On the ratchet effect

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Understanding the ratchet effect induced by symmetry breaking of temporal forces is a fundamental issue that has remained unresolved for decades. While the dependence of the directed transport on each of the ratchet-controlling parameters has been individually investigated experimentally, theoretically, and numerically, there is still no general criterion to apply to the whole set of these parameters to optimally control directed transport in general systems. We report that, to optimally enhance directed transport, there exists a universal force waveform which can be understood as the result of two competing fundamental mechanisms: the increase of the degree of breaking of the aforementioned temporal symmetries and the decrease of the force impulse. We demonstrate that this universal waveform explains all the previous experimental, theoretical, and numerical results for a great diversity of systems, including motor enzymes.

Consider a general system (classical or quantum, dissipative or non-dissipative, one- or multi-dimensional, noisy or noiseless) where a ratchet effect (*1-3*) is induced by solely violating temporal symmetries of a T-periodic zero-mean ac force $f(t)$ which drives the system (*4-18*). This effect is important because of its clear applicability to such diverse fields as biology, and micro- and nano-technology (*2,19-23*). In particular, it applies to electronic transport through molecules (*12*), smoothing surfaces (*24*), controlling vortex density in superconductors (*22*), separating particles (*25,26*), controlling directed current in semiconductors (*6,9*), and rectifying the phase across a SQUID (*27*). A popular choice would be the simple case of a biharmonic force,

$$f(t) = \epsilon_1 har_1 \left(\omega_1 t + \varphi_1\right) + \epsilon_2 har_2 \left(\omega_2 t + \varphi_2\right), \tag{1}$$

where $har_{1,2}$ represents indistinctly sin or cos, and $p\omega_1 = q\omega_2$, p, q coprime integers. In this case, the aforementioned symmetries are the shift symmetry ($\mathbf{S}_s : f(t) = -f(t + T/2), T \equiv 2\pi q/\omega_1 = 2\pi p/\omega_2$) and the time-reversal symmetries ($\mathbf{S}_{tr,\pm}$: $f(-t) = \pm f(t)$). Now a general unsolved problem is to find the regions of the parameter space $(\epsilon_i, \varphi_i), \epsilon_1 + \epsilon_2 = const.$, where the ratchet effect is optimal in the sense that the average of relevant observables is maximal, the remaining parameters being held constant. We show in the following that such regions are those where the *effective* degree of symmetry breaking is maximal. Without loss of generality, we shall illustrate the degree of symmetry breaking (DSB) mechanism by using the following working model for the driving force:

$$\begin{aligned} f_{ellip}(t) &= \epsilon f(t; T, m, \theta) \\ &\equiv \epsilon \, \mathrm{sn}\left(\Omega t + \Theta; m\right) \mathrm{cn}\left(\Omega t + \Theta; m\right), \end{aligned} \tag{2}$$

where $\mathrm{cn}(\cdot; m)$ and $\mathrm{sn}(\cdot; m)$ are Jacobian elliptic functions of parameter m, $\Omega \equiv 2K(m)/T$, $\Theta \equiv K(m)\theta/\pi$, $K(m)$ is the complete elliptic integral of the first kind, T is the period of the force, and θ is the (normalized) initial phase ($\theta \in [0, 2\pi]$). Fixing ϵ, T, and θ, the force waveform changes as the shape parameter m varies from 0 to 1 (see Fig. 1). Physically, the motivation of the choice represented by Eq. 2 is that $f_{ellip}(t; T, m = 0, \theta) = \epsilon \sin\left(2\pi t/T + \theta\right)/2$, and that $f_{ellip}(t; T, m = 1, \theta)$ vanishes except on a set of instants that has Lebesgue measure zero, i.e., in these two limits directed transport is not possible, while it is expected for $0 < m < 1$. Thus, one may expect *in general* the average of any relevant observable $\Re$ to exhibit an extremum at a certain critical value $m = m_c$ as the shape parameter m is varied, the remaining parameters being held constant. In this work, we demonstrate that such a value m_c is universal, i.e., there exists a universal force waveform which optimally enhances the ratchet effect in any system. Clearly, there are two competing fundamental mechanisms which allow one to understand the appearance of such extremum: the increase of the degree of breaking of the shift symmetry as m is increased, which increases the absolute value of the average, and the effective narrowing of the force pulse as m is increased, which decreases the absolute value of the average. The former mechanism arises from the fact that a broken symmetry is a structurally stable situation (Curie's principle) and can be quantitatively characterized by noting that

