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How many premises can an argument have?

G C. Goddu
University of Richmond

David Hitchcock

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G.C. GODDU

Department of Philosophy
University of Richmond
28 Westhampton Way
USA
ggoddu@richmond.edu

ABSTRACT: Is it possible for an argument to have either zero premises or an infinite number of premises? I shall argue that regardless of how you conceive of arguments you should accept that an argument could have an infinite number of premises. The zero case is more complicated since the matter seems to depend not only on the metaphysics of arguments, but also the nature and function of arguing. I shall argue that at least a plausible case can be made for the possibility of zero premise arguments.

KEYWORDS: argument, infinite, premise, reasons, regress, zero

1. INTRODUCTION

Most definitions of argument are noncommittal when it comes to how many premises an argument can have. For example, consider what Douglas Walton calls the typical or conventional definition of argument: "An argument is a set of propositions, one of which is the conclusion and the remainder are premises." (Walton 1996: 3) For all this definition says, the remainder could be anything from zero to infinity. But some definitions or articulations do make a commitment. For example, Mark Vorobej writes: "We'll stipulate that each argument has a single conclusion and any finite number of premises greater than or equal to one." (Vorobej 2006: 8) Many authors of definitions of argument would, I suspect, accept Vorobej's exclusion of at least one, if not both, of zero and infinity as possible numbers of premises. Indeed, very few, if any, definitions exist that explicitly identify zero or infinity as allowable numbers of premises.[1]

In this paper I explore whether or not it is possible for an argument to have either zero premises or an infinite number of premises. I shall argue, in Section 2, that regardless of how you conceive of arguments you should accept that an argument could have an infinite number of premises. The zero case, which I turn to in Section 3, is more complicated since the matter seems to depend not only on the metaphysics of arguments, but also on how one answers questions about (a) the relationship between arguing and arguments and (b) whether the function of arguments or arguing is necessary for distinguishing arguments from non-arguments. I shall argue that certain plausible answers to (a) and (b) allow for the possibility of zero premise arguments.

[1] Terence Parsons (1996) explicitly describes what he calls a successful argument with an infinite number of steps, even though his definition of argument is noncommittal. Similarly, Roy Sorensen, in Sorensen 1999, explicitly argues for zero premise arguments, but never provides a definition of argument.

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2. CAN AN ARGUMENT HAVE INFINITE PREMISES?

Suppose arguments are composed of propositions. Define a *simple* argument as a set of a set of propositions (the premise set) and another proposition (the conclusion). Define a *complex* argument as a set of simple arguments. [Clearly this definition of complex arguments is inadequate for capturing what most people think of when they think of complex arguments, i.e., arguments composed of simple arguments joined in some way, since the definition says nothing about whether the arguments in the set are joined or not. For my purposes here it turns out to be irrelevant how the arguments are joined—it merely matters how many there are, so I sidestep the thorny issue of trying to articulate an adequate notion of how simple arguments may be joined to form complex arguments.] Given these definitions there are two questions. Firstly, is there any good reason to hold that the premise set of a simple argument cannot have an infinite number of propositions as members? Secondly, even if the answer to the first question is 'yes', is there any good reason to suppose that a complex argument could not have an infinite number of members? If either answer is 'no', then given that there are infinite sets of propositions and infinite sets of sets of propositions, there will be no good reason to say that an argument cannot have an infinite number of premises.

We can certainly *stipulate* that the sets in question be finite, but is there any reason to prohibit the infinite cases? Considered merely as abstract objects, the answer has to be 'no'. There *are* sets of premise sets and conclusions in which the premise set contains zero members, some finite number of members, and at least denumerably infinite members. Logicians and mathematicians make use of these sets to prove various theorems such as the completeness of various logical systems. If there is a reason for excluding the infinite case, then that reason must concern something external to the nature of propositions and sets.

Why might one claim that the finite cases are arguments, but the infinite cases are not? Perhaps, to count as an argument we must be able to express it, and we cannot express infinite sets of propositions. Alternatively, perhaps to count as an argument there must be a corresponding act of arguing, and we, finite beings that we are, just cannot argue with an infinite number of premises.

Consider the latter claim—we restrict the sets of propositions that count as arguments because we cannot argue using an infinite set of premises. Presumably such an appeal would also be a reason for someone who holds that arguments are complex acts, rather than sets of propositions, to restrict which potential complex acts could count as arguments. Given that arguing is a kind of act, what we can and cannot argue is an internal matter with respect to the position that arguments are complex acts. Hence, if the reason can be shown to fail, even in the case in which arguments are taken to be complex acts, then it will certainly fail as a reason to restrict the sets of propositions that count as arguments. Similarly for the former claim—if what can and cannot be expressed fails to exclude infinite premise arguments when arguments are taken to be sentences or expressions in some language, then it will certainly fail as a reason for restricting which sets of propositions count as arguments.

