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An Elective Mathematics Course for College-Bound Students

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AN ELECTIVE MATHEMATICS COURSE FOR COLLEGE-BOUND STUDENTS

by
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a project submitted to the Division of Curriculum and Instruction in partial fulfillment of the requirements for the degree of
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Abstract

The intent of this project was to research and analyze the changes in college mathematics curricula and to establish the need for a change in the current college-preparatory mathematics program. The research indicates that colleges are emphasizing computer applications, statistics, and discrete mathematics. One response to the change is the course designed in this project.

Presented in this work is a one-semester course in mathematical modeling and statistical analysis of data. It also includes topics in probability, sampling, and algorithms. The methods of instruction include discussion, cooperative learning, lecture, projects, labs, computer investigations, problem solving, and writing. Students of this course will learn to value mathematics, reason and communicate mathematically, and gain confidence in their abilities to solve problems.

Chapter 1: Introduction

The college-preparatory curriculum in mathematics has remained virtually unchanged during the past thirty years. In most high schools, students who intend to enter college are required to take Algebra I, Geometry, and Algebra II. They are encouraged to take Pre-Calculus and advanced placement Calculus if they are mathematically gifted and if time permits. These courses have served for many years as a preparation for college for those students who are adept and achievement-oriented in mathematics.

However, there are many secondary school students who are not mathematically gifted but who are still planning on a college career(Leitzel & Osborne, 1985). Once these students complete the college-preparatory requirement of Geometry and Algebra II, they find their choices are quite limited. They can pursue the honors track and take Pre-Calculus or Calculus, which is intended to prepare them for the possibility of a career in mathematics, science, or engineering. They can take basic courses such as consumer math, general math, business math, or liberal arts math which are not college-preparatory. Or they can choose the third, increasingly popular, option of taking no mathematics in their senior year.

The omission of a mathematics course from the senior's schedule can have a devastating effect on the student's performance in freshman college mathematics courses. Terrel Bell, the former Secretary of Education admits that:

> Too often American students don't take enough mathematics courses in high school, effectively shutting them out of many courses in science and other fields which depend on a mathematical background. Without a strong background in mathematics, students face limited opportunities in postsecondary studies and in many career areas (Romberg, 1984, p. v).

Tomorrow's college students will need to be able to think algorithmically in order to maximize their use of computers(Steen, 1989). The infusion of calculators and computers into the workplace has dramatically changed the mathematical needs of most professions over the last decade. Tedious calculations have virtually been eliminated due to these revolutionary tools. "To help today's students prepare for tomorrow's world, the goals of school mathematics must be appropriate for the demands of a global economy in an age of information"(Steen, 1989, p. 19).

Colleges are presently in a state of transition in their mathematics curricula. They are adding courses in discrete mathematics and statistics as well as in computer applications and programming (Steen, 1989). Many colleges are de-emphasizing the role of calculus as the sole keystone of advanced mathematics. Computers are having an impact on all areas of the college curriculum. These changes at the collegiate level raise the question of how high schools can best provide for students' mathematical needs. Is it important or even necessary to introduce topics in high school that students will encounter in college outside of the traditional calculus courses? Is a new course needed or should the algorithmic approach be incorporated into the existing curriculum? What are the alternatives for students who do not excel in mathematics but are headed for college nonetheless?

In An Agenda for Action (1980), the National Council of Teachers of Mathematics (NCTM) recommended that "a flexible curriculum with a greater range of options should be designed to accommodate the diverse needs of the student population" (p. 1). We are currently in a period of major curricular reforms in mathematics. The focus of this project assumes that certain changes need to be made in the current mathematics curriculum to better prepare students for the future. On the basis of recommendations from the literature, it will propose a practical program for incorporating new topics and methods of thinking into the traditional college-preparatory mathematics program, especially applicable to high school students who are college-bound but do not intend to major in mathematics, science, or engineering.

Chapter 2: Review of Related Literature

The decade of the 1980s has produced volumes of research and opinion papers about educational reform. The U. S. Department of Education's report: A Nation at Risk: The Imperative for Education Reform (1983) shocked the general public into the realization that serious deficiencies existed in the nation's schools. It proved to be a catalyst for much of the research conducted in the ensuing six years. The focus of this paper is the changing role of mathematics in the world in general and specifically in secondary schools.

An Historical Perspective

Beginning with Plato's Academy and the Roman quadrivium, students have been required to study mathematics to learn to think clearly (Steen, 1989). About 500 years ago, arithmetic and algebra became part of the educational system in response to expanding commerce. Geometry and arithmetic are now paradigms of school mathematics although they may not be especially relevant in today's society.

Mathematics educators have a long history of reforms. At the turn of the century, E. H. Moore urged that schools abolish the separation of algebra, geometry, and physics (Cooney, 1988). At the same time, John Perry emphasized the importance of applications and laboratory teaching techniques in teaching mathematics. After World War II, the Commission on Post-War Plans pointed to serious shortcomings of Americans' mathematical knowledge. Some claimed considerable success in curricular reform during the post-war era of "modern mathematics". However, others have noted that the "modern" approach was sterile and notation-bound. Surprisingly few studies were conducted to analyze the effects of the reform movement of the 1950s and 1960s.

In the 1960s there was considerable belief that intellectualism could lead society toward the resolution of all its problems (Cooney, 1988). Mathematics teachers were told to teach the structure of mathematics and all else would fall into place. In the 1970s the buzzword was "relevant." We saw the emergence and proliferation of competency-based educational programs with their behavioral objectives. The era of anti-intellectualism was manifested eventually in the "back to basics" movement.

Today the pendulum of educational reform is swinging back toward an emphasis on conceptualization and analysis (Steen, 1989). In mathematics, the trend is toward algorithmic thinking and the use of technology. Our economy is increasingly reliant on the processing and transfer of information. Computers and calculators are essential tools for many occupations. Students need a heightened awareness of mathematical concepts, but not necessarily the ones we are currently teaching in our secondary schools.

Mathematical sciences are no longer a requirement for future scientists only. Mathematics should be an essential ingredient in the education of all Americans. Yet many reports (NCTM, 1985; Robitaille & Garden, 1989; United States Department of Education [USDOE], 1983) cite serious deficiencies in the mathematical performance of U. S. students compared to other nations and to our own expectations. Students in this country drop out of mathematics at alarming rates, averaging about 50% each year after mathematics becomes an elective subject (Steen, 1989). Goldenstein, Ronning, and Walter (1988) found that today's students are taking more courses, but less rigorous academic courseloads. In the senior year, the average American student earns only 0.4 credit in mathematics and 0.3 credit in science (Brodinsky, 1985). This is substantially less coursework than their counterparts take in Japan, West Germany, and the U.S.S.R. In those countries, all students take at least one course each in science and mathematics each year in the upper secondary school.

