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Throughput enhancement with parallel redundancy in multi-product flow line system

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THROUGHPUT ENHANCEMENT WITH PARALLEL REDUNDANCY IN MULTI-PRODUCT FLOW LINE SYSTEM

by
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ABSTRACT

We develop a new analytic approximation method to replace a set of parallel machines by an equivalent machine in a series-parallel flow line with finite buffer. We develop our method based on discrete state Markov chain. The proposed technique replaces a set of parallel machines at a work centre by an equivalent machine in order to obtain a traditional flow line with machines in series separated by intermediate buffers. We derive equations for the parameters of the equivalent machine when it operates in isolation as well as in flow line. The existing analytic methods for series-parallel systems can tract only lines with a maximum of two machines in series and a buffer in-between them. The method we propose in this thesis can be used in conjunction with an approximation method or simulation to solve flow lines of any length.

We also model and evaluate the performance of series-parallel systems manufacturing more than one product types with predefined sequence and lot size. We address this issue for a considerable longer flow line system with finite buffer which is common in industry. We consider the set-up time of the machines as the product type changes, deterministic processing times and operation dependent failures of the machines. We analyze the effects of buffer and number of machines in parallel on the performance of series-parallel systems.

DEDICATION

I dedicate this thesis

To my beloved parents,

two of the best people I know.

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LIST OF ABBREVIATIONS

MTTF	Mean Time To Failure
MTTR	Mean Time To Repair
TDF	Time Dependent Failure
ODF	Operation Dependent Failure
WIP	Work In Process
EWIso	Estimation With Isolated (equivalent machine)
EWIn	Estimation With Inline (equivalent machine)
OSL	Original Serial Line
SPL	Series-Parallel Line
ESL	Equivalent Serial Line

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CHAPTER 1
INTRODUCTION

A manufacturing flow line is a common type of production system. It is also known as transfer line or production line and can be represented as a finite buffer tandem queueing system. In this thesis, we mainly use the term 'flow line'. We also use the terms 'work centre' and 'stage' interchangeably in this thesis. A flow line consists of a number of work centres or machines in series with intermediate buffers between successive work centres or machines. Figure 1.1 illustrates a serial flow line made of m work centres and (m-1) buffers. Examples of flow lines can be observed in electronic components assembly and high-volume automotive parts production industries.

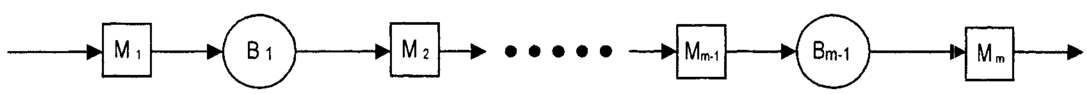

Figure 1.1: Serial flow line

In a flow line system, materials flow from work centre to buffer to work centre in a fixed sequence visiting each work centre only once. Raw materials from outside enter the system through the first work centre M_1, flow through the system in the direction of arrows and receive processing at each work centre. The final product leaves the system at the last work centre M_m. The work centres may be made of one machine or a set of machines in parallel. The terms 'work centre' and 'machine' are used interchangeably for work centres made of one machine. When there are multiple machines at a work centre, the system may be referred to as a series-parallel system (Burman, 1995). We focus on series-parallel systems in this thesis.

In a synchronized flow line system such as automatic transfer line and automatic assembly line, materials transfer from one work centre to the next at the same instant. In this system, the machines in the line are highly coupled and as soon as any machine breaks down all other machines in the line will be forced down at the next transfer instant. Unlike synchronized system, in an asynchronized flow line system, transfer of materials from one work centre to the next begins as soon as its processing at a work centre is complete and the next work centre has become free. In this system, a machine failure is considered to be equivalent to an extended processing time and eventually causes all other machines in the line to be forced down (Buzacott, 1969). The amount of time the material spends in a work centre may be considered as deterministic if it does not vary from one part to the next and stochastic if it varies randomly from part to part. This randomness may be due to random processing times, random failure and repair events, or both (Dallery and Gershwin, 1992). The series-parallel systems dealt with in this thesis are asynchronous in nature, the processing times are deterministic and failures and repairs are random with operation dependent failures. Parts are transferred to the next work centre via the intermediate buffer on a unit-by-unit basis.

In a flow line system, intermediate buffers are used to decouple the machines and to provide them with some independence in their operations. When a machine in a flow line fails or has a longer operation time, the machine upstream of it can still operate until the upstream buffer fills up, and the downstream machine can still operate until the downstream buffer becomes empty. If the buffer downstream of a machine fills up, the machine has to temporarily stop production until the buffer has space for more output. In this case, the machine is said to be blocked. A machine may also have to stop temporarily if the buffer upstream of it is empty; now the machine is said to be starved. The introduction of buffer minimizes the blocking and starvation times of the machines.

Manufacturing flow lines are designed with the objective of achieving a greater production rate. There are different ways to improve the performance of a flow line system:

(a) Increase intermediate storage capacity (Buzacott, 1971), because such buffers may allow the machines to continue working when one of them is down

(b) Improve the availability of machines by providing some degree of parallelism at the stage level, because parallel machines can continue the function when one or more (not all) of the working machines fails

(c) Increase the working capacity of machines

(d) Perform preventive maintenance of the machines to improve machine reliability and hence availability by reducing failure rate

Every solution described above has merits as well as limitations. An increase of intermediate storage capacity and addition of parallel machines could be expensive and requires valuable floor space at the plant. There are also other cost factors involved such as holding cost and material handling cost associated with the in-process inventories on the buffer. Increasing the working capacity of machines is restricted by the design of the machines. Every machine has its designed maximum and rated speed. Running a machine beyond this recommended speed decreases the reliability and eventually the life of the machine. Failures of the machines may not be completely eliminated in a manufacturing system. However, they can be reduced with the implementation of preventive maintenance in a planned manner. The benefit of preventive maintenance depends on the frequency of use and costs involved. So, for the selection among the above solutions a performance evaluation model that shows the economic tradeoffs is needed.

However, use of parallel machines at work centres is common in industries. There are two main reasons why machines are used in parallel at work centres: (a) to achieve a greater production rate or (b) to achieve greater reliability (Dallery and Gershwin, 1992). The first case is often observed when some machine is inherently much slower than the others. The latter is considered when some machine is much less reliable than the others.

There are three basic types of redundancy scheme: standby, parallel and splitting. The choice of the scheme depends on the characteristics of the machine to which redundancy is applied. Among them, the parallel redundancy scheme maintains the production rate at a stage where all the machines perform the same function. When one of the machines fails, other machines continue working, but the production rate is reduced to the rate of the working machines. If the maximum production rate of the machine cannot meet the required system production rate, then only parallel redundancy can be used (Buzacott, 1968). In this thesis, we apply parallel redundancy.

Manufacturing industries are facing extreme challenges and competition due to the globalization of markets. In addition, the continuously changing customer needs dictate the launch of new products. Product introductions and changes are occurring so rapidly that the engineers are faced with increasing pressure to reduce development cycles. But flow lines are of significant economic importance and involve large investments for most companies. So, there is a need for designing more flexible and efficient production lines that can produce more than one product type in the same line or could be used for other products with no reengineering or with minimum reengineering of the existing production lines. We discuss this issue in this thesis.

To summarize, this thesis addresses the issue of incorporating parallel machines to improve the performance of production systems. It also focuses on the issue of processing multiple products in the same series-parallel system where products are processed in batches with a predefined sequence and lot size. As the product type changes, machines require set-up. Due to the complicated nature of the formulated problem, the analysis of solution is split in two folds. First, an analytical model is developed to approximate a set of parallel machines at a stage of a series-parallel system by an equivalent machine. The system is then converted into a traditional serial system, a solution of which already exists. So, the developed method in conjunction with an approximation method or simulation can be used to evaluate the performance of a series-parallel system of any length. Secondly, a simulation-based model is developed to analyze the performance of a series-parallel system that processes multiple products in the same line. The simulation approach is adopted due to the intractability of analytical models for large production line system. Moreover, the flow line of interest processes more than one product type in batches, which necessitates multiple machine set-ups. This makes finding an analytical solution an intractable problem. This use of simulation is believed to be appropriate mainly for two reasons (Baker *et al.,* 1990). First, it is necessary in the development of a basic science of line design, because even relatively simple questions cannot be answered mathematically. Secondly, representative of the emerging practice of line design, because the advent of software for factory simulation has radically changed the way design problems can be approached. Moreover, simulation provides visual representation with the aid of three-dimensional graphics that helps to understand a real system with greater ease. Detailed and more representative model provided by simulation allows the analysis of many parameters that impact the performance of the system and thus provides insights about the system improvement and the best strategies to implement.

The remainder of the thesis is organized as follows. Chapter 2 gives a general review of literature of manufacturing systems based on the model development techniques. A review of research papers related to series-parallel systems and systems that process multiple products is also presented in this chapter. Chapter 3 introduces the notation, a brief description of the formulated problem, working assumptions, and performance measures. Chapter 4 develops a methodology to replace a set of parallel machines by an equivalent machine to solve a series-parallel system. Chapter 5 presents the application of the developed analytic approximation method on numerous series-parallel configurations to check its accuracy and sensitivity. As well, the method is compared with the other existing approximation methods. A simulation-based model incorporated with the proposed analytic approximation method is presented in Chapter 6 to evaluate the performance of a series-parallel system employing multiple products. Finally, conclusions and future research directions are discussed in Chapter 7.

CHAPTER 2
LITERATURE REVIEW

Manufacturing systems performance can be categorized according to many criteria such as the reliability of the system, the type of processing times, the mode of operation, etc. In section 2.1 a general review of the literature of manufacturing systems based on model development technique is presented. Section 2.2 shows the literature review of production systems with multiple products and parallel machine stages.

2.1 General Review of Literature

2.1.1 Analytical Models

2.1.1.1 Exact Models

Analytical models represent a mathematical abstraction of the real system based primarily on certain mathematical relationships, heuristics and approximations. Exact analytical results are difficult to obtain due to the mathematical complexities, and are only available for short production lines mainly with two work centres with a finite buffer between them. They are analyzed using basic formulae and Markov processes, and in general via the use of transform methods. Buzacott (1968) developed basic formulae for the prediction of efficiency in terms of reliability of constituent machines. He considered basic connections of the production systems, with and without redundancy and with no internal storage. He also showed how the efficiency of more complex systems could be predicted using the basic formulae. Buzacott (1969) addressed the issue of reliability measured in terms of efficiency or availability of production systems subject to breakdowns. He developed simple models of fixed cycle systems with and without buffer based on renewal theory and Markov chain. He also derived formulae of efficiency for different line configurations using combinations of operating and repair time distributions. Buzacott (1971) presented some simple models of production-inventory systems and demonstrated how production capacity and flexibility is enhanced with the use of inventory banks. The models were developed with different sets of assumptions of breakdowns, processing times, repair times and times between failures.

Buzacott (1967) was one of the earliest researchers to investigate the effect of buffer stocks on the performance of transfer lines modeling the system as a discrete time Markov chain. He analyzed the extreme (zero and infinity) buffering configurations, based on which one could at least decide whether buffering would be worth considering at all. He assumed geometric distributions of the life times, repair times and time dependent failures of the machines. Sheskin (1976) studied the same type of systems using discrete Markov chain with some different assumptions. He considered that all events would only occur at epochs when processing of parts was finished.

Buzacott and Hanifin (1978) provided a review and comparison of previous models of transfer lines. They studied the assumptions and derivations of the previous models analyzing the nature of the lines and their stoppages and found inconsistencies with some models with reality. Based on the extensive study they commented that a model with operation dependent failure is more appropriate than a model with time dependent failure.

Among the few recent models based on Markov analysis are the papers of Alves (1990), Burman (1995), Tan (1998), Patchong and Willaeys (2001), Jeong *et al.* (2005).

2.1.1.2 Approximation Models

2.1.1.2.1 Heuristic Methods

The problem size and the absence of efficient solution schemes often necessitate the use of a heuristic approach to find a good solution to the problem. Heuristic rules are easier to code and understand (Askin and Standridge, 1993). Ignall and Silver (1977) developed a heuristic procedure to estimate the output of a two stage series-parallel system with a finite buffer. Their method of approximating the system output allows system designers to evaluate the economic tradeoffs of buffer storage cost and increased system output value.

Hillier and So (1991) presented a heuristic method based on empirical results to estimate the amount of storage space required to compensate the negative effect of machine breakdowns on throughput. They used an exact analytical model to conduct a detailed study of how the machine breakdowns and inter-stage storage capacity can effect the throughput of production lines with more than three (four and five) machines. The operation times at all machines were assumed identical two-stage Coxian distribution.

In theory, systems can be modeled via Markov chains for any number of stages; but in practice, it is very difficult to obtain exact analytical solutions of transfer lines with more than three machines. Because the number of system states in the Markov chain increases exponentially with the increase of machines and the interstage buffer capacity. For example, a line with four machines and interstage buffers of capacity 3 gives rise to a Markov chain with 19,402 states (Hillier and So, 1991). The intractability of analytic models for large production line systems has prompted researchers to propose two main approximation techniques: *decomposition* methods and *aggregation* methods. Dallery and Gershwin (1992) provided a good review and comparison of the approximation methods in the analysis of production and transfer lines.

2.1.1.2.2 Aggregation Methods

The basic idea of the aggregation technique is to reduce the system dimension by replacing a two-machine-one-buffer sub-line by one single equivalent machine in the system. Then this equivalent machine is combined with a buffer and a machine of the original line to form a new two-machine one-buffer sub-line, which is then aggregated into a single equivalent machine. This process is repeated until the last or first machine is reached, depending on the direction the aggregation is performed (Dallery and Gershwin 1992). The major cause of inaccuracy of the aggregation methods is that they only account for a unidirectional propagation of events. For example, when the first two stages are aggregated, there is no accounting for the effects that downstream blocking might have on the parameters of a previously aggregated stage. Buzacott (1967) used aggregation types of approximations to carry out a preliminary analysis of optimization issues when the number of machines is relatively small. In his model, the machines indexed on a common cycle and the operating time had a geometric distribution. Analysis of continuous flow lines using aggregation was proposed by Ancelin and Semery (1987) and Terracol and David (1987) for lines with operation dependent failures. They assumed exponentially distributed up-times and down-times where processing rates at machines might be different.

De Koster (1987 b) used repeated aggregation to multi-stage continuous flow lines of several unreliable machines separated by buffers for the prediction of line efficiency. He considered time dependent failure of the machines with exponentially distributed life and repair times. De Koster (1988) improved the work of De Koster (1987 b) by developing a substantially improved algorithm.

2.1.1.2.3 Decomposition Methods

The idea of the decomposition technique is to decompose the analysis of a multi-stage line into the analysis of a set of two-machine lines, which are much easier to analyze. The set of two-machine lines is assumed to have equivalent or similar behaviours to the original system. The decomposition method is generally more accurate than aggregation method as it accounts for both the up and down stream propagation of events.

Sevast'yanov (1962) introduced the decomposition concept that generalizes aggregation. He extended a two-stage model to longer lines using the same processing and repair rates but different failure rates for all the machines in the line.

Buzacott (1969) used an approximate procedure based on Sevast'yanov (1962) to analyze three-stage fixed cycle systems with unreliable machines and finite buffer. This method has limitation as it assumes repair rates are equal for all the three stages.

Gershwin (1987a) extended the work of Schick and Gershwin (1978) on two-stage model using decomposition technique. He assumed a homogeneous processing rate but different repair and failure rates for all the machines in the line. Gershwin's model demonstrated the tradeoffs between buffer capacity and throughput. The algorithm was slow, lacked robustness and failed to converge at times. The efficiency of Gershwin (1987a) algorithm was improved upon by Dallery *et al.* (1988) who proposed an efficient algorithm, known as DDX algorithm that solved the Gershwin equations in substantially less time.

Gershwin (1987b) made an attempt to treat three-parameter continuous material systems where he combined two equal processing rate machines together. The conception was, one machine would be accountable for the failure rate and the other would modify the machine speed. But this idea was proved unrealistic after the simulation experiments.

Choong and Gershwin (1987) were the first to present decomposition method to three parameter discrete material systems. They used the two-stage, discrete material, exponential processing time model of Gershwin and Berman (1981) as the basis of their analysis. They developed an algorithm that had difficulties in converging. This problem was solved by Gershwin (1989) with a DDX-type algorithm.

The first continuous material decomposition equations for non-homogeneous lines were developed by Glassey and Hong (1993) which was based on their earlier two-stage continuous material model (Glassey and Hong 1986).

Dallery and Le Bihan (1997) extended the applicability of the decomposition methods by homogenization techniques. The accuracy of the results was improved again by Le Bihan and Dallery (2000).

2.1.2 Simulation Models

Methodologies other than Markov chain analysis and approximation techniques have also been used for model development. Simulation models are experimental and mimic the events that occur in the real system, allowing experimentation with operating parameters or control logic (Askin and Standridge, 1993).

Barten (1962) simulated serial production lines with normal distributions and equal buffer sizes in order to estimate throughput. El-Rayah (1979) also simulated serial lines with normal distributions and found no incentive to deploy unequal buffers. A review of this literature was given by Smunt and Perkins (1985).

Conway *et al.* (1988) used simulation to investigate the behaviour of serial buffered lines due to lack of synchronization and explored the distribution and accumulation of work-in-process (WIP) inventory. Based on the results obtained, they presented rules about the optimal buffer allocation/ location.

Baker *et al.* (1990) used simulation to study the effects of buffers, focusing on the optimal placement of buffers, the allocation of buffer capacity, and the tradeoff between adding buffers and reducing operation times in buffered and unbuffered balanced assembly systems with feeder lines. They assumed independent and random operation times and no breakdowns of the machines.

Powell and Pyke (1996) presented a simulation based approach to analyze reliable and asynchronous serial lines with variable processing times. They focused on the allocation of optimum buffer in the production lines with bottleneck machine and showed how the optimal allocation depends on the location and severity of the bottleneck, imbalances in the mean processing times, length of the line, as well as the number of buffers available.

Among other few recent models based on simulation are the papers of Amin and Altiok (1997), Abdul-Kader and Gharbi (2002), Noseworthy (2003).

2.1.3 Hybrid Models

The hybrid models combine different methods to evaluate the performance of production systems. A simulation model of a complex system could be built where modules of the real system are replaced by simple analytic models. The logic of the simulation model relates the individual modules together to replicate overall system occurrences (Askin and Standridge, 1993). Blumenfeld (1990) combined the result of prior theoretical and simulation studies into an approximate expression for the production rate of serial lines with identical random processing time distributions at each stage and identical buffer capacities between each pair of stages. Baker *et al.* (1994) adopted the same technique to predict the throughput of unbalanced three-machine serial lines without intermediate buffer. Kaplan and Unal (1993) presented a method to accurately estimate the flow time of a job in the shop in combination of simulation and statistical analysis.

2.2 Review of Literature of Production Systems with Parallel Machines and Multi-Product

There is a substantial literature on the analysis of production systems of various categories as shown in the general literature review in Section 2.1. Most of this research has focused on the effect of buffer on the performance of serial systems. In contrast, there exists a few studies to improve the unreliable systems with deterministic processing times having multiple parallel machines at stages. This type of production system is referred to as series-parallel flow lines (Burman, 1995). Moreover we are not aware of any study on series-parallel flow line systems manufacturing more than one product types in the same line. This is one of the main motivations of this thesis.

The issue of parallelism was probably first considered by Buzacott (1968) who studied the impact of the three basic redundancy schemes (parallel, stand-by, and splitting) on the efficiency. He developed basic formulae for the prediction of efficiency in terms of reliability of constituent machines. For convenience of model development, he did not consider any internal storage and considered identical parallel machines.

Ignall and Silver (1977) developed a heuristic procedure to estimate the hourly output as a function of buffer storage capacity of a two stage, synchronous flow line system with a finite buffer, where each stage has one or more identical and unreliable parallel machines. However, their work was limited to a short production line.

Iyama and Ito (1987) investigated the effects of the allocation of multi-server stages on the maximum production rate in a relatively short flow line system with buffers. They assumed exponentially distributed mean time to failure and service times. An approximate method was developed from a Markov model that could solve a line of maximum three stages in length.

Alves (1990) addressed the issue of performance evaluation of flow-type series-parallel systems having identical machines at stages and no buffer. He modeled the system using continuous time Markov chains. He considered single product type and assumed deterministic processing times and operation dependent failures of the machines. The mean time to failure and the repair time were assumed to be exponentially distributed.

Tan (1998) presented closed-form exact formulae for the asymptotic mean and variance of the throughput of series, parallel and series-parallel production systems. The methodology used a general result derived for continuous time Markov chains. The model considered continuous materials flow, unreliable machines with exponential failure and repair times, time dependent failures, and deterministic processing times. The study is limited to lines with all identical machines at stages and with no interstage buffers.

Short flow line models of Ignall and Silver (1977), and Iyama and Ito (1987) effectively reduced each set of parallel machines to an equivalent single machine that operated according to a complicated Markov process and hence are limited to the restrictions on the complexity of solvable cases. Models of Buzacott (1968), Alves (1990), and Tan (1998) are restricted to zero buffer capacity.

Ancelin and Semery (1987) was probably first to approximate a set of parallel machines of a stage by an equivalent machine in a series-parallel system with unreliable machines and finite buffers. The equivalent parameters yielded by their proposed method were often unrealistic and did not give satisfaction in their application to performance diagnosis.

Burman (1995) developed an analytical model to approximate a set of parallel machines at stages of series-parallel flow line system by a single equivalent machine. The line considered was composed of unreliable machines and finite buffers where failures were operation dependent. The model was based on a continuous time Markov process that uses a continuous material approximation to represent the movement of parts. But the use of independence-of-machines assumption in determining the variance of the set of parallel machines was not practical for a continuous flow material system. Also, selection of time associated with his derived time-dependent equation of variance was rather complicated. The method was limited to lines only with relatively large sized buffer.

Patchong and Willaeys (2001) presented an analytical method for modeling and analyzing series-parallel flow lines composed of unreliable machines. Machines' failures were assumed to be operation dependent. The technique they used was to replace each parallel-machine stage by an equivalent machine and they submitted equations for computing the parameters of equivalent machine. The proposed method could be applied with any random distributions of uptime, downtime and processing time. However, the method was not accurate for flow lines where parallel machines of a stage or stages are less reliable than the single machine at other stages.

All the papers reviewed so far assumed single product flow lines. De Koster (1987 a) was one among the few who considered multiple products in his proposed model. He studied two-stage automated production lines with an intermediate buffer to establish the relationship between the throughput and buffer capacity. The products were processed in batches that required set-up of machines. Machines were prone to failure and the life and repair times were assumed to be exponentially distributed.

Johri (1987) studied a highly automated manufacturing line similar to that of De Koster (1987 a) considering relatively small buffer. He used a linear programming approach to estimate the production rate of the line for a given machines configuration, buffer sizes, and product mix and sequence. Each machine needed to be set-up before a batch of new product type could be processed on it. Processing times were assumed to be deterministic and failures of the machines were not addressed explicitly.

Amin and Altiok (1997) presented a simulation based analysis of a multi-product, multi-stage pull-type manufacturing system having finite buffer between successive machines with the objective of identifying a set of control policies that were plausible for a given production-inventory environment. A set-up was considered every time a product type changed at a stage. The set-up times and the processing times were assumed exponentially distributed and the machines were reliable.

Abdul-Kader and Gharbi (2002) extended the work of Johri (1987) addressing explicitly the phenomena of a machine's failure and repair and evaluating the impact of buffer on the overall capacity of the production line. They proposed a simulation-based experimental design methodology to improve the performance (in terms of cycle time) of a multi-product, multi-stage, serial production line composed of unreliable machines and finite buffers. The set-up and processing times were assumed deterministic; the breakdown of machines was operation dependent.

A linear programming model was proposed by Abdul-Kader (2006) by exploiting Johri (1987)'s multi-product model and further modified by replacing machines' repair and downtimes with the insertion of fictive products to address the issue of capacity estimation/ improvement of a multi-stage, unreliable serial production line with finite buffers. The line could process a variety of products in batch according to a predefined sequence. Buffer contribution to minimize the cycle time of the production line was also addressed through experimental optimization.

