A FIXED-INVERSE BINARY MISCLASSIFICATION MODEL UNDER POSSIBLE FALSE-POSITIVE MISCLASSIFICATION

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Abstract

- In this project, we develop a particular statistical model for binary data that allows for the possibility of false-positive misclassification. To account for the misclassification, the model incorporates a two-stage sampling scheme.
- Next, we apply maximum likelihood methods to find estimators of the primary prevalence parameter p as well as the false-positive misclassification rate parameter ϕ. In addition, we derive confidence intervals for p based on inverting Wald, score and likelihood ratio statistics.
- Also, we graphically compare coverage and width properties of the Wald-based, score-based, and likelihood ratio-based confidence intervals for p through a Monte Carlo simulation. The simulation study is done under different parameter and sample size configurations. Also, we apply the newly-derived confidence intervals for p to a real data set.

Introduction

- Due to practical reasons such as cost and time savings, fallible classifiers which are prone to error are used to classify binary data.
- Misclassification may result in false-negatives or false-positives.
- Misclassification errors may distort results of statistical analysis.
- Models that account for misclassification have been developed to compensate for the effect of errors.
- The misclassification rate parameter is a measurable feature of a statistical model that accounts for misclassification.
- Different applications requires different statistical models, which have specific advantages and limitations.
- Better estimation can be done by an infallible device, but at a higher cost.
- A double sampling scheme using both fallible and infallible devices may be used at a reasonable cost , while properly accounting for misclassification.
- We consider a model that allows only for false-positive misclassification, which treats the first sample of a two-stage sampling scheme as fixed and the second stage of the scheme as random (inverse sampling).

Two-Stage Sampling Scheme

The double sampling scheme involves the use of a fallible (cheap) and infallible (expensive) classifier in two stages in an effort to appropriately estimate p and the rate(s) of misclassification. The first stage involves the use of a fallible classier that is prone to producing false-positives under fixed sampling. The second stage involves using an infallible and fallible classifier under inverse sampling technique. For insight into this two-stage scheme consider the following example (* denotes false-positive):

Stage	Pop.	0	1	1	0	0	0	0	0	1	1	0	1	0	1	0	1
First	Fallible	0	1	1	1*	1*	0	0	1								
Sec.	Fallible									1	1	1*	1	0	1	1*	1
	Infallible									1	1	0	1	0	1	0	1

Fixed-Inverse Binary Misclassification Model

For the two stage sampling scheme, define the following counts:

y = # of observations labeled success after m trials of the fallible device in stage 1,

n_{00} = # of observations labeled failure by both fallible and infallible devices in stage 2,

n_{10} = # of observations labeled "success" by fallible device but "failure" infallible device in stage 2,

n_{11} = # of observations labeled success by both fallible and infallible devices in stage 2.

For example above, $y = 5$, $n_{00} = 1$, $n_{10} = 2$, and $n_{11} = 5$.

- **Distributional Assumptions**

The Binomial distribution and Negative Multinomial distribution are used to model the counts y, n_{00}, n_{10} :

$$f_1(y) = \binom{m}{y} \pi^y (1-\pi)^{m-y}$$

and

$$f_2(n_{00}, n_{10}) = \frac{(n_{11}+n_{00}+n_{10}-1)!}{(n_{11}-1)!\,n_{00}!\,n_{10}!}((1-p)(1-\phi))^{n_{00}}(\phi(1-p))^{n_{10}}p^{n_{11}}$$

where,

$n = n_{00} + n_{10} + n_{11}$ is the random sample size needed to observe n_{11} successes labeled by both fallible and infallible classifiers in stage 2,

p = probability infallible device yield success,

ϕ = probability fallible device yield false-negative,

π = probability fallible device labels an observation as success.

- **Maximum Likelihood Estimators**

$$\hat{p} = \frac{n_{11}(n_{11}+n_{10}+y)}{(n_{11}+n_{10})(n_{00}+n_{10}+n_{11}+m)}$$

and

$$\hat{\phi} = \frac{n_{10}(n_{10}+n_{11}+y)}{(n_{11}+n_{10})(n_{00}+n_{10}+n_{11}+m)(1-\hat{p})}$$

- **Large Sample Confidence Intervals for p**

  - **Wald CI**: $\hat{p} \pm Z_{\frac{\alpha}{2}}\sqrt{I^{11}(\hat{p},\hat{\phi})}$
  - **Score CI:** values of p that satisfy :
  $$[u_p(\hat{\phi}_p)]^2\, I^{11}(p,\hat{\phi}_p) \le \chi^2{}_1(\alpha)$$
  - **Likelihood CI:** values of p that satisfy :
  $$2\left(l(\hat{p},\hat{\phi}) - l(p,\hat{\phi}_p)\right) \le \chi^2{}_1(\alpha)$$

where,

- $Z_{\frac{\alpha}{2}}$ is $(1-{}^{\alpha}/_{2})$ percentile for the standard normal distribution
- $u_p(\hat{\phi}_p) = \frac{\partial l}{\partial p}$ at $\hat{\phi}_p$
- $\chi^2{}_1(\alpha)$ is $(1-\alpha)$ percentile of a chi-squared distribution with one degree of freedom
- $I^{11}(p,\hat{\phi}_p)$ is the $(1,1)$ element of the inverse of Fisher's Information Matrix
- l is the log likelihood function

