COUNTING THE NUMBER OF NEUTRINO SPECIES

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COUNTING THE NUMBER OF NEUTRINO SPECIES

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Abstract : We discuss the methods of determining the number of neutrino species N_ν, one of the most important numbers in particle physics and playing an important role in the Big Bang nucleosynthesis. As useful measures for the N_ν, we consider the reactions : $e^+e^-\to\gamma\nu\bar{\nu}$, $e^+e^-\to\mu^+\mu^-\nu\bar{\nu}$, $e^+e^-\to e^+e^-\nu\bar{\nu}$, $\mu Z\to\mu\nu\bar{\nu}Z$, $\mu Z\to\mu\Psi Z$ ($\Psi\to\nu\bar{\nu}$), $B(\pi^\pm, K^\pm, p, \bar{p})\ Z\to h\Psi$ ($\Psi\to\nu\bar{\nu}$) + anything, $\bar{p}p$, $pp\to\Psi\Psi$ ($\Psi\to\nu\bar{\nu}$) + anything, e^+e^-, e^-p, pp, $\bar{p}p\to Z^\circ Z^\circ$ ($Z^\circ\to\nu\bar{\nu}$) + anything, e^-p, pp, $\bar{p}p\to\gamma Z^\circ$ ($Z^\circ\to\nu\bar{\nu}$) + anything, and νe, pp, $\bar{p}p\to W^\pm Z^\circ$ ($Z^\circ\to\nu\bar{\nu}$) + anything, most of which are discussed for the first time in this article. Particular emphasis is placed upon the transverse momentum spectra in the reaction : $\bar{p}p\to Z^\circ(\to\sum_\nu\nu\bar{\nu})$+jet+anything, accompanying a hadronic jet with large transverse momentum.

§ 1. Introduction

One of the most important open problems in particle physics is that of the number of fermion generations. This number is equal to the one of neutrino species N_ν if each generation has one and only one neutrino. Besides having important implications in particle physics, the number of light neutrino plays an important role in the cosmological models of the development of the early universe. Cosmogical arguments from the standard Big Bang nucleosynthesis place an upper limit on N_ν; $N_\nu\leqslant 4$[1]. The constraints imposed by our understanding of the late stages of stellar evolution, including red giants, carbon-burning stars and cooling neutron stars have been examined in Ref. 2. These astrophysical considerations restrict $N_\nu\lesssim 10^{2\pm1}$. In particle physics laboratory experiments the limit $N_\nu<1400$ has been obtained from the nonobservation of the decay $K\to\pi\nu\bar{\nu}$[3], $N_\nu<10^5$ from the decay $J/\Psi\to\sum_\nu\nu\bar{\nu}$[4] and $N_\nu<137$ from a study of the neutrino effects in the radiative corrections to the ρ-parameter and the $e^+e^-\to\mu^+\mu^-$ cross-section[5]. Recently the best particle physics bound $N_\nu<67$ have been derived[6] from the experimental results of UA1 and UA2 experiments[7] on

$$R_{exp}=\frac{\sigma_Z\, B(Z^\circ\to e^+e^-)}{\sigma_{W^+}\, B(W^+\to e^+\nu_e)+\sigma_{W^-}\, B(W^-\to e^-\bar{\nu}_e)} \tag{1.1}$$

Two other methods have been proposed for determining N_ν. One is the method of studying the reaction, accompanying the production of a single hard photon[8].

$$e^+e^-\to\gamma\ Z^\circ,\quad Z^\circ\to\sum_\nu\nu\bar{\nu}. \tag{1.2}$$

The other one is to use the weak production of (nearly massless) $\sum_\nu\nu\bar{\nu}$ pairs in $p\bar{p}$ collision via[9]

$$\bar{p}p\to Z^\circ+\text{anything},\quad Z^\circ\to\sum_\nu\nu\bar{\nu} \tag{1.3}$$

The feasibility of using the missing hadronic energy spectra $d\sigma/dE_m$ and/or the missing hardronic transverse momentum spectra $d\sigma/dp_T$ of reaction (1.3) as a measure of the number of light neutrino flavours, assuming the standard model for the fermion couplings has been discussed in Ref. 9.

In the present article we study several methods of determining N_ν, accessible in particle physics laboratory experiments. In particular we reexamine the method proposed by Dunbar who studied the reaction (1.3) within the Drell-Yan model, introducing the transverse momenta of the produced pairs $\sum_\nu \nu\bar{\nu}$ in a phenomenological way. We propose the Z°-boson production and its subsequent decay into the pairs $\sum_\nu \nu\bar{\nu}$ through

$$\bar{p}p,\ pp \to Z^\circ \underset{\hookrightarrow \sum_\nu \nu\bar{\nu}}{} + \text{jet} + \text{anything} \tag{1.4}$$

in which the transverse momenta are produced by the high p_T jet[10]. The other methods we propose or review as the measure of the N_ν are the reactions: $e^+e^- \to \gamma\nu\bar{\nu}$, $e^+e^- \to \mu^+\mu^-\nu\bar{\nu}$, $e^+e^- \to e^+e^-\nu\bar{\nu}$, $\mu Z \to \mu\nu\bar{\nu}Z$, $\mu Z \to \mu\Psi Z$, $B(\pi^\pm, K^\pm, p, \bar{p})\ Z \to h\Psi_{\hookrightarrow \nu\bar{\nu}}$ + anything, $\bar{p}p$, $pp \to \Psi_{\hookrightarrow \nu\bar{\nu}}\Psi$ + anything, e^+e^-, e^-p, $\bar{p}p$, $pp \to Z^\circ Z^\circ_{\hookrightarrow \nu\bar{\nu}}$ + anything, e^-p, pp, $\bar{p}p \to \gamma Z^\circ_{\hookrightarrow \nu\bar{\nu}}$ + anything and νe, pp, $\bar{p}p \to W^\pm Z^\circ_{\hookrightarrow \nu\bar{\nu}}$ + anything, most of which are discussed for the first time in this article.

We organize the paper as follows: In Sect. 2, we describe the proposed reactions for determining N_ν. Sect. 3 is devoted to the concluding remarks. In Appendix, we present the formulas of the cross-section for the reaction (1.4) in which we are particularly interested.

