1985

Maximizing expected profits in competitive bidding

Steven H. Bullard
Stephen F. Austin State University, Arthur Temple College of Forestry and Agriculture, bullardsh@sfasu.edu

Maximizing expected profits in competitive bidding

Steven H. Bullard

Abstract

Forest products firms often buy much of their raw material through competitive bidding. The bidding process is vital to such companies, yet models are often used which merely help predict winning bids. Managers should consider expected returns from potential timber buying contracts—the product of profit and the probability of realizing that profit. A general approach is summarized for maximizing expected profit in competitive bidding. For timber buying, profits are net returns minus stumpage costs. The probability of obtaining the profit is the probability a given bid will be accepted, and can be represented by a probability density function. The product of profit and the probability of acceptance is then maximized with respect to bid price. The approach is demonstrated for a simplified case, but can be adapted to meet the needs of individual firms.

Forest products firms frequently purchase timber cutting rights by offering competitive bids. The *science* of estimating bid prices for stumpage involves processing physical and economic sale data, and deriving a reference point or approximate bid. The *art* of estimating bid prices, however, is adjusting the final bid using past experience and detailed knowledge of the important factors for each sale.

This paper summarizes an approach to estimating bid prices which will maximize expected profits from potential timber purchases. Such prices may then be adjusted for factors not explicitly reflected by the model. The approach is adapted from an example for optimizing bid prices on chemical contracts (8).

Although maximizing utility is the theoretically correct objective under risk (4, 7), in many cases utility measures are not available and cannot be readily estimated. For these reasons, expected profit is considered in the present paper, rather than the expected utility of profit. The general approach is unaffected, however, and can be applied for either objective.

Symbols

The following symbols are used:

A = bid price with a 50 percent chance of acceptance (dollars per unit of volume),
B = bid price (dollars per unit of volume),
S = range of bid prices above and below A,
L = $A - S$ = lowest possible bid (0% chance of acceptance),
H = $A + S$ = highest possible bid (100% chance of acceptance),
R = revenue from the sale of final products (dollars per unit of volume), net of processing, harvesting, and other costs except stumpage, and
EP = expected profit from a potential timber sale (dollars per unit of volume).

Model

Tradeoffs occur in competitive bidding. For a particular sale, bids with relatively high chances of being accepted result in relatively low profits. Low bids result in higher profits but may have little probability of acceptance. On many potential timber sales, buyers should evaluate potential profit, as well as the probability their bid will be accepted (the probability that profit will be realized). Most timber buyers currently make such assessments implicitly.

This paper presents a general approach for recognizing potential tradeoffs directly. The approach is designed to estimate bid prices which maximize expected profits: (Profits)*(Probability of Acceptance). The general model can be simply stated as

$$\underset{\{B\}}{Maximize:}\; EP = (R - B) \int_{L}^{B} \delta(B)\, d(B).$$

The author is an Assistant Professor of Forest Economics, Mississippi State Univ., P.O. Drawer FR, Mississippi State, MS 39762. This is Paper 5759 of the Mississippi Agri. and Forestry Expt. Sta. It was received for publication in March 1984.

Forest Prod. J. 35(2):58-60.

We therefore solve for the bid price (B) which maximizes expected profit (EP). Expected profit from each sale is potential profit ($R - B$) times the probability the bid offered will be accepted (6(B) is the probability density for bid price acceptance). For timber contracts involving extended periods, the above objective can be modified to represent the present value of expected future profits.

Example

To demonstrate the modeling approach, a simple triangular distribution is used to represent probabilities of acceptance [6(B)]. The distribution allows the general approach to be demonstrated without data, and can be solved for optimal results. The procedure for maximizing expected profit from potential contracts is general, however, and can be used with other probability density functions to meet the assumptions, data, and needs of individual firms. The example is not intended as a realistic model for any particular firm, but is presented to demonstrate the general approach of determining bids which maximize expected profits. In practice, of course, applications may involve more complex distributions and optimization methods.