As recently as 1980, most states required only one year of mathematics and one year of science for high school graduation, with many states having no requirements for specific curricular courses (Brodinsky, 1985). A Nation at Risk (USDOE, 1983) provided a turning point for curricular reform. One year after its release, 48 states were considering new high school graduation requirements and 35 had already approved changes (USDOE, 1984). Florida now requires three years of mathematics and three years of science for graduation. Many recent reports (Henningsen, 1985; NCTM, 1985, 1989; Ralston & Young, 1983) have stressed the need for four years of mathematics for college-bound students.

However, raising the graduation requirements will not necessarily improve the achievement of students in mathematics and sciences nor result in a better education for all students (Brodinsky, 1985). It will increase the number of courses offered in these areas in which there is already a shortage of qualified teachers. It will also force non-college-bound students to take college-preparatory-level mathematics and science courses, which they and their teachers will find frustrating. Simply increasing the number of courses required for high school graduation will not solve the mathematical and scientific deficiencies of American society. More useful alternatives to improving the quality and quantity of mathematics and science education include: improving the quality of science and math instruction in elementary schools, increasing the supply of mathematics and science teachers, improving science lab facilities, and developing appropriate mathematics and science classes for vocational and other non-college-bound students.

What is Currently Offered

Secondary mathematics programs today generally consist of two groups of courses: the formal math track and the "remedial" track (Brodinsky, 1985). The formal math track includes theoretical treatments of concepts and skills at the abstract level, commonly presented by chalkboard-lectures and computational assignments. Remedial-type courses involve review of previously taught concepts, usually with the same method of presentation, but with more emphasis on paper-and-pencil skills. Both tracks are often deficient in concept development and emphasize memorization and manual calculation.

The traditional college-preparatory curriculum consists of separate year-long courses in Algebra I, Geometry, Algebra II, and Pre-Calculus (Conference Board of the Mathematical Sciences [CBMS], 1983). This sequence is adequate for many college-bound students. However, it is not preparing the majority of students with the types of skills and modes of thinking which they will need in the college curriculum. It ignores significant topics that are rather common in the real world, such as probability and statistics, estimation and approximation, and uses of computers.

There is much debate about what is included and what should be included in the pre-calculus course. Maurer (1983) notes that the content of the course varies widely but usually contains lots of review of topics from algebra and geometry; coordinate geometry, especially conics and functions; exponential, logarithmic, and circular functions; vector geometry; permutations, combinations, and elementary probability; sequences and series; and elementary theory of equations. Topics which are generally not taught in pre-calculus or anywhere else in high school include algebraic proof; algebraic structures; mathematical induction; combinatorics, including set notation, Sigma and Pi notation; recursive methods; matrix algebra; and graph theory.

Computers have been introduced into most schools today (Maurer, 1983). High school mathematics departments are very gradually incorporating them as teaching tools in traditional math classes. However, the computer is not being utilized for its ability to present a new mode of thought or as an object of mathematical study.

For very capable mathematics students, advanced placement (AP) calculus is the zenith of high school mathematics training. The advanced placement exams, first offered in the late 1950s, were taken in 1981 by about 33,000 students (Maurer, 1983). That calculus should be the culmination of high school mathematics was a fine idea when the AP program began, but calculus is no longer the sole keystone of advanced mathematics, argues Maurer. However, at the high school level the old idea is still prevalent and is reinforced by the current AP program.

The College Board has recently developed a new AP course and exam in computer science which was first given in 1984. It is equivalent to a full-year introductory college course in computer science (Maurer, 1983). The emphasis of the course is how to think and write well algorithmically, using block-structured programs and data structures. AP computer science is a viable alternative or supplement to AP calculus for mathematically-oriented students.

Changes in the College Mathematics Curriculum

Many experts (Maurer, 1983; Ralston, 1983; Steen, 1981) agree that the current curriculum for mathematics in the first two years of college is not serving as large a proportion of students as it should. They also feel that the mathematics community is ripe for change. Arguments in favor of a change in curriculum are motivated by changes wrought in the fabric of science, technology, and education by computers and computer science. Computer scientists have more use for discrete mathematics than for the continuous mathematics of calculus. The second industrial revolution, which focuses on the immaterial (knowledge, communications, and information) will force upon all scientists and educators a new world view which will require an emphasis on discrete mathematics at least equal to the emphasis on classical analysis (Ralston, 1983).

There are some arguments against a major change in college mathematics curriculum (Ralston, 1983). The current curriculum has served the mathematics, science, engineering, and other communities reasonably well for a long time. Unlike calculus, discrete mathematics has no fundamental theorems or unifying ideas. It is harder to grasp, so should be avoided by weaker college freshmen. Certain mathematics educators feel that discrete mathematics would not reinforce high school mathematics lessons as well as the current curricula of calculus and linear algebra.

Among mathematicians who agree that discrete mathematics should become a major emphasis in the undergraduate curriculum, there is some disagreement on how to incorporate it with calculus. Ralston (1983) believes in a single integrated curriculum including topics from discrete mathematics and calculus in both years to serve all disciplines including mathematics itself. Maurer (1983) argues in favor of two separate one-year sequences, one calculus and one discrete mathematics. This plan would not interfere with the content of the advanced placement calculus course presently taught in thousands of high schools throughout the country. After successful completion of that needed to meet the goals: raise expectations, increase breadth of courses, use calculators, engage students, encourage teamwork, assess objectives, require mathematics, demonstrate connections, stimulate creativity, reduce fragmentation, require writing, and encourage discussion.

Will high schools utilize the new "standards" to implement changes in course content, presentation, emphasis, or course offerings? Or will they continue with the traditional college-preparatory track through Algebra II, Pre-Calculus, and Calculus? Ernest Boyer, president of the Carnegie Foundation for the Advancement of Teaching, thinks that we need to take "a creative look at curriculum in relation to the future and not to the past" (Brandt, 1988, p. 9).

Many experts (CBMS, 1983; Maurer, 1983; Romberg, 1984; Steen, 1989) recommend a streamlining of the current traditional college-preparatory curriculum to make room for new topics and approaches to mathematics instruction. The advent of calculators and computers has made tedious manipulative drill practically obsolete (CBMS, 1983). Teachers can reduce the time they spend on fractions, long division, graphing by hand, pencil-and-paper algorithms, reading and interpolating from tables. In geometry, many authorities advocate a de-emphasis of two-column proofs. In pre-calculus, less time could be spent reviewing topics from algebra and geometry (CBMS, 1983; Steen, 1981).