In a recent paper, Colledani *et al.* (2005) presented an approximate analytical method to evaluate the performance of a serial production line with finite buffer, multiple failure modes and multiple part types. Unlike the above mentioned models, the parts processed were in certain ratios, not in batches; hence no set-up of the machines was considered. They developed an algorithm inspired by the DDX algorithm, assumed deterministic processing times scaled to unity and operation dependent failures of the machines. MTTF and MTTR were assumed to be geometrically distributed.

Based on the research papers reviewed, there are no known studies that evaluate series-parallel systems manufacturing more than one product types. This thesis introduces a new approach in modeling and evaluating the performance of manufacturing systems with multiple products and parallel machines at stages. We address this issue for a considerably longer flow line system with finite buffer which is common in industry. We consider deterministic processing times, operation dependent failures of the machines with exponentially distributed failures and repairs. We also analyze the effects of number of machines in parallel at stages. Our model can employ more than one product types with predefined sequence and lot size. We consider set-up times of the machines involved as the product type changes.

CHAPTER 3
PROBLEM FORMULATION

3.1 Introduction

This chapter introduces the notation, features describing machine behaviour, description of the problem, working assumptions and performance measures that we apply and deal with in this thesis.

3.2 Notation

In this section, we present the notations we frequently use in this thesis:

Subscript i denotes Machines/ Work centres in series

Where $i = 1, 2, 3,\ldots\ldots\ldots\ldots, m$

Subscript j denotes Number of machines in parallel in a work centre

Where $j = 1, 2, 3,\ldots\ldots\ldots\ldots, n$

Subscript k denotes Products comprising the product mix

Where $k = 1, 2, 3,\ldots\ldots\ldots\ldots, l$

M_i = Machines in series

λ_i = Failure rate of M_i

μ_i = Repair rate of M_i

u_i = Processing rate of M_i

q_i = Production rate of M_i when it operates in isolation

Q_i = Production rate of M_i when it operates in flow line

S_{ik} = Set-up time of product k on M_i

P_{ik} = Processing time of product k on M_i

M_{ij} = jth machine of ith work centre

λ_{ij} = Failure rate of machine M_{ij}

μ_{ij} = Repair rate of machine M_{ij}

u_{ij} = Processing rate of machine M_{ij}

e_{ij} = Efficiency of M_{ij} when it operates in isolation

q_{ij} = Production rate of M_{ij} when it operates in isolation

Pu_{ij} = Probability that machine M_{ij} is up (up= operating + idle)

Po_{ij} = Probability that machine M_{ij} is operating

Pd_{ij} = Probability that machine M_{ij} is down

Pi_{ij} = Probability that machine M_{ij} is idle

Ps_{ij} = Probability that machine M_{ij} is starved

Pb_{ij} = Probability that machine M_{ij} is blocked

S_{ijk} = Set-up time of product k on M_{ij}

P_{ijk} = Processing time of product k on M_{ij}

M_i' = Equivalent machine of a set of parallel machines at work centre i

λ_i' = Failure rate of M_i'

μ_i' = Repair rate of M_i'

u_i' = Processing rate of M_i'

e_i' = Efficiency of M_i' when it operates in isolation

q_i' = Production rate of M_i' when it operates in isolation

Q_i' = Production rate of M_i' when it operates in a flow line

Pu_i' = Probability that machine M_i' is up (up= operating + idle)

Po_i' = Probability that machine M_i' is operating

Pd_i' = Probability that machine M_i' is down

Pi_i' = Probability that machine M_i' is idle

S_{ik}' = Set-up time of product k on M_i'

P_{ik}' = Processing time of product k on M_i'

L_k = Lot size of product k

B_i = Buffer between M_i and M_{i+1} work centres (Except the last)

C_i = Capacity of buffer B_i

b_i = Content of buffer B_i = 0, 1, 2, 3,............., C_i

Q = Average production rate of a flow line

3.3 Features Describing Machine Behaviour

In this section, we define some of the important features that describe the behaviour of a machine operating in a flow line. This would help to understand the system we study in this thesis.

Set-up Time

Set-up time is the time a job spends waiting for the machine to be set up. It may include change of tools, fixtures and reprogramming of the machine.

Processing Time

Processing time is defined as the time to transfer the part from the previous machine plus the actual time required for processing the part at the machine.

Cycle Time

The cycle time of a machine is the time from the instant when a part begins to transfer from the previous machine until the instant when the next part begins to transfer from the previous machine. Thus the cycle time exceeds the processing time by the amount of time that the machine is either blocked or starved.

Operating Time

The length of time a machine operates between successive failures of itself.

A machine is said to be **up** if, it is either working/ operating i.e. carrying out its function or idle. An idle machine is in working order but unable to operate because of being either blocked or starved.

Blocking of Machine

A machine is said to be blocked if, the machine has no space to discharge a completed part. The blocked machine is stopped from processing the next part until a space becomes available in the downstream buffer. This blocking mechanism is also known as blocking after service (Dallery and Gershwin, 1992).

Starvation of Machine

A machine is said to be starved if, the machine has no part to process. This situation arises when the upstream buffer is empty. The starved machine is prevented from processing until a part arrives in the upstream buffer.

A machine is said to be **down** if it becomes unavailable for a certain amount of time due to failure.

There are two important types of single stage failures. (1) ***Operation dependent failures*** (ODFs) occur as a function of the number of parts produced by a machine or total working/ operating time of a machine since the last time the machine was repaired. In this case, the machine experiences failures only when a job is being processed, (2) ***Time dependent failures*** (TDFs) occur as a function of the time a machine runs since its last failure. In this case, machines may fail in time regardless of the machine status even if they are idle (Dallery and Gershwin, 1992). There is a significant impact on flow line model estimate based on whether one assumes ODFs or TDFs.

3.4 Description of the Problem

The series-parallel flow line being studied in this thesis is shown in Figure 3.1. The squares represent the machines and the circles are the buffers. The production line consists of m work centres in series and (m-1) buffers separating each consecutive pair of work centres. A work centre i (i = 1, 2, 3, ..., m) may be composed of a single machine or a set of j (j = 1, 2, 3,..........., n) machines in parallel. The parallel machines may be identical or non-identical in type. The system can process a variety of products k (k = 1, 2, 3, ..., l) with a predefined lot size for each product type.

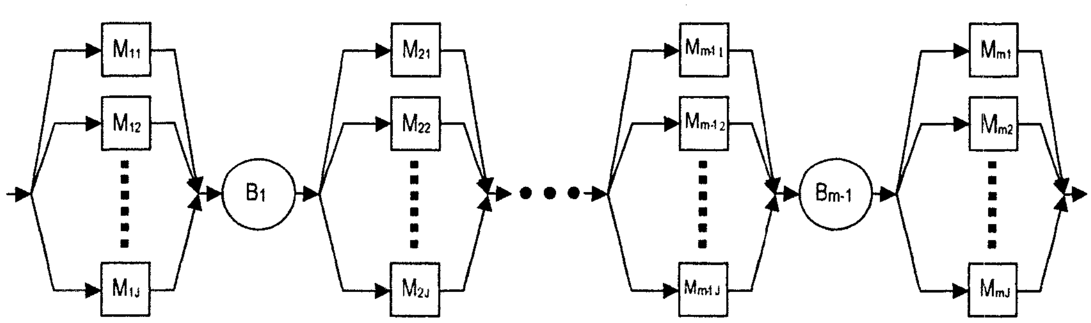

Figure 3.1: Series-parallel flow line made of m work centres

All the products of the mix enter the system from outside through the first work centre and flow through the system in the direction of arrows. The product mix follows a predetermined sequence, visits each work centre once and receives processing there. It begins with the processing of the first product type and ends with the processing of the last product type of the mix. Set-up of the machines is required when the product type changes. Products are transferred to the longest unoccupied machine in a set of parallel machines at a work centre where each machine in a set operates independently. Each unit of any product of the mix is transferred to the next work centre via the intermediate buffer on a unit-by-unit basis. The final product exits from the last work centre.

The processing time and the set-up time of a product type is fixed for a machine but varies from one work centre to another in the production line. Also, they are not the same from one product type to another for all the machines. Moreover, each machine of the production line is subject to random failure and repair. Due to this lack of synchronization of the machines, not only in performing operations, but also in failing and getting repaired, machines are forced to be starved or blocked. This affects the performance of the system. Intermediate buffers are deployed to decouple successive work centres and mitigate the loss due to a disruption (a failure or a long processing time). Parallel machines are added to achieve a greater availability and hence a greater performance.

In order to simplify the problem of interest, we split it in two folds. First, as the existing analytic methods for series-parallel systems are generally limited to no more than two machines in series and the existing approximation methods to solve longer series-parallel lines are not accurate enough, we develop a relatively accurate analytic approximation method in Chapter 4. The technique is to replace a set of parallel machines at a work centre by an equivalent machine in order to obtain a traditional serial flow line, the solution of which is within our reach. We derive equations for the parameters of the equivalent machine when it operates in isolation as well as in flow line. The main motivation of this approach is that it provides relative ease in modeling the system and brings mathematical convenience in numerical manipulations. We validate our proposed method in Chapter 5 with numerous experiments.

The second fold of the problem is the performance evaluation of a multi-product multi-stage series-parallel system with the objective of improving the performance of the system. To be more specific, our goal is to evaluate and reduce the cycle time of the production line. A simulation-based experimental design methodology is proposed in Chapter 6 to improve the performance of this flexible production line. We also address the impact of buffer and addition of parallel machines on cycle time.

3.5 Modeling Assumptions

In this section, we describe the set of assumptions considered in developing our model of series-parallel production system.

Assumptions related to the approximation model:

Following assumptions are considered for the approximation model:

1. Each work centre consists of either one machine or may contain a set of parallel machines that are either identical or non-identical.

2. Each machine in a set of parallel machines at a work centre operates independently.

3. The first machine is assumed never starved as there is an infinite supply and the last machine is never blocked as there is an infinite storage to unload the finished products.

4. The processing times are deterministic.

5. Each machine of the production line is subject to random failure, which is assumed to be operation dependent. Extensive study has shown that most failures of transfer lines are operation dependent rather than time dependent (Buzacott and Hanifin, 1978).

6. When a machine failure occurs, the part that is being processed remains at the same machine during repair and the remaining operations are done when the machine is up. So, there is no rework or scrap when a machine breaks down.

7. The failure rate and the repair rate of the machines are exponentially distributed. We would not argue that these exponential assumptions are adequate or accurate descriptions of the real world. But the memoryless property of the exponential distribution provides theoretical grounds for hypothesizing its use for modeling situations with a constant rate change, e.g., time between failures with a constant hazard rate (Papadopoulos *et al.*, 1993).

8. Repairs at different machines are independent of one another. It follows a first down-first repaired strategy.

9. Intermediate buffers have finite capacity.

Assumptions related to the simulation model:

In addition to the above assumptions, following assumptions are considered for the simulation models:

10. The system can process a variety of products and set-up time is considered when a product type changes.

11. Products are transferred to the longest unoccupied machine in a set of parallel machines at a work centre.

12. Each machine of the production line can process only one product type at a time. The processing sequence is predetermined, i.e., Product 1, Product 2 .....up to Product l.

13. There is an unlimited maintenance resource. So maintenance is available as soon as a machine fails.

3.6 Performance Measures

The performance measures of interest to validate our proposed method for series-parallel flow lines are:

- Production rate
- Average buffer level

Again, for the analysis of multi-product multi-stage series-parallel systems, the performance measures considered are:

- Cycle time of the production line
- % of the time the machines are in operation
- % of the time the machines are blocked
- % of the time the machines are starved
- % of the time the machines are down
- % of the time the machines are in set-up

The motivation is that by judiciously interpreting this information, a manager can take one of the following steps to improve the cycle time:

- Increase the buffer size selectively
- Add new machines in parallel

CHAPTER 4
SOLUTION METHODOLOGY

4.1 Introduction

Series-parallel flow lines are becoming popular for their higher reliability and increased productivity. Due to the complexity of a series-parallel line, existing analytical models fall far short in capturing the full extent of its behaviour. In this chapter, we propose a new approximation method to replace a set of parallel machines by an equivalent machine in a series-parallel flow line with finite buffer. We develop our model based on discrete state Markov chain. The proposed technique replaces a set of parallel machines at a work centre by an equivalent machine in order to obtain a traditional flow line with machines in series separated by intermediate buffers. We derive equations for the parameters of the equivalent machine when it operates in isolation as well as in flow line.

4.2 Description of the Proposed Method

The performance of a flow line is defined generally by its effectiveness or availability:

$$E = \lim_{t \to \infty} \frac{q(t)}{Q(t)} \tag{4.1}$$

where,

E = Effectiveness

$q\ (t)$ = Actual output over time t

$Q\ (t)$ = Output that would have been produced in absence of any work stoppages over time t

We rewrite equation (4.1) in the following way to make it more practical.

$$E = \frac{E(\text{uptime})}{E(\text{uptime} + \text{downtime})} \tag{4.2}$$

Where operator *E (.)* refers to the expected value, uptime refers to the interval during which production occurs and downtime refers to the period during which finished product does not leave the line (Askin and Standridge, 1993).

4.2.1 Machine Behaviour in Isolation

We consider an unreliable machine M_i. First, we analyze the behaviour of machine M_i when it operates in a flow line without being idle (neither starved nor blocked). We consider this situation as M_i is performing in isolation. Then we observe the change of behaviour as M_i operates in a flow line subject to idleness. This situation is referred to as M_i is performing in a flow line. When a machine operates in isolation, it can be in either of two states: up (also known as operational or working) or down (failed or under repair). The status of the machine can be described by a discrete time, discrete state Markov chain where the states of the Markov chain are (up, down). This can be more formally represented as (1, 0) where 1 represents up and 0 represents down.

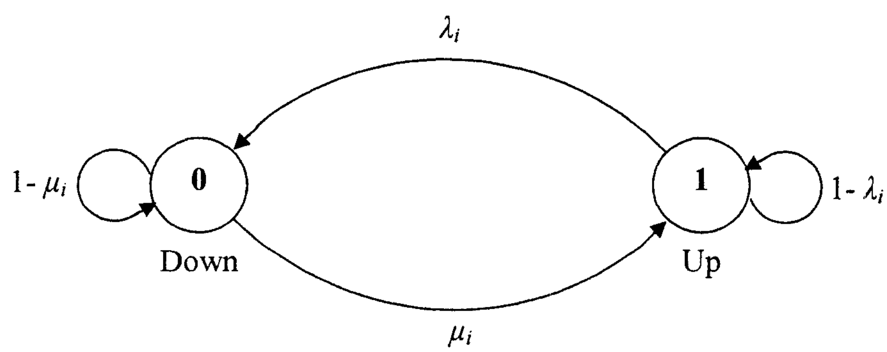

Figure 4.1: Graphical representation of Markov chain for an isolated unreliable machine.

For operation dependent failures, a machine can fail only when it is operating. When a machine M_i is operating, we assume that the transition rate from the operating state to the down state is λ_i. Again, when a machine M_i is down, the transition rate from the down state to the operating state is μ_i. The graph of this Markov chain is shown in Figure 4.1.

For this Markov chain, we can write the probability distributions as follows:

$$\mathrm{p_i}(0,t+1) = \mathrm{p_i}(0,t)(1-\mu_i) + \mathrm{p_i}(1,t)\lambda_i \tag{4.3}$$

$$\mathrm{p_i}(1,t+1) = \mathrm{p_i}(1,t)(1-\lambda_i) + \mathrm{p_i}(0,t)\mu_i \tag{4.4}$$

where $\mathrm{p_i}$ refers to the probability distribution of the state of M_i and t refers to time.

We can write equations (4.5) and (4.6) by solving equations (4.3) and (4.4). For detailed solution procedure the reader is referred to (Gershwin, 1994).

$$\mathrm{p_i}(0,t) = \mathrm{p_i}(0,0)(1-\lambda_i-\mu_i)^t + \frac{\lambda_i}{\mu_i+\lambda_i}\left[1-(1-\lambda_i-\mu_i)^t\right] \tag{4.5}$$

$$\mathrm{p_i}(1,t) = \mathrm{p_i}(1,0)(1-\lambda_i-\mu_i)^t + \frac{\mu_i}{\mu_i+\lambda_i}\left[1-(1-\lambda_i-\mu_i)^t\right] \tag{4.6}$$

At steady-state (as $t \to \infty$), equations (4.5) and (4.6) can be reduced to:

$$\mathrm{p_i}(0) = \frac{\lambda_i}{\mu_i+\lambda_i} \tag{4.7}$$

and

$$\mathrm{p_i}(1) = \frac{\mu_i}{\mu_i+\lambda_i} \tag{4.8}$$

Equations (4.7) and (4.8) give the probability that machine M_i is down and up respectively.

Since M_i has only two states, from equations (4.7) and (4.8), we can write:

$$p_i(0) + p_i(1) = \frac{\lambda_i}{\mu_i + \lambda_i} + \frac{\mu_i}{\mu_i + \lambda_i} = 1 \tag{4.9}$$

We get the balance equation for this Markov chain by solving equations (4.7) and (4.8).

$$p_i(0)\mu_i = p_i(1)\lambda_i \tag{4.10}$$

To determine the relationship between the basic parameters of a machine operating in isolation, we use the following notation:

M_i = Work centres/ machines in series ($i = 1, 2, 3, \ldots\ldots\ldots, m$)

λ_i = Mean failure rate of M_i

$MTTF_i$ = Mean time to failure of M_i

μ_i = Mean repair rate of M_i

$MTTR_i$ = Mean time to repair of M_i

u_i = Mean processing rate on M_i

P_i = Mean processing time on M_i

e_i = Efficiency of M_i when it operates in isolation

q_i = Production rate of M_i when it operates in isolation

We write the following basic equations for this set of parameters:

$$\lambda_i = \frac{1}{MTTF_i} \tag{4.11}$$

$$\mu_i = \frac{1}{MTTR_i} \tag{4.12}$$

$$u_i = \frac{1}{P_i} \tag{4.13}$$

As M_i has only two states, we can rewrite equation (4.8) as follows:

$$\begin{aligned} \mathrm{p_i}(1) = \mathrm{p_i}(up) &= \frac{\mu_i}{\mu_i + \lambda_i} \\ &= e_i \\ &= \text{Probability that } M_i \text{ is operating} \\ &= \text{Efficiency of machine } M_i \end{aligned} \tag{4.14}$$

Again, from the definition of production rate we can write:

$$\begin{aligned} \text{Production rate in isolation, } q_i &= \text{Mean processing rate} \times \text{efficiency in isolation} \\ &= u_i \times e_i \\ &= \frac{u_i \mu_i}{\mu_i + \lambda_i} \end{aligned} \tag{4.15}$$

4.2.2 Machine Behaviour in Flow Lines

As stated before, an isolated machine M_i can be either in two states, up or down. But when this machine M_i performs in a flow line with finite buffer capacity, it needs a third state to define it. Because an up machine in a flow line can be in one of two states: working (also called operating) or idle (forced down). An idle machine is in working order but unable to operate because it has no workpiece to process or cannot transfer its completed workpiece to the downstream machine or buffer.

The status of machine M_i can also be described by a discrete time, discrete state Markov chain where the states of the Markov chain are (down, operating, idle). The graph of this Markov chain is shown in Figure 4.2.

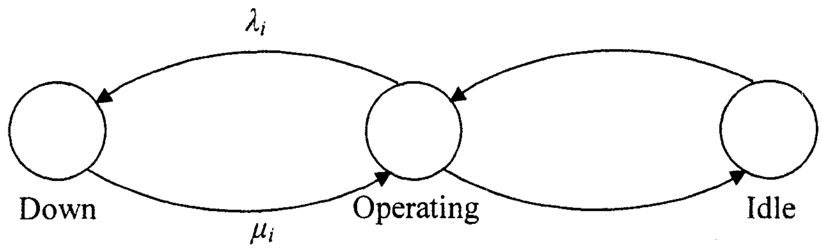

Figure 4.2: Graphical representation of Markov chain for an unreliable machine in flow line.

The steady-state equation for machine M_i at 'Down' state for this Markov chain is:

$$\mathrm{p}(operating)_i \times \lambda_i = \mathrm{p}(down)_i \times \mu_i \tag{4.16}$$

We denote,

Pu_i = Probability that machine M_i is up (operating or idle)

Po_i = Probability that machine M_i is operating

Pd_i = Probability that machine M_i is down

Pi_i = Probability that machine M_i is idle

Ps_i = Probability that machine M_i is starved

Pb_i = Probability that machine M_i is blocked

Q_i = Production rate of M_i when it is in flow line

We rewrite equation (4.16) as:

$$Po_i \times \lambda_i = Pd_i \times \mu_i \tag{4.17}$$

As machine M_i can be in any of the three states (down, operating, idle), we write:

$$Po_i + Pd_i + Pi_i = 1 \tag{4.18}$$

An idle machine can be in one of two states (starved or blocked). We can write:

$$Pi_i = Ps_i + Pb_i \tag{4.19}$$

Combining equations (4.18) and (4.19), we get:

$$Po_i + Pd_i + (Ps_i + Pb_i) = 1 \tag{4.20}$$

If machine M_i is not starved or blocked (not idle), equation (4.20) reduces to:

$$Po_i + Pd_i = 1 \tag{4.21}$$

which is the equation of an isolated machine as given in equation (4.9).

Again, if machine M_i is reliable, that is $Pd_i = 0$, then equation (4.20) becomes:

$$Po_i + Ps_i + Pb_i = 1 \tag{4.22}$$

The efficiency of machine M_i when it is in a flow line with finite buffer is given by:

$$= \frac{Pu_i - Pi_i}{Pu_i + Pd_i}$$

$$= \frac{Po_i}{Pu_i + Pd_i}$$

$$= Po_i$$

From equation (4.18), we can write:

$$Po_i = 1 - Pd_i - Pi_i \tag{4.23}$$

From equation (4.17), we get:

$$Po_i \times \lambda_i = Pd_i \times \mu_i$$

$$\Rightarrow Pd_i = \frac{\lambda_i}{\mu_i} Po_i \tag{4.24}$$

Combining equations (4.23) and (4.24) we get:

$$Po_i = 1 - \frac{\lambda_i}{\mu_i} Po_i - Pi_i$$

$$\Rightarrow Po_i(1 + \frac{\lambda_i}{\mu_i}) = 1 - Pi_i$$

$$\Rightarrow Po_i = \frac{1 - Pi_i}{\frac{\mu_i + \lambda_i}{\mu_i}}$$

$$= \frac{\mu_i}{\mu_i + \lambda_i}(1 - Pi_i)$$

$$= e_i(1 - Pi_i) \tag{4.25}$$

Equation (4.25) shows the relationship between the expressions of efficiency of machine M_i when it operates in isolation and when it operates in a flow line.

From the definition of production rate, we can write:

Production rate in flow line, Q_i = Mean processing rate × efficiency (Inline)

$$= u_i \times Po_i \tag{4.26}$$

4.2.3 Solution Approach

The study of a machine or even small lines with Markov chain models is easy and does not involve much complexity. But when the lines get longer, it becomes difficult to study such systems with Markov chain models because of the prohibitively large number of state spaces and their indecomposability. When the system is modeled as a discrete-space Markov chain, the total number of distinct states is the product of the number of different machine states and the number of distinct buffer levels. For instance, a line with m stages and (m-1) intermediate buffers of size C each requires 2^m $(C+1)^{m-1}$ states to analyze. A 20 machine serial line with 19 buffers each of size 10, for example, has over 6.41×10^{25} states (Gershwin, 1994).

Multiple machines in stages of flow lines are a common scenario in today's industry. The addition of parallel machines in stages makes the configuration of flow lines more complex and as such the study of this type of lines with Markov chain models become more difficult. For example, a simple 4-stage line with 3 machines at each stage and buffer with capacity of 100 would have $2^{4\text{x}3}$ x 100^3 or approximately 4 billion states (Burman, 1995). In real life, the number of stages in a line is usually much higher than 4 and the buffer size may go remarkably higher especially in the production of high volume consumer products.

