Self-Consistent Dual Unitarization Scheme with Several Trajectories

著者名(英)	Masaki HAYASHI, Kazuo KATSUURA
journal or publication title	埼玉医科大学進学課程紀要
volume	1
page range	1-11
year	1980-06-30
URL	http://id.nii.ac.jp/1386/00000039/

Self-Consistent Dual Unitarization Scheme with Several Trajectories

Masaki HAYASHI and Kazuo KATSUURA

Department of Physics

(Received 1 September, 1979)

In the self-consistent dual unitarization model with several input Regge trajectories we prove that: i) the intercept of output Pomeron pole is exactly equal to one, i.e. $\alpha_P(0)=1$, ii) equal spacing rules hold for the intercepts of the Regge trajectories, i.e. $2\alpha_{K^*}(0)=\alpha_\rho(0)+\alpha_\phi(0)$, $2\alpha_{D^*}(0)=\alpha_\rho(0)+\alpha_\psi(0)$, $2\alpha_{F^*}(0)=\alpha_\phi(0)+\alpha_\psi(0)$, iii) SU(4) symmetric coupling constant g^2 satisfies the relation

$$1/g^2=1/(1-\alpha_\rho(0))+1/(1-\alpha_\phi(0))+1/(1-\alpha_\psi(0)).$$

The proof for these consequences is given by allowing in the loops of the unitarity sum all the possible Regge trajectories and assuming exact SU(4) symmetry for the Reggeon coupling constants, while broken SU(4) symmetry for the intercepts of the Regge trajectories.

The positions of the daughter trajectories for the Pomeron and the Regge trajectories are also discussed numerically.

§1. Introduction

Considerable interest has been devoted recently to a dual unitary program (dual unitarization) initiated originally by Veneziano[1] and Huan Lee[2] aiming at the construction of the topological Pomeron and self-consistent Regge pole generation in the absorptive part of $2\to2$-body scattering amplitudes, using unitarity and the quark topology structure of the multi-Regge amplitudes. Much of the subsequent investigations along this line has proved that this program provides rather encouraging results, thus reproducing the basic features of high-energy scatterings such as calculations of Pomeron effects, the OZI rule and its violation, exotic exchanges, the breaking of exchange degeneracy both for meson and baryon trajectories and so on[3]. Chaichian and one of the present author[4] have studied recently the general case of the dual unitarization model in which several input Regge trajectories are present. In the present paper following the general idea of Ref 4) we consider the bootstrap for the Regge trajectories by studying the set of equations in which all the trajectories are included. We investigate the consequences following from the self-consistency requirement between the Pomeron sector and the Reggeon sector. As in I, we consider the case of several input trajectories by introducing sets of coupled integral equations and studying their Mellin transforms. We distinguish two different cases: the approximate case a) and the exact case b). In the approximate case a) which could be called the "leading trajectory approximation", we put in the loops of unitarity sum only the (one) highest-lying Regge trajectory which is allowed by duality diagram. In the exact case b) we allow all the Regge trajectories, which are permitted by duality diagrams, to be exchanged in the unitarity sum.

In both cases, as the consequences of the self-consistency between the dual unitarity bootstrap for the Pomeron sector and the Reggeon sector we prove that:

i) the intercept of the output Pomeron pole is exactly equal to one, i.e.

$$\alpha_P(0)=1.$$

ii) equal spacing rules hold for the SU(4) broken intercepts of the Regge trajectories:

$$2\alpha_{K^*}(0)=\alpha_\rho(0)+\alpha_\phi(0),$$

$$2\alpha_{D^*}(0)=\alpha_\rho(0)+\alpha_\psi(0),$$

$$2\alpha_{F^*}(0)=\alpha_\phi(0)+\alpha_\psi(0). \quad (1.1)$$

iii) SU(4) symmetric coupling constant g^2 satisfies the relation:

$$1/g^2=1/(1-\alpha_\rho(0))+1/(1-\alpha_\phi(0))+1/(1-\alpha_\psi(0)). \quad (1.2)$$

In deriving these consequences we assume the exact SU(4) symmetry for the Reggeon coupling constants while broken SU(4) symmetry for the intercepts of the Regge trajectories.

The plan of the paper is as follows: In § 2, after clarifying the notations used specifically in the present paper we discuss the self-consistency between the Reggeon sector and the Pomeron sector in the leading trajectory approximation. In § 3, we treat the problem exactly allowing all the Regge trajectories, which are permitted by duality diagrams, to be exchanged in the unitarity sum. We first consider the SU (3) symmetry case and then extend our discussions to SU(4) case. We also discuss numerically the positions of the daughter trajectories for the Pomeron and the Reggeons.

§2. Leading Trajectory Approximation

2.1 Notations

We use the following notations throughout the paper.

$A^R_{ij\to kl}$:
the imaginary part of the Mellin transformed amplitude for the two-body Reggeon scattering of $i+j \to k+l$ in which the Reggeon $R(\rho, K^*, \phi, D^*, F^*, \psi)$ is exchanged (Fig. 1)

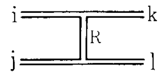


Fig.1. Quark line diagram for $i+j\to k+l$

$\beta_{ij\to kl}$:
the product of two coupling constants $g_{ij\sigma}g_{\sigma kl}$ in the Born term for $i+j\to\sigma\to k+l$ (Fig. 2)

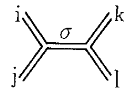


Fig.2. Quark line diagram for Born term $i+j\to\sigma\to k+l$.

