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# The formal failure and social success of logic

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# The formal failure and social success of logic

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**ABSTRACT:** Is formal logic a failure? It may be, if we accept the context-independent limits imposed by Russell, Frege, and others. In response to difficulties arising from such limitations I present a Toulmin-esque social recontextualization of formal logic. The results of my project provide a positive view of formal logic as a success while simultaneously reaffirming the social and contextual concerns of argumentation theorists, critical thinking scholars, and rhetoricians.

**KEYWORDS:** argumentation, dispute resolution, formal, Frege, Gödel, Leibniz, logic, peace, Russell

## 1. INTRODUCTION

Is formal logic a failure? If we judge the success of formal logic using criteria that are imposed and evaluated from within that very field of inquiry, it seems that we are likely to run into a number of difficulties. The aspirations of logicians have traditionally been lofty, and their methods for achieving their many goals have typically been austere. Logicians have developed systems that serve their attempts to transcend context, and to reach towards the unchanging and eternal. The nature of the goals and methods of logicians tend to obviate the consideration of formal logic as a social or historical entity. But developments in anthropology, sociology, and history—as well as work in new fields such as argumentation theory and critical thinking—may demonstrate a value in placing increased emphasis on those social and historical details that are normally considered irrelevant in accounts of the development of logic (e.g. Kneale & Kneale 1962). If we abandon attempts to offer context-independent evaluations and begin to consider the history of formal logic in light of contextual concerns, it becomes possible to see—and more importantly, to describe—logic as a success. To this end we will begin by investigating some of the origins of formal logic in the 1600s, when logic held promise as an endeavour of social utility. We will then investigate the deterioration of social concerns within the field of logic during the 1879-1931 period, address the all-too-common view of logic as a ‘failure’, and conclude by seeing how the work of this period can be regarded as a success when social factors are reintroduced.

## 2. LEIBNIZ AND HIS INFLUENCE

Toulmin (1989: 98-103) makes a compelling case for Leibniz’s development of a *characteristica universalis* and *calculus ratiocinator* as a socially motivated activity. As evidenced by many of the some 15,000 or more letters written during his lifetime, Leibniz

was deeply troubled by the religious, economic, and political disputes of his day. While it is possible to consider Leibniz and his contributions to logic in a relatively ahistorical fashion, Toulmin argues that it is more sensible to see Leibniz as a “German intellectual who accepted his responsibility to do whatever he could do to remedy the situation of Europe in his time” (1989: 103). Leibniz makes clear reference to the resolution of real-world disputes when proposing his universal calculus:

Whenever controversies arise, there will be no more need for arguing among two philosophers than among two mathematicians. For it will suffice to take the pens into the hand and to sit down by the abacus, saying to each other (and if they wish also to a friend called for help): *Let us calculate.* (in Gabbay & Woods 2004: 1)

Properly formalized, details from philosophical, political, economic, and religious disputes could be fed into a calculating system. Such a system would be supported by the establishment of a global standard of communication, and would not only serve as a tool of dispute resolution, but also as a universal solution to the barriers of language.

Leibniz’s proposed system may exhibit a “characteristic blend of genius and insanity” (Coffa 1993: 14), but the influence of this idea in the development of logic has been immense. In 1882, Ernst Schröder became involved in a journal dispute with Gottlob Frege, who alleged that Schröder had misunderstood the purpose of the *Begriffsschrift*. Frege attempted to clarify this by responding: “I was trying, in fact, to create a ‘*lingua characteristicistica*’ in the Leibnizian sense” (1968: 89). Soon thereafter, Peano and his followers took up the habit of recalling Leibniz’s “project of creating a universal script ... in the opening sentences of their general writings on logic” (Grattan-Guinness 2001: 237). Bertrand Russell—himself a notable Leibniz scholar—also wrote that “Leibniz foresaw the science which Peano has perfected ... Leibniz’s dream has become fact” (1901; 1918: 79). The influences of Leibniz, Frege, and Peano eventually saw Russell, along with Alfred North Whitehead, produce that great project of formal logic, the *Principia Mathematica*.

