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Compatible Cubic-Foot Stem Volume and Upper-Stem Diameter Equations for Semi-intensive Plantation Grown Loblolly Pine Trees in East Texas

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ABSTRACT: *The Max-Burkhart taper equation was used to develop compatible taper and volume equations for semi-intensive plantation grown loblolly pine (*Pinus taeda L.*) trees in East Texas. Semi-intensive plantations in East Texas are characterized by some form of mechanical site preparation, a burn, possibly chemical weed control, improved seedlings if planted after 1985, and, possibly, a midrotation thinning and/or fertilization. The equations in this study were compared with those of Lenhart et al. [Lenhart, J.D., T.L. Hackett, C.J. Laman, T.J. Wiswell, and J.A. Blackard. 1987. Tree content and taper functions for loblolly and slash pine trees planted on non-old field in east Texas. South J. Appl. For. 10(2)109–112.] and Baldwin and Feduccia [Baldwin, V.C. Jr., and D.P. Feduccia. 1991. Compatible tree-volume and upper-stem diameter equations for loblolly and slash pines in the West Gulf Region. South. J. Appl. For. 10(2)109–112.] using independent data. The new equations ranked first (sum of ranks = 9) in terms of percent bias and percent SEE for inside-bark predictions of upper-stem diameters and cubic-foot volumes, while the Lenhart et al. [Lenhart, J.D., T.L. Hackett, C.J. Laman, T.J. Wiswell, and J.A. Blackard. 1987. Tree content and taper functions for loblolly and slash pine trees planted on non-old field in east Texas. South J. Appl. For. 10(2)109–112.] ranked second (sum of ranks = 17) and Baldwin and Feduccia [Baldwin, V.C. Jr., and D.P. Feduccia. 1991. Compatible tree-volume and upper-stem diameter equations for loblolly and slash pines in the West Gulf Region. South. J. Appl. For. 10(2)109–112.] ranked third (sum of ranks = 22). For outside-bark predictions of volumes and diameters, Baldwin and Feduccia [Baldwin, V.C. Jr., and D.P. Feduccia. 1991. Compatible tree-volume and upper-stem diameter equations for loblolly and slash pines in the West Gulf Region. South. J. Appl. For. 10(2)109–112.] ranked first (sum of ranks = 12), while the new equations ranked a close second (sum of ranks = 13) and Lenhart et al. [Lenhart, J.D., T.L. Hackett, C.J. Laman, T.J. Wiswell, and J.A. Blackard. 1987. Tree content and taper functions for loblolly and slash pine trees planted on non-old field in east Texas. South J. Appl. For. 10(2)109–112.] ranked third (sum of ranks = 20). We recommend using the new equations for loblolly pine trees up to a 16-in. dbh and provide examples to illustrate their use. South. J. Appl. For. 30(3):132–141.*

Key Words: *Pinus taeda*, taper functions, volume prediction, segmented polynomial, simultaneous equations, inventory.

Pine plantations represent 22% of the private ownership in East Texas. Most existing plantations were converted from natural mixed pine-hardwood forests beginning in the early 1970s, and this conversion process has continued to the present. Over 1 million ac of industrial forestland has been sold or exchanged within the last few years. To properly value this timber resource, managers require reliable tree content estimation equations that are applicable to East Texas pine plantations. Lenhart et al. (1987) developed the first volume, weight, and taper equations applicable to loblolly (*Pinus taeda*) and slash (*Pinus elliotti*) pine plantations growing on converted forestland in East Texas. Although an improvement over existing equations developed for old fields in East Texas (Hasness and Lenhart 1972), Lenhart et al. (1987) did not include larger diameter

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Table 1. Numbers of East Texas loblolly pine trees sampled in this study by diameter class (in.) and height class (ft).

Diameter class	Height class 20	30	40	50	60	70	80
4	5 (1)	3	1				
5		9 (2)	8 (1)	4	(1)		
6		3	7 (1)	16	9 (1)	1	
7		1	5	11 (1)	10 (1)	6	
8			2	17 (3)	13 (1)	6	
9			1	7	9 (2)	8 (1)	2
10				5 (1)	11	9 (1)	1
11				3	12 (2)	6 (1)	2
12					5 (1)	1 (1)	3
13					3	3	2 (1)
14					2	3	
15					(1)	1	

Values outside parentheses represent the numbers of trees used for model development and values inside parentheses represent the number of trees used for model evaluation.

trees (dbh > 13 in.) and they did not produce compatible volume/weight and taper equations. Baldwin and Feduccia (1991) developed compatible volume and taper equations for plantation-grown loblolly pine in the West Gulf region, in which East Texas is included. Although they sampled larger trees (dbh < 21 in.), all trees were sampled in central Louisiana and not East Texas.

The purpose of this study was to develop compatible volume and taper equations applicable to trees found in semi-intensive management loblolly pine plantations in East Texas. Semi-intensive plantations in East Texas are characterized by some form of mechanical site preparation, a burn, possibly chemical weed control, improved seedlings if planted after 1985, and, possibly, some midrotation activity such as a thinning and/or fertilization. These plantations are not highly intensively managed, but they are typical of much of forest industry land in East Texas. This study also includes data from larger trees in older plantations in East Texas. The equations developed in this study were applied to an independent data set along with the equations of Lenhart et al. (1987) and Baldwin and Feduccia (1991) to examine bias trends.

Data Description

This study used 261 loblolly pine trees ranging in diameter from 4 to 15 in. (Table 1) sampled in four separate felled-tree studies on industrial forestland in East Texas. Sample trees from all studies were combined and analyzed collectively because the equations developed in this study were designed for use in semi-intensive management plantations. We did not create separate equations for all the different combinations of cultural practices (or lack thereof) found in these plantations because we want the equations to have widespread applicability to typical industrial plantations found in the East Texas region. In many situations, especially in land sales/exchanges, plantation records are not always conveyed to the new owners, so there are no records about past management activities. Thus, we need equations that can accommodate these increasingly common situations faced by forest managers and owners, as well as procurement foresters.