$$\frac{-f\left(t+T/2\right)}{f\left(t\right)}=\frac{\sqrt{1-m}}{\mathrm{dn}^{2}\left(\Omega t+\Theta;m\right)}\equiv D\left(t;T,m,\theta\right), \tag{3}$$

where $\mathrm{dn}\left(\cdot;m\right)$ is the Jacobian elliptic function. Equation 3 indicates that the degree of deviation from the shift symmetry condition $\left(D\left(t;T,m,\theta\right)\equiv 1\right)$ increases on average as $m\rightarrow 1$, irrespective of the values of the *period* and *initial phase* (see Fig. 2). Thus, while increasing the shape parameter from m $(0<m<m_c)$ improves the directed transport yielding a higher average, it simultaneously narrows the force pulse lowering the driving effectiveness of the force. Indeed, the latter effect becomes dominant for sufficiently narrow pulses $(m>m_c)$. Next, one exploits the universality of the waveform corresponding to m_c to deduce the optimal values of the parameters of the biharmonic force of Eq. 1. To this end, notice that we chose the function of Eq. 2 to satisfy the requirement that m_c be sufficiently far from 1 so that the elliptic force is effectively approximated by its first two harmonics. One thus obtains a relationship between the amplitudes of the two harmonics in parametric form: $\epsilon_{1,2}=\epsilon_{1,2}(m)$. This relationship does not depend on the initial phase θ, and hence neither does it depend on the breaking of time-reversal symmetries of the biharmonic approximation corresponding to the elliptic force. For a general biharmonic force (Eq. 1), this means according to the DSB mechanism that the optimal ratchet-inducing values of the initial phases φ_1,φ_2 should be those giving a maximal breaking of one of the two time-reversal symmetries of the force, while the relationship $\epsilon_2=\epsilon_2(\epsilon_1;p,q)$ should control solely the degree of breaking of the shift symmetry. Note that this symmetry is *not* broken when p,q are both odd integers. Consequently, if the DSB mechanism is right, the relationship $\epsilon_2=\epsilon_2(\epsilon_1;p,q)$ $(p+q=2n+1, n=1,2,...)$ controlling the degree of breaking of the shift symmetry should be independent of the particular system in which directed transport is induced. This implies that any averaged observable $<\Re>$ should be proportional to a certain function $g\left(\epsilon_1,\epsilon_2\right)\equiv g\left(\epsilon_1,\epsilon_2;p,q\right)$ which is $\sim p_1\left(\epsilon_1\right)p_2\left(\epsilon_2\right)$ in leading order with $p_1(\epsilon_1)\sim\epsilon_1^r, p_2(\epsilon_2)\sim\epsilon_2^s$, r,s positive integers. Since the aforementioned extremum m_c is independent of the driving amplitude, one defines $\epsilon_1=\epsilon\left(1-\alpha\right),\epsilon_2=\epsilon\alpha$

($\alpha \in [0,1]$), so that $g(\epsilon_1, \epsilon_2) \sim (1-\alpha)^r \alpha^s$ taking $\epsilon = 1$ without loss of generality. Since the extremum m_c is also independent of the driving period, one has the symmetry relationship $g(\epsilon_1, \epsilon_2; p, q) = g(\epsilon_2, \epsilon_1; q, p)$. The problem thus reduces to finding the relationship between (r, s) and (p, q). From Maclaurin's series, one sees that the *simplest* function satisfying all these requirements in leading order is $(1-\alpha)^p \alpha^q$, and hence $g(\epsilon_1, \epsilon_2; p, q) \sim \epsilon_1^p \epsilon_2^q$. Indeed, previous theoretical analyses of every kind on a great diversity of systems (*5-11,14,15,17,18*) have found that the averaged observable is always proportional to such a factor in leading order. In particular, one thus obtains