### 3. THE DEPARTURE OF THE SOCIAL

Leibniz had a definite influence on Frege’s logic. Frege’s *Begriffsschrift* was “influenced by two of Leibniz’s ideas: a *calculus ratiocinator* (a formal calculus of reasoning) and a *lingua characteristicistica* (a universal language)” (Moore, in Aspray & Kitcher 1988: 102). According to Eike-Henner Kluge, the logical projects of Leibniz and Frege share “such extensive and close correspondence on so many central issues that the hypothesis of a fundamental influence on the part of Leibniz [is] very difficult to deny” (1980: 154). Despite similarities, Leibniz’s social motivations for his *characteristica* were not carried forward by Frege. In the preface to his *Begriffsschrift*, Frege regards Leibniz’s “conception of a universal characteristic, a calculus philosophicus or ratiocinator, [as] too grandiose for the attempt to realize it to go further than the bare preliminaries” (Heijenoort 1999: 5). In recognizing the scope and ambition of Leibniz’s project, Frege is also conscious that some two centuries have passed since Leibniz first envisioned his ‘dream’. Yet Frege does not give up on Leibniz’s project altogether:

Even if this great aim cannot be achieved at the first attempt, one need not despair of a slow, step by step approach. If the problem in its full generality appears insoluble, it has to be limited provisionally; it can then, perhaps, be dealt with by advancing gradually. Arithmetical,

geometrical, and chemical symbols can be regarded as realizations of the Leibnizian conception in particular fields. The *Begriffsschrift* offered here adds a new one to these—indeed, the one located in the middle, adjoining all the others. (Heijenoort 1999: 6-7)

Frege can therefore be seen as continuing the work of Leibniz’s project, albeit in a limited manner. Frege saw clearly that diverse communities of academics such as geometers and chemists had produced fragments of Leibnizian systems within their various disciplines. Taking care to describe his project as existing alongside, and not above or below the piecemeal developments made in other disciplines, Frege plainly does not regard his own project as a complete realization of Leibniz’s ‘dream’. He seems encouraged, though, that the ‘central location’ of his own decidedly mathematical project might make it the most promising path for timely advancement towards Leibniz’s goal.

The tremendous advancements made possible by Frege’s work during the fifty years following the publication of the *Begriffsschrift* are testament to the effectiveness of this limitation of scope. In particular, Frege’s project paved the way for that two-thousand page behemoth rumoured to have arrived at Cambridge University Press in a wheelbarrow, the *Principia Mathematica*. While the *Principia* made great inroads in the logicist project, the social concerns of Leibniz were nowhere to be seen. Like Leibniz, Russell was deeply concerned with peace. This concern was certainly present following the Boer war, and persisted throughout the remainder of Russell’s life. But for Russell, logic had become *l’art pour l’art*. Above all else, the Russell of this period was committed to the strictly academic improvement of intellectual discipline and the pursuit of clarity, objectivity, and truth. It seems possible that Russell might have conceived of such goods as intrinsic benefits that might stem from study of the *Principia*. However, Russell is not recalled for any insistence that some socially pragmatic value might be attached to his efforts in logic, and, most certainly, not in the sense suggested by Leibniz.

#### 4. GÖDEL: MATHEMATICS AND MISAPPROPRIATION

The path of advancement that was opened by Frege’s focus on logicism and continued by the work of Russell slowed in 1931, when Kurt Gödel presented his incompleteness theorem. The projects of Frege and Russell sought to place mathematics on firm logical foundations, but Gödel’s incompleteness theorem showed that such systems were unable to capture all of the “truths of basic arithmetic” (Smith 2007: 122). Among modern academics, the popular picture that is painted of the 1879-1931 period in formal logic is often unfavourable. The rationale for these views seems to arise from a kind of narrative in which Frege plays the role of ‘alpha’, while Gödel figures as ‘omega’. For example, Ian Hacking characterizes logicism as an “arithmetical revolution [that] failed to take place” (2000: 45) and describes the *Principia* as a project that “did not pan out, for very famous reasons, connected with the name of Kurt Gödel” (2000: 45). Similar views are not restricted to philosophers:

In the *Principia* [Whitehead and Russell] took up, among other tasks, the project of establishing two theses about logic and mathematics: first, that the disciplines can be complete and, second, that they can be consistent. ... The *Principia*, a logocentric enterprise par excellence, seeks to establish the self-sufficiency and integrity of such concepts as identity, contradiction, proof, and system. Whitehead and Russell failed in this effort ... it remained to Gödel to show them why. (Thomas 1995: 250)

Gödel's result showed that the logical systems of Frege and Russell were incapable of providing axiomatic foundations from which the entirety of mathematics could be derived. But quotations such as the preceding make too much of Gödel's result, and, in so doing, run the risk of excessively devaluing the importance of formal logic. Such views are not always discouraged by the language use of mathematicians and logicians, who have described the impact of Gödel's result with dramatic adjectives. Michael Friedman characterizes Gödel's theorem as a "fatal blow" (in Aspray & Kitcher 1988: 93) for the logical projects of the 1879-1931 period. Similarly, Lindstrom and Palmgren suggest that Gödel's result "shattered" (2010: 19) such projects, while Sinaceur uses the term "destroyed" (2010: 376). The efforts of logicians through the 1879-1931 period have even been branded as a quest for the "Holy Grail of logic" (Girle 2003: 192; Hunter 1996: 93); in such a narrative, the incompleteness theorem produced by Gödel stands as the central obstacle preventing the acquisition of the 'grail'.