$\gamma_{im\to kn}$:
the product of the coupling constants $g_{im\sigma}g_{\sigma kn}$ of the loop diagram in the unitarity sum (Fig. 3)

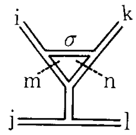


Fig.3. The loop diagram in the unitarity sum.

The contribution of this diagram in the Mellin-transformed integral equation reads as:

$$\gamma_{im\to kn}/(j-(\alpha_m^{in}+\alpha_n^{in}-1))A^R_{mj\to nl} \quad (2.1)$$

$\alpha_i^{in}(0)=\alpha_i$ denotes the intercept of the (input) Regge trajectories with the notations $\rho=1,\ K^*=2,\ \phi=3, D^*=4, F^*=5,\ \psi=6.$

For simplicity, we also use the notations:

$$A^R_{ii\to jj}\equiv A^R_{ij}\ ,$$

$$\beta_{ii\to jj}\equiv\beta_{ij}, \quad (2.2)$$

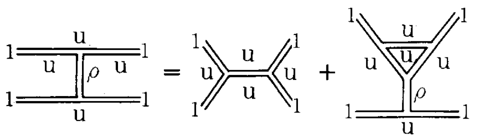
$$\gamma_{ii\to jj}\equiv\gamma_{ij}.\quad (i, j, k, l=1,2,3\ldots.6)$$

For the Pomeron sector (§2.3 and §3.2) we use the similar notations using the superscript P instead of R. Other notations are the same as in I.

2.2 Reggeon Sector

We consider first the planar bootstrap for Reggeons in the leading trajectory approximation. In the following we consider several representative vector meson-vector meson scatterings in which one definite Reggeon R is exchanged. For each process we write the integral equation diagramatically for the planar unitarity sum in terms of dual-quark lines and then rewrite it in the Mellin transformed form.

(i) $\rho\rho\to\rho\rho$



(2.3)

In terms of the Mellin transformed scattering amplitude this equation reads as:

$$A^\rho_{11}=\beta_{11}+\{\gamma_{11}/(j-\alpha_{c_1})\}A^\rho_{11}. \quad (2.4)$$

Hence one has

$$A^\rho_{11}=(j-\alpha_{c_1})\beta_{11}/(j-\alpha_{c_1}-\gamma_{11}). \quad (2.5)$$

The output $\rho-$ trajectory is generated as the singularity position of the scattering amplitudes A^ρ_{11}:

$$\alpha_1^{out}=\alpha_{c_1}+\gamma_{11}. \quad (2.6)$$

Imposing the bootstrap condition that the output pole should be equal to the input one, i.e.

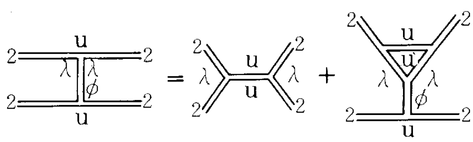
$$\alpha_1^{out}=\alpha_1^{in}=\alpha_1 \quad (2.7)$$

we obtain

$$\gamma_{11}=1-\alpha_1. \quad (2.8)$$

One can treat the following process ($K^*K^*\to K^*K^*$) and all other ones in a similar manner as (i).

(ii) $K^*K^*\to K^*K^*$



(2.9)

$$A^\phi_{22}=\beta_{22}+\gamma_{22}/(j-\alpha_{c2})A^\phi_{22}\ , \quad (2.10)$$

$$A^\phi_{22}=(j-\alpha_{c2})\beta_{22}/(j-\alpha_{c2}-\gamma_{22}). \quad (2.11)$$

For the output pole ϕ one has:

$$\alpha_3^{out}=\alpha_{c2}+\gamma_{22}. \quad (2.12)$$

We assume that all the coupling constants $g_{ij\sigma}$ preserve the exact SU(4) symmetry, i.e. all possible quark diagrams have equal weight irrespective to whether they contain p, n, λ (or c) quarks or not. Thus,

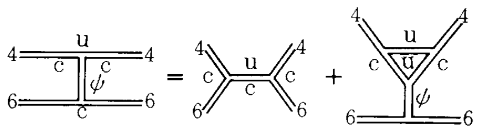
$$\gamma_{ij\to kl}\equiv g^2,$$

$$\beta_{ij\to kl} \equiv \beta \text{ (for any } i, j, k, l). \qquad (2.13)$$

Then this exact symmetry assumption and the condition that the output α_3^{out} should satisfy $\alpha_3^{out} = \alpha_3$ $(=\alpha_3^{in})$ together with eq. (2.8) yield the equal spacing relation:

$$2\alpha_2 = \alpha_1 + \alpha_3 \qquad (2.14)$$

(iii) $D^*\psi \to D^*\psi$



(2.15)

$$A_{46}^{\psi} = \beta_{46} + \gamma_{44}/(j-\alpha_{c4})A_{46}^{\psi}, \qquad (2.16)$$

$$A_{46}^{\psi} = (j-\alpha_{c4})\beta_{46}/(j-\alpha_{c4}-\gamma_{44}). \qquad (2.17)$$

