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SIMULTANEOUS TOLERANCE SYNTHESIS FOR MANUFACTURING AND QUALITY

by

BING YE

A Thesis
submitted to
the College of Graduate Studies and Research
through
the Department of Industrial and Manufacturing System Engineering
in Partial Fulfillment of the requirements for
the Degree of Master of Applied Science
at
the University of Windsor

Windsor, Ontario, Canada
1998

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ABSTRACT

Tolerance synthesis attempts to minimize the manufacturing cost of a product by specifying an optimal set of tolerances without compromising product performance. Concurrent engineering is a new approach in product design. Tolerancing is a bridge among design, manufacturing and quality engineers and plays a key role in concurrent engineering. Since cost and quality are fundamental product issues, simultaneous tolerance synthesis for manufacturing and quality is an excellent way to implement concurrent engineering.

In this thesis, a new tolerance synthesis method, *Simultaneous Tolerance Synthesis for Manufacturing and Quality*(STSMQ), is presented. A nonlinear optimization model is constructed to implement this method. In this model, the manufacturing cost and quality loss are combined together into a single objective function. Both process tolerance and design tolerance are chosen as decision variables. The manufacturing cost decreases as tolerance is loosened while the quality loss increases as tolerance is loosened. The purpose of this model is to balance both manufacturing cost and quality loss to achieve optimum design tolerances and process tolerances under the minimum total cost of manufacturing and quality loss. A procedure of implementation of this model is also included.

The proposed method is tested by some examples. The results show that a significant reduction in total cost in manufacturing and quality loss is obtained compared to other techniques.

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DEDICATION

To My Parents and Brothers

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NOTATIONS

$f(x_i)$ = design function of dimension chain

x_i = ith component dimension in the dimension chain

$\frac{\partial f}{\partial x_i}$ = the partial derivative of the design function with respect to component dimension i

C_{total} = total cost of manufacturing cost and quality

$C(tp_{ij})$ = manufacturing cost of producing dimension i with process j

tp_{ij} = process tolerance for process j for producing dimension i

n = total number of component dimension in the dimension chain

p_i = total number of processes for producing dimension i

σ_y^2 = variance of functional dimension of dimension chain

w_i = weighting factors between manufacturing cost and quality

k = quality loss coefficient

t_{id} = design tolerance of component dimension i

T_f = assembly tolerance of dimension chain

Tp_{ij} = allowable variation of the stock removal for process j of producing dimension i

$tp_{ij}^{\min}$ = lower bound of process j for producing dimension i

$tp_{ij}^{\max}$ = upper bound of process j for producing dimension i

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tp_{ip_i} = last process for producing dimension i

C_{pi} = last process capability index for producing dimension i

A = cost of loss caused by defective product

A_{ij} , B_{ij} , C_{ij} , D_{ij} = coefficients of cost-process tolerance function

$\forall$ = each

C_m = sum of manufacturing cost

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CHAPTER 1

INTRODUCTION

In tolerance synthesis, the design engineer seeks the set of tolerances which minimizes cost without compromising product performance. The specification of tolerance on the dimensions of manufactured parts has a significant impact on final production cost and quality.

Traditionally, tolerance synthesis was performed without considering the manufacturing processes and quality, resulting in tolerances that were far from optimal. It is desirable to develop a new tolerance synthesis method, *Simultaneous Tolerance Synthesis for Manufacturing and Quality* (STSMQ), in order to achieve truer optimum tolerance allocation. The definition of tolerance synthesis is described in Section 1.2.

Concurrent engineering is a new approach to product development which has emerged in recent years. Section 1.3 examines the relationship between tolerance synthesis and concurrent engineering. Section 1.4 presents the objective of this thesis. Section 1.5 describes the organization of this document. Before these aspects are discussed, it is important to introduce the definition of the terminology that will be used in subsequent sections.

1.1 Terminology

Some of the terms that will be used in this document are defined below.

Design Function: A characteristic of a part or assembly that measures how well the part or assembly can perform its intended function.

Design Tolerance: A limit on the geometry of a part or assembly intended to ensure a minimum functionality. Design tolerances are derived from design functions but there is not a

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one to one correspondence between them. A design tolerance applies to a dimension that can be controlled directly during manufacturing. In general, a design function is controlled only indirectly through one or more design tolerances.

Process Tolerance: A limit (allowance range) on variations of a manufacturing operation. Process tolerances are related to, but different from, design tolerances. Section 2.3.2.1 discusses the relationship between design tolerance and process tolerance.

1.2 Tolerance Synthesis

Tolerance synthesis or tolerance design is the assignment of values to tolerances so that the limits of variation are defined. Tolerance synthesis attempts to minimize the manufacturing cost of a product by specifying an optimal set of tolerances. The specification of tolerances on the dimensions of manufactured parts has a significant impact on final production cost and product performance. Manufacturing costs decrease as tolerances increase, so large tolerances are favored. However, if tolerances are too large, a product may function poorly or not at all. Therefore, improper tolerance specification may result in a loss of market share.

1.3 Concurrent Engineering and Tolerance Synthesis

Concurrent design attempts to organize the product realization process so as to have as much information and knowledge about all the issues in a product's life available at all stages of the design process. Sometime this is also referred to as Design for 'X', where X stands for some key issue, such as the customer's requirements, manufacturing, quality, etc. In any manufacturing company, three separate groups are constantly concerned with the problems of determining the magnitude of tolerances to be assigned to dimensions. The first group, the design engineers, have the responsibility of issuing designs

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that incorporate the maximum possible working tolerances compatible with the functional requirements of the design; they focus on functional requirements. The second group, the manufacturing engineers, have the responsibility of issuing operation sheets that prescribe the best and most economical sequence capable of producing the part; they worry about cost. The third group, the quality engineers, have the responsibility of measuring and evaluating the quality of parts and products; they are concerned with quality. Tolerance is a key in connecting these three groups (Figure1.1). *Tolerance synthesis* tries to find the optimal tradeoff among them. *Simultaneous tolerance synthesis or design* is one of the best ways to implement concurrent engineering.

To explain this further, section 1.3.1 discusses tolerance design for manufacturing, and section 1.3.2 discusses tolerance design for quality.

1.3.1 Tolerance Design for Manufacturing

Traditionally, tolerance synthesis is carried out at two separate stages: the design stage and the process planning stage. It is called sequential method. At the design stage, design engineers distribute assembly tolerances or functional tolerances to component tolerances by considering functional requirements on the basis of certain standards and the designer's experiences. These are design tolerances. At process planning stage, process engineers specify process tolerances for each manufacturing operation based on design tolerances and process plan in order to produce parts. Unfortunately, in such separate efforts, design engineers allocating design tolerances are often not aware of manufacturing processes and their capabilities to produce the product. This may be due to either lack of communication between the design engineers and the process engineers, or the design engineer's lack the knowledge of manufacturing processes. Such a design often results in a process plan that cannot be executed effectively, or that can only be executed at undesirably high manufacturing costs. In that case,

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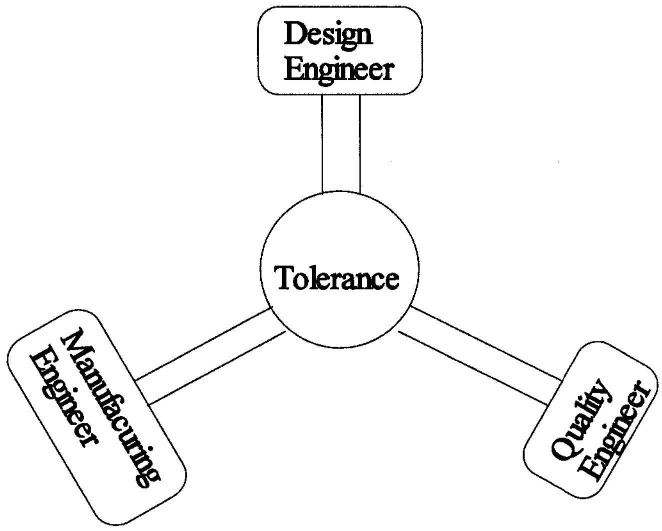

Figure 1.1 Tolerance: The bridge among Design, Manufacturing and Quality

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redesign and modifying design tolerances are required by process engineers. The overhead of redesign and modification significantly increases lead time and lowers design process efficiency.

A new tolerance synthesis approach is needed to incorporate manufacturing processes into the design stage. The integration of design and manufacturing is aimed at breaking the barrier between design and manufacturing, reducing the need for redesign and modification. Because the intermediate tolerance synthesis is deleted, the assembly tolerance can be allocated directly to each process tolerance. Therefore, it can greatly loosen the process tolerances and reduce the manufacturing cost.

It is important in manufacturing that the tolerances be as loose as possible, since tight tolerances imply higher costs. However, loosening tolerances degrades the quality of product. For product performance, it is important that tolerance be kept as tight as possible, since performance degrades as parts deviate from the nominal. So, if the tolerance is loosened too much, the quality loss thus incurred will be bigger than the reduction in the manufacturing cost. Therefore, there is a limit to the amount which tolerances can be loosened without considering quality constraints even if the loosening does not compromise the functional requirements.

1.3.2 Tolerance Design for Quality

The concept of *quality loss* is introduced by Taguchi (1989), "Quality means less trouble, less power consumption and longer life for the same function. Better quality imparts less trouble to the consumer. The quality of the product is the minimum loss imparted by the product to the society from the time the product is shipped." He proposed that performance degradation can be measured as a deviation from some target value, and he asserted that the degradation can be related to a loss in value to the consumer that he called a *quality loss*. Taguchi's quality viewpoint is different from a traditional quality viewpoint, based on a pass/fail methodology.

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In the pass/fail methodology, minimum performance requirements are specified. Sets of tolerance values are rejected when they allow products to be made that do not meet the minimum requirements. In the terminology of optimization, they are *unfeasible*. The pass/no-pass methodology results in a constrained optimization problem. The performance requirements define a subset of tolerance values that forms a feasible region. Tolerance values within the feasible region meet the performance requirements. Thus, the optimum set of tolerance values is constrained to lie within the feasible region defined by the performance requirements or constraints. An assumption of the pass/fail methodology is that a product that performs better than the minimum acceptable level has no additional value. Constraints define the boundaries of a feasible region but have no meaning within the region. Once within the feasible region, only the manufacturing cost is considered. This assumption that improved performances add no additional value is not borne out by consumer behavior. Products with superior performance command higher prices. By this reckoning, an optimum set of tolerances is, therefore, not only the cheapest set of tolerances for a particular level of *performance,* but also the one which provides the optimum level of performance. Here the optimum level is the one where the manufacturing cost of improved product performance is balanced by the cost of the quality loss. The goal of the author's work is to discover how this optimum level can be established.

In order to find the point where increased manufacturing cost is balanced by the cost of the quality loss, it is necessary to measure the value of the performance to the consumer. One method of doing so is the quality loss function of Taguchi (1989). Taguchi emphasizes that quality level does not mean the percentage of defective products, but it means the level indicating the magnitude of societal losses. Even if a product is well within specifications, the product has a quality loss if its quality characteristic value is not at the ideal performance target. This loss is defined in monetary terms so it can be compared to the cost of manufacturing a product.

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An important result of the quality loss method is that a simpler class of algorithms can be used. Under the pass/fail methodology, manufacturing cost is minimized by loosening tolerances until a performance constraint is violated. This is a constrained optimization problem. However, under the quality loss method, loosening a tolerance also causes an increase in the quality loss. Tolerances are naturally constrained by the increase in quality loss and explicit constraints can be eliminated. The result is an unconstrained optimization problem.

1.4 Statement of Thesis

As noted above, it is desirable to develop a concurrent engineering technique to deal with tolerance synthesis. The thesis of this work is that it is possible to develop an optimization model for simultaneous tolerance synthesis that combines quality loss and manufacturing cost as elements of equal importance, and that such a model will improve our ability to design. At the design stage, design engineers can use this model to specify the optimum design tolerances and process tolerances simultaneously. The procedure of implementation of this method is also described in this thesis. Some examples are used to test this approach which suggest the approach can yield substantial savings.

1.5 Organization of Thesis

This thesis is organized as follows: Chapter 2 reviews the current literature in tolerance synthesis. Chapter 3 presents STSMQ method. Chapter 4 discusses the development of synthesis the model of simultaneous tolerance. Test examples and results are given in Chapter 5. The procedure of implementation of simultaneous tolerance synthesis method is described in Chapter 6. Conclusions and suggestions for further research are given in Chapter 7.

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CHAPTER 2

LITERATURE REVIEW

Tolerance synthesis usually is modeled as an optimization problem. Optimization theory is concerned with finding a point where the value of some functions $f(x)$ which is called the objective function. The space which contains this point is defined by the variables of the function $f(x)$, called decision variables. This space may be unbounded or it may be restricted to a subset of decision variable values called a feasible region. The boundaries of this feasible region are defined by additional functions called constraints. Research in tolerance synthesis can be classified as treating the objective function $f(x)$, the constraint functions $g_i(x)$ or the algorithm used to find the optimum tolerance. The constraint function is discussed in the first two sections. Section 2.1 describes the development of design functions, which described the performance characteristics of a product. Design functions are defined in terms of the actual deviations of the parts from the nominal. How do the different tolerances affect the range of deviation of the design function? This is discussed in Section 2.2 which considers the techniques used to determine the relationship between tolerances and part deviations. Then the development of objective functions, optimization models and their algorithms is explored in Section 2.3. Section 2.4 describes the work on tolerance synthesis as part of parameter design and optimization. Section 2.5 describes some problems need to be solved and further research

2.1 Design Function

In most cases, the characteristics which determined the ability of a product or assembly to

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perform its intended function cannot be controlled directly during manufacturing. Instead they must be controlled indirectly through other characteristics over which the manufacturer has control. The design function then defines the relationship between those characteristics which determine product performance and those which the manufacturer can control. A typical design function might be the clearance between the diameter of a pin and its mating hole in an assembly. In this case, the ability of the parts to be assembled is determined by the amount of clearance between the outer surface of the pin and the inner surface of the hole. The manufacturer cannot control the clearance directly , but can control the pin and hole diameters. The tolerances on these diameters are specified as an indirect means of controlling the clearance.

The simplest form of design function is the one dimensional tolerance 'stack-up'. As its name implies, the part dimensions of interest stack up end to end. The design function is a simple sum or difference of the part dimensions. In determining the total variation of the basic tolerance stack-up, the range of variation of the design function is just the sum of the tolerances.

In one dimensional design functions, the tolerance is a limit on the value of a dimension, its value automatically becomes the worse case value of the variation. But in two or three dimensional functions, the identity between worst case variation and tolerance values break down when design functions are generalized to more than one dimension. The reason is that more variations can occur in the three dimensional case than can be modeled as changes in dimension values.

Gupta and Turner (1993) proposed using additional parameters called model variables to describe the additional variations possible in three dimensional geometric model. Several model variables are assigned to each face of the model. Each model variable describes a different deviation from the nominal geometry. Deviations are characterized by type as either size, location, orientation, or form.

ElMaraghy, Wu, ElMaraghy (1991) have proposed a method of evaluating assembly tolerances

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that is based on a hierarchical datum system in their TASS system. Each part is represented as a separate datum tree. The nodes within the tree represent local datums and features. Each node may has, as its ancestors, the features and datums that were used to define the datum or feature it represents. The leaves represent features being dimensioned. The datum trees for individual parts are joined by relating globe datums at the roots of the trees. Design dimensions are analyzed by finding a path through the tree. TASS also contain feature based tolerance assembly models which is superior to the point based mode. She assumed that part datums will be aligned during assembly. However, the features used as part datums are not necessarily those used for assembly alignment.

Design functions are typically used as constraints in the tolerance optimization problem. An acceptable range of performance for the design function is determined from product or assembly functional requirements. Deviation outside this range is not allowed. All performance values of the design function that fall within the acceptable range of performance are considered equally desirable. Most work in tolerance synthesis has used this pass/no-pass approach. An alternative is to make the design function part of the objective function.

2.2 Tolerance Analysis

The design function equation defines a relationship between a set of part deviations and a product performance characteristic, however , tolerance synthesis is concerned with optimizing tolerances, which are the limits placed on the part deviations. The design engineer therefore needs to know how different tolerance values affect the range of deviations of the design function. This is the function of tolerance analysis.

In mass production, large numbers of parts are produced and each exhibits different variations. The statistical nature of these variations may enable a designer to relax the tolerances of a part with the

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assurance that unacceptable combinations of variations will occur only rarely. Other applications may require that there be no possibility of an unacceptable combination.

Section 2.2.1 discusses worst case analyses, where it is assumed that all dimensions are at their maximum tolerance. Section 2.2.2 discusses statistical analysis , which attempts to reduce the total cost by assuming a statistical distribution of part variations.

2.2.1 Worst Case Analysis

The worst case model relies on the adage "the whole is equal to the sum of its parts" and assumes that each part has taken on its worst possible value. In this case all component dimensions are situated at the one or other extreme of the tolerance range. Assuming that the dimensions are independent and have bilateral tolerances , the resulting assembly tolerance T_s would be equal to the sum of the component tolerances, tol_i . Using the Taylor series approximation yields for the general case,

$$T_s = \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| tol_i , \tag{2.1}$$

where T_s is the assembly tolerance and tol_i is the component tolerance.