The first study (Lenhart et al. 1987) sampled 65 trees to develop their volume, weight, and taper equations. In the second study, Lapongan et al. (1993) sampled 36 trees and then combined their data with those of Lenhart et al. (1987) to develop updated volume and taper equations. The field sampling procedures for both studies were identical. Trees were selected to represent a wide geographic distribution of growing conditions in unthinned, low-intensity management loblolly pine plantations. Although these plantations received little or no midrotation management, they received intensive mechanical site preparation that included a burn. Trees were destructively sampled adjacent to research plots of the East Texas Pine Plantation Research Project (ETPPRP; Lenhart et al. [1985]). The dbh (nearest 0.1 in.) was measured with a diameter tape before felling the sample tree. After felling, total tree height was measured from the base of the stem to the tip of the terminal leader to the nearest 0.1 ft with a fiberglass tape. Stump height was added to this measurement. Then, the stem was cut into 3-ft bolts, and diameter inside bark (dib), as well as diameter outside bark (dob), was measured at each was cut to the nearest 0.1 in. with calipers. The total number of trees in the two studies, (65 + 36 = 101) was reduced to 67 when 34 trees with dbh < 4.0 in. were eliminated from further analysis in this study (these trees were too small to be considered merchantable).

In the third study, Clark et al. (2000) sampled 54 trees in East Texas as part of a study to examine wood properties of loblolly pine across the South (see Jordan et al. [2005]). These trees also were destructively sampled adjacent to ETPPRP plots. The dbh was measured and marked on the stem before felling. After felling, stem length was measured from the dbh mark to the tree tip. Total height was found by adding 4.5 ft to this measured stem length. The stem was cut

Table 2. Observed tree dbh (in.), THT (ft), TPA, and BA/A (ft^2) for east Texas loblolly pine plantation development and evaluation data sets for the compatible taper and volume equations (Equations 1 and 2, respectively).

	Model development data set (n = 236 trees)				Model evaluation data set (n = 25 trees)			
Variables	Mean	SD.	Minimum	Maximum	Mean	SD	Minimum	Maximum
DBH	8.3	2.4	4.0	15.3	8.6	2.9	4.5	14.8
THT	54.7	13.0	18.6	83.0	54.9	13.7	20.9	78.0
TPA	392.6	161.3	81.0	701.0	312.0	159.0	81.0	623.0
BA/A	90.7	35.1	12.0	186.0	101.9	33.2	56.0	186.0

BA/A, basal area per acre; THT, total height; TPA, trees per acre.

Table 3. Parameter estimates and fit statistics of East Texas loblolly pine plantation compatible taper and volume equations.

Equation	Parameter	Parameter estimate	SE	Pr(b_i or a_I = 0)	R^2	RMSE
Ob	b_1	−3.7178	1.4622	0.0111	0.95—Taper	0.09—Taper
	b_2	1.7805	0.8052	0.0271	0.97—Volume	0.81—Volume
	b_3	−1.4659	0.7879	0.0629		
	b_4	74.1804	5.7580	<0.0001		
	a_1	0.7986	0.0679	<0.0001		
	a_2	0.0895	0.0034	<0.0001		
Ib	b_1	−2.6676	0.4258	<0.0001	0.94—Taper	0.08—Taper
	b_2	1.2357	0.2447	<0.0001	0.98—Volume	0.89—Volume
	b_3	−1.3667	0.2269	<0.0001		
	b_4	61.4260	6.9728	<0.0001		
	a_1	0.7019	0.0395	<0.0001		
	a_2	0.0789	0.0043	<0.0001		

Note: R^2 and RMSE are reported for the taper and volume equations separately as part of the SUR fitting procedure.
RMSE, room means square error.

at 4.5 and 10 ft and in 5-ft increments thereafter to 50 ft. The dib and dob were measured with calipers at each cut point, except that stump dib was not measured. To find stump dib, the 101 trees from the studies by Lenhart et al. (1987) and Lapongan et al. (1993) were used to build a stump dob/dib ratio estimator (Shiver and Borders 1996), because the plot of dib over dob was linear and passed through the origin (ratio = 0.84868, SE = 0.00492268594). Stump dob was multiplied by this ratio to find stump dib.

In the fourth study, a total of 140 loblolly pine trees were destructively sampled in semi-intensive plantations managed by Molpus Timberlands Management. In Trinity and Polk counties, Texas, 42 of the 140 trees were selected from nine separate plantations in August 2003. Within each plantation, up to five trees were selected for sampling. Before being felled, trees were measured and classified for dbh and crown class. Once felled, trees were measured for stump height, stem length (total height − stump height), and height to live crown. Diameters were measured at heights above stump at 0, 2.5, and 5 ft and every 5 ft thereafter. Diameters were measured with calipers by taking two perpendicular measurements. At each diameter measurement, two single bark thicknesses were taken perpendicular to one another using a bark gauge. In December 2003–January 2004, 98 of the 140 trees were selected from 21 separate plantations in Hardin, Newton, Jasper, Trinity, Polk, and San Jacinto counties, Texas, and Allen Parish, Louisiana. The same sampling protocol described previously was used for these trees.

For all combined data sets, sample trees tended to be free of stem damage, forks, and disease. No conscious effort was made to sample severely damaged or diseased trees. Smalian's formula was used to calculate the cubic-foot volume of each bolt, because this volume estimation technique works well for short log segments (Husch et al. 1982).

From the total 261 trees, about 10% (i.e., 25 trees) were randomly selected and removed from the data set used for model fitting. They were reserved for model evaluation. Thus, a total of 236 trees were used for model fitting (Table 2).

Table 4. Rank of %bias and %SEE for overall predictions and predictions by RHC of cubic-foot volume (volume) and upper-stem diameters (diameter, in.) from three compatible taper and volume equations (this study, Baldwin and Feduccia [1991], and Lenhart et al. [1987]) for ib, wood only, and ob, wood and bark.

Statistic			This study	Baldwin and Feduccia (1991)	Lenhart et al. (1987)
Ib, Wood only					
%Bias	Overall	Volume	1	2	3
		Diameter	2	3	1
	RHC	Volume	1	3	2
		Diameter	1	3	2
%SEE	Overall	Volume	1	2	3
		Diameter	1	3	2
	RHC	Volume	1	3	2
		Diameter	1	3	2
Sum of ranks			9—first	22—third	17—second
ob, Wood and bark					
%Bias	Overall	Volume	2	1	3
		Diameter	1	3	2
	RHC	Volume	2	1	3
		Diameter	3	1	2
%SEE	Overall	Volume	1	2	3
		Diameter	1	2	3
	RHC	Volume	2	1	3
		Diameter	1—tied	1—tied	1—tied
Sum of ranks			13—second	12—first	20—third

Table 5. Overall bias, mean absolute bias, %bias, SEE, %SEE, number of samples (*n*), and rank by %bias and %SEE for both ob and ib predicted upper-stem diameters (diameter, in.) and segment cubic-foot volume (volume) from three taper equations: this study, Baldwin and Feduccia (1991), and Lenhart et al. (1987).