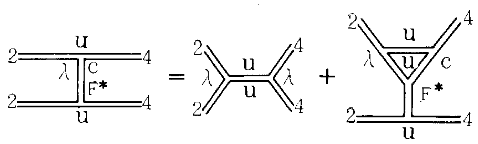
For the output Reggeon α_6^{out}, we have

$$\alpha_6^{out} = \alpha_{c4} + \gamma_{44}. \qquad (2.18)$$

The bootstrap condition $\alpha_6^{out} = \alpha_6$ together with eqs (2.8) and (2.13) yields:

$$2\alpha_4 = \alpha_1 + \alpha_6. \qquad (2.19)$$

(iv) $K^*K^* \to D^*D^*$



(2.20)

$$A_{24}^{F^*} = \beta_{24} + \gamma_{24}/\{j-(\alpha_2^{in}+\alpha_4^{in}-1)\}A_{24}^{F^*}, \qquad (2.21)$$

$$A_{24}^{F^*} = (j-\alpha_2^{in}-\alpha_4^{in}+1)\beta_{24}/\{j-(\alpha_2^{in}+\alpha_4^{in}-1) -\gamma_{24}\}. \qquad (2.22)$$

For the position of α_5^{out} we have

$$\alpha_5^{out} - (\alpha_2^{in} + \alpha_4^{in} - 1) - \gamma_{24} = 0 \qquad (2.23)$$

Eqs. (2.8) and (2.13) together with $\alpha_5^{out} = \alpha_5$ yield:

$$\alpha_5 = \alpha_2 + \alpha_4 - \alpha_1. \qquad (2.24)$$

Then using eqs. (2.14) and (2.19) we obtain:

$$2\alpha_5 = \alpha_3 + \alpha_6. \qquad (2.25)$$

Discussions on the other processes lead either to eq. (2.8) or to one of the relations (2.14), (2.19) and (2.25) and do not add any new results.

2.3 Pomeron Sector

In contrast to the planar bootstrap for Reggeons, the (topological) Pomeron is generated from the sum of planar (untwisted) and cylinder (twisted, crossed) contributions in the loops of the unitarity sum. For the Pomeron sector we have discussed in I in rather detail the derivations of the integral equations for (imaginary part of) VN-diffractive scattering amplitudes and hence we do not repeat here their derivations. Here we summarize the main expressions for the Pomeron sector in the leading trajectory approximation.

$$\bar{A}_1^P = 2\gamma_{11}/(j-\alpha_{c1})\bar{A}_1^P, \qquad (2.26)$$

$$\bar{A}_2^P = \gamma_{22}/(j-\alpha_{c2})\bar{A}_2^P + \gamma_{12}/(j-\alpha_{c1})\bar{A}_1^P, \qquad (2.27)$$

$$\bar{A}_3^P = 2\gamma_{23}/(j-\alpha_{c2})\bar{A}_2^P, \qquad (2.28)$$

$$\bar{A}_4^P = \gamma_{44}/(j-\alpha_{c2})\bar{A}_4^P + \gamma_{41}/(j-\alpha_{c4})\bar{A}_1^P, \qquad (2.29)$$

$$\bar{A}_5^P = \gamma_{52}/(j-\alpha_{c2})\bar{A}_2^P + \gamma_{45}/(j-\alpha_{c4})\bar{A}_4^P, \qquad (2.30)$$

$$\bar{A}_6^P = 2\gamma_{64}/(j-\alpha_{c4})\bar{A}_4^P, \qquad (2.31)$$

where $\bar{A}_i^P$ denote the imaginary part of the

Mellin-transformed physical amplitudes $\bar{A}_i^P = 1/2(A_i + B_i)$ which are expressed in terms of the untwisted (A_i) and the twisted scattering amplitudes (B_i). From eq. (2.26) we have for the intercept of the Pomeron:

$$\alpha_P = 2\alpha_1 - 1 + r_{11}. \qquad (2.32)$$

If one imposes the self-consistency requirement with the planar Reggeon bootstrap (i.e. eq. (2.8)) on this equation, then the Pomeron intercept becomes equal to one:

$$\alpha_P(0) = 1 \qquad (2.33)$$

In eqs. (2.26) – (2.31), taking $\lim_{j\to 1}$ and denoting the Pomeron residues as

$$\beta_i^P = \lim_{j\to 1} (j-1)\bar{A}_i^P \qquad (2.34)$$

and using the results of the planar Reggeon bootstrap, i.e. eq. (2.8) and the equal spacing relations (2.14), (2.19) and (2.25), we obtain the ratios between the total cross-section for the VN diffractive scattering as follows[4]:

$$R_2 = \beta_2^P/\beta_1^P = \sigma(K^*N)/\sigma(\rho N) = 1/2\,(1+r_1), \qquad (2.35)$$

$$R_3 = \beta_3^P/\beta_1^P = \sigma(\phi N)/\sigma(\rho N) = r_1, \qquad (2.36)$$

$$R_4 = \beta_4^P/\beta_1^P = \sigma(D^*N)/\sigma(\rho N) = 1/2\,(1+r_2), \qquad (2.37)$$

$$R_5 = \beta_5^P/\beta_1^P = \sigma(F^*N)/\sigma(\rho N) = 1/2\,(R_3 + R_5), \qquad (2.38)$$

$$R_6 = \beta_6^P/\beta_1^P = \sigma(\psi N)/\sigma(\rho N) = r_2, \qquad (2.39)$$

where

$$r_1 = (1-\alpha_1(0))/(1-\alpha_3(0)), \qquad (2.40)$$

$$r_2 = (1-\alpha_1(0))/(1-\alpha_6(0)). \qquad (2.41)$$

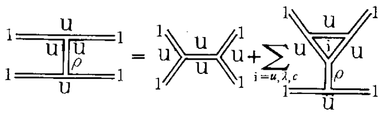
The relations (2.35) – (2.41) coincide with those obtained in the framework of f-dominated Pomeron model of Carlitz, Green and Zee[5].