The major benefits of worst case tolerance are its computational simplicity and intuitive appeal. If all components in the stack are manufactured to specification, the assembly is guaranteed to be within specified limits. The major drawback of the worst case model is that it assumes the production processes fluctuate such that all components are produced at the positive or negative tolerance extremes . It should be evident that the probability of this event is very small . It approaches zero as the number of components in the assembly increases . The result is an assembly tolerance that is too loose or component

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tolerances that are too tight . Because most assemblies have a large number of components, worst case stack up analysis is not recommended for this reason: the method results in wide assembly tolerance specifications , overly conservative component feature specifications, and costly production processes. However, worst case analysis is still quite common in practice because it protects the designer at the expense of manufacturing, since assemblies are guaranteed to be within their specification if components are produced within their specification.

2.2.2 Statistical Analysis

With mass produced products, worst case deviations are rarely encountered. Instead the deviations form a random pattern about the nominal. The designer can take advantage of this to relax tolerances but he must know two things: what is the mature of this random patten (i.e. the distribution of the deviations) and how do tolerance constraints affect this patten.

Mansoor (1963) was one of the first researchers to study the manufacturing variations of actual machined parts. He found that the variation of a part dimension consisted of an offset from the normal dimension and a normal distribution about that offset. Mansoor measures the relative importance of the offset and the distribution with a *relative precision index*, the ratio of the specified *design tolerance* to the 6 times the standard deviation of the process distribution. The relative precision index has generally been replaced by the capability index, *Cp*. The capability index uses a natural process tolerance that is six times the standard deviation. It is defined by the equation

$$C_p = \frac{t_{upper} - t_{lower}}{6\sigma} \tag{2.2}$$

where t_{upper} and t_{lower} are the limits of the tolerance zone and σ is the standard deviation of the

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process. The capability index relates the variance of the process deviations to the tolerance specification. The index C_{pk} relates the mean shift of the process to the tolerance specification. It is defined by the formula

$$C_{pk} = \min\left(\frac{t_{upper} - \mu}{3\sigma}, \frac{\mu - t_{lower}}{3\sigma} \right) \tag{2.3}$$

where μ is the mean of the distribution. The index C_{pk} is basically a measure of the distance between the mean of the distribution and the closer tolerance limit.

The models of Mansoor (1963), Chase and Greenwood (1988), and the other researchers that depend on summing the variances of the part distributions assume that the underlying distributions are normal. The variances of the design function distribution are still valid if the part distributions are not normal but the portion of the distribution which lies within the acceptance region(i.e., the yield) cannot be determined for other than normal distributions. The method of D'Errico and Zaino (1988) provides a better estimate since it can compute higher moments of the distribution but it is still limited. For some types of distributions, different methods are used.

Lee and Johnson (1993) use the Monte Carlo technique. While it is popular in tolerance analysis, the Monte Carlo technique has disadvantages when applying the results of the analysis to an optimization algorithm. The most significant of these is the lack of gradient information. A Monte Carlo calculation builds its answer from thousands of random points. The effect of a small change on the distribution of these points cannot usually be determined without redoing the calculation. Furthermore, the random nature of the point selection introduces a small error in the final result. For most analyses, this is not a problem, but in a gradient, the difference between two similar numbers must be computed. With the random error

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of the underlying analysis, the value of the gradient is easily overwhelmed by the random error of the underlying analyses. Lee and Johnson avoid the problems by using a generic algorithm for their optimization . This class of algorithms is generally less sensitive to error that gradient number of points.

2.3 Optimization Models and Methods

Tolerance synthesis can be formed as an optimization problem. Traditionally, tolerance synthesis is carried out at design stage and process planning stage separately. It's called sequential method. The first two sections discusses the proposed optimization models of traditional method. Section 2.3.1 describes the proposed tolerance synthesis models at design stage. Section 2.3.2 discusses tolerance synthesis at the process planning stage. Some researchers have presented some models which deal with design and process tolerance synthesis simultaneously. They are described in Section 2.3.3. Quality loss based models are discussed in Section 2.3.4.

2.3.1 Design Tolerance Synthesis Models

At the design stage, design engineers need to specify the tolerance of each part according to the functional requirement. Tolerance synthesis is formed as optimization problem. The most obvious quantity to optimize is manufacturing cost. The cost-based models are discussed in Section 2.3.1.1. Some researchers have proposed some criteria other than cost. They are discussed in Section 2.3.1.2.

2.3.1.1 Cost-Based Models

The goal of an optimization process is to find the best feasible values for a set of decision variables. The criteria that determining this best are quantified and combined to form an objective function $f(x)$. Finding the best values for the decision variables now becomes a search for those

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values that minimize (or maximize) the value of the objective function $f(x)$. In a tolerance synthesis, the most obvious quantity to optimize is manufacturing cost. As a consequence, much research has been devoted to determining the relationship between the manufacturing cost and tolerances of a part. Many cost models for tolerances have been proposed. All have several features in common: the cost of an infinite tolerance (no tolerance constraint) is zero; the cost of a zero tolerance is infinite; and cost is a non-increasing function of tolerance. Six cost-tolerance models were found in the literature. They are list in the Table 2.1.

Wu, ElMaraghy, and ElMaraghy (1988) compared the above models. They concluded that the combined model was the most accurate, with the exponential model next in accuracy. The reciprocal power model was third in accuracy. Dong, Hu and Xue (1994) proposed several cost models using polynomial curves. They compared their models with the models already described using the same data as Wu et al plus additional cost data published for particular processes. They concluded that their polynomial models provided a better fit to the cost data than the previous models. They also noted that the model with the best fit could be different for different processes.

All of the above studies are vulnerable to the reliability of the actual cost-tolerance data used. Ostwald and Blake (1989) used a previously published cost estimating method to create their reference data. This method computers cost as machining time plus setup time divided by the lot size. Dong, Hu and Xue state that "In practice, the empirical cost-tolerance data should be directly obtained from machine shops through experiments or observations." All of the above cost models have been empirical. They attempt to find a relation between cost and tolerance based on experimental data. J. He (1991) has proposed a cost model of the manufacturing process, where cost is defined as the sum of material cost plus the machining, inspection, rework, and scrap costs for each operation. He divides this cost by the fraction of acceptable units produced to determine the unit cost.

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Models	$c_i(tol_i)$	Stack-up Method	Solution Method	Reference
Reciprocal Square	$C_i = A_i + \frac{B_i}{tol_i^2}$	Worst case	Lagrange Multiplier	Hillier (1967) Spotts(1973)
Reciprocal	$C_i = A_i + \frac{B_i}{tol_i}$	Worst case	Lagrange Multiplier	Willey et al.(1983)
Reciprocal	$C_i = A_i + \frac{B_i}{tol_i}$	Statistical	Lagrange Multiplier Path of steepest descent	Chase et al.1988 Sayed et al. 1985
Exponential	$C_i = A_i e^{B_i tol_i}$	Worst case	Lagrange Multiplier Geometric programmin Nonlinear programming	Speckhart (1972) Wide et al.(1975) Zhang et al.1993
Exponential/ reciprocal power	$C_i = A_i tol^{k_i} e^{B_i tol_i}$	Statistical	Nonlinear programming	Michael & Siddall (1981)
Sutherland	$C_i = A_i + \frac{B_i}{tol^{k_i}}$	Statistical	Nonlinear programming	Lee & Woo1986 Sutherland 1975

Table 2.1 Summary of Proposed Cost-Tolerance Function

Note: These cost-tolerance function was obtained by using line fitting technique. These function represent the relationship between cost and design tolerance. All of them have several features in common: the cost of an infinite tolerance is zero; the cost of a zero tolerance is infinite; and cost is a non-increasing function of tolerance.

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2.3.1.2 Criteria Other Than Cost

He (1991) has proposed a method involving the process capability . He assumes that cost will be approximately proportional to how tight the tolerance is with respect to the capability of the process used to make it. When tolerance are tight, He proposes minimizing the scrap percentage, particularly among later processes where value has already been added by previous operations. When tolerances are loose compared to capability, scrap is negligible and the percentage defective dose not provided a meaningful measure. In these cased, He suggests that the ratio of tolerance to process capability be the same for all tolerances. This gives each tolerance the same relative margin as the others.

Lee and Woo (1990) chose the distances of each face of the polytope from the nominal design point as their criterion. These distances are computed by through the reliability index which is used in civil engineering work. The reliability index is defined as he minimum distance from the origin to a limit-state surface formed by a design function in an independent standardized coordinate system, called the standard system. They scaled their state space according to process capability. This results in fixed ratios between tolerances and no ability to trade off one tolerance for another.

2.3.2 Process Tolerance Synthesis Models

After the design tolerance is specified at the design stage, the process engineers will distribute design tolerance to a set of manufacturing operations at the process planning stage in order to produce the parts. Firstly, the relationship between design tolerance and process tolerance is discussed in Section 2.3.2.1. Section 2.3.2.2 discusses the process tolerance allocation at the process planning stage.

2.3.2.1 Design Tolerance and Process Tolerance

The tolerances found on drawings are design tolerances. Design tolerances are derived from

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design functions , which measure how well a part or assembly performs its function. However, a design function typically measures a characteristic, such as the clearance between a pin and a hole, that cannot be controlled directly. Design tolerances are specified for the underlying dimensions of the assembly , in this case the diameters of the pin and hole, so that the variation of the design function is within acceptable limits.

Process tolerances are related to, but different from, design tolerances. The relationship between design tolerances and process tolerances is complex and ambiguous. The manufacturer determines the process tolerance as part of the process plan that will be used to make the part. The process tolerance must be such that the design tolerances are met. However, there is not a direct correspondence between the design and process tolerance. For example, all the dimensions of a part formed by stamping have the same process tolerance, since they are formed by the same process. They do not necessarily have the same design tolerances since these are determined by how the part will be used. One dimension may also be manufactured by multiple processes. In this case, the design tolerance is a combination of several process tolerances. A single value of the design tolerance could have many compatible combinations of process tolerances.

2.3.2.2 Tolerance Synthesis in Process Planning

Process planning is the determination of the sequence of operations that will be used to create a part. Tolerance charts are used to specify the tolerances for each operation and to verify that the tolerances specified by the designer will be met by the process plan. The steps involved in preparing a process chart are outlined in Mittal, Irani, and Lehtihet (1990), and in Ji (1993) (see Figure 2.1 and Figure 2.2 for a typical process plan and tolerance chart). The identification and analysis of tolerance loops is important in process planning and tolerance charting. Finished surfaces are produced through multiple

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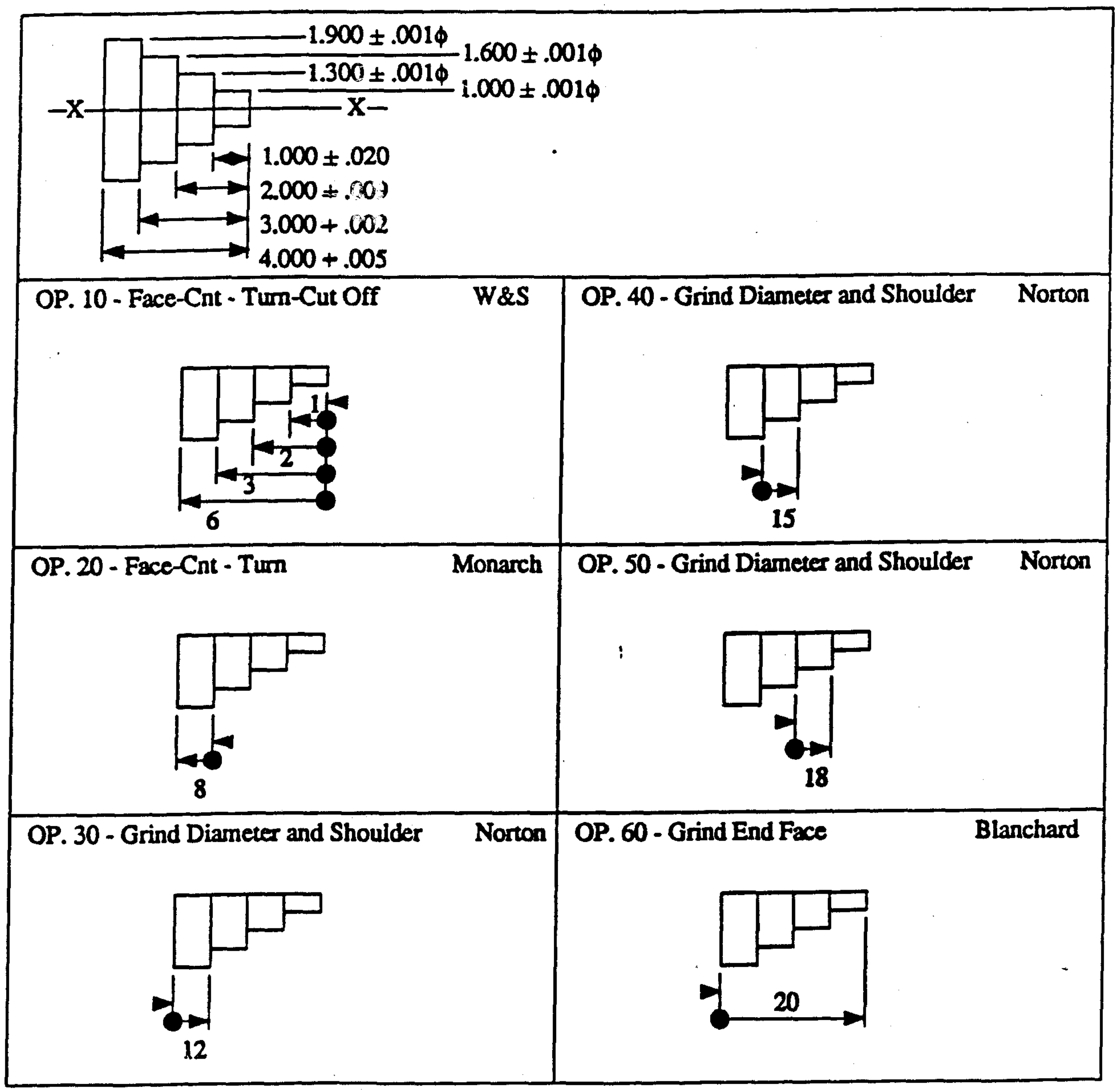

Figure 2.1 Typical Process Plan (from Mittal et al. (1990))

Note: This is typical process plan. The drawing of a cylindrical part is on the top of picture. Six operation are needed to produce this part. The sequence of operations are Turning and Cutting off; Turning; Grinding diameters and shoulders; Grinding end face

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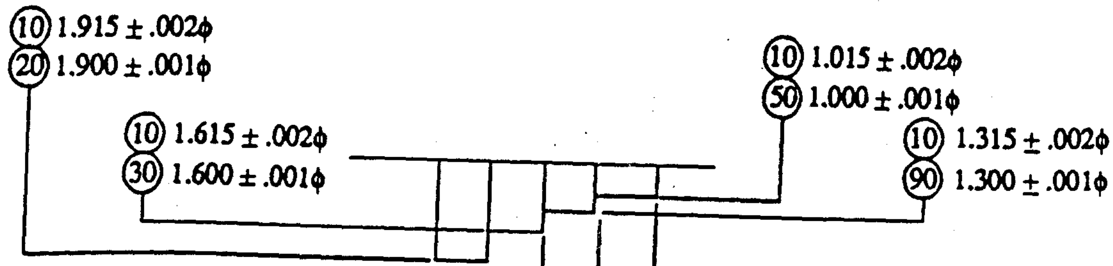

Line No.	Oper No.	Rev. No.	Machine Used	Machine to Mean	Machine to +Tol	A B C D E	Balance Dim. Mean	Balance Dim. +Tol	Lines	Stock Removal Mean	Stock Removal +Tol
1	10		W+S	0.979	.003	1				Solid	
2	10			1.994	.003	2				Solid	
3	10			3.003	.003	3				Solid	
4						4	1.009	.006	3-2		
5						5	2.024	.006	3-1		
6	10			4.031	.010	6				Solid	
7						7	1.025	.013	6-3		
8			Monarch	1.008	.004	8			7-8	.020	.017
9						9	2.017	.010	4+8		
10						10	3.032	.010	5+8		
11						11	4.011	.007	3+8		
12	30		Norton	1.000	.001	12			8-12	.008	.005
13						13	1.017	.011	9-12		
14						14	2.032	.011	10-12		
15	40		Norton	1.000	.003	15			13-15	.017	.014
16						16	2.000	.004	12+15		
17						17	1.032	.014	14-15		
18	50		Norton	1.000	.015	18			17-18	.032	.029
19						19	3.000	.019	16+18		
20	60		Blanchard	4.000	.001	20 Z			11-20	.011	.008
21						21 W	3.000	.002	20-12		
22						22 X	2.000	.005	20-16		
23						23 Y	1.000	.020	20-19		
24											
25				Blueprint			Resultants				
26				4.000	.005	20 Z	4.000	.001			
27				1.000	.020	23 W	1.000	.020			
28				2.000	.009	22 X	2.000	.005			
29				3.000	.002	21 Y	3.000	.002			
30											

Figure 2.2 Typical Tolerance Chart (from Mittal et al. (1990))

Note: Tolerance is a tool to analyze the process plan. It forms a tolerance loop to ensure that early operations do not remove so much material that the final dimension and tolerance cannot be achieved in the later step.

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steps, and the process planner must ensure that early steps do not remove so much material that the final dimension cannot be achieved in the later step. The dimensions affected by each step and tolerance loop for that step must be identified in order to produce a correct process plan. In optimizing the tolerances within their process plan, Ji (1993) proposed a tree and generating tolerance chart and provided algorithms for creating the tree. He proposed that the sum of the process plan tolerance values be maximized. This is equivalent to minimizing slack as used by Mittal et al., since Ji does not permit any tolerance to exceed its specification. The above method make the assumption that changing one tolerance has the same effect on cost as changing any other. Problems arise with this assumption when making tradeoffs between tolerances in a tolerance stack-up or loop.