To these characterizations we must add the realization that Gödel's result has become the subject of frequent and problematic misappropriations in more popular venues of discourse. Generally speaking, these troubling examples involve the misapplication of Gödel's result to subjects beyond the realm of mathematics. Examples of these misuses can be found in philosophy: as early as 1958, William Barrett wrote, "if human reason can never reach complete systematization in mathematics, it is not likely to reach it anywhere else" (p. 39). In *The Postmodern Condition*, Jean-François Lyotard sees Gödel's result as necessitating a "reformulation of the question of the legitimation of knowledge" (1984: 43). Elsewhere, Gödel's result has been compared to Zen Buddhist kōans (Hofstadter 1980: 246-272), and used to offer an explanation of the nature of human consciousness (Penrose 1996: 64-126). Each of these examples takes Gödel's result far beyond the realm of mathematics. In a world where Gödel's result is inexpertly invoked in discussions ranging from theology to politics, one academic has even seen fit to produce a guide to the use and abuse of Gödel's theorems. Another introductory philosophy text suggests that people who cite Gödel's result should be "assumed guilty until proven innocent" (Baggini & Fosl 2010: 252).

## 5. WHEN 'FAILURE' IS SUCCESS

From a mathematical standpoint, Gödel's theorem is a particularly valuable result. In and of itself, it offers no reason to regard formal logic—or the *Principia*, as the paradigmatic representative of efforts in formal logic—as failures. Indeed, on formal grounds, it seems more correct to say that the projects of formal logic are enriched, and not destroyed or shattered, by Gödel's result. B.J. Sokol writes: "in mathematics, a negative conclusion has always been as richly interesting as a positive one" (1995: 1054). This positivity has been embraced by some writers, who offer logical or mathematical descriptions of the impact of Gödel's result that do not engage in overly dramatic exaggerations:

There's an old hope (which goes back to Leibniz) that can be put in modern terms like this: we might one day be able to mechanize mathematical reasoning to the point that a suitably primed computer could solve all mathematical problems in a domain by deciding theoremhood in an appropriate formal theory. What [Gödel has] shown is that this is a false hope. (Smith 2007: 45)

While the preceding description does not overstep the scope of Gödel's result, it serves as an example of the inaccessibility of logical or mathematical discourse. Regrettably, accurate descriptions of Gödel's result that are presented by mathematicians and logicians seem to fall largely upon deaf ears. When such accounts are compared with the easily accessible and hyperbolic alternatives offered by other writers, it is perhaps unsurprising, although unfortunate, that Gödel's result is so widely misunderstood.

Understanding formal logic as a socially situated and pragmatically motivated activity allows us to generate a semantic alternative to the normal mathematical and logical explanations for Gödel's result and its implications for the projects of formal logic. This semantic alternative makes use of the vocabulary of argumentation theorists and critical thinking scholars, and centres on the idea of viewing formal logic as a socio-political attempt at dispute resolution and peacemaking. If we invoke Toulmin's understanding of Leibniz as a socially motivated individual who sought a system by which peace and understanding could more easily be established, then Gödel's result should not be regarded as the end of the road for Leibniz's vision, nor as dealing a 'death blow' to the 'failed' projects of formal logic. Rather, what Gödel's result has shown is that Leibniz's dream of a system by which international peace could be established and maintained will not be fulfilled in a singularly mathematical manner; it does not preclude the possibility that Leibniz's goal might be achieved in some other fashion:

Gödel's theorem, far from undermining the project of *Principia*, provided its culminating achievement and glory, by showing that mechanical decidability, the mere grinding out, of significant mathematical proofs is impossible. Unaided computers, for instance, will never be able to prove (or disprove) weighty mathematical theorems, no more than ruler and compass will be able to trisect angles. (Sokol 1995: 1054)

If anything, Gödel's theorem should leave us uncertain as to the pragmatic status of mathematical problem solving. Along with the efforts of Frege and Russell, Gödel's theorem shows us that we would be well advised to maintain diligence and rigor in any analysis of argumentation and in the resolution of disputes, even—and perhaps especially—if no mathematical 'autopilot' exists for the resolution of disagreements. This very fact of knowing that mathematical reasoning may be insufficient for the successful resolution of disputes is incredibly significant, and recommends the investigation of other, more socially and contextually robust, forms of analysis and reasoning. At core, an explanation using these semantics does not differ significantly from the mathematical and scientific idea that negative and positive results can be of equal value. It may be the case, however, that these semantics are able to communicate the efforts of formal logicians and the result found by Gödel with greater explanatory power.

