1963

Potentials of tidal power on the North Atlantic coast.

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POTENTIALS OF TIDAL POWER ON THE NORTH ATLANTIC COAST

A THESIS
Submitted to the Faculty of Graduate Studies through the Department of Civil Engineering in Partial Fulfillment of the Requirements for the Degree of Master of Applied Science at University of Windsor

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1963

ABSTRACT

In this thesis cosmic potential which manifests itself in the tides is investigated. The introduction deals with the necessity of introducing new sources of energy supply, and it is followed by historical discussion of the previous utilization of kinetic energy of the tidal flow, also the latest developments in the field of tidal power projects.

Chapter III and IV are dealing with waves and tides, describing difference between them and introducing tide producing forces. This is followed by general description and classification of the modern tidal power plants and latest developments in the field of turbogenerators.

Finally, the last three chapters are describing the possible locations of the tidal power projects on the North Atlantic coast, classifying them in respect to the pool arrangements, and the possible power output. In order to supplement the varying output of the tidal power projects, some auxiliary power sources are also discussed.

ACKNOWLEDGMENT

The author expresses gratitude to Dr. H. P. Herbich, Professor and Head of the Department of Civil Engineering for encouragement and advice and for reading the draft manuscript of the thesis.

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NOMENCLATURE

A	area of any section
A_d	cross-sectional area of the draft tube
A_s	cross-sectional area of the scroll case
a	distance, acceleration
B	width
B_t	top width
b	distance
C	any coefficient
D	distance from moon's center, diameter
D_o	diameter of the orifice
D_s	specific diameter of the turbine runner, diameter of generator stator
D_3	the runner discharge diameter
d	depth, derivative
E	energy, mass of the earth
E_T	energy output of the turbines
E_p	energy consumed by the pumping
e	efficiency, base of Napierian logarithms
F	force
f	electrical frequency
G	gravitatational constant
g	acceleration of gravity
H	wave amplitude or height
H_w	set-up of the water surface elevation
h	water head
h_s	draft head
h_v	vapor pressure head
I	moment of inertia
k	number of nodes
L	distance, length
L_f	fetch
M	mass of the moon
m	mass of the body, model ratio, direction cosine
N	normal speed
N_s	specific speed

n	number of items
P	power
P_o, P_1, P_2	mathematical terms
p	direction cosine
Q	volume rate of flow or discharge
Q_a	average discharge
q	discharge per unit width
R	radius, radius of the earth, radius of gyration
r	distances between centers of the masses
r_o	radial distance
S	mass of the sun
s	distance
T	period of the tide
t	time
t_o	period of the wave
U	potential at the point
u	wave celerity
V	tide producing potential, volume
v	velocity
v_w	wind velocity
W	weight
w	unit weight of water
x	horizontal coordinate or distance
y	vertical coordinate or depth
α	angle, coefficient
Σ	symbol of summation
θ	angle
λ	wave length
π	constant = 3.1416
ρ	density, radial distance from center of the earth
σ	surface tension
ω	angular velocity

CHAPTER I

INTRODUCTION

To the seashore dweller, the ebb and flow of tides is a familiar phenomenon of nature. Tides are, as it were, the breathing of the sea, the pulse of the cosmos, which gives life to the oceans of the world and sets them in motion; and since the remotest ages, this mystery has haunted the mind of man and stimulated his heart and brain to meditate and search.

The ocean tides have long been envisioned as a source of power. "Tidal mills" were constructed in Europe as early as the 11th century and in America as early as the 17th century. Such small tidal hydro-mechanical power developments were practical for grinding corn or spices as the work could be adjusted to the periodic and varying power from the tides. Electrification and the rapid industrial and economic growth of the 20th century, however, have established a heavy demand for electric power, and it is expected that the world's present need for energy will double within the next ten years.

With numerous harnessable rivers already developed to the full and the increasing scarcity of convenient sites for the setting up of further medium and high-head plants, with thermal plants paying now for their coal four times as much as in 1939 and hardly able to improve their efficiency, and with the economical production of atomic power still far distant, conditions are veering more and more in favour of tidal power.

The cosmic potential which manifests itself in the tides is so large that, in comparison, the totality of the energy won up to the present from the water appears to be trivial. A rough computation proceeding from a mean tide height of 2.3 ft. arrives at an approximate total output of some 54,000 million h.p. whereas the possible output of the continental water resources of the whole world does not exceed 7,600 million h.p. - a mere seventh of the tidal potential.

A closer investigation of the possibility of harnessing tidal power reveals an amazing multiplicity of overlapping problems for which practical solutions are by no means as simple as appears at first sight. Further, traditional hydro-electric practice applies only conditionally to tidal plants, in the working of which, cosmic forces, intervene to a very wide extent. We must therefore realize that no economical and practical solution of the problem can be arrived at without a close co-operation of the astronomer with the hydrographer, and of the civil with mechanical and electrical engineer, not to mention the economist.

At the present time there is not a single tidal project in operation, but the French engineers have promised to have one in operation by 1966. Other nations also are very energetically studying possibilities of the tidal potentials along their sea coasts. Within recent years tidal power on a large scale has been considered as a practical addition to a supply network. This has been due to the possibility, above all, of finding means to regulate and control the supply of the tidal energy, which owes its origin, to another phenomenon of the universe, the rotation of the earth and the interaction of the attraction of the sun and the moon. The tides remained independent of the atmospheric movements, although the latter might affect their amplitude in certain regions. Tidal energy could not however, be directly harnessed wherever it presented itself, as in the open sea, and it was the reflection of tidal movements on the continental plateau, and the incidental resonances which caused these to be amplified up to 25 times, which gave the possibility for their energy to be harnessed.

Another important quality posed by tidal energy was its fidelity to the calculated figures of time and tidal range, apart from slight changes due to atmospheric action. Tidal energy could be predicted exactly for many years in advance and this was a vastly different picture from that presented by river flows.

Utilisation of the tides to generate a substantial quantity of power requires provision for the storage of large quantities of water so that discharge may be made from a higher to a lower elevation through hydraulic turbines. A large single pool may be built to entrap water from the ocean, but generation of power is limited to those times in the tidal cycle when the differential in elevation between the ocean and the pool is sufficient for operation of the turbines. A combination of two storage pools for simultaneous entrapment and exclusion of water from the ocean can, however, provide for some generation at all times. As compared to most river hydroelectric projects the potential average hydraulic head of tidal projects is quite small; but the very large quantities of water available for power production are accurately predictable for many years in the future.

The work embodied in this thesis consists of a few interconnected problems.

Firstly, origin and tide producing forces are analyzed and differences between ordinary and tidal waves described.

Secondly, the most suitable locations for erection of the tidal power plants on the North Atlantic coast are reviewed and chosen.

Thirdly, the actual utilization of the tide's energy to generate power in various layouts is considered. The best pool arrangement for a given location is selected, and other problems connected with generation of energy from the sea are given consideration.

CHAPTER II

HISTORICAL DISCUSSION

Man has for centuries devised methods of putting the ocean tides to work. Attempts to utilize the potential and kinetic energy offered by perennial rhythm of ebb and flow goes very far back in history. We meet with the first instance of a practical realization of the problem on a very small scale in the shape of so-called tide mills, which made their appearance as early as the eleventh century in England and other Western European countries. Grinding corn or gypsum, or sawing timber, these floating contraptions in which an undershot waterwheel utilized only the kinetic energy of the tidal flow.

The oldest work on the utilisation of tides was written in 1438 by the Italian Mariano, of Siena, and includes, in addition to sketches, constructional hints.

In Chelsea, Massachusetts, in 1734, "Slade's Mill" was built to grind spices. This mill developed about 50 horsepower from four water wheels driven by the head created by damming a small estuary to trap water at high tide.

Tidal power in England was first used centuries ago, and the early water supply of London was pumped to a water tower by a mill operated by the tidal flow; it consisted of a large paddle wheel mounted on a raft and moored between two of the piers of old London Bridge, and this worked until the old bridge was replaced by the present structure in 1824. In 1790 a tidal basin water-wheel installation was operating on the River Tamar, Devon, the wheels being later replaced by turbines.

In 1912, the German engineer Pein made some stir with his proposal to set up a tidal power plant at a particularly favourable spot on the North Sea coast. In Great Britain, a few years after World War I, preliminary projects came out for the Severn, followed by schemes for two tidal plants in the vicinity of Liverpool. In France as well, the import of this heretofore untapped source of energy was promptly recognized, and suggestions were made to harness tides to power generation with the help of the technical means available at the time. The first major project, also worked out after World War I, is based on the very favourable conditions obtained in the Bay of Rotheneuf at Cape Benard. Subsequently, all these tidal schemes sank more or less into oblivion, and it appeared from various publications on the subject that an economical harnessing of tides to power generation was still very far away.

This brief historical survey is proof enough that some significance had been granted at early juncture to the problem of tidal power, and at the present period of intensive search for new sources of energy, it is perfectly natural that we should try again to bring the problem to a satisfactory solution. After all, as proved by the evolution of public and industrial consumption during the past forty years, the saturation point of energy requirements is still very far ahead.

Latest developments in different countries can be described as follows:

France. French engineers promised that St. Malo Tidal Plant on the Rance River will be in operation by 1965. This is the first and only tidal-power scheme in the world actually under construction. The Rance scheme is in the long estuary which culminates opposite Dinard and St. Malo, appears to be the most promising, having a tidal range of 10.8 ft. at neaps and 37.9 ft. at springs.

When the scheme will be in full operation sometime in 1966, (first power in 1965) the gross output of its twenty-four 10,000-kw units will be 608.5 million kwh annually. Of this, 537 million kwh will be generated when water is flowing from the basin to the sea and 71.5 million kwh when it is flowing from sea to basin.

Power for pumping, estimated at 64.5 million kwh annually, is a necessary deduction from this total. Taking pumping into account, the net output of the Rance plant is estimated at 544 million kwh. There will also be several sluices to help in lowering or raising the water level at periods of valving as rapidly as possible.

Great Britain. (Proposed Severn Barrage). In 1926 a tidal model was made at Manchester University and a series of experiments carried out in connection with the proposed Severn Barrage under the direction of Professor A. H. Gibson. The effect of the proposed scheme on such matters as the deposit of silt and the erosion of the shore were carefully studied, and lessons of great value were derived from these experiments. Some of the dock authorities in the Bristol area were still not satisfied with the results of previous model experiments, and in 1947 the government sanctioned a further tidal model investigation to be carried out by the Hydraulics Research Organization under Sir Claude Inglis.

Some aspects of the Severn Barrage are thus still being investigated. One of the most important points in its favour at the present time is that it would save about a million tons of coal every year. It has been suggested that the most suitable application of the power generated could be for the heating for large district heating schemes, and that in the case of this Barrage the power could be transmitted at high voltage to centers where such schemes are required. The hot water could be readily stored in large insulated containers. By this method the difficult problem of the intermittent nature of tidal power would be overcome.

The Severn Barrage is a single-basin scheme. At the start of the operation, the head of the turbines is about 47 ft, and when the station is closed down the head is only 7 ft. At this point the tide will be rising in the estuary, and the gates in the sluice dam will be opened when the water-level is the same on both sides; at high tide the sluice gates will be closed, and the cycle will start all over again, beginning 50 minutes later each day. Ship locks are provided so that shipping can pass up and down the river even when the power station is operating, and there will have to be a number of fish passes provided so that fish can also pass up or down the river.

Canada and U.S.A. Another interesting tidal power project location could be the Bay of Fundy, New Brunswick. Here the tidal range is the greatest in the world, varying from 21.1 feet on neap tides to 52.2 feet on springs; corresponding figures for the Severn scheme are 22.2 and 47.6 feet.

In 1935 the Passamaquoddy Bay Tidal Project commission of the United States' Federal Emergency Administration of Public Works recommended construction of an initial Cobscook Bay project, all within the State of Maine, at an estimated cost of $30 million. An ultimate project was contemplated to embrace the Passamaquoddy Bay. The tidal range on this site is 11 feet on neap tides and 26 feet on springs. The recommended initial project was approved by the United States Government and $10 million (later reduced to $7 million) was allotted to the U. S. Army Corps of Engineers to start construction.

The Corps of Engineers estimated that the initial project with a modified design would cost $61,500,000. A review board raised this estimate to $68,158,000. Work progressed until August, 1936, with an expenditure of about $7 million, and was discontinued due to lack of further appropriations by the Congress.

A report by the U. S. Federal Power Commission, published in 1941, reached the conclusion that the above-mentioned tidal power scheme could not at present compete with ordinary hydro-electric projects, or even with large modern steam plants.

This underestimating of the new resources of tidal energy was mostly due to lack of progressive thinking in the field of turbo-machinery, which should be designed to meet specific requirements of tidal projects.

Latest French experimental results on the model study of the new bulb-type turbine designed for the Rance project and the interest in the energy from the sea by the Government of New Brunswick, brought a new interest in tidal projects and threw a new light on the old pessimistic report published by the U. S. Federal Power Commission in 1941.

In 1961 the International Joint Commission (Canada and U. S. A.) investigated once again the economical aspects of the tidal power energy of the project dropped by the U.S.A. government in 1936, and produced a more favourable report on this matter.

In 1962, Premier Robichaud, of New Brunswick, after consultation with three consulting firms regarding the possibility of producing an abundant supply of low cost electric power generated by tides at the head of the Bay of Fundy, said: "It is considered that further studies of the potential power development at the head of the Bay of Fundy is justified". He also mentioned that three years would be required for preliminary explorations and design before construction of the tidal power plant could begin.

U.S.S.R. From publications of L. B. Bernstein, Gosenergdoisdat, Moscow, and Leningrad, 1961, western readers can learn about tidal power in Russia.

Although much smaller in installed capacity than the French Rance scheme, the Russian Kislogubskoy project, which is the only other tidal power development under construction, is worth mentioning on account of the unorthodox construction methods being used. The site is on the coast between Murmansk and the Norwegian frontier, and the whole power-station block will be assembled in the dry near Archangel from prefabricated sections and then floated to its final position.

The machines chosen for the above mentioned project will be French bulb units. Two French firms Neyrpic of Grenoble and Alsthom of Belfort, are to supply bulb units which the Soviets have ordered for their tidal power development. The three horizontal-shaft turbines to be manufactured by Neyrpic, will develop 400 kw each under a head of 1.28 m, at 72 rpm. with 3.3 m diameter runner. The synchronous generators, to be manufactured by Alsthom are rated at 415 kVA-380V each.

The machines are to be suitable for operation as turbines or pumps, and it was this factor more than any other that made the latest design acceptable rather than earlier proposals involving conventional Kaplan turbines in conjunction with a pumped-storage plant.

Australia. In Western Australia considerable interest is now being focused on the vast tidal-power resources of the Kimberley coast in the far north of the State. The tides in the area range up to 40 ft and the coastline is steep and rugged containing many natural inlets with comparatively narrow openings to the sea.

In March, 1962, the Government received a report from the Public Works Planning and Design Engineer, J. G. Lewis, outlining the possibilities of some 45 schemes along the Kimberley coastline. Mr. Lewis showed that some of these proposals compared very favourably with others throughout the world including the present scheme under construction at La Rance. At about the same time the Government received independent information from the French firm, France Technique, which also reported very favourably on the general possibilities.

It is not yet possible to assess the economics of the proposals with any accuracy because of the lack of survey data, but the Government is expected to announce plans for an extensive survey and investigation very soon. The incentive for power development in this very remote area would be to develop the large mineral resources now being discovered right across the north of Australia. The Lewis report draws attention to the possibilities of very high-voltage direct-current transmission in exploiting these minerals.

CHAPTER III

WAVES

Waves are fluctuations in the surface level of a fluid. These variations in level may be periodic, fluctuating without period, or transient. They may be segregated into two major classes, waves of translation and waves of oscillation.

Translatory waves are most commonly encountered in hydrology. The translatory wave form advances, and the water over which it passes is also moved forward in the direction of movement. It can be shown that the translatory wave is a problem in unsteady flow.

Oscillatory waves, on the other hand, do not result in appreciable displacement of the water particles in the direction of motion, the particles oscillating in an orbit about a mean position. Translatory waves include flood waves in natural channels, surges, and seiches. Wind-generated ocean waves are typical oscillatory waves.

The hydraulic jump is sometimes called a standing wave. Standing waves may form in flowing streams under the proper flow conditions. Here the wave form remains essentially stationary, and the water flows through it. Standing waves may be important factors in hydraulic design.

Surges are of occasional interest in hydraulic design as a flood wave due to sudden change in discharge.

Tides, which affect streamflow of many coastal streams, are the result of seiches and frequently induce bores, a translatory wave form.

TRANSLATORY WAVES

Uniformly Progressive Flow. A special case of unsteady flow possible in uniform channels is that in which the wave form moves down the channel without undergoing any change in shape. This is uniformly progressive flow, (Fig. 3-1), though not directly applicable to an irregular natural channel will shed some light on certain aspects of natural flood waves.

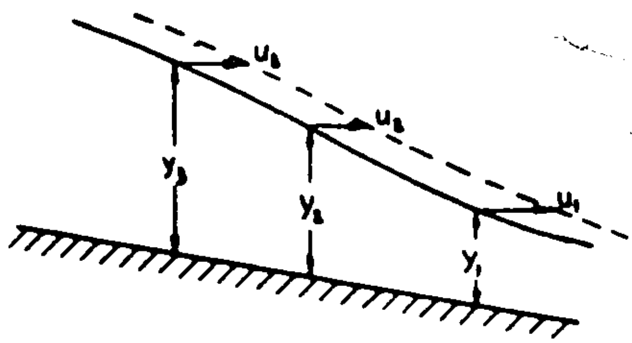


Fig. 3-1. Uniformly progressive flow.

From the definition of uniformly progressive flow, it follows that successive positions of the wave front must be parallel, i.e.

$$u_1 = u_2 = u_3 \; . \; . \; . \; . \; .$$

and the dotted line in Fig. 3-1 represents the position of the wave front 1 second after the time of the initial position. The wave configuration travels downstream with celerity u, but mean water velocities in the cross section vary from section to section as the hydraulic radius and surface slope change.

Monoclinal rising wave. Fig. 3-2 shows a special case of uniformly progressive flow which is called the monoclinal rising flood wave. Such a wave is approximately similar to the flood waves in natural streams. It can be generated by introducing into a channel, where steady uniform flow at depth d_1 and velocity v_1 is occuring, a new steady discharge Q_2 which requires the stage d_2 and velocity v_2 for uniform flow.

A uniform channel of constant slope is assumed. The two regions of steady flow are separated by the wave configuration abdc in which a condition of unsteady flow exists. This configuration tends to develop a definite and unchanging shape and to travel with a constant celerity u, greater than either v_1 or v_2.

Since the wave celerity u is greater than the mean velocity of flow preceding or following the wave, a volume of water equal to $(u - v_1) A_1$ must enter the front of the wave at ab, where A_1 is the cross-sectional area.

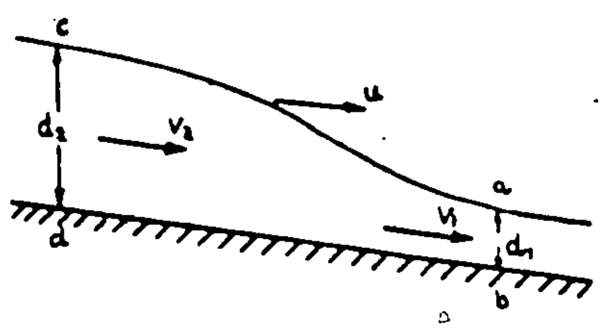


Fig. 3-2. Monoclinal rising wave.

However, because the wave configuration has a constant shape and volume, an equal quantity of water must be discharged through the section cd. The water which flows through the wave shape is called the overrun, Q'. It follows that

$$Q' = (u - v_1)A_1 = (u - v_2)A_2 \tag{3-1}$$

Solving this equation for u

$$u = \frac{A_1v_1 - A_2v_2}{A_1 - A_2} \tag{3-2}$$

Since Q = Av, Eq. (3-2) becomes

$$u = \frac{Q_1 - Q_2}{A_1 - A_2} \tag{3-3}$$

Combining Eqs. (3-1) and (3-3), the overrun is found to be

$$Q' = \frac{Q_1A_2 - Q_2A_1}{A_1 - A_2} \tag{3-4}$$

Inspection of Eq. (3-3) discloses that if there is no initial flow (Q_1 and A_1 equal zero) then the celerity is Q_2/A_2 or v_2. When there is initial flow, the velocity of the wave front is greater than the velocity of the water in the wave, because some water ahead of the wave is gathered up by it. The wave configuration thus moves along more rapidly than the water particles making up its volume at any instant.

It is evident from Eq. (3-3) that the velocity of a monoclinal wave is a function of the area-discharge relation for the channel. Figure 3-3 shows such a curve. Velocity usually increases with stage, therefore discharge, (the product of area and velocity) increases at a more rapid rate than area, and the area-discharge curve is concave upward for most stations.

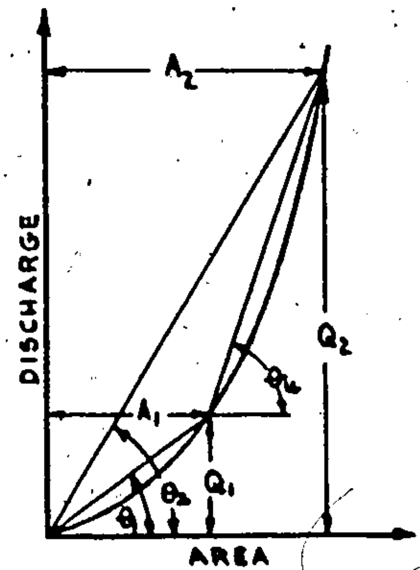


Fig. 3-3. Velocity relations in a monoclinal wave.

From the figure it may be seen that

$$v_1 = \frac{Q_1}{A_1} = \tan \theta_1 \quad \text{and} \quad v_2 = \frac{Q_2}{A_2} = \tan \theta_2$$

and

$$u = \frac{Q_2 - Q_1}{A_2 - A_1} = \tan \theta_u \tag{3-5}$$

Since the curve is concave upward, the celerity u, must always be greater than v_1 and v_2. It follows also that as Q_1 approaches any given value of Q_2 the celerity increases, approaching a maximum as $Q_2 - Q_1$ approaches zero. Therefore

$$u_{max} = \frac{dQ}{dA} \tag{3-6}$$

Letting B_t designate top width, $dA = B_t dy$, and

$$u_{max} = \frac{1}{B_t} \frac{dQ}{dy} \tag{3-7}$$

The term dQ/dy is the slope of the rating curve of the section for uniform flow.

Seddon's Principle. Eq. (3-7) states the principle Seddon developed by study of gage heights on the Mississippi and Missouri Rivers. He determined wave celerities by noting the time at which the point of rise occurred at each successive gaging station and found the celerity to be quite constant for small rises. On the Mississippi River below Cairo the wave celerity was found to be about 92 miles/day at all stages over a reach 277 miles long. For a 333 mile reach of the Missouri River below Kansas City, the celerity was found to be

$$u = 4.28 + 0.20G \tag{3-8}$$

where u is in feet per second and G is the average stage in the reach. Seddon pointed out that the celerity in a reach is dependent on the mean width of the river between the stations. He even used the determination of u to compute B_t. For the reach to which Eq. (3-8) is applicable, the average rating curve could be expressed as

$$Q = 0.15\,(1.17G + 19)^4$$

Solving Eq. (3-7) for B_t

$$B_t = \frac{5.62\,(G + 16.25)^3}{G + 21.4}$$

The principle outlined by Seddon is applicable only to small rises. As shown in Fig. 3-3, the celerity of large waves is less than that for very small rises reaching the same crest height ($\tan\theta_u > \tan\theta_2$).

It is interesting to note that Seddon determined the celerity on the Mississippi between Carrollton and Baton Rouge to be more than

400 miles/day. This is to be expected from Eq. (3-7) since dQ/dy is necessarily very large in the lower reaches of the Mississippi, approaching infinity at the outlet where stage is controlled by sea level.

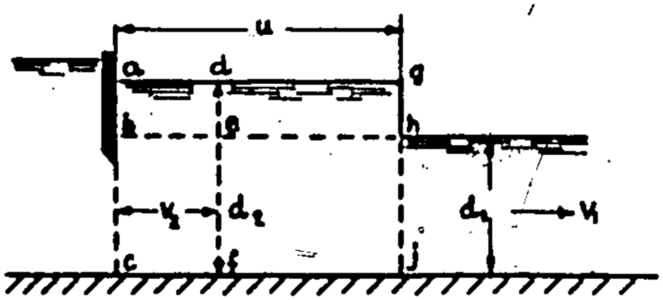


Fig. 3-4. Abrupt translatory wave.

Abrupt translatory waves. An abrupt translatory wave is the result of an instantaneous increase in discharge. Fig. 3-4 indicates conditions one second after the instantaneous opening of a gate in a channel has generated an abrupt translatory wave. The volume of water entering the channel in this time is $Q_2 = A_2 v_2$ (area acfd).

The increased volume abhg is given by

$$Q_2 - Q_1 = u(A_2 - A_1)$$

Substituting $Q = Av$

$$v_2 = (A_1 v_1 + A_2 u - A_1 u) \frac{1}{A_2} \tag{3-9}$$

The volume dfjg has been accelerated from v_1 to v_2.

The force required to produce this change in momentum is

$$F = m(v_2 - v_1) = \frac{(u - v_2)(v_2 - v_1)\, A_2\, w}{g} \tag{3-10}$$

where m is mass and w is the unit weight of water. Since F also equals the difference in hydrostatic pressure on areas A_1 and A_2

$$F = w\,A_2\bar{y}_2 - w\,A_1\bar{y}_1 \qquad (3\text{-}11)$$

where $\bar{y}$ is the depth to the center of gravity of the section. Equating these values of F, eliminating w, and inserting v_2 from Eq. (3-9)

$$u = v_1 \pm \sqrt{g\,\frac{A_2\bar{y}_2 - A_1\bar{y}_1}{A_1\left[1 - \frac{A_1}{A_2}\right]}} \qquad (3\text{-}12)$$

Considering a unit width of rectangular channel we may substitute d for A and d/2 for $\bar{y}$

$$u = v_1 \pm \sqrt{\frac{gd_2}{2d_1}\,(d_2 + d_1)} \qquad (3\text{-}13)$$

As the height of the wave decreases, d_1 approaches d_2

$$u = v_1 \pm \sqrt{gd} \qquad (3\text{-}14)$$

Equation (3-12) is a general equation applying to any channel and any height of wave, Eq. (3-13) applies to a rectangular channel, and Eq. (3-14) is limited to waves of very small magnitude. The direction of flow is assumed to be positive downstream. If u is zero, Eq. (3-12) reduces to the general equation of the hydraulic jump. Thus, the hydraulic jump may be considered a standing wave with celerity zero.

The abrupt wave or moving hydraulic jump is known as the hydraulic bore. A bore can form in a rectangular channel only when

$$\frac{(u - v_1)^2}{2g} > \frac{d_1}{2}$$

In other words, the bore can form only when the initial depth in the channel is subcritical with respect to the net wave velocity $(u - v_1)$. This is analogous to the conditions under which a jump may form.

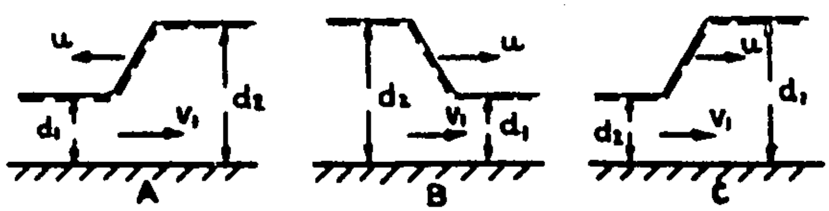


Fig. 3-5. Types of bores.

Bores occur frequently and sometimes regularly in some tidal estuaries. As the tide rises rapidly, a bore is formed and moves swiftly upstream. Such bores are to be observed on streams entering the Bay of Fundy, the Severn River in England, and the Tsientang River in China. Tidal bores are of type A, Fig. 3-5. Floods due to intensive rainstorm or due to failure of a dam can be accompanied by bores of type B. Type C bore can result only from a decrease in flow, as by the closing of a gate in a canal or powerhouse tailrace.

Surges. Sudden changes in discharge caused by the opening or closing of a gate result in the transmission of a surge wave upstream from the gate. Surges are considered as positive or negative depending upon whether they are above or below the original still-water level. A positive surge, Fig. (3-6) may be caused by the sudden closing of a gate, and its celerity may be determined by an analysis similar to that applied to the abrupt translatory wave Eq. (3-12).

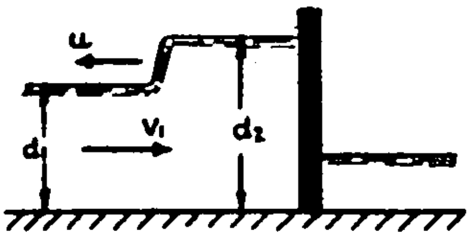


Fig. 3-6. Positive surge.

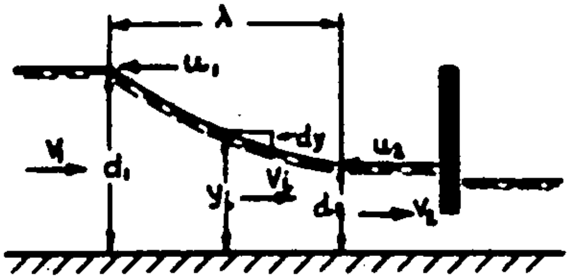


Fig. 3-7. Negative surge.

