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Firm-level productivity and international expansion of firms from the Lodz Voivodeship

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Abstract

The paper investigates the link between firm-level productivity and internationalization (through exports, imports and FDI) in the Lodz Voivodeship, Poland. Two hypotheses have been tested –self-selection and learning by internationalization. It has been found that productivity may affect import and FDI decisions of firms, while there is no evidence of such an effect regarding exports. At the same time, there is no proof for learning, suggesting that within the timeframe of the analysis firms from the Lodz Voivodeship do not experience productivity gains due to international trade or investment.

JEL: F12, F23, D22

Keywords: international trade, foreign direct investment, internationalization, productivity, self-selection, learning

1. Introduction

Traditional theories apply to the macroeconomic approach to international trade or investment. It took decades before theorists were able to find a way to explain the simple observation that, in fact, each firm has to take its own decision on whether to internationalize operations or not. The final breakthrough, leading to the establishment of the so called New New Trade Theory (NNTT), took place in 2003, when Marc Melitz (2003) presented his simple model of individual export decisions among heterogeneous firms. The basis for diversification of firms was their productivity, a factor that plays a key role in contemporary economic theory, especially when engines for economic growth are discussed (see e.g. Hulten 2001;Aiyar&Dalgaard 2004).

Since these theories are still considered new, there is a strong need for their empirical testing in different national economies, performing in diversified conditions. Poland is a transition country which evolved from Soviet bloc central planning to a West-oriented economy, gradually converging with "old" EU member-states. And Lodz Voivodeship is an average region of Poland[1]. It is therefore interesting to test the NNTT postulates on that particular ground.

The aim of the paper is to examine productivity differences between internationalized and non-internationalized enterprises and test possible causal linkages between productivity and firms' decisions about engaging in three primary forms of international operations. The main body of this article is accordingly divided into three sections. Section 2 focuses on exporters and non-exporters. Section 3 refers to intermediate and capital goods importers and non-importers. It is worth stressing that whenever imports and importers are mentioned, it is strictly related to the import of intermediate and capital goods, as only those sorts of imports seem to be a source of long-term economic development. Section 4 concentrates on foreign direct investment (FDI) as another form of internationalization. Section 5 concludes.

2. Exporters

According to Melitz (2003) firm-level productivity is the most important factor affecting decisions whether to export or not and only the most productive firms are competitive enough to enter foreign markets. Low marginal costs enable those firms to earn such a big revenue that they can cover the sunk fixed costs of exporting (e.g. costs of the formation of distribution channels or promotion activities). It became clear that in every industry there exists a productivity threshold and if a firm is below that value then it is unable to begin and maintain exporting. Similar results were obtained in other theoretical models (Bernard, Eaton, Jensen & Kortum 2003; Melitz & Ottaviano 2008).

This mechanism of productivity influencing decisions about exporting is known as self-selection. This hypothesis is based on an empirically proved correlation between firms' efficiency and their export status. However, correlation does not indicate causality, hence the opposite direction of the link between productivity and exporting has to be considered. In the so called learning-by-exporting (LBE) hypothesis, it is assumed that engagement in international trade boosts a firm's efficiency. Yeaple (2005) proved that in order to export, firms must adopt better technology and more skilled workers, which results in an

[1]Poland consists of 16 Voivodeships, which means that, on average, one region should produce 6.25% of GDP. Lodz Voivodeship produces about 6.17% of Polish GDP. Moreover, it is the geographical centre of Poland, with a similar distance from Western (EU) and Eastern (CIS) trade partners of Poland.

increase in productivity. Verhoogen (2008) showed that foreign customers' preference for high quality forces exporters to look for more skilled labour. Bernard, Redding and Schott (2010) presented a model in which engagement in export results in concentration on the firm's core competence, which again leads to an increase in productivity.

