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Lamb shifts and fine-structure splittings for the muonic ions μ^--Li, μ^--Be, and μ^--B: A proposed experiment

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Detailed calculations are presented for the energy splittings of the states $2s_{1/2}$-$2p_{1/2}$ and $2s_{1/2}$-$2p_{3/2}$ for the muonic ions μ^--Li, μ^--Be, and μ^--B obtained by numerical integration of the Dirac equation. It is shown that there is severe cancellation between the vacuum polarization and finite nuclear size contributions to the energy differences, leading to transition frequencies which lie in the visible region of the spectrum. As a consequence of the cancellation, a measurement of the transition frequency would provide a sensitive probe of nuclear size and structure. The system μ^--^{7}Li appears to offer particularly good possibilities for performing such an experiment.

I. INTRODUCTION

High-precision measurements of the $2s_{1/2}$-$2p_{1/2}$ and $2s_{1/2}$-$2p_{3/2}$ transition frequencies in the muonic system μ^--He^{2+} (Refs. 1 and 2) have stimulated extensive theoretical work on the various quantum electrodynamic and finite nuclear size contributions to the energy levels involved.[3] Theory and experiment have now developed to the point that if the QED contributions (primarily vacuum polarization) are taken as correct, then a precise value for the nuclear radius can be extracted for use in calculations of the corresponding electronic $2s_{1/2}$-$2p_{1/2}$ Lamb shift transition[4] (primarily electron self-energy). The absorption wavelengths for the μ^--He^{2+} system are $\lambda(2s_{1/2}\text{-}2p_{3/2})\simeq 8116$ Å and $\lambda(2s_{1/2}\text{-}2p_{1/2})\simeq 8975$ Å. Since the QED contributions to the energy nominally scale as Z^4, one might expect the corresponding transitions in heavier muonic atoms such as μ^--Li^{3+} and μ^--Be^{4+} to lie in the vacuum-ultraviolet region of the spectrum. However, order-of-magnitude estimates for these heavier systems[5] show that there is severe cancellation between the vacuum polarization and finite nuclear size contributions to the $2s_{1/2}$ energy. As a result, the wavelengths remain in the visible or infrared regions of the spectrum where tunable laser sources are available. The severe cancellation also means that a measurement of the transition wavelength would provide a very sensitive probe of nuclear size and structure.

The purpose of this paper is to provide more detailed calculations of the transition frequencies for the systems μ^--Li^{3+}, μ^--Be^{4+}, and μ^--B^{5+} as a function of the nuclear size. Section II presents a review of the orders of magnitude of the lowest-order contributions, Sec. III contains the details of more precise numerical calculations, and Sec. IV presents the results together with a discussion of possible experiments.

II. ORDERS OF MAGNITUDE FOR DOMINANT CONTRIBUTIONS

In this section, the orders of magnitude of the most important contributions to the energies of light muonic atoms are discussed in order to make clear the physical significance of the detailed numerical calculations described in Sec. III.

In the nonrelativistic limit, the energy levels of a muon of mass m in the field of a point nucleus of charge Z and mass M are given by the simple Rydberg formula

$$E_n = -(Z^2/n^2)\mathscr{R}_\mu \,, \tag{1}$$

where $\mathscr{R}_\mu = e^2/a_\mu$ is the Rydberg constant for a particle of reduced mass $\mu = m_\mu M/(m_\mu + M)$ and $a_\mu = \hbar^2/\mu e^2$ is the corresponding Bohr radius. Since $m_\mu/m_e \simeq 200$, $\mathscr{R}_\mu$ is about 200 times larger than the corresponding Rydberg for an electron, and a_μ is about 200 times smaller than the electronic Bohr radius a_0.