Fig. (3-7) shows a negative surge such as might result from the sudden opening of a gate. The negative surge differs from the wave forms previously discussed in that it does not have constancy of form, because the upper elements of the wave travel faster than the lower elements. During one second the force required to change the momentum of the vertical element intercepted by the increment of height dy is

$$F = m\ dv = \frac{(v - u)\ A\ w}{g}\ dv \qquad (3\text{-}15)$$

The value of u is negative since we are considering a wave moving upstream. The force F is also equal to the difference in pressure across the distance (v - u)

$$F = \frac{wB}{2}(y - dy)^2 - \frac{wBy^2}{2} \qquad (3\text{-}16)$$

where B is the width of the channel. Neglecting $(dy)^2$ this equation reduces to

$$F = -w\,B\,y\,dy = -w\,A\,dy \qquad (3\text{-}17)$$

The negative sign indicates a positive force since y is decreasing, therefore, dy is negative. Combining Eqs. (3-15) and (3-17)

$$dy = -\frac{(v - u)}{g}\,dv$$

since $v - u = \sqrt{gy}$, [Eq. (3-14)]

$$\frac{dy}{\sqrt{y}} = -\frac{dv}{\sqrt{g}}$$

Integrating between d_1 and d_2 and v_1 and v_2

$$2\sqrt{d_2} - 2\sqrt{d_1} = -\frac{1}{\sqrt{g}}(v_2 - v_1)$$

which can also be written

$$v_2 - v_1 = 2\sqrt{gd_1} - 2\sqrt{gd_2}$$

The water velocity where the depth is y_i, Fig. (3-7) is

$$v_i = v_1 + 2\sqrt{gd_1} - 2\sqrt{gy_i} \qquad (3\text{-}18)$$

and since $v = u + \sqrt{gy}$

$$u_i = v_1 + 2\sqrt{gd_1} - 3\sqrt{gy_i} \qquad (3\text{-}19)$$

At the crest of the wave where $y_i = d_1$

$$u_1 = v_1 - \sqrt{gd_1}$$

and at the trough of the wave

$$u_2 = v_1 + 2\sqrt{gd_1} - 3\sqrt{gd_2} \qquad (3\text{-}20)$$

By setting $u_2 = 0$ in Eq. (3-20), a value of $d_1 - d_2$ can be derived in terms of d_1 and v_1, namely

$$d_1 - d_2 = \frac{5}{9} d_1 - \frac{v_1}{9g} (v_1 + 4\sqrt{gd_1}) \qquad (3\text{-}21)$$

The rate of lengthening of the surge is

$$\frac{d\lambda}{dt} = u_2 - u_1 = 3\sqrt{g}\ (\sqrt{d_1} - \sqrt{d_2})$$

and integrating for the length of the wave

$$\lambda = 3\sqrt{g}\ (\sqrt{d_1} - \sqrt{d_2})t$$

Since $u_1 t = x$, where x is measured from the point at which $\lambda = 0$, we can express λ in terms of x, the distance from the origin as

$$\lambda = \frac{3\sqrt{g}\,(\sqrt{d_1} - \sqrt{d_2})x}{v_1 - \sqrt{gd_1}} \qquad (3\text{-}22)$$

<u>Flood wave due to dam failure.</u> If the dam of Fig. (3-8) is assumed to fail suddenly, a negative surge travels upstream in the reservoir with celerity u_1. Simultaneously, an advancing wave moves downstream with celerity u_o. The celerity will be zero at some point A at which v_1 also is zero, and hence, from Eq. (3-21)

$$s_A = (d_1 - d_A) = 5/9\ d_1$$

Since $d_A = d_1 - z_A$, d_A becomes $4/9\ d_1$. From Eq. (3-18)

$$v_A = 0 + 2\sqrt{gd_1} - 2\sqrt{gd_A} = 2/3\sqrt{gd_1} \qquad (3\text{-}23)$$

At the toe of the wave, where $d = 0$, the celerity u_o must equal the water velocity v_o and, from Eq. (3-18)

$$u_o = 0 + 2\sqrt{gd_1} - 0 = 2\sqrt{gd_1} \qquad (3\text{-}24)$$

The derivations above are of theoretical interest but of limited practical application. The complete, instantaneous failure of any dam is most improbable, and frictional resistance would rapidly reduce any wave celerity well below that indicated by Eq. (3-24). Furthermore, Eq. (3-18) and Eq. (3-21) were derived for a surge of very small amplitude.

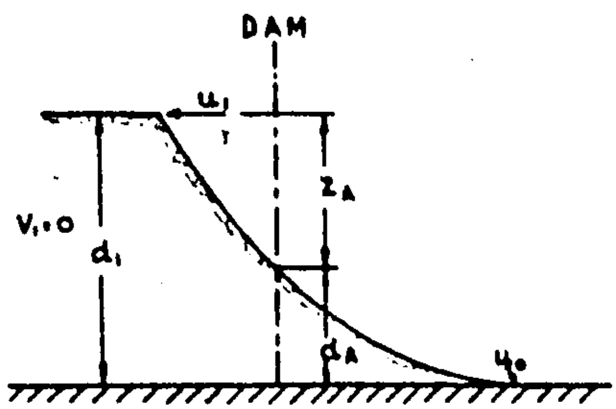


Fig. 3-8. Flood wave caused by failure of a dam.

Seiches. Oscillations of water bodies above and below their mean level are called seiches. These oscillations have a natural period depending upon the physical features of the water body. A disturbing force with the same period of oscillation as a lake or pool builds up a seiche to the point where the energy dissipated by friction equals the rate of application of energy. Seiches may be caused also by strong winds or differences in barometric pressure which cause an initial displacement of a water surface.

When the force causing the displacement ceases or changes in intensity, a series of pulsations follow at the natural frequency until damped by frictional forces. Fluctuations in excess of 2 ft. have been observed on the Great Lakes as a result of wind.

A seiche may be considered to be composed of two waves traveling in opposite directions and in such phase relation that $z_a = 0$ at the nodes, and $v = 0$ at the boundaries of the basin. Fig. (3-9) represents a uninodal seiche in a basin of uniform cross section. The elevation at N is constant, and the water surface at the ends fluctuates between the limits a, b and c, j. From Eq. (3-14), with $v_1 = 0$, the period of the seiche is

$$t_o = \frac{2L}{u} = \frac{2L}{\sqrt{gd}} \qquad (3\text{-}25)$$

The wave length is

$$\lambda = 2L = ut_o \qquad (3\text{-}26)$$

Fig. (3-10) presents two cases of multinodal seiches in a closed basin of length L. If k is the number of nodes, the general expressions of Eqs. (3-25) and (3-26) are

$$t_o = \frac{2L}{k\sqrt{gd}} \qquad (3\text{-}27)$$

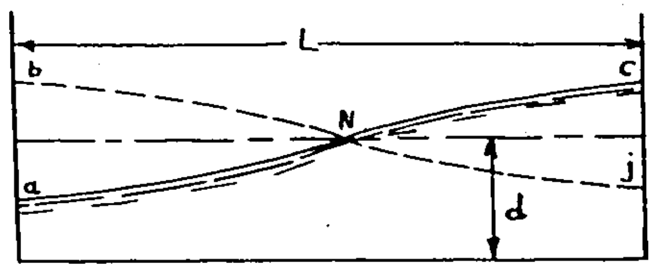


Fig. 3-9. Uninodal seiche.

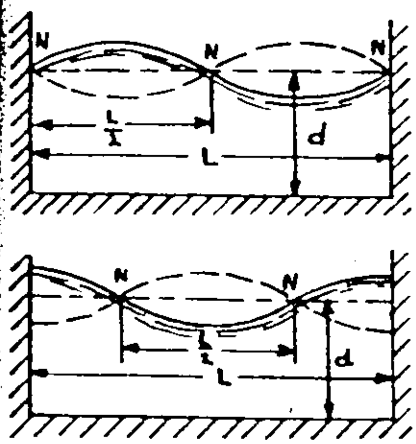


Fig. 3-10. Multinodal seiches.

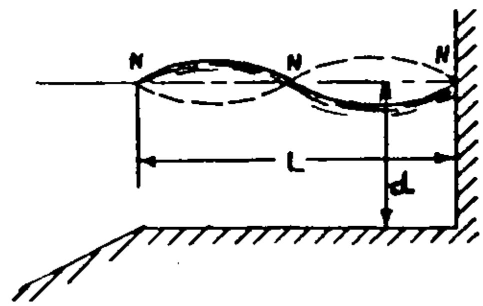


Fig. 3-11. Seiche in a basin open at one end.

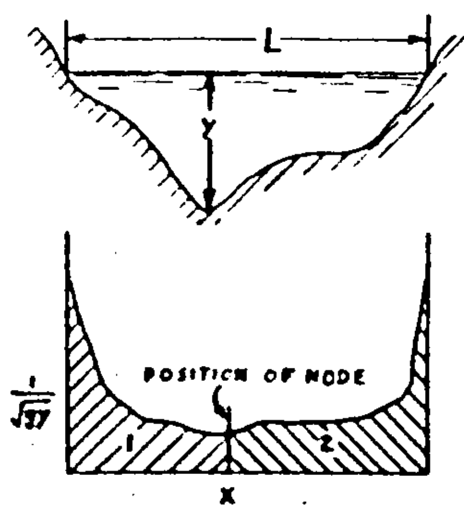


Fig. 3-12. Seiche in an irregular basin.

$$\lambda = \frac{2L}{k} \tag{3-28}$$

In no case should more than two nodes be counted per wave length. Thus, both cases shown in Fig. (3-10) have $k = 2$.

If the basin is open at one end Fig. (3-11) the water-surface elevation remains constant at this end and the seiche takes the form of that in a closed basin twice the length of the open basin. Thus, Eq. (3-27) becomes

$$t_o = \frac{4L}{k\sqrt{gd}} \tag{3-29}$$

Equations (3-25) to (3-28) have all been based on the assumption of a uniform and constant cross section in the basin. In a basin of irregular section, the period is given by integrating

$$t_o = 2\int_o^L \frac{dx}{\sqrt{gy}} \tag{3-30}$$

Because of the usually complex relation between x and y in a natural lake or other basin, it is impossible to integrate Eq. (3-30) by analytical means. Such a basin may be treated readily by plotting $1/\sqrt{gy}$ against x and measuring the area under the resulting curve. The node will occur at the value of x dividing the area into two equal parts ($A_1 = A_2$, Fig. 3-12).

OSCILLATORY WAVES

An example of the oscillatory wave is the wind-generated ocean wave. Observation of a floating object in fairly deep water will demonstrate that there is little forward motion of the water but rather that the particle tends to oscillate in a circular orbit.

Waves differ from one another not only in height (H) but also in steepness, and hence in length (λ). The ratio λ/H is often used as a measure of wave steepness. The steepest possible wave has a value of 7 for λ/H, but values this low are not commonly observed. Wave form is unstable at any lesser value.

Waves grow in length as well as in height under the continued action of a constant-velocity wind. The effect of wind duration is to raise wave velocity in relation to wind velocity. When the wind shifts in direction or intensity the new generation of waves will differ from the older waves, which still persist in their original course. The more recent waves may intercept the earlier waves. In any such mingling of different wave trains some waves will be reinforced and others reduced, depending upon the extent to which the two waves are in phase. The effect of these conflicts is the very confused wave pattern which one usually observes in any detailed inspection of a storm at sea. Of course, these troubled waters are a natural accompaniment to the gusts and eddies that characterize the generating winds. The confusion of detail should not be permitted to obscure the underlying pattern, which also deserves attention.

From observation as well as from theoretical analyses, it

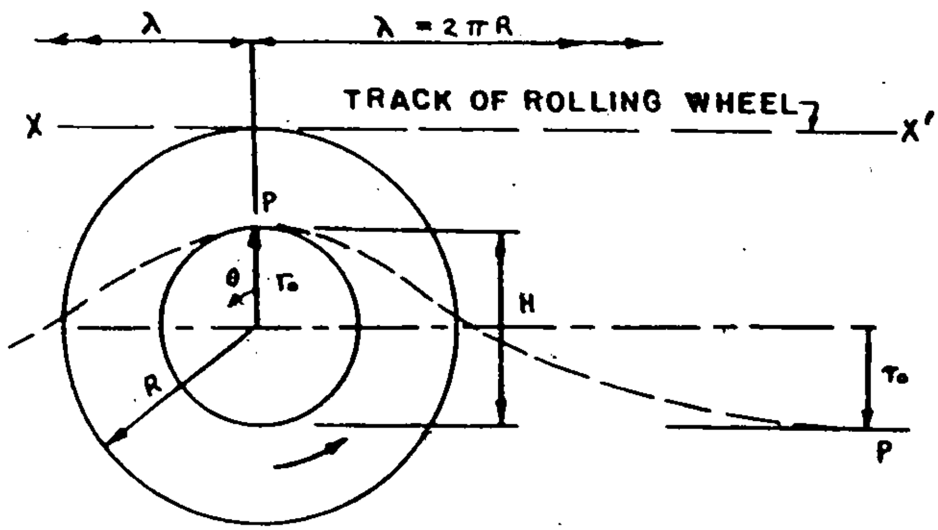


Fig. 3-13. Trochoidal wave form.

appears that the trace of an oscillatory wave on a vertical transverse plane closely resembles a trochoid, with the circumference of the rolling circle equal to $\lambda = 2\pi R$ and the length of the tracing arm equal to the radius of the orbit of the surface particles r_o.

A trochoid is the curve traced by a point on a wheel which is rolling along a straight line. Fig. (3-13) shows a wheel of radius R rolling (upside down) on the horizontal line X-X'.

The trochoidal form results in sharper crests and flatter troughs than indicated by simple harmonic motion. When $r_o = \lambda/2\pi$, (then $r_o = R$), the trochoidal form results in sharp cusps at the wave crests, a limiting case not observed in nature, the curve is cycloid.

The general equations for trochoids are

$$x = R\theta - r_o \sin\theta$$

$$y = r_o (1 - \cos\theta)$$

Velocity and Celerity of the Waves. A wave may be defined as temporal variation in fluid velocity which is propagated through a fluid medium. The speed of propagation of such a pressure disturbance relative to the fluid is known as wave celerity. The celerity is quite distinct from the local change in fluid velocity and in general of much greater magnitude.

The wave celerity (u) should be distinguished from the absolute wave velocity (v_a) which is the vector sum of the celerity and the undisturbed velocity v of the fluid through which the disturbance is propagated.

$$v_a = \bar{v} + \bar{u}$$

In open channel waves these vectors are as a general rule parallel to the channel axis.

then $v_a = v + u$ (scalars) velocity

when $c = -v$ the absolute velocity = 0 and a standing wave results.

The wave celerity depends on the magnitude of the velocity change and on the fluid properties, mainly on modulus of elasticity and density. There are three basic types of waves.

(1) Capillary waves, when surface tension is a motivating factor.

(2) Gravity waves, when fluid weight is an essential property.

(3) Elastic waves, where compressibility is an important factor.

In both (1) and (2) the existence of a free surface between liquid and gas, or between two liquids is required, and both are usually called the surface waves.

The celerity of a surface wave may be expressed by general equation (3-31), introduced by G. H. Keulegan, National Hydraulic Laboratory Bureau of Standards, Washington, D.C., as

$$u = \sqrt{\frac{\sigma 2\pi}{\rho \lambda} + \frac{w \lambda}{\rho\, 2\pi} \tanh \frac{2\pi d}{\lambda}} \qquad (3\text{-}31)$$

where λ = wave length

d = depth of the flow

σ = surface tension

w = unit weight of water

ρ = density = w/g

For very small wave lengths where capillary action is predominant

$$u = \sqrt{\frac{\sigma}{\rho}\ \frac{2\pi}{\lambda}} \qquad (3\text{-}32)$$

For relatively large wave lengths, the gravitational term becomes predominant.

$$u = \sqrt{\frac{w}{\rho}\frac{\lambda}{2\pi}\ \tanh\frac{2\pi d}{\lambda}} \qquad (3\text{-}33)$$

Gravity waves are sometimes classified as the deep water waves and the shallow water waves, the dividing or critical value of

$\frac{d}{\lambda}$ being $\frac{1}{2}$

When $\frac{d}{\lambda} > \frac{1}{2}$, the waves are called deep water.

When $\frac{d}{\lambda} < \frac{1}{2}$, the waves are called shallow water.

The expression for wave celerity of gravity wave may be approximated as follows.

(1) As long as $\frac{d}{\lambda} > \frac{1}{2}$, $\tanh\frac{2\pi d}{\lambda} \approx 1$

and $$u = \sqrt{g\lambda/2\pi} \qquad (3\text{-}34)$$

(2) As $\frac{\lambda}{d} \longrightarrow \infty$, $\tanh\frac{2\pi d}{\lambda} \to \frac{2\pi d}{\lambda}$

and $$u = \sqrt{gd} \qquad (3\text{-}35)$$

<u>Deep-water Waves</u>. Waves generated in deep water are usually of the type known as oscillatory waves, in which the particles of water making up the wave oscillate in a circular orbit about some mean position. In shallow depth of water the orbits are ellipsoidal in shape. The amplitude of oscillation of a trochoidal wave becomes negligible at depths greater than one-half the wave length. Thus waves occurring in

water with a depth greater than $\lambda/2$ may be considered as deep-water waves

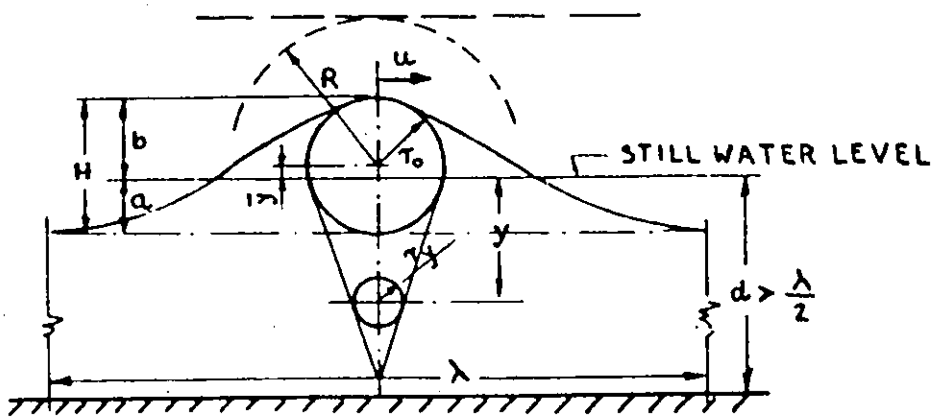


Fig. 3-14. Elements of an oscillatory wave.

Fig. (3-14) shows the various elements of the trochoidal wave. For deep water, as in the ocean, where d is large compared to the wave-length, Eq. (3-34 becomes

$$u = \sqrt{g\,\lambda/2\pi} = 2.27\sqrt{\lambda}$$

The wave celerity with which an oscillatory wave progresses is related to the wave period (t) and the length (λ) by

$$\lambda = u\,t$$

so period in seconds

$$t = \frac{\lambda}{u} = \frac{\sqrt{\lambda}}{2.27}$$

Radius of the orbit has been found to be

$$r_y = r_o e^{-y/R} = \frac{r_o}{e^{2\pi y/\lambda}}$$

Amplitude $H = 2r_o$

Examination of the wave profile of Fig. (3-14) reveals that wave crests are much steeper than wave troughs and that the trochoid is not symmetrical about a level line through the surface orbit center. Actually, there is not enough water in the crest above such a level line to fill the space above the depressed trough below the level of the surface orbit center. Consequently, the total volume of sea water being the same in storm as in calm, the still-water level must lie below the level of surface-orbit centers. The distance by which the orbit center of a particle is raised above its still-water position is given by the expression

$$\bar{y} = \frac{r_o^2}{2R} = \frac{\pi}{4}\frac{H^2}{\lambda} = 0.78\frac{H^2}{\lambda}$$

where $r_o = \frac{H}{2}$

and $R = \frac{\lambda}{2\pi}$

so $a = \frac{H}{2} - 0.78\frac{H^2}{\lambda}$

$$b = \frac{H}{2} + 0.78\frac{H^2}{\lambda}$$

The existence of the mass of water comprising the wave crest at an elevation above still-water level constitutes a potential energy source, similar in nature to the pool behind a power dam. Likewise, the revolving water particles have kinetic energy like the jet striking an impulse wheel. The potential energy, of course, moves on with the wave form itself, but the kinetic energy imparted to the water particles is not so transferred. In a given deep-water wave equal quantities of energy are stored as potential and as kinetic energy. The total energy in a wave per 1-foot width of crest is expressed by the equation

$$E = \frac{1}{8} w \lambda H^2 \left[1 - \frac{\pi^2}{2} \left(\frac{H}{\lambda} \right)^2\right] \quad (3\text{-}36)$$

or

$$E = \frac{1}{8} w \lambda H^2 \text{ approximately}$$

Deep-water waves are considered to be of moderate height when the ratio H/λ is between 1/100 and 1/25 and of great height when this ratio exceeds 1/25. Waves of great height tend to have narrower and steeper crests and broader and flatter troughs than moderate waves. When the ratio H/λ exceeds 1/7, the wave form becomes unstable and breaks up in numerous irregular waves of lesser height.

Shallow-Water Waves. When the still-water depth d is less than $\lambda/2$, the influence of the bottom on particle orbits becomes appreciable. Most of the equations presented in the preceding section must be modified for use with shallow-water waves.

The orbital paths of the water particles are assumed generally to be elliptical. According to this view, the actual position of the particle at any instant is taken to be on the intersection of an ellipse, with a vertical line through the position the particle would otherwise occupy on a circumscribed circle. The wave thus becomes more nearly a translatory wave. When $d < \lambda/25$, the wave can be considered a translatory wave with celerity

$$u = \sqrt{gd}$$

In the intermediate range, when the depth is between $\lambda/2$ and $\lambda/25$, the celerity is given by Eq. (3-33)

$$u = \sqrt{\frac{g\lambda}{2\pi} \tanh \frac{2\pi d}{\lambda}}$$

Wind and Waves. Numerous empirical relations between wind and waves have been developed. Their verification is difficult because of the inaccuracies in estimating wave heights and because waves, once generated, tend to persist for some time, even though wind velocity changes. Stevenson found that where the fetch L_f, the distance through which the wind acts on the water surface, exceeds 30 miles, the maximum wave height to be expected with high winds is given in feet

$$H = 1.5 \sqrt{L_f}$$

For short fetches (under 30 miles) H appears to conform to

$$H = 1.5\sqrt{L_f} + 2.5 - \sqrt[4]{L_f}$$

Waves of maximum height are not generated with the onset of a given wind. Evidence indicates that the time required to develop waves of maximum height corresponding to a certain velocity increases with velocity. In general, the time required is less than 12 hours.

Molitor modified Stevenson's formulas to introduce wind velocity as follows

$$H = 0.17\sqrt{v_w L_f}, \quad \text{for } L_f > 20 \text{ miles} \qquad (3\text{-}37)$$

$$H = 0.17\sqrt{v_w L_f} + 2.5 - \sqrt[4]{L_f}, \quad \text{for } L_f < 20 \text{ miles} \qquad (3\text{-}38)$$

Molitor points out that empirical formulas are much less satisfactory than actual observations for a particular site because of the strong influence of local conditions.

Ride-up and Set-up. In determining the proper freeboard above maximum still-water level for design of dams or levees, a safety factor must be introduced into Eqs. (3-37) and (3-38) to allow for the ride-up of waves on the sloping surface of the structure. This factor is usually taken as about 1.5.

A persistent wind will result in a shift of water from one side of a basin or reservoir to the other, increasing the water-surface elevation on the leeward side. This increase in elevation is sometimes referred to as set-up H_w, and it must be considered in calculating freeboard on levees and dams. The formula proposed by the Lorentz Zuyder Zee Commission of Holland for this effect is

$$H_w = \frac{v_w^2 L_f}{1400d} \cos \theta \qquad (3\text{-}39)$$

where H_w = set-up in feet above still pool level, v_w = wind velocity in miles per hour, L_f = fetch in miles, d = average depth of water in feet, and θ = angle of wind and fetch (measured between the fetch of the wind and a normal to the shore line where the set-up is to be computed).

CHAPTER IV

TIDES

The principal movements of the sea may be divided into three classes: Ordinary or wind waves, tidal movements, and ocean currents. The essential feature of any tidal movement is, as the name implies, its periodicity. The period may not be of constant length, but if variable it must follow some conceivable law. In conformity with this notion, the word tide may be defined as the periodic alternate rising and falling of oceanic and other large bodies of water, due mainly to the attraction of the moon and the sun as the earth rotates upon its axis. This rising and falling is accompanied by and depends upon a lateral or horizontal movement of the waters: such movements are called tidal currents. Their periodic character distinguishes them from ocean currents. Remarkable stages of the water level at a given place, whether due to earthquakes, gales, or other causes which probably have no definite law of recurrance although popularly known as "tidal waves" cannot be regarded as belonging to tidal phenomena. On the other hand, the stages of a river, if periodic in their nature, may with propriety be included in its tides.

The tide rises until it reaches a maximum height called high water, and then falls until it reaches a minimum height called low water. These two phases of the tide may be spoken of as the tides. The difference between a high and a low water is called a range of tide, and so is independent of absolute heights: its average value is called the mean range (Mn). For a few minutes before and after high or low water, it is difficult to observe any vertical motion in the tide. While thus apparently stationary, the tide is said to stand.

When the water is rising, it is flood tide; when it is falling, it is ebb tide. High water occurs at the moment when the water level is highest, and low water when it is lowest. Successive high waters come about 12 1/2 hours apart; more precisely, the average interval between corresponding high waters on successive days is $24^h\ 50^m$. This is the same as the average interval between two successive passages of the moon across the meridian; and the coincidence, maintained indefinitely, makes it certain that there must be some close connection between the moon and the tides.

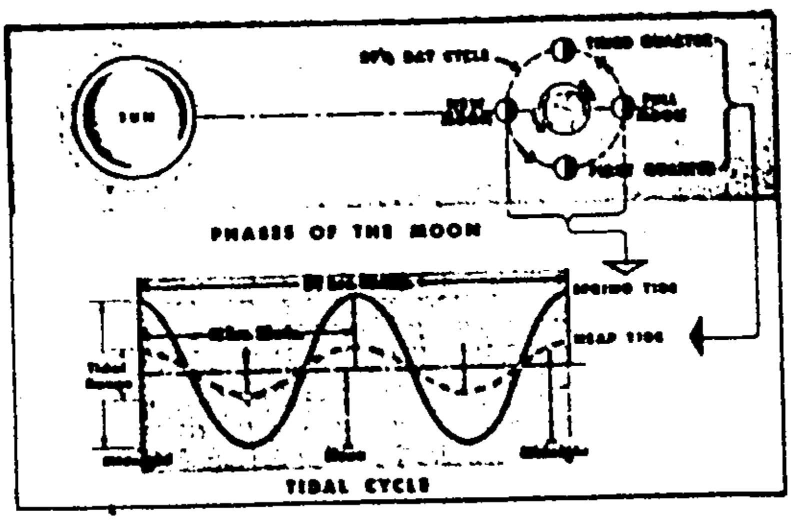


Fig. 4-1.

When the moon is new or full, the range of the tides is considerably greater than the average, high water rising higher and low water falling lower, without affecting mean sea-level. These are called spring tides. The neap tides, at the first and third quarters of the moon, have the smallest range, -- usually rather less than half that of the spring tides.

Further evidence that the moon is largely responsible for the tides is found in the fact that when the moon is in perigee (nearest the earth) their range is nearly 20 per cent greater than when it is in apogee. The greatest range of all happens when the moon is new or full at the time when it is in perigee.

Based upon the fact that the tides are due chiefly to the difference between the moon's attraction upon the enveloping sea and the earth as a whole, one would expect that at most tidal stations two high waters and two low waters would occur each lunar day; in other words, to each transit of the moon there would correspond one high water and one low water. On the average, the time of high water at a given station follows the time of transit by a certain number of hours and minutes, called the high-water interval (HWI), or high-water lunitidal interval, or corrected establishment. In like manner the low-water interval (LWI), or low-water lunitidal interval, indicates, unless otherwise specified, the average number of hours and minutes between the time of transit and the time of low water.

If intervals at some particular time are meant, they should be properly distinguished by name or otherwise; or the average values

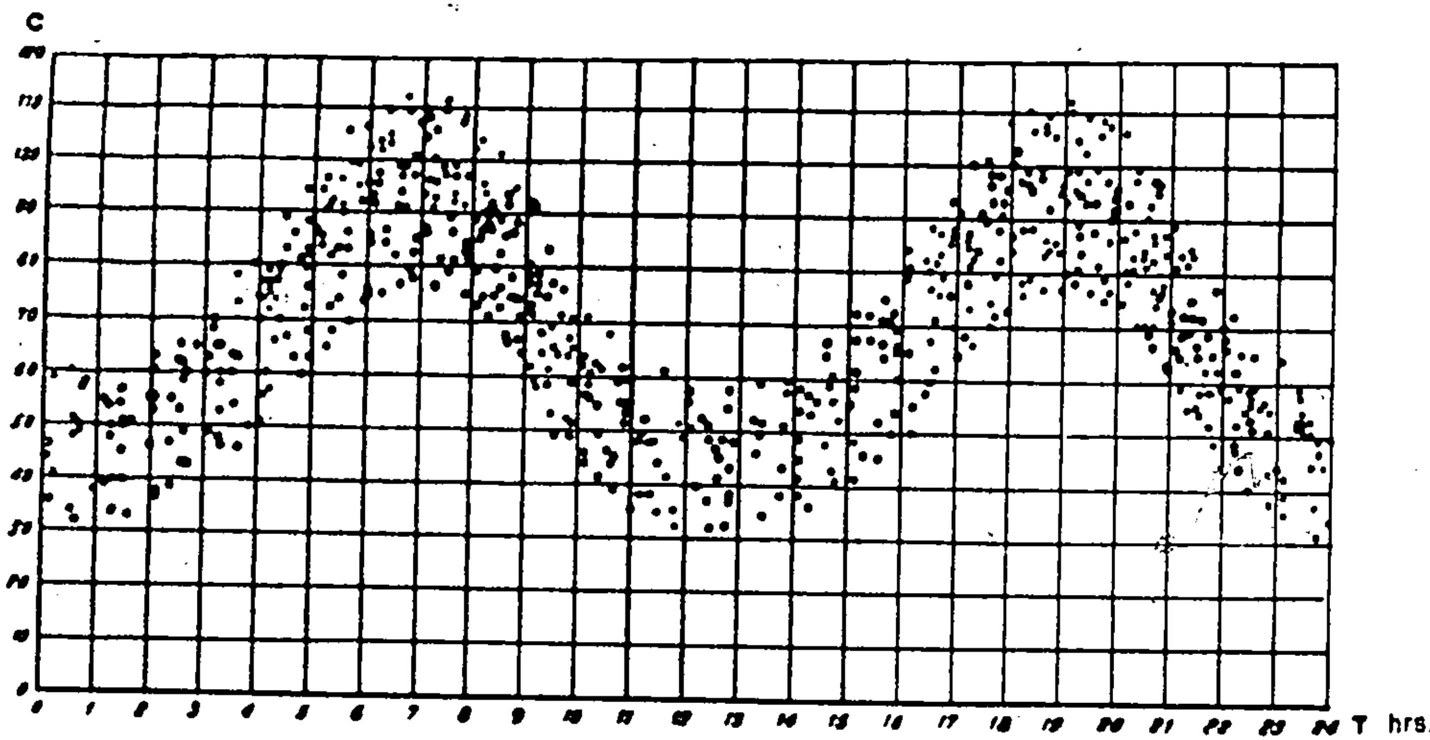


Fig. 4-2. Variation in times of high tide at St. Malo during 1943.

c = Coefficients of tide. (For normal high tide, c = 100)

T = Times.

may, for distinction, have the word mean prefixed to their name. The establishment of a port is the mean interval between the time of high water at that port and the next preceding passage of the moon across the meridian. The establishment of New York, for instance, is 8^h 13^m, ---on the average, high water occurs there 8^h 13^m after the moon has crossed the meridian; but the actual interval varies fully half an hour on each side of this.

An inequality in interval, range, or height is a systematic departure of the same from its mean value. The extreme amount of this regular departure is sometimes called the coefficient of the inequality. Diurnal inequality in height is the difference in height between two consecutive high waters or low waters. Diurnal inequality in time or interval is the difference in length of two consecutive high water intervals or low water intervals.

In North Atlantic waters the morning and afternoon high tides are about equally high, and the low tides equally low; but in many seas, as in the Gulf of Mexico and the North Pacific, there is a marked diurnal inequality in the heights, coupled with considerable irregularity in the times of high and low water. Indeed, in extreme cases there is but one high water and one low water a day. This inequality occurs only when the moon is far north or south of the equator. When the declination is zero, there are two equal tides daily.

All of these complicated phenomena are readily explicable by the effects of the disturbing forces, due to the sun and moon, on the water of the oceans.

Not long after new or full moon, the tidal effect of the sun is added to that of the moon. When this effect upon the range is greatest the tides are called spring tides. At any given place the retard, or interval between new or full moon and spring tides, may be regarded as constant. Not long after the moon is in quadrature, the tidal effect of the sun is taken away from that of the moon, and when the range becomes a minimum from this cause, neap tides occur. Their retard at a given place may be regarded as constant, and it does not differ much from the retard of the spring tides; unless the water is shallow, in which case the retard (spring or neap), as derived from the high waters alone, will differ from that derived from the low waters.

The inequality, or apparent irregularity, in time or height introduced by the sun, and so dependent upon the moon's phase, is variously styled the semimenstrual, semimensual, semimonthly, or phase inequality; the last seems preferable for most purposes, because there are several kinds of month in common use, especially in tidal work, and the word "phase" suggests a connection with the age of the moon.

