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Commentary on Girle

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COMMENTS ON GIRLE'S "BELIEF SETS AND COMMITMENT STORES"

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I am awkwardly positioned to offer a critique of Rod Girle's paper. The awkwardness is that I agree with its assertions and I like the case he makes for them. If we academics were disposed to do our work efficiently, I would in present circumstances end my remarks here, so that we could move on to the enlightenment of true contention elsewhere, perhaps in the bar.

I am not confident enough to make a break with a tradition as hallowed as this one, so I have tried to cast around for something useful to say about this system **DL3**, apart from what we already know about it. I myself am interested in the nature and very existence of strategies for conflict resolution in the abstract sciences. Among the matters that I am currently looking into are such old chestnuts as *ex falso quodlibet* and the set theoretic and semantic paradoxes. These issues are instructive in their way, but differently. From the point of view of conflict resolution, the principal fact about *ex falso* is the sheer awfulness of the arguments made by its detractors and its proponents. And the chief moral to be learned from this fact is that logicians have developed scant expertise for handling such contentions in their own backyard. *Ex falso* says that from a contradiction everything follows. There is a proof of this dating from the first year of the thirteenth century but, in an act of crass American cultural appropriation, it is known these days as the Lewis-Langford proof.[1] Here it is:

1. $\Phi \wedge \neg\Phi$ assumption

2. Φ 1, simplification

3. $\Phi \vee \Psi$ 2, addition

4. $\neg\Phi$ 1, simplification

5. Ψ 3,4, disjunctive syllogism

Conditionalizing, we have it that $\lceil \Phi \wedge \neg\Phi \rceil$ entails arbitrary Ψ.

We get a whiff of the awfulness of arguments which rage over *ex falso* by noting that, in the submission of Anderson and Belnap, the Lewis-Langford proof is "self-evidently preposterous", that the move to line 5 from lines 3 and 4 is "fallacy", indeed that disjunctive syllogism commits a "fallacy of relevance".[2] For his part, Stephen Read finds that there is no single sense of "or" that will concurrently deliver the goods for the rules of addition and disjunctive syllogism. Speaking for the side opposite, Quine complains that giving up on *ex falso* amounts to giving up on classical negation and that it would be difficult to abandon something so basic. This is not argument. This is whimsy.

However, it is hopeful to note that **DL3** gives the appearance of possessing the technical wherewithal for significant improvement upon this bad record, especially in its challenge and resolution rules. Let us see.

Revisiting the proof, S is the classical supporter of *ex falso* and the Lewis-Langford proof; H is the other side. Though **DL3** doesn't offer specific guidance about how to make and share assumptions, let us take it that S and H agree to assume 1, and that in so doing $\lceil \Phi \wedge \neg\Phi \rceil$ enters the commitment store (*CS*) of both parties, with its provisional status suitably flagged. Subsequent steps come by way of **DL3**'s standard rules. Thus line 2 enters the *CS* of both S and H by way of the rule of simplification together with **DL3**'s rule, *LogChall*. Similarly for lines 3 and 4. It is the same way with line 5, or so it would appear. Girle's *Imcon* and *LogChall* "prevent the withdrawal or challenge of logical principles". The prevention embeds an ambiguity. Does it, in the case before us, disenjoin the challenge or abandonment of DS *because* it is a principle of logic? Or does it forbid this *if* DS is a principle of logic? Or yet again, does it forbid it when *S and H agree* that *DS* is a principle of logic? Bearing in mind that S is our classicist and H our relevantist and that they disagree about whether *DS* is a principle of logic, we see that *Imcom* and *LogChall* fail to be usable rules precisely where a usable rule is needed. It is important to emphasize that the Lewis-Langford problem stops cold any prospect of **DL3**-resolution. We might think that the dispute now moves to the question of whether *DS* is a principle of logic. That is not an askable challenge until it is established that *DS* is *not* a rule of logic. Girle's rules oblige us not to challenge *DS* unless it is invalid. But its invalidity is precisely what S and H are deadlocked over. The present result easily generalizes. **DL3** is unable to resolve any disagreement about any "logical principle". This is a disappointment, of course. We should not, however, rush to judgement. **DL3** is no good for disputes about logical principles. But it is not good for nothing, as I shall now show.

Let me now mimic the (epistolary) conversation between Russell and Frege in 1902. Russell is S and Frege is H.

    1. S: If R (the set of all non-self-membered sets) exists then $R \in R$ and $R \notin R$.
    2. H: Yes.
    3. S: By axioms we both accept, R exists.
    4. H: Agreed.
    5. S: So we're in trouble.
    6. H: You can say that again!

At this juncture, I break away from the actual, historical conversation between Russell and Frege. It is well-known, and requires no elaboration here, that in the view of the historical Russell and Frege, the paradox of sets, at a *minimum*, destroyed the intuitive concept of set, the very *idea* of set, so to speak. (This is another matter about which, as I suggested at the beginning, instruction is required). Elsewhere I have called this interpretation of the import of the Russell paradox "*Frege's Sorrow*".

Frege's Sorrow would not have been the conclusion had this conversation been regulated by Girle-rules. To see how this comes to pass we re-start the conversation at step 7.

    7. S: Since our resolution rules tell us to drop a conjunct if a statement in our *CS* is an immediate contradiction, let's drop "$R \in R$".
    8. H: But there is also a rule about honouring immediate consequences of what's left, i.e., "$R \notin R$". The trouble is that "$R \notin R$" immediately restores "$R \in R$"; and we're right back where we started.
    9. S: Worse still, the rules drive us into an endless cycle of resolution and paradox-rebirth.
    10. H: Of course, there is no prospect under the rules of wriggling out of Excluded Middle, is there?
    11. S: No; it's a principle of logic.
    12. H: But look, S. You've shown that if R exists then $R \in R$ and $R \notin R$.

13. S: Unfortunately.

14. H: Now the consequent of that conditional is a logical falsehood, *n`est ce pas*?

15. S: Yes, and of course its negation is a logical truth.

16. H: Right, *And* we can't give up that logical truth and we can't give up your fateful conditional.

17. S: Nor can we give up modus tollens, another principle of logic.

18. H: Which, together with the conditional and our logical truth produces as an immediate consequence the negation of its antecedent.

19. S: You mean, that R doesn't exist, after all?

20. H: Yip.

21. S: So arithmetic isn't toppling ?

22. H: Yip.

What is so striking about the Girle apparatus is that it imposes, perhaps inadvertently, the Barber solution on the paradox of sets. Whether it is right to do so is not here at issue. The point rather is that it takes a radically deflationary stand against paradoxical hysteria. This is so not only for the Russell paradox but also for the paradox of the Liar, putting "If L is a statement, then L is true and L is not true" in place of "If R is a set, then $R \in R$ and $R \notin R$" and repeating the Girle conversation.

To sum up: **DL3** is of no consequence for disputes about what are and aren't principles of logic. But it is tailor-made for paradoxers, and should be welcomed by them.

Notes

1. C.I. Lewis and C.H. Langford, *Symbolic Logic*, New York: Dover 1932, 252.

2. Alan Ross Anderson and Nuel D. Belnap, Jr., *Entailment*, vol. 1, Princeton: University of Princeton Press, 1975.