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Introduction to forest valuation and investment analysis

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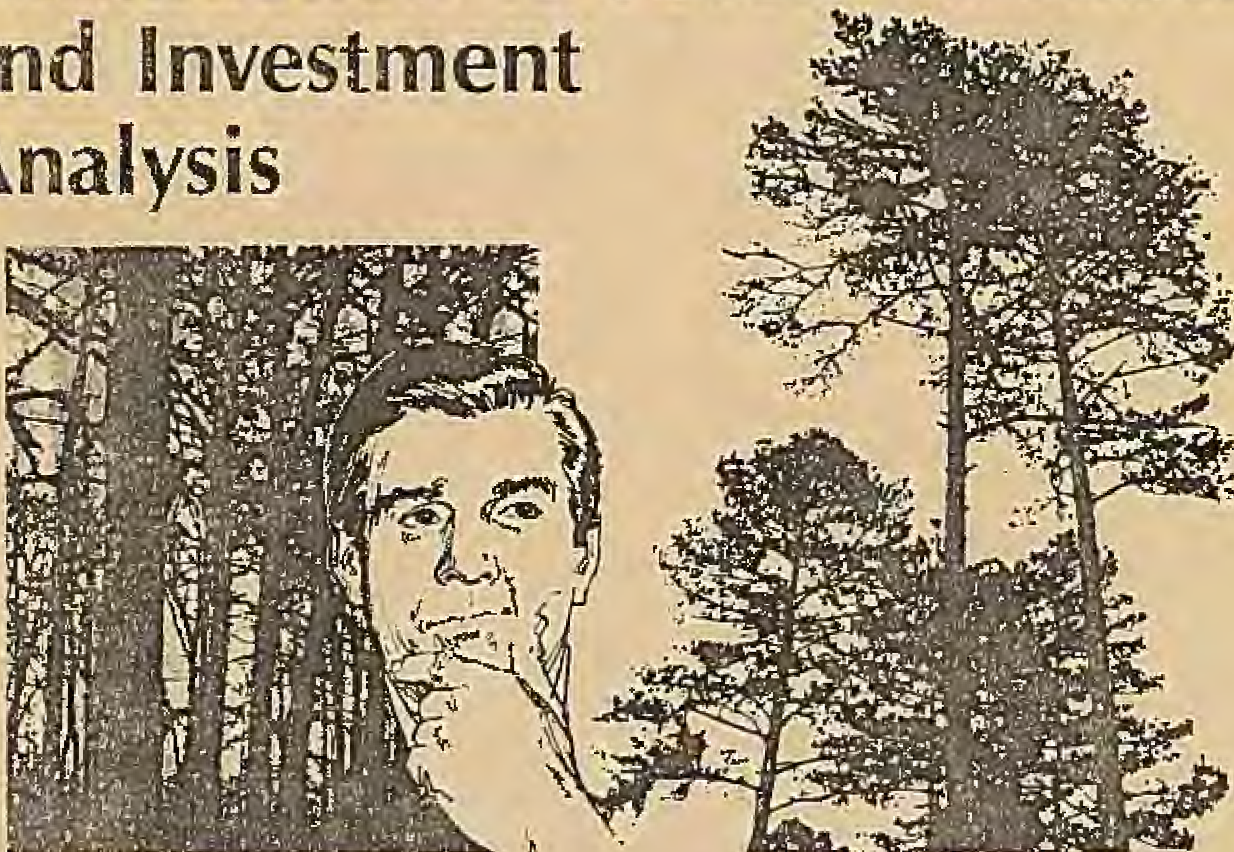
Introduction to Forest Valuation and Investment Analysis



by

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Introduction to Forest Valuation and Investment Analysis

By

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Introduction to Forest Valuation

and Investment Analysis

I. INTEREST AND THE TIME VALUE OF MONEY

Most foresters and forest landowners are aware that money has a time value. A dollar today is worth more than a dollar tomorrow. If you borrow $1,000 from the bank today, you would have to pay back more than $1,000 in 90 days. The term forest economists use for this concept is the time value of money: the closer to today you receive a sum of money, the greater its present value.

Two aspects of forestry investments require that we understand the time value of money: high investment costs and the long period of time often involved. The first aspect, high investment costs, means that we often invest quite a lot of money in stand establishment or other forestry practices in anticipation of future profit or other benefits. The second aspect, the long period of time involved, means a period will pass before most forestry investments produce cash returns. Together, these aspects of forestry force us to carefully consider the time value of money in our management decisions.

Interest is used to equate values of money over time. Interest is the "rent" paid for the use of money. If you borrow $1,000 from the bank today, you will expect to pay back $1,000 plus an interest payment in 90 days. The interest added to the $1,000 makes the value of the repayment in 90 days exactly equal in terms of value to the original $1,000 (i.e., the interest accounts for the time value of money).

II. CASH FLOW DIAGRAMS AND EQUIVALENCE

Cash Flow Diagrams

A forestry investment usually consists of more than one payment or more than one receipt. For example, if you borrowed $1,000 and paid it back with three monthly payments, you would have one receipt and three payments. The four cash transactions represent a cash flow. The cash flow diagram is a useful tool for analyzing costs and revenues, by providing a handy means of representing their timing. The basis of a cash flow diagram is a time line, identifying each interest period (usually a year). Arrows pointing upward at a period indicate income and arrows pointing downward at a period indicate costs. Figure 1 shows a time line for an initial investment of $5,000, costs of $1,000 for each of the next four years, and a $16,000 income at the end of year 5.

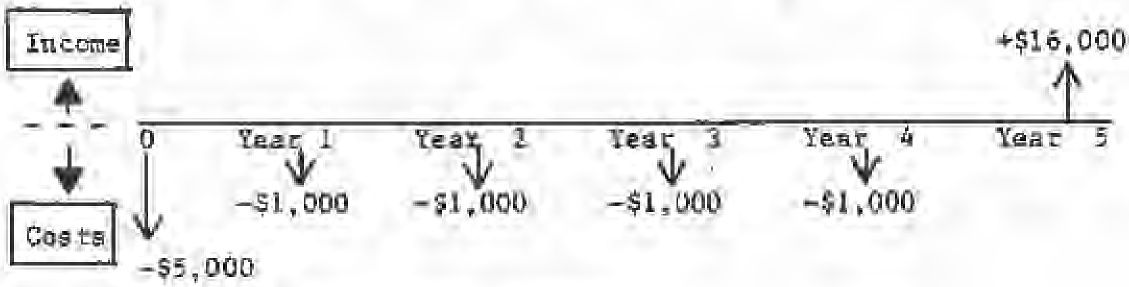

Figure 1. Example cash flow diagram.

Equivalence

Two amounts of money can be equated if the proper interest rate is used. The equivalent value of the amounts of money must be defined at a specific point in time. So investment analysis requires two values for each income and each cost: the dollar amount and when it occurs. When both values are known, equivalence between amounts of money can be established by using an interest rate and the proper equation or formula.

Example 1

You borrow \$100 today and promise to repay the principal (\$100) in one year, plus 10 percent interest. The future value of the \$100 to the lender is:

Future Value = \$100 + (\$100 x 0.10) = \$110.

Thus, \$100 today and \$110 in one year are equivalent, at a 10 percent interest rate.

III. PRESENT AND FUTURE VALUES

Interest is the device that equates sums of money over time. It is used to equate a sum of money today with a future sum of money. For example, suppose you placed $100.00 in a savings account for 5 years at 8 percent interest. How much money will be in the account after 5 years? On a year-to-year basis, the solution is:

After Year 1: $100.00 + $100.00(0.08) = $108.00

After Year 2: $108.00 + $108.00(0.08) = $116.64

After Year 3: $116.64 + $116.64(0.08) = $125.97

After Year 4: $125.97 + $125.97(0.08) = $136.05

After Year 5: $136.05 + $136.05(0.08) = $146.93.

This is an example of equivalence. At 8 percent interest, $100.00 today is equivalent to $146.93 in five years. If you look closely at the calculations above, you'll probably notice a pattern to the steps used in solving the problem. It is possible to develop a formula that combines these steps. First, we'll need to define a few terms; let:

V_0 = the present value of a sum of money (or the value in year 0),

V_n = the future value of a sum of money (or the value after year n),

i = the interest rate expressed as a decimal (for example, 8% = .08), and

n = the number of interest bearing periods (usually years).

Notice in the above calculation that the value at the end of any year can be obtained by multiplying the beginning value by (1 + .08) or (1.08). That is, on a year-to-year basis:

After Year 1: $100.00(1.08) = $108.00

After Year 2: $108.00(1.08) = $116.64

After Year 3: $116.64(1.08) = $125.97

After Year 4: \$125.97(1.08) = \$136.05

After Year 5: \$136.05(1.08) = \$146.93

Note that the beginning value for any year is the ending value for the prior year. The year 5 value (\$146.93) could be derived by multiplying the initial \$100 by a series of (1.08)'s:

$$\$100.00(1.08)(1.08)(1.08)(1.08)(1.08) = \$146.93$$

To simplify the math:

$$\$100.00(1.08)^5 = \$146.93$$

This example illustrates the effect of compound interest. Similar calculations can be performed in a general manner using mathematical notation, rather than actual numbers. In terms of our earlier definitions:

V_0 = the present value of a sum of money = \$100.00

V_n = the future value of a sum of money = \$146.93

i = the interest rate expressed as a decimal = 0.08

n = number of years = 5

We want to develop a relationship between the present value of a sum of money (V_0) and the future value of the same sum of money (V_n). This relationship is defined by the expression $(1 + i)^n$. The future value of a sum of money is related to the present value by the <u>future value of a single sum formula</u>:

$$V_n = V_0(1 + i)^n \qquad (1)$$

Note that the interest rate in formula 1 must be expressed as a decimal (10% = 0.10). Figure 2 shows the relationship between the present value of a single sum and the future value of a single sum. This cash flow diagram shows a future value (V_n) occurring "n" periods after a present value (V_o).

Compound interest multipliers are listed in separate columns of Appendix A. Each table in Appendix A lists multipliers for a different interest rate (tables A1 to A18 correspond to interest rates of from 1 to 18 percent). Column 1 gives the value of $(1 + i)^n$ for selected values of "i" and "n". Note that the factor for $(1.08)^5$ is 1.46933, and using equation (1) for the previous problem:

$$V_5 = \$100(1.46933) = \$146.93$$

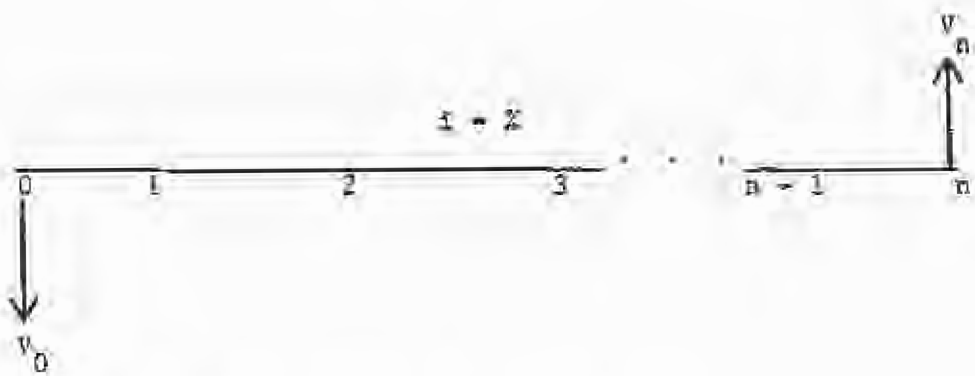

Figure 2. Cash flow diagram for single sums.

The diagram on the last page of the manual, titled COMPOUND INTEREST FORMULAS, shows formulas used for different types of cash flows. The diagram also refers you to the appropriate Appendix table to get the multiplier for each formula. The diagram is intended as a handy guide for solving problems, and is presented on the last page for quick reference.

The alternative to using the tabulated values, of course, is to calculate them directly with a hand-held calculator. Many calculators are programmed to determine present and future values, and other investment criteria automatically. Any calculator with a y^x key can be used, however, to determine multiplier values (for $(1.08)^5$, for example, enter 1.08 y^x 5 = and the calculator should display 1.4693281).

Example 2

What is the future value of $100.00 compounded for 10 years at 8 percent interest?

$$\begin{aligned} V_n &= V_0(1 + i)^n \\ &= \$100.00(1 + .08)^{10} \\ &= \$100.00(1.08)^{10} \\ &= \$100.00(2.15892) \text{ from column 1, Appendix Table A8} \\ &= \$215.89 \end{aligned}$$

Equation 1 can also be used to solve for the present value of a future sum of money. Solving Equation 1 for V_0 gives the present value of a single sum formula:

$$V_0 = \frac{V_n}{(1 + i)^n} = V_n\left[\frac{1}{(1 + i)^n}\right] \qquad (2)$$

Column 2 of the tables in Appendix A gives the multiplier (the bracketed term above) to discount the future value of a single sum of money to its present value. That is, it gives the value of $1/(1 + i)^n$ for selected values of "i" and "n". Note that the multiplier in Example 3 can be obtained from column 2 of Table A11. The value of $1/(1.11)^{12}$ is 0.28584. Also note that the formula table on the last page of the manual refers you to column 2 of Appendix A for the present value of a single sum.

Calculations involving formula 1 are called compounding; calculations involving formula 2 are called discounting. The interest rate used in formulas 1 and 2 is also called the discount rate, the cost of capital, or the alternative rate of return.

Example 3

An investment will return $10,000 in 12 years. You use an 11 percent interest rate to evaluate investments. What can you afford to pay for this investment today and earn 11 percent over the 12 year period (i.e., what is the present value of the investment)?

$$V_0 = V_n \left[\frac{1}{(1 + i)^n}\right]$$

$$= 10,000\left[\frac{1}{(1.11)^{12}}\right]$$

$= 10,000\ (0.28584)$ from column 2, Appendix Table A11

$= \$2,858.40$

Problems

1. If $800.00 is placed in a savings account earning 11 percent annually, how much will be in the account in 7 years?

2. If you hold a $100,000.00 bond due in 9 years, what is its present value at a 5 percent interest rate?

3. You are considering an investment in forest fertilization that will increase yield by 10 cords to the acre in 11 years. If a cord of pulpwood is expected to be worth $16.00 in 11 years, how much could you pay for fertilization today and earn 7 percent on the investment?

4. You are offered $6,500.00 today for your loblolly pine plantation that you expect to be worth $10,000 in 6 years. If your cost of capital is 7 percent, should you accept the offer?

5. You invest $2,500 in a money market account that pays 10 percent, compounded annually. How much will be in the account in 8 years?

6. What is the present value of $100,000 that is due in 10 years? Use an 8 percent interest rate.

7. You are considering an investment of $10.00 per acre in timber stand improvement. The stand will be harvested in 14 years. Using a 6 percent interest rate, how much additional harvest value must be generated to justify the investment?

8. You have cruised a private tract and determined that $380,000 worth of timber is on the tract. You will have to wait two years to harvest the tract due to the landowner's restrictions. If the value remains constant and your cost of capital is 10 percent, how much could you pay for the timber today?

IV. MONTHLY COMPOUNDING

Monthly interest is more familiar to many people than any other type of interest. Although often stated on an annual basis, compounding interest on a monthly basis is very common.

If an annual interest rate is given for monthly compounding, the monthly interest rate is the annual rate divided by 12. For example, 18 percent interest compounded monthly is 1½ percent per month. The term "n" in our equations represents the number of compounding periods, or months in this case. All of our formulas assume interest is compounded annually, but they are easily modified to non-annual compounding periods: simply use "i" divided by the number of compounding periods per year as the interest rate, and multiply the number of years by the number of compounding periods per year to get "n."

Example 4

You place $100 in a savings account that pays 12 percent interest, compounded monthly. How much will be in the account in 2 years?

The account will pay 1 percent per month (12 percent/12 months) for 24 months (n = 2*12). In terms of equation 1, the final value will be:

$$V_{24} = \$100(1.01)^{24} = \$126.97$$

The multiplier can be obtained from column 1 of Appendix Table A1.

The 12 percent interest rate is called a nominal interest rate or annual percentage rate (APR). This is the rate a bank or loan agency will quote. But, isn't the effective interest rate bound to be greater than 12 percent? Compounding takes place from month-to-month and interest is paid on accumulated interest as well as the unpaid balance. The effective rate for monthly payments is given by:

$$i_{effective} = (1 + i_{monthly})^{12} - 1 \quad (3)$$

Example 5

What is the effective annual interest rate in Example 4?

$$i_{effective} = (1.01)^{12} - 1 = 12.7\%$$

See column 1 of Appendix Table A1 for the multiplier. Note that n = 12 can be used to compute effective annual rates for any monthly interest charge.

In many cases with non-annual compounding periods, the interest rate will have a fractional component. For example, an APR of 8.8 percent is a monthly rate of 8.8/12 = 0.73 percent. Appendix tables are not included for such interest rates, but present and future values can still be calculated by using the proper formula and the y^x key on your calculator.

Problems

9. You borrow $1,000.00 at a 12 percent interest rate, compounded monthly. You will repay the principal and interest in $2\frac{1}{2}$ years. How much will be due?

10. If you have a credit card, it probably charges 18 or 21 percent interest on an annual basis. Of course you receive monthly statements and you pay monthly interest charges. What is the effective interest rate on credit card purchases?

V. SERIES OF CASH FLOWS

The formulas for the present value and future value of a single sum can be used to evaluate any series of cash flows. However, if the cash flow series is long, the calculations could be quite tedious. Formulas have therefore been developed to reduce the calculations necessary for most types of cash flow series.

Before presenting these formulas, a few definitions are needed. An <u>annual series</u> is a uniform series of costs or revenues which are due each year. A <u>periodic series</u> is due on a non-annual basis (e.g., every six months or every two years). A <u>terminating series</u> is a series of costs or revenues that ends after a specified period of time. A <u>perpetual series</u> is due indefinitely. Since series of costs or revenues may be annual or periodic, and terminating or perpetual, four combinations are needed:

Terminating Annual Series

Terminating Periodic Series

Perpetual Annual Series, and

Perpetual Periodic Series

Simple derivations for the formulas are presented in Appendix D. An important characteristic of all of the formulas is that the <u>first</u> cost or revenue in each series occurs at the <u>end of the first period</u>, and the <u>last</u> cost or revenue occurs at the <u>end of the last period</u>.

<u>Terminating Annual Series</u>

<u>Present Value</u>. Consider an investment that yields $1,000 per year for 8 years. Since the time period is finite and the payments are annual, the investment represents a terminating annual series. What is

the present value of the investment at 5 percent interest? Or, phrased another way, how much could you afford to pay for the investment and earn 5 percent? Figure 3 shows the cash flow diagram for the investment. Note from the diagram that the terminating annual series begins at the end of the first year and ends at the end of the eighth year.

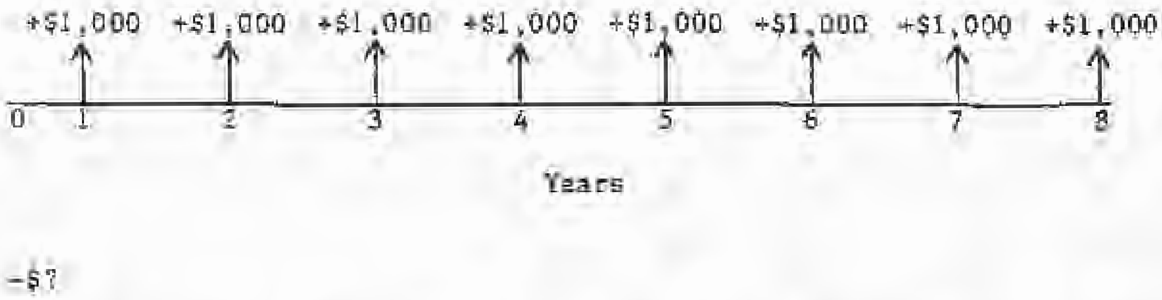

Figure 3. Cash flow diagram for a $1,000 8-year terminating annual series of revenues.

This problem can be solved by using formula 2 (present value of a single sum). Each of the 8 cash flows is discounted by 5 percent and the eight results are summed. Table 1 shows the calculations necessary to determine the present value of this terminating annual series.

The present value of a $1,000 8-year terminating annual series at a 5 percent interest rate is $6,463.21. The calculations were time-consuming and could be quite tedious for longer series. Fortunately, generalized formulas for calculations like these can be easily developed. Let us add a definition:

a = the dollar amount of a uniform, periodic or annual cost or revenue (annuity).

= $1000 in the example in Table 1.

Table 1. The present value of a \$1,000 8-year terminating annual series at a 5 percent interest rate.

Year of Payment	Discount Period (years) (n)	Present Value Single Sum Factor (5%) (Appendix Table A5)	Annual Series (a)	Present Value (V_0)
1	1	0.95238	\$1,000	\$ 952.38
2	2	0.90703	1,000	907.03
3	3	0.86384	1,000	863.84
4	4	0.82270	1,000	822.70
5	5	0.78353	1,000	783.53
6	6	0.74621	1,000	746.21
7	7	0.71068	1,000	710.68
8	8	0.67684	1,000	676.84
				\$6,463.21

A general formula exists to calculate the present value of a terminating annual series of costs or revenues:

$$V_0 = a\left[\frac{(1 + i)^n - 1}{i(1 + i)^n}\right] \tag{4}$$

This term is shown in the COMPOUND INTEREST FORMULA diagram (back cover) as PRESENT VALUE, TERMINATING ANNUAL SERIES, and multipliers or values for the term in brackets are found in column 4 of Appendix A (Tables A1 through A18).

Equation 4 can be used to find the present value of the example cash flow series in Table 1:

$$V_0 = \$1,000.00\left[\frac{(1.05)^8 - 1}{.05(1.05)^8}\right]$$

$$= \$1,000.00\left[\frac{1.47746 - 1}{.05(1.47746)}\right]$$

$$= \$1,000.00\ (6.46323) \text{ from column 4, Appendix Table A5}$$

$$= \$6,463.23 \text{ (small difference due to rounding).}$$

Example 6

A hunting club offers to lease a 900 acre forest tract from you for $6.00 per acre per year ($5,400 annually). The lease would terminate in 50 years. Using 7 percent as an alternative rate of return, what is the present value of the hunting lease?

$$V_0 = a\left[\frac{(1 + i)^n - 1}{i(1 + i)^n}\right]$$

$$= \$5,400\left[\frac{(1.07)^{50} - 1}{.07(1.07)^{50}}\right]$$

$$= \$5,400\ (13.8074) \text{ from column 4, Appendix Table A7}$$

$$= \$74,524.00$$

Future Value. The COMPOUND INTEREST FORMULA diagram also shows the formula for the FUTURE VALUE, TERMINATING ANNUAL SERIES:

$$V_n = a\left[\frac{(1 + i)^n - 1}{i}\right] \tag{5}$$

Values for the term in brackets are listed in column 3 of the tables in Appendix A.

Example 7

The future value, or value after year 50, of the hunting lease payments in example 6 would be:

$$V_{50} = \$5,400 \left[\frac{(1.07)^{50} - 1}{0.07} \right]$$

$= \$5,400\ (406.5300)$ from column 3, Appendix Table A7

$= \$2,195,262.00$

Problems

11. Timber rights on a 40 acre tract are purchased by a firm with a 6 percent cost of capital. The timber will be cut in 20 years. The firm agrees to pay the property tax of $3.50 per acre on the tract until the timber is cut. What is the present value of the tax payments?

12. Operating costs for a pickup truck are expected to be $750 per year. If you own the truck for 5 years, what is the present value of the costs at a 10% interest rate?

13. A hunting club leases a 1,750 acre tract for 20 years. The club will pay $3.00 per acre per year for the entire 20 years due today. The lessor will use a 5 percent interest rate to compound payments. What will the value of the lump sum payment be?

14. What if the lease revenue in problem 13 is not due until the end of the 20 years? What will be the future value of the annual lease payments, with interest?

<u>Sinking Fund Accounts</u>. Two types of terminating annual series deserve special treatment. Sinking fund accounts are simply a modification of the <u>future</u> value of a terminating annual series, and capital recovery problems are similar to the <u>present</u> value of a terminating annual series.

<u>Sinking fund accounts</u> are designed to accumulate a given sum of money within a certain number of years. We make yearly payments into an account that earns interest, so that at the end of "n" years we will have accumulated a given amount, V_n. For the formula, we simply solve the future value of a terminating annual series formula for "a", the annual payment:

$$a = V_n\left[\frac{i}{(1 + i)^n - 1}\right] \tag{5}$$

Values for the term in brackets are listed in column 5 of the tables in Appendix A.

<u>Example 8</u>

You want to pay cash for a new pickup truck 4 years from now. If you think you will need \$9,000 to purchase the truck, how much would you have to deposit each year into an account earning 5 percent interest? Before looking at the solution, will the amount be more or less than \$9,000/4 = \$2250 per year?

$$a = \$9{,}000\left[\frac{.05}{(1.05)^4 - 1)}\right]$$

$$= \$9{,}000\ (0.23201)\ \text{column 5, Appendix Table A5}$$

$$= \$2{,}088.09$$

Sinking fund accounts are most commonly used in forestry to calculate annual savings needed to replace logging or other equipment. In a sense, by saving for a future expense, you are making payments to yourself and accumulating interest rather than paying interest to someone else. In the next section, we discuss capital recovery through installment payments, and with the pickup truck example we'll see the difference it makes when interest is allowed to accumulate in your own account.