Some topics currently taught in high school could be moved to the junior high mathematics curriculum (Maurer, 1983).

A plethora of topics could be added to the present college-preparatory mathematics curriculum in order to make it relevant to tomorrow's college students (Brodinsky, 1985; CBMS, 1983). In algebra, emphasis needs to be placed on understanding functions. Computers can be utilized to evaluate and to graph functions. Probability and statistics, patterns, data collection, observation, estimation, and conjecture could supplement the present algebra curriculum. Because too often word problems in algebra are contrived and unrealistic, problems could be devised which reflect actual applications in science and business.

Brodinsky (1985) and CBMS (1983) further note that geometry could be taught with a transformational approach, incorporating computer graphics packages to help students visualize geometric relationships. It could also include algebraic methods, analytic geometry, and vector algebra, especially in three dimensions.

Computers can be incorporated into all areas of the mathematics curriculum (CBMS, 1983). In pre-calculus, computers can enable students to perform qualitative analysis of the graphs of functions. Students can develop algorithms to solve problems, then program computers to do it. Even those who are not mathematically gifted can benefit from access to computers.

Computers offer a fresh window into mathematics for students who are not headed for careers in science or technology.

In addition to incorporating new topics, mathematics educators need to re-examine their approach and emphasis to the traditional courses (CBMS, 1983). Calculators and computers should be included in all levels of mathematics instruction, kindergarten through grade twelve. Rote skills are not a prerequisite to learning problem-solving processes, which should be the primary focus of mathematics instruction. Higher order thinking skills can be taught to large numbers of students (Brodinsky, 1985).

Extensive inservice training needs to be provided for current teachers to keep them abreast of the changes in focus of mathematics. The retraining problem is of paramount importance (Maurer, 1983). If this were a period when a flood of new young teachers were entering the profession, already versed in algorithmics, then the problem might not be so serious; but of course, exactly the opposite is the case. It is a monumental task to get experienced teachers to look at their subject areas in a new light.

Teachers and publishers need to confer on the content of new textbooks (Brodinsky, 1985; Romberg, 1984). New materials emphasizing problem solving and the algorithmic approach will need to be supplemented with detailed, usable curriculum guidelines. Computer courseware needs to be developed that is user-friendly and compatible with the curriculum guidelines.

Non-Traditional Mathematics Curricula

What creative mathematical alternatives have been developed and implemented during the years since An Agenda for Action (NCTM, 1980) was published? New courses for senior year electives for college-bound students fall into three main categories: discrete mathematics, elementary statistics, and computer science. The Conference Board of the Mathematical Sciences (CBMS, 1983) considers all of these topics to be more important than what is now taught in trigonometry.

Discrete mathematics is appearing gradually and experimentally on the high school level as a reaction to the new emphasis in college mathematics. The content of the course varies widely but usually includes basic combinatorics, graph theory, discrete probability, recursion, and development of algorithms (Brodinsky, 1985; Steen, 1989). Where it is offered, it is generally an honors level course and an alternative to pre-calculus or calculus.

Brodinsky (1985) recommends that college-bound students take at least one semester of statistics in their junior or senior year, as well as trigonometry and computer usage. Elementary statistics involves collection and analysis of data, probability, interpretation of tables, graphs, surveys, and sampling (CBMS, 1983).

Computer courses can emphasize applications or programming and algorithms (NCTM, 1980). Both approaches are important and students should have the opportunity to take either or both before they graduate from high school. Computer courses should be taken in addition to, not instead of, the usual pre-college mathematics courses (Brodinsky, 1985).

In summary, teachers should strive for a fresh, new approach in their teaching of all math courses. They can emphasize algorithmic thinking as an essential part of problem solving (CBMS, 1983). They can facilitate learning through discovery by involving students in data gathering and investigation of mathematical ideas. They can utilize a variety of instructional methods, active learning, written expression, and continual assignments. Teachers can minimize rote memorization, lecture, one method/one answer, and routine worksheets (Steen, 1989). Maurer (1983) feels that an algorithmic frame of mind should become pervasive throughout secondary mathematics.

Chapter 3: Research Procedures

Determining the Needs of Students

The related literature has shown that the typical traditional college-preparatory curriculum is not serving the mathematical needs of a majority of college-bound students (CBMS, 1983; Henningsen, 1985; NCTM, 1980). A large percentage of seniors are taking no math courses at all.

Consultation with college mathematics instructors will help to identify areas of mathematical deficit they have observed in incoming freshmen; such information will indicate those areas of mathematics appropriate for high school study. The course titles that college admissions officers consider to be academically sound and relevant to college mathematics will direct the design of a senior elective mathematics course. The perceptions of college freshmen and sophomores regarding the adequacy of their mathematical backgrounds will also suggest how such a course might be structured.

Determining Objectives and Selecting Content

As with many mathematics courses, the objectives of this senior elective mathematics course will focus on mastering the content of the course. The content will be derived from curriculum guides of established courses and pilot programs currently being offered throughout the country as alternatives to calculus and pre-calculus. The Florida Academic Scholars Program offers honors scholarships to Florida residents at state universities (USDOE, 1984). They require at least four years of high school mathematics, and they specify particular courses. The curriculum frameworks for these courses can be obtained from the Florida Department of Education when determining what content to include in this new course. The International Baccalaureate Program offers an alternative to the advanced placement program that could provide options which may be appropriate to include in the content selected for this course.

Textbooks and computer courseware provide a major source of content. Publishers will be contacted for examination copies of their latest materials relevant to alternative mathematics curricula. Decisions regarding the appropriateness of materials will be made based on professional experience, recommendations from colleagues, and professional reviews in educational journals.

How to Organize Content and Learning Experiences

This course is to be an alternative to traditional mathematics instruction because the present curriculum is not appropriate for all college-bound students. Logic and professional experience dictate that innovative approaches should be tried. Students are often bored and unmotivated with the lecture/drill format so common in secondary mathematics courses. Every effort will be made to present the topics of this course in active, unusual, and relevant ways. This will be a point in the curriculum where computer science and software can be utilized effectively because they provide a means to reveal to students the living, evolving character of mathematics (Steen, 1986). The availability of applicable software will be a determining factor in organizing the content of the course with appropriate learning experiences.

Evaluation

This new course will be evaluated before, during, and after its implementation by several colleagues in secondary mathematics. A checklist will be developed so that they may easily report their perceptions of the appropriateness of the content and the likely effectiveness of the course as a whole. It is extremely important to evaluate new programs so that objectives, content, and learning experiences can be revised to better serve the students.