When the sun's tidal effect shortens the lunitidal intervals, causing the tides to occur earlier than usual, there is said to be a priming of the tide; when, from the same cause, the interval is larger than usual, there is said to be a lagging.

A tidal day is the variable interval (24^h 50^m on an average) between two alternate high or low waters. A more accurate definition is the interval between the mean of four consecutive tides and the mean of the succeeding or preceding group of four consecutive tides.

The amount by which the tidal day exceeds $24^h\ 50^m$ is sometimes called the "lagging of the tide", and the amount which it falls short the "priming".

The amount by which corresponding tides grow later day by day (i.e. the amount by which the tidal day exceeds $24^h\ 00^m$) may be called the daily retardation.

The retard, expecially spring and more especially spring high-water, has been called the age of the tides. If this term is to be retained, it seems desirable to suppose the age to have one value, and that such as to suit the neap as well as the spring tides, the low waters as well as the high. Moreover, instead of age of the tide, the expression age of the phase inequality will generally be used in what follows. It will subsequently appear that, for deep water at least, the lunitidal interval of such tides as happen to occur as many hours after syzygy as represent the age of the phase inequality, have their mean values. In other words, the spring and neap intervals are about equal to the mean intervals. Because of this fact the times of such tides as give mean intervals may be used in determining the age. Experience has shown that the ages as determined from heights or ranges do not agree with those determined from times (intervals).

Since successive transits of the moon occur on an average $12^h\ 25^m$ apart, the age can be approximately expressed by stating the number of the transit preceding the tide to which the lunitidal intervals are to be applied. The effect of selecting an earlier transit is to increase the lunitidal interval by $12^h\ 25^m$. Of course, by adapting the transits to a suitable terrestrial meridian, any age can be allowed for.

Another way of reckoning the age is by the hour of the moon's transit. The time of transit increases on an average 50^m daily, so that if the transit used for spring tides occur at $0^h\ 50^m$, such transit follows syzygy by 24^h. But the tide follows the transit by the lunitidal interval

$$\text{age} = \frac{24 \times 60}{50}\left[\begin{array}{c}\text{hour of transit for}\\ \text{maximum range}\end{array}\right] + 1/2\ (\text{HWI} + \text{LWI}) \qquad (4\text{-}1)$$

Whenever the hour of transit exceeds 12^h, 12^h must be rejected. The same formula is adapted to neap tide, by replacing the word "maximum" by the word "minimum" and always discarding 6^h or 18^h from the hour of transit.

To infer the age from the time when the interval has its mean value, replace maximum range by mean lunitidal interval = 1/2 (HWI + LWI).

Some writers prefer to increase the age or retard, as defined above, by the high-water interval, because of the fanciful notion that they thereby obtain the interval between the transit of the moon and the appearance of the resulting high water.

<u>Height of the Tides</u>. In the open ocean the range of the tides (as observed on isolated islands) averages about two and a half feet, but is very different on on different islands, owing to the complicated mode of oscillation of the ocean waters. On the coast the range is usually greater, for the oscillations tend to grow higher as they run into shallow water. In shallow water the wave moves slowly, so that the time of high water

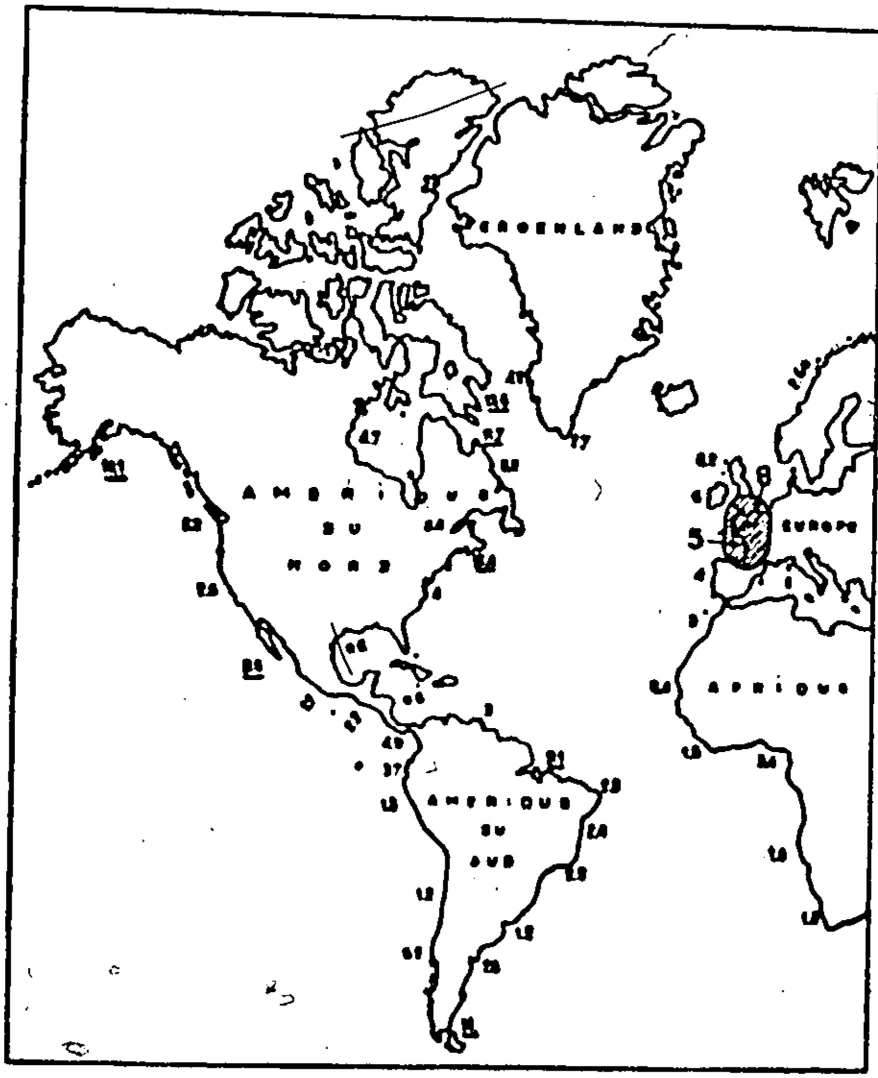


Fig. 4-3. Average spring-tide ranges (in metres).

differs widely at stations only a few score miles apart.

In shallow estuaries where the range of tide is considerable, the high water is propagated inward faster than the low water, for at high water the greater depth prevails. The high water thus gaining upon the low water causes the duration of rise of tide to become shorter as the wave proceeds; and so further the wave goes without breaking, the more abrupt its front becomes. Finally, it becomes so steep that the top of the wave falls forward (not in the middle of the stream but near the shelving shores) something like the crest of a breaker. This phenomenon, usually accompanied by much noise, is called a bore. Other names for the bore, boor, or boar's head are eager (England), mascaret or barre (France). The following rivers and arms of the sea have bores: The Amazon, Tsien-tang, Brahmaputra, Ganges, Indus, Garrone, Seine, Severn, and Wye rivers.

Where the configuration of the coast forces the tide into a corner, or where the natural period of oscillation of the water in a bay or gulf is nearly the same as that of the tides, the range becomes very great. At the head of the Bay of Fundy the mean range of spring tides is fifty feet, and on the eastern coast of Patagonia it is almost as great. It is also very great in the Bristol Channel, on the coast of Normandy, and in the Hudson Strait.

Datum planes. The average height of all low waters at any place over a sufficiently extended period of time is called mean low water and is the official reference plane for the depth shown on navigation charts, and of improved channels, in the waters of the Atlantic and Gulf coasts of the United States. The average height of the lower of the two daily low waters is called the mean lower low water and is the official reference datum in the waters of the Pacific coast of the United States. In British waters the datum is usually the mean low water of spring tides, or low-water springs. This reference plane is also used at the Pacific entrance to the Panama Canal. The average height of the sea, as determined usually by the average of the observed hourly heights over an extended period of time, is called mean sea level, and is the standard datum to which elevations on land are referred.

DISTINCTION BETWEEN ORDINARY AND TIDAL WAVES.

Short Waves. (The depth of the water is supposed to exceed the length of the wave, and the rise to be several times less than the wave length.)

(1) Particles move in ellipses which are very nearly circles at the surface.

(2) The horizontal and vertical displacements of the particles diminish rapidly below the surface.

(3) Particles originally in the same vertical line are, at any given instant, in the same phase of oscillation.

(4) The wave profile approaches a curtate cycloid. (The marigram, or record of a self-registering gauge, is a curtate cycloid.)

(5) The period or wave length assumed, the other becomes fixed, regardless of the depth of the water or the rise and fall of the surface.

(6) The celerity of propagation depends upon the wave length only,

$$u = \sqrt{g\lambda/2\pi} = 2.27\sqrt{\lambda}$$

This follows from Eq. (3-32) that the period and celerity of short waves in deep water vary as the square root of the wave length and are independent of depth of the water.

Long Tidal Waves. (The depth of the water is supposed to exceed, by a considerable amount, the rise and fall of the tide, and the length of the wave exceeds the depth of the water.)

(1) Particles move in ellipses approaching horizontal straight lines.

(2) The horizontal displacements of the particles are about the same at the bottom as at the surface; the vertical displacements are proportional to the heights of the particles above the bottom.

(3) Particles once in the same verticle line remain so for a long time.

(4) The wave profile approaches a cosine curve. (The marigram, or record of a self-registering gauge, is a cosine curve.)

(5) Two of the quantities period, wave length, and depth of water, assumed, the remaining one becomes fixed, regardless of the rise and fall of the surface.

(6) The celerity of propagation depends upon the depth only

$$u = \sqrt{gd} = 5.67\sqrt{d}$$

This follows from Eq. (3-33) that the celerity of a very long wave to compare to the depth of the water (as in the free tidal wave) varies as the square root of the depth, and is independant of the wave length.

Characteristics based on observations only are mentioned below:

Wind Waves.

(1) The period of a short wave at a given place depends upon the velocity, continuance and (in limited bodies of water) direction of the wind.

(2) The amount of rise and fall at a given place depends upon the velocity, continuance, and direction of the wind.

(3) Wind waves do not arise unless the velocity of the wind exceeds a certain value: 0.45 miles per hour for capillary waves, 2 miles for gravity waves.

(4) The period, as well as the amount of rise and fall, may vary rapidly from place to place, as can be seen in passing around a breakwater.

(5) Wind waves are very confused, and their period uncertain.

(6) Wind waves are soon destroyed by the viscosity of the water.

(7) Storm waves at sea (wind 30 or 40 knots) have a rise and fall of 15 or 20 feet, a period of about 10 seconds, and, by (43), a length of about 500 feet.

Tidal Waves.

(1) The period of a tidal oscillation does not depend upon the given place, but upon the astronomical forces to which it is due.

(2) The amount of rise and fall of the tide at a given place depends upon, or rather varies with, the direction, and intensity of the astronomical forces to which the tide is due.

(3) Tidal oscillations of same periods are very nearly proportional to the disturbing causes, however small these latter may be.

(4) The period is fixed around the world; the amount of rise and fall

changes slowly from place to place.

(5) Tidal waves recur with remarkable regularity.

(6) Tidal waves move on as free waves through long distances.

(7) The rise and fall of the tide at sea, by the equilibrium theory is about 1.8 feet x (cosine of latitude)^2 and the length of the tidal wave is hundreds, or even thousands, of miles.

THE ORIGIN OF TIDES

As previously mentioned, the wave has been regarded as an existing phenomenon without special reference to its astronomical cause. We will now consider the origin of the tide under several conditions. This will indicate some of the difficulties with which a general theory has to contend in the case of nature.

All particles of the earth (including the seas) will occupy positions fixed relatively to one another if no other forces act upon them than the following: The earth's attraction, its centrifugal force of axial rotation, and an extraneous force acting upon all of its particles alike. If the extraneous force does not act upon all particles alike, then motions will be set up in the yielding parts.

Suppose the earth to consist chiefly of a spheroidal nucleus either rigid throughout or rigid in its outward layers. Suppose this nucleus to be covered in whole or in part by one or more seas either shallow or not, in comparison with the earth's radius. The attraction of the moon upon any given particle near the surface, say, is along a line drawn (at any given instant) from the particle to the moon's center;

its intensity, which is inversely proportional to the square of the distance, and local direction (i.e., direction with respect to the earth's surface) continually change as the earth rotates upon its axis. The attraction of the moon upon a particle at the earth's center is along a line drawn from the earth's center to that of the moon; its intensity is independent of the earth's axial rotation.

The difference between these two forces may be called the tide-producing force at the surface point in question.

Just what this force will do to the water as the earth rotates upon its axis, cannot be clearly seen except for very special cases. This force being very small in comparison with the earth's attraction, its vertical component, which slightly alters the intensity but not the direction of terrestrial gravity, cannot set up in seas shallow in comparison with the earth's radius any considerable motion amongst the fluid particles. The horizontal component of the tide-producing force may, however, impart a sensible horizontal motion to the waters of an extended sea, and, because the fluid is incompressible and continuous within a given basin, indirectly create a slight rising and falling of the surface, whether or not the period of the earth's axial rotation were sufficiently long to enable the surface to approach a level surface; i.e. to arrange itself normal to disturbed gravity at each point.

<u>The Tide-producing Force</u>. The system of arrows in Figs. (4-4) and (4-5) are intended to represent the horizontal component of the moon's tide-producing force at various places on the earth's surface. The arrows

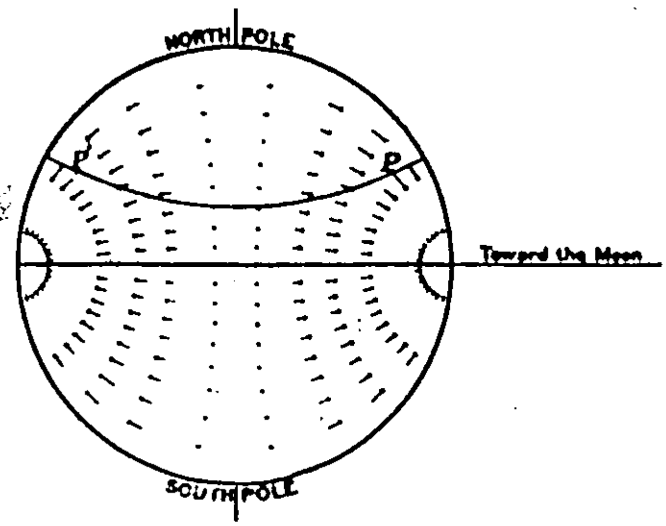

NORTH POLE
P
P
Toward the Moon
SOUTH POLE

Fig. 4-4

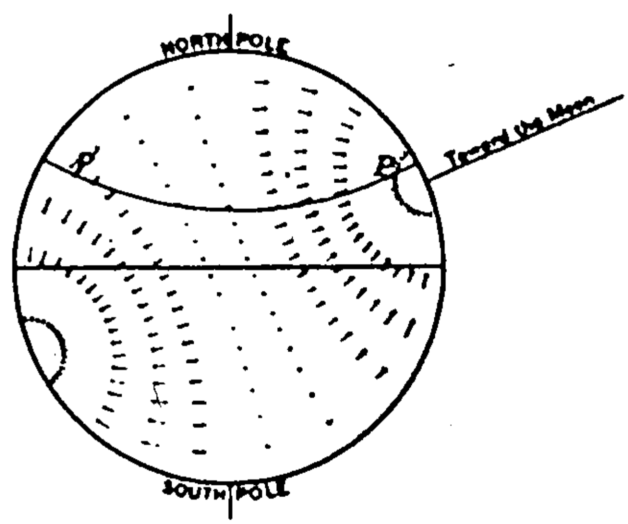

NORTH POLE
P
P
Toward the Moon
SOUTH POLE

Fig. 4-5

located upon the same small circle (isodynamic line) are supposed to be of equal length, and all arrows are supposed to lie in a system of great circles which meet in a point directly under the moon, and, of course, in a point 180° therefrom. At these two points the length of the arrows is zero; for, the horizontal component of the moon's disturbing force must there vanish - the force itself being vertical. The length of the arrows is likewise zero along a great circle midway between these two points; for, all points along this circle are very nearly as far from the moon as is the earth's center.

The system of arrows is fixed with respect to the moon, and so sweeps over the surface of the earth as the moon performs her apparent daily revolution, or shifts somewhat as she declines north or south from the celestial equator. At any point P on the earth's surface, the moon being upon the equator, the horizontal forces are equal in magnitude and direction to the horizontal forces at P', a point upon the same parallel of latitude as P, but 180° distant in longitude; or, what amounts to the same thing, they repeat themselves at any given point P every half lunar day. But when the moon is not upon the equator, the forces are not generally the same at P and P', either in magnitude or in direction, and so do not exactly repeat themselves every half lunar day. This alternation of the forces gives rise to a diurnal inequality in the tides.

It will be noticed that for places situated upon either side of the equator, the forces have, when the moon is upon the equator, a meridional component directected from the poles toward the equator, consequently the existence of the moon causes the water (half-tide level) at the equator to be higher than it would otherwise have been.

The moon's movement in declination causes a fortnightly fluctuation in half-tide level.

It should be noted also, that the accelerations, and not the forces, are what really count. The force of the sun's attraction on the earth is more than eighty times as great as that on the moon, owing to the difference of mass; but the accelerations at the same distance are equal. This use of the term 'force' instead of 'acceleration' is common in celestial mechanics. It is noteworthy that exact calculations can be made of the motion of a body (for example, a comet) when we have no idea what its mass is and consequently do not know the force of attraction ($F = G\,m\,E/r^2$) but the acceleration ($F/m = G\,E/r^2$) is accurately known,

where F = force of gravitation between masses (planets)

E = mass of the earth

m = mass of the body

r = distance between centers of the masses

G = gravitational constant = 6.67×10^{-8} dyne-cm^2/gm^2

Calculation of the tide-producing potential.

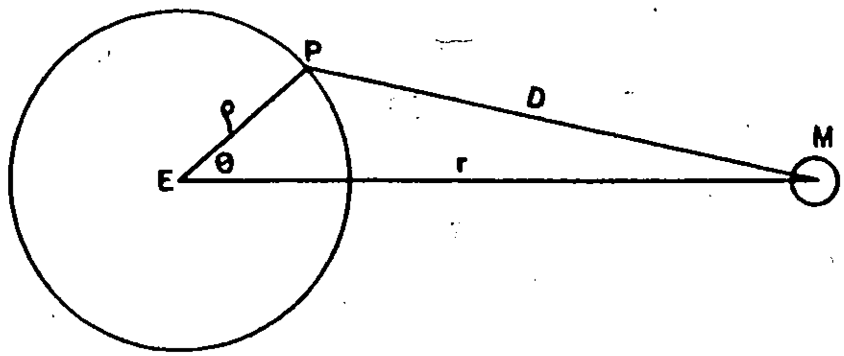


Fig. 4-6.

The attracting force of the moon upon any particle of unit mass whose distance is D from the moon's center is

$$\frac{G\,M}{D^2} \qquad (4\text{-}2)$$

where M is the moon's mass and G the attraction between unit masses unit distance apart.

$$g = \frac{E}{R^2}\,G \quad \text{and} \quad G = g\,\frac{R^2}{E}$$

where R is a mean radius of the earth, and E is earth's mass.
Now if W is such a function that

$$\frac{dW}{dD} = -\frac{GM}{D^2} \qquad (4\text{-}3)$$

it is, by definition, the gravitational potential of the moon at the point where the particle is situated; for, in the direction of D increasing, the force is negative. From this equation, it is seen that the moon's potential decreases as the distance of the particle from the moon increases.

If

$$W = \frac{GM}{D} + \text{constant} \tag{4-4}$$

equation (4-3) is satisfied. Let

r = the distance of the moon's center from the center of the earth

ρ = the distance of the disturbed particle from the center of the earth

θ = the angle at the earth's center between the disturbed particle and the moon's center

In the plane triangle defined by the earth's center, the moon's center, and the disturbed particle, the lengths of two sides are r and ρ , while the included angle is θ. Consequently the remaining side, whose length is D, has the value

$$D = \sqrt{r^2 - 2r\rho\cos\theta + \rho^2}$$

Replacing D by this value and making the constant zero, Eq. (4-4) becomes

$$W = \frac{GM}{\sqrt{r^2 - 2r\rho\cos\theta + \rho^2}}$$

Suppose the earth and the particle P to constitute a system not a subject to deformation by the moon. The whole system is urged toward the moon just as if each unit particle of the system had applied to it the force

$$\frac{GM}{r^2}$$

acting in a direction parallel to the line joining the centers of the earth and moon. The components of this force are

$$m_1 \frac{GM}{r^2}, \quad m_2 \frac{GM}{r^2}, \quad m_3 \frac{GM}{r^2}$$

where m_1, m_2, m_3 are direction-cosines of the line joining the centers of the earth and moon referring to axes fixed in the earth, the origin being the earth's center. If U denotes the potential at P of this force, it must be such a function of x, y, z, the co-ordinates of P, that its partial derivations shall be the above component forces.

Such a function is

$$U = \frac{GM}{r^2}(m_1 x + m_2 y + m_3 z) + \text{constant}$$

If p_1, p_2, p_3 denote the direction-cosines of P referring to the axes mentioned in connection with m_1, m_2 m_3, we have

$$x = p_1 \rho$$

$$y = p_2 \rho$$

$$z = p_3 \rho$$

$$\cos\theta = p_1 m_1 + p_2 m_2 + p_3 m_3 =$$

$$= \sin\lambda \sin\delta + \cos\lambda \cos\delta \cos (\text{moon's hour angle})$$

where

λ = the latitude of P

δ = the declination of the moon

$$U = \frac{GM\rho}{r^2}\cos\theta + \text{constant}$$

Now let the system be subject to deformation; in other words, let there be an opportunity for P to move relatively to the earth's center or to a rigid nucleus which may surround the center. The force causing this movement has for its potential

$$W - U = V,$$

or

$$\frac{GM}{\sqrt{r^2 - 2r\rho\cos\theta + \rho^2}} - \frac{GM\rho}{r^2}\cos\theta - \text{constant} = V$$

At the earth's center the potential of the tide-producing force must be zero because W and U are there equal.

Making $\rho = 0$ and $V = 0$, the constant becomes equal to

$$+\frac{GM}{r}$$

$$V = \frac{GM}{\sqrt{r^2 - 2r\rho\cos\theta + \rho^2}} - \frac{GM}{r} - \frac{GM\rho}{r^2}\cos\theta \qquad (4\text{-}5)$$

The expression

$$\frac{1}{\sqrt{r^2 - 2r\rho\cos\theta + \rho^2}}$$

may be written

$$\frac{1}{r}\left(1 - 2\frac{\rho}{r}\cos\theta + \frac{\rho^2}{r^2}\right)^{-\frac{1}{2}}$$

This expanded in powers of $\frac{\rho}{r}$ becomes

$$\frac{1}{r}\left(P_o + P_1\frac{\rho}{r} + P_2\frac{\rho^2}{r^2} + P_3\frac{\rho^3}{r^3} + \ldots\ldots\right)$$

where

$$P_o = 1$$

$$P_1 = \cos\theta$$

$$P_2 = \frac{3 \cos^2 \theta - 1}{2}$$

$$P_3 = \frac{5 \cos^2 \theta - 3 \cos \theta}{2}$$

The P's are functions of θ alone and are called zonal harmonics or Legendre's coefficients. Eq. (4-5) now becomes

$$V = \frac{GM}{r} \left[P_2 \frac{\rho^2}{r^2} + P_3 \frac{\rho^3}{r^3} + \ldots\ldots \right] \qquad (4\text{-}6)$$

neglecting higher powers of the moon's parallax, the required tide-producing potential is

$$V = GM \left[\frac{\rho^2}{r^3} \left(\frac{3 \cos^2 \theta - 1}{2} \right) \right] \qquad (4\text{-}7)$$

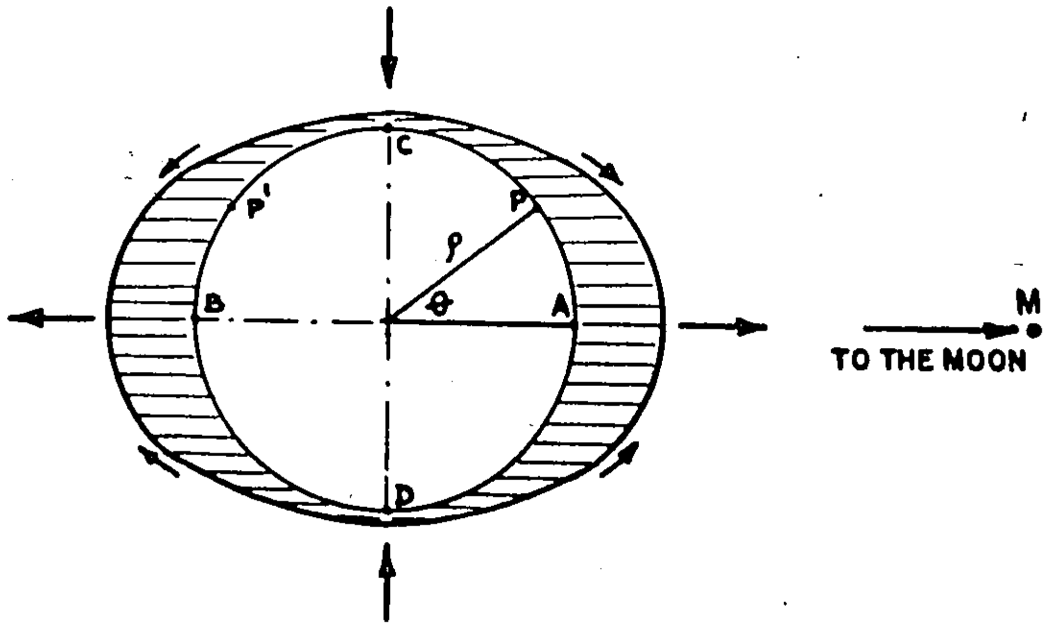


Fig. 4-7. The tide-raising force.

Components of the tide-raising force. Differentiating the tide-producing potential along the direction of the required force, the vertical and tangential (horizontal) components are:

(1) Vertical component $\frac{\partial V}{\partial \rho} = \frac{GM\rho}{r^3}(3\cos^2\theta - 1)$ (4-8)

(2) Tangential component $-\frac{\partial V}{\rho\,\partial\theta} = 3\,\frac{GM\rho}{r^3}\cos\theta\sin\theta$ (4-9)

$$= \frac{3}{2}\,\frac{GM\rho}{r^3}\sin 2\theta$$

The magnitude of the tide-raising force is very small, this can be obtained from using Eqs. (4-8) and (4-9) representing forces per unit mass (or acceleration) in vertical and tangential directions.

The moon attracts the ocean at point A, Fig. (4-7) directly beneath it and nearer to it, more powerfully than it attracts the solid mass of the earth, and so tends to pull the two apart. The ocean at B, on the far side, is less strongly attracted than the mass of the earth, and so the disturbing force again tends to separate the two. At points C and D the moon's attraction pulls the earth and the ocean along converging lines, and draws them together. At intermediate points, such as P and P', a combination of the two effects take place, and the resultant forces are indicated by the arrows.

Investigating the extreme values of the tide-raising force, let us select points A and C where tangential components are equal zero.

Force (acceleration) in vertical direction is:

$$a = \frac{GM\rho}{r^3}(3\cos^2\theta - 1) \qquad (4\text{-}8)$$

If, $G = g\frac{R^2}{E}$, and $R = \rho$ where, R is a mean radius of the earth and E its mass.

Combining Eq. (4-8) and the value of G we will have

$$a = g\frac{M}{E}\left(\frac{R}{r}\right)^3 (3\cos^2\theta - 1) \qquad (4\text{-}10)$$

If M is the moon's mass (in terms of the earth's) r its distance in radii of the earth, and g the acceleration of gravity at the earth's surface, the tide-raising force (on the unit mass) at point A, where angle $\theta = 0$ is

$$a = 2\frac{M}{E}\left(\frac{R}{r}\right)^3 g \qquad (4\text{-}11)$$

For the moon-earth ratio $M/E = 1/81$ and $r/R = 60.3$

$$a = 1/8{,}800{,}000\ g = 0.000{,}000{,}112\ g$$

or

$$a = 0.000{,}003{,}594\ \text{ft/sec}^2,\ \text{if } g = 32.09\ \text{ft/sec}^2$$

This is the maximum tide-raising force(acceleration), it is directed toward the moon when A lies between E and M and from the moon on the other side of E.

The tide producing force at point C, when $\theta = 90^\circ$ is half as great, this follows from Eq. (4-10), $a = 0.000{,}000{,}056\ g$, and acts vertically downwards.

The entire range of the disturbing force, as the angle between A and C varies from 0° to 90°, is:

$$3\frac{M}{E}\left(\frac{R}{r}\right)^3 g = 0.000{,}000{,}168\ g \qquad (4\text{-}12)$$

The influence of the sun on the tide-raising force is much less than the moon's, using Eq. (4-11) we have for the sun-earth ratio $S/E = 333,420$, where S = sun's mass, and $r/R = 23,466$, so $a/g = 1/19,400,000$. The sun's tide-raising force is thus nearly 5/11 of the moon's.

Equations (4-8) and (4-9) show that for most positions of P the vertical and horizontal components of the tide-producing force are not very unequal in magnitude.

Since the horizontal acceleration $(\frac{d^2s}{dt^2})$ is $\frac{3}{2}\frac{GMp}{r^3}\sin 2\theta$, or $0.000,002,71 \sin 2\theta$ feet/sec^2., we have upon integration over 90° or three lunar hours (with relation $\theta = \omega t$),

$$s = 0.000,002,71 \frac{1}{\omega^2} = 137 \text{ feet} \qquad (4\text{-}13)$$

where ω is 0.000,1405 radian per second^2, instead of 28.984 degrees per hour. This gives 137 feet for the maximum excursion of a particle at the equator east or west from its mean position, due to the moon.

In order to see that the vertical force can have little or nothing to do with the tides, let us suppose that, in a sea of uniform depth, the density of the water be increased or decreased from place to place in such proportions as the force of gravity is altered by the vertical disturbing force of the moon. But the extreme variation in density over the globe would then be only 0.000,000,168 and so, returning to the consideration of water of constant density, it follows that the extreme variation in the height of the free surface of a sea of uniform depth would be but a 0.000,000,168 part of the depth.

The vertical force being generally about the same magnitude as the horizontal, and acting nearly perpendicularly to the free surface of the fluid, cannot create a horizontal motion comparable with that created by the horizontal force.

The deviation of the plumb line is evidently due wholly to the horizontal force. It is supposed to be practically independent of the depth or mass of the water, the topography of the continents, etc. Any surface normal to the disturbed plumb line is a level surface.

If a liquid surface coincide with an instantaneous level surface while the latter undergoes changes, the forces responsible for the behavior of the liquid must be horizontal and not vertical.

An Hypothetical Equatorial Canal of Uniform Depth Surrounding the Earth.

The present illustration is given for the purpose of showing that the surface of the sea does not of necessity arrange itself normal to the plumb line as disturbed by the moon; and here also it is necessary to consider the force to deviate the same, that is, the horizontal component of the moon's tide producing force. All particles of the canal in the hemisphere toward the moon have imparted to them horizontal accelerations, urging them toward a point of the canal where the moon is on the meridian. All particles in the other hemisphere are at the same time urged toward a point 180° distant in longitude. There is no acceleration (east or west) at these two points, or at points 90° distant where the moon is in the horizon. Consequently, at any given place, from moonrise to upper local transit, the acceleration is eastward, because the particle is continually approaching the moon; from transit to moonset the acceleration is westward; from moonset to lower transit it is eastward; and from lower transit to moonrise it is westward. Now, if the fluid be heavy and frictionless, the maximum eastward velocity will occur after all the eastward acceleration has been imparted, that is, at lunar moon or after midnight; the greatest westward velocity, at moonrise; and zero velocity at the third, ninth, fifteenth, and twenty-first lunar hours.

