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# Estimating Optimal Thinning and Rotation for Mixed-Species Timber Stands Using a Random Search Algorithm

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# *Estimating Optimal Thinning and Rotation for Mixed-Species Timber Stands Using a Random Search Algorithm*

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ABSTRACT. The problem of optimal density over time for even-aged, mixed-species stands is formulated as a nonlinear-integer programming problem with numbers of trees cut by species and diameter class as decision variables. The model is formulated using a stand-table projection growth model to predict mixed-species growth and stand-structure. Optimal thinning and final harvest age are estimated simultaneously using heuristic random search algorithms. For sample problems with two species, random search methods provide near-optimal cutting strategies with very little computer time or memory. Optimal solutions are estimated for problems with eight initial species/diameter class groups, projected for up to three discrete growth periods. Such solution methods merit further study for evaluating complex stand- and forest-level decisions. FOREST SCI. 31:303–315.

ADDITIONAL KEY WORDS. Combinatorial optimization, heuristics, integer programming, nonlinear programming, random search, stand-level optimization, stand-table projection.

INCREASING COMPETITION for forest resources is resulting in more emphasis on stand-level decisions in forest management. Harvesting decisions can be especially complex for the mixed-species forests common throughout the United States because species groups often have pronounced differences in growth rates, abilities to respond to release, and values by size classes.

Previous single-species harvest optimizations have used marginal analysis (Chappelle and Nelson 1964), calculus of variations (Naslund 1969, Schreuder 1971), dynamic programming (Amidon and Akin 1968, Chen and others 1980, Brodie and Kao 1979, Riitters and others 1982), and nonlinear programming (Kao and Brodie 1980). These studies have not, however, recognized distinct size classes for individual species groups. Here, the problem of optimal density over time is formulated as a nonlinear-integer programming problem. Random search methods are applied to *estimate* optimal thinning regimes and rotations for existing, even-aged, mixed-species stands. Lack of mixed-species growth information precluded an empirical application of the model. The approach is demonstrated for a two-species stand using assigned growth model parameters.

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## NOTATION

$a$ = subregion or area under the objective value probability density relation in which at least one solution is desired,
$b_w^{i,j}$ = growth model upgrowth ($w = 1, \ldots, S + 1$) and mortality ($w = S + 2, \ldots, 2S + 2$) parameter estimates by species and diameter class,
$d_i$ = initial number of diameter classes by species,
EXP = exponential function with base $e$,
FC = fixed harvesting costs (sale administration, marking, etc.) per unit area,
$G$ = number of growth periods projected,
$i$ = index for species group,
$j$ = index for diameter class,
$k$ = index for growth period,
ln = natural logarithm,
$L$ = value of cash flows expected after existing stand removal, discounted to time of final harvest,
$m$, $q$ = indexes used for species and diameter class, respectively, in relations where $i$ or $j$ is fixed,
$w$ = index for growth model parameters,
$n$ = number of solutions evaluated in simple random search, or number of solutions evaluated per stage in multistage random search,
$N_{i,j,k}^{R,I,C,U,M}$ = number of trees per unit area by species, diameter class, and growth period. Superscripts $R$, $I$, $C$, $U$, and $M$, denote residual, initial, cut, upgrowth, and mortality numbers, respectively,
$P_{i,j}$ = value per tree (price per unit volume times average volume per tree, by species and diameter class),
$\mathrm{PP}_{i,j,k}$ = upper bound on upgrowth by species, diameter class, and growth period, expressed as a potential proportion for trees advancing one diameter class,
Pr = probability that at least one of a set of randomly generated solutions will have an objective value within a specified upper region of the objective value probability density relation,
PV = present value of future cash flows, per unit area,
$r$ = real discount rate for calculating present values,
$S$ = number of species groups,
$t$ = number of years per growth period,
$V_{i,j}$ = average volume per tree (merchantable volume is used in order to determine per tree values),
$V_{m,\geq j,k-1}$ = volume per unit area of species $m$ in diameter classes greater than or equal to $j$, at the beginning of growth period $k$, and
$V_{T,k-1}$ = total volume per unit area at the beginning of growth period $k$.
$X_k$ = a binary variable equal to 1 if thinning occurs and 0 otherwise.

## THINNING MODEL DEVELOPMENT

A growth model was developed to provide post-harvest data by species and diameter class, which could be integrated with an optimization procedure to form a thinning model. The species/diameter level of resolution reflects interspecific growth rates and size-class value differences often found in mixed-species stands.