So suppose we take arguments to be sets or groups of sentences.[2] If arguments are taken to be sets of sentences, then, just as in the case of sets of propositions, there will be no reason, with respect to the nature of sets, to restrict arguments to having finite premises. Just as there are infinite sets of propositions, there are infinite sets of sentences (at least sentence types, which are just another kind of abstract object). So any reason to restrict which sets of sentences count as arguments would have to appeal to restrictions on sentence tokens or to restrictions on what we can or cannot do with sentences.

Sentences, at least sentence tokens, are not abstract objects. As finite beings, at best we can only produce texts composed of finite sentence tokens, so it is just impossible for us to provide or use a group of sentences that has an infinite number of members. But is this mere practical limitation enough to exclude the case of an infinite number of sentences being part of or constituting an argument? God has no such limitation, and so God could express or write down an argument with an infinite number of sentences as premises. (God might have to alter the structure of our universe first though—our current best estimates for the number of atoms in the universe is 10^{80}, which, while quite large, is far from infinite. So, even assuming God could use a single atom to write a premise, at the very least God could not have all the premises written down at the same time—though as long as God could perform supertasks, he could reuse atoms and still write the entire argument down in a finite amount of time.) Regardless, I will not pursue this line of defence further since I am going to challenge the claim that we do not in fact provide or make use of texts that express arguments composed of an infinite number of sentences.

Being finite beings we do not actually write down infinite premise arguments—at least such arguments in which we token every premise individually. But we do have the expressive means to make clear that we are in fact deploying an argument with an infinite number of sentences. As Terence Parsons puts it: "Even if a text containing an argument must, by the nature of a text, be finite, this would not extend to refined arguments embodied in the text." (Parsons 1996: 172, n9) Consider, for example, the following argument:

(A) 1 can be paired with 2.
2 can be paired with 4.
3 can be paired with 6.
.
.
. ______________________________________
Hence, the natural numbers can be in one-to-one correspondence with the even numbers.

Why think (A) is an argument with an infinite number of premises? Because it is the same argument as both:

[2] Some might include statements as a possibility here. I avoid the use of 'statement' since it is ambiguous between what is stated (which could be the actual words used, or the proposition expressed by those words), or the act of uttering or writing the words.

(B) 1 can be paired with 2.
2 can be paired with 4.
3 can be paired with 6.
4 can be paired with 8.
.
.
. ______________________________________
Hence, the natural numbers can be in one-to-one correspondence with the even numbers.

and

(C) 1 can be paired with 2.
2 can be paired with 4.
3 can be paired with 6.
4 can be paired with 8.
5 can be paired with 10.
.
.
. ______________________________________
Hence, the natural numbers can be in one-to-one correspondence with the even numbers.

Why think (A), (B), and (C) are all the same argument? All three arguments are successful or unsuccessful together and if they are successful or unsuccessful it will be for the exact same reason in each case. All three arguments appeal to the same number of premises—they just differ on how many of those premises they make explicit rather than implicit in the ellipsis. And surely the decision about when to stop listing premises and put in the ellipsis does not make a difference to the argument being made.

Suppose, however, that one insists that the three arguments are in fact distinct. But on what basis? We cannot rely solely on the explicitly presented premises or else we will not be able distinguish (A) from:

(A-) 1 can be paired with 2.
2 can be paired with 4.
3 can be paired with 6.
Hence, the natural numbers can be in one-to-one correspondence with the even numbers.

But (A-) by itself is an awful argument, and whether (A) is ultimately good or bad, it will not be bad for the same obvious reason as (A-). For example, I suspect many would grant that (A-) is an example of hasty generalization, whereas (A) is not. If (A) and (A-) are distinct arguments, then, following the same pattern, (B) and (B-), and (C) and (C-), and so on are distinct arguments. But by similar, though not exactly the same, reasoning we can also show that (A) is not the same as any of the finite (-) arguments.[3] The only plausible option left is that (A), (B), (C), etc., all are shorthand expressions of the infinite premise argument.

Suppose one admits that (A), (B), (C), and so on are the same argument, but not in virtue of expressing an argument with an infinite number of premises, but in virtue of being in an equivalence class together. Whatever the equivalence relation is that puts them all in the same class, it presumably has to exclude (A-) and so on from being in the class. But then what could this equivalence relation be other than the infinite premise case? Indeed, what could exclude the infinite case from being in the class itself? But if the infinite premise case satisfies the equivalence relation, then it is the same argument as the others, in which case it is an argument, and so some arguments have an infinite number of premises.

To sum up: Absent some sort of *ad hoc* stipulation about what the ellipsis stands for in arguments (A), (B), (C) etc., the most plausible account of these arguments is that they all stand in for an argument with an infinite number of premises. Hence, the fact that we are limited to finite texts to express arguments does not entail that the arguments themselves are finite. So far then, those who hold that arguments are composed of sentences do not have a good reason to reject the possibility of infinite premise arguments.

Suppose we take arguments to be composed of acts such as utterances. We cannot perform an infinite number of utterances and so cannot perform the complex act that would be an argument with an infinite number of premises.

