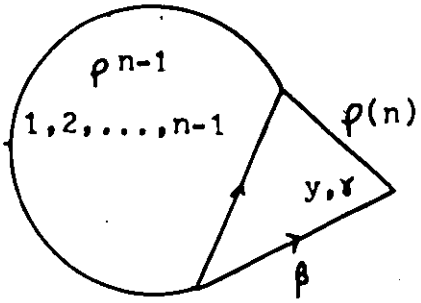
If we suppose that the antisymmetric wave function

of n-1 equivalent nucleons is known, and is given by



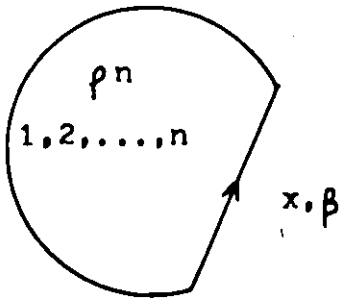
(1-4-3)

The functions

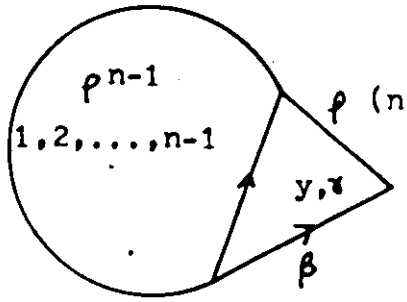


(1-4-4)

are antisymmetric in the particles number 1 to n-1, but not totally antisymmetric in the particles number 1 to n. The antisymmetric function (1-4-2) belongs to a restricted subspace of the linear vector space spanned by the function (1-4-4). Therefore



$$= \sum_{y,\gamma} \langle \rho^{n} ; x\beta \,|\, \rho^{n-1} ; y\gamma \rangle$$



(1-4-5)

The expansion coefficients $\langle \rho^n ; x\beta \mid \rho^{n-1} ; y\gamma \rangle$ are the fractional parantage coefficients (c.f.p.). The orthonomality of the functions described in (1-4-5) with different values of the additional quantum number x yield the following sum rule

$$\sum_{y\gamma} \langle \rho^n ; x\beta \mid \rho^{n-1} ; y\gamma \rangle \langle \rho^n ; x'\beta \mid \rho^{n-1} ; y\gamma \rangle = \delta(xx'). \tag{1-4-6}$$

A shell in the nucleus is characterized by the quantum numbers ρ of its constituent nucleons. The states available for each nucleon in a given shell are defined by the different z-projections m_ρ of ρ. Since there are $(2\rho+1)$ available states for each nucleon characterized by the quantum numbers ρ, states of ρ^n can be constructed by distributing the n equivalent nucleons considered among $N = (2\rho+1)$ availabe states. Let us call a given choice of n states out of N possible states a distribution of ρ^n. A distribution determines one and only one antisymmetric state of n particles. This antisymmetric state is a unique linear combination of the n! distinct product functions obtained by permuting the order of the nucleons associated with the occupied states. Thus every distribution of ρ^n can be labelled, in the m_ρ representation, by a distribution index λ and posesses a specified value of the total projection quantum number $B = \sum m_\rho$.

The number of distributions of n particles among N states gives the binomial coefficient

$$\binom{N}{n} \qquad (1\text{-}4\text{-}7)$$

of allowed antisymmetric states of ρ^n.

A closed shell will be one which contains $N = 2\rho + 1$ nucleons. Then according to (1-4-7), there is only one distribution ρ^n and therefore only one antisymmetric state. This closed shell state is unique within a sign and has all angular momentum quantum numbers 0. We will denote it by $\Psi(S^*)$.

The N-n states left unoccupied in any distribution λ of ρ^n determine a complementary distribution λ^c of ρ^{N-n}. Thus, to every antisymmetric state $\Phi^B_\lambda(n)$ of ρ^n, corresponds one and only one complementary antisymmetric state $\Phi^{-B}_{\lambda^c}(N-n)$ of ρ^{N-n}. The one to one correspondence comes from the fact that

$$\binom{N}{n} = \binom{N}{N-n} \quad . \qquad (1\text{-}4\text{-}8)$$

Here B is the z-projection of the quantum number β. The ρ^n states refer to particle states whereas the ρ^{N-n} states to hole states.

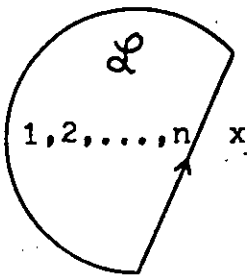
The allowed antisymmetric states of ρ^n span a linear vector space $\mathcal{L}$; $\mathcal{R}$ will be the corresponding space of ρ^{N-n}. The closed shell function $\Psi(S^*)$ can be expressed as a linear superposition of products

$$\Phi^B_\lambda(n)\ \Phi^{-B'}_{\lambda^c}(N-n) \quad , \qquad (1\text{-}4\text{-}9)$$

of vectors of $\mathcal{L}$ and $\mathcal{R}$ spaces. Only the products in which $\lambda^{c'} = \lambda^{c}$ can occur in this expansion, in order to satisfy Pauli's exclusion principle. If this was not the case, we would have the possibility of two particles in the same state. Taking directly the result of the expansion from Macfarlane and French[10], we have

$$\Psi(S^*) = \binom{N}{n}^{-\frac{1}{2}} \sum_{B}{}^{\circ}(-)^{B-Bm} \sum_{\{\lambda ; B_\lambda = B\}} \Phi^{B}_{\lambda}(n)\ \Phi^{-B}_{\lambda^c}(N-n) \quad . \tag{1-4-10}$$

Where $\{\lambda ; B_\lambda = B\}$ stands for the set of all distributions λ such as $B_\lambda = B$ and $Bm = \beta$, the largest value of B. Note that the sum over $\{\lambda^c ; B_{\lambda^c} = -B\}$ is implicit since λ^c represents the complementary distribution of the distribution λ. The equation (1-4-10) presents the closed shell wave function $\Psi(S^*)$ as an invariant product of two vectors, one from the space $\mathcal{L}$ and the other from the space $\mathcal{R}$. The product is invariant in the sense that since the closed shell wave function is unique, it must retain the same form under a change of basis in $\mathcal{L}$ and $\mathcal{R}$. We thus perform a unitary change of basis in $\mathcal{L}$

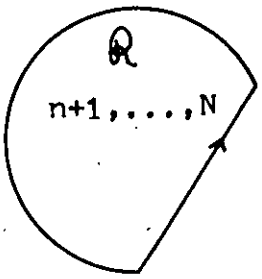


$$x,\beta,B = \sum_{\{\lambda ; B_\lambda = B\}} C_{x\beta ; \lambda}\ \Phi^{B}_{\lambda}(n) \quad . \tag{1-4-11}$$

Since the matrix of transformation is unitary,

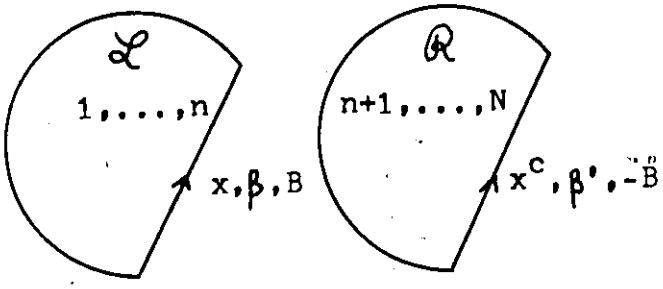
$$\sum_{\{\lambda;B_\lambda=B\}} C^{\dagger}_{x\beta;\lambda} \; C_{\lambda;x'\beta'} = \delta(xx') \; \delta(\beta\beta') \quad . \qquad (1\text{-}4\text{-}12)$$

Similarly in $\mathcal{R}$



$$x^{c'},\beta',-B = \sum_{\{\lambda^c;B_{\lambda^c}=-B\}} C'_{x^{c'}\beta';\lambda^c} \overline{\Phi}^{-B}_{\lambda^c}(N-n). \qquad (1\text{-}4\text{-}13)$$

The invariance of the product implies



$$= \sum_{\substack{\{\lambda;B_\lambda=B\} \\ \{\lambda^c;B_{\lambda^c}=-B\}}} C_{x\beta;\lambda} \, C'_{x^{c'}\beta';\lambda^c} \, \overline{\Phi}^{B}_{\lambda}(n) \, \overline{\Phi}^{-B}_{\lambda^c}(N-n) \quad ,$$

$$= \overline{\Phi}^{B}_{\lambda}(n) \, \overline{\Phi}^{-B}_{\lambda^c}(N-n) \quad ,$$

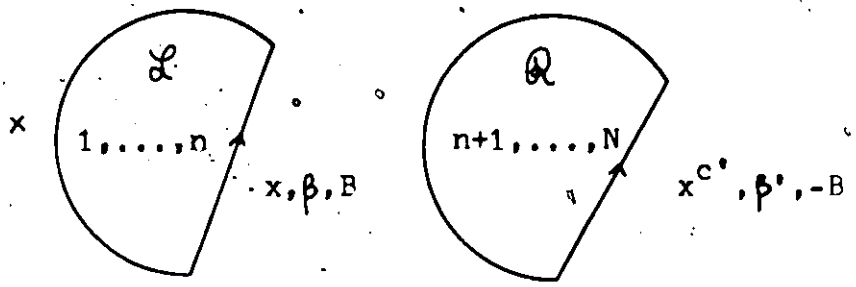
which is verified if and only if in the matrix representation

$$C^T \, C' = I \quad ,$$

or $C' = (C^T)^{-1} = C^* \quad$, since C is unitary.

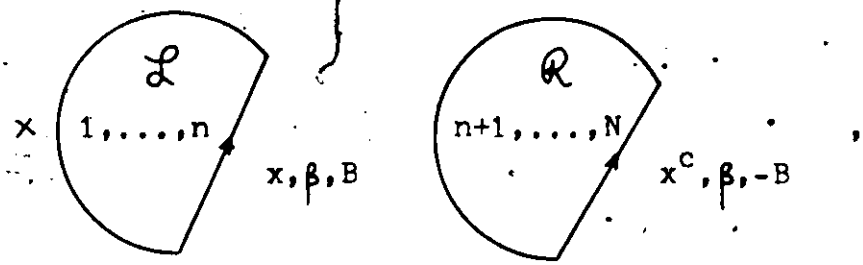
Inverting (1-4-12) and (1-4-13) and substituting it into (1-4-10), we obtain

$$\Psi(S^*) = \binom{N}{n}^{-\frac{1}{2}} \sum_B (-)^B \sum_{\substack{x\beta \\ x'\beta'}} (-)^{\beta} C_{x\beta;\lambda}\, (C_{x'\beta';\lambda})^{\dagger}$$

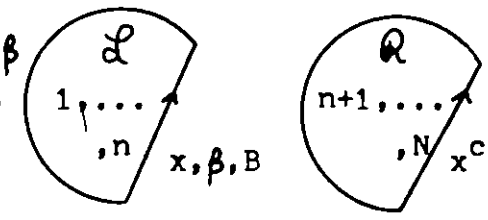


From the relation (1-4-12), this reduces to

$$\Psi(S^*) = \binom{N}{n}^{-\frac{1}{2}} \sum_B (-)^B \sum_{\substack{x\beta \\ x'\beta'}} (-)^{\beta}\, \delta(xx')\, \delta(\beta\beta')$$

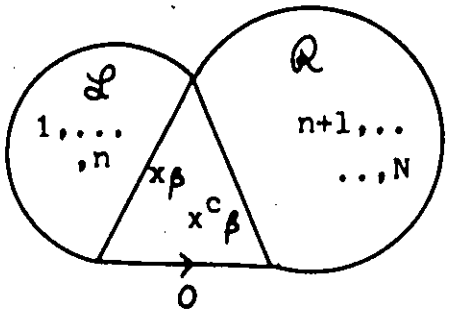


$$\Psi(S^*) = \binom{N}{n}^{-\frac{1}{2}} \sum_{x\beta B} (-)^{B-\beta}$$



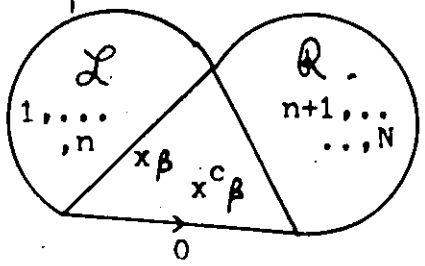
In a coupled representation

$$\Psi(S^*) = \binom{N}{n}^{-\frac{1}{2}} \sum_{x\beta B} (-)^{B-\beta} C \begin{matrix} B & -B & 0 \\ \beta & \beta & 0 \end{matrix}$$



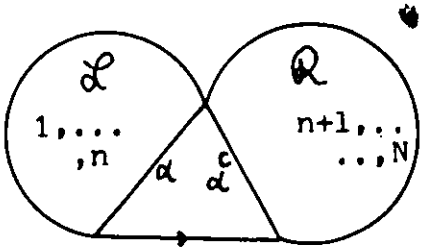
Evaluating the Clebsch-Gordan coefficient and carrying out the summation over $B = -\beta, -\beta+1, \ldots \beta-1, \beta$, we obtain

$$\Psi(S^*) = \binom{N}{n}^{-\frac{1}{2}} \sum_{x\beta} (2\beta+1)^{\frac{1}{2}}$$



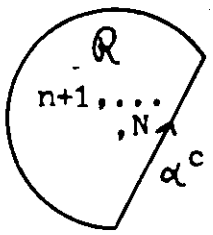
Reducing the quantum numbers $x\beta$ and $x^c\beta$ into $\alpha \equiv x\beta$ and $\alpha^c \equiv x^c\beta$, we obtain the Racah hole-particle correspondence formula

$$\Psi(S^*) = \binom{N}{n}^{-\frac{1}{2}} \sum_{\alpha} (2\alpha+1)^{\frac{1}{2}}$$

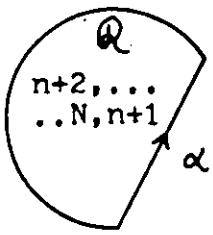


(1-4-14)

Since



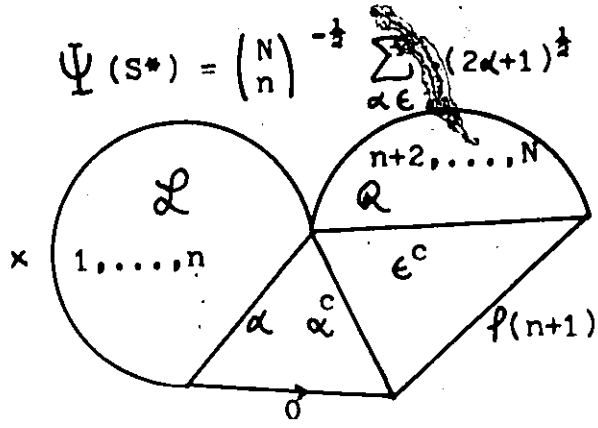
$= (-)^{n+1}$



,

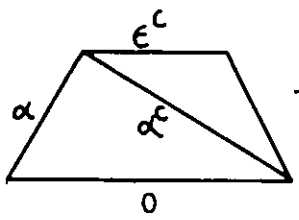
and making use of the equation (1-4-5), (1-4-14) may be written

$$\Psi(S^*) = \binom{N}{n}^{-\frac{1}{2}} \sum_{\alpha\epsilon} (2\alpha+1)^{\frac{1}{2}} \langle \rho^{N-n}, \alpha^c \mid \rho^{N-n-1}, \epsilon^c \rangle (-)^{n+1}$$