§ 2. The proposed reactions for determining N_ν

$$\bar{p}p,\ pp \to Z^\circ \underset{\hookrightarrow \sum_\nu \nu\bar{\nu}}{} + \text{jet} + \text{anything} \tag{2.1}$$

In such reaction the neutrino pairs produced can be identified by the presence of a gluon or quark jet with a transverse momentum p_{jT}, which is equal to the missing (antiparallel) transverse of the $\nu\bar{\nu}$ pairs[11]. We calculate the transverse momentum spectra of the produced jet $d\sigma/dp_{jT}$ within the first order (in α_s) of the perturbative QCD, taking into account two types of sub-processes (quark-antiquark annihilation and a gluon-(anti) quark Compton processes, see Fig. 1), contributing to the reaction (2.1). The resulting formulas are presented in Appendix.

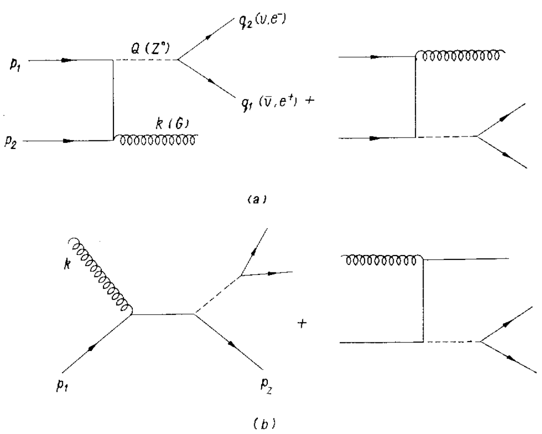

Fig. 1 (a) The quark-antiquark annihilation diagrams.
(b) The quark-gluon Compton diagrams.

We have

$$\frac{d\sigma}{dp_{jT}}(pp,\ \bar{p}p \to Z^\circ_{\hookrightarrow \sum_\nu \nu\bar{\nu}} + \text{jet} + \text{anything}) = N_\nu \frac{B(Z^\circ \to \nu\bar{\nu})}{B(Z^\circ \to e^+e^-)} \times \frac{d\sigma}{dp_{jT}}(pp,\ \bar{p}p \to Z^\circ_{\hookrightarrow e^+e^-} + \text{jet} + \text{anything}), \tag{2.2}$$

where in the Glashow-Weinberg-Salam model[12] with $\sin^2\theta_W=0.226$[13]

$$\frac{B(Z^\circ\to\nu\bar{\nu})}{B(Z^\circ\to e^+e^-)}=\frac{1}{1-4\sin^2\theta_W+8\sin^4\theta_W}$$
$$=1.98. \qquad (2.3)$$

Adopting the standard method[10], and employing the parametrization of Baier et al.[14] for the parton structure functions we have calculated $d\sigma/dp_{jT}$ for $\bar{p}p$ collisions at $\sqrt{s}=540$ GeV, $\bar{p}p\to Z^\circ(\to\sum_\nu \nu\bar{\nu})$ + jet + anything with $N_\nu=3$, 4 and 10, the resulting curves of which are presented as a function of p_{jT} in Fig. 2. For the sake of comparison, the curves obtained by Dunbar[9] and the one for $\bar{p}p\to Z^\circ(\to e^+e^-)$ + jet + anything are also shown. In the calculations the primordial parton transverse momenta as well as the soft-gluon effects are neglected[15]. We conclude from the figure that the measurements of missing transverse momentum spectra should be feasible and can provide important information on determining the number of neutrino species. The more detailed numerical studies on reaction (2.1) would be needed before organizing the experiment at the $\bar{p}p$ collider[16].

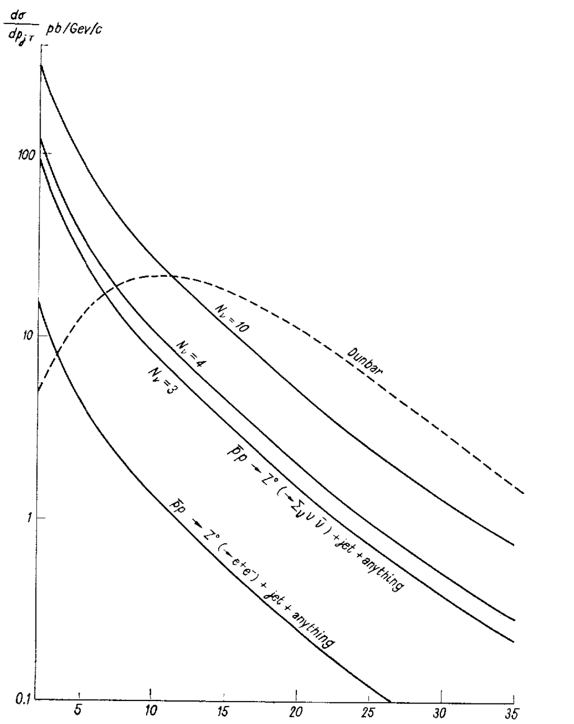

Fig. 2 Transverse momentum spectra $d\sigma/dp_{jT}$ ($\bar{p}p\to Z^\circ(\to\sum_\nu \nu\bar{\nu})$ + jet + anything) with $N_\nu=3$, 4, 10 and $d\sigma/dp_{jT}$ ($\bar{p}p\to Z^\circ(\to e^+e^-)$ + jet + anything) at $\sqrt{s}=540$ GeV as a function of jet transverse momentum p_{jT}. The curve obtained by Dunbar[9] for $d\sigma/dp_{jT}$ ($\bar{p}p\to Z^\circ(\to\sum_\nu \nu\bar{\nu})$ + jet + anything) with $N_\nu=3$ is also shown by dotted line.

