Article

Money Demand Function of $M_2 + CD$ Decomposed into M_1 and Quasi Currency+CD through Cointegration Analysis

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abstract

We analyze a money demand function in long equilibrium relation defined by a cointegration property among (money, gdp, interest rate). A wide sense of money "$M_2 + CD$" consists of narrow money "M_1" and wide one "*quasi currency* + *CD*" (denoted by *q-money*). Regarding money as an asset, a rigorous correspondence of money to interest rate requires that M_1 and *q-money* should be coupled with short-term interest rate and spread interest rate (long-term interest rate minus short-term one) respectively. Hence, The cointegration between $M_2 + CD$ and gdp should be described by two kinds of interest rates stated above. Due to a deflationary economy in Japan, cointegration property is said to break down, because for future anxiety people save money and the balance between gdp and total amount of money is disturbed. Such saved money is called "precautionary demand". Based on the fact that precautionary demand is increased in recession periods, we assume that precautionary demand is proportional to the magnitude of recession. Denoting $v(t)$ as the business condition given by TANKAN DI, we define $v_-(t) = |\min\{v(t), 0\}|$ and $v_+(t) = \max\{v(t), 0\}$. Adjusted moneys are defined as $M_{1adj} = M_1 - k_1 * v_- - k_2 * v_+$ and $q\text{-}money_{adj} = q\text{-}money - \tilde{k}_1 * v_- - \tilde{k}_2 * v_+$ respectively. The cointegration property can be shown to hold among (M_{1adj}, $q\text{-}money_{adj}$, gdp, interest rates), where parameters k_1, k_2, $\tilde{k}_1$ and $\tilde{k}_2$ are estimated under the criterion such that the log-likelihood of gdp in VEC model shuld be maximized.

Keywords: money demand function, cointegration, estimation of precautionary demand

1 Introduction

When the central bank wants to do an adequate monetary policy, the stable relationship is necessary among macro economic variables like the real money demand, real income and interest rate. That is, the stable money demand function should be obtained in the form of $M/p = f(y, r)$, where real money demand(M/p) is a function of the real income(y) and the opportunity cost of holding money(r). y is positively related to the money demand, because real income means the transactions and wealth effect. One of the most important aspects of modeling the money demand is the choice of opportunity cost variables. The previous literatures measure the opportunity cost by the difference between return on money (own rate) and returns on alternative assets (rival rate).

One of famous money demand models was given by Goldfeld (1973)[1], where M_1/p is regressed by regressors of real GNP, short-term interest rate and M_1/p itself with the first lag. Since variables under consideration are in many cases nonstationary, Goldfeld's model was criticized as a spurious regression. Although nonstationary variables are differenced in order to generate a stationary model, Engle and Granger (1987)[2] showed that cointegration property has to be taken into consideration for long-run equilibrium relationship among nonstationary variables. Their model is called "Error Correction Model". However, in their model, a number of cointegrating vectors is assumed to be 1. Johansen (1988)[3] and Johansen and Juselius (1990)[4] detected a rank of matrix in order to find out multiple cointegrating vectors and formulated Vector Error Correction Model. The money demand function in long-run equilibrium relation is given by a cointegration property, using either Engle and Granger's method or Johansen's one.