Problems

15. It will cost $25,000.00 to replace a logging truck in 4 years. If a 9 percent sinking fund is established to pay for the truck in 4 years, what will be the annual payments into the fund?

16. A $220,000.00 tractor must be replaced in 4 years. If the firm's cost of capital is 12 percent, how much is the payment into an annual sinking fund?

Capital Recovery. Often, it is desirable to compute the annual payment that is equal to a certain present value at a given interest rate. A good example is installment payments (i.e., paying off a loan, with interest charged on the unpaid balance). The annual payment would be the amount necessary to exactly recover (repay) an initial capital investment within a specified time period (hence the name capital recovery). The annual series of payments needed to repay a capital investment within a specific time period is:

$$a = V_0\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right] \qquad (6)$$

The capital recovery multiplier is listed in column 6 of Appendix A, for different values of "i" and "n". As you may notice, the capital recovery formula is simply the present value of a terminating annual series formula written to solve for "a" rather than V_0.

Example 9

Suppose you borrow $9,000 to buy the pickup truck in example 8. For comparison, assume you could borrow at 5 percent interest, and you will make 4 annual payments, beginning in one year.

$$a = \$9,000\left[\frac{.05(1.05)^4}{(1.05)^4 - 1}\right]$$

$= \$9,000\ (0.28201)$ column 6, Appendix Table A6

$= \$2538.09$

In example 8, where you accumulated the $9,000 before you spent it, only $2088.09 was needed each year. The difference would be greater, of course, for higher interest rates.

Since many people make monthly installment payments on borrowed funds, examples 10 and 11 are presented to illustrate the steps involved (see the section on Monthly Compounding for more discussion).

Example 10

You want to borrow $9,000 for the truck in the previous example, and the dealer quotes you an annual percentage rate (APR) of 8.8%. What would your monthly payments be for 48 months?

Two modifications are needed: use the number of months for "n", and use the monthly interest rate ($APR/12 = i_{monthly}$) for "i" in the capital recovery formula.

Substituting into equation 6:

$$a = (\text{Amount Borrowed})\left[\frac{\frac{APR}{12}\left(1+\frac{APR}{12}\right)^{Years * 12}}{\left(1+\frac{APR}{12}\right)^{Years * 12} - 1}\right]$$

$$= \$9,000\left[\frac{\frac{.088}{12}\left(1+\frac{.088}{12}\right)^{48}}{\left(1+\frac{.088}{12}\right)^{48} - 1}\right]$$

$$= \$233.11$$

Tables are not presented for all possible values of $i = APR/12$, so the above example is a good opportunity to check your hand-held calculator results.

Example 11

Perhaps the second most common type of capital recovery is home mortgage loans. Let's calculate the monthly payment on a 30-year, $60,000 mortgage with a fixed-rate loan at 10½ percent:

$$a = (\$60,000)\left[\frac{\frac{.105}{12}\left(1+\frac{.105}{12}\right)^{360}}{\left(1+\frac{.105}{12}\right)^{360}-1}\right]$$

$$= \$548.84$$

Home mortgage payments are very important to foresters and forest landowners. Residential construction is the greatest single outlet for wood products. Can you relate high or low interest rates to forestry employment or stumpage prices? Prices and employment are affected by the <u>demand</u> for forest products, and the demand results from end-product demands such as new home construction. Compare the monthly payments below and you can easily see how interest rates can influence stumpage prices and forestry employment:

Annual Percentage Rate	Monthly Payment on a Fixed-Rate, 30-Year Mortgage of $60,000
4%	$ 286.45
7	399.18
10	526.54
13	663.72
16	806.85
19	953.34
22	1,101.59

Figure 4 illustrates how monthly payments vary for different interest rates and principal amounts.

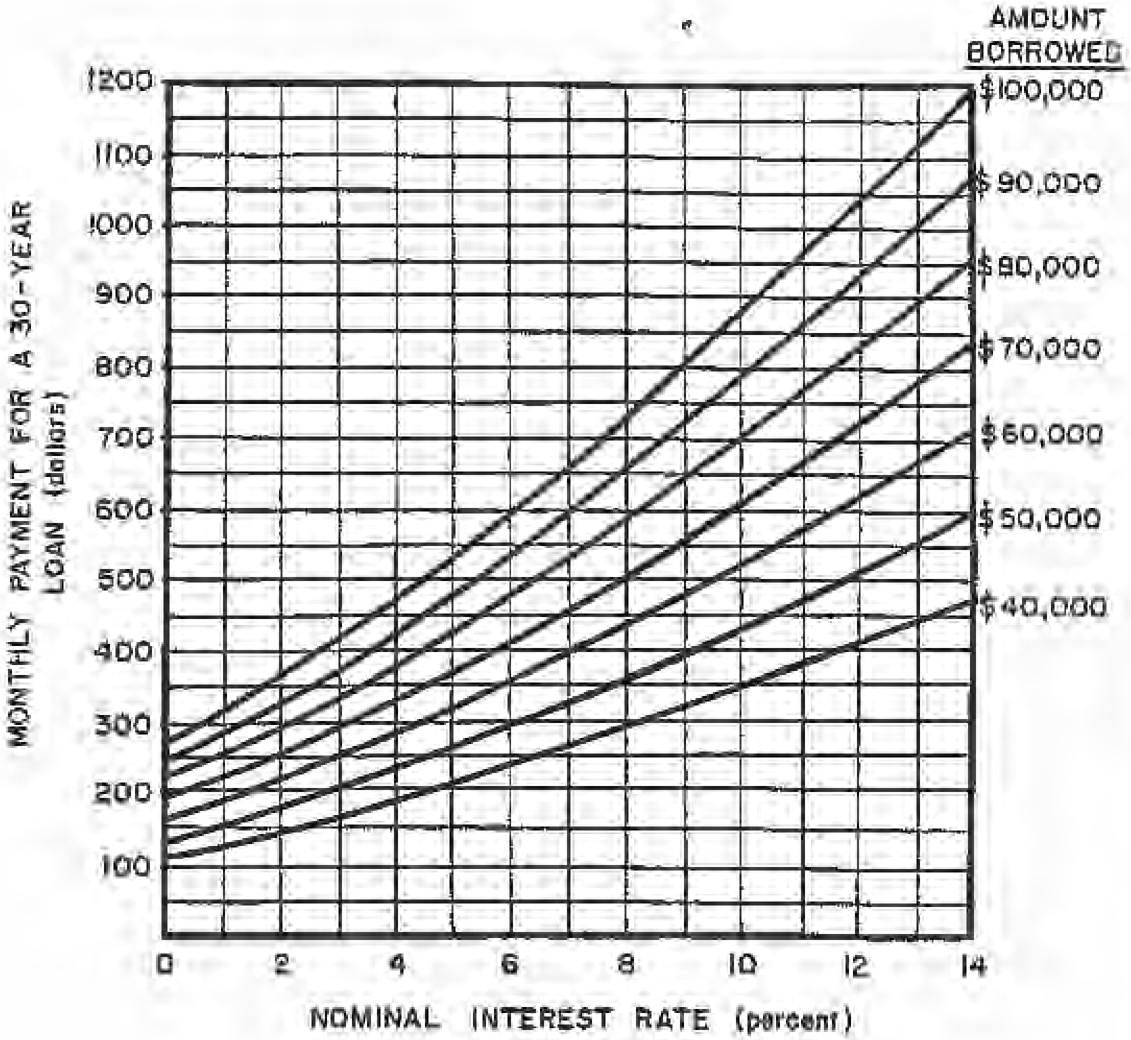

Figure 4. Monthly payments for a given nominal interest rate and various principal amounts.

Problems

17. A firm purchases a site prep tractor for $120,000 at a 9 percent interest rate. The firm will make six uniform annual payments, beginning in a year. What will be the amount of the payment?

18. You borrow $88,000.00 to purchase a tract of forest land. If payments are spread over 20 years and your interest rate is 11 percent, what is your annual payment to retire the loan?

Terminating Periodic Series

Present Value. A series of costs or revenues is periodic if the values occur on a non-annual but uniform basis. The diagram below defines "t" as the period between the "a" costs or revenues.

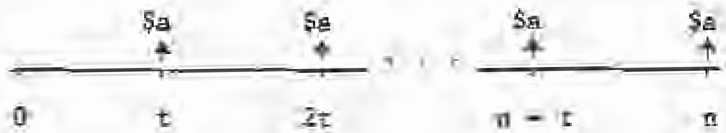

Figure 5. Terminating periodic series of costs or revenues.

The formula for the present value of a terminating periodic series is:

$$V_0 = a\left[\frac{(1 + i)^n - 1}{((1 + i)^t - 1)(1 + i)^n}\right] \tag{7}$$

The formula is referred to in the COMPOUND INTEREST FORMULA diagram, and values for the term in brackets are listed for i = 4, 8 and 12% in columns 2a, 2b, and 2c of Appendix B.

Example 12

What is the present value of prescribed burning costs of \$5.50/ac, if they occur every 10 years through year 50? i = 12%.

$$V_0 = \$5.50\left[\frac{1.12^{50} - 1}{((1.12)^{10} - 1)(1.12)^{50}}\right]$$

$= \$5.50\ (0.4732)$ from column 2c, Appendix B

$= \$2.60$

Future Value. Periodic series of costs and revenues can also be compounded. The formula for the future value of a terminating periodic series is:

$$V_n = a\left[\frac{(1 + i)^n - 1}{(1 + i)^t - 1}\right] \quad (8)$$

The formula is listed in the COMPOUND INTEREST FORMULA diagram, and values for the term in brackets are listed in columns 1a, 1b, and 1c of Appendix B (1a, 1b, and 1c correspond to interest rates of 4, 8, and 12 percent, respectively).

Example 13

Your forest yields approximately $6000 every 5 years. If you put the $6000 into an account earning 8 percent annual interest, how much money would be in the account 25 years from now?

In this case, t = 5 years per period, and there are 5 periods in the 25-year time span. The future value is:

$$V_{25} = \$6000\left[\frac{1.08^{25} - 1}{1.08^{5} - 1}\right]$$

$$= \$6000\ (12.4614) \text{ from column 1b, Appendix B.}$$

$$= \$74,768.40$$

Perpetual Annual Series (Present Value). A perpetual annual series is a series of costs or revenues ("a") occurring one year apart for an infinite number of years. A common forestry example of such an asset is an endless flow of raw material from a regulated forest. The formula for the present value of a perpetual annual series is given by:

$$V_0 = \frac{a}{i} \tag{9}$$

Formula (9) is shown in the COMPOUND INTEREST FORMULA diagram as the PRESENT VALUE, PERPETUAL ANNUAL SERIES. Values are not tabulated, however, since the formula is simply "a" divided by "i".

Example 14

You manage a 1,000 acre tract of bottomland hardwood on a 5-year cutting cycle. The forest is regulated and you expect to harvest 7 cords per acre from 200 acres each year. Hardwood stumpage is worth $4 per cord. What is the present value of this forest investment if you intend to hold it in perpetuity? Your cost of capital is 8 percent.

$$V_0 = \frac{a}{i}$$

$$V_0 = \frac{7 \text{ cds./ac.} \times \$4/\text{cd.} \times 200 \text{ ac.}}{.08}$$

$$V_0 = \frac{\$5600}{.08} = \$70,000$$

Problems

19. If a fund is established to pay a $2.00 per acre property tax on a forest tract in perpetuity, how much money must be deposited in an 8 percent account to cover the payment?

20. A Douglas-fir forest is fully regulated and produces $100,000 of timber revenue annually. What would this forest be worth today at 4 percent interest?

Perpetual Periodic Series (Present Value). A perpetual periodic series is a common type of cash flow series in forest regulation. The formula for the present value of a perpetual periodic series of costs or revenues is:

$$V_0 = a\left[\frac{1}{(1+i)^n - 1}\right] \quad (10)$$

Values for the term in brackets are listed in Appendix C for different values of "i" and "n".

From the COMPOUND INTEREST FORMULA diagram, note that only present values are listed for the perpetual annual and periodic series. Future values are not appropriate for such series since we assume the costs or revenues do not end.

Example 15

A loblolly pine plantation is expected to yield $1,290 per acre every 28 years in perpetuity. What is the present value per acre of the plantation's cash flow series at a 6 percent discount rate?

$$V_0 = \$1,290\left[\frac{1}{(1.06)^{28} - 1}\right]$$

$= \$1,290\ (0.24321)$ from Appendix C

$= \$313.70$

Example 16

What is the present value of a $1,000 payment every five years in perpetuity at a 12 percent interest rate? The multiplier from Appendix C for a 5-year period at a 12 percent interest rate is 1.31175, and

$$\$1,000\ (1.31175) = \$1,311.75$$

Does this make sense? An infinite series of payments worth only a few hundred dollars more than the original payment? Let's discount the first 8 payments using column 2 of Appendix Table A12:

Payment	Year	Factor	Present Value
1	5	.56743	$ 567.43
2	10	.32197	321.97
3	15	.18270	182.70
4	20	.10367	103.67
5	25	.05882	58.82
6	30	.03338	33.38
7	35	.01894	18.94
8	40	.01075	10.75
Total			$1,297.66

Almost 99 percent of the present value is accounted for in the first 40 years. This shows the power of compound interest. The payment in year 45, and all remaining payments, are worth a total of only $44.09 in present value terms.

Problems

21. A forestry investment is expected to yield $118,900.00 at the end of every 28 year rotation. The tract is bare and needs to be planted. The discount rate is 4 percent. What is the value of the investment in perpetuity?

22. What is the present value of bare land which could produce $200,000.00 of net revenue at 35 year intervals? The interest rate is 8 percent.

VI. DECISION CRITERIA

Decision criteria are used to evaluate forestry investment alternatives. Different criteria may be appropriate for different investment situations. Often the choice of a particular criterion is just a matter of personal preference. We discuss six major decision criteria used in forestry investment analysis

(i) payback period,

(ii) net present value,

(iii) equivalent annual income,

(iv) benefit/cost ratio,

(v) internal rate of return, and

(vi) land expectation value.

Payback Period

Payback period is a common measure of the attractiveness of forestry investments. It is the number of years required to recover the initial cash investment in a project. If the annual returns from an investment are equal, the formula for the payback period is:

$$\text{Payback Period} = \frac{\text{Initial Investment}}{\text{Annual Return}} \qquad (11)$$

The shorter the payback period, the better the investment.

Example 17

An attachment to a planting machine costs $600. Two companies produce models, both at the same cost. Company A's model will reduce planting costs by $200 per year. Company B's model will reduce planting costs by $300 in year 1, $200 in year 2, and $100 in its remaining years. Which model is preferable using the payback period decision criterion?

Company	Year 0	1	2	3	Payback Period (Years)
A	-$600	$200	$200	$200	3
B	-$600	$300	$200	$100	3

Since the payback period for each model is the same, the investor should be indifferent between models based on this decision criterion.

The payback period criterion has several shortcomings. Most importantly, it does not consider the time value of money. That is, it does not consider the interest cost of the invested capital. Also, it does not consider cash flows after the payback period. What if Company A's model reduced costs by $200 for years 4 to 7 and Company B's model reduced costs by $100 for years 4 to 7? This would not have been considered by the payback period.

The payback period criterion does have several advantages. First, it is simple and easy to use. Second, usually cash flows must be estimated for only the first few years of an investment. Many managers feel uncomfortable estimating cash flows over long periods of time. Third, when investment capital is tight, a company might be interested in an investment's payback period. Also, investments with short payback periods are usually considered less risky. If two potential investments are similar in terms of other economic criteria, for example, the one with the shorter payback period may be the best choice. Less uncertainty is usually involved with shorter investment periods.

Problems

23. A centrally located service center for your area has been proposed. Total cost would be $175,000. The center should reduce service costs by $25,000 annually. What is the payback period for this investment?

24. Three regional seedling storage facilities have been proposed. Total cost is expected to be $120,000. The facilities would reduce regeneration costs by $40,000 annually for the first two years, then by $20,000 annually. What is the payback period for this investment?

Net Present Value

The net present value (NPV) criterion is a very popular decision criterion. It is also commonly called present net value (PNV), present net worth (PNW), and net present worth (NPW). It is simple to use and it does consider the time value of money. Net present value is the discounted value of all revenues minus the discounted value of all costs associated with an investment. In mathematical terms:

$$NPV = \sum\left(\frac{R_n}{(1 + i)^n}\right) - \sum\left(\frac{C_n}{(1 + i)^n}\right) \quad (12)$$

where:

NPV = net present value,

$\sum$ = sum of all values in parentheses

R_n = revenue in year n,

C_n = costs in year n,

n = year in which cash flow occurs, and

i = interest rate.

If the interest rate used in the calculation is your cost of capital, any investment with a positive NPV will yield a rate of return greater than your cost of capital. The decision rule used with this criterion is to accept investments with positive NPV's.

Example 18

A landowner asks you to determine the net present value of regenerating 40 acres. Site preparation and regeneration will cost $160 per acre. Property taxes and management costs will be $2.50 per acre per year. Thinnings will occur in years 16 and 22 and will yield 5 cords and 8 cords per acre, respectively. Harvest will occur at year 27 and will yield 66 cords per acre. Pulpwood is worth $19.50 per cord. The landowner's alternative rate of return is 4 percent (see Appendix A for i = 4%). What is the investment's NPV?

Revenues

Year	Item	Amount	Multiplier	Present Value
16	Thin	$ 97.50	.53391	$ 52.06
22	Thin	156.00	.42196	65.83
27	Harvest	1,287.00	.34682	446.36

Present Value of Revenues Per Acre = $564.25

Costs

Year	Item	Amount	Multiplier	Present Value
0	Site Prep	$160.00	1.00000	$160.00
1-27	Annual Costs	2.50	16.32958	40.82

Present Value of Costs Per Acre = $200.82

Per Acre NPV = $564.25 - $200.82 = $363.43

In Example 18 the investment earned a 4 percent rate of return, plus $363.43. If the NPV had been 0, the rate of return on the investment would have been exactly 4 percent. If the NPV was less than zero, the rate of return would have been less than 4 percent.

Example 19

A firm is considering an investment in fertilization that will cost $50 per acre now and $50 per acre in 10 years. The fertilization is expected to result in an additional dollar yield in 20 years of $251. What is the NPV of this investment for various interest rates?

Interest Rate	Present Value of Costs	Present Value of Revenue	NPV
0	$100.00	$251.00	$151.00
2	91.02	168.92	77.90
4	83.78	114.55	30.77
6	77.92	78.26	0.34
8	73.16	53.85	-19.31
10	69.28	37.31	-31.97
12	66.10	26.02	-40.08

Example 19 illustrates the relationship between interest rates and NPV. The higher the interest rate, the lower NPV (see Figure 6). As the interest rate is raised, the rent for the use of money over time is higher, lowering the NPV. When NPV equals zero, the investment is earning just the interest rate. That is, the rate of return on the fertilization investment is 6 percent.

Notice in Figure 6 that the present value of revenue decreases more quickly (as "i" increases) than the present value of costs. Why? Because in Example 19, as in most forestry investments, revenues occur farther in the future than costs, and are therefore discounted more heavily to yield present values.

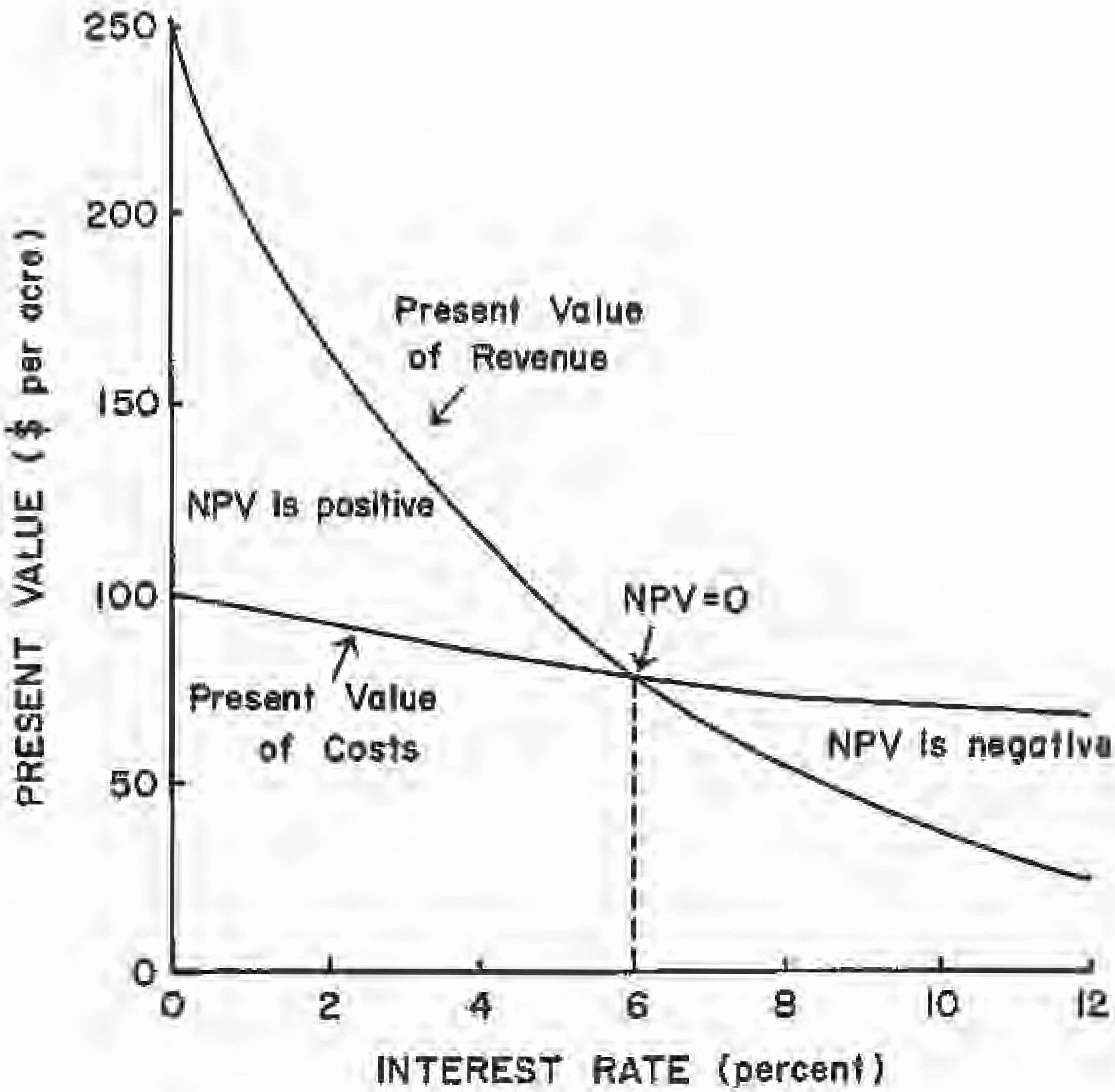

Figure 6. Present value of revenue and costs for the problem in Example 19.

Problems

25. An investment of $25,000 today will produce revenues of $9,000 for each of the next three years. Using the NPV decision criterion and a 7 percent interest rate, should you accept the investment?

26. Precommercial thinning a pine plantation at age 8 is expected to produce additional revenue of $36 per acre and $150 per acre at years 17 and 24, respectively. How much can you afford to spend on the precommercial thinning using the NPV decision criterion and a 5 percent interest rate?

Equivalent Annual Income (EAI)

Equivalent annual income (EAI) is the annual cash flow that is equivalent to another specified cash flow at a particular interest rate. This criterion is also referred to as equal annual income and equal annual equivalent. It is especially useful in comparing forestry investments to agricultural investments since agriculture yields annual income, while forests often yield periodic income. By converting the periodic income into an equivalent annual cash flow, one can easily compare an agricultural alternative for land, like soybeans or annual pasture rental, to a forestry investment.

The procedure for calculating EAI is simple. First, calculate the NPV for one cycle (i.e., rotation) of the forestry investment. Second, convert NPV to EAI using the capital recovery multiplier:

$$EAI = NPV\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right] \qquad (13)$$

Recall that capital recovery multipliers (values for the term in brackets) are listed in column 6 of the tables in Appendix A, for different values of "i" and "n".

Example 20

What is the EAI of the investment in Example 18?

$$EAI = \$363.43\left[\frac{.04(1.04)^{27}}{(1.04)^{27} - 1}\right]$$

$= \$363.43\ (0.06124)$ from column 6, Appendix Table A4

$= \$22.26$

The investment yields a net income equivalent to an annual income of \$22.26/ac./yr. over the 27 year rotation.

Problem

27. You have a tract of forest land that you are considering converting to soybeans. Soybeans yield $80/ac./yr. The timber stand yields $350/ac. every 5 years. At 6% interest, compare the investments using the EAI criterion.