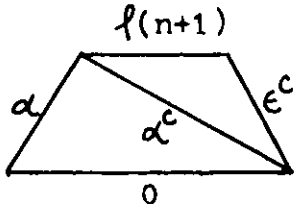


(1-4-15)

We have that

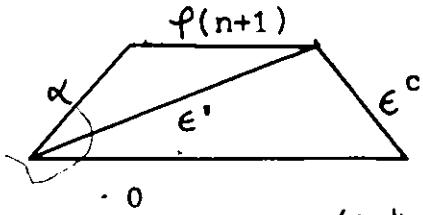


$$\rho(n+1) = (-)^{\rho+\epsilon-\alpha}$$



,

$$= (-)^{\rho+\epsilon-\alpha} \sum_{\epsilon'} U(\alpha\rho 0\epsilon;\epsilon'\alpha)$$



,

(1-4-16)

where

$$U(\alpha\rho\ 0\epsilon;\ \epsilon'\alpha) = \left[(2\epsilon'+1)(2\alpha+1)\right]^{\frac{1}{2}} (-)^{\alpha+\rho+\epsilon} \begin{Bmatrix} \alpha & \rho & \epsilon' \\ \epsilon & 0 & \alpha \end{Bmatrix} ,$$

the symbol in braces is a Wigner 6-j coefficient. Since

$$\begin{Bmatrix} \alpha & \rho & \epsilon' \\ \epsilon & 0 & \alpha \end{Bmatrix} \overset{13)}{=} \begin{Bmatrix} \alpha & 0 & \alpha \\ \epsilon & \rho & \epsilon' \end{Bmatrix} \overset{13)}{=} \begin{Bmatrix} \alpha & \alpha & 0 \\ \epsilon & \epsilon' & \rho \end{Bmatrix} ,$$

$$\overset{14)}{=} (-)^{\alpha+\epsilon+\rho} \left[(2\alpha+1)(2\epsilon+1)\right]^{-\frac{1}{2}} \delta(\epsilon,\epsilon') ,$$

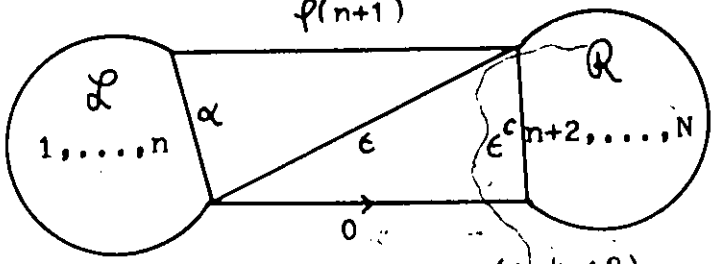
then the 6-j symbol reduces to

$$U(\alpha\, \rho 0\, \epsilon\,;\, \epsilon' \alpha) = \delta(\epsilon, \epsilon') \quad . \qquad (1\text{-}4\text{-}17)$$

Substituting (1-4-16,17) into (1-4-15),

$$\Psi(S^*) = \binom{N}{n}^{-\frac{1}{2}} \sum_{\alpha\epsilon} (2\alpha+1)^{\frac{1}{2}} \langle \rho^{N-n},\, \alpha^c \mid \rho^{N-n-1},\, \epsilon^c \rangle^{\mathcal{R}}$$

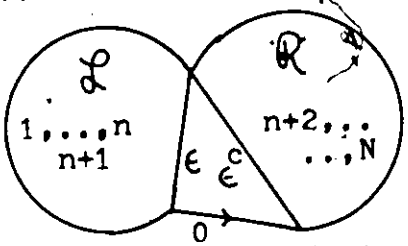
$$\times (-)^{\rho+\epsilon-\alpha+n+1}$$



(1-4-18)

If in the equation (1-4-14), we replace n by (n+1),

$$\Psi(S^*) = \binom{N}{n+1}^{-\frac{1}{2}} \sum_{\epsilon} (2\epsilon+1)^{\frac{1}{2}}$$

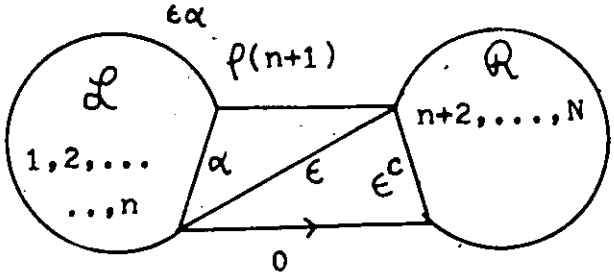


(1-4-18)

and if we decompose (1-4-18) as in equation (1-4-5),

$$\Psi(S^*) = \binom{N}{n+1}^{-\frac{1}{2}} \sum_{\epsilon\alpha} (2\epsilon+1)^{\frac{1}{2}} \langle \rho^{n+1},\, \epsilon \mid \rho^{n},\, \alpha \rangle^{\mathcal{L}}$$

$\times$



. (1-4-19)

Comparing the equations (1-4-18 and 19) we obtain

$$\frac{\langle \rho^{N-n},\, \alpha^c \mid \rho^{N-n-1},\, \epsilon^c \rangle^{\mathcal{R}}}{\langle \rho^{n+1},\, \epsilon \mid \rho^{n},\, \alpha \rangle^{\mathcal{L}}} = (-)^{\alpha-n-1-\rho-\epsilon}$$

$$\times \left[\frac{(n+1)(2\epsilon+1)}{(N-n)(2\alpha+1)} \right]^{\frac{1}{2}} . \qquad (1\text{-}4\text{-}20)$$

We have that each state of ρ^n in the $\mathcal{L}$ space is characterized by a set of quantum numbers $\alpha \equiv (x\ \beta)$ and the complementary state of ρ^{N-n} in the $\mathcal{R}$ space has the quantum numbers $\alpha^c = (x^c\ \beta)$. We can now use the extra quantum number x introduced in (1-4-2) to require that it be defined such that $(x^c\ \beta)$ will also label a state ρ^{N-n} in the $\mathcal{L}$ space. Then all the allowed states of the ρ shell will fall into two classes according to whether

$$\bar{\Psi}^{\mathcal{L}}_{\alpha}(n) = \Psi^{\mathcal{R}}_{\alpha}(n) ,$$

or $$\Psi^{\mathcal{L}}_{\alpha}(n) = - \Psi^{\mathcal{R}}_{\alpha}(n) .$$

The evaluation of the spectroscopic factor S consists in calculating the overlap integral between the initial and final nucleus. It describes the probability of the nucleus B in its final state of containing the target nucleus A in its ground state and the transferred nucleon in a specified single particle state. This is essentially what has been written in the equations (1-2-54, 55 and 56). The spectroscopic factor is defined[10] by

$$S(\ell j) = n \sum_{j} \left| \mathcal{J}(\ell j) \right|^2 . \qquad (1\text{-}4\text{-}21)$$

Where $\mathfrak{J}(\ell j)$ is the nuclear overlap integral

$$\mathfrak{J}(\ell j) = \left\langle \; J_B \; \middle| \; \ell(p), J_A, s(p), j(p), J_B \; \right\rangle \quad . \qquad (1\text{-}4\text{-}22)$$

J_B and J_A are the total angular momentum of the residual and target nucleus respectively, $\ell(p)$, s(p) and j(p) are the orbital angular momentum, spin and total angular momentum of the transferred proton. We obviously have

$$\underline{j}(p) = \underline{\ell}(p) + \underline{s}(p) \quad ,$$

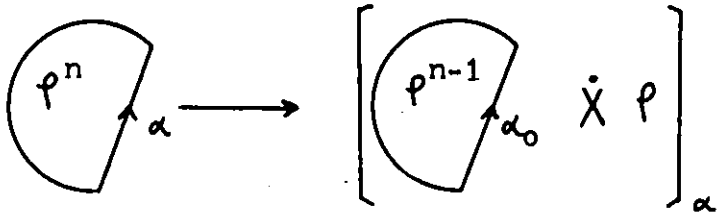
and $$\underline{J}_B = \underline{J}_A + \underline{j}(p) \quad .$$

n is the number of antisymmetrically coupled nucleons in the final nucleus equivalent to the transferred nucleon. Note that in the case we include isospin formalism, n will become the number of protons in the residual nucleus.

Since closed shells constitute inert groups of nucleons whose total angular momentum are zero, and therefore have no influence on the reduced width, the spectroscopic factor (1-4-21) reduces to[10)]

$$S(\rho) = n \left\langle \rho^n \, ; \alpha \; \middle| \; \rho^{n-1} \, ; \alpha_o \right\rangle^2 \quad , \qquad (1\text{-}4\text{-}23)$$

which is precisely the reduced width for the transition

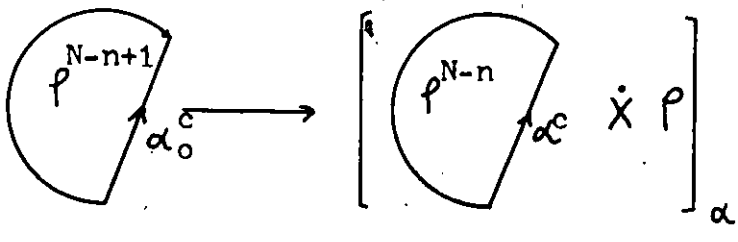


(1-4-24)

where $[\dot{x}]_\alpha$ indicates the vector coupling to a resultant angular momentum α. Let us rewrite (1-4-23), considering (1-4-24), as

$$S(n,\alpha \longrightarrow n-1, \alpha_o) = n \langle \rho^n ; \alpha | \rho^{n-1} ; \alpha_o \rangle^2 \quad . \tag{1-4-25}$$

The hole-particle correspondence described in the equation (1-4-14), leads us to expect a simple connection between the relative reduced width for the transition (1-4-24) and for the complementary transition



(1-4-26)

Similarly to (1-4-25), we have

$$S(N-n+1, \alpha_o^c \longrightarrow N-n, \alpha^c) = (N-n+1) \langle \rho^{N-n+1} ; \alpha_o^c | \rho^{N-n} ; \alpha^c \rangle^2 \quad . \tag{1-4-27}$$

Taking the ratio (1-4-25) ÷ (1-4-27) and making use of (1-4-40), one gets

$$\frac{S(n,\alpha \rightarrow n-1,\alpha_o)}{S(N-n+1,\alpha_o^c \rightarrow N-n,\alpha^c)} = \frac{n}{(N-n+1)} \frac{\langle \rho^n;\alpha \mid \rho^{n-1};\alpha_o\rangle^2}{\langle \rho^{N-n-1};\alpha_o^c \mid \rho^{N-n};\alpha^c\rangle^2} ,$$

$$= \frac{n}{(N-n+1)} \frac{(N-n+1)}{n} \frac{(2\alpha_o+1)}{(2\alpha+1)} .$$

Then

$$S(N-n+1,\alpha_o^c \rightarrow N-n,\alpha^c) = \frac{2\alpha+1}{2\alpha_o+1} S(n,\alpha \rightarrow n-1,\alpha_o) . \qquad (1\text{-}4\text{-}28)$$

The condition that the wave function $\rho^n;\alpha$ expressed in the equation (1-4-5), be normalized to unity, yield

$$\sum_{\alpha_o} \langle \rho^n;\alpha \mid \rho^{n-1};\alpha_o\rangle^2 = 1 . \qquad (1\text{-}4\text{-}29)$$

Then summing over α_o in the equation (1-4-25), be normalized to unity, yields

$$\sum_{\alpha_o} S(n,\alpha \rightarrow n-1,\alpha_o) = \sum_{\alpha_o} n \langle \rho^n;\alpha \mid \rho^{n-1};\alpha_o\rangle^2 ,$$

$$= n . \qquad (1\text{-}4\text{-}30)$$

The same way carrying out the summation over α in the equation (1-4-27),

$$\sum_{\alpha} S(N-n+1,\alpha_o^c \rightarrow N-n,\alpha^c) = (N-n+1) . \qquad (1\text{-}4\text{-}31)$$

Therefore if we sum over α in (1-4-28) we yield the expression

$$\sum_{\alpha} \left(\frac{2\alpha+1}{2\alpha_0+1}\right) S(n, \alpha \rightarrow n-1, \alpha_0) = (N-n+1) \quad . \quad (1\text{-}4\text{-}32)$$

We note that the right-hand side of this equation gives an expectation value of the number of proton holes in the p shell of the target nucleus A.

We now generalize from the jj to the jjT representation by simply giving the expected result. One is referred to the articles of Macfarlane and French[10,24] for justification of the result.

The expression is similar to (1-4-32), we only include the isospin coupling Clebsch-Gordan coefficient derived precedingly in the section (1-2),

$$\sum \left(\frac{2J_B+1}{2J_A+1}\right) C^2S(\ell j) = \langle \text{proton holes} \rangle_{\ell j} \quad . \quad (1\text{-}4\text{-}33)$$

The summation extends over all levels of the residual nucleus we wish to consider and include only one value of ℓ and j (if we make use of spin-orbit coupling in our DW calculations). From the equation (1-2-98) one immediately has

$$\sum \frac{(2j+1)}{4.42} \frac{2}{(2s+1)} \frac{\frac{d\sigma_{\ell sj}}{dw}}{\sigma_{DW}(\theta)} = \langle \text{proton holes} \rangle_{\ell j} \quad .$$

(1-4-34)

CHAPTER II

ENERGY LEVELS AND PROTON HOLES

2-1 Energy Levels of ^{70}Ge and Experimental Cross-Sections

The data for the ^{69}Ga (^{3}He, d) ^{70}Ge reaction were taken using a 22.5 MeV helion beam from the McMaster University FN Tandem Van de Graaff. The reaction product deuterons were analysed with a magnetic spectrograph and detected with photographic emulsions at several angles between 8 and 55 degrees. The spectrum at each angle was obtained from counting the number of tracks on the plate as a function of the plate position. The scanning of the nuclear emulsions was done at intervals of 0.25 mm at the Instytut Badan Jadhowych.

The results were afterwards supplied as data to the peak fitting program SPECTRA VII written by Von Egily and further modified by R. O'Neil. This program fits gaussians with exponential tails on the experimental data. Knowing the energy of the incident beam, the Q-value of the reaction which was 3.027 MeV and the calibration of the spectrograph, SPECTRA VII extracts, with relativistic kinematics, the excitation energy of the ^{70}Ge nucleus corresponding to each peak. It also integrates the surface under each fitted peak which allows us to calculate directly the experimental cross-sections $\frac{d\sigma_{lsj}}{dw}$ in the centre of mass system.

The shape of the fitting curve is given by

$$y = H \left[\exp - \left[\frac{(x-x_o)^2 \; 4 \; \ln 2}{G^2} \right] + S \exp \left[\frac{(x-x_o-GG) \; \ln 2}{A} \right] \right.$$

$$\left. \left[1 - \exp - \left[\frac{(x-x_o)^2 \; 4 \; \ln 2}{GG^2} \right] \right] \right] \qquad (2\text{-}1\text{-}1)$$

Where H is the height of the peak, the first term is a gaussian centered at x_o and of FWHM given by G. The second term is an exponential tail which has a decay rate such that it falls to half of its height in a distance A. The exponential tail is multiplied by a reversed gaussian so that there is no contribution from the exponential at x_o. In the figures 4, 5 and 6, one can notice the effects of the different parameters appearing in the equation (2-1-1).