Chapter 4: Design of the Course

Need for the Course

The present college-preparatory curricula in mathematics are not adequately preparing all students for the math they will encounter in their college courses. Colleges are placing more emphasis on statistics, computer applications, and discrete mathematics while de-emphasizing the role of calculus in many majors. The review of related literature affirms the need for college-bound students to take four years of math in high school. However, many high school seniors wishing to take math must choose between the rigorous, honors level calculus or pre-calculus courses and the remedial or review courses which are not college-preparatory.

This course is designed for high school seniors who do not necessarily want or need calculus but who do intend to go to college. The projects, cooperative group activities, and labs should appeal to students with a wide range of mathematical abilities and interests.

In their Standards (1989), the NCTM emphasizes that mathematics teachers need to take a fresh approach in their teaching. They should try to incorporate a variety of instructional methods to cultivate students' abilities to investigate and find meaning in new situations, to make conjectures, and to use flexible strategies to solve problems. In addition to lecture and teacher-led discussions, NCTM recommends that students be provided the opportunity for small group work, individual explorations, peer instruction, and teacher-moderated class discussions.

Prerequisites

Students enrolled in the course must have completed Algebra II and be able to read and use formulas. They should be juniors or seniors in high school. The course is intended for college-bound students, but students of all ability levels could be admitted.

Course Overview

The course will encompass an 18-week semester and will consist of four broad topics: statistics, sampling, probability, and algorithms. Many instructional methods will be employed including discussion, cooperative learning, lecture, projects (with student-generated or collected data), labs, computer investigations, textbook problems and exercises, reading, and writing. Students will have the opportunity to work with their peers in cooperative problem solving but will also learn to think for themselves.

Students of this course will investigate problems from many perspectives. They will explore, formulate and test conjectures, and discuss and apply the results of their investigations. The students will also use technology to enhance many of their explorations.

Goals and Objectives

After completing this course, students will:

-understand statistical reasoning

-use statistical methods to analyze data

-use appropriate graphs to represent data

-use powerful tools of technology

-find mathematical solutions to everyday problems

-reason and communicate mathematically.

Curriculum Units

The traditional algebra and geometry topics that the student has studied up to this point were probably dominated by memorization of facts and procedures. This course will give the student a view of mathematics as a problem-solving tool. It will emphasize conceptual understandings, multiple representations and connections, and mathematical modeling. The course will begin with a two week introduction to one-variable statistics. The emphasis of this unit will be the graphical representations of lists of data. Students will generate several types of graphs including line plots, histograms, stem-and-leaf plots, and pie charts using real data. Most of the data used in the course will concern current events and topics of interest to teenagers such as sports, music, cars, SAT scores, and male/female ratios.

Collecting, representing, and processing data are activities of major importance to contemporary society. To enhance their social awareness and career opportunities, students should learn to use statistical techniques in solving problems and in evaluating statistical claims they encounter in their daily lives. Two-variable statistics will be studied for five to six weeks. In order to recognize the relationships between two variables, the students need to have a thorough understanding of functions. They will analyze linear, quadratic, and exponential functions and learn how to recognize them from their graphs or from data points. Unlike traditional algebraic word problems where students insert values into formulas and solve for the unknown, statistical problems require the student to study a list of data points and recognize the function or relationship between them.

Students will create scatter plots from real world data, fit lines to their data, and analyze the results. The students will learn methods of linear regression, correlation, and standard error of estimate. They will do a research project to see whether there is an association between two variables. The data can be collected from almanacs or other resources, or it can be collected by the students.

The class discussions generated by the research projects will lead naturally into the next unit on sampling. Students need to acquire intuitive notions of randomness, representativeness, and bias in sampling to enhance their ability to evaluate statistical claims. Random samples can be generated by rolling ten-sided dice, by drawing numbers out of a box, by a computer or calculator, or by looking them up in a table. Sampling will be explored for approximately four weeks. Students will learn about sampling distributions and will see the relationships among various samples from one population by graphing them as box plots. Their familiarity with box plots should also help them to visualize the concept of a confidence interval. The students will complete a research project in which they create a random sample of their peers, choose a random variable, collect and analyze the data from their sample, and predict the mean of the population based on their sample.

Probability provides concepts and methods for dealing with uncertainty and for interpreting predictions based on uncertainty. Formal concepts of theoretical probability will be developed only after a conceptual base is established through an intuitive approach. Probability will be introduced through simulations using dice, coins and random numbers. Students will learn the differences between experimental and theoretical probability and will solve problems using both methods. Students will learn how to calculate and use permutations and combinations. They will explore the fascinating world of Pascal's triangle and the binomial theorem. They will use Venn diagrams to find probabilities of events within a sample space. By the end of the three to four week unit on probability, the students will be calculating and analyzing binomial distributions.

The development and analysis of algorithms are the basis for computer methods of solving problems. Students should be given the opportunity to construct mathematics from an algorithmic point of view. An algorithm is a step-by-step procedure that tells a person or a computer how to perform a task or solve a problem. By this time students should be familiar with the algorithmic approach to problem solving since the statistical models they create are actually algorithms. In the last two weeks of the term, students will be exposed to mathematical problems and real world situations for which they can create algorithms to produce solutions.

Various types of algorithms will be discussed, including direct computation, enumeration, iteration, and recursion. Iteration is a useful technique in solving problems involving sequences and series. At this point in the course, students should be familiar with summation notation, combinations and permutations, the binomial theorem, and set theory. Students can create algorithms utilizing some or all of these concepts. They can use mathematical induction to prove their algorithms correct. Finally, the students will translate some of their algorithms into BASIC computer programs and run them on computers.

Weekly Plan

Week 1: One-variable Statistics: line plots, histograms, stem-and-leaf plots, pie charts of frequency distributions.

Week 2: Mean, median, quartile, box plots, outliers.

Week 3: (test 1) Two-variable Statistics: graphing data

Week 4: Scatter plots: fitting lines to data

Week 5: linear regression, error, correlation

Week 6: (test 2) recognizing functions from graphs

Week 7: analyzing linear, quadratic, and exponential functions

Week 8: applications (lab/project)

Week 9: (test 3) Sampling: random numbers

Week 10: sampling distributions, box plots

Week 11: confidence intervals

Week 12: (test 4) applications (lab/project)

Week 13: Probability: counting (permutations & combinations), binomial theorem, Pascal's triangle

Week 14: sample space, events, Venn diagrams

Week 15: conditional probability, binomial distributions (test 5)

Week 16: Algorithms: sequences and series, iteration

Week 17: induction, programming (group project)

Week 18: review, exam

Learning Experiences

The following is a sample of the activities included in this course. One lesson is presented for each instructional week of the course, corresponding to the weekly plan on the previous page.