Motivating Example

- The western blot procedure (WBP) is one diagnostic test of the herpes simplex virus. Out of 693 women tested, the WBP yielded that 375 had the virus. (Hildeshiem and Boese)
- Under the binomial model (not accounting for misclassification), the maximum likelihood estimator (MLE) and Wald confidence interval for p is

Parameter	Estimate	Wald C.I
p	0.541	(0.504,0.578)

It turns out that WBP is a fallible detector of the virus, hence the estimate above is biased.

- Another procedure called the Refined Western Blot Procedure (RWBP) is accurate and we will consider it as an infallible classifying device.
- Also, from Hildesheim's data we will only consider the false positives from stage 2 and allow the false negatives to be absorbed into n_{11}.
- From the two stages, the following counts were observed:
  $y = 375$, $m = 693$, $n_{00} = 13$, $n_{10} = 3$, and $n_{11} = 23$.
- Under the Fixed-Inverse Binary Misclassification Model, the MLE's for p and ϕ and confidence intervals for p are

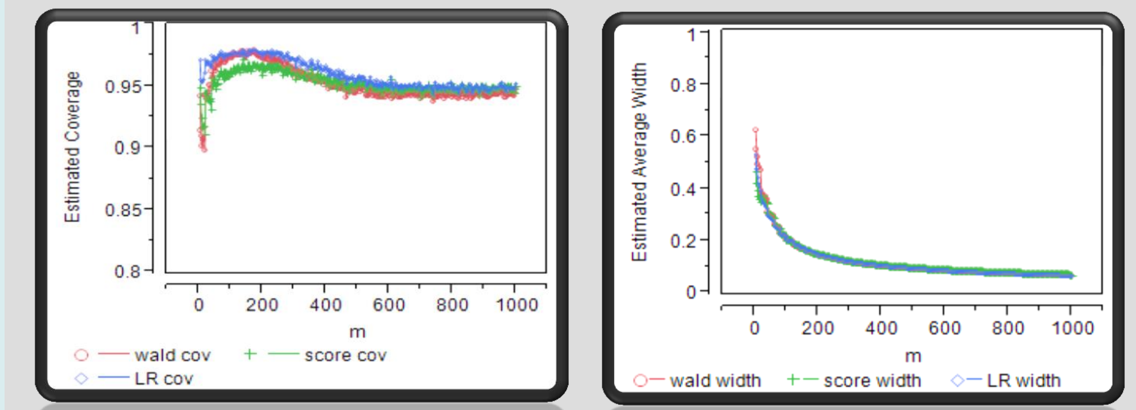
Parameter	Estimates	Wald C.I *	Score C.I.*	Lik.-Ratio C.I *
p	0.485	(0.4102,0.5589)	(0.3899,0.5793)	(0.4279,0.5413)
ϕ	0.123	Not yet	Not yet	Not yet

- The estimate of p under Fixed-Inverse Binary Misclassification Model is smaller than under the binomial model, which is intuitive because the misclassification rate (ϕ) is around 12% . Hence, the estimate of p under the binomial model is likely overestimated due to false-positives generated by using only the fallible classifier.