But the reasoning for the case of sentences carries over to the case of acts. Firstly, just because we are so limited does not prohibit God from performing such acts, and so God could perform the act that is arguing with an infinite number of premises. Secondly, let us say that the arguments (A), (B), etc. are not the groups of sentences, but rather my utterances of those sentences. What then is the force of the utterance of the phrase ‘and so on’ in each of the arguments? I argue that it is a finite act that is a stand in for performing the infinite number of acts in the same pattern as the already uttered premises that remain. But if so, then while (A), (B), and (C) are different finite acts, they are all stand-ins for the same complex infinite act, i.e., an act that contains an infinite number of premise utterances.

Suppose, however, one insists that (A), (B), (C), and so on are not just stand-ins for an infinite act, but rather just distinct complex finite acts, and so distinct arguments. But now consider the following *new* argument:

(A*) 1 can be paired with 2.
2 can be paired with 4.
3 can be paired with 6.
.
.
.
Hence, the natural numbers can be in one-to-one correspondence with the even numbers.

[3] The reasoning is the same for showing that (A) is not the same as (B-) and (C-), but after perhaps a million or so premises one might reasonably hold that whatever the (-) argument’s fault is, it is no longer hasty generalization. Now the problem becomes giving a reason other than mere stipulation that (A) is shorthand for the million premise argument rather than the million and one or million and twenty premise arguments, etc.

New? Isn't (A*) just (A)? No. (A*) occurred later than (A), and acts are differentiated in part by the time of their occurrence, so (A*) is not the same argument as (A). We could avoid this problem by resorting to act types. But now the question becomes what makes (A) and (A*) both acts of the same type such that we would say they are the same argument. The obvious answer is that they have the same content. But if, when identifying different act tokens as the same argument, we are relying on content, then (A), (B), (C), etc. once again count as different act tokens which are of the same act type—an act type with infinite premises.

An act theorist can stay the course and just insist that (A) and (A*) are different arguments. The price tag, however, is high. On such a fine-grained view of distinct arguments, every argument happens exactly once. Arguments cannot be repeated, though new arguments with the same content might occur numerous times. I cannot re-present your arguments—I can present my arguments with the same content as yours. Perhaps some are willing to pay the price. I find it prohibitive.

Granted the price tag is high, but not as high, some might argue, as accepting arguments with infinite premises. Regardless of the metaphysics of arguments, that an argument might have infinite premises is just absurd. For example, Daniel Bonevac, in his textbook *Simple Logic* defines an argument, in part, as a finite string of statements. He goes on to write: "it is important that the string of premises be finite. If the premises never end, the conclusion is never established." (Bonevac 1999: 3)

There are two problems with Bonevac's claim. Firstly, the fact that an argument with infinite premises would, if Bonevac is right, fail to establish its conclusion does not mean the string is not an argument—it merely means it could not be a *good* argument. Classically, I know that an argument with contradictory premises is valid, but automatically unsound. That does not disqualify arguments with contradictory premises as arguments—it merely means that such arguments will classically fail. Arguments that cannot succeed are certainly poor choices for convincing others, but that does not mean they are not arguments.

Secondly, it is just false that a string of infinite premises cannot establish a conclusion. (A) has an infinite number of premises and establishes its conclusion. Indeed, one can plausibly argue that any finite number of premises of the given form will not be sufficient to establish (A)'s conclusion. Many other mathematical examples like (A) exist.

Perhaps Bonevac's point fails for simple arguments, but what of complex arguments? After all, the whole point of distinguishing simple arguments from complex arguments was to make sure we are not conflating an infinite set of reasons with an infinite chain of reasoning. Clearly, the latter, in the form of an infinite regress, has been vilified. Richard Fumerton, for example, writes: "finite minds cannot complete an infinitely long chain of reasoning, and so, if all justification were inferential, no one would be justified in believing anything at all to any extent whatsoever." (Fumerton 2006: 40) Hence, one might argue that at the very least we should prohibit complex arguments with infinite members, for such a complex argument would involve an infinite chain of reasoning.

Despite the almost universal condemnation of infinite regresses, we must take care to distinguish regresses that cannot be calculated from those that can. Under some fairly plausible restrictions on the conditional probabilities involved, it turns out that many infinite chains of reasoning (infinitely many in fact) have calculable values for the probability of the conclusion given all the previous steps. Not only is it false that all infinite chains of reasoning provide no justification for their conclusions, in some cases we can calculate precisely what the level of justification, understood probabilistically, is (see Peijnenburg 2007).

So far then, I see no reason to prohibit arguments from having an infinite number of premises. In addition, given that there appear to be clear examples of such arguments, regardless of what one takes the underlying metaphysics to be, we ought to accept that arguments could have infinite premises.

Suppose however one argues that the examples are inconclusive, since we do not need such arguments to do the required job. For example, any infinite premise argument could be rewritten as a single premise argument in which the premise is just an infinitely long conjunction. Hence, (A) could be rewritten as:

(A1) 1 can be paired with 2, and 2 can be paired with 4, and 3 can be paired with 6, and …
Hence, the natural numbers can be in one-to-one correspondence with the even numbers.