Benefit/Cost Ratio

Benefit/cost (B/C) ratios are closely related to NPV. For NPV, the sum of all discounted costs is subtracted from the sum of all discounted revenues. For B/C ratios, total discounted revenues (benefits) are simply divided by total discounted costs.

$$B/C = \sum \frac{R_n}{(1+i)^n} \Big/ \sum \frac{C_n}{(1+i)^n} \tag{14}$$

B/C ratios are often used to evaluate public projects, with regulations and guidelines on how benefits and costs are measured and what discount rates should be used. The guideline for evaluating investment projects with this criterion is:

If $B/C \geqslant 1$, Accept (benefits exceed costs).

If $B/C < 1$, Reject (benefits less than costs).

Note that if $B/C \geqslant 1$, then $NPV \geqslant 0$, and if $B/C < 1$, $NPV < 0$ (see Figure 6). The decision to accept or reject an investment will be the same whether you use B/C or NPV as a criterion. When accepted projects are ranked by NPV and B/C, however, the order of ranking may be different. Note that the B/C ratio of Example 18 is:

B/C = \$564.25 / \$200.82 = 2.81.

Internal Rate of Return

The internal rate of return (IRR) is the average rate of capital appreciation for an investment, or more simply, the interest rate that makes the net present value of an investment equal to zero. If an investment's NPV equals zero, the investment is earning a return exactly equal to the interest rate. This requires that the sum of the investment's discounted revenues equal the sum of the discounted costs. The IRR is the interest rate that causes the following relationship to be true:

$$\sum \frac{R_n}{(1+i)^n} = \sum \frac{C_n}{(1+i)^n} \qquad (15)$$

where:

R_n = revenue in year n,

C_n = costs in year n,

n = year in which cash flow occurs, and

i = interest rate = IRR when relationship is true.

Note that both terms appear in the formula for NPV, and that when the relationship is true, NPV must equal zero. The IRR for the problem in Example 19 is easily identified in Figure 6 as 5 percent.

Example 21

Calculate the IRR of the investment in Example 18, to the nearest percent. First, we note the NPV of the investment is $363.43 at a 4 percent interest rate. Therefore, the IRR is greater than 4 percent. But, how much greater than 4 percent? This answer requires us to repeat the process of calculating NPV using different interest rates. As you will see, common sense will help. First, let's calculate the NPV at 8 percent:

Year	Amount	Factor	Present Value
0	-$160.00	1.00000	-$160.00
16	97.50	0.29189	28.46
22	156.00	0.18394	28.69
27	1,287.00	0.12519	161.12
1-27	-2.50	10.93516	- 27.34
			NPV = $ 30.93

Since the NPV is positive at 8 percent, the IRR is greater than 8 percent. Now let's try a 10 percent interest rate:

Year	Amount	Factor	Present Value
0	-$160.00	1.00000	-$160.00
16	97.50	0.21763	21.22
22	156.00	0.12285	19.16
27	1,287.00	0.07628	98.17
1-27	-2.50	9.23723	- 23.09
			NPV = -$ 44.54

Since the NPV is negative at 10 percent, the IRR is less than 10 percent. Since the NPV is negative at 10 percent and positive at 8 percent we know that the IRR is between 8 and 10 percent. The NPV at 9 percent is:

Year	Amount	Factor	Present Value
0	-$160.00	1.00000	-$160.00
16	97.50	0.25187	24.56
22	156.00	0.15018	23.43
27	1,287.00	0.09761	125.62
1-27	-2.50	10.02659	- 25.07
			NPV = -$ 11.46

Since NPV < 0, we now know that the IRR is less than 9 percent. If you do the calculations, the actual IRR is about 8.7 percent. The NPV at 8.7 percent is 0.17, or for practical purposes, zero.

This example illustrates the typical repetitive process and the reasoning involved in the IRR of an investment. Few people actually go through these tedious calculations. For detailed problems a computer package will be used. Most financial calculators can also handle basic IRR problems.

The decision rule used with the IRR decision criterion is based on comparing the IRR with a minimum acceptable rate of return, usually the cost of capital for a firm. Private landowners may compare IRR's for forestry investments with their cost of capital if they borrow money, or with their highest possible alternative rate of return. The decision guideline is:

If IRR > minimum acceptable rate, Accept

If IRR < minimum acceptable rate, Reject.

The NPV and IRR decision criteria are the two most widely used and accepted investment criteria. A major advantage of the IRR criterion is that the answer provided is an interest rate. Many investors, especially nonindustrial private forest landowners, are most comfortable with rate of return information. Both criteria yield the same answer when used to answer the question "Is this investment profitable?". That is, when NPV is greater than zero, IRR is greater than the discount rate and vice versa. As with B/C ratios, however, project rankings with NPV and IRR do not always agree.

Example 22

A firm is considering an investment in fertilization that will cost $50 per acre. The fertilization is expected to result in an additional dollar yield in 20 years of $160.36. What is the IRR of the investment?

Simple problems with one cost and one revenue can be solved directly using equation 1. Solving equation 1 for "i":

$$V_n = V_0(1 + i)^n \quad (1)$$

$$(1 + i)^n = V_n/V_0$$

$$(1 + i) = (V_n/V_o)^{1/n}$$

$$i = (V_n/V_o)^{1/n} - 1 \quad (16)$$

Equation 16 can be solved using your hand-held calculator, or by using the tables in Appendix A. In our example,

$$V_n/V_0 = 160.36/50.00 = 3.2072.$$

This is the value by which V_0 must be multiplied (at "i", the interest rate) to obtain V_n, i.e., \$50.00 x 3.2072 = \$160.36. Or, this is the future value, single sum multiplier from Appendix A. Since we know the factor (3.2072), and we know that n = 20, we can go to Appendix A and scan the n = 20 row of column 1 for each table until we locate 3.2072. We find 3.20716 for i = 6 percent. Therefore, IRR is approximately 6 percent.

Problems

28. What is the IRR of an investment with the following cash flow pattern?

Year	Amount
0	-$801.23
1	200.00
3	800.00

29. An investment in timber stand improvement (TSI) that costs $30 per acre at year 20 will yield 6 additional cords in 10 years worth $10.80 per cord. What is the IRR of the TSI investment?

Land Expectation Value

The land expectation value (L_e) decision criterion is also widely used in forestry. It is also called the Faustmann Formula and the bare land value or soil expectation value formula (since the value of bare land in perpetual forest production is calculated). The standard formula for a perpetual periodic series (Appendix C) is used for the calculation. This is actually a standard NPV calculation, but with several critical assumptions:

1. The values of all costs and revenues are identical for all rotations. All costs and revenues are compounded to the end of the rotation to get the future value of one rotation. This value will be the amount received every "n" years.
2. The land will be forested in perpetuity.
3. The land requires regeneration costs at the beginning of the rotation.
4. Land value does not enter into the calculation. Land value is what you are calculating.

The calculation is quite easy and involves two steps. First, each cost and revenue is compounded to the end of the first rotation. The net value at rotation represents the dollar amount available at the end of each rotation in perpetuity. Second, the present value of the dollar amount is calculated on a perpetual periodic basis, and multipliers for the calculation are therefore listed in Appendix C.

Example 23

You need to determine the bare land value of a forest tract which presently has no merchantable timber. Following reforestation, the tract will be managed on a 30-year rotation and your cost of capital is 6 percent. Site preparation and regeneration will occur in year 0 at a cost of $80.00 per acre. Annual management costs and property taxes will be $1.50 per acre. Thinnings will occur at ages 18 and 25 and will yield 6 and 10 cords per acre, respectively. Final harvest will yield 57 cords per acre. Pulpwood is worth $16 per cord. If you intend to follow the above management sequence and you want to earn at least 6 percent on your investment, how much can you afford to pay for the bare land?

Revenues

Year	Item	Amount	Factor	Future Value
18	Thin	$ 96.00	2.0122	$ 193.17
25	Thin	160.00	1.3382	214.11
30	Harvest	912.00	1.00000	912.00

Future Value (V_{30}) of Revenue = $1,319.28

Costs

Year	Item	Amount	Factor	Future Value
0	Site Prep	$80.00	5.7435	$459.48
1-30	Ann. Costs	1.50	79.0580	118.59

Future Value (V_{30}) of Costs = $578.07

Net Future Value = $1,319.28 - $578.07 = 741.21

The L_e assumptions treat the $741.21 as a perpetual periodic series (paid every 30 years).

$$L_e = 741.21\left[\frac{1}{(1.06)^{30} - 1}\right]$$

Using Appendix C,

L_e = $741.21 (.21082)

= $156.26 per acre

L_e represents the maximum amount that could be paid for a tract of land and still earn the required interest rate. You could pay $156.26 per acre for the tract and earn 5 percent on your investment, assuming you use the land to grow timber according to the management schedule outlined.

Problem

30. You are considering the purchase of a non-forested tract of land. For a 28-year rotation you expect the following costs and revenues:

Year	Item	Amount (per acre)
0	Site Prep	-$50.00
1	Plant	-20.00
22	Burn	-3.50
1-28	Annual Hunting Lease	1.50
1-28	Annual Cost	-1.50
10	Thin	150.00
21	Thin	210.00
28	Harvest	1,120.00

How much can you afford to pay for the tract, per acre, if you would like to earn at least 4 percent on your invested capital?

Application of Criteria

Rotation Determination. Optimal rotation age is commonly determined using one of the decision criteria we've discussed (usually NPV, IRR, or L_e). The best rotation age is largely determined by the timber growth and yield since future revenues depend on expected yields. Consider the following simple yield relationship:

Age	Yield (Cords)
10	13.4
15	38.4
20	54.0
25	67.9
30	76.8

You may be familiar with the term mean annual increment. Mean annual increment (MAI) is simply the average volume of wood grown each year (average annual growth). Or, in formula form:

$$MAI = \frac{Y}{r} \quad (17)$$

where: MAI = mean annual increment,

Y = yield at rotation age, and

r = rotation age.

The rotation age that maximizes MAI will maximize wood yield from a stand over time. It is often used by public agencies in rotation determination. For our simple example, MAI is:

Age	Yield (Cords)	MAI (Cords)
10	13.4	1.34
15	38.4	2.56
20	54.0	2.70
25	67.9	2.72*
30	76.8	2.56

In the above example, maximum MAI occurs at rotation age 25. If one's goal is simply to maximize average annual timber growth, this is a satisfactory criterion. As you can see, however, MAI does not consider the time value of money or production costs. When financial considerations, as well as growth and yield relationships, are taken into account, a rotation length <u>shorter</u> than maximum MAI is usually calculated. This results because financial criteria reflect initial costs and the financial advantage of receiving early harvest revenues.

For example, if pulpwood can be sold for $16 per cord, site preparation/regeneration costs are $80 per acre, and annual management costs are $1 per acre per year, at a 6 percent interest rate the various rotations have the following net present values:

Age	Yield (Cords)	Money Yield	Discounted Money Yield	Site Prep/ Regeneration Costs	Discounted Annual Costs	Net Present Value
10	13.4	$ 214.40	$119.72	$80.00	$ 7.36	$ 32.40
15	38.4	614.40	256.37	80.00	9.71	166.66
20	54.0	864.00	269.40	80.00	11.47	177.93*
25	67.9	1,086.40	253.13	80.00	12.78	160.35
30	76.8	1,228.80	213.95	80.00	13.76	120.19

Internal rates of return for the various rotation lengths are:

Age	IRR (%)
10	9.5
15	14.0*
20	11.9
25	10.3
30	9.1

Land expectation values for the various rotation lengths are:

Age	Yield (Cords)	Money Yield	Compounded Establishment Costs	Compounded Annual Costs	Value at Rotation End (a)	L_e
10	13.4	$ 214.40	$143.27	$13.18	$ 57.95	$ 73.27
15	38.4	614.40	191.72	23.28	399.40	285.99*
20	54.0	864.00	256.57	36.79	570.64	258.55
25	67.9	1,086.40	343.35	54.86	688.19	209.06
30	76.8	1,228.80	459.48	79.06	690.26	145.52

To summarize the results:

Age	Yield (Cords)	MAI (Cords)	NPV	IRR (%)	L_e
10	13.4	1.34	$ 32.40	9.5	$ 73.27
15	38.4	2.56	166.66	14.0*	285.99*
20	54.0	2.70	177.93*	11.9	258.55
25	67.9	2.72*	160.35	10.5	209.06
30	76.8	2.56	120.19	9.1	145.52

The rotation which maximizes MAI is longer than that which maximizes economic criteria. The NPV, IRR, and L_e criteria all consider the time value of money and produce shorter rotations than MAI. In the above example, L_e is maximized with a 15 year rotation, while the best rotation according to NPV is 20 years. L_e is the most valid criterion since <u>all</u> future revenues and costs are considered. NPV considers only one rotation, and therefore does not consider the opportunity to grow subsequent stands. Such stands can only be grown after the first stand is harvested.

Each economic criterion reflects different management objectives. NPV's objective is to maximize the net present value of the future cash flows from one rotation. IRR's objective is to maximize the rate of return on investment, and L_e's objective is to maximize bare land value, the present value of all future net income.

Problem

31. Below is a yield table for planted loblolly pine on an average site in eastern Virginia. Calculate the best rotation length using the MAI, NPV, IRR, and L_e decision criteria. Assume establishment costs for a loblolly pine plantation in eastern Virginia are $100 per acre and annual management costs and property taxes are $2 per acre per year. Stumpage price is $0.20 per cubic foot. Cost of capital is 3%.

Rotation Age	Yield per acre (cubic foot)
15	1,217
20	2,135
25	2,968
30	3,715
35	4,379
40	4,958

Reforestation and Sensitivity Analysis. Before discussing taxes and inflation, let's analyze the variables affecting site preparation and regeneration decisions. The major variables can be seen in a simple net present value calculation. Consider only the front-end costs of reforestation and the harvest value of the forest yield. The net present value of one rotation for this simple example is given by:

$$NPV = \frac{HV}{(1 + i)^n} - RC \qquad (18)$$

where:

NPV = net present value,

HV = harvest value,

RC = site prep./regeneration costs,

i = interest rate, and

n = rotation length, in years.

This relation merely says that the net present value of a reforestation investment is the discounted harvest value minus the cost of site preparation and regeneration. Our simple example includes the four major variables that affect the economics of reforestation (i, n, HV, and RC).

The interest rate, "i", is one of the most important variables affecting reforestation decisions. When compounding or discounting over a rotation length, a small change in the interest rate can make a large difference in an investment's net present value. The choice of an appropriate interest rate is therefore a key decision affecting forestry investment analyses. If the interest variable changes, through a change in time preference (how soon you need cash), market rates, or land ownership, forestry investment decisions may change dramatically.

Likewise, the rotation length, n, or the length of the investment, will have a major impact on the compounding and discounting of investment dollars. The present value of revenues is inversely related to the interest rate, and will also decrease as "n" increases, unless increased stand age brings quality or product changes whose value differences more than offset the discounting effects of interest.

Site preparation and regeneration costs occur at the beginning of the rotation. In terms of the NPV of the forestry investment, site preparation and regeneration undergo little discounting. If they occur in year 0, of course, they are not discounted at all. Front-end costs can therefore be very critical in determining net present values for forestry investments.

The major timber yield under even-aged management occurs at the time of final harvest. The anticipated cash flow at harvest is the expected timber yield times the price per unit volume. Yield can be predicted with some degree of accuracy, but the price per unit volume involves critical assumptions. Will timber prices in 30 years be the same as today? Will they change only with inflation, or will increases or decreases occur after inflation is accounted for? Price projections may be uncertain, but they are also heavily discounted and have much less influence (per dollar) on NPV's or other economic criteria than do front-end costs or revenues.

Intermediate costs and revenues (e.g., prescribed burning costs and thinning revenues) have been omitted from the example. They also have a much smaller effect on economic decisions than front-end costs, and usually have less effect on present values than the large revenues at harvest (although their effect per dollar is greater). If they were added to the example, each cost and revenue would be discounted to year 0 and added to or subtracted from the total NPV. Also omitted were the annual costs and revenues. Each series of annual management costs, annual property taxes, hunting lease revenues, etc., should be discounted as a terminating annual series of costs or revenues.

Sensitivity Analysis

How sensitive is NPV to changes or possible errors in each of the four major variables (i, n, HV, and RC)? A simple example illustrates their potential influence.

Assume a 25 year rotation of slash pine. The regeneration cost is $100 per acre and the harvest yield is 40 cords per acre. Pulpwood is worth $15 per cord and the interest rate is 4 percent. The net present value of one rotation is:

$$NPV = \frac{\$600.00}{(1.04)^{25}} - \$100.00$$

$$= \$225.07 - \$100.00$$

$$= \$125.07$$

What if the "i" or "n" change by 10 percent? A 10 percent change in the interest rate appears to be trivial; 3.6 percent and 4.4 percent appear to be very close to 4 percent. However, a 10 percent decrease in "i" causes NPV to increase by 18.2 percent (to $147.83) and a 10 percent increase in "i" causes NPV to decrease by 16.5 percent (to $104.47). A 10 percent decrease in "n" causes NPV to increase by 18.5 percent (to $148.26) and a 10 percent increase in n causes NPV to decrease by 16.8 percent (to $104.05). We can see by this simple analysis that choosing an appropriate interest rate is critical. While the rotation length is very important, a small change in "i" can affect NPV as much as a large change in "n".

The effect of a change in reforestation costs on our example is easy to see. These costs occur at year 0 and are subtracted directly from NPV. If RC increases by 10 percent, NPV decreases by 8 percent (to $115.07); or if RC decreases by 10 percent, NPV increases by 8 percent (to $135.07).

Harvest values are subject to discounting, and a 10 percent increase in HV (due to an increase in yield and/or price) causes NPV to increase by 18 percent (to $147.57) and a 10 percent decrease in HV causes NPV to decrease by 18 percent (to $102.56). Does this mean that HV changes or errors affect NPV more than RC changes? A 10 percent change in HV changed NPV by 18 percent, yet a 10 percent change in RC changed NPV by only 8 percent. Reforestation costs are more important on a <u>per dollar</u> basis than estimated final harvest values: a one dollar error in RC creates a one dollar error in NPV, but a one dollar error in HV at age 25 causes only a 38¢ error in NPV.

The sensitivity analysis shows the choice of "i" has a critical impact on NPV calculations. We also have to make a critical assumption on intermediate and final harvest stumpage prices. Reforestation costs, rotation length, and harvest yields probably require the least guesswork. Foresters or landowners who analyze forestry investments should be aware of the importance of assumptions in their analyses. Simple sensitivity analysis often helps evaluate the possible effects of key assumptions on forestry decisions.

See new pages inserted behind p. 80

VII. INVESTMENT ANALYSIS - ACCOUNTING FOR TAXES

Up until now, our discussions on investment analysis have been on a before-tax basis; that is, they did not consider the impact of taxation on the investment. This section will develop a framework for after-tax investment analysis based on the federal income tax treatment of timber. Before getting into the investment analysis discussion, we first review the basics of federal tax treatment of timber. Our review is necessarily limited to an introductory level.

Income is assigned to one of two federal income tax categories: ordinary income and capital gains income. Ordinary income is the net profit that comes from the economic activity of a corporation or individual. Capital gains (or losses) result when a capital asset is sold for more (or less) than its book value. A capital asset is any asset that is not normally bought or sold in the business of an individual or firm. Internal Revenue Service (IRS) regulations define which assets may be considered capital assets, including a minimum length of time an asset must be held by an individual or firm for capital gains treatment.

Capital gains income is taxed at a lower rate than ordinary income. Individuals are allowed to exempt 60 percent of long-term capital gains from taxation; corporations are subject to a maximum capital gains income tax rate of 28 percent. Individuals are currently subject to a maximum federal income tax rate of 50 percent and corporations are subject to a maximum tax rate of 46 percent on ordinary income over $100,000. This is the advantage of capital gains; it reduces the taxes paid by an individual or firm.

CHAPTER VII: INVESTMENT ANALYSIS - ACCOUNTING FOR TAXES

Taxes are an important part of forestry investment decisions. Taxes must be considered to accurately reflect revenues, costs, and rates of return for forestry activities. Our purpose in this chapter is to present correct methods for after-tax analysis, rather than to describe details of current tax laws and provisions that relate to forestry. Correct methods do not change with changes in tax laws, and this chapter is therefore relevant regardless of specific tax provisions and changes from year to year. See Hoover et al. (1989) and Haney and Siegel (1988) for detailed descriptions of current federal income tax provisions that relate to forestry costs and revenues.

In an after-tax investment analysis, all revenues should be placed on an after-tax basis, all cost-related tax savings (deductions and credits) should be accounted for, and an after-tax discount rate should be used. This chapter therefore has separate sections for revenues, costs, and interest rates.

After-tax Revenues

To calculate after-tax revenues, simply subtract taxes due from revenues received. The tax rate which applies is the marginal tax rate--the rate paid on additional or marginal income. Our examples use 15 and 28 percent tax rates for private nonindustrial landowners, and 34 percent for corporations. Tax rates have varied through the years, and in some years special provisions have been made for income from timber sales and for other "capital gains." Our examples do not include special provisions, and in general we include federal income taxes only. Other taxes, such as self-employment taxes or state income taxes, may also be subtracted from income received.

After-tax income is simply the income remaining after taxes have been subtracted:

$$\begin{bmatrix}\text{After-tax}\\ \text{Income}\end{bmatrix} = \begin{bmatrix}\text{Before-tax}\\ \text{Income}\end{bmatrix} - (\text{tax rate})\begin{bmatrix}\text{Before-tax}\\ \text{Income}\end{bmatrix} \quad (1)$$

Equation (1) can be reduced to:

$$\begin{bmatrix}\text{After-tax}\\ \text{Income}\end{bmatrix} = \begin{bmatrix}\text{Before-tax}\\ \text{Income}\end{bmatrix}(1 - \text{tax rate}) \quad (2)$$

Example 1

After subtracting all costs of the sale, your timber sale income last year was $22,000. If you pay 28 percent of the income in taxes, the after-tax revenue from the timber sale is:

$22,000 (1 - .28) = $15,840

After-tax Costs

Costs that relate to forestry investments are generally deductible for income tax purposes. Some costs are deducted entirely in the year they occur (they are expensed) while other costs are deducted when income is realized from the investment, or they are deducted over a period of years (they are capitalized). We first describe the correct way to calculate after-tax costs for items that can be expensed, and then we consider after-tax costs where the expenditure must be capitalized. With changes in tax laws, changes occur in the types of costs that can be expensed versus those that must be capitalized. Our examples are general, however, and are intended to demonstrate how tax savings from deductions should be accounted for in after-tax investment analysis.

Expensed Costs. Costs that can be expensed, i.e., deducted entirely in the year they occur, save you money by reducing the amount of income tax due at the end of the current year. Taxes due are calculated by applying the appropriate tax rate to income after deducting allowable costs:

$$[\text{Taxes Due}] = (\text{tax rate})\ [\text{Income} - \text{Deductions}] \quad (3)$$

A deductible expense therefore reduces your income tax by (tax rate)*(deduction). To place the expense on an after-tax basis, simply subtract the tax savings from the original expense incurred:

$$\begin{bmatrix}\text{After-tax}\\ \text{Cost}\end{bmatrix} = \begin{bmatrix}\text{Before-tax}\\ \text{Cost}\end{bmatrix} - (\text{tax rate})\begin{bmatrix}\text{Before-tax}\\ \text{Cost}\end{bmatrix} \qquad (4)$$

Equation (4) can be reduced to:

$$\begin{bmatrix}\text{After-tax}\\ \text{Cost}\end{bmatrix} = \begin{bmatrix}\text{Before-tax}\\ \text{Cost}\end{bmatrix}(1 - \text{tax rate}) \qquad (5)$$

Example 2

Property taxes on your 100-acre tract of timberland are $300 per year. What is the cost on an after-tax basis if your marginal tax rate is 15 percent? Since property taxes can be expensed, the actual or effective cost is only:

$$\$300\ (1 - .15) = \$255$$

If you had not incurred the $300 property tax expense, your tax bill would have been $45 higher, and the property taxes therefore have an actual cost of $255.

Terms such as "actual" cost and "effective" cost are often used to denote after-tax costs. After-tax costs reflect the true cost of an item or service, since all potential tax savings are subtracted from the initial expense incurred. In the next sub-section, capitalized costs are considered, and phrases like "actual" or "effective" cost refer to the after-tax present value, i.e., where all tax savings have been discounted to the present and subtracted from the initial expense.