The empirical literature shows that self-selection is more common than LBE. For example, Hagemejer (2006) used a VAR model and found no evidence of LBE but at the same time proved the existence of self-selection among Polish manufacturing firms. With probit regression he found that firm-level productivity has an effect on the probability of exporting. In our research we addressed those issues according to their evolution in theory. Firstly, we proved differences between firm-level productivity distribution among exporters and non-exporters. Secondly we tested for both self-selection and LBE.

2.1 Exporters and non-exporters – differences in productivity distribution

Productivity in general describes how well a given company performs. In particular, the so called Total Factor Productivity (TFP) is a measure of how effectively all inputs are transformed into economic outcome. Technically it is difficult to estimate TFP, as simple measures are often biased which distorts conclusions based on them.

We incorporated a semiparametric method proposed by Olley and Pakes (1996) – see Appendix 2. Their method is suitable for estimating firm-level productivity as it resolves two main problems connected with panel data: simultaneity and selection bias (Yasar, Raciborski & Poi 2008: 221). The first one refers to the situation in which observed inputs, such as labour or capital, may be correlated with unobserved inputs or productivity shocks, e.g. quality of materials, management skills, technical wear of capital, etc. The second one concerns the problem of firms dropping out of the data set. Furthermore, such endogenous exits are usually correlated with other variables, e.g. with firm size (Aguirregabiria 2009: 2). The Olley-Pakes algorithm (OPA) deals with those issues using investment as a proxy for the unobserved, time-varying productivity shocks and probit estimates of survival probability (Yasar, Raciborski & Poi 2008: 222). The OPA was used by Hagemejer (2006) and Hagemejer&Kolasa (2008), hence utilising the same method facilitates comparison of the results.

Using OPA and Central Statistical Office data, we measured firm-level productivity of companies from the Lodz Voivodeship (Poland). At first, we divided our subjects into two groups: exporters and non-exporters. We repeated our calculations for three years: 2005 (the consequences of Polish EU accession), 2008 (the verge of the *subprime* crisis) and 2011 (latest available data, global crisis entering its fadeout). We expected to obtain higher productivity estimates for exporters in each year. Table 1 shows the results.

Table 1. Results of TFP estimates

TFP estimates	Number of companies					
	Exporters			Non-exporters		
	2005	2008	2011	2005	2008	2011
(0.0 - 5.5]	17	21	26	59	52	58
(5.5 - 6.0]	140	97	96	162	148	134
(6.0 - 6.5]	170	165	163	173	180	143
(6.5 - 7.0]	120	108	107	80	78	66
(7.0 - 7.5]	56	58	85	17	28	23
(7.5 - 8.0]	34	46	33	4	11	15
(8.0 - 8.5]	7	13	17	1	4	3
(8.5 - 9.0]	6	4	6	0	0	1
(9.0 - 9.5]	0	0	1	0	0	0
9.5 <	0	1	4	0	1	1
Total number (% of all comp.)	550 (53%)	513 (51%)	538 (55%)	496 (47%)	502 (49%)	444 (45%)
Avg. TFP	6.45	6.53	6.59	6.09	6.18	6.18
St. deviation	0.66	0.72	0.78	0.51	0.59	0.65

Source: own calculations

The most important observation is the fact that the average productivity of exporters was in fact higher in all years. Our calculation was based on a full set of all production companies from the Lodz region, so our averages are in fact the directly given expected values of the complete discrete distributions shown in Table 1. However, should one wish to widen our research scope and treat our sets of firms as subsets of exporters and non-exporters in general (e.g. in Poland) and our averages as estimates of expected values, then, based on simple statistical testing, one would find the differences between our estimates significant at $\alpha = 0.01$ (both the cross-section and in the time dimension).