§3. Exact Treatment

By the exact treatment we mean that we keep in the loops of the unitarity sum all the possible Regge trajectories. We first discuss in the SU(3) symmetry and then in SU(4). Similarly to the previous section we restrict ourselves to several representative processes. We first write the integral equation in terms of quark lines and then rewrite it in the Mellin-transformed form. Exact symmetry eq. (2.13) is assumed.

3.1 Reggeon Sector SU(3)

(i) $\rho\rho \to \rho\rho$



(3.1)

$$A_{11}^{\rho} = \beta_{11} + r_{11}/(j-\alpha_{c1})A_{11}^{\rho} + r_{21}/(j-\alpha_{c2})A_{21}^{\rho}. \qquad (3.2)$$

Exact symmetry for coupling constants eq. (2.13) implies:

$$A_{11}^{\rho} = A_{21}^{\rho}. \qquad (3.3)$$

Consequently one has

$$A_{11}^{\rho} = \beta_{11}/\{1 - r_{11}/(j-\alpha_{c1}) - r_{21}/(j-\alpha_{c2})\}. \qquad (3.4)$$

The output position for ρ can be determined from the equation

$$1/g^2 = 1/(j-\alpha_{c1}) + 1/(j-\alpha_{c2}). \qquad (3.5)$$

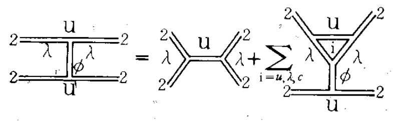
Imposing the condition

$$j = \alpha_1^{out} = \alpha_1 (= \alpha_1^{in}) \qquad (3.6)$$

we obtain

$$1/g^2=1/(1-\alpha_1)+1/(1+\alpha_1-2\alpha_2). \tag{3.7}$$

(ii) $K^*K^*\to K^*K^*$



(3.8)

$$A_{22}^{\phi}=\beta_{22}+r_{22}\Big/(j-\alpha_{c2})\,A_{22}^{\phi}+r_{23}\Big/(j-\alpha_{c3})\,A_{32}^{\phi}. \tag{3.9}$$

Similarly to eq. (3.3) the equality

$$A_{22}^{\phi}=A_{32}^{\phi} \tag{3.10}$$

holds under eq . (2.13) and hence one has:

$$A_{22}^{\phi}=\beta_{22}/\{1-r_{22}/(j-\alpha_{c2})-r_{23}/(j-\alpha_{c3})\}. \tag{3.11}$$

The equation which determines the singularity position for ϕ reads as:

$$1/g^2=1/(j-\alpha_{c2})+1/(j-\alpha_{c3}). \tag{3.12}$$

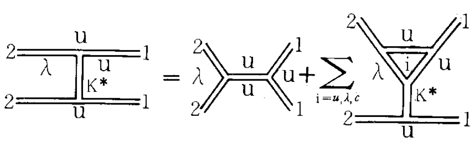
Then imposing the condition

$$j=\alpha_3^{out}=\alpha_3(=\alpha_3^{in}) \tag{3.13}$$

we obtain

$$1/g^2=1/(1+\alpha_3-2\alpha_2)+1/(1-\alpha_3). \tag{3.14}$$

(iii) $K^*K^*\to\rho\rho$



(3.15)

$$A^{K*}_{22\to11}=\beta_{22\to11}+r_{22\to11}/(j-(\alpha_2+\alpha_1-1))\,A^{K*}_{22\to11}+r_{23\to12}/(j-(\alpha_3+\alpha_2-1))\,A^{K*}_{23\to12}. \tag{3.16}$$

Using the relation

$$A^{K*}_{22\to11}=A^{K*}_{23\to12} \tag{3.17}$$

one obtains

$$A^{K*}_{22\to11}=1/\{1-r_{22\to11}/(j-(\alpha_2+\alpha_1-1))-r_{23\to12}/(j-(\alpha_3+\alpha_2-1))\}. \tag{3.18}$$

Consequently the equation which determines the position of K^* reads as:

$$1/g^2=1/\{j-(\alpha_2+\alpha_1-1)\}+1/\{j-(\alpha_3+\alpha_2-1)\}. \tag{3.19}$$

It follows from eqs. (3.7) and (3.14) that:

$$2\alpha_2=\alpha_1+\alpha_3. \tag{3.20}$$

Thus in the exact treatment as well the equal spacing relation holds. For the coupling constant g^2one has

$$1/g^2=1/(1-\alpha_1)+1/(1-\alpha_3) \tag{3.21}$$

which is also consistent with eq. (3.19) if one puts therein $j=\alpha_2^{out}=\alpha_2$.One can solve the quadratic (with respect to j) equations (3.5), (3.12) and (3.19) in which eq. (3.21) is used for $1/g^2$. Each equation contains as its roots the corresponding Reggeon α_i and its daughter $(\alpha_i)_D$. Using the equal spacing relations (3.20) we obtain the following relation between the Reggeon α_i and its daughter $(\alpha_i)_D$:

$$\alpha_i-(\alpha_i)_D=1-\alpha_2+(\alpha_1-\alpha_3)^2/4(1-\alpha_2)^2. \quad (i=1,2,3) \tag{3.22}$$

Note that the discussions on other scattering processes do not add any new results other than eqs. (3.5), (3.12) and (3.19).