Because of these difficulties, most practical work on estimating flow line behaviour is done by rules of thumb or by simulation (Gershwin, 1994). However, approximation methods developed so far have also proved to be efficient ways to analyze the performance parameters of traditional long serial lines. Decomposition techniques among them require a machine in a line to be described in term of three parameters: failure rate, repair rate and processing rate.

To reduce the complexity of the series-parallel system of our interest (Figure 4.4), we propose an approach to replace a work centre M_i (composed of a set of parallel machines) by an equivalent machine M_i'. The series-parallel system (Figure 4.4) then changes to a serial system (Figure 4.5) having single machine at each stage which is exactly the same as the traditional serial flow line illustrated in Figure 4.3.

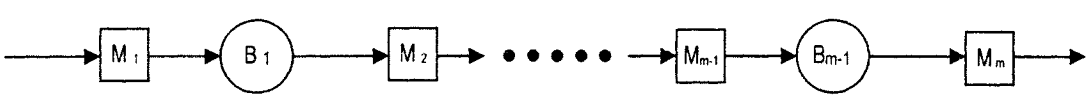

Figure 4.3: Traditional serial flow line made of *m* work centres

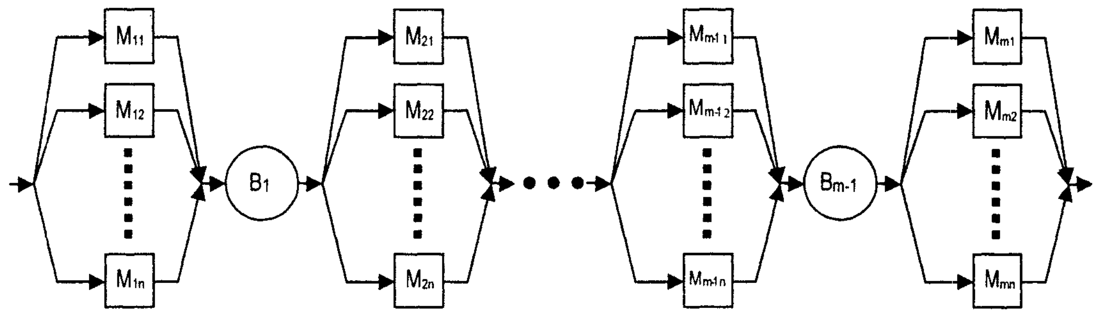

Figure 4.4: Series-parallel flow line made of m work centres

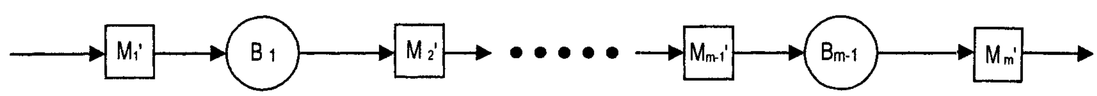

Figure 4.5: Serial flow line (replacing each set of parallel machines by an equivalent machine)

To solve the problem of series-parallel systems, our approach requires an equivalent machine M_i ' operating in isolation to be described by three parameters: λ_i', μ_i' and u_i' (failure rate, repair rate and processing rate of M_i ' respectively) and the estimation of these parameters (Figure 4.6). We develop the required equations to estimate them. The development of these equations requires knowing or being able to calculate independent parameters (u_{ij}, $\lambda_{ij,}$ and μ_{ij}) of each parallel machine M_{ij} at a work centre i. When the equivalent machine operates in a flow line, its behaviour should match with that of the set of parallel machines. So, at least one more parameter that describes the effect of idleness in a flow line must be known to define the equivalent machine M_i '. We assume that parameters u_{ij}, $\lambda_{ij,}$ μ_{ij} and Po_{ij} are known. In fact, in this case metrology should be performed for the performance diagnosis of series-parallel system. Metrology is the measurement of times necessary to computing needed data. We use the following set of notation to estimate the failure and repair parameters and probabilities of each machine in a set being at different states.

Tt_{ij} = Total time required for machine M_{ij}

Tu_{ij} = Uptime of machine M_{ij} in Tt_{ij}

To_{ij} = Operating time of machine M_{ij} in Tt_{ij}

Td_{ij} = Downtime of machine M_{ij} in Tt_{ij}

Ts_{ij} = Starvation time of machine M_{ij} in Tt_{ij}

Tb_{ij} = Blocking time of machine M_{ij} in Tt_{ij}

f_{ij} = Number of failures of machine M_{ij} in Tt_{ij}

Based on the results from metrology, failure and repair parameters and probabilities of each machine in a set being at different states can be estimated in the following manner:

Pu_{ij} = Probability that machine M_{ij} is up = $\dfrac{Tu_{ij}}{Tt_{ij}}$

Po_{ij} = Probability that machine M_{ij} is operating = $\dfrac{To_{ij}}{Tt_{ij}}$

Pd_{ij} = Probability that machine M_{ij} is down = $\dfrac{Td_{ij}}{Tt_{ij}}$

Ps_{ij} = Probability that machine M_{ij} is starved = $\dfrac{Ts_{ij}}{Tt_{ij}}$

Pb_{ij} = Probability that machine M_{ij} is blocked = $\dfrac{Tb_{ij}}{Tt_{ij}}$

λ_{ij} = Failure rate of machine M_{ij} = $\dfrac{f_{ij}}{To_{ij}}$

μ_{ij} = Repair rate of machine M_{ij} = $\dfrac{f_{ij}}{Td_{ij}}$

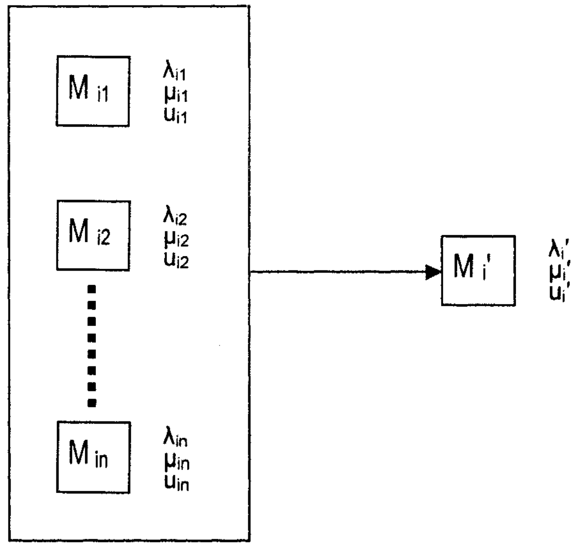

Figure 4.6: A set of parallel machines replaced by an equivalent machine

First, we develop the equations to determine the parameters of the equivalent machine M_i' in isolation. Then, we place this machine in a flow line with finite buffer and adjust the parameters to account for the effect of idleness in a flow line.

4.2.3.1 Set of Parallel Machines in Isolation

Processing Rate of Equivalent Machine:

The maximum isolated processing rate u_i' corresponding to the equivalent machine M_i' may be reasonably defined as the sum of the individual isolated processing rates u_{ij} of the set of parallel machines M_{ij} at work centre i when all the machines are up and not starved or blocked:

$$u_i' = \sum_{j=1}^{n} u_{ij} \tag{4.27}$$

Production Rate of Equivalent Machine:

Similarly, we define the expected production rate q_i' of the equivalent machine M_i' as the sum of the individual production rates q_{ij} of the set of parallel machines M_{ij} at work centre i:

$$\begin{aligned} q_i' &= \sum_{j=1}^{n} q_{ij} \\ &= \sum_{j=1}^{n} (u_{ij} e_{ij}) \\ &= \sum_{j=1}^{n} \left(u_{ij} \frac{\mu_{ij}}{\mu_{ij} + \lambda_{ij}} \right) \end{aligned} \tag{4.28}$$

Efficiency of Equivalent Machine:

Efficiency (e_i') of the equivalent machine M_i' can be derived as follows:

$$\begin{aligned} q_i' &= u_i' \times e_i' \\ \Rightarrow e_i' &= \frac{q_i'}{u_i'} \\ &= \frac{\sum_{j=1}^{n} \left(u_{ij} \frac{\mu_{ij}}{\mu_{ij} + \lambda_{ij}} \right)}{\sum_{j=1}^{n} u_{ij}} \end{aligned} \tag{4.29}$$

Failure Rate of Equivalent Machine:

For operation dependent failures, a machine can fail only when it is operating. As an isolated machine has two states: up and down, it can fail only when it is up. We assume that each of the machines in a set of parallel machines at a work centre operates independently which means that failures of the machines are independent and has no effect on the other machines in the same set.

We approximate the probability of transitions from the *up state* (probability of failures) of an equivalent machine M_i' to be equal to the sum of probability of all the transitions from the *up state* (probability of failures) of a set of parallel machines M_{ij} at a work centre *i*.

So, we write:

$$\lambda_i' Pu_i' = \sum_{j=1}^{n} \lambda_{ij} Pu_{ij} \tag{4.30}$$

From equation (4.30), it can be stated that the equivalent machine M_i' is considered to be down if at least one of the *n* parallel machines at stage *i* is down while the other (*n* -1) machines at the same stage remain operational. Subsequent failure of a parallel machine at stage *i* is not considered as the equivalent machine M_i' is already down.

As the parallel machines operate in isolation, we can write:

$Pu_{ij} = e_{ij}$ and $Pu_i' = e_i'$ [See equation (4.14)]

So, equation (4.30) can be rewritten in the following way:

$$\lambda_i' e_i' = \sum_{j=1}^{n} \lambda_{ij} e_{ij} \tag{4.31}$$

So, the expression for the failure rate of the equivalent machine is:

$$\lambda_i' = \frac{1}{e_i'} \sum_{j=1}^{n} \lambda_{ij} e_{ij} \tag{4.32}$$

Repair Rate of Equivalent Machine:

We approximate the probability of transitions from the *down state* (probability of repairs) of an equivalent machine M_i' to be equal to the sum of probability of all the transitions from the *down state* (probability of repairs) of a set of parallel machines M_{ij} at a work centre i.

So, we write:

$$\mu_i' Pd_i' = \sum_{j=1}^{n} \mu_{ij} Pd_{ij}$$

$$\Rightarrow \mu_i' \left(1 - Pu_i'\right) = \sum_{j=1}^{n} \mu_{ij} \left(1 - Pu_{ij}\right)$$

$$\Rightarrow \mu_i' = \frac{\sum_{j=1}^{n} \mu_{ij} \left(1 - Pu_{ij}\right)}{\left(1 - Pu_i'\right)}$$

$$\Rightarrow \mu_i' = \frac{\sum_{j=1}^{n} \mu_{ij} \left(1 - \frac{\mu_{ij}}{\mu_{ij} + \lambda_{ij}}\right)}{\left(1 - e_i'\right)}$$

$$\Rightarrow \mu_i' = \frac{\sum_{j=1}^{n} \mu_{ij} \left(\frac{\lambda_{ij}}{\mu_{ij} + \lambda_{ij}}\right)}{\left(1 - e_i'\right)}$$

$$\Rightarrow \mu_i' = \frac{\sum_{j=1}^{n} \lambda_{ij} Pu_{ij}}{\left(1 - e_i'\right)}$$

$$\Rightarrow \mu_i' = \frac{\lambda_i' Pu_i'}{\left(1-e_i'\right)}$$

$$\Rightarrow \mu_i' = \lambda_i' \frac{e_i'}{\left(1-e_i'\right)} \tag{4.33}$$

Repair rate of equivalent machine = Failure rate of equivalent machine $\times \dfrac{e_i'}{\left(1-e_i'\right)}$

As the parallel machines operate in isolation, we replaced Pu_i' by e_i' in deriving equation (4.33) [See equation (4.14)].

This expression of repair rate of the equivalent machine could also be derived in two other different ways using equation (4.14) and (4.17).

4.2.3.2 Set of Parallel Machines in Flow Line

We consider the case when the set of parallel machines operates in a flow line with finite buffer. Machines in flow line behave in a different way than when they are isolated. An isolated machine can never be idle (starved or blocked). Unlike isolated machine, a machine in a flow line can be starved or blocked and for operation dependent failure, a machine cannot fail when it is idle though it is up. Due to this idleness, parameters describing a machine are affected. We derive the equations of these parameters considering the effect of idleness.

Production Rate of Equivalent Machine:

We define the expected production rate Q_i' of the equivalent machine M_i' as the sum of the individual production rates q_{ij} of the set of parallel machines M_{ij} at work centre i :

$$Q_i' = \sum_{j=1}^{n} q_{ij}$$
$$= \sum_{j=1}^{n} (u_{ij} Po_{ij}) \tag{4.34}$$

Efficiency of Equivalent Machine:

We can also define equivalent production rate Q_i' of the machine M_i' as follows:

$$Q_i' = u_i' \times Po_i'$$
$$\Rightarrow Po_i' = \frac{Q_i'}{u_i'}$$
$$= \frac{\sum_{j=1}^{n} \left(u_{ij} Po_{ij}\right)}{\sum_{j=1}^{n} u_{ij}} \tag{4.35}$$

Idleness of Equivalent Machine:

From equation (4.25), we write:

$$Po_{ij} = e_{ij}(1 - Pi_{ij})$$
$$\Rightarrow (1 - Pi_{ij}) = \frac{Po_{ij}}{e_{ij}}$$
$$\Rightarrow Pi_{ij} = 1 - \frac{Po_{ij}}{e_{ij}} = 1 - \frac{\text{Efficiency of } M_{ij} \text{ (In Line)}}{\text{Efficiency of } M_{ij} \text{ (Isolated)}} \tag{4.36}$$

From equation (4.36), we write for the equivalent machine:

$$Pi_i' = 1 - \frac{Po_i'}{e_i'} = 1 - \frac{\text{Efficiency of } M_i' \text{(In Line)}}{\text{Efficiency of } M_i' \text{(Isolated)}} \tag{4.37}$$

Failure Rate of Equivalent Machine:

For operation dependent failures, a machine can fail only when it is operating. As a machine in flow line has three states: operating, idle and down, it can fail only when it is at its operating state. We assume that each of the machines in a set of parallel machines at a work centre operates independently which means that failures of the machines are independent and has no effect on the other machines in the same set.

We approximate the probability of transitions from the *operating state* (probability of failures) of an equivalent machine M_i' to be equal to the sum of probability of all the transitions from the *operating state* (probability of failures) of a set of parallel machines M_{ij} at a work centre i.

So, we write:

$$\lambda_i' Po_i' = \sum_{j=1}^{n} \lambda_{ij} Po_{ij}$$

$$\Rightarrow \lambda_i' = \frac{1}{Po_i'} \sum_{j=1}^{n} \lambda_{ij} Po_{ij} \tag{4.38}$$

Repair Rate of Equivalent Machine:

From equation (4.17) we write:

$$Po_i' \times \lambda_i' = Pd_i' \times \mu_i'$$

$$\Rightarrow \mu_i' = \frac{Po_i' \times \lambda_i'}{Pd_i'}$$

$$\Rightarrow \mu_i' = \frac{Po_i' \times \lambda_i'}{1 - Pu_i'}$$

$$\Rightarrow \mu_i' = \frac{Po_i' \times \lambda_i'}{1-(Po_i' + Pi_i')}$$

$$\Rightarrow \mu_i' = \frac{Po_i' \times \lambda_i'}{1- Po_i' -1+\frac{Po_i'}{e_i'}}$$

$$\Rightarrow \mu_i' = \frac{Po_i' \times \lambda_i'}{Po_i'(\frac{1}{e_i'}-1)}$$

$$\Rightarrow \mu_i' = \frac{\lambda_i' e_i'}{(1-e_i')} \qquad (4.39)$$

Repair rate of equivalent machine = Failure rate of equivalent machine x $\frac{e_i'}{\left(1-e_i'\right)}$

Note that, equations (4.33) and (4.39) show that the relationship between the failure rate and the repair rate of an equivalent machine is exactly the same whether it operates in a flow line or in isolation.

4.3 Conclusion

In this chapter, we derived equations to estimate the parameters of an equivalent machine that approximate a set of parallel machines at a work centre of a series-parallel flow line. We demonstrate an improved and reliable method to replace a set of parallel machines by an equivalent machine that operates in isolation and also in flow line. Simulation experiments will be conducted to check the accuracy of the proposed method in Chapter 5.

The literature concerning series-parallel manufacturing systems is limited. Ancelin and Semery (1987), Burman (1995), and Patchong and Willaeys (2001) are among the few papers that used the technique of replacing a set of parallel machines at a stage by an equivalent machine to tackle series-parallel flow lines. These are the main existing papers related to our work. In section 4.3.1, we present the equations derived in these papers to estimate the failure rate, repair rate and processing rate of the equivalent machine using the same notation used in this thesis. We do this in order to compare as well as to show the differences between these existing methods and the proposed method.

We conclude with the important differences between this work and the work of Patchong and Willaeys (2001) in Section 4.3.1. A summary of the equations derived in this chapter is presented in Section 4.3.2.

4.3.1 Analyses of the Main Existing Methods

In this section, we mainly describe the interpretations and expressions of processing rate, failure rate and repair rate required to define an equivalent machine found in existing literature. We also perform the critical analyses of some of the existing methods in this section, but the important criticism that can be made about these methods is illustrated in Section 5.4.

All the methods compared below define the equivalent processing rate of the set of the parallel machines at a stage as follows:

$$u_i' = \sum_{j=1}^{n} u_{ij}$$

4.3.1.1 Ancelin and Semery (1987)

Ancelin and Semery (1987) were probably the first to approximate a set of parallel machines of a stage by an equivalent machine in a series-parallel system. They defined the failure of the equivalent machine if at least one of the parallel machines in a stage was down while the other machines in the set remained up. A subsequent failure of the parallel machines was not counted as a failure of the equivalent machine; rather the resulting slowing down of the processing rate was taken into account. Their derived equations to estimate the failure and repair rates are:

Equivalent Failure Rate:

$$\lambda_i' = \sum_{j=1}^{n}\left(\lambda_{ij}\prod_{\substack{k=1\\k\neq j}}^{n}\frac{\mu_{ik}}{\mu_{ik}+\lambda_{ik}}\right)$$

$$= \sum_{j=1}^{n}\left(\lambda_{ij}\prod_{\substack{k=1\\k\neq j}}^{n}e_{ik}\right)$$

Equivalent Repair Rate:

They derived the equivalent repair rate μ_i' from the following relations:

$$q_i' = u_i' e_i'$$

$$= u_i' \frac{\mu_i'}{\mu_i' + \lambda_i'}$$

where

$$q_i' = \sum_{j=1}^{n} q_{ij}$$

also q_{ij}, u_i' and λ_i' are given.

4.3.1.2 Burman (1995)

Burman (1995) applied a different approach to determine the failure and repair rates of the equivalent machine. He estimated the failure and repair rates of the equivalent machine solving the following two equations. These equations are based on matching the variance of the production output of the equivalent machine to the variance of the set of parallel machines. The final form of the first equation is:

$$u_i'^2\left(\frac{2\mu_i'\lambda_i'}{\left(\mu_i'+\lambda_i'\right)^3}\right)=\sum_{j=1}^{n}u_{ij}^2\left(\frac{2\mu_{ij}\lambda_{ij}}{\left(\mu_{ij}+\lambda_{ij}\right)^3}\right)$$

The second equation is:

$$u_i\left(\frac{\mu_i'}{\mu_i'+\lambda_i'}\right)=\sum_{j=1}^{n}u_i\left(\frac{\mu_{ij}}{\mu_{ij}+\lambda_{ij}}\right)$$

4.3.1.3 Patchong and Willaeys (2001)

The work presented in this chapter deals with a system which is similar to that described in Patchong and Willaeys (2001). The main differences between these two works are:

- Selection of independent parameters based on which the models are developed
- Interpretation of the equivalent failure rate
- Interpretation of the equivalent repair rate
- Formulation of the probability of idleness

Equivalent Failure Rate:

Patchong and Willaeys (2001) followed the similar strategy of Ancelin and Semery (1987) to define the failure of the equivalent machine. But unlike Ancelin and Semery (1987), they considered the probability of working of all the parallel machines in the set and their equivalent machine in the expression of failure rate. Their derived equation to estimate the equivalent failure rate when the parallel machines operate in a flow line is:

$$\lambda_i' = \frac{1}{Po_i'} \sum_{j=1}^{n} \left(\lambda_{ij} Po_{ij} \prod_{k \neq j} (1 - Pd_{ik}) \right) \tag{a}$$

We further solve equation (a) to show a clear relationship between λ_i' and λ_i, μ_i and *n*. When the parallel machines operate in isolation, equation (a) changes to the following form:

$$\lambda_i' = \frac{1}{Po_i'} \sum_{j=1}^{n} \left(\lambda_{ij} \prod_{k=1}^{n} Po_{ik} \right) \tag{b}$$

As the parallel machines operate in isolation, we can write:

$Po_{ik} = e_{ik}$ and $Po_i' = e_i'$ [See equation (4.14)]

So, equation (b) can be rewritten in the following way:

$$\lambda_i' = \frac{1}{e_i'} \sum_{j=1}^{n} \left(\lambda_{ij} \prod_{k=1}^{n} e_{ik} \right) \tag{c}$$

If the parallel machines at a stage are identical to each other in all respect, we can write:

$\lambda_{i1} = \lambda_{i2} = \lambda_{i3} = \dots\dots = \lambda_{in} = \lambda$ (Assume)

$\mu_{i1} = \mu_{i2} = \mu_{i3} = \dots\dots = \mu_{in} = \mu$ (Assume)

$u_{i1} = u_{i2} = u_{i3} = \dots\dots = u_{in} = u$ (Assume)

$e_{i1} = e_{i2} = e_{i3} = \dots\dots = e_{in} = e$ (Assume) where the efficiency of each identical parallel machine:

$$e = \frac{\mu}{\mu + \lambda}$$

From equation (4.29), we can write the efficiency of the equivalent machine:

$$e_i' = \frac{\sum_{j=1}^{n} u_{ij} e_{ij}}{\sum_{j=1}^{n} u_{ij}}$$

$$= \frac{nue}{nu} = e$$

Putting all these values in equation (c), we get:

$$\lambda_i' = \frac{1}{e} \sum_{j=1}^{n} \left(\lambda_{ij} e^n \right)$$

$$= \frac{1}{e} \left(\lambda_{i1} e^n + \lambda_{i2} e^n + \dots\dots\dots\dots + \lambda_{in} e^n \right)$$

$$= \frac{1}{e} \left(n \lambda e^n \right)$$

$$= n \lambda e^{n-1}$$

$$= n\lambda \left(\frac{\mu}{\mu + \lambda} \right)^{n-1}$$

$$= AB^{n-1} \qquad \text{(d)}$$

where,

$$A = n\lambda$$

$$B = \left(\frac{\mu}{\mu + \lambda} \right) = \text{Efficiency of each parallel machine}$$

and $0 < B < 1$

Equation (d) determines the failure rate of the equivalent machine when at least one of the n identical parallel machines at a stage fails and the remaining (n-*1*) machines of the stage remain operational. This equation does not give enough insight in terms of accountability for slowing down of the processing rate.

From equation (d), it is observed that the equivalent failure rate (λ_i') is a function of the repair rate (μ) and the failure rate (λ) of the parallel machines as well as their number (n) at a stage. The equivalent failure rate changes geometrically with A and exponentially with B. At lower λ and smaller n, the effect of A on λ_i' is greater than B, and as a result λ_i' increases. But as λ and n increase, eventually λ_i' decreases and becomes even lower than the failure rate of a single machine in a set of parallel machines (See Figure 5.3). This implies that the equivalent machine does not fail even when the parallel machines at a stage fail, which is difficult to admit and is not supported by the existing literature. We illustrate some examples in Section 5.4 to make it clear.