All the time from moonrise till transit, more water flows toward the east than enters from the west at a given place, because the particles of moving water are continually acted upon by a force imparting to them an eastward acceleration. The reverse is true from transit moonset. Similarly for the other half of the lunar day. During one of these periods the tide must be continually falling, and during the other continually rising. Low and high waters occur at the close of these periods, that is, at the transits and when the moon is in the horizon. The tides having a fixed position with respect to the moon, it follows that the wave-form travels westward around the earth twice during each lunar day. Of course even the horizontal displacements of the fluid particles are small in comparison with the earth's radius. But it is obvious that if the wave-form advance westward, the orbital direction of the fluid particles must be such that the particles at high water are moving westward and at low water eastward; and so, as just stated, high water must occur when the moon is in the horizon, and low water when on the meridian.

<u>The Equilibrium Theory of Tides</u>. The equilibrium theory begins by assuming;

(1) That the nucleus of the earth is comparatively rigid (or that at least its outer layer is a rigid shell), and that it is composed of concentric spherical layers, each layer having a constant density.

(2) That the nucleus is covered by a fluid of uniform depth, shallow as compared to the radius of the nucleus, but deep as compared to the rise and fall of the tide.

(3) That this fluid has neither inertia nor viscosity, nor is there friction between the fluid layer and the nucleus or the enveloping atmosphere.

As these conditions are far from being realised in the case of nature, observations will show at best only certain approximations toward ideal values. Before introducing the modifications necessary to adapt the theory to the tides, it seems desirable to ascertain what the tendencies are in the ideal case.

Since the angular velocity of the moon in its orbit and the rotary motion of the earth's surface are finite, while the particles of fluid are supposed to respond immediately to the forces acting upon them, we may consider the earth's surface as stationary during any given instant, and treat the surface assumed by the water as a case of static equilibrium.

Because of hypothesis (1), the attraction of the moon upon the nucleus is the same as it would have been had the entire mass been concentrated at the earth's center.

At any given place the tide-producing tendencies depend chiefly upon the distance and direction of the disturbing body, and are governed by what may be referred to as Laws I and II.

Law I. The tendency to produce tides at a given station varies directly as the mass of the disturbing body and inversely as the cube of the body's distance from the earth's center.

In consequence of this law the amplitude of the solar tide ought to be about 0.455 times that of the lunar tide. For, the mass of the sun = 333,420/1, and the mass of the moon = 1/81, the mass of the earth being unity, while the sun's distance = 92,800,000 miles and the moon's distance = 239,000 miles, so that:

$$\text{solar tide} : \text{lunar tide} = \frac{333{,}420 \times 81}{(92{,}800{,}000)^3} : \frac{1}{(239{,}000)^3} ;$$

$$\text{solar tide} = 0.455 \text{ lunar tide}$$

Law II. The tendencies to produce tide for various relative positions of the tide-producing body are proportional to

$$3 \cos^2 \theta - 1,$$

where θ is the zenith distance of the body corrected for parallax. In other words, θ is the angle at the earth's center defined by the given station and the center of the disturbing body.

If h denote the height of tide expressed in terms of the earth's radius, R, then it is proportional to $3 \cos^2 \theta - 1$, or equal say $\alpha(3 \cos^2 \theta - 1)$. The equation of the surface of the sea at any given instant is:

$$\rho = R(1 + h)$$

$$\rho = R + \alpha R(3 \cos^2 \theta - 1) \qquad (4\text{-}14)$$

which is the equation of an ellipsoid whose semiaxes are

$$R(1 + 2\alpha), \quad R(1 - \alpha), \quad R(1 - \alpha)$$

That is, forces acting according to this law cause the surface of the sea to assume the form of an ellipsoid of revolution whose longest axis points toward the tide-producing body.

It will be observed that when the moon, say, is in the zenith the elevation of the sea is 2 $R\alpha$ higher because of the existance of the moon; but when in the horizon, the elevation of the sea is $R\alpha$ lower.

For a given place the height of the tide will vary from hour to hour of the day chiefly on account of the variations in θ; but, as already noted, it varies somewhat on account of the variation in r, the moon's distance.

For a given place the angle θ depends upon the moon's hour angle and its declination, both of which are functions of time.

<u>The Height of Lunar and Solar Tides.</u> Imagine that the moon is divided into two equal parts, the one occupying the moon's position, the other a position diametrically opposite but at an equal distance from the earth's center. (See Fig. 4-8).

The potential of the entire moon at any point P now becomes

$$W = \frac{1}{2} GM \left(\frac{1}{D'} + \frac{1}{D}\right) = \frac{\frac{1}{2} GM}{\sqrt{r^2 + 2r\rho \cos \theta + \rho^2}} + \frac{\frac{1}{2} GM}{\sqrt{r^2 - 2r\rho \cos \theta + \rho^2}}$$

$$= \frac{1}{2} GM \frac{1}{r} \left(P_o - P_1 \frac{\rho}{r} + P_2 \frac{\rho^2}{r^2} - \ldots\ldots + P_o + P_1 \frac{\rho}{r} + P_2 \frac{\rho^2}{r^2} + \ldots\ldots\right) =$$

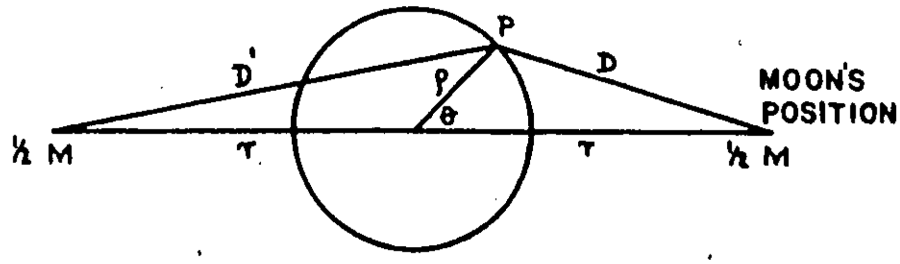

P
D'
D
ρ
θ
MOON'S
POSITION
½ M
T
T
½ M

Fig. 4-8

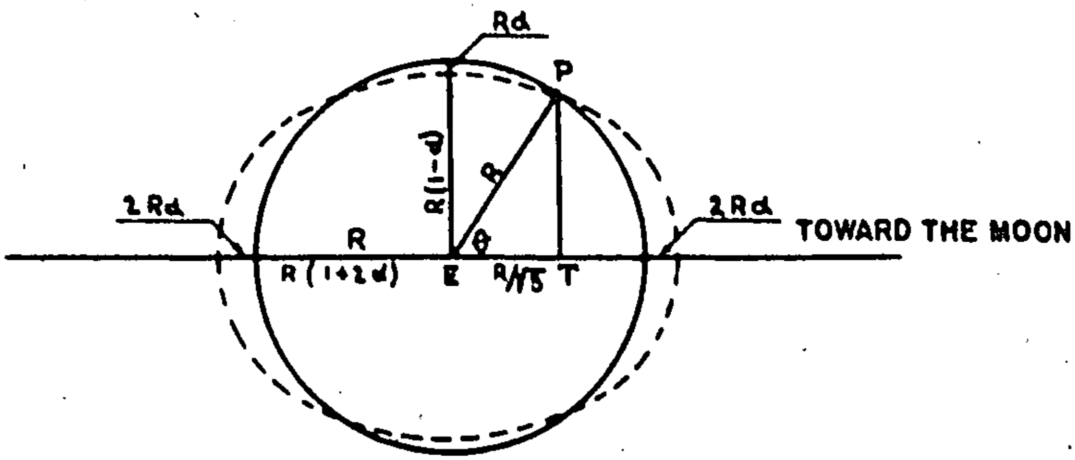

Rd
P
R(1-d)
R
2Rd
2Rd
TOWARD THE MOON
R
θ
R(1+2d)
E
R/√3
T

Fig. 4-9

$$= \frac{GM}{r} \left(P_o + P_2 \frac{\rho^2}{r^2} + P_4 \frac{\rho^4}{r^4} + \ldots\ldots \right) \qquad (4\text{-}15)$$

A surface of equilibrium or a level surface is one which has the same potential at all its points. Supposing the earth to be a sphere without rotation, and the moon divided as in Fig. (4-8); any surface of equilibrium has for its equation

$$\frac{GE}{\rho} + \frac{1}{2} GM \left[\frac{1}{D'} + \frac{1}{D} \right] = \text{constant} \qquad (416)$$

E denoting the earth's mass. Because the water of the sea is incompressible and because the action of the moon is symmetric about a line joining the centers of earth and moon, the surface whose equation is Eq. (4-16) must cut a perfect or undisturbed sphere in two small circles whose planes are perpendicular to the line joining the centers. Let R = the radius vector of the surface Eq. (4-16) at these small circles, = the mean radius of undisturbed sphere. If now we write

$$\rho = R\ (1 + h) \qquad (4\text{-}17)$$

R h is a very small quantity in comparison with R and represents the variation of ρ from a constant value R. Since Eq. (4-16) is true for all possible values of ρ, it is true when $\rho = R$ or $h = 0$. Developing Eq. (4-16) and putting $\rho = R$ we have, as in Eq. (4-15)

$$\frac{GE}{R} + \frac{GM}{r} \left[P_o + P_2 \frac{R^2}{r^2} + P_4 \frac{R^4}{r^4} + \ldots\ldots \right] = \text{constant} \qquad (4\text{-}18)$$

Now $2 P_2 = 3 \cos^2 \theta - 1$,....if we make $\cos \theta = 1/\sqrt{3}$, $P_2 = 0$. Hence, if we omit all terms beyond $P_2 R^2/r^2$ in the brackets as being comparatively small, Eq. (4-18) becomes

$$\frac{GE}{R} + \frac{GM}{r} = \text{constant}$$

Writing this value for the constant in Eq. (4-16) we have;

$$\frac{E}{\rho} + \frac{1}{2} M \left[\frac{1}{D'} + \frac{1}{D}\right] = \frac{E}{R} + \frac{M}{r} \qquad (4\text{-}19)$$

Putting $\rho = R(1 + h)$, Eq. (4-19) gives upon development

$$\frac{E}{R}(1 - h + h^2 - \dots) + \frac{M}{r} \quad P_o + P_2 \frac{R^2(1+h)^2}{r^2} + \dots = \frac{E}{R} + \frac{M}{r}$$

(4-20)

Omitting the second and higher powers of h as being very small, we have from Eq. (4-20),

$$h = \frac{M}{E}\frac{R^3}{r^3} P_2 = 2\alpha P_2 = \alpha(3 \cos^2 \theta - 1) \qquad (4\text{-}21)$$

when $\theta = 0$,

$$h = \frac{M}{E}\frac{R^3}{r^3} = 2\alpha \qquad (4\text{-}22)$$

when $\theta = 90^\circ$

$$h = -\frac{1}{2}\frac{M}{E}\frac{R^3}{r^3} = -\alpha \qquad (4\text{-}23)$$

Now Rh represents the inequality in the radius vector of the surface of equilibrium. If the surface be an ellipsoid of revolution, the section made by a plane passing through the centers of the earth and moon must be an ellipse having semiaxes $R(1 + 2\alpha)$ and $R(1 - \alpha)$, respectively, as shown on Fig. (4-9).

For the lunar tide,

$$R\alpha = \frac{1}{2}\ \frac{\text{mass of moon}}{\text{mass of earth}} \times \frac{R^4}{(\text{moon's distance})^3} = 0.59 \text{ feet}$$

and for the solar tide,

$$R\alpha = \frac{1}{2}\ \frac{\text{mass of sun}}{\text{mass of earth}} \times \frac{R^4}{(\text{sun's distance})^3} = 0.27 \text{ feet}$$

The equation of such an ellipse is

$$\frac{x^2}{R^2(1 + 2\alpha)^2} + \frac{y^2}{R^2(1 - \alpha)^2} = 1 \qquad (4\text{-}24)$$

Now writing:

$$x = \rho \cos \theta$$

$$y = \rho \sin \theta$$

$$\rho = R (1 + h)$$

and Eq. (4-24) becomes

$$\frac{(1 + h)^2 \cos^2 \theta}{(1 + 2\alpha)^2} + \frac{(1 + h)^2 \sin^2 \theta}{(1 - \alpha)^2} = 1$$

$$h = \alpha(3 \cos^2 \theta - 1) \qquad (4\text{-}25)$$

which agrees with Eq. (4-21), and shows that the variation in ρ is such that the section made by the plane passing through the center of the earth and moon is an ellipse.

To show that the condition of continuity is fulfilled, that is, that no volume has been lost or gained by the deforming action of the moon, it is only necessary to remember that the volume of the ellipsoid is:

$$\frac{4}{3} \pi R^3 (1 + 2\alpha) (1 - \alpha) (1 - \alpha)$$

$$= \frac{4}{3} \pi R^3 (1 + 0\alpha - 3\alpha^2 + \ldots.)$$

$$= \frac{4}{3} \pi R^3$$

when α is so small that all powers beyond the first may be neglected. But this is the volume of a sphere whose radius is R.

In obtaining Eq. (4-20), cos θ was arbitrarily put equal to $1/\sqrt{3}$ in order that P_2 might be equal to zero. The reason for this is that for some value of θ we know that the tide-producing potential must be zero; and to the degree of approximation here assumed, this potential is

$$\frac{GM\rho^2}{r^3} P_2$$

Explanation of the Principal Tidal Phenomena. At new and full moon the lunar and solar high tides would come together, and the range of level would be unusally great. At the quarters the solar high tide would coincide with the lunar low tide, and would partly fill up the depression, producing a decreased range. This explains the spring and neap tides. The greatest and least ranges should be in the ratio of (1 + 0.455) to (1 - 0.455) or 2.66:1. The observed ratio is usually not so great.

The maximum calculated spring tide would be 2 x 0.86 = 1.72 ft., which corresponds closely to observed height in the open ocean under the ideal conditions, where other disturbing forces are at their minimum.

Effect of Wind and Barometric Pressure. In quiet weather the predictions of the tide tables are usually very accurate, but a strong wind may drive in the water before it, raising the level sometimes by several feet. A northeast wind, for example, raises the level at the western end of Long Island Sound. In such a case the tide runs in longer than usual, and high water comes late. A wind in the opposite direction produces the reverse effect.

When the barometer is lower than usual, the level of the water is usually higher than it would otherwise be, at the rate of about a foot for each inch of barometric height. In a hurricane, where the barometer is very low and the wind drives the water onto the shore, the tide may rise very high, 13.7 feet above normal high water at Providence on September 21, 1938.

Complexity of the Actual Problem. The simple equilibrium theory (worked out by Newton) accounts for the general behavior of the tides very well, but it breaks down in details. It does not explain why high tide, at different points, comes at all sorts of different intervals after the moon's transit, or why the range of the tides is many times greater in some places than in others, or why there is a marked diurnal inequality of the tides in some places near the equator (like Manila) and very little at high latitudes in the North Atlantic.

There are two reasons for these discrepancies. First, the earth rotates rapidly, so that the tide-raising forces in a given region of the ocean change too fast for the water-level to adjust itself fully to them; second, the ocean basins are very irregular in form and depth.

The first of these facts makes it a very difficult problem to calculate the tides theoretically even in an ocean of uniform depth, bounded by parallels of latitude and longitude; the second destroys all hope of general theoretical calculation.

For bodies of water as large, as irregular, and as much cut up by land barriers as the oceans are, theoretical calculation is hopeless, except in one very important particular, it enables exact prediction of the periods of the actual tidal oscillations.

Consider for example, the sun's influence at the time when it is on the celestial equator. At any given point the tide-raising force varies in amount and direction from hour to hour, but returns to the same value at intervals of twelve hours. After such a force has acted long enough for a 'steady state' to be established, the surface of the ocean will be thrown into oscillations. The character of these oscillations, whether they are simple motions, like those in a small lake, or complicated ones, with several regions in which the water rises at the same time, separated by others in which it falls, will depends on the size, shape, and depth of the sea and on the range of level. The time of high water will differ from place to place, but the period will be exactly that of the impressed force.

Similarly, the lunar tide-raising force, if acting alone, would set up a series of oscillations at intervals of $12^h\ 25^m$ with a greater range and a somewhat different 'pattern' of regions of high and low water. These two oscillations are called the solar and lunar semi-diurnal tides. Under the combined action of the moon and the sun they are superposed without sensibly modifying one and another, and their combination produces the spring and neap tides.

TIDE PREDICTION

When a record of the actual fluctuations of the water-level at a given port for a year or more has been obtained (which may readily be done by a self-registering tide-gauge), the ranges and phases of the separate periodic changes of level are not hard to find. For example, taking readings made at noon on every day of the year, the lunar tide will sometimes be high and sometimes low, and its effects will practically disappear in the average, while the solar tide will always be the same. Taking averages of the readings at 1 p.m., 2 p.m., etc., the course of the solar tides can be found. Tides of other periods can be treated similarly.

Having thus found the various component tides at the given port, the level of the water at any time may be predicted by adding together their effects. This may be done mechanically, with the aid of an automatic tide-gauge recorder which, when once set with the tidal constants for a given port, will automatically draw a curve on a long roll of paper, showing the oscillations of the tide-level for a whole year. The United States Coast Survey uses such a machine to prepare the Tide Tables which give the predicted times and heights of every high and low water for about seventy of the principal ports of the world.

The sun, moon and earth occupy almost identically the same relative positions in the heavens every 18 years and 11 days. Consequently tidal phenomena are reproduced under similar conditions at the end of successive periods of this duration, which were known to the ancient Chaldeans and were called by them a saros. If, therefore, there be obtained the records of a series of tides during one complete saros, they will serve to establish a suitable prediction for any subsequent saros.

If these records are not available, the predictions have generally to be made from certain local observations over a short interval of time expanded by a system of harmonic analysis.

The observations should extend over at least a fortnight, so as to include a spring and a neap tide. If there is no automatic tide-gauge recorder available for the purpose, the tidal readings should be booked, generally at intervals of an hour, but every 5 or 10 minutes about the time of high and low water, so as to obtain a fairly accurate representation of a curve, which should be drawn through a series of points set out on squared paper. The times of high and low water have then to be referred to the time of the moon's transit, and any irregularities in the interval between transit and high water should be carefully noted, the average interval between transit and high water being also noted. The average interval from the moon's transit to high water of spring tides gives the mean or corrected establishment, and the average interval from full and change of the moon to spring tides gives the age of the tide.

If the results for eight or nineteen years are all plotted on the same annual basis, the generalised curve will be a very nearly correct curve of mean level, showing the regular seasonal effects of barometer change, wind, and fresh water run-off or ocean currents. For rough purposes of prediction, half the average spring or neap ranges (which may be picked out each half-moon and averaged), added or subtracted from this mean level, will give a fair indication of high or low water level at springs or neaps.

Any record of the rise and fall of tides plotted on a time basis can be subjected to harmonic analysis, and an irregular curve can be always shown to be a combination of a number of simple sine curves which can be plotted on square-ruled graph paper. Most of these curves are recognizable from one tidal graph to another and so can be identified by a series of letters. Each one can be related to a particular tide-producing force, the sum of which produces the irregular tidal curve expressing the combined effects of all the forces exerted on the Earth.

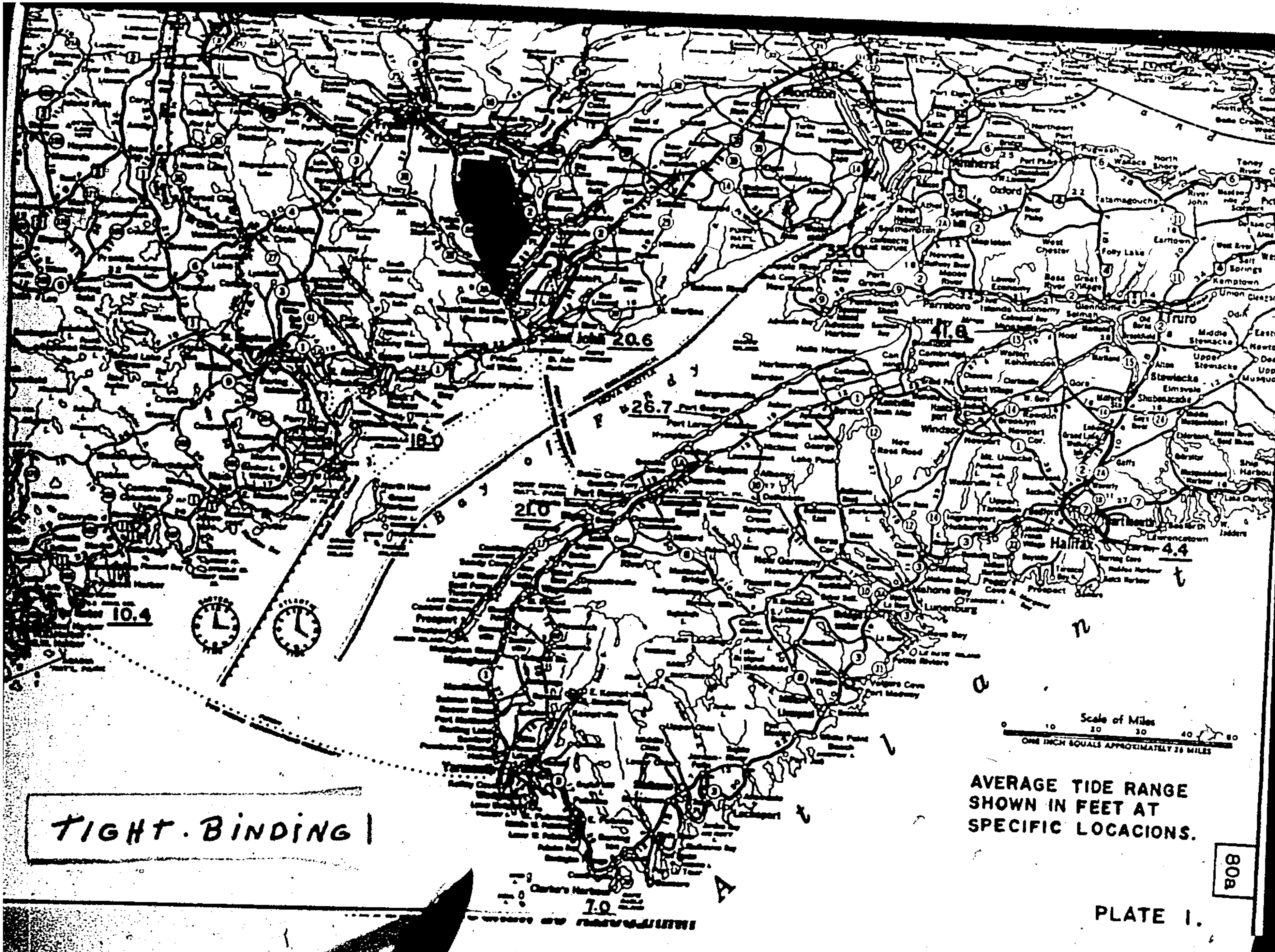
In other words, in order to arrive at a basis of prediction for future tides, the mathematician finds it convenient to dissect the tidal curve (or wave) and resolve it into a series of independent curves (or waves) so proportioned as to produce the actual result by cumulative effect.

CHAPTER V

POWER FROM THE TIDES

Tidal hydroelectric power, similar to river hydro power, can be produced by a flow of water from a higher to a lower level through hydraulic turbines. A single pool equipped with gates may be built to trap water at high tide and discharge through turbines to the ocean at low tide, or the pool may be emptied at low tide to receive turbine discharge from the ocean at high tide. Two separate pools equipped with emptying and filling gates may be used, one pool filled at high tide and the other emptied at low tide, with the high pool discharging through the turbines into the low pool.

The advantages of a tidal power plant are that the tides, which can be predicted with accuracy for many years in the future, can produce power unaffected by droughts, floods, ice jams, or silting -- adverse factors which decrease the output and limit the life of river hydroelectric plants. An inherent disadvantage of the tides as a source of power is that the tides, following gravitational pull of the moon as it passes overhead every 24 hours and 50 minutes, are out of phase with the 24-hour solar day. This 50 minute daily lag is fundamental to the economics of tidal power for, since power output varies with the tides, tidal power is completely out of step with the normal patterns of daily use of electricity. Therefore, unless the tidal plant is supplemented by an auxiliary power plant,



AVERAGE TIDE RANGE
SHOWN IN FEET AT
SPECIFIC LOCACIONS.

80a

PLATE I.

such varying power would be of little value.

The height the tide will reach is affected by the sun, moon and to a high degree, by the coastline. In the Gulf of Maine, however, which opens toward the deep areas of the Atlantic Ocean as the continental shelf drops off beyond Georges and Browns banks, the tides are greatly amplified by the size and configuration of the shore and bottom of the ocean. As shown on plate I, the mean tidal ranges become progressively greater as the tides move into the Gulf of Maine toward the mouth of the Bay of Fundy. The funnel-shaped Bay of Fundy again amplifies the tidal range, producing the highest tides in the world at the head of the Bay. To devise a workable and feasible scheme to harness these tides for the economical production of uninterrupted power constitutes the essence of tidal power engineering.

The extremely ingenious solution to the problem of providing tidal power at practically all times of the day investigated by Electricite de France, and now to be put into practical operation, hinges on a combination of tidal power generation and pumped storage.

It was not possible to consider such a scheme until the designers of water turbines had produced a unit which could operate both as a pump and a turbine under variable heads, and which could generate or pump with the water flow in either direction. This problem has been the subject of research and experiment by French designers for a number of years, and after tests with machines of this type in different conditions, it was decided that sufficient experience had been gained to warrant the inauguration of the Rance tidal power scheme.

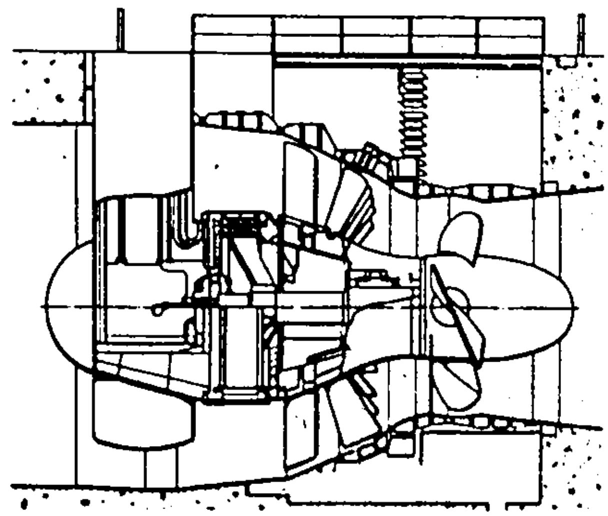


Fig. 5-1. Diagram of 10,000 kW 88.2 r.p.m. turbogenerator for St. Malo.

The method of operation is as follows: assume first that high-tide level exists on both sides of the dam. If energy can be borrowed from the supply system for pumping purposes, it is possible to raise the level within the pool (basin) behind the dam, by turning the generators into pumps; as the tide falls away outside the dam a head becomes available which is obviously greater at any given state of the tide than would have been the case if pumping had not taken place.

Generation can start as soon as the minimum head is available; this period in relation to the tide occurs sooner than would have been the case if pumping had not raised the head within the pool. Generation continues until sufficient water has been evacuated from within the pool to reduce the head to minimum generating levels. The turbines, with their feathering blades, have the additional advantage that they can be turned into valves.

By turning all turbines into straight-through valves and also opening all the sluices, the water within the pool can be rapidly evacuated at about the time of low tide. The units being totally submerged, it is possible, when all the water that can be evacuated under gravity flow has been removed, to pump out further water again if energy for pumping is available. That is, if this period, in relation to the tide, is an off-peak period.

By this reverse pumping operation it becomes possible to produce an artificially low water level within the pool; and as the tide rises on the outside of the dam again a generating head is reached earlier, in relation to the tide, than would have been the case if the water level had not been artificially lowered.

As the tide continues to rise generation takes place until sufficient water has flowed into the pool to destroy the minimum generating head. The turbines are again turned into straight-through valves and the sluice valves are opened, thus allowing rapid inflow of water until high tide is again reached. The cycle can then restart.

One-way operating tidal plant. Let us consider the case of a pool separated from the sea by a dam, the water level in the pool following the rise and fall of the sea, and let us call H the tide height, and V the capacity of the pool (basin) at the corresponding water level. For a pool area A(y) as a dependent variable of water level y in the pool, the work performed during the emptying period will be

$$E_1 = \int_{y=0}^{H} wA(y)y\,dy \qquad (5\text{-}1)$$

and during the filling period

$$E_2 = \int_{y=0}^{H} wA(y)(H-y)\,dy \qquad (5\text{-}2)$$

emptying and filling the pool occuring necessarily within a tide interval during which the level of the sea rises from 0 to H. The available energy capacity during a whole tidal cycle would then be

$$E_1 + E_2 = E_B = wH \int_{y=0}^{H} A(y)\,dy = wHV_B \qquad (5\text{-}3)$$

Actually, however, the practical utiliseable energy is but a fraction of the above ideal value, so that the theoretical extent to which the energy capacity can be utilized is limited to

$$E_B = CwHV_B, \qquad \text{where } C < 1 \tag{5-4}$$

With the help of artificial means, however, it is possible to arrive at a much higher degree of utilization.

The above considerations apply exclusively to plants operating with turbines only, and utilization possibilities are substantially modified by the addition of a pumping system. If, for instance we have to deal with the case of a one-way plant operating with turbines and pumps, then we may have, in addition to the main pool, an auxiliary pool $A_1(y)$, $(y \geq H)$, with its bottom lying at the level of the tide height H. While the incoming tide fills the main pool, the pumps coupled to the turbines lift water into the auxiliary pool to level B. If the turbines and pumps are of equal capacity, we then have

$$\int_{y=0}^{H} wA(H-y)dy + \int_{y=H}^{B} wA_1\,(y-H)\,dy \tag{5-5}$$

and if we express the pumped volume of the water by V_p, Eq. (5-5) expands to

$$E_B = \int_{y=0}^{H} wAydy + \int_{y=H}^{B} wA_1ydy = wH(V_p + V_B) \tag{5-6}$$

the theoretical energy produced during the ebb period by the water flowing out of the two pools being thus greater by wHV_p than in the case of Eq. (5-3) of a plant operating with turbines only.

The simplest tidal project arrangement is the single pool created by a barrage enclosing an estuary or a bay. The potential energy of the water is utilized when the pool is emptied and this is known as the single high-pool emptying cycle, Fig. (5-2) shows a typical example.

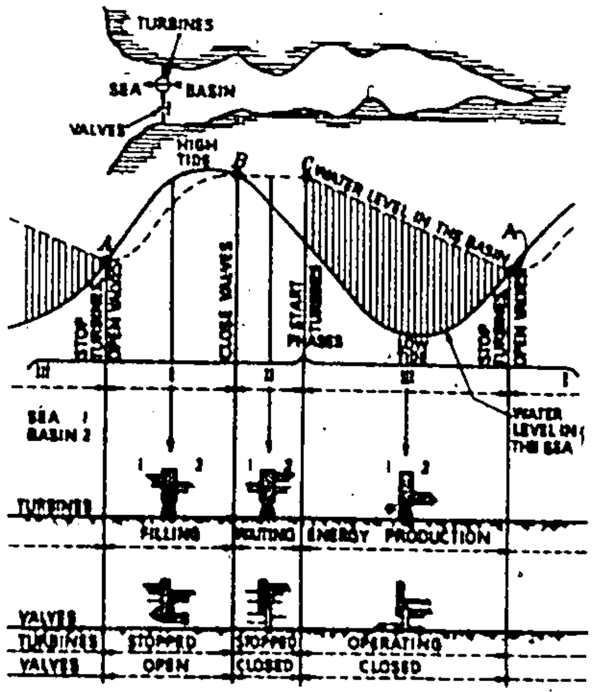


Fig. 5-2. One - way operating tidal plant.