Ngoi (1991) presented a mathematical model for the tolerance balancing process and made use of linear programming to solve the model. The model used a simplified weighing system to represent the relative importance of various process instead of the more complex approach adopted by Irani et al. (1989).

Mittal et al. (1990) provided algorithms for constructing the graph from the chart and for identifying tolerance loops within the chart. They showed that the tolerance chart is equivalent to a graph where the nodes are machined surfaces and the arcs are the cuts made to produce one surface relative to another. Loops within this graph are tolerance loops .

Dong and Hu (1991) and Tang et al. (1994) have proposed methods for optimizing the machining tolerance for the intermediate steps of a process plan. Dong and Hu (1991) add the cost of all the steps. They propose that the cost of a machining step is a function of the tolerances of the surface being machined both before and after the machining step is performed. Tang's method is similar. However, their cost function use the tolerance achieved and the amount of material removed by the step.

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2.3.3 Integrated Tolerance Synthesis

Most previous research works dealt with tolerance synthesis at design stage and process stage separately. Some researchers have studied tolerance allocation problem with simultaneous manufacturing processes. This section will discuss their research works.

Section 2.3.3.1 introduces some cost based models. Section 2.3.3.2 discusses the models other than cost based models.

2.3.3.1 Cost Based Models

In order to obtain the minimal manufacturing cost , the machining process sequence needs to be considered before tolerances are assigned . It is therefore difficult to assign optimal tolerances so that minimum manufacturing cost is achieved. The optimal tolerance allocation over multiple processes alternative has been discussed in some research works.

Ostwald and Huang (1977) firstly formulated the optimal tolerance allocation with multiple process alternative as linear 0-1 integer programming with variables assuming the value of zero or one. Some specific tolerance values are determined by some available machining process sequences. The cost associated with the tolerance values and process sequences are also determined . The minimal cost is the objective function , and the design requirements are used as constraints. By solving the zero-one algorithm, the minimal cost can be achieved by selecting the available tolerance values. Because the tolerance values are discrete and the process sequences are also determined , it is suitable in the situation where the operation sequence and the tolerance of the operation is fixed.

Another model is presented by Lee and Woo (1989). Their cost-tolerance model is similar to that of Huang and Ostwald (1977). They also treat a tolerance as unique to a given process. However, they simplify the complex stack-up condition using the reliability index, which was previously advocated by

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Hasofer and Lind (1974), and also develop a branch and bound algorithm for efficient tree enumeration. Monotonicity between the tolerance and reliability index is employed to make the enumeration tree small. Because of the simplification of the stack-up condition and the efficient tree enumeration method, the applicability of this model is significantly increased.

Chase et al (1990) presented three methods, exhaustive search , univariate search and sequential quadratic programming (SQP), to solve the model of Huang and Ostwald. The exhaustive search method generates all feasible process combinations and uses nonlinear programming technique to determine the optimal component tolerances for each process combination. The process combination that leads to the least total cost will define the optimal solution. While the exhaustive search technique achieves optimum tolerance, it is not practical since the number of process combinations increase exponentially as a function of the total number of processes being considered. The univariate search method fixes a process for each of the last *(n-1)* components (n is the number of components) and determines the best process for the first component by enumerating over the possible choices and solving a nonlinear programming problem to determine the optimal tolerance and the associated cost for each choice. Then , the process is fixed for the first component , and the last *(n-2)* components varied for the second component, and similarly , the best process is determined for the second component. The methodology is repeated until all components are treated. . Although the univariate search method produces a good result, it requires solving a series of nonlinear programming problems in order to reach the result . Since the enumeration is not exhaustive the resulting solution is not the global minimum. Therefore , the time spent in optimization is not well justified. Similarly, SQP technique relaxes the 0, 1 restriction and let the binary variable vary continuously from 0 to 1. Then nonlinear programming methods are used to solve this problem.

Nagarwala et.al. (1994) proposed a new method, slope-based method, to deal with tolerance

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allocation with process selection problem. They presented a new cost model for the combined process. The new tolerance - cost function can be referred to as the *efficient* (or *non-dominating*) tolerance-cost curve since each point on the cost curve refers to a unique process which is the most cost-effective process with the associated tolerance value. This method eliminates process selection for each component, thereby also eliminating the selection for each component and hence eliminates the generation or process combinations.

All of the above models and methods assume each component dimension is produced by only one process. The tolerance obtained from the process has a single fixed value. A cost is associated with each tolerance value. In practice, one dimension is usually produced by several manufacturing processes. Such assumptions hinder the application of the above models. Therefore, above models are less used.

Zhang and Wang (1993) presented an analytical model for simultaneously allocating design and machining tolerances based on the least-manufacturing-cost criterion. In their model, tolerance allocation is formulated as a nonlinear optimization problem based on the cost-tolerance relationship. Simulated annealing algorithm is employed to solve the nonlinear programming problem.

Al-Ansary and Deiab (1997) proposed the same model as Zhang and Wang's model. They used the genetic algorithm method to solve their model. They conclude the genetic algorithm is better than simulated annealing algorithm in solving nonlinear programming problems.

Zhang's model is more practical than the other models. Because the assumption of each component dimension can be produced by only one process is removed in the their model and they treat the manufacturing cost-tolerance function as continuous function rather than discrete function. Zhang's model had been shown a great advantage in reducing the manufacturing cost. The design and process tolerances can be loosened greatly. But their model fails to consider the quality. Loosening design and process tolerances will degrade the quality of product and increase the quality loss.

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2.3.3.2 Non-Cost Based Models

The other ways to deal with tolerance-process optimization problem are Taguchi's method and design of experiments. Taguchi applied statistical experimental design methods to the quality engineering. He has advocated some novel methods of statistical data analysis and some approaches to the design of experiments. Both Taguchi's method and design of experiments are based on a statistical planned experiments. Although the Taguchi's method and design of experiments are statistical tools for the design and analysis of experiments, they are useful in discrete tolerance synthesis.

Gadallah and ElMaraghy (1995) present an algorithm which is base on Taguchi's system of experimental design and orthogonal arrays to solve tolerance-process optimization problem. The inner orthogonal array is used to allocate the magnitude of tolerance to each design dimension and the outer array is used to select the corresponding manufacturing process . Several combinations of orthogonal arrays are coded and used to search for the tolerance combinations and the corresponding processes that yield the minimum production cost. The search graph techniques are used to present the assignment of either the design dimensions or the cost-process curves. Kusiak and Feng (1995) applied both design of experiments and Taguchi's method approaches to this process-tolerance selection problem . They compared these both approaches. Their comparative study show that the design of experiments and Taguchi's method can easily be applied to probabilistic and nonlinear tolerance synthesis problems, while the design of experiments approach is superior to the Taguchi's method .

2.3.4 Quality Loss Based Models

A part of the increased awareness of the importance of quality in manufacturing , Taguchi (1989) and Kapur, Raman, and Pulat (1990) have proposed that quality loss be treated as a cost along with the manufacturing cost. This quality loss represents the loss to society that occurs when a product deviates

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form the optimum set of design parameters. The deviations are controlled by the tolerances. Thus loose tolerances have costs in term of excessive quality loss and tight tolerances have excessive manufacturing costs. If these costs are balanced, the result should be an optimum set of tolerance values where the loss to society from degraded performance is balanced by cost of manufacturing. A problem with the concept of quality loss is that it is difficult to measure . The value of a product to a particular user is somewhat subjective. Also different users have different expectations as to how a product should perform.

The quality loss can also be considered a performance degradation. The quality loss function transforms the degradation into a cost to society that can then be included in the objective function $f(x)$ along with the manufacturing cost (Kapur, Raman, and Pulat (1990)).

Phadke (1989) and Kapur et al (1990) have both derived formulas to calculate quality loss as a function of deviation from the nominal. It is

$$Q = k(y - y_0)^2 \tag{2.4}$$

where k is the quality loss coefficient y is the actual value of the design function and y_0 is its target value.

Determining the value of the constant k in the quality loss function remains a problem. Often the exact dollar value of loss is a subjective judgment. Several methods are described below. In Soderberg's (1993) method the quality loss is determined directly from performance data. The methods of Cook and DeVor (1991), and Vasseur, Kurfess, and Cagan (1993) attempt to interpolate form information about the complete product.

Soderberg (1993) developed a quality loss function for a roller bearing from data relating the life of the bearing to the clearance within the bearing assembly. If the quality loss for the maximum lifetime is taken as zero, then a clearance that results in only half the maximum life has a cost equal to Cn, the cost of replacing the assembly. This is because there is one additional replacement over the maximum life of

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the bearing assembly. Other points on the quality loss curve can be constructed in a similar manner. The resulting curve can be used directly. Soderberg chose to approximate the curve using an asymmetrical form of the quality loss formula

$$Q = \begin{cases} 0 & \mathit{for}\ y = 0 \\ k_1 y^2 & \mathit{otherwise} \end{cases} \tag{2.5}$$

Cook and DeVor (1991) have proposed a means of computing the quality loss function from their S-model of manufacturing. In this model, quality is defined as the value added by a product , less its cost to society, The equation describing this relationship is

$$Q = K(V - C)^2 \tag{2.6}$$

where V is the product's value to the customer. C is a variable cost. K is a constant. Quality is proportional to the square of the difference because of two effects. An increase in value over cost increases the unit value of the product. Also, by increasing value for both consumer and producer , it increases demand for the product. The increased value per product times the increased number of products results in quality is thus proportional to the square of the product value. To determine V, Cook and DeVor note that the maximum return on investment occurs when the increased value to consumer equals the gain to the producer

$$V - P = P - C \tag{2.7}$$

or

$$P = (V + C)/2 \tag{2.8}$$

where P is the selling price of the product. Thus if price and cost are known, the value of the product can be computed assuming that the product is being sold at its optimum price. The constant K is determined by

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finding a point on the performance-quality curve where the value of the product is zero. At this point the quality loss is equal to the cost of manufacturing the product . The quality loss at smaller values of performance degradation can then be determined by interpolation.

Vasseur, Kurfess, and Cagan (1993) use the Cook and DeVor S-model to develop a procedure for allocating tolerance based on the maximization of profit. The quality loss function is used to determine the reduction in value due to an off-target product and this is balanced against the reduction in manufacturing cost. They determined that optimum profit occurred when the derivatives for the quality loss and manufacturing cost functions were equal. The same result is also obtained when the objective function is the sum of manufacturing cost and quality loss. Taguchi (1989) has proposed an alternative to balancing quality loss against manufacturing cost, wherein the cost of accepting the part (quality loss) is compared to against the cost of rejecting it (scrap or rework cost). When the quality loss is greater, it is more economic to scrap or rework the part. Thus tolerances are set at the point where the quality loss is equal to the cost of scrapping or rejecting the part.

Quality loss based models deal with design tolerance allocation problem without considering manufacturing process. They treat tolerance synthesis like traditional method. They impose quality loss limits on the trend of loosening design tolerance. The quality loss base models tend to allocate tight design tolerances to each component dimension in order to achieve the least quality loss. This will result in tighter design tolerances than the traditional method. These tighter design tolerances will be distributed to several manufacturing processes. The process tolerances will be considerable tight. As a result, the manufacturing cost will be extremely high.

2.4 Robust Design

Robust design for manufacturing is a method for making a product or manufacturing process

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less sensitive to manufacturing variations. As it reduces manufacturing variation by diminishing the influence of sources of variation rather than controlling them , robust design is a cost effective technique for improving quality . In robust design , variables that affect performances of a product or a process are classified into two categories: 1) design (control) parameters , and 2) noise (random uncertainty) parameters. Design parameters are the product characteristics whose nominal settings can be specified by the product designer . A vector of the settings of design parameters defines product design specifications and vice versa . Noise parameters are variables that cause performance variations during a product's life span. Design of a product or a process includes system design, parameter design, and tolerance design. Robust design is closely applicable to parameter design and tolerance design. The current literature on robust design has focused on parameter design while the problem of tolerance design has not been adequately covered. By making a design more tolerant of variation , it is possible to reduce the number of rejected parts or to use less expensive parts. Such a design is said to be robust. Taguchi's methodology for model development and interpretation relies on direct experimentation.

Previous related research falls into three areas. The first area involves implementing Taguchi concepts using nonlinear programming. Second area is the methods for including the effects of tolerance during design optimization. The third area , which will be heavily relied on the statistical analysis, is stochastic optimization , i.e. incorporating statistically distributed variables during design optimization.

2.5 Areas for Further Research

Research in tolerance optimization is continuing in the areas of design function development, statistical tolerance analysis, algorithm development, and robust design.

Three dimensional tolerance analysis and synthesis have not been fully explored. There are currently two approaches to this problem, the model variable approach of Gupta and Turner(1993), the

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datum tree approach of ElMaraghy, Wu, and ElMaraghy (1991). The advantages and disadvantages of these techniques need to be further investigate.

Geometric tolerance analysis and synthesis also need to be further developed. Nassef and ElMaraghy (1993) have proposed a method to allocate the geometric tolerance types and values which is formed as a combinatorial optimization problem by using genetic algorithm. The problem here is that for each feature the number of combinations of tolerance types is large. Therefore, Nassef and ElMaraghy (1997) proposed a new non-cost based criterion, *mismatch*, rather than cost criterion to solve this problem.

The development of quality loss as part of the total manufacturing cost needs further exploration. In particular the accuracy of the quality loss function needs to be as good as that of manufacturing cost.

The relationships between tolerance and process planning, production scheduling and control , and throughput are not clear and need to be further explored.

Robust design is also a fertile area for research. It holds the promise of combining tolerance design and manufacturing design with product design. However, it is not clear whether the additional complexity in the resulting problem is compensated by superior design.

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CHAPTER 3

THE STSMQ METHOD

From literature review, we can see that in both the sequential tolerance synthesis method and the quality loss based method, functional tolerance cannot be fully allocated to each manufacturing process due to the two step's allocation. Both will allocate the tight tolerances which result in high manufacturing cost and low quality loss. Although design and process tolerances is loosened and manufacturing cost is reduced by integrated method , the quality loss is increased. Therefore, it is necessary to develop a simultaneous tolerance synthesis technique to deal with tolerance allocation problem. Section 3.1 discusses the drawbacks of these three methods. Section 3.2 presents a new tolerance synthesis method, *Simultaneous Tolerance Synthesis for Manufacturing and Quality* (STSMQ).

3.1 Shortcomings of Previous Methods

Sequential methods and quality based methods carry out tolerance synthesis in two stages, design and manufacturing. In design, tolerance assignment requires considerations of the interrelated dimensions and components of an assembly. Tolerance values are often specified on a trial and error basis, or using a simple tolerance analysis tool to ensure consistency. In manufacturing, a design tolerance is formed by a manufacturing process of several sequentially arranged production/machining operations. The design tolerance is an important factor in determining the manufacturing processes of a part, and process tolerance of every operation. Assignment of the process tolerance is always an activity driven by pure experience, with the assistance of tolerance charts. Such a procedure is illustrated in Figure 3.1. This practice is contrary to the spirit of concurrent engineering, and has the following shortcomings:

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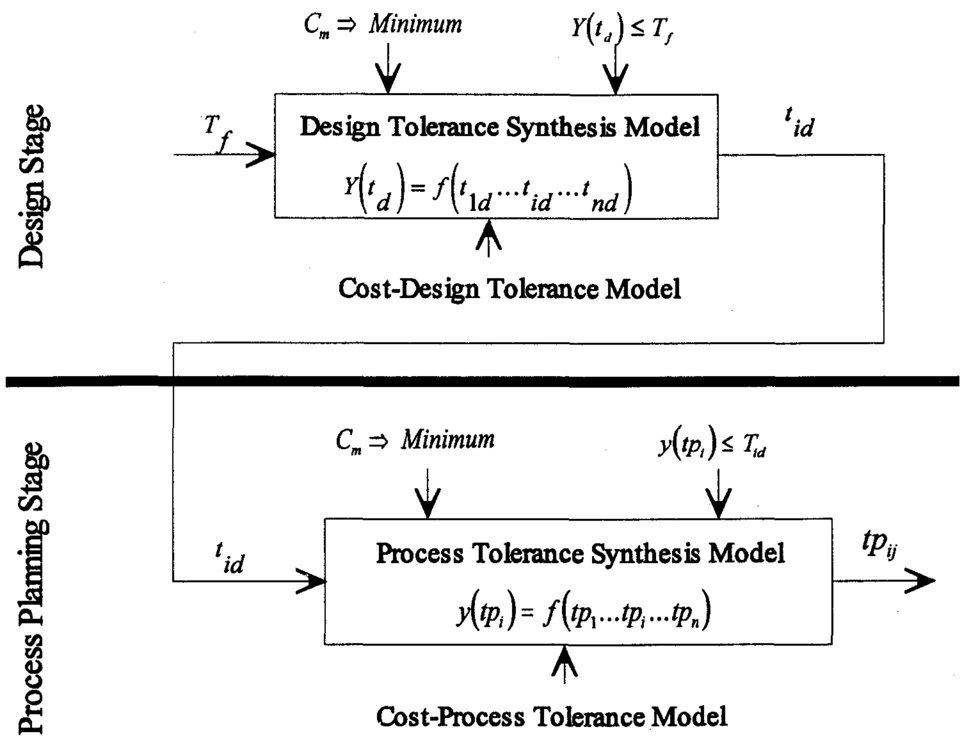

Figure 3.1 Sequential Tolerance Synthesis

Note: Sequential tolerance synthesis is carried out in two stages. In design stage, the input of design tolerance synthesis function, assembly tolerance is allocated to each component under the controls of minimum manufacturing cost and assembly tolerance. The outputs of the function are design tolerances. In the process planing stage, the input of process tolerance synthesis function, design tolerance is distributed to each operation under the controls of minimum manufacturing cost and design tolerance. The outputs are process tolerances.