Variable	Component	Equation	*n*	Bias	Mean absolute bias	%Bias	Rank %bias	SEE	%SEE	Rank %SEE
Diameter	Wood and bark (ob)	This study	310	0.16	0.37	0.67	1	0.49	7.96	1
		Baldwin and Feduccia (1991)	310	−0.17	0.42	−6.33	3	0.55	8.86	2
		Lenhart et al. (1987)	310	0.26	0.46	4.63	2	0.62	10.67	3
Diameter	Wood only (ib)	This study	310	0.05	0.35	−3.15	2	0.47	8.51	1
		Baldwin and Feduccia (1991)	310	−0.35	0.49	−12.66	3	0.63	11.28	3
		Lenhart et al. (1987)	310	0.08	0.43	−2.64	1	0.58	10.48	2
Volume	Wood and bark (ob)	This study	310	0.48	0.57	6.42	2	0.94	10.19	1
		Baldwin and Feduccia (1991)	310	−0.49	0.63	−3.04	1	1.01	10.91	2
		Lenhart et al. (1987)	310	−1.08	1.34	−21.85	3	1.85	22.36	3
Volume	Wood only (ib)	This study	310	0.30	0.49	3.31	1	0.94	12.67	1
		Baldwin and Feduccia (1991)	310	−0.70	0.78	−10.22	2	1.19	16.12	2
		Lenhart et al. (1987)	310	−0.93	1.24	−26.54	3	2.04	27.83	3

Note: Ranks are scored within each combination of variable (diameter and volume) and component (ob and ib) for the three equations.

Data Analysis

Although many different equations were considered and explored, the Max-Burkhart segmented polynomial equation (Max and Burkhart 1976) was used in this study. This equation is well known and widely used in loblolly pine taper and volume prediction. It is composed of three subcomponents grafted into one equation at two points called "join points." Each subcomponent describes the stem profile for that section of the tree. One subcomponent represents the lower section of the tree where butt swell occurs. Another subcomponent represents the middle of the tree. The other subcomponent represents the upper section of the tree. By dividing the tree into three segments, this equation attempts to better accommodate different trees shapes than

Table 6. Average cubic-foot volume (volume, wood only—ib), number of samples (*n*), bias (cubic-feet), mean absolute bias (cubic-feet), percent bias (%bias), SEE (cubic-feet), %SEE, and rank by %bias and %SEE for predicted wood only cubic-foot volume by relative height class from three volume equations: this study, Baldwin and Feduccia (1991), and Lenhart et al. (1987).

Equation	RHC	*n*	Volume	Bias	Bias	%Bias	Rank%bias	SEE	%SEE	Ranks %SEE
This study	0.05	57	2.17	0.05	0.12	3.63	1	0.16	7.59	1
	0.15	29	4.33	0.10	0.20	3.35	1	0.30	7.00	1
	0.25	29	6.47	0.23	0.36	3.43	1	0.56	8.58	1
	0.35	32	7.47	0.19	0.43	3.20	1	0.68	9.12	1
	0.45	33	8.37	0.40	0.59	3.88	1	1.03	12.30	1
	0.55	29	9.00	0.35	0.59	2.69	1	1.11	12.32	2
	0.65	30	10.27	0.58	0.80	3.85	1	1.56	15.18	3
	0.75	35	10.73	0.37	0.69	2.66	1	1.31	12.17	2
	0.85	27	10.12	0.49	0.77	3.76	1	1.52	15.02	2
	0.95	9	13.86	0.63	1.15	0.43	1	3.24	23.35	3
Sums of ranks							10 [1]			17 [1]
Baldwin and Feduccia (1991)	0.05	57	2.17	−0.25	0.27	−9.68	2	0.41	18.95	2
	0.15	29	4.33	−0.56	0.56	−11.77	2	0.78	18.09	2
	0.25	29	6.47	−0.67	0.67	−10.55	2	0.95	14.69	2
	0.35	32	7.47	−0.79	0.80	−9.94	3	1.23	16.53	2
	0.45	33	8.37	−0.64	0.71	−8.69	3	1.08	12.93	3
	0.55	29	9.00	−0.78	0.88	−10.04	3	1.26	14.00	3
	0.65	30	10.27	−0.72	0.98	−9.14	3	1.38	13.40	2
	0.75	35	10.73	−1.09	1.22	−11.18	3	1.74	16.23	3
	0.85	27	10.12	−0.92	1.12	−10.32	3	1.64	16.18	3
	0.95	9	13.86	−1.30	1.69	−14.19	3	2.49	18.00	2
Sums of ranks							27 [3]			24 [3]
Lenhart et al. (1987)	0.05	54	2.14	−2.39	2.58	−99.70	3	3.60	168.32	3
	0.15	29	4.22	−2.13	2.17	−48.79	3	3.01	71.17	3
	0.25	29	6.30	−1.32	1.39	−18.89	3	2.04	32.31	3
	0.35	32	7.36	−0.66	0.85	−8.83	2	1.34	18.19	3
	0.45	33	8.21	−0.09	0.60	−2.78	2	0.99	12.00	2
	0.55	29	8.90	−0.03	0.56	−1.85	2	0.78	8.74	1
	0.65	30	10.09	0.10	0.74	−0.40	2	1.02	10.10	1
	0.75	35	10.64	−0.39	0.72	−4.31	2	1.05	9.84	1
	0.85	27	10.01	−0.45	0.70	−5.11	2	1.07	10.66	1
	0.95	9	13.62	−0.95	1.00	−8.19	2	1.63	11.99	1
Sums of ranks							23 [2]			19 [2]

Note: Sums of ranks are compared within columns for %bias and %SEE separately. Numbers in square brackets represent scores based on rank sums: 1 = first place, 2 = second place, and 3 = third place.

a single-segment equation (e.g., Ormerod [1973]). Upper-stem diameter (D_U in inches, both ib and ob) was predicted from the Max-Burkhart taper equation that was fit to the East Texas loblolly pine data

$$D_U^2 = D^2\{b_1(h_U - 1) + b_2(h_U^2 - 1) + b_3(a_1 - h_U)^2 J_1 + b_4(a_2 - h_U)^2 J_2\} \quad (1)$$

where

J_1 = 1, $h_U \leq a_1$; 0 otherwise;
J_2 = 1, $h_U \leq a_2$; 0 otherwise;
h_U = H_U/H;
H = total height (ft);
H_U = upper-stem height (ft);
D = dbh (in.; 4.5 ft);
b_i = regression coefficients; i = 1, 2, 3, 4 (see Table 3);
a_i = inflection or join point; i = 1, 2 (see Table 3).