For the $2s_{1/2}$, $2p_{1/2}$, and $2p_{3/2}$ states of the $n=2$ level, the most important corrections are the $2p_{3/2}$-$2p_{1/2}$ fine-structure splitting,

$$\Delta E_{\rm FS} = \frac{\alpha^2 Z^4}{16}\left[1 + \frac{5(\alpha Z)^2}{8}\right]\mathscr{R}_\mu \,, \tag{2}$$

and the finite nuclear size correction to the $2s_{1/2}$ state,

$$\Delta E_R = (Z^4/6)(R^2/a_\mu^2)\mathscr{R}_\mu \,, \tag{3}$$

where R is the rms nuclear radius, and the vacuum polarization correction to the $2s_{1/2}$-$2p_{1/2}$ splitting given in the nonrelativistic limit by[5,6]

$$\Delta E_{\rm VP} = -\frac{\alpha Z^4}{15\pi}(\lambdabar/a_\mu)^2 I(Z\lambdabar/2a_\mu)\mathscr{R}_\mu \,, \tag{4}$$

where

$$I(\beta) = \tfrac{5}{2}\int_0^1 \frac{(1+z^2/2)(1-z^2)^{1/2}}{(1+\beta z)^4} z\,dz \tag{5}$$

and $\lambdabar = \alpha a_0$ is the Compton wavelength. For $\beta < 1$, $I(\beta)$ can be expanded in a convergent power series as

$$I(\beta) = \sum_{n=0}^{\infty} T_n \beta^n \,, \tag{6}$$

with $T_0 = 1$, $T_1 = -25\pi/32$, and the remaining T_n are given by the recurrence relation

TABLE I. Values of $I(\beta)$ and the lowest-order nonrelativistic vacuum-polarization correction for muonic ions from Eq. (4). Here $\beta = Z\lambda\!\!\!^{-}/2a_\mu$ and $I(\beta)\to 1$ in the limit $\beta\to 0$.

Ion	μ/m_e	β	$I(\beta)$	$\Delta E_{\rm VP}$ (meV)
^{1}H	185.841	0.678 076	0.284 695	−205.02
^{4}He	201.069	1.467 26	0.114 154	−1655.77
^{6}Li	202.940	2.221 39	0.061 416	−4665.0
^{7}Li	203.478	2.227 27	0.061 158	−4682.4
^{9}Be	204.198	2.980 20	0.037 833	−9255.8
^{11}B	204.659	3.733 67	0.025 579	−15 376.0

$$T_n = (n+3)(n+4)/[(n-1)(n+5)]T_{n-2}\,. \quad (7)$$

For an electron, $\beta \simeq \alpha Z/2$ is much less than unity and $I(\beta)\simeq 1$. However, for a muon, $\beta \gtrsim 1$ and, as shown in Table I, $I(\beta)$ is much less than unity. The consequent suppression of $\Delta E_{\rm VP}$ plays a crucial role in the near cancellation between $\Delta E_{\rm VP}$ and ΔE_R.

Counting only the terms considered thus far, the energy splittings are given by

$$E(2s_{1/2}) - E(2p_{1/2}) = \Delta E_{\rm VP} + \Delta E_R \quad (8)$$

and

$$E(2s_{1/2}) - E(2p_{3/2}) = \Delta E_{\rm VP} - \Delta E_{\rm FS} + \Delta E_R\,. \quad (9)$$

The ratio

$$\Delta E_{\rm VP}/\Delta E_R = -(2\alpha/5\pi) I(\beta)(\lambda\!\!\!^{-}/R)^2 \quad (10)$$

with $\beta = Z\lambda\!\!\!^{-}/2a_\mu$ is -1.30 for μ^--^{6}Li and -0.82 for μ^--^{9}Be. Since the ratio would be -1 if the $2p_{1/2}$ and $2s_{1/2}$ states were degenerate, this explains the reversal in sign of the $2p_{1/2}$-$2s_{1/2}$ energy splitting for these two cases. Similarly, the ratio

$$(\Delta E_{\rm VP} - \Delta E_{\rm FS})/\Delta E_R = -[(2\alpha/5\pi)I(\beta) + 3/8(m_e/\mu)^2](\lambda\!\!\!^{-}/R)^2 \quad (11)$$

is -1.04 for μ^--^{10}Be and -0.91 for μ^--^{11}B. This explains the reversal in sign of the $2p_{3/2}$-$2s_{1/2}$ energy splitting. The closer the ratio is to -1, the more sensitively the energy splittings depend upon ΔE_R.

The above does not include a number of small corrections discussed in the following section. As will be shown there, they are too small to affect the qualitative features of the energy splittings, but must be included in a quantitative comparison with experiment.