Equivalent Repair Rate:

They derived an equation to estimate the equivalent repair rate when the parallel machines operate in a flow line. The basis of this equation is that the loss due to a failing machine leads to the shortfall of an equivalent machine, in proportion with its speed.

$$\mu_i' = \frac{\sum_{j=1}^{n}\left(\lambda_{ij} Po_{ij} \prod_{k \neq j}(1 - Pd_{ik})\right)}{\sum_{j=1}^{n}\left(Pd_{ij} \times \frac{u_{ij}}{u_i'}\right)} \tag{e}$$

When the parallel machines operate in isolation, equation (e) changes to a form:

$$\mu_i' = \frac{\sum_{j=1}^{n}\left(\lambda_{ij} \prod_{k=1}^{n} Po_{ik}\right)}{\sum_{j=1}^{n}\left(Pd_{ij} \times \frac{u_{ij}}{u_i'}\right)} \tag{f}$$

From equation (b) and equation (21a) of Patchong and Willaeys (2001), equation (f) can be rewritten as follows:

$$\mu_i' = \frac{\lambda_i' Po_i'}{Pd_i'} \tag{g}$$

As the parallel machines operate in isolation, we can write:

$Po_i' + Pd_i' = 1$ and $Po_i' = e_i'$ [See equation (4.14)]

So, equation (g) can be rewritten in the following way:

$$\begin{aligned}
\mu_i' &= \frac{\lambda_i' Po_i'}{1 - Po_i'} \\
&= \frac{\lambda_i' e_i'}{1 - e_i'} \\
&= n\lambda e^{n-1} \frac{e}{1-e} \\
&= n\lambda e^{n-1} \frac{\frac{\mu}{\mu+\lambda}}{1 - \frac{\mu}{\mu+\lambda}} \\
&= n\lambda e^{n-1} \frac{\frac{\mu}{\mu+\lambda}}{\frac{\mu+\lambda-\mu}{\mu+\lambda}} \\
&= n\mu e^{n-1} \\
&= n\mu \left(\frac{\mu}{\mu+\lambda} \right)^{n-1}
\end{aligned} \tag{h}$$

From equation (h), it is observed that the equivalent repair rate (μ_i') is a function of the repair rate (μ) and the failure rate (λ) of the parallel machines as well as their number (n) at a stage. As λ increases μ_i' obtained from this equation decreases even when μ and n do not change.

4.3.2 Summary of Equations Derived in This Thesis

$$u_i' = \sum_{j=1}^{n} u_{ij}$$

4.3.2.1 Set of Parallel Machines in Isolation

$$e_{ij} = \frac{\mu_{ij}}{\mu_{ij} + \lambda_{ij}}$$

$$q_i' = \sum_{j=1}^{n} (u_{ij} e_{ij})$$

$$e_i' = \frac{q_i'}{u_i'}$$

$$\lambda_i' = \frac{1}{e_i'} \sum_{j=1}^{n} \lambda_{ij} e_{ij}$$

$$\mu_i' = \lambda_i' \frac{e_i'}{\left(1 - e_i'\right)}$$

4.3.2.2 Set of Parallel Machines in Flow Line

$$Po_{ij} = e_{ij}(1 - Pi_{ij})$$

$$Q_i' = \sum_{j=1}^{n}(u_{ij} Po_{ij})$$

$$Po_i' = \frac{Q_i'}{u_i'}$$

$$\lambda_i' = \frac{1}{Po_i'}\sum_{j=1}^{n}\lambda_{ij} Po_{ij}$$

$$\mu_i' = \frac{\lambda_i' e_i'}{(1 - e_i')}$$

$$Pi_{ij} = 1 - \frac{Po_{ij}}{e_{ij}} = 1 - \frac{\text{Efficiency of } M_{ij} \text{ (In Line)}}{\text{Efficiency of } M_{ij} \text{ (Isolated)}}$$

$$Pi_i' = 1 - \frac{Po_i'}{e_i'} = 1 - \frac{\text{Efficiency of } M_i' \text{(In Line)}}{\text{Efficiency of } M_i' \text{(Isolated)}}$$

CHAPTER 5
NUMERICAL ANALYSIS

5.1 Introduction

This chapter presents numerical examples of our proposed method in order to evaluate its accuracy. Unfortunately, there is no universal example that we can use to check the accuracy of our method. Moreover, the possible number of configurations of series-parallel flow line is inordinate. So, we have selected a few specific examples similar to those illustrated by Burman (1995) and Patchong and Willaeys (2001) to make a comparison in Section 5.2.1. Then, we present some additional examples of series-parallel flow lines with non-identical parallel machines in Section 5.2.2 and parallel machines at more than one stage in Section 5.2.3 and validate them by simulation. Proofs of some derived equations using data from simulation results are illustrated in Section 5.3. Section 5.4 presents a comparative study with the existing methods of series-parallel systems and the conclusion is given in Section 5.5.

5.2 Numerical Experiments

We examine three types of series-parallel flow line configurations: (a) Series-parallel flow lines with identical machines in parallel (Section 5.2.1), (b) Series-parallel flow lines with non-identical machines in parallel (Section 5.2.2), (c) Series-parallel flow lines with identical and non-identical parallel machines at multiple stages (Section 5.2.3). We experiment with the configurations of (a) and (b) for two buffer levels, 10 and 2 and for configuration (c), for four buffer levels, 2, 7, 15 and 20. We apply different buffer levels to examine their impact on the accuracy of our proposed method. All the machines in the line are considered having deterministic processing rates and exponentially distributed failure and repair rates. We simulate both the original series-parallel line and the equivalent serial line derived from the original series-parallel line replacing the set(s) of parallel machines by the corresponding equivalent machine(s), in determining the accuracy of our method. We adapt this approach due to two reasons: first, the absence of exact analytical method to evaluate relatively longer flow lines and secondly, the use of approximation method may cause additional and unexpected error in estimating performance parameters.

We use ProModel, a commercial simulation software, to simulate each experiment. Twenty different cases (Case 1 to Case 20) are examined where each case is simulated for three types of estimation. First, we simulate the original series-parallel flow line and refer to the results as 'Original' (Estimation of original series-parallel flow line). Then we simulate equivalent serial line derived from the original series-parallel line where the equivalent machine(s) operates in isolation. We use the set of equations shown in Section 4.3.2.1 to determine the parameters of the equivalent machine(s). We refer to the results of this simulation as 'EWIso' (Estimation With Isolated equivalent machines). The third simulation was performed with equivalent serial line derived from the original series-parallel line where the equivalent machine(s) operates in a flow line. In this case, the machines in the flow line can be blocked and starved. We obtain the parameter (Po_{ij}) from the simulation experiments of series-parallel lines (Case 1 to Case 20) that describes the effect of idleness when the equivalent machine(s) operates in a flow line .We use the set of equations shown in Section 4.3.2.2 to determine the other parameters of the equivalent machine(s). We refer to the results obtained in this manner as 'EWIn' (Estimation With Inline equivalent machines). For all the cases (Case 1 to Case 20) illustrated in this chapter, we obtain the same value for the parameters of the equivalent machine(s) when it operates in isolation (using the set of equations shown in Section

4.3.2.1) and also in flow line (using the set of equations shown in Section 4.3.2.2). So, the results of simulation for 'EWIso' and 'EWIn' are the same and we report both 'EWIso' and 'EWIn' in the same column (Table 5.1 to Table 5.10). Unit of time used in these experiments is, unless otherwise specified, Time unit. Percentage difference in estimation is calculated as follows all over this thesis:

$$\% \text{ Difference (EWIso)} = \frac{\text{EWIso - Original}}{\text{Original}} \times 100$$

$$\% \text{ Difference (EWIn)} = \frac{\text{EWIn - Original}}{\text{Original}} \times 100$$

Quantities of interest are the production rate of the system (Q), equivalent processing rate (u_i'), equivalent failure rate (λ_i') and equivalent repair rate (μ_i') of machines M_i' and average buffer levels ($\bar{b}_i$) of the intermediate buffers B_i. Parameters used in all the cases (Case 1 to Case 20) referred to in this chapter can be found in Appendix - A.

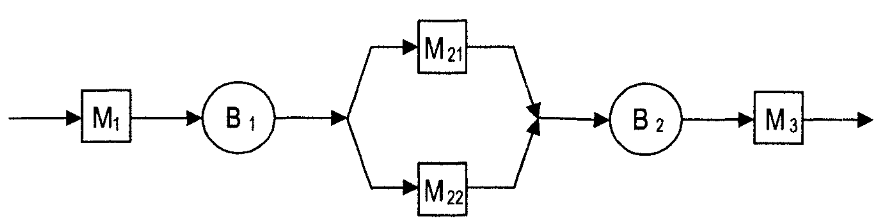

Figure 5.1: Series-parallel flow line made of three work centres

5.2.1 Identical Machines in Parallel *(Case 1 to Case 12)*

Case 1 to Case 12 are the examples similar to those presented by Patchong and Willaeys (2001) and Burman (1995). We follow the same line configurations, parameters and a similar experimental environment as illustrated in these papers to make a close comparison with them. For these experiments, a three stage series-parallel system (See Figure 5.1) is considered. The first and third stages consist of one machine each whereas the second stage may contain two or five identical machines. The first and third machines are identical in all respect. But the design of the second work centre is different for each category to illustrate different flow line attributes and parameters. These cases (Case 1 to Case 12) are simulated for a longer period of 400,000 hours with a transient period of 50,000 hours for one replication. No statistics are collected during the transient period to ensure steady-state behaviour of the system. Following three fundamental categories are examined.

5.2.1.1 Redundant Machines in Parallel

We examine the series-parallel systems where all the machines at stage two are identical. They are also identical in all respect to the first and third stage machines. We replace the redundant machines at stage two by an equivalent machine. The parameters used in these examples are presented in Appendix - A where Case 1 (Table A.1) and Case 2 (Table A.2) correspond to two-machine second stage flow lines and Case 7 (Table A.7) and Case 8 (Table A.8) correspond to five-machine second stage flow lines. Results obtained from simulation for 'Original', 'EWIso' and 'EWIn' are reported in Table 5.1 and Table 5.2.

2 Machines in parallel	Case 1			Case 2		
	Buffer Size = 10			Buffer Size = 2		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.872	0.863	-1.03	0.846	0.825	-2.48
$\overline{b}_1$	3.543	3.347	-5.53	0.776	0.730	-5.93
$\overline{b}_2$	6.327	6.542	3.40	1.124	1.241	10.41
λ_2'	n/a	0.02	n/a	n/a	0.02	n/a
μ_2'	n/a	0.2	n/a	n/a	0.2	n/a
u_2'	n/a	2	n/a	n/a	2	n/a

Table 5.1: Results of 3-stage flow lines with two redundant parallel machines at second stage and equal buffer sizes of 10 and 2

5 Machines in parallel	Case 7			Case 8		
	Buffer Size = 10			Buffer Size = 2		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.876	0.871	-0.57	0.856	0.843	-1.52
$\overline{b}_1$	3.270	3.448	5.44	0.683	0.721	5.56
$\overline{b}_2$	6.541	6.459	-1.25	1.207	1.216	0.75
λ_2'	n/a	0.05	n/a	n/a	0.05	n/a
μ_2'	n/a	0.5	n/a	n/a	0.5	n/a
u_2'	n/a	5	n/a	n/a	5	n/a

Table 5.2: Results of 3-stage flow lines with five redundant parallel machines at second stage and equal buffer sizes of 10 and 2

5.2.1.2 Slow Machines in Parallel

We examine the series-parallel systems where all the machines at stage two are identical. They are also identical in all respect to the first and third stage machines except that they operate at a lower speed than the machines at first and third stages. In these cases, the second stage machines are so slow that although they are two or five in numbers, the flow lines are still balanced, which implies that their isolated production rates are equal. For two-machine second stage flow lines, each of the parallel machines operates at half the speed of the first and third stage machines. In the cases of five-machine second stage flow lines, each of the parallel machines operates at one-fifth the speed of the first and third stage machines. The parameters used in these examples are presented in Appendix - A where Case 3 (Table A.3) and Case 4 (Table A.4) correspond to two-machine second stage flow lines and Case 9 (Table A.9) and Case 10 (Table A.10) correspond to five-machine second stage flow lines. Results obtained from simulation for 'Original', 'EWIso' and 'EWIn' are reported in Table 5.3 and Table 5.4.

2 Machines in parallel	**Case 3**			**Case 4**		
	Buffer Size = 10			Buffer Size = 2		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.832	0.832	0.00	0.791	0.790	-0.13
$\overline{b_1}$	6.500	6.616	1.78	1.338	1.416	5.83
$\overline{b_2}$	3.489	3.409	-2.29	0.660	0.582	-11.82
λ_2'	n/a	0.02	n/a	n/a	0.02	n/a
μ_2'	n/a	0.2	n/a	n/a	0.2	n/a
u_2'	n/a	1	n/a	n/a	1	n/a

Table 5.3: Results of 3-stage flow lines with two slow parallel machines at second stage and equal buffer sizes of 10 and 2

5 Machines in parallel	Case 9			Case 10		
	Buffer Size = 10			Buffer Size = 2		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.838	0.837	-0.12	0.795	0.793	-0.25
$\overline{b_1}$	6.606	6.989	5.80	1.279	1.556	21.66
$\overline{b_2}$	3.363	2.989	-11.12	0.709	0.447	-36.95
λ_2'	n/a	0.05	n/a	n/a	0.05	n/a
μ_2'	n/a	0.5	n/a	n/a	0.5	n/a
u_2'	n/a	1	n/a	n/a	1	n/a

Table 5.4: Results of 3-stage flow lines with five slow parallel machines at second stage and equal buffer sizes of 10 and 2

5.2.1.3 Machines in Parallel with Higher Failure Rates

We examine the series-parallel systems where all the machines at stage two are identical. They are also identical in all respect to the first and third stage machines except that each parallel machine is less available than the machines at first and third stages due to higher failure rate. In these cases, the second stage machines are half as available as the first and third stage machines. The parameters used in these examples are presented in Appendix - A where Case 5 (Table A.5) and Case 6 (Table A.6) correspond to two-machine second stage flow lines and Case 11 (Table A.11) and Case 12 (Table A.12) correspond to five-machine second stage flow lines. Results obtained from simulation for 'Original', 'EWIso' and 'EWIn' are reported in Table 5.5 and Table 5.6.

2 Machines in parallel	Case 5			Case 6		
	Buffer Size = 10			Buffer Size = 2		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.771	0.744	-3.50	0.706	0.642	-9.07
$\overline{b_1}$	5.499	5.261	-4.33	1.033	0.926	-10.36
$\overline{b_2}$	4.431	4.686	5.75	0.926	1.056	14.04
λ_2'	n/a	0.24	n/a	n/a	0.24	n/a
μ_2'	n/a	0.2	n/a	n/a	0.2	n/a
u_2'	n/a	2	n/a	n/a	2	n/a

Table 5.5: Results of 3-stage flow lines with two unreliable parallel machines at second stage and equal buffer sizes of 10 and 2

5 Machines in parallel	Case 11			Case 12		
	Buffer Size = 10			Buffer Size = 2		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.873	0.861	-1.37	0.851	0.798	-6.23
$\overline{b_1}$	3.052	2.988	-2.10	0.465	0.531	14.19
$\overline{b_2}$	6.444	6.814	5.74	1.169	1.406	20.27
λ_2'	n/a	0.6	n/a	n/a	0.6	n/a
μ_2'	n/a	0.5	n/a	n/a	0.5	n/a
u_2'	n/a	5	n/a	n/a	5	n/a

Table 5.6: Results of 3-stage flow lines with five unreliable parallel machines at second stage and equal buffer sizes of 10 and 2

In Section 5.2.2 and Section 5.2.3 below, we present some experiments (Case 13 to Case 20) of series-parallel systems that are different from the existing literature. We simulate these cases for 40,000 hours with a transient period of 3,500 hours. The transient period is identified using the Welch Moving Average Technique (Harrell *et al.*, 2003). Each experiment is simulated for four replications to ensure the independence of the observations. Performance measures are obtained at the 95% confidence level. We do not collect any statistics during the transient period to ensure steady-state behaviour of the system.

5.2.2 Non-Identical Machines in Parallel *(Case 13 to Case 16)*

We consider a three stage series-parallel system for these experiments (See Figure 5.1). The first and third stages consist of one machine each whereas the second stage may contain two or five non-identical parallel machines. The second stage machines are non-identical in such a manner that either they are two or five in numbers, the flow lines are unbalanced (Isolated production rate of the stages are not equal). For two-machine second stage flow lines, one of the parallel machines is less available and the other one is more available than the first and third stage machines. In the cases of five-machine second stage flow lines, parallel machines are either less available or more available or even equally available as the first and third stage machines. This availability is varied by varying the failure rate and repair rates of the machines. The parameters used in these examples are presented in Appendix - A where Case 13 (Table A.13) and Case 14 (Table A.14) correspond to two-machine second stage flow lines and Case 15 (Table A.15) and Case 16 (Table A.16) correspond to five-machine second stage flow lines. Results obtained from simulation for 'Original', 'EWIso' and 'EWIn' are reported in Table 5.7 and Table 5.8.

2 Machines in parallel	Case 13			Case 14		
	Buffer Size = 10			Buffer Size = 2		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.871	0.863	-0.92	0.848	0.822	-3.07
$\overline{b}_1$	3.529	3.292	-6.72	0.743	0.690	-7.13
$\overline{b}_2$	6.341	6.549	3.28	1.126	1.281	13.77
λ_2'	n/a	0.029	n/a	n/a	0.029	n/a
μ_2'	n/a	0.244	n/a	n/a	0.244	n/a
u_2'	n/a	2	n/a	n/a	2	n/a

Table 5.7: Results of 3-stage flow lines with two non-identical parallel machines at second stage and equal buffer sizes of 10 and 2

5 Machines in parallel	Case 15			Case 16		
	Buffer Size = 10			Buffer Size = 2		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.876	0.871	-0.57	0.856	0.845	-1.29
$\overline{b}_1$	3.284	3.448	4.99	0.634	0.679	7.10
$\overline{b}_2$	6.583	6.400	-2.78	1.200	1.211	0.92
λ_2'	n/a	0.089	n/a	n/a	0.089	n/a
μ_2'	n/a	0.865	n/a	n/a	0.865	n/a
u_2'	n/a	5	n/a	n/a	5	n/a

Table 5.8: Results of 3-stage flow lines with five non-identical parallel machines at second stage and equal buffer sizes of 10 and 2

5.2.3 Parallel Machines at Multiple Stages *(Case 17 to Case 20)*

We consider a five stage series-parallel system for these experiments (See Figure 5.2). The first, third and fifth stages consist of one machine each and they are identical. The second and fourth stages contain three machines each. Machines of stage two are non-identical to each other and those of stage four are identical to each other. However, in these examples, all the machines at stage two and four are non-identical to the other stage machines. The presence of non-identical machines makes the flow lines unbalanced (Isolated production rates are not equal). The parameters used in these examples are presented in Appendix - A where Case 17 (Table A.17), Case 18 (Table A.18), Case 19 (Table A.19) and Case 20 (Table A.20) correspond to these examples. The results obtained from the simulation model for 'Original', 'EWIso' and 'EWIn' are reported in Table 5.9 and Table 5.10.

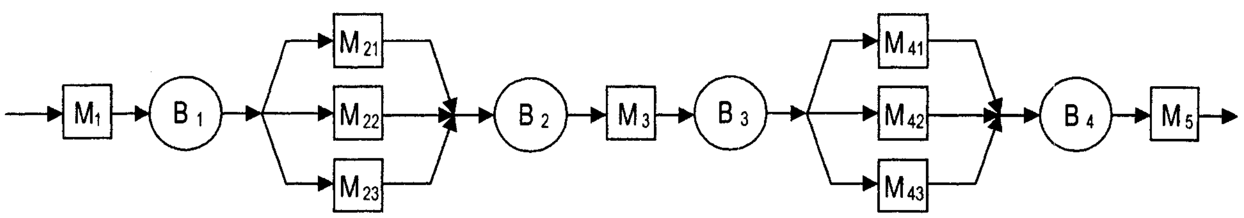

Figure 5.2: Series-parallel flow line made of parallel machines at multiple stages

	Case 17			Case 18		
	Buffer Size = 2			Buffer Size = 7		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.805	0.773	-3.98	0.839	0.825	-1.67
$\overline{b_1}$	0.944	0.964	2.12	3.226	3.109	-3.63
$\overline{b_2}$	1.379	1.489	7.98	4.983	5.124	2.83
$\overline{b_3}$	0.531	0.467	-12.05	1.909	1.768	-7.39
$\overline{b_4}$	0.866	0.846	-2.31	3.574	3.612	1.06
λ_2'	n/a	0.049	n/a	n/a	0.049	n/a
μ_2'	n/a	0.375	n/a	n/a	0.375	n/a
u_2'	n/a	2.1	n/a	n/a	2.1	n/a
λ_4'	n/a	0.03	n/a	n/a	0.03	n/a
μ_4'	n/a	0.3	n/a	n/a	0.3	n/a
u_4'	n/a	2.4	n/a	n/a	2.4	n/a

Table 5.9: Results of 5-stage flow lines with three non-identical parallel machines at second stage, three identical parallel machines at fourth stage and equal buffer sizes of 2 and 7

	Case 19			Case 20		
	Buffer Size = 15			Buffer Size = 20		
Parameter	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)	Original	EWIso/ EWIn	% Difference (EWIso/ EWIn)
Q	0.863	0.858	-0.58	0.872	0.868	-0.46
$\overline{b_1}$	6.314	6.134	-2.85	7.965	7.926	-0.49
$\overline{b_2}$	11.093	11.132	0.35	14.861	15.070	1.41
$\overline{b_3}$	3.872	3.686	-4.80	4.872	4.879	0.14
$\overline{b_4}$	8.566	8.594	0.33	11.702	11.850	1.26
λ_2'	n/a	0.049	n/a	n/a	0.049	n/a
μ_2'	n/a	0.375	n/a	n/a	0.375	n/a
u_2'	n/a	2.1	n/a	n/a	2.1	n/a
λ_4'	n/a	0.03	n/a	n/a	0.03	n/a
μ_4'	n/a	0.3	n/a	n/a	0.3	n/a
u_4'	n/a	2.4	n/a	n/a	2.4	n/a

Table 5.10: Results of 5-stage flow lines with three non-identical parallel machines at second stage, three identical parallel machines at fourth stage and equal buffer sizes of 15 and 20

5.3 Proofs of Derived Equations with Simulation Results

The set of equations shown in Section 4.3.2.2 should be used to evaluate the performance of an existing flow line based on actual data collected on it. They should also be used when one needs to compute the probability of idleness of the parallel machines in a set or their equivalent machine. We introduce a new experiment (Case 21) to collect the simulation data as a sample to prove the equations derived in this thesis by using them. For this case, we collect the data only for the original line configuration. For Case 21, the first and third stages of a three stage series-parallel system consist of one machine each whereas the second stage contains two non-identical parallel machines(See Figure 5.1). Each consecutive pair of stages is separated by a buffer of size 10. One of the parallel machines at the second stage is less available and the other one is more available than the first and third stage machines. This availability is varied by varying the failure and repair rates of the machines. As we are interested in tracking the activity at each individual machine, we define the machines at second stage as a multi-unit location. The parameters used in this example are presented in Table A.21 in Appendix - A and the collected simulation results are presented in Table 5.11. M_2' represents the machines (M_{21} and M_{22}) at the second work centre and is generated by the software.

We simulate Case 21 for 40,000 hours with a transient period of 3,500 hours. The transient period is identified using the Welch Moving Average Technique (Harrell *et al.*, 2003). Each experiment is simulated for four replications to ensure the independence of the observations. Performance measures are obtained at the 95% confidence level.

Work centre	Po_{ij}	Ps_{ij}	Pb_{ij}	Pd_{ij}
M_1	0.8718	0.0	0.0411	0.0871
M_{21}	0.6435	0.0699	0.2544	0.0322
M_{22}	0.2282	0.4754	0.2510	0.0453
M_2'	0.4359	0.2727	0.2527	0.0387
M_3	0.8718	0.0411	0.0	0.0871

Table 5.11: Simulation results of machine performance for Case 21

Following are the simulation results taken from Table 5.11:

$$Po_{21} = 0.6435$$

$$Po_{22} = 0.2282$$

$$Po_2' = 0.4359$$

Now we put the value of Po_{21} and Po_{22} in equation (4.35). So, we write:

$$Po_2' = \frac{\sum_{j=1}^{2}\left(u_{2j} Po_{2j}\right)}{\sum_{j=1}^{2} u_{2j}}$$

$$= \frac{u_{21}Po_{21} + u_{22}Po_{22}}{u_{21} + u_{22}}$$ (See Table A.21 in Appendix - A)

$$= 0.4359$$

This computed value of Po_2' is exactly the same as given by the simulation model and so proves the correctness of equation (4.35). As equation (4.35) is based on equation (4.27) and equation (4.34), it implies that these equations are correct.