Three operating phases may be distinguished: (A) A filling phase, with the turbine stopped and the valves open; with the tide rising, water flows into the pool through the valves. (B) A waiting phase, with the turbines stopped and the valves closed. The sea level falls but the level of the water in the pool does not change. The operators then wait for the most favourable moment for the commencing of energy production. (C) A production phase, with the turbines operating and the valves closed. The turbines produce energy by the reason of the head created by the difference in the level between the pool and the sea. In this arrangement, when the turbines are operating the head is always in the same direction, that is, from the pool toward the sea.

A similar type of operation is a single low-pool, in which the turbines operate during the filling of the pool. As the banks of estuaries are irregular in shape, and the volume of water moved for the same difference in level are considerably greater in relation to the upper layer of the pool than in relation to the lower layers, the filling-cycle type of operation (single low-pool) gives less energy than emptying cycle (single high-pool). For example, for the French Rance project the ratio is approximately as 2/3:1.

It would be an advantage in both types of single pool projects to use turbines that also operate as pumps. This would require energy from an outside source. In the high-pool project, for example, the pool level can be raised further by pumping for a short period after the filling gates or valves are closed. The energy used in pumping at low head increases the pool level, and increases the head during the subsequent generating period. The energy gain in the generating cycle exceeds the energy used in pumping to the benefit of the project.

<u>Two-way operating plant</u>. In a plant operating with double-admission turbines and two-way pumping, and in the general case of a single pool without higher auxiliary pool, the pumps drawing their power from the network, we have the following four phases:-

(1) The water level in the pool, equal to low water level, being further lowered to -B by pumping, the pumps will draw from the network energy

$$E_{p1} = \int_{y=0}^{-B} wA(y)y\,dy \tag{5-7}$$

(2) The rising tide fills the pool, and the energy output of the turbine is

$$E_{T1} = \int_{y=-B}^{H} wA(y)(H-y)\,dy \tag{5-8}$$

(3) Upon high water level being reached, the pumps raise the water level in the pool to level C with an energy consumption of

$$E_{p2} = \int_{y=H}^{C} wA(y)(y-H)\,dy \tag{5-9}$$

(4) The turbines utilize the water in the pool down to the level 0, during which period their energy output is

$$E_{T2} = \int_{y=0}^{C} wA(y)(y-H)\,dy \tag{5-10}$$

This, after deduction of the requirements of the pumps, leaves for an operating cycle an energy production of

$$E_B = E_{T1} + E_{T2} - E_{p1} - E_{p2} = wH \int_{y=-B}^{C} A(y)dy =$$

$$= wH(V_B + V_{p1} + V_{p2}) \quad (5\text{-}1\text{I}')$$

Therefore, if turbines and pumps operate at both flow and ebb periods, the energy gain to be expected amounts to

$$wH(V_{p1} + V_{p2})$$

as against turbines exclusively in operation Eq. (5-3) and

$$wHV_{p2}$$

in the case of one-way pumps delivering into a higher auxiliary pool.

The judicious selection of the sea-dam site in a bay or estuary makes it possible, by taking advantage of resonance, to increase the tide range during a tidal cycle, since each pool has a natural frequency of its own depending on its size and proportions. In a pool of a length L metres or feet, with a rectangular cross-section and a depth d, the natural frequency is expressed by $t_o = 2L/\sqrt{gd}$. When the natural frequency of the pool approaches the frequency of the tide, the tidal impulses which set the water in the pool into oscillation are reinforced. There is then the possibility, as proved by computation, of tide-range increases up to 10 per cent.

It follows that engineers should be anxious to achieve a double-effect cycle, as shown in Fig. (5-5). Energy is produced both during the filling and emptying of the pool. With a single barrage, as shown on the left at the top of Fig. (5-5), the head producing the energy is sometimes operating from the sea towards the pool, and sometimes from the pool toward the sea. Using conventional turbines, it would have been necessary, in this case to pay special attention to the intake and evacuation passages within the dam and arrange them in such

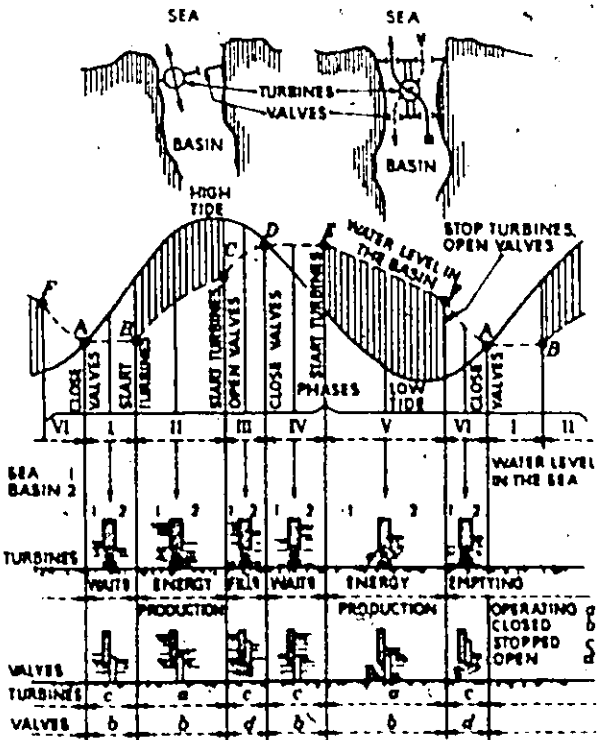


Fig. 5-3. Two-way operating tidal plant.

a way that the flow of water through the turbines is always in the same direction.

Another double-effect arrangement could consist of an H-shaped design as shown to the right of Fig. (5-3). The pool is closed by two lines of valves between which, and parallel to the bank, would be situated the power station itself. Suitable operation of the valves enable the water always to be passed through the turbines in the same direction.

Both of these solutions would entail supplementary expenditure on a very large scale, when compared with the one-way operating scheme. It is therefore particularly fortunate that the modern feathering-blade turbine enables all these complications to be avoided, and allows an extremely simple scheme to be adopted.

Pumping. To improve still further the operation of the double-effect cycle, the newer turbines can be made to operate as pumps at slack water periods and the utilizable volume of water and the operating head thus increased. The net gain in power production, in spite of the energy consumed by pumping, is substantial and a further economic advantage is secured by the differencial in value which exists between peak-load and off-peak energy.

All the utilization cycles so far mentioned refer to the period of a single tide. There is, however, a wide field open for research to investigate the economics of combining generating and pumping operations within the cycle of one tide and a number of tides combined together. This could produce continuous energy, so the tidal power could always be made available to the system operator at the time it is most needed - the peak period.

Figure (5-3) shows that a tide can be considered as including four possible movements, two generating movements and two pumping movements. This gives rise to 16 possible combinations, each representing a practicable cycle for the utilization for a tide. A single tide can thus be studied with the object of yielding energy of constant value, i.e., with the price per kWh independent of the time of production and the date. In the case of two tides, of a duration rather greater than 24 hours, the calculations become more difficult and consideration can then be given to the essential case where the value of energy varies with the time of production, i.e., if this relates to a peak hour or to an off peak period. The value of the energy is obviously different in these two cases, a point which was neglected until recent times.

It would be convenient to consider the case of 27 tides spread over 14 days, since this allows for consideration being given to peak periods and also to the off peak times on Saturday and Sunday when energy can be considered as having the lowest value. But each tide produces four movements, and we have seen that for one tide the number of combinations is 16. For two tides, we have the possibility of $16^2 = 256$ cycles, and this large figure obviously introduces some difficulty in the calculations. For 27 tides we have 16^{27} different cycles, which is about 300,000 milliards of milliards of milliards. Obviously there is no hope of calculating all these combinations. But the studies started by Electricite de France in regard to one tide and two tides, and the extrapolation of these calculations to the case of three tides could provide fundamental necessary data of the extraordinary operational flexibility of the tidal power project.

Certain terminology has been evolved in consideration for the above problem, and includes what is called the "cycle of the first order" relative to variations of value per kWh over the period of one tide, and then "cycles of the second order" relating to two tides.

<u>Cycles of the First Order (One Tide)</u>. The cycles here considered have a single tide as their basic period. Let (a) relate to the pumping period, (b) the reverse pumping, (1) direct generating, (2) reverse generating, so that with this notation double effect double pumping (represented in Fig. 5-4) could be described as a1 b2. According to the periodicity indicated, the movements would follow each other:

a1 b2, a1 b2, a1 b2, ...

The starting point and the end of the cycle being on the same side. Table I shows clearly the make-up of this series of 16 cycles.

The cycle without movement indicated by z-zero corresponds to the case where hydro-electric power stations have to spill excess water, and where the value of energy is therefore reduced during the whole period of time.

TABLE I

	No pumping	Direct pumping	Reverse pumping	Two pumping operations
No generating	z	a	b	ab
Generating on emptying	1	a1	1b	a1b
Generating on filling	2	a2	b2	ab2
Two generating periods	12	a12	1b2	a1 b2

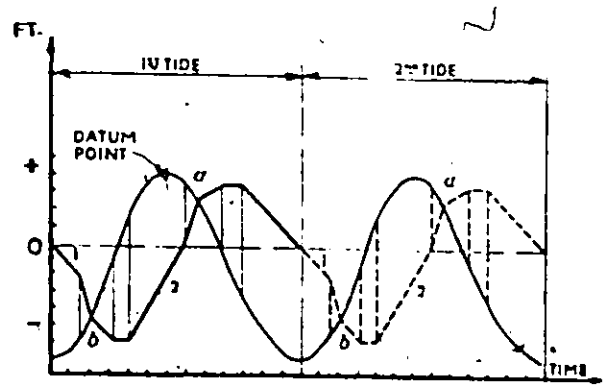


Fig. 5-4. First order cycle a1 b2 positive (double generating, double pumping).

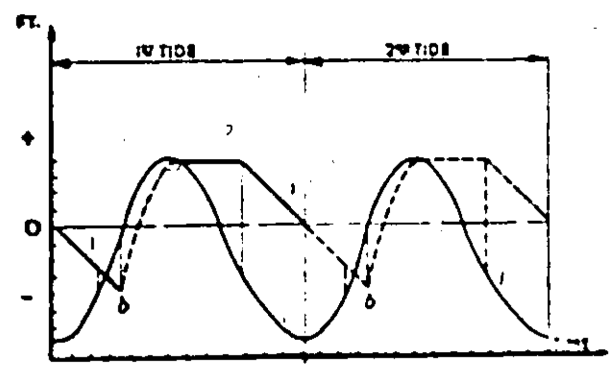


Fig. 5-5. Cycle 1b (pumping undesirable) negative.

Certain cycles, as for example 1b in Fig. (5-5), appear to indicate that pumping is undesirable, since it lowers the level in the pool at the time when it is desirable to open the valves in order to prepare for the new single effect emptying operation. For this reason, pumping would consume energy for a useless purpose.

But when atomic power is established, to avoid shutting down a nuclear reactor and causing some degree of poisoning by xenon 135, it will perhaps be useful to arrange for some of its energy to be usefully employed. In fact, the preponderance of atomic generation may in time be such that shutting down at nights of all thermal and water-power stations would still not leave sufficient load for the nuclear plants to deal with, and, in this case, it may be useful to pay a small figure for energy consumption, i.e., instead of the consumer paying for energy, he would be paid to receive it. In this instance the pumping operations on tidal power plants, which would involve almost negligible cost, would undoubtedly be preferable to the installation of liquid resistances on a large scale, or setting up other artificial consuming devices. Cycles such as Fig. (5-5) may thus have some economic interest at a later date.

Returning to the present-day conditions, we may call positive cycles those which relate to instances where the energy produced never has a negative value. We find immediately that there are eight positive cycles, apart from the zero cycle. These are as follows: Four single-effect cycles which include, without pumping, (1) emptying (2) filling; and two with one pumping operation (a1) emptying (b2) filling. Then there are four double-effect cycles which include one without pumping (1,2); two with one pumping operation (a12) (1b2); and one with two pumping operations (a1 b2).

The determination of the useful cycles for the exploration of a single tide is thus complete.

Turbines for tidal projects. The conditions to be met by tidal turbines differ essentially in some respects from those obtaining for run-of-river turbines operating on inland waters. In consequence, the types of turbines generally used in run-off river plants are not fully suitable for tidal duties. The turbines shaft can be either horizontal, as in the tubular turbine, or vertical. The tubular turbine has the advantage of favourable flow both at entry and exit, resulting not only in better discharge conditions but also in lower cost. Leaving aside a survey of the evolution of this turbine type, we will discuss here the most recent designs.

As a result of the experimental work carried out, it was considered that for the Rance scheme the bulb-type unit as shown on Fig. 5-1 would be superior, and this type of unit was finally chosen. Its advantage rested mainly on the monobloc design of each unit, the elements of which could be assembled and tested on test beds. In spite of a total weight exceeding 300 tons, the unit could be placed in position or removed, practically in a single piece in a very short time.

The rapidity with which handling operations could be carried out, and the ease of maintenance was a particularly important factor when considered in relation to units which were to operate in totally

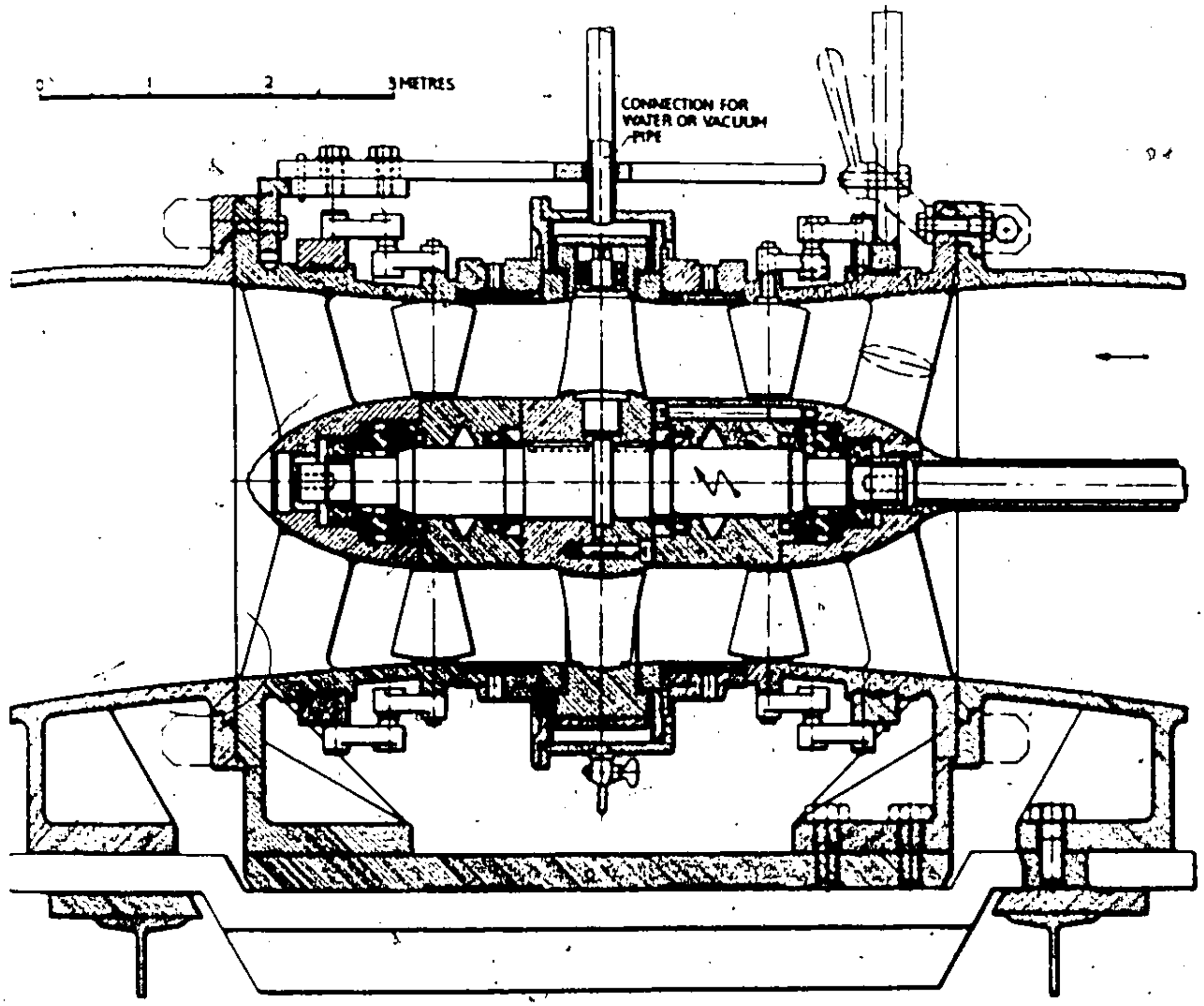


Fig. 5-6. Longitudinal section of two-way tubular turbine.

submerged conditions. It was thought, that each unit would have to be removed for overhaul about every three years, and this would imply, on a scheme like the Rance project, the dismantling and reassembly of one unit every month. This type of unit with horizontal shaft comprises of a Kaplan turbine with adjustable runner blades and an alternator placed in a bulb.

The monobloc design was found to be simple and robust, and to give reliable service without direct supervision. Automatic operation methods were devised so that each unit could, in effect, operate as an independent entity. Many problems had to be solved: the evacuation of the heat losses, the design of electrical connections, the air circulation system and the protection of the alternator against fire risks, being typical problems which the designers had to discuss and to which solutions had to be found.

In Fig. (5-6) we have the double-discharge type of turbine design and model-tested by the Escher-Wyss concern, in which two sets of movable guide vanes, fitted one on either side of the runner, regulate admission to the latter; at an appropriate position of the vanes, this turbine can also act as a pump.

The type introduced by the Maier Engineering Works and A. Fischer Fig. (5-7) is an adjustable two-wheel turbine-pump, also operating in either direction of the flow as a turbine or a pump, as required. The turbine-pump and its hydraulically coupled generator are enclosed in a cylindrical tube widening at one end to a bulb-shaped section designed in accordance with wind-tunnel tests. The respective duties

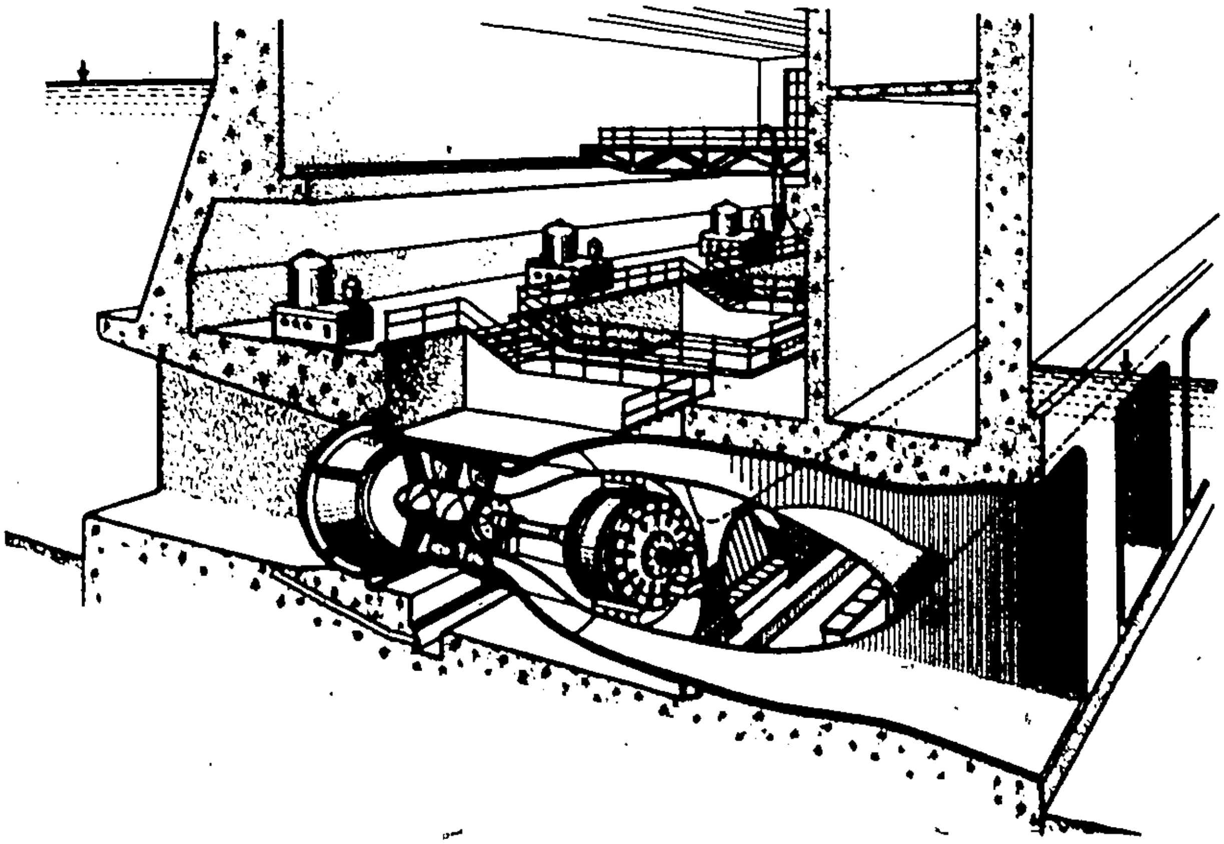


Fig. 5-7. Perspective view of tidal power house showing one of the two-runner turbine-pump units.

of the guide vane and runner are fulfilled either by the front or by the rear wheel according to the direction of the flow. These have adjustable and reversable vanes, and can be thrown in or out independently of each other. At a given position of the vanes, it is even possible to stop the flow through the turbine altogether. Determining the correct shape of the blades in this type of turbine-pump proved a difficult proposition, since, at normal turbine operation, it is the front of the blade which is acted upon by the flow, and at pumping periods, the back. Separation of the water-layer from the guide vanes and cavitation at the runner occurred to some extent during the initial tests, but practically disappeared after some improvements had been made in the construction of the model turbine.

In vertical-shaft turbines, conditions are much more complex. If we take, for instance, a Kaplan turbine with alternate flow admission from two directions, the inlet half-scroll common to most low-head plants is ruled out, owing to its asymmetric form, but even a heart-shaped symmetrical inlet would not substantially improve flow conditions. The adoption of an inlet chamber has the advantage of leading the flow to the guide vanes symmetrically in either direction, if the flow velocities are kept adequately small, losses do not exceed admissible limits. Considered from the aspect of engineering technique, the advantages of this pattern of turbine-pump should in no way be disregarded.

SELECTED LOCATIONS FOR THE TIDAL POWER PLANTS

There are three main requirements governing selection of a suitable location for a tidal power plant. The first requirement is the economical justification for such a project in a chosen area and distance to the possible present and future markets for the produced energy. Second and third requirements will be, sufficient height of the tides and favourable configuration of the shore line. The higher the tide range, the larger the amount of energy that could be produced. A shore line having a number of natural bays and estuaries create an ideal location for the tidal power scheme, as this will help engineers to build artificial pools with minimum required length of the enclosing dams and therefore makes the project more economical.

The study of the tide heights on the Atlantic coast indicates that the tide range increases along the United States coastline from south to north, reaching the following average heights; in Newport 3.5 ft., Boston 9.5 ft., Portland 8.9 ft., Bar Harbor 10.4 ft., and in Cobscook Bay between the United States and New Brunswick, Canada, 18.0 ft.

In the Bay of Fundy along the New Brunswick coast, the tide range steadily increases towards the head of the bay reaching at St. John 20.6 ft and at the inlet of Chignecto Bay to Cumberland Basin, 35 ft. Along the Nova Scotia shore the process is repeated and the average tide height at Burntcoat Head (Minas Basin) is equal to 41.6 ft. At Port George near Middleton, 26.7 ft., Digby 21 ft., Yarmouth 17.5 ft., Cape Sable 7 ft., and at Halifax 4.4 ft. only.

From the above discussion it seems also, that the Bay of Fundy offers the best possibilities for development of tidal energy than any other place on the Atlantic coast.

The most suitable places for erecting the tidal power plant could be located in the following places along the shore of the Bay of Fundy:

(1) On the border between United States and Canada, the configuration of the shore-line creates two large bays, they are Cobscook Bay and Passamaquoddy Bay with the St. Croix River estuary. A number of islands located close to the shore at the inlets to the above mentioned bays create an ideal location for a tidal power plant. Even the average tide range in this area is only 18.0 ft., the place itself offers large possibilities and flexibilities to the designer to produce a continuous supply of power, using more than one pool system.

(2) At the head of Chignecto Bay where the average tide range is 35 ft., two bays, Shepody Bay with the Petitcodiac River estuary and Cumberland Basin creates also a very interesting prospect for producing energy from the sea. In respect to the economical aspects of this project, the above two basins could be designed as two single two-way operating plants, or if outside pumping energy is available to combine generating and pumping operations in such a manner that two separate tide cycles, so called cycles of the second order, can be developed. This could be done in such a way that the time of the peak energy production from the two basins be staggered and supply more continuous

power output.

The other and probably better layout would be to combine operation of the two bays into a two-pool scheme by using one bay as a high pool and the other as a low. The power-house in this case could be constructed on the land peninsula between two basins and water could be discharged through turbines from the high to the lower pool through an artifically excavated canal.

(3) The next possible place would be Minas Basin where the height of the tides is the highest in the world, the average height is over 50 ft. The barrage could be built in the narrowest part of the Minas Channel at the entrance to the basin, somewhere to the west of Parrsboro. Such a dam would cut off a large volume of the water at the high tide and a large amount of energy could be produced. This project would be a typical one-pool scheme (one or two-way operating plant) and could not produce alone a continuous supply of power until supported by auxiliary power sources, such as pump storage plant, river hydro-electric, or steam electric plant.

(4) Another two possible places where energy from tides could be produced are located along the shores of Nova Scotia. They are St. Mary Bay and Annapolis Basin, where average tide height reaches about 21 ft. These two bays could also be developed into two separate single high-pool schemes, but much smaller in area than those previously mentioned. In line with more attractive and available previously mentioned locations, these two places are certainly worth considering as future reserves of tidal power.

CHAPTER VI

INTERNATIONAL TIDAL POWER PLANT

In the preceding chapter, a few suitable places for location of the tidal power plant were described.

First, let us consider the St. Croix River estuary with Passamaquoddy Bay and Cobscook Bay, as, for this area investigations including area mapping, deep and shallow water drilling, under water mapping and tide gaging were completed. This could be easily understood as Cobscook Bay which is the area where in 1935 the U.S. Army Corps of Engineers started construction of the ill-fated tidal project.

This also looks attractive as compared with the other areas because at the present time the Department of Mines and Technical Surveys does not have coast charts prepared for such a promising area as Minas Bay at the head of the Bay of Fundy.

The project arrangment thus selected for design includes 100 square miles of Passamaquoddy Bay and 40 square miles of Cobscook Bay. This would involve both New Brunswich (Canada) and Main (U.S.A.) interests and therefore would be an international project. The range of tides in the above mentioned areas near the mouth of the Bay of Fundy at the site of the proposed tidal project, varies from a minimum of 11.3 ft. at neap tide to a maximum of 25.7 ft. at spring tide, averaging 18.0 ft. During each tidal cycle, an average volume of approximately 70 billion cubic feet of water regularly enters and leaves both bays.

Considering design possibilities including both bays as a part of one project, this would be far superior than separate development of either bay by each nation. Independent separate projects also would involve international complications concerning navigation, fish and wild life and other aspects.

A power project using tidal pools is essentially the same as a river hydroelectric project. The amount of power generated by both methods is proportional to the amount of water flowing and the head through which it drops. Because the head at a tidal power project is considerably less than at most conventional hydroelectric projects, large quantities of water must be used to generate the same amount of power. Dams, channels, gates and a powerhouse are needed for a tidal project as for a power project on a river. Other factors such as a rapidly varying head and problems of salt water corrosion must also be considered. For use in a tidal project, however, these structures must be built to extract power from a smaller head and greater flow. The components of a tidal project may be arranged in many different ways to generate power. These methods are described in the following paragraphs.

PRELIMINARY INVESTIGATION OF THE TIDAL POWER OUTPUT

Before the best layout of the tidal project arrangement can be chosen, let us consider a few possible layouts which can be applied to Passamaquoddy and Cobscook Bay.

For this preliminary investigation purpose some assumption will have to be made simplifying the actual complicated problem so that the power generated for each different layout could be compared.

The assumptions are:

(1) The average height of the tide for the investigated area is 18 ft.

(2) The tide wave profile is a cosine curve having equation

$$y = z \cos \frac{2\pi}{T} t$$

where amplitude $z = 9$ ft.

period $T = 12^h\ 25^m = 12.416$ hrs.

variable t = time

(3) Each considered pool has vertical sides and the constant cross sectional area.

(4) Water level in the pool changes (drops) following straight line relationship between drop and time (dy_1/dt = constant), when the turbines are in operation. This will indicate also that the flow through the turbines (discharge) Q is constant in time t, (dQ/dt = constant). Such theoretical assumption is justified because the amount of the water-flow through the turbine can be regulated to be constant under variable head.

(5) Minimum operational head for the turbines is assumed to be 6 ft.

Single Pool Arrangements. (1) The simplest tidal project arrangement is the single-pool plan. To operate the single high-pool plan, as illustrated on Fig.(6-1), the pool is filled during high tide up to 8 ft. above m.s.l., after which the filling gates are closed. As soon as the head from the pool to the ocean is large enough, power is generated during each tide cycle, over a time period in which the water level in the pool drops 6 ft. No power is generated during the pool filling on the high tide.

LEGEND

—— Operation without pumping.

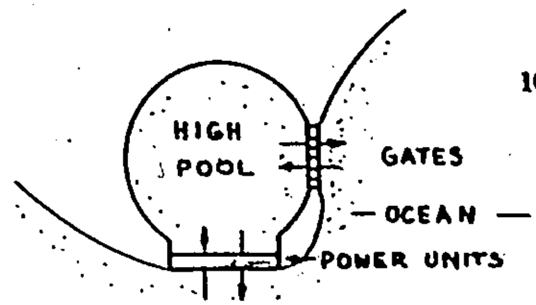
---- Operation with pumping.

A , Power units as turbines.

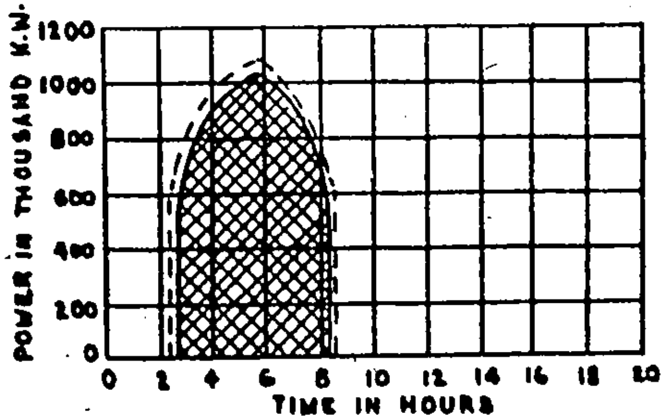
B , Power units as pumps.

D , Power units inoperative.