### Growth Model

Adams and Ek (1974) used a modified form of a stand-table projection model presented by Ek (1974) to derive optimal cutting policies for uneven-aged northern hardwoods. To provide volume and stand structure information, we used a stand-table projection approach related to the above model and consisting of upgrowth and mortality equations for each species/diameter class, after each growth period projected. The equations differ from those of Adams and Ek in that ingrowth is omitted for even-aged stands and individual species are recognized. For growth model details see Bullard (1983, 1985).

*Upgrowth.*—The number of trees of species ($i$) advancing from diameter class ($j$) to ($j+1$) during growth period ($k$) may be expressed as a potential proportion of residual trees in period ($k-1$), reduced to an actual proportion based on stand density:

$$N_{i,j,k}{}^{U} = (N_{i,j,k-1}{}^{R})(\mathrm{PP}_{i,j,k})\left\{\mathrm{EXP}\left[b_1{}^{i,j}(V_{T,k-1}) + \sum_{w=2}^{S+1} b_w{}^{i,j}(V_{m,\geq j,k-1})\right]\right\} \tag{1}$$

where

$$b_w{}^{i,j} \leq 0 \qquad (w = 1, \ldots, S + 1),$$

and

$$m = w - 1.$$

Potential upgrowth is a function of diameter, species, age, and site quality and may be projected separately. The last bracketed term is the proportion of potential realized during a growth period. The negative exponential function of volume ensures that estimated proportions will be between 0 and 1, and will be inversely related to density. Density terms include total stand volume at the beginning of the growth interval ($V_{T,k-1}$) and terms for each species indicating volume in diameter classes greater than or equal to ($j$) at the beginning of the growth period ($V_{m,\geq j,k-1}$). Such terms reflect the relative position of each diameter class within the stand.

*Mortality.*—The number of trees dying in each species/diameter class during a growth period may also be estimated as a function of stand density:

$$N_{i,j,k}{}^{M} = (N_{i,j,k-1}{}^{R})\left\{1 - \mathrm{EXP}\left[b_{S+2}{}^{i,j}(V_{T,k-1}) + \sum_{w=S+3}^{2S+2} b_w{}^{i,j}(V_{m,\geq j,k-1})\right]\right\} \tag{2}$$

where

$$b_w{}^{i,j} \leq 0 \qquad (w = S + 2, \ldots, 2S + 2),$$

and

$$m = w - 1.$$

Mortality for each species/diameter class is the number of trees at the beginning of the growth period ($N_{i,j,k-1}{}^{R}$) multiplied by the projected mortality proportion. The second term on the right side of (2) is the projected proportion dying. The specification ensures projections between 0 and 1, and is an increasing function of stand density.

### Thinning Formulation

The two-equation stand-table projection system was used in formulating the thinning problem as a nonlinear-integer programming model, i.e., one with nonlinear terms in the objective function and/or constraints and integer valued decision variables. Decision variables are cutting prescriptions, or numbers of trees harvested from each species/diameter class, after each growth period projected ($N_{i,j,k}{}^{C}$).

A disadvantage of the nonlinear programming formulation is that inexact solution techniques are required. Such methods are *only* advocated when exact methods such as linear or dynamic programming are inappropriate, unavailable, or impractical (see Bullard and Klemperer 1984). In the present problem, dynamic programming was not used because state-space dimensionality becomes a problem in stand-level applications which recognize diameter classes (Hann and Brodie 1980). The difficulties multipy when diameter classes are modeled for each species group represented in a mixed-species stand. Simply relaxing grid size does not abate the problem, since species diameter combinations expand the dynamic programming state-space to an impractical number of dimensions.

*Constraints.*—Constraints are used to restrict the numbers of cut trees to the numbers projected after each growth period. In general, constraint development follows the methods presented by Adams and Ek (1974). Equation sets (3) through (6) define the residual stand after each growth period, while (7) ensures non-negativity for all terms in the formulation.