Capitalized costs. In general, legitimate costs that cannot be expensed are capitalized for tax purposes. Capitalized costs are added to a capital account--a specific record of costs to be deducted from income in future years. There are four basic types of forest-related expenses that must be capitalized. They represent four different types of capital assets:

a. Assets like land that generally do not depreciate--costs are deducted from income when the asset is sold.

b. Assets like buildings and equipment that generally depreciate with time--costs are deducted over a number of years. The number of years and the schedule of depreciation (percentage of costs deducted each year) are specified by the IRS for structures and for different types of equipment.

c. Assets like timber--costs of certain resource-based assets like timber, oil, and gas are deducted as the resource is used (depleted). A "depletion allowance" is the dollar amount that can be deducted in a given year, and is based on the percentage of the resource that was depleted in that year. If 30 percent of a timber stand is harvested, for example, 30 percent of the capitalized costs may be deducted from income received; for a clearcut timber sale, 100 percent of capitalized costs are deducted. Specialized terms such as depletion rate, depletion unit, basis for depletion, etc., are often used, but the basic procedure is simply to deduct costs as the resource is depleted.

d. Reforestation expenses--since 1980, special tax incentives have been provided to encourage private landowners to invest in reforestation. There are specific limits, options, and guidelines, but the most common tax treatment for qualifying expenditures is a 10 percent tax credit, and deduction of 95 percent of the total expense (up to $10,000 per year). Costs are recovered by deducting 1/14th of the costs on the first year's tax return, 1/7th of the costs on each of the next six tax returns, and the remaining 1/14th on the eighth tax return after the reforestation expense.

In general, landowners must keep separate capital accounts (records) for each of the above types of forestry costs. In some cases, subaccounts are necessary to accurately record capital expenses--subaccounts are kept for premerchantable and merchantable timber, for example. For land costs or timber costs, the simplest approach for after-tax analysis is to deduct the appropriate cost from the income generated by the land sale or timber sale. In this manner, costs should be deducted from the revenue generated by the land or timber sale, and the remainder (net revenue) multiplied by (1 - tax rate) to determine the after-tax net revenue.

The following examples show how "effective" costs are determined for equipment purchases and reforestation expenses. In general, the time value of money must be accounted for, and the after-tax cost is determined by subtracting the present value of current and future tax savings from the initial expense.

Example 3

<u>After-tax present value of equipment cost</u>. If a pickup truck is to be purchased for $14,000, and the buyer plans to deduct the costs using straight-line depreciation over 5 years (20 percent per year), what is the effective cost of the truck? (tax rate = .34, before-tax discount rate* = 10 percent).

Five deductions of $2,800 each are taken, and each deduction saves ($2,800)(.34) = $952 in taxes. The present value of the tax savings is:

Year	Tax Savings	Present Value*
0	$952	$952.00
1	$952	$893.06
2	$952	$837.77
3	$952	$785.90
4	$952	$737.24
		$4,205.97

The total present value of all tax savings is $4,205.97, and the effective cost of the truck is therefore $14,000 - $4,205.97 = $9,794.03.

This example illustrates how after-tax present values for equipment are calculated. For actual depreciation schedules, current tax information should be consulted.

*A before-tax discount rate of 10 percent is equal to an after-tax discount rate of 6.6 percent if the marginal tax rate is .34. The present values in example 3 were calculated using 6.6 percent. After-tax discount rates are discussed in the next section (following Example 4).

Example 4

After-tax present value of reforestation costs. Landowners who qualify for reforestation tax incentives receive a credit and 8 separate deductions. A landowner who spends $10,000 on reforestation, who claims a 10 percent tax credit, and who deducts 95 percent of the expense on the next eight tax returns has the following tax savings: (tax rate = .28, before-tax discount rate* = 10 percent)

Year	Item	Tax Savings	Present Value*
0	10% credit	$1,000	$1,000.00
0	(1/14)($9500)(.28)	190	190.00
1	(1/7)($9500)(.28)	380	354.48
2	(1/7)($9500)(.28)	380	330.67
3	(1/7)($9500)(.28)	380	308.46
4	(1/7)($9500)(.28)	380	287.72
5	(1/7)($9500)(.28)	380	268.42
5	(1/7)($9500)(.28)	380	250.39
7	(1/14)($9500)(.28)	190	116.79

Total Present Value of Tax Savings = $3,106.93

Effective Cost = $10,000 - $3,106.93 = $6,893.07

For landowners who receive government cost-shares for reforestation, the effective cost is further reduced to: (1 - s)($6,893.07), where s is the percentage of costs paid by a federal or state program.

*An after-tax discount rate of 7.2 percent was used, as discussed in the following section.

After-tax Discount Rates

For interest income that is taxable, your actual or after-tax interest income is [from equation (2)]:

$$\begin{bmatrix}\text{After-tax interest}\\ \text{Income}\end{bmatrix} = \begin{bmatrix}\text{Before-tax interest}\\ \text{Income}\end{bmatrix}(1 - \text{tax rate}) \qquad (6)$$

Interest income, of course, is often expressed as a percent, and the after-tax interest rate earned is the before-tax rate multiplied by (1 - tax rate):

$$\begin{bmatrix}\text{After-tax rate}\\ \text{of return}\end{bmatrix} = \begin{bmatrix}\text{Before-tax rate}\\ \text{of return}\end{bmatrix}(1 - \text{tax rate}) \qquad (7)$$

If taxes have been subtracted from forestry or other revenues, the after-tax alternative rate of return is the appropriate discount rate. The after-tax rate is also the appropriate rate for discounting the tax savings from deductions of capitalized costs. The tax savings are comparable to after-tax revenues since they are savings that are not subject to additional taxes.

Example 5

Your before-tax alternative rate of return is 9 percent, and your marginal tax rate is 28 percent. What is your after-tax alternative rate of return?

$$(.09)(1 - .28) = 6.48 \text{ percent}$$

Summary of After-tax Investment Analysis

To account for taxes in forestry or other investment analyses, place all costs and revenues on an after-tax basis, and calculate all present values using an after-tax alternative rate of return. Specifically:

1. Convert taxable revenues to after-tax revenues:
   (After-tax revenue) = (Before-tax revenue) (1 - tax rate).

2. If the taxable revenue is from a timber sale and a depletion allowance applies, the depletion allowance should be subtracted from the timber sale income before multiplying by (1 - tax rate).

3. If the taxable revenue is from the sale of land, or land and timber together, deduct the cost from revenues before multiplying by (1 - tax rate).

4. For all costs other than the capitalized costs of land and/or timber, convert costs to after-tax costs:

   a. If the cost can be expensed,
      (After-tax cost) = (Before-tax cost) (1 - tax rate).

   b. If the cost must be capitalized (like buildings, equipment, or reforestation costs), calculate the after-tax present value of costs by discounting all future tax savings to the present and subtracting from the original expense incurred.

5. Use an after-tax discount rate:
   (After-tax discount rate) = (Before-tax rate) (1 - tax rate).

VIII. INVESTMENT ANALYSIS - ACCOUNTING FOR INFLATION

Inflation averaged about 7 percent during the 1970's. That means prices in general doubled over the 10-year period. Something that cost $100 in 1971 was likely to cost around $200 in 1980.

Inflation is a general rise in prices over time. These price changes are measured by price indexes (e.g., Consumer Price Index or Wholesale Price Index). Prices and costs in investment analysis can be expressed in current dollar prices or constant dollar prices.

Current dollar prices are the actual marketplace prices charged in any particular year. They include inflation. Constant dollar prices are fixed purchasing power dollars relative to a base year. Often, constant dollars are expressed in terms of 1967 prices. The effects of inflation are removed from constant dollars by indexing. Indexing is accomplished by dividing the current price in a given year by the appropriate index for that year. For example, the producer price index (PPI) for selected years and current and constant dollar stumpage prices for southern pine sawtimber are given below (from Southern Journal of Applied Forestry (6):195-200).

Year	PPI	Sawtimber Stumpage Prices Current	Sawtimber Stumpage Prices Constant
1967	100.0	38.3	38.3
1970	110.4	44.1	39.9
1973	134.7	93.4	69.3
1976	183.0	87.0	47.5

Constant dollars are obtained by dividing current dollars by the price index factor. The factor is the PPI for the future year divided by the PPI for the base year. For example, the constant dollar price for 1976 is calculated by 87.0/(183.0/100.0) = 47.5.

A real price change occurs when a particular price changes relative to other prices in the economy. That is, the price must change at a different rate than the general price level (general rate of inflation). For example, assume that hardwood sawtimber stumpage prices have been increasing at an average rate of 8.5 percent per year since 1960. For the same period, assume hardwood pulpwood stumpage prices increased at an average annual rate of 2.8 percent, and that the general inflation rate over that period was 5 percent. In this example, sawtimber stumpage prices increased at a faster rate than the general price level, for a real price increase of over 3 percent. Pulpwood stumpage prices increased, but real prices decreased by about 2 percent per year.

Let's look at the mathematics of inflation; let:

i = market interest rate,

r = real interest rate, and

f = inflation rate.

To see how the market interest rate is related to the real interest rate and inflation, consider a value today (V_0) and what the value would be one year from today (V_1). If a real increase occurs, the value in one year is:

$$V_1 = V_0(1 + r)$$

If inflation also occurs, however, the value in one year would be:

$$V_1 = V_0 \,(1 + r)(1 + f), \text{ or}$$

multiplying the terms in parentheses,

$$V_1 = V_0(1+r + f + rf).$$

We know that $V_1 = V_0$ (1 + market interest rate), so the market rate of interest would be:

$$i = r + f + rf \qquad (19)$$

For example, if r = 3 percent and f = 5 percent,

$$i = .03 + .05 + (.03)(.05)$$

$$= .0815$$

$$= 8.15 \text{ percent}$$

If you want to solve for the real rate or the rate of inflation:

$$r = \frac{1 + i}{1 + f} - 1, \quad \text{and } f = \frac{1 + i}{1 + r} - 1$$

For example, if i = 8.15 percent and f = 5 percent,

$$r = \frac{1.0815}{1.05} - 1$$

$$= .03$$

$$= 3 \text{ percent}$$

How does all this affect investment analyses? The rule which must be followed in order to account for inflation is:

> If the discount rate includes an inflation factor, so must the estimated cash flows. If constant dollar values are used in expressing future cash flows, however, then the discount rate used should be adjusted to remove the effect of inflation.

Example 33

You are considering buying land and converting it to a pine plantation. Site preparation and regeneration will cost $50 per acre. At age 35 you will harvest 10 MBF and 10 cords per acre. The stumpage price for sawtimber is $240/MBF and $16/cord for pulpwood. Annual management costs and property taxes will be $1.50 per acre per year. What is the bare land value? The current interest rate is 11 percent.

Year	Item	Amount	Factor	Value at Year 35
0	Site Prep.	$50.00	38.57443	-$1,928.72
35	Harvest	2,560.00	1.00000	2,560.00
1-35	Annual Costs	1.50	341.58571	-512.38
				+$118.90

$$L_e = \frac{\$118.90}{(1.11)^{35} - 1} = \$3.16$$

It is not unusual to get answers like this for bare land value calculations. What's wrong with the example? Constant 1984 prices were used, but a market interest rate that included inflation was used to discount cash flows. Real interest rates have been three to four percent over the last few decades. Let's resolve the problem using a real interest rate of 4 percent.

Year	Item	Amount	Factor	Value at Year 35
0	Site Prep.	$50.00	3.94608	-$197.30
35	Harvest	2560.00	1.00000	2,560.00
1-35	Annual Costs	1.50	73.65210	-110.48
				$2,252.22

$$L_e = \frac{\$2,252.22}{(1.04)^{35} - 1} = \$764.48$$

The correct bare land value is $764.48.

Problems

33. An investment in a forest property is expected to return 3 percent in real terms over the next 7 years. Inflation is expected to average 7 percent over the period. What is the market rate or nominal rate of return expected from the investment?

34. A money market account will pay you 10 percent over the next 9 years. You expect inflation to average 5 percent over the same period. What will be your real rate of return on the account?

IX. SPECIAL TOPICS

Several topics require further development. Equivalence is an important concept that was only given passing coverage in an earlier section. Continuous compounding will be briefly discussed. Loan problems will finish out this section.

Gradient Cash Flow Series

Not all cash flows are uniform. A common situation is a gradient cash flow series, where the cash flow is expected to increase or decrease by a uniform amount each compounding period. In effect, two separate cash flow series are present in gradient cash flow series problems: a uniform series and a gradient series. The gradient series increases or decreases by a constant amount, "g", each compounding period.

Consider the operation and maintenance expense for a small logging crew. The expense for year 1 is $50,000. Beginning with year 2, due to equipment deterioration, the expense is expected to increase by $5,000 a year until the end of year 6, when the equipment is retired. The cost of capital is 8 percent.

Figure 7 illustrates the cash flow series. Notice that Figure 7 represents a composite of a uniform series and a gradient series. Figure 8 shows the components of the composite cash flow series.

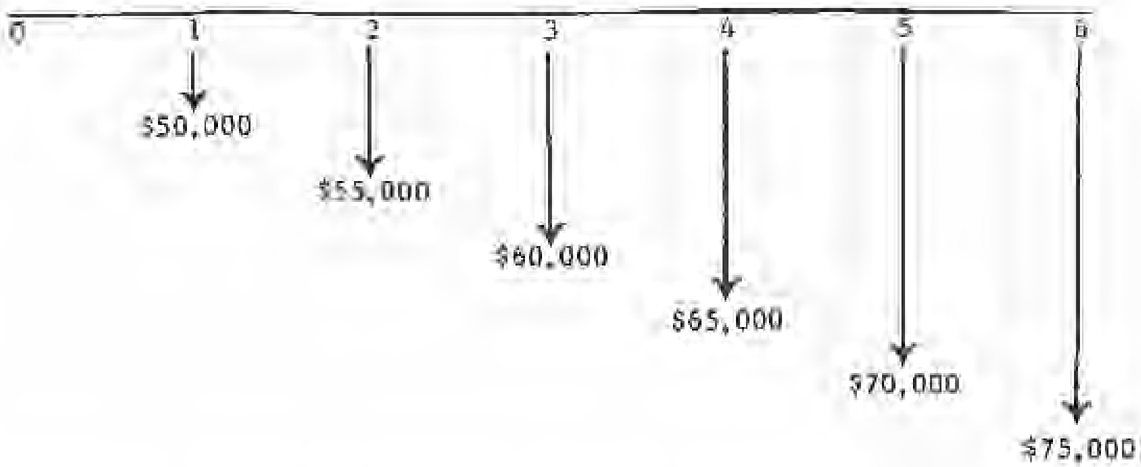

Figure 7. Example of gradient cash flow series (composite of a uniform series and a gradient series).

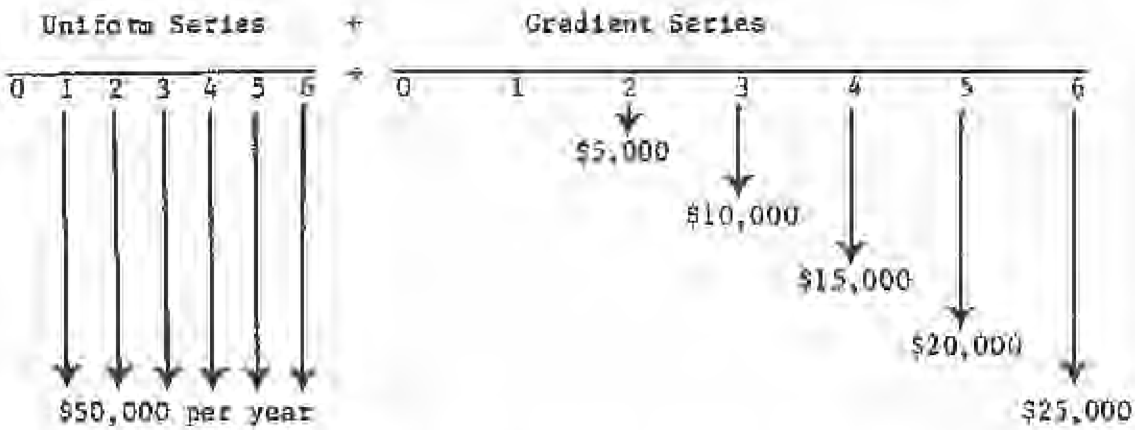

Figure 8. Composite cash flow series broken down into its component uniform and gradient series.

Let:

g = the amount of increase or decrease in a cash flow gradient.

The present value of a gradient cash flow series is given by:

$$V_0 = g\left[\frac{1.0 - (1 + ni)(1 + i)^{-n}}{i^2}\right] \qquad (20)$$

The future value of a gradient cash flow series is given by:

$$V_n = g\left[\frac{(1 + i)^n - (1 + ni)}{i^2}\right] \qquad (21)$$

The uniform series gradient conversion factor, the factor that converts a gradient series to a uniform series, is given by:

$$a = g\left[\frac{1}{i} - \frac{n}{(1 + i)^n - 1.0}\right] \quad (22)$$

Using formula 20, the present value of the example gradient cash flow is:

$$V_0 = \$5,000\left[\frac{1.0 - (1.48)(0.63017)}{0.0064}\right]$$

$$= \$52,616.37$$

Using formula 21, the future value of the example gradient cash flow is:

$$V_6 = \$5,000\left[\frac{1.5869 - 1.48}{0.0064}\right]$$

$$= \$83,495.56$$

Using formula 22, the uniform series that is equivalent to the example gradient series is:

$$a = \$5,000\left[\frac{1}{0.08} - \frac{6}{(1.08)^6 - 1.0}\right]$$

$$= \$5,000\ (12.5 - 10.2237)$$

$$= \$11,381.73$$

The present value of the \$50,000 uniform series can be determined from formula 4:

$$V_0 = \$50,000\left[\frac{(1.08)^6 - 1.0}{0.08(1.08)^6}\right]$$

$$= \$231,143.98$$

The future value of the \$50,000 uniform series can be determined from formula 5:

$$V_6 = \$50,000\left[\frac{(1.08)^6 - 1.0}{0.08}\right]$$

$$= \$366,796.45$$

The present value of the composite cash flows series is \$52,616.37 + \$231,143.98 = \$283,760.35. The future value of the composite cash flow series is \$83,495.56 + \$366,796.45 = \$450,292.01. The composite equivalent uniform cash flow is \$11,381.73 + \$50,000 = \$61,381.73

As a check, a uniform series of \$61,381.73 should be equivalent to the composite series present value of \$283,760.35.

$$V_0 = \$61,381.73\left[\frac{(1.08)^6 - 1.0}{0.08(1.08)^6}\right]$$

$$= \$283,760.35$$

Example 34

You purchase 40 acres of land. The purchase price is a series of payments of \$20,000, \$15,000, \$10,000, and \$5,000 in years t = 3, 4, 5, and 6, respectively. What is the present value of the cash flow at a 9 percent interest rate? Figure 9 illustrates the cash flows of the example.

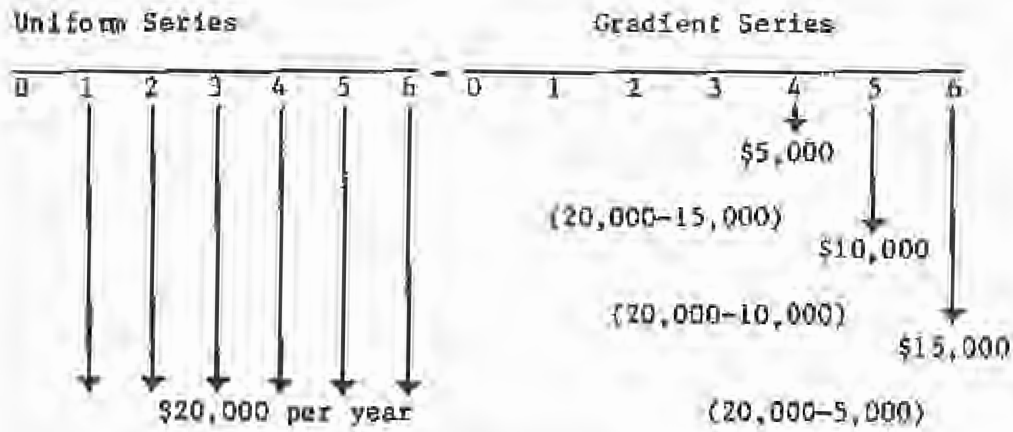

Figure 9. Example cash flow series.

This problem can be solved by determining the present value of a uniform and gradient series. The present value of a uniform series of \$20,000 payments at a 9 percent interest rate is given by formula 4:

$$V_0 = \$20,000\left[\frac{(1.09)^4 - 1.0}{0.09(1.09)^4}\right]$$

$$= \$64,794.40$$

Since the first payment of the annuity occurs at $t = 3$, the value of "V_0" is actually at $t = 2$. Thus, the value above must be discounted for 2 years to obtain the actual value of V_0:

$$V_0 = \frac{64,794.40}{(1.09)^2} = \$54,536.15$$

The present value of the gradient series is given by:

$$V_0 = \$5,000\left[\frac{1.0 - (1.36)(0.708425)}{0.0081}\right]$$
$$= \$22,556.61$$

V_0 in the gradient series also occurs at $t = 2$, so the above value must be discounted for 2 years:

$$V_0 = \frac{\$22,556.61}{(1.08)^2} = \$18,985.45$$

The present value of the investment is the difference between the present value of the uniform and gradient series, or, \$54,536.15 − \$18,985.45 = \$35,550.70.

Geometric Cash Flow Series

White et al. (1977) developed the formulas necessary to analyze geometric cash flows. A geometric cash flow increases or decreases by a fixed percentage each compounding period. Figure 10 illustrates such a cash flow.

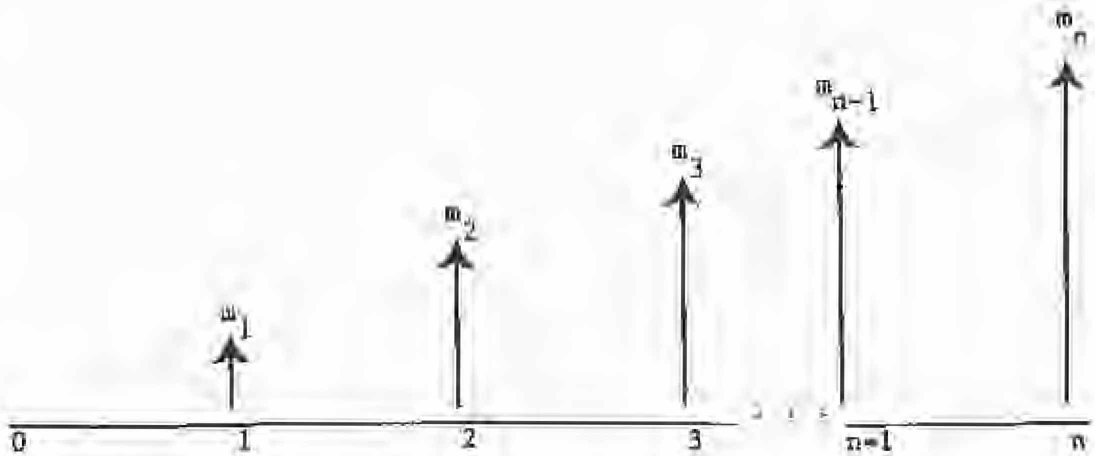

Figure 10. Cash flow diagram of a geometric cash flow series.

The present value of a geometric cash flow series, where "m" is the first payment in the series, and "j" equals the percentage increase or decrease in the cash flow between periods, is given by:

$$V_0 = \begin{cases} m\left[\dfrac{1 - (1 + j)^n (1 + i)^{-n}}{i - j}\right] & \text{if} \quad i \neq j \\ \text{or} & \\ \dfrac{n\ m}{1 + i} & \text{if} \quad i = j \end{cases} \tag{23}$$

The future value of a geometric cash flow series is given by:

$$V_n = \begin{cases} m\left[\dfrac{(1 + i)^n - (1 + j)^n}{i - j}\right] & \text{if} \quad i \neq j \\ \text{or} & \\ n\ m(1 + i)^{n - 1} & \text{if} \quad i = j \end{cases} \tag{24}$$

Example 35

Regeneration costs have been increasing 3 percent per year. A woodlands desires to set aside a fund to pay regeneration costs for the next 10 years. Regeneration costs next year will be $320,000.00. The firm's cost of capital is 11 percent. How much should be placed in the account today?

$$V_0 = \$320{,}000.00\left[\frac{1.0 - (1.03)^{10}(1.11)^{-10}}{0.11 - 0.03}\right]$$

$$= \$320{,}000.00\left[\frac{0.5267}{0.08}\right]$$

$$= \$2{,}106{,}774.00$$

Example 36

Assume an annual payment of \$1,000.00 that increases by 10 percent annually, beginning with the second year. The payments are deposited for five years into an account earning 10 percent per year. What amount will be in the account at t = 5?

$$V_5 = (5)(\$1{,}000.00)(1.10)^4$$

$$= \$7{,}320.50$$

Equivalence

Two cash flow series are equivalent at a specified interest rate if their present values are equal at the specified rate. If two cash flow series have equal present values at a specified interest rate, then their values will be equal at any point in time at the specified interest rate. Also, equivalent cash flow series will have equal uniform cash flow series over the same time period.