The extensive papers estimated the money demand function through a cointegration property of (money, gdp, interest rate). A wide sense of money "$M_2 + CD$" consists of narrowly defined money "M_1" and widely defined one "*q-money*", where the former money includes only currency and demand deposits and the latter includes time deposits and certificate of deposits. For M_1 money, Nakashima and Saito (2002)[5] showed a cointegration property with a structural change, where call rate was adopted as an interest rate. Fujiki and Watanabe (2004) [6] also showed a cointegration property using ln(call rate) in the interval till 1996. For $M_2 + CD$ money, Kimura and Fujita (1999) [7] insisted that the financial shock in the end of 1997 broke a cointegration property and that a new set of variables (money, gdp, interest rate, financial anxiety) makes it possible to hold a cointegration property. Although recognizing the influence of financial shock, Hosono et al. (2001) [8] insisted that a cointegration property still holds till 1999 without a new variable of financial anxiety. Rahman et al.(2006) [9] improved the derivation of financial anxiety and found out a cointegration property in a wider range of time intervals by estimating precautionary demand due to financial anxiety. In three cases stated above, spread interest rate is adopted by a difference between long and short-term interest rates. Miyao(2006)[10] showed that a cointegration holds for M_1 and does not hold for $M_2 + CD$, where call rate and ln(call rate) are used in both kinds of money. Morita et al(2008)[11] insisted that M_1 should be considered with short-term interest rate while *q-money* should be with spread interest rate, and showed that a cointegration property holds among $M_2 + CD$, gdp and two kinds of interest rates in (1980q4,2002q1). This paper extends Morita's result and shows the cointegration property even in the interval (1980,2005), by introducing adjusted money associated with the estimation of precautionary demand.

2 Cointegration Analysis

2.1 Data Description

We consider $M_2 + CD$, M_1, *q-money*, GDP and several kinds of interest rates to analyze the monetary system in Japan. These data are obtained from the Nikkei Needs Database over the period (1980q1,2005q2). All variables except interest rates and business condition are seasonally adjusted.

Symbolic notations are given:

$rm2(t)$	$= \ln(M_2 + CD/p)$	$rm1(t)$	$= \ln(M_1/p)$
$rqm(t)$	$= \ln(q\text{-}money/p)$	$y(t)$	$= \ln(\text{nominal GDP}/p)$
r_{cd}	= average of CD interest rates	$lnr_{cd}(t)$	$= \ln(r_{cd})$
$r_{call}(t)$	= call rate	$r_{GB}(t)$	= interest rate of 10 years gov. bond
$r_{spd}(t)$	$= r_{GB} - r_{cd}$	$p(t)$	= GDP deflator
$v(t)$	= business condition	$v_-(t)$	$= \lvert\min\{v(t), 0\}\rvert$
$v_+(t)$	$= \max\{v(t), 0\}$		

2.2 Unit root test

Two kinds of unit root test are carried out. One is DF-GLS (ERS) test with unit root as the null hypothesis and the other is KPSS test with stationarity as the null hypothesis. The results are shown in Table 1.

Table 1: Unit Root Test $[1980q1, 2005q2]$

var.	ERS	*lag*	KPSS	*trend*
$rm2$	0.537	2	$1.195{*}{*}{*}$	*intercept*
$rm1$	-0.459	1	$0.290{*}{*}{*}$	*trend+intercept*
rqm	0.259	1	$0.977{*}{*}{*}$	*intercept*
y	1.005	3	$1.173{*}{*}{*}$	*intercept*
lnr_{cd}	1.748	3	$1.058{*}{*}{*}$	*intercept*
r_{spd}	$-1.992{*}{*}$	0	$0.527{*}{*}$	*intercept*

$*{*}{*}$, $*{*}$ and $*$ denote significance levels of 1%, 5% and 10% respectively. A lag length is decided by AIC.

Every variable in Table 1 exhibits unit root property except r_{spd} whose unit root (i.e. nonstationary property) as well as stationarity is rejected. For convenience, we regard r_{spd} to be nonstationary. We also checked 1st differenced processes to be stationary. It should be noted that the call rate r_{call} has been fixed as 0.001% since 2001 due to the zero interest rate policy, while an interest rate r_{cd} varies similarly as r_{call} and is not fixed even after 2001. Therefore, r_{cd} is adopted as short-term interest rate instead of r_{call}. See Figures 1, 2 and 3.