Again from Table 5.11, we write the following simulation results:

$$Ps_{21} = 0.0699 \qquad Pb_{21} = 0.2544$$

$$Ps_{22} = 0.4754 \qquad Pb_{22} = 0.2510$$

$$Ps_2' = 0.2727 \qquad Pb_2' = 0.2527$$

We compute from the above data:

$$Pi_{21} = Ps_{21} + Pb_{21} = 0.3243$$

$$Pi_{22} = Ps_{22} + Pb_{22} = 0.7264$$

$$Pi_2' = Ps_2' + Pb_2' = 0.5254$$

From equation (4.36), that defines the probability of idleness of a parallel machine in a set, we write:

$$Pi_{ij} = 1 - \frac{Po_{ij}}{e_{ij}} = 1 - \frac{\text{Efficiency of } M_{ij} \text{ (In Line)}}{\text{Efficiency of } M_{ij} \text{ (Isolated)}}$$

$$\Rightarrow Pi_{22} = 1 - \frac{Po_{22}}{e_{22}}$$

$$= 1 - \frac{0.2282}{0.8333}$$

$$= 0.7262$$

where $e_{22} = \dfrac{\mu_{22}}{\mu_{22} + \lambda_{22}} = 0.8333$ (See Table A.21 in Appendix - A)

This computed value of Pi_{22} is the same as given by the simulation model and so proves the correctness of equation (4.36). As equation (4.36) is based on equation (4.25), it implies that this equation is also correct. We can prove this for the other parallel machine in the same way.

Probability of idleness of an equivalent machine is defined by equation (4.37). From equation (4.37), we write:

$$Pi_i' = 1 - \frac{Po_i'}{e_i'} = 1 - \frac{\text{Efficiency of } M_i' \text{(In Line)}}{\text{Efficiency of } M_i' \text{(Isolated)}}$$

$$\Rightarrow Pi_2' = 1 - \frac{Po_2'}{e_2'}$$

$$= 1 - \frac{0.4359}{0.8929}$$

$$= 0.5121$$

where $$e_{21} = \frac{\mu_{21}}{\mu_{21} + \lambda_{21}} = 0.9524$$ (See Table A.21 in Appendix - A)

and $$e_2' = \frac{u_{21}e_{21} + u_{22}e_{22}}{u_2'} = 0.8929$$

This computed value of Pi_2' is close to that given by the simulation model and so it proves the correctness of equation (4.37).

5.4 Comparative Study with Existing Methods

Both Burman (1995) and Patchong and Willaeys (2001) have examined the cases of series-parallel flow lines composed of only identical machines in parallel. In Table 5.12, the results for the proposed method and those of Patchong and Willaeys (2001) are compared for Cases 1 to 6 and those of Burman (1995) are compared for Cases 1, 3 and 5. We also compare the results for the proposed method and those of Patchong and Willaeys (2001) for Cases 7 to 10 in Table 5.13. Percentage difference in estimation for this comparative study is calculated as follows:

For Patchong and Willaeys (2001):

$$\% \text{ Difference (OLE)} = \frac{\text{OLE - ORLE}}{\text{ORLE}} \times 100$$

where,

OLE = Out-Line Estimation

ORLE = Original Line Estimation

For Burman (1995):

$$\% \text{ Difference (EWV)} = \frac{\text{EWV - Original}^{\text{B}}}{\text{Original}^{\text{B}}} \times 100$$

where,

EWV = Estimate With Variance

$\text{Original}^{\text{B}}$ = Estimate of the original system

Case #	Parameter	Original Line			Equivalent Machine Operating in Isolation			% Difference		
		Thesis (Original)	P&W* (ORLE)	Burman ($\text{Original}^{\text{B}}$)	Thesis (EWIso)	P&W* (OLE)	Burman (EWV)	EWIso (Thesis)	OLE (P& W*)	EWV (Burman)
Case 1	Q	0.872	0.872	0.870	0.863	0.863	0.861	-1.03	-1.03	-1.03
	$\overline{b_1}$	3.543	3.717	3.724	3.347	3.567	3.437	-5.53	-4.04	-7.71
	$\overline{b_2}$	6.327	6.382	6.246	6.542	6.647	6.543	3.40	4.15	4.76
Case 2	Q	0.846	0.834	n/a	0.825	0.826	n/a	-2.48	-0.96	n/a
	$\overline{b_1}$	0.776	0.786		0.730	0.784		-5.93	-0.25	
	$\overline{b_2}$	1.124	1.250		1.241	1.293		10.41	3.44	
Case 3	Q	0.832	0.830	0.831	0.832	0.832	0.830	0.00	0.24	-0.12
	$\overline{b_1}$	6.500	6.672	6.619	6.616	6.540	6.603	1.78	-1.98	-0.24
	$\overline{b_2}$	3.489	3.519	3.407	3.409	3.265	3.397	-2.29	-7.22	-0.29
Case 4	Q	0.791	0.761	n/a	0.790	0.794	n/a	-0.13	4.34	n/a
	$\overline{b_1}$	1.338	1.318		1.416	1.396		5.83	5.92	
	$\overline{b_2}$	0.660	0.673		0.582	0.601		-11.82	-10.70	
Case 5	Q	0.771	0.757	0.756	0.744	0.680	0.728	-3.50	-10.17	-3.70
	$\overline{b_1}$	5.499	5.961	5.803	5.261	5.275	5.393	-4.33	-11.51	-7.07
	$\overline{b_2}$	4.431	4.214	4.215	4.686	4.766	4.598	5.75	13.10	9.09
Case 6	Q	0.706	0.624	n/a	0.642	0.586	n/a	-9.07	-6.09	n/a
	$\overline{b_1}$	1.033	1.130		0.926	1.003		-10.36	-11.24	
	$\overline{b_2}$	0.926	0.788		1.056	1.005		14.04	27.54	

(P&W* refers to Patchong and Willaeys 2001)

Table 5.12: Comparison of results of 3-stage flow lines with two identical parallel machines at second stage with Burman (1995) and Patchong and Willaeys (2001)

From this comparative study, it is not easy to conclude about the superiority of the methods compared. They provide almost the same results for all the cases except when the failure rate of the parallel machines and their number in a set are high; see the Cases 5, 6, 11 and 12. It is evident that when the machines of second stage are less available than the first and third stages due to higher failure rate, the accuracy of the method proposed by Patchong and Willaeys (2001) is remarkably low. The accuracy gets worse when the number of unreliable machines in parallel goes higher. We focus on this weakness of Patchong and Willaeys (2001) and explain it in the following manner.

Case #	Parameter	Original Line		Equivalent Machine Operating in Isolation		% Difference	
		Thesis (Original)	P& W* (ORLE)	Thesis (EWIso)	P& W* (OLE)	EWIso (Thesis)	OLE (P& W*)
Case 7	Q	0.876	0.884	0.871	0.873	-0.57	-1.24
	$\overline{b_1}$	3.270	3.502	3.448	3.495	5.44	-0.20
	$\overline{b_2}$	6.541	6.794	6.459	6.467	-1.25	-4.81
Case 8	Q	0.856	0.859	0.843	0.844	-1.52	-1.75
	$\overline{b_1}$	0.683	0.659	0.721	0.783	5.56	18.82
	$\overline{b_2}$	1.207	1.287	1.216	1.226	0.75	-4.74
Case 9	Q	0.838	0.847	0.837	0.838	-0.12	-1.06
	$\overline{b_1}$	6.606	7.054	6.989	6.869	5.80	-2.62
	$\overline{b_2}$	3.363	3.549	2.989	3.162	-11.12	-10.90
Case 10	Q	0.795	0.746	0.793	0.797	-0.25	6.84
	$\overline{b_1}$	1.279	1.180	1.556	1.494	21.66	26.61
	$\overline{b_2}$	0.709	0.844	0.447	0.501	-36.95	-40.64

(P&W* refers to Patchong and Willaeys 2001)

Table 5.13: Comparison of results of 3-stage flow lines with five identical parallel machines at second stage with Patchong and Willaeys (2001)

Patchong and Willaeys (2001) Case 5 and Case 6					
	i	1	2_1	2_2	3
Failure Rate	λ_i	0.010	*0.120*	*0.120*	0.010
	λ_2'	n / a	*0.109*		n / a

Table 5.14: Failure rate of equivalent machine presented by Patchong and Willaeys (2001) for Case 5 and Case 6

For Cases 5 and 6, when the availability of the two machines of the second stage is reduced and made half as available as those in the first and third stages, the failure rate of each machine of second stage increases to 0.120 (See Table 5.14). The calculated failure rate of the equivalent machine applying Patchong and Willaeys (2001) method is 0.109. We notice that the failure rate of the equivalent machine for these cases is even lower than that of a single machine in a set of parallel machines. We illustrate an example to make it clear and unambiguous where a stage is composed of five identical parallel machines. The failure and repair rates of each machine in the set are initially set to 0.0100 and 0.10 respectively. Then, we vary the failure rate of the machines in the set from 0.0100 up to 0.1000 and down to 0.0003 leaving the repair rate unchanged. We restrict the upper limit of the failure rate of each machine to 0.1000 to ensure a steady-state system. The results of failure rate of the equivalent machine computed by Patchong and Willaeys (2001) and the proposed method are presented in Figure 5.3 and the repair rate of the equivalent machine for both the methods are presented in Figure 5.4.

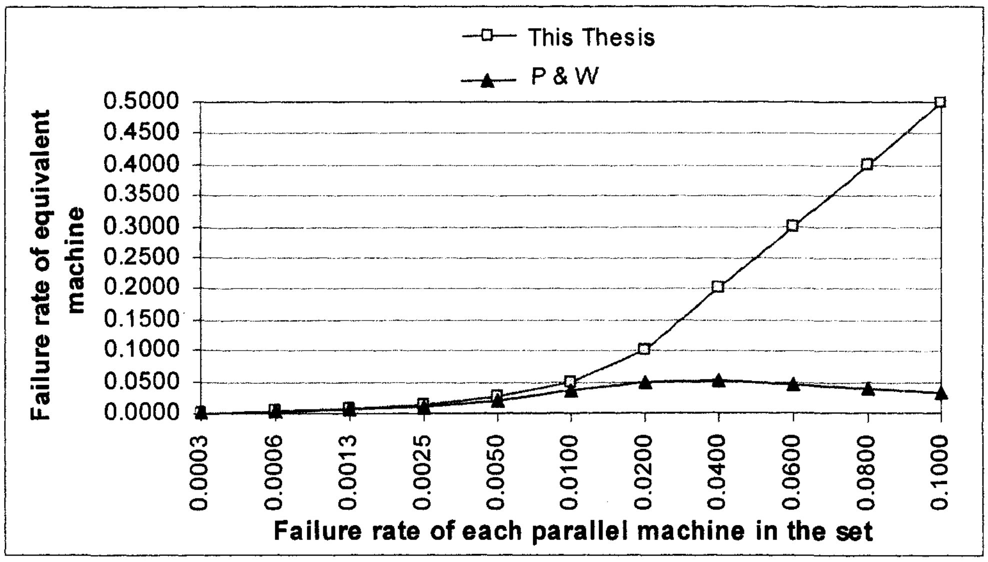

Figure 5.3: Failure rate of equivalent machine of a five-identical- parallel-machine stage

According to Patchong and Willaeys (2001), as shown in Figure 5.3, the failure rate of the equivalent machine increases as that of the parallel machines increases and reaches the maximum when the failure rate of each parallel machine is 0.0400. The trend of the graph changes then and the failure rate of the equivalent machine decreases though the failure rate of the parallel machines increases (See Section 4.3.1.3 for explanation). It implies that the failure rate of each parallel machine at a stage either increases or decreases from 0.0400, the failure rate of the equivalent machine decreases which means that the equivalent machine becomes more available. It can be interpreted from the trend of the graph as "The equivalent machine of a set of highly reliable or unreliable machines is highly reliable" which is very difficult to admit and not easy to explain in real production system. From Figure 5.3, we notice that the trend of the graph for both the methods is similar when the failure rate of each parallel machine is low and they differ as the failure rate goes higher. Unlike Patchong and Willaeys (2001), the failure rate of the equivalent machine, according to the proposed method, is directly related to the failure rate and the number of parallel machines in a set. The higher the failure rate and the higher the number of machines in a set, the higher the failure rate of the equivalent machine and hence lower the production output and vice versa. This interpretation seems more logical and is supported by the existing literature [Kalir and Arzi (1997), Mustapha *et al.*, (2004)].

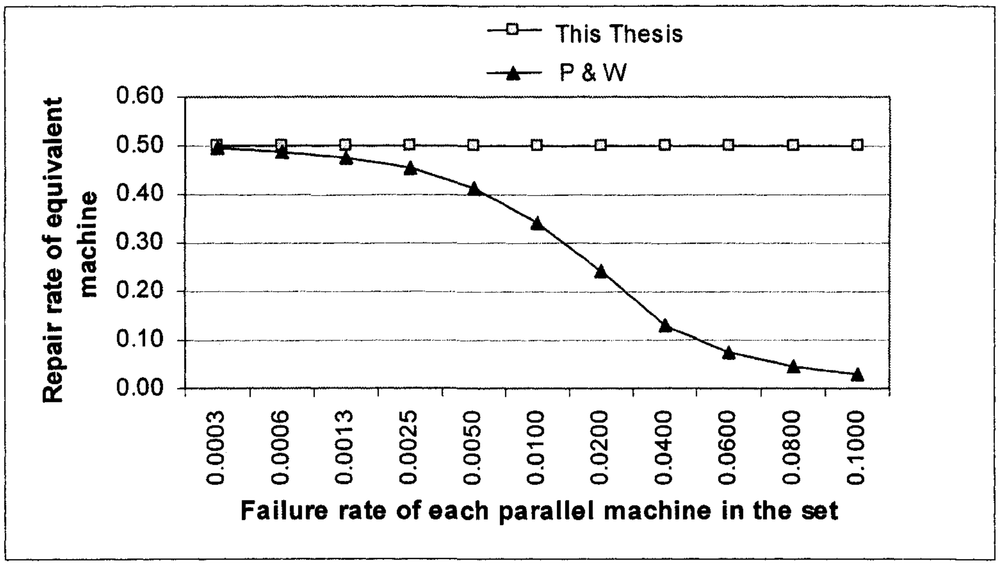

Figure 5.4: Repair rate of equivalent machine of a five-identical- parallel-machine stage

Another important criticism that can be made about the method of Patchong and Willaeys (2001) is that unlike the proposed method, the repair rate of the equivalent machine decreases as the failure rate of the set of parallel machines increases (Figure 5.4). It means that the higher the failure of the machines the higher it takes to repair them [See Equation (h) in Section 4.3.1.3 for the explanation]. But the fact is, as we do not change the repair rate of the set of parallel machines, the repair rate of the equivalent machine should remain the same irrespective of change of failure rate. As the interpretation of failure rate used by Ancelin and Semery (1987) is similar to that of Patchong and Willaeys (2001), is also difficult to admit.

However, Cases 5, 6, 11 and 12 are the cases similar to the situations explained above. When the failure rate of each parallel machine is high but the number of machines at a stage is low (Cases 5 and 6), our proposed method works better than that of Patchong and Willaeys (2001). The proposed method also performs well when the failure rate of each parallel machine as well as the number of machines at a stage is high (Cases 11 and 12). It is to be mentioned that the accuracy of the proposed method is limited to flow lines with smaller buffer size. We notice that among these four cases, Patchong and Willaeys (2001) did not examine Cases 11 and 12 commenting that these are unrealistic situations. Moreover, their presented results for Case 6 are not consistent with that of Case 5. However, we consider all the cases in order to test the accuracy and sensitivity of the proposed method.

It is also noticed that the results presented by Patchong and Willaeys (2001) for the production rate either underestimate or overestimate the actual result. We do not have any explanation if it is possible for a method that provides such results. Unlike Patchong and Willaeys (2001), the method of Burman (1995) and the proposed method always underestimate the true value of production rate.

There is no research known to us that has examined the cases when there are non-identical machines at a stage or there are multiple-machines at more than one stage for series-parallel flow lines. We explore these situations in this thesis and validate our results by simulation. These results along with the findings are presented in section 5.2.2 and section 5.2.3 and this is a contribution of this research work.

5.5 Conclusion

We have executed numerous experiments with various configurations of series-parallel flow lines assigning identical, non-identical or a combination of identical and non-identical parallel machines at one or even more than one stage. It is evident from the results that the accuracy of the proposed method to estimate the production rate (Q) for almost all the cases is high with a variation of less than 4%. The less accurate results are observed when the availability of the multiple machines at a stage is less than that of the other stages due to higher failure rate and the adjacent buffers are too small to accommodate as much material as produced during an average failure (Case 6 and Case 12). This small buffer also impacts the accuracy of the method in estimating the average buffer contents to some extent. The deployment of higher buffer contributes to the production rate for all the cases which is in conformity to the findings of Abdul-Kader and Gharbi (2002). The impact of buffer contribution is reduced with the incorporation of parallel machines at stages. However, the number of parallel machines either identical or non-identical at a stage does not reduce the accuracy of the proposed method. The accuracy of the method is not also affected by the length of the flow line and the assignment of multiple machines at more than one stage. We also notice that the results of EWIso and EWIn are exactly the same for the flow lines with either identical or non-identical machines in parallel. This proves the correctness of the derived equations and the accuracy of the proposed method. Based on the analysis and comparative study, we can conclude that the proposed method is accurate and comparable with the existing methods.

When all the parallel machines at a work centre are identical, our derived equations for equivalent failure rate, repair rate and processing rate reduce to a form similar to the heuristic proposed by Burman (1995). Kalir and Arzi (1997) and Mustapha *et al.*, (2004) have also applied a similar strategy to define the parameters of the equivalent machine in their papers. This implies the accuracy of our derived equations.

The existing analytic methods for series-parallel systems can tract only lines with a maximum of two machines in serial. The method we propose in this thesis can be used in conjunction with an approximation method or simulation to solve flow lines of any length. The set of equations shown in Section 4.3.2.1 should be used to design manufacturing systems and those shown in Section 4.3.2.2 should be used to evaluate the performance of an existing flow line based on actual data collected on it. Also they can be used to compute the probability of idleness of the parallel machines in a set when needed.

CHAPTER 6
PARALLEL REDUNDANCY IN MULTI-PRODUCT FLOW LINE SYSTEM

6.1 Introduction

This chapter presents an automated multi-product manufacturing system to address the issue of throughput enhancement. The production line is made of m work centres in series with (m-1) intermediate buffers where each buffer separates a consecutive pair of work centres. A work centre may have a single machine or a set of machines in parallel. A simulation-based model is developed that incorporates the proposed analytical approximation method to improve the performance of this system in terms of cycle time. The key aspect of studying this production system is to show how to make use of the approximation method we developed in Chapter 4. We show how the change of buffer sizes and the incorporation of parallel machines significantly impact the cycle time of the line. We also present some performance measures such as the percentages of the time machines are in operation, blocked, starved, down or in set-up, which provide insights about the system improvement. Judicious interpretation of this information will help the managers to adopt the appropriate strategy to reduce the cycle time.

6.2 Multi-Product Series-Parallel System - A Solution Approach

Production systems are often organized with the work centers connected in series, separated by buffers and limited to the analysis of a single product. Parallel machines are introduced into the systems to achieve a greater availability and hence a greater performance. A production line with machines in parallel at work centres is referred to as series-parallel system (Burman, 1995). In Chapter 4, we proposed an analytical approximation method to solve the problem of series-parallel system manufacturing a single product. The proposed technique replaces a set of parallel machines at a work centre by an equivalent machine in order to obtain a traditional flow line with machines in series separated by buffers. This traditional flow line could be solved by approximation methods or by simulation. We derived equations for the parameters defining an equivalent machine when it operates in isolation as well as in flow line. The parameters considered to define an equivalent machine in isolation are equivalent processing rate (u_i'), equivalent failure rate (λ_i') and equivalent repair rate (μ_i'). Then in Chapter 5, we applied this developed method in conjunction with simulation on various line configurations to estimate the production rate of the series-parallel system manufacturing a single product.

In this chapter, we tackle an automated multi-product manufacturing system to minimize the cycle time. The production system is made of *m* work centres in series with (*m*-1) intermediate buffers. A work centre may be composed of a single machine or a set of machines in parallel as shown in Figure 6.1. Unlike the production systems presented in Chapter 5, this system is capable of producing more than one product type in a batch production environment. Each product has a predefined lot size and the product mix follows a predetermined sequence. The set-up of the machines is considered as the product type changes.

We use the analytical approximation method proposed in Chapter 4 to solve this multi-product series-parallel system. The machines at a work centre are replaced by an equivalent machine and this equivalent machine is defined with the parameters such as equivalent processing rate (u_i'), equivalent failure rate (λ_i') and equivalent repair rate

(μ_i'). But in this case, the equations derived in Chapter 4 to define these parameters are used in a multi-product manufacturing environment as shown below. In addition, as the line considers set-up of the machines as the product type changes, the set-up time of the equivalent machine for a product type is also estimated. We use the same notation shown in Section 3.2 to describe the system as well as the parameters. The cycle time is determined with a simulation-based model that incorporates the parameters of the equivalent machine estimated by the proposed method.

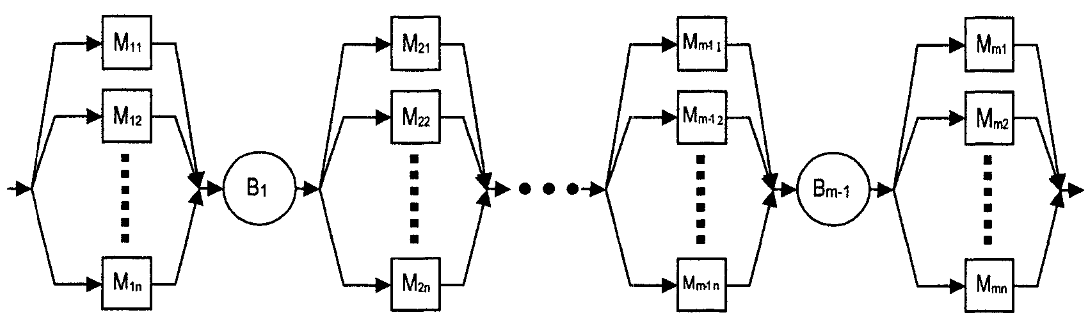

Figure 6.1: Series-parallel production line made of m work centres and (m-1) buffers

Processing Rate on the Equivalent Machine

The maximum isolated processing rate of product type k corresponding to the equivalent machine M_i' may be reasonably defined as the sum of the individual isolated processing rates of product type k on the set of parallel machines M_{ij} at work centre i when all the machines are up and not starved or blocked:

$$u_{ik}' = \sum_{j=1}^{n} u_{ijk} \tag{6.1}$$

Where,

u_{ik}' = Isolated processing rate of product type k on the equivalent machine M_i'

u_{ijk} = Isolated processing rate of product type k on parallel machines M_{ij}

Subscript j denotes the number of machines in parallel at work centre i

Where $j = 1, 2, 3, \ldots\ldots, n$

Failure Rate and Repair Rate of the Equivalent Machine

We use equation (4.32) and equation (4.33) to estimate the failure and repair rates of the equivalent machine.