Alternatively, we could rewrite at least some such arguments as two premise mathematical inductions, such as:

(AMI) 1 can be paired with 2.
For any n, if n can be paired with 2n, then n +1 can be paired with 2n+2.
Hence, the natural numbers can be in one-to-one correspondence with the even numbers.

Why should the fact that we could perhaps do the same job with a finite premise argument mean that the infinite premise arguments were not in fact arguments? After all, every single finite multi-premise argument can be rewritten as a single premise argument in which the premise is just the conjunction of the original argument's premises. But surely no one wants to argue that therefore the original structure was not really an argument after all.

Additionally, neither (A1) nor (AMI) make clear that we can do without infinite premise arguments. For example, since the premise of (A1) is a conjunction I should be able to conclude any of the conjuncts individually, such as:

2 can be paired with 4,

and

4 can be paired with 8,

and so on. But from this infinite collection of statements I should be able to conclude that the even numbers can be put in one-to-one correspondence with the whole number multiples of 4. Without a rather *ad hoc* restriction on what can be concluded from a conjunction, we can easily generate other infinite premise arguments from the premise of (A1). Disallowing infinite conjunctions just relocates the original problem—now we need a reason, other than mere stipulation, for disallowing infinite conjunctions.

Similarly, either the universal premise in (AMI) is itself an infinite conjunction and so subject to the same sort of reasoning just made in the case of (A1), or the universal premise itself is really just shorthand for an infinite number of conditional statements, in which case (AMI) itself contains an infinite number of premises. (A) represents the additional premises with the ellipsis; (AMI) represents them via the variable ranging over an infinite domain.

Since it is not at all clear that the proposed arguments in fact avoid an appeal to infinite premise arguments, we cannot yet use these arguments as a reason to reject the possibility of infinite premise arguments. So far then, all argumentation theorists ought to accept that arguments could have an infinite number of premises. But what of the other end of the spectrum—could an argument have no premises at all?

3. CAN AN ARGUMENT HAVE ZERO PREMISES?

Suppose arguments are composed of propositions. Could a simple argument have an empty premise set? There certainly are sets of a set of propositions and another proposition in which the initial set is empty. Is there any reason to exclude these sets from the class of arguments? Considered as sets of propositions the answer is 'no'. Logicians make use of the fact that any set in which the premise set is non-empty can, given certain assumptions about the support relation involved, be turned into a set in which the premise set is empty with no change in the validity status of either set. Hence, to deny that the sets with empty premise sets are not arguments one would need to deny that validity (i.e., its not being possible for all the premises to be true and the conclusion false), is solely a property of arguments.[4] Additionally, many proofs concerning the various properties of logical systems become much easier once the conversion has taken place. To deny that structures with empty premises are arguments one would effectively need to hold that these logical systems say nothing about arguments at all. However, neither denial seems plausible, at least given that we stay focused on arguments as propositions. Whatever the limitations of our logical systems in other regards, these systems are an extremely useful means of modeling numerous relations that hold amongst propositions and sets of propositions. If there is a reason to deny that empty premise set structures are arguments, it must come from some feature external to the nature of propositions.

Suppose then that arguments are composed of sentences. Could a group of sentences be an argument even if no members of the group count as premises? Again, if we take the groups to be sets, then there is no real difference from the case of sets of propositions. But if we take the groups to be sentences written on the page, one might wonder how we could possibly distinguish a sentence that is a mere claim for a position from a sentence that is an argument for that claim. Here though is an example from Roy Sorensen:

> Many sensible people think that an argument must have premises. Not me. My re buttal:
>
> MT: ______________
> Therefore, there are arguments without premises. (Sorensen 1999: 498)

Even though Sorensen's primary concern in presenting this example is circularity and question-begging, one might still point to Sorensen's example and say that MT is not just

[4] For some more reasons logicians give for premiseless arguments, see Sorensen (1999: 499).

the claim that arguments have no premises, rather it is an argument for that claim—one might even say an argument by example.[5] We can tell it is an argument, rather than a mere claim, because it is presented in a standard argument form—list the premises, draw a line, write the conclusion. Given this standard means of presenting arguments, then for any sentence at all we have a means of distinguishing when the sentence is merely a claim, and when it is the conclusion of a premiseless argument.

Perhaps however whether a sentence is an argument depends not on the other structures around it, but what we can or cannot do with it. We cannot argue merely by uttering or presenting sentences such as:

> Hence, every even number is the sum of two prime numbers,

or

> Therefore, arguments can have no premises.

If we cannot argue via these sorts of sentences then we should not count them as arguments.

More generally, one might plausibly argue that what makes something an act of arguing is that it is, at the very least, an act of reason-giving.[6] If there is no reason-giving then there is no arguing, and if there is no arguing then there is no argument. So one might claim that those who take arguments to be composed of acts have a very good reason to reject premiseless arguments—you cannot argue without giving reasons.