$e^+e^-\to\gamma\nu\bar{\nu}$ (2.4)

This reaction proceeds by the Feynman diagrams of Fig. 3[8),17]. At energies near $\sqrt{s}\sim M_Z$, one can safely calculate the amplitudes of the W-boson exchange diagrams (3c)~(3e) by taking the W-boson mass M_W to be infinite, thus approximating this part by the four fermion theory. The relevant four-lepton couplings appearing in Fig. 3 can be written in terms of the following two effective Hamiltonians:

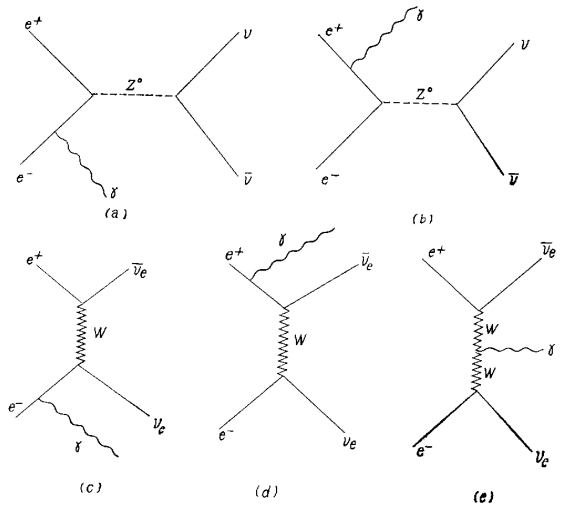

Fig. 3 Lowest order Feynman diagrams contributing to the process $e^+e^-\to\gamma\nu\bar{\nu}$.

$$H^Z=4\sqrt{2}\,G_F\,\bar{\Psi}_l\gamma^\alpha(a_l-b_l\gamma_5)\Psi_l$$

$$[\sum_{\nu} \bar{\Psi}_{\nu}\gamma_{\alpha}(a_{\nu}-b_{\nu}\gamma_5)\Psi_{\nu}], \tag{2.5a}$$

$$H^W = G_F/\sqrt{2}\ \bar{\Psi}_l\gamma^{\alpha}(1-\gamma_5)\Psi_l\cdot\Psi_{\nu_l}\gamma_{\alpha}(1-\gamma_5)\Psi_{\nu_l}. \tag{2.5b}$$

We have the following formula for the cross-section, neglecting the electron mass[18),

$$\frac{d\sigma}{dxdy} = \frac{G_F^2\alpha}{3\pi^2}\frac{Q^2}{x(1-y^2)}\left[(1-\frac{x}{2})^2+\frac{1}{4}x^2y^2\right]\left[2\left\{N_{\nu}(a_l^2+b_l^2)+(a_l+b_l)(1-\frac{Q^2}{M_Z^2})\right\}|D_Z(Q^2)|^2M_Z^4+1\right], \tag{2.6}$$

where

α=the QED fine structure constant,

$x=E_{\gamma}/E$=photon energy in unit of the energy,

$y=\cos\theta_{\gamma}$, (θ_{γ}=the photon angle with respect to the incident beam direction)

$s=4E^2$=the square of the e^+e^- c.m. energy.

$Q^2=(1-x)s$,

G_F=the Fermi coupling constant$=\dfrac{\pi\alpha}{\sqrt{2}\,M_Z^2\sin^2\theta_W\cos^2\theta_W}$.

The definitions of a_l, b_l, a_{ν} and b_{ν}, and $|D_Z(Q^2)|^2$ are given in Eqs (A.1) and (A.7), resp. in Appendix. In Eq (2.6), the $(a_l^2+b_l^2)$ term comes from the square of the Z° amplitudes, the "1" term comes from the square of the W-exchange term and the (a_l+b_l) term comes from the $W-Z^{\circ}$ interference. Fig. 4 shows the cross-section $d\sigma/dx$ for the e^+e^- c.m. energy E_{cm}=105 GeV and $|y_{max}|$ = 0.94[18). In the figure a peak appears at E_{γ}=14 GeV, corresponding to a Z° missing mass recoiling against the photon. The contributions to the cross-section of the W-exchange term and the interference term are also shown.

From the experiment in which a photon is observed with an energy such that the recoil system has an energy about equal to the Z° mass, it is possible to determine the number N_{ν}, contributing to Z°-decay. Such

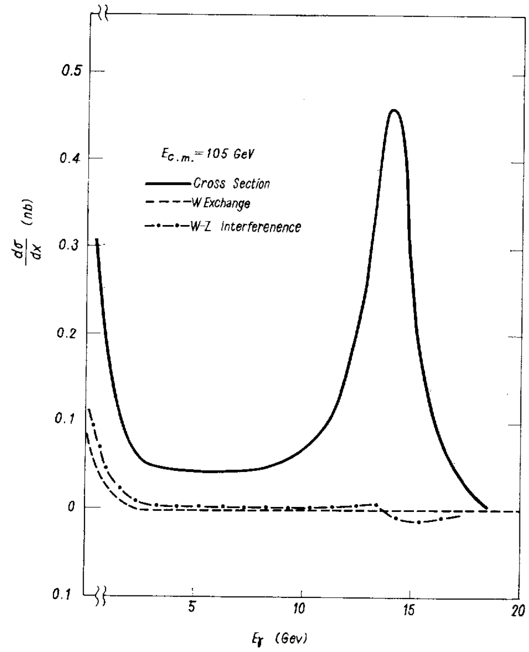

Fig. 4 Differential cross-section $d\sigma/dx$ versus the photon energy E_{γ}. The three curves are for the total cross-section, the contribution from W-exchange, and the W-Z interference term.

experiments should be accessible in electron-positron colliding beam facilities now under construction or to be constructed in future: TRISTAN, SLC and LEP etc.

$\underline{e^+e^-\to\mu^+\mu^-\nu\bar{\nu}}$ (2.7)

This reaction can be treated similarly as $e^+e^-\to\gamma\nu\bar{\nu}$ by replacing γ in diagrams of Fig. 3 by $(\gamma, Z^{\circ})\to\mu^+\mu^-$, i.e. by attaching the $\mu^+\mu^-$ (γ, Z°) vertex. However, in this case, the number of experimentally accessible observables increases and thus one can study various characteristics of the single muon as those of muon pairs.

$\underline{e^+e^-\to e^+e^-\nu\bar{\nu}}$ (2.8)

Although this resembles to the reaction (2.7), additional diagrams shown in Fig. 5 should be taken into account. On performing calculations one can make use of the equivalent photon (Williams-Weiszäcker) method for the vertex containing photon exchange.

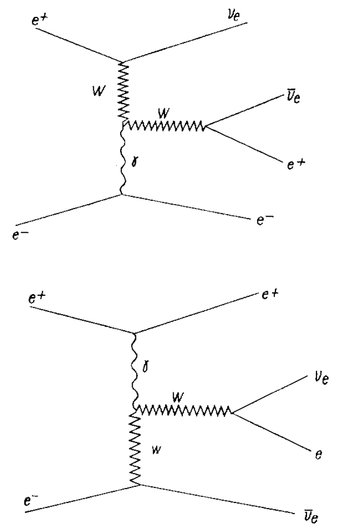

Fig. 5 Additional Feynman diagrams contributing to the process $e^+e^- \to e^+e^- \nu\bar{\nu}$.