3.2 Pomeron Sector SU(3)

In the SU(3) symmetry we have for *VN*-diffractive scattering (physical) amplitudes (in which Pomeron is exchanged) the following set of equations[4]:

$$\bar{A}_1^P = 2\gamma_{11}/(j-\alpha_{c1})\bar{A}_1^P + 2\gamma_{12}/(j-\alpha_{c2})\bar{A}_2^P, \quad (3.23)$$

$$\bar{A}_2^P = \gamma_{21}/(j-\alpha_{c1})\bar{A}_1^P + 2\gamma_{22}/(j-\alpha_{c2})\bar{A}_2^P + \gamma_{23}/(j-\alpha_{c3})\bar{A}_3^P, \quad (3.24)$$

$$\bar{A}_3^P = 2\gamma_{32}/(j-\alpha_{c2})\bar{A}_2^P + 2\gamma_{33}/(j-\alpha_{c3})\bar{A}_3^P. \quad (3.25)$$

The singularity position for the Pomeron is determined from the following equations:

$$\begin{vmatrix} \frac{1}{2g^2}-\frac{1}{j-\alpha_{c1}} & -\frac{1}{j-\alpha_{c2}} & 0 \\ \frac{-1}{2(j-\alpha_{c1})} & \frac{1}{2g^2}-\frac{1}{j-\alpha_{c2}} & \frac{-1}{2(j-\alpha_{c3})} \\ 0 & -\frac{1}{j-\alpha_{c2}} & \frac{1}{2g^2}-\frac{1}{j-\alpha_{c3}} \end{vmatrix} = 0 \quad (3.26)$$

Here eq. (2.13) is used. Imposing on the eq. (3.26) the self-consistency condition with the planar bootstrap, i.e. eqs. (3.20) and (3.21) we obtain the solutions for the Pomeron singularity:

$$j_1^P = 1,$$

$$j_2^P = 1 - \Delta,$$

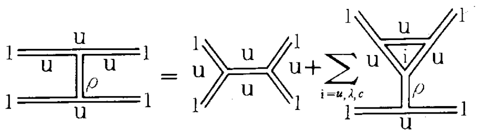
$$j_3^P = 1 - 2\Delta, \quad (3.27)$$

with $\Delta = \{(1-\alpha_1)^2 + (1-\alpha_3)^2\}/2(1-\alpha_2)$. Thus, in the exact treatment as well the Pomeron has the intercept exactly equal to one: $\alpha^P(0)=1$. [Note that we have $\sum_{i=1}^{3}(j_i^P) = 3(1-\Delta) < 3$].

3.3 Reggeon Sector SU(4)

We extend our discussions to the SU(4) symmetry case. As in the previous section for each representative scattering process we write, in order, the integral equation in terms of dual-quark lines, its Mellin-transformed form, the equation which determines the singularity position of the corresponding Reggeon R assuming the exact symmetry for the coupling constants (2.13), and finally we impose the bootstrap condition $\alpha_R^{out} = \alpha_R (= \alpha_R^{in})$.

(i) $\rho\rho \to \rho\rho$



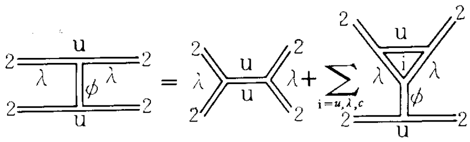
(3.28)

$$A_{11}^\rho = \beta_{11} + \sum_{i=1,2,4} \{\gamma_{1i}/(j-\alpha_{ci})\} A_{i1}^\rho, \quad (3.29)$$

$$1/g^2 = 1/(j-\alpha_{c1}) + 1/(j-\alpha_{c2}) + 1/(j-\alpha_{c4}) \quad (3.30)$$

$$1/g^2 = 1/(1-\alpha_1) + 1/(1+\alpha_1-2\alpha_2) + 1/(1+\alpha_1-2\alpha_4). \quad (3.31)$$

(ii) $K^*K^* \to K^*K^*$



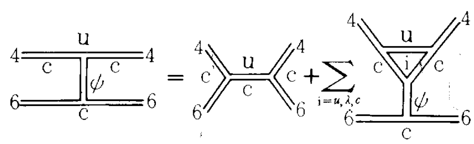
(3.32)

$$A_{22}^\phi = \beta_{22} + \sum_{i=2,3,5} \{\gamma_{2i}/(j-\alpha_{ci})\} A_{2i}^\phi, \quad (3.33)$$

$$1/g^2 = 1/(j-\alpha_{c2}) + 1/(j-\alpha_{c3}) + 1/(j-\alpha_{c5}), \quad (3.34)$$

$$1/g^2 = 1/(1-\alpha_2) + 1/(1+\alpha_2-2\alpha_3) + 1/(1+\alpha_2-2\alpha_5). \quad (3.35)$$