$$N_{i,j,k}{}^{R} = N_{i,j,k}{}^{I} - N_{i,j,k}{}^{C} \quad \{i = 1, \ldots, S; j = 1, \ldots, d_i; k = 0\} \tag{3}$$

$$N_{i,j,k}{}^{R} = N_{i,j,k-1}{}^{R} - N_{i,j,k}{}^{U} - N_{i,j,k}{}^{M} - N_{i,j,k}{}^{C} \quad \{i = 1, \ldots, S; j = 1; k = 1, \ldots, G\} \tag{4}$$

$$N_{i,j,k}{}^{R} = N_{i,j,k-1}{}^{R} - N_{i,j,k}{}^{U} - N_{i,j,k}{}^{M} - N_{i,j,k}{}^{C} + N_{i,j-1,k}{}^{U} \quad \{i = 1, \ldots, S; j = 2, \ldots, d_i + k - 1; k = 1, \ldots, G\} \tag{5}$$

$$N_{i,j,k}{}^{R} = N_{i,j-1,k}{}^{U} - N_{i,j,k}{}^{C} \quad \{i - 1, \ldots, S; j = d_i + k; k = 1, \ldots, G\} \tag{6}$$

$$N_{i,j,k}{}^{R,I,C,U,M} \geq 0 \quad \{i = 1, \ldots, S; j = 1, \ldots, d_i + k; k = 0, \ldots, G\}. \tag{7}$$

Harvesting may occur immediately and after each growth period, with clear-cutting assumed after the last growth period projected. Equation set (3) defines the residual stand-table only after the initial growth period ($k = 0$), while (4), (5), and (6) give residuals after each of the ($G$) growth periods projected. For the smallest diameter class ($j = 1$), (4) represents the number of trees at the beginning of the growth interval, minus the numbers projected as upgrowth and mortality, minus the number of trees cut. Set (5) is the number of trees in all intermediate diameter classes ($j = 2, \ldots, d_i + k - 1$). For each of these diameters, an upgrowth term is added for trees advancing from the class just smaller. The number of trees in the largest diameter class that is reached ($j = d_i + k$) during each growth period is defined by (6) as projected upgrowth into the class, minus the number of trees harvested.

Finally, relations (1) and (2) were substituted into (4), (5), and (6). Equations (8) and (9) translate volume terms to numbers of trees, based on average volumes per tree by species and diameter.

$$V_{T,k-1} = \sum_{i=1}^{S} \sum_{j=1}^{d_i+k-1} (V_{i,j} N_{i,j,k-1}{}^{R}) \tag{8}$$

$$V_{m,\geq j,k-1} = \sum_{q=j}^{d_i+k-1} (V_{i,q} N_{m,q,k-1}{}^{R}). \tag{9}$$

The stand-table projection model is thereby integrated with the thinning model constraints, with numbers of trees cut by species and diameter as decision variables. The interface is achieved by using relations (8) and (9) for the volume terms in the growth model equations, using the resulting upgrowth and mortality functions in constraint sets (4), (5), and (6), and simplifying and collecting terms. The formulation reflects alternative cutting strategies, since harvesting trees reduces stand density, resulting in higher proportions of upgrowth and lower proportions of mortality in subsequent growth periods.

Other constraints may also be included in modeling mixed-species thinning. For example, constraints could ensure minimum thinning volumes if small quantities are not marketable. For certain forest types, maximum thinning volumes could ensure sufficient residual stand density to prevent quality losses from epicormic branching, enlarged lower limbs, etc. Constraints may also reflect wildlife, recreation, or other management considerations.

*Objective Function.*—Assuming final harvest after the last growth period projected, optimal cutting policies are estimated for 1 growth period, 2 growth periods, etc. The optimal harvest schedule maximizes the present value of current and future income[1]:

$$\underset{\{N_{i,j,k}{}^{C}\}}{\text{Maximize:}} \quad \text{PV} = \left\{ \sum_{k=0}^{G} \left[ \sum_{i=1}^{S} \sum_{j=1}^{d_i+k} [(P_{i,j}/(1 + r)^{kt})N_{i,j,k}{}^{C}] - X_k\text{FC}/(1 + r)^{kt} \right] + [L/(1 + r)^{Gt}] \right\}. \tag{10}$$

Summation terms are present values of timber harvested after each growth period, by diameter class and species. When thinning occurs, $X_k$ is 1 and otherwise is 0. Binary values may also be used in constraints representing minimum removals. Thinnings, for example, might be constrained to be greater than or equal to some minimum volume multiplied by $X_k$, and less than or equal to some upper limit multiplied by $X_k$. Hence, if thinning occurs, volume removed must be between the range of prescribed values. Various implicit enumeration methods may be used to ensure binary decision values in mathematical programming models. However, for the models considered here, it is more convenient to assign binary values explicitly. In the solution procedures to be discussed, IF statements (FORTRAN or BASIC) are used to test if thinning is selected, thereby defining $X_k$.