The fitting parameters for a whole spectrum are usually obtained from a best least squares fit on the ground state peak. One has to approximate values of the parameters H, x_o, G, A, S and GG. The program SPECTRA VII then searches for better parameters in order to minimize the square of the deviation from the experimental values. This operation must be repeated quite often before obtaining satisfying results since many combinations of the parameters may yield local minimas in the square deviation function. An example of such a fit on a ground state peak is shown in figure 7.

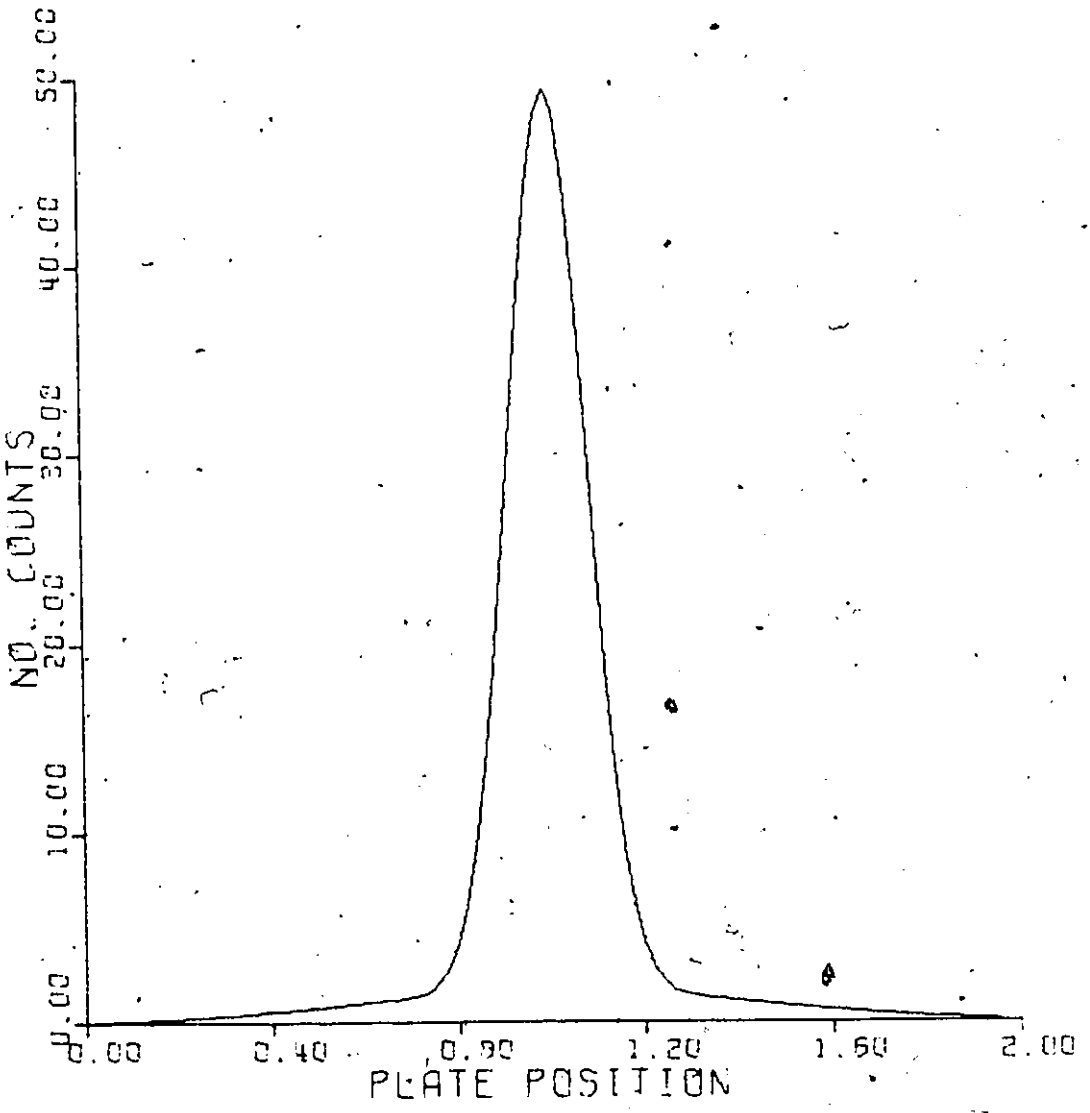


Fig. 4. Fitting curve with G=0.2, S=0.04, A=0.4 and GG=0.3.

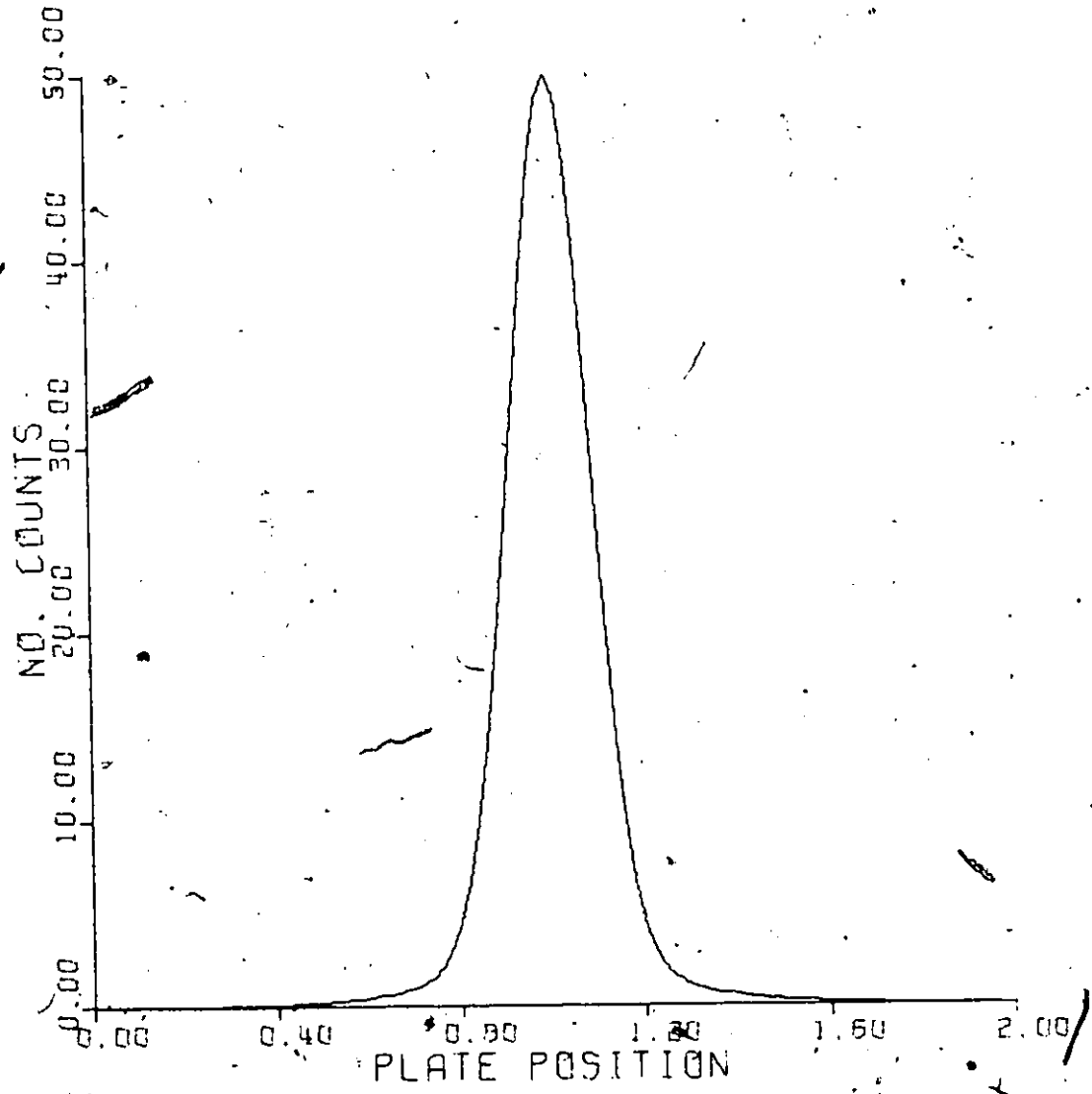


Fig. 5. Fitting curve with G=0.2, S=0.02, A=0.1 and GG=0.3.

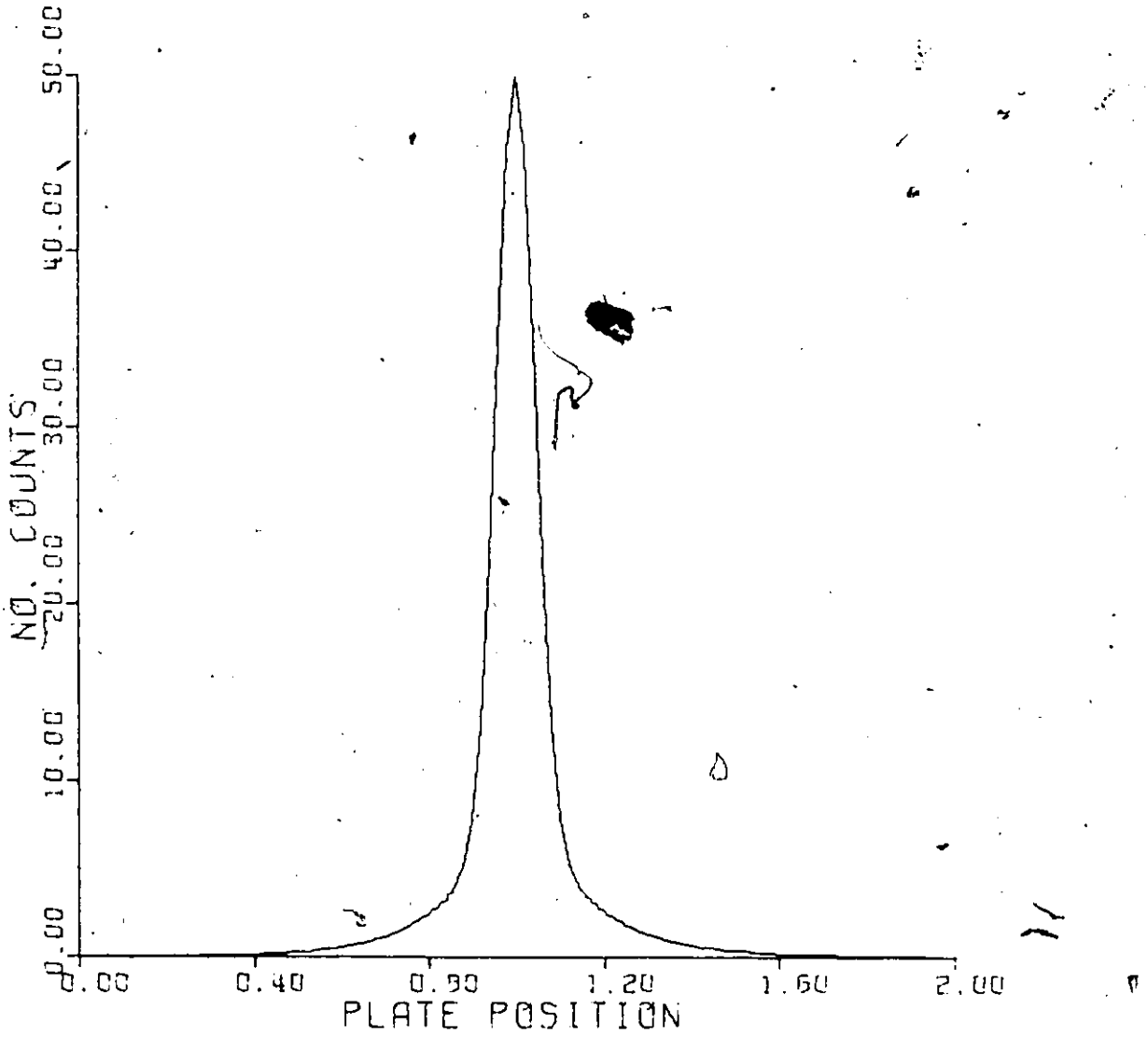


Fig. 6. Fitting curve with G=0.1, S=0.1, A=0.1 and GG=0.1.