Set-up Time of the Equivalent Machine

We consider the following set of assumptions to estimate the set-up time of the equivalent machine:

(a) Set-up times of the machines are reasonably larger than the processing times on the machines

(b) Set-up of a machine at a work centre begins when the first unit of the new product enters that machine

(c) There is an unlimited number of operators to set-up the machines. So, set-up of a machine can be started as soon as it receives the new product

We use the following set of notation to define the set-up time of the machines:

$S_{ijk_{act}}$ = Actual time elapsed for setting up machine M_{ij} for product k

S_{ijk} = Set-up time of machine M_{ij} for product k

S_{ik}' = Set-up time of the equivalent machine M_i' for product k

$P_{(i-1)k}$ =Processing time of product k on the machine upstream of M_i'

Subscript j denotes the number of machines in parallel in a work centre i

Where $j = 1, 2, 3,\ldots\ldots\ldots\ldots, n$

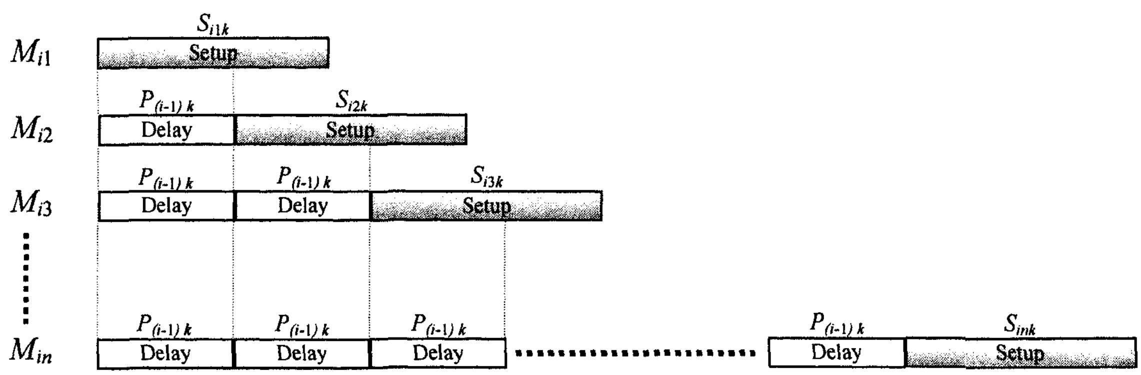

Figure 6.2: Graphical representation of the set-up process for n parallel machines at a work centre i for a product type k

In a multi-product manufacturing system, the set-up of the machines is considered as the product type changes. There are various ways to define the set-up of the machines. In this thesis, we consider that the products are transferred to the longest unoccupied machine in a multi-machine work centre and the set-up of a machine at a work centre begins when the first unit of the new product enters that machine. We also consider that the set-up times of the machines are reasonably larger than the processing times on the machines in a line. So, when the first unit of the new product enters the first machine in a set, the set-up of this machine can readily be started. But, the rest of the machines of the set wait for set-up (starve) for the new product when the first machine is being set-up. The set-up of the second machine is delayed from that of the first machine by the processing time of one unit of the new product on the upstream work centre. Similarly, the set-up of the third machine is delayed from that of the first machine by the processing time of two units of the new product on the upstream work centre and so on. If a work centre i has n machines in parallel, the set-up process of the machines for a product type k can be graphically represented as shown in Figure 6.2.

As described above, the set-up of the first machine of a multi-machine work centre can start as soon as the new product enters the machine. So, the actual time elapsed for setting up this machine M_{i1} for a product type k can be defined as follows:

$$S_{i1k_{act}} = 0 \times P_{(i-1)k} + S_{i1k} \tag{i}$$

The set-up of the second machine is delayed from that of the first machine by the processing time of one unit of the new product on the upstream work centre. So, the actual time elapsed for setting up this machine M_{i2} for a product type k can be defined as follows:

$$S_{i2k_{act}} = 1 \times P_{(i-1)k} + S_{i2k} \tag{ii}$$

The set-up of the third machine is delayed from that of the first machine by the processing time of two units of the new product on the upstream work centre. So, the actual time elapsed for setting up this machine M_{i3} for a product type k can be defined as follows:

$$S_{i3k_{act}} = 2 \times P_{(i-1)k} + S_{i3k} \tag{iii}$$

Similarly, the set-up of the n th machine is delayed from that of the first machine by the processing time of (n-1) units of the new product on the upstream work centre. So, the actual time elapsed for setting up this machine M_{in} for a product type k can be defined as follows:

$$S_{ink_{act}} = (n-1) \times P_{(i-1)k} + S_{ink} \tag{iv}$$

So, the set-up time of the equivalent machine M_i' for product type k may be reasonably approximated as the average of the actual time elapsed for setting up the set of parallel machines M_{ij} at work centre i for product type k:

$$\begin{aligned} S_{ik}' &= \frac{\sum_{j=1}^{n} S_{ijk_{act}}}{n} \\ &= \frac{\sum_{j=1}^{n} \left[S_{ijk} + (j-1) P_{(i-1)k} \right]}{n} \end{aligned} \tag{6.2}$$

In this section, we develop a solution methodology based on the approximation method proposed in Chapter 4 to solve a series-parallel system producing multiple products. This methodology is applied to a case study presented in Section 6.3.

6.3 Multi-product Manufacturing System – A Case Study

We present a case study for a multi-product manufacturing system of realistic size to show how to make use of the solution approach developed in Section 6.2. This section is organized as follows: Section 6.3.1 provides a detailed description of the production system considered in the case study; Section 6.3.2 presents the definitions of the important terms related to the performance analysis of the production system and Section 6.3.3 develops the simulation model for the production system of the case study.

6.3.1 Description of the System

The production system we tackle includes five work centres in series and four intermediate buffers. Figure 6.3 depicts this production system. This system is able to process two product types (Product 1 and Product 2) in a batch production environment. Each product has a predefined lot size which is given in Table 6.1. The product mix follows a predetermined sequence when it is processed which is Product 1 then Product 2 and again back to Product 1 for the next cycle. Set-up of the machines is required when the product type changes. Data of the set-up times for the two product types on five machines is presented in Table 6.2. Unit of time used in this chapter is, unless otherwise specified, Time unit.

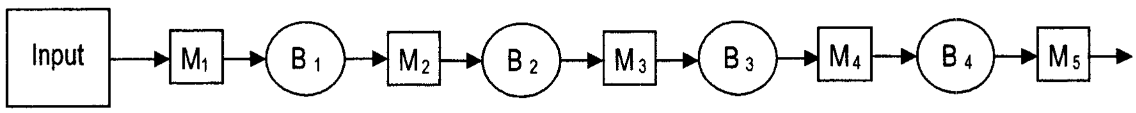

Figure 6.3: Production line made of five work centres and four intermediate buffers

	Product	Unit
Lot Size	1	60
	2	75

Table 6.1: Lot sizes (Unit)

	Product	Work centre				
		1	2	3	4	5
Processing Time	1	80	70	60	75	65
	2	60	90	70	55	85
Set-up Time	1	300	225	275	250	200
	2	300	225	275	250	200

Table 6.2: Processing times and set-up times (Time unit)

The products are assumed to be available and stored in an input storage which is placed before the first work centre M_1. This storage is assigned with infinite capacity to support the assumption that the first work centre M_1 cannot be starved. The products are then transferred to the first work centre M_1 of the production line. In the line, products flow from work centre to buffer to work centre in a fixed sequence in the direction of arrows. They visit each work centre once and receive processing. Each product of the mix is transferred to the next work centre via the intermediate buffer on a unit-by-unit basis. The final product leaves the system from the last work centre M_5.

The processing time and the set-up time of a product type are assumed deterministic for a machine but vary from one machine to another in the production line. Also, they vary from one product type to another for all the machines. Data for the processing times of the two products on different machines is shown in Table 6.2.

Each machine of the production line is subject to random failure and repair. The failure and repair rates are exponentially distributed. These uncontrollable parameters of the five machines are presented in Table 6.3. Due to the lack of synchronization of the machines, not only in performing operations, but also in failing, getting repaired, and setting up, machines can be starved or blocked. This impacts the performance of the system and increases the cycle time. We tackle this important problem of the system in this chapter as well.

	Work centre				
	1	2	3	4	5
Failure rate	0.00059	0.00065	0.00062	0.00074	0.00055
Repair rate	0.0047	0.0037	0.0055	0.0067	0.0042

Table 6.3: Failure and repair rates of work centres (Per time unit)

To summarize, the production system we examine in this case study is representative of a real world automated production system. The objective of this research is to improve the performance of this system in terms of cycle time. There are different ways to do that (See Page 3); we adopt two among them. First, we analyze the contribution of adding buffers to compensate for the loss due to a disruption (a failure or a long processing time) in improving the performance. Secondly, we analyze the impact of incorporating parallel machines at work centres to achieve a greater availability and hence a greater performance. However, it is worth noting that our main focus is on the second strategy as we apply the analytical model that we developed in Chapter 4 to solve these series-parallel lines.

6.3.2 Important Definitions

Before developing a model for the production line used in the case study, it is necessary to define and clarify some of the important terms related to the performance analysis of the production system that we use in this chapter.

Bottleneck Work Centre

The overall bottleneck work centre of a production line is the one that takes the maximum amount of time (including blocked/ starved time) to process a given product mix. As the bottleneck depends on the buffer sizes, the product mix, as well as the sequence in which the products are produced, it may shift if any of the contributing factors is altered (Johri, 1987).

Production Line Cycle Time

The cycle time is defined by researchers in various ways. A definition of machine cycle time for a single-product system is presented in Section 3.3. But when the system produces more than one product type, the cycle time may be defined as the time that the slowest work centre takes to process the product mix (Martinich, 1997). This slowest work centre is referred to as the bottleneck work centre of the line. In other words, the cycle time of the bottleneck work centre is the line cycle time. We use this definition in this thesis. This way of defining cycle time permits us to identify the overall bottleneck work centre for each combination of buffer sizes, the product mix and the sequence in which the products are produced. The cycle time we defined may be expressed in the following way:

T_{ik} = Cycle time of work centre M_i for a product mix composed of k product types

$T_{ik}^{C}\_in$ = Clock time, when the first unit of the first product (k = 1) of the mix enters work centre M_i

$T_{ik}^{C}\_out$ = Clock time, when the last unit of the last product (k = l) of the mix leaves work centre M_i

T = Line cycle time for a product mix composed of k product types

Where $i = 1, 2, 3, \ldots\ldots\ldots\ldots, m$

$k = 1, 2, 3, \ldots\ldots\ldots\ldots, l$

So, the cycle time of work centre M_i is:

$$T_{ik} = T_{il}^{C}\_out - T_{i1}^{C}\_in \tag{6.3}$$

and the line cycle time is:

$$T = Max\ \left[T_{ik}\right]_{i=1}^{m}; \qquad for\ k = 1,2,3,\ldots\ldots\ldots,l \tag{6.4}$$

6.3.3 Model Development

The simplest form of a production line is a two machine - one buffer line. It has received a significant amount of attention from researchers such as Buzacott (1972), Gershwin and Berman (1981) and Johri (1987). The reason for this attention was to understand the mechanism and performance of simple systems, so that the models of longer and more complex systems could be developed based on that.

So, before developing a model for the case study which is a longer line, it is better to apply our modeling approach to the examples presented in Johri (1987). This allows us to compare and validate our results to those obtained from Johri's examples.

Johri (1987) presented some illustrative examples of production lines composed of two work centres and one buffer. He also developed a linear programming method to estimate the cycle time of a production line producing a variety of products and compared the results of the developed method to those of the examples. The product mix used in these examples is composed of two product types, and failures and repairs of the work centres are not addressed explicitly. Input data used in these examples are presented in Table 6.4 to Table 6.7.

Data used in the examples presented in Johri (1987):

Johri's example 1:

Buffer capacity = 6 units

	Product	Unit
Lot Size	1	60
	2	75

Table 6.4: Lot sizes (Unit)

	Product	Work centre	
		1	2
Processing Time	1	80	20
	2	40	100
Set-up Time	1	300	200
	2	300	200

Table 6.5: Processing times and set-up times (Time unit)

Johri's example 3:

Buffer capacity = 10 units

	Product	Unit
Lot Size	1	12
	2	15

Table 6.6: Lot sizes (Unit)

	Product	Work centre	
		1	2
Processing Time	1	80	20
	2	40	100
Set-up Time	1	300	200
	2	300	200

Table 6.7: Processing times and set-up times (Time unit)

We simulate these two examples of Johri (1987) with the objective of verifying the correctness of the codes that we write to compute the cycle time. We use the simulation software ProModel to model the systems presented in the examples and write a small program using built-in logic building elements of the software in order to compute the cycle time. To debug the developed model, we set 'Counter' and their corresponding 'Clock' before and after all the work centres for all the product types and capture the time for various conditions. These recorded times are then compared with those obtained from the manual calculation for the same conditions. Many scenarios are visualized and evaluated with graphical animation to verify the proper functioning of the system. We also verify and analyze the output statistics of the simulation experiments. As all the input parameters of the model are deterministic, we run the experiments for a period until the results of the cycle time become stable. Results of these experiments and that of Johri (1987) are shown in Table 6.8.

Experiments	**Cycle Time** (Time Unit)	
	Johri (1987)	**Thesis**
Example 1 (Johri, 1987)	12400	12400
Example 3 (Johri, 1987)	2220	2220

Table 6.8: Comparison of cycle time with Johri (1987)

It is observed from Table 6.8 that the cycle times obtained from our simulation results for both the cases are exactly the same as those given in Johri's examples. This validates our results and confirms the correctness of the model we developed to compute the cycle time. Then we apply our modeling approach to the longer lines of five work centres and four buffers described in Section 6.3.1 (case study).

6.4 Experiments

In this section, we demonstrate experiments on the multi-product production lines in order to minimize the cycle time. We start with the production line illustrated earlier in Figure 6.3. The buffer size and the number of parallel machines for this configuration are varied to analyze their impacts on the cycle time. For the convenience of description, we refer to the line described in Figure 6.3 as the Original Serial Line or OSL for short. When a machine is added in parallel at the first bottleneck work centre of OSL, the series-parallel line is referred to as the Series-Parallel Line-1 or SPL-1 (See Figure 6.5) and its equivalent serial line (replacing parallel machines) is referred to as the Equivalent Serial Line-1 or ESL-1 (See Figure 6.6) where '1' represents that the first bottleneck work centre of OSL is composed of two machines in parallel. Similarly, when the first two bottleneck work centres of OSL are composed of two parallel machines each, they are referred to as 'SPL-2' (See Figure 6.8) and 'ESL-2' (See Figure 6.9) respectively and so on. Simulations are run for 75,000 hours for all the experiments with a transient period of 13,400 hours. The transient period is identified using the Welch Moving Average Technique (Harrell *et al.*, 2003). Each experiment is simulated for two replications to ensure the independence of the observations. Performance measures are obtained at the 95% confidence level. We do not collect any statistics during the transient period as our focus is on the steady-state behaviour of the system.

6.4.1 Contribution of Buffer

This section presents the analysis of the buffer contributions on the line cycle time. The use of buffer space is an efficient tool that contributes to enhancing the capacity of the production line (Buzacott, 1971). Introducing buffer space decouples successive work centres by allowing them to work independently for a certain period of time. When a work centre in a line fails or has a longer processing time, the work centre upstream of it can still operate until the upstream buffer fills up, and the downstream work centre can still operate until the downstream buffer becomes empty. The use of buffers also helps to reduce the blocking and starvation times of the work centres and thus lowers the cycle time. For example, imagine a ten-stage line in which each work centre has a 90% isolated efficiency and each processes at a rate of one part per time unit. Under the assumption of TDFs, if there were no buffers, the average line throughput would be approximately 0.9^{10} or 0.35. With infinite buffers, average line throughput would be 0.9 (Burman, 1995). A similar effect can also be observed for ODFs.

A designed experiment composed of four buffers of equal sizes and their many levels is conducted on the serial line composed of five work centres and four buffers, which is termed as 'OSL'. The goal is to find the minimum cycle time for this line configuration. The smallest buffer size considered for these experiments is 1 unit. This buffer size of 1 unit practically represents the lowest buffer level for a finite buffer system and is used to help a unit of production transiting from one work centre to another. From the preliminary experiments it is observed that the change of buffer size at a lower level has a greater impact on the cycle time than that at a higher level. We consider another buffer size of 2 units with the intention to check the impact of smaller buffer size. This value is then incremented in two steps; first by 2 units up to 10 and then by 4 units from 10 up to 42. The parameters applied in these experiments are outlined in Section 6.3.1 (Table 6.1 to Table 6.3). Simulation results for the cycle times are shown in Table 6.9. For the convenience of result presentation, we assume that a minute is equal to 60 time units. A curve showing the trade-offs between the cycle times and the buffer sizes is plotted in Figure 6.4. Additional information of the work centre performance and the percentage of the time the buffers are at different states are presented in Table 6.10 and Table 6.11 respectively.

Buffer Size (Units)	Cycle Time (Min)	Bottleneck Work centre	Buffer Size (Units)	Cycle Time (Min)	Bottleneck Work centre
1	281.96	M_1	18	222.22	M_2
2	262.98	M_1	22	221.34	M_2
4	243.29	M_2	26	221.22	M_2
6	235.36	M_2	30	221.13	M_2
8	230.98	M_2	**34**	**220.92**	$\boldsymbol{M_2}$
10	227.65	M_2	38	220.93	M_2
14	223.99	M_2	42	220.99	M_2

Table 6.9: Cycle times of five work centre and four buffer production line (OSL) for different buffer sizes

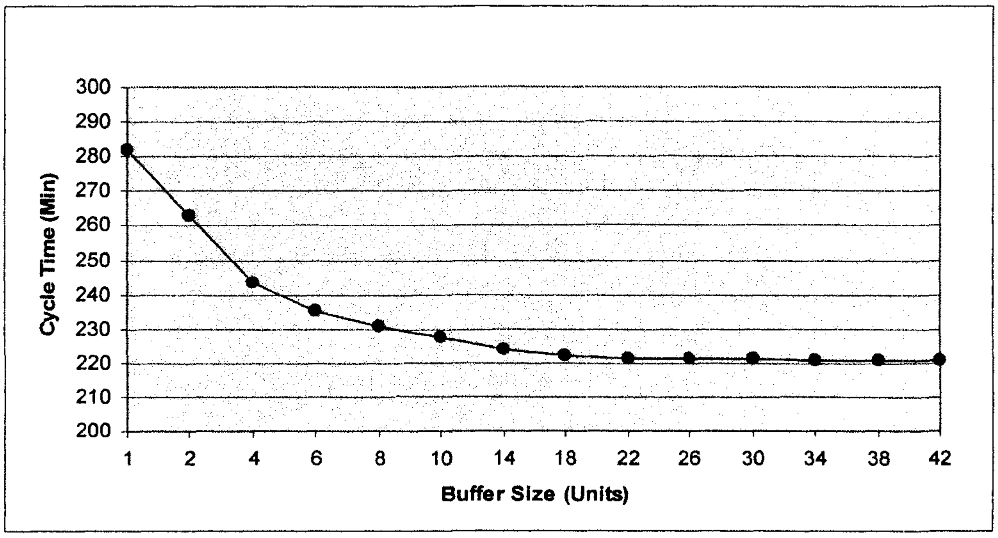

Figure 6.4: Cycle time versus buffer size for five work centre and four buffer production line (OSL)

The experiments are conducted with the objective of determining the contribution of equal buffers to minimize the cycle time. The curve in Figure 6.4 shows that the cycle time is the maximum at the smallest buffer size of 1 unit and decreases as the buffer size increases. But it is well known that the buffers have a limited contribution to the performance of the production system beyond a certain point. When the buffer capacity approaches 22 units, the curve starts flattening, which means that the buffer is large enough to significantly contribute to the decoupling of the work centres. The minimum cycle time for this system is 220.92 minutes corresponding to the buffer size of 34 units. The addition of further buffer space beyond this point will not reduce the cycle time, rather this buffer space will remain unused, which is never expected. An alternative way to reduce the cycle time of this production system is, first to identify the bottleneck work centre and then to make it either faster or more available or both. The incorporation of parallel machines is a means to attain this goal. It is evident from Table 6.9 that work centre M_2 takes the maximum amount of time to process the product mix and hence it is the bottleneck work centre. So, a machine may be added in parallel to the existing machine in work centre M_2 in order to further reduce the cycle time.

Work centre	Scheduled Hours	% Operation	% Set-up	% Starved	% Blocked	% Down
M_1	75,000	67.00	4.32	0.00	20.45	8.23
M_2	75,000	78.88	3.24	**3.05**	**1.23**	13.60
M_3	75,000	63.75	3.96	21.04	4.22	7.03
M_4	75,000	62.13	3.60	21.11	6.48	6.68
M_5	75,000	74.02	2.88	13.63	0.00	9.46

Table 6.10: Percentage of work centre performance of OSL for buffer sizes of 8 units

Table 6.9 also shows that M_1 is the bottleneck work centre for buffer sizes up to 2 units. But, as the buffer size increases, bottleneck shifts from work centre M_1 to work centre M_2 at buffer size 4 units which comply with the findings of Johri (1987).

Intermediate Buffer	Scheduled Hours	% Empty	% Partially Occupied	% Full
B_1	75,000	9.41	41.29	**49.30**
B_2	75,000	**57.09**	39.26	3.65
B_3	75,000	42.95	43.06	13.99
B_4	75,000	35.41	48.06	16.53

Table 6.11: Percentage of buffer states of OSL for buffer sizes of 8 units

Table 6.10 provides additional information such as the percentages of the time the work centres are at their different states (Operation, Set-up, Starvation, Blocking and Down) in a cycle for OSL with 8 units of equal buffer sizes. This choice of buffer size is simply for illustration purposes. In addition, Table 6.11 presents simulation results of buffer states in percentage. The additional information may guide us to improve the performance of the system. For example, work centre M_2 is starved (3.05%) and blocked (1.23%) for the minimum percentages of the time in a cycle as compared to the other work centres; and the upstream buffer (B_1) and the downstream buffer (B_2) of work centre M_2 is full (49.30%) and empty (57.09%) for the maximum percentages of the time. In addition, M_2 is down (13.60%) for the maximum percentages of the time. It means that this work centre M_2 produces less as compared to the adjacent work centres and makes the line slow. It also gives an indication that this work centre might be the bottleneck work centre for this configuration. As seen from Table 6.9, this work centre takes the maximum amount of time to process the product mix; this justifies the analysis based on the additional information. Considering more space for the upstream buffer of work centre M_2 will reduce the blocking of the upstream work centre M_1. Similarly, the reduction of space of the downstream buffer may reduce the associated cost and free up valuable space on the factory floor. Another option to improve the performance is to increase the capacity of this work centre.

6.4.2 Contribution of Parallel Machine

This section is dedicated to the analysis of adding parallel machines. The experiments are conducted on two different line configurations to check the contribution of the parallel machine. Section 6.4.2.1 presents the experiments on a series-parallel line with a work centre composed of two machines in parallel. In Section 6.4.2.2, the experiments on a series-parallel line with two of its work centres composed of two machines each are presented. These series-parallel lines are tackled following the solution approach shown in Section 6.2. The multiple machines at a work centre are replaced by an equivalent machine. Parameters defining an equivalent machine are estimated. In addition, the set-up of the equivalent machine for a product type is also estimated. We use equation (6.1) to estimate the equivalent processing rate corresponding to a product type. Equation (6.2) is used to estimate the set-up time on the equivalent machine for a product type. The failure and repair rates of the equivalent machine are estimated using Equation (4.32) and (4.33) respectively. Finally, simulation is conducted to determine the cycle time.

6.4.2.1 Series-Parallel Line with a Work Centre Composed of Two Machines in Parallel

From Section 6.4.1, it is observed that for the serial system composed of five work centres and four intermediate buffers (OSL), M_2 is the bottleneck work centre. In order to minimize the cycle time of OSL, we add an identical machine at the bottleneck work centre M_2, in parallel to the existing machine. This system is then converted to a series-parallel system with two machines in parallel at work centre M_2. We refer to this series-parallel system as 'SPL-1' (Figure 6.5) and the parallel machines as M_{21} and M_{22}.

When the parallel machines at work centre M_2 of SPL-1 are replaced by an equivalent machine M_2', the series-parallel system (SPL-1) turns back to a traditional serial system. We refer to this serial system as Equivalent Serial Line-1 or ESL-1 (Figure 6.6). The parameters defining the equivalent machine M_2' are determined by the equations (6.1), (4.32), (4.33) and (6.2). Calculations of the equivalent processing rate and the set-up time are shown below:

Calculation of the Equivalent Processing Rate

From Table B.1 of Appendix - B:

Processing time of product 1 on M_{21}, P_{211} = 70 Time units

Processing time of product 1 on M_{22}, P_{221} = 70 Time units

So,

Processing rate of product 1 on M_{21}, $u_{211} = \dfrac{1}{70}$ per time units

Processing rate of product 1 on M_{22}, $u_{221} = \dfrac{1}{70}$ per time units

From equation (6.1), processing rate of product 1 on the equivalent machine M_2' is:

$$\begin{aligned} u_{21}' &= \sum_{j=1}^{2} u_{2j1} \\ &= u_{211} + u_{221} \end{aligned}$$

$$= \frac{1}{70} + \frac{1}{70}$$
$$= \frac{1}{35} \text{ per time units}$$

So, the processing time of product 1 on the equivalent machine M_2', $P_{21}' = 35$ Time units. The processing time of product 2 on the equivalent machine M_2', P_{22}' can be calculated in the same way.