A cubic-foot volume equation was derived by integrating the Max and Burkhart taper Equation 1 between H_U and H_L:

$$V = fD^2H \begin{bmatrix} \frac{b_1}{2}(h_U^2 - h_L^2) + \frac{b_2}{3}(h_U^3 - h_L^3) - (b_1 + b_2)(h_U - h_L) \\ - \frac{b_3}{3}((a_1 - h_U)^3 I_1 - (a_1 - h_L)^3 I_1') \\ - \frac{b_4}{3}((a_2 - h_U)^3 I_2 - (a_2 - h_L)^3 I_2') \end{bmatrix} \quad (2)$$

where

V = cubic-foot volume between H_L and H_U;
f = $\pi/(4 * 144)$ = 0.00545415;
I_1 = 1, $h_U \leq a_1$; 0 otherwise;
I_2 = 1, $h_U \leq a_2$; 0 otherwise;
I_2' = 1, $h_L \leq a_1$; 0 otherwise;
I_2' = 1, $h_L \leq a_2$; 0 otherwise;
h_L = H_L/H;
H_L = lower-stem (e.g., stump) height (ft)

and all other variables and coefficients are defined as mentioned previously.

Because Equations 1 and 2 are related, seemingly unrelated regression (SUR) was used to estimate the coefficients for both

Table 7. Average cubic-foot volume (volume, wood and bark—ob), number of samples (*n*), bias (cubic-feet), mean absolute bias (cubic-feet), %bias, SEE (cubic-feet), %SEE, and rank by %bias and %SEE for predicted wood and bark cubic-foot volume by RHC from three volume equations: this study, Baldwin and Feduccia (1991), and Lenhart et al. (1987).

Equation	RHC	*n*	Volume	Bias	Mean absolute bias	%Bias	Rank %bias	SEE	%SEE	Rank %SEE
This study	0.05	57	2.91	0.08	0.14	5.27	2	0.19	6.37	1
	0.15	29	5.66	0.20	0.24	6.41	2	0.33	5.90	1
	0.25	29	8.18	0.27	0.33	5.17	2	0.43	5.31	1
	0.35	32	9.41	0.34	0.49	6.16	2	0.63	6.71	1
	0.45	33	10.37	0.56	0.65	6.81	2	0.95	9.20	2
	0.55	29	11.18	0.61	0.73	6.71	2	1.14	10.19	2
	0.65	30	12.60	0.83	0.93	7.06	2	1.57	12.42	2
	0.75	35	13.31	0.76	0.88	7.06	2	1.38	10.38	1
	0.85	27	12.45	0.78	0.88	8.09	2	1.51	12.16	2
	0.95	9	17.11	1.10	1.32	6.67	2	3.41	19.91	3
Sum of ranks							20 [2]			16 [2]
Baldwin and Feduccia (1991)	0.05	57	2.91	−0.10	0.19	0.32	1	0.28	9.60	2
	0.15	29	5.66	−0.37	0.42	−3.37	1	0.60	10.59	2
	0.25	29	8.18	−0.57	0.61	−4.89	1	0.85	10.38	2
	0.35	32	9.41	−0.63	0.69	−3.95	1	1.09	11.60	2
	0.45	33	10.37	−0.51	0.60	−3.40	1	0.95	9.14	1
	0.55	29	11.18	−0.57	0.70	−3.78	1	1.07	9.55	1
	0.65	30	12.60	−0.52	0.80	−3.67	1	1.19	9.48	1
	0.75	35	13.31	−0.73	0.91	−3.98	1	1.43	10.76	2
	0.85	27	12.45	−0.63	0.87	−3.04	1	1.36	10.94	1
	0.95	9	17.11	−0.84	1.27	−4.71	1	2.15	12.59	1
Sum of ranks							10 [1]			15 [1]
Lenhart et al. (1987)	0.05	33	2.20	−1.27	1.40	−66.71	3	1.82	82.74	3
	0.15	29	4.05	−1.62	1.63	−37.38	3	2.21	54.53	3
	0.25	29	6.48	−1.22	1.36	−20.18	3	1.85	28.60	3
	0.35	32	7.76	−0.98	1.20	−14.85	3	1.81	23.27	3
	0.45	33	8.76	−0.56	1.14	−10.60	3	1.69	19.28	3
	0.55	29	9.67	−0.58	0.95	−8.53	3	1.41	14.57	3
	0.65	30	10.93	−0.68	1.28	−9.85	3	1.75	16.00	3
	0.75	35	11.64	−1.28	1.45	−13.09	3	1.96	16.81	3
	0.85	27	10.87	−1.40	1.50	−14.98	3	2.02	18.63	3
	0.95	9	15.15	−1.78	1.99	−16.37	3	2.71	17.90	2
Sum of ranks							30 [3]			29 [3]

Note: Sums of ranks are compared within columns for %bias and %SEE separately. Numbers in square brackets represent scores based on rank sums: 1 = first place, 2 = second place, and 3 = third place.

Table 8. Average ib diameter (in.), number of samples (n), bias (in.), mean absolute bias (in.), %bias, SEE (in.), %SEE, and rank by %bias and %SEE for predicted upper-stem ib diameters by RHC from three taper equations: this study, Baldwin and Feduccia (1991), and Lenhart et al. (1987).