Example 37

Figure 11 shows two equivalent cash flow series. At 10 percent interest both have the same present value (\$302.92). Note also that the top cash flow series, since it is equivalent to the bottom uniform series, can also be expressed as a uniform 5-year cash flow series of \$79.91. The present value of the top cash flow series is:

$$V_0 = \$100.00 + \frac{\$100.00}{(1.10)^1} + \frac{\$250.00}{(1.10)^3} + \frac{\$200.00}{(1.10)^5}$$

$$= \$302.92.$$

The equivalent uniform cash flow series can be obtained via Equation 13:

$$a = \$302.92\left[\frac{0.10(1.1)^5}{(1.10)^5 - 1.0}\right]$$

$$= \$79.91.$$

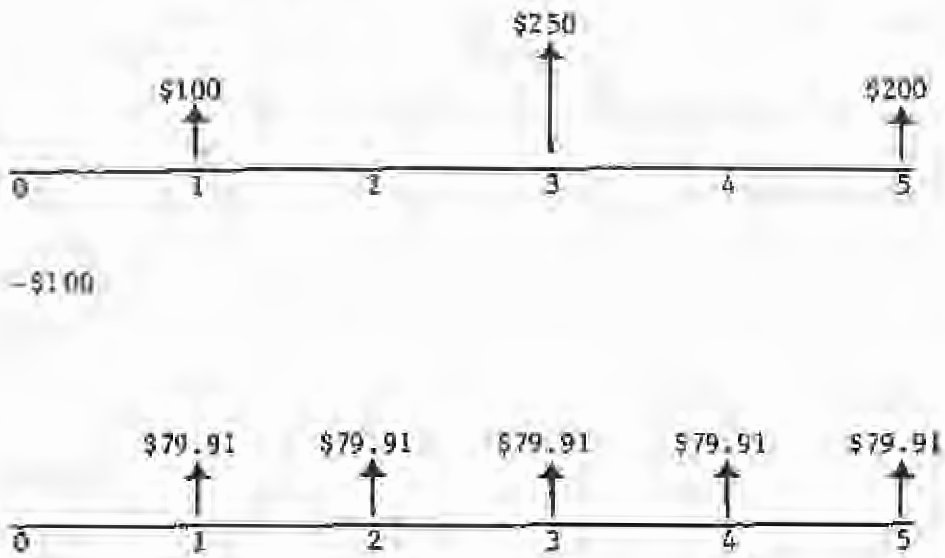

Figure 11. Example equivalent cash flow series.

<u>Continuous Compounding</u>

Throughout our previous discussions, we have assumed that interest is compounded at the end of discrete periods. However, continuous compounding is also common. Figure 12 illustrates that most interest is accumulated on a discrete basis. However, some institutions, as a competitive tactic, do compound on a continuous basis. For completeness, the formulas for continuous compounding will be listed, without derivation.

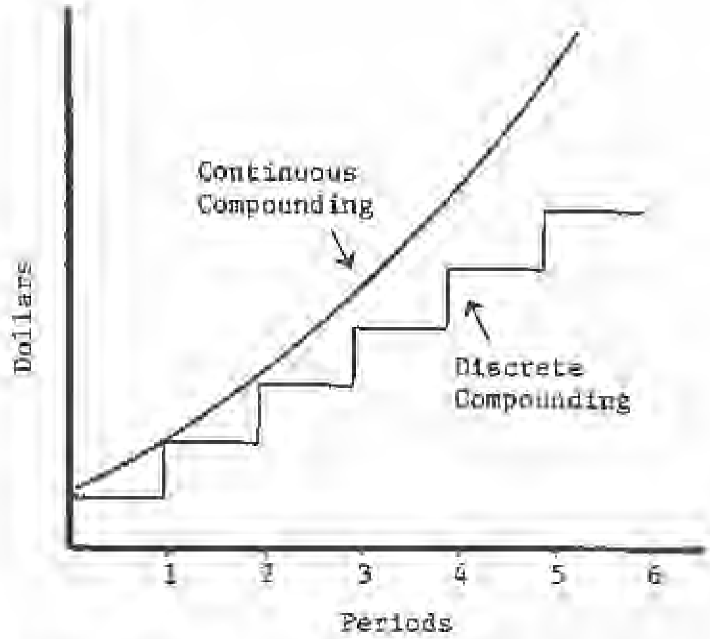

Figure 12. Discrete versus continuous compounding.

Let:

q = the nominal annual interest rate, expressed as a decimal. The present value of a single sum under continuous compounding is given by:

$$V_0 = V_n e^{-qn} \tag{25}$$

The future value of a single sum under continuous compounding is given by:

$$V_n = V_0 e^{qn} \tag{26}$$

The effective interest rate under continuous compounding is given by:

$$i_{effective} = e^q - 1 \tag{27}$$

Example 38

If $1,000 is placed in a savings account that earns 9 percent interest, compounded continuously, how much will be in the account after 6 years?

$$V_n = \$1000[e^{(0.09)(6)}] = 1{,}000e^{(0.54)}$$
$$= 1{,}716.01$$

Verify the result using formula 25.

$$V_0 = \$1{,}716.01\ e^{-(0.09)(6)} = \$1{,}716.01e^{(.5827483)}$$
$$= \$1{,}000.00$$

What was the effective interest rate in this example?

$$i_{effective} = e^{0.09} - 1 = 1.0941743 - 1.0$$
$$= 0.0942$$

Add-on Interest

Add-on interest is frequently used in financing loans. The interest charge is computed at $t = 0$, so that the loan amount plus the added-on interest are repaid in equal installments.

Example 39

You obtain a \$20,000 loan from a bank. The finance charge is 20 percent, and is added-on to the original amount. The total amount, loan plus add-on, is to be repaid in 12 equal monthly installments. What are your monthly payments?

$$\text{Payments} = \frac{\text{Loan + Finance Charge}}{12}$$
$$= \frac{\$20{,}000 + 0.20(\$20{,}000)}{12}$$
$$= \frac{24{,}000}{12} = \$2{,}000 \text{ per month}$$

If you borrowed \$20,000 and paid \$2,000 per month for a year, the actual monthly interest rate could be derived from the capital recovery formula (Equation 6):

$$a = V_0\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

In terms of the add-on interest example,

a = $2,000 (payments per month), and

V_0 = $20,000 (received as a loan).

Substituting those values into the capital recovery formula:

$$\$2,000 = \$20,000\left[\frac{i(1+i)^{12}}{(1+i)^n - 1}\right],$$

where "i" is the monthly interest rate actually being paid on the loan. We can estimate "i" by trial and error, or by looking for the value $2,000/$20,000 = .10 in column 6 of Appendix A, for n = 12. The value .10046 is found for i = 3 percent (Appendix Table A3); the actual monthly interest rate is about 2.92 percent.

What is the <u>nominal</u> rate of interest?

$$2.92 \times 12 = 35.04\%$$

What is the <u>effective</u> annual rate of interest?

$$i_{effective} = (1.0292)^{12} - 1$$

$$= 41.25\%$$

Amortization

When an investment is financed by a loan, the amount of interest paid in particular time periods will affect income taxes. Amortization is a common financial calculation and a forest investment analyst should be aware of the principles involved in determining the principal amount and interest amount in loan payments.

If a loan is to be repaid in equal installments, the amount of the payment can be determined with Equation 6:

$$a = V_0\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

The amortization schedule for a loan of $10,000 to be repaid in 3 equal annual payments at 6 percent annual interest would be as follows:

Year	Annual Interest Charge	Amount Owed	Equal Annual Payment	Balance, End-of-Year
1	$600	$10,000	$3,741	$6,859
2	412	7,271	3,741	3,530
3	212	3,742	3,742	-0-

The equal annual payment is:

$$a = \$10{,}000\left[\frac{0.06(1.06)^3}{(1.06)^3 - 1.0}\right] = \$3{,}741.$$

The amortization table for this loan would be:

Year	Payments to Principal	Payments to Interest	Balance
1	$3,141	$600	$6,859
2	3,329	412	3,530
3	3,529	212	-0-

The amount by which payment number "p" reduces the unpaid principal is given by:

$$\text{Reduction in Principal} = \left[\frac{a}{(1+i)^{n-p+1}}\right] \qquad (28)$$

For example, the first loan payment reduces the unpaid principal by:

$$\text{Reduction in Principal} = \left[\frac{\$3,741}{(1.06)^{3-1+1}}\right]$$

$$= \$3,141$$

The amount of payment "p" that corresponds to interest on the loan is given by:

$$\text{Interest Payment} = a\left[1 - \frac{1}{(1+i)^{n-p+1}}\right] \qquad (29)$$

$$= \$600$$

Example 40

Develop an amortization table for the first 4 payments on the following home mortgage:

Total Cost = \$60,000

Down Payment = \$20,000

Mortgage Rate = 12¼% compounded monthly

Terms: 30 years, equal monthly payments

$$\text{Payment} = \$40,000\left[\frac{(\frac{.1225}{12})(1+\frac{.1225}{12})^{360}}{(1+\frac{.1225}{12})^{360} - 1}\right]$$

$$= \$40,000(0.010479) = \$419.16.$$

Month	Payments to: Principal	Interest	Balance
1	\$10.83	\$408.33	\$39,989.12
2	10.94	408.22	39,978.23
3	11.05	408.11	39,967.18
4	11.15	408.00	39,956.02

XI. REFERENCES

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XII. APPENDICES

Table A1. Compound Interest Multipliers, i = 1%

n	SINGLE SUM Future Value 1	Present Value 2	ANNUAL SERIES Future Value 3	Present Value 4	Sinking Fund 5	Capital Recovery 6	n
1	1.0100	0.9901	1.0000	0.9901	1.0000	1.0100	1
2	1.0201	0.9803	2.0100	1.9704	0.4975	0.5075	2
3	1.0303	0.9706	3.0301	2.9410	0.3300	0.3400	3
4	1.0406	0.9610	4.0604	3.9020	0.2463	0.2563	4
5	1.0510	0.9515	5.1010	4.8534	0.1960	0.2060	5
6	1.0615	0.9420	6.1520	5.7955	0.1625	0.1725	6
7	1.0721	0.9327	7.2135	6.7282	0.1386	0.1486	7
8	1.0829	0.9235	8.2857	7.6517	0.1207	0.1307	8
9	1.0937	0.9143	9.3685	8.5660	0.1067	0.1167	9
10	1.1046	0.9053	10.4622	9.4713	0.0956	0.1056	10
11	1.1157	0.8963	11.5668	10.3676	0.0865	0.0965	11
12	1.1268	0.8874	12.6825	11.2551	0.0788	0.0888	12
13	1.1381	0.8787	13.8093	12.1337	0.0724	0.0824	13
14	1.1495	0.8700	14.9474	13.0037	0.0669	0.0769	14
15	1.1610	0.8613	16.0969	13.8650	0.0621	0.0721	15
16	1.1726	0.8528	17.2579	14.7179	0.0579	0.0679	16
17	1.1843	0.8444	18.4304	15.5622	0.0543	0.0643	17
18	1.1961	0.8360	19.6147	16.3983	0.0510	0.0610	18
19	1.2081	0.8277	20.8109	17.2260	0.0481	0.0581	19
20	1.2202	0.8195	22.0190	18.0455	0.0454	0.0554	20
21	1.2324	0.8114	23.2392	18.8570	0.0430	0.0530	21
22	1.2447	0.8034	24.4716	19.6604	0.0409	0.0509	22
23	1.2572	0.7954	25.7163	20.4558	0.0389	0.0489	23
24	1.2697	0.7876	26.9735	21.2434	0.0371	0.0471	24
25	1.2824	0.7798	28.2432	22.0231	0.0354	0.0454	25
26	1.2953	0.7720	29.5256	22.7952	0.0339	0.0439	26
27	1.3082	0.7644	30.8209	23.5596	0.0324	0.0424	27
28	1.3213	0.7568	32.1291	24.3164	0.0311	0.0411	28
29	1.3345	0.7493	33.4504	25.0658	0.0299	0.0399	29
30	1.3478	0.7419	34.7849	25.8077	0.0287	0.0387	30
31	1.3613	0.7346	36.1327	26.5423	0.0277	0.0377	31
32	1.3749	0.7273	37.4941	27.2696	0.0267	0.0367	32
33	1.3887	0.7201	38.8690	27.9897	0.0257	0.0357	33
34	1.4026	0.7130	40.2577	28.7027	0.0248	0.0348	34
35	1.4166	0.7059	41.6603	29.4086	0.0240	0.0340	35
40	1.4889	0.6717	48.8864	32.8347	0.0205	0.0305	40
45	1.5648	0.6391	56.4811	36.0945	0.0177	0.0277	45
50	1.6446	0.6080	64.4632	39.1961	0.0155	0.0255	50
55	1.7285	0.5785	72.8524	42.1472	0.0137	0.0237	55
60	1.8167	0.5504	81.6696	44.9550	0.0122	0.0222	60
65	1.9094	0.5237	90.9366	47.6266	0.0110	0.0210	65
70	2.0068	0.4983	100.6763	50.1685	0.0099	0.0199	70

APPENDIX A

Table A2. Compound Interest Multipliers, i = 2%

n	SINGLE SUM: Future Value (1)	SINGLE SUM: Present Value (2)	ANNUAL SERIES: Future Value (3)	ANNUAL SERIES: Present Value (4)	ANNUAL SERIES: Sinking Fund (5)	ANNUAL SERIES: Capital Recovery (6)	n
1	1.0200	0.9804	1.0000	0.9804	1.0000	1.0200	1
2	1.0404	0.9612	2.0200	1.9416	0.4951	0.5151	2
3	1.0612	0.9423	3.0604	2.8839	0.3268	0.3468	3
4	1.0824	0.9238	4.1216	3.8077	0.2426	0.2626	4
5	1.1041	0.9057	5.2040	4.7134	0.1922	0.2122	5
6	1.1262	0.8880	6.3081	5.6014	0.1585	0.1785	6
7	1.1487	0.8706	7.4343	6.4720	0.1345	0.1545	7
8	1.1717	0.8535	8.5829	7.3255	0.1165	0.1365	8
9	1.1951	0.8368	9.7546	8.1622	0.1025	0.1225	9
10	1.2190	0.8203	10.9497	8.9826	0.0913	0.1113	10
11	1.2434	0.8043	12.1687	9.7868	0.0822	0.1022	11
12	1.2682	0.7885	13.4120	10.5753	0.0746	0.0946	12
13	1.2936	0.7730	14.6803	11.3483	0.0681	0.0881	13
14	1.3195	0.7579	15.9739	12.1062	0.0626	0.0826	14
15	1.3459	0.7430	17.2934	12.8492	0.0578	0.0778	15
16	1.3728	0.7284	18.6392	13.5777	0.0537	0.0737	16
17	1.4002	0.7142	20.0120	14.2918	0.0500	0.0700	17
18	1.4282	0.7002	21.4122	14.9920	0.0467	0.0667	18
19	1.4568	0.6864	22.8405	15.6784	0.0438	0.0638	19
20	1.4859	0.6730	24.2973	16.3514	0.0412	0.0612	20
21	1.5157	0.6598	25.7832	17.0112	0.0388	0.0588	21
22	1.5460	0.6468	27.2989	17.6580	0.0366	0.0566	22
23	1.5769	0.6342	28.8449	18.2922	0.0347	0.0547	23
24	1.6084	0.6217	30.4218	18.9139	0.0329	0.0529	24
25	1.6406	0.6095	32.0302	19.5234	0.0312	0.0512	25
26	1.6734	0.5976	33.6708	20.1210	0.0297	0.0497	26
27	1.7069	0.5859	35.3442	20.7069	0.0283	0.0483	27
28	1.7410	0.5744	37.0511	21.2812	0.0270	0.0470	28
29	1.7758	0.5631	38.7921	21.8443	0.0258	0.0458	29
30	1.8114	0.5521	40.5679	22.3964	0.0247	0.0447	30
31	1.8476	0.5412	42.3793	22.9377	0.0236	0.0436	31
32	1.8845	0.5306	44.2269	23.4683	0.0226	0.0426	32
33	1.9222	0.5202	46.1114	23.9885	0.0217	0.0417	33
34	1.9607	0.5100	48.0336	24.4985	0.0208	0.0408	34
35	1.9999	0.5000	49.9943	24.9986	0.0200	0.0400	35
40	2.2080	0.4529	60.4017	27.3554	0.0166	0.0366	40
45	2.4378	0.4102	71.8924	29.4901	0.0139	0.0339	45
50	2.6916	0.3715	84.5790	31.4236	0.0118	0.0318	50
55	2.9717	0.3365	98.5861	33.1747	0.0101	0.0301	55
60	3.2810	0.3048	114.0510	34.7608	0.0088	0.0288	60
65	3.6225	0.2761	131.1255	36.1974	0.0076	0.0276	65
70	3.9995	0.2500	149.9771	37.4986	0.0067	0.0267	70

Table A3. Compound Interest Multipliers, i = 3%

n	SINGLE SUM		ANNUAL SERIES				n
	Future Value	Present Value	Future Value	Present Value	Sinking Fund	Capital Recovery	
	1	2	3	4	5	6	
1	1.0300	0.9709	1.0000	0.9709	1.0000	1.0300	1
2	1.0609	0.9426	2.0300	1.9135	0.4926	0.5226	2
3	1.0927	0.9151	3.0909	2.8286	0.3235	0.3535	3
4	1.1255	0.8885	4.1836	3.7171	0.2390	0.2690	4
5	1.1593	0.8626	5.3091	4.5797	0.1884	0.2184	5
6	1.1941	0.8375	6.4684	5.4172	0.1546	0.1846	6
7	1.2299	0.8131	7.6625	6.2303	0.1305	0.1605	7
8	1.2668	0.7894	8.8923	7.0197	0.1125	0.1425	8
9	1.3048	0.7664	10.1591	7.7861	0.0984	0.1284	9
10	1.3439	0.7441	11.4639	8.5302	0.0872	0.1172	10
11	1.3842	0.7224	12.8078	9.2526	0.0781	0.1081	11
12	1.4258	0.7014	14.1920	9.9540	0.0705	0.1005	12
13	1.4685	0.6810	15.6178	10.6350	0.0640	0.0940	13
14	1.5126	0.6611	17.0863	11.2961	0.0585	0.0885	14
15	1.5580	0.6419	18.5989	11.9379	0.0538	0.0838	15
16	1.6047	0.6232	20.1569	12.5611	0.0496	0.0796	16
17	1.6528	0.6050	21.7616	13.1661	0.0460	0.0760	17
18	1.7024	0.5874	23.4144	13.7535	0.0427	0.0727	18
19	1.7535	0.5703	25.1169	14.3238	0.0398	0.0698	19
20	1.8061	0.5537	26.8704	14.8775	0.0372	0.0672	20
21	1.8603	0.5375	28.6765	15.4150	0.0349	0.0649	21
22	1.9161	0.5219	30.5368	15.9369	0.0327	0.0627	22
23	1.9736	0.5067	32.4529	16.4436	0.0308	0.0608	23
24	2.0328	0.4919	34.4265	16.9355	0.0290	0.0590	24
25	2.0938	0.4776	36.4593	17.4131	0.0274	0.0574	25
26	2.1566	0.4637	38.5530	17.8768	0.0259	0.0559	26
27	2.2213	0.4502	40.7096	18.3270	0.0246	0.0546	27
28	2.2879	0.4371	42.9309	18.7641	0.0233	0.0533	28
29	2.3566	0.4243	45.2188	19.1885	0.0221	0.0521	29
30	2.4273	0.4120	47.5754	19.6004	0.0210	0.0510	30
31	2.5001	0.4000	50.0027	20.0004	0.0200	0.0500	31
32	2.5751	0.3883	52.5027	20.3888	0.0190	0.0490	32
33	2.6523	0.3770	55.0778	20.7658	0.0182	0.0482	33
34	2.7319	0.3660	57.7302	21.1318	0.0173	0.0473	34
35	2.8139	0.3554	60.4621	21.4872	0.0165	0.0465	35
40	3.2620	0.3066	75.4012	23.1148	0.0133	0.0433	40
45	3.7816	0.2644	92.7198	24.5187	0.0108	0.0408	45
50	4.3839	0.2281	112.7968	25.7298	0.0089	0.0389	50
55	5.0821	0.1968	136.0716	26.7744	0.0073	0.0373	55
60	5.8916	0.1697	163.0534	27.6756	0.0061	0.0361	60
65	6.8300	0.1464	194.3327	28.4529	0.0051	0.0351	65
70	7.9178	0.1263	230.5940	29.1234	0.0043	0.0343	70

APPENDIX A

Table A4. Compound Interest Multipliers, i = 4%

n	SINGLE SUM		ANNUAL SERIES				n
	Future Value	Present Value	Future Value	Present Value	Sinking Fund	Capital Recovery	
	1	2	3	4	5	6	
1	1.0400	0.9615	1.0000	0.9615	1.0000	1.0400	1
2	1.0816	0.9246	2.0400	1.8861	0.4902	0.5302	2
3	1.1249	0.8890	3.1216	2.7751	0.3203	0.3603	3
4	1.1699	0.8548	4.2465	3.6299	0.2355	0.2755	4
5	1.2167	0.8219	5.4163	4.4518	0.1846	0.2246	5
6	1.2653	0.7903	6.6330	5.2421	0.1508	0.1908	6
7	1.3159	0.7599	7.8983	6.0021	0.1266	0.1666	7
8	1.3686	0.7307	9.2142	6.7327	0.1085	0.1485	8
9	1.4233	0.7026	10.5828	7.4353	0.0945	0.1345	9
10	1.4802	0.6756	12.0061	8.1109	0.0833	0.1233	10
11	1.5395	0.6496	13.4863	8.7605	0.0741	0.1141	11
12	1.6010	0.6246	15.0258	9.3851	0.0666	0.1066	12
13	1.6651	0.6006	16.6268	9.9856	0.0601	0.1001	13
14	1.7317	0.5775	18.2919	10.5631	0.0547	0.0947	14
15	1.8009	0.5553	20.0236	11.1184	0.0499	0.0899	15
16	1.8730	0.5339	21.8245	11.6523	0.0458	0.0858	16
17	1.9479	0.5134	23.6975	12.1657	0.0422	0.0822	17
18	2.0258	0.4936	25.6454	12.6593	0.0390	0.0790	18
19	2.1068	0.4746	27.6712	13.1339	0.0361	0.0761	19
20	2.1911	0.4564	29.7781	13.5903	0.0336	0.0736	20
21	2.2788	0.4388	31.9692	14.0292	0.0313	0.0713	21
22	2.3699	0.4220	34.2479	14.4511	0.0292	0.0692	22
23	2.4647	0.4057	36.6179	14.8568	0.0273	0.0673	23
24	2.5633	0.3901	39.0826	15.2470	0.0256	0.0656	24
25	2.6658	0.3751	41.6459	15.6221	0.0240	0.0640	25
26	2.7725	0.3607	44.3117	15.9828	0.0226	0.0626	26
27	2.8834	0.3468	47.0842	16.3296	0.0212	0.0612	27
28	2.9987	0.3335	49.9675	16.6631	0.0200	0.0600	28
29	3.1186	0.3207	52.9662	16.9837	0.0189	0.0589	29
30	3.2434	0.3083	56.0849	17.2920	0.0178	0.0578	30
31	3.3731	0.2965	59.3283	17.5885	0.0169	0.0569	31
32	3.5081	0.2851	62.7014	17.8735	0.0159	0.0559	32
33	3.6484	0.2741	66.2095	18.1476	0.0151	0.0551	33
34	3.7943	0.2636	69.8578	18.4112	0.0143	0.0543	34
35	3.9461	0.2534	73.6521	18.6646	0.0136	0.0536	35
40	4.8010	0.2083	95.0254	19.7928	0.0105	0.0505	40
45	5.8412	0.1712	121.0294	20.7200	0.0083	0.0483	45
50	7.1067	0.1407	152.6669	21.4822	0.0066	0.0466	50
55	8.6464	0.1157	191.1589	22.1086	0.0052	0.0452	55
60	10.5196	0.0951	237.9903	22.6235	0.0042	0.0442	60
65	12.7987	0.0781	294.9679	23.0467	0.0034	0.0434	65
70	15.5716	0.0642	364.2898	23.3945	0.0027	0.0427	70