III. DETAILED CALCULATIONS

The results of Sec. II were obtained by using nonrelativistic wave functions, and treating the finite nuclear size and vacuum-polarization potentials as small perturbations. In the present section, these terms are included explicitly in numerical solutions to the Dirac equation, and the remaining small corrections due to the muon self-energy, the higher-order Källén-Sabrey vacuum-polarization term, and nuclear polarization are included by perturbation theory. The calculations of Borie and Rinker[3] for muonic helium show that all of these effects are required for a precise comparison with experiment.

A. Numerical solution of the Dirac equation

In units with $\hbar = c = 1$, the Dirac equation for a muon of reduced mass μ is

$$[\boldsymbol{\alpha}\cdot\mathbf{p} + \beta\mu + V_N(r) + V_{\rm VP}(r)]\psi = E\psi\,, \quad (12)$$

where $V_N(r)$ is the potential due to a nucleus of finite size and $V_{\rm VP}(r)$ is the lowest-order Uehling vacuum-polarization potential. Following Rinker[7] we assume a Gaussian charge distribution for the nucleus so that

$$V_N(r) = -(2Ze^2/\sqrt{\pi})\int_0^{r/r_0} e^{-\mu}du\,, \quad (13)$$

with $r_0 = (2R^2/3)^{1/2}$. For a nucleus with charge density $\rho(r)$, the vacuum-polarization potential is[3]

$$V_{\rm VP}(r) = -(2\alpha^2/3\pi)\int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} X_1(2\,|\mathbf{r}-\mathbf{r}'|)\,, \quad (14)$$

with

$$X_n(x) = \int_1^\infty dt \frac{(t^2-1)^{1/2}}{t^{n+1}}\left[1+\frac{1}{2t^2}\right]e^{-2xt}\,.$$

Since the finite nuclear radius changes the contribution from $V_{\rm VP}$ by only a small amount, we assumed a uniform charge distribution

$$\rho(r) = \begin{cases} 3Z/4\pi r_c^3 & \text{for } r < r_c\,, \\ 0 & \text{for } r > r_c \end{cases} \quad (15)$$

with $r_c = (\frac{5}{3})^{1/2}R$. Approximate numerical methods described by Huang,[8] and Fullerton and Rinker[9] were then used to evaluate (14). Finally, the eigenvalues of the radial equation associated with (12) were located by standard methods of numerical integration. Values of the physical constants used are given in Table II. The results for several values of R are listed in Table III. The individual eigenvalues were located to an accuracy of 1 part in 10^8. The errors quoted in Table III are the resulting uncertainty after subtraction of the nearly degenerate eigenvalues. For the reasons outlined in Sec. II, many of the predicted energy splittings change dramatically in response to small changes in the assumed value of the nuclear radius. The values obtained for ^{4}He at $R = 1.674$ fm are in good agreement with the sum of the Uehling vacuum polarization (first iteration plus higher iterations), fine structure, and finite nuclear size contributions calculated by Borie and Rinker.[3]

TABLE II. Values of physical constants used.

Constant	Value
α^{-1}	137.035 96
a_0	$0.529\,177\times 10^{-8}$ cm
$\lambda\!\!\!^{-}$	386.159 fm
$m_\mu c^2$	105.659 46 MeV
m_μ/m_e	206.769

TABLE III. Calculated $2s$-$2p$ splittings of muonic ions by numerical integration of the Dirac equation (12) including Uehling vacuum polarization and finite nuclear size terms (in meV). Numbers in parentheses indicate the uncertainty in the final figure quoted.

Ion	R (fm)	$2p_{1/2}$-$2s_{1/2}$	$2p_{3/2}$-$2s_{1/2}$
^{4}He	1.644	1387.1(1)	1533.0(1)
	1.674	1376.9(1)	1522.8(1)
	1.704	1366.4(1)	1512.4(1)
^{6}Li	2.460	1483.0(1)	2229.4(1)
	2.560	1225.7(1)	1972.1(1)
	2.660	959.1(1)	1705.4(1)
^{7}Li	2.290	1894.2(1)	2642.9(1)
	2.390	1651.3(1)	2399.9(1)
	2.490	1398.6(1)	2147.3(1)
^{9}Be	2.420	−525.0(2)	1850.4(2)
	2.520	−1320.9(2)	1052.7(2)
	2.620	−2147.2(2)	223.0(2)
^{10}Be	2.350	−6976.7(2)	−1160.9(2)
	2.450	−8832.7(2)	−3017.6(2)
	2.550	−10 755.1(2)	−4940.6(2)
^{11}B	2.320	−6478.5(2)	−656.6(2)
	2.420	−8319.6(2)	−2498.4(2)
	2.520	−10 227.7(2)	−4407.1(2)