The issue, however, is not so clear cut for the advocates of arguments as acts. At the very least one cannot restrict acts of arguing to speech acts. Consider for example, how I might gesture at pictures of the Columbine, Virginia Tech, and Congresswoman Giffords shooters and then say, "thus, the United States needs more stringent gun laws." No speech act has occurred that is a reason-giving act, yet it is quite plausible to say that an act of arguing occurred. More generally, one might plausibly claim that the gesture itself is not any act of premising, and so I can perform acts that do not involve explicit premising acts.

Of course, one might deny that my example is an act of arguing, though one then needs to say why. One might claim that the gesture (or the pictures or both) makes the audience think of relevant premises, and so the audience at least performs a relevant act of premising. But still *my* act of arguing does not appear to involve any premise acts. One might claim that the gesture is a sort of premising act via the pictures or the context of the pictures. But if the pictures or the context of the pictures are what is truly relevant, then one is no longer holding that arguments are composed of acts, but more likely falling back on arguments as some sort of content (propositional or otherwise) that is merely being presented via the act of arguing.

Resolving these possibilities carefully is well beyond the scope of this paper. For the moment I will grant that arguing requires reason-giving. But the question remains—does that mean there are no zero-premise arguments? If we commit to the claim that for every argu-

[5] Note that Sorensen does not take arguments to be sentences, but rather propositions. Regardless, someone who does take arguments to be sentences still might point to Sorensen's example as an example of an argument with no premises.

[6] J.A. Blair, in Blair 2003, for example suggests that what is common to all understandings of argument and arguing is the giving of reasons.

ment there is a corresponding act of arguing, then the answer appears to be 'yes'. But must we commit to the claim that for every argument there is a corresponding act of arguing?

At the very least we already have a reason to reject the claim that for every argument there is a corresponding act of arguing that makes every element of the original argument explicit—the fact that we can use finite acts of arguing as stand-ins for infinite acts of arguing refutes that. But one might still insist that the zero case is different than the infinite case. In the infinite case, God could at least perform the infinite act, but not even God could argue using just a conclusion.

Suppose arguing with just a conclusion is impossible. Consequently, those who hold that arguments are composed of acts might deny that there are any premiseless arguments. But should the supposition that arguing with just a conclusion is impossible also commit those who hold that arguments are composed of say, propositions, to denying zero-premise arguments? Certainly some logicians and philosophers have let what arguing is dictate what arguments are. For example, according to Sorensen, Augustus De Morgan held that arguing was the deriving of a conclusion from the combining of information. Since combining requires two things to start with, "De Morgan believed that an argument must have at least two premises" (Sorensen 1999: 500).

Despite the fact that some theorists let the nature of arguing dictate the nature of arguments, we are certainly not compelled to do so. One can grant that arguing requires reason-giving, but hold that some arguments are not (and perhaps even cannot) be used to argue or convince, etc. Instead, some arguments, such as the zero-premise ones, can be used to help us understand the possible support relations amongst propositions or to define the notion of logical truth.

But surely the function of arguing, of providing an argument, just is to convince someone of, or at least alter one's attitude toward, some claim on the basis of reasons. Again this claim can be granted. Arguing may have that function[7]—but the arguments which are not used to argue do not. That does not mean they are not arguments—they are merely arguments that we do not use for the purpose of arguing. Advocates of arguments as propositions who do not hold that an appeal to function helps distinguish arguments from non-arguments will just not find an appeal to the function of arguing compelling.

Even if one holds that distinguishing arguments from non-arguments requires an appeal to function, one can still accept that the function of arguing is to convince via reasons and accept zero-premise arguments. One need only deny that such a function is the way, or the only way, to distinguish arguments from non-arguments.[8] As long as there is at least one function that can be served by zero-premise structures, such as defining the concept of logical truth, then advocates of arguments as propositions, who also hold that function distinguishes arguments from non-arguments, can still accept arguments with no premises.

4. CONCLUSION

If what I have argued here is correct, then everyone should accept the possibility of infinite premise arguments. On the other hand, whether we should accept zero-premise arguments seems to depend upon the resolution of other highly controversial options in argumentation

[7] See Goodwin (2007) for an argument that arguing and arguments may have no function.

[8] See Hitchcock (2009) for an example of the view that arguments can have multiple functions.

theory such as—does every argument need a corresponding act of arguing? Does defining argument require an appeal to function? Trying to resolve these issues is a project for another time. At the very least, however, I hope that I have sketched out a position according to which it is straightforwardly possible for there to be zero-premise arguments.

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Commentary on “HOW MANY PREMISES CAN AN ARGUMENT HAVE?” by Geoff Goddu

DAVID HITCHCOCK

Department of Philosophy
McMaster University
Hamilton, Ontario
Canada L8S 4K1
hitchckd@mcmaster.ca

1. INTRODUCTION

Stripped down, Geoff Goddu’s core argument seems to run as follows: A simple argument is a set consisting of a set of premisses and a conclusion. Whether you take the premisses and conclusion to be propositions or sentences or speech act types, the set of premisses can in principle be denumerable or empty. Since logicians use both denumerably-premissed and zero-premissed arguments in their theorizing, and both sorts can be expressed by sentences and used in acts of arguing, and indeed we can construct examples of each sort, then arguments can have infinitely many premisses and can also have zero premisses, regardless of how one conceives of arguments—except that there cannot be zero-premiss arguments if one conceives of arguments as complex speech acts in which reasons are given.