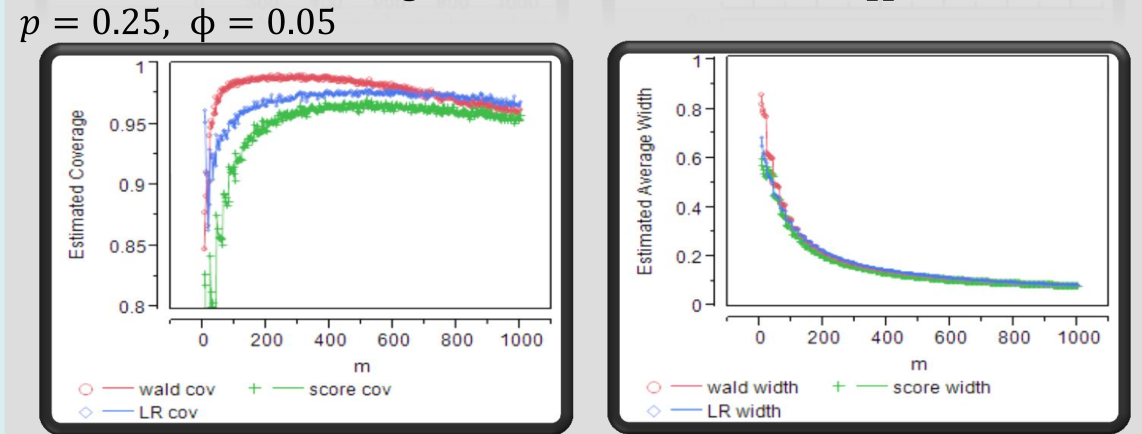
Simulation Results

- Next, we consider the coverage and width properties of the three confidence intervals when estimating p.
- Here we consider two configurations of ratios of the fallible data to the infallible data: $n_{11} = 0.05m$ and $n_{11} = 0.4m$, while varying the parameters p and ϕ.
- All simulations were performed in SAS IML with a simulation size of 10,000 iterations.
- The nominal confidence level was 95%.

Simulation Results When $n_{11} = 0.05m$

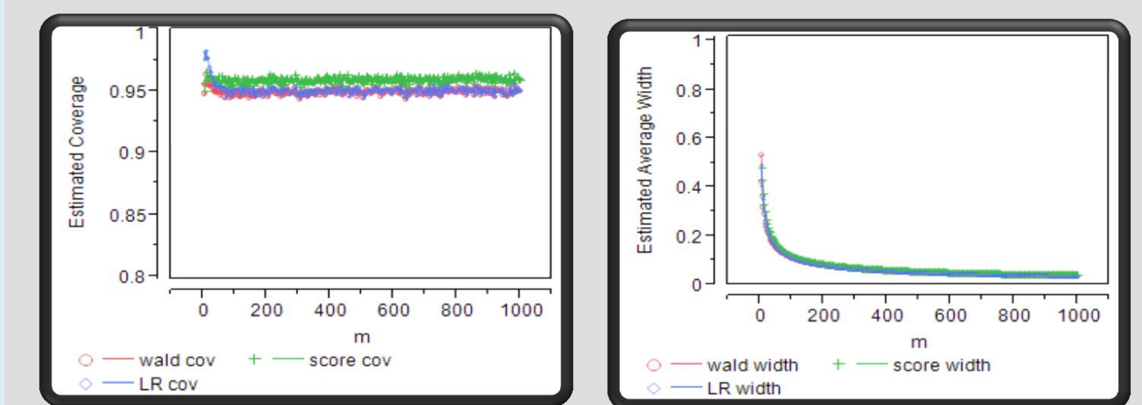


Estimated Actual Coverages and Actual Widths when $n_{11} = 0.05m$ and $p = 0.25,\ \phi = 0.05$

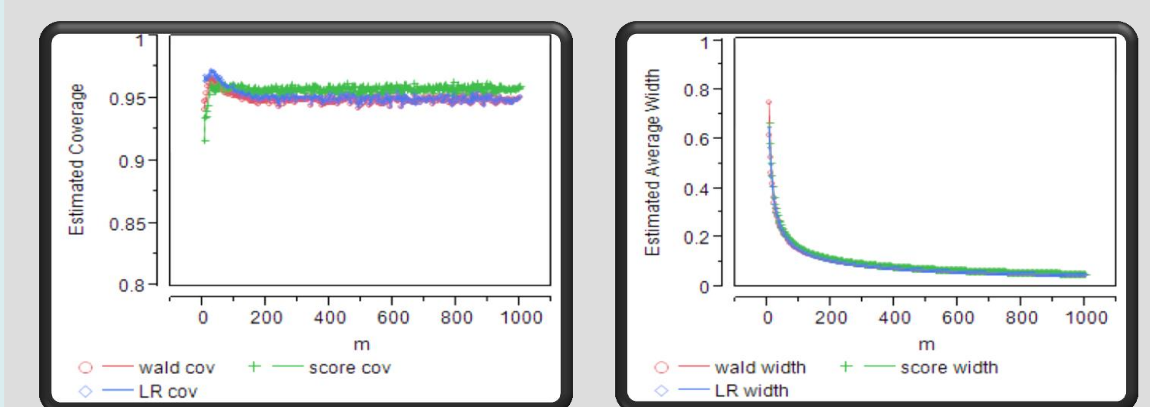


Estimated Actual Coverages and Actual Widths when $n_{11} = 0.05m$ and $p = 0.5,\ \phi = 0.05$

Simulation Results When $n_{11} = 0.4m$



Estimated Actual Coverages and Actual Widths when $n_{11} = 0.4m$ and $p = 0.25,\ \phi = 0.05$



Estimated Actual Coverages and Actual Widths when $n_{11} = 0.4m$ and $p = 0.5,\ \phi = 0.05$

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