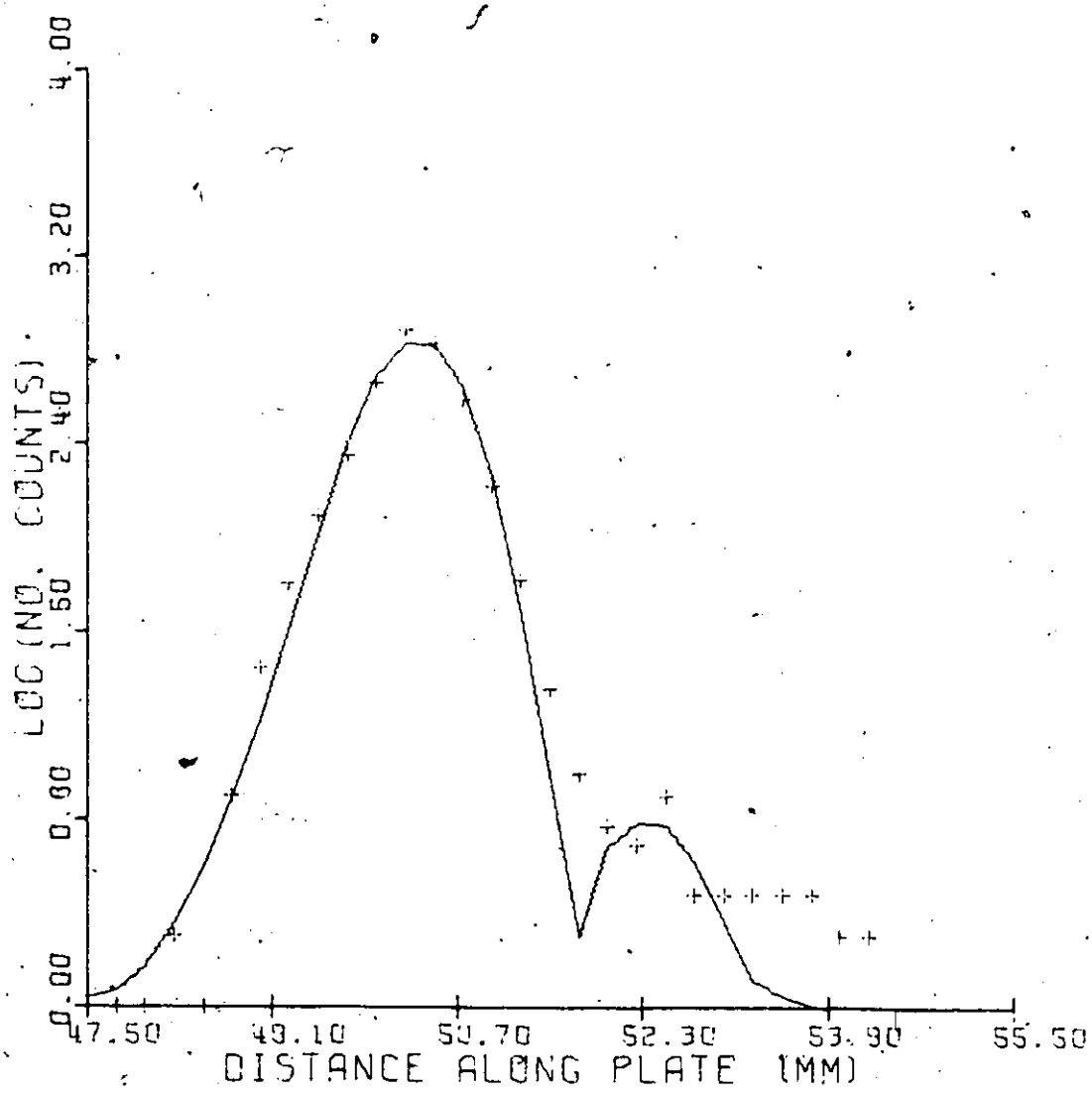
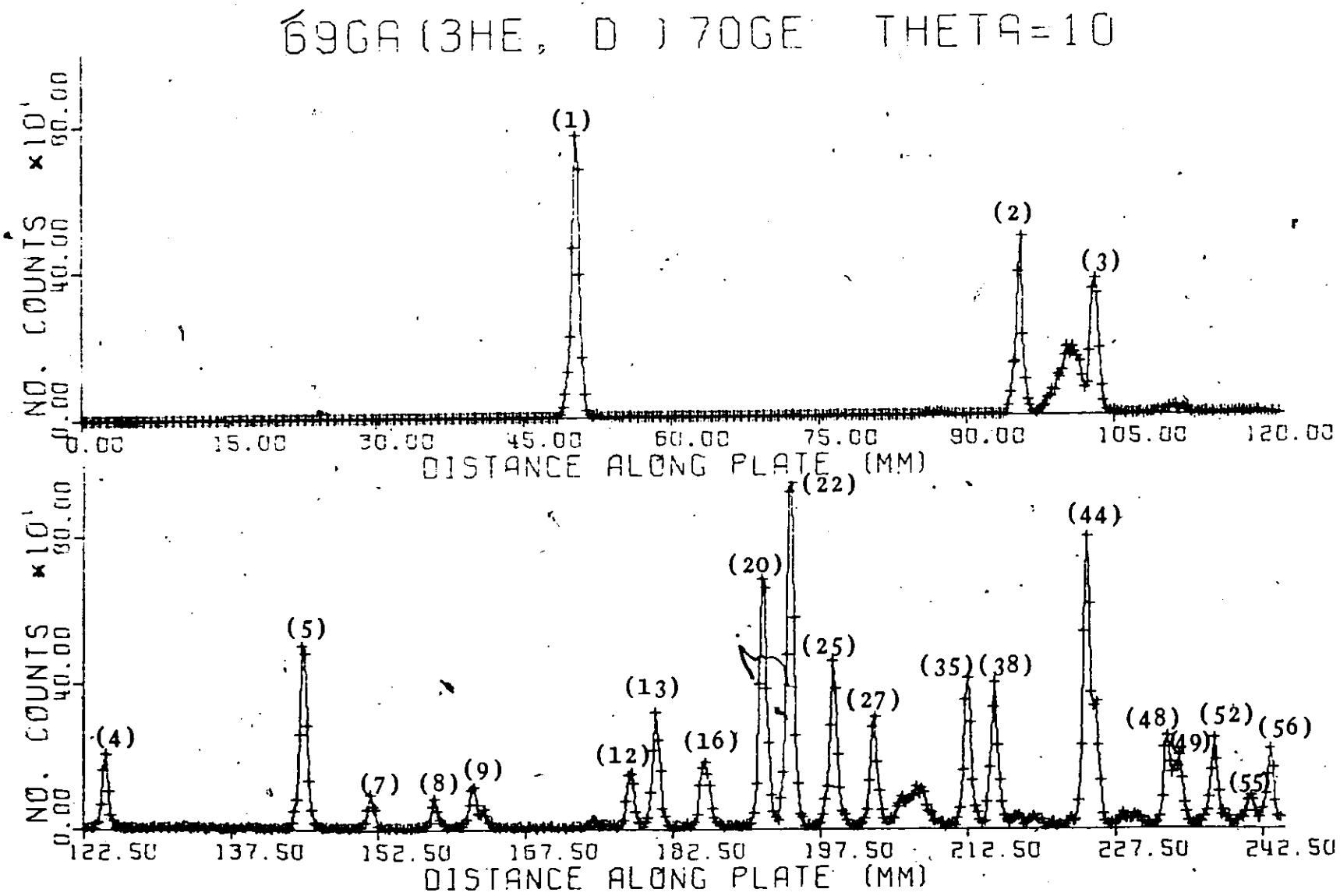


Fig. 7. Fit on the ground state peak at 10° (lab). The + indicate the data points and the continuous line is the fitting curve.

Fig. 8a)


69GA(3HE, D)70GE THETA=10
NO. COUNTS ×10¹
80.00
40.00
0.00
0.00
15.00
30.00
45.00
60.00
75.00
90.00
105.00
120.00
DISTANCE ALONG PLATE (MM)
(1)
(2)
(3)
NO. COUNTS ×10¹
80.00
40.00
0.00
122.50
137.50
152.50
167.50
182.50
197.50
212.50
227.50
242.50
DISTANCE ALONG PLATE (MM)
(4)
(5)
(7)
(8)
(9)
(12)
(13)
(16)
(20)
(22)
(25)
(27)
(35)
(38)
(44)
(48)
(49)
(52)
(55)
(56)

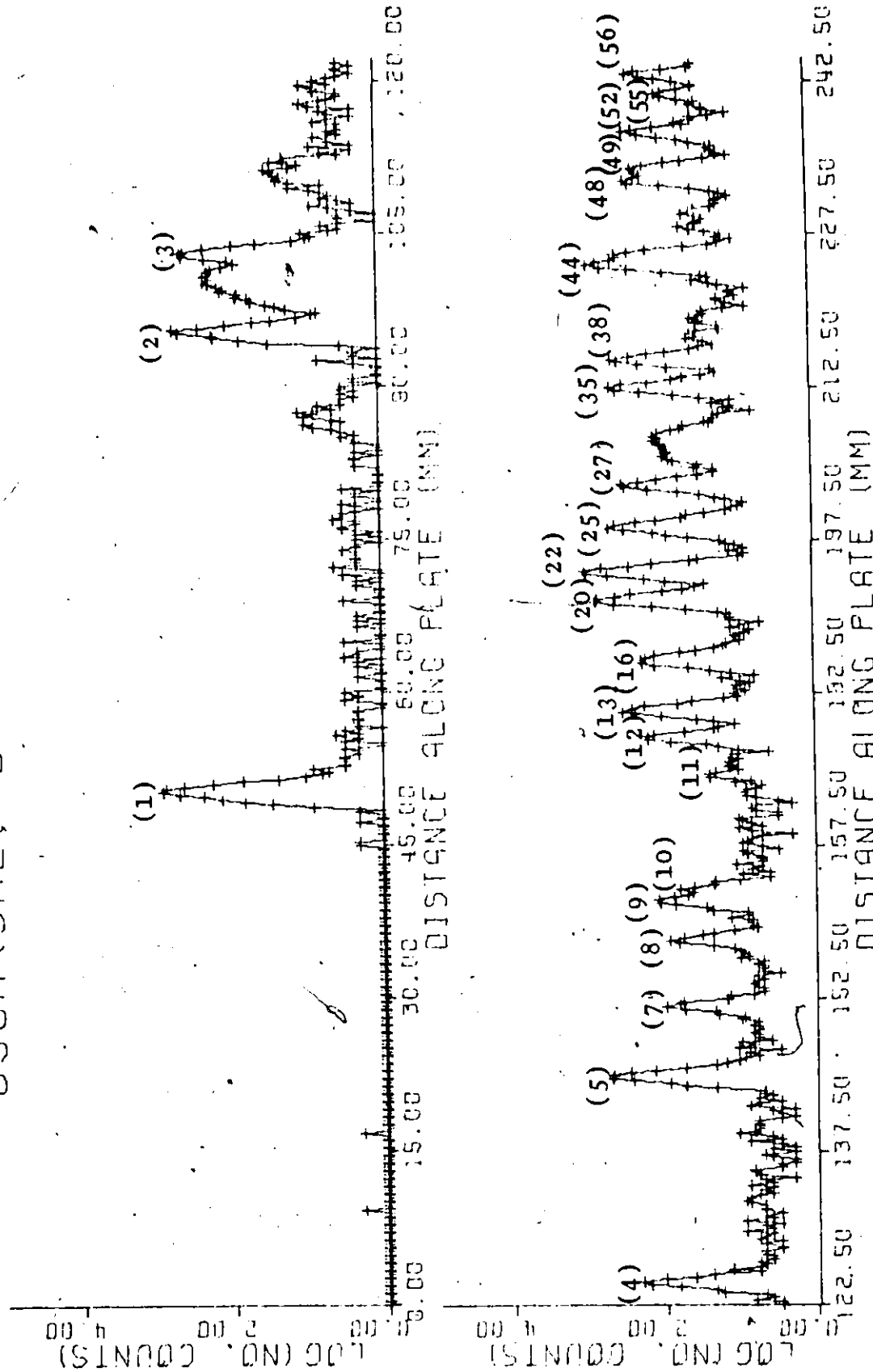

69GA(3HE, D)70GE ,THETA=10
LOG (NO. COUNTS)
DISTANCE ALONG PLATE (MM)
(1)
(2)
(3)
(4)
(5)
(7)
(8)
(9)
(10)
(11)
(12)
(13)
(16)
(20)
(22)
(25)
(27)
(35)
(38)
(44)
(48)
(49)
(52)
(55)
(56)

Fig. 8b)

A typical spectrum obtained at 10 degrees (laboratory system) is displayed on a linear scale in figure 8a) and on a semi-logarithmic scale in figure 8b). The number over each peaks correspond with the peak numbers in table 1.

The excitation energy of the ^{70}Ge nucleus for each peak has been averaged out over all the measured angles. The results are summarized in table 1, where we compare our results with the ones obtained by G. Brown et al.[25] and P. F. Hinrichsen et al[32] through the ^{70}Ge (p,p') ^{70}Ge reaction. Such a reaction excites most of the levels. We have identified most of the levels reported by G. Brown and P. F. Hinrichsen and the agreement is generaly satisfactory. Several new levels were observed but these were weakly excited.

As stated previously, SPECTRA VII integrates the surface under the fitted peaks. To extract the experimental cross-sections from these quantities, we must do a solid angle correction. Let us consider the figure 9.

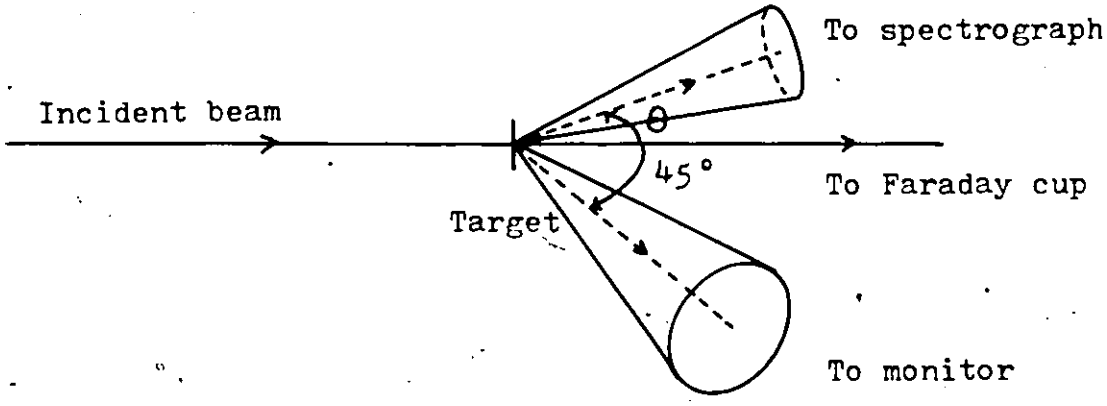


Fig. 9.

TABLE 1

ENERGY LEVELS OF ^{70}Ge

Peak	Present work Energy(MeV)	$(p,p')^{a)}$ Energy(MeV)	$(p,p')^{b)}$ Energy(MeV)
1	0.000	0.000	0.000
2	1.041	1.039	1.041
3	1.217	1.215	1.218
4	1.709	1.707	1.711
		2.156	2.155
5	2.159	2.160	
6	2.185^{W}		
7	2.309	2.310	
8	2.455	2.454	2.450
9	2.539	2.538	2.537
10	2.565	2.565	2.562
11	2.811	2.809	2.807
12	2.892	2.890	2.887^{W}
13	2.950	2.948	2.945
14	2.973^{W}		
15	3.015^{W}		
16	3.054	3.049	3.053
17	3.066^{W}	3.062	
18	3.086^{W}		
19	3.102^{W}	3.109	3.107
20	3.187	3.182	3.182
		3.195	3.194
21	3.214^{W}		
22	3.246	3.242	3.242
23	3.272^{W}		
		3.296	3.297
24	3.318	3.316	3.315
		3.336	3.334

ENERGY LEVELS OF ^{70}Ge (con't)

Peak	Present work Energy(MeV)	$(p,p')^{a)}$ Energy(MeV)	$(p,p')^{b)}$ Energy(MeV)
25	3.342	3.345	
		3.351	
26	3.365^{W}		
		3.419	3.149
27	3.430	3.428	3.429
		3.432	
28	3.456	3.456	3.458
29	3.466^{W}		
		3.483	
30	3.488	3.489	3.488
31	3.517^{W}		
32	3.563^{W}	3.563	3.563
33	3.573^{W}	3.570	
		3.581	3.584
34	3.592^{W}	3.593	
35	3.638	3.633	3.632
36	3.660^{W}	3.667	
37	3.672	3.678	3.673
38	3.694	3.691	3.685
39	3.716	3.710	3.712
40	3.745	3.740	3.740
		3.777	3.777
41	3.783	3.783	
		3.850	
		3.857	3.855
42	3.866	3.871	3.866
43	3.898	3.891	3.892
		3.904	3.902
44	3.916	3.911	
		3.928	3.929
			3.959

ENERGY LEVELS OF ^{70}Ge (con't)

Peak	Present work Energy(MeV)	$(p,p')^{a)}$ Energy(MeV)	$(p,p')^{b)}$ Energy(MeV)
45	3.975	3.976	3.975
		3.990	
46	4.008	4.003	4.002
47	4.042	4.037	4.038
		4.054	4.058
48	4.071	4.062	4.068
49	4.097	4.097	4.097
		4.107	
50	4.116	4.119	4.119
		4.132	4.131
51	4.147	4.144	4.146
		4.153	
		4.166	4.160
52	4.173		4.176
53	4.194		4.196
			4.219
54	4.228		4.227
			4.242
55	4.253		
			4.261
			4.282
56	4.293		
			4.334

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a) P. F. Hinrichsen et al, Nucl. Phys. A123 (1968) 250

b) G. Brown et al, Nucl. Phys. A101 (1967) 163

w indicates weakly exited

Since our problem is φ independent, the differential cross-section is simply the ratio of the number of scattered particles in the direction θ over the number of incident particles N_o per unit of solid angle. The differential cross-section for elastic scattering at 45 degrees into the monitor is given by

$$\sigma_M = \frac{N_M}{N_o \, \Omega_{M.}} \quad , \qquad (2\text{-}1\text{-}2)$$

where N_M is the number of counts in the monitor and Ω_M is the solid angle subtended by the monitor. In the same way we have

$$\frac{d\sigma_{lsj}}{dw} = \frac{N_{peak}}{N_o \, \Omega_{spectr.}} \quad , \qquad (2\text{-}1\text{-}3)$$

where N_{peak} is the number of counts under the peak fitted by SPECTRA VII and $\Omega_{spectr.}$ is the solid angle subtended by the spectrograph.