The first step in using the triangular distribution is to estimate a bid price which has a 50 percent chance of acceptance (A). Bid prices greater than A will have greater than 50 percent chances, while bids lower than A have less than 50 percent probabilities.[1]

The probability that a given bid price will be accepted is illustrated in Figure 1. For bids greater than or equal to A, the probability of acceptance is 1 minus the area of triangle B_HZH. For bids less than or equal to A, the probability of acceptance is the area of triangle B_LXL. H represents the highest bid of interest, with a 100 percent probability of acceptance. L is the lowest bid considered and has a zero percent probability. In the following discussion, the distribution is assumed to be symmetric ($H - A = A - L = S$) with a height of $Y - A = 1/S$. The areas of AYL and AYH therefore sum to 1,[2] and Figure 1 represents the probability density assumed for stumpage bid prices.

In maximizing expected profit with the triangular distribution, two cases must be considered:

1. For bids greater than or equal to A,
   $$EP_H = (R - B_H)(1 - B_HZH).$$
2. For bids less than or equal to A,
   $$EP_L = (R - B_L)(B_LXL).$$

Areas for B_HZH and B_LXL are functions of bid price (B_H or B_L). The slope of lines LY and HY is $(1/S)/S = 1/S^2$. The area of B_HZH is thus $(½)((H - B_H)/S^2)(H - B_H) = (H - B_H)^2/(2S^2)$,[3] and the area of B_LXL is $(B_L - L)^2/(2S^2)$. For the two cases, therefore, expected profits are as follows:

$$1.\quad EP_H = (R - B_H)(1 - [(H - B_H)^2/(2S^2)]), \qquad [1]$$

and

$$2.\quad EP_L = (R - B_L)((B_L - L)^2/(2S^2)). \qquad [2]$$

To maximize expected profit, derivatives with respect to bid price are considered. Setting the first derivative of relation [1] with respect to B_H equal to zero yields:

$$B_H^* = \frac{(2H + R) \pm [(H - R)^2 + 6S^2]^{1/2}}{3}. \qquad [3]$$

The second derivative of [1] is negative and B_H^* represents the bid price which maximizes expected profit for bids greater than or equal to A.

For bids less than or equal to A, the first derivative of relation [2] with respect to B_L equals zero for $B_L = L$ and $B_L = (2R + L)/3$. The second derivative of [2] is positive for $B_L = L$ and expected profit is minimized when the bid offered has a zero percent chance of acceptance. The second derivative is negative, however, and expected profit is maximized when:

$$B_L^* = \frac{2R + L}{3}. \qquad [4]$$

B_H^* and B_L^* are calculated from parameters H, L, R, and S, and substituted into Equations [1] and [2], respectively. Infeasible values result for B_L^* when $R > H - ½S$.[4] For the assumed distribution, if revenues net of all costs except stumpage are relatively high, bids with little chance of acceptance are non-optimal. Bids with

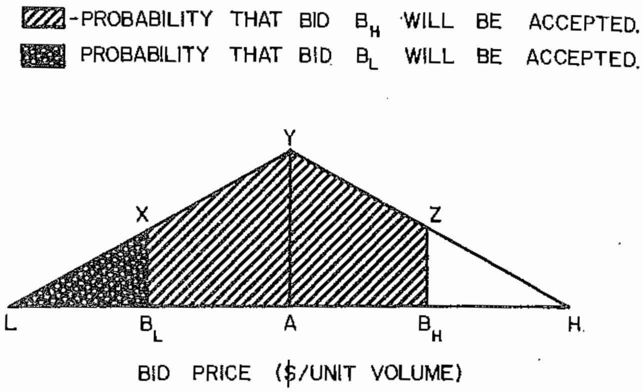

Figure 1. — Probability density assumed for bid price acceptance on a potential timber sale.

[1]Although the triangular distribution is used merely to demonstrate an approach, in practice A could be estimated in several ways, including multiple linear regression (see (1, 2, and 3)).

[2]$$AYL + AYH = ½(A - L)(1/S) + ½(H - A)(1/S)$$
$$= ½(A - L)(1/S) + ½(A - L)(1/S) = (A - L)/S$$
$$= (A - L)/(A - L) = 1.$$

[3]The height of B_HZH for any B_H is:

$$Slope = Height/Base$$
$$1/S^2 = Height/(H - B_H)$$
$$Height = (H - B_H)/S^2$$

[4]$$R > H - ½S,$$
$$R > (3H - 3S - H + 2S)/2,$$
$$R > (3A - L)/2,$$
$$\frac{2R + L}{3} > A.$$