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Table A5. Compound Interest Multipliers, i = 5%

n	SINGLE SUM: Future Value (1)	SINGLE SUM: Present Value (2)	ANNUAL SERIES: Future Value (3)	ANNUAL SERIES: Present Value (4)	ANNUAL SERIES: Sinking Fund (5)	ANNUAL SERIES: Capital Recovery (6)	n
1	1.0500	0.9524	1.0000	0.9524	1.0000	1.0500	1
2	1.1025	0.9070	2.0500	1.8594	0.4878	0.5378	2
3	1.1576	0.8638	3.1525	2.7232	0.3172	0.3672	3
4	1.2155	0.8227	4.3101	3.5459	0.2320	0.2820	4
5	1.2763	0.7835	5.5256	4.3295	0.1810	0.2310	5
6	1.3401	0.7462	6.8019	5.0757	0.1470	0.1970	6
7	1.4071	0.7107	8.1420	5.7864	0.1228	0.1728	7
8	1.4775	0.6768	9.5491	6.4632	0.1047	0.1547	8
9	1.5513	0.6446	11.0265	7.1078	0.0907	0.1407	9
10	1.6289	0.6139	12.5779	7.7217	0.0795	0.1295	10
11	1.7103	0.5847	14.2068	8.3064	0.0704	0.1204	11
12	1.7959	0.5568	15.9171	8.8632	0.0628	0.1128	12
13	1.8856	0.5303	17.7129	9.3936	0.0565	0.1065	13
14	1.9799	0.5051	19.5986	9.8986	0.0510	0.1010	14
15	2.0789	0.4810	21.5785	10.3796	0.0463	0.0963	15
16	2.1829	0.4581	23.6574	10.8378	0.0423	0.0923	16
17	2.2920	0.4363	25.8403	11.2741	0.0387	0.0887	17
18	2.4066	0.4155	28.1323	11.6896	0.0355	0.0855	18
19	2.5269	0.3957	30.5389	12.0853	0.0327	0.0827	19
20	2.6533	0.3769	33.0659	12.4622	0.0302	0.0802	20
21	2.7860	0.3589	35.7192	12.8211	0.0280	0.0780	21
22	2.9253	0.3419	38.5051	13.1630	0.0260	0.0760	22
23	3.0715	0.3256	41.4304	13.4886	0.0241	0.0741	23
24	3.2251	0.3101	44.5019	13.7986	0.0225	0.0725	24
25	3.3863	0.2953	47.7270	14.0939	0.0210	0.0710	25
26	3.5557	0.2812	51.1133	14.3752	0.0196	0.0696	26
27	3.7334	0.2678	54.6690	14.6430	0.0183	0.0683	27
28	3.9201	0.2551	58.4024	14.8981	0.0171	0.0671	28
29	4.1161	0.2429	62.3225	15.1411	0.0160	0.0660	29
30	4.3219	0.2314	66.4386	15.3724	0.0151	0.0651	30
31	4.5380	0.2204	70.7606	15.5928	0.0141	0.0641	31
32	4.7649	0.2099	75.2986	15.8027	0.0133	0.0633	32
33	5.0032	0.1999	80.0635	16.0025	0.0125	0.0625	33
34	5.2533	0.1904	85.0667	16.1929	0.0118	0.0618	34
35	5.5160	0.1813	90.3200	16.3742	0.0111	0.0611	35
40	7.0400	0.1420	120.7995	17.1591	0.0083	0.0583	40
45	8.9850	0.1113	159.6995	17.7741	0.0063	0.0563	45
50	11.4674	0.0872	209.3470	18.2559	0.0048	0.0548	50
55	14.6356	0.0683	272.7113	18.6335	0.0037	0.0537	55
60	18.6791	0.0535	353.5818	18.9293	0.0028	0.0528	60
65	23.8398	0.0419	456.7954	19.1611	0.0022	0.0522	65
70	30.4262	0.0329	588.5249	19.3427	0.0017	0.0517	70

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Table A6. Compound Interest Multipliers, $i = 6\%$

n	SINGLE SUM Future Value 1	SINGLE SUM Present Value 2	ANNUAL SERIES Future Value 3	ANNUAL SERIES Present Value 4	ANNUAL SERIES Sinking Fund 5	ANNUAL SERIES Capital Recovery 6	n
1	1.0600	0.9434	1.0000	0.9434	1.0000	1.0600	1
2	1.1236	0.8900	2.0600	1.8334	0.4854	0.5454	2
3	1.1910	0.8396	3.1836	2.6730	0.3141	0.3741	3
4	1.2625	0.7921	4.3746	3.4651	0.2286	0.2886	4
5	1.3382	0.7473	5.6371	4.2124	0.1774	0.2374	5
6	1.4185	0.7050	6.9753	4.9173	0.1434	0.2034	6
7	1.5036	0.6651	8.3938	5.5824	0.1191	0.1791	7
8	1.5938	0.6274	9.8975	6.2098	0.1010	0.1610	8
9	1.6895	0.5919	11.4913	6.8017	0.0870	0.1470	9
10	1.7908	0.5584	13.1808	7.3601	0.0759	0.1359	10
11	1.8983	0.5268	14.9716	7.8869	0.0668	0.1268	11
12	2.0122	0.4970	16.8699	8.3838	0.0593	0.1193	12
13	2.1329	0.4688	18.8821	8.8527	0.0530	0.1130	13
14	2.2609	0.4423	21.0150	9.2950	0.0476	0.1076	14
15	2.3966	0.4173	23.2759	9.7122	0.0430	0.1030	15
16	2.5403	0.3936	25.6725	10.1059	0.0390	0.0990	16
17	2.6928	0.3714	28.2128	10.4773	0.0354	0.0954	17
18	2.8543	0.3503	30.9056	10.8276	0.0324	0.0924	18
19	3.0256	0.3305	33.7599	11.1581	0.0296	0.0896	19
20	3.2071	0.3118	36.7855	11.4699	0.0272	0.0872	20
21	3.3996	0.2942	39.9927	11.7641	0.0250	0.0850	21
22	3.6035	0.2775	43.3922	12.0416	0.0230	0.0830	22
23	3.8197	0.2618	46.9957	12.3034	0.0213	0.0813	23
24	4.0489	0.2470	50.8155	12.5504	0.0197	0.0797	24
25	4.2919	0.2330	54.8644	12.7834	0.0182	0.0782	25
26	4.5494	0.2198	59.1563	13.0032	0.0169	0.0769	26
27	4.8223	0.2074	63.7057	13.2105	0.0157	0.0757	27
28	5.1117	0.1956	68.5280	13.4062	0.0146	0.0746	28
29	5.4184	0.1846	73.6397	13.5907	0.0136	0.0736	29
30	5.7435	0.1741	79.0580	13.7648	0.0126	0.0726	30
31	6.0881	0.1643	84.8015	13.9291	0.0118	0.0718	31
32	6.4534	0.1550	90.8896	14.0840	0.0110	0.0710	32
33	6.8406	0.1462	97.3430	14.2302	0.0103	0.0703	33
34	7.2510	0.1379	104.1835	14.3681	0.0096	0.0696	34
35	7.6861	0.1301	111.4345	14.4982	0.0090	0.0690	35
40	10.2857	0.0972	154.7616	15.0463	0.0065	0.0665	40
45	13.7646	0.0727	212.7436	15.4558	0.0047	0.0647	45
50	18.4201	0.0543	290.3351	15.7619	0.0034	0.0634	50
55	24.6502	0.0406	394.1708	15.9905	0.0025	0.0625	55
60	32.9876	0.0303	533.1263	16.1614	0.0019	0.0619	60
65	44.1448	0.0227	719.0803	16.2891	0.0014	0.0614	65
70	59.0757	0.0169	967.9294	16.3845	0.0010	0.0610	70

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Table A7. Compound Interest Multipliers, *i* = 7%

n	SINGLE SUM: Future Value 1	SINGLE SUM: Present Value 2	ANNUAL SERIES: Future Value 3	ANNUAL SERIES: Present Value 4	ANNUAL SERIES: Sinking Fund 5	ANNUAL SERIES: Capital Recovery 6	n
1	1.0700	0.9346	1.0000	0.9346	1.0000	1.0700	1
2	1.1449	0.8734	2.0700	1.8080	0.4831	0.5531	2
3	1.2250	0.8163	3.2149	2.6243	0.3111	0.3811	3
4	1.3108	0.7629	4.4399	3.3872	0.2252	0.2952	4
5	1.4026	0.7130	5.7507	4.1002	0.1739	0.2439	5
6	1.5007	0.6663	7.1533	4.7665	0.1398	0.2098	6
7	1.6058	0.6227	8.6540	5.3893	0.1156	0.1856	7
8	1.7182	0.5820	10.2598	5.9713	0.0975	0.1675	8
9	1.8385	0.5439	11.9780	6.5152	0.0835	0.1535	9
10	1.9672	0.5083	13.8165	7.0236	0.0724	0.1424	10
11	2.1049	0.4751	15.7836	7.4987	0.0634	0.1334	11
12	2.2522	0.4440	17.8885	7.9427	0.0559	0.1259	12
13	2.4098	0.4150	20.1407	8.3577	0.0497	0.1197	13
14	2.5785	0.3878	22.5505	8.7455	0.0443	0.1143	14
15	2.7590	0.3624	25.1291	9.1079	0.0398	0.1098	15
16	2.9522	0.3387	27.8881	9.4467	0.0359	0.1059	16
17	3.1588	0.3166	30.8403	9.7632	0.0324	0.1024	17
18	3.3799	0.2959	33.9991	10.0591	0.0294	0.0994	18
19	3.6165	0.2765	37.3790	10.3356	0.0268	0.0968	19
20	3.8697	0.2584	40.9955	10.5940	0.0244	0.0944	20
21	4.1406	0.2415	44.8652	10.8355	0.0223	0.0923	21
22	4.4304	0.2257	49.0058	11.0612	0.0204	0.0904	22
23	4.7405	0.2109	53.4362	11.2722	0.0187	0.0887	23
24	5.0724	0.1971	58.1768	11.4693	0.0172	0.0872	24
25	5.4274	0.1842	63.2491	11.6536	0.0158	0.0858	25
26	5.8074	0.1722	68.6766	11.8258	0.0146	0.0846	26
27	6.2139	0.1609	74.4840	11.9867	0.0134	0.0834	27
28	6.6488	0.1504	80.6978	12.1371	0.0124	0.0824	28
29	7.1143	0.1406	87.3467	12.2777	0.0114	0.0814	29
30	7.6123	0.1314	94.4609	12.4090	0.0106	0.0806	30
31	8.1451	0.1228	102.0732	12.5318	0.0098	0.0798	31
32	8.7153	0.1147	110.2184	12.6466	0.0091	0.0791	32
33	9.3254	0.1072	118.9336	12.7538	0.0084	0.0784	33
34	9.9781	0.1002	128.2590	12.8540	0.0078	0.0778	34
35	10.6766	0.0937	138.2371	12.9477	0.0072	0.0772	35
40	14.9745	0.0668	199.6355	13.3317	0.0050	0.0750	40
45	21.0025	0.0476	285.7500	13.6055	0.0035	0.0735	45
50	29.4571	0.0339	406.5300	13.8007	0.0025	0.0725	50
55	41.3151	0.0242	575.9302	13.9399	0.0017	0.0717	55
60	57.9466	0.0173	813.5228	14.0392	0.0012	0.0712	60
65	81.2731	0.0123	1146.7590	14.1099	0.0009	0.0709	65
70	113.9898	0.0088	1614.1400	14.1604	0.0006	0.0706	70

APPENDIX A

Table A8. Compound Interest Multipliers, i = 8%

n	SINGLE SUM		ANNUAL SERIES				n
	Future Value	Present Value	Future Value	Present Value	Sinking Fund	Capital Recovery	
	1	2	3	4	5	6	
1	1.0800	0.9259	1.0000	0.9259	1.0000	1.0800	1
2	1.1664	0.8573	2.0800	1.7833	0.4808	0.5608	2
3	1.2597	0.7938	3.2464	2.5771	0.3080	0.3880	3
4	1.3605	0.7350	4.5061	3.3121	0.2219	0.3019	4
5	1.4693	0.6806	5.8666	3.9927	0.1705	0.2505	5
6	1.5869	0.6302	7.3359	4.6229	0.1363	0.2163	6
7	1.7138	0.5835	8.9228	5.2064	0.1121	0.1921	7
8	1.8509	0.5403	10.6366	5.7466	0.0940	0.1740	8
9	1.9990	0.5002	12.4876	6.2469	0.0801	0.1601	9
10	2.1589	0.4632	14.4866	6.7101	0.0690	0.1490	10
11	2.3316	0.4289	16.6455	7.1390	0.0601	0.1401	11
12	2.5182	0.3971	18.9771	7.5361	0.0527	0.1327	12
13	2.7196	0.3677	21.4953	7.9038	0.0465	0.1265	13
14	2.9372	0.3405	24.2149	8.2442	0.0413	0.1213	14
15	3.1722	0.3152	27.1521	8.5595	0.0368	0.1168	15
16	3.4259	0.2919	30.3243	8.8514	0.0330	0.1130	16
17	3.7000	0.2703	33.7503	9.1216	0.0296	0.1096	17
18	3.9960	0.2502	37.4503	9.3719	0.0267	0.1067	18
19	4.3157	0.2317	41.4463	9.6036	0.0241	0.1041	19
20	4.6610	0.2145	45.7620	9.8181	0.0219	0.1019	20
21	5.0338	0.1987	50.4230	10.0168	0.0198	0.0998	21
22	5.4365	0.1839	55.4568	10.2007	0.0180	0.0980	22
23	5.8715	0.1703	60.8933	10.3711	0.0164	0.0964	23
24	6.3412	0.1577	66.7648	10.5288	0.0150	0.0950	24
25	6.8485	0.1460	73.1060	10.6748	0.0137	0.0937	25
26	7.3964	0.1352	79.9545	10.8100	0.0125	0.0925	26
27	7.9881	0.1252	87.3509	10.9352	0.0114	0.0914	27
28	8.6271	0.1159	95.3389	11.0511	0.0105	0.0905	28
29	9.3173	0.1073	103.9660	11.1584	0.0096	0.0896	29
30	10.0627	0.0994	113.2833	11.2578	0.0088	0.0888	30
31	10.8677	0.0920	123.3460	11.3498	0.0081	0.0881	31
32	11.7371	0.0852	134.2137	11.4350	0.0075	0.0875	32
33	12.6761	0.0789	145.9508	11.5139	0.0069	0.0869	33
34	13.6901	0.0730	158.6269	11.5869	0.0063	0.0863	34
35	14.7854	0.0676	172.3170	11.6546	0.0058	0.0858	35
40	21.7245	0.0460	259.0569	11.9246	0.0039	0.0839	40
45	31.9205	0.0313	386.5062	12.1084	0.0026	0.0826	45
50	46.9017	0.0213	573.7711	12.2335	0.0017	0.0817	50
55	68.9140	0.0145	848.9247	12.3186	0.0012	0.0812	55
60	101.2573	0.0099	1253.2160	12.3766	0.0008	0.0808	60
65	148.7802	0.0067	1847.2520	12.4160	0.0005	0.0805	65
70	218.6069	0.0046	2720.0860	12.4428	0.0004	0.0804	70

APPENDIX A

Table A9. Compound Interest Multipliers, i = 9%

n	SINGLE SUM		ANNUAL SERIES				n
	Future Value	Present Value	Future Value	Present Value	Sinking Fund	Capital Recovery	
	1	2	3	4	5	6	
1	1.0900	0.9174	1.0000	0.9174	1.0000	1.0900	1
2	1.1881	0.8417	2.0900	1.7591	0.4785	0.5685	2
3	1.2950	0.7722	3.2781	2.5313	0.3051	0.3951	3
4	1.4116	0.7084	4.5731	3.2397	0.2187	0.3087	4
5	1.5386	0.6499	5.9847	3.8897	0.1671	0.2571	5
6	1.6771	0.5963	7.5233	4.4859	0.1329	0.2229	6
7	1.8280	0.5470	9.2004	5.0330	0.1087	0.1987	7
8	1.9926	0.5019	11.0285	5.5348	0.0907	0.1807	8
9	2.1719	0.4604	13.0210	5.9952	0.0768	0.1668	9
10	2.3674	0.4224	15.1929	6.4177	0.0658	0.1558	10
11	2.5804	0.3875	17.5603	6.8052	0.0569	0.1469	11
12	2.8127	0.3555	20.1407	7.1607	0.0497	0.1397	12
13	3.0658	0.3262	22.9534	7.4869	0.0436	0.1336	13
14	3.3417	0.2992	26.0192	7.7862	0.0384	0.1284	14
15	3.6425	0.2745	29.3609	8.0607	0.0341	0.1241	15
16	3.9703	0.2519	33.0034	8.3126	0.0303	0.1203	16
17	4.3276	0.2311	36.9737	8.5436	0.0270	0.1170	17
18	4.7171	0.2120	41.3014	8.7556	0.0242	0.1142	18
19	5.1417	0.1945	46.0185	8.9501	0.0217	0.1117	19
20	5.6044	0.1784	51.1602	9.1285	0.0195	0.1095	20
21	6.1088	0.1637	56.7646	9.2922	0.0176	0.1076	21
22	6.6586	0.1502	62.8734	9.4424	0.0159	0.1059	22
23	7.2579	0.1378	69.5320	9.5802	0.0144	0.1044	23
24	7.9111	0.1264	76.7899	9.7066	0.0130	0.1030	24
25	8.6231	0.1160	84.7010	9.8226	0.0118	0.1018	25
26	9.3992	0.1064	93.3241	9.9290	0.0107	0.1007	26
27	10.2451	0.0976	102.7233	10.0266	0.0097	0.0997	27
28	11.1672	0.0895	112.9684	10.1161	0.0089	0.0989	28
29	12.1722	0.0822	124.1355	10.1983	0.0081	0.0981	29
30	13.2677	0.0754	136.3077	10.2737	0.0073	0.0973	30
31	14.4618	0.0691	149.5754	10.3428	0.0067	0.0967	31
32	15.7634	0.0634	164.0372	10.4062	0.0061	0.0961	32
33	17.1821	0.0582	179.8006	10.4644	0.0056	0.0956	33
34	18.7284	0.0534	196.9827	10.5178	0.0051	0.0951	34
35	20.4140	0.0490	215.7111	10.5668	0.0046	0.0946	35
40	31.4095	0.0318	337.8831	10.7574	0.0030	0.0930	40
45	48.3274	0.0207	525.8598	10.8812	0.0019	0.0919	45
50	74.3577	0.0134	815.0853	10.9617	0.0012	0.0912	50
55	114.4085	0.0087	1260.0950	11.0140	0.0008	0.0908	55
60	176.0318	0.0057	1944.7970	11.0480	0.0005	0.0905	60
65	270.8468	0.0037	2998.2970	11.0701	0.0003	0.0903	65
70	416.7314	0.0024	4619.2380	11.0844	0.0002	0.0902	70

Table A10. Compound Interest Multipliers, i = 10%

n	SINGLE SUM Future Value 1	Present Value 2	ANNUAL SERIES Future Value 3	Present Value 4	Sinking Fund 5	Capital Recovery 6	n
1	1.1000	0.9091	1.0000	0.9091	1.0000	1.1000	1
2	1.2100	0.8264	2.1000	1.7355	0.4762	0.5762	2
3	1.3310	0.7513	3.3100	2.4869	0.3021	0.4021	3
4	1.4641	0.6830	4.6410	3.1699	0.2155	0.3155	4
5	1.6105	0.6209	6.1051	3.7908	0.1638	0.2638	5
6	1.7716	0.5645	7.7156	4.3553	0.1296	0.2296	6
7	1.9487	0.5132	9.4872	4.8684	0.1054	0.2054	7
8	2.1436	0.4665	11.4359	5.3349	0.0874	0.1874	8
9	2.3579	0.4241	13.5795	5.7590	0.0736	0.1736	9
10	2.5937	0.3855	15.9374	6.1446	0.0627	0.1627	10
11	2.8531	0.3505	18.5312	6.4951	0.0540	0.1540	11
12	3.1384	0.3186	21.3843	6.8137	0.0468	0.1468	12
13	3.4523	0.2897	24.5227	7.1034	0.0408	0.1408	13
14	3.7975	0.2633	27.9750	7.3667	0.0357	0.1357	14
15	4.1772	0.2394	31.7725	7.6061	0.0315	0.1315	15
16	4.5950	0.2176	35.9497	7.8237	0.0278	0.1278	16
17	5.0545	0.1978	40.5447	8.0216	0.0247	0.1247	17
18	5.5599	0.1799	45.5992	8.2014	0.0219	0.1219	18
19	6.1159	0.1635	51.1591	8.3649	0.0195	0.1195	19
20	6.7275	0.1486	57.2750	8.5136	0.0175	0.1175	20
21	7.4003	0.1351	64.0025	8.6487	0.0156	0.1156	21
22	8.1403	0.1228	71.4028	8.7715	0.0140	0.1140	22
23	8.9543	0.1117	79.5431	8.8832	0.0126	0.1126	23
24	9.8497	0.1015	88.4974	8.9847	0.0113	0.1113	24
25	10.8347	0.0923	98.3471	9.0770	0.0102	0.1102	25
26	11.9182	0.0839	109.1818	9.1609	0.0092	0.1092	26
27	13.1100	0.0763	121.1000	9.2372	0.0083	0.1083	27
28	14.4210	0.0693	134.2100	9.3066	0.0075	0.1075	28
29	15.8631	0.0630	148.6310	9.3696	0.0067	0.1067	29
30	17.4494	0.0573	164.4941	9.4269	0.0061	0.1061	30
31	19.1944	0.0521	181.9435	9.4790	0.0055	0.1055	31
32	21.1138	0.0474	201.1379	9.5264	0.0050	0.1050	32
33	23.2252	0.0431	222.2517	9.5694	0.0045	0.1045	33
34	25.5477	0.0391	245.4768	9.6086	0.0041	0.1041	34
35	28.1023	0.0356	271.0245	9.6442	0.0037	0.1037	35
40	45.2593	0.0221	442.5928	9.7791	0.0023	0.1023	40
45	72.8905	0.0137	718.9053	9.8628	0.0014	0.1014	45
50	117.3909	0.0085	1163.9090	9.9148	0.0009	0.1009	50
55	189.0593	0.0053	1880.5930	9.9471	0.0005	0.1005	55
60	304.4819	0.0033	3034.8190	9.9672	0.0003	0.1003	60
65	490.3712	0.0020	4893.7120	9.9796	0.0002	0.1002	65
70	789.7478	0.0013	7987.4780	9.9873	0.0001	0.1001	70

APPENDIX A

Table A11. Compound Interest Multipliers, i = 11%

	SINGLE SUM		ANNUAL SERIES				
n	Future Value	Present Value	Future Value	Present Value	Sinking Fund	Capital Recovery	n
	1	2	3	4	5	6	
1	1.1100	0.9009	1.0000	0.9009	1.0000	1.1100	1
2	1.2321	0.8116	2.1100	1.7125	0.4739	0.5839	2
3	1.3676	0.7312	3.3421	2.4437	0.2992	0.4092	3
4	1.5181	0.6587	4.7097	3.1024	0.2123	0.3223	4
5	1.6851	0.5935	6.2278	3.6959	0.1606	0.2706	5
6	1.8704	0.5346	7.9129	4.2305	0.1264	0.2364	6
7	2.0762	0.4817	9.7833	4.7122	0.1022	0.2122	7
8	2.3045	0.4339	11.8594	5.1461	0.0843	0.1943	8
9	2.5580	0.3909	14.1640	5.5370	0.0706	0.1806	9
10	2.8394	0.3522	16.7220	5.8892	0.0598	0.1698	10
11	3.1518	0.3173	19.5614	6.2065	0.0511	0.1611	11
12	3.4985	0.2858	22.7132	6.4924	0.0440	0.1540	12
13	3.8833	0.2575	26.2116	6.7499	0.0382	0.1482	13
14	4.3104	0.2320	30.0949	6.9819	0.0332	0.1432	14
15	4.7846	0.2090	34.4054	7.1909	0.0291	0.1391	15
16	5.3109	0.1883	39.1900	7.3792	0.0255	0.1355	16
17	5.8951	0.1696	44.5008	7.5488	0.0225	0.1325	17
18	6.5436	0.1528	50.3959	7.7016	0.0198	0.1298	18
19	7.2633	0.1377	56.9395	7.8393	0.0176	0.1276	19
20	8.0623	0.1240	64.2028	7.9633	0.0156	0.1256	20
21	8.9492	0.1117	72.2652	8.0751	0.0138	0.1238	21
22	9.9336	0.1007	81.2143	8.1757	0.0123	0.1223	22
23	11.0263	0.0907	91.1479	8.2664	0.0110	0.1210	23
24	12.2392	0.0817	102.1742	8.3481	0.0098	0.1198	24
25	13.5855	0.0736	114.4133	8.4217	0.0087	0.1187	25
26	15.0799	0.0663	127.9988	8.4881	0.0078	0.1178	26
27	16.7386	0.0597	143.0786	8.5478	0.0070	0.1170	27
28	18.5799	0.0538	159.8173	8.6016	0.0063	0.1163	28
29	20.6237	0.0485	178.3972	8.6501	0.0056	0.1156	29
30	22.8923	0.0437	199.0209	8.6938	0.0050	0.1150	30
31	25.4105	0.0394	221.9132	8.7331	0.0045	0.1145	31
32	28.2056	0.0355	247.3237	8.7686	0.0040	0.1140	32
33	31.3082	0.0319	275.5292	8.8005	0.0036	0.1136	33
34	34.7521	0.0288	306.8375	8.8293	0.0033	0.1133	34
35	38.5749	0.0259	341.5896	8.8552	0.0029	0.1129	35
40	65.0009	0.0154	581.8261	8.9511	0.0017	0.1117	40
45	109.5303	0.0091	986.6387	9.0079	0.0010	0.1110	45
50	184.5649	0.0054	1668.7710	9.0417	0.0006	0.1106	50
55	311.0025	0.0032	2818.2050	9.0617	0.0004	0.1104	55
60	524.0573	0.0019	4755.0670	9.0736	0.0002	0.1102	60
65	883.0671	0.0011	8018.7920	9.0806	0.0001	0.1101	65
70	1488.0190	0.0007	13518.3600	9.0848	0.0001	0.1101	70