2. TWO OBJECTIONS

Goddu’s provocative question and challenging response raise fundamental questions about how we are to conceive of arguments and what evidence we should accept for the existence and content of a particular argument.

Let me begin by raising an objection to Goddu’s general conception of a simple argument. Suppose that we think that an argument is composed of sentences. Then a simple argument is not just a set consisting of a set of sentences and a sentence. For the set {{‘snow is white’, ‘grass is green’}, ‘birds sing’} is no argument unless ‘birds sing’ is being drawn as a conclusion from the set {‘snow is white’, ‘grass is green’}. It is fine to say that an argument is a pair consisting of a set of premisses and a conclusion. But a set consisting of a set of sentences and a sentence does not consist of a set of premisses and a conclusion unless there is some indication that the sentence is being inferred from the set of sentences. If we are to conceive of an argument as an abstract object that can be expressed in different ways and on different occasions and used for different purposes, we must include in our characterization of such an abstract object its illative component, the part signified in English by inferential uses of such particles as ‘because’, ‘since’, ‘so’ and ‘therefore’. (Adding an illative component to the basic conception of argument, as far as I can see, does not affect the main line of Goddu’s argument, since he attends to the question whether a set whose component set is infinite or empty can be expressed or used using an illative.)

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Another objection: The terminology of mathematical logic is as far as I can see irrelevant to the question how many premisses an argument can have. Mathematical logic does not theorize about arguments. The word ‘argument’ does not appear in the lengthy index to the Whiggish history of logic written by William and Martha Kneale (1962), in which the history of logic is selectively raided for those bits that can be construed as leading up to contemporary mathematical logic. Nor does the word ‘argument’ appear in the index of Forbes (1994), the textbook that I use when teaching basic symbolic logic to graduate students in philosophy, although the book starts with two pages about arguments before getting down to the real business. Boolos, Burgess and Jeffrey (2007), the textbook that I used most recently for teaching the metatheory of first-order logic and first-order arithmetic, does contain one occurrence of the word ‘argument’ in the index, but in the sense of an argument of a function. The word ‘argument’ does not appear at all in the index of Jeffrey (2006), another textbook I have used in the past for teaching metalogic. It appears in the index to Church’s (1956) classic only in the sense of an argument as input to a function. In its first 71 volumes (1936 to 2006), *The Journal of Symbolic Logic*, the leading journal in mathematical logic, published exactly one article (Chong and Yang 1998) with the word ‘argument’ in its title. By comparison, the journal *Informal Logic* in its first 30 volumes (up to 2010) published 75 articles with the word ‘argument’ in the title. (*The Journal of Symbolic Logic* did publish 11 reviews of books with the word ‘argument’ in their titles. Of these book titles, four refer to particular arguments such as the ontological argument or the master argument of Diodorus, two describe the book itself as an argument, two refer generally to legal argument, one (Peter Geach’s *Reason and Argument*) refers to argument in general, one uses the phrase ‘for the sake of the argument’, and one refers to argument in the sense of an input to a function.)

To repeat: Mathematical logic does not theorize about arguments. One method of proving the completeness of first-order logic does involve the construction of an infinite set of sentences of the formal language of the logic. Mathematical logicians can happily use this method without committing themselves to the existence of arguments with infinitely many premisses. They can even note that the set of sentences is consistent, in which case no logical falsehood follows from it. But remarks about what follows from an infinite set of sentences do not imply that there are arguments with that set as a premiss. Likewise, mathematical logicians have the bad habit of using the word ‘valid’ not only for arguments that cannot, logically speaking, have true premisses and a false conclusion but also for sentences that must be true. But they don’t actually refer to valid sentences as arguments, and nothing other than a certain theoretical neatness obliges them to do so. It would be less misleading for them to use the term ‘logically true’ for sentences that must (logically speaking) be true.

We are left with Goddu’s examples of a simple argument with infinitely many premisses and a simple argument with zero premisses.

3. CAN AN ARGUMENT HAVE INFINITELY MANY PREMISSES?

As far as I can see, Goddu has argued very nicely in terms of his example of an argument with infinitely many premisses that a finite sequence of inscriptions can express through such devices as ellipsis or the phrase ‘and so on’ an argument with a denumerable set of premisses, and similarly that a finite sequence of speech act types can use such an argument. I would like to push the envelope even further by asking whether there can be arguments with a non-enumerably infinite set of premisses. After all, non-constructivist mathematicians recognize the existence of non-enumerably infinite sets, such as the set of real numbers. So, unless one is a constructivist, there is no objection in principle to arguments with a non-enumerable infinity of premisses. Can one express such an argument in sentences and use it by uttering a series of speech acts? Well, consider the proof by means of a geometrical diagram (Fig. 1) that there is a one-to-one correspondence between the set of real numbers properly between 0 and 1 and the entire set of real numbers.