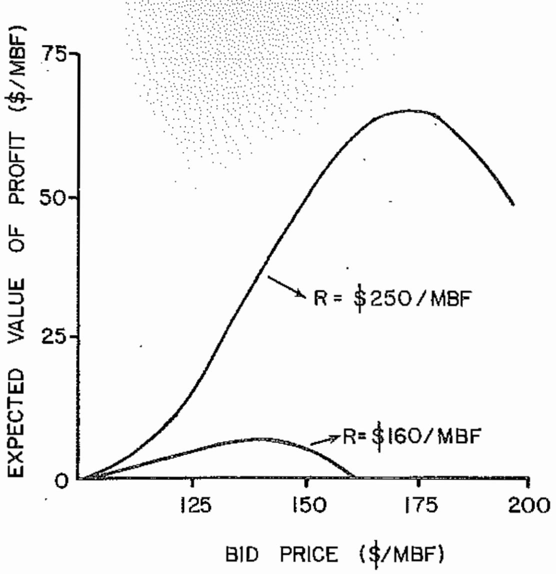

Figure 2. — Expected profit from a potential contract, with high and low potential for profit.

less than 50 percent probabilities are only optimal for sales with little potential for profit.

The effect of profit potential on optimal bid prices for the assumed relationships is illustrated in the following example. Let A = $150/thousand board feet (MBF) and S = $50. Therefore, L = $100 and H = $200/MBF. Expected profit relationships are illustrated in Figure 2 for two net revenue possibilities. If revenue from the sale of final products (net of all costs except stumpage) is $250/MBF, the optimal bid (from Equation [3]) is $172.57. If net revenue is only $160, however, the optimal bid price (from Equation [4]) is $140.

Discussion

In theory, stumpage prices are determined by the difference between the value of processed products and the costs of obtaining them from standing timber, including a margin for the processor's profit and risk (5, 6). In bidding for timber, the price offered directly affects not only the potential profit but the probability that profit will be realized. This tradeoff is reflected by expected profit, but it is not considered in models which merely predict winning bid prices.

The procedure illustrated with the assumed triangular distribution is general since other probability densities can be used to maximize expected profits. Individual firms may vary in how the bid price-probability of acceptance relationship is viewed. Another factor in using the approach is the degree to which profit can be isolated as a function of stumpage prices. For highly integrated firms, for example, problems may arise in determining net revenues per unit of volume input. For a producer with a single product and one sawmill, however, revenues net of all costs other than stumpage would be much easier to estimate.

Competitive bidding in forestry involves many factors difficult to reflect in mathematical models. Relative accessability in wet weather, the time period over which harvesting would occur, and the number of competitors expected are examples. Bidding models of varying complexity are used by forest products firms, however, providing prices from which consideration of other factors begins. For a particular sale, the estimated bid price which maximizes expected profit provides a basis for further adjustment. In general, differences between actual and estimated optimal bid prices should reflect the premium which managers are willing to pay for omitted factors and/or strategic reasons.

Literature cited

1. Anderson, W.C. 1976. A method for appraising multiple-product sales of southern pine stumpage. USDA Forest Serv. Res. Pap. SO-126, Southern Forest Expt. Sta., New Orleans, La.
2. ————. 1976. Appraising southern pine pulpwood stumpage. USDA Forest Serv. Res. Pap. SO-129, Southern Forest Expt. Sta., New Orleans, La.
3. Barge, C.D. 1977. An appraisal model for bidding on lump-sum multiple-product stumpage sales. Unpubl. M.F. Professional Pap., Mississippi State Univ.
4. Chang, S.J. 1980. Discounting under risk: comments on adjusting interest rates. J. of Forestry 78(10):634-635.
5. Davis, K.P. 1966. Forest Management: Regulation and Valuation (Second Ed.), McGraw-Hill Book Co., New York, N.Y.
6. Duerr, W.A. 1960. Fundamentals of Forestry Economics. McGraw-Hill Book Co., New York, N.Y.
7. Kapur, J.N. 1982. Optimization in decision analysis, marketing and financial investments. Int. J. of Math. Educ. in Sci. and Technol. 13(1):25-33.
8. Lial, M.L., and C.D. Miller. 1974. Mathematics: With Applications in the Management, Natural, and Social Sciences. Scott, Foresman and Co., Glenview, Ill. and Brighton, England.