The final term in (10) is the present value of expected cash flows *after* final harvest. If the existing stand were replaced by a perpetual series of similarly managed stands, for example, $L$ would be the soil expectation value for the proposed series. Otherwise $L$ is projected land sale income, or the value in year $G(t)$ of an alternative use.

*Convexity and Program Size.*—Convexity of the objective function and the constraint set are desirable properties in nonlinear programming. Convex programs lack local optima which are not globally optimal, and the first-order Kuhn-Tucker local optimality conditions are necessary and sufficient to assure a global optimum (under certain constraint qualifications). Ignoring the discrete nature of the problem for the time being, and following the growth model substitutions outlined, the mixed-species constraints represent nonlinear equalities and therefore a nonconvex feasible region. It can also be shown, however, that the constraints have no equivalent convex structure, since they may be expressed as additions and subtractions of convex relationships. Therefore, even if we overlook the problem's

[1] A discussion of valuing existing stands is presented by Clutter and others (1983, p. 226).

discrete character, the possibility of locally optimal solution arises in solving for optimal thinning and rotation with the mixed-species formulation. Although program size can be a computational feasibility issue in nonlinear programming, it is not a practical limitation with the solution methods used here.

## THINNING MODEL SOLUTION

The nonlinear-integer programming thinning formulation is difficult to solve with exact methods.[2] The number of possible integer solutions can be astronomical for problems recognizing diameter classes by species over time. For large discrete problems, iterative methods without proven convergence to an optimum are often advocated. These are termed "heuristics" (see Silver and others 1980, or Muller-Merbach 1981).

Several optimization approaches use random sampling in selecting and evaluating discrete options (see Brooks 1958, Karnopp 1963, Luus and Jaakola 1973, Mabert and Whybark 1977, Conley 1980, 1981, and Solis and Wets 1981). We have applied and compared two such methods—simple random search and multistage random search.

### SIMPLE RANDOM SEARCH

To randomly select the number of trees to cut from each species/diameter class, a random number between 0 and 1 is generated and transformed to an integer number of trees in the species/diameter group.[3] After obtaining random variates for numbers to cut from all classes recognized, cutting combinations not meeting volume or other constraints are rejected. If a high proportion of the cutting strategies generated is infeasible, random numbers may be modified so that each set yields a feasible combination (see Conley 1980). This problem did not arise in our applications. For multiple growth period problems, the random solutions are generated sequentially, period by period. In this manner, potential numbers of trees to cut after each period are limited to projections of those remaining after previous harvests.

Figure 1 shows major steps in the simple random search. For each feasible thinning plan, the objective value is calculated and compared with the highest value obtained thus far. After each comparison, the higher present value (and associated thinning regime) is stored. The sampling process continues until a specified number of random solutions have been evaluated.

Given a finite number of solutions, the relative frequency distribution of ob-

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[2] Numbers of trees harvested over time are integer-valued decision variables, and selecting and evaluating integer thinning alternatives is a combinatorial problem. Such problems involve the "arrangement, grouping, ordering, or selection of discrete objects, usually finite in number" (Lawler 1976). Solving large combinatorial problems with exact methods can be difficult because the solution effort increases very rapidly (usually exponentially) with certain problem characteristics, such as the number of discrete variables (Kovacs 1980).

[3] We generated uniformly distributed (pseudo-) random numbers between 0 and 1, implying that numbers of trees within diameter classes are uniformly distributed. Random numbers (and subsequent variates) may be generated from other relative frequency distributions, however, depending on diameter and diameter class size, species, or other factors affecting the distribution within diameter classes. To ensure equal probabilities for each integer number of trees, the maximum number is incremented by one and the new range is multiplied by the random number, then rounded to the nearest integer. The artificially high number and the lowest number each have one-half the proper probability. Therefore, if the upper number results after rounding, the lowest number is assigned (doubling its probability). In this manner, the artificial number is never recommended for cutting, and all integer numbers of trees have an equal probability of selection.

```
 _________________________________________________
|                                                 |
| Assign the number of growth periods to be       |
| considered, and the number of thinning          |
| schedules to be evaluated.                      |
|_________________________________________________|
                        |
                        |<- - - - - - - - - - - - - - -
                        V                             |
 _________________________________________________    |
|                                                 |   |
| Assign input values for the initial stand.      |   |
|_________________________________________________|   |
                        |                             |
                        V                             |
 _________________________________________________    |
|                                                 |   |
| Generate (randomly) a feasible thinning         |   |
| schedule and project the residual stand for     |   |
| the next growth period.  Repeat until the       |   |
| specified number of growth periods has been     |   |
| considered.                                     |   |
|_________________________________________________|   |
                        |                             |
                        V                             |
 _________________________________________________    |
|                                                 |   |
| Calculate PV and compare with the optimum       |   |
| thus far.  Store the solution with the          |   |
| greater PV.  Has the specified number of        |>_N_|
| thinning schedules been evaluated?              |
|_________________________________________________|
                        | Y
                        V
 _________________________________________________
|                                                 |
| Write the highest PV obtained, and the          |
| associated thinning regime.   STOP.             |
|_________________________________________________|
```