T_f = functional tolerance of dimension chain

t_{id} = design tolerance of component dimension *i*

tp_{ij} = process tolerance for process *j* for producing dimension *i*

C_m = manufacturing cost

$Y(t_d) = f(t_{1d} \ldots t_{id} \ldots t_{nd})$: design function

$y(tp_i) = f(tp_1 \ldots tp_i \ldots tp_n)$: process function

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1. It is impossible to obtain optimal design tolerances in tolerance synthesis in design stage, because the manufacturing cost cannot be precisely determined without information about the manufacture of a workpiece. Quite often, the predicted manufacturing cost at the design stage is not the same as the *actual cost* of manufacturing .

2. The manufacturing cost depends on the selected process which produces the workpiece which is unknown to the design engineer. That is why the design engineer must interact with the manufacturing engineer in order to modify design tolerances.

3. Assembly tolerances cannot be allocated to process tolerances directly. There is no direct relation between the deviation of product specification and the process tolerances. The lack of direct relation between the deviation of product specification and the process tolerances hinders the application of dynamic tolerance control in manufacturing.

4. It needs more lead time, because we need to do the same things twice: to form a cost model and to solve a constrained optimization problem.

Integrated method (1993) allocates tolerances as large as possible without violating the functional requirements. It comes from a traditional quality viewpoint: if a product is within its specifications, everything is fine and no quality is lost. But Taguchi (1989) pointed out that the quality level does not indicate the level of percent of defective items, it indicates the level indicating the magnitude of societal losses; therefore , "quality losses must be defined as deviation from target not conformance to arbitrary specifications." According to Taguchi's theory, even if a product is well within its specifications , the product has a quality loss if its quality characteristic value is not at the ideal performance target. Therefore, loosening the tolerance reduces the manufacturing cost, but it degrades the quality of the product and increases the quality loss. Integrated method loosens tolerances without the restriction of quality loss. This method will cause the big quality loss when it loosens tolerances.

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3.2 The STSMQ Method

In order to overcome the above drawbacks, it is necessary to develop a concurrent tolerance synthesis technique, *Simultaneous Tolerance Synthesis for Manufacturing and Quality* (STSMQ). The STSMQ method is shown in Figure 3.2.

STSMQ method takes design tolerance and process tolerance as decision variables. The purpose is to break the barrier between design engineer and manufacturing engineer. After performing the synthesis function, both design tolerance and process tolerances can be allocated simultaneously. The benefit of this choice is that the tolerance can be loosen and the manufacturing cost can be reduced. At meanwhile, quality loss is associated with manufacturing cost as a control on the synthesis function. The purpose is to limit the loosening of tolerance, balance the manufacturing cost and quality loss to achieve optimum product cost.

Tolerance synthesis problem can be formed as an optimization problem. STSMQ can be formed an optimization problem too. In the model of STSMQ, both design tolerance and process are chosen as decision variables. The combination of manufacturing cost *and* quality loss is taken as objective function. The purpose is to balance the manufacturing cost and quality loss and make them at their optimum levels in order to achieve the minimum product cost.

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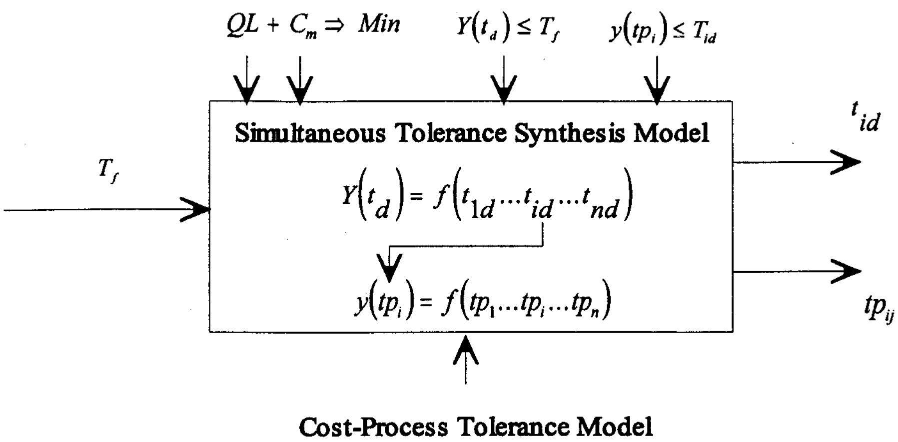

Figure 3.2 Simultaneous Tolerance Synthesis

Note: Simultaneous tolerance synthesis integrates both design and process planning into one model. The input of simultaneous tolerance synthesis function, assembly tolerance is distributed to each component and operation simultaneously under the controls of the minimum of manufacturing cost and quality loss, assembly tolerance and design tolerance. After performing simultaneous tolerance synthesis function, we can get design tolerance and process tolerance simultaneously.

T_f = functional tolerance of dimension chain

t_{id} = design tolerance of component dimension *i*

tp_{ij} = process tolerance for process *j* for producing dimension *i*

C_m = sum of manufacturing cost

QL = quality loss

$Y(t_d) = f(t_{1d} \cdots t_{id} \cdots t_{nd})$: design function

$y(tp_i) = f(tp_1 \ldots tp_i \ldots tp_n)$: process function

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CHAPTER 4

MODEL DEVELOPMENT

As mentioned above, STSMQ method can be formed as an optimization problem. This chapter discusses the development of the STSMQ optimization model. Before developing the model, some assumptions are addressed in Section 4.1. A mathematical model for STSMQ and its detail description is given in Section 4.2.

4.1 Assumptions

1. Design function can be retrieved and formulated from assembly drawings. The design functions define a relationship between a set of dimension deviations in the dimension chain and a product performance characteristic. In this thesis, design function is one dimensional.
2. The resultant tolerance of a dimension chain is given. This is a functional tolerance which comes from functional requirements or customer requirements.
3. A process plan for each dimension is already known.
4. Each process has a normal distribution and is under statistical control.
5. The dimensions in a dimension chain and the manufacturing processes for each dimension are independent.
6. All tolerances are dimensional tolerances. Up to now, the mapping from cost to tolerance only exists to dimensional tolerances. Other mappings, such as the function of heat-treating tolerance and cost, don't exist.

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4.2 Formulation of Optimization Model

STSMQ can be formed as optimization problem. As we know, an optimization model consists objective function and constraints. In the STSMQ model, the objective function is the combination of manufacturing cost and quality loss. This is discussed in Section 4.2.1. Constraints are discussed in Section 4.2.2. The summary of STSMQ model is given in Section 4.2.3.

4.2.1 Objective Function

The objective of the author's model is to optimize the cost of product by setting tolerances to help balance manufacturing costs and quality losses. Therefore, the combined manufacturing cost and quality loss is taken as objective function. The weights are introduced to represent the importance between manufacturing cost and quality loss. The objective function is

$$\sum_{i=1}^{n}\sum_{j=1}^{p_i} w_1 C\left(tp_{ij}\right) + w_2 k \sigma_y^2 \tag{3.1}$$

where, the first item is manufacturing cost and the second item is quality loss. w_i are the weight factors.

Section 4.2.1.1 discusses the manufacturing cost model. Section 4.2.1.2 describes the quality loss function. Section 4.2.1.3 discusses how to decide either the manufacturing cost or quality loss is more important.

4.2.1.1 Manufacturing Cost

It is known that in manufacturing the smaller the tolerance of a machining process, the higher (in general) the machining cost. In previous research, tolerance synthesis has been based on the cost-design tolerance model. In that model, several manufacturing processes are combined to model a single cost-

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tolerance curve which describe the relationship between total machining cost and a design tolerance, or simply assume only one process to produce the design tolerance. Several modeling methods have been reported to describe this cost-design tolerance relationship.

Since a design tolerance is usually achieved by performing a series of operations, a simple curve may not be sufficient to depict the cost-tolerance relationship of a design tolerance . It is especially not appropriate for a design tolerance which is obtained by a set of interrelated process tolerances, *a operation dimension chain.* In other hand, It is difficult to get the cost-design tolerance models. Firstly, the model is design-dependent. Each feature-tolerance combination would have a different model. Secondly , in manufacturing , each mechanical feature is produced through a sequence of production or machining operations called a manufacturing process. Different features with different tolerance require difference manufacturing processes. The cost-design tolerance model is a reflection of the cost-to-accuracy relation of all related production operations. At the design stage, without a prior knowledge of the manufacturing process of the part, it is not feasible to form an accurate cost-to-accuracy relation model determined by the downstream production operations. The unavailability of the cost-design tolerance models is a severe obstacle to the practical application of tolerance synthesis.

In manufacturing , each mechanical feature of a designed part is modified from its raw material form to the designed shape and accuracy through a manufacturing process. Because the cost-process tolerance model are built directly from empirical data for commonly-used machining operations, these models can be obtained very accurately.

In author's optimization model, cost-process tolerance function is adopted, with the manufacturing cost function occurring in the objective function, to model each manufacturing process. Firstly, this model method is more accurate than cost-design tolerance model. Secondly, functional tolerance can be distributed to each process tolerance directly. The total manufacturing cost, C_m, is the sum of

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manufacturing cost of each machining process of each component dimension:

$$C_m\left(tp_{ij}\right) = \sum_{i}^{n} \sum_{j=1}^{p_i} C\left(tp_{ij}\right) \tag{3.2}$$

where n is number of dimensions in the dimension chain, p_i is number of processes to produce dimension i, and $C(tp_{ij})$ is the cost-process tolerance function of machining process.

An exponential function is used to as cost-process tolerance modeling function in the author's thesis. For a particular manufacturing process, the following relationship is found

$$C_m(tp_{ij}) = A_{ij} e^{-B_{ij}(tp_{ij} - C_{ij})} + D_{ij} \tag{3.3}$$

4.2.1.2 Quality Loss

Taguchi(1989) suggests that given an ideal target value, a loss function associated with deviations from that target can be developed. This loss function is a quadratic expression estimating the cost of the average versus target and the variability of the product characteristic in terms of the monetary loss due to product failure in the eyes of the customer. As shown in the following equation, the loss function, L(Y), indicates a monetary measure for the product characteristic value versus its target values

$$L(Y) = k(y - m)^2 \tag{3.4}$$

where $k = \dfrac{A}{T_f}$ (3.5)

A is the cost of replacement or repair if the dimension does not meet functional tolerance requirements,

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T_f is functional tolerance requirement, m is the target value of the functional dimension, and y is the design characteristic.

It is assumed that the functional dimension has a normal distribution and a mean at the target value. In this case, the quality loss can be conveniently described in terms of the standard deviation of the functional dimension, and the expected value of the loss function $L(Y)$ can be written as

$$QL = E(L(Y)) = k[(\mu - m)^2 + \sigma_y^2] \quad (3.6)$$

where μ is the mean of Y and σ_y^2 is the variance of Y.

Equation (3.5) Contains two components. The first term, $(\mu - m)^2$ is the difference of the mean of Y, μ from the target, m. The second term, σ_y^2 consists of the results from the variance of Y.

Obviously, the loss function is a way to show the economic value of reducing variation and staying closer to the target. Hence, the quality engineer has to establish the design target for the lowest cost and to reduce process variation through process design. μ can be adjusted to the target m in parameter design and this adjustment will not affect the value of process variability σ_y, so the expected quality loss $L(Y)$ can be diminished by reduction of σ_y for the quality characteristics Y. That is

$$QL = E(L(Y)) = k\sigma_y^2 \quad (3.7)$$

Based on the design function, the resultant overall quality characteristic can be estimated from the set of individual quality characteristics in the design function. These approximation functions can be found by resorting to the Taylor series expansion. Therefore, the resultant variance, σ_y^2, of Y can be

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expressed approximately in terms of variances, σ_{xi}^2, of the individual quality characteristics as

$$\sigma_y^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{xi}^2 \tag{3.8}$$

Tolerances are always related to manufacturing processes, and they must be designed in conjunction with the application of a specific manufacturing process. If a tolerance is determined without considering a specific manufacturing process, there is great risk in having a mismatch between the required tolerance and capability of a given process. One way to express the relationship between tolerance and manufacturing process capability is process capability index, *Cp*. which is the ratio of design tolerance boundaries to the measured variability of the manufacturing process output response. The process capability index is a measure of the ability of the process to manufacture product that meets specifications. We have

$$C_p = \frac{t_d}{3\sigma} \tag{3.9}$$

Solving for σ for each quality characteristic gives

$$\sigma_{xi} = \frac{t_{id}}{3C_{pi}} \tag{3.10}$$

Substituting equation (3.9)into equation (3.7), we have

$$\sigma_y^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \left(\frac{t_{id}}{3C_p} \right)^2 \tag{3.11}$$

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The total quality loss is now

$$QL(t_{id}) = \frac{A}{T_f^2} \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \left(\frac{t_{id}}{3C_p} \right)^2 \qquad (3.12)$$

4.2.1.3 Weighting Factors

Traditionally, cost has been considered of paramount importance, and no one would argue that cost is unimportant. However, in the 1980's, especially as a result of Japanese examples, quality --- defined very broadly--- became equally or possibly more important. Cost and quality are the two most important effects for an industrial company. Sometimes, they conflict: high quality mostly implies high manufacturing cost. It is not easy to weigh the importance of manufacturing cost against quality loss; it depends on the application. For example, in today's automotive market, high competitiveness forces companies to constantly improve the quality of their products. In this case the quality should be more important than manufacturing cost. The weight factor of quality loss(w_2 in equation 3.1)is higher than the weight of the manufacturing cost(w_1 in equation 3.1). The determination of weight factors should be integrated into the company management system since they reflect corporate priorities.

4.2.2 Constraints

In author's model, constraints come from both design function and manufacturing process. Design function constraints are described in Section 4.2.2.1. Section 4.2.2.2 discusses stock removal in machining processes. Process capability constraints are described in Section 4.2.2.3. Section 4.2.2.4 discusses how both manufacturing cost and quality loss are integrated into one model.

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4.2.2.1 Design Function Constraint

Design function is always used as a constraint to guarantee the assembly tolerance won't exceed the function tolerance. The resultant tolerance of design function must be equal to or less than the assembly functional tolerance limits. In the dimension chain, different dimensions contribute to assembly tolerance differently. Partial derivatives are used to estimate their contribution. We have for worst case,

$$\sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| t_{id} \leq T_f \tag{3.13}$$

where T_f is assembly functional tolerance limit.

for statistical case

$$\sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right|^2 t_{id}{}^2 \leq T_f^2 \tag{3.14}$$

4.2.2.2 Machining Allowance Constraints

In machining tolerance allocation, consideration should be given not only to process capability but also to the amount of machining allowance for operation. The machining allowance is the layer of material that is to be removed from the surface of a workpiece in machining, in order to obtain the required accuracy and surface quality. The determination of machining allowance greatly influences the quality and the production efficiency of the machined part. Excessive machining allowance will increase the consumption of material, machining time, tool and power , and thus increase the manufacturing cost. On the other hand, if there is not enough machining allowance then the surface roughness and defective surface layer caused by a preceding operation cannot be removed completely from the workpiece

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surface. Thus it will influence the surface quality of the part.

The amount of a machining allowance is the difference between the machining dimension obtained in the preceding operation and that of the current operation. Due to the existence of operation error, the actual stock removals cut from workpiece surfaces vary in a certain range. The variation of stock removal is the sum of the manufacturing tolerances in current and proceeding process. In practice, an appropriate amount of stock removal should be provided for each process. Thus the variation of stock removal should be controlled in a certain amount. This amount is usually found in machine manuals and handbooks. Therefore, we have machining allowance constraints

$$tp_{ij} + tp_{ij-1} \leq Tp_{ij} \qquad \forall\, i, \quad \forall\, j \tag{3.15}$$

4.2.2.3 Process Capability Constraints

Various process operations have their own process accuracies. The process capability for various operations can be gotten from manufacturing handbooks. Each process operation must be performed within its process capability. Therefore, we have

$$tp_{ij}^{\min} \leq tp_{ij} \leq tp_{ij}^{\max} \tag{3.16}$$

4.2.2.4 Integration of Manufacturing Cost and Quality Loss

In the author's model, manufacturing cost is a function of process tolerances while quality loss is a function of design tolerances. However, they need to be combined together. Since the intermediate process tolerances are not final tolerances on a manufactured dimension, they don't affect the functional performance and quality. Therefore, no quality loss is associated with these intermediate process tolerances. The final process tolerances are the final tolerances for the manufactured dimension. They

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are also design tolerances. The quality loss are associated with design tolerances, the final process tolerances. The fact that the final process tolerances equal the design tolerances serves as the link between manufacturing cost and the quality loss. Therefore, we have

$$tp_{ip_i} = t_{id} \tag{3.17}$$

4.2.3 Summary of STSMQ Model

Combination of the equation from 3.1 to 3.17 , we get the STSMQ model:

Minimize
$$C_{total} = \sum_{i=1}^{n} \sum_{j=1}^{p_i} w_1 C\left(tp_{ij}\right) + w_2 k \sigma_y^2 \tag{3.1}$$

Subject to

Design function constraints

$$\sum_{i=1}^{n} \left|\frac{\partial f}{\partial x_i}\right|^2 t_{id}^{\ 2} \le T_f^2 \tag{3.14}$$

Operation constraints

$$tp_{ij} + tp_{ij-1} \le Tp_{ij} \qquad \forall\ i, \quad \forall\ j \tag{3.15}$$

Process capability constraints

$$tp_{ij}^{\min} \le tp_{ij} \le tp_{ij}^{\max} \tag{3.16}$$

Relationship of design tolerance and process tolerance

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$$tp_{ip_i} = t_{id} \tag{3.17}$$

where $$\sigma_y^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \left(\frac{t_{id}}{3C_{pi}} \right)^2 \tag{3.11}$$

$$C_m(tp_{ij}) = A_{ij} e^{-B_{ij}(tp_{ij} - C_{ij})} + D_{ij} \tag{3.3}$$

$$k = A / T_f^2 \tag{3.5}$$

This is a nonlinear programming optimization model. It can be solved by some methods of nonlinear programming, and some commercial software such as LINDO, MatLab or other similar software package.