The relation between the differential cross-section in the laboratory system and the centre of mass system is given by[4)]

$$\sigma_{lab} = \frac{(1 + \gamma^2 + 2\gamma \cos\theta_{C.M.})^{3/2}}{|1 + \gamma \cos\theta_{C.M.}|} \, \sigma_{C.M.} \quad , \qquad (2\text{-}1\text{-}4)$$

where

$$\gamma = \frac{\text{mass of the helion nucleus}}{\text{mass of the target}}$$

Since this ratio is small, we put in our calculation $\sigma_{M_{lab}} \propto \sigma_{M_{C.M.}}$. Then from the equations (2-1-2 and 3), we have

$$\frac{d\sigma_{\ell sj}}{dw} \quad \frac{\Omega_M}{\Omega_{spectr.}} \frac{N_{peak}}{N_M} \sigma_M \quad . \qquad (2\text{-}1\text{-}5)$$

The differential cross-section for elastic scattering of the helion ions into the monitor is calculated by the DW program DWUCK (see section 2-2). The solid angle of the monitor was 0.8804×10^{-4} sr and for the spectrograph 1.950×10^{-3} sr. The table 2 gives the experimental cross-sections in the centre of mass system for each peak found at the different measured angles.

TABLE 2

ENERGY KeV	$\left(\frac{d\sigma}{d\Omega}\right)_{C.M.}$ μb/sr													
	8.3°	10.3°	12.4°	14.5°	16.5°	18.6°	20.7°	22.7°	24.9°	26.9°	28.9°	31.0°	51.5°	56.6°
Ground State	1876.	1774.	1269.	755.	385.	286.	674.	302.	483.	618.	412.	286.	70.	71.
1041.	1343.	901.	915.	591.	302.	218.	449.	211.	323.	337.	258.	196.	34.	44.
1217.	1057.	998.	1189.	699.	237.	169.	331.	143.	307.	295.	234.	278.	37.	38.
1709.	554.	359.	415.	285.	153.	111.	219.	107.	191.	191.	136.	96.	11.	18.
2159.	1175..	1151.	873.	580.	348.	308.	470.		390.	367.	286.	227.	46.	48.
2185.			25.			6.	3.		12.	2.	3.	5.		
2309.	161.	179.	142.	98.	40.	30.	50.	34.	47.	55.	45.	32.	6.	7.
2455.	105.	154.	156.	168.	185.	226.	290.	182.	187.	145.	106.	88.	13.	28.
2539.	223.	247.	215.	165.	132.	135.	217.	103.	133.	134.	85.	63.	22.	11.
2565.	70.	100.	103.	108.	119.	113.	131.	79.	117.	101.	94.	92.	23.	16
2811.	21.	36.	49.	73.	91.	144.	141.	92.	90.	68.	51.	39.	12.	14.
2892.	362.	337.	215.	187.	83.	76.	116.	48.	70.	72.	93.	73.	16.	12.
2950.	624.	673.	594.	449.	428.	480.	573.	333.	412.	350.	260.	204.	60.	47.
2973.		14.			6.	10.	18.	10.	13.					
3015.		8.			6.						2.		3.	
3054.	154.	375.	570.			821.	590.	727.	513.	549.	444.	304.	164.	179.
3066.	214.	124.	130.			72.			299.	301.	165.	197.		
3086.		64.				20.	42.					5.		
3102.				12.	46.			11.		5.		2.		

TABLE 2 (continued)

ENERGY KeV	$\left(\frac{d\sigma}{d\Omega}\right)_{C.M.}$ μb/sr													
	8.3°	10.3°	12.4°	14.5°	16.5°	18.5°	20.5°	22.7°	24.9°	26.9°	28.9°	31.0°	51.5°	56.6°
3187.	1512.	1436.	1234.	888.	453.	463.	422.	327.	485.		428.	337.	82.	15.
3214.	101.	79.	42.		82.	48.		45.	76.		36.			
3246.	2059.	2077.	1631.	1176.	687.	532.	817.	444.	710.	510.	590.	453.	103.	67.
3272,	106.		21.		20.	19.		100.	28.	9.	12.	7.	11.	
3318.			47.	46.	57.	58.	27.		26.	20.	22.	23.		4.
3342.	986.	960.	738.	552.	293.	210.	330.		284.	335.	278.	231.	46.	35.
3365.		60.			17.	12.	22.	8.	20.	8.	14.	8.		
3430.	650.	641.	523.	374.	200.	163.	278.	146.	247.	275.	219.	187.	45.	8.
3456.		30.				28.	34.	21.	24.	26.	23.	17.		
3466.	132.	31.			19.	32.	17.	10.	7.					
3488.	292.	189.	81.	74.	36.	39.	57.	27.	39.	43.	36.	25.	10.	
3517.	154.	204.	83.								3.			5.
3563.		49.	137.	36.					5.	17.	6.			
3573.					28.	22.	38.	23.	12.		13.	14.		
3592.			133.			19.		9.	7.	9.	5.			5.
3638.	829.	845.	731.	642.	323.	184.	268.	136.	256.	320.	246.	193.	34.	
3660.		36.	38.								22.	11.		30.
3672.				26.	71.	69.	75.	34.	19.	20.	12.			
3694.	884.	793.	652.	522.	321.	280.	336	204.	286.	332.	260.	194.	50.	10.

TABLE 2 (continued)

ENERGY KeV	$\left(\frac{d\sigma}{d\Omega}\right)_{C.M.}$ μb/sr													
	8.3°	10.3°	12.4°	14.5°	16.5°	18.5°	20.5°	22.7°	24.9°	26.9°	28.9°	31.0°	51.5°	56.6°
3716.		42.	22.		64.	58.	29.	29.	31.	8.	14.	8.		33.
3745.	56.	77.	67.	41.	19.	58.	36.	29.	30.	39.	28.	22.		
3783.	24.	42.	61.	62.	40.	59.	39.	47.	32.	39.	36.	30.		
3866.			31.	28.	41.	59.	74.	55.	51.		35.	27.	17.	8.
3898.	1649.	1520.	1291.	1143.	624.	476.	691.	363.	573.		491.	393.	124.	
3916.	583.	605.	466.		264.	220.	285.	216.	259.		208.	161.		97.
3975.		73.	73.	62.	47.	61.	110.	46.	77.	87.	76.	54.	15.	
4008.	55.	73.	57.	57.	46.	60.	65.	43.	46.	50.	50.	38.	11.	13.
4042.			15.			10.			11.		9.	10.		
4071.	555.	505.	480.	343.	155.	159.	188.	100.	145.	214.	181.	147.	32.	13.
4097.	336.	442.	324.	219.	127.	151.	157.	36.	153.	222.	159.	122.	24.	25.
4116.					22.		13.		14.		20.	82.		
4147.			28.	42.	34.	37.	61.	50.	52.	48.	49.	44.	11.	
4173.	491.	485.	373.	291.	105.	107.	163.	87.	162.	174.	156.	118.	22.	10.
4194.		52.	37.		72.	16.	13.		17.	24.	33.	14.		15.
4228.		28.	23.	5.	11.	39.	33.	30.	26.	25.	29.	17.	7.	
4253.	173.	192.	170.	102.	90.	73.	83.	45.	61.	78.	86.	26.	14.	6.
4293.	379.	434.	333.	282.	79.	145.	117.	53.	150.	172.	113.	118.	28.	7.

2-2 Optical-Model Parameters

The optical-model potential parameters from which the distorted waves in the incoming and outgoing channels are calculated, are determined from the appropriate elastic scattering analysis[26,27,28]. The general form of the potential used is a sum of the following potentials:

1 - Central potential

real part $\quad - V_s \, f(r, r_{os}, a_s)$

imaginary part $\quad - W_s \, f(r, r_{oI}, a_I) + 4a_I W_D \frac{d}{dr} f(r, r_{oI}, a_I)$

2 - Spin-orbit potential

real part $\quad \left[\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{V_{soR}}{r} \frac{d}{dr} f(r, r_{os}, a_s)\right] \underline{L}\cdot\underline{S}$

imaginary part $\quad \left[\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{V_{soI}}{r} \frac{d}{dr} f(r, r_{oI}, a_I)\right] \underline{L}\cdot\underline{S}$

3 - Coulomb potential

Preceding works[8,9,23,26,27,28] have used a uniform charge distribution and calculated the Coulomb potential from it. The effect of the more realistic Fermi charge distribution, using the parameters in the table 3, was to lower the DW cross-sections by less than 2 %.

The function $f(r,r_o,a)$ is the usual Saxon form factor

$$f(r,r_o,a) = \left[1 + \exp\left[\frac{r-r_o A^{1/3}}{a}\right]\right]^{-1},$$

where A is the mass number. The imaginary part is composed of a volume part W_s and a surface part W_D, with the factor $4a_I$ being introduced so that the surface form factor $4a_I\left(\frac{d}{dr}f\right)$ has unity for its maximum value.

The different potential parameters used are presented in table 3.

As pointed out by P. E. Hogson[30], the inclusion of a spin-orbit potential mostly affects the cross-section in the backward direction ($\theta > 50$). Since our data are concentrated at forward directions, it was not necessary to include such a potential in our calculations and it was only used to calculate the proton bound state wave function $\phi^A_{\ell j}$ (equation 1-3-4).

TABLE 3

Optical-model parameters used in the entrance and exit channels

Particle	V_s (MeV)	W_s (MeV)	W_D (MeV)	V_{SOR} (MeV)	V_{SOI} (MeV)	r_{os} (fm)	a_s (fm)	r_c (fm)	a_c (fm)	r_{oI} (fm)	a_I (fm)
^3He [a]	170.	17.5	0.	0.	0.	1.14	0.723	1.4	0.723	1.6	0.81
d [b]	86.	0.	82.	0.	0.	1.15	0.81	1.15	0.859	1.34	0.68
p [c]	d)	0.	0.	8.0	0.	1.20	0.65	1.25	0.65	0.	0.

a) R. H. Bassel and R. N. Drisko, Proceeding of the Symposium on Direct Reaction with ^3He, IPCR Cyclotron Progress Report, Supplement 1, 1967.

b) G. M. Perey and F. G. Perey, Phys. Rev. 132, 755 (1963).

c) F. G. Perey, Phys. Rev. 131, 745 (1963).

d) adjusted to reproduce the binging energy of the transferred proton.

2-3 Distorted Wave Analysis, Spectroscopic Strengths and Proton Holes in ^{69}Ga

By comparing the angular distribution of each level with the predictions of the DWBA theory, we were able to determine the transferred orbital angular momentum and the spectroscopic strength of several excited levels of ^{70}Ge.

The range of the spin of the final state in the residual nucleus may be obtained from the vector sum of the initial spin of the target and the transferred angular momentum $\underline{j} = \underline{\ell} + \underline{s}$:

$$\left| J_A - j \right| \leq J_B \leq J_A + j \quad , \qquad (2\text{-}3\text{-}1)$$

The ^{69}Ga nucleus has 31 protons and 38 neutrons. In the single-particle shell model, the neutrons can be considered as part of the core and thus contribute in the reaction only by their total effect. In this model, all the proton levels are filled up to the $2p_{3/2}$ subshell which has a hole in it.

From the figure 10, we can expect to populate the $2p_{3/2}$, $1f_{5/2}$ and $2p_{3/2}$ subshells, corresponding to a transferred proton with orbital angular momentum $\ell = 1, 3$ and 1 respectively.

The DWBA calculations were made using the computer

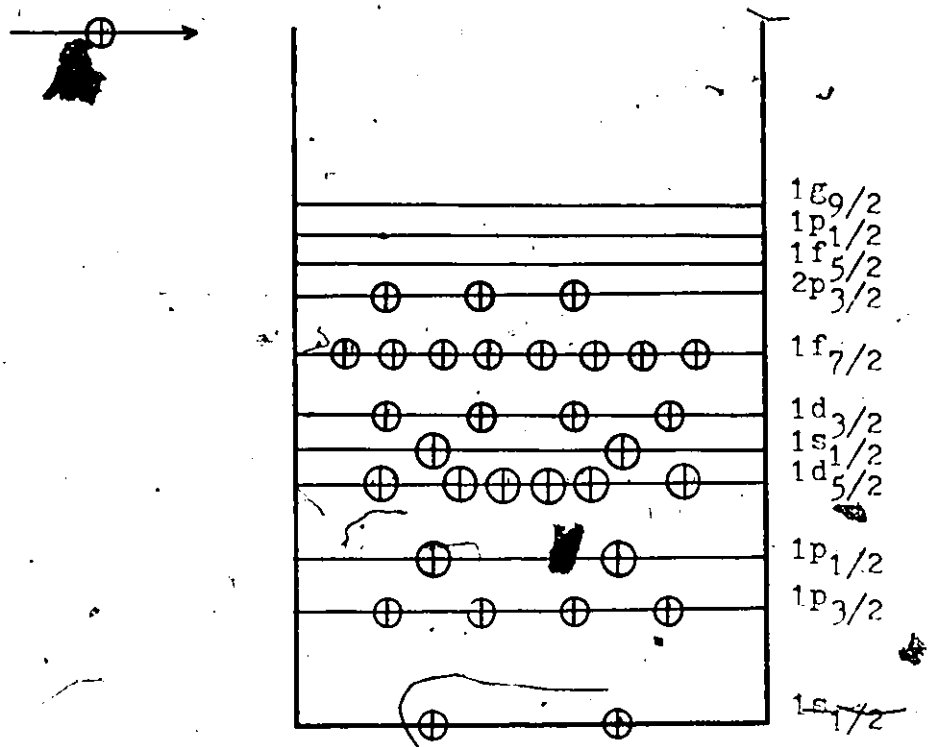


Fig. 10. Proton levels of ^{69}Ga in the single-particle model.

code DWUCK for Q-values of 3.027 to -2.027 MeV at intervals of 1 MeV, corresponding to excited levels of ^{70}Ge from 0 to 5 MeV. The relative shapes and magnitudes of the predicted cross-sections are shown in figure 11 for various ℓ-values, the potential parameters of table 3 and a Q-value of 2.027 MeV.