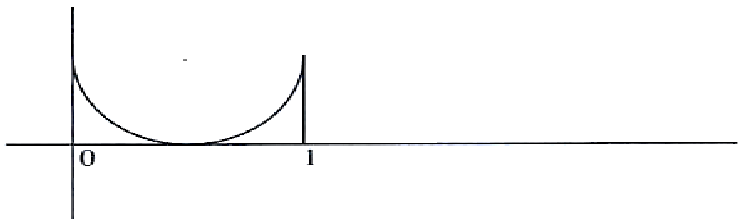

Fig. 1. Geometrical diagram

The proof involves setting up a correspondence between the points on the open interval (0, 1) and the points on the entire *x*-axis. If from a point A on the open interval one draws a perpendicular straight line that meets the semi-circle in the figure at some point B, then draws a straight line from the centre of the semi-circle through B to some point C on the *x*-axis, and one does the same for every point on the open interval, then there will be a one-to-one correspondence between the set of points on the open interval and the set of points on the *x*-axis, i.e. between the set of real numbers properly between 0 and 1 and the entire set of real numbers. One might argue for the left-to-right part of this correspondence as follows:

(1) A point A on the open interval will be connected to exactly one point on the x–axis by means of a perpendicular to the x–axis at that point and a straight line from the centre of the semi-circle through the point on its circumference intersected by that perpendicular to the x-axis.
(2) Similarly, another point A′ on the open interval will be connected to exactly one point on the x–axis by means of a perpendicular to the x–axis at that point and a straight line from the centre of the semi-circle through the point on its circumference intersected by that perpendicular to the x-axis.
(3) And so on.
(4) Hence every point on the open interval will be connected to exactly one point on the x–axis by means of a perpendicular to the x–axis at that point and a straight line from the centre of the semi-circle through the point on its circumference intersected by that perpendicular to the x-axis.

As far as I can see, Goddu's arguments against attempts to deny that his example expresses an argument with infinitely many premisses work equally well against attempts to deny that this example expresses an argument with infinitely many premisses. But, by Cantor's method of diagonalization, the premisses of this argument are not enumerable.

Hence, even with unlimited time it would be impossible to enumerate all its premisses. Even a being that uttered each sentence of the argument in half the time of its immediate predecessor would not complete the utterance of all the premisses. In fact, if the premisses are ordered in the same order as the real numbers in the open interval (0, 1), then no premiss has an immediate predecessor or an immediate successor. So perhaps even God cannot state all the premisses of this argument. But a finite human being can indicate them.

What about the worries of intuitionists and other constructivists about the postulation of actual infinities? Iemhoff (2009) reminds us that "in intuitionism all infinity is considered to be potential infinity. In particular this is the case for the infinity of the natural numbers." Intuitionists do however talk about infinite sets, as Iemhoff's article exemplifies. But, as she remarks, "Since for the intuitionist all infinity is potential, infinite objects can only be grasped via a process that generates them step-by-step." (Iemhoff 2009) The premisses of Goddu's example can be generated step by step. So it seems unexceptionable from an intuitionist point of view. There is however no process that generates the set of real numbers step-by-step, so my example is problematic from an intuitionist perspective. Nevertheless, Brouwer, the father of intuitionism, found a way to accommodate the continuum in intuitionist mathematics. So perhaps intuitionists and other constructivists can accommodate an argument with a continuum of premisses.

My sample argument for the connection of each member of the set of real numbers between 0 and 1 to exactly one real number establishes, we might suppose, that an argument can have a set of premisses whose cardinality is that of the power set of the set of natural numbers. Can there be arguments whose set of premisses has a cardinality even greater? Well, consider the power set of the set of real numbers. It has a cardinality greater than that of the set of real numbers (by a variant of Cantor's diagonalization method). And its cardinality is the same as that of the power set of the set of real numbers between 0 and 1. Presumably one can construct an argument that each member of the power set of the set of real numbers between 0 and 1 is connected by some specified method of connection to exactly one member of the power set of the set of real numbers. This argument would have even more infinitely many premisses than the argument for the connection of each point on the (0, 1) interval with exactly one real number.

And this process can be repeated for the power set of that power set. And so on. Thus there seems to be no upper limit to the non-denumerable infinities of premisses that an argument can have. (Parenthetically, I note that I have just used an argument with a denumerable infinity of premisses, thus giving an actual case supporting Goddu's claim that there can be arguments with infinitely many premisses.)

4. CAN AN ARGUMENT HAVE ZERO PREMISSES?

As to Goddu's example, borrowed from Roy Sorensen, of an argument with zero premisses, here we are on shakier ground. Sorensen argues by ostension that there are arguments with zero premisses by producing one, written with a label 'MT' followed by a colon followed by an underlining with nothing above it followed by another line on which is written the sentence "Therefore, there are arguments without premises". This example is a parody of an argument. The word 'therefore' in its inferential use indicates as part of its meaning that what follows it is being inferred from what precedes it. Any conclusion introduced by such an illative must be preceded by at least one premiss, which may perhaps be indicated contextually rather than actually stated. Without such a contextual indication, the sequence of words 'Therefore there are arguments without premisses' is not an argument with zero premisses, because it is not an argument. It is an ill-formed piece of discourse, because there is nothing preceding the word 'therefore' from which the statement is being inferred.