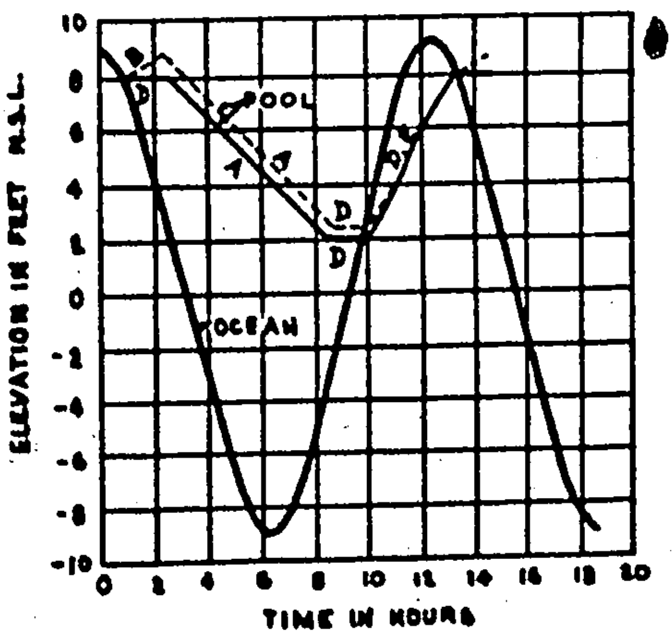
E , Filling gates open.



SCHEMATIC LAYOUT



POWER OUTPUT



POOL ELEVATIONS

SINGLE-HIGH-POOL PLAN

Fig. 6-1.

Fig.(6-1) indicates that applying equation of the tide wave oscillation $y = 9 \cos \frac{2\pi}{12.416} t$, minimum required 6 ft. head for turbine operation will occur 2.66 hours after high water, and the power generating period would be carried out over 5.74 hours when again the water head will drop to 6 ft. at time t = 8.4 hours. The maximum operational head 13.53 ft. occurs at t = 5.75 hours., or 0.46 hours before low water.

Since, during the period 5.74 hours water level in the pool drops 6 ft., the average flow through the turbines will be

$$Q_a = \frac{V}{t} = \frac{6A}{5.74} = 4.075 \text{ billion cu ft/hr.} = 1{,}130{,}000 \text{ cu ft/sec.}$$

where: V = volume of the water generating power

t = time

A = area of the pool = 140 sq. miles = 3.9 billion sq.ft.

If steady discharge of Q_a cu ft./sec. is available with a net head of h feet, the power P that can be developed from this quantity of water passing through a power generating installation, expressed in kilowatt, will be:

$$P\ (\text{kW}) = \frac{Q_a hwe}{737} \qquad (6\text{-}1)$$

where: w = the unit weight of water (in this case mixed fresh and sea water) = 63.25 lb/cu ft.

e = average efficiency of a power plant,

737 is the number of foot-pounds per second in one kilowatt.

With an assumed average efficiency of a power plant at 88 per cent, the Eq. (6-1) becomes:

$$P\ (\text{kW}) = \frac{Q_a h}{13.2} \qquad (6\text{-}2)$$

Table II shows a relationship between time, operational head and generated power, for one-way operating plant.

TABLE II

Time t (hrs)	Water Level in pool (ft)	Operational head h (ft)	Generated power P (kW)
2.66	+ 8.00	6.00	514,000
4.00	+ 6.59	10.51	904,000
5.75	+ 4.78	13.53	1,160,000
7.50	+ 2.92	10.10	865,000
8.40	+ 2.00	6.00	514,000

Note: Water level in respect to m.s.l.

It would be an advantage in a single pool project to use turbines that also operate as pumps. This would require energy from an outside source. In the high-pool project, for example, the pool level can be raised further by pumping for a short period after the filling gates are closed. The energy used in pumping at a low head increases the pool level, and increases the head during the subsequent generating period.

(2) Using turbines which can generate power from flow in either direction, a single mean pool can be operated as a high-pool during low tides, and as a low pool during high tides as shown on Fig. (6-2), where the mean pool is filled and drained respectively to ± 2 ft.

This arrangement results in two separate generating periods in each tide cycle. Since, the average pool level is about the same as mean sea level, the generating head is considerably less than for high-pool

LEGEND

——— Operation without gates.

---- Operation with gates.

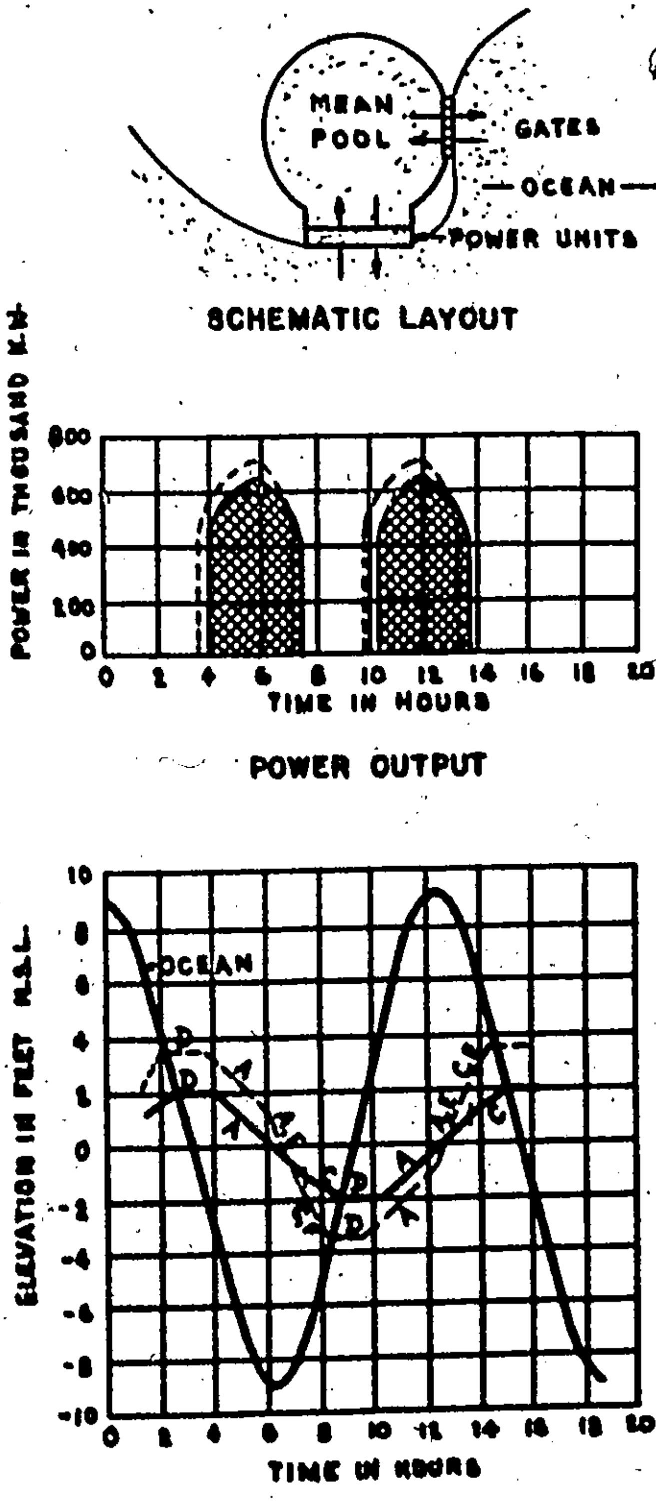
A , Power units as turbines.

C , Power units as orifices.

D , Power units inoperative.

E , Filling gates open.

F , Emptying gates open.



SINGLE -MEAN - POOL PLAN

Fig. 6-2.

arrangement. Because of the smaller head, total power generation for the mean-pool plan is less than for the previous layout.

The required minimum operational head of 6 ft. will occur in the pool 4.02 hrs. after high water and the power generating period will last for 3.53 hrs. until the operational head will drop to 6 ft. at t = 7.55 hrs. and during this time the water level in the pool will drop 3 ft. Then the water from the pool will drain through the turbines which are changed to orifices and at the time t = 8.87 hrs. after high water, level in the pool will drop to 2 ft. below m.s.l.

The reverse process will start again at t = 10.23 hrs. when the required operational 6 ft. head will be available, but this time water from the ocean will flow to the pool through the turbines, which will again generate power for 3.53 hrs. until ocean level in respect to the pool will drop below 6 ft. Then turbines again will be changed to orifices and the cycle will be repeated.

The maximum operational head 9.31 ft. will occur twice during one tide period, 0.37 hrs. before the time of the high and low water respectively.

Since, during both 3.53 hrs. generating periods, water level in the pool rises and falls 3 ft. each time, the average flow through the turbines will be:

$$Q_a = \frac{V}{t} = \frac{3A}{3.53} = 3.32 \text{ billion cu ft/hr.} = 922{,}000 \text{ cu ft/sec.}$$

where the pool area A as before = 3.9 billion sq. ft.

Developed power can be expressed for each of two generating periods as:

$$P\ (kW) = \frac{Q_a h}{13.2}$$

Table III shows a relationship between time, operational head and generated power, for two-way operating plant.

TABLE III

Time t (hrs)	Water Level in pool (ft)	Operational head h (ft)	Generated power P (kW)
4.02	+ 2.00	6.00	418,000
5.84	+ 0.46	9.31	650,000
7.55	- 1.00	6.00	418,000
8.87	- 2.00	0.00	0
10.23	- 2.00	6.00	418,000
12.05	- 0.46	9.31	650,000
13.76	+ 1.00	6.00	418,000
15.08	+ 2.00	0.00	0

An interesting aspect of the mean-pool arrangement is the possibility of wasting water from the pool to gain energy. This apparent contradiction is diagrammed by the dotted lines on the single-mean-pool illustration. It is accomplished by use of auxiliary gates. The gates are opened near the end of a generating period to supplement the flow through the turbines, this accelerates the change in the pool level. The change in pool level increases the head and consequently the energy during the following generating period.

A further modification of the single-mean-pool plan, may include both auxiliary gates and additional pumping at selected periods. For this arrangement, the power units are operated as turbines or pumps with flow in both directions through the powerhouse. Even with these auxiliary features, however, power generation with this arrangement remains intermittent.

Two-Pool Arrangements. The disadvantage of intermittent generation is overcome by the simple two-pool plan as illustrated on Fig.(6-3). The high pool, having the area of 100 square miles is filled during high tide through one set of gates, and the low pool, having area of 40 square miles emptied during low tide through a separate set of gates. Since one pool is operated at a high level and the other at a low level, conventional turbines which permit flow in one direction can be used. The two-pool plan produces varying but continuous power.

To compare this operation with the previously described one-pool arrangement, let us assume that maximum water level in the high pool will reach 8 ft.. Then, during the period of 9.54 hours, the water level in the high pool will drop 3 ft. and the filling gates for the high pool will be open at time t = 10.47 hours, as the high water is approaching. Filling process will take 2.88 hours and the water level again will reach 8 ft. at the time t = 13.35 hours, after the first high water.

The low pool, having 2.5 times smaller area, will drain its water volume at low tide to the lowest level 8.5 ft. below m.s.l., and then the emptying gates will be closed. As there is a continuous flow of the constant volume of the water from the high to low pool, the water level in the low pool will increase 2.5 times faster than it decreases in the high

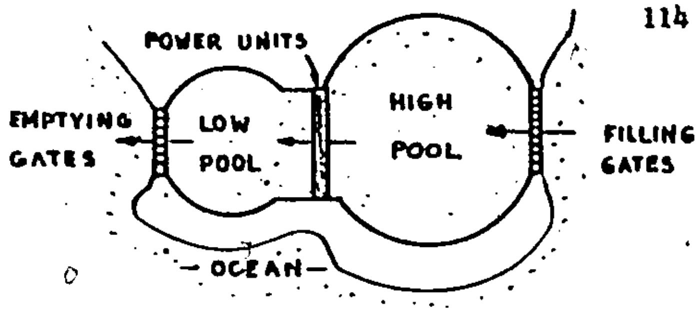
LEGEND

A , Power units as turbines.

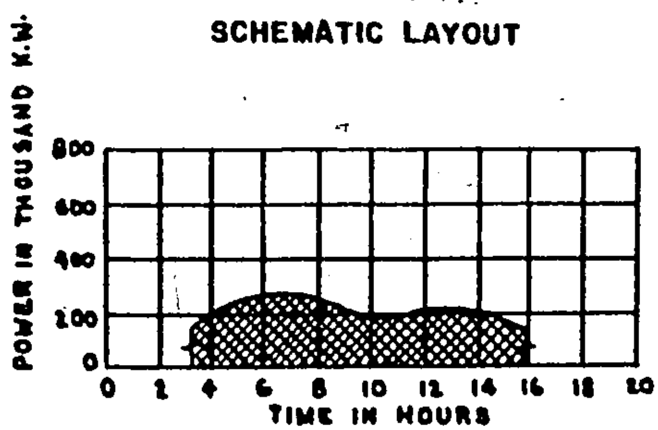
E , Filling gates open.

F , Emptying gates open.

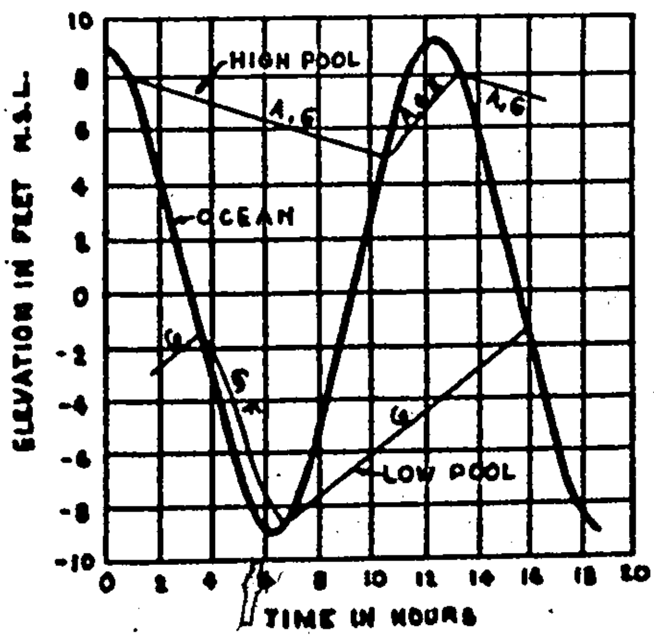
G , Flow from upper pool to lower pool.



SCHEMATIC LAYOUT



POWER OUTPUT



POOL ELEVATIONS

SIMPLE TWO - POOL PLAN

Fig. 6-3.

pool. This will last for 8.95 hours and at t = 15.85 hours, the water level in the low pool will reach its peak, 1.5 ft. below m.s.l.

Since the high pool level is always above m.s.l. and the low pool is always below and the difference between these two levels is creating operational head for the turbines which never drops below the minimum required height of 6 ft., a continuous supply of power is produced.

The average discharge from the high pool to low is given as:

$$Q_a = \frac{V}{t} = \frac{3A}{9.54} = 880 \text{ million cu ft/hr.} = 244{,}000 \text{ cu ft/sec.}$$

where: A = area of the high pool = 100 sq. miles = 2.7878 billion sq. ft.

Developed power can be expressed for this continuous generating period by Eq. (6-2).

Table IV shows a relationship between time, operational head and generating power for the two-pool arrangement.

TABLE IV

Time t (hrs)	High Pool level (ft)	Low Pool level (ft)	Operational head h (ft)	Generated power P (kW)
0.93	+ 8.00	- 3.42	11.42	211,000
3.43	+ 7.21	- 1.50	8.71	161,000
6.89	+ 6.12	- 8.50	14.62	270,000
10.47	+ 5.00	- 5.68	10.68	196,000
13.35	+ 8.00	- 3.42	11.42	211,000
15.85	+ 7.21	- 1.50	8.71	161,000

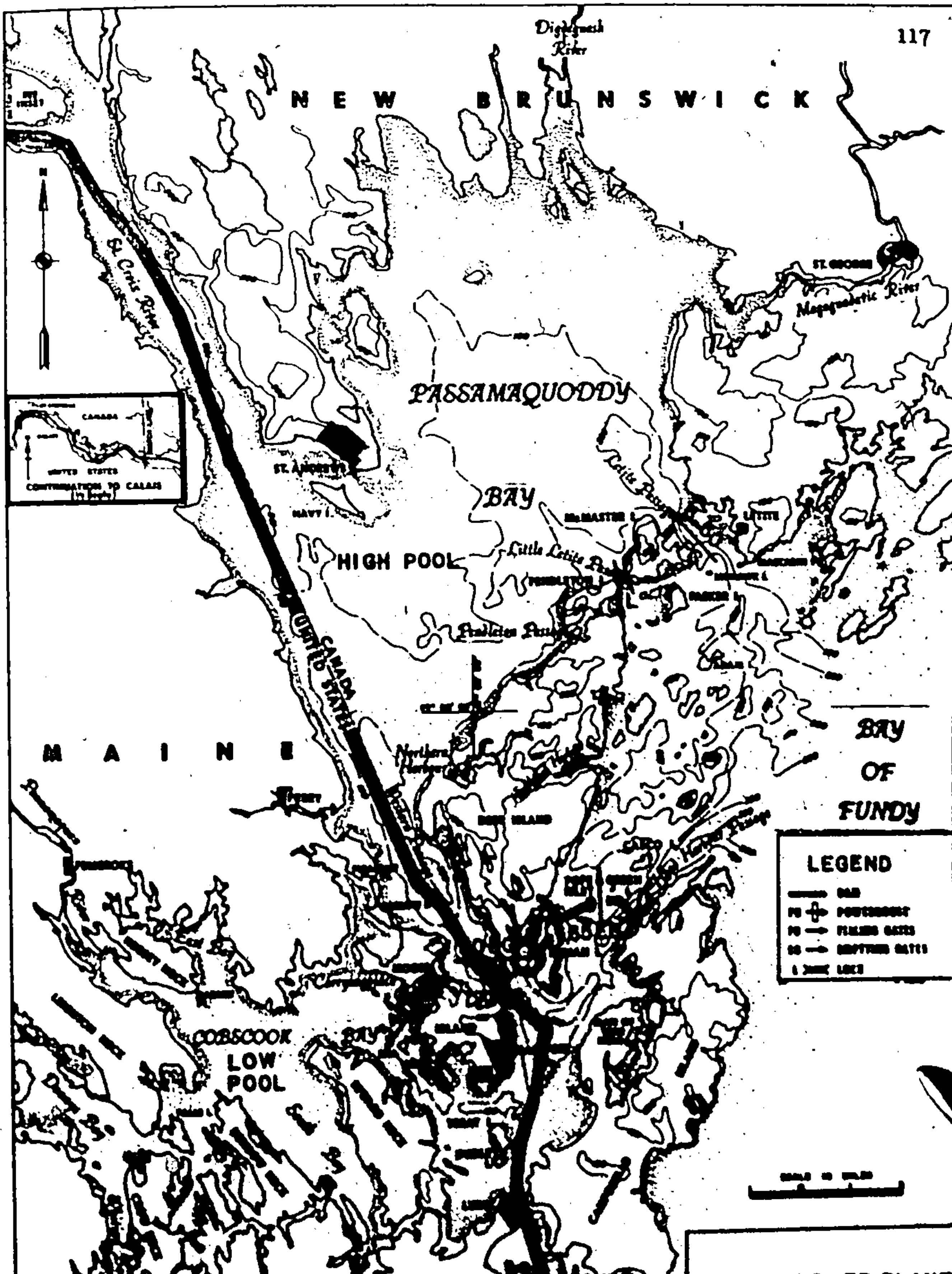
Conclusion. A single high pool has the serious disadvantage of producing discontinuous power, because no power can be generated without a sufficient difference between the level of the pool and the level of the ocean. Thus no generation is possible until the ocean has receded sufficiently to obtain the difference in water levels, or power head; nor is generation possible on the rising tide after the level of the ocean becomes too high to provide this minimum necessary head. For similar reasons, a single low pool will produce interrupted power. Also, a single mean pool, even when with auxiliary features such as gates and pumps, is not able by itself to deliver an uninterrupted supply of power. This disadvantage is avoided in the two-pool plan, which generates varying but continuous amounts of power. This continuous power is achieved in the two-pool plan by emptying and filling gates so that the level of one pool is always sufficiently higher than the other.

As the steady supply of power is the most important factor, the two-pool arrangement is the most attractive one and will be discussed in the following.

THE SELECTED TIDAL POWER PROJECT

From comparative analyses of a number of single-pool and double-pool schemes, it became evident that conditions at Passamaquoddy-Cobscook were particularly well suited to a two-pool tidal power project.

The project arrangement thus selected for design includes 100 square miles of Passamaquoddy Bay as the high pool and the 40 square miles of Cobscook Bay as the low pool, with a powerhouse located on Moose Island, northwest from Eastport. The narrowest part of the Island will be excavated to provide the headrace for the powerhouse. The location of the



TIDAL POWER PLANT LAYOUT.

PLATE 2.

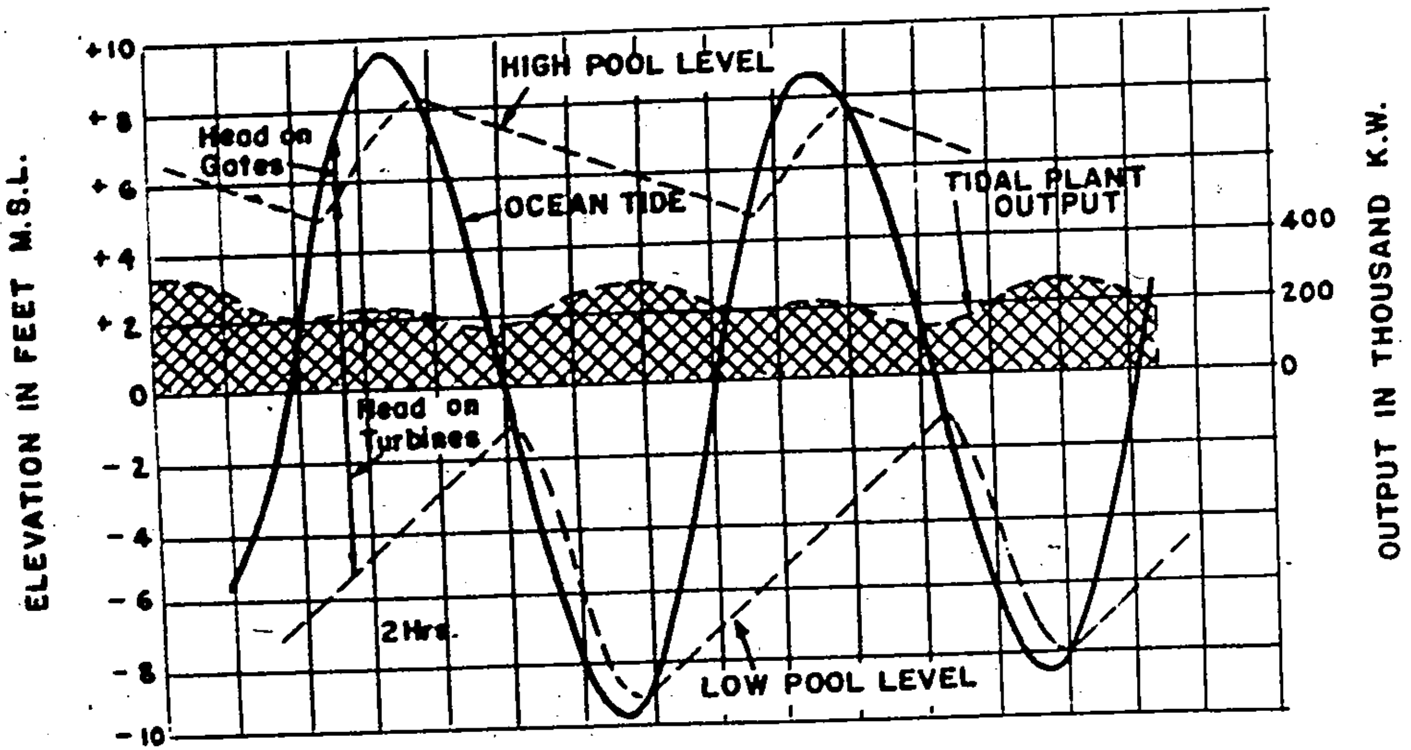
tidal power project and the general arrangement of the selected layout is shown on Plate 2, where the arrows indicate the direction of the flow. With 34 generating units rated at 10,000 kW each, operated at 15 per cent above rating capacity for short periods during spring tides, the output of the tidal power plant would range from 72,000 to 391,000 kW. Average energy generation would be 2,025 million kilowatt-hours a year.

About 33,000 linear feet of tidal dams will be required, which will be composed of clay core supported by flanking dumped-rock fills.

The selected plan calls for 100 filling gates, 50 in Letite Passage and 50 at Deer Island Point. North east from Indian Island, 65 emptying gates, and 5 more at Quoddy Roads similar to the filling gates but set at lower elevation would be needed to empty the lower pool. A vertical lift gate, 30 ft x 30 ft set in the venturi throat would be recommended for this project, as the venturi throat permits maximum discharge for a given gate area.

Four navigation locks would be required for this project. Two locks, approximate dimensions, 100 ft x 25 ft and 12 ft, one in the passage between McMaster and Pendleton Islands and the second at Quoddy Roads to pass fishing vessels. Another two locks, approximate dimensions 500 ft x 60 ft x 25 ft, one at Head Harbour Passage and the second at Western Passage north of Eastport, would pass the large seagoing vessels.

The topography of the selected area permits many different arrangements of the components of a large scale two-pool project. Cobscook Bay has much flatter shores in the tide range than Passamaquoddy Bay for this reason the area of the tidal pools would be larger if Cobscook



TYPICAL TIDAL PLANT OPERATION

Fig. 6-4.

Bay would be used as the high pool and Passamaquoddy as the low pool, contrary to the selected layout. An important advantage of the selected plan over the Cobscook Bay high pool plan is that all Passamaquoddy Bay and Western Passage would be in the upper pool where the water surface would vary between el. + 2.21 ft and +11.85 ft, instead of the present maximum variation between el. ± 12.85 ft in respect to m.s.l. This would improve navigation and harbour depth for ports in Canada and United States. The controlling depth for navigation in St. Croix River to Calais, Maine, and St. Stephen, New Brunswick, would be about 22 ft at mean low stage of the upper pool, instead of the existing 7 ft at mean low tide. For this reason, the Passamaquoddy Bay high plan was adopted.

Fig. (6-4) illustrates the typical cycle of the tidal plant operation, where the power output of the selected two-pool plant would vary with the ebb and flood of the tides.

FILLING AND EMPTYING GATES

For the above project, a 30 ft x 30 ft vertical-lift steel gate set in a reinforced concrete venturi throat would be selected. The venturi throat was selected because, among other important advantages, it permits maximum discharge for a given gate area. Since the filling and emptying gates are submerged, the flow through them can be expressed by formula:

$$Q = CA_g \sqrt{2gh_1} \qquad (6\text{-}3)$$

where: Q = discharge through one gate, cu ft/sec

A_g = cross-sectional area of the venturi throat in sq ft

h_1 = water head on the gate in ft.

C = coefficient of discharge (representing the product of the experimental coefficient C_v due to frictional effect and coefficient of contraction C_c)

Assuming that C = 0.9 and A_g = 900 sq ft

$$Q = 810 \sqrt{2gh_1} = 6,500 \sqrt{h_1}$$

The head on the gates h_1, which is the difference between water level in the pool and the ocean, changes with time. Since the water level in the pool is not only a function of discharge Q, but also of the pool area A, which varies with configuration of the pool, the variability in time of h_1 can be accurately obtained only by performing model study.

However, in this case, the initial and final elevation of the pool level is known when the gates are open and when they are closed, also the time in which this difference will occur, so by using the average values of h_1 and Q, this problem can be solved.

Filling Gates. Fig. (6-5) shows seven ordinates representing different values of h_1, using the average tide height 18 ft. The solid line AB indicates the average change of the water level in the high pool, and the dotted line A C D B shows more accurately the change of the water level.

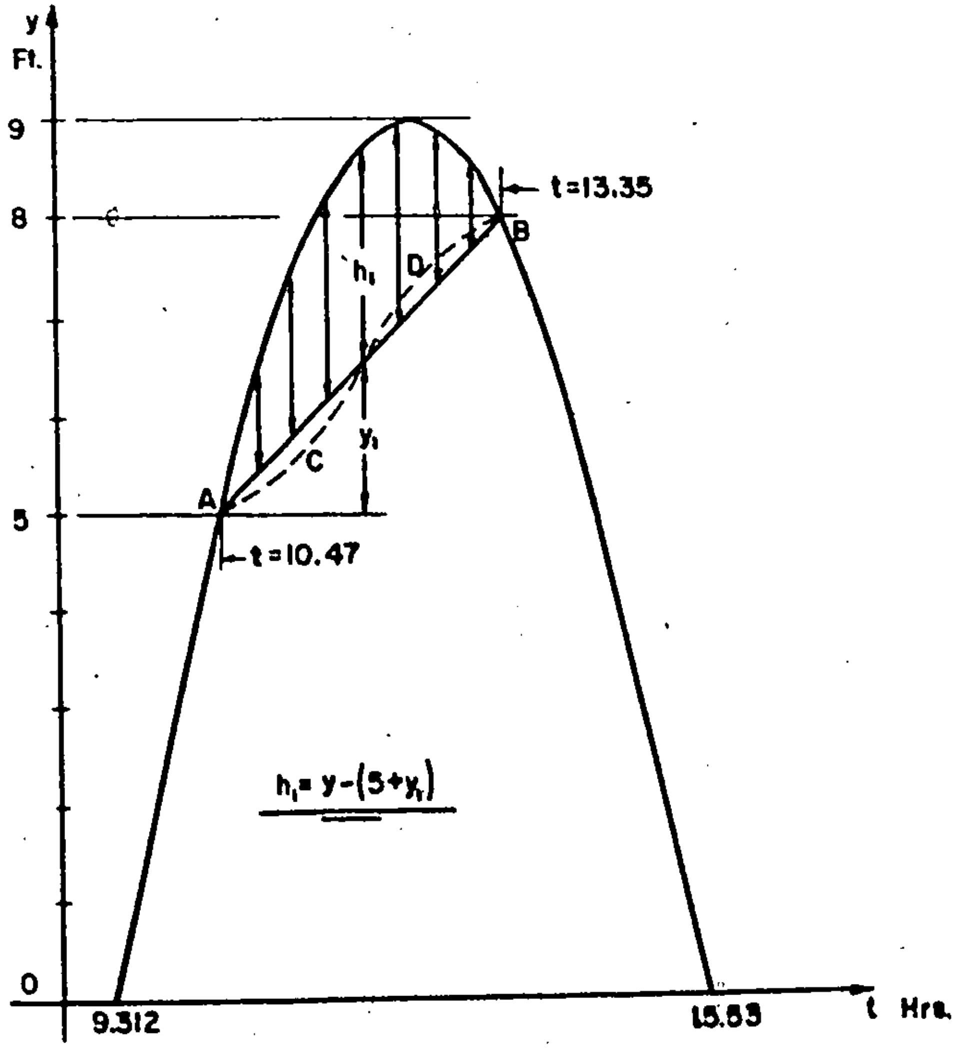


Fig. 6-5. Head on the Filling Gates.