The computed angular distributions fitting the experimental data are shown in the figures 12a) and 12b). Since the ground state spin and parity of ^{69}Ga and ^{70}Ge are $3/2^-$ and 0^+ respectively, the only possible ℓ- and j-

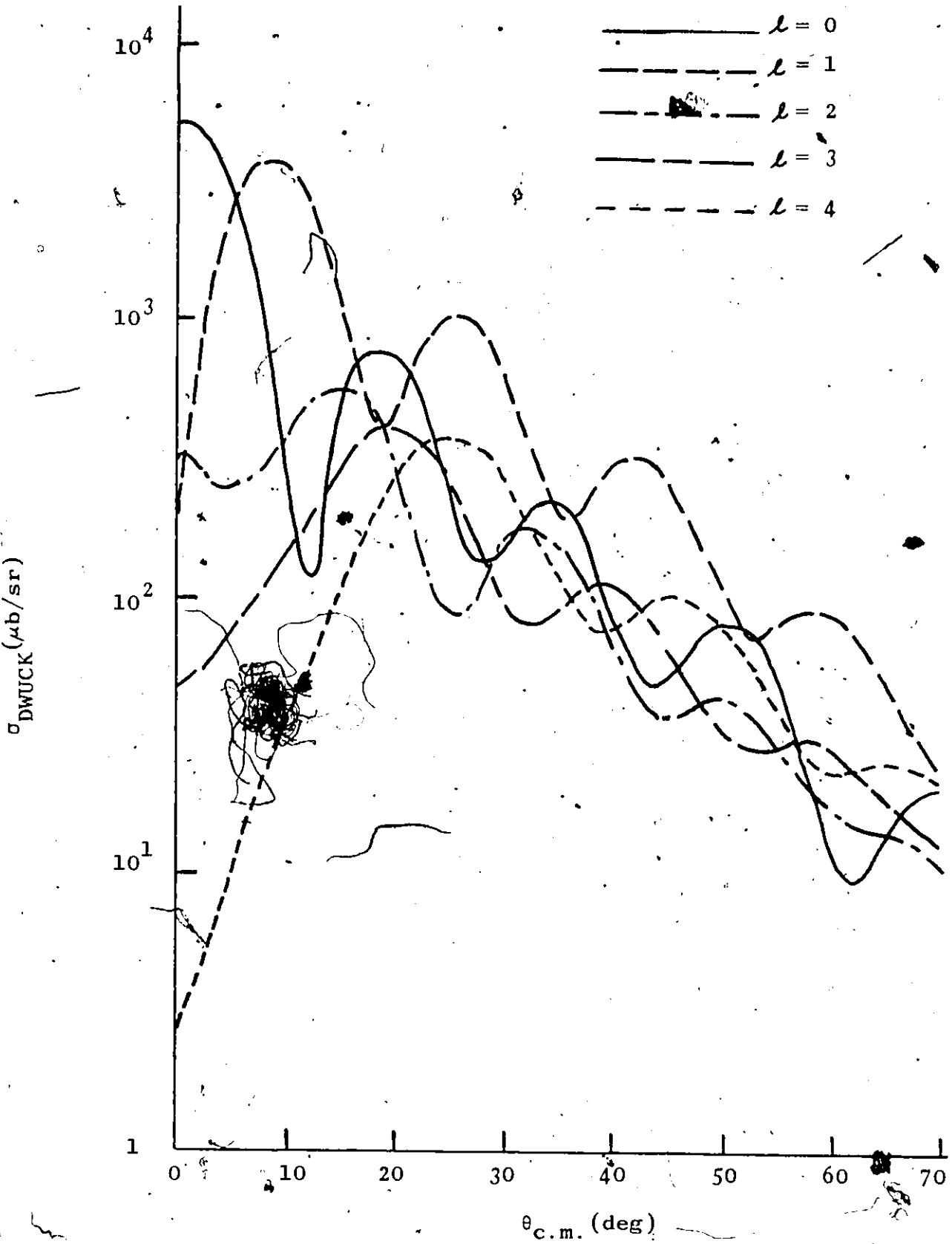


Fig. 11. A comparison of some of the shapes and magnitudes predicted by the DW code DWUCK.

values contributing to the formation of the ground state of ^{70}Ge are $\ell = 1$ and $j = 3/2$; which was observed on the curve No. 1 of figure 12a).

Several energy levels corresponding to $\ell = 1$ and $\ell = 3$ transfers were identified. By combining the DWBA predictions for $\ell = 1$ and $\ell = 3$ transfers, we were able to identify mixed levels. Such a mixture is shown in figure 13.

The relation between the predicted and experimental cross-sections is given by the equation (1-2-95). For energy levels corresponding to intermediate Q-values, the DWBA cross-sections were extracted directly by interpolation on a linear plot. The spectroscopic strengths of each level can be calculated from the equations (1-2-97 and 98)

$$\left[\frac{2J_B+1}{2J_A+1}\right] C^2S(\ell_j) = \frac{2j+1}{4.42} \frac{2}{2s+1} \frac{\frac{d\sigma_{\ell sj}}{dw}}{\sigma_{DWUCK}(\theta)}$$

where the right-hand side of this equation is the spectroscopic strength as defined in section 1-4.

The total transition strength associated with a given single-particle state is then the sum of the strengths of all the levels which have been identified as resulting from the transfer of a proton in this state (ℓ_j). The sum of the spectroscopic strengths for a given set ℓ_j, may be compared with the limit obtained from the sum rule (1-4-34):

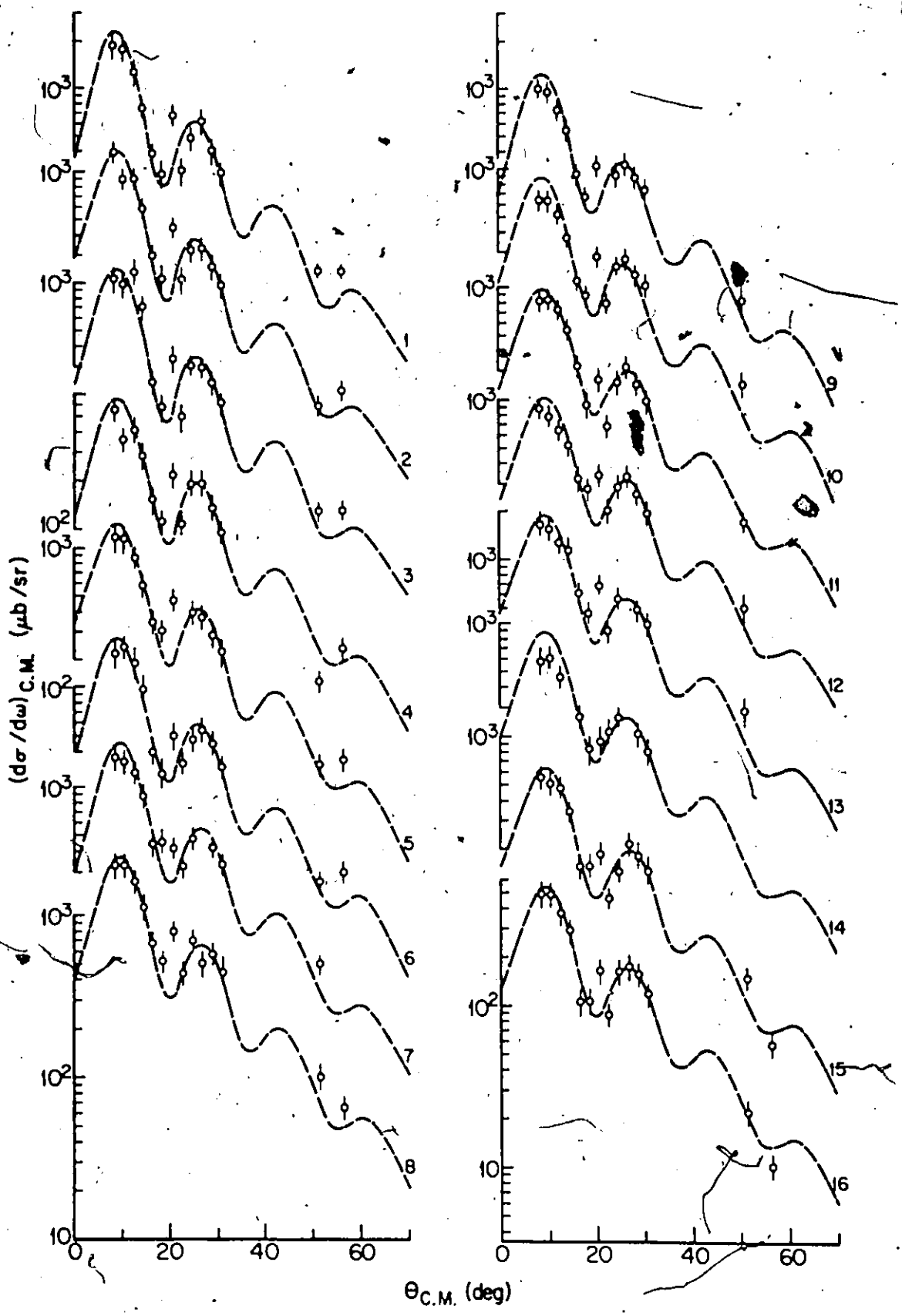


Fig. 12a). The angular distributions compared with the DW predictions for $\ell = 1$ transfers.

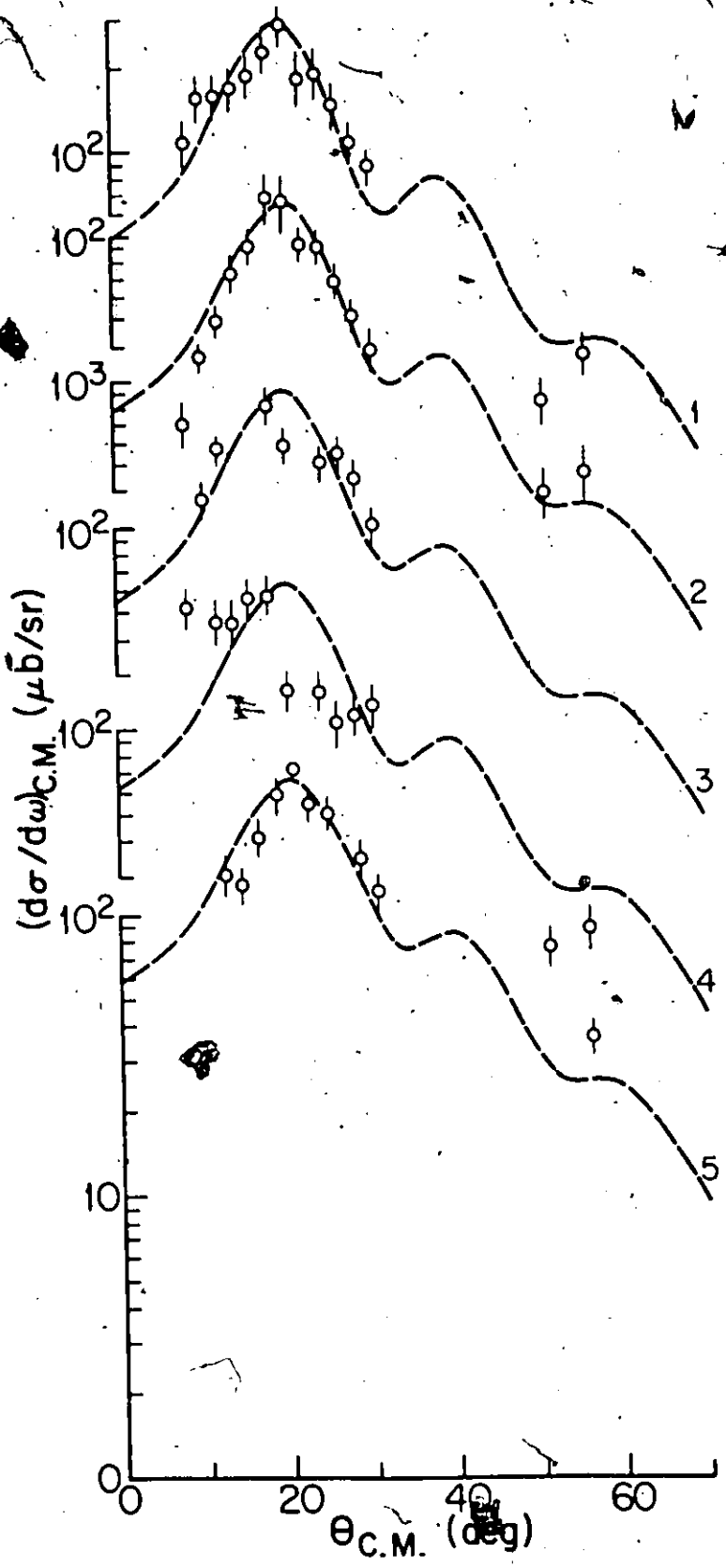


Fig. 12b). The angular distributions compared with the DW predictions for ℓ = 3 transfers.

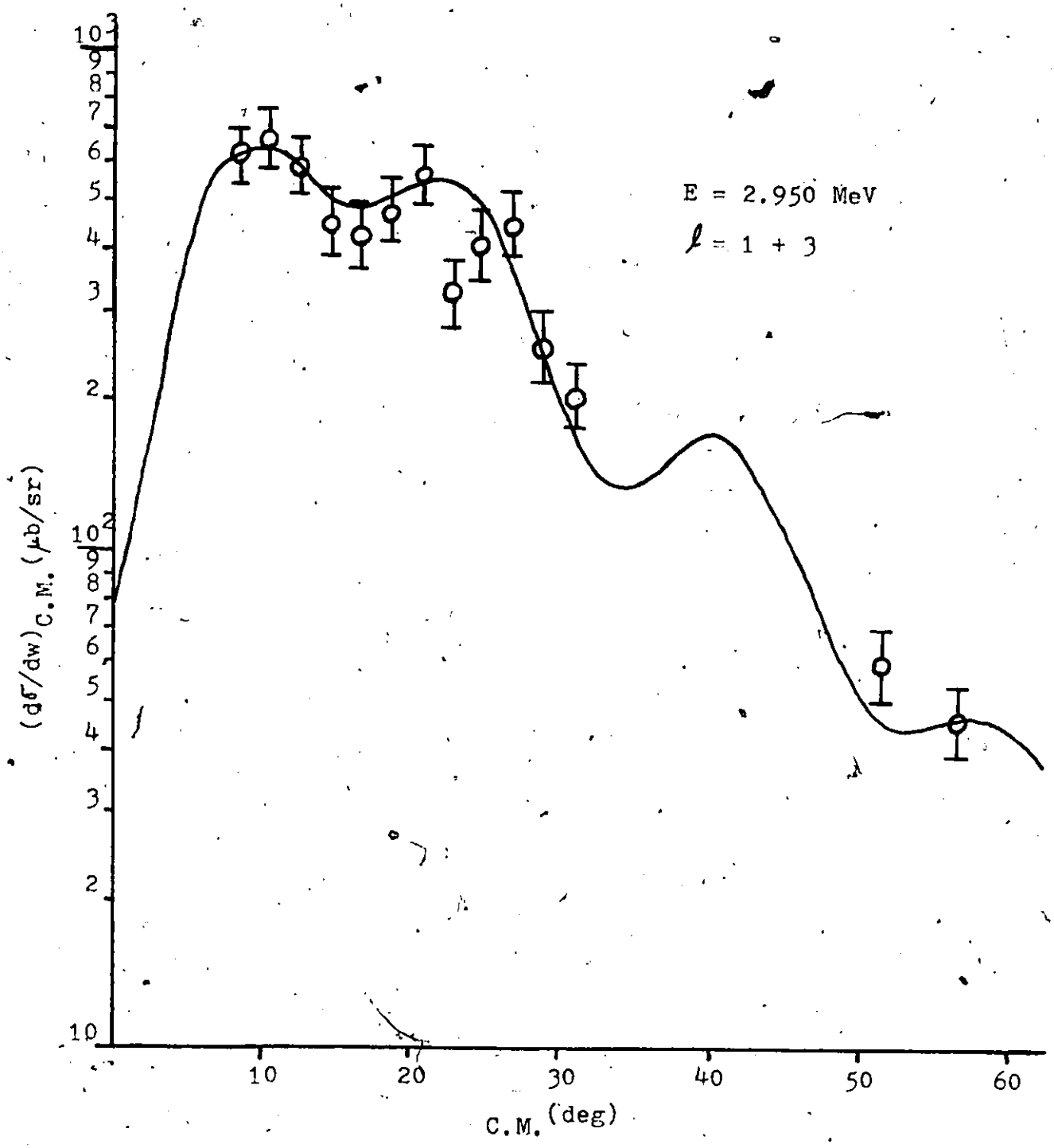


Fig. 13. The fitting curve was obtained from $0.1 \times \sigma_{DWUCK}$ for $\ell = 1$ + $0.9 \times \sigma_{DWUCK}$ for $\ell = 3$.

$$\sum \frac{2j+1}{4.42} \quad \frac{2}{2s+1} \quad \frac{\frac{d\sigma_{\ell sj}}{dw}}{\sigma_{DWUCK}(\theta)} = \langle \text{proton holes} \rangle \ell_j \qquad (2\text{-}3\text{-}2)$$

The calculations are summarized in table 4 and 5.