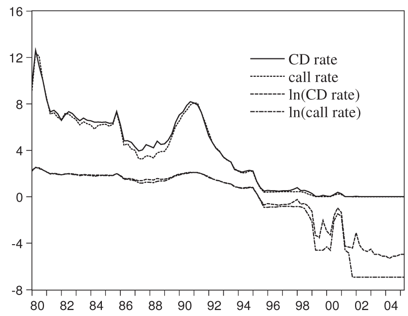

Figure 1: Behavior of Interest Rates $r_{cd}(t)$, $r_{call}(t)$, $\ln(r_{cd}(t))$ and $\ln(r_{call}(t))$ in $[1980, 2005]$

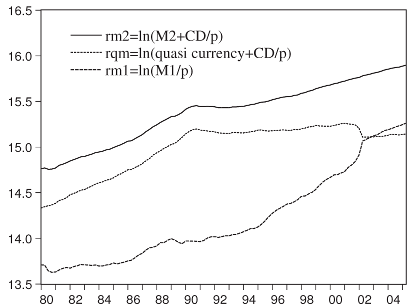

Figure 2: Behavior of Money Supply $rm2(t)$, $rm1(t)$ and $rqm(t)$ in $[1980, 2005]$, where $M_2 + CD = M_1 + q\text{-}money$

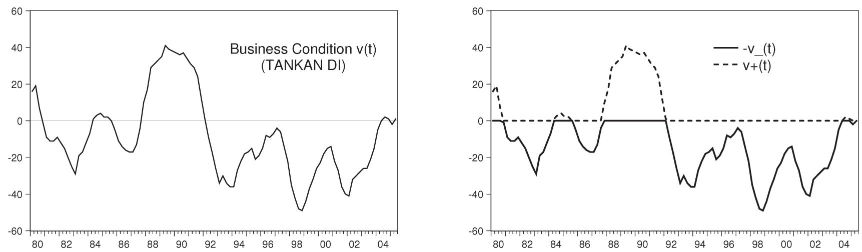

Figure 3: Behavior of Business Condition $v(t)$ which is decomposed into $-v_{-}(t)$ and $v_{+}(t)$ in $[1980, 2005]$

2.3 Cointegration Property in Vector Error Correction Model

Consider the p-dimensional autoregressive process $x(t)$ defined by the equations(Johansen [3]).

$$x(t) = \sum_{i=1}^{k} \Pi_i x(t-i) + \Phi D(t) + \varepsilon(t), \tag{1}$$

where the deterministic terms $D(t)$ can contain constant, a linear term, seasonal dummies and so on. Assume that $x(t)$ belongs to $I(1)$ class, that is, $x(t)$ is nonstationary and $\triangle x(t)$ is stationary. Equation(1) can be written in error correction form:

$$\triangle x(t) = \Pi x(t-1) + \sum_{i=1}^{k-1} \Gamma_i \triangle x(t-i) + \Phi D(t) + \varepsilon(t), \tag{2}$$

where $\Pi = \sum_{i=1}^{k} \Pi_i - I$ and $\Gamma_i = -\sum_{j=i+1}^{k} \Pi_j$. When the rank of Π is $r < p$, then Π has a representation of

$$\Pi = \alpha\beta' \quad \text{with} \quad \alpha, \beta(p \times r). \tag{3}$$

The above relation implies that $\beta' x$ becomes stationary, while $x(t)$ itself is nonstationary, that is, there are r kinds of linear combinations each of which is stationary although every element of x is nonstationary.

Setting $x(t) = (rm1, rqm, y, lnr_{cd}, r_{spd})'$, we rewrite Eq.(2) in the case of two cointegrating vectors denoted by $ect1(t-1)$ and $ect2(t-1)$:

$$\triangle rm1(t) = c^0_{m1} + \alpha^1_{m1} ect1(t-1) + \alpha^2_{m1} ect2(t-1) + \sum_{i=1}^{p} c^i_{m1} \triangle rm1(t-i) + \sum_{i=1}^{p} d^i_{m1} \triangle rqm(t-i)$$
$$+ \sum_{i=1}^{p} e^i_{m1} \triangle y(t-i) + \sum_{i=1}^{p} f^i_{m1} \triangle lnr_{cd}(t-i) + \sum_{i=1}^{p} g^i_{m1} \triangle r_{spd}(t-i) + \varepsilon_{m1}(t), \tag{4}$$