$\underline{\mu Z \to \mu \ \nu\bar{\nu} Z}$ (2.9)

The cross-section and scattered muon distributions for the neutrino pair production by muons in the Coulomb field of a heavy nucleus Z (see Fig. 6) can be a measure of the number of light neutrino flavours. This reaction has been studied in Ref. 19 and shown that the predicted cross-section is very small :

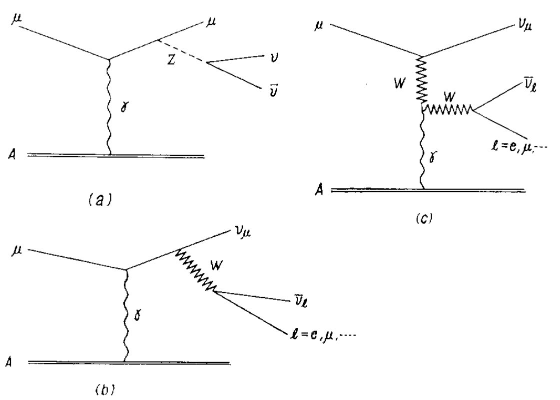

Fig. 6 Feynman diagrams for neutrino pair production by muons in the Coulomb field of a heavy nucleus.

At $E_\mu = 280$ GeV,

$$\sigma(\mu N \to \mu \sum_\nu \nu\bar{\nu} N) \sim 3.10^{-42}\ \text{cm}^2 \text{ with } N_\nu = 3. \qquad (2.10)$$

Also the final muon energy peaks near zero so that the practical acceptance is reduced. Thus a measurement of this cross-section is not foreseen in the near future despite its interest.

$$\mu Z \to \mu V^\circ Z^\circ \quad \llcorner\!\!\to \sum_\nu \nu\bar{\nu} \qquad (2.11)$$

The diffractive production of vector mesons $\mu Z \to \mu V^\circ Z$ with $V^\circ \to \sum_\nu \nu\bar{\nu}$ ($V^\circ = \rho^\circ$, ω, ϕ, Ψ,$\cdots$) decay can also offer an independent measure of the N_ν. This reaction has been studied in Ref. 19, using for the diffractive Ψ production the photon-gluon model and for the production of ρ°, ω and ϕ mesons the vector dominance model. It has been shown that in the case of Ψ production at $E_\mu = 280$ GeV we have

$$\sigma(\mu N \to \mu \Psi N)\ B(\Psi \to \sum_\nu \nu\bar{\nu}) = 5.1\, N_\nu \cdot 10^{-42}\ \text{cm}^2, \qquad (2.12)$$

giving thus a factor of 3 greater than the direct neutrino pair production (2.9) signal. The background from ρ°, ω, and ϕ productions does not compete. Since the Ψ contribution can cleanly be separated it can be regarded as an alternative and independent measure of the number N_ν.

$$\underline{B(\pi^\pm,\ K^\pm,\ pp)\ Z \to h\Psi\ (\to \sum_\nu \nu\bar{\nu}) + \text{anything}} \qquad (2.13)$$

From the Ψ-production and its subsequent decay $\Psi \to \sum_\nu \nu\bar{\nu}$ with a low p_T (almost leading) hadron $h\,(=\pi^\pm,\ K^\pm)$ in the target fragmentation region one has :

$$\frac{d\sigma\,(BZ \to h\Psi\ (\to \sum_\nu \nu\bar{\nu}) + \text{anything})}{d\sigma\,(BZ \to h\Psi\ (\to \mu^+\mu^-) + \text{anything})} = \frac{B(\Psi \to \sum_\nu \nu\bar{\nu})}{B(\Psi \to \mu^+\mu^-)} \qquad (2.14)$$

Various kinematical characteristics for the reaction $BZ \to h\Psi\ (\to \mu^+\mu^-)$ + anything have been investigated in Ref. 20, using the recombination model. We present in Fig. 7 curves[20] of associated hadron spectrum for $\pi^- W \to h\Psi$+anything at 90°, $s = 752\ \text{GeV}^2$.

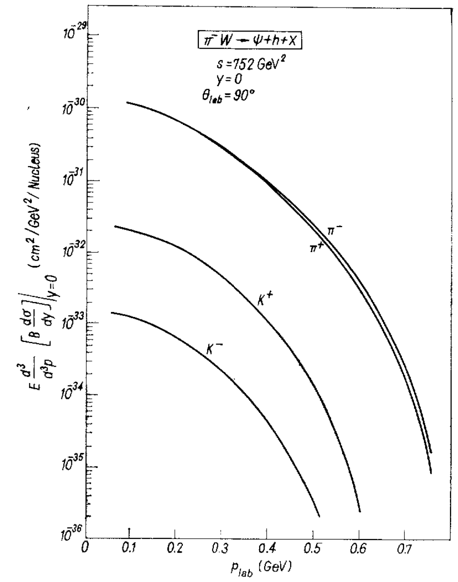

Fig. 7 Associated hadron spectrum for $\pi^- W \to \Psi + h +$anything at 90° in the lab. system with fixed Ψ rapidity $y=0$, and $s=752$ GeV^2.

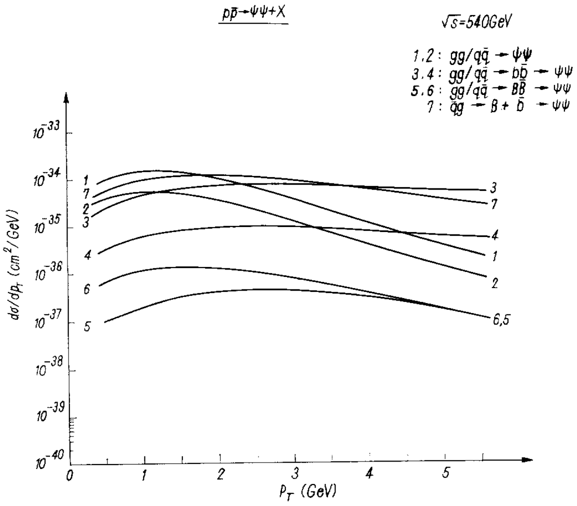

Fig. 8 p_T distribution for the constituent processes (1)–(7) of $\bar{p}p \to \Psi\Psi +$any-thing.