When referring to the dynamics of average productivity, two facts stand out. Firstly, exporters' TFP grew in the entire period. Non-exporters' productivity, on the other hand, grew in 2008 in comparison with2005, but in 2011 the average TFP remained precisely the same as in 2008. This proves that the economic crisis was better managed by exporters. Secondly, the crisis brought a drop in the number of both exporters and non-exporters. This even led to an increase of the non-exporters' share in all companies. However, in 2011 the number of exporters began to rise again, while more and more non-exporters fell out of the market. That is why again one could note that engagement in exports might have been a good way to recover more efficiently after the first symptoms of the crisis.

2.2 The self-selection hypothesis

This hypothesis has been supported by numerous empirical studies (Bernard, Jensen 2004; Mayer, Ottaviano 2007 to name but a few). Hagemejer (2006) even investigated the Polish case and found

evidence for self-selection[2]. With that in mind, we tried to test if such evidence can be found among firms from the Lodz Voivodeship.

Our research was based on a logit estimation on panel data with export status as the dependent variable, where 1 means exporter and 0 means non-exporter. The sample consisted of 5373 observations. Table 2 contains the independent variables exploited in that estimation.

Table 2. List of variables

Symbol	Description
h_t	Olley-Pakes estimation of firm-level TFP
l_t	log of *i*-th firm workforce
a_t	log of *i*-th firm age
s_t	dummy for Treasury in ownership structure
j_t	dummy for local government in ownership structure
z_t	dummy for foreign capital in ownership structure
p_{t1}	dummy for PKD divisions 10, 11 or 12 (Food, beverages and tobacco)
p_{t2}	dummy for PKD divisions 13, 14 or 15 (Textiles and clothes)
p_{t3}	dummy for PKD divisions 16, 17 or 31 (Wood processing)
p_{t4}	dummy for PKD divisions 18, 26 or 32 (IT equipment)
p_{t5}	dummy for PKD divisions 19, 22 or 23 (Non-metallic raw materials processing)
p_{t6}	dummy for PKD divisions 20 or 21(Chemicals and pharmaceuticals)
p_{t7}	dummy for PKD divisions 24 or 25 (Metal processing)
p_{t8}	dummy for PKD divisions 27, 28 or 33 (Electrical equipment)
p_{t9}	dummy for PKD divisions 29 or 30 (Vehicles and transportation equipment)
x_t	dummy for being an exporter
m_t	dummy for being an importer of intermediate and capital goods
b_t	dummy for being a foreign direct investor

PKD stands for Polish Business Classification: http://stat.gov.pl/Klasyfikacje/

Source: own calculations

The results of our estimation are shown in Table 3. Surprisingly, we found no proof of self-selection. The marginal effect of change in productivity turned out to be negligible. To make matters worse, firm-level efficiency entered the equation with a negative coefficient, although it was insignificant at $\alpha = 0.1$. This result could be viewed as evidence that the competitive edge of firms from the Lodz region is based not on productivity, but other characteristics. It can be speculated that these could be e.g. the high quality or the uniqueness of the products, high financial liquidity or access to vast networks of contacts. The low values of R-squared (McFadden R-squared or adjusted R-squared) indicate that those other factors play a main role when firms decide if they should enter foreign markets through export or not. This result might also indicate that the productivity threshold for export is low enough for the majority of firms from the region to cross it. This means that the necessary condition of exporting formulated by NNTT is met by most companies.

[2]Our research differs from Hagemejer (2006) in terms of research period (2005-2011 – post-EU-accession period), geographical unit (regional scale) and the set of independent variables (inclusion of other forms of internationalization).