TABLE 4

Spectroscopic Strengths

	E_x(MeV)	$\ell(j)^a$	$\frac{2J_B+1}{2J_A+1} C^2S$
(1)	0.000	1(3/2)	0.537
(2)	1.041	1(3/2)	0.246
(3)	1.217	1(3/2)	0.226
(4)	1.709	1(3/2)	0.096
(5)	2.159	1(3/2)	0.229
(6)	2.309	1(3/2)	0.033
(1)	2.455	3(5/2)	0.785
		1(1/2)	0.025
	2.539	+ 3(5/2)	0.305
(2)	2.811	3(5/2)	0.284
	2.892	1(1/2)	0.061
		1(1/2)	0.068
	2.950	+ 3(5/2)	1.185
(3)	3.054	3(5/2)	2.408
(7)	3.187	1(1/2)	0.266
	3.214	0?	
(8)	3.246	1(1/2)	0.350
(4)	3.318	3(5/2)	0.162
(9)	3.342	1(1/2)	0.175
(10)	3.430	1(1/2)	0.139
	3.488	1(1/2)	0.021
	3.573	3(5/2)	0.091
(11)	3.638	1(1/2)	0.136
(12)	3.694	1(1/2)	0.131
	3.745	2?	
(5)	3.866	3(5/2)	0.166
(13)	3.898	1(1/2)	0.249
(14)	3.916	1(1/2)	0.101
	4.008	1(1/2)	0.013
		+ 3(5/2)	0.357

TABLE 4 (continued)

Spectroscopic Strengths

	E_x(MeV).	$\ell(j)^a$	$\frac{2J_B+1}{2J_A+1} C^2S$
(15)	4.071	1(1/2)	0.086
	4.097	1(1/2)	0.073
	4.147	4?	
(16)	4.173	1(1/2)	0.074
	4.253	1(1/2)	0.075
	4.293	1(1/2)	0.073

the j-value assumed in the calculations are given in parenthesis.

TABLE 5

Proton Holes

Single Particle State	Total Transition Strength	Expected Total
$2p_{3/2}$	1.367	1
$1f_{5/2}$	5.743	6
$2p_{1/2}$	2.116	2

2-4 Discussion and Conclusion

The results of the investigation of the ^{69}Ga nucleus are presented in the figure 14. Since the DW calculations are insensitive to spin-orbit coupling at forward angles, we are unable to distinguish between $j = 3/2$ and $j = 1/2$ transfers, except that we expect the latter to lie at higher energies than the $j = 3/2$ transfers.

From the figure 14, we can see that the $\ell = 1 (3/2^-)$ transfers are characterized by a strong transition to the ground state which carries about 40 % of the total strength of all the transitions to the $2p_{3/2}$ state. Transfers to the $1f_{5/2}$ subshell are characterized by two strong transitions neighbouring in energy. They carry about 60 % of the total strength of all the transition to this state.

The vacancy probability[29] U_j^2 of each subshell were obtained from the relation

$$U_j^2 = \frac{\sum S(\ell j)}{S^{max}(\ell j)} \quad , \qquad (2\text{-}4\text{-}1)$$

where the $S(\ell j)$ are now defined as the spectroscopic strength, $S^{max}(\ell j)$ is the maximum possible strength of the subshell (ℓj) obtained from the single-particle shell model and $\sum S(\ell j)$ is the total experimental strength of this subshell.

The centre-of-gravity energy of each subshell was

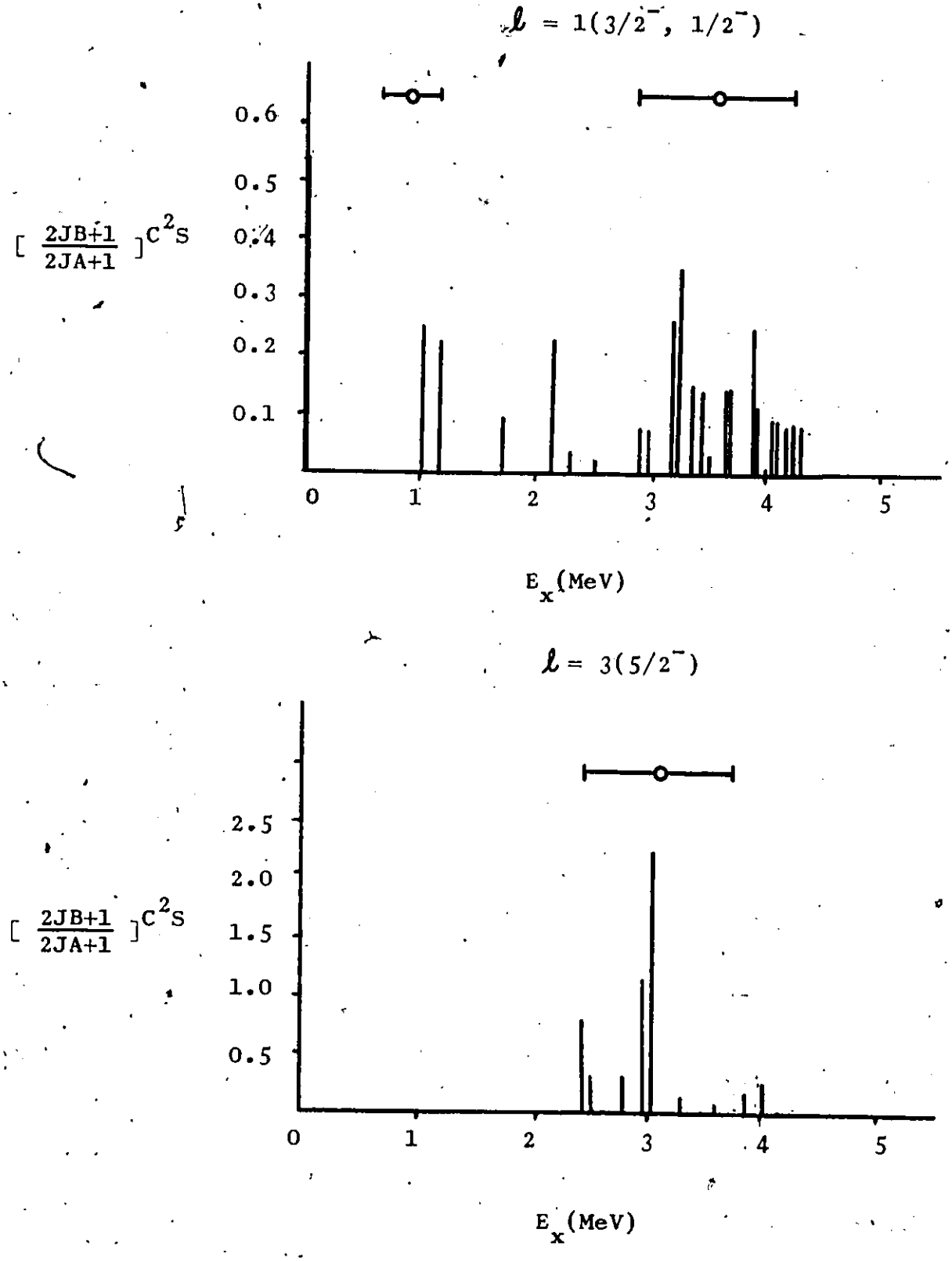


Fig. 14. The distribution of the spectroscopic strengths to the levels populated by $\ell = 1$ and $\ell = 3$ transfers.

deduced from the relation

$$\varepsilon_j = \frac{\sum S(\ell j) E_j}{\sum S(\ell j)} \qquad (2\text{-}4\text{-}2)$$

where $S(\ell j)$ is the spectroscopic strength corresponding to the energy level E_j. These are shown in the figure 14.

The vacancy probability and the centre-of-gravity energies for the $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ subshells are listed in the table 6 where they are compared with the pairing model calculations. The comparaison is repeated in figure 15.

The overall error in our experimental results can be estimated to be of the order of 20 %.

For the vacancy probability, the discrepancy between our results and the pairing model calculations is of about

40 % for the $2p_{3/2}$ subshell

13 % for the $1f_{5/2}$ subshell

10 % for the $2p_{1/2}$ subshell.

The pairing model calculations do not predict the right magnitude and order of the separation between these subshells.

Similar disagreement between experimental data and the pairing model calculations have been previously noted by Dr. Habib[31] in the investigation of ^{71}Ga nucleus, which has the same number of protons as ^{69}Ga, with a (^3He, d) reaction.

TABLE 6

Vacancy (U_j^2) and quasi-particle energy (E_j)

Single-particle state	Experimental		Pairing model calculation	
	U_j^2	E_j(MeV)	U_j^2	E_j(MeV)
$2p_{3/2}$	0.342	0.926	0.585	1.517
$1f_{5/2}$	0.957	3.008	0.832	1.784
$2p_{1/2}$	1.058	3.566	0.936	2.726

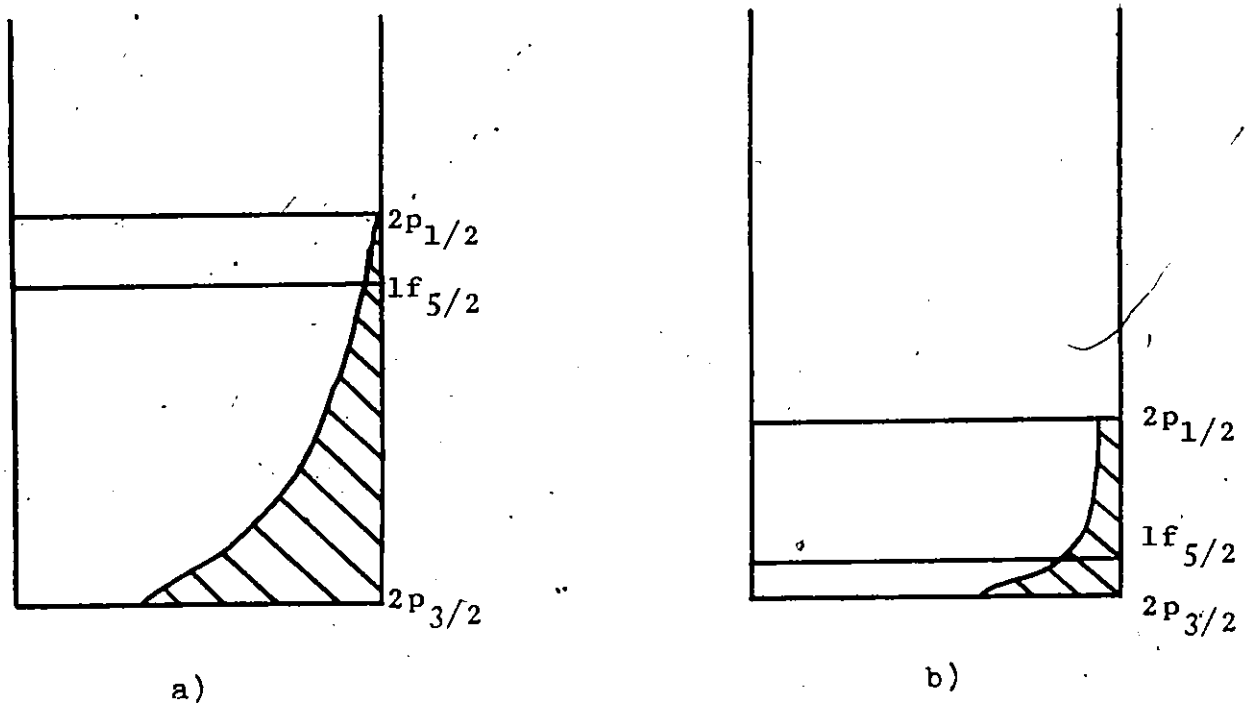
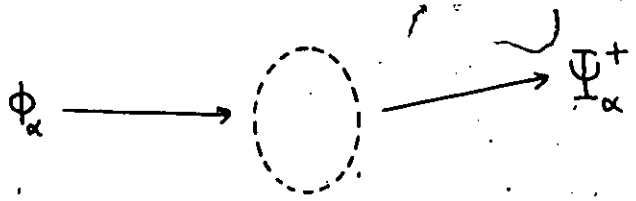


Fig. 15. Vacancy and center of gravity energy of the $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ subshells of ^{69}Ga.
a) experimental, b) calculated

APPENDIX A

We shall now discuss briefly the scattering theory in order to derive a general expression for the transition matrix element and for the differential cross-section.

Let us consider a particle of mass μ incident on a scattering region that can be represented by a potential $V(r)$. We thus have an incident plane wave $\Phi_\alpha(\underline{k}_\alpha, \underline{r})$ that evolves into $\Psi_\alpha^+(\underline{k}_\alpha, \underline{r})$ as the system enters the scattering region.



The Ψ_α^+ must satisfy the Schrödinger equation

$$\left[\frac{-\hbar^2}{2\mu}\nabla^2 + V(\underline{r})\right]\Psi_\alpha^+ = E\Psi_\alpha^+ \quad , \qquad \text{(A-1)}$$

and the plane wave Φ_α is solution of the homogeneous equation. The solution of (A-1) is given by

$$\Psi_\alpha^+(\underline{k}_\alpha, \underline{r}) = i\int G^+(\underline{r}, \underline{r}')\ \Phi_\alpha(\underline{k}_\alpha, \underline{r}')\, d\underline{r}' \quad , \qquad \text{(A-2)}$$

where $G^+(\underline{r}, \underline{r}')$ is the total Green's function which satisfies

$$\frac{1}{\hbar}\left[E - H(\underline{r})\right] G^+(\underline{r}, \underline{r}') = \delta(\underline{r}-\underline{r}') \quad , \qquad \text{(A-3)}$$

where the Hamiltonian H is given by

$$H(\underline{r}) = -\frac{\hbar^2}{2\mu}\nabla^2 + V(\underline{r}) \quad , \tag{A-4}$$

and μ is the reduced mass in the channel α .