From the study on the reaction (2.13), one gains insight into the N_ν.

$$\bar{p}p,\ pp \to \Psi\Psi + \text{anything} \qquad (2.15)$$

The integrated hadron cross-section and various characteristics of the kinematical distributions have been investigated in Ref. 21 in the framework of the perturbative QCD with the nonrelativistic approximation for the heavy quark bound states. The integrated cross-section for $\bar{p}p \to \Psi\Psi +$anything at $\sqrt{s}=540$ GeV is above 10^{-33} cm^2. The p_T distributions for the constituent processes of $\bar{p}p \to \Psi\Psi +$angthing are presented in Fig. 8. The processes in which one of the Ψ-mesons decays via $\Psi \to \sum_\nu \nu\bar{\nu}$ should give us useful information on N_ν.

$$e^+e^-,\ e^-p,\ \bar{p}p,\ pp \to Z^\circ Z^\circ + \text{anything} \qquad (2.16)$$

$$e^-p,\ pp,\ \bar{p}p \to \gamma Z^\circ + \text{anything} \qquad (2.17)$$

$$\nu e,\ pp,\ \bar{p}p \to W^\pm Z^\circ + \text{anything} \qquad (2.18)$$

These gauge boson pair productions proceed in the Born approximation via (sub) processes shown in Fig. 9. These processes have been studied extensively in Ref. 22,

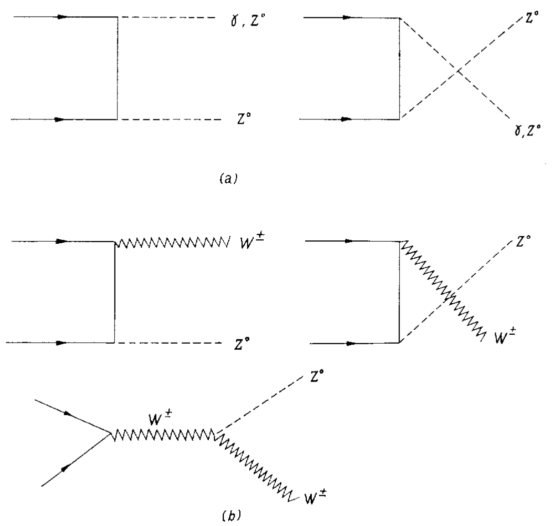

Fig. 9 Feynman diagrams for the subprocesses contributing to : (a) the processes (2.16) and (2.17), (b) the processes (2.18).

originally advocated as useful processes to study the trilinear gauge boson couplings, the renormalizability in gauge theories and the anomalous magnetic moments of the gauge bosons, etc. Obviously the processes in which the decay of (one of) the Z°-boson (s) can be utilized as providing additional independent measures on the neutrino species.

§ 3. Concluding remarks

The number of neutrino species N_ν plays such important role in particle physics that it is desirable to determine it experimentally, performing as many independent tests as possible. The reactions we considered are presented as:

$$e^+e^-,\ \bar{p}p,\ pp,\cdots\rightarrow A+Z^\circ+\text{anything},\quad (A=\gamma,\ Z^\circ,\ W^\pm,\cdots) \tag{3.1}$$

$$pp,\ \bar{p}p,\ hZ,\cdots\rightarrow B+\Psi+\text{anything},\quad (B=\gamma,\ \Psi,\cdots) \tag{3.2}$$

with subsequent decays $Z^\circ\rightarrow\sum_\nu \nu\bar{\nu}$ and $\Psi\rightarrow\sum_\nu \nu\bar{\nu}$. Since the $\nu\bar{\nu}$ pairs are invisible in the experiments their presence can be identified by detecting the particle A or B. Among the kinematical characteristics for particle A or B, the transverse momentum spectra should be especially useful. In extracting the number N_ν from the experiments of the proposed reactions, one has to consider carefully the background contributions in each reaction which in most cases can be discriminated from the study on the kinematical characteristics of various distributions.

We conclude the paper by emphasizing that, among the reactions proposed the reaction:

$$pp\rightarrow Z^\circ\ (\rightarrow\sum_\nu \nu\bar{\nu})+\text{jet}+\text{anything} \tag{3.3}$$

is particularly interesting. The experiment for determining the N_ν through such reaction is sufficiently realistic[11], which we hope to be performed in the $\bar{p}p$ collider at CERN in the near future.

Appendix

We summarize here the formulas of the cross-section $p_{j0}\dfrac{d\sigma}{d^3p_j}$ and $d\sigma/dp_{jT}dz$ for the reaction

$$\bar{p}p,\ pp\rightarrow Z^\circ(\rightarrow e^+e^-)+\text{jet}+\text{anything} \tag{A.1}$$

derived within the $O(\alpha_s)$ perturbative QCD together with the Glashow-Weinberg-Salam model[12]. The cross-section for

$$\bar{p}p,\ pp\rightarrow Z^\circ(\rightarrow\sum_\nu \nu\bar{\nu})+\text{jet}+\text{anything} \tag{A.2}$$

can be obtained immediately using Eq (2.2) and the cross-section for (A.2). The following interaction Lagrangian is used.

$$L_{int}=e\bar{l}\gamma_\mu lA^\mu+\bar{\nu}\gamma_\mu(a_\nu-b_\nu\gamma_5)\nu Z^\mu+\bar{l}\gamma_\mu(a_l-b_l\gamma_5)lZ^\mu+\bar{q}_i\gamma_\mu(A_i-B_i\gamma_5)q_iZ^\mu, \tag{A.3}$$

where A_μ, l, q_i and Z^μ are photon field, lepton field, the quark field with flavour i and the neutral gauge boson, respectively. A_i, B_i, a_l, b_l, a_ν and b_ν are the vector and axial-vector couplings of the Z° to fermions:

$A_i=I_3/2-e_i\sin^2\theta_W,\ B_i=I_3/2$

(I_3: the third component of weak isospin, e_i: the charge of quark q_i in unit of e)

$a_l=\sin^2\theta_W-\frac{1}{4},\ b_l=-\frac{1}{4}$ for $l=e,\ \mu,\ \tau$,

$a_\nu=\frac{1}{4},\ b_\nu=\frac{1}{4}$, for $\nu=\nu_e,\ \nu_\mu,\ \nu_\tau$.