$$\triangle rqm(t) = c^0_{qm} + \alpha^1_{qm} ect1(t-1) + \alpha^2_{qm} ect2(t-1) + \sum_{i=1}^{p} c^i_{qm} \triangle rm1(t-i) + \sum_{i=1}^{p} d^i_{qm} \triangle rqm(t-i)$$
$$+ \sum_{i=1}^{p} e^i_{qm} \triangle y(t-i) + \sum_{i=1}^{p} f^i_{qm} \triangle lnr_{cd}(t-i) + \sum_{i=1}^{p} g^i_{qm} \triangle r_{spd}(t-i) + \varepsilon_{qm}(t), \tag{5}$$

$$\triangle y(t) = c^0_{y} + \alpha^1_{y} ect1(t-1) + \alpha^2_{y} ect2(t-1) + \sum_{i=1}^{p} c^i_{y} \triangle rm1(t-i) + \sum_{i=1}^{p} d^i_{y} \triangle rqm(t-i)$$
$$+ \sum_{i=1}^{p} e^i_{y} \triangle y(t-i) + \sum_{i=1}^{p} f^i_{y} \triangle lnr_{cd}(t-i) + \sum_{i=1}^{p} g^i_{y} \triangle r_{spd}(t-i) + \varepsilon_{y}(t), \tag{6}$$

$$\triangle lnr_{cd}(t) = c^0_{cd} + \alpha^1_{cd} ect1(t-1) + \alpha^2_{cd} ect2(t-1) + \sum_{i=1}^{p} c^i_{cd} \triangle rm1(t-i) + \sum_{i=1}^{p} d^i_{cd} \triangle rqm(t-i)$$
$$+ \sum_{i=1}^{p} e^i_{cd} \triangle y(t-i) + \sum_{i=1}^{p} f^i_{cd} \triangle lnr_{cd}(t-i) + \sum_{i=1}^{p} g^i_{cd} \triangle r_{spd}(t-i) + \varepsilon_{cd}(t), \tag{7}$$

$$\triangle r_{spd}(t) = c^0_{spd} + \alpha^1_{spd} ect1(t-1) + \alpha^2_{spd} ect2(t-1) + \sum_{i=1}^{p} c^i_{spd} \triangle rm1(t-i) + \sum_{i=1}^{p} d^i_{spd} \triangle rqm(t-i)$$
$$+ \sum_{i=1}^{p} e^i_{spd} \triangle y(t-i) + \sum_{i=1}^{p} f^i_{spd} \triangle lnr_{cd}(t-i) + \sum_{i=1}^{p} g^i_{spd} \triangle r_{spd}(t-i) + \varepsilon_{spd}(t), \tag{8}$$

with error correction terms $ect1(t)$ and $ect2(t)$:

$$ect1(t) = rm1(t) + \beta_0 + \beta_1 y(t) + \beta_2 lnr_{cd}(t) + \beta_3 r_{spd}(t), \tag{9}$$
$$ect2(t) = rqm(t) + \tilde{\beta}_0 + \tilde{\beta}_1 y(t) + \tilde{\beta}_2 lnr_{cd}(t) + \tilde{\beta}_3 r_{spd}(t). \tag{10}$$

We introduce the restriction on cointegrating vector β.

[Assumption] Assume that $\beta_3 = 0$ and $\tilde{\beta}_2 = 0$;

$$rm1(t) + \beta_0 + \beta_1 y(t) + \beta_2 lnr_{cd}(t) = 0, \tag{11}$$
$$rqm(t) + \tilde{\beta}_0 + \tilde{\beta}_1 y(t) + \tilde{\beta}_3 r_{spd}(t) = 0. \tag{12}$$

The first assumption implies that in a long-run state $rm1$ is ruled out only by the short term interest rate, while in the second assumption time deposit rqm is ruled out by the spread interest rate. Cointegration test is carried out under the above assumptions on interest rates.