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CHAPTER 5

EXAMPLES AND DISCUSSION OF RESULTS

In this chapter, three examples are used to test the STSMQ model. Section 5.1 discusses the first example in detail. Section 5.2 briefly discusses the example 2 and example 3.

5.1 Example 1

An assembly of piston and cylinder of a diesel engine is used to test the author's model. Section 5.1.1 describes the example. The result of its application is given in Section 5.1.2. Section 5.1.3 discusses the results of the example.

5.1.1 Description of Example 1

This is an assembly of piston and cylinder of a diesel engine which is used by Zhang and Wang(1993) and Al-Ansary and Deiab(1997). The critical dimension is clearance between the piston and the cylinder (see Figure 5.1). The piston must fit closely with the cylinder bore, but not too closely as it must be free to slide and expand as temperature changes, and not too loosely or it will knock. Therefore the variation of the clearance should be controlled within a certain range. The given diameter of the piston D_p is 50.8mm, the cylinder bore diameter D_c is 50.856 mm, the clearance is 0.056+-0.025 mm.

Assume the following process plans for piston and cylinder:

- Piston: rough turning, finish turn, rough grinding, and finally finish grinding
- Cylinder bore: drilling, boring , semi-finish boring, and finally grinding

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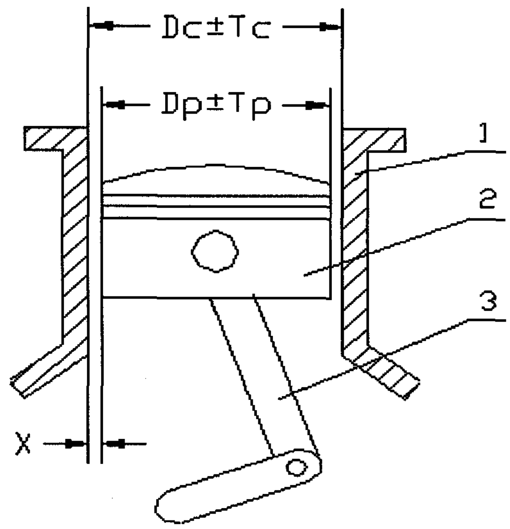

Figure 5.1 Assembly Drawing of Example 1 (from Zhang and Wang (1993))
(Note: 1—Cylinder, 2—Piston, 3—Crank)

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The coefficient A, is \$100 which comes from Taguchi(1989). The following are the data of process capabilities, machining allowance and coefficient of manufacturing cost-tolerance function for each process which come from Al-Ansary and Deiab(1997).

Piston		
Sequence	Operations	Process Capabilities
1	rough turn	0.125=<T11<=0.50
2	finish turn	0.050=<T12<=0.3
3	rough grind	0.0125=<T13<=0.075
4	finish grind	0.005=<T14<=0.025

Table 5.1 Data of process capabilities for piston of example 1 (mm)

Note: There are four operations which are required to produce the piston. Each operation has a process capability range

Cylinder		
Sequence	Operations	Process Capabilities
1	drill	0.175=<T21<=0.50
2	bore	0.075=<T22<=0.3
3	semi-finish bore	0.015=<T23<=0.125
4	final grind	0.0075=<T24<=0.050

Table 5.2 Data of process capabilities for cylinder of example1 (mm)

Note: There are four operations which are required to produce the cylinder. Each operation has process capability range

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Coefficient	$C_{m11}(T_{11})$	$C_{m12}(T_{12})$	$C_{m13}(T_{13})$	$C_{m14}(T_{14})$
A	5	9	13	18
B	309	790	3196	8353
C	0.005	0.00204	0.00053	0.000219
D	1.41	4.36	7.48	11.99

Table 5.3 Cost-tolerance coefficients for piston of example 1

Note: A, B, C, D are the coefficients of cost function, $C_m(tp_{ij}) = A_{ij}e^{-B_{ij}(tp_{ij}-C_{ij})} + D_{ij}$, for each machining process to produce the piston

Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$	$C_{m24}(T_{24})$
A	4	8	10	2
B	299	986	3206	9428
C	0.00702	0.00297	0.0006	0.00036
D	2.35	5.29	9.67	13.12

Table 5.4 Cost-tolerance coefficients for cylinder of example 1

Note: A, B, C, D are the coefficients of cost function, $C_m(tp_{ij}) = A_{ij}e^{-B_{ij}(tp_{ij}-C_{ij})} + D_{ij}$, for each machining process to produce the cylinder

5.1.2 Result of Example 1

In piston -cylinder bore assembly, there is only one resultant dimension, namely the clearance between the piston and the cylinder, and the two dimensions that form the dimension chain which are the

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piston diameter and the cylinder bore diameter. The design function is as following

$$X = D_p - D_c \tag{5.1}$$

The proposed model is tested at four cases:

Case 1 $w_1 = w_2 = 1,\ C_{p1} = C_{p2} = 1$

Case 2 $w_1 = 1,\ w_2 = 2,\ C_{p1} = C_{p2} = 1$

Case 3 $w_1 = w_2 = 1,\ C_{p1} = C_{p2} = 0.5$

Case 4 $w_1 = w_2 = 1,\ C_{p1} = C_{p2} = 1.5$

The LINGO software package is used to solve the models. STSMQ model refers to Appendix A.1. Lingo programming for STSMQ method refers to Appendix A.2. Lingo programming for integrated method refers to Appendix A.3. Lingo programming for sequential method refers to Appendix A.4. The results of each case are given below.

Piston diameter			
Tolerance	Proposed Method	Integrated Method	Sequential method
T11	0.407	0.4	0.388
T12	0.093	0.096	0.113
T13	0.032	0.029	0.013
T14	0.013	0.016	0.007
T1d	0.013	0.016	0.007

Table 5.5 Optimum tolerances of piston of example 1 for case 1($w_1 = w_2 = 1,\ C_{p1} = C_{p2} = 1$) (mm)

Note: The optimum design tolerance, T_{1d}, and process tolerances, T_{ij}, for each machining processes to produce piston are obtained by using three methods

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Cylinder bore diameter			
Tolerance	Proposed Method	Integrated Method	Sequential Method
T21	0.407	0.405	0.384
T22	0.093	0.095	0.012
T23	0.032	0.03	0.009
T24	0.011	0.015	0.008
T2d	0.011	0.015	0.008

Table 5.6 Optimum tolerances of cylinder of example 1 for case 1 ($w_1 = w_2 = 1$, $C_{p1} = C_{p2} = 1$) (mm)

Note: The optimum design tolerance, T_{1d}, and process tolerances, T_{ij}, for each machining processes to produce piston are obtained by using three methods

	ProposedMethod	Integrated Method	Sequential Method
Manufacturing cost	67.27	65.82	111.89
Quality loss	4.97	8.96	1.79
Total	72.24	74.78	113.68

Table 5.7 Optimum costs of three methods for example 1: Case 1 ($w_1 = w_2 = 1$, $C_{p1} = C_{p2} = 1$) (dollars)

Note: This table lists the optimum costs of three methods for the case 1. From table, the total cost saving of the author's model is 3.4% compared to Integrated method and 36.4% compared to sequential method

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	ProposedMethod	Integrated Method	Sequential Method
Manufacturing cost	68.90	65.82	111.89
Quality loss	7.66	17.92	3.58
Total	76.56	83.74	115.47

Table 5.8 Optimum costs of three methods for example 1: Case 2 (w_1 =1, w_2 =2, $C_{p1} = C_{p2}$ =1)(dollars)

Note: This table lists the optimum costs of three methods for the case 2. From table, the total cost saving of the author's model is 8.6% compared to Integrated method and 33.7% compared to sequential method

	Proposed Method	Integrated Method	Sequential Method
Manufacturing cost	71.77	65.82	111.89
Quality loss	11.31	35.84	7.14
Total	83.08	101.66	119.03

Table 5.9 Optimum costs of three methods for example 1:Case 3 ($w_1 = w_2$ =1, $C_{p1} = C_{p2}$ =0.5)(dollars)

Note: This table lists the optimum costs of three methods for the case 3. From table, the total cost saving of the author's model is 18.2% compared to Integrated method and 30.2% compared to sequential method

	ProposedMethod	Integrated Method	Traditional Method
Manufacturing cost	66.31	65.82	111.89
Quality loss	2.83	3.98	0.8
Total	69.14	69.8	112.69

Table 5.10 Optimum costs of three methods for example1:Case 4($w_1 = w_2$ =1, $C_{p1} = C_{p2}$ =1.5)(dollars)

Note: This table lists the optimum costs of three methods for the case 4. From table, the total cost saving of the author's model is 1.0% compared to integrated method and 38.6% compared to sequential method

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5.1.3 Analysis of Results

In case 1, the manufacturing cost is as important as the quality loss. That is to say, w1 =w2. Most north American companies choose C_p =1 as their quality standard. From Table 5.7, we can see that the total cost saving of the author's model is 3.4% compared to integrated method, and 36.4% compared to sequential method.

Sequential method deals with tolerance allocation in two steps. Because of these iterations of allocation, it results in the tight process tolerance(see Table 5.5 and Table 5.6), thus incurring the highest manufacturing cost and lowest quality loss among the three methods. Integrated method loosens the tolerance greatly(see Table 5.5 and Table 5.6) and achieves the lowest manufacturing cost, but it results in the highest quality loss. The proposed method shows the good balance between manufacturing cost and quality loss. In the proposed method, the trend of loosening tolerance is limited by quality loss. Manufacturing cost is higher than integrated method and is lower than the sequential method. Quality loss is lower than integrated method and is higher than the sequential method. However, the total cost is the lowest among the three methods(see Table 5.7).

Quality loss is defined as deviation from the target The quality of the product is the minimum loss imparted by the product to the society form the time the product is shipped. Taguchi (1989) also introduced a quality loss function to measure the deviation from the target of the quality characteristics. The quality loss function is a mathematical way to transfer deviation from target of quality characteristics into cost and quantify this cost. Because the deviations are controlled by tolerances which are specified by design engineer, the quality loss function is a function of design tolerances. Therefore, quality loss which is presented by cost can be associated with manufacturing cost as an objective function which is minimized in the STSMQ model. In case 1, the design tolerances of three methods for piston are 0.0005106, 0.0006506, 0.0002659 in inch respectively. We can get the value of quality losses of these

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three design tolerances by using quality loss function (equation 3.11). They are 2.90, 4.70, 0.79 in dollars respectively. For cylinder, the quality losses for three method are 2.07, 4.29, 1.0 in dollars. The total quality losses of three method are 4.97, 8.96, 1.79 in dollars. These quality losses values are associated with manufacturing cost as a part of product cost.

In case 2, quality loss is as twice important as manufacturing cost w_1 =1 , w_2 =2, and $C_{p1}=C_{p2}$ =1. The total cost saving of the author's method compared to integrated method is 8.6% and the total cost saving compared to sequential method is 34%(see Table .8). It demonstrates proposed method is more useful in an environment which requires high quality.

In the case 3, $w_1=w_2$ =1, $C_{p1}=C_{p2}$ =0.5, the quality is at a low level. The total saving of the propose method compared to integrated method is 18% and 30% compared to sequential method(see Table 5.9). This shows the proposed method can save more money in an environment in which quality is a low priority. It also implies that there is a need to improve the quality.

In case 4, $w_1=w_2$ =1, $C_{p1}=C_{p2}$ =1.5, the quality is at a high level. Few quality losses occur. Despite this, the total cost saving of the author's method compared to integrated method is still 0.9% and 39% compared to sequential method(see Table 4.9). For mass production, it means remarkable savings.

5.2 Other Examples

In this section, two more complicated examples are used to test the proposed method. Section 5.2.1 describes the example 2. The results of example 2 are given in Section 5.2.2. Section 5.2.3 describes the example 3. The results of example 3 are given in Section 5.2.4. Section 5.2.5 discusses the results of both examples.

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5.2.1 Description of Example 2

This example involves the assembly of speed reducer(refer to Figure 5.2). Between the left bearing and the shaft there is a gap(L_0 in Figure 5.2). If this gap becomes too small there is a risk for jamming when the shaft heats up. On the other hand, if the gap becomes too big it may cause axial motions between the gears, which may damage the gears. To allow interchangeability among parts and to make sure the gap stays within specified limits, tolerances have to be assigned to all dimensions. The tolerances on the gap constrain all other dimensions in the chain. The design function is retrieved from drawing. It is

$$L_0 = L_1 + L_2 - L_3 - L_4 - L_5 - L_6 - L_7 \tag{5.2}$$

The process plan for each dimension is given in Appendix B.1. The data that comes from machining handbooks, Trucks (1989), Machinability Data Center (1980), Dieter (1983) and Dong (1997) are listed in Appendix B.1.

5.2.2 Results of Example 2

The LINGO software package is used to solve the models. Lingo programming for STSMQ method refers to Appendix B.2. Lingo programming for integrated method refers to Appendix B.3. Lingo programming for sequential method refers to Appendix B.4. The results of each case are given below.

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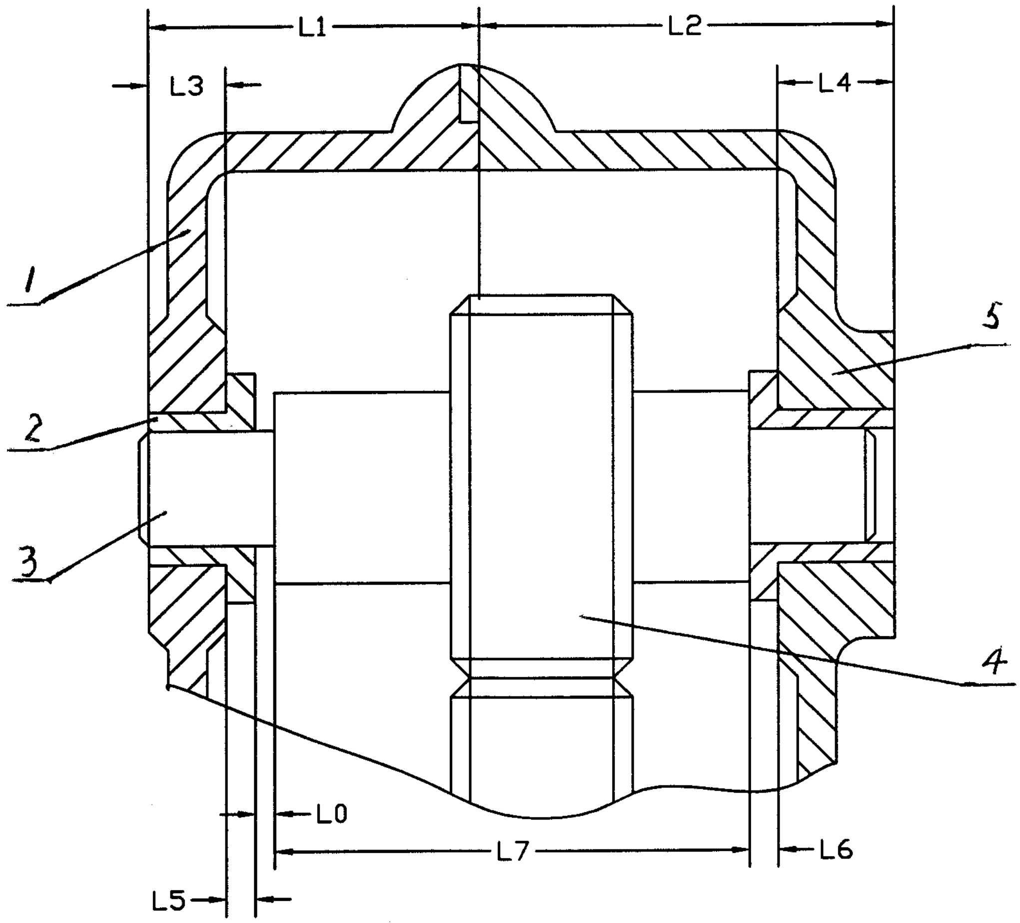

Figure 5.2 Assembly Drawing of Speed Reducer (Example 2)
(Note: 1— Left body, 2— Bore, 3— Shaft, 4— Gear, 5— Right body)

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	Proposed Method	Integrate Method	Sequential Method
Manufacturing cost	161.14	157.24	292.51
Quality loss	24.76	34.67	11.81
Total	185.90	191.91	304.32