Activity week 1: Baseball Cards (graphing one variable data)

The following are the number of home runs that Hank Aaron hit in each of his 21 years with the Braves (1954 to 1974): 13, 27, 26, 44, 30, 39, 40, 34, 45, 44, 24, 32, 44, 39, 29, 44, 38, 47, 34, 40, 20. How can we make an interesting chart or graph to represent the data? In how many years did he hit 40 or more home runs? How many years was the number of home runs in the 30s, 20s, and 10s?

Students' responses can lead to discussion about various means of graphing frequency distributions. They will probably be familiar with line plots, histograms, and pie charts. The data should be graphed by all methods suggested by the students as well as stem-and-leaf plots. The graphs can be compared and the purposes and effects of each should be discussed.

Students will choose which graphing technique they prefer and baseball cards will be distributed. Each student will use the chosen technique to make a graph to represent the number of hits in the career of his or her chosen player.

A follow-up activity to this exercise is to use computer software to generate graphs of various baseball statistics for professional players or teams. Students also enjoy analyzing sports statistics of their school's teams.

Activity week 2: Estimating Time (median, quartile, box plots)

The class is divided into two large groups and students within the groups work in pairs. Each pair will need a watch or school clock with a second hand or digital display. One student in each pair estimates when one minute has passed while the other student watches the clock and records the actual time.

The students in one group concentrate quietly on the timing task, while half the students in the second group exert constant effort to distract their partners. The partners then switch roles so that data is collected on all students. The estimates for both groups will be listed and students will be asked which group gave the better estimates. Students will suggest finding the mean of the numbers, which is a useful measure of central tendency and should be examined.

Are there any other ways to find a central number of a list of data? When the numbers are listed in order, the median can be found. In this experiment, the means and the medians may be quite close, but the distracted group should have a greater variation in their answers. This provides the opportunity to discuss quartiles and range. Box-and-whisker plots can be created for the two groups to show that there is a difference in the data even though the two medians might not vary by a significant amount. Also, any numbers falling outside the "normal" range can be examined as outliers.

This activity was adapted from an exercise in NCTM's Standards (1989, p. 107). The Standards contain many useful suggestions and exercises for incorporating statistics into the existing middle school and high school curricula.

Activity week 3: Education Statistics (two variable graphing)

The chart on the following page represents the number of special education students in Florida enrolled in each exceptionality by age for the 1988-89 school year. The data are from the Florida Department of Education (1990, p. 36). How would one display this information graphically? After class discussion, each student will draw a graph to represent one exceptionality. They will then exchange graphs and each will write a legend and description of someone else's graph. They can speculate on reasons for the trends and patterns they observe.

As a group activity, students can create graphs comparing related exceptionalities (mental or physical or emotional) and examine them for evidence of when students tend to enter and drop out of programs, or switch to more severe programs. The class can discuss reasons for the trends they discover.

STATE TOTALS

PROGRAM NO.	PROGRAM NAME	AGE 3	AGE 4	AGE 5	AGE 6	AGE 7	AGE 8	AGE 9	AGE 10	AGE 11	AGE 12	AGE 13
201	EDUCABLE MENTALLY HANDICAPPED	102	180	267	487	872	1090	1315	1296	1335	1340	1286
202	TRAINABLE MENTALLY HANDICAPPED	69	113	200	224	238	253	236	261	275	288	314
203	PHYSICALLY HANDICAPPED	126	163	190	178	173	139	158	153	159	130	100
204	PHYSICAL/OCCUPATIONAL THERAPY PT	4	5	17	36	48	34	25	26	17	13	10
205	SPCH/LANG & HRNG PT	298	1164	6263	10562	11415	10141	7892	5633	3949	2481	1742
206	SPCH/LANG & HRNG	189	396	422	298	288	249	248	204	183	139	129
207	VISUALLY HANDICAPPED PT	9	6	15	32	42	46	31	44	37	44	31
208	VISUALLY HANDICAPPED	15	32	27	13	10	20	8	20	19	7	10
209	EMOTIONALLY HANDICAPPED PT	5	9	22	116	314	647	687	890	896	924	891
210	EMOTIONALLY HANDICAPPED	22	86	118	267	495	719	887	1066	1176	1244	1194
211	SPECIFIC LEARNING DISABILITY PT	1	3	44	409	2037	4822	6832	7315	7376	5962	6043
212	SPECIFIC LEARNING DISABILITY	25	69	99	199	558	1230	2093	2681	2925	2961	3109
213	GIFTED PT	0	0	83	1216	3310	5860	7225	7428	7688	7193	6325
214	HOSPITAL/HOMEBOUND PT	72	13	46	60	58	46	60	66	105	119	169
215	PROFOUNDLY HANDICAPPED	77	121	181	191	223	246	309	335	408	367	440
AGE TOTALS		1015	2350	7994	14278	20081	25442	27995	27418	26548	23202	20783
016	205 & 206 DEAF STUDENTS	60	112	119	95	100	89	107	76	116	88	90
017	215 PROFOUNDLY MENTALLY H.C.	60	70	95	93	89	79	86	83	112	75	83
018	215 DEAF-BLIND	0	0	1	0	1	0	2	0	0	0	2
019	215 AUTISTIC	13	22	42	39	50	31	27	30	27	21	26
020	215 SEV. EMOTIONALLY DISTURBED	14	29	43	59	83	136	184	212	269	271	379

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PROGRAM NO.	PROGRAM NAME	AGE 14	AGE 15	AGE 16	AGE 17	AGE 18	AGE 19	AGE 20	AGE 21	PROGRAM TOTAL	AGE 22+
201	EDUCABLE MENTALLY HANDICAPPED	1229	1266	1161	1159	849	359	82	30	15695	6
202	TRAINABLE MENTALLY HANDICAPPED	335	363	370	359	333	258	241	94	4823	20
203	PHYSICALLY HANDICAPPED	132	107	107	125	74	45	24	8	2291	0
204	PHYSICAL/OCCUPATIONAL THERAPY PT	8	7	4	5	1	1	1	0	262	0
205	SPCH/LANG & HRNG PT	1277	802	557	381	192	52	5	2	64808	2
206	SPCH/LANG & HRNG	111	90	87	72	57	30	9	0	3201	0
207	VISUALLY HANDICAPPED PT	29	33	37	33	11	6	2	1	489	0
208	VISUALLY HANDICAPPED	13	7	6	9	9	3	1	0	230	0
209	EMOTIONALLY HANDICAPPED PT	804	817	550	368	200	39	5	0	8084	0
210	EMOTIONALLY HANDICAPPED	1090	966	648	397	219	80	15	2	10681	1
211	SPECIFIC LEARNING DISABILITY PT	4544	3801	3002	2484	1634	370	46	2	55616	2
212	SPECIFIC LEARNING DISABILITY	2938	2583	2100	1777	1086	340	45	5	26813	0
213	GIFTED PT	4584	2998	2329	1937	116	4	0	0	58296	0
214	HOSPITAL/HOMEBOUND PT	299	366	326	304	114	30	8	3	2264	1
215	PROFOUNDLY HANDICAPPED	439	448	323	283	183	107	75	32	4788	20
AGE TOTALS		17832	14664	11597	9693	4978	1724	558	179	258331	52
016	205 & 206 DEAF STUDENTS	83	69	70	63	54	21	8	0	1420	0
017	215 PROFOUNDLY MENTALLY H.C.	80	81	72	72	70	69	44	24	1427	16
018	215 DEAF-BLIND	2	1	2	0	3	1	3	0	18	0
019	215 AUTISTIC	18	16	19	18	16	12	14	3	444	1
020	215 SEV. EMOTIONALLY DISTURBED	339	350	230	193	94	36	14	6	2899	4