Table 3. Results of estimations of export status

Variable	Logit estimation			
	Coefficient	**Standard error**	**z**	**Marginal effect**
const.	-1.816	0.458	-3.964	---
h_t	-0.012	0.076	-0.163	-0.003
l_t	0.469	0.045	10.508	0.112
a_t	0.116	0.045	2.607	0.028
s_t	-0.176	0.186	-0.945	-0.042
j_t	-1.632	0.983	-1.660	-0.390
z_t	1.323	0.102	12.923	0.316
p_{t1}	-1.517	0.241	-6.306	-0.362
p_{t2}	-0.562	0.236	-2.379	-0.137
p_{t3}	-0.864	0.251	-3.441	-0.212
p_{t4}	-1.248	0.259	-4.827	-0.301
p_{t5}	-0.929	0.243	-3.823	-0.228
p_{t6}	-1.304	0.264	-4.939	-0.313
p_{t7}	-0.847	0.248	-3.419	-0.208
p_{t8}	-0.921	0.245	-3.762	-0.226
b_t	0.627	0.287	2.182	0.137
m_t	1.337	0.071	18.843	0.315
Statistics	Dependent variable – mean: 0.571 Dependent variable – SE: 0.495 McFadden R-squared: 0.226 Adjusted R-squared: 0.222		Log likelihood: -2842.816 Akaike criterion: 5719.633 Schwarz criterion: 5831.667 Hannan-Quinn criterion: 5758.754	

Source: own calculations

The lack of self-selection should be very important for policymakers, as without it there are no intra-industry reallocations towards the most productive firms. Those reallocations are considered to be serious contributors to aggregate productivity gains (Pavcnik 2002; Melitz 2003).

The results also indicate that, *caeteris paribus*, the probability of exporting increases with firm size (proxied by workforce) as well as with age, both significantly. It seems that the bigger the firm, the more human capital can be assigned to international operations. The result for age can be interpreted in two ways. Firstly, according to sequential internationalization theories (like the Uppsala model) only firms which are successful enough on their domestic market can begin international activities. Building such a position requires time, which is why older firms are more likely to become exporters. Secondly, this could be a premise for the occurrence of the so called hysteresis effect, which means that past incentives may affect the present state. Subjects with more experience, especially those which actively exported in previous periods[3], are more likely to export. Thus, age can be seen as a proxy for experience.

[3]However, many companies are only incidentally engaged in export, which, despite being less effective than permanent export activity, also generates some experience. That is why we decided to use age as a proxy for the hysteresis effect rather that lagged values of export status.

Our results also show that ownership is another important aspect of a firm's decision about exporting. Having local government among the owners seems to make firms less prone to export (the coefficient with Treasury was insignificant at $\alpha = 0.1$). On the other hand, foreign ownership increases the probability of exporting. What is more, if the firm is an importer of capital goods or a foreign direct investor, then it is also more likely to explore external markets via export. The observed effects of foreign ownership and engagement in other forms of internationalization led to the conclusion that being a part of an international production network increases the probability of becoming an exporter.

Furthermore, our results indicate the importance of the industry that the firm operates in. We divided industries into nine categories and used eight of them in estimations (omitting one to avoid collinearity). However, each of the coefficients was negative and significant. This suggests that belonging to the non-included (ninth) sector, which is the production of transport vehicles, increases the probability of exporting. It seems that the Lodz Voivodeship has a (static) comparative advantage in that sector.

2.3 The learning-by-exporting hypothesis

Supporters of the LBE hypothesis believe that the productivity shift is a result of exposure to foreign markets and international partners. In our research we tried to determine if firms from the Lodz region that enter foreign markets (new exporters) benefit from any productivity increases during the first three years of their international activities. We analysed increments of the h_t variable (see Table 3). The results are presented in Table 4.

Table 4. Productivity gains in the first three years after engaging in exports

	Productivity (h_t) gain after		
	1 year	**2 years**	**3 years**
Mean	0.019	0.029	0.032
St. deviation	0.085	0.101	0.121
25^{th} percentile	-0.018	-0.025	-0.037
50^{th}percentile	-0.003	0.005	0.014
75^{th}percentile	0.037	0.066	0.087

Source: own calculations

The average increases of productivity are positive and grow over time, however, with declining dynamics. They are also surprisingly low. With an average h_t value of nearly 6.5, the gain of magnitude order of 0.03 is only about 0.5%. Those results are statistically insignificant at $\alpha = 0.1$.