Equation	RHC	n	Diameter	Bias	Mean absolute bias	%Bias	Rank %bias	SEE	%SEE	Rank %SEE
This study	0.05	57	8.40	0.05	0.32	0.51	1	0.44	5.22	1
	0.15	29	7.14	0.03	0.25	0.31	1	0.37	5.23	1
	0.25	29	6.98	0.16	0.29	2.08	2	0.41	5.88	2
	0.35	32	6.31	0.12	0.26	1.70	2	0.39	6.13	2
	0.45	33	5.59	0.11	0.33	0.82	1	0.54	9.61	2
	0.55	29	4.89	0.11	0.30	0.64	1	0.46	9.43	2
	0.65	30	4.37	0.16	0.44	1.48	1	0.68	15.61	2
	0.75	35	3.21	−0.02	0.47	−4.61	2	0.63	19.63	2
	0.85	27	1.82	−0.20	0.46	−19.80	1	0.65	35.49	2
	0.95	9	1.09	−0.41	0.42	−58.00	2	0.97	88.78	3
Sum of ranks							14 [1]			19 [1]
Baldwin and Feduccia (1991)	0.05	57	8.40	−0.41	0.61	−5.27	3	0.73	8.68	3
	0.15	29	7.14	−0.52	0.56	−7.47	3	0.67	9.41	3
	0.25	29	6.98	−0.23	0.33	−3.58	3	0.42	6.01	3
	0.35	32	6.31	−0.17	0.30	−2.97	3	0.39	6.11	1
	0.45	33	5.59	−0.14	0.37	−3.81	3	0.49	8.73	1
	0.55	29	4.89	−0.16	0.33	−5.04	3	0.42	8.62	1
	0.65	30	4.37	−0.19	0.48	−6.82	3	0.61	13.88	1
	0.75	35	3.21	−0.52	0.60	−20.78	3	0.81	25.29	3
	0.85	27	1.82	−0.64	0.66	−45.83	3	0.90	49.37	3
	0.95	9	1.09	−0.68	0.68	−85.49	3	0.89	81.08	2
Sum of ranks							30 [3]			21 [3]
Lenhart et al. (1987)	0.05	54	8.39	0.12	0.54	0.65	2	0.69	8.17	2
	0.15	29	7.14	−0.29	0.37	−4.68	2	0.45	6.32	2
	0.25	29	6.98	0.04	0.29	−0.01	1	0.37	5.25	1
	0.35	32	6.31	0.14	0.27	1.42	1	0.40	6.40	3
	0.45	33	5.59	0.26	0.39	2.92	2	0.61	10.91	3
	0.55	29	4.89	0.32	0.45	4.39	2	0.59	11.99	3
	0.65	30	4.37	0.38	0.56	6.16	2	0.76	17.38	3
	0.75	35	3.21	0.06	0.50	−2.52	1	0.61	19.02	1
	0.85	27	1.82	−0.20	0.47	−21.04	2	0.59	32.60	1
	0.95	9	1.09	−0.39	0.41	−56.33	1	0.58	52.67	1
Sum of ranks							16 [2]			20 [2]

Note: Sums of ranks are compared within columns for %bias and %SEE separately. Numbers in square brackets represent scores based on rank sums: 1 = first place, 2 = second place, and 3 = third place.

equations (Table 3), because it accounts for correlation across the equations (Borders 1989, Robinson 2004). For SUR to work properly, the number of observations in both equations should be equal (Van Deusen 1988). In this study, fewer observations exist for the volume equation than for the taper equation. We used Van Deusen's procedure to create equal sample sizes by using a partial stem volume that represents the total stem volume minus the cumulative volume between the stump and an upper-stem height. Thus, each bolt in the volume equation represents a portion of the tree volume.

Taper and Volume Equation Evaluation

The Max-Burkhart taper and volume equations for both dib and dob developed in this study were compared with the taper and volume equations of Lenhart et al. (1987) and Baldwin and Fedducia (1991). The equation forms for Lenhart et al. (1987) are

$$V = b_0 D^{b_1} H^{b_2} - b_4 D_U^{b_3} D^{b_6} (H - 4.5),$$

$$D_U = D\left[\frac{H - H_U}{H - 4.5}\right]^{b_7}, \qquad (3)$$

where all variables and coefficients are defined as aforementioned.

The volume equation for Baldwin and Feduccia (1991), which is not shown, was found by integrating their taper equation of the form

$$D_U = D\left\{b_1 + b_2 \ln\left[1 - (1 - e^{-b_3/b_4})\left(\frac{H_U}{H}\right)^{1/3}\right]\right\}, \qquad (4)$$

where ln = natural logarithm and all other variables and coefficients are defined as aforementioned.

All three sets of equations were evaluated with the independent 10% evaluation data set (25 trees) at two levels of resolution: (1) overall diameter and cubic-foot volume, and (2) diameter and volume by relative height class (RHC; RHC = upper-stem height/total height). Four criteria (Kozak and Smith 1993) were used to rank the three sets of equations:

$$\text{Mean Bias} = \text{Bias} = \frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)}{n}, \qquad (5)$$

$$\text{Mean Percent bias} = \%\text{Bias} = \frac{\sum_{i=1}^{n}\left[100\left(\frac{(Y_i - \hat{Y}_i)}{Y_i}\right)\right]}{n}, \qquad (6)$$

Table 9. Average ob diameter (in.), number of samples (*n*), bias (in.), mean absolute bias (in.), %bias, SEE (in.), %SEE, and rank by %bias and %SEE for predicted upper-stem ob diameters by RHC from three taper equations: this study, Baldwin and Feduccia (1991), and Lenhart et al. (1987).

Equation	RHC	*n*	Diameter	Bias	Mean absolute bias	%Bias	Rank %bias	SEE	%SEE	Rank %SEE
This study	0.05	57	9.83	0.11	0.36	1.50	3	0.47	4.83	2
	0.15	29	8.09	0.15	0.26	2.29	1	0.35	4.31	2
	0.25	29	7.69	0.20	0.28	2.75	3	0.40	5.16	2
	0.35	32	6.88	0.21	0.31	3.10	3	0.43	6.28	2
	0.45	33	6.10	0.28	0.36	3.91	2	0.59	9.70	2
	0.55	29	5.35	0.32	0.40	5.34	2	0.57	10.60	2
	0.65	30	4.76	0.32	0.48	5.27	2	0.71	15.01	2
	0.75	35	3.52	0.05	0.48	−1.77	1	0.66	18.69	1
	0.85	27	2.06	−0.07	0.44	−9.31	2	0.63	30.52	2
	0.95	9	1.28	−0.23	0.35	−28.03	2	0.77	60.12	3
Sums of ranks							21 [3]			20 [1]
Baldwin and Feduccia (1991)	0.05	57	9.83	−0.02	0.59	−0.26	1	0.67	6.83	3
	0.15	29	8.09	−0.38	0.44	−4.29	3	0.54	6.69	3
	0.25	29	7.69	−0.23	0.32	−2.74	2	0.42	5.43	3
	0.35	32	6.88	−0.16	0.29	−2.19	1	0.40	5.77	1
	0.45	33	6.10	−0.06	0.31	−1.72	1	0.45	7.39	1
	0.55	29	5.35	−0.02	0.29	−0.99	1	0.39	7.35	1
	0.65	30	4.76	−0.02	0.38	−2.09	1	0.54	11.31	1
	0.75	35	3.52	−0.31	0.49	−12.17	3	0.71	20.05	2
	0.85	27	2.06	−0.39	0.50	−26.00	3	0.73	35.59	3
	0.95	9	1.28	−0.43	0.44	−44.22	3	0.65	50.80	2
Sums of ranks							19 [1]			20 [1]
Lenhart et al. (1987)	0.05	33	9.25	−0.11	0.26	−1.45	2	0.31	3.40	1
	0.15	29	8.09	−0.22	0.24	−2.73	2	0.31	3.83	1
	0.25	29	7.69	0.04	0.24	0.31	1	0.33	4.24	1
	0.35	32	6.88	0.20	0.31	2.39	2	0.43	6.30	3
	0.45	33	6.10	0.45	0.50	6.09	3	0.71	11.64	3
	0.55	29	5.35	0.62	0.66	10.42	3	0.79	14.83	3
	0.65	30	4.76	0.74	0.79	13.77	3	1.01	21.15	3
	0.75	35	3.52	0.48	0.69	10.44	2	0.81	22.98	3
	0.85	27	2.06	0.22	0.52	4.97	1	0.61	29.66	1
	0.95	9	1.28	0.03	0.32	−5.61	1	0.41	32.05	1
Sums of ranks							20 [2]			20 [1]