Table 2: Cointegration Test of $(rm1, rqm, y, lnr_{cd}, r_{spd})$ in (1980q1,2005q2), with restriction on β

Test for the number r_c of cointegrating vectors					
E.values	0.380	0.232	0.087	0.075	
Hypo.	$r_c = 0$	$r_c \leq 1$	$r_c \leq 2$	$r_c \leq 3$	
λ_{max}	$47.3**$	$26.1*$	8.98	7.74	
λ_{trace}	$90.6**$	43.3	17.2	8.21	
$p(\lambda_{max})$	0.0007	0.077	0.834	0.406	
$p(\lambda_{trace})$	0.0005	0.126	0.626	0.444	
Adjustment Coefficients α					
$\triangle rm1$	-0.087	[-2.2]	-0.005	[-0.83]	
$\triangle rqm$	0.041	[1.55]	-0.0008	[-0.21]	
$\triangle y$	-0.014	[-0.74]	-0.003	[-1.20]	
$\triangle lnr_{cd}$	-4.47	[-4.63]	-0.48	[-3.4]	
$\triangle lnr_{spd}$	-1.11	[-1.40]	-0.36	[-3.1]	
Restricted cointegrating coefficients β'					
	$rm1$	rqm	y	lnr_{cd}	r_{spd}
coint1	1.00	0	-0.923	0.165	0.00
coint2	0.00	1.00	0.116	0.00	0.415

$**$ $(*)$ denotes rejection of hypothesis at 5 % (10 %) significance level. $p(\lambda_{max})$ and $p(\lambda_{trace})$ are p-values of λ_{max} and λ_{trace} respectively. A lagged difference is decided as $p = 2$. Adjustment coefficients α are shown with t-statistics denoted by $[\cdots]$.

In Table 2, signs of β and α are not satisfactory at all. In order to make the monetary system stable, β and $\alpha(5 \times 2)$ should satisfy the following conditions:

$$\begin{aligned} \beta_1 \geq 0, \quad \beta_2 \leq 0 \\ \tilde{\beta}_1 \geq 0, \quad \tilde{\beta}_3 \leq 0 \\ \alpha_{11} \leq 0, \quad \alpha_{22} \leq 0 \\ \beta_1 \alpha_{31} + \tilde{\beta}_1 \alpha_{32} \leq 0, \\ \alpha_{41} \leq 0, \quad \alpha_{52} \leq 0, \end{aligned}$$

where $\alpha_{41} \leq 0$ does not necessarily hold, because this term is concerned with the monetary policy of the central bank.

In Table 3 we can see that restrictions by Assumption are rejected.

Table 3: LR Test for Binding Restrictions of Cointegration in (1980q1,2005q2)

Hypothesized No. of CE(s)	Restricted Log-Likeli.	LR Stat.	Dgr. of Freedom	Prob.
2	871.52	8.97	2	0.0113

2.4 Estimation of precautionary demand and its related cointegration property

From the end of 1997 to 1998 we have experienced serious financial shocks and Japan economy was said to enter into a deflationary economy. It was about 2002 that our economy was taking off from deflation. However, cointegration property cannot be shown even in the period of (1980,2005) which contains the recovering periods 2003,2004 and 2005. This is because during recessions the central bank supplies a large amount of money but people save money for future anxiety without consumption, and so a balance of money and gdp is not kept in cointegration relation. Money demand is usually classified by transaction demand and precautionary one, and the latter containing saved money increases in recession period. In this subsection, we estimate precautionary demand in order to analyze the relation between money and gdp.

Business condition is reported from the central bank of Japan as "Tankan DI (Diffusion Index)", and we denote it by $v(t)$. If $v(t)$ is positive, the economy is in a good condition and otherwise in a recession. We define magnitudes of recession and heated economy respectively as

$$v_-(t) = |\min\{v(t), 0\}|, \tag{13}$$

$$v_+(t) = \max\{v(t), 0\}. \tag{14}$$

Assuming that precautionary demand is proportional to magnitudes of recession and heated economy, we define precautionary demand as

$$\text{precautionary demand} = c_0 + k_1 * v_-(t) + k_2 * v_+(t). \tag{15}$$

Adjusted money is defined as

$$\text{money}_{adj} = \text{money} - \{c_0 + k_1 * v_-(t) + k_2 * v_+(t)\}. \tag{16}$$