Table 5.11 Optimum costs of example 2 for case 1 ($w_1 = w_2 = 1$, $C_{pi} = 1$) (dollars)

Note: This table lists the optimum costs of three methods for the case 1. From table, the total cost saving of the author's model is 3.1% compared to integrated method and 38.9% compared to sequential method

	Proposed Method	Integrated Method	Sequential Method
Manufacturing cost	187.61	157.24	292.51
Quality loss	41.84	69.34	23.62
Total	208.53	226.58	316.13

Table 5.12 Optimum costs of example 2 for case 2 ($w_1 = 1$ $w_2 = 2$, $C_{pi} = 1$) (dollars)

Note: This table lists the optimum costs of three methods for the case 2. From table, the total cost saving of the author's model is 8.0% compared to integrated method and 34% compared to sequential method

	Proposed Method	Integrated Method	Sequential Method
Manufacturing cost	178	157.24	292.51
Quality loss	67.99	138.66	47.26
Total	245.99	295.9	339.77

Table 5.13 Optimum costs of example 2 for case 3 ($w_1 = w_2 = 1$, $C_{pi} = 0.5$) (dollars)

Note: This table lists the optimum costs of three methods for the case 3. From table, the total cost saving of the author's model is 16.9% compared to integrated method and 28% compared to sequential method

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	Proposed Method	Integrated Method	Sequential Method
Manufacturing cost	158.44	157.24	292.51
Quality loss	12.73	15.41	5.25
Total	171.17	172.65	297.76

Table 5.14 Optimum costs of example 2 for case 4 ($w_1 = w_2 = 1$, $C_{pi} = 1.5$) (dollars)

Note: This table lists the optimum costs of three methods for the case 4. From table, the total cost saving of the author's model is 0.9% compared to integrated method and 42.5% compared to sequential method

Dimensions	Tolerances	Proposed Method	Integrated Method	Sequential method
L1	T11	0.5	0.5	0.5
	T12	0.109	0.097	0.095
	T13	0.058	0.070	0.031
	T1d	0.058	0.070	0.031
L2	T21	0.5	0.5	0.5
	T22	0.109	0.097	0.095
	T23	0.058	0.070	0.031
	T2d	0.058	0.070	0.031
L3	T31	0.384	0.375	0.375
	T32	0.241	0.25	0.25
	T33	0.134	0.12	0.05
	T34	0.041	0.05	0.033
	T3d	0.041	0.05	0.033

Table 5.15 Optimum tolerances from dimension L1 to L7 for example 2 (mm)

Note: There are seven dimensions in the design function, from L1 to L7. Each dimension is produced by several operations. The optimum design tolerance, T_{1d}, for each dimension and process tolerances, T_{ij}, for each machining processes to produce each dimension are obtained by using three methods

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Dimensions	Tolerances	Proposed Method	Integrated Method	Traditional method
L4	T41	0.384	0.375	0.375
	T42	0.241	0.25	0.25
	T43	0.134	0.125	0.05
	T44	0.041	0.050	0.033
	T4d	0.041	0.050	0.033
L5	T51	0.375	0.375	0.375
	T52	0.125	0.125	0.125
	T53	0.066	0.064	0.0375
	T54	0.034	0.036	0.028
	T5d	0.034	0.036	0.028
L6	T61	0.375	0.375	0.375
	T62	0.125	0.125	0.125
	T63	0.066	0.064	0.0375
	T64	0.034	0.036	0.028
	T6d	0.034	0.036	0.028
L7	T71	0.888	0.893	0.955
	T72	0.111	0.106	0.45
	T73	0.038	0.043	0.03
	T7d	0.038	0.043	0.03

Table 5.15 Optimum tolerances from dimension L1 to L7 for example 2(continue)

Note: There are seven dimensions in the design function, from L1 to L7. Each dimension is produced by several operations. The optimum design tolerance, T_{1d}, for each dimension and process tolerances, T_{ij}, for each machining processes to produce each dimension are obtained by using three methods

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5.2.3 Description of Example 3

This example involves the assembly of a stamping die (see Figure 5.3). The clearance(L_0 L_0 in Figure 5.3) is the major factor determining the shape and quality of the sheared edge. As clearance increases, the edges become rougher and the zone of deformation become larger. The burr height increases with increasing clearance. In turn, if the clearance is too small, it will shorten the life of dies. In practice, clearance usually range between 2 and 10 percent of the thickness of the sheet. From assembly drawing, we have the design function

$$L_0 = \frac{1}{2}\left(L_5 + L_7 - L_3 - L_4\right) - L_1 - L_6 + L_2 \qquad (5.3)$$

The process plan for each dimension is given in Appendix C.1. The data that comes from machining handbooks, Trucks (1989), Machinability Data Center (1980), Dieter (1983) and Dong (1997) are listed in Appendix C.1.

5.2.4 Results of Example 3

The LINGO software package is used to solve the models. Lingo programming for STSMQ method refers to Appendix C.2. Lingo programming for integrated method refers to Appendix C.3. Lingo programming for sequential method refers to Appendix C.4. The results of each case are given below.

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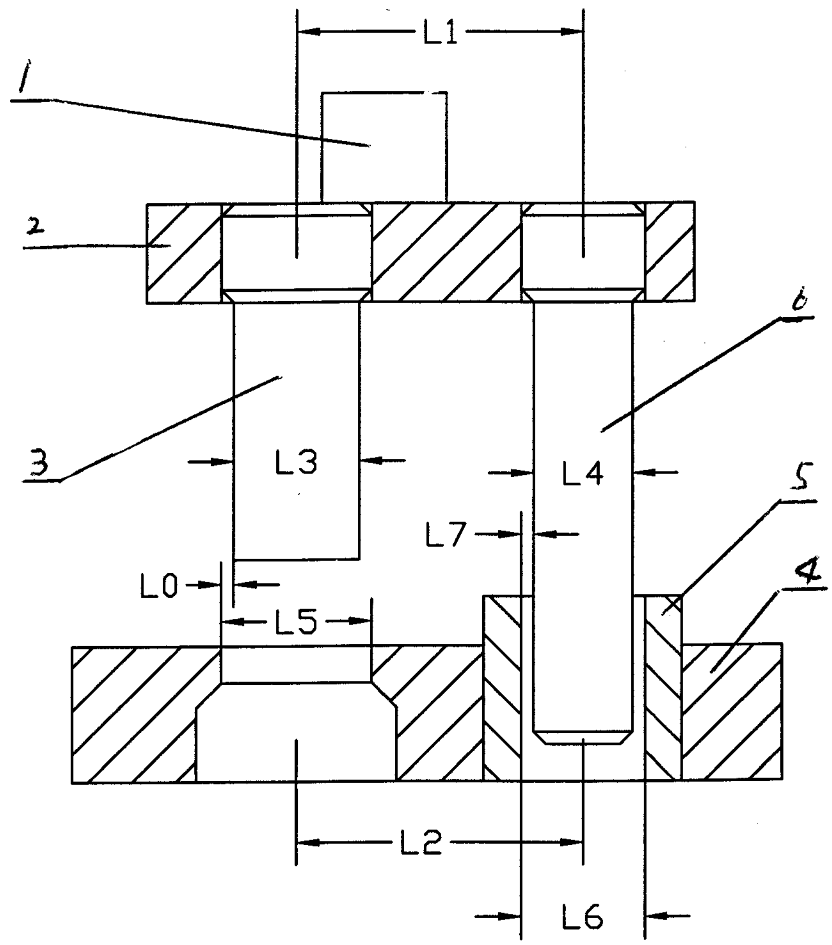

Figure 5.3 Assembly Drawing of Stamping Die (Example 3)
(Note: 1— Holder, 2— Upper plate, 3— Punch,
4— Die shoe, 5— Guide bush, 6— Guide)

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	Proposed Method	Integrated Method	Sequential Method
Manufacturing cost	237.04	231.33	302.8
Quality loss	23.57	38.68	10.15
Total	260.61	270.01	312.95

Table 5.16 Optimum costs of example 3 for case 1 ($w_1 = w_2 = 1$, $C_{pi} = 1$) (dollars)

Note: This table lists the optimum costs of three methods for the case 1. From table, the total cost saving of the author's model is 3.5% compared to integrated method and 17% compared to sequential method

	Proposed Method	Integrated Method	Sequential Method
Manufacturing cost	244.47	231.33	302.8
Quality loss	36.83	67.36	20.30
Total	281.30	298.69	323.1

Table 5.17 Optimum costs of example 3 for case 2 ($w_1 = 1, w_2 = 2$, $C_{pi} = 1$) (dollars)

Note: This table lists the optimum costs of three methods for the case 2. From table, the total cost saving of the author's model is 5.8% compared to integrated method and 13% compared to sequential method

	Proposed Method	Integrated Method	Sequential Method
Manufacturing cost	258.28	231.33	302.8
Quality loss	54.39	154.70	40.62
Total	312.67	386.03	343.42

Table 5.18 Optimum costs of example 3 for case 3 ($w_1 = w_2 = 1$, $C_{pi} = 0.5$) (dollars)

Note: This table lists the optimum costs of three methods for the case 3. From table, the total cost saving of the author's model is 19% compared to integrated method and 9% compared to sequential method

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	Proposed Method	Integrated Method	Sequential Method
Manufacturing cost	233.14	231.33	302.8
Quality loss	12.98	17.19	4.51
Total	246.12	248.52	307.31

Table 5.19 Optimum costs of example 3 for case 4 ($w_1 = w_2 = 1$, $C_{pi} = 1.5$) (dollars)

Note: This table lists the optimum costs of three methods for the case 4. From table, the total cost saving of the author's model is 1.0% compared to integrated method and 20% compared to sequential method

Dimensions	Tolerances	Proposed Method	Integrated Method	Sequential method
L1	T11	0.916	0.890	0.875
	T12	0.584	0.609	0.625
	T13	0.166	0.140	0.125
	T14	0.084	0.11	0.05
	T1d	0.084	0.11	0.05
L2	T21	0.916	0.89	0.875
	T22	0.583	0.609	0.625
	T23	0.166	0.14	0.125
	T24	0.083	0.109	0.05
	T2d	0.083	0.109	0.05

Table 5.20 Optimum tolerances from dimension L1 to L6 for example 3 (mm)

Note: There are six dimensions in the design function, from L1 to L6. Each dimension is produced by several operations. The optimum design tolerance, T_{1d}, for each dimension and process tolerances, T_{ij}, for each machining processes to produce each dimension are obtained by using three methods

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Dimensions	Tolerances	Proposed Method	Integrated Method	Sequential method
L3	T31	0.148	0.149	0.167
	T32	0.052	0.051	0.033
	T33	0.023	0.024	0.021
	T3d	0.023	0.024	0.021
L4	T41	0.279	0.279	0.288
	T42	0.221	0.222	0.212
	T43	0.029	0.028	0.038
	T44	0.021	0.022	0.024
	T4d	0.021	0.022	0.024
L5	T51	0.34	0.34	0.319
	T52	0.16	0.161	0.180
	T53	0.065	0.064	0.045
	T54	0.022	0.023	0.017
	T5d	0.022	0.023	0.017
L6	T61	0.15	0.15	0.16
	T62	0.098	0.098	0.089
	T63	0.026	0.026	0.035
	T64	0.024	0.023	0.024
	T6d	0.024	0.023	0.024

Table 5.20 Optimum tolerances from dimension L1 to L6 for example 3(continue)

Note: There are six dimensions in the design function, from L1 to L6. Each dimension is produced by several operations. The optimum design tolerance, T_{1d}, for each dimension and process tolerances, T_{ij}, for each machining processes to produce each dimension are obtained by using three methods

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5.2.5 Analysis of the Results of Example 2 and 3

The above two examples show results similar to those of Example 1. The total cost savings compared to integrated method ranges from 0.9% to 19%. The total cost savings compared to sequential method ranges from 10% to 38%.

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CHAPTER 6

IMPLEMENTATION IN INDUSTRIAL ENVIRONMENT

In order to apply the simultaneous tolerance synthesis technique in industrial environment, Following procedures should be followed.

I. Setting up a manufacturing database for each machine tool.

The data which are used in this thesis come from other papers and some machining handbooks. In practical applications, a manufacturing process is executed by a specific machine tool. Each machine tool has its own operation features and process capability. First of all, we need to build a manufacturing database for every machine tool. Some of the manufacturing process parameters which will be used in simultaneous tolerance synthesis are process capabilities, cost-process tolerance function, process capability index Cp and maximum machining allowance . All these data can be obtained through experiments. The cost-manufacturing function can be determined from experiment data by means of curve-fitting techniques.

II. System design and Parameter design

As the first step of product design, system design denotes the development of a basic prototype design that performs the desired and required functions of the product with minimum deviation from the desired and required functions of the product with minimum deviation from target performance values. Once the system design is established, the next step is to ascertain the optimal levels for the parameters of each element in the system so that

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the functional deviations of the product are minimized. During these two steps, the functional tolerances are specified.

III. Identifying the functional dimension chains

After system and parameter design, the drawing of product can be obtained and the functional tolerance is specified. According to drawing of product, we can retrieve the functional dimension chains.

IV. Process planning

Process planning is strongly affected by design tolerances. Although the design tolerance has not been decided, we use interim tolerances, an average tolerance value which comes from dividing functional tolerance equally among the number of dimension in a tolerance chain, as a guide to select process. It should be noted that once a feasible process plan is selected, this interim tolerance becomes useless.

V. Sensitivity analysis

The purpose of sensitivity analysis is to see how each component dimension contributes to functional tolerance.

VI. Building optimization model

After all information are obtained, the proposed model is used to build an optimization model.

VII. Solving the optimization model

Once the optimization model has been built, we need to solve this optimization model. There are some commercial software which can be used to solve this model, including LINDO, MatLab, etc. After solving this optimization model, we can get the optimum design tolerance and process tolerances. Theses tolerances will be used as a guide for process planning later.

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CHAPTER 7

CONCLUSION AND FURTHER WORK

7.1 Conclusion

This dissertation presents a simultaneous tolerance synthesis method. It integrates both manufacturing cost and quality loss into one model. It uses process tolerances and design tolerances as decision variables. A nonlinear optimization model is constructed to implement this method. The proposed method is tested by three examples. The results show a significant reduction in the combined manufacturing cost and quality loss was obtained compared to other methods. Savings in total cost ranges from 0.9% to 19% with respect to integrated method, from 10% to 39% over sequential method in their various application cases. Particularly significant savings can be attained in environments in which high quality is a major concern or where quality is at low levels.

Besides the direct saving of total cost, The proposed method has the following benefits:

1. It is a concurrent engineering technique. It breaks the barrier between the design engineer, manufacturing engineer, and quality engineers. Using this model, optimum design tolerances and process tolerances can be allocated at an early stage of design.
2. Since STSMQ method takes both manufacturing and quality into account simultaneously, it eliminates the many potential causes of redesign and re-planning for manufacturing. It shortens the lead time of product development.
3. Using a cost-process tolerance function provides a more accurate model of manufacturing cost than a cost-design tolerance function.

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4. Using quality loss functions allows design engineers to compare the increased performance from tight tolerances with their associated increased manufacturing costs. In this model, loosening the tolerances is accompanied by an increase in quality loss. The purpose of this model is to strike a balance between decreased manufacturing cost and increased quality loss to achieve optimum tolerances.

5. Using process capability indices provides the relationship between the design tolerance and deviation of manufacturing process. It shows the balance of responsibility for quality between design and manufacturing engineers.

This research has identified a problem with concurrent techniques in the design of tolerances. The proposed solution is shown to allow noticeable and sometimes significant savings in the combined manufacturing cost and quality loss of a product. This method, if applied in industry, will allow engineers to design better products. Although the applicability of the proposed method is currently somewhat limited, there is no reason to believe these shortcomings cannot be overcome with further research.

7.2 Further Research

1. The current model can only deal with dimensional tolerances. It is necessary to expend the proposed model to include geometric tolerances. One of the roadblocks in achieving this is how to establish a cost-geometric tolerance function.

2. Robust design is also an area for further research. The goal of robust design is to find the optimum design under conditions of manufacturing variability, such that the design parameters are set so that the effects of the tolerances are minimized. One question here is: can robust design principles be incorporated directly into the proposed model? The relationship between the proposed method and robust design needs further exploration.

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3. The proposed model needs to be expanded to include the selection of machine tools associated with each manufacturing process. In this case, a mixed integer, nonlinear programming model will be formulated. The problem is that there is no sufficient method available to solve this model.