Calculation of the Equivalent Set-up Time

From Table B.1 of Appendix - B:

Set-up time of machine M_{21} for product 1, $S_{211} = 225$ Time units

Set-up time of machine M_{22} for product 1, $S_{221} = 225$ Time units

Processing time of product 1on M_1, $P_{11} = 80$ Time units

From equation (6.2), set-up time of the equivalent machine M_2' for product 1 is:

$$S_{ik}' = \frac{\sum_{j=1}^{n}\left[S_{ijk} + (j-1)P_{(i-1)k}\right]}{n}$$
$$\Rightarrow S_{21}' = \frac{\sum_{j=1}^{2}\left[S_{2j1} + (j-1)P_{(2-1)1}\right]}{2}$$
$$= \frac{S_{211} + (1-1)P_{(2-1)1} + S_{221} + (2-1)P_{(2-1)1}}{2}$$
$$= \frac{225 + 0 + 225 + 80}{2}$$
$$= 265 \text{ Time units}$$

The set-up time of the equivalent machine M_2' for product 2, S_{22}' can be calculated in the same way.

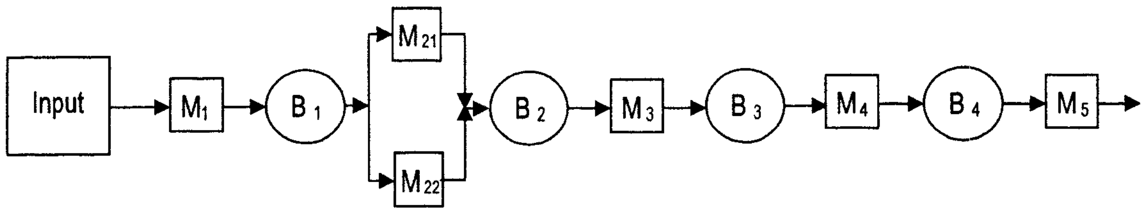

Figure 6.5: Series-parallel line made of five work centres and four buffers with two parallel machines at the first bottleneck work centre (SPL-1)

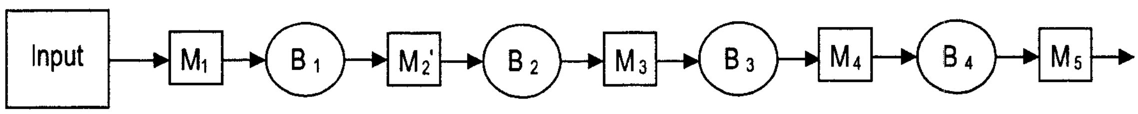

Figure 6.6: Equivalent serial line of SPL-1 replacing parallel machines by an equivalent machine (ESL-1)

We extend the simulation model developed for the serial system (OSL) to the series-parallel system (SPL-1) and its equivalent serial system (ESL-1). A designed experiment composed of four buffers of equal sizes and their many levels as applied in Section 6.4.1 is conducted on SPL-1 and ESL-1 to find the minimum cycle time. ESL-1 incorporates the proposed approximation method with simulation and SPL-1 represents the simulation of the original series-parallel line. So, we simulate both the SPL-1 and ESL-1 to compare the results of the proposed method (ESL-1) to those obtained from the SPL-1. The parameters applied in the experiments of SPL-1 and ESL-1 are presented in Section B.1.1 and B.1.2 respectively of Appendix - B. Simulation results for the cycle times are shown in Table 6.12. The trade-offs between the cycle times and the buffer sizes are plotted in Figure 6.7. Additional information of the work centre performance is presented in Table 6.13.

Buffer Size (Units)	Cycle Time (Min)			Bottleneck Work centre	Buffer Size (Units)	Cycle Time (Min)			Bottleneck Work centre
	SPL-1	ESL-1	% Difference			SPL-1	ESL-1	% Difference	
1	252.81	257.76	-1.96	M_1	18	200.66	200.47	0.09	M_5
2	237.49	240.56	-1.29	M_1	**22**	**199.91**	**199.78**	0.07	M_5
4	220.60	222.06	-0.66	M_1	26	199.85	199.87	-0.01	M_5
6	212.81	213.29	-0.23	M_5	30	199.92	199.98	-0.03	M_5
8	208.26	208.56	-0.14	M_5	34	199.97	199.87	0.05	M_5
10	205.53	205.44	0.04	M_5	38	199.89	199.88	0.01	M_5
14	202.02	201.93	0.04	M_5	42	199.79	199.78	0.01	M_5

Table 6.12: Comparison of cycle times of SPL-1 and ESL-1 for different buffer sizes

Table 6.12 shows that the cycle time has the maximum value at the smallest buffer size of 1 unit and decreases as the buffer sizes increase. When the buffer capacity reaches 22 units, the reduction in cycle time becomes insignificant and a further addition of buffer space does not contribute in minimizing the cycle time substantially. The minimum cycle time that could be achieved for this system is 199.78 minutes corresponding to the buffer size of 22 units. So, the contribution due to the incorporation of a parallel machine at the first bottleneck work centre of OSL are (a) reduction in cycle time = 220.92 –199.78 = 21.14 minutes, which is about 10% and (b) saving in buffer space = 34 – 22 = 12 units per buffer, which is about 35% of those of OSL (See Table 6.9 and Table 6.12).

Table 6.12 and Figure 6.7 compare the simulation results of cycle time as obtained from SPL-1 and ESL-1. As observed, the results are close and the maximum percentage difference is just below 2%. It is to be noted that this difference is high at a lower buffer level and as the buffer size increases the percentage difference approaches zero.

Simulation results of the additional information such as the percentages of the time the work centres are at their different states (Operation, Set-up, Starvation, Blocking and Down) in a cycle for SPL-1 and ESL-1 are also compared and found to be close (See Table B.13 and Table B.14 of Appendix - B).

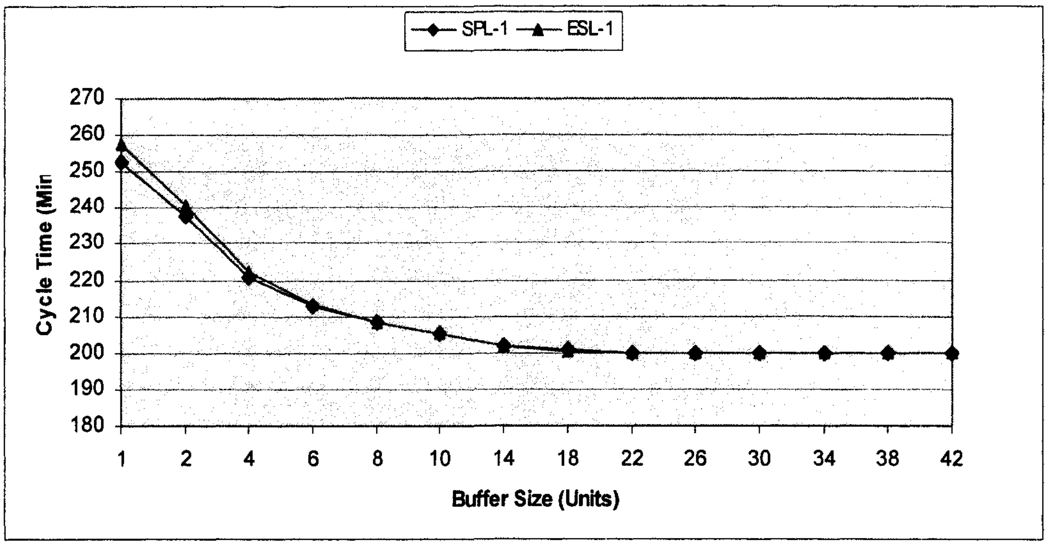

Figure 6.7: Cycle time versus buffer size (SPL-1 and ESL-1)

Work centre	Scheduled Hours	% Operation	% Set-up	% Starved	% Blocked	% Down
M_1	75,000	74.20	4.79	0.00	11.86	9.15
M_2'	75,000	43.69	3.59	**8.26**	**37.00**	7.46
M_3	75,000	70.62	4.39	1.63	15.55	7.81
M_4	75,000	68.82	3.99	2.44	17.37	7.38
M_5	75,000	81.98	3.19	4.26	0.00	10.56

Table 6.13: Simulation results of work centre performance for ESL-1 for buffer sizes of 8 units

The minimum cycle time that could be achieved for this system (ESL-1) is 199.78 minutes corresponding to the buffer size of 22 units. An addition of buffer space beyond this point will not reduce the cycle time, rather this buffer space will remain unused. A further reduction in the cycle time of this system requires adding a machine in the next bottleneck work centre in parallel to the existing machine. Simulation results show that work centre M_5 takes the maximum amount of time to process the product mix and hence it is the bottleneck work centre (Table 6.12). So, a machine may be added in parallel to the existing machine in the work centre M_5 in order to further reduce the cycle time.

6.4.2.2 Series-Parallel Line with Two Work Centres Composed of Two Machines in Parallel

It is observed from Table 6.10 that the bottleneck work centre M_2 in OSL is starved and blocked only for 3.05% and 1.23% respectively. As a machine is incorporated at this work centre, they both increase to 8.26% and 37.00% which is shown in Table 6.13. Table 6.13 also shows that the equivalent machine M_2' operates only for 43.69%, which is low as compared to the other work centres in the line. This implies that the capacity of the equivalent machine M_2' is much higher than that is required for this line. As a result, a portion of the capacity is used and the rest is wasted in blocking and starvation. To minimize the further wastage of machine capacity, a smaller machine is added at the second bottleneck work centre M_5 that processes at half the speed of the other machine of the work centre. We refer to this new line as SPL-2 (Figure 6.8) and its equivalent serial line as ESL-2 (Figure 6.9).

Parallel machines at work centre M_5 of SPL-2 are replaced by an equivalent machine M_5'. The parameters defining the equivalent machine M_5' are determined by the equations

(6.1), (4.32), (4.33) and (6.2). Calculations of the equivalent processing rate and the set-up time are shown below:

Calculation of the Equivalent Processing Rate

From Table B.7 of Appendix - B:

Processing time of product 1 on M_{51}, P_{511} = 65 Time units

Processing time of product 1 on M_{52}, P_{521} = 130 Time units

So,

Processing rate of product 1 on M_{51}, $u_{511} = \dfrac{1}{65}$ per time units

Processing rate of product 1 on M_{52}, $u_{521} = \dfrac{1}{130}$ per time units

From equation (6.1), processing rate of product 1 on the equivalent machine M_5' is:

$$\begin{aligned} u_{51}' &= \sum_{j=1}^{2} u_{5j1} \\ &= u_{511} + u_{521} \\ &= \frac{1}{65} + \frac{1}{130} \\ &= \frac{3}{130} \text{ per time units} \end{aligned}$$

So, the processing time of product 1 on the equivalent machine M_5', P_{51}' = 43.34 time units. The processing time of product 2 on the equivalent machine M_5', P_{52}' can be calculated in the same way.

Calculation of the Equivalent Set-up Time

From Table B.7 of Appendix - B:

Set-up time of machine M_{51} for product 1, S_{511} = 200 Time units

Set-up time of machine M_{52} for product 1, S_{521} = 200 Time units

The processing time of product 1on M_4, P_{41} = 75 Time units

From equation (6.2), the set-up time of the equivalent machine M_5' for product 1 is:

$$
\begin{aligned}
S_{ik}' &= \frac{\sum_{j=1}^{n}\left[S_{ijk} + (j-1)P_{(i-1)k}\right]}{n} \\
\Rightarrow S_{51}' &= \frac{\sum_{j=1}^{2}\left[S_{5j1} + (j-1)P_{(5-1)1}\right]}{2} \\
&= \frac{S_{511} + (1-1)P_{(5-1)1} + S_{521} + (2-1)P_{(5-1)1}}{2} \\
&= \frac{200 + 0 + 200 + 75}{2} \\
&= 237.5 \text{ Time units}
\end{aligned}
$$

Set-up time of the equivalent machine M_5' for product 2, S_{52}' can be calculated in the same way.

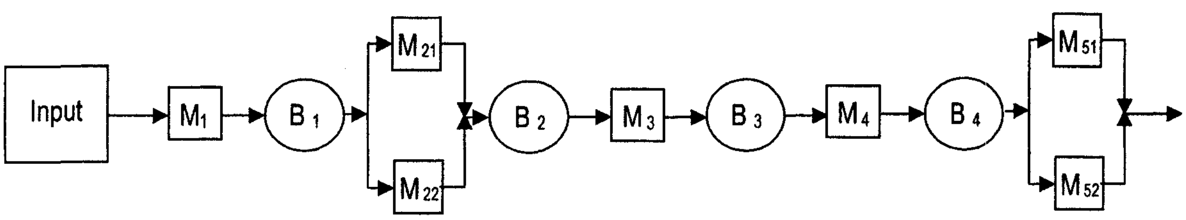

Figure 6.8: Series-parallel line made of five work centres and four buffers with the first two bottleneck work centres composed of two parallel machines each (SPL-2)

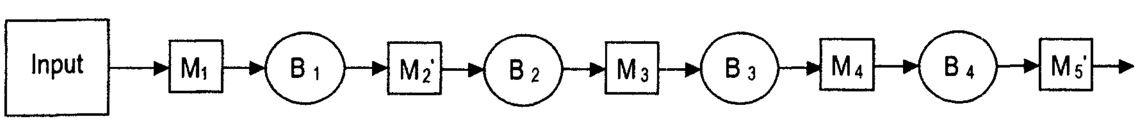

Figure 6.9: Equivalent serial line of SPL-2 replacing parallel machines by equivalent machines (ESL-2)

We apply the same approach and follow the same experimental design as described in Section 6.4.2.1 to tackle the lines SPL-2 and ESL-2 and to perform related experiments. The parameters applied in the experiments of SPL-2 and ESL-2 are presented in Section B.2.1 and B.2.2 respectively of Appendix - B. Simulation results for the cycle times are shown in Table 6.14. The trade-offs between the cycle times and the buffer sizes are plotted in Figure 6.10.

Table 6.14 shows that the cycle time is the maximum at the smallest buffer size of 1 unit and decreases as the buffer size increases. When the buffer capacity reaches 22 units, the reduction in cycle time stops and a further addition of buffer space does not contribute to cycle time reduction. The minimum cycle time that could be achieved for this system is 184.18 minutes corresponding to the buffer size of 22 units. So, the contribution due to the incorporation of parallel machines at work centres M_2 and M_5 of OSL are (a) reduction in cycle time = 220.92 – 184.18 = 36.74 minutes, which is about 17% and (b) reduction in buffer capacity = 34 – 22 = 12 units, which is about 35% of those of OSL (See Table 6.9 and Table 6.14).

Buffer Size (Units)	**Cycle Time** (Min)			**Buffer Size** (Units)	**Cycle Time** (Min)		
	SPL-2	**ESL-2**	**% Difference**		**SPL-2**	**ESL-2**	**% Difference**
1	230.97	238.95	-3.45	18	184.48	184.55	-0.04
2	217.45	222.45	-2.30	**22**	**184.11**	**184.18**	**-0.04**
4	202.72	205.04	-1.14	26	184.01	184.09	-0.04
6	196.52	197.72	-0.61	30	183.99	183.99	0.00
8	192.31	193.12	-0.42	34	183.96	183.95	0.01
10	189.44	189.87	-0.23	38	183.93	184.00	-0.04
14	185.91	186.20	-0.16	42	184.02	184.01	0.01

Table 6.14: Comparison of cycle times of SPL-2 and ESL-2 for different buffer sizes

Table 6.14 and Figure 6.10 compare the simulation results of the cycle time as obtained from SPL-2 and ESL-2. As observed, the results are close and the maximum difference is 3.45%. It is to be noted that this difference is high at lower buffer level and as the buffer size increases the percentage difference approaches zero. Simulation results of the additional information such as the percentages of the time the work centres are at their different states (Operation, Set-up, Starvation, Blocking and Down) in a cycle for SPL-2 and ESL-2 are also compared and found to be close (See Table B.15 and Table B.16 of Appendix - B).

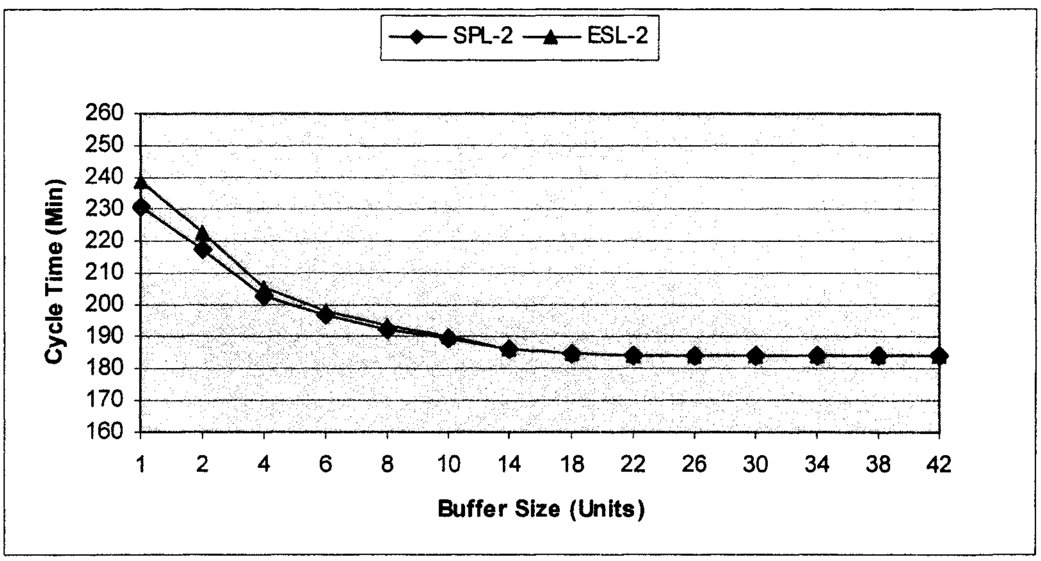

Figure 6.10: Cycle time versus buffer size (SPL-2 and ESL-2)

Based on the experiments performed so far, it is clear that the incorporation of parallel machine(s) contributes in reducing the cycle time as well as the buffer capacity. This gives us the motivation to perform some analyses of the results which is presented in the Section 6.5 below.

6.5 Analysis of Results

The results of Section 6.4.1 as presented in Table 6.9 show that the variation of the buffer size significantly impact the cycle time of multi-product production lines. Introducing buffer space decouples successive work centres, reduces their blocking and starvation times and thus lowers the cycle time. The maximum cycle time that could be reduced for a five work centre and four buffer serial line (OSL) is (281.96 – 220.92) = 61.04 minutes and this is the maximum level that the equal buffer can contribute to this configuration. The reduction in cycle time is higher at lower buffer level and decreases as the buffer capacity increases which is in conformity to the existing literature. For example, for OSL, the reduction in cycle time is (281.96 – 262.98) = 18.98 minutes, as the buffer size is increased from 1 unit to 2 units. On the other hand, the reduction is only (223.99 – 222.22) = 1.77 minutes as the buffer size increases from 14 units to 18 units. The minimum cycle time for this system is 220.92 minutes corresponding to the buffer size of 34 units. It means that the buffer capacity of 34 units is large enough to decouple the work centres.

The results of Section 6.4.2 show that the incorporation of parallel machine also contributes in reducing the cycle time of multi-product production lines. When a machine is added to the first bottleneck work centre of OSL and buffer capacity is varied between 1 unit and 42 units, the average reduction in cycle time is 22 minutes which is approximately 9.5 % of that of OSL (Calculated based on the results presented in Table 6.9 and Table 6.12). Similarly, as a parallel machine is added to the second bottleneck work centre of OSL, the average reduction in cycle time increases to 38 minutes which is approximately 16.5% of that of OSL (Calculated based on the results presented in Table 6.9 and Table 6.14).

The cycle time comparison between the series-parallel system and their equivalent serial system shows that the maximum percentage difference in cycle time between SPL-1 and ESL-1 is less than 2 % (Table 6.12) and between SPL-2 and ESL-2 is 3.45 % (Table 6.14). The higher difference is observed at a lower buffer level. As the buffer size increases the percentage difference approaches zero. It is to be mentioned that the equivalent serial lines, ESL-1 and ESL-2 are derived from the series-parallel lines SPL-1 and SPL-2 respectively using the proposed approximation method. We recall that the proposed method is limited in accuracy when the availability of the multiple machines at a work centre is less than that of the other work centres and the adjacent buffers are low in capacity. This is one of the possible reasons of the high percentage difference at lower buffer capacity.

Adding identical machine(s) is common in industries. It gives flexibility in control, operation and as well as in maintenance. If the work centres in a production line have approximately the equal capacity among them, adding identical machines at some work centres may result in excessive or lost capacity that does not meet the requirements of efficiency as explained in Section 6.4.2.2. Moreover, the cost associated with a work centre is very high. On the other hand, if the machines are non-identical, there exists a control issue in allocating the work among the non-identical machines at a work centre.

We conclude this section with the analysis of a practical problem. For example, a manager requires that the cycle time of a multi-product production system composed of five work centres and four buffers to be 222 minutes. To get the appropriate solution of the problem from the experiments we have conducted in this chapter, we plot three curves showing the trade-offs between the cycle times and the buffer sizes for the three configurations OSL, ESL-1 and ESL-2. Figure 6.11 shows these curves.

From Figure 6.11, we find that there are three possible solutions for the problem described above.

Solution 1: OSL with 18 units of buffer capacity

Solution 2: ESL-1 with 4 units of buffer capacity

Solution 3: ESL-2 with 2 units of buffer capacity

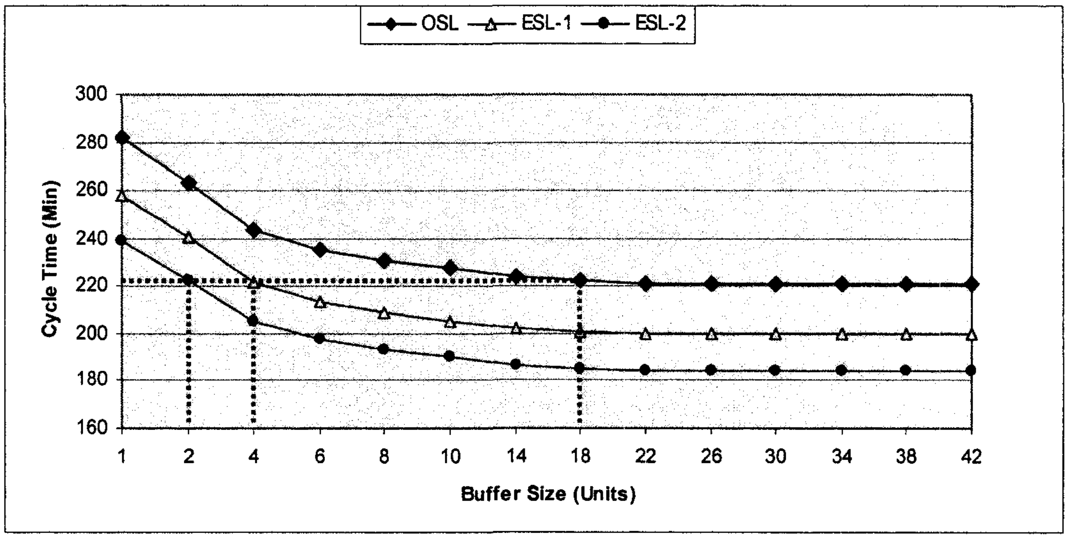

Figure 6.11: Cycle time versus buffer size (OSL, ESL-1 and ESL-2)

The selection of the appropriate solution among these three is not very straightforward and rather depends on many issues such as the cost of buying machines or buffers, availability of floor space to locate the larger buffers or the new machine(s), cost related to the control, operation, and maintenance of the new machine(s), cost related to work-in-process inventory etc. So, a long term economic analysis considering all possible factors will allow the manager to come to the best solution among the three.