TABLE V

t (hrs)	$\Sigma \Delta t$ (hrs)	y (ft)	y_1 (ft)	h_1 (ft)
10.47	0	5.00	0	0
10.83	0.36	6.14	0.375	0.765
11.19	0.72	7.29	0.75	1.54
11.55	1.08	8.10	1.125	1.975
11.91	1.44	8.74	1.50	2.24
12.27	1.80	8.99	1.87	2.12
12.63	2.16	8.95	2.25	1.70
12.99	2.52	8.65	2.62	1.03
13.35	2.88	8.00	3.00	0
				$\Sigma h_1 = 11.37$

From Table V

h_a (average) $= \frac{11.37}{7} = 1.63$ ft and $\sqrt{h_a} = 1.275$ ft.

When the gates are open the water level in the high pool will rise 3 ft from minimum to maximum level in 2.88 hours, so that the average flow through the gates will be:

$$Q_a = \frac{3A}{2.88 \times 3,600} = 803,000 \text{ cu ft/sec}$$

where average high pool area A = 2.7878 billion sq ft.

The required number of n gates can be obtained from:

$$n = \frac{Q_a}{6,500 \sqrt{h_a}} = 97.0$$

Because of the constant and rugged use of the gates, some will have to be repaired periodically and therefore three additional gates will be added, bringing the total amount to 100 filling gates.

Emptying Gates. The same procedure will be followed for finding the required number of emptying gates. The solid line A B, Fig. 6-6, represents the average change of the water level in the low pool, and the dotted line A C D B shows more accurately the actual change of the water level.

TABLE VI

t (hrs)	$\sum \Delta t$ (hrs)	y (ft)	y_1 (ft)	h_1 (ft)
3.430	0	1.50	0	0
3.862	0.4325	3.15	0.88	0.77
4.295	0.865	5.16	1.74	1.92
4.727	1.297	6.60	2.62	2.50
5.160	1.730	7.75	3.50	2.75
5.592	2.162	8.56	4.37	2.63
6.025	2.595	8.95	5.24	2.21
6.457	3.027	8.90	6.10	1.30
6.890	3.460	8.50	7.00	0
				$\sum h_1 = 14.08$

From Table VI

$$h_a \text{ (average)} = \frac{14.08}{7} = 2.01 \text{ ft and } \sqrt{h_a} = 1.418 \text{ ft.}$$

When the gates are open, the water level in the low pool will change from maximum to minimum level in 3.46 hours, so that the average flow through the gates will be:

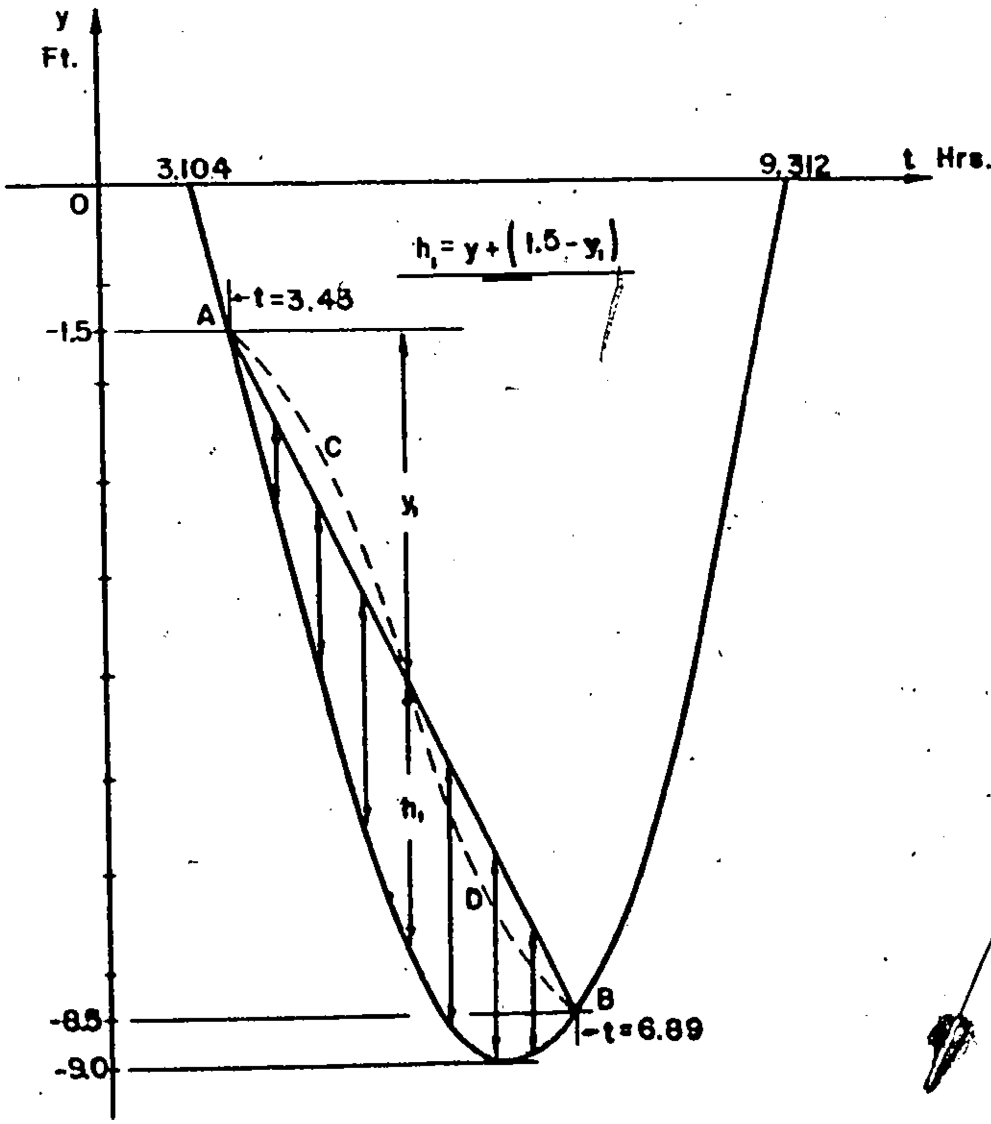


Fig. 6-6. Head on the Emptying Gates.

$$Q_a = \frac{7\ A_1}{3.46 \times 3,600} = 624,000 \text{ cu ft/sec.}$$

where the average low pool area A_1 = 1.11 billion sq ft.

The required number of emptying gates will be:

$$n = \frac{Q_a}{6,500\ \sqrt{h_a}} = 67.7$$

For the reasons previously mentioned, total amount of filling gates will be increased to 70.

POWER OUTPUT OF THE SELECTED TIDAL POWER PLANT

Fig. (6-4) introduced before, indicated that power output of the tidal plant varies and changes with the height of the tide. The maximum power output will be during the spring tide, when the tide height is maximum, and minimum output during the neap tide, as the generating head on the turbines will be minimum.

Because of the small in height, but continuous variation of the generating head, it would be advantageous for the tidal power plant to have automatic control of the water volume flowing through the turbines. This would make it possible to have a constant rate of discharge, in time and volume, passing through the powerhouse and generating power.

The average discharge calculated previously for an 18 ft tide, Q_a = 244,000 cu ft/sec, can be assumed as the average for all tides and automatically controlled to remain constant under variable water heads.

Furthermore, during the generating period when the gates are closed, changes of the water level in the pools are rather small, so the area of the pools will remain almost unchanged. Based on the above, it can be assumed with a certain degree of approximation that the water level in the high and low pool will follow a straight line relationship in respect to the time. This is particularly true in the case of the high pool, Passamaquoddy Bay.

Since the object of this thesis is to determine the possible range of the power output only, exact analysis of one or more tidal cycles, each 29.6 days, will not be necessary. Only the maximum and minimum tide will be considered, as the average tide was already investigated.

<u>Spring Tide</u>. The height of the spring tide is 25.7 ft, the preceding and following tide have an approximate height of 24.7 ft. Equation $y = z \cos (2\pi/12.416)t$ represents the tide wave profile of each tide wave separately. The amplitude z is equal to half of the tide height and varies 6 in. with each successive tide.

The high pool water level will reach its maximum height 11.85 ft at the time t = 0.81 hrs (assuming that high water of the spring tide occurs at t = 0), and then the filling gates will be closed and the water level will begin to drop down. The maximum generating head $h_{max} = 21.24$ ft will occur at t = 7.03 hrs. The water level in the low pool will reach its minimum level 11.35 ft below m.s.l. in 0.82 hrs after low water.

The maximum power output using the average discharge $Q_a = 244,000$ cu ft/sec will be

$$P_{max} = \frac{Q_a h_{max}}{13.2} = 393,000 \text{ kW}$$

<u>Neap Tide</u>. Height of the neap tide is 11.3 ft and is preceded and followed by a tide having the height of 12.3 ft. To obtain the absolute minimum value of the generating head which can occur during the neap tide, it will

be assumed that the low water of this tide occurs at the time $t = 0$ hrs., and is followed by the high water of the same tide at $t = 6.208$ hrs.

The low pool will reach its minimum level 5.16 ft below m.s.l. at $t = 0.83$ hrs, and then the emptying gates will be closed. The low pool will rise 6.35 ft in 8.07 hrs, reaching its maximum level 1.2 ft above m.s.l. at $t = 8.89$ hrs. The absolute minimum generating head $h_{min} = 3.9$ ft will occur at the same time, because the high pool on the other hand, reached its maximum level 5.15 ft above m.s.l. at $t = 7.03$ hrs and since that time was dropping down.

The minimum possible power output using the average previously mentioned discharge will be:

$$P_{min} = \frac{Q_a h_{min}}{13.2} = 72{,}000 \text{ kW}$$

Table VII shows comparison between maximum average and minimum tidal power output.

TABLE VII

	Max. head (ft)	Min. head (ft)	P max. (kW)	P min. (kW)
Maximum tide	21.24	15.65	393,000	290,000
Average tide	14.62	8.71	270,000	161,000
Minimum tide	8.41	3.9	155,500	72,000

Selecting 34 generating units rated at 10,000 kW each, operated at 15 per cent above rated capacity for short periods during spring tides, the output of the tidal power plant would range from 72,000 to 391,000 kW. Average energy E generated during one year would be:

$E = 231{,}500 \times 365 \times 24 = 2{,}025$ million kilowatt-hours a year.

TURBINES

Design of an economical 34 unit power house of minimum length to fit the available site and capable of handling a tremendous volume of discharge, indicated the use of turbines as large as possible. The turbines selected for this project are the European Kaplan, and due to low and variable operating head, the large turbines would be connected to generators with a relatively low rating of 10,000 kW.

In this case the selection of the following dimensions of the turbines and power house will be investigated under average conditions only.

The operating head will vary from 3.9 ft to 21.24 ft, but the average power generating head is 11 ft. This value represents an average of ten ordinates taken during the tide height of 18 ft. For the same tide height, the average flow through one turbine from high to low pool will be:

$$Q_a = \frac{244{,}000}{34} = 7{,}180 \text{ cu ft/sec}$$

and the power generated by one turbine under an 11 ft head will be:

$$P = 5{,}980 \text{ kW} = 8{,}040 \text{ hp}$$

<u>Normal Speed</u>. For design head h = 11 ft, Kaplan turbine with $N_s = 180$ will be selected.

$$N \text{ rpm} = \frac{N_s h^{5/4}}{\sqrt{P}} = \frac{180 \times 20}{89.7} = 40$$

where: N = normal or operational speed of a turbine runner.

N_s = specific speed which is defined as the speed of a homologous runner when it has been so reduced in size that it develops 1 hp under 1 ft head.

P = generated power = 8,040 hp.

Synchronous Speed. In a modern hydroelectric plant the turbine is directly connected to an alternating-current generator of a given frequency. Every alternating-current generator must have an even number of poles, and preferably a number of poles divisable by 4. For the sake of synchronization, the turbine speed must conform to the speed of the generator. In other words, the following relation will hold:

$$N \text{ rpm} = \frac{120\ f}{p} = \frac{7,200}{p}$$

where: f = electrical frequency = 60 cycles/sec.

p = number of generator field poles

$$p = \frac{7,200}{N} = \frac{7,200}{40} = 180$$

Using 180 poles, $N = \frac{7,200}{180} = 40$ rpm

Corrected specific speed $N_s = \frac{N\sqrt{P}}{h^{5/4}} = 179.4 \cong 180$

Unit Discharge. A turbine may be considered as a kind of orifice to which the laws of discharge through orifices under variable head can be applied. The following formula for discharge through an orifice can therefore be applied as well to a turbine:

$$Q = \frac{C\pi}{4} D_o^2 \sqrt{2gh} \qquad (6\text{-}4)$$

where: C = coefficient of discharge

D_o = diameter of the orifice or of the turbine runner

<u>Specific Diameter and Model Ratio.</u> Eliminating Q from the following relations

$P \propto Qh$ see Eq. (6-1)

$Q \propto D_o^2 \sqrt{h}$ see Eq. (6-4)

solving for D_o,

$$D_o \propto \frac{\sqrt{P}}{h^{3/4}} \text{ or } D_o = D_s \frac{\sqrt{P}}{h^{3/4}}$$

Where D_s to be called specific diameter which is the diameter of homologous runner developing 1 hp under 1 ft head.

Let

$$m = \frac{\sqrt{P}}{h^{3/4}} \qquad (6\text{-}5)$$

in which m is the model ratio, then

$$D_o = mD_s \qquad (6\text{-}6)$$

This means that the specific diameter multiplied by the model ratio should give the nominal diameter of the runner.

<u>Determination of the Runner Discharge Diameter.</u> The outlet or discharge diameter of the Kaplan turbine is measured at the throat or the top of the draft tube, and is only slightly larger than the nominal runner diameter. It will be indicated by D_3, (to distinguish it from the general case), and it will provide a very useful base figure which can be

used to determine preliminary key dimensions of the scroll case and draft tube and also the over-all dimensions of the substructure of a powerhouse.

The model ratio m, of the given turbine to a homologous turbine which will deliver 1 hp under 1 ft head may be calculated from Eq. (6-5).

$$m = \frac{\sqrt{P}}{h^{3/4}} = \frac{89.7}{11^{3/4}} = 14.85$$

The specific diameter D_s in inches may be calculated by the following experience formula:

$$D_s = \frac{113}{N_s^{0.34}} = \frac{113}{180^{0.34}} = 19.3 \text{ in.}$$

The required runner discharge diameter

$$D_3 = mD_s = 287 \text{ in.} \cong 24 \text{ ft.}$$

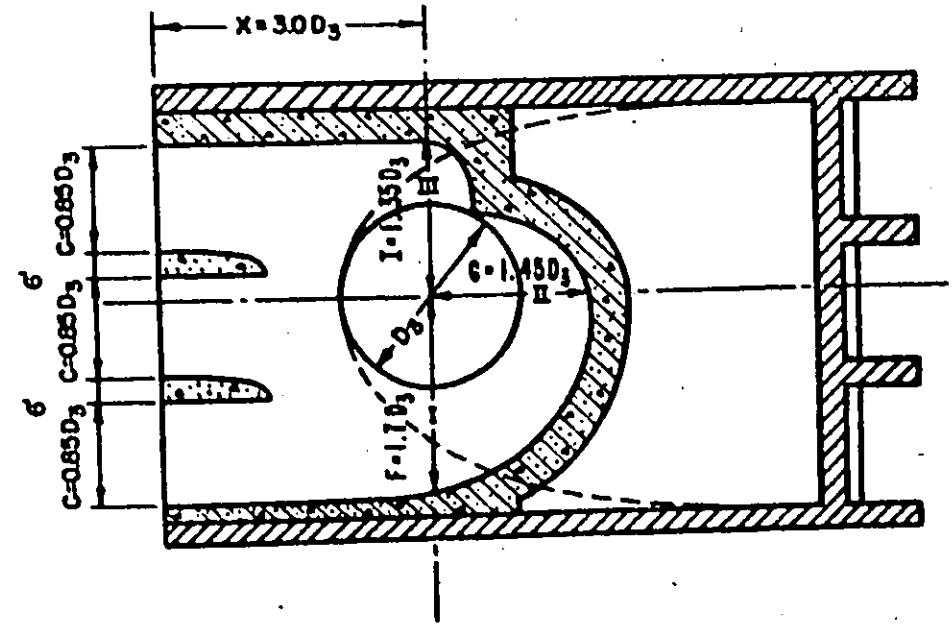
Solving Eq. (6-4) for C, using orifice diameter 24 ft. and average discharge Q_a = 7,180 cu ft/sec, the coefficient of discharge C will be equal to 0.6. This value appears to be in the range proved by experiments.

<u>The Scroll Case.</u> Having a rated turbine at 8,040 hp and variable head from 3.9 ft to 21.24 ft, a concrete type spiral scroll case should be selected. Fig. (6-7) shows the main scroll case dimensions for the propeller-type runner, recommended by the U.S. Bureau of Reclamation.

Intake width c = 0.85 D_3 = 20.4 ft; assumed 20 ft and 4 in.

Intake entrance total = 3c + 12 ft = 73 ft.

The net entrance cross-sectional area of the rectangular scroll case



Range of Values

X : $2.0D_3 - 4.0D_3$
C : $0.76D_3 - 1.12D_3$
F : $1.2D_3 - 1.8D_3$
G : $1.3D_3 - 1.7D_3$
I : $1.2D_3 - 1.5D_3$

Fig. 6-7. Setting dimensions for preliminary estimates, propeller-type concrete scroll.

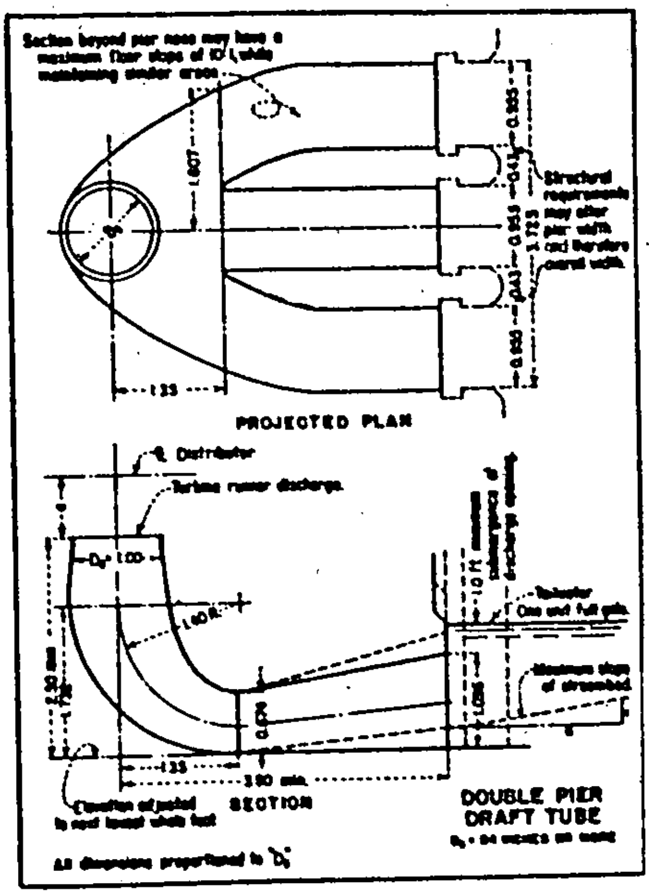


Fig. 6-8. Reaction turbine draft tube.
(Bureau of Reclamation).

will be A_s = 61 x 47 = 2,860 sq ft.

Water velocity:

$$v = \frac{Q_a}{A_s} = \frac{7,180}{2,860} = 2.5 \text{ ft/sec}$$

which is less than the permissible water velocity for the concrete scroll.

The Draft Tube. Fig. (6-8) shows the preliminary recommended basic outlines of the draft tube suitable for the Kaplan turbine. The approximate vertical distance from the center line of the distributor to the center line of the propeller runner will be a = 0.41 D_3 = 9.85 ft; assumed 10 ft. The minimum draft tube height from the center line of the runner to the bottom of the tube = 2.5 D_3 = 60 ft. The minimum length of the draft tube from the center line of the turbine = 3.8 D_3 = 91.4 ft; assumed 100 ft. The overall width of the draft tube will be altered to 3.1 D_3=75 ft, and the height at the outlet of the draft tube will be 1.415 D_3 = 34 ft. The cross-sectional area of the draft tube at the distance 1.35 D_3 from the center line of the turbine will be

A_d = 2.68 D_3 x 0.87 D_3 = 1,340 sq ft.

Water outflow velocity:

$$v = \frac{Q_a}{A_d} = \frac{7,180}{1,340} = 5.35 \text{ ft/sec.}$$

The recommended outflow velocity may vary from 4 to 8 ft/sec.

Powerhouse Arrangement. The powerhouse would be the outdoor type, with 34 main unit bays, having width 175 ft, as shown on Fig. (6-9). The powerhouse arrangement will be based on size of the water intake sections,

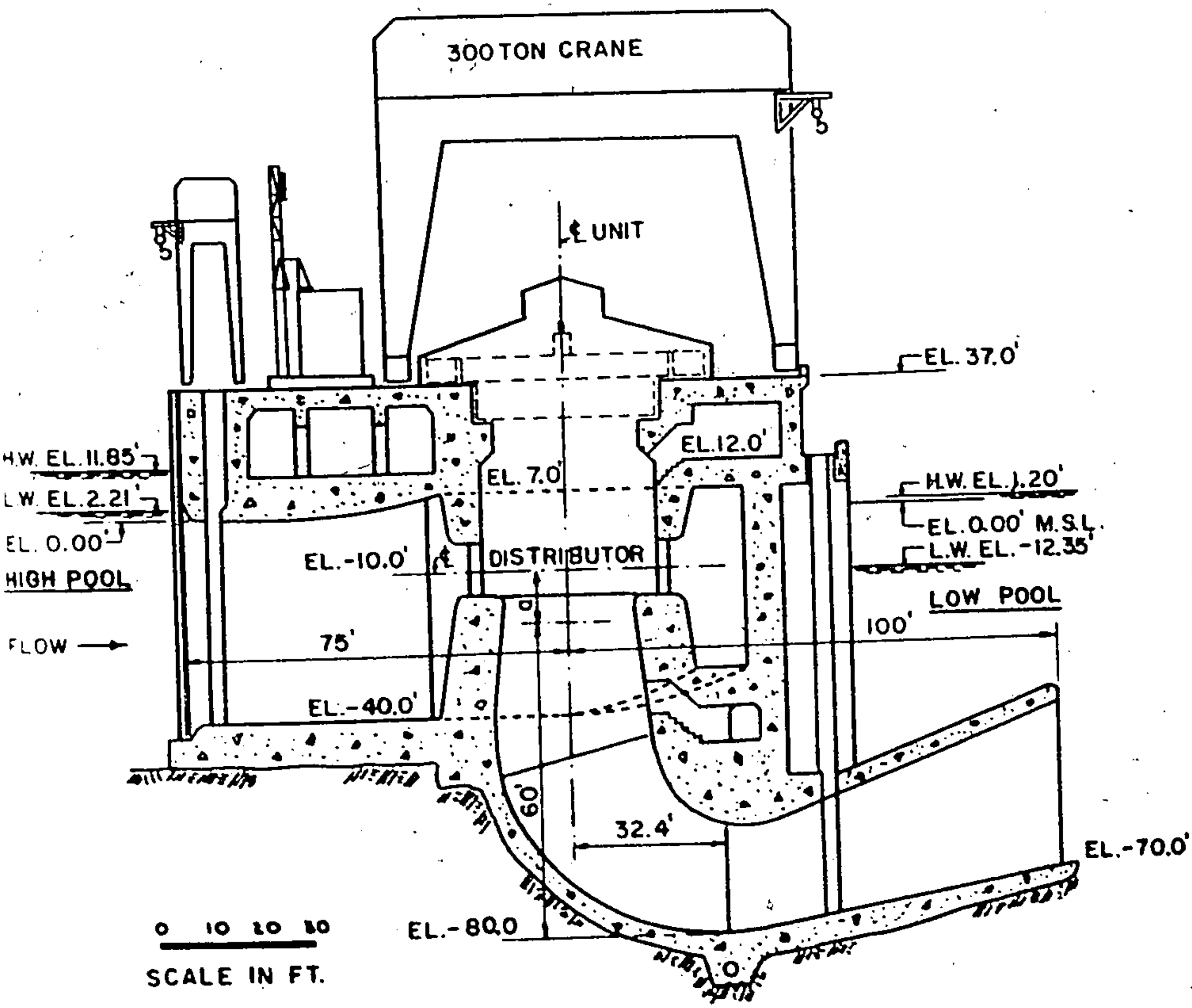


Fig. 6-9. Typical section through powerhouse.

each having three water passages 20 ft and 4 in., with two interior and two end piers, each 6 ft wide, making a total bay width of 85 ft, this would bring a total length of powerhouse to 2,900 ft.

Cavitation. The absolute pressure at a point of minimum pressure in a turbine may be expressed in feet of water, as follows:

$$h_{min} = h_{atm} - h_s - \frac{v^2}{2g} = h_{atm} - h_s - \sigma h \qquad (6\text{-}7)$$

where: h_{atm} = atmospheric pressure

h_s = draft head, or distance from tailwater to the point of minimum pressure, which point is usually taken as the center of the propeller runners.

$h\sigma$ = velocity head proportional to the total head h, in which σ is a cavitation coefficient.

Cavitation will occur when the absolute pressure is equal to the vapour pressure h_v, or

$$h_{atm} - h_s - \sigma h = h_v \qquad (6\text{-}8)$$

substituting $h_b = h_{atm} - h_v$

$$h_s = h_b - \sigma h \qquad (6\text{-}9)$$

For the average year round water temperature of 50°F and plant location on the sea level h_b = 33.5 ft, and for N_s = 180, σ = 0.85, therefore

$$h_s = 33.5 - 0.85 \times 11 = 24.15 \text{ ft}$$

This indicates that cavitation will occur when the turbine runner is set at 24.15 ft or more above the low pool water level. There is no danger of cavitation in this case as the turbine runner will be set below the low pool water level, and the absolute pressure in the turbine will never be low enough to reach the vapour pressure value.

Runaway Speed. The runaway speed of turbines is the maximum speed that the runner could rotate under a given head and gate opening with no external load (i.e., no torque on the turbogenerator shaft). For Kaplan turbines, it is generally 2.5 to 3 times the normal speed.

It would be recommended that the runaway speed N_R would be:

$$N_R = 2.75\ N_s = 110 \text{ rpm}$$

Governor. A hydraulic turbogenerator installation must operate as closely as possible to a constant speed to maintain control over the frequency of the electric current produced. For conventional type governors it is required to control the speed mechanically within a maximum increase or decrease of 30 per cent of normal speed.

The speed change due to variation in head can be computed from:

$$\text{Speed in per cent of Normal Speed} = \left(1 \pm \frac{\Delta h}{h}\right)^{1/2} 100$$

where h = normal head, and Δh = rise or drop in head.

$$h_{max} = 21.24 \text{ ft}$$

$$h_{design} = 11.00 \text{ ft}$$

$$h_{min} = 3.90 \text{ ft}$$

(1) The head increase $\Delta h = 21.24 - 11.0 = 10.24$ ft. Speed in % of Normal Speed $= \left(1 + \frac{10.24}{11}\right)^{1/2} 100 = 139\%$. Speed increased = 39%.

(2) The head decrease $\Delta h = 11.0 - 3.9 = 7.1$ ft. Speed in % of Normal Speed $= \left(1 - \frac{3.9}{11}\right)^{1/2} 100 = 59.5\%$. Speed decreased = 40.5%.

A specially designed governor for the tidal power plant must be used, as decrease and increase of the speed exceeds 30%.

Diameter of the Generator Stator. Approximate diameter of the generator stator can be determined from the formula:

$$D_s = 4.68\ p^{0.466}\ kVA^{0.233}$$

where: p = number of generator field poles

kVA = capacity of generator

D_s = diameter of stator in inches

Capacity of the generator $= \frac{10,000}{0.8} = 12,500$ kVA

where value 0.8 is the so called power factor (ratio of the effective to the impressed voltage).

For p = 180 and capacity of generator = 12,500 kVA,

D_s = 476 in = 39.7 ft.

Weight of the Turbogenerator. Weight of the turbogenerator can be calculated from the following relationship:

$$WR^2 = \frac{700,000\ (kVA)^{1.25}}{(rpm)^2}$$

where: W = the weight of the revolving turbogenerator in pounds

R = the radius of gyration of the revolving masses in feet.

Radius of gyration for a circular turbogenerator, having diameter D_s will be:

$$R = \sqrt{\frac{I}{A}} = \frac{D_s}{4} = 9.94 \text{ ft}$$

For the normal speed of the turbine = 40 rpm, and capacity of generator = 12,500 kVA,

$$WR^2 = 56,800,000 \text{ lb-ft}^2$$

$$W = \frac{56,800,000}{98.9} = 575,000 \text{ lb.}$$

Cranes. Two 300 ton double-trolley gantry cranes having an 80 ft span will be required to operate along the full length of the powerhouse and erection bays. The largest single load lifted by the crane will be the turbogenerator, having the weight of 575,000 lb.

Corrosion. Prevention of corrosion of metals exposed to sea water is a considerable problem in any marine installation. Corrosion would be particularly serious in a tidal power plant because the turbines, for example, would be immersed in sea water at all times. Corrosion can be controlled to some extent by using metals with a high degree of natural resistance, or by using protective coatings. Stainless steel having a content of nickel and chromium is known to be very resistant to corrosion. As this metal is costly, it cannot be used in every place where corrosion resistance is needed, except for the most essential parts, such as turbine blades, discharge rings, wicket top and bottom plates, and topmost 4 to 6 ft of the draft tube liner. The less important elements such as trashracks, guides, and gates could be fabricated from structural grade carbon steel and can be coated with a coal tar or enamel primer coat.

AUXILIARY POWER PROJECTS

For load carrying purposes, power from the tidal project is limited to the capacity it can furnish under most adverse conditions (in this case 72,000 kW). All output in excess of this capacity is not dependable for serving loads and would have value only as a non-firm energy. This excess generation is considerable and to make it dependable for serving loads, it must be firmed up by the use of an auxiliary source of power.

<u>Comparison of Tidal Power Output with Normal Load Pattern.</u> Each tide will fill the upper pool of the tidal project regardless of the amount of water used to generate energy in the previous cycle. Water left unused would be wasted. Somewhat similar in this respect to a "run-of-the-river" hydro-electric plant where no storage is available, energy generated by a two-pool tidal plant must take full advantage of the ebb and flood of the tides. Power must be generated from each tide before the next tide occurs or it is forever lost.

The tides follow closely the cycles of the moon as it circles the earth at varying distances. There are two high tides in a lunar day of 24 hours and 50 minutes. Therefore, high and low water tides occur 50 minutes later each day. If a two-pool tidal plant is operated to extract as much energy as possible from the tides, maximum power production would occur shortly after low tide. Minimum power production would occur 2 to 4 hours before low tide.