B. Small corrections

The first of the small corrections not included in the preceding section is the muon self-energy given for a point nucleus by

$$\Delta E_{\rm SE}(2p_j)-\Delta E_{\rm SE}(2s_{1/2})=\frac{-\alpha(Z\alpha)^4}{6\pi}m_\mu c^2\{(1+m_\mu/M)^{-3}[\ln(Z\alpha)^{-2}-\ln(\epsilon_{20}/\epsilon_{21})+\tfrac{11}{24}+\tfrac{3}{8}-\tfrac{1}{5}(1+0.66)]$$
$$-(1+m_\mu/M)^{-2}c_j/8+3\pi\alpha Z(\tfrac{139}{128}-\tfrac{1}{2}\ln2+\tfrac{5}{192})+O(\alpha/\pi)+O(\alpha^2Z^2)\}\ , \qquad (16)$$

where

$$c_j=\begin{cases}\frac{1}{2} & \text{for } 2p_{3/2}\ ,\\ -1 & \text{for } 2p_{1/2}\ .\end{cases}$$

The above includes the anomalous magnetic moment and muon vacuum-polarization contributions. The term containing 0.66 in (16) arises from the estimate of Borie and Rinker[3] of the correction due to hadronic vacuum polarization given by

$$\Delta E_{\rm HVP}\simeq 0.66\Delta E_{\mu{\rm VP}}\ .$$

Higher-order vertex corrections of order $\alpha^2(Z\alpha)^4$ and

TABLE IV. Summary of small corrections to the $2s$-$2p$ splittings of muonic ions (in meV). Column A: Sum of muon self-energy, anomalous magnetic moment, muon vacuum-polarization, and hadronic vacuum-polarization terms in Eq. (16). Column B: Second-order (Källén-Sabry) vacuum-polarization term.

	$2p_{1/2}$-$2s_{1/2}$		$2p_{3/2}$-$2s_{1/2}$	
Ion	A	B	A	B
^{4}He	−10.46	11.55	−10.13	11.55
^{6}Li	−45.2	32.27	−43.5	32.27
^{7}Li	−45.9	32.44	−44.2	32.44
^{9}Be	−128.8	65.3	−123.3	65.3
^{10}B	−283.8	111.3	−270.5	111.3
^{11}B	−285.3	111.1	−272.0	111.1

$\alpha(Z\alpha)^6$ are negligibly small. For the Bethe logarithm, we used the nonrelativistic point nucleus value

$$\ln(\epsilon_{20}/\epsilon_{21})=\ln(\epsilon_{20}/Z^2R_\infty)-\ln(\epsilon_{21}/Z^2R_\infty)\simeq 2.841\,79\ .$$

We also include the finite nuclear radius correction to the muon self-energy discussed by Borie.[10] For this term, one replaces $\langle\nabla^2V\rangle$ by $\langle -4\pi\rho(\mathbf{r})\rangle$ to obtain

$$\delta E_{\rm SE}(2s_{1/2})=\frac{-(Z\alpha)^5}{6\pi}\left[\frac{35R}{8(3)^{1/2}\lambda\!\!\!^{-}_\mu}\right]\times[\ln(Z\alpha)^{-2}+\ln(R_\infty/\epsilon_{20})+\tfrac{5}{6}]\mu c^2\ , \quad (17)$$

where $\lambda\!\!\!^{-}_\mu=\lambda\!\!\!^{-} m_e/\mu$ is the muon Compton wavelength. Lepage *et al.*[11] have argued that (17) greatly overestimates the size of the correction in the case of electrons because $R \ll \lambda\!\!\!^{-}$ and the nonrelativistic approximations used to obtain (16) are no longer valid in this region. However, for muons, $R\simeq\lambda\!\!\!^{-}_\mu$ and the nonrelativistic approximation is valid in lowest order. The effect of this term is to reduce the magnitude of the self-energy contribution to the $2s$-$2p$ splittings by about $2Z\%$. The shifts obtained from (16) and (17) are listed in Table IV.