$$< \Re > \sim \epsilon^3 S(m) \equiv \epsilon^3 \frac{\operatorname{sech}^2 \left[\frac{\pi K(1-m)}{K(m)}\right] \operatorname{sech} \left[\frac{2\pi K(1-m)}{K(m)}\right]}{m^3 K^6(m)}, \tag{4}$$

for the biharmonic approximation corresponding to the elliptic force (i.e., $p = 2, q = 1$). Therefore, the shape function $S(m)$ is a universal function which controls the breaking of the shift symmetry in leading order for the resonance $(p, q) = (2, 1)$. It presents a single maximum at $m = 0.960057 \simeq m_c$ for which $\epsilon_2 = \epsilon_2(\epsilon_1) \simeq 0.396504\epsilon_1$ (note that $\epsilon_2 = \epsilon_1/2$ for $m = 0.983417$; see Fig. 3). Since the DSB mechanism is *scale-independent*, the critical value m_c could well be defined by a purely geometric condition (Occam's razor) which takes into account the two aforementioned competing mechanisms (degree of breaking of symmetries and transmitted impulse): $A(m = m_c)/A(m = 0) = \Phi/2$, where $A(m) \equiv \int_0^{T/2} f_{ellip}(t; T, m, \theta = 0) dt$ and $\Phi = \left(\sqrt{5} + 1\right)/2$ is the golden ratio. This gives $m_c = 0.9830783...$. The waveform corresponding to this value can be very accurately approximated by a sawtooth of unit height ($h = 1$), unit period ($\lambda = \lambda_1 + \lambda_2 = 1$), critical asymmetry parameter $a_c \equiv \lambda_1/\lambda = 1/4$, and critical tangents $\tan\theta_{c,1} \equiv h/\lambda_1 = 4$, $\tan\theta_{c,2} = h/\lambda_2 = 4/3$ with $\theta_{c,2} \equiv 4 \arctan \Phi$.

We now discuss the application of the DSB mechanism to time-reversal symmetries. For the sake of clarity, consider first the case with no dissipation. According to the above discussion, the optimal ratchet-inducing values of the initial phases φ_1, φ_2 should be those yielding a maximal interference-type effect of the two harmonic functions in Eq. 1, i.e., *in general*, $|q\varphi_2 - p\varphi_1|_c$ can solely adopt some of the values $\{0, \pi/2, \pi, 3\pi/2\}$ depending upon the particular harmonic functions $har_{1,2}$ and the particular values p, q. When dissipation is not negligible, two effects are expected: The average $< \Re >$ will decrease as dissipation is increased, and the critical values of the initial phases will be shifted from their values at the limiting dissipationless case ($\varphi_{c,i} \rightarrow \varphi_{c,i} + \varphi_0$). Note that the DBS mechanism implies that the dissipation phase $\varphi_0 \to 0$ as dissipation vanishes.