Possible discussion questions include:

How many variables are shown on the table? (three)

What would make an interesting graph?

How can one graph three variables in two dimensions?

For what ages are the total numbers of special education students increasing? decreasing?

What are some reasons why the change occurs when it does?

This exercise can lead to the examination and graphical representation of other real world data collected from almanacs or textbooks. After graphing sets of data by hand, students can be introduced to computer software that will produce various types of graphs for data they enter.

Activity week 4: Car Mileage (scatter plots)

Students will investigate the possible relationship between car age and mileage. They can collect data from the school parking lot when people are arriving or departing. For as many cars as possible, they will ask the driver for the model year and the odometer reading rounded to the nearest thousand miles. Back in class, students will list their collected data together as one sample.

The class will work in small groups to graph the data on a grid, preferably on graph paper. The horizontal axis will represent the model years from oldest to newest. The vertical axis will represent miles in thousands. After they construct the graphs they should see why these graphs are called scatter plots. Points will be scattered on the graph, not in a line or exactly on a curve. Real world data rarely is. However, there should be a pattern to the arrangement of the points. Students should notice the negative correlation (the higher the model year, the lower the number of miles). They can see the descending nature of the points and guess where to put a "best" line that would represent the trend of the data points.

Students should find the equation of the line which they think best describes the data and use the equation to predict, for example, the expected mileage of a 1982 car. They will then write a summary paragraph about the information displayed in the graph and include inferences they believe are supported by their analyses of the data.

Activity week 5: Car Mileage (regression analysis)

The class can discuss the variety of equations generated by the previous week's experiment on car mileage. Different groups may have used different strategies in determining the "best" line. There are lots of ways to generate a line of linear regression.

The car mileage graphs can be examined in terms of the "errors" between the actual data points and the lines generated by the students. By minimizing these errors (or actually their squares), the least squares line can be determined for a given set of points. This continuing activity can also lead to a discussion of the correlation coefficient which is closely related to the slope of the regression line and the size of the errors. Computer software and scientific calculators can be utilized in calculating equations of regression lines as well as correlation coefficients.

Activity week 6: Pendulum Experiment (recognizing functions from graphs)

After learning how to fit lines to data, students will conduct an experiment to discover a relationship between the length of a pendulum and its period. A pendulum device can be created using fishing line with a fishing weight at the end. For various lengths of line, students will use a stopwatch to measure the length of time it takes the pendulum to complete one period. Due to the small amounts of time involved, it is best to measure the duration of ten periods and divide by ten. Also, to get a good model of the data, students should be sure to make measurements for very short lengths of line.

After collecting their data, students will construct scatter plots of the data points comparing length of pendulum (in cm.) to length of period (in sec.). As they try to fit a line to the data, they will discover that the relationship between the two variables is not linear but quadratic. This provides the opportunity to explore and discuss methods of determining what type of function best describes the data.

Activity week 7: Green Globs (recognizing equations from graphs)

Computers or graphics calculators can enhance the study of functions and their graphs. Many public domain programs will graph a function entered by the student. This saves much time and tedium usually associated with graphing by plotting points, and allows the student to see very quickly how various equations are related. Sunburst Communications produces two programs which require the student to recognize or analyze a graph shown on the monitor.

Interpreting Graphs familiarizes the student with the graph as a functional relationship. A student is given a situation and asked to choose which of three graphs best represents the relationship between two variables. Another Sunburst program, Green Globs and Graphing Equations contains four excellent activities. In the first, "Linear and Quadratic Graphs", the student is provided with a graph and must write an equation for it. The program then displays the graph of the student's equation as well as the original graph. The student can revise the equation until it matches the computer's. The level of difficulty is adjusted automatically depending on the accuracy of the student's answers.

In "Green Globs", students must enter equations to create graphs that will hit 13 green globs scattered randomly on a grid. "Tracker" requires students to locate linear and quadratic graphs that are hidden in a coordinate plane, then determine their equations. "Equation Plotter" is a general utility program that can be used to graph any general functions entered by the student. The first two activities should prove very useful in this course. Students will gain valuable experience recognizing and identifying functions from their graphs. They will also acquire some awareness of the capabilities of microcomputers.

Activity week 8: Project (two variable statistics)

This project is the culmination of the unit on statistics. Each student will collect numerical data from an almanac or statistical abstract about one sample (cities or states or people) concerning two variables. Each will construct a scatter plot to represent the relationship between the two variables. Students will analyze their graphs to determine whether there is a correlation and if there is a linear, quadratic, or exponential relationship between the variables. The results will be presented in a written report including a detailed explanation of the findings.

Format:

1. Title of report. The title should accurately describe the study.
   Example: "Comparison of average annual salary and percent of college graduates by state, 1987"
2. List the data. Tables of data may be reproduced but sources must be cited. Example: "Source: Statistical Abstract of the United States, 1989, p. 405"
3. Graph the data. Make a clear and accurate scatter plot of all the data points in the sample. Label the graph and both axes. Identify the units that the numbers represent.
4. Calculate the coefficient of correlation. Explain the meaning of the resulting number.

5. Determine the relationship between the two variables if one exists. Does it appear to be linear, quadratic, or exponential? Explain the process you follow to make the determination. If there is a relationship, fit a curve to the data and find its equation.
6. Conclusion. Interpret the results, speculate on reasons for the relationship (or lack of one), describe your expectations and any surprises you encountered.

Activity week 9: Sampling the Class

Have each student answer five "yes or no" type questions on a small piece of paper. Collect the papers and have one student calculate the proportion of yeses. Possible questions include: "Are you an only child?; Did you watch the Super Bowl on TV this year?; Do you think abortion should be illegal?; Is your father older than your mother?; Do you have a computer at home?".