The next small correction is the second-order vacuum-polarization term derived by Källén and Sabry.[12] Their result can be expressed in terms of a correction to the Coulomb field of the form[8,9,13]

$$V_{\rm VP2}(r)=-(Z\alpha/r)R_{21}(r)\ , \quad (18)$$

where

$$R_{21}(r)=-\frac{\alpha^2}{\pi^2}\int_1^\infty dt\, e^{-2tr}\Big\{(\tfrac{13}{54}t^{-2}+\tfrac{7}{108}t^{-4}+\tfrac{2}{9}t^{-6})(t^2-1)^{1/2}+(\tfrac{44}{9}t^{-1}+\tfrac{2}{3}t^{-3}+\tfrac{5}{4}t^{-5}+\tfrac{2}{9}t^{-7})\ln[t+(t^2-1)^{1/2}]+(\tfrac{4}{3}t^{-2}+\tfrac{2}{3}t^{-4})(t^2-1)^{1/2}\ln[8t(t^2-1)]+(-\tfrac{8}{3}t^{-1}+\tfrac{2}{3}t^{-5})\int_t^\infty dx\, f(x)\Big\} \quad (19)$$

and

$$f(x)=\frac{3x^2-1}{x(x^2-1)}\ln[x+(x^2-1)^{1/2}]-\frac{1}{(x^2-1)^{1/2}}\ln[8x(x^2-1)]\ .$$

The energy shift for each state was obtained by numerical integration of the first-order perturbation expression

$$\Delta E_{\rm VP2}=\frac{\int_0^\infty[\,|f(r)|^2+|g(r)|^2]V_{\rm VP2}(r)r^2dr}{\int_0^\infty[\,|f(r)|^2+|g(r)|^2]r^2dr}\ ,$$

where $g(r)$ and $f(r)$ are the large and small components of the radial Dirac wave functions obtained from the numerical integration of (12). The resulting contributions to the energy splittings are given in Table IV. The values for μ^--^{4}He agree with the calculations of Borie and Rinker.[3] Higher-order vacuum-polarization corrections of order $\alpha^2(Z\alpha)^2$ and $\alpha(Z\alpha)^3$ are small compared with the nuclear polarization uncertainties discussed below, and so are not included.

The remaining small correction is the energy shift due to the polarization of the nucleus by the electric field of the muon. This is the least certain part of the calculation because dynamic nuclear polarizabilities are not well known, and accurate calculations depend upon a detailed knowledge of nuclear structure.[7,14–17] In fact, for $Z\gtrsim 10$, the Coulomb energy of the muon is no longer small compared with typical nuclear excitation energies.[17] Detailed calculations of energy shifts have been done by Rinker[7] only for the systems μ^--^{3}He and μ^--^{4}He, and even these are accurate to only $\pm 20\%$ because of various approximations made.[14] The accuracy of models for the nuclear response function has also been checked by Rosenfelder[18] for the case of ^{12}C. However, the reliability of different computational methods is still a matter of controversy in the literature.

Our approach here is to use a semiempirical formula based upon an approximate expression for the energy shift obtained by Rinker,[7] adjusted to reproduce his results for μ^--^{3}He and μ^--^{4}He. He shows that in the nonrelativistic closure approximation, the energy shift of the $2s$ state due to virtual dipole excitations of the nucleus is given by

$$\Delta E^{2s}=\frac{\alpha^4Z^3}{8\pi^2}\frac{(m_\mu c^2)^3}{hc\tilde{R}}\sigma_{-2}\ , \quad (20)$$

where σ_{-2} is related to the nuclear photoabsorption cross section $\sigma(E)$ by

$$\sigma_{-2}=\int_0^\infty dE\, E^{-2}\sigma(E) \quad (21)$$

and $\tilde{R}$ is an arbitrary cutoff of the order of the nuclear radius R which must be introduced to avoid a divergence at the origin. Rinker[7] shows that setting $\tilde{R}=R$ yields the correct order of magnitude for ΔE^{2s}, but the numerical value is too large by about a factor of 2. However, the choice $\tilde{R}=2.36R$, together with the experimentally determined values for R and σ_{-2}, yields values for ΔE^{2s} which are in close agreement with Rinker's detailed calculations for both μ^--^{3}He and μ^--^{4}-He. With the above choice of R, Eq. (20) becomes

TABLE V. Estimates of the nuclear polarization energy shift for the $2s_{1/2}$ state of muonic ions.