We found that the present theory confirms and explains all the previous experimental, theoretical, and numerical results for a great diversity of systems subjected to a biharmonic force (Eq. 1) (*5-11,14,15,17,18*). In particular, it explains recent experimental results of directed diffusion in a symmetric optical lattice (*13*), where the force $F = (\text{constant})\left[(1-B)\cos(\omega t) + B\cos(2\omega t - \phi)\right]$ yielded the maximum velocity of the centre-of-mass of the atomic cloud at $B \simeq 0.33$, $\phi = \pi/2$, and the complete data series fitted the functional form $(1-B)^2 B$ (i.e., $\epsilon_1^2\epsilon_2$), in confirmation of the predictions above. The analysis of the problem of electron tunneling through a one-dimensional potential barrier in the presence of an externally applied ac force $f_1 \sin(\omega t) + f_2 \sin(2\omega t + \varphi)$ (*11*) showed that the directed current is proportional to $2Af_1^2 f_2 \cos\varphi$, in confirmation of the present theory. Also, the DSB mechanism works powerfully in the case of electron transport through a molecular wire weakly coupled to two leads and subjected to a biharmonic $(A_1 \sin(\Omega t) + A_2 \sin(2\Omega t + \phi))$ laser field (*12*). Quantum calculations showed (*12*) that only when the symmetry breaking is maximal (i.e., $A_2 = A_1/2, \phi = \pi/2$) does the average current reach its maximum value, and that the average current is proportional to the coupling strength. For weak coupling, this maximum is about five orders of magnitude higher than the corresponding value at $\phi = 0$. We also found that the universality of the present theory is confirmed by previous studies on systems subjected to sawtooth-shaped excitations (*26,28).* Indeed, a study of horizontal transport of granular layers on a vertically vibrated sawtooth-shaped base (*26*) showed that the horizontal transport $\langle v_x \rangle$ versus asymmetry parameter a has an extremum at $a = 0.25$ only when the sawtooth waveform approximated the above universal waveform (i.e., $\lambda = 6$ mm, $h = 5$ mm, cf. (*26*); note that the universal waveform is recovered for $h = 6$ mm). Although we have here limited ourselves to temporal symmetries, the DSB mechanism applies to spatial symmetries as well. In this regard, one should expect that the ratchet potential underlying biological motor proteins might be optimized according to the DSB mechanism. A clear experimental confirmation of this prediction appears in the context of the actomyosin dynamics in the presence of external loads (*28*), where the mean first-exit time versus external applied force was studied for a sawtooth potential profile with different values of the asymmetry parameter. The experimental data were optimally fitted only when the sawtooth waveform approximated the above universal waveform (i.e., $\tan\theta_1 \equiv h/\lambda_1 \simeq 4.55$, cf. (*28*); recall that $\tan\theta_{c,1} = 4$ for the universal waveform).

We here studied the example of kink-asisted directed energy transport in a driven, damped, sine-Gordon (sG) equation (see Fig. 4), where directed energy transport is predicted for a non-zero topological charge, implying the existence of sG solitons (kinks) in the system. As Fig. 5 shows, the theoretical predictions are confirmed by numerical simulations even in the presence of noise. We should stress that the consequences of the present theory extend beyond the problem of directed transport. It applies, for example, to the phenomenon of synchronization of arrays of coupled limit-cycle oscillators (*29*), where the maximal symmetry breaking of a homogeneous, time-delayed, periodic coupling gave the maximum decrease of the synchronization frequency (see Fig. 6). Also, the DBS mechanisms explains the effectiveness of harmonic mixing signals (with frequencies $\omega, 2\omega$ and amplitudes $\eta_\omega, \eta_{2\omega}$, respectively) in controlling (enhancing and suppressing) stochastic resonance phenomena (*30,31*). In particular, it explains the dependence of the output power at the signal frequency ω ($P_2 \sim \eta_\omega^2 \eta_{2\omega}$, cf. (*30*)) in a modified Schmitt trigger electronic circuit, and the dependence of the time- and noise-averaged mean value of the response ($\gamma_0 \sim \eta_\omega^2 \eta_{2\omega}$, cf. (*31*)) of an overdamped two-well Duffing oscillator. We should stress that the present theory can be directly applied to optimizing the effectiveness of a two-colour laser field for strong harmonic generation [32] as well as to optimally designing synthetic molecular motors [33].

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Figure Captions

Figure 1. Function $f(t;T,m,\theta)$ (Eq. 2) vs t/T for $\theta=0$ and three shape parameter values, $m=0, 1-10^{-6}$ (cyan), and 0.96 (magenta), showing an increasing symmetry-breaking sequence as the pulse narrows, i.e., as $m\rightarrow 1$.

Figure 2. Deviation from the shift symmetry condition $D\,(t;T,m,\theta=0)=1$ (Eq. 3) showing an increasing deviation as $m\rightarrow 1$.