Outside the range of the potential $V(\underline{r})$, Ψ_α^+ evolves into a stationnary linear combination of ϕ_β's. Thus we can write Ψ_α^+ as

$$\Psi_\alpha^+ = \sum_\beta (\phi_\beta, \Psi_\alpha^+)\, \phi_\beta \quad . \tag{A-5}$$

Then the probability amplitude of the transition from the state α to the state β is given by the scattering matrix element

$$S_{\beta\alpha} = (\phi_\beta, \Psi_\alpha^+) \quad . \tag{A-6}$$

Substituting (A-2) into (A-6)

$$S_{\beta\alpha} = i \iint \phi_\beta^* (\underline{k}_\beta, \underline{r})\, G^+(\underline{r}, \underline{r}')\, \phi_\alpha (\underline{k}_\alpha, \underline{r}')\, d\underline{r}'\, d\underline{r} \quad . \tag{A-7}$$

If we write the total Green's function in terms of the free particle Green's function G_o

$$G^+(\underline{r},\underline{r}') = G_o^+ (\underline{r},\underline{r}') + \frac{1}{\hbar}\int G_o^+ (\underline{r},\underline{r}_1)\, V(\underline{r}_1)\, G^+(\underline{r},\underline{r}')\, d\underline{r}_1 \quad . \tag{A-8}$$

Substituting into (A-7)

$$S_{\beta\alpha} = i \iint \phi^*_\beta (\underline{k}_\beta, \underline{r}) G_o^+(\underline{r},\underline{r}') \phi_\alpha(\underline{k}_\alpha, \underline{r}') d\underline{r}' \, d\underline{r}$$

$$+ \frac{i}{\hbar} \iiint d\underline{r}_1 \, d\underline{r}' \, d\underline{r} \, \phi^*_\beta (\underline{k}_\beta, \underline{r}) \, G_o^+(\underline{r},\underline{r}_1) \, V(\underline{r}_1)$$

$$\times G^+(\underline{r}_1, \underline{r}') \, \phi_\alpha(\underline{k}_\alpha, \underline{r}') \quad . \qquad \text{(A-9)}$$

We note that since

$$\phi_\beta(\underline{k}_\beta, \underline{r}') = -i \int G_o^- (\underline{r}',\underline{r}) \, \phi_\beta(\underline{k}_\beta, \underline{r}) \, d\underline{r} \quad ,$$

taking the complex conjugate of this equation and with the following property of the free particle Green's function

$$G_o^+ (\underline{r},\underline{r}') = G_o^- (\underline{r}', \underline{r}) \quad ,$$

$$\phi^*_\beta (\underline{k}_\beta,\underline{r}') = i \int \phi^*_\beta (\underline{k}_\beta, \underline{r}) \, G_o^+ (\underline{r},\underline{r}') \, d\underline{r} \quad . \qquad \text{(A-10)}$$

Therefore with (A-2 and 10), the scattering matrix element given in (A-9) becomes

$$S_{\beta\alpha} = \int \phi^*_\beta (\underline{k}_\beta,\underline{r}') \, \phi_\alpha(\underline{k}_\alpha,\underline{r}') \, dr'$$

$$- \frac{i}{\hbar} \int \phi^*_\beta(\underline{k}_\beta,\underline{r}_1) \, V(\underline{r}_1) \, \Psi^+_\alpha(\underline{k}_\alpha,\underline{r}_1) \, d\underline{r}_1 \quad ,$$

which can be re-written as

$$\langle \beta | S | \alpha \rangle = \langle \beta | \alpha \rangle - \frac{i}{\hbar} \int d\underline{r}_1 \, \phi^*_\beta(\underline{k}_\beta,\underline{r}_1) V(\underline{r}_1) \Psi^+_\alpha(\underline{k}_\alpha,\underline{r}_1) \, . \qquad \text{(A-11)}$$

In a direct reaction process, $\phi_\beta \neq \phi_\alpha$. Thus the probability amplitude of a transition from the state α to the state β is given by the second term on the right-hand side of the equation (A-11). We then define the transition matrix element as

$$T_{\beta\alpha} = \langle \phi_\beta | V | \Psi_\alpha^+ \rangle \quad . \tag{A-12}$$

From the definition of the transition matrix element, the cross-section for the reaction will be proportional to the square of the transition amplitude.

If we define $\rho(E)\, dE$ to be the number of final states within the energy interval dE, the transition rate λ is given by Fermi's golden rule

$$\lambda = \frac{1}{(2J_A+1)(2s_h+1)} \frac{2\pi}{\hbar} \rho(E) |T_{\beta\alpha}|^2 \quad , \tag{A-13}$$

where we have averaged out on the initial states of our system; J_A and s_h being the spin of the target and incident helion nucleus respectively. Since the flux of particles in the incident beam is given by $v = \frac{k_\alpha \hbar}{\mu_\alpha}$, the relationship between the transition rate λ and the cross-section σ is

$$\lambda = v\sigma = \frac{k_\alpha \hbar}{\mu_\alpha} \sigma \quad . \tag{A-14}$$

We are interested in the differential cross-section, therefore for the outgoing particles in a solid-angle dw

$$\frac{d\sigma}{dw} = \frac{\mu_\alpha}{\hbar k_\alpha} \frac{d\lambda}{dw} ,$$

from (A-13)

$$\frac{d\sigma}{dw} = \frac{1}{(2J_A+1)(2s_h+1)} \frac{2\pi\mu_\alpha}{\hbar^2 k_\alpha} |T_{\beta\alpha}|^2 \frac{d}{dw} \rho(E) .$$

(A-15)

Normalizing our wave functions in a box of volume L^3,

$$\frac{d\sigma}{dw} = \frac{\mu_\alpha \mu_\beta}{(2\pi\hbar^2)^2} \frac{k_\beta}{k_\alpha} \frac{1}{(2J_A+1)(2s_h+1)} |\langle \phi_\beta | V | \Psi_\alpha^+ \rangle|^2 .$$

(A-16)

APPENDIX B

In order to calculate the distorted waves, we must solve the Schrödinger equation (1-2-4)

$$\left[\frac{-\hbar^2}{2\mu}\nabla^2 + U(\underline{r}) + V_s(\underline{r})\,\underline{L}\cdot\underline{S} + V_{Coul}(\underline{r})\right]\Phi(\underline{k},\underline{r}) = E\Phi(\underline{k},\underline{r}) \quad . \tag{B-1}$$

Where $U(\underline{r})$ is the central optical-model potential, $V_{Coul}(\underline{r})$ the Coulomb potential and $V_s(\underline{r})$ the value of the spin-orbit coupling for the transferred proton in an LJ orbit around A. The target A is taken to be a zero spin nucleus in order to avoid the possibility of spin-spin interactions. There is no evidence that such interactions are required to fit data[3,7,9] and we will not include them in the optical-model calculations. μ is the reduced mass in the channel.

In the absence of the nuclear field, the Schrödinger equation becomes

$$\left(\nabla^2 + k^2 - 2\eta\frac{k}{r}\right)\Phi_c(\underline{k},\underline{r}) = 0 \quad . \tag{B-2}$$

Where

$$\eta = \frac{\mu Z_1 Z_2}{\hbar^2 k} \quad ,$$

$Z_1 + Z_2$ is the number of protons of the reduced particle in the channel, and

$$k^2 = \frac{2\mu E}{\hbar^2} \quad . \tag{B-3}$$

If we do a partial-wave expansion of Φ_c

$$\Phi_c = \sum_{L=0}^{\infty} \frac{u_L(r)}{r} P_L(\cos\Theta) \quad . \tag{B-4}$$

The radial wave function must satisfy

$$\frac{d^2}{dr^2} u_L(r) + (k^2 - \frac{L(L+1)}{r^2} - \frac{2\eta k}{r}) u_L(r) = 0 \quad . \tag{B-5}$$

This equation has two independent solutions[16]. The regular solution which vanishes at r=0 has the asymptotic form

$$F_L(r) \simeq \sin(kr - \frac{L\pi}{2} - \eta \ln(2kr) + \sigma_L) \quad , \tag{B-6}$$

and the irregular solution with the asymptotic form

$$G_L(r) \simeq \cos(kr - \frac{L\pi}{2} - \eta \ln(2kr) + \sigma_L) \quad . \tag{B-7}$$

Where

$$\sigma_L = \arg \Gamma(L + 1 + i\eta) \quad , \tag{B-8}$$

is the Coulomb phase shift.

The total wave function will be a linear combination of $F_L(r)$ and $G_L(r)$[17]. We may define on outgoing wave

$$u_L^+(r) = e^{-i\sigma_L} (G_L(r) + i F_L(r)), \tag{B-9a}$$

and an ingoing wave

$$u_L^-(r) = e^{i\sigma_L}\,(G_L^*(r) - i\,F_L^*(r)) \quad . \qquad \text{(B-9b)}$$

The radial wave function in the outside region can always be written as[17]

$$u_L(r) = a\,u_L^-(r) + b\,u_L^+(r) \quad . \qquad \text{(B-10)}$$

Then in the region outside the Coulomb field,

$$u_L(r) \simeq a\,e^{-i(kr-\frac{L\pi}{2})} + b\,e^{i(kr-\frac{L\pi}{2})} \quad . \qquad \text{(B-11)}$$

This equation was obtained from (B-6,7,9 and 10). In this region, the wave function is a plane wave $e^{i\underline{k}\cdot\underline{r}}$. If we expand the plane wave into partial waves,

$$e^{i\underline{k}\cdot\underline{r}} = \sum_{L=0}^{\infty} i^L\,(2L+1)\,j_L(kr)\,P_L(\cos\Theta) \quad , \qquad \text{(B-12)}$$

where $j_L(kr)$ is the spherical Bessel function which has the asymptotic form

$$j_L(kr) \simeq \frac{1}{kr}\sin(kr - \frac{L\pi}{2}) \quad ,$$

$$= \frac{1}{2ikr}\,(\exp i\,(kr - \frac{L\pi}{2}) - \exp -i\,(kr - \frac{L\pi}{2})) \quad . \qquad \text{(B-13)}$$

Then

$$e^{i\underline{k}\cdot\underline{r}} = \frac{1}{2kr} \sum_{L=0}^{\infty} i^{(L+1)} (2L+1)(\exp - i(kr - \frac{L\pi}{2})$$

$$- \exp i\,(kr - \frac{L\pi}{2})) P_L(\cos\Theta) \quad . \qquad \text{(B-14)}$$

Only the outgoing wave which is proportional to e^{ikr} is changed by the Coulomb interaction. Hence, the actual wave function in the incident channel has an asymptotic behavior differing from (B-14) in the coefficient of e^{ikr} only. Therefore we may write the wave function Φ_c for $kr \gg 1$

$$\Phi_c \approx \sum_{L=0}^{\infty} \frac{1}{2kr} i^{(L+1)} (2L+1)(\exp - i\,(kr - \frac{L\pi}{2})$$

$$- C_L \exp i\,(kr - \frac{L\pi}{2})) P_L(\cos\Theta) \quad . \qquad \text{(B-15)}$$

Comparing (B-11) with (B-4 and 15), one gets

$$a = \frac{1}{2k} i^{(L+1)} (2L+1) \quad , \qquad \text{(B-16a)}$$

$$b = - C_L\, a \quad . \qquad \text{(B-16b)}$$

Let us define Φ_{LJ} such as

$$\Phi_{LJ}(k,r) = \frac{i}{2} (H_L^*(kr) - \eta_{LJ} H_L(kr))\, e^{i\sigma_L} \quad . \qquad \text{(B-17)}$$

Then the equation (B-4) becomes

$$\Phi_c = \frac{1}{kr} \sum_{L=0}^{\infty} i^L (2L+1)\, \Phi_{LJ}(k,r)\, P_L(\cos\Theta) \quad ,$$

$$= \frac{4\pi}{kr} \sum_{L=0}^{\infty} i^L \Phi_{LJ}(k,r)\, Y_L^{M*}(\Omega_k)\, Y_L^{M}(\Omega_r) . \quad \text{(B-18)}$$

In the equation (B-17), η_{LJ} are the reflection coefficients, $\eta_{LJ} = C_L\, e^{-2i\sigma_L}$. They are computed by integrating numerically (B-2) and matching the function and its derivates to (B-17) at large r. The H_L are defined as[7]

$$H_L = G_L + i\, F_L \quad , \qquad \text{(B-19)}$$

which corresponds to the outgoing Coulomb wave function.

The equation (B-18) which is for the Coulomb region only, must also include an internal structure wave function. As discussed earlier, we take it to be only the projectile spin function. Then in the channel α, (B-18) becomes

$$\Phi_{c_\alpha} = \frac{4\pi}{k_\alpha r_\alpha} \sum_{L=0}^{\infty} i^{L_\alpha} \frac{i}{2}\left(H^*_{L_\alpha}(k_\alpha r_\alpha) - \eta_{L_\alpha J_\alpha} H_{L_\alpha}(k_\alpha r_\alpha)\right)$$

$$\times e^{i\sigma_{L_\alpha}} Y_{L_\alpha}^{M*_\alpha}(\theta_{k_\alpha}, \varphi_{k_\alpha})\, Y_{L_\alpha}^{M_\alpha}(\theta_{r_\alpha}, \varphi_{r_\alpha})\quad \psi_h(\xi_h) \, . \qquad \text{(B-20)}$$

ψ_h is the spin wave function of the helion nucleus. It can

be divided into

$$\psi_h(\xi_h) = \sum_{m_{s_h}} a_{m_{s_h}} \psi_{m_{s_h}} , \qquad (B\text{-}21)$$

where m_{s_h} are the different z-components of the spin s_h of ^3He, $\psi_{m_{s_h}}$ are the normalized eigenfunctions of S_z and $a_{m_{s_h}}$ the corresponding amplitudes.

At large values of r, $\chi(\underline{k},\underline{r})$ becomes equal to $\Phi_c(\underline{k},\underline{r})$ defined by (B-20). We note that $Y_L^M(\Omega_r)\psi(\xi)$ is a simultaneous eigenfunction of the operators $\underline{L}^2$, L_z, $\underline{S}^2$, S_z but not of $\underline{L}\cdot\underline{S}$. Thus we introduce a function[18] $\mathcal{Y}_{JLS}^{M_J}(\Omega_r)$ which is a simultaneous eigenfunction of $\underline{L}^2$, $\underline{S}^2$, $\underline{J}^2$ and $\underline{L}\cdot\underline{S}$.

$$\mathcal{Y}_{JLS}^{M_J} = \sum_{m_s} C_{L\ S\ J}^{M\ m_s\ M_J}\ Y_L^M(\Omega_r)\psi(\xi) . \qquad (B\text{-}22)$$

If we calculate the matrix element

$$\langle L\ J\ M_{J_1} \mid \underline{L}\cdot\underline{S} \mid L\ J\ M_{J_2}\rangle = \delta_{12}\ \tfrac{1}{2}\ (J(J+1) - L(L+1) - S(S+1))\ \hbar^2 . \qquad (B\text{-}23)$$

Then for the different values of J, $\underline{J} = \underline{L}+\underline{S}$, we will have a particular solution of the Schrödinger equation (B-1). So that the total solution $\Phi(\underline{k},\underline{r})$ will be a linear combination of uncoupled particular solutions of (B-1). This may

be written in a more general form as matrix elements in the space-spin space for the outgoing-wave as[5,7]

$$\langle m_s' | \underline{\Phi}^+(\underline{k},\underline{r}) | m_s \rangle = \underline{\Phi}^+(\underline{k},\underline{r})_{m_s' m_s} =$$

$$\frac{4\pi}{kr} \sum_{L=0}^{\infty} C^{M\ m_s\ M+m_s}_{L\ S\ J} \; C^{M+m_s-m_s'\ m_s'\ M+m_s}_{L\ S\ J}$$

$$\times i^L \underline{\Phi}_{LJ}(k,r) \; Y_L^{M^*}(\theta_k, \varphi_k) \; Y_L^{M+m_s-m_s'}(\theta_r, \varphi_r) \; . \qquad \text{(B-24)}$$

This equation is obtained by combining (B-9a, 17, 18, 20, 21 and 22).

The ingoing wave is related to the outgoing wave by time reversal invariance[2,7], so that

$$\underline{\Phi}^{-*}_{m_s' m_s}(\underline{k},\underline{r}) = (-)^{m_s - m_s'} \underline{\Phi}^{+}_{-m_s' - m_s}(-\underline{k},\underline{r}) \; . \qquad \text{(B-25)}$$

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