For each question, the class will discuss whether they are a representative sample of the U. S. population. Why or why not? If they are not a representative sample, how would one go about finding a representative sample? This should lead to suggestions about ways to generate random samples.

The class can be treated as a population and random samples of various sizes can be compared. At this point, random numbers should be generated by a hands-on method, either by rolling 10-sided dice, drawing numbered chips out of a container, or spinning spinners in circles that are divided into 10 equal sectors numbered 0 through 9. After students see random numbers produced in this tangible way, they can use and appreciate random number tables or generate random numbers by computer or calculator.

Students in groups will generate samples of 10, 20 30, and 40% of their class. First they will predict the range of the number of yeses in each trial for one of the five questions already answered by the class. Then each group will conduct 12 trials for a particular percentage of the population. After the groups finish the experiment, results from all four groups will be recorded on separate number lines. The class can compare and discuss the differences in the distributions and what effect sample size has on the accuracy of one's predictions.

<u>Activity week 10:</u> Green M&Ms (sampling distributions)

This activity should be planned to coincide with a candy sale conducted by some club on campus if possible. The teacher presents the class with a large bag of M&Ms and asks what the probability is of picking a green M&M if one were to close his or her eyes and grab one out of the bag. Students can estimate the probability from their previous experience eating the product.

After recording their estimates, the teacher will empty the bag into a glass jar or bowl and have the students estimate the probability while looking at the large quantity of candy. The teacher will then generate discussion about how to calculate the probability. Students may suggest counting all the M&Ms in the jar and comparing the total number to the number of green ones. Some will probably come up with the idea of sampling. They can take samples from the big jar or they can each buy a box or bag of M&Ms. Each box is a sample.

Students will count and record the number and percentage of green M&Ms in their boxes. The data from their samples can be used to generate discussion about sampling distributions and standard error of the mean. The mean of the samples can be compared to the hypothetical mean (their estimates) as well as the established mean (10% green).

The results of this activity can be carried over into the next unit on confidence intervals. When one uses sample means to predict the mean of a population, how accurate is the estimate?

Activity week 11: Sampling the School (confidence intervals)

In a small school, it is possible to survey the entire school. In a large school, it might be more practical to survey just the senior class. Either one would comprise a relatively large population. Devise a questionnaire with no more than 10 questions, some yes-or-no, some numerical. Have all students answer the questions anonymously during homeroom or English class in order to have the largest possible response.

Select the most interesting or controversial question from the survey to analyze. Students work in groups to generate random samples of the population and extract the data for their sample. Compare the sample means or proportions for all the groups. Can we use the sample data to predict the mean (or proportion of yeses) for the whole population? Would you predict a number (point estimate) or a range of numbers (interval estimate)? If you use a range how wide should it be? These questions and the students' previous experiences with sampling distributions will lead to a discussion of confidence intervals.

Once a confidence interval has been established, the actual population mean (or proportion) can be calculated. This is quite a tedious job since the population is large. The students should gain an appreciation for sampling to determine statistics for large populations.

Activity week 12: Project (sampling)

For this project, students will take random samples of their peers, ask a yes-no question on a topic of their choice, and report on the results.

Format:

1. Title of the project. It should be descriptive. Example: "Seniors' Opinions of School Cafeteria Food"
2. State the survey question. It should be a yes-no question for a survey of 30 students.
3. State the population you are sampling. Examples: "all girls in my school; all students taking French at my school"

Hint: As a pretest, ask your question to some of your classmates to see if they interpret it exactly as you intend. Is it possible they may not tell you the truth? Will you ask for a verbal answer or a secret ballot? Change your procedures or the wording of your question, if necessary.

4. Explain how you will use random sampling to select 30 students.
5. State the names of those in your sample. Ask them your question and record the results.
6. What is your sample proportion?
7. Construct a 95% confidence interval for the percentage of yeses in the population.
8. Report the results of your survey in the form of an article for the school newspaper. Be sure to explain the meaning of the confidence interval in your article.

Activity week 13: The Remote Control (counting)

At home, I have a remote control which is supposed to control both my VCR and my television. However, one day I found that it had no effect on the VCR. It wouldn't play, rewind, or fast forward a tape. I knew the batteries were not dead because it worked fine on all functions of the TV. When I turned the remote control over, I discovered that the cover of the battery compartment was missing. Switches were exposed that set the control to be compatible with various brands of TVs and VCRs. Assuming that someone had flipped one or more of the VCR switches, and unable to locate the instruction manual, I began flipping switches to see if I could find the right combination to work my brand of VCR. How many settings would I need to try to assure that I tried them all? There are five switches and each has two settings (on or off).

This problem can be discussed in small groups. The group members can present various strategies for determining the answer. The ensuing class discussion should include strategies for efficiency in testing all the possibilities. Related examples include guessing on a multiple choice or true-false test, lottery numbers, and number of license plates possible for particular configurations of letters and numbers. Also, telephone company officials in New York City have recently announced that it will soon be necessary to start another area code for the city because they have used almost all the numbers possible for one area code. How many seven digit numbers are possible in each area code? Numbers cannot begin with zero. Are there any other restrictions?

Activity week 14: Free Throws (probability simulation)

Scott has just learned a new way to shoot free throws in basketball. Using his old method, his average of shots made was 60%. Using the new method, he scored 9 out of his first 10 shots. Can he conclude that the new method really is better than the old method? The class can discuss the question and identify the problem: What are the chances of shooting at least 9 out of 10 if you normally shoot 60%?

Students will model the problem using their spinners or polyhedral dice numbered 0-9. To simulate the 60% probability, they will assign six digits (4-9) to the event that a basket is made, and four digits (0-3) to the event that it is missed. Students work in pairs rolling or spinning ten times. If nine or more "baskets" occur, the trial counts as a success. They repeat the experiment nine more times and determine the percentage of successes. This figure is their estimate of the probability of Scott making 9 or more baskets in 10 attempts.

The results of this activity should be pooled for the whole class to get a more accurate estimate of the probability. This simulation exercise can be expanded to even larger numbers of trials using a random number table or a computer program.

Activity week 15: Language Students (conditional probability)

Who gets better grades: students who take Spanish or students who take Latin? The class can generate a random sample of students who take Spanish and another sample of those who take Latin. They can compare the list of students in each sample to the honor roll list for the previous term. The result will be paired qualitative (nonnumerical) data. How can we represent the data in a graph or chart? Students will suggest Venn diagrams since they were studied the previous week. They may also come up with the idea of a two-way table to summarize the findings. Each cell of the table contains the number of students who are in two categories (i.e. Latin students who are on the honor roll).