6.6 Conclusions

Production systems are often organized with the work centers connected in series, separated by buffers and limited to the analysis of a single product. Much of the analysis is aimed at evaluating the performance of a system with specified work centers and buffers. Analyses are performed to select the minimal buffer space to achieve a higher performance. They all assume that the number of machines is specified, and the only parameters to find are buffers' sizes (Conway et al., 1988; Powell and Pyke, 1996).

This chapter solves a design problem of a series-parallel production system where parallel machines and buffers are considered to achieve a greater performance in terms of cycle time. This system can produce more than one product type and the set-up time of the machines is considered as the product type changes. The series-parallel production system is tackled with simulation by incorporating an analytical approximation method. This approach allows buffers and machines of different parameters to be included in the production system and the design aims at the selection of both buffers and machines simultaneously in reducing the cycle time.

Results show that the accuracy of the proposed approximation method is reasonable and the developed model can be applied to other multi-product line configurations. Both the increase of buffer capacity and the incorporation of parallel machine contribute in minimizing the cycle time. But the use of parallel redundancy should be restricted to the situations where work centres have a largely unequal capacity or are performing almost at their full capacity to avoid any excessive or lost capacity.

CHAPTER 7
CONCLUSIONS AND FUTURE WORK RECOMMENDATIONS

7.1 Conclusions

Manufacturing industries are facing extreme challenge and competition due to the globalization of markets. To face this challenge, manufacturers are striving to maximize the production output applying various appropriate strategies. The incorporation of multiple machines at stages of flow line system is a way that helps to increase the productivity and hence is becoming common in industries. This type of production system is known as series-parallel system. But the solution of series-parallel system is not easy and the developed methods are observed to be not accurate. So, in this thesis, we developed a new analytical approximation method to replace a set of parallel machines by an equivalent machine in a series-parallel flow line with finite buffer. We developed our method based on discrete state Markov chain. The proposed technique replaces a set of parallel machines at a work centre by an equivalent machine in order to obtain a traditional flow line with machines in series separated by intermediate buffers, solutions of which already exist. We derived equations for the parameters of the equivalent machine when it operates in isolation as well as in flow line.

We have conducted numerous experiments with various configurations of series-parallel flow lines in order to check the accuracy and sensitivity of our proposed method. It is evident from the results that the accuracy of the proposed method to estimate the production rate (Q) for almost all the cases is high with a variation of less than 4%. The less accurate results are observed when the availability of the multiple machines at a stage is less than that of the other stages and the adjacent buffers are too small to accommodate as much material as produced during an average failure period. However, the number of parallel machines either identical or non-identical at a stage does not reduce the accuracy of the proposed method. Accuracy of the method is not also affected by the length of the flow line and assignment of multiple machines at more than one stage. We also notice that the results of EWIso and EWIn are exactly the same for the flow lines with either identical or non-identical machines in parallel. This proves the correctness of the derived equations and the accuracy of the proposed method. Based on the analysis and comparative study, we can conclude that the proposed method is easy to implement and comparable with the existing methods.

The existing analytical methods for series-parallel systems are in general limited to lines with a maximum of two machines in series. Our proposed method can be used in conjunction with an approximation method or simulation to solve flow lines of any length. The proposed method can be used to design manufacturing systems as well as to evaluate the performance of an existing flow line based on actual data collected on it. Also they can be used to compute the probability of idleness of the parallel machines in a set when needed.

Again, the continuously changing and refined customer needs dictate and control the launch of new products. Nowadays, product introductions and changes are occurring so rapidly that the engineers are faced with increasing pressure to reduce development cycles to keep the companies profitable. But flow lines are of significant economic importance and involve large investments for most companies. So, there is a need for designing more flexible and efficient production lines that can produce more than one product type on the same line.

We have proposed an approximation method that can be used for multi-product series-parallel system to improve its performance. The system is capable of manufacturing more than one product type with predefined sequence and lot size. Work centres of the line have operation dependent failures, deterministic processing times and require set-up as the product type changes. The series-parallel system is represented with a simulation-based model that incorporates the proposed analytical approximation method to solve the problem of multiple machine stage. Both parallel machines and interstage buffers are considered to achieve a greater output. The specific goal is to minimize the cycle time. This approach allows buffers and machines of different parameters to be included in the production system and the design aimed at the selection of both buffers and machines simultaneously to reduce the cycle time.

We have analyzed the effects of buffer and number of machines in parallel on the performance of series-parallel systems. Results show that both the increase of buffer capacity and the incorporation of parallel machine contribute in minimizing the cycle time. The accuracy of the proposed approximation method is affected by the buffer capacity. The accuracy is low at lower buffer level and high at upper buffer level.

This multi-product series-parallel model outlined in this thesis is representative of a real world production system and can be easily applied in the design and analysis of other real manufacturing situations in determining the most economical production line configuration.

7.2 Future Work Recommendations

The proposed method is limited in accuracy when the availability of the multiple machines at a stage is less than that of the other stages due to higher failure rate and the adjacent buffers are too small to accommodate as much material as produced during an average failure. This small buffer also impacts the accuracy of the method in estimating the average buffer contents to some extent. So, there is a need for further research to develop a model that could overcome these limitations. Moreover, there is a scope for further studies of a series-parallel system considering the impact of preventive maintenance to minimize the cycle time.

It has been observed from the research that the performance of a manufacturing system improves as more and more machines in parallel are considered. Also, the addition of intermediate buffers provides similar benefits. So, practical design questions may arise such as (a) if an increase in machine redundancy or an addition of intermediate buffers provides the same benefits to a line, what would be the appropriate decision to adopt or (b) a combination of machine redundancy and buffer will be more effective? So, an optimal performance evaluation model that focuses on the economic tradeoffs of adding buffer and increasing the degree of parallelism of the system is needed.

There exists a lot of research in developing approximation methods such as deterministic processing time model (deterministic processing time, geometrically distributed MTBF and MTTR), exponential model (exponentially distributed processing time, MTTF and MTTR) and continuous model (processed material is treated as a continuous fluid) for various manufacturing systems (Gershwin, 1994). The accuracy of the methods is somewhat limited because of a trade-off between complexity and accuracy. It would be useful to have a systematic study of the accuracy of the different approximation methods. Moreover, the methods developed so far are for the flow lines with work centres in series and separated by buffers. There is no such approximation method that could be directly used for the evaluation of a serial line having multiple machines in parallel at stages. So, there is a need for developing an approximation method for series- parallel system.

APPENDIX - A

We performed a number of experiments to test the accuracy and sensitivity of the model developed in Chapter 4 under various specific conditions. In Appendix - A, we present the parameters used in the examples illustrated in Chapter 5.

A.1 Identical Machines in Parallel

The following (Case 1 to Case 10) is a listing of each of the test cases provided in Patchong and Willaeys (2001). Case 1, Case 3 and Case 5 among them are the test cases provided in Burman (1995). Case 11 and Case 12 are the additional cases used in this thesis.

A.1.1 Two Identical Machines in Parallel

In Section A.1.1, we present the parameters used in the examples shown in Section 5.2.1 where the second stage of the three stage flow line consists of two identical parallel machines.

Case 1					
	i	1	21	22	3
Parameters of M_i	μ_i	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.010	0.010
	u_i	1.000	1.000	1.000	1.000
	Po_i	n / a	0.4361	0.4357	n / a
	C_i	10	n / a	10	n / a

Table A.1: Parameters used in Case 1

Case 2					
	i	1	21	22	3
Parameters of $\boldsymbol{M_i}$	μ_i	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.010	0.010
	u_i	1.000	1.000	1.000	1.000
	Po_i	n / a	0.4230	0.4231	n / a
	C_i	**2**	n / a	**2**	n / a

Table A.2: Parameters used in Case 2

Case 3					
	i	1	21	22	3
Parameters of $\boldsymbol{M_i}$	μ_i	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.010	0.010
	u_i	1.000	**0.500**	**0.500**	1.000
	Po_i	n / a	0.8323	0.8322	n / a
	C_i	10	n / a	10	n / a

Table A.3: Parameters used in Case 3

Case 4					
	i	1	21	22	3
Parameters of $\boldsymbol{M_i}$	μ_i	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.010	0.010
	u_i	1.000	**0.500**	**0.500**	1.000
	Po_i	n / a	0.7911	0.7912	n / a
	C_i	**2**	n / a	**2**	n / a

Table A.4: Parameters used in Case 4

Case 5						
	i	1	21	22	3
Parameters of M_i	μ_i	0.100	0.100	0.100	0.100
	λ_i	0.010	**0.120**	**0.120**	0.010
	u_i	1.000	1.000	1.000	1.000
	Po_i	n / a	0.3855	0.3855	n / a
	C_i	10	n / a	10	n / a

Table A.5: Parameters used in Case 5

Case 6					
	i	1	21	22	3
Parameters of M_i	μ_i	0.100	0.100	0.100	0.100
	λ_i	0.010	**0.120**	**0.120**	0.010
	u_i	1.000	1.000	1.000	1.000
	Po_i	n / a	0.3535	0.3529	n / a
	C_i	**2**	n / a	**2**	n / a

Table A.6: Parameters used in Case 6

A.1.2 Five Identical Machines in Parallel

In Section A.1.2, we present the parameters used in the examples shown in Section 5.2.1 where the second stage of the three stage flow line consists of five identical parallel machines.

Case 7								
	i	1	21	22	23	24	25	3
Parameters of M_i	μ_i	0.100	0.100	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.010	0.010	0.010	0.010	0.010
	u_i	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	Po_i	n / a	0.1752	0.1751	0.1751	0.1751	0.1752	n / a
	C_i	10	n / a	n / a	n / a	n / a	10	n / a

Table A.7: Parameters used in Case 7

Case 8								
	i	1	21	22	23	24	25	3
Parameters of M_i	μ_i	0.100	0.100	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.010	0.010	0.010	0.010	0.010
	u_i	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	Po_i	n / a	0.1711	0.1711	0.1711	0.1711	0.1711	n / a
	C_i	**2**	n / a	n / a	n / a	n / a	**2**	n / a

Table A.8: Parameters used in Case 8

Case 9								
	i	1	21	22	23	24	25	3
Parameters of M_i	μ_i	0.100	0.100	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.010	0.010	0.010	0.010	0.010
	u_i	1.000	**0.200**	**0.200**	**0.200**	**0.200**	**0.200**	1.000
	Po_i	n / a	0.8377	0.8378	0.8380	0.8381	0.8375	n / a
	C_i	10	n / a	n / a	n / a	n / a	10	n / a

Table A.9: Parameters used in Case 9

Case 10								
	i	1	21	22	23	24	25	3
Parameters of M_i	μ_i	0.100	0.100	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.010	0.010	0.010	0.010	0.010
	u_i	1.000	**0.200**	**0.200**	**0.200**	**0.200**	**0.200**	1.000
	Po_i	n / a	0.7949	0.7944	0.7949	0.7950	0.7946	n / a
	C_i	**2**	n / a	n / a	n / a	n / a	**2**	n / a

Table A.10: Parameters used in Case 10

Case 11								
	i	1	21	22	23	24	25	3
Parameters of M_i	μ_i	0.100	0.100	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	**0.120**	**0.120**	**0.120**	**0.120**	**0.120**	0.010
	u_i	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	Po_i	n / a	0.1748	0.1749	0.1747	0.1746	0.1744	n / a
	C_i	10	n / a	n / a	n / a	n / a	10	n / a

Table A.11: Parameters used in Case 11

Case 12								
	i	1	21	22	23	24	25	3
Parameters of M_i	μ_i	0.100	0.100	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	**0.120**	**0.120**	**0.120**	**0.120**	**0.120**	0.010
	u_i	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	Po_i	n / a	0.1702	0.1703	0.1700	0.1701	0.1701	n / a
	C_i	**2**	n / a	n / a	n / a	n / a	**2**	n / a

Table A.12: Parameters used in Case 12

In addition to the 10 Patchong and Willaeys (2001) experimental cases, we tested some other cases in Chapter 5 to validate our method. These are listed below:

A.2 Non-Identical Machines in Parallel

A.2.1 Two Non-Identical Machines in Parallel

In Section A.2.1, we present the parameters used in the examples shown in Section 5.2.2 where the second stage of the three stage flow line consists of two non-identical parallel machines.

Case 13					
	i	1	21	22	3
Parameters of M_i	μ_i	0.100	0.200	0.100	0.100
	λ_i	0.010	0.010	0.020	0.010
	u_i	1.000	1.000	1.000	1.000
	Po_i	n / a	0.4617	0.4097	n / a
	C_i	10	n / a	10	n / a

Table A.13: Parameters used in Case 13

Case 14					
	i	1	21	22	3
Parameters of M_i	μ_i	0.100	0.200	0.100	0.100
	λ_i	0.010	0.010	0.020	0.010
	u_i	1.000	1.000	1.000	1.000
	Po_i	n / a	0.4491	0.3987	n / a
	C_i	**2**	n / a	**2**	n / a

Table A.14: Parameters used in Case 14

A.2.2 Five Non-Identical Machines in Parallel

In Section A.2.2, we present the parameters used in the examples shown in Section 5.2.2 where the second stage of the three stage flow line consists of five non-identical parallel machines.

Case 15								
	i	1	21	22	23	24	25	3
Parameters of M_i	μ_i	0.100	0.200	0.100	0.100	0.200	0.400	0.100
	λ_i	0.010	0.010	0.020	0.010	0.020	0.030	0.010
	u_i	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	Po_i	n / a	0.1772	0.1715	0.1752	0.1753	0.1765	n / a
	C_i	10	n / a	n / a	n / a	n / a	10	n / a

Table A.15: Parameters used in Case 15

Case 16								
	i	1	21	22	23	24	25	3
Parameters of M_i	μ_i	0.100	0.200	0.100	0.100	0.200	0.400	0.100
	λ_i	0.010	0.010	0.020	0.010	0.020	0.030	0.010
	u_i	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	Po_i	n / a	0.1731	0.1677	0.1712	0.1713	0.1724	n / a
	C_i	**2**	n / a	n / a	n / a	n / a	**2**	n / a

Table A.16: Parameters used in Case 16

A.3 Parallel Machines at Multiple Stages

In Section A.3, we present the parameters used in the examples shown in Section 5.2.3 where the second and fourth stages of the five stage flow line consist of three non-identical and three identical parallel machines respectively.

Case 17										
	i	1	21	22	23	3	41	42	43	5
Parameters of M_i	μ_i	0.100	0.100	0.100	0.200	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.020	0.020	0.010	0.010	0.010	0.010	0.010
	u_i	1.000	0.700	0.700	0.700	1.000	0.800	0.800	0.800	1.000
	Po_i	n / a	0.3822	0.3618	0.3831	n / a	0.3489	0.3488	0.3489	n / a
	C_i	2	n / a	n / a	2	2	n / a	n / a	2	n / a

Table A.17: Parameters used in Case 17

Case 18										
	i	1	21	22	23	3	41	42	43	5
Parameters of M_i	μ_i	0.100	0.100	0.100	0.200	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.020	0.020	0.010	0.010	0.010	0.010	0.010
	u_i	1.000	0.700	0.700	0.700	1.000	0.800	0.800	0.800	1.000
	Po_i	n / a	0.3985	0.3766	0.3991	n / a	0.3635	0.3639	0.3631	n / a
	C_i	7	n / a	n / a	7	7	n / a	n / a	7	n / a

Table A.18: Parameters used in Case 18

Case 19										
	i	1	21	22	23	3	41	42	43	5
Parameters of M_i	μ_i	0.100	0.100	0.100	0.200	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.020	0.020	0.010	0.010	0.010	0.010	0.010
	u_i	1.000	0.700	0.700	0.700	1.000	0.800	0.800	0.800	1.000
	Po_i	n / a	0.4098	0.3875	0.4108	n / a	0.3739	0.3741	0.3738	n / a
	C_i	**15**	n / a	n / a	**15**	**15**	n / a	n / a	**15**	n / a

Table A.19: Parameters used in Case 19

Case 20										
	i	1	21	22	23	3	41	42	43	5
Parameters of M_i	μ_i	0.100	0.100	0.100	0.200	0.100	0.100	0.100	0.100	0.100
	λ_i	0.010	0.010	0.020	0.020	0.010	0.010	0.010	0.010	0.010
	u_i	1.000	0.700	0.700	0.700	1.000	0.800	0.800	0.800	1.000
	Po_i	n / a	0.4143	0.3914	0.4153	n / a	0.3780	0.3778	0.3779	n / a
	C_i	**20**	n / a	n / a	**20**	**20**	n / a	n / a	**20**	n / a

Table A.20: Parameters used in Case 20

A.4 Proofs of Derived Equations

In Section A.4, we present the parameters used in the example shown in Section 5.3 where the second stage of the three stage flow line consists of two non-identical parallel machines.

Case 21					
	i	1	21	22	3
Parameters of M_i	μ_i	0.100	0.200	0.100	0.100
	λ_i	0.010	0.010	0.020	0.010
	u_i	1.000	1.000	1.000	1.000
	C_i	10	n / a	10	n / a

Table A.21: Parameters used in Case 21

APPENDIX - B

In Appendix - B, we present the parameters used in the experiments of case study illustrated in Chapter 6.

B.1 Series-Parallel Line with Parallel Machines at One Work Centre

B.1.1 SPL-1: Series-Parallel Line with Two Machines in Parallel at the First Bottleneck Work Centre

In this section, we present the parameters used in the experiments shown in Section 6.3.2.1, where the line is composed of five work centres and four buffers with two machines in parallel at the first bottleneck work centre. Buffer capacity is varied between 1 unit and 42 units.

	Product	Work centre					
		1	**21**	**22**	**3**	**4**	**5**
Processing Time	**1**	80	70	70	60	75	65
	2	60	90	90	70	55	85
Set-up Time	**1**	300	225	225	275	250	200
	2	300	225	225	275	250	200

Table B.1: Processing times and set-up times (Time unit)

	Product	Unit
Lot Size	**1**	60
	2	75

Table B.2: Lot sizes (Unit)

	Work centre					
	1	**21**	**22**	**3**	**4**	**5**
Failure rate	0.00059	0.00065	0.00065	0.00062	0.00074	0.00055
Repair rate	0.0047	0.0037	0.0037	0.0055	0.0067	0.0042

Table B.3: Failure and repair rates of work centres (Per time unit)

B.1.2 ESL-1: Equivalent Serial Line of SPL-1

In Section B.1.2, we present the parameters used in the experiments shown in Section 6.3.2.1, where the line is composed of five work centres and four buffers. In this case, the parallel machines at the first bottleneck work centre are replaced by an equivalent machine. Buffer capacity is varied between 1 unit and 42 units.

	Product	Work centre				
		1	**2′**	**3**	**4**	**5**
Processing Time	**1**	80	**35**	60	75	65
	2	60	**45**	70	55	85
Set-up Time	**1**	300	**265**	275	250	200
	2	300	**255**	275	250	200

Table B.4: Processing times and set-up times (Time unit)

	Product	**Unit**
Lot Size	**1**	60
	2	75

Table B.5: Lot sizes (Unit)

	Work centre				
	1	2′	3	4	5
Failure rate	0.00059	**0.0013**	0.00062	0.00074	0.00055
Repair rate	0.0047	**0.0074**	0.0055	0.0067	0.0042

Table B.6: Failure and repair rates of work centres (Per time unit)

B.2 Series-Parallel Line with Parallel Machines at Two Work Centres

B.2.1 SPL-2: Series-Parallel Line with Two Parallel Machines Each at the First Two Bottleneck Work Centres

In this section, we present the parameters used in the experiments shown in Section 6.3.2.2, where the line is composed of five work centres and four buffers with first two bottleneck work centres made of two parallel machines each. Buffer capacity is varied between 1 unit and 42 units.

	Product	Work centre						
		1	21	22	3	4	51	52
Processing Time	1	80	70	70	60	75	65	**130**
	2	60	90	90	70	55	85	**170**
Set-up Time	1	300	225	225	275	250	200	**200**
	2	300	225	225	275	250	200	**200**

Table B.7: Processing times and set-up times (Time unit)

	Product	Unit
Lot Size	1	60
	2	75

Table B.8: Lot sizes (Unit)

	Work centre						
	1	21	22	3	4	51	52
Failure rate	0.00059	0.00065	0.00065	0.00062	0.00074	0.00055	**0.00055**
Repair rate	0.0047	0.0037	0.0037	0.0055	0.0067	0.0042	**0.0042**

Table B.9: Failure and repair rates of work centres (Per time unit)

B.2.2 ESL-2: Equivalent Serial Line of SPL-2

In Section B.2.2, we present the parameters used in the experiments shown in Section 6.3.2.2, where the line is composed of five work centres and four buffers. In this case, the parallel machines at the first two bottleneck work centres are replaced by one equivalent machine each. Buffer capacity is varied between 1 unit and 42 units.

	Product	Work centre				
		1	2′	3	4	5′
Processing Time	1	80	35	60	75	**43.34**
	2	60	45	70	55	**56.67**
Set-up Time	1	300	265	275	250	**237.50**
	2	300	255	275	250	**227.50**

Table B.10: Processing times and set-up times (Time unit)

	Product	Unit
Lot Size	1	60
	2	75

Table B.11: Lot sizes (Unit)

	Work centre				
	1	**2′**	**3**	**4**	**5′**
Failure rate	0.00059	0.0013	0.00062	0.00074	**0.0011**
Repair rate	0.0047	0.0074	0.0055	0.0067	**0.0084**

Table B.12: Failure and repair rates of work centres (Per time unit)

B.3 Simulation Results of Work Centre Performance

B.3.1 SPL-1:

Work centre	Scheduled Hours	% Operation	% Set-up	% Starved	% Blocked	% Down
M_1	75,000	77.47	5.00	0.00	7.99	9.54
M_{21}	75,000	45.62	3.75	3.12	39.72	7.78
M_{22}	75,000	45.59	3.75	3.12	39.73	7.80
M_3	75,000	73.72	4.58	0.08	13.48	8.14
M_4	75,000	71.84	4.17	0.20	16.05	7.73
M_5	75,000	85.59	3.33	0.12	0.00	10.96

Table B.13: Simulation results of work centre performance for SPL-1 for buffer sizes of 22 units

B.3.2 ESL-1:

Work centre	Scheduled Hours	% Operation	% Set-up	% Starved	% Blocked	% Down
M_1	75,000	77.51	5.00	0.00	7.89	9.60
M_2'	75,000	45.63	3.75	3.59	39.22	7.81
M_3	75,000	73.76	4.58	0.09	13.43	8.13
M_4	75,000	71.88	4.17	0.23	15.96	7.76
M_5	75,000	85.64	3.33	0.11	0.00	10.91

Table B.14: Simulation results of work centre performance for ESL-1 for buffer sizes of 22 units

B.3.3 SPL-2:

Work centre	Scheduled Hours	% Operation	% Set-up	% Starved	% Blocked	% Down
M_1	75,000	84.13	5.43	0.00	0.09	10.35
M_{21}	75,000	49.54	4.07	31.90	6.03	8.46
M_{22}	75,000	49.52	4.07	31.91	6.03	8.48
M_3	75,000	80.06	4.97	5.65	0.48	8.84
M_4	75,000	78.02	4.52	9.04	0.02	8.39
M_{51}	75,000	58.02	3.62	30.95	0.00	7.41
M_{52}	75,000	69.87	3.62	17.72	0.00	8.79

Table B.15: Simulation results of work centre performance for SPL-2 for buffer sizes of 22 units

B.3.4 ESL-2:

Work centre	Scheduled Hours	% Operation	% Set-up	% Starved	% Blocked	% Down
M_1	75,000	84.09	5.43	0.00	0.11	10.37
M_2'	75,000	49.50	4.07	31.67	6.31	8.45
M_3	75,000	80.03	4.97	5.63	0.50	8.86
M_4	75,000	77.99	4.52	9.03	0.01	8.44
M_5'	75,000	61.98	3.62	26.49	0.00	7.90

Table B.16: Simulation results of work centre performance for ESL-2 for buffer sizes of 22 units

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