Use of electricity follows the pattern of the 24-hour solar day. Maximum and minimum demands for energy occur at about the same time each day, minimum demand occurs on weekends and holidays, and the pattern of use during each week is approximately the same. Therefore, the maximum demand can occur at times of minimum tidal power production with the two continually shifting with respect to each other so that the

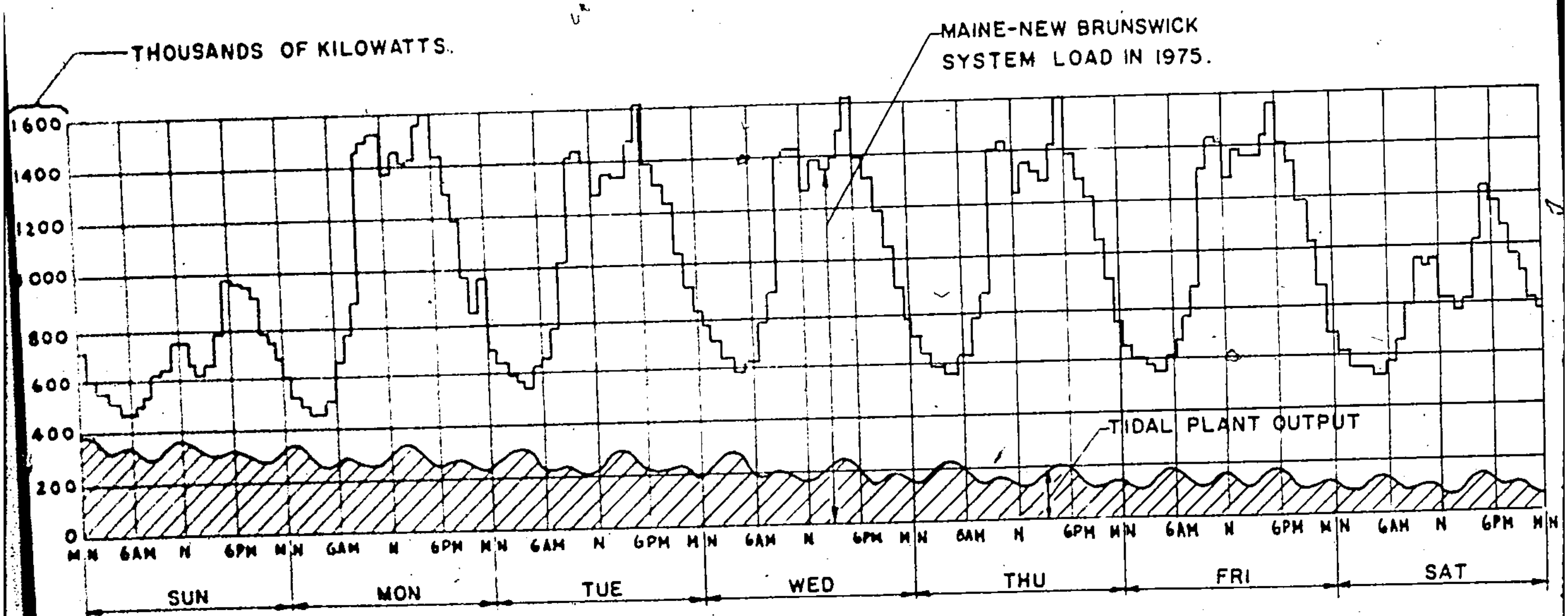


Fig. 6-10. Load pattern and output of the tidal plant alone.

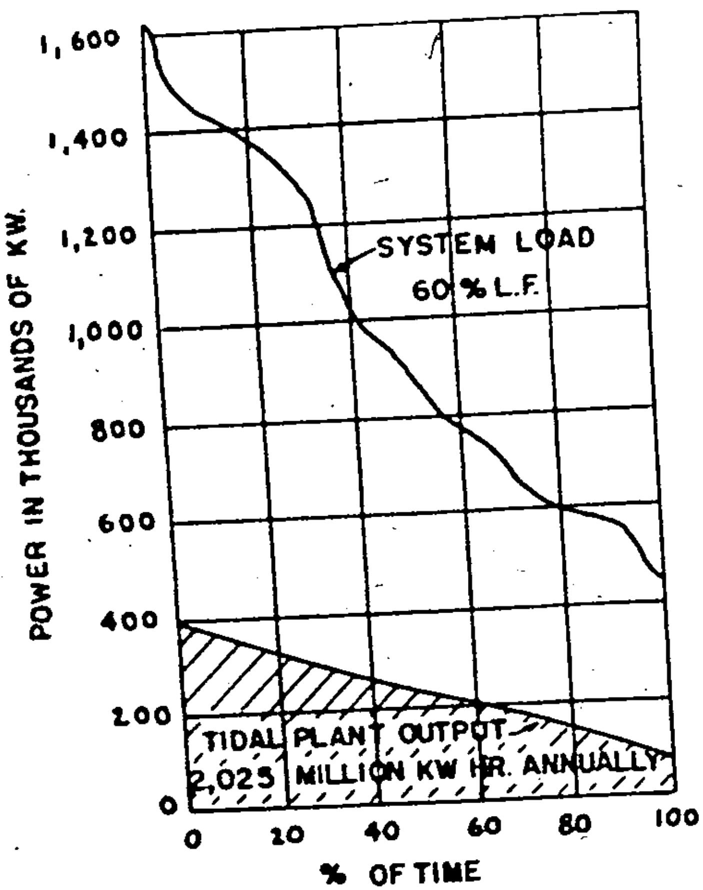


Fig. 6-11. Duration curves of system load and tidal plant output.

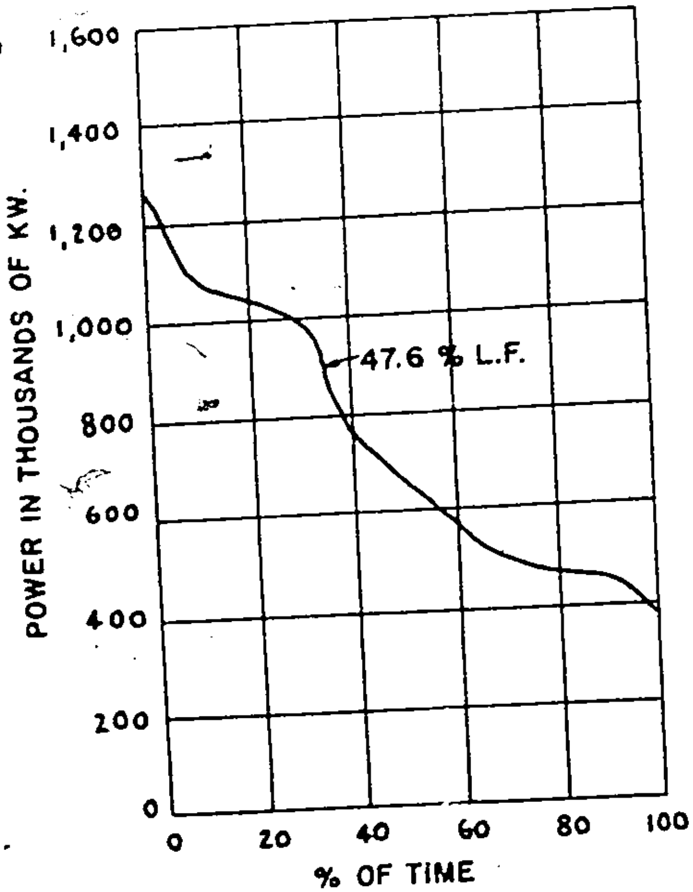


Fig. 6-12. Duration curve of system load minus tidal plant output.

L.F. = Load Factor.

minimum demand would also occur at times of maximum tidal generation. In addition to the day-to-day variation in the tides, they also vary through a 14.8-day cycle from spring tide (high ranges) to neap tides (low ranges) and back to spring tides.

Fig. (6-10) shows the output of the tidal plant for one week compared with a typical load curve. In this particular case, the spring tide and maximum power output occurred Sunday after midnight. The load curve shown has a total annual energy of 8,450 million kW-hr and a load factor of 60 per cent. It is representative of the combined utility loads in Maine and New Brunswick expected in about the year 1975.

The load factor is defined as the ratio of the average load over a designated period (one year in this case) to the peak load occurring with the same period. Fig. (6-11) shows duration curves of system load and tidal plant output, and Fig. (6-12) shows duration curve of system load minus tidal plant output.

Tidal Power in a Large System. If the tidal power were a small portion of the total power required by the market, the fluctuations in output due to the varying tides would not be a serious problem. The other generating plants in the power system could adjust their output to supply the difference between the tidal plant output and the demand. The proposed tidal power project would, however, furnish a sizeable amount of power to the market of the region. Thus, the other plants in the system would be required to generate power at varying rates according to the difference between the tidal plant output and the demand. On a long term basis, and using the typical load illustrated on Fig. 6-10, the other plants would have to operate on a 47.6 per cent load factor. The continually changing tidal output would cause the load on the remaining plants to change continually. The generating pattern would also change, and the pattern for two consecutive days would not be the same. These variations would be greater than are normally experienced by a generating plant. Since these variations would be less acceptable than regulating for the load alone, the addition of an auxiliary source of power to the tidal plant would be essential.

Tidal Power Supplemented by Auxiliary Power. The primary purpose of an auxiliary to the tidal plant is to supplement the tidal plant output during periods of low generation. The combined output would supply a portion of the system load so that the pattern of the remaining load would be acceptable to the other generating plants of the system.

The design of an auxiliary power source must take into consideration both capacity and energy. The capacity problem can be solved simply by designing the auxiliary plant for a given dependable capacity; the combined dependable capacity is then the sum of the individual capacities. The energy poses a greater problem, since the auxiliary power generation requirements would be high during neap tides and considerably less during spring tides.

One method of firming the tidal plant output would be to store a portion of the tidal plant energy by pumping water to a storage reservoir during times of high tidal plant output. The stored water would then be used to generate power when the tidal plant output is low. By alternately pumping and generating, the pumped-storage auxiliary would regulate the tidal plant output to meet the varying load demand. The pumped-storage operation would actually result in a decrease in total energy because of pumping and regeneration losses.

Another method of firming the tidal plant output would be to add energy to the tidal plant output from an auxiliary power source such as hydroelectric or steam-electric plant. The auxiliary energy would be added so that the combined tidal plant and auxiliary energy would form a pattern similar to the system load. As the energy provided by the auxiliary is increased, the more closely could the combined tidal plant and auxiliary output form a constant proportion of the system load.

CHAPTER VII

INTERPROVINCIAL TIDAL POWER PLANT

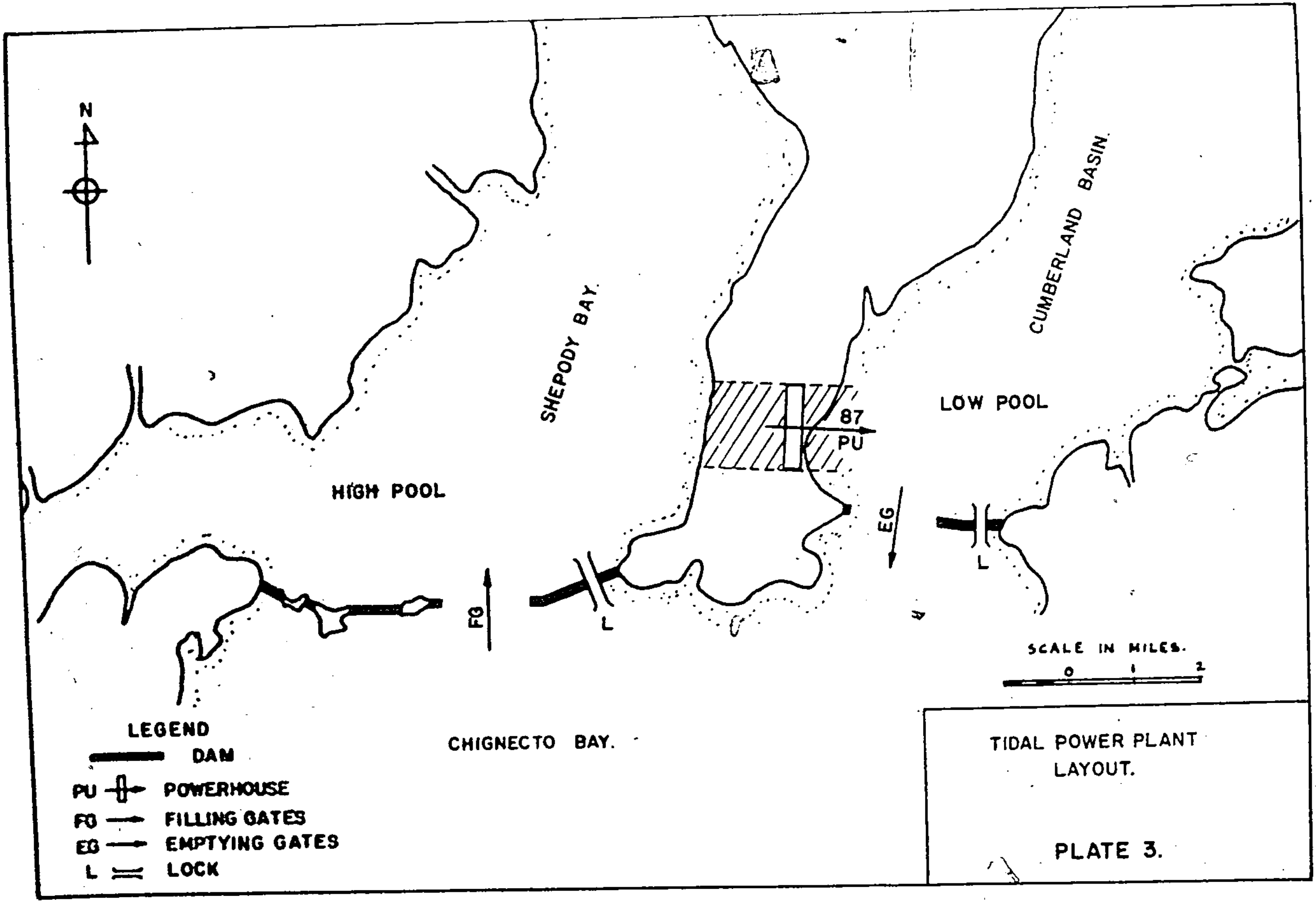
A very promising and purely Canadian tidal power project could be created by combined use of Shepody Bay and Cumberland Basin, located at the head of the Bay of Fundy, between New Brunswick and Nova Scotia.

Shepody Bay, with an area of 75 sq. miles could be used as the high pool, which also would improve navigation on the Petitcodiac River. Cumberland Basin with an area equal to 50 sq. miles (after some improvements), could be used as the low pool.

Plate 3 shows proposed Interprovincial Tidal Power Project layout. A number of filling gates and a navigation lock to pass sea-going vessels, could be located at the entrance to Shepody Bay. The emptying gates, also a small navigation lock to pass fishing vessels, could be located at the entrance to Cumberland Basin. The filling gates and the navigation locks, should have the shape and size as previously described for the Passamaquoddy Tidal Power Project.

The best location for the power house could be at the narrowest point of the peninsular, separating two pools at Peck's Cove, with excavated area for the head and tailrace. With 87 generating units rated at 10,000 kW each, operated at 15 per cent above rated capacity for short periods during spring tides, the output of the tidal power plant would range from 318,000 kW to 1,000,500 kW. Average energy generation would be 5,780 million kWh a year.

Height of Tides. The tides at the head of the Bay of Fundy are propagated up the Petitcodiac and other rivers. The lower part of the tidal oscillation is gradually cut off with progress up the rivers by the upward



TIDAL POWER PLANT LAYOUT.

PLATE 3.

slope of the river beds. The upper part of the oscillation is, however, gradually increased, partly by friction and partly by the decreasing cross-section of the river. This rise in the high-water heights levels-off as the upper limit of tidal waters is approached. At Cape Hopewell in the lower reaches of Petitcodiac River, the largest tides rise to about 24 ft above the level plane of mean sea level, on the other hand, the low water of this tide falls to about 23 ft below mean sea level.

Maximum tide height for the above project will be assumed as 47 ft, average tide height 35 ft, and minimum tide height 23 ft.

Two Pool Arrangement. To produce a continuous supply of power, a two pool plant must be selected. The investigation of the possible power output will start (as in Chapter VI) on the average tide height. By using different volumes of flow through the turbines (discharge), different ratios between maximum and minimum power output could be obtained.

Figure (7-1) shows typical operation and power output of the Interprovincial Power Plant, using the average tide height of 35 ft. In this case, assumed elevation zero is located 6 in.above m.s.l..

The filling gates will be closed when water level in the high pool will reach its maximum level + 16.5 ft at time t = 0.67 hours, after high water. The water level in the high pool will drop 6 ft in 10.47 hours, and at t = 11.14 hours the filling gates will be open again.

The low pool having smaller area will drain its water volume at low tide, to the lowest level - 17.0 ft, and then the emptying gates will be closed. As there is a continuous flow of the constant volume of the water from high to low pool, water level in the low pool will increase

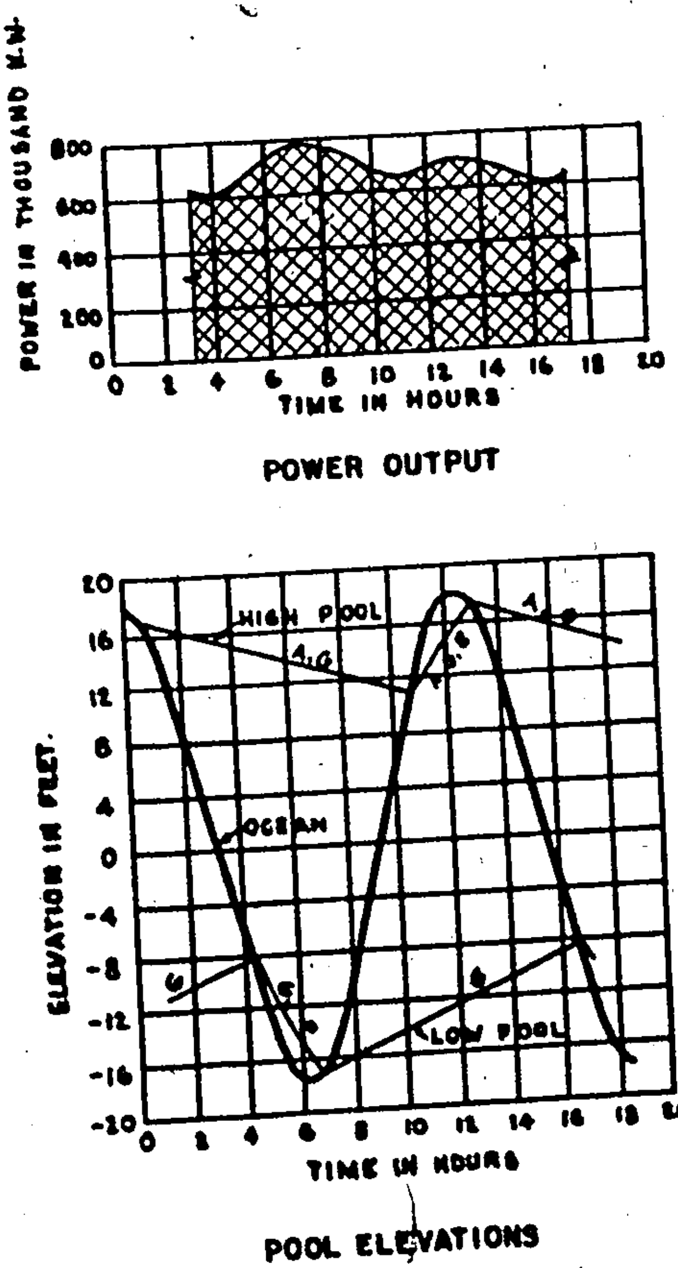
LEGEND

A , Power units as turbines.

E , Filling gates open.

F , Emptying gates open.

G , Flow from upper pool to lower pool.



POWER OUTPUT FROM THE AVERAGE TIDE HEIGHT. INTERPROVINCIAL TIDAL POWER PLANT.

Fig. 7-1

1.5 times faster than it decreases in the high pool. This will last for 9.85 hours and at t = 16.52 hours, the water level in the low pool will reach its highest level, and then the emptying gates will be open.

The average discharge from high pool to low pool is given as:

$$Q_a = \frac{V}{t} = \frac{6A}{10.47} = 1.26 \text{ billion cu ft/hr.} = 350{,}000 \text{ cu ft/sec.}$$

where: A = average area of the high pool = 75 sq. miles = 2.1 billion sq. ft.

With an assumed average efficiency of a power plant at 88 per cent, developed power can be expressed by previously used Eq.(6-2).

$$P\ (\text{kW}) = \frac{Q_a h}{13.2}$$

Table VIII shows comparison between maximum, average and minimum power output.

TABLE VIII

	Max. head (ft)	Min. head (ft)	P max. (kW)	P min. (kW)
Maximum tide	40.50	----	1,075,000	----
Average tide	29.94	23.09	783,000	610,000
Minimum tide	----	12.01	----	318,000

<u>Comparison Between Interprovincial (Shepody Bay) and International (Passamaquoddy Bay) Tidal Power Plant</u>. From the comparison of Plates 2 and 3, also Tables VII and VIII, the superiority of the Interprovincial Tidal Power Plant over the International can be expressed as follows:

(1) Dependable power supply would be 4.42 times higher.

(2) Average energy generated per year would be 2.86 times more.

(3) The ratio of the high pool area to the low pool area is closer to unity, which creates a more economical operating plant.

(4) Average design head for selection of the turbines is 26 ft, as compared to 11 ft in the case of the International Tidal Power Plant, this will result in selecting smaller sized turbines to produce the same amount of energy.

(5) Only two navigation locks are required to pass the sea-going and fishing vessels.

Powerhouse. To save on the amount of excavation for the powerhouse, and for the head and tailrace, the tidal powerhouse, equipped with 87 tubular turbines designed to permit flow in one direction only, would be selected. The perspective view of a similar powerhouse, but with turbines operating in both directions was shown in Chapter V, Fig.(5-7) on page 99.

Turbines. The turbines selected for the project are one-way operating tubular turbines, having the following properties:

Design head h = 26 ft.

Average discharge through one turbine $Q_a = \frac{350,000}{87} = 4,025$ cu ft/sec.

Specific speed $N_s = 156$.

Normal (operating) speed $N = 89.9 \cong 90$ rpm.

Runner discharge diameter $D_3 = 175$ in. = 14.6 ft.

Conclusion. From the above, it is evident that a purely Canadian tidal power development at Shepody Bay and Cumberland Basin is more attractive in every respect than the Passamaquoddy tidal power project. Both the developments would have nearly the same average length and depth of earth dams. A larger amount of turbogenerators will be required for the Shepody

Bay project, but as they are smaller in size, they will cost less per unit and be easier in handling and removing for overhaul.

The Interprovincial Tidal Power Project as the most promising, should be given first consideration over all other tidal power developments.

CHAPTER VIII

MINAS BASIN TIDAL POWER PLANT

The tide is also propagated from the Bay of Fundy into Minas Basin. It is off Burntcoat Head, in the upper part of Minas Basin that the largest tides in the world have been recorded. On July 16th, 1916, tidal ranges of over 53 ft were measured. However, for the whole area of the basin, maximum tide heights of 49 ft, average tide height of 40 ft, and minimum tide height of 31 ft could be assumed.

As previously mentioned, there are no coast charts for this very promising area available at the present time.

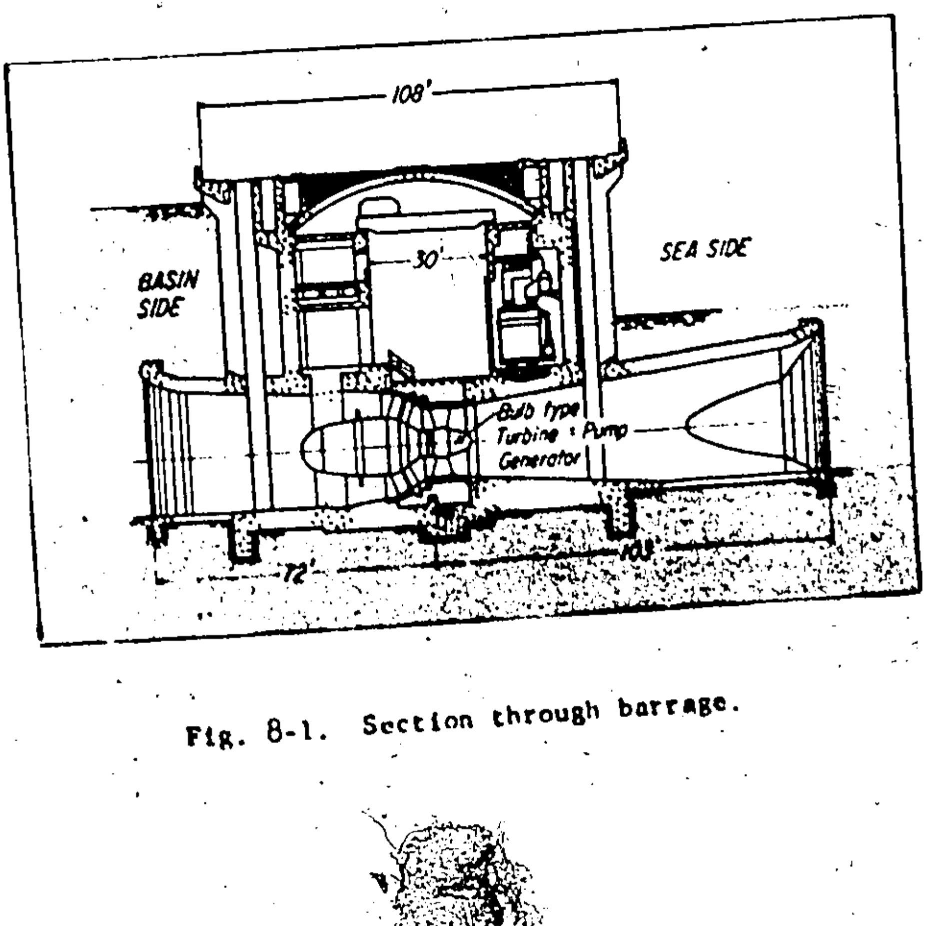


Fig. 8-1. Section through barrage.

The Minas Basin Tidal Power Plant could be a typical one pool scheme, with two-way operating plant. The 3.5 mile long barrage could be built in the narrowest part of the passage, between Minas Channel and Minas Basin, at Cape Sharp. The powerhouse would be part of the barrage and could be similar to that used for the French St. Malo project, equipped with bulb type turbines as shown in Fig. (8-1).

It is beyond the scope of this thesis, to arrive at some definite conclusions of the possible values of the tidal power generation for this large water area of 325 sq. miles approximately. This would require eleborate comparison study between available volume of the water discharge, and the economical number of turbines which could be installed to generate power. It is no doubt that the cost of installation and available length of barrage to accomodate navigation locks, turbogenerators, and probably sluice gates, would limit the number of turbines which could be used.

The other aspects, such as the use of turbines which operate as pumps also, and the use of sluice gates could also influence the amount of power output from Minas Basin. This is a wide field, open for research to investigate the economics of combining generating and pumping operation within the cycle of one tide.

In this chapter however, to point out the tremendous potential energy reserves accumulated in the water of Minas Basin, we will investigate the energy which can be produced by average tide height only.

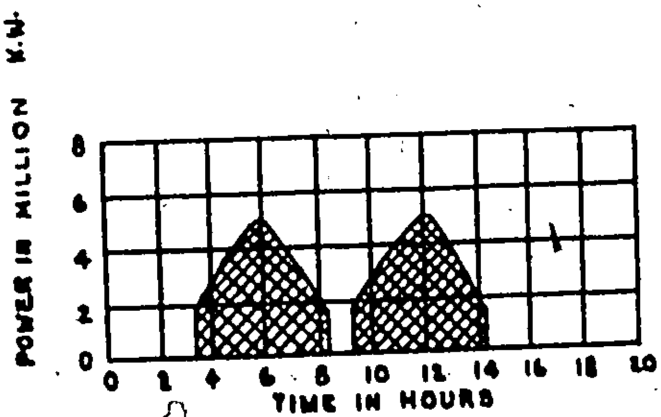
Comparison between single-mean-pool energy output for Passamaquoddy and Cobscook Bays (described in chapter VI) with Minas Basin, can be obtained by comparing Fig.(6-2) and Fig.(8-2).

LEGEND

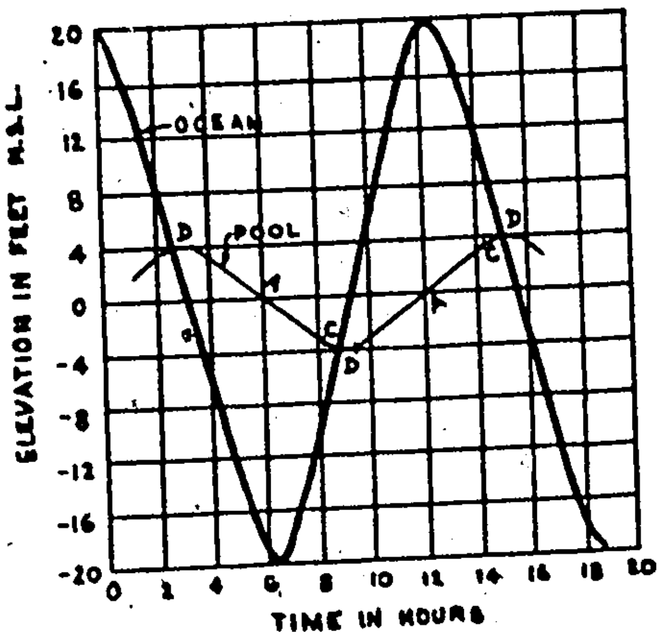
A , Power units as turbines.

C , Power units as orifices.

D , Power units inoperative.



POWER OUTPUT



POOL ELEVATIONS

POWER OUTPUT FROM THE AVERAGE TIDE HEIGHT. MINAS BASIN TIDAL POWER PLANT.

Fig. 8-2.

Using turbines which can generate power from flow in either direction, a single mean pool can be operated as a high-pool during low tides, and as a low-pool during high tides as shown on Fig.(8-2), where the mean pool is filled and drained respectively to $\pm$ 4 ft above or below m.s.l.

This arrangement results in two separate generating periods in each tide cycle. The selected required minimum operational head of 6 ft will occure in the pool 3.3 hrs. after high water and the power generating period will last for 5.08 hrs. until the operational head will drop to 6 ft at t = 8.38 hrs., and during this time the water level in the pool will drop 7 ft. The water from the pool will drain through the turbines which are changed to orifices,and at time t = 8.9 hrs.after high water, level in the pool will drop to 4 ft below m.s.l.

The reverse process will start again at 9.5 hrs., when the required operational 6 ft head will be available, but this time water from the ocean will flow to the pool through the turbines, and the whole generating period will be repeated.

The maximum operational head 20.16 ft will occur twice during one tide period, 0.272 hrs. before the time of the low and high water respectively.

Since, during both 5.08 hrs. generating periods, water level in the pool rises and falls 7 ft each time the average flow through the turbines will be:

$$Q_a = \frac{7A}{5.08} = 12.4 \text{ billion cu ft/hr.} = 3,460,000 \text{ cu ft/sec.}$$

where the pool average area A = 9.025 billion sq ft.

Developed power can be expressed for each of two generating periods by Eq.(6-2).

Table IX shows a relationship between time, operational head and generated power for two-way operating plant. Pool area A is assumed to remain constant during the generating period.

TABLE IX

Time t (hrs)	Water Level in pool (ft)	Operational head h (ft)	Generated power P (kW)
3.30	+ 4.00	6.00	1,576,000
5.94	+ 0.36	20.16	5,270,000
8.38	- 3.00	6.00	1,576,000
8.90	- 4.00	0.00	0
9.50	- 4.00	6.00	1,576,000
12.24	- 0.36	20.16	5,270,000
14.58	+ 3.00	6.00	1,576,000
15.10	+4.00	0.00	0

Conclusion. Minas Basin could produce large quantities of intermittent energy, using the average tide height for a comparison basis, the Minas Basin for the same pool arrangement, could produce about eight times more power than the International Tidal Power Plant during each generating cycle.

The most economical use of energy supplied, could be decided upon after thorough examination of the future decade load pattern for the Canadian maritime provinces which is not available at the present time.

However, some part of the energy output of Minas Basin could be stored by pumping water to the elevated storage reservoir. The stored water could be utilized to generate power where there is no power supply from Minas Basin, and to meet varying load demands. It could also supplement the Interprovincial Tidal Power Project, increasing its dependable capacity, and make its power production more acceptable to the other generating plants of the network system.

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