Ion	R (fm)[a]	σ_{-2} (fm^2/MeV)	ΔE_2^{2s} (meV)
^{3}He	1.87	0.013[b]	-4.9 ± 1.0
^{4}He	1.67	0.0074[b]	-3.1 ± 0.6
^{6}Li	2.56	0.016[c]	-15 ± 4
^{7}Li	2.39	0.0210[d]	-21 ± 4
^{9}Be	2.52	0.0366[d]	-82 ± 16
^{10}B	2.45	0.023[c]	-103 ± 21
^{11}B	2.42	0.027[c]	-122 ± 24

[a]Reference 19.
[b]Quoted by Rinker (Ref. 7).
[c]Value interpolated from the data of Ahrens *et al.* (Ref. 20) assuming an $A^{5/3}$ dependence on atomic weight.
[d]Reference 20.

$$\Delta E^{2s} \simeq -87.8 Z^3 \sigma_{-2}/R \text{ meV} , \tag{22}$$

with R expressed in fm and σ_{-2} in fm^2/MeV. The numerical values of R, σ_{-2}, and ΔE^{2s} are summarized in Table V for ions up to $Z=5$.[19,20] As discussed by Ahrens *et al.*,[20] the value of σ_{-2} is anomalously high for ^{9}Be because of a large magnetic dipole contribution at low energies. The values of ΔE^{2s} for both ^{3}He and ^{4}He coincide with the two significant figures quoted by Rinker.[7] However, all the results are assigned the same $\pm 20\%$ uncertainty already present in Rinker's calculation.

In addition to the above, there are further small relativistic nuclear recoil corrections denoted by Borie and Rinker[21] as

$$\Delta B_{2S} \simeq \tfrac{9}{70}(Z\alpha)^5 (r_c/\lambda\!\!\!^{-})(\mu^2/Mm_e)\mu c^2 \tag{23}$$

for a uniform nucleus of radius r_c [cf. Eq. (15)] and

$$\Delta S_{1,2} = \frac{-(Z\alpha)^5}{6\pi}(\mu/M)\mu c^2[\tfrac{97}{12} + 2\ln(\epsilon_{21}/\epsilon_{20}) - \tfrac{1}{2}\ln(Z\alpha)] . \tag{24}$$

Although the individual contributions from ΔB_{2s} and $\Delta S_{1,2}$ are comparable in size to the nuclear polarization uncertainties listed in Table V, they almost exactly cancel one another, leaving a net contribution of less than 0.2 meV in all cases studied. These terms are therefore not tabulated separately. As an example, the numerical values for ^{11}B are $\Delta B_{2s} = 14.9$ meV and $\Delta S_{1,2} = -15.0$ meV.

IV. RESULTS AND DISCUSSION

The total predicted values for the $2p_{1/2}$-$2s_{1/2}$ and $2p_{3/2}$-$2s_{1/2}$ transition energies are listed in Table VI, along with the assumed value R_0 for the nuclear radius. Also listed are values of the coefficients a and b in the empirical expansion

$$\Delta E(R) = \Delta E(R_0) + a(R-R_0) + b(R-R_0)^2 \tag{25}$$

to permit calculation of energy differences for other values of R near R_0. The uncertainties listed are only those due to the nuclear polarization contributions. The results for ^{4}He agree to within 0.1 meV with the calculations of Borie and Rinker.[21] The transition wavelengths and the corresponding uncertainties due to both nuclear polarization and nuclear size are given in Table VII. It is clear that for all the ions studied beyond ^{4}He, the wavelength uncertainty due to the assumed value of the nuclear radius is much greater than that due to nuclear polarization. All of the ions have $2s_{1/2}$-$2p_{1/2}$ or $2s_{1/2}$-$2p_{3/2}$ transitions in the visible (or near-ir) regions of the spectrum. Assuming that the theoretical description is correct, then even a crude measurement of the transition wavelength would determine a much improved value for the nuclear radius. For ions heavier than ^{11}B, the corresponding tran-

TABLE VI. Calculated transition energies for muonic ions. R_0 is the assumed rms nuclear radius, and a and b are the parameters appearing in Eq. (25).