Another important point in the discussion of LBE in the Lodz region is its sectoral diversification. Table 5 shows our results of average productivity gains in nine considered sectors.

Table 5. Sectoral differences in averages of the productivity gains within the first three years of exporting

	Avg. productivity (h_t) gain after		
Sector	**1 year**	**2 years**	**3 years**
p_{t1}	0.018	0.035	0.043
p_{t2}	0.012	0.025	0.018
p_{t3}	0.026	0.017	0.014
p_{t4}	0.026	0.022	0.013
p_{t5}	0.040	0.055	0.072
p_{t6}	0.028	0.027	0.039
p_{t7}	0.006	0.006	0.001
p_{t8}	0.001	0.012	0.014
p_{t9}	0.019	0.058	0.077

Source: own calculations

Unfortunately, all productivity increments turned out to be insignificant. Our results prove, however, that historically, and primarily for the Lodz region, the textile industry, marked as p_{t2}, turned out to be one of the least promising in terms of LBE perspectives. Moreover, one of the local government's strategies to stimulate the region's development was to encourage BPO services in Lodz.However, the production of IT technology, marked as p_{t4}, which is an industry most complementary to BPO services, was one of the sectors with the lowest LBE effects. The very lowest export driven productivity gains were noted for the production of metallic raw materials, however, marked as p_{t7}. These sectors definitely should not be promoted by local government or business support institutions.

3. Importers

The recent theoretical and empirical literature focuses on export. However, a positive correlation between firm-level efficiency and engagement in import was also found. With this in mind, in this section we explore that field. We start by characterizing productivity distributions of importers and non-importers. Then, we study whether self-selection and learning-by-importing describe firms from the Lodz Voivodeship.

3.1 Importers and non-importers – differences in productivity distribution

OPA estimations allowed us to compare not only exporters and non-exporters but also importers and non-importers. It is worth highlighting once again that by import we only mean foreign supplies of capital and intermediate goods (not final goods). The results are presented in Table 6.

Table 6. Results of TFP estimates

TFP estimates	Number of companies					
	Importers			Non-importers		
	2005	2008	2011	2005	2008	2011
(0.0 - 5.5]	13	21	21	63	52	63
(5.5 - 6.0]	119	95	95	183	150	135
(6.0 - 6.5]	177	164	161	166	181	145
(6.5 - 7.0]	133	125	116	67	61	57
(7.0 - 7.5]	61	64	86	12	22	22
(7.5 - 8.0]	36	54	39	2	3	9
(8.0 - 8.5]	8	16	19	0	1	1
(8.5 - 9.0]	6	4	7	0	0	0
(9.0 - 9.5]	0	0	1	0	0	0
9.5 <	0	2	5	0	0	0
Total number (% of all comp.)	553 (53%)	545 (54%)	550 (56%)	493 (47%)	470 (46%)	432 (44%)
Avg. TFP	6.51	6.59	6.65	6.03	6.09	6.10
St. deviation	0.66	0.73	0.79	0.47	0.51	0.57

Source: own calculations

One may observe that firms engaged in import were more productive than those sourcing only domestically. The difference, both between average productivity of the two groups of firms and in time, was statistically significant. Moreover, that difference slightly widened during the 2005-2011 period. Although non-importers gradually improved their efficiency, the productivity changes among importers were more evident.

3.2 The self-selection hypothesis

Although self-selection has been aphenomenon widely studied in the context of export, several models were presented to explain similar behaviour characterizing importers. Gibson and Graciano (2011) and Ramanarayanan (2007; 2012) are examples of that line of research. Hagemejer and Kolasa (2008) proved that the phenomenon is existent among Polish importers of capital goods.