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APPENDIX A

A.1 STSMQ Model of Example 1

MINIMIZE

$$C_{total} = 5e^{-309(T11-0.005)} + 1.51 + 9e^{-790(T12-0.00204)} + 4.36 +$$
$$13e^{-3196(T13-0.00053)} + 7.48 + 18e^{-8353(T14-0.000219)} + 11.99 +$$
$$4e^{-299(T21-0.00702)} + 2.35 + 8e^{-986(T22-0.00297)} + 5.29 +$$
$$10e^{-3206(T23-0.0006)} + 9.67 + 2e^{-9428(T11-0.00036)} + 13.12$$

SUBJECT TO

Design function constraints
T14^2+T24^2<=0.0001^2

Machining operation constraints
T11+T12<=0.02
T12+T13<=0.005
T13+T14<=0.0018
T21+T22<=0.02
T22+T23<=0.005
T23+T24<=0.0018

Process capability constraints
T11>=0.005
T11<=0.02
T12>=0.002
T12<=0.012
T13>=0.0005
T13<=0.003
T14>=0.0002
T14<=0.001
T21>=0.007
T21<=0.02
T22>=0.003
T22<=0.012
T23>=0.0003

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T23<=0.005
T24>=0.0003
T24<=0.002

The equation of manufacturing and design tolerance
T14=T1d
T24=T2d

Quality Loss Coefficient
K=100000000

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A2. Lingo Programming of STSMQ Model for Example 1

```
!OBJECTIVE FUNCTION;
MIN=
!FOR PISTON;
5/@EXP(309*(T11-0.005)) + 1.51 +
9/@EXP(790*(T12-0.00204)) + 4.36+
13/@EXP(3196*(T13-0.00053))+7.48+
18/@EXP(8353*(T14-0.000219))+11.99+k*(T1d/3)^2+
!FOR CYLINDER;
4/@EXP(299*(T21-0.00702))+2.35+
8/@EXP(986*(T22-0.00297))+5.29+
10/@EXP(3206*(T23-0.0006))+9.67+
2/@EXP(9428*(T24-0.00036))+13.12+k*(T2d/3)^2;

!CONSTRAINTS;

!DESIGN FUNCTION CONSTRAINT;
T1d^2+T2d^2<=0.001^2;

!MACHINING ALLOWANCE CONSTRAINTS;

!FOR PISTON;
T11+T12<=0.02;
T12+T13<=0.005;
T13+T14<=0.0018;

!FOR CYLINDER;
T21+T22<=0.02;
T22+T23<=0.005;
T23+T24<=0.0018;

!PROCESS CAPABILITY CONSTRAINTS;

!FOR PISTON;
T11>=0.005;
T11<=0.02;
T12>=0.002;
T12<=0.012;
T13>=0.0005;
T13<=0.003;
T14>=0.0002;
T14<=0.001;

!FOR CYLINDER;
T21>=0.007;
```


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```
T21<=0.02;
T22>=0.003;
T22<=0.012;
T23>=0.0003;
T23<=0.005;
T24>=0.0003;
T24<=0.002;

!RELATIONSHIP  OF DESIGN TOLERANCE AND
MANUFACTURING TOLERANCE CONSTRAINTS;
T14=T1d;
T24=T2d;

!QUALITY LOSS COEFFICIENT
K=100000000
```


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A3.Lingo Programming of Integrated Model for Example 1

```
!OBJECTIVE FUNCTION;
MIN=
!FOR PISTON ( FOUR PROCESSES);
5/@EXP(309*(T11-0.005))+1.51+
9/@EXP(790*(T12-0.00204))+4.36+
13/@EXP(3196*(T13-0.00053))+7.48+
18/@EXP(8353*(T14-0.000219))+11.99+
!FOR CYLINDER ( FOUR PROCESSES);
4/@EXP(299*(T21-0.00702))+2.35+
8/@EXP(986*(T22-0.00297))+5.29+
10/@EXP(3206*(T23-0.0006))+9.67+
2/@EXP(9428*(T24-0.00036))+13.12;

!CONSTRAINTS;

!MACHINING ALLOWANCE  CONSTRAINTS;
!FOR PISTON;
T11+T12<=0.02;
T12+T13<=0.005;
T13+T14<=0.0018;

!FOR CYLINDER;
T21+T22<=0.02;
T22+T23<=0.005;
T23+T24<=0.0018;

!PROCESS CAPABILITY CONSTRAINTS;
T11>=0.005;
T11<=0.02;
T12>=0.002;
T12<=0.012;
T13>=0.0005;
T13<=0.003;
T14>=0.0002;
T14<=0.001;
T21>=0.007;
T21<=0.02;
T22>=0.003;
T22<=0.012;
T23>=0.0006;
T23<=0.005;
T24>=0.0003;
T24<=0.002;
```


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A4. Lingo Programming of Sequential Method for Example 1

```
!Step 1: Tolerance Synthesis in Design Stage;
!OBJECTIVE FUNCTION;
MIN=
18/@EXP(8353*(T1d-0.000219))+11.99+
2/@EXP(9428*(T2d-0.00036))+13.12;
!DESIGN FUNCTION CONSTRAINTS;
T1d^2+T2d^2<=0.001^2;

!Step 2: Tolerance Synthesis in  Process Planning Stage;

! FOR PISTON;
!OBJECTIVE FUNCTION;
MIN=5/@EXP(309*(T11-0.005))+1.51+
9/@EXP(790*(T12-0.00204))+4.36+
13/@EXP(3196*(T13-0.00053))+7.48+
18/@EXP(8353*(T14-0.000219))+11.99;

!MACHINING ALLOWANCE CONSTRAINTS;
T11+T12<=0.02;
T12+T13<=0.005;
T13+T14<=0.7659299E-03;

!PROCESS CAPABILITY CONSTRAINTS;
T11>=0.005;
T11<=0.02;
T12>=0.002;
T12<=0.012;
T13>=0.0005;
T13<=0.003;
T14>=0.0002;
T14<=0.001;

!FOR CYLINDER;
!OBJECTIVE FUNCTION;
MIN=4/@EXP(299*(T21-0.00702))+2.35+
8/@EXP(986*(T22-0.00297))+5.29+
10/@EXP(3206*(T23-0.0006))+9.67+
2/@EXP(9428*(T24-0.00036))+13.12;

!MACHINING ALLOWANCE CONSTRAINTS;
T21+T22<=0.02;
T22+T23<=0.005;
T23+T24<=0.6429241E-03;
```


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```
!PROCESS CAPABILITY CONSTRAINTS;
T21>=0.007;
T21<=0.02;
T22>=0.0003;
T22<=0.012;
T23>=0.0003;
T23<=0.005;
T24>=0.0003;
T24<=0.002;
```


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APPENDIX B

B1. The Data for Example 2

1. Process sequence, process capability, and machining allowance

L1 and L2			
Sequence	Operations	Process Capabilities	Machining Allowance
1	rough shape	0.011=<T11<=0.02	
2	finish shape	0.0038=<T12<=0.009	0.04
3	broaching	0.0007=<T13<=0.005	0.0067

L3 and L4			
Sequence	Operations	Process Capabilities	Machining Allowance
1	rough mill	0.009=<T11<=0.02	
2	finish mill	0.005=<T12<=0.01	0.025
3	rough grind	0.002=<T13<=0.008	0.015
4	finish grind	0.0005=<T14<=0.005	0.007

L5 and L6			
Sequence	Operations	Process Capabilities	Machining Allowance
1	rough turn	0.015=<T11<=0.03	
2	finish turn	0.003=<T12<=0.025	0.02
3	rough grind	0.0015=<T13<=0.008	0.008
4	finish grind	0.0003=<T14<=0.003	0.004

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L7			
Sequence	Operations	Process Capabilities	Machining Allowance
1	rough turn	0.015=<T11<=0.04	
2	finish turn	0.0018=<T12<=0.025	0.04
3	grind	0.0004=<T13<=0.007	0.006

2. The coefficients of cost-process tolerance function for each process

Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$
A	2.5	6	11
B	105	400	2043
C	0.013	0.0043	0.0018
D	1.1	2.0	6.1

Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$	$C_{m24}(T_{24})$
A	3	5	8	13
B	133	309	640	3196
C	0.0093	0.005	0.0028	0.00093
D	1.21	1.51	3.54	7.48

Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$	$C_{m24}(T_{24})$
A	2	7	9	15
B	85	510	790	4985
C	0.017	0.0036	0.002	0.0009
D	1.0	2.79	6.13	9.34

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Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$
A	2	7	14
B	85	510	3970
C	0.017	0.0036	0.00096
D	1.0	2.79	8.2

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B2. Lingo Programming of STSMQ Model for Example 2

```
!OBJECTIVE FUNCTION;
MIN=
2.5/@EXP(105*(T11-0.013)) + 1.1 +
6/@EXP(400*(T12-0.0043)) +2.0+
11/@EXP(2043*(T13-0.00018))+6.1+k*(T1d/3)^2+

2.5/@EXP(105*(T21-0.013)) + 1.1 +
6/@EXP(400*(T22-0.0043)) +2.0+
11/@EXP(2043*(T23-0.00018))+6.1+k*(T2d/3)^2+

3/@EXP(133*(T31-0.0096))+1.21+
5/@EXP(309*(T32-0.005))+1.51+
8/@EXP(640*(T33-0.00284))+3.54+
13/@EXP(3196*(T34-0.00093))+7.48+k*(T3d/3)^2+

3/@EXP(133*(T41-0.0096))+1.21+
5/@EXP(309*(T42-0.005))+1.51+
8/@EXP(640*(T43-0.00284))+3.54+
13/@EXP(3196*(T44-0.00093))+7.48+k*(T4d/3)^2+

2/@EXP(85*(T51-0.017))+1.0+
7/@EXP(510*(T52-0.0036))+2.79+
9/@EXP(790*(T53-0.00204))+6.13+
15/@EXP(4985*(T54-0.0009))+9.34+k*(T5d/3)^2+

2/@EXP(85*(T61-0.017))+1.0+
7/@EXP(510*(T62-0.0036))+2.79+
9/@EXP(790*(T63-0.00204))+6.13+
15/@EXP(4985*(T64-0.0009))+9.34+k*(T6d/3)^2+

2/@EXP(85*(T71-0.017))+1.0+
7/@EXP(510*(T72-0.0036))+2.79+
14/@EXP(3970*(T73-0.00096))+8.2+k*(T7d/3)^2;

!CONSTRAINTS;

!DESIGN FUNCTION CONSTRAINT;
T1d^2+T2d^2+T3d^2+T4d^2+T5d^2+T6d^2+T7d^2<=0.0098^2;

!MACHINING ALLOWANCE CONSTRAINTS;
T11+T12<=0.04;
T12+T13<=0.0067;

T21+T22<=0.04;
```


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```
T22+T23<=0.0067;

T31+T32<=0.025;
T32+T33<=0.015;
T33+T34<=0.007;

T41+T42<=0.025;
T42+T43<=0.015;
T43+T44<=0.007;

T51+T52<=0.02;
T52+T53<=0.008;
T53+T54<=0.004;

T61+T62<=0.02;
T62+T63<=0.008;
T63+T64<=0.004;

T71+T72<=0.04;
T72+T73<=0.006;

!PROCESS CAPABILITY CONSTRAINTS;
T11>=0.011;
T11<=0.02;
T12>=0.0038;
T12<=0.009;
T13>=0.0007;
T13<=0.005;

T21>=0.011;
T21<=0.02;
T22>=0.0038;
T22<=0.009;
T23>=0.0007;
T23<=0.005;

T31>=0.009;
T31<=0.020;
T32>=0.005;
T32<=0.01;
T33>=0.002;
T33<=0.008;
T34>=0.0005;
T34<=0.005;

T41>=0.009;
```


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```
T41<=0.020;
T42>=0.005;
T42<=0.01;
T43>=0.002;
T43<=0.008;
T44>=0.0005;
T44<=0.005;

T51>=0.015;
T51<=0.03;
T52>=0.003;
T52<=0.025;
T53>=0.0015;
T53<=0.008;
T54>=0.0003;
T54<=0.003;

T61>=0.015;
T61<=0.03;
T62>=0.003;
T62<=0.025;
T63>=0.0015;
T63<=0.008;
T64>=0.0003;
T64<=0.003;

T71>=0.015;
T71<=0.04;
T72>=0.0018;
T72<=0.025;
T73>=0.0004;
T73<=0.007;

!Relationship of Design tolerance and Manufacturing tolerance;
T13=T1d;
T23=T2d;
T34=T3d;
T44=T4d;
T54=T5d;
T64=T6d;
T73=T7d;

!QUALITY LOSS COEFFICIENT;
K=10000000;
```


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B3. Lingo Programming of Integrated Method for Example 2

```
!OBJECTIVE FUNCTION;
MIN=
2.5/@EXP(105*(T11-0.013)) + 1.1 +
6/@EXP(400*(T12-0.0043)) +2.0+
11/@EXP(2043*(T13-0.00018))+6.1+

2.5/@EXP(105*(T21-0.013)) + 1.1 +
6/@EXP(400*(T22-0.0043)) +2.0+
11/@EXP(2043*(T23-0.00018))+6.1+

3/@EXP(133*(T31-0.0096))+1.21+
5/@EXP(309*(T32-0.005))+1.51+
8/@EXP(640*(T33-0.00284))+3.54+
13/@EXP(3196*(T34-0.00093))+7.48+

3/@EXP(133*(T41-0.0096))+1.21+
5/@EXP(309*(T42-0.005))+1.51+
8/@EXP(640*(T43-0.00284))+3.54+
13/@EXP(3196*(T44-0.00093))+7.48+

2/@EXP(85*(T51-0.017))+1.0+
7/@EXP(510*(T52-0.0036))+2.79+
9/@EXP(790*(T53-0.00204))+6.13+
15/@EXP(4985*(T54-0.0009))+9.34+

2/@EXP(85*(T61-0.017))+1.0+
7/@EXP(510*(T62-0.0036))+2.79+
9/@EXP(790*(T63-0.00204))+6.13+
15/@EXP(4985*(T64-0.0009))+9.34+

2/@EXP(85*(T71-0.017))+1.0+
7/@EXP(510*(T72-0.0036))+2.79+
14/@EXP(3970*(T73-0.00096))+8.2;

!CONSTRAINTS;

!DESIGN FUNCTION CONSTRAINT;
T13^2+T23^2+T34^2+T44^2+T54^2+T64^2+T73^2<=0.0098^2;

!MACHINING ALLOWANCE CONSTRAINTS;
T11+T12<=0.04;
T12+T13<=0.0067;

T21+T22<=0.04;
```


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```
T22+T23<=0.0067;

T31+T32<=0.025;
T32+T33<=0.015;
T33+T34<=0.007;

T41+T42<=0.025;
T42+T43<=0.015;
T43+T44<=0.007;

T51+T52<=0.02;
T52+T53<=0.008;
T53+T54<=0.004;

T61+T62<=0.02;
T62+T63<=0.008;
T63+T64<=0.004;

T71+T72<=0.04;
T72+T73<=0.006;

!PROCESS CAPABILITY CONSTRAINTS;
T11>=0.011;
T11<=0.02;
T12>=0.0038;
T12<=0.009;
T13>=0.0007;
T13<=0.005;

T21>=0.011;
T21<=0.02;
T22>=0.0038;
T22<=0.009;
T23>=0.0007;
T23<=0.005;

T31>=0.009;
T31<=0.020;
T32>=0.005;
T32<=0.01;
T33>=0.002;
T33<=0.008;
T34>=0.0005;
T34<=0.005;

T41>=0.009;
```


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```
T41<=0.020;
T42>=0.005;
T42<=0.01;
T43>=0.002;
T43<=0.008;
T44>=0.0005;
T44<=0.005;

T51>=0.015;
T51<=0.03;
T52>=0.003;
T52<=0.025;
T53>=0.0015;
T53<=0.008;
T54>=0.0003;
T54<=0.003;

T61>=0.015;
T61<=0.03;
T62>=0.003;
T62<=0.025;
T63>=0.0015;
T63<=0.008;
T64>=0.0003;
T64<=0.003;

T71>=0.015;
T71<=0.04;
T72>=0.0018;
T72<=0.025;
T73>=0.0004;
T73<=0.007;
```


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B4. Lingo Programming of Sequential Method for Example 2

Step 1 Tolerance Synthesis in Design Stage

```
!OBJECTIVE FUNCTION;
MIN=
11/@EXP(2043*(T1d-0.00018))+6.1+
11/@EXP(2043*(T2d-0.00018))+6.1+
13/@EXP(3196*(T3d-0.00093))+7.48+
13/@EXP(3196*(T4d-0.00093))+7.48+
15/@EXP(4985*(T5d-0.0009))+9.34+
15/@EXP(4985*(T6d-0.0009))+9.34+
14/@EXP(3970*(T7d-0.00096))+8.2;

!CONSTRAINTS;

!DESIGN FUNCTION CONSTRAINT;
T1d^2+T2d^2+T3d^2+T4d^2+T5d^2+T6d^2+T7d^2<=0.0098^2;
```

Step 2: Tolerance Synthesis in Process Planning

```
!FOR DIMENSION L1;

!OBJECTIVE FUNCTION;
MIN=
2.5/@EXP(105*(T11-0.013)) + 1.1 +
6/@EXP(400*(T12-0.0043)) +2.0+
11/@EXP(2043*(T13-0.00018))+6.1;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T11+T12<=0.04;
T12+T13<=0.4961270E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T11>=0.011;
T11<=0.02;
T12>=0.0038;
T12<=0.009;
T13>=0.0007;
T13<=0.005;

!FOR DIMENSION L2;

!OBJECTIVE FUNCTION;
```


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```
MIN=
2.5/@EXP(105*(T21-0.013)) + 1.1 +
6/@EXP(400*(T22-0.0043)) +2.0+
11/@EXP(2043*(T23-0.00018))+6.1;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T21+T22<=0.04;
T22+T23<=0.5055428E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T21>=0.011;
T21<=0.02;
T22>=0.0038;
T22<=0.009;
T23>=0.0007;
T23<=0.005;

!FOR DIMENSION L3

!OBJECTIVE FUNCTION;
MIN=
3/@EXP(133*(T31-0.0096))+1.21+
5/@EXP(309*(T32-0.005))+1.51+
8/@EXP(640*(T33-0.00284))+3.54+
13/@EXP(3196*(T34-0.00093))+7.48;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T31+T32<=0.025;
T32+T33<=0.015;
T33+T34<=0.3532754E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T31>=0.009;
T31<=0.020;
T32>=0.005;
T32<=0.01;
T33>=0.002;
T33<=0.008;
T34>=0.0005;
T34<=0.005;