Note: Sums of ranks are compared within columns for %bias and %SEE separately. Numbers in square brackets represent scores based on rank sums: 1 = first place, 2 = second place, and 3 = third place.

$$\text{Standard error of the estimate} = \text{SEE} = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-k}}, \tag{7}$$

$$\text{Percent SEE} = \%\text{SEE} = \left(\frac{\text{SEE}}{\bar{x}}\right)100, \tag{8}$$

where

Y_i = observed diameter or volume for observation i;
$\hat{Y}_i$ = predicted diameter or volume for observation i;
$\bar{x}$ = mean diameter or volume;
n = number of bolts;
k = number of estimated parameters in equation.

Note that $k = 6$ for this study, $k = 1$ for Lenhart et al. (1987) study, and $k = 2$ for Baldwin and Feduccia (1991) study.

Based on these four criteria, each of the three sets of equations was ranked from lowest (best predictions) to highest (worst predictions). Therefore, a low overall rank value corresponds to the best predictions (lowest bias and SEE; Kozak and Smith [1993]). Also, note that a negative mean bias value corresponds to an overprediction and a positive mean bias value corresponds to an underprediction. Kozak and Smith (1993) also added as a note of caution that the values for percent bias and percent SEE can be misleading because the values of these statistics increase for dib's close to the tops of trees, although they used these criteria in their study.

We also reported mean absolute bias as an additional evaluation criterion in the tables, but did not use it to rank the equations. Mean absolute bias is the absolute value of the mean bias. Mean absolute bias provides a measure of how far away from zero the differences actually were, on the average, because the absolute value does not allow the negative and positive bias values to cancel. Although SEE addresses this issue (Kozak and Smith 1993), some readers may prefer mean absolute bias because the units are equivalent to those for diameter and volume, whereas those for SEE are squared.

Evaluation Results

Based on the comparison using the 10% evaluation data set, the dib taper and volume equation in this study ranked

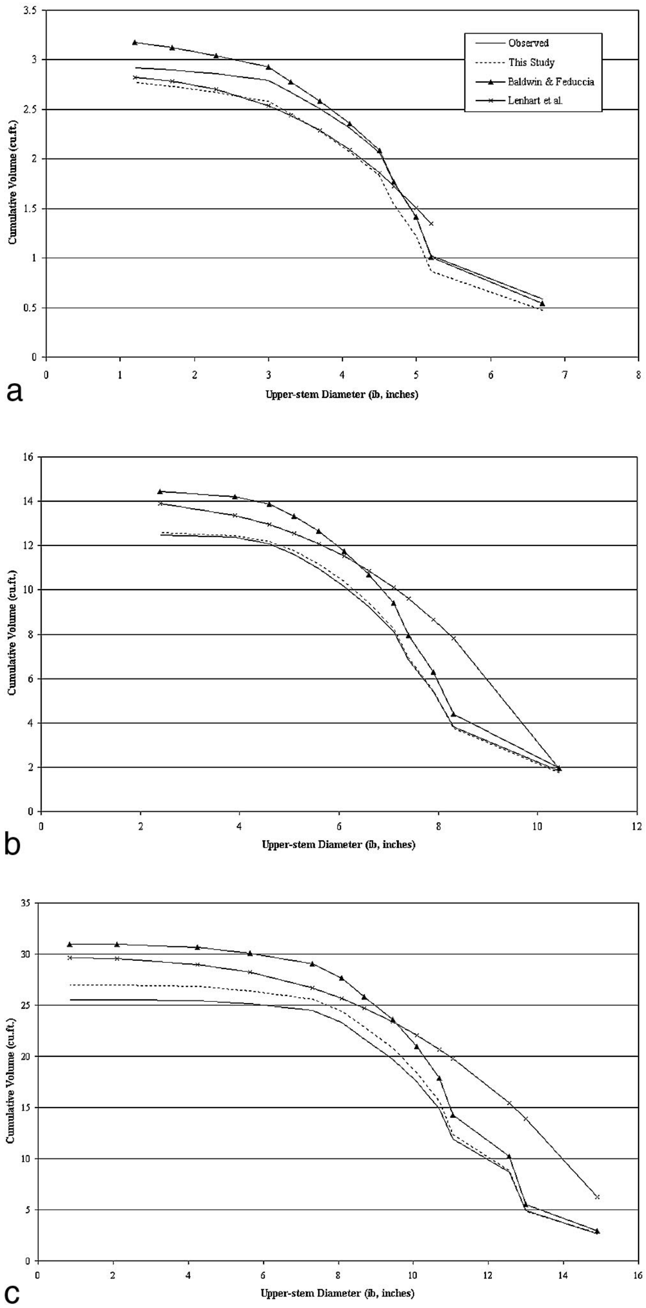

Figure 1. Predictions of cumulative wood only volume by ib upper-stem diameter from three compatible taper and volume equations (this study, Baldwin and Feduccia [1991]), and Lenhart et al. [1987]) for three East Texas loblolly pine trees: (a) dbh = 6.0 in. and total height (THT) = 40 ft, (b) dbh = 10.0 in. and THT = 66 ft, and (c) dbh = 14.8 in. and THT = 64 ft.