FIGURE 1. Major steps in solving thinning model formulations with simple random search.

jective function values is bounded on the right by the maximum. The goal in simple random search is to obtain at least one solution yielding an objective value within a specified subregion of the optimum or extreme value. The relative size of the desired subregion ($a$) also represents the probability that a single randomly selected solution will have an objective value within the specified area ($a$) under the relative frequency plot (Brooks 1958). For any given problem, therefore, the probability (Pr) that at least one of $n$ random solutions yields an objective value within the desired upper region of the probability density relation is (Conley 1980)

$$\Pr = 1 - (1 - a)^n. \tag{11}$$

For problems where large numbers of samples can be selected and evaluated easily, the probability that at least one will yield an objective value within a small upper region of the density plot may be forced arbitrarily close to 1. For example, given a sample of ($n$ = 10,000) feasible solutions, the probability that at least one is within the upper ($a$ = 0.001) region of the objective value density relation is

$$\Pr = 1 - (1 - 0.001)^{10,000} = 0.9999548. \tag{12}$$

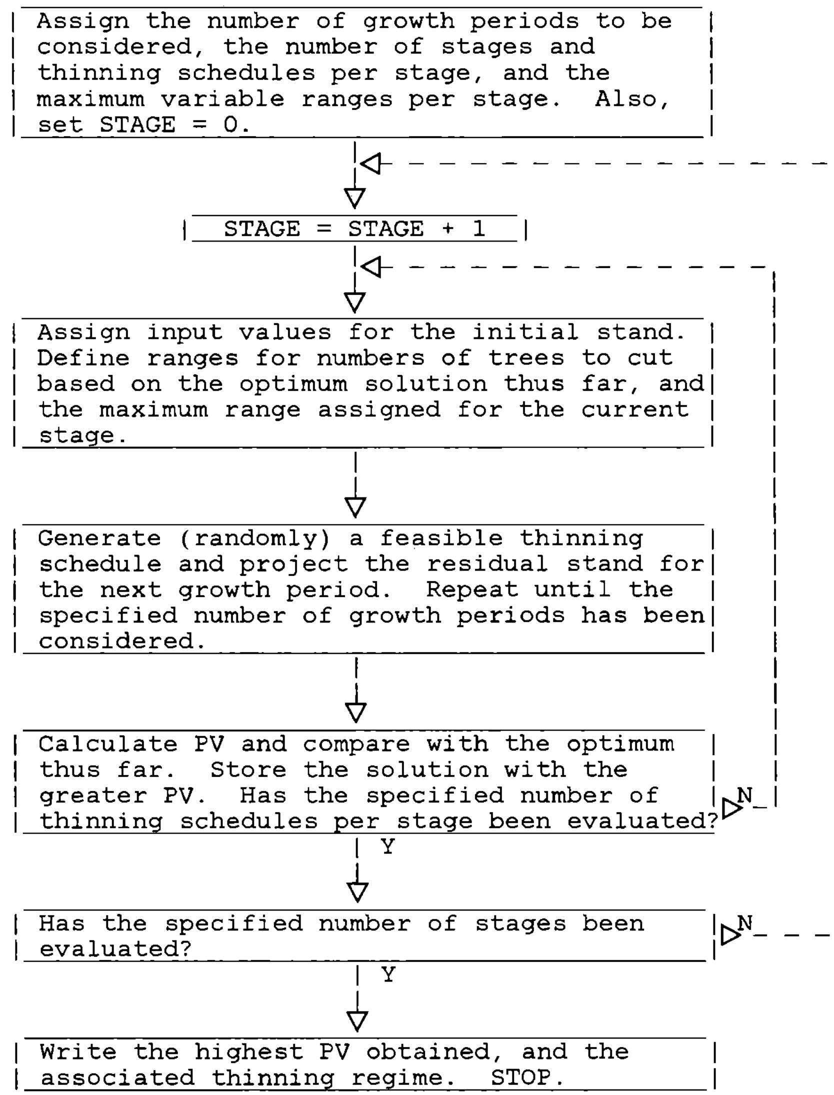


FIGURE 2. Major steps in solving thinning model formulations with multistage random search.