Applying similar methodology as in the case of exporters, we were able to detect self-selection in importing activities of firms from the Lodz Voivodeship. The results presented in Table 7 indicate that firm-level productivity affects a firm's decision whether to import. It may be so for several reasons, such as additional sunk costs of importing (hence only firms with low marginal costs do not lose their competitiveness when they include those costs in price calculations) or the ability to absorb foreign technology (very productive firms may be able to make the most of foreign – and better – capital goods). At the same time, one may bear in mind that values of McFadden R-squared are rather low. It indicates that other factors influence decisions whether to start importing. These might be variables unobservable to us, such as access to external funds or the level of standardization of inputs.

Table 7. Results of logit estimation of import status

Variable	Logit estimation			
	Coefficient	**Standard error**	**z**	**Marginal effect**
const.	-9.440	0.508	-18.570	---
h_t	1.255	0.084	14.891	0.294
l_t	0.224	0.046	4.864	0.053
a_t	0.212	0.047	4.513	0.050
s_t	-0.989	0.200	-4.949	-0.232
j_t	-0.970	0.880	-1.102	-0.227
z_t	0.883	0.105	8.419	0.207
p_{t1}	-1.395	0.223	-6.242	-0.335
p_{t2}	-0.024	0.215	-0.113	-0.006
p_{t3}	-0.517	0.234	-2.215	-0.126
p_{t4}	0.172	0.246	0.699	0.039
p_{t5}	-0.333	0.227	-1.466	-0.080
p_{t6}	0.715	0.271	2.640	0.150
p_{t7}	-0.591	0.228	-2.585	-0.144
p_{t8}	-0.399	0.227	-1.761	-0.096
x_t	1.309	0.071	18.372	0.304
b_t	0.288	0.296	0.972	0.065
Statistics	Dependent variable – mean: 0.571 Dependent variable – SE: 0.495 McFadden R-squared: 0.290 Adjusted R-squared: 0.285		Log likelihood: -2609.729 Akaike criterion: 5253.459 Schwarz criterion: 5365.493 Hannan-Quinn criterion: 5292.580	

Source: own calculations

The probability of importing increases (*caeteris paribus*) with firm size and age with a similar interpretation as in the case of export (with the exception that internationalization theories are best suited to describe export and FDI). The results also indicate that ownership is an important determinant of the decision about engagement in importing. Firms with the Treasury among the owners are less likely to import (this time, the coefficient with local government is insignificant). Foreign-owned enterprises are more prone to expand abroad via import. Engagement in international production networks – the status of international exporter – also increases the probability of becoming an importer. FDI status, on the other hand, turned out to be insignificant.

As far as sectoral differences are concerned, it seems that the biggest likelihood of importing characterizes firms operating in the chemical and pharmaceutical industries (p_{t6}). That sector is heavily dependent on foreign inputs and it translates into a high probability of importing when an enterprise operates in that industry. Three sectoral variables, p_{t2}, p_{t4} and p_{t5}, were statistically insignificant with $\alpha = 0.1$. Those variables describe industries which are considered as either typical for the Lodz region (textiles) or with high growth potential in the region (IT due to BPO). This means that the probability of importing among companies from these three sectors was statistically no different than in the benchmark

sector (production of transport vehicles). However, it was still higher than in sectors with significant and negative coefficients.

3.3 The learning-by-importing hypothesis

To the best of our knowledge there is no term such as learning by importing which exists in the literature, but we find it convenient to adopt it because it mirrors the notion of the LBE effect. Assuming that importers of capital goods may naturally raise their productivity, we study the impact of that import on a firm's TFP. The results are summarized in Table 8.

Table 8. Productivity gains in the first three years after engaging in imports

	Productivity (h_t) gain after		
	1 year	**2 years**	**3 years**
Mean	0.025	0.045	0.064
St. deviation	0.092	0.115	0.135
25^{th}percentile	-0.014	-0.023	-0.024
50^{th} percentile	-0.003	0.010	0.028
75^{th}percentile	0.038	0.090	0.137

Source: own calculations

According to the results, engagement in import increases firm-level productivity, but those gains are small and they are parallel to those associated with export. The increases are slightly higher than in export, but still insignificant at $\alpha = 0.1$.