Goddu's own example, of someone pointing at pictures of mass killers in the United States and then asserting, "thus, the United States needs more stringent gun laws," strikes me as an argument, but as one with three premisses, namely the indications of the killers at Columbine High School, Virginia Tech University and the Tucson Safeway store where Congresswoman Giffords was shot. The fact that the arguer adduces these premisses by pointing rather than making statements is irrelevant to their status as premisses. Arguments can (and do) have visual premisses, such as photographs or drawings.

As Goddu points out, if there were arguments with zero premisses, then their expression in sentences and their use as complex speech acts would have to be distinguished from the mere assertion of their conclusion by the use of an illative, such as 'so' preceding the conclusion or 'because' following it. But such illatives have as part of their meaning a connection between two items, a premiss and a conclusion. (The use of 'because' or 'just because' with no ensuing clause, as a response to a request to justify a claim, is a refusal to give a supporting premiss, not the introduction of an empty set of premisses.) The empty set cannot be a premiss linked by an illative to a conclusion. So one can neither express nor use an argument with zero premisses. And that's a pretty strong reason for thinking that there cannot be any such arguments.

5. CONCLUSION

Thus Goddu has established by his example that there are arguments with infinitely many premisses. Indeed, it appears that there are analogous examples with non-denumerably infinitely many premisses and that there is no upper limit to how great an infinity of premisses an argument can have. But there are, as far as I can see, no arguments with zero premisses.

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Reply to David Hitchcock

GEOFFREY C. GODDU

Department of Philosophy
University of Richmond
28 Westhampton Way, University of Richmond, VA 23173
United States
ggoddu@richmond.edu

1. DEFINING 'ARGUMENT'

Hitchcock writes of my articulation:

> A simple argument is a set consisting of a set of premisses and a conclusion. Whether you take the premisses and conclusion to be propositions or sentences or speech act types, the set of premisses can in principle be denumerable or empty. (p. 1)

These two sentences misrepresent my articulation. I define arguments in terms of sets only for propositions. I do not define arguments in terms of sets for sentences or speech acts. I consider the possibility that those who prefer arguments to be composed of sentences are talking about sets, but quickly dismiss it on the grounds that the arguments from the proposition discussion will apply and go on to consider further reasons why an argument composed of sentences could have infinite members. I never consider the possibility that arguments are sets of acts—merely that arguments are composed of acts or are complex acts.

As a result, Hitchcock's first objection, that my definition of simple argument is missing a needed 'illative' component is off the mark. Perhaps arguments composed of sentences or acts need an illative component, but arguments construed as sets of propositions do not. Though a full defense would require more space than available in a short reply, the main idea is that capturing hypothetical or possible, but non-actual arguments, in terms of propositions will make appeal to the illative unnecessary.

Hitchcock's second objection is that "Mathematical logic does not theorize about arguments" (p. 2). Perhaps he is correct. At the very least he is correct that mathematical logicians rarely use the term 'argument' within their discussions. But they do talk of logical truths, well-formed formulas, sets of propositions, etc. in such a way that the defender of the view that arguments are a certain sort of set of propositions can claim that, at least indirectly, the discussions of mathematical logicians do concern those sets, even if they do not call them arguments.

2. THE ILLATIVE AND ZERO PREMISES

While Hitchcock is perfectly willing to accept arguments with infinite premises, he resists the case for arguments with zero premises. Concerning my possible example of pointing at pictures of mass killers, he writes that it strikes him as argument, but one with premises, namely the pictures. Perhaps he is correct, but speech act theorists, the target of this particular example, cannot appeal to the pictures. The point of my example was to argue that speech act theorists face a dilemma—either, on their view, there is no act of arguing

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going on in the example, or some arguments composed of speech acts can have no premises. Speech act theorists however cannot appeal to the pictures as premises, since arguments, on their view, are composed of speech acts.

Concerning Sorensen's argument MT he writes:

> [T]his example is a parody of an argument. The word 'therefore' in its inferential use indicates as part of its meaning that what follows it is being inferred from what precedes it. Any conclusion introduced by such an illative must be preceded by at least one premise. (p. 4)

As it stands, Hitchcock's second sentence does not follow from the first. If we infer the conclusion from what precedes it, namely nothing, then we satisfy the meaning of the inferential use of the illative without it being the case that the illative must be preceded by at least one premise. Later Hitchcock provides a stronger meaning when he writes: "such illatives have as part of their meaning a connection between two items, a premise and a conclusion."(p. 5) But does the illative have the stronger meaning rather than merely the weaker one (or the weaker one at all)? Even if it does, those who do not hold the illative as part of the argument need not be moved by Hitchcock's appeal to the meaning of the illative.