$\bar{p}p\rightarrow Z^\circ\ (\rightarrow e^+e^-)+\text{jet}+\text{anything}$

The cross-section can be calculated by standard trace techniques and finally reduced to the following form[23]

$$p_{j0}\frac{d\sigma}{d^3p_j}=\left(p_{j0}\frac{d\sigma}{d^3p_j}\right)^{\text{annih.}}+\left(p_{j0}\frac{d\sigma}{d^3p_j}\right)^{\text{Compton}} \tag{A.4}$$

where the cross-section $(p_{j0}\,d\sigma/d^3p_j)^{\text{annih.}}$ by the annihilation diagrams of Fig. 1(a) is:

$$\left(p_{j0}\frac{d\sigma}{d^3p_j}\right)^{\text{annih.}}=-(2\pi)^{-2}\frac{32}{9}\alpha_s\alpha^2\int_0^{2\pi}d\phi\int_{-1}^{1}d\cos\theta\int_0^{q_1\max}p_jdp_j\int_{x_1\min}^{1}\frac{dx_1}{x_1x_2}\times\sum_i\{F_i(x_1,\ Q^2)F_{\bar{i}}(x_2,\ Q^2)+(x_1\leftrightarrow x_2)\}\frac{K_{ij}(\hat{w})X_A+L_{ij}(\hat{w})Y_A}{s\sqrt{\hat{s}}\,q_0|x_1-u_2-u_{j2}|}, \tag{A.5}$$

where

$$X_A=-\frac{1}{\hat{u}_{j1}}\left(1-\frac{1}{\hat{u}_{j2}}\right)\left(\frac{\hat{h}_2\eta}{\hat{s}}+\frac{\hat{u}_2\hat{\zeta}}{2}\right)$$

$$-\frac{1}{\hat{u}_{j2}}\left(1-\frac{1}{\hat{u}_{j1}}\right)\left(\frac{\hat{h}_1\eta}{\hat{s}}+\frac{\hat{u}_1\hat{\zeta}}{2}\right)$$
$$+\left(\frac{1}{\hat{u}_{j1}}+\frac{1}{\hat{u}_{j2}}-\frac{2}{\hat{u}_{j1}\hat{u}_{j2}}\right)(\hat{u}_2\hat{h}_1+\hat{u}_1\hat{h}_2)$$
$$-2\frac{\hat{h}_1\hat{u}_1}{\hat{u}_{j1}}-2\frac{\hat{h}_2\hat{u}_2}{\hat{u}_{j2}},$$
$$Y_A=\frac{1}{\hat{u}_{j1}}\left(1-\frac{1}{\hat{u}_{j2}}\right)\left(\frac{\hat{u}_2\hat{\zeta}}{2}-\frac{\hat{h}_2\eta}{\hat{s}}\right)$$
$$+\frac{1}{\hat{u}_{j2}}\left(1-\frac{1}{\hat{u}_{j1}}\right)\left(\frac{\hat{h}_1\eta}{\hat{s}}-\frac{\hat{u}_1\hat{\zeta}}{2}\right)$$
$$-\left(\frac{1}{\hat{u}_{j1}}+\frac{1}{\hat{u}_{j2}}-\frac{1}{\hat{u}_{j1}\hat{u}_{j2}}\right)(\hat{u}_2\hat{h}_1-\hat{u}_1\hat{h}_2). \tag{A. 6}$$

and the cross-section $(p_{j0}\, d\sigma/d^3p_j)^{\text{Compton}}$ by the Compton diagrams of Fig. 1(b) is :

$$\left(p_{j0}\frac{d\sigma}{d^3p_j}\right)^{\text{Compton}}=(2\pi)^{-2}\frac{3}{4}\alpha_s\alpha^2\int_0^{2\pi}d\phi$$
$$\int_{-1}^{1}d\cos\theta\int_0^{q_1\max}p_jdp_j\int_{x_1\min}^{1}\frac{dx_1}{x_1x_2}$$
$$\times\sum_i\{F_{\text{gluon}}(x_1,Q^2)F_i(x_2,Q^2)+(x_1\leftrightarrow x_2)\}$$
$$\frac{\{K_{ij}(\hat{w})X_c+L_{ij}(\hat{w})Y_c\}}{s\sqrt{\hat{s}}\,q_0|x_1-u_2-u_{j2}|}, \tag{A. 7}$$

where

$$X_c=\left(1+\frac{\hat{u}_{j2}}{\hat{u}_{j1}}\right)\left(\frac{\eta\hat{h}_1}{\hat{s}}+\frac{\hat{\zeta}\hat{u}_1}{\hat{u}_{j1}}\right)+\frac{1}{\hat{u}_{j1}}(1-\hat{u}_{j2})$$
$$(\hat{u}_2\hat{h}_1+\hat{h}_2\hat{u}_1)+\left(1-\frac{1}{\hat{u}_{j1}}+\frac{2\hat{u}_{j2}}{\hat{u}_{j1}}\right)$$
$$\left(\frac{\eta\hat{h}_2}{\hat{s}}+\frac{\hat{\zeta}\hat{u}_2}{2}\right)+2\hat{u}_2\hat{h}_2-\frac{\eta\hat{\zeta}}{\hat{s}\hat{u}_{j1}},$$
$$Y_c=\left(1+\frac{\hat{u}_{j2}}{\hat{u}_{j1}}\right)\left(-\frac{\eta\hat{h}_1}{\hat{s}}+\frac{\hat{\zeta}\hat{u}_1}{2}\right)+\frac{1}{\hat{u}_{j1}}$$
$$(1-\hat{u}_{j2})(-\hat{u}_1\hat{h}_2+\hat{u}_2\hat{h}_1)$$
$$+\left(1-\frac{1}{\hat{u}_{j1}}+\frac{2\hat{u}_{j2}}{\hat{u}_{j1}}\right)\left(-\frac{\eta\hat{h}_2}{\hat{s}}+\frac{\hat{u}_2\hat{\zeta}}{2}\right). \tag{A. 8}$$

and

$$K_{ij}(\hat{w})=\frac{(A_i^2+B_i^2)(a_j^2+b_j^2)|D_Z(\hat{w})|^2}{\sin^4\theta_W\cos^4\theta_W}$$
$$L_{ij}(\hat{w})=\frac{4a_jb_j\,A_iB_i|D_Z(\hat{w})|^2}{\sin^4\theta_W\cos^4\theta_W}$$

where $D_Z(\hat{w})$ is the Z^0-propagator :

$$|D_Z(\hat{w})|^2=\frac{1}{(\hat{w}-M_Z^2)^2+M_Z^2\Gamma_Z^2}$$

In the hadronic c.m. system (Fig. 10), we have :