	Latin	Spanish	Total
Honor Roll	8	8	16
Not Honor Roll	10	14	24
Total	18	22	40

What is the probability that a student takes Spanish? What is the probability that a student takes Latin and is not on the honor roll? What is the probability that a student takes Latin or is not on the honor roll? These are questions that could be answered with the aid of Venn diagrams. They concern probabilities of unions and intersections of sets.

This discussion should lead to conditional probability questions. If a student takes Latin, what is the probability that he or she is not on the honor roll? If a student is not on the honor roll, what is the probability that he or she takes Latin? Students can understand and compute conditional probabilities from two-way tables. This activity can lead to derivation of formulas if the studetns fill in the two-way table with probabilities of the intersections of events and compare them to the probabilities of the events themselves.

Activity week 16: Number of Ancestors (sequences and series)

The concept of an algorithm can be introduced by having the students write detailed, step-by-step instructions for completing familiar tasks. Imagine that they are instructing a robot or an alien from another world how to complete the task. It is imperative that they include every step. Possible tasks to describe include brushing teeth, going to the grocery store, or calculating the number of minutes in your life.

After students have some experience writing algorithms, they can be introduced to sequences and series and develop algorithms to find a particular term or the sum of terms. How many ancestors have you had in the last n generations (for n = any whole number)? Class discussion should yield the sequence 2, 4, 8, 16, ... to represent one's ancestors. Students can work in small groups to develop algorithms to find the sum of any number of terms. Sequences should be examined from an explicit perspective, based on the positions of the terms, as well as in terms of recurrence relations, where each term is expressed as a function of previous terms. The Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, ...) is one which can be defined recursively. It also is fascinating to study because of its frequent occurrences in nature.

Activity week 17: Making Change (mathematical induction)

Some people have proposed that the U.S. penny should be abolished. Others argue that replacing the penny with a 2-cent coin would not give enough flexibility for pricing merchandise. What prices could still be paid with exact change if the penny were abolished and a coin worth 2 cents were introduced? Students can check to see what values can be obtained using only 2-cent and 5-cent coins. To model the problem, they could use chips in two colors to represent the two denominations of coins.

After trying several numbers, the students should discover that the only prices that would be impossible are 1 cent and 3 cents. How can they prove their hypothesis for very large numbers, or indeed all integers greater than or equal to 4? Within their groups, they can look for patterns and arrangements of the coins. If they can find a way to show that for any (kth) number, they can exchange some coins to make the next (k+1 th) number, they will have discovered inductive proof. Once the students internalize the principle of mathematical induction, they can use it to prove relationships and formulas, especially in algebraic and geometric series.

Textbooks

Topics from several textbooks will be studied. This table indicates which chapters will provide materials during each unit of instruction.

Unit	Textbooks
Introduction	Exploring Data, chapters 1-5
Statistics	Exploring Data, chapters 6-7 Statistics, chapter 11 Pre-calculus, chapters 2, 3, 11, 18
Sampling	Exploring Surveys, chapters 1-6 Statistics, chapters 8-10
Probability	Statistics, chapters 4-6 Pre-calculus, chapter 9
Algorithms	Pre-calculus, chapter 9 Discrete Structures, chapters 1, 7, 8, 10

Freund, J. E., & Smith, R. M. (1986). Statistics: A first course. Englewood Cliffs, NJ: Prentice-Hall.

Landwehr, J. M., & Watkins, A. E. (1986). Exploring data. Palo Alto: Dale Seymour.

Landwehr, J. M., Swift, J., & Watkins, A. E. (1986). Exploring surveys and information from samples. Palo Alto: Dale Seymour.

Shanks, M. E., Fleenor, C. R., & Brumfiel, C. F. (1981). Pre-calculus mathematics. Menlo Park, CA: Addison-Wesley.

Van Iwaarden, J. L. (1988) Discrete structures: Essential computer mathematics. San Diego: Harcourt Brace Jovanovich.

Evaluating Student Progress

Two goals of this course are for students to understand statistical reasoning and to reason and communicate mathematically. Too often in high school mathematics classes, students get by on memorization and being able to manipulate formulas. They do not gain any true insight into mathematical thinking. To ensure that students are gaining true understanding, they will write about their statistical experiences throughout the course By elaborating on processes and explaining the results of their studies, students will be thinking mathematically.

Acquisition of knowledge will be assessed by means of five tests, a semester exam, projects, group assignments, individual homework assignments, and participation in class. The tests and exam will contain a variety of questions reflecting the various teaching and learning modes of the course. Assessments should yield information about students' understanding of concepts and procedures, ability to solve problems and interpret the results, ability to reason and communicate mathematically, and overall disposition toward mathematics.

Notes to Prospective Instructors

It is hoped that this work will inspire mathematics teachers to incorporate some of the concepts of data analysis into their courses. Most of the included activities are applicable to younger age groups. One variable statistics could be introduced in Algebra I. Probability and mathematical modeling can be incorporated into the current Algebra II curriculum.

It is beyond the scope of this work to include every activity to be completed in a semester course. The weekly outline and suggested activities should provide the instructor with a starting point in developing and refining a new course. Encourage students to discover, discuss, analyze, and enjoy mathematics.

Chapter 5: Conclusions

The course presented in this work is one answer to the dilemma of how to prepare high school students mathematically for the future. It includes many topics and teaching methods that were recommended in NCTM's Standards (1989). It challenges the students to reason algorithmically and to model mathematical problems. The hands-on activities should intrigue students of all ability levels.

As this course is taught, it will emerge and evolve. The curriculum and time schedule are flexible to allow for exploration and discovery. Some of the activities and experiments may lead to explorations of related topics. The course may dictate more group and individual projects and fewer tests. Depending on the composition of the class, more theoretical concepts could be examined.

This course is not the only alternative course possible for college-bound seniors. A case could be made for a course in discrete mathematics or a traditional (theoretical) course in probability and statistics. Some schools offer SAT preparation courses for math credit. An advanced geometry course could be designed to explore three- and four-dimensional geometry, transformations, and an introduction to topology. A mathematical exploration course could be devised to examine exciting new mathematical topics like fractals and chaos. Alternative courses should be tried in the secondary mathematics curriculum so that students will learn to approach problems from various perspectives.

One problem I have to resolve is selection of an appropriate title for this new course. The main thrust of the course is collecting and analyzing data. My first inclination was to call the course "Data Analysis". However, in taking an informal survey of some juniors and seniors in high school, I found that they would be reluctant to sign up for courses called "Data Analysis" or "Algorithms" because they sound too difficult. Some students indicated they would be interested in courses called "Probability and Statistics", "Discrete Math", "Algebra III", "Geometry II", and "Statistics Lab". Of these choices, I feel that "Statistics Lab" best describes the course, so it is currently my most likely course title.

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Vita

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