The impact of import on firm-level productivity might also be dependent on the industry. The results can be seen in Table 9.

Table 9. Sectoral differences in averages of the productivity gains within the first three years of importing

	Avg. productivity (h_t) gain after		
Sector	**1 year**	**2 years**	**3 years**
p_{t1}	0.026	0.050	0.064
p_{t2}	0.013	0.023	0.047
p_{t3}	0.017	0.021	0.060
p_{t4}	0.030	0.045	0.041
p_{t5}	0.061	0.110	0.123
p_{t6}	0.061	0.067	0.083
p_{t7}	0.026	0.040	0.061
p_{t8}	0.003	0.026	0.038
p_{t9}	0.013	0.051	0.057

Source: own calculations

Unfortunately, again all average productivity growths following engagement in foreign expansion in different sectors turned out to be insignificant at $\alpha = 0.1$.

4. Foreign Investors (FDI)

FDI is seen by many as the most mature form of international expansion of firms (especially by the adherents of theories of sequential internationalization). Due to the high financial, human and organizational costs of establishing a foreign affiliate, FDI should be characterized by self-selection. At the same time, physically appearing on the foreign market creates opportunities to raise productivity. Whether that intuition is supported among firms from the Lodz Voivodeship remained an open question. The structure of this section is comparable to the parts describing export and import – we present productivity distributions and test whether the above mentioned phenomena are present in the region.

4.1 Investors and non-investors – differences in productivity distribution

As in the case of export and import, we applied the OPA to compare the productivity distributions of firms engaged in international investment (in the form of FDI) and firms which do not conduct such an activity (Table 10).

Table 10. Results of TFP estimates

TFP estimates	Number of companies					
	Investors			Non-investors		
	2005	2008	2011	2005	2008	2011
(0.0 - 5.5]	0	1	1	76	72	83
(5.5 - 6.0]	0	3	1	302	242	229
(6.0 - 6.5]	1	5	9	342	340	297
(6.5 - 7.0]	0	3	3	200	183	170
(7.0 - 7.5]	0	5	6	73	81	102
(7.5 - 8.0]	0	2	5	38	55	43
(8.0 - 8.5]	1	1	5	7	16	15
(8.5 - 9.0]	0	0	1	6	4	6
(9.0 - 9.5]	0	0	1	0	0	0
9.5 <	0	0	1	0	2	4
Total number (% of all comp.)	2 (0%)	20 (2%)	33 (3%)	1044 (100%)	995 (98%)	949 (97%)
Avg. TFP	7.01	6.75	7.25	6.28	6.35	6.38
St. deviation	1.43	0.80	1.04	0.62	0.68	0.72

Source: own calculations

The results followed the same patterns as in other forms of internationalization. Firms with FDI were more productive than enterprises without foreign affiliates. The differences of averages were statistically significant across the groups and the years of observations (apart from the average of 2 for investors in 2005, which should not be included in the analysis because of the extremely low sample size).

4.2 The self-selection hypothesis

According to the literature, self-selection is not limited to international trade. Helpman, Melitz and Yeaple (2004) prove its existence in the context of horizontal FDI. Antras and Helpman (2004) explained

the phenomenon in their analysis of vertical FDI. Grossman, Helpman and Szeidl (2006) studied complex FDI and proved that firm-level productivity affects the investment strategy of an enterprise.
Using a logit model, we tried to test that hypothesis. Table 11 presents the results. Firm-level productivity positively influenced the decision whether to invest abroad. However, that impact was negligible in terms of its marginal effect, even though the variable itself was statistically significant with $\alpha = 0.1$. This could suggest that the productivity advantage may be crucial above a certain level, but not when considering productivity close to the average. One should remember that the reported marginal effects were calculated for average levels of independent variables. If direct investment requires productivity much higher than average, such marginal effects are in fact close to zero. This would not only mean that self-selection is present among FDI investors, but also that it is strong and concerns high levels of economic efficiency.