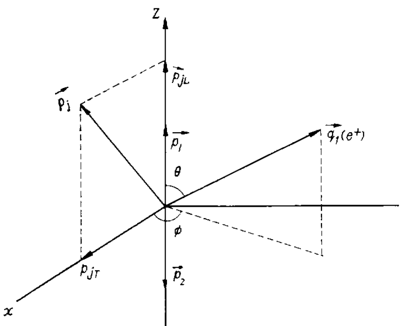

Fig. 10 Kinematics in the hadronic c.m. system.

$q_1=(q_1,\ q_1\sin\theta\cos\phi,\ q_1\sin\theta\sin\phi,$
$q_1\cos\theta)$ with $q_1=|\vec{q_1}|$,
$P_1=(\sqrt{s}/2,\ 0,\ 0,\ \sqrt{s}/2)$,
$P_2=(\sqrt{s}/2,\ 0,\ 0,\ -\sqrt{s}/2)$,
$p_j=(p_{j0},\ p_{jT},\ 0,\ p_{jL})$=the momentum of the detected gluon or quark jet.

The longitudinal (transverse) momentum $p_{jL}\,(p_{jT})$ is determined as :

$p_{jL}=|\vec{p_j}|\tanh y_j,\ p_{jT}=|\vec{p_j}|/\cosh y_j,$

where y_j is the jets' rapidity in the hadronic c.m. system.

We have (caret refers to quantities of sub-processes)

$$\hat{s}=x_1x_2s,\ \hat{u}_1=u_1/x_2,\ \hat{u}_2=u_2/x_1,$$
$$u_1=q_1(1-\cos\theta)/\sqrt{s},\ u_2=q_1(1+\cos\theta)/\sqrt{s}$$
$$\hat{u}_{j1}=u_{j1}/x_2,\ u_{j1}=(p_{j0}-p_{jL})/\sqrt{s},$$
$$\hat{u}_{j2}=u_{j2}/x_1,\ u_{j2}=(p_{j0}+p_{jL})/\sqrt{s},$$
$$\hat{h}_1=\frac{\hat{s}}{2}(1-\hat{u}_1-\hat{u}_{j1}),\ \hat{h}_2=\frac{\hat{s}}{2}(1-\hat{u}_2-\hat{u}_{j2}),$$
$$\eta=2q_1(p_{j0}-p_{jT}\sin\theta\cos\phi-p_{jL}\cos\theta),$$
$$q_2=\frac{\sqrt{s}}{2}(x_1+x_2)-q_1-p_{j0}$$
$$\hat{w}=\hat{s}(1-\hat{u}_{j1}-\hat{u}_{j2}),\ \hat{\zeta}=\hat{s}(1-\hat{u}_1-\hat{u}_2),$$
$$x_2=\frac{(u_1+u_{j1})x_1-\eta/s}{x_1-u_2-u_{j2}},\ x_{1\min}=\frac{u_2+u_{j2}-\eta/s}{1-u_1-u_{j1}}$$
$$p_j^{\max}=\frac{s-2p_{j0}\sqrt{s}}{2\{\sqrt{s}-(p_{j0}-p_{jT}\sin\theta\cos\phi-p_{jL}\cos\theta)\}}.$$

For α_s, the QCD running coupling, we use

$$\alpha_s=\frac{12\pi}{(33-2N_f)\ln p_{jT}^2/\Lambda^2},\ \Lambda=0.4\ \text{GeV/c}. \tag{A. 9}$$

In α_s and in Eqs (A. 5) and (A. 7), the evolution scale Q^2 for the parton structure functions we take $Q^2=p_{jT}^2$. For the decay width of the Z°-boson[24] we have the following formula, in which the heavy quark mass effects and the QCD radiative corrections are taken into account. In the formula all fermions are assumed to be massless, except m_b (=5 GeV) and m_t (=35 GeV)*)

$$\Gamma_Z=\frac{\alpha M_Z}{\sin^2\theta_W \cos^2\theta_W}\Big[N_\nu(a_\nu^2+b_\nu^2)+N_l(a_l^2+b_l^2)+3\ C_{QCD}\ [2\{(A_u^2+B_u^2)+(A_d^2+B_d^2)\}+\sqrt{1-4\mu_b^2}\ \{(1-\mu_b^2)(A_b^2+B_b^2)+3\mu_b^2(A_b^2-B_b^2)\}+\sqrt{1-4\mu_t^2}\ (1-4\mu_t^2)(A_t^2+B_t^2)+3\mu_t^2(A_t^2-B_t^2)\}]\Big] \quad (A.10)$$

where

$C_{QCD}=1+\alpha_s/\pi$,

N_l=number of lepton species,

$\mu_b=5/M_Z$, $\mu_t=35/M_Z$

$M_Z=92.1\ \mathrm{GeV/c^2}$ (obtained with $\sin^2\theta_W=0.226$ taking into account the radiative corrections)

$pp\to Z^\circ(e^+e^-)$+jet+anything

The cross-section for this case can be obtained by replacing trivially the factor: $\sum_i\{F_i(x_1,Q^2)F_{\bar i}(x_2,Q^2)+(x_1\leftrightarrow x_2)\}$ in Eq (A. 5) and $\sum_i\{F_{\mathrm{gluon}}(x_1,Q^2)F_i(x_2,Q^2)+(x_1\leftrightarrow x_2)\}$ in Eq (A. 7), correspondingly.