The above calculation does not depend on the total number of possible solutions to a problem. The success of simple random search in providing near-optimal solutions depends on the *magnitude* of the objective function values for a problem, and on the *shape* of the right-hand tail of the probability density of objective values.

Objective value magnitude is important since being within an upper fraction of the possible solutions to a problem is little consolation if the absolute difference between optimum and near-optimum values is large (Golden and Assad 1981). The necessary characteristic for the right-hand tail of the objective value density for a problem is that the maximum (extreme right-hand) value should not be isolated or at the end of an extended tail (Conley 1980). In problems with extended tails, obtaining a solution with an objective value within a specified region does not guarantee the solution will be near (in actual value to) the optimum.

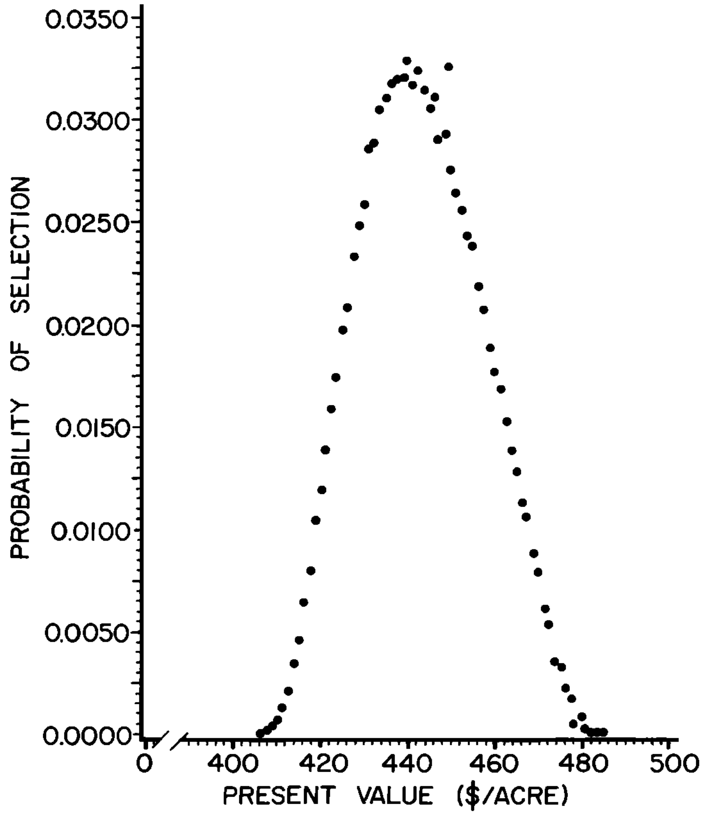


FIGURE 3. Probability density of objective function values for all feasible solutions to problem 1.

MULTISTAGE RANDOM SEARCH

Multiple sets (or stages) of random solutions are selected and evaluated in the multistage approach. After each set of solutions, variable ranges are reduced, based on the best solution from the previous sets, thus narrowing the potential region in which the optimum may lie. Candidate solutions are thereby concentrated in a region centered around the optimum thus far. Multistage approaches differ by how many sets of samples are examined, the total number evaluated and their distribution among stages, and the degree to which variable ranges are reduced from stage to stage.

Figure 2 outlines the multistage procedure used here. A uniform number of thinning alternatives is evaluated at each stage. In the initial stage, the ranges from which thinning options are randomly selected are set equal to the initial number of trees in each species/diameter class. For the first set of solutions, present values are calculated and compared with the greatest value obtained thus far. The first stage incumbent (or optimum thus far) is the solution with the greatest present value generated by $n$ randomly selected thinning schedules.