Table 11. Results of logit estimation of investment status

Variable	**Logit estimation**			
	Coefficient	**Standard error**	**z**	**Marginal effect**
const.	-9.787	1.025	-9.544	---
h_t	0.333	0.184	1.807	0.000
l_t	0.495	0.135	3.682	0.001
a_t	0.512	0.152	3.367	0.001
s_t	-2.398	1.173	-2.044	-0.007
j_t	-703.665	3355.040	-0.210	-1.990
z_t	-0.273	0.229	-1.191	-0.000
p_{t1}	-1.140	0.532	-2.142	-0.002
p_{t2}	-0.222	0.421	-0.527	-0.000
p_{t3}	-0.119	0.471	-0.253	-0.000
p_{t4}	-0.678	0.559	-1.212	-0.001
p_{t5}	0.0713	0.421	0.169	0.000
p_{t6}	-0.501	0.544	-0.920	-0.001
p_{t7}	-0.614	0.505	-1.216	-0.001
p_{t8}	-0.910	0.512	-1.776	-0.001
m_t	0.393	0.288	1.363	0.001
x_t	0.772	0.287	2.686	0.002
Statistics	Dependent variable – mean: 0.023 Dependent variable – SE: 0.150 McFadden R-squared: 0.137 Adjusted R-squared: 0.108		Log likelihood: -509.152 Akaike criterion: 1052.304 Schwarz criterion: 1164.338 Hannan-Quinn criterion: 1091.425	

Source: own calculations

Just as in export and import, firm size and age resulted in a higher likelihood of international operation (FDI). In contrast to trade, foreign-owned firms seemed to be less likely to conduct FDI than "domestic" enterprises. It is reasonable because when a firm is itself a foreign affiliate, it is rarely a decision-maker when it comes to FDI. However, the coefficient with z_t was not statistically significant. In terms of the

signs of the coefficients, other results concerning ownership and engagement in international production networks mirror the results presented in sections 2.2 and 3.2. As in the case of import, local government ownership was insignificant, as well as import status, which shows that export is more effective in pushing firms towards new forms of international activities than import or FDI. The latter are usually the results of increased expansion.

When analysing sectoral heterogeneity, one may find that most variables decomposing branch effects were insignificant at $\alpha = 0.1$. Yet, firms operating in the production of food, beverages and tobacco products as well as those producing electrical equipment turned out to be significantly less likely to invest in FDI. However, since these companies are most often affiliates of international corporations, so not fully independent foreign investments themselves, it is understandable.

4.3 The learning-by-investing hypothesis

Apart from identifying the possibility of learning from international trade, we were also interested whether a firm investing abroad increases its productivity (learning-by-investing, using terminology similar to LBE). The results are summarized in Table 12.

Table 12. Productivity gains in the first three years after engaging in FDI

	Productivity (h_t) gain after		
	1 year	**2 years**	**3 years**
Mean	0.004	0.003	0.003
St. deviation	0.035	0.040	0.044
25^{th} percentile	-0.007	-0.021	-0.029
50^{th} percentile	-0.001	0.007	-0.007
75^{th} percentile	0.022	0.022	0.022

Source: own calculations

The striking feature is the fact that investors improve their productivity only negligibly, even when compared to companies engaged in foreign trade. It may be so because they are the most productive firms and hence their scope for learning is limited. Another possibility is that FDIs of firms from the Lodz Voivodeship are driven mostly by market- or resource-seeking motives (and not efficiency-seeking ones which may be the motive generating the highest increase in productivity). It is also possible that positive effects of FDI require more time to materialize and cannot be observed when only the first three years are considered. All in all, the learning-by-investing effects in the first years after engaging in FDI were statistically