The formulas of the cross-section $d\sigma/dzd\,p_{jT}^2$ for the process $\bar pp\to Z^\circ(\to e^+e^-)$+jet+anything are also presented below[25].

$$\frac{d\sigma}{dzd\,p_{jT}^2}=\sum_{q=u,d,\cdots}\left\{\frac{d^2\sigma^{q\bar q}}{dzd\,p_{jT}^2}+\frac{d^2\sigma^{qG}}{dzd\,p_{jT}^2}+\frac{d^2\sigma^{\bar qG}}{dzd\,p_{jT}^2}\right\}, \quad (A.11)$$

where $z=\cos\theta$; $\theta(\phi)$ is the polar (azimuthal, resp.) angle in the $\bar pp$ c.m. system between e^- and the p.

*) The heavier quarks belonging to the higher fermion generations if any, such as b', t' are assumed to have masses $m_{b'}$, $m_{t'}\gtrsim M_z/2$ and not to contribute to the Γ_z.

$$\frac{d^2\sigma^{q\bar q}}{dzd\,p_{jT}^2}=\int dx_1\int dx_2\Big\{F_q^p(x_1,Q^2)\,F_{\bar q}^{\bar p}(x_2,Q^2)\textstyle\sum^{q\bar q}+F_{\bar q}^p(x_1,Q^2)\,F_q^{\bar p}(x_2,Q^2)\sum^{\bar qq}\Big\} \quad (A.12)$$

Adopting the narrow-width approximation:

$$\frac{1}{(Q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\to\frac{\pi}{M_Z\Gamma_Z}\delta(Q^2-M_Z^2) \quad (A.13)$$

$\sum^{q\bar q}$ can be written as:

$$\textstyle\sum^{q\bar q}=\frac{1}{2\hat s}\cdot\frac{1}{(2\pi)^3}\,\frac{M_Z}{\Gamma_Z}\int_0^{2\pi}d\phi\,\frac{\theta(\cdots)}{A^2}\{\cdots\}\,|M_{q\bar q}|^2 \quad (A.14)$$

where

$A=2(\hat s+p_{\rm II}^2)^{1/2}-2p_{\rm II}\cos\theta-2[(p_{jT}^2+p_{jL}^2)^{1/2}-p_{jT}\sin\theta\cos\phi-p_{jL}\cos\theta]$

$\theta(\cdots)=\theta[(\hat s+p_{\rm II}^2)^{1/2}-(p_{jT}^2+p_{jL}^2)^{1/2}-M_Z^2/A]$,

$$\{\cdots\}=\int dp_{jL}\left\{\frac{\delta(p_{jL}-p_{jL+})}{(p_{jT}^2+p_{jL+}^2)^{1/2}\,|f'(p_{jL+})|}\,\theta[(\hat s+p_{\rm II}^2)^{1/2}-(p_{jT}^2+p_{jL+}^2)^{1/2}]+(p_{jL+}\to p_{jL-})\right\},$$

$p_{\rm II}$=the c.m. momentum of the subprocess $=p_{10}-p_{20}$

$$p_{jL\pm}=\frac{(\hat s-M_Z^2)\,p_{\rm II}\pm(\hat s+p_{\rm II}^2)^{1/2}\sqrt{\Delta}}{2\hat s},\quad \Delta=(\hat s-M_Z^2)^2-4\hat s\,p_{jT}^2,\quad f'(p_{jL\pm})=2p_{\rm II}-2(\hat s+p_{\rm II}^2)^{1/2}.\ \frac{p_{jL\pm}}{(p_{jT}^2+p_{jL\pm}^2)^{1/2}}.$$

The explicit formula of $|M_{q\bar q}|^2$ reads as:

$$|M_{q\bar q}|^2=\frac{1}{9}G^4g^2\frac{M_Z^4}{(kp_2)(kp_1)}\Big\{(R-T)[(p_1q_1)^2+(p_2q_2)^2]+(R+T)[(p_1q_2)^2+(p_2q_1)^2]\Big\}, \quad (A.15)$$

where

$G^2=4\sqrt{2}\,G_FM_Z^2$, $g^2=4\pi\alpha_s$,

$R=(a_l^2+b_l^2)(A_q^2+B_q^2)$, $T=4a_lb_l\,A_qB_q$.

The expression of $|M_{\bar qq}|^2$ is deduced from that of $|M_{q\bar q}|^2$ by the exchange $p_1\leftrightarrow p_2$ in Eq (A. 13). Similarly in the Compton case, the corresponding formulas are simply found by crossing from Eq (A. 13) and taking into account the color factor properly.

The various scalar products appearing in the integrals are listed below :

$$(p_1p_2)={}^1/_2\,\hat{s},$$
$$(kp_1)={}^1/_2\{(\hat{s}+p_{\text{II}}{}^2)^{1/2}-p_{\text{II}}\}\{(p_{jT}{}^2+p_{jT}{}^2)^{1/2}+p_{jL}\},$$
$$(kp_2)={}^1/_2\{(\hat{s}+p_{\text{II}}{}^2)^{1/2}+p_{\text{II}}\}\{(p_{jT}{}^2+p_{jL}{}^2)^{1/2}-p_{jL}\},$$
$$(p_1q_2)={}^1/_2\{(\hat{s}+p_{\text{II}}{}^2)^{1/2}-p_{\text{II}}\}\frac{M_Z{}^2}{A}(1+z),$$
$$(p_2q_2)={}^1/_2\{(\hat{s}+p_{\text{II}}{}^2)^{1/2}+p_{\text{II}}\}\frac{M_Z{}^2}{A}(1-z),$$
$$(p_1q_1)=(p_1p_2)-(kp_1)-(p_1q_2), \qquad \text{(A. 16)}$$
$$(p_2q_1)=(p_1p_2)-(kp_2)-(p_2q_2),$$
$$(kq_1)=(kp_1)+(kp_2)-(kq_2),$$
$$(kq_2)=(p_1q_2)+(p_2q_2)-{}^1/_2\,M_Z{}^2.$$

The momentum fractional integral can be expressed as

$$\int dx_1\int x_2=\int_{\tau_{\min}}^{1}d\tau\int_{(\tau-1)/2}^{(1-\tau)/2}dX\frac{1}{(X^2+\tau)^{1/2}} \qquad \text{(A. 17)}$$

where

$$\tau=x_1x_2=\hat{s}/s,\ \ X={}^1/_2\,(x_1-x_2),$$
$$\tau_{\min}=\frac{(p_{jT}+\sqrt{M_Z{}^2+p_{jT}{}^2})^2}{s}$$

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