(iii) $D^*\psi \to D^*\psi$



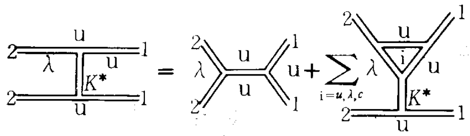
(3.36)

$$A^{\psi}_{46}=\beta_{46}+\sum_{i=4,5,6}\{\gamma_{4i}/(j-\alpha_{ci})\}A^{\psi}_{i6}, \quad (3.37)$$

$$1/g^2=1/(j-\alpha_{c4})+1/(j-\alpha_{c5})+1/(j-\alpha_{c6}), \quad (3.38)$$

$$1/g^2=1/(1-\alpha_6)+1/(1+\alpha_6-2\alpha_4)+1/(1+\alpha_6-2\alpha_5). \quad (3.39)$$

(iv) $K^*K^*\to\rho\rho$



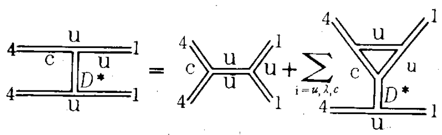
(3.40)

$$A^{K^*}_{22\to11}=\beta_{22\to11}+\gamma_{22\to11}/\{j-(\alpha_2+\alpha_1-1)\}A^{K^*}_{22\to11}+\gamma_{23\to12}/\{j-(\alpha_3+\alpha_2-1)\}A^{K^*}_{32\to21}+\gamma_{25\to14}/\{j-(\alpha_5+\alpha_4-1)\}A^{K^*}_{52\to41} \quad (3.41)$$

$$1/g^2=1/\{j-(\alpha_2+\alpha_1-1)\}+1/\{j-(\alpha_2+\alpha_3-1)\}+1/\{j-(\alpha_5+\alpha_4-1)\}, \quad (3.42)$$

$$1/g^2=1/(1-\alpha_1)+1/(1-\alpha_3)\quad 1/\{1-(\alpha_5+\alpha_4-\alpha_2)\}. \quad (3.43)$$

(v) $D^*D^*\to\rho\rho$



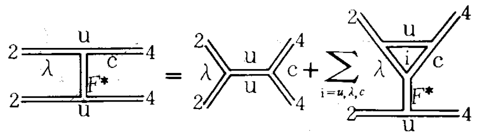
(3.44)

$$A^{D^*}_{44\to11}=\beta_{44\to11}+\gamma_{44\to11}/\{j-(\alpha_4+\alpha_1-1)\}A^{D^*}_{44\to11}+\gamma_{45\to12}/\{j-(\alpha_5+\alpha_2-1)\}A^{D^*}_{54\to21}+\gamma_{46\to14}/\{j-(\alpha_6+\alpha_4-1)\}A^{D^*}_{64\to41}, \quad (3.45)$$

$$1/g^2=1/\{j-(\alpha_4+\alpha_1-1)\}+1/\{j-(\alpha_5+\alpha_2-1)\}+1/\{j-(\alpha_6+\alpha_4-1)\}, \quad (3.46)$$

$$1/g^2=1/(1-\alpha_1)+1/\{1-(\alpha_5+\alpha_2-\alpha_4)\}+1/(1-\alpha_6). \quad (3.47)$$

(vi) $K^*K^*\to D^*D^*$



(3.48)

$$A^{F^*}_{22\to44}=\beta_{22\to44}+\gamma_{22\to44}/\{j-(\alpha_2+\alpha_4-1)\}A^{F^*}_{22\to44}+\gamma_{23\to45}/\{j-(\alpha_3+\alpha_5-1)\}A^{F^*}_{32\to54}+\gamma_{25\to46}/\{j-(\alpha_5+\alpha_6-1)\}A^{F^*}_{52\to64}, \quad (3.49)$$

$$1/g^2=1/\{j-(\alpha_2+\alpha_4-1)\}+1/\{j-(\alpha_3+\alpha_5-1)\}+1/\{j-(\alpha_5+\alpha_6-1)\}, \quad (3.50)$$

$$1/g^2=1/\{1-(\alpha_2+\alpha_4-\alpha_5)\}+1/(1-\alpha_3)+1/(1-\alpha_6). \quad (3.51)$$

Discussions on all other VV-scattering channels do not yield any new results and can be reduced to one of the relations obtained above. Using the notations $x_i=1-\alpha_i$, we obtain from eqs. (3.47) and (3.51)

$$(x_4-x_5)^2-2c(x_4-x_5)-x_2^2=0 \quad (3.52)$$

with $c=x_1x_3/(x_3-x_1)$.

Hence

$$x_4-x_5=c-\sqrt{c^2+x_2^2}\,. \quad (3.53)$$

A minus sign in the r.h.s. of eq. (3.53) was chosen by taking into account that $\alpha_4>\alpha_5$ and hence $x_4-x_5<0$. From eqs. (3.31) and (3.35) we obtain:

$$(x_3-x_1)2x_2(2x_2-x_1-x_3)/\{x_1x_3(2x_2-x_1)(2x_2-x_3)\}+\{2(x_5-x_4)-(x_3-x_1)\}/\{(2x_4-x_1)(2x_5-x_3)\}=0. \quad (3.54)$$

Using eq. (3.53) we get

$$2(x_5-x_4)-(x_3-x_1)=2\sqrt{c^2+x_2^2}-2c^2-(x_3-x_1)$$

$$=\frac{(2x_2-x_3-x_1)(2x_2+x_3+x_1)}{2\sqrt{c^2+x_2^2}+2c+x_3-x_1} \qquad (3.55)$$