Let us remain sceptical, though, and proceed with care, so as to ensure that the use of these semantics does not easily result in the abuse of Gödel's theorem. With these semantics, let us hypothesize three highly generalized possible scenarios in which we might consider the immediate comprehensibility of likely and potential implications for our socio-political view of formal logic. First, suppose that a system of peacemaking and dispute-resolution such as the one envisioned by Leibniz has in fact become a reality. In this scenario, every individual might have a handheld device with an 'app' for the near-instantaneous resolution of all matters of disputes, philosophical or otherwise. In a second, far more negative situation, we hypothesize that a fundamental meta-theorem impacting all areas of human activity and thought has proven beyond any doubt that all sys-

tems of dispute-resolution are doomed to failure. In a third, neutral situation, we can imagine that a theorem specific to one field of human inquiry has strongly suggested that a particular mode of human activity might not be able to provide a concrete, failsafe system for the resolution of disputes, but that other modes of human activity might still be used to achieve peace and prosperity. By analogy, it seems that the second, negative scenario has thus far been the most common interpretation of Gödel's result. However, this negative scenario is not supported by our intuitions: personal experience readily suggests that most people are clearly able to use systems—such as computers—to solve problems. The positive situation, too, finds little in the way of empirical support: it is uncontroversial that people do not have portable handheld applications that are capable of resolving *all* problems; indeed, our lives are riddled with ongoing disputes. The third scenario, then, seems to not only best describe Gödel's result, but also appears to be the only intuitively appealing option in terms of real-world practice.

## 6. CONCLUSION

Abandoning an ahistorical view of formal logic encourages the consideration of socio-political motivations in the study of notable logicians and their works. Such studies permit the usage of a new set of socially germane semantics, which may allow the efforts of formal logicians to be more accurately communicated to non-logicians. The value of a socially, historically, and contextually situated view of formal logic may be contested. However, it seems naïve to ignore the many social factors that motivated various developments in the works and goals of formal logicians. As Toulmin suggests, Leibniz might well have devised his *characteristica* in order to address deeply troubling social concerns. It seems clear, too, that such social concerns fall out of sight in the work of Frege and Russell, whose aspirations for logic shifted towards those abstract and decontextualized goals with which we are now so familiar. But Russell and Frege need not have offered explicit endorsements regarding the pragmatic values of their works in order for us to acknowledge their social and real-world applications. Even intensely abstract and seemingly asocial efforts are of social consequence: consider the physicists of the Manhattan Project, whose protracted unawareness of the profound and inescapable implications of their work did not lessen the real-world impact of atomic weapons.

Formal logicians may experience some measure of discomfort with the presented view of formal logic as an effort of socio-political dispute resolution. They may prefer to preserve a pure view of their field of study, and maintain that their goals for formal logic transcend context. If, in time, it becomes apparent that viewing formal logic as a socially situated and pragmatic activity will allow other academics to have a more accurate—and more positive—view of formal logic, then perhaps such semantics will curry favour with formal logicians. For philosophers concerned with argumentation theory and the development of critical thinking skills—i.e., those subfields broadly regarded as ‘informal logic’—such an account recommends a more positive view of the history of formal logic, as well as the herculean efforts of formal logicians. At core, the idea of seeing formal logic as an effort in dispute-resolution is not at odds with the values of ‘informal’ practitioners. In conclusion, we may therefore be mindful that formal and informal practitioners share many of the same goals, and that, if these groups disagree, it is perhaps mainly in their opinions regarding how those goals might be reached.