!FOR DIMENSION L4;
```


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```
!OBJECTIVE FUNCTION;
MIN=
3/@EXP(133*(T41-0.0096))+1.21+
5/@EXP(309*(T42-0.005))+1.51+
8/@EXP(640*(T43-0.00284))+3.54+
13/@EXP(3196*(T44-0.00093))+7.48;
!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T41+T42<=0.025;
T42+T43<=0.015;
T43+T44<=0.3541106E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T41>=0.009;
T41<=0.020;
T42>=0.005;
T42<=0.01;
T43>=0.002;
T43<=0.008;
T44>=0.0005;
T44<=0.005;

!FOR DIMENSION L5;

!OBJECTIVE FUNCTION;
MIN=
2/@EXP(85*(T51-0.017))+1.0+
7/@EXP(510*(T52-0.0036))+2.79+
9/@EXP(790*(T53-0.00204))+6.13+
15/@EXP(4985*(T54-0.0009))+9.34;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T51+T52<=0.02;
T52+T53<=0.008;
T53+T54<=0.2499848E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T51>=0.015;
T51<=0.03;
T52>=0.003;
T52<=0.025;
T53>=0.0015;
T53<=0.008;
```


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```
T54>=0.0003;
T54<=0.003;

!FOR DIMENSION L6;

!OBJECTIVE FUNCTION;
MIN=
2/@EXP(85*(T61-0.017))+1.0+
7/@EXP(510*(T62-0.0036))+2.79+
9/@EXP(790*(T63-0.00204))+6.13+
15/@EXP(4985*(T64-0.0009))+9.34;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T61+T62<=0.02;
T62+T63<=0.008;
T63+T64<=0.2494072E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T61>=0.015;
T61<=0.03;
T62>=0.003;
T62<=0.025;
T63>=0.0015;
T63<=0.008;
T64>=0.0003;
T64<=0.003;

!FOR DIMENSION L7;

!OBJECTIVE FUNCTION;
MIN=
2/@EXP(85*(T71-0.017))+1.0+
7/@EXP(510*(T72-0.0036))+2.79+
14/@EXP(3970*(T73-0.00096))+8.2;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T71+T72<=0.04;
T72+T73<=0.2894656E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T71>=0.015;
T71<=0.04;
```


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T72>=0.0018;
T72<=0.025;
T73>=0.0004;
T73<=0.007;

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APPENDIX C

C1. Data of Example 3

1. Process sequence, process capability, and machining allowance (inch)

L1 and L2			
Sequence	Operations	Process Capabilities	Machining Allowance
1	drill	0.009=<T11<=0.06	
2	bore	0.007=<T12<=0.04	0.06
3	semi-bore	0.005=<T13<=0.01	0.03
4	finish bore	0.001=<T14<=0.008	0.01

L3			
Sequence	Operations	Process Capabilities	Machining Allowance
1	rough turn	0.002=<T11<=0.02	
2	semi-turn	0.0008=<T12<=0.005	0.008
3	finish-turn	0.0004=<T13<=0.002	0.003

L4			
Sequence	Operations	Process Capabilities	Machining Allowance
1	rough turn	0.005=<T11<=0.02	
2	finish turn	0.002=<T12<=0.012	0.02
3	rough grind	0.0005=<T13<=0.003	0.01
4	finish grind	0.0002=<T14<=0.002	0.002

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L5			
Sequence	Operations	Process Capabilities	Machining Allowance
1	drill	0.007=<T11<=0.02	
2	bore	0.003=<T12<=0.012	0.02
3	semi-bore	0.0006=<T13<=0.005	0.009
4	finish bore	0.0003=<T14<=0.002	0.0035

L6			
Sequence	Operations	Process Capabilities	Machining Allowance
1	rough bore	0.003=<T11<=0.01	
2	semi bore	0.0008=<T12<=0.005	0.01
3	rough grind	0.0004=<T13<=0.0012	0.005
4	finish grind	0.0002=<T14<=0.00095	0.002

2. The coefficients of cost-process tolerance function for each process

Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$	$C_{m24}(T_{24})$
A	2	4	8	14
B	75	156	408	978
C	0.02	0.01	0.0053	0.00028
D	1.20	2.5	4.9	9.1

Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$
A	4	8	14
B	205	640	3970
C	0.006	0.0028	0.00046
D	1.38	3.54	8.2

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Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$	$C_{m24}(T_{24})$
A	3	5	9	13
B	133	309	790	3196
C	0.0096	0.005	0.00204	0.00053
D	1.21	1.51	4.36	7.48

Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$	$C_{m24}(T_{24})$
A	4	6	8	10
B	299	510	986	3206
C	0.00702	0.0041	0.00297	0.0006
D	2.35	3.5	5.29	9.67

Coefficient	$C_{m21}(T_{21})$	$C_{m22}(T_{22})$	$C_{m23}(T_{23})$	$C_{m24}(T_{24})$
A	5	7	10	13
B	381	935	2170	4240
C	0.0061	0.0029	0.0012	0.0008
D	2.97	7.31	10.9	12.2

3. The tolerance of dimension L7 is given by 0.0005 inch.

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C2. Lingo Programming of STSMQ Model for Example 3

```
!OBJECTIVE FUNCTION;
MIN=
2/@EXP(75*(T11-0.02))+1.2+
4/@EXP(156*(T12-0.01))+2.5+
8/@EXP(408*(T13-0.0053))+4.9+
14/@EXP(978*(T14-0.0028))+9.1+k*(T1d/3)^2+

2/@EXP(75*(T21-0.02))+1.2+
4/@EXP(156*(T22-0.01))+2.5+
8/@EXP(408*(T23-0.0053))+4.9+
14/@EXP(978*(T24-0.0028))+9.1+k*(T2d/3)^2+

4/@EXP(205*(T31-0.006))+1.38+
8/@EXP(640*(T32-0.0028))+3.54+
14/@EXP(3970*(T33-0.00046))+8.2+k*(T3d/3)^2+

3/@EXP(133*(T41-0.0096))+1.21+
5/@EXP(309*(T42-0.005))+1.51 +
9/@EXP(790*(T43-0.00204)) + 4.36+
13/@EXP(3196*(T44-0.00053))+7.48+k*(T4d/3)^2+

4/@EXP(299*(T51-0.00702))+2.35+
6/@EXP(510*(T52-0.0041))+3.5+
8/@EXP(986*(T53-0.00297))+5.29+
10/@EXP(3206*(T54-0.0006))+9.67+k*(T5d/3)^2+

5/@EXP(381*(T61-0.0061))+2.97+
7/@EXP(935*(T62-0.0029))+7.31+
10/@EXP(2170*(T63-0.0012))+10.9+
13/@EXP(4240*(T64-0.0008))+12.2+k*(T6d/3)^2;

!CONSTRAINTS;

!DESIGN FUNCTION CONSTRAINT;
T1d^2+T2d^2+T3d^2+T4d^2+T5d^2+T6d^2+0.0005^2<=0.011^2;

!MACHINING ALLOWANCE CONSTRAINTS;
!FOR DIMENSION L1;
T11+T12<=0.06;
T12+T13<=0.03;
T13+T14<=0.01;

!FOR DIMENSION L2;
T21+T22<=0.06;
```


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```
T22+T23<=0.03;
T23+T24<=0.01;


T31+T32<=0.008;
T32+T33<=0.003;

T41+T42<=0.02;
T42+T43<=0.01;
T43+T44<=0.002;

T51+T52<=0.02;
T52+T53<=0.009;
T53+T54<=0.0035;

T61+T62<=0.01;
T62+T63<=0.005;
T63+T64<=0.002;

!PROCESS CAPABILITY CONSTRAINTS;

T11>=0.009;
T11<=0.06;
T12>=0.007;
T12<=0.04;
T13>=0.005;
T13<=0.01;
T14>=0.001;
T14<=0.008;

T21>=0.009;
T21<=0.06;
T22>=0.007;
T22<=0.04;
T23>=0.005;
T23<=0.01;
T24>=0.001;
T24<=0.008;

T31>=0.002;
T31<=0.02;
T32>=0.0008;
T32<=0.005;
T33>=0.0004;
T33<=0.002;
```


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```
T41>=0.005;
T41<=0.020;
T42>=0.002;
T42<=0.012;
T43>=0.0005;
T43<=0.003;
T44>=0.0002;
T44<=0.002;

T51>=0.007;
T51<=0.02;
T52>=0.003;
T52<=0.012;
T53>=0.0006;
T53<=0.002;
T54>=0.0003;
T54<=0.002;

T61>=0.003;
T61<=0.01;
T62>=0.0008;
T62<=0.005;
T63>=0.0004;
T63<=0.0012;
T64>=0.0002;
T64<=0.00095;

!RELATIONSHIP OF DESIGN TOLERANCE AND
MANUFACTURING TOLERANCE
T14=T1d;
T24=T2d;
T33=T3d;
T44=T4d;
T54=T5d;
T64=T6d;

!QUALITY LOSS COEFFICIENT;
K=8260000;
```


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C3. Lingo Programming of Integrated Method for Example 3

```
!OBJECTIVE FUNCTION;
MIN=
2/@EXP(75*(T11-0.02))+1.2+
4/@EXP(156*(T12-0.01))+2.5+
8/@EXP(408*(T13-0.0053))+4.9+
14/@EXP(978*(T14-0.0028))+9.1+

2/@EXP(75*(T21-0.02))+1.2+
4/@EXP(156*(T22-0.01))+2.5+
8/@EXP(408*(T23-0.0053))+4.9+
14/@EXP(978*(T24-0.0028))+9.1+

4/@EXP(205*(T31-0.006))+1.38+
8/@EXP(640*(T32-0.0028))+3.54+
14/@EXP(3970*(T33-0.00046))+8.2+

3/@EXP(133*(T41-0.0096))+1.21+
5/@EXP(309*(T42-0.005))+1.51 +
9/@EXP(790*(T43-0.00204)) + 4.36+
13/@EXP(3196*(T44-0.00053))+7.48+

4/@EXP(299*(T51-0.00702))+2.35+
6/@EXP(510*(T52-0.0041))+3.5+
8/@EXP(986*(T53-0.00297))+5.29+
10/@EXP(3206*(T54-0.0006))+9.67+

5/@EXP(381*(T61-0.0061))+2.97+
7/@EXP(935*(T62-0.0029))+7.31+
10/@EXP(2170*(T63-0.0012))+10.9+
13/@EXP(4240*(T64-0.0008))+12.2;

!CONSTRAINTS;

!DESIGN FUNCTION CONSTRAINT;
T14^2+T24^2+T33^2+T44^2+T54^2+T64^2+0.0005^2<=0.011^2;

!MACHINING ALLOWANCE CONSTRAINTS;
!FOR DIMENSION L1;
T11+T12<=0.06;
T12+T13<=0.03;
T13+T14<=0.01;

!FOR DIMENSION L2;
T21+T22<=0.06;
```


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```
T22+T23<=0.03;
T23+T24<=0.01;

T31+T32<=0.008;
T32+T33<=0.003;

T41+T42<=0.02;
T42+T43<=0.01;
T43+T44<=0.002;

T51+T52<=0.02;
T52+T53<=0.009;
T53+T54<=0.0035;

T61+T62<=0.01;
T62+T63<=0.005;
T63+T64<=0.003;

!PROCESS CAPABILITY CONSTRAINTS;
T11>=0.009;
T11<=0.06;
T12>=0.007;
T12<=0.04;
T13>=0.005;
T13<=0.01;
T14>=0.001;
T14<=0.008;

T21>=0.009;
T21<=0.06;
T22>=0.007;
T22<=0.04;
T23>=0.005;
T23<=0.01;
T24>=0.001;
T24<=0.008;

T31>=0.002;
T31<=0.02;
T32>=0.0008;
T32<=0.005;
T33>=0.0004;
T33<=0.002;

T41>=0.005;
T41<=0.020;
```


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```
T42>=0.002;
T42<=0.012;
T43>=0.0005;
T43<=0.003;
T44>=0.0002;
T44<=0.002;

T51>=0.007;
T51<=0.02;
T52>=0.003;
T52<=0.012;
T53>=0.0006;
T53<=0.005;
T54>=0.0003;
T54<=0.002;

T61>=0.003;
T61<=0.01;
T62>=0.0008;
T62<=0.005;
T63>=0.0004;
T63<=0.002;
T64>=0.0002;
T64<=0.00095;
```


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C4. Lingo Programming of Sequential Method for Example 3

Step 1: Tolerance Synthesis in Design Stage

```
!OBJECTIVE FUNCTION;
MIN=
14/@EXP(978*(T14-0.0028))+9.1+
14/@EXP(978*(T24-0.0028))+9.1+
14/@EXP(3970*(T33-0.00046))+8.2+
13/@EXP(3196*(T44-0.00053))+7.48+
10/@EXP(3206*(T54-0.0006))+9.67+
13/@EXP(4240*(T64-0.0008))+12.2;

!CONSTRAINTS;

!DESIGN FUNCTION CONSTRAINT;
T1d^2+T2d^2+T3d^2+T4d^2+T5d^2+T6d^2+0.0005^2<=0.011^2;
```

Step 2: Tolerance Synthesis in Process Planning

```
!FOR DIMENSION L1;
!OBJECTIVE FUNCTION;
MIN=
2/@EXP(75*(T11-0.02))+1.2+
4/@EXP(156*(T12-0.01))+2.5+
8/@EXP(408*(T13-0.0053))+4.9+
14/@EXP(978*(T14-0.0028))+9.1;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T11+T12<=0.06;
T12+T13<=0.03;
T13+T14<=0.7009450E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T11>=0.009;
T11<=0.06;
T12>=0.007;
T12<=0.04;
T13>=0.005;
T13<=0.01;
T14>=0.001;
T14<=0.008;

!FOR DIMENSION L2;
```


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```
!OBJECTIVE FUNCTION;
MIN=
2/@EXP(75*(T21-0.02))+1.2+
4/@EXP(156*(T22-0.01))+2.5+
8/@EXP(408*(T23-0.0053))+4.9+
14/@EXP(978*(T24-0.0028))+9.1;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T21+T22<=0.06;
T22+T23<=0.03;
T23+T24<=0.7009450E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T21>=0.009;
T21<=0.06;
T22>=0.007;
T22<=0.04;
T23>=0.005;
T23<=0.01;
T24>=0.001;
T24<=0.008;

!FOR DIMENSION L3;
!OBJECTIVE FUNCTION;
MIN=
4/@EXP(205*(T31-0.006))+1.38+
8/@EXP(640*(T32-0.0028))+3.54+
14/@EXP(3970*(T33-0.00046))+8.2;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T31+T32<=0.008;
T32+T33<=0.2147821E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T31>=0.002;
T31<=0.02;
T32>=0.0008;
T32<=0.005;
T33>=0.0004;
T33<=0.002;

!FOR DIMENSION L4;
```


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```
!OBJECTIVE FUNCTION;
MIN=
3/@EXP(133*(T41-0.0096))+1.21+
5/@EXP(309*(T42-0.005))+1.51 +
9/@EXP(790*(T43-0.00204)) + 4.36+
13/@EXP(3196*(T44-0.00053))+7.48;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T41+T42<=0.02;
T42+T43<=0.01;
T43+T44<=0.2489356E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T41>=0.005;
T41<=0.020;
T42>=0.002;
T42<=0.012;
T43>=0.0005;
T43<=0.003;
T44>=0.0002;
T44<=0.002;

!FOR DIMENSION L4;
!OBJECTIVE FUNCTION;
MIN=
3/@EXP(133*(T41-0.0096))+1.21+
5/@EXP(309*(T42-0.005))+1.51 +
9/@EXP(790*(T43-0.00204)) + 4.36+
13/@EXP(3196*(T44-0.00053))+7.48;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T41+T42<=0.02;
T42+T43<=0.01;
T43+T44<=0.2489356E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T41>=0.005;
T41<=0.020;
T42>=0.002;
T42<=0.012;
T43>=0.0005;
T43<=0.003;
```


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```
T44>=0.0002;
T44<=0.002;

!FOR DIMENSION L5;
!OBJECTIVE FUNCTION;
MIN=
4/@EXP(299*(T51-0.00702))+2.35+
6/@EXP(510*(T52-0.0041))+3.5+
8/@EXP(986*(T53-0.00297))+5.29+
10/@EXP(3206*(T54-0.0006))+9.67;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T51+T52<=0.02;
T52+T53<=0.009;
T53+T54<=0.2474279E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T51>=0.007;
T51<=0.02;
T52>=0.003;
T52<=0.012;
T53>=0.0006;
T53<=0.005;
T54>=0.0003;
T54<=0.002;

!FOR DIMENSION L6;
!OBJECTIVE FUNCTION;
MIN=
5/@EXP(381*(T61-0.0061))+2.97+
7/@EXP(935*(T62-0.0029))+7.31+
10/@EXP(2170*(T63-0.0012))+10.9+
13/@EXP(4240*(T64-0.0008))+12.2;

!CONSTRAINTS;

!MACHINING ALLOWANCE CONSTRAINTS;
T61+T62<=0.01;
T62+T63<=0.005;
T63+T64<=0.2356510E-02;

!PROCESS CAPABILITY CONSTRAINTS;
T61>=0.003;
T61<=0.01;
```


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```
T62>=0.0008;
T62<=0.005;
T63>=0.0004;
T63<=0.002;
T64>=0.0002;
T64<=0.00095;
```


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