From eqs. (3.54) and (3.55) we get

$$(2x_2-x_1-x_3)[(x_3-x_1)2x_2/\{x_1x_3(2x_2-x_1)$$
$$(2x_2-x_3)\}+(2x_2+x_3+x_1)/\{(2x_4-x_1)(2x_5-x_3)$$
$$(2\sqrt{c^2+x_2^2}+2c+x_3-x_1)\}]=0. \qquad (3.56)$$

Since $1>\alpha_1>\alpha_2>\alpha_3>\alpha_4>\alpha_5$, one has in the l. h.s. of eq. (3.56)$[\cdots]>0$ and hence

$$2x_2-x_1-x_3=0 \qquad (3.57)$$

or

$$2\alpha_2=\alpha_1+\alpha_3. \qquad (3.58)$$

Then it follows from eqs. (3.31) and (3.51), and eqs. (3.35) and (3.47) together with eq. (3.58) that:

$$2\alpha_4=\alpha_1+\alpha_6, \qquad (3.59)$$

$$2\alpha_5=\alpha_3+\alpha_6, \qquad (3.60)$$

$$1/g^2=1/(1-\alpha_1)+1/(1-\alpha_3)+1/(1-\alpha_6). \qquad (3.61)$$

One can solve the cubic (with respect to j) equations (3.30), (3.34), (3.38), (3.42), (3.46) and (3.50) in each of which eq. (3.61) is assumed for $1/g^2$ and determine the positions of the daughter trajectories. These equations contain as the roots the corresponding Reggeon α_i and its two daughters $(\alpha_i)_{D1}$ and $(\alpha_i)_{D2}$. $((\alpha_i)_{D1}>(\alpha_i)_{D2})$. Using the relations (3.58) – (3.60), we obtain the following "equal spacing relations" between α_i, $(\alpha_i)_{D1}$ and $(\alpha_i)_{D2}$.

$$\alpha_i-(\alpha_i)_{D1}=\Delta_1, \qquad (3.62)$$

$$(\alpha_i)_{D1}-(\alpha_i)_{D2}=\Delta_2 \quad (\text{for } i=1,2,3,\ldots,6) \qquad (3.63)$$

where

$$\Delta_1=g^2\left(\frac{1-\alpha_5}{1-\alpha_1}+\frac{1-\alpha_4}{1-\alpha_3}+\frac{1-\alpha_2}{1-\alpha_6}\right)-\frac{\Delta_2}{2}, \qquad (3.64)$$

$$\Delta_2=g^2\sqrt{\frac{(\alpha_3-\alpha_6)^2}{(1-\alpha_1)^2}+\frac{(\alpha_6-\alpha_1)^2}{(1-\alpha_3)^2}+\frac{(\alpha_1-\alpha_3)^2}{(1-\alpha_6)^2}-2\frac{(\alpha_3-\alpha_6)(\alpha_6-\alpha_1)}{(1-\alpha_1)(1-\alpha_3)}-2\frac{(\alpha_1-\alpha_3)(\alpha_3-\alpha_6)}{(1-\alpha_1)(1-\alpha_6)}-2\frac{(\alpha_6-\alpha_1)(\alpha_1-\alpha_3)}{(1-\alpha_3)(1-\alpha_6)}}. \qquad (3.65)$$

In table 1 we summarize the numerical values g^2, Δ_1 and Δ_2 for several typical values of the Reggeon intercepts, using eqs. (3.58) – (3.61) and (3.64) – (3.65).

Table 1

α_1	0.5					
α_3	0			0.3		
α_2	0.25			0.4		
α_6	−2	−4	−6	−2	−4	−6
α_4	−0.75	−1.75	−2.75	−0.75	−1.75	−2.75
α_5	−1.00	−2.00	−3.00	−0.85	−1.85	−2.85
g^2	0.300	0.313	0.318	0.266	0.276	0.280
Δ_1	0.830	0.832	0.833	0.616	0.617	0.617
Δ_2	1.939	3.898	5.880	2.170	4.140	6.127

3.4 Pomeron Sector SU(4)

For *VN*-diffractive scattering amplitudes (in which Pomeron is exchanged) we have the following set of equations[4]:

$$\{1-2r_{11}/(j-\alpha_{c1})\}\bar{A}_1^P=\sum_{i=2,4}2r_{1i}/(j-\alpha_{ci})\bar{A}_i^P, \qquad (3.66)$$

$$\{1-2r_{22}/(j-\alpha_{c2})\}\bar{A}_2^P=\sum_{i=1,3,4,5}r_{2i}/(j-\alpha_{ci})\bar{A}_i^P, \qquad (3.67)$$

$$\{1-2r_{33}/(j-\alpha_{c3})\}\bar{A}_3^P=\sum_{i=2,5}2r_{3i}/(j-\alpha_{ci})\bar{A}_i^P, \quad (3.68)$$

$$\{1-2r_{44}/(j-\alpha_{c4})\}\bar{A}_4^P=\sum_{i=1,2,5,6}r_{4i}/(j-\alpha_{ci})\bar{A}_i^P, \quad (3.69)$$

$$\{1-2r_{55}/(j-\alpha_{c5})\}\bar{A}_5^P=\sum_{i=2,3,4,6}r_{5i}/(j-\alpha_{ci})\bar{A}_i^P, \quad (3.70)$$

$$\{1-2r_{66}/(j-\alpha_{c6})\}\bar{A}_6^P=\sum_{i=4,5}2r_{6i}/(j-\alpha_{ci})\bar{A}_i^P. \quad (3.71)$$