Ion	R_0 (fm)	$\Delta E(R_0)$ (meV)	a (meV/fm)	b (meV/fm^2)
		$\Delta E(2p_{1/2}$-$2s_{1/2})$		
^{4}He	1.674	1381.0 ± 0.6[a]	344.0	114.0
^{6}Li	2.560	1228.0 ± 4.0	2620.0	468.0
^{7}Li	2.390	1659.0 ± 4.0	2478.0	483.0
^{9}Be	2.520	-1302.0 ± 16.0	8111.0	1518.0
^{10}B	2.450	-8902.0 ± 21.0	18 893.0	3322.0
^{11}B	2.420	-8372.0 ± 24.0	18 748.0	3347.0
		$\Delta E(2p_{3/2}$-$2s_{1/2})$		
^{4}He	1.674	1527.3 ± 0.6	344.0	114.0
^{6}Li	2.560	1976.0 ± 4.0	2620.0	468.0
^{7}Li	2.390	2409.0 ± 4.0	2478.0	483.0
^{9}Be	2.520	1077.0 ± 16.0	8137.0	1594.0
^{10}B	2.450	-3074.0 ± 21.0	18 890.0	3316.0
^{11}B	2.420	-2537.0 ± 24.0	18 753.0	3342.0

[a]Uncertainty due to nuclear-polarization contribution.

TABLE VII. Calculated absorption wavelengths (in Å) for transitions in muonic ions. The first uncertainty listed for the wavelengths is that due to nuclear polarization and the second is that due to the rms nuclear radius R.

Ion	R (fm)	$\lambda(2s_{1/2}\text{-}2p_{1/2})$	$\lambda(2s_{1/2}\text{-}2p_{3/2})$
^{4}He	1.674±0.012	8978.0± 4±27	8118.0± 3±22
^{6}Li	2.56 ±0.05	10097.0± 33±1072	6275.0± 13±414
^{7}Li	2.39 ±0.03	7473.0± 18±334	5147.0± 9±159
^{9}Be	2.520±0.012	−9520.0±116±703	11512.0±173±1048
^{10}B	2.45 ±0.12	−1393.0± 3±354	−4033.0± 27±2947
^{11}B	2.42 ±0.12	−1481.0± 4±397	−4887.0± 46±4286

sitions all lie in the far ultraviolet or x-ray regions of the spectrum.

An experimental measurement could be done as for helium[1,2] by tuning a dye laser to the $2s_{1/2}$-$2p_{1/2}$ or $2s_{1/2}$-$2p_{3/2}$ transition frequency and detecting the prompt $2p$-$1s$ x ray that is emitted at resonance. The $2s_{1/2}$ state is metastable with a two-photon decay rate to the ground state of $1.65\times10^3Z^6$ sec^{-1}. A likely candidate would be the $2s_{1/2}$-$2p_{3/2}$ transition of ^{7}Li near 5150 Å. For this ion, the predicted wavelength uncertainty is not so large as to make the resonance difficult to find and the $2s_{1/2}$ state radiative lifetime is 0.83 μsec, which is about the same as for the muon itself. The $2p\rightarrow1s$ transition energy is 18.7 keV. A major problem concerns devising a suitable source. Bertin *et al.*[1] had to use a pressure vessel containing 40 atm of helium gas in order to capture a significant fraction of the muons in the detection region. It is probably impractical to achieve such pressures with pure lithium vapor, but it may be possible to add sufficient lithium to a high-pressure helium buffer gas in a heat-pipe apparatus[22,23] for an experiment to be feasible. The helium would thermalize the muons, but they would be preferentially captured by the more highly charged Li nuclei. As the muons cascade down, they would tend to eject the atomic electrons through Auger transitions.[24,25] This is important because otherwise the atomic electrons would induce a rapid depopulation of the $2s_{1/2}$ muonic state through monopole Auger-electron emission.

Another possibility one could consider is a crystalline source such as LiH or LiF. Although the unperturbed crystal field of cubic symmetry would not depopulate the $2s_{1/2}$ muonic state, it seems likely that the violent vibrational excitations that would accompany the processes of muon capture and electron ejection would produce a rapid quenching of the metastable state. It would seem that a gas-phase experiment has the best initial chance of success if the other technical problems can be overcome. However, if a time-resolved signal can be observed from μ^--Li ions imbedded in a crystal, it would provide a valuable probe of vibrational relaxation effects.

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