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# Commentary on “THE FORMAL FAILURE AND SOCIAL SUCCESS OF LOGIC” by William Brooke

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## 1. INTRODUCTION

I am grateful to the organizers for giving me the opportunity to reply to William Brooke’s scholarly and thought-provoking paper. Brooke traces the influence of Leibniz’s concept of a universal language and mechanical system of rational dispute resolution, the *characteristica universalis*, to Frege and Russell, and ultimately to Gödel. Gödel’s incompleteness results are often, but erroneously, seen as exploding Leibniz’s project. Brooke argues that they only have this effect if the *characteristica universalis* is understood in formal, mathematical terms. He contends that an informal *characteristica universalis*—a source of dispute resolution through non-mathematical means—is still open to us; and as informal logicians we should be well placed to take up where Leibniz left off. This narrative has consequences in several fields. I shall concentrate on three: politics, mathematics, and logic.

## 2. POLITICS

Gottfried Wilhelm Leibniz was born in 1646, two years before the end of the Thirty Years War, by far the most destructive European conflict prior to the twentieth century. Throughout his life he was an indefatigable diplomatist. The pursuit of peace is, as Brooke observes, an unsurprising priority for his philosophical work. The means by which he thought it could be achieved, a system of reasoning in which all controversies might be resolved by mechanical means, is surprising. Nonetheless, Brooke follows Toulmin in arguing persuasively that these social concerns were the key motivation for Leibniz’s logical work.

Bertrand Russell’s work in logic far surpassed that of Leibniz in technical sophistication. And Russell is also remembered as an outspoken advocate of peace, from his pacifism in World War I, to his advocacy of nuclear disarmament. However, as Brooke notes, ‘Russell is not recalled for any insistence that some socially pragmatic value might be attached to his efforts in logic, and, most certainly, not in the sense suggested by Leibniz’ (Brooke 2011: 3). Indeed, Russell is scathing about the prospects for the *characteristica universalis*, ascribing to Leibniz a fundamental confusion about the location of philosophical problems:

For the business of philosophy is just the discovery of those simple notions, and those primitive axioms, upon which any calculus or science must be based. ... And thus Leibniz supposed that the great requisite was a convenient method of deduction. Whereas, in fact, the problems of philosophy should be anterior to deduction. (Russell 1900: 201)

Thus, for Russell, the hard problem of philosophy is finding the right principles; reasoning from these principles is easy by comparison. Russell's position is echoed by John Woods in his identification of *Philosophy's Most Difficult Problem*: 'that of adjudicating in a principled way the conflict between supposing that [a valid argument] is a sound demonstration of a counterintuitive truth, as opposed to seeing it as a counterexample of its premisses' (Woods 2003: 14). This problem remains, and remains difficult, no matter what progress we make in realizing Leibniz's *characteristica universalis*, whether formally or informally.

Nonetheless, Russell does acknowledge the practical utility of informal reasoning. For example, in a letter published in the February 16th, 1941 edition of the *New York Times*, Russell offered a closely reasoned explanation for his support for the British war effort, and argued that the United States should enter World War II. In the course of his argument, he reflects upon the style of reasoning it employs:

The whole argument, either way, is necessarily based on hypotheses and probabilities, as to which no certainty is possible; but as immediate decisions are forced upon us, we have to act upon such data as can be obtained, knowing that even the most careful consideration may lead us astray. (Russell 2002: 182)

Hence, although Russell may have seen no application for his own formal logic in arguments over war and peace, he does acknowledge a role for what we now call informal logic.

The famously unworldly Kurt Gödel had little interest in politics. However, he was more sanguine about the prospects for the *characteristica universalis* than Russell. Indeed, he seems to have believed that Leibniz had actually perfected the scheme:

But there is no need to give up hope. Leibniz did not in his writings about the *Characteristica universalis* speak of a utopian project; if we are to believe his words he had developed this calculus of reasoning to a large extent, but was waiting with its publication till the seed could fall on fertile ground. (Gödel 1944: 152 f.)

Taken literally, as Martin Davis notes, this seems quite mad: Leibniz's logic is not remotely sophisticated enough to accomplish this goal (Davis 2000: 134). However, Gödel's interest in the *characteristica universalis* does seem to have been restricted to its mathematical workings, rather than its political ambitions. It is the relationship between Gödel's own mathematical work, specifically his incompleteness results, and Leibniz's project that most concerns Brooke.