Since this adjusted money has a close relation to gdp, we can expect the cointegration property among (money_{adj},y,interest rates) in the period (1980,2005). Using this idea, we introduce adjusted moneys for both $rm1$ and rqm:

$$rm1_{adj}(t) = rm1(t) - (c_0 + k_1 * v_-(t) + k_2 * v_+(t)), \tag{17}$$

$$rqm_{adj}(t) = rqm(t) - (\tilde{c}_0 + \tilde{k}_1 * v_-(t) + \tilde{k}_2 * v_+(t)). \tag{18}$$

With $rm1$ and rqm replaced by $rm1_{adj}$ and rqm_{adj} for fixed parameter values of k_1, k_2, $\tilde{k}_1$ and $\tilde{k}_2$, the VEC model in Eqs.(4) to (10) can be derived. It should be noted that the constant terms c_0 and $\tilde{c}_0$ in Eqs.(17) and (18) cannot be identified in VEC model (4) to (10), because $\triangle rm1_{adj} = \triangle rm1 - k_1 \triangle v_-(t) - k_2 \triangle v_+(t)$ and $\triangle c_0 = 0$. Without loss of generality, we set $c_0 = 0$ and $\tilde{c}_0$ in the formulation of adjusted moneys. The criterion of estimating k_1, k_2, $\tilde{k}_1$ and $\tilde{k}_2$ is given so as to maximize the log-likelihood function of $\triangle y(t)$-process in Eq. (5), because monetary policy caused by financial shocks sometimes changes drastically and produces relatively large residuals. As a result, relatively small change of $y(t)$ cannot be detected well under the criterion of maximizing the log-likelihood function of the whole VEC model containing money and interest rates.

[Estimation procedures of VEC model associated with $(k_1, k_2, \tilde{k}_1, \tilde{k}_2)$]

1. Set initial estimates of k_1, k_2, $\tilde{k}_1$ and $\tilde{k}_2$.
2. Generate $rm1_{adj} = rm1 - k_1 v_- - k_2 v_+$ and $rqm_{adj} = rqm - \tilde{k}_1 v_- - \tilde{k}_2 v_+$.
3. Estimate the cointegrated system (3) to (9) by the maximum likelihood method [3].

4. Calculate the likelihood function of $\Delta y(t)$-process in the above Procedure 3. Set the value as $C^{(0)}$.

5. Select another set of $(k_1, k_2, \tilde{k}_1, \tilde{k}_2)$ and calculate Procedures 1-4.

6. Set the value of the likelihood function of $\Delta y(t)$ as $C^{(1)}$.

7. Compare $C^{(0)}$ with $C^{(1)}$, and adopt a larger $C^{(i)}$, $(i = 0, 1)$.

8. Iterate above procedures till $C^{(i)}$ takes the maximum value.

[Estimation results] Adjusted moneys are estimated as

$$rm1_{adj} = rm1 - (c_0 + 0.022v_- - 0.024v_+), \tag{19}$$

$$rqm_{adj} = rqm - (\tilde{c}_0 - 0.0045v_- + 0.0084v_+) \tag{20}$$

where constant terms c_0 and $\tilde{c}_0$ in precautionary demands are set to be zero.

In Tables 4 and 5, we show the result of cointegration test in (1980q2,2005q2).