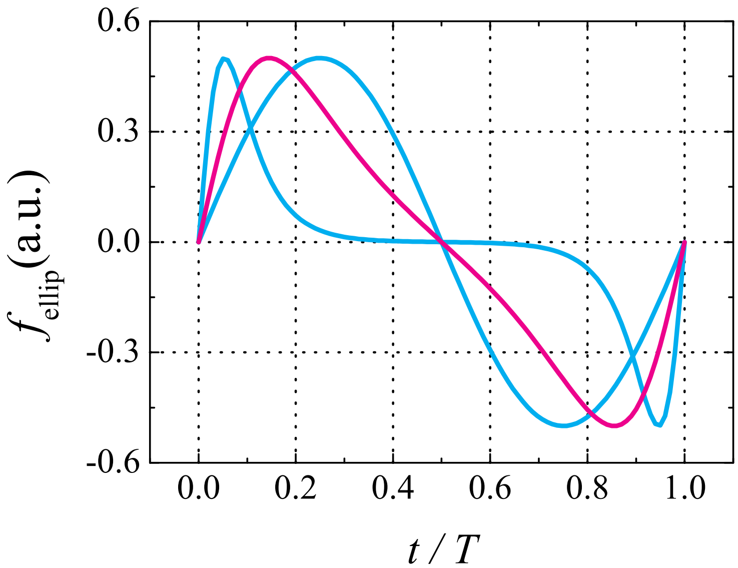
Figure 3. Universal shape function $S(m)$ in leading order for the resonance $(p,q)=(2,1)$ (Eq. 4) exhibiting a maximum at $m=0.960057$.

Figure 4. Average velocity of the kink centre-of-mass versus shape parameter for the sG equation $U_{tt}-U_{xx}+\sin(U)=-\beta U_t+f_{ellip}(t)$ with $\epsilon=0.1, \beta=0.05, T=20\pi$, and $\theta=0$. Previous results (*14*) from a collective coordinate approach with two degrees of freedom, $X(t)$ and $l(t)$ (respectively, position and width of the kink), can be directly applied to obtain an ODE governing the dynamics of these two collective coordinates in the presence of the elliptic force (Eq. 2): $\dot{P}=-\beta P-qf_{ellip}(t)$, $\ddot{l}=\dot{l}^2/(2l)+1/(2\alpha l)-\beta\dot{l}-(\Omega_R^2 l/2)(1+M_0^{-2}P^2)$, where the momentum $P(t)=M_0 l_0\dot{X}/l(t)$, $\Omega_R=\sqrt{12}/(\pi l_0)$ is the Rice frequency, $\alpha=\pi^2/12$, and $M_0=8$, $q=2\pi$, and $l_0=1$ are, respectively, the dimensionless kink mass, topological charge, and unperturbed width. For the biharmonic approximation corresponding to the elliptic force (2), one straightforwardly obtains the following estimate for the average velocity of the kink: $\left\langle \dot{X}\,(t)\right\rangle=\epsilon^3 F(\beta,T)S\,(m)$, where $S(m)$ is the shape function (Eq. 4) and $F(\beta,T)$ provides the dependence upon the dissipation and the period (*14*). The solid, long-dashed, and dot-dashed lines represent, respectively, the results from the numerical solution of the sG PDE, the results from the numerical solution of the collective coordinates ODE, and the above analytical estimate $\left\langle \dot{X}\,(t)\right\rangle$. The curves corresponding to the two kinds of numerical solution present minima at $m=0.983417$ (solid line; recall that $m_c=0.9830783$) and $m=0.97904$ (long-dashed line), respectively, while the curve corresponding to the analytical approximation has its minimum at $m=0.960057$.

Figure 5. Average velocity of the kink centre-of-mass versus shape parameter for the sG equation $U_{tt}-U_{xx}+\sin(U)=-\beta U_t+f_{ellip}(t)+\sqrt{D}\eta\,(x,t)$ with $\epsilon=0.1, \beta=0.05, T=$

$20\pi, D = 0.001$ and $\theta = 0$, where $\eta(x,t)$ is a Gaussian white noise of zero mean and correlations $\langle \eta(x,t)\,\eta(x',t')\rangle = \delta(x-x')\,\delta(t-t')$. The lines represent the noiseless cases described in Fig. 4, while circles represent the results from the numerical solution of the noisy sG PDE which presents a minimum at $m = 0.986525$ (recall that $m_c = 0.9830783$).