### 3. MATHEMATICS

Gödel's incompleteness results are of profound importance, but precisely stating that importance can be a challenge. Brooke rightly deprecates the 'application of a misunderstood rendition of Gödel's result to subjects beyond the realm of mathematics', and observes that there is a great temptation to 'make too much of Gödel's result, and, in so doing, run the risk of excessively devaluing the importance of formal logic' (Brooke 2011: 4). As Jon Barwise put it, 'Karl Marx is supposed to have said, "Every time the train of history goes around a corner, the thinkers fall off." Gödel's Theorem was a very sharp corner on the logic line' (Barwise 1981: 99). Brooke itemizes some of the logic line's more accident prone passengers. But this sort of critique is infectious. Brooke tells us that 'what Gödel's result has shown is that Leibniz's dream of a system by which international peace could be established and maintained will not be fulfilled in a solely mathematical

manner' (Brooke 2011: 5). While international peace may be unattainable for many reasons, Gödel's result does not strictly place the *characteristica universalis* beyond the scope of mathematics, but rather of any *single* formal system within mathematics. As Martin Davis summarizes,

Gödel's incompleteness theorem shows that ... [f]or any specific given formalism there are mathematical questions that will transcend it. On the other hand, in principle, each such question leads to a more powerful system which enables the resolution of that question. One envisions hierarchies of ever more powerful systems each making it possible to decide questions left undecidable by weaker systems. (Davis 2000: 124)

On this picture, mathematics is unbounded, but at the expense of outgrowing every formal system. As Raymond Smullyan concludes,

In the prophetic words of the logician Post, this means that mathematics is, and must remain, essentially creative. Or, as commented by the mathematician Rosenbloom, it means that man can never eliminate the necessity of using his own intelligence, regardless of how cleverly he tries. (Smullyan 2001: 88)

In other words, mathematics can never fully transcend its informal aspects. This suggests that the study of informal reasoning should play an indispensable role in the analysis of mathematical practice (as I have argued elsewhere, for other reasons: see, for example, Aberdein 2009).

#### 4. LOGIC

Brooke concludes with the salutary reminder 'that formal and informal practitioners share many of the same goals, and that, if we disagree, it is mainly in our opinions regarding how those goals might be reached' (Brooke 2011: 5). I want to conclude with a few words about the nature of that disagreement. Elsewhere I have characterized the broader context required for a system of logic to be advocated as applicable to natural argumentation as a *logical theory*, a quadruple comprising

⟨system, parsing theory, inferential goal, background theories⟩

(Aberdein and Read 2009: 618; cf. Thagard 1982: 37). Each of these components needs a little explication. The system comprises the syntax, semantics, and metatheory of the logic, or whatever other such details its presentation requires. The parsing theory consists of a method for translating between the system and natural, unsystematized usage. The inferential goal determines what the use of this logic is intended to achieve and what its valid inferences are expected to preserve. The background theories are broader philosophical or psychological theories which inform the choices made in putting together the rest of the theory.

This account gives rise to a hierarchy of moves by which a logical theory may be revised in response to a problem:

- I Indifference;
- II Non-revisionary responses:
  - (a) *Delimitation of the subject matter of logic*;
  - (b) *Novel paraphrase*;
  - (c) *Semantic innovation*;
- III Conservatively revisionary responses;
- IV Non-conservatively revisionary responses:
  - (a) *Restriction of the logic*;
  - (b) *Wholesale revision*;
- V Change of subject matter

(cf. Aberdein and Read 2009: 635).

Brooke draws attention to a problem which became increasingly acute for formal logic in the mid-twentieth century: its inability to contribute significantly to such applied problems as socio-political dispute resolution. One response was the development of argumentation theory by such pioneers as Toulmin and Perelman. In terms of the above hierarchy, this exemplifies the last and most radical of these moves: a change of subject matter. This occurs when a change of inferential goal is ‘precipitated by a non-conservative revision of the background theories. Typically this will alter the motivation of the whole logical enterprise, move the problem into a different area, and change the subject matter of logic’ (Aberdein and Read 2009: 640). This coincides with, for example, the stance of Ralph Johnson and Tony Blair, who ‘distinguish informal logic from formal logic, not only by methodology but also by its focal point ... the cogency of the support that reasons provide for the conclusions they are supposed to back up’ (Johnson and Blair 1997: 161). In my terminology this focal point is the inferential goal. A different response emerged somewhat later with the development of systems of non-classical formal logic, such as non-monotonic logic, which were better adapted to the vicissitudes of natural argumentation. These systems exemplify the previous level of the hierarchy: wholesale, non-conservative revision of the system, accompanied by appropriate adaptations to the rest of the logical theory. For several decades these two responses to the same problem proceeded independently, and indeed, in apparent ignorance of each other. But more recently, we have begun to see a rapprochement between the two. It is perhaps in such hybridization that Leibniz’s goal stands the best hope of some form of fulfilment.

## COMMENTARY

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