first for all criteria combined, followed by the equations of Baldwin and Feduccia (1991) and then those of Lenhart et al. (1987; Table 4). The dob taper and volume equations of Baldwin and Feduccia (1991) ranked first for all criteria combined, followed closely by those in this study and then those of Lenhart et al. (1987; Table 4). These results represent the rank sums of the four evaluation criteria for both diameter and volume at the two levels of resolution: overall (Table 5) and RHC (Tables 6–9). For ib predictions of diameter and volume, the equations in this study ranked first in every category except percent bias for overall diameter estimation (Table 4). The second and third place ranks varied between the equations of Baldwin and Feduccia (1991) and Lenhart et al. (1987), except for Lenhart et al. (1987) ranking first in percent bias for overall diameter estimation (Table 4). However, the ob predictions of diameter and volume were very similar in rank between this study and Baldwin and Feduccia (1991; Table 4). The equations in this study ranked first or second for all categories, except for diameter predictions by RHC, in which they ranked third (Table 4).

In terms of volume by RHC (Tables 6 and 7), the equations in this study consistently underpredicted volumes to the same degree for all RHCs. This underprediction was higher for ob volumes than ib volumes. Baldwin and Feduccia (1991) consistently overpredicted volumes for all RHCs. This overprediction was higher for ib volumes than outside volumes. Compared with the equations in this study, Baldwin and Feduccia (1991) performed much better (sum of ranks = 10 versus 20, Table 7) for ob volumes than ib volumes (sum of ranks = 27 versus 10, Table 6), likely because of bark thickness differences in the data. Baldwin and Feduccia (1991) fit their equations to data collected in Louisiana. Perhaps those trees had different bark thickness than the East Texas trees used in this study. Lenhart et al. (1987) consistently overpredicted volumes for all RHCs. However, predictions were most overpredicted in the lower sections of the tree (i.e., butt logs). We believe the model form of Lenhart et al. (1987) lacks flexibility to more accurately account for the taper in the lower bole sections of the evaluation trees compared with the other two sets of equations. In fact, for RHC = 0.05, Lenhart et al. (1987) was unable to calculate volumes for some of these bole sections (note that $n = 54$ in Table 6 and $n = 33$ in Table 7 versus $n = 57$ for the other two equations).

In terms of diameter by RHC (Tables 8 and 9), the equations in this study underpredicted diameters for all RHCs in the lower three-quarters of the tree (RHC < 0.75) and diameters were overpredicted in the top quarter of the tree. Diameters in the upper-most sections (RHC = 0.95) were the most overpredicted. Baldwin and Feduccia (1991) consistently overpredicted diameters for all RHCs, with the worst overpredictions in the upper sections of the tree (RHC > 0.65). Similar to volumes by RHC, Baldwin and Feduccia (1991) performed slightly better (sum of ranks = 19 versus 21) for ob diameters than ib diameters (sum of ranks = 30 versus 14), likely because of bark thickness differences in the data. Interestingly, all three sets of equations performed similarly for ob diameters (sum of ranks between 19 and 21), except that Lenhart et al. (1987) did not poorly predict ob diameters in the top sections such as the other two sets of equations. As with volumes, for RHC = 0.05, Lenhart et al. (1987) was unable to calculate diameters for some of these bole sections (note that $n = 54$ in Table 8 and $n = 33$ in Table 9 versus $n = 57$ for the other two equations).

To further compare the three sets of equations, cumulative ib volume was predicted then plotted over upper-stem ib diameter for three trees in the evaluation data set that spanned a range of sizes: (1) dbh = 14.8 in. and total height = 64 ft (Figure 1a), (2) dbh = 10 in. and total height = 66 ft (Figure 1b), and (3) dbh = 6 in. and total height = 40 ft (Figure 1c). For the largest dbh tree (Figure 1a), the cumulative ib volume predicted by equations in this study most closely follow the observed cumulative ib volume, especially for the larger upper-stem ib diameters, which corresponds to the lower sections of the tree (i.e., butt log). The trend is similar for the 10-in. dbh tree, except the predicted cumulative ib volumes even more closely follow the observed cumulative ib volumes at all upper-stem ib diameters (Figure 1b). For the smallest dbh tree, cumulative ib volumes predicted by Baldwin and Feduccia (1991) most closely follow the observed cumulative ib volumes, except at the smaller upper-stem ib diameters, which corresponds to the upper sections of the tree. Also, the cumulative ib volume predictions from the equations in this study closely follow those of Lenhart et al. (1987), except for the larger upper-stem ib diameters. This result is also likely caused by the inflexibility of Lenhart et al.'s (1987) model to accommodate the lower-stem taper of this tree. In fact, Lenhart et al. (1987) was unable to calculate a nonnegative number for the largest upper-stem diameters for this small tree.

Applications

To show how the new volume and taper equations work, we will show how to calculate several tree attributes.

Calculating a Total Stem Volume

Using Equation 2, calculate the total cubic-foot volume (V) of wood only above the stump for a loblolly pine tree with dbh = D = 11 in., total height = H = 62 ft, and stump height = H_L = 0.5 ft. First, calculate

$$h_U = \frac{H_U}{H} = \frac{62}{62} = 1, \quad h_L = \frac{H_L}{H} = \frac{0.5}{62} = 0.00806, \quad (9)$$

I_1 = 0 because 1 > 0.7019 = a_1,
I'_1 = 1 because 0.00806 < 0.7019 = a_1,
I_2 = 0 because 1 > 0.0789 = a_2,
I'_2 = 1 because 0.00806 < 0.0789 = a_2.