Figure 6. Symmetry-breaking-induced frequency suppression versus shape parameter for the system of N coupled oscillators $d\phi_i(t)/dt = \omega_0 + 2\epsilon \sum_j f_{ellip}\left[\phi_j(t-\tau) - \phi_i(t); T, m, \theta\right]$ with $T = 2\pi, \theta = 0$, and where ϵ is the coupling constant, τ is the delay, and ω_0 is the intrinsic frequency of the oscillators. The lowest stable frequency associated with the synchronization states ($\phi_i = \Omega t + \Omega_0$) for the biharmonic approximation corresponding to the elliptic coupling is given by $\Omega_{\min} \approx \omega_0/[1+2\pi^2 n\tau\epsilon g(m)]$, where n is the number of neighbours and $g(m) \equiv \frac{1}{mK^2(m)}\left\{\operatorname{sech}\left[\frac{\pi K(1-m)}{K(m)}\right] + 4\operatorname{sech}\left[\frac{2\pi K(1-m)}{K(m)}\right]\right\}$. The curve exhibits a minimum at $m = 0.9845$ (recall that $m_c = 0.9830783$) for $n = 4, \tau = 0.1$, and $\epsilon = 3$. Note that this minimum is the same for all values of τ, n, and ϵ, in confirmation of the present theory.



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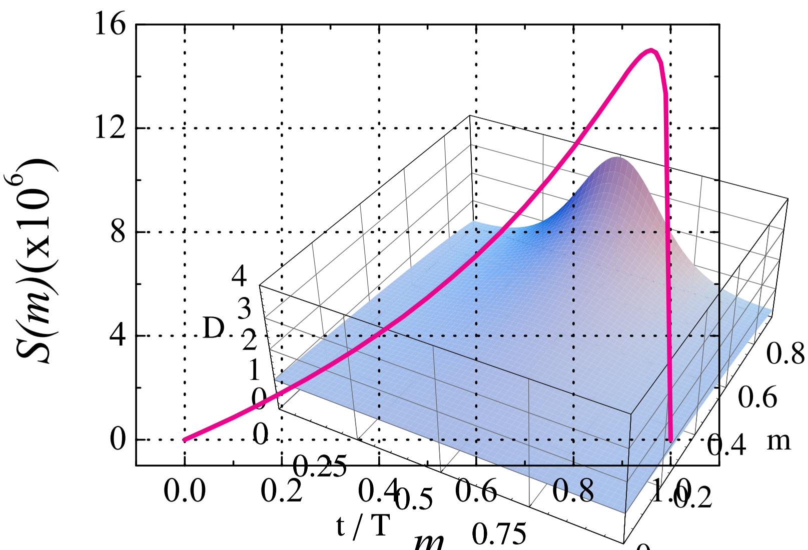