Assuming eq. (2.13), we obtain the following equation which determines the singularity position for the Pomeron:

$$f(j)=\begin{vmatrix} \frac{1}{2g^2}-\frac{1}{j-\alpha_{c1}} & -\frac{1}{j-\alpha_{c2}} & 0 & -\frac{1}{j-\alpha_{c4}} & 0 & 0 \\ -\frac{1}{j-\alpha_{c1}} & \frac{1}{g^2}-\frac{2}{j-\alpha_{c2}} & -\frac{1}{j-\alpha_{c3}} & -\frac{1}{j-\alpha_{c4}} & -\frac{1}{j-\alpha_{c5}} & 0 \\ 0 & -\frac{1}{j-\alpha_{c2}} & \frac{1}{2g^2}-\frac{1}{j-\alpha_{c3}} & 0 & -\frac{1}{j-\alpha_{c5}} & 0 \\ -\frac{1}{j-\alpha_{c1}} & -\frac{1}{j-\alpha_{c2}} & 0 & \frac{1}{g^2}-\frac{2}{j-\alpha_{c4}} & -\frac{1}{j-\alpha_{c5}} & -\frac{1}{j-\alpha_{c6}} \\ 0 & -\frac{1}{j-\alpha_{c2}} & -\frac{1}{j-\alpha_{c3}} & -\frac{1}{j-\alpha_{c4}} & \frac{1}{g^2}-\frac{2}{j-\alpha_{c5}} & -\frac{1}{j-\alpha_{c6}} \\ 0 & 0 & 0 & -\frac{1}{j-\alpha_{c4}} & -\frac{1}{j-\alpha_{c5}} & \frac{1}{2g^2}-\frac{1}{j-\alpha_{c6}} \end{vmatrix}=0 \quad (3.72)$$

The equation (3.72) is a 6-th order equation (with respect to j) and hence we do not try to solve it analytically. However, requiring the self-consistency with the planar bootstrap, i.e. the results of the Reggeon sector: eqs. (3.59), (3.60) and (3.61), one can show:

$$f(j=1)=0 \quad (3.73)$$

irrespective to the input Reggeon values α_1, α_3 and α_6. This means that $j = 1$ is one of the roots of the equation and hence the singularity position for the Pomeron $\alpha_P(0)$ generated from eq. (3.72) is exactly equal to 1 even in the exact treatment if the self-consistency with the planar bootstrap is imposed under the exact SU(4) symmetry. We have computed the remaining roots of eq. (3.72) numerically using a computer, which give the positions of the daughter trajectories of the Pomeron. In table 2, we summarize the positions for j_{P_i} ($i=1, 2, \cdots, 6$) for several representative values of $\alpha_3(0)$ and $\alpha_6(0)$ as the input (for $\alpha_1(0)$ we assume $\alpha_1(0)=0.5$). The values α_2, α_4, α_5 and g^2 are determined from the relations (3.59), (3.60) and (3.61) and are given in Table 1.

Table 2

α_3	0.0			0.3		
α_6	−2	−4	−6	−2	−4	−6
j_{P1}	1	1	1	1	1	1
j_{P2}	0.170	0.168	0.167	0.384	0.383	0.383
j_{P3}	-0.661	-0.665	-0.666	-0.233	-0.233	-0.233
j_{P4}	-1.770	-3.730	-5.713	-1.786	-3.757	-5.743
j_{P5}	-2.600	-4.563	-6.545	-2.403	-4.373	-6.360
j_{P6}	-4.539	-8.460	-12.425	-4.573	-8.513	-12.487

For the total cross-section ratios $R_i = \sigma(V_i N)/\sigma(\rho N)$ as already pointed out in I, one can prove analytically that the exact treatment yields the identical values (except for the coupling constant g^2) with the case when only one leading diagram is kept in the unitarity sum (i.e. as in the CGZ-model). From the numerical analysis obtained by varying the input values $\alpha_3(0)$ and $\alpha_6(0)$($\alpha_1(0)$ is fixed at 0.5) while for $\alpha_2(0)$, $\alpha_4(0)$, $\alpha_5(0)$, and g^2 using the values determined from eqs. (3.59), (3.60) and (3.61), we see that the following equal-spacing relations hold among the intercepts of the Pomeron daughters.

$$j_{P1} - j_{P2} = j_{P2} - j_{P3} = j_{P4} - j_{P5} \quad (3.74)$$

$$j_{P3} - j_{P5} = j_{P5} - j_{P6}. \quad (3.75)$$

One of the authors (M.H.) would like to thank the Research Institute for Theoretical Physics, Helsinki University for the hospitality where a part of this work has been done. M.H. is also very grateful to Professor M. Chaichian for many useful discussions.

Reference

1) G. Veneziano, Phys. Letters 43B, 413 (1973), see also Nucl. Phys. B74, 365 (1974), Phys. Letters 52B, 220 (1974).
2) Huan Lee, Phys. Rev. 30, 719 (1973).
3) For relevant references, see, e.g. Ref 4).
4) M. Chaichian and M. Hayashi, Helsinki University Preprint of HU-TFT-77-23 (1977), Phys. Rev. D20, 326 (1979). We refer this paper as I.
5) R. Carlitz, M.B. Green and A. Zee, Phys. Rev. Lett. 26. 1515 (1971), Phys. Rev. D4, 3439 (1971).