Table 4: Cointegration Test of Adjusted Money: $(rm1_{adj}, rqm_{adj}, y, lnr_{cd}, r_{spd})$ in (1980q1,2005q2), with restriction on β

Test for the number r_c of cointegrating vectors					
E.values	0.380	0.269	0.081	063	
Hypo.	$r_c = 0$	$r_c \leq 1$	$r_c \leq 2$	$r_c \leq 3$	
λ_{max}	47.4$**$	31.0$**$	8.37	6.46	
λ_{trace}	95.4$**$	48.0$**$	16.9	8.62	
$p(\lambda_{max})$	0.0007	0.017	0.879	0.555	
$p(\lambda_{trace})$	0.0001	0.0484	0.641	0.401	
Adjustment Coefficients α					
$\triangle rm1_{adj}$	-0.15	[-2.1]	-0.20	[-1.2]	
$\triangle rqm_{adj}$	0.017	[0.89]	-0.014	[-0.31]	
$\triangle y$	0.014	[2.6]	0.018	[1.35]	
$\triangle lnr_{cd}$	0.333	[0.93]	0.288	[0.33]	
$\triangle lnr_{spd}$	-1.37	[-5.7]	-3.26	[-5.6]	
Restricted cointegrating coefficients β'					
	$rm1_{adj}$	rqm_{adj}	y	lnr_{cd}	r_{spd}
coint1	1.00	0	-2.78	0.048	0.00
coint2	0.00	1.00	-0.976	0.00	0.084

$**$ $(*)$ denotes rejection of hypothesis at 5 % (10 %) significance level. $p(\lambda_{max})$ and $p(\lambda_{trace})$ are p-values of λ_{max} and λ_{trace} respectively. A lagged difference is decided as $p = 2$. Adjustment coefficients α are shown with t-statistics denoted by $[\cdots]$.

Table 5: LR Test for Binding Restrictions of Cointegration in (1980q1,2005q2)

Hypothesized No. of CE(s)	Restricted Log-Likeli.	LR Stat.	Dgr. of Freedom	Prob.
2	693.37	4.76	2	0.0924

The number of cointegrating vectors is detected to be 2. Property concerning α and β are satisfactory except for $\alpha_{41} \geq 0$. However, this sign is due to the monetary policy by the bank of Japan. The assumption restricted on β cannot be rejected.

2.5 Cointegration tests in periods (1980q1,2000q1), ..., (1980q1,2005q2)

We fix the precautionary demand, that is, adjusted moneys in Eqs.(19) and (20) given in (1980q1,2005q2), and carry out cointegration tests in other periods (1980q1,2000q1), ..., (1980q1,2005q2) so as to check the robustness of cointegration property.

Table 6: Cointegration Test in (1980q1,2000q1), ..., (1980q1,2005q2)