Maximum ranges for numbers of trees to cut are specified for each stage. The ranges are narrowed in later stages and are centered around the decision variable values in the incumbent solution. For example, if the maximum range is specified as 10 trees per acre for a given species/diameter class at a particular stage, and the incumbent solution calls for 15 trees to be harvested, the range used for that class in subsequent selections will be from 10 to 20 trees per acre. If a solution

*TABLE 1. Thinning model formulation for Problem 1.**

```
Maximize PV = $201.45+0.65X(2)+4.66X(4)+0.22X(6)+0.45X(8)+0.44X(9)+3.50X(10)
              +5.03X(11)+0.15X(12)+0.31X(13)+2.75X(14)-4X(0)
Subject to:

  X(1) + X(2) = 49          X(3) + X(4) = 39
  X(5) + X(6) = 49          X(7) + X(8) = 19

  X(9)-X(1)(EXP(-.0000973X(1)-.0001515X(3)-.0000192X(5)-.0000802X(7))
      -.6EXP(-.007072X(1)-.0110169X(3)-.0015261X(5)-.0058289X(7))) = 0

  X(12)-X(5)(EXP(-.0000768X(1)-.0001196X(3)-.0000302X(5)-.0000633X(7))
      -.25EXP(-.0062733X(1)-0097726X(3)-.0024701X(5)-.0051706X(7))) = 0

  X(10)-X(3)(EXP(-.0000384X(1)-.0001137X(3)-.0000151X(5)-.0000317X(7))
      -.7EXP(-.002944X(1)-.0095712X(3)-.0011592X(5)-.0024265X(7)))
      -.6X(1)EXP(-.007072X(1)-.00110169X(3)-.0015261X(5)-.0058289X(7)) = 0

  X(13)-X(7)(EXP(-.000023X(1)-.0000359X(3)-.0000091X(5)-.000019X(7))
      -.35EXP(-.0061082X(1)-.0095154X(3)-.0024051X(5)-.0050345X(7)))
      -.25X(5)EXP(-.0062733X(1)-.0097726X(3)-.0024701X(5)-.0051706X(7))= 0

  X(11)-.70X(3)EXP(-.002944X(1)-.0095712X(3)-.0011592X(5)-.0024265X(7))= 0

  X(14)-.35X(7)EXP(-.006108X(1)-.0095154X(3)-.0024051X(5)-.0050345X(7))= 0

  12.8X(2)+19.94X(4)+5.04X(6)+10.55X(8) ≤ 926X(0)
  12.8X(2)+19.94X(4)+5.04X(6)+10.55X(8) ≥ 370X(0)

  X(i) ≥ 0   (i=1,...,14)
```

*For ease of presentation, the following notation is used in Table 1:

$X(0) = X_k$, where k=0 (binary variable)

| | |
|---|---|
| $X(1) = N^R_{1,2,0}$ | $X(8) = N^C_{2,2,0}$ |
| $X(2) = N^C_{1,2,0}$ | $X(9) = N^R_{1,2,1}$ |
| $X(3) = N^R_{1,3,0}$ | $X(10) = N^R_{1,3,1}$ |
| $X(4) = N^C_{1,3,0}$ | $X(11) = N^R_{1,4,1}$ |
| $X(5) = N^R_{2,1,0}$ | $X(12) = N^R_{2,1,1}$ |
| $X(6) = N^C_{2,1,0}$ | $X(13) = N^R_{2,2,1}$ |
| $X(7) = N^R_{2,2,0}$ | $X(14) = N^R_{2,3,1}$ |

is obtained with a higher objective value and calls for 19 trees to be harvested from the combination, the new range will be from 14 to 24 trees per acre, etc.

In this manner, each time a specified number of solutions per stage has been evaluated, new (narrower) maximum ranges are used. In all cases the ranges used are feasible: i.e., ranges are centered around current decision variable values, but are restricted to lie between 0 and the number of trees existing or projected for a given species/diameter class. Sampling continues until the specified number of stages has been completed.

### Sample Problem Results.

*Problem 1.*—As an initial case, a stand was defined with two diameter classes for each of two species. The number of trees in each class resulted in a total of 2,000,000 possible thinning alternatives. Thinnings were constrained to be between 30 and 50 percent of initial stand volume. Economic and growth model parameters were assigned to project the 40-year-old stand for one 5-year growth period. The nonlinear-integer programming formulation for Problem 1 is presented in Table 1.

A FORTRAN program was used to evaluate all possible solutions to Problem 1, and the overall optimum objective value was $485.76. Figure 3 shows the complete probability density of objective function values for Problem 1. The distribution is obtained by dividing the difference between maximum and minimum objective values into histogram intervals, counting the number of solutions whose present values lie in each interval, and dividing each sum by the total number of feasible solutions.