Table A12. Compound Interest Multipliers. i = 12%

n	SINGLE SUM		ANNUAL SERIES				n
	Future Value	Present Value	Future Value	Present Value	Sinking Fund	Capital Recovery	
	1	2	3	4	5	6	
1	1.1200	0.8929	1.0000	0.8929	1.0000	1.1200	1
2	1.2544	0.7972	2.1200	1.6901	0.4717	0.5917	2
3	1.4049	0.7118	3.3744	2.4018	0.2963	0.4163	3
4	1.5735	0.6355	4.7793	3.0373	0.2092	0.3292	4
5	1.7623	0.5674	6.3528	3.6048	0.1574	0.2774	5
6	1.9738	0.5066	8.1152	4.1114	0.1232	0.2432	6
7	2.2107	0.4523	10.0890	4.5638	0.0991	0.2191	7
8	2.4760	0.4039	12.2997	4.9676	0.0813	0.2013	8
9	2.7731	0.3606	14.7757	5.3283	0.0677	0.1877	9
10	3.1058	0.3220	17.5487	5.6502	0.0570	0.1770	10
11	3.4785	0.2875	20.6546	5.9377	0.0484	0.1684	11
12	3.8960	0.2567	24.1331	6.1944	0.0414	0.1614	12
13	4.3635	0.2292	28.0291	6.4235	0.0357	0.1557	13
14	4.8871	0.2046	32.3926	6.6282	0.0309	0.1509	14
15	5.4736	0.1827	37.2797	6.8109	0.0268	0.1468	15
16	6.1304	0.1631	42.7533	6.9740	0.0234	0.1434	16
17	6.8660	0.1456	48.8837	7.1196	0.0205	0.1405	17
18	7.6900	0.1300	55.7497	7.2497	0.0179	0.1379	18
19	8.6128	0.1161	63.4397	7.3658	0.0158	0.1358	19
20	9.6463	0.1037	72.0524	7.4694	0.0139	0.1339	20
21	10.8038	0.0926	81.6987	7.5620	0.0122	0.1322	21
22	12.1003	0.0826	92.5026	7.6446	0.0108	0.1308	22
23	13.5523	0.0738	104.6029	7.7184	0.0096	0.1296	23
24	15.1786	0.0659	118.1552	7.7843	0.0085	0.1285	24
25	17.0001	0.0588	133.3339	7.8431	0.0075	0.1275	25
26	19.0401	0.0525	150.3339	7.8957	0.0067	0.1267	26
27	21.3249	0.0469	169.3740	7.9426	0.0059	0.1259	27
28	23.8839	0.0419	190.6989	7.9844	0.0052	0.1252	28
29	26.7499	0.0374	214.5828	8.0218	0.0047	0.1247	29
30	29.9599	0.0334	241.3327	8.0552	0.0041	0.1241	30
31	33.5551	0.0298	271.2926	8.0850	0.0037	0.1237	31
32	37.5817	0.0266	304.8477	8.1116	0.0033	0.1233	32
33	42.0915	0.0238	342.4295	8.1354	0.0029	0.1229	33
34	47.1425	0.0212	384.5210	8.1566	0.0026	0.1226	34
35	52.7996	0.0189	431.6635	8.1755	0.0023	0.1223	35
40	93.0510	0.0107	767.0914	8.2438	0.0013	0.1213	40
45	163.9876	0.0061	1358.2300	8.2825	0.0007	0.1207	45
50	289.0022	0.0035	2400.0180	8.3045	0.0004	0.1204	50
55	509.3206	0.0020	4236.0050	8.3170	0.0002	0.1202	55
60	897.5969	0.0011	7471.6410	8.3240	0.0001	0.1201	60
65	1581.8720	0.0006	13173.9400	8.3281	0.0001	0.1201	65
70	2787.8000	0.0004	23223.3300	8.3303	*	0.1200	70

* smaller than .0001

Table A13. Compound Interest Multipliers, i = 13%

n	SINGLE SUM Future Value 1	Present Value 2	ANNUAL SERIES Future Value 3	Present Value 4	Sinking Fund 5	Capital Recovery 6	n
1	1.1300	0.8850	1.0000	0.8850	1.0000	1.1300	1
2	1.2769	0.7831	2.1300	1.6681	0.4695	0.5995	2
3	1.4429	0.6931	3.4069	2.3612	0.2935	0.4235	3
4	1.6305	0.6133	4.8498	2.9745	0.2062	0.3362	4
5	1.8424	0.5428	6.4803	3.5172	0.1543	0.2843	5
6	2.0820	0.4803	8.3227	3.9975	0.1202	0.2502	6
7	2.3526	0.4251	10.4047	4.4226	0.0961	0.2261	7
8	2.6584	0.3762	12.7573	4.7988	0.0784	0.2084	8
9	3.0040	0.3329	15.4157	5.1317	0.0649	0.1949	9
10	3.3946	0.2946	18.4197	5.4262	0.0543	0.1843	10
11	3.8359	0.2607	21.8143	5.6869	0.0458	0.1758	11
12	4.3345	0.2307	25.6502	5.9176	0.0390	0.1690	12
13	4.8980	0.2042	29.9847	6.1218	0.0334	0.1634	13
14	5.5348	0.1807	34.8827	6.3025	0.0287	0.1587	14
15	6.2543	0.1599	40.4174	6.4624	0.0247	0.1547	15
16	7.0673	0.1415	46.6717	6.6039	0.0214	0.1514	16
17	7.9861	0.1252	53.7390	6.7291	0.0186	0.1486	17
18	9.0243	0.1108	61.7251	6.8399	0.0162	0.1462	18
19	10.1974	0.0981	70.7494	6.9380	0.0141	0.1441	19
20	11.5231	0.0868	80.9468	7.0248	0.0124	0.1424	20
21	13.0211	0.0768	92.4699	7.1015	0.0108	0.1408	21
22	14.7138	0.0680	105.4909	7.1695	0.0095	0.1395	22
23	16.6266	0.0601	120.2048	7.2297	0.0083	0.1383	23
24	18.7881	0.0532	136.8314	7.2829	0.0073	0.1373	24
25	21.2305	0.0471	155.6194	7.3300	0.0064	0.1364	25
26	23.9905	0.0417	176.8500	7.3717	0.0057	0.1357	26
27	27.1093	0.0369	200.8404	7.4086	0.0050	0.1350	27
28	30.6335	0.0326	227.9497	7.4412	0.0044	0.1344	28
29	34.6158	0.0289	258.5831	7.4701	0.0039	0.1339	29
30	39.1159	0.0256	293.1990	7.4957	0.0034	0.1334	30
31	44.2009	0.0226	332.3148	7.5183	0.0030	0.1330	31
32	49.9470	0.0200	376.5157	7.5383	0.0027	0.1327	32
33	56.4402	0.0177	426.4627	7.5560	0.0023	0.1323	33
34	63.7774	0.0157	482.9029	7.5717	0.0021	0.1321	34
35	72.0684	0.0139	546.6803	7.5856	0.0018	0.1318	35
40	132.7814	0.0075	1013.7030	7.6344	0.0010	0.1310	40
45	244.6410	0.0041	1874.1620	7.6609	0.0005	0.1305	45
50	450.7352	0.0022	3459.5020	7.6752	0.0003	0.1303	50
55	830.4503	0.0012	6380.3870	7.6830	0.0002	0.1302	55
60	1530.0500	0.0007	11761.9300	7.6873	0.0001	0.1301	60
65	2819.0190	0.0004	21677.0700	7.6896	*	0.1300	65
70	5193.8580	0.0002	39945.0600	7.6908	*	0.1300	70

* smaller than .0001

APPENDIX A

Table A14. Compound Interest Multipliers, i = 14%

n	SINGLE SUM Future Value	Present Value	ANNUAL SERIES Future Value	Present Value	Sinking Fund	Capital Recovery	n
	1	2	3	4	5	6	
1	1.1400	0.8772	1.0000	0.8772	1.0000	1.1400	1
2	1.2996	0.7695	2.1400	1.6467	0.4673	0.6073	2
3	1.4815	0.6750	3.4396	2.3216	0.2907	0.4307	3
4	1.6890	0.5921	4.9211	2.9137	0.2032	0.3432	4
5	1.9254	0.5194	6.6101	3.4331	0.1513	0.2913	5
6	2.1950	0.4556	8.5355	3.8887	0.1172	0.2572	6
7	2.5023	0.3996	10.7305	4.2883	0.0932	0.2332	7
8	2.8526	0.3506	13.2328	4.6389	0.0756	0.2156	8
9	3.2519	0.3075	16.0853	4.9464	0.0622	0.2022	9
10	3.7072	0.2697	19.3373	5.2161	0.0517	0.1917	10
11	4.2262	0.2366	23.0445	5.4527	0.0434	0.1834	11
12	4.8179	0.2076	27.2708	5.6603	0.0367	0.1767	12
13	5.4924	0.1821	32.0887	5.8424	0.0312	0.1712	13
14	6.2613	0.1597	37.5811	6.0021	0.0266	0.1666	14
15	7.1379	0.1401	43.8424	6.1422	0.0228	0.1628	15
16	8.1372	0.1229	50.9804	6.2651	0.0196	0.1596	16
17	9.2765	0.1078	59.1176	6.3729	0.0169	0.1569	17
18	10.5752	0.0946	68.3941	6.4674	0.0146	0.1546	18
19	12.0557	0.0829	78.9692	6.5504	0.0127	0.1527	19
20	13.7435	0.0728	91.0249	6.6231	0.0110	0.1510	20
21	15.6676	0.0638	104.7684	6.6870	0.0095	0.1495	21
22	17.8610	0.0560	120.4360	6.7429	0.0083	0.1483	22
23	20.3616	0.0491	138.2971	6.7921	0.0072	0.1472	23
24	23.2122	0.0431	158.6587	6.8351	0.0063	0.1463	24
25	26.4619	0.0378	181.8708	6.8729	0.0055	0.1455	25
26	30.1666	0.0331	208.3328	6.9061	0.0048	0.1448	26
27	34.3899	0.0291	238.4994	6.9352	0.0042	0.1442	27
28	39.2045	0.0255	272.8893	6.9607	0.0037	0.1437	28
29	44.6931	0.0224	312.0938	6.9830	0.0032	0.1432	29
30	50.9502	0.0196	356.7869	7.0027	0.0028	0.1428	30
31	58.0832	0.0172	407.7371	7.0199	0.0025	0.1425	31
32	66.2148	0.0151	465.8203	7.0350	0.0021	0.1421	32
33	75.4849	0.0132	532.0351	7.0482	0.0019	0.1419	33
34	86.0528	0.0116	607.5200	7.0599	0.0016	0.1416	34
35	98.1002	0.0102	693.5728	7.0700	0.0014	0.1414	35
40	188.8836	0.0053	1342.0250	7.1050	0.0007	0.1407	40
45	363.6792	0.0027	2590.5650	7.1232	0.0004	0.1404	45
50	700.2331	0.0014	4994.5230	7.1327	0.0002	0.1402	50
55	1348.2390	0.0007	9623.1370	7.1376	0.0001	0.1401	55
60	2595.9200	0.0004	18535.1400	7.1401	0.0001	0.1401	60
65	4998.2210	0.0002	35694.4300	7.1414	*	0.1400	65
70	9623.6490	0.0001	68733.2100	7.1421	*	0.1400	70

* smaller than .0001

APPENDIX A

Table A15. Compound Interest Multipliers, i = 15%

n	SINGLE SUM		ANNUAL SERIES				n
	Future Value	Present Value	Future Value	Present Value	Sinking Fund	Capital Recovery	
	1	2	3	4	5	6	
1	1.1500	0.8696	1.0000	0.8696	1.0000	1.1500	1
2	1.3225	0.7561	2.1500	1.6257	0.4651	0.6151	2
3	1.5209	0.6575	3.4725	2.2832	0.2880	0.4380	3
4	1.7490	0.5718	4.9934	2.8550	0.2003	0.3503	4
5	2.0114	0.4972	6.7424	3.3522	0.1483	0.2983	5
6	2.3131	0.4323	8.7537	3.7845	0.1142	0.2642	6
7	2.6600	0.3759	11.0668	4.1604	0.0904	0.2404	7
8	3.0590	0.3269	13.7268	4.4873	0.0729	0.2229	8
9	3.5179	0.2843	16.7858	4.7716	0.0596	0.2096	9
10	4.0456	0.2472	20.3037	5.0188	0.0493	0.1993	10
11	4.6524	0.2149	24.3493	5.2337	0.0411	0.1911	11
12	5.3503	0.1869	29.0017	5.4206	0.0345	0.1845	12
13	6.1528	0.1625	34.3519	5.5831	0.0291	0.1791	13
14	7.0757	0.1413	40.5047	5.7245	0.0247	0.1747	14
15	8.1371	0.1229	47.5804	5.8474	0.0210	0.1710	15
16	9.3576	0.1069	55.7175	5.9542	0.0179	0.1679	16
17	10.7613	0.0929	65.0751	6.0472	0.0154	0.1654	17
18	12.3755	0.0808	75.8364	6.1280	0.0132	0.1632	18
19	14.2318	0.0703	88.2118	6.1982	0.0113	0.1613	19
20	16.3665	0.0611	102.4436	6.2593	0.0098	0.1598	20
21	18.8215	0.0531	118.8101	6.3125	0.0084	0.1584	21
22	21.6447	0.0462	137.6316	6.3587	0.0073	0.1573	22
23	24.8915	0.0402	159.2764	6.3988	0.0063	0.1563	23
24	28.6252	0.0349	184.1679	6.4338	0.0054	0.1554	24
25	32.9190	0.0304	212.7930	6.4641	0.0047	0.1547	25
26	37.8568	0.0264	245.7120	6.4906	0.0041	0.1541	26
27	43.5353	0.0230	283.5688	6.5135	0.0035	0.1535	27
28	50.0656	0.0200	327.1041	6.5335	0.0031	0.1531	28
29	57.5755	0.0174	377.1697	6.5509	0.0027	0.1527	29
30	66.2118	0.0151	434.7452	6.5660	0.0023	0.1523	30
31	76.1436	0.0131	500.9570	6.5791	0.0020	0.1520	31
32	87.5651	0.0114	577.1005	6.5905	0.0017	0.1517	32
33	100.6998	0.0099	664.6655	6.6005	0.0015	0.1515	33
34	115.8048	0.0086	765.3653	6.6091	0.0013	0.1513	34
35	133.1755	0.0075	881.1701	6.6166	0.0011	0.1511	35
40	267.8635	0.0037	1779.0900	6.6418	0.0006	0.1506	40
45	538.7693	0.0019	3585.1280	6.6543	0.0003	0.1503	45
50	1083.6580	0.0009	7217.7170	6.6605	0.0001	0.1501	50
55	2179.6220	0.0005	14524.1500	6.6636	0.0001	0.1501	55
60	4383.9990	0.0002	29219.9900	6.6651	*	0.1500	60
65	8817.7870	0.0001	58778.5800	6.6659	*	0.1500	65
70	17735.7200	0.0001	118231.50	6.6663	*	0.1500	70

* smaller than .0001

APPENDIX A

Table A16. Compound Interest Multipliers, i = 16%

n	SINGLE SUM Future Value	Present Value	ANNUAL SERIES Future Value	Present Value	Sinking Fund	Capital Recovery	n
	1	2	3	4	5	6	
1	1.1600	0.8621	1.0000	0.8621	1.0000	1.1600	1
2	1.3456	0.7432	2.1600	1.6052	0.4630	0.6230	2
3	1.5609	0.6407	3.5056	2.2459	0.2853	0.4453	3
4	1.8106	0.5523	5.0665	2.7982	0.1974	0.3574	4
5	2.1003	0.4761	6.8771	3.2743	0.1454	0.3054	5
6	2.4364	0.4104	8.9775	3.6847	0.1114	0.2714	6
7	2.8262	0.3538	11.4139	4.0386	0.0876	0.2476	7
8	3.2784	0.3050	14.2401	4.3436	0.0702	0.2302	8
9	3.8030	0.2630	17.5185	4.6065	0.0571	0.2171	9
10	4.4114	0.2267	21.3215	4.8332	0.0469	0.2069	10
11	5.1173	0.1954	25.7329	5.0286	0.0389	0.1989	11
12	5.9360	0.1685	30.8502	5.1971	0.0324	0.1924	12
13	6.8858	0.1452	36.7862	5.3423	0.0272	0.1872	13
14	7.9875	0.1252	43.6720	5.4675	0.0229	0.1829	14
15	9.2655	0.1079	51.6595	5.5755	0.0194	0.1794	15
16	10.7480	0.0930	60.9250	5.6685	0.0164	0.1764	16
17	12.4677	0.0902	71.6730	5.7487	0.0140	0.1740	17
18	14.4625	0.0691	84.1407	5.8178	0.0119	0.1719	18
19	16.7765	0.0596	98.6032	5.8775	0.0101	0.1701	19
20	19.4607	0.0514	115.3797	5.9288	0.0087	0.1687	20
21	22.5745	0.0443	134.8404	5.9731	0.0074	0.1674	21
22	26.1864	0.0382	157.4148	6.0113	0.0064	0.1664	22
23	30.3762	0.0329	183.6012	6.0442	0.0054	0.1654	23
24	35.2364	0.0284	213.9774	6.0726	0.0047	0.1647	24
25	40.8742	0.0245	249.2138	6.0971	0.0040	0.1640	25
26	47.4141	0.0211	290.0880	6.1182	0.0034	0.1634	26
27	55.0003	0.0182	337.5020	6.1364	0.0030	0.1630	27
28	63.8004	0.0157	392.5023	6.1520	0.0025	0.1625	28
29	74.0084	0.0135	456.3027	6.1656	0.0022	0.1622	29
30	95.8498	0.0116	530.3111	6.1772	0.0019	0.1619	30
31	99.5857	0.0100	616.1608	6.1872	0.0016	0.1616	31
32	115.5195	0.0087	715.7466	6.1959	0.0014	0.1614	32
33	134.0026	0.0075	831.2660	6.2034	0.0012	0.1612	33
34	155.4430	0.0064	765.2686	6.2098	0.0010	0.1610	34
35	180.3138	0.0055	1120.7110	6.2153	0.0009	0.1609	35
40	378.7206	0.0026	2360.7540	6.2335	0.0004	0.1604	40
45	795.4424	0.0013	4965.2650	6.2421	0.0002	0.1602	45
50	1670.7010	0.0006	10435.6300	6.2463	0.0001	0.1601	50
55	3509.0410	0.0003	21925.2600	6.2482	*	0.1600	55
60	7370.1830	0.0001	46057.4000	6.2492	*	0.1600	60
65	15479.9000	0.0001	96743.1200	6.2496	*	0.1600	65
70	32513.0700	*	203200.50	6.2498	*	0.1600	70

* smaller than .0001

APPENDIX A

Table A17. Compound Interest Multipliers, i = 17%

n	SINGLE SUM Future Value	SINGLE SUM Present Value	ANNUAL SERIES Future Value	ANNUAL SERIES Present Value	ANNUAL SERIES Sinking Fund	ANNUAL SERIES Capital Recovery	n
	1	2	3	4	5	6	
1	1.1700	0.8547	1.0000	0.8547	1.0000	1.1700	1
2	1.3689	0.7305	2.1700	1.5852	0.4608	0.6308	2
3	1.6016	0.6244	3.5389	2.2096	0.2826	0.4526	3
4	1.8739	0.5337	5.1405	2.7432	0.1945	0.3645	4
5	2.1924	0.4561	7.0144	3.1993	0.1426	0.3126	5
6	2.5652	0.3898	9.2068	3.5892	0.1086	0.2786	6
7	3.0012	0.3332	11.7720	3.9224	0.0849	0.2549	7
8	3.5115	0.2848	14.7733	4.2072	0.0677	0.2377	8
9	4.1084	0.2434	18.2847	4.4506	0.0547	0.2247	9
10	4.8068	0.2080	22.3931	4.6586	0.0447	0.2147	10
11	5.6240	0.1778	27.1999	4.8364	0.0368	0.2068	11
12	6.5801	0.1520	32.8239	4.9884	0.0305	0.2005	12
13	7.6987	0.1299	39.4040	5.1183	0.0254	0.1954	13
14	9.0075	0.1110	47.1027	5.2293	0.0212	0.1912	14
15	10.5387	0.0949	56.1101	5.3242	0.0178	0.1878	15
16	12.3303	0.0811	66.6488	5.4053	0.0150	0.1850	16
17	14.4265	0.0693	78.9791	5.4746	0.0127	0.1827	17
18	16.8789	0.0592	93.4056	5.5339	0.0107	0.1807	18
19	19.7484	0.0506	110.2845	5.5845	0.0091	0.1791	19
20	23.1056	0.0433	130.0329	5.6278	0.0077	0.1777	20
21	27.0335	0.0370	153.1384	5.6648	0.0065	0.1765	21
22	31.6292	0.0316	180.1720	5.6964	0.0056	0.1756	22
23	37.0062	0.0270	211.8012	5.7234	0.0047	0.1747	23
24	43.2973	0.0231	248.8074	5.7465	0.0040	0.1740	24
25	50.6578	0.0197	292.1047	5.7662	0.0034	0.1734	25
26	59.2696	0.0169	342.7625	5.7831	0.0029	0.1729	26
27	69.3455	0.0144	402.0321	5.7975	0.0025	0.1725	27
28	81.1342	0.0123	471.3775	5.8099	0.0021	0.1721	28
29	94.9270	0.0105	552.5117	5.8204	0.0018	0.1718	29
30	111.0646	0.0090	647.4386	5.8294	0.0015	0.1715	30
31	129.9455	0.0077	758.5032	5.8371	0.0013	0.1713	31
32	152.0363	0.0066	888.4488	5.8437	0.0011	0.1711	32
33	177.8825	0.0056	1040.4850	5.8493	0.0010	0.1710	33
34	208.1225	0.0048	1218.3670	5.8541	0.0008	0.1708	34
35	243.5033	0.0041	1426.4900	5.8582	0.0007	0.1707	35
40	533.8683	0.0019	3134.5190	5.8713	0.0003	0.1703	40
45	1170.4780	0.0009	6879.2830	5.8773	0.0001	0.1701	45
50	2566.2130	0.0004	15089.4900	5.8801	0.0001	0.1701	50
55	5626.2860	0.0002	33089.7200	5.8813	*	0.1700	55
60	12335.3400	0.0001	72554.9400	5.8819	*	0.1700	60
65	27044.5900	*	159079.70	5.8821	*	0.1700	65
70	59293.8500	*	348781.50	5.8823	*	0.1700	70

* smaller than .0001

Table A18. Compound Interest Multipliers, i = 18%

n	SINGLE SUM: Future Value (1)	SINGLE SUM: Present Value (2)	ANNUAL SERIES: Future Value (3)	ANNUAL SERIES: Present Value (4)	ANNUAL SERIES: Sinking Fund (5)	ANNUAL SERIES: Capital Recovery (6)	n
1	1.1800	0.8475	1.0000	0.8475	1.0000	1.1800	1
2	1.3924	0.7182	2.1800	1.5656	0.4587	0.6387	2
3	1.6430	0.6086	3.5724	2.1743	0.2799	0.4599	3
4	1.9388	0.5158	5.2154	2.6901	0.1917	0.3717	4
5	2.2878	0.4371	7.1542	3.1272	0.1398	0.3198	5
6	2.6996	0.3704	9.4420	3.4976	0.1059	0.2859	6
7	3.1855	0.3139	12.1415	3.8115	0.0824	0.2624	7
8	3.7589	0.2660	15.3270	4.0776	0.0652	0.2452	8
9	4.4355	0.2255	19.0859	4.3030	0.0524	0.2324	9
10	5.2338	0.1911	23.5213	4.4941	0.0425	0.2225	10
11	6.1759	0.1619	28.7552	4.6560	0.0348	0.2148	11
12	7.2876	0.1372	34.9311	4.7932	0.0286	0.2086	12
13	8.5994	0.1163	42.2187	4.9095	0.0237	0.2037	13
14	10.1473	0.0985	50.8181	5.0081	0.0197	0.1997	14
15	11.9738	0.0835	60.9653	5.0916	0.0164	0.1964	15
16	14.1290	0.0708	72.9391	5.1624	0.0137	0.1937	16
17	16.6723	0.0600	87.0681	5.2223	0.0115	0.1915	17
18	19.6733	0.0508	103.7404	5.2732	0.0096	0.1896	18
19	23.2145	0.0431	123.4137	5.3162	0.0081	0.1881	19
20	27.3931	0.0365	146.6281	5.3527	0.0068	0.1868	20
21	32.3238	0.0309	174.0212	5.3837	0.0057	0.1857	21
22	38.1421	0.0262	206.3450	5.4099	0.0048	0.1848	22
23	45.0077	0.0222	244.4872	5.4321	0.0041	0.1841	23
24	53.1091	0.0188	289.4949	5.4509	0.0035	0.1835	24
25	62.6687	0.0160	342.6039	5.4669	0.0029	0.1829	25
26	73.9491	0.0135	405.2727	5.4804	0.0025	0.1825	26
27	87.2599	0.0115	479.2219	5.4919	0.0021	0.1821	27
28	102.9667	0.0097	566.4817	5.5016	0.0018	0.1818	28
29	121.5007	0.0082	669.4485	5.5098	0.0015	0.1815	29
30	143.3709	0.0070	790.9493	5.5168	0.0013	0.1813	30
31	169.1776	0.0059	934.3202	5.5227	0.0011	0.1811	31
32	199.6296	0.0050	1103.4980	5.5277	0.0009	0.1809	32
33	235.5630	0.0042	1303.1280	5.5320	0.0008	0.1808	33
34	277.9645	0.0036	1538.6910	5.5356	0.0006	0.1806	34
35	327.9979	0.0030	1816.6550	5.5386	0.0006	0.1806	35
40	750.3900	0.0013	4163.2220	5.5482	0.0002	0.1802	40
45	1716.6880	0.0006	9531.5990	5.5523	0.0001	0.1801	45
50	3927.3680	0.0003	21813.1500	5.5541	*	0.1800	50
55	8984.8660	0.0001	49910.3900	5.5549	*	0.1800	55
60	20555.2100	*	114190.00	5.5553	*	0.1800	60
65	47025.3500	*	261246.40	5.5554	*	0.1800	65
70	107582.600	*	597675.60	5.5555	*	0.1800	70