Next, use Equation 2 with the appropriate coefficient and variable values to calculate the volume:

$$V = 0.005454 * 11^2 * 62 * \begin{bmatrix} \frac{-2.6676}{2}(1^2 - 0.00806^2) + \frac{1.2357}{3}(1^3 - 0.00806^3) \\ - (-2.6676 + 1.2357)(1 - 0.00806) \\ - \frac{-1.3667}{3}((0.7019 - 1)^3 * 0 - (0.7019 - 0.00806)^3 * 1) \\ - \frac{61.4260}{3}((0.0789 - 1)^3 * 0 - (0.0789 - 0.00806)^3 * 1) \end{bmatrix} = 14.5 \text{ ft}^3. \quad (10)$$

Calculating a Butt Log Volume

For the same tree in the previous example, use Equation 2 to calculate the cubic-foot volume (V) of wood only in the butt log, where the log length = 16 ft and the trim allowance = 0.3 ft. First, calculate

$$H_U = 16 + 0.3 + 0.5 = 16.8 \quad h_U = \frac{H_U}{H} = \frac{16.8}{62} = 0.27097, \quad (11)$$

I_1 = 1 because 0.27096 < 0.7019 = a_1
I_2 = 0 because 0.27097 > 0.0789 = a_2

Next, use Equation 2 with the appropriate coefficient and variable values to calculate the volume:

$$V = 0.005454 * 11^2 * 62 * \begin{bmatrix} \frac{-2.6676}{2}(0.27097^2 - 0.00806^2) \\ + \frac{1.2357}{3}(0.27097^3 - 0.00806^3) \\ - (-2.6676 + 1.2357)(0.27097 - 0.00806) \\ - \frac{-1.3667}{3}((0.7019 - 0.27097)^3 \\ * 1 - (0.7019 - 0.00806)^3 * 1 \\ - \frac{61.4260}{3}((0.0789 - 0.27097)^3 \\ * 0 - (0.0789 - 0.00806)^3 * 1 \end{bmatrix} = 7.3 \text{ ft}^3. \quad (12)$$

Calculating an Upper-Stem Diameter

Using Equation 1, calculate the upper-stem dib (D_U) at the upper-stem height = H_U = 48 ft for a loblolly pine tree with dbh = D=12 in. and total height = H = 61 ft. First, calculate some necessary variables:

$$h_U = \frac{H_U}{H} = \frac{48}{61} = 0.78689, \quad (13)$$

J_1 = 0 because 0.78689 > 0.7019
J_2 = 0 because 0.78689 > 0.0789

Next, again use Equation 1 with the appropriate coefficient and variable values to calculate the upper-stem dib:

$$D_U^2 = 12^2 \begin{Bmatrix} -2.6676 * (0.78689 - 1) \\ +1.2357 * (0.78689^2 - 1) \\ -1.3667 * (0.7019 - 0.78689)^2 * 0 \\ +64.4260 * (0.0719 - 0.78689)^2 * 0 \end{Bmatrix} = 14.10229. \quad (14)$$

Then, take the square root to find D_U:

$$D_U = \sqrt{14.10229} = 3.8 \text{ in.} \quad (15)$$

Conclusions

Overall, the taper and volume equations developed in this study better estimate upper-stem diameters and cubic-foot volumes of East Texas loblolly pine trees growing in semi-intensive management plantations than the equations of Baldwin and Feduccia (1991) and Lenhart et al. (1987). Baldwin and Feduccia (1991) performed marginally better than the new equations (sum of ranks = 12 versus 13, respectively [Table 4]) for dob and volumes but much more poorly than the new equations (sum of ranks = 22 versus 9, respectively [Table 4]) for dib and volumes. We attribute this difference to Baldwin and Feduccia (1991) using data from central Louisiana trees that may have different bark thicknesses than East Texas trees to develop their equations. Lenhart et al. (1987) used mostly small trees to develop their equations as well as noncompatible volume and taper equations, which lacked flexibility in describing the lower sections (i.e., butt logs) of the tree. Based on these results, we recommend using the new equations for loblolly pine trees that have up to 16-in. dbh growing in semi-intensive plantations in East Texas. We further recommend forest managers use taper and volume equations developed and/or calibrated for the region in which they are implemented.

Literature Cited

Baldwin, V.C. Jr., and D.P. Feduccia. 1991. Compatible tree-volume and upper-stem diameter equations for plantation loblolly pines in the West Gulf Region. South. J. Appl. For. 15(2):92–97.

Borders, B.E. 1989. Systems of equations in forest stand modeling. For. Sci. 35(2):548–556.

Clark, A. III, R.F. Daniels, and G. Peter. 2000. Effects of tree, stand, silvicultural, and environmental variables on wood properties of planted loblolly pine: Field sampling procedures. Unpublished report on file at the Arthur Temple College of Forestry and Agriculture, Stephen F. Austin State University, Nacogdoches, Texas. 20 p.

Hasness, J.R., and J.D. Lenhart. 1972. Cubic-foot volumes for loblolly pine trees in old-field plantations in the Interior West Gulf Coastal Plain. Texas Forestry Pap. 12, School of Forestry, Stephen F. Austin State University, Nacogdoches, TX. 7 p.

Husch, B., C.I. Miller, and T.W. Beers. 1982. Forest Mensuration, 3rd Ed. John Wiley and Sons, Inc., New York. 402 p.

Jordan, L., R.F. Daniels, A.C. Clark III, and R. He. 2005. Multilevel nonlinear mixed-effects models for the modeling of earlywood and latewood microfibril angle. For. Sci. 51(4):357–371.

Kozak, A., and J.H.G. Smith. 1993. Standards for evaluating taper estimating systems. For. Chron. 69(4):438–444.

Lapongan, J., A.B. Vaughn, and J.D. Lenhart. 1993. Tree content and taper functions for planted loblolly and slash pine trees in East Texas. East Texas Pine Plantation Res. Project Rep. 28, Arthur Temple College of Forestry and Agriculture, Stephen F. Austin State University, Nacogdoches, TX. 9 p.

Lenhart, J.D., E.V. Hunt Jr., and J.A. Blackard. 1985. Establishment of permanent growth and yield plots in loblolly and slash pine plantations in east Texas. P. 436–437 *in* Proc. 3rd Bienn. South. Silvic. Res. Conf., Shoulders, E. (ed.), USDA For. Serv. Gen. Tech. Rep. SO-54.

Lenhart, J.D., T.L. Hackett, C.J. Laman, T.J. Wiswell, and J.A. Blackard. 1987. Tree content and taper functions for loblolly and slash pine trees planted on non-old field in east Texas. South. J. Appl. For. 10(2):109–112.

Max, T.A., and H.E. Burkhart. 1976. Segmented polynomial regression applied to taper equations. For. Sci. 22(3):283–289.

Ormerod, D.W.. 1973. A simple bole model. For. Chron. 49(2):136–138.

Robinson, A. 2004. Preserving correlation while modelling diameter distributions. Can. J. For. Res. 34(1):221–232.

Shiver, B.D., and B.E. Borders. 1996. Sampling techniques for forest resource inventory. John Wiley and Sons, New York. 356 p.

Van Deusen, P.C. 1988. Simultaneous estimation with a squared error loss function. Can. J. For. Res. 18(8):1093–1096.