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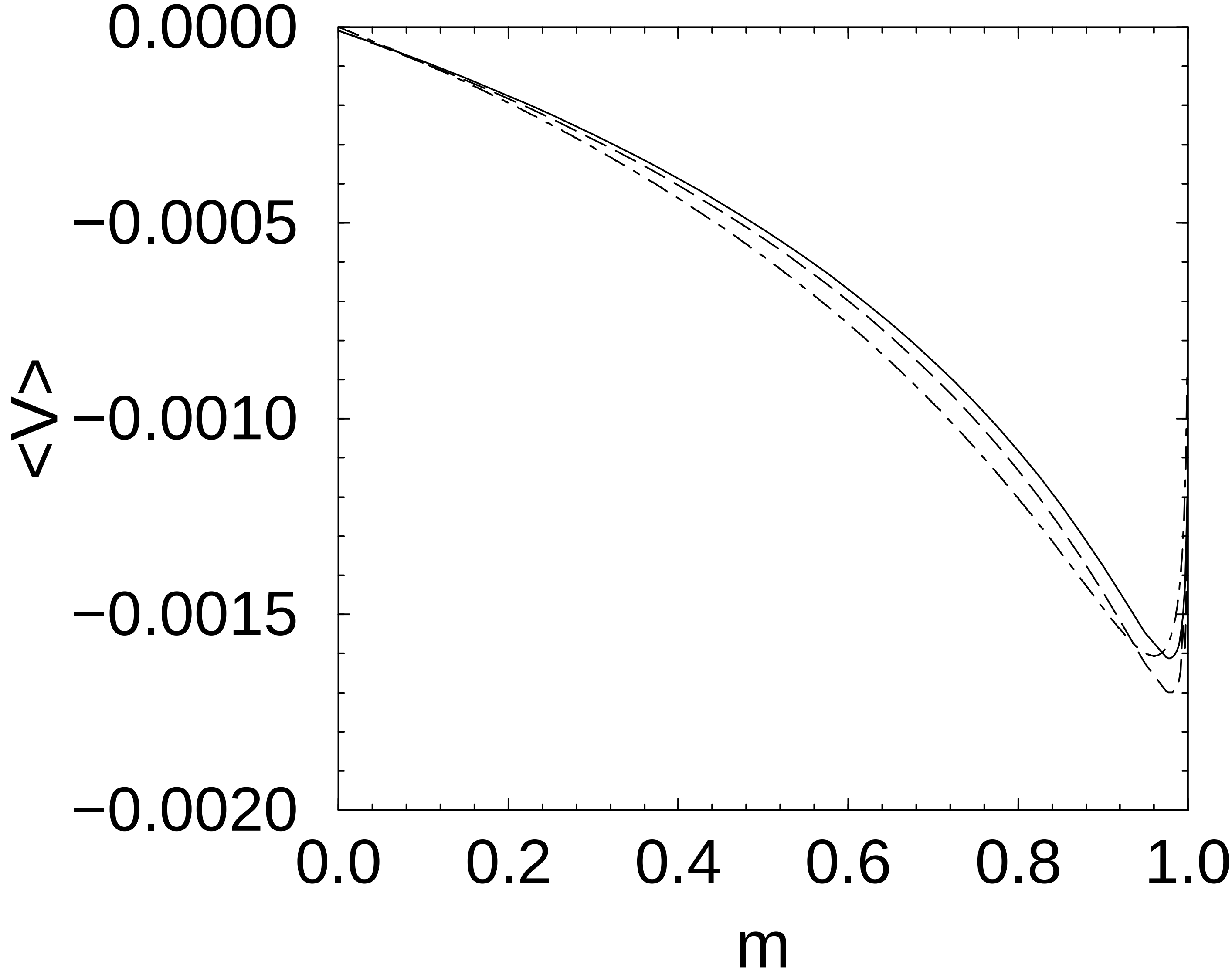
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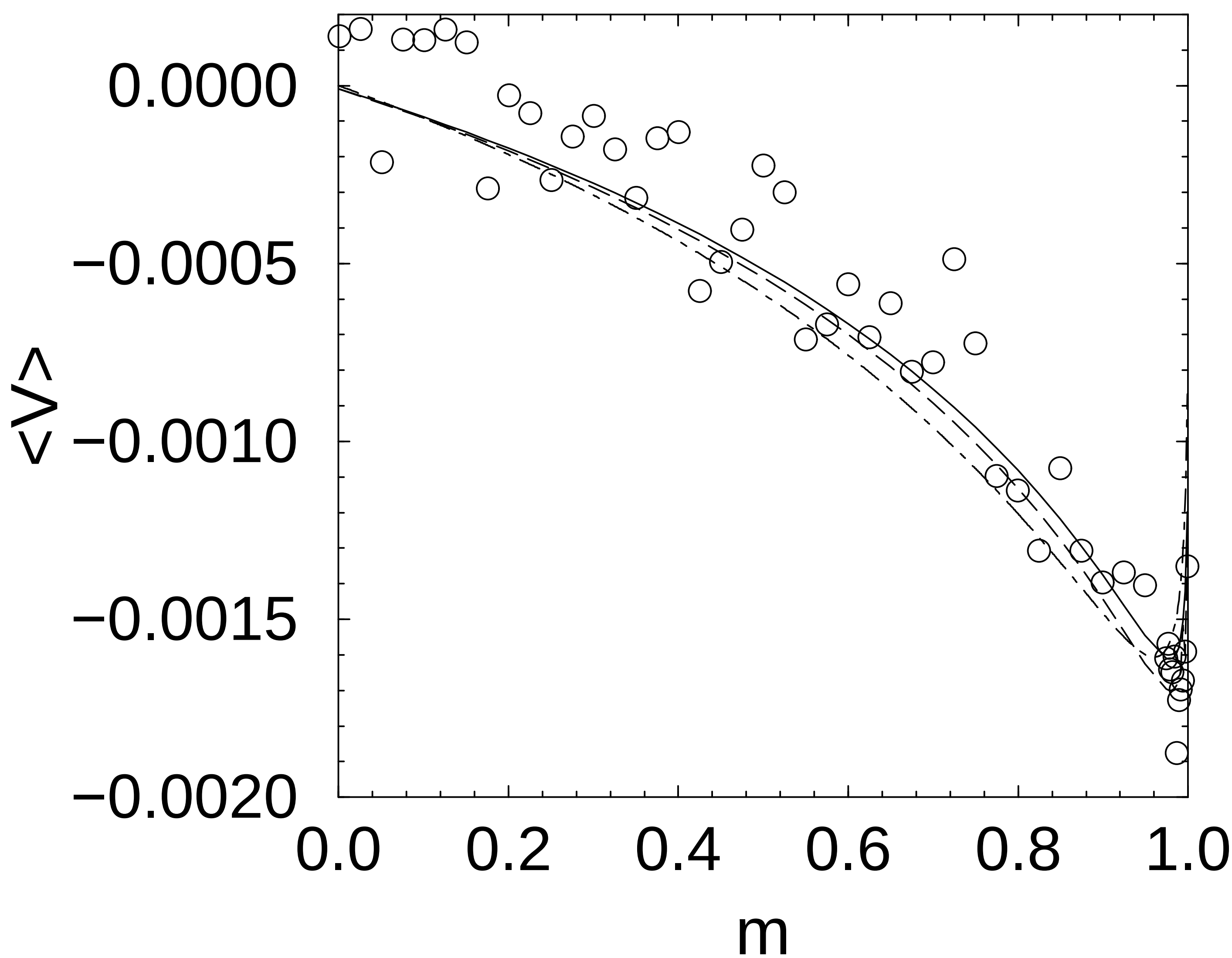