		Test for the number r_c of cointegrating vectors					
Period	*Hypo.*	$r_c = 0$	$r_c \leq 1$	$r_c \leq 2$	$r_c \leq 3$	*restrictions(prob.)*	signs of α and β
(1980q1,2000q1)	$p(\lambda_{max})$	0.0005	0.037	0.694	0.322	0.776	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0001	0.0349	0.369	0.257		
(1980q1,2000q2)	$p(\lambda_{max})$	0.0004	0.0398	0.775	0.601	0.908	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0003	0.105	0.722	0.672		
(1980q1,2000q3)	$p(\lambda_{max})$	0.0003	0.046	0.793	0.591	0.724	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0002	0.109	0.700	0.613		
(1980q1,2000q4)	$p(\lambda_{max})$	0.0003	0.0485	0.771	0.560	0.594	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0001	0.081	0.576	0.449		
(1980q1,2001q1)	$p(\lambda_{max})$	0.0002	0.047	0.808	0.496	0.494	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0001	0.0655	0.506	0.319		
(1980q1,2001q2)	$p(\lambda_{max})$	0.0003	0.022	0.789	0.655	0.714	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0001	0.083	0.767	0.732		
(1980q1,2001q3)	$p(\lambda_{max})$	0.0003	0.02	0.783	0.657	0.736	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0001	0.076	0.762	0.731		
(1980q1,2001q4)	$p(\lambda_{max})$	0.0004	0.018	0.773	0.633	0.871	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0001	0.064	0.724	0.679		
(1980q1,2002q1)	$p(\lambda_{max})$	0.0011	0.023	0.712	0.657	0.542	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0002	0.047	0.566	0.502		
(1980q1,2002q2)	$p(\lambda_{max})$	0.0009	0.085	0.742	0.534	0.0023	$\alpha_{41} \geq 0$, $\alpha_{52} \geq 0$
	$p(\lambda_{trace})$	0.0003	0.085	0.45	0.314		$\tilde{\beta}_3 \leq 0$
(1980q1,2002q3)	$p(\lambda_{max})$	0.0011	0.033	0.738	0.589	0.234	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0002	0.057	0.542	0.438		
(1980q1,2002q4)	$p(\lambda_{max})$	0.0021	0.032	0.72	0.54	0.189	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0004	0.054	0.53	0.442		
(1980q1,2003q1)	$p(\lambda_{max})$	0.0023	0.031	0.649	0.599	0.0066	$\alpha_{41} \geq 0$, $\alpha_{52} \geq 0$
	$p(\lambda_{trace})$	0.0004	0.0511	0.521	0.506		$\tilde{\beta}_3 \leq 0$
(1980q1,2003q2)	$p(\lambda_{max})$	0.0028	0.03	0.703	0.557	0.010	$\alpha_{41} \geq 0$, $\alpha_{52} \geq 0$
	$p(\lambda_{trace})$	0.0005	0.059	0.581	0.536		$\tilde{\beta}_3 \leq 0$
(1980q1,2003q3)	$p(\lambda_{max})$	0.0018	0.027	0.862	0.53	0.127	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0004	0.066	0.644	0.431		
(1980q1,2003q4)	$p(\lambda_{max})$	0.0017	0.029	0.902	0.587	0.165	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0006	0.097	0.761	0.546		
(1980q1,2004q1)	$p(\lambda_{max})$	0.0016	0.026	0.896	0.726	0.15	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0008	0.115	0.841	0.711		
(1980q1,2004q2)	$p(\lambda_{max})$	0.0007	0.017	0.879	0.555	0.128	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0004	0.096	0.797	0.652		
(1980q1,2004q3)	$p(\lambda_{max})$	0.0012	0.025	0.885	0.528	0.104	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0003	0.064	0.658	0.417		
(1980q1,2004q4)	$p(\lambda_{max})$	0.0012	0.022	0.838	0.477	0.086	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0002	0.047	0.581	0.377		
(1980q1,2005q1)	$p(\lambda_{max})$	0.0008	0.023	0.858	0.487	0.116	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0002	0.049	0.579	0.351		
(1980q1,2005q2)	$p(\lambda_{max})$	0.0007	0.017	0.879	0.555	0.092	$\alpha_{41} \geq 0$
	$p(\lambda_{trace})$	0.0001	0.048	0.641	0.401		

In Table 6, we can see two cointegrating vectors in every interval. The assumption on restricted β is rejected in (1980q1,2002q2), (1980q1,2003q1) and (1980q1,2003q2). In the same three intervals, we can also see that the signs of α_{52} and $\tilde{\beta}_3$ are unsatisfactory. In the other intervals, cointegration property can be seen to hold among $(rm1_{adj}, rqm_{adj}, y, lnr_{cd}, r_{spd})$.

3 Implications

Precautionary demand for M_1-money increases in a recession and decreases in a heated economy, while for Q-money precautionary demand decreases in a recession and increases in a heated economy. In this sense, M_1-money and Q-money is in a relation of substitution.

By considering adjusted moneys, we can see cointegration property even in (1980q1,2005q2) containing zero interest rate period.

4 Conclusions

A money demand function of $M_2 + CD$ is considered in the long equilibrium relationship of cointegration. $M_2 + CD$ is decomposed into M_1 and $quasi\ currency + CD$, and the interest rate corresponding to each decomposed money is determined as lnr_{cd} and r_{spd} respectively. Cointegration among 5 variables is tested in (1980q1,2005q2) containing deflationary economy, where restrictions are taken into consideration such that $rm1$ should not be coupled with r_{spd} and that rqm should not with lnr_{cd}. We introduced the concept of adjusted money which is defined by $money_{adj}$=money - precautionary demand, and we can show cointegration property among ($money_{adj}$,gdp,interest rates).

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