The relative frequency plot of present values for Problem 1 lacks an extended right-hand tail, and simple random search was expected to provide solutions near the optimal value of $485.76. Using sample sizes of 1,000 and 10,000, simple random search provided solutions with present values within 1 percent of the optimum, using several initial seed numbers. The FORTRAN program used for the simple random search was less than 100 lines and required under 10 seconds of execution time on an IBM 3081.

We also solved Problem 1 with a multistage algorithm. The program was approximately 150 lines and required up to 30 seconds of execution time for samples of 1,000 and 10,000 solutions for each of 6 stages. With 1,000 solutions per stage, the true optimal solution was generated in 3 out of 10 trials. The worst of the remaining estimates had a present value of $483.17. With 10,000 solutions per stage, the true optimum was generated once. The other 9 trials yielded a locally optimal solution with a present value of $483.46.

*Problem 2.*—The second problem was defined for a 30-year-old stand with 2 species groups, each with 4 diameter classes, and thinning again constrained between 30 and 50 percent of prethinning volumes. Economic and growth model parameters were assigned to project the stand for 1, 2, and 3 growth periods of 5 years each. Optimal thinning policies were therefore estimated for 1-, 2-, and 3-growth period subproblems. Exhaustive search was impractical. The 1-growth period subproblem alone had a total of over 8 trillion possible thinning alternatives.

Optimal thinning policies were estimated for each subproblem using both solution methods. Sample sizes of 1,000 and 10,000 were used with several seed numbers for starting the random processes. Seven stages were used for the multistage algorithm. The FORTRAN programs ran up to 3.5 minutes, the longest being the multistage approach with 3 growth periods. For every trial of each subproblem, the multistage approach yielded higher present values than simple random search.

We evaluated the sensitivity of present values and harvest schedules to changes in interest rates and prices, and noted that the present value relationship is not necessarily concave with respect to the number of growth periods projected. Depending on assumed values by size classes, for example, the present value may increase, decrease, and increase again as sufficient growth periods are projected to allow growth into higher valued sizes. Optimal final harvest age, therefore, cannot always be determined simply by projecting increasing numbers of growth periods until present value decreases.

## Further Study

The promising results obtained with sample thinning model problems indicate the need for further research in estimating solutions to these and other stand- and forest-level problems with heuristic algorithms. Other multistage approaches, including those of Luus and Jaakola (1973) and Solis and Wets (1981) may be considered, as well as the "biased sampling" and "improvement sampling" techniques presented by Mabert and Whybark (1977). Further analyses of mixed-species thinning would benefit from statistically estimated growth model parameters for actual stands.

The greatest shortcoming of random search is that the optimum objective value remains unknown. Hence, the degree to which heuristically estimated values approach the optimum is unknown. For a given problem, the shape of the right-hand tail of the objective value density relation may be examined for large random samples of solutions. This process may eliminate random sampling for some problems (with obviously extended tails), but *cannot* result in confidence in using the methods since the exact tail behavior is not determined.

Further research should improve ways to quantify how close random search methods can come to an estimated true optimum, as discussed by Clough (1969), Dannenbring (1977), and Golden and Alt (1979). From a random sample of solutions, they derive point and/or interval estimates of the optimum value, for which the management plan is unknown. Sampling continues until the best solution is within a given range of the estimated optimum, as demonstrated for a discrete problem by McRoberts (1971). Dannenbring (1977) has evaluated several reduced-bias and best-fit truncation point estimators, including their performance in problems with adverse tail behavior. Several previously published estimators, and a very simple approach to confidence interval estimation, are briefly summarized by Zanakis and Evans (1981). Further random search applications in forestry should include optimal value estimation in developing effective stopping criteria for the sampling procedures.

## CONCLUSIONS

Thinning and rotation alternatives for existing mixed-species forests were estimated using a stand-table projection growth model in a nonlinear-integer programming thinning formulation. Harvest options began at present and continued with fixed-length growth projections until final harvest. Final harvest age was selected by projecting the stand for increasing numbers of growth periods, choosing the alternative with the highest present value. Optimal thinning regimes and final harvest age were thus simultaneously estimated.

Random search methods for solving forestry problems can be viable when exact solution methods are unavailable, when growth model or price and cost predictions are inexact, or when problems must be solved many times at low cost. The approach presented for mixed-species can model highly complex relationships using very little computer memory. Heuristic methods do not require restrictive assumptions such as linearity, and systems can be modeled as accurately as data permit. Random search algorithms require little storage and can be implemented on microcomputers. The type of models presented here could therefore be widely applied to thinning/rotation or other complex decisions in forestry.

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