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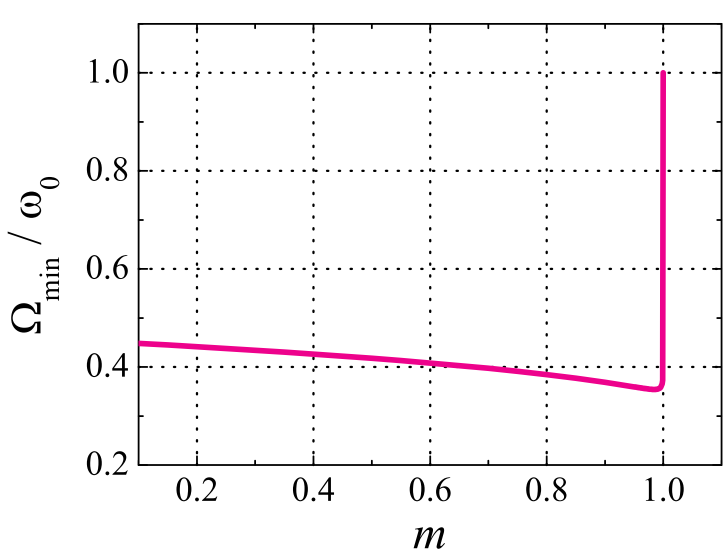

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