* smaller than .0001

APPENDIX B

Interest Multipliers, Terminating Periodic Series.

		i=4%		i=8%		i=12%	
k	t	Future Value 1a	Present Value 2a	Future Value 1b	Present Value 2b	Future Value 1c	Present Value 2c
2	2	2.0816	1.7794	2.1664	1.5924	2.2544	1.4327
2	3	2.1249	1.6793	2.2597	1.4240	2.4049	1.2184
2	4	2.1699	1.5855	2.3605	1.2753	2.5733	1.0374
2	5	2.2167	1.4975	2.4693	1.1438	2.7623	0.8894
2	10	2.4802	1.1320	3.1589	0.6777	4.1058	0.4256
2	20	3.1911	0.6647	5.6610	0.2606	10.6463	0.1144
2	25	3.6658	0.5159	7.8485	0.1673	18.0001	0.0623
2	30	4.2434	0.4034	11.0627	0.1093	30.9599	0.0345
2	35	4.9461	0.3176	15.7854	0.0722	53.7996	0.0193
2	40	5.8010	0.2517	22.7245	0.0481	94.0510	0.0109
3	2	3.2515	2.5697	3.5269	2.2225	3.8279	1.9393
3	3	3.3902	2.3819	3.8466	1.9243	4.3788	1.5790
3	4	3.5384	2.2101	4.2114	1.6724	5.0495	1.2961
3	5	3.6969	2.0528	4.6283	1.4590	5.8682	1.0721
3	10	4.6714	1.4403	7.8199	0.7771	13.7521	0.4590
3	20	7.9921	0.7597	27.3855	0.2705	103.6973	0.1155
3	25	10.7725	0.5686	54.7502	0.1705	307.0022	0.0625
3	30	14.7630	0.4327	112.3199	0.1102	928.5568	0.0345
3	35	20.5177	0.3339	234.3923	0.0725	2841.5990	0.0195
3	40	28.8508	0.2607	494.6806	0.0482	8752.5330	0.0109
4	2	4.5168	3.3004	5.1138	2.7628	5.8017	2.3432
4	3	4.8135	3.0065	5.8456	2.3214	7.1518	1.8357
4	4	5.1395	2.7440	6.7296	1.9643	8.9455	1.4592
4	5	5.4978	2.5091	7.8004	1.6736	11.3418	1.1759
4	10	7.9148	1.6486	17.8825	0.8231	43.7121	0.4698
4	20	18.5117	0.8031	128.6429	0.2726	1001.2940	0.1156
4	25	29.7177	0.5884	575.9555	0.1709	5220.0590	0.0625
4	30	48.8823	0.4417	1131.2380	0.1103	27820.4900	0.0345
4	35	81.9645	0.3380	3466.5740	0.0725	150036.4000	0.0193
4	40	139.5130	0.2626	10747.7100	0.0483	814432.8000	0.0109
5	2	5.8853	3.9759	6.9647	3.2260	8.2777	2.6652
5	3	6.4145	3.5618	8.3638	2.6366	11.0478	2.0184
5	4	7.0124	3.2004	10.1555	2.1789	15.0759	1.5629
5	5	7.6890	2.8843	12.4614	1.8196	20.9880	1.2346
5	10	12.7158	1.7893	39.6071	0.8445	136.7631	0.4732
5	20	41.5615	0.8229	600.5988	0.2730	9659.7780	0.1157
5	25	80.2225	0.5958	2575.7240	0.1710	88742.3200	0.0625
5	30	159.5445	0.4445	11384.2700	0.1105	853500.6000	0.0345
5	35	324.4390	0.3391	51255.5500	0.0725	7921864.0000	0.0193
5	40	670.8041	0.2630	253490.2000	0.0485	75783750.00	0.0109

APPENDIX C

Interest Multipliers, Perpetual Periodic Series.

n	i=4%	i=6%	i=8%	i=10%	i=12%	i=14%	i=16%	n
2	12.25491	8.09062	6.00961	4.76190	3.93082	3.33778	2.89352	2
3	8.00872	5.23517	3.85042	3.02115	2.46758	2.07665	1.78296	3
4	5.88725	3.80986	2.77401	2.15471	1.74362	1.45146	1.23359	4
5	4.61568	2.95661	2.13070	1.63797	1.31175	1.08060	0.90881	5
6	3.76905	2.38938	1.70394	1.29607	1.02688	0.83684	0.69619	6
7	3.16524	1.98559	1.40090	1.05405	0.82598	0.66566	0.54758	7
8	2.71320	1.68395	1.17518	0.87444	0.67752	0.53977	0.43890	8
9	2.36233	1.45037	1.00100	0.73641	0.56399	0.44406	0.35677	9
10	2.08228	1.26447	0.86287	0.62745	0.47487	0.36938	0.29313	10
11	1.85372	1.11322	0.75095	0.53963	0.40346	0.30996	0.24288	11
12	1.66381	0.98795	0.65869	0.46763	0.34531	0.26192	0.20259	12
13	1.50359	0.88267	0.58152	0.40779	0.29731	0.22260	0.16990	13
14	1.36673	0.79308	0.51621	0.35746	0.25726	0.19007	0.14311	14
15	1.24853	0.71605	0.46037	0.31474	0.22354	0.16292	0.12098	15
16	1.14550	0.64920	0.41221	0.27817	0.19492	0.14011	0.10259	16
17	1.05496	0.59075	0.37037	0.24664	0.17047	0.12082	0.08720	17
18	0.97483	0.53928	0.33378	0.21930	0.14948	0.10444	0.07428	18
19	0.90347	0.49368	0.30160	0.19547	0.13136	0.09045	0.06339	19
20	0.83954	0.45308	0.27315	0.17460	0.11566	0.07847	0.05417	20
21	0.78200	0.41674	0.24790	0.15624	0.10200	0.06818	0.04635	21
22	0.72997	0.38409	0.22540	0.14005	0.09009	0.05931	0.03970	22
23	0.68273	0.35464	0.20528	0.12572	0.07967	0.05165	0.03404	23
24	0.63967	0.32798	0.18722	0.11300	0.07053	0.04502	0.02921	24
25	0.60030	0.30378	0.17098	0.10168	0.06250	0.03927	0.02508	25
26	0.56419	0.28174	0.15634	0.09159	0.05543	0.03429	0.02155	26
27	0.53096	0.26162	0.14310	0.08258	0.04920	0.02995	0.01852	27
28	0.50032	0.24321	0.13111	0.07451	0.04370	0.02617	0.01592	28
29	0.47200	0.22633	0.12023	0.06728	0.03884	0.02289	0.01370	29
30	0.44575	0.21082	0.11034	0.06079	0.03453	0.02002	0.01179	30
31	0.42138	0.19654	0.10134	0.05496	0.03072	0.01752	0.01014	31
32	0.39872	0.18337	0.09314	0.04972	0.02734	0.01533	0.00873	32
33	0.37759	0.17122	0.08565	0.04499	0.02434	0.01343	0.00752	33
34	0.35787	0.15997	0.07880	0.04074	0.02167	0.01176	0.00647	34
35	0.33943	0.14956	0.07254	0.03690	0.01931	0.01030	0.00558	35
40	0.26309	0.10769	0.04825	0.02259	0.01086	0.00532	0.00265	40
45	0.20656	0.07834	0.03234	0.01391	0.00614	0.00276	0.00126	45
50	0.16376	0.05740	0.02179	0.00859	0.00347	0.00143	0.00060	50
55	0.13078	0.04228	0.01472	0.00532	0.00197	0.00074	0.00029	55
60	0.10505	0.03126	0.00997	0.00330	0.00112	0.00039	0.00014	60
65	0.08475	0.02318	0.00677	0.00204	0.00063	0.00020	0.00006	65
70	0.06863	0.01722	0.00460	0.00127	0.00036	0.00010	0.00003	70

APPENDIX D

Derivations of Compound Interest Formulas

Recall the earlier notation:

V_0 = the present value of a sum of money

V_n = the future value of a sum of money

i = the interest rate, expressed as a decimal

n = the number of compounding periods

a = the amount of a uniform periodic or annual cost or revenue

t = the number of years between periodic costs or revenues

Present Value of a Terminating Annual Series

In general, the procedure for calculating the present value of a terminating annual series of costs or revenues is to discount each payment to the present, or:

$$V_0 = \frac{a}{(1+i)^1} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \ldots + \frac{a}{(1+i)^{n-1}} + \frac{a}{(1+i)^n} \qquad \text{(D1)}$$

Multiply both sides of equation D1 by $(1 + i)$:

$$V_0(1 + i) = a + \frac{a}{(1 + i)} + \frac{a}{(1+2)^2} + \ldots + \frac{a}{(1+i)^{n-1}} \qquad \text{(D2)}$$

Notice that equations D1 and D2 have numerous terms in common and that if equation D1 is subtracted from equation D2 the expression will be simplified to:

$$V_0(1 + i) - V_0 = a - \frac{a}{(1 + i)^n} \qquad \text{(D3)}$$

Simple factoring produces:

$$V_0[(1 + i) - 1] = a\left[1 - \frac{1}{(1 + i)^n}\right]$$

$$V_0 i = a\left[1 - \frac{1}{(1 + i)^n}\right] \qquad \text{(D4)}$$

Dividing both sides of equation D4 by "i" produces:

$$V_0 = a\left[\frac{(1 + i)^n}{i(1 + i)^n} - \frac{1}{i(1 + i)^n}\right] \qquad (D5)$$

Combining terms produces the formula for the present value of a terminating annual series:

$$V_0 = a\left[\frac{(1 + i)^n - 1}{i(1 + i)^n}\right] \qquad (D6)$$

Future Value of a Terminating Annual Series

In general, the procedure for calculating the future value of a terminating annual series of costs or revenues is to compound each payment to year n (except the last payment which occurs at year "n"). Note that the first payment is received at the end of year 1, so that it is compounded for n - 1 years. Then, the future value of a terminating annual series is:

$$V_n = a + a(1 + i)^1 + a(1 + i)^2 + \ldots + a(1 + i)^{n - 1} \qquad (D7)$$

Multiplying both sides of equation D7 by (1 + i) produces:

$$V_n(1 + i) = a(1 + i) + a(1 + i)^2 + \ldots + a(1 + i)^n \qquad (D8)$$

To simplify, substract (D7) from (D8):

$$V_n = (1 + i) - V_n = -a + a(1 + i)^n \qquad (D9)$$

Simple factoring produces:

$$V_n[(1 + i) - 1] = a[-1 + (1 + i)^n] \qquad (D10)$$

Combining terms:

$$V_n(i) = a[(1 + i)^n - 1] \qquad (D11)$$

Solving for V_n produces the formula for the future value of a terminating annual series:

$$V_n = a\left[\frac{(1 + i)^n - 1}{i}\right] \qquad (D12)$$

Sinking Fund Factor

The annual savings needed to accumulate a specific capital sum "n" years in the future can be derived from the formula for the future value of a terminating annual series. Equation D12 is:

$$V_n = a\left[\frac{(1 + i)^n - 1}{i}\right]$$

Solving for a:

$$a = \frac{V_n}{\frac{(1 + i)^n - 1}{i}}$$

$$a = V_n\left[\frac{i}{(1 + i)^n - 1}\right] \qquad \text{(D13)}$$

Capital Recovery Formula

The annual series of payments needed to repay a given sum within a specific time period can be derived from the formula for the present value of a terminating annual series, formula D6:

$$V_0 = a\left[\frac{(1 + i)^n - 1}{i(1 + i)^n}\right]$$

Solving for a:

$$a = \left[\frac{V_0}{\frac{(1 + i)^n - 1}{i(1 + i)^n}}\right]$$

$$a = V_0\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right] \qquad \text{(D14)}$$

Present Value of a Terminating Periodic Series

The procedure for calculating the present value of a terminating periodic series is to discount each periodic payment to the present.

or:

$$V_0 = \frac{a}{(1 + i)^t} + \frac{a}{(1 + i)^{2t}} + \frac{a}{(1 + i)^{3t}} + \ldots + \frac{a}{(1 + i)^n} \tag{D15}$$

Multiply both sides of equation D15 by $(1 + i)^t$:

$$V_0(1 + i)^t = a + \frac{a}{(1 + i)^t} + \frac{a}{(1 + i)^{2t}} + \ldots + \frac{a}{(1 + i)^{n - t}} \tag{D16}$$

Notice that equations (D15) and (D16) have numerous terms in common and that if (D15) is subtracted from (D16) the expression is simplified. Subtract (D15) from (D16):

$$V_0(1 + i)^t - V_0 = a - \frac{a}{(1 + i)^n} \tag{D17}$$

Simple factoring produces:

$$V_0[(1 + i)^t - 1] = a(1 - \frac{1}{(1 + i)^n}) \tag{D18}$$

Solving for V_0:

$$V_0 = a\,\frac{1 - \frac{1}{(1 + i)^n}}{(1 + i)^t - 1} \tag{D19}$$

Multiply the fraction in (D19) by $\frac{(1 + i)^n}{(1 + i)^n}$, or 1:

$$V \;\; = a\left[\frac{(1 + i)^n - 1}{[(1 + i)^t - 1](1 + i)^n}\right] \tag{D20}$$

Equation D20 is the formula for the present value of a terminating periodic series.

Future Value of a Terminating Periodic Series

To obtain the future value of a terminating periodic series of costs or revenues, each payment or receipt (except the last one which occurs at year "n") must be compounded to year "n". Note that the first payment is received at the end of the t'th period, so that it is compounded for n-t years. Then, the future value of a terminating periodic series is:

$$V_n = a + a(1 + i)^t + a(1 + i)^{2t} + \ldots + a(1 + i)^{n - t} \qquad \text{(D21)}$$

Multiplying both sides of (D21) by $(1 + i)^t$ produces:

$$V_n(1 + i)^t = a(1 + i)^t + a(1 + i)^{2t} + \ldots + a(1 + i)^n \qquad \text{(D22)}$$

To simplify, subtract (D21) from (D22):

$$V_n(1 + i)^t - V_n = -a + a(1 + i)^n \qquad \text{(D23)}$$

Simple factoring produces:

$$V_n = [(1 + i)^t - 1] = a[-1 + (1 + i)^n]$$

$$V_n[(1 + i)^t - 1] = a[(1 + i)^n - 1] \qquad \text{(D24)}$$

Solving for V_n gives the formula for the future value of a periodic series:

$$V_n = a\left[\frac{(1 + i)^n - 1}{(1 + i)^t - 1}\right] \qquad \text{(D25)}$$

<u>Present Value of a Perpetual Annual Series</u>

A perpetual annual series consists of a series of costs or revenues (a) occurring one year apart for an infinite number of years ( ).

Recall the formula for the present value of a terminating annual series (equation D6):

$$V_0 = a\left[\frac{(1 + i)^n - 1}{i(1 + i)^n}\right]$$

In the case of a perpetual annual series, n equals infinity (n =). When n = , equation D6 can be expressed as:

$$V_0 = \frac{a}{i}\left[\frac{(1 + i) - 1}{(1 + i)}\right]$$

or:

$$V_0 = \frac{a}{i} \lim_{n \to \infty} \frac{(1 + i)^n - 1}{(1 + i)^n} \qquad \text{(D26)}$$

As n approaches infinity, the term $\frac{(1 + i)^n - 1}{(1 + i)^n}$ approaches 1, or $\lim_{n}$

$\frac{(1 + i)^n - 1}{(1 + i)^n} = 1$. Thus, the formula for the present value of a perpetual annual series is:

$$V_0 = \frac{a}{i} \tag{D27}$$

Formula D27 gives the present value of a single sum that is equivalent to a perpetual annual income or cost stream at a specified interest rate. Notice that as the time horizon is increased, the value of the residual income stream becomes negligible.

<u>Present Value of a Perpetual Periodic Series</u>

Recall equation D20, the formula for the present value of a terminal periodic series:

$$V_0 = a\left[\frac{(1 + i)^n - 1}{[(1 + i)^t - 1](1 + i)^n}\right]$$

In this case n goes to infinity and equation D20 can be written as:

$$V_0 = a\left[\frac{(1 + i) - 1}{[(1 + i)^t - 1](1 + i)}\right] \tag{D28}$$

or:

$$V_0 = a\left[\lim_{n\to\infty} \frac{(1 + i)^n - 1}{[(1 + i)^t - 1](1 + i)^n}\right] \tag{D29}$$

Separating the terms that contain n:

$$V_0 = \left[\frac{a}{(1 + i)^t - 1}\right]\left[\lim_{n\to\infty} \frac{(1 + i)^n - 1}{(1 + i)^n}\right] \tag{D30}$$

As n approaches infinity, $\frac{(1 + i)^n - 1}{(1 + i)^n}$ approaches 1, so (D30) reduces to the formula for a perpetual periodic series:

$$V_0 = \left[\frac{a}{(1 + i)^t - 1}\right] \tag{D31}$$

Appendix E. Solutions to Problems

1. $V_7 = V_0(1 + i)^n$

$= \$800.00(1.11)^7$

$= \$800.00\ (2.0762)$ from column 1, Appendix Table A11

$= \$1,660.96$

2. $V_0 = \$100,000 \times \frac{1}{(1.05)^9}$

$= \$100,000\ (0.6446)$ from column 2, Appendix Table A5

$= \$64,460$

3. $V_0 = \$160 \times \frac{1}{(1.07)^{11}}$

$= \$160\ (0.4751)$ from column 2, Appendix Table A7

$= \$76.02$

4. $V_6 = \$6,500(1.07)^6$

$= \$6,500\ (1.5007)$ from column 1, Appendix Table A7

$= \$9,754.55$

5. $V_8 = \$2,500(1.10)^8$

$= \$2,500\ (2.1436)$ from column 1, Appendix Table A10

$= \$5,359.00$

6. $V_0 = \$100,000 \times \frac{1}{(1.08)^{10}}$

$= \$100,000\ (0.4632)$ from column 2, Appendix Table A8

$= \$46,320$

7. $V_{14} = \$10.00 \times (1.06)^{14}$

$= \$10.00\ (2.2609)$ from column 1, Appendix Table A6

$= \$22.61$

8. $V_0 = \$380{,}000 \times \frac{1}{(1.10)^2}$

$= \$380{,}000\ (0.8264)$ from column 2, Appendix Table A10

$= \$314{,}032$

9. $V_{2\frac{1}{2}} = \$1{,}000(1.01)^{30}$

$= \$1{,}000\ (1.3478)$ from column 1, Appendix Table A1

$= \$1{,}347.80$

10. For 18% APR:

$i_{effective} = (1.015)^{12} - 1$

$= 19.56\%$

For 21% APR:

$i_{effective} = (1.0175)^{12} - 1$

$= 23.14\%$

11. $V_0 = \$3.50\ (11.4699)$ from column 4, Appendix A6

$= \$40.14$ per acre

12. $V_0 = \$750.00\ (3.7908)$ from column 4, Appendix Table A10

$= \$2{,}843.09$

13. $V_0 = (1{,}750\text{ ac.})(\$3.00/\text{ac./yr.})(12.4622)$ from column 4, Appendix Table A5

$= \$65{,}426.55$

14. $V_{20} = (1{,}750\text{ ac.})(3.00/\text{ac./yr.})(33.0659)$ from column 3, Appendix Table A5

$= \$173{,}595.98$

15. $a = (\$25{,}000)(0.2187)$ from column 5, Appendix Table A9

$= \$5{,}467.50$

16. (\$220,000)(0.2092) from column 5, Appendix Table A12

    = \$46,024.00

17. a = \$120,000 (0.22292) from column 6, Appendix Table A9

    = \$26,748.00

18. a = \$88,000 (0.1256) from column 6, Appendix Table A11

    = \$11,052.80

19. $V_0 = \frac{\$2.00}{.08} = \$25.00$

20. $V_0 = \frac{\$100,000}{.04} = \$2,500,000$

21. V_0 = \$118,900 (0.50032) from Appendix C

    = \$59,489.24

22. V_0 = \$200,000 (.07254) from Appendix C

    = \$14,508.00

23. $\text{Payback Period} = \frac{\$175,000}{\$25,000} = 7 \text{ years}$

24. Payback Period = 4 years

25. $NPV = -\$25,000 + \frac{\$9,000}{1.07} + \frac{9,000}{(1.07)^2} + \frac{9,000}{(1.07)^3}$

    = -\$25,000 + \$9,000(.9346) + \$9,000(.8734) + \$9,000(.8163) from column 2, Appendix Table A7

    = -\$25,000 + \$8,411.40 + \$7,860.60 + 7,346.70

    = -\$1,381.30

26. $\text{NPV} = \dfrac{\$36.00}{(1.05)^9} + \dfrac{\$150.00}{(1.05)^{16}}$

$= (\$36)(0.6446) + (\$150)(0.4581)$ from column 2, Appendix Table A5

$= \$23.21 + \$68.72$

$= \$91.93$

27. NPV = (\$350)(0.7473) from column 2, Appendix Table A6, = \$261.55

EAI = (NPV)(Capital Recovery Multiplier) = (\$261.55)(0.2374)

= \$62.09

Timber yields \$62.09 on an equivalent annual basis at 6% interest.

28. 9 percent

29. $i = 8\%$

30.

Revenues			Costs		
Year	Amount	Future Value	Year	Amount	Future Value
10	\$ 150.00	\$ 303.88	0	\$50.00	\$149.96
21	210.00	276.35	1	20.00	57.67
28	1,120.00	1,120.00	22	3.50	4.43
		\$1,700.23			\$212.04

$L_e = (1700.23 - 212.04)(.50032)$ from Appendix C

= \$744.57 per acre

31.

Rotation Age	MAI (cu.ft.)	NPV	IRR	L_e
15	81.13	\$ 32.35	4.8%	\$ 90.34
20	106.75	106.66	6.55*	238.98
25	118.72	148.68	6.4%	284.61*
30	123.83	166.91	6.1%	283.85
35	125.11*	168.27*	5.6%	261.04
40	123.95	157.75	5.1%	227.49

32. a. \$110.08

b. \$498.88

c. \$0.54

d. \$1.08 each

e. \$200.00

f. \$48.65

33. $i = r + f + rf$

$= 0.03 + 0.07 + (0.03)(0.07)$

$= 10.21\%$

34. $r = \frac{1 + i}{1 + f} - 1$

$= \frac{1.10}{1.05} - 1$

$= 4.76\%$

COMPOUND INTEREST FORMULAS

For A ↓ / To Calculate →	FUTURE VALUE (V_n)	PRESENT VALUE (V_o)
SINGLE SUM	$V_n = V_o\left[(1+i)^n\right]$ Bracketed term in column 1 of Appendix A	$V_o = V_n\left[\frac{1}{(1+i)^n}\right]$ Bracketed term in column 2 of Appendix A
TERMINATING ANNUAL SERIES	$V_n = a\left[\frac{(1+i)^n - 1}{i}\right]$ Bracketed term in column 3 of Appendix A	$V_o = a\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$ Bracketed term in column 4 of Appendix A
TERMINATING PERIODIC SERIES	$V_n = a\left[\frac{(1+i)^n - 1}{(1+i)^t - 1}\right]$ Bracketed term in column 1a, 1b, or 1c of Appendix B	$V_o = a\left[\frac{(1+i)^n - 1}{((1+i)^t - 1)(1+i)^n}\right]$ Bracketed term in column 2a, 2b, or 2c of Appendix B
PERPETUAL ANNUAL SERIES		$V_o = \frac{a}{i}$
PERPETUAL PERIODIC SERIES		$V_o = a\left[\frac{1}{(1+i)^n - 1}\right]$ Bracketed term in Appendix C

where:

V_o = present value (value in period o)

V_n = future value (value after period n)

a = annual or periodic cost or income

i = interest rate

n = number of interest bearing periods (usually years)

t = interval between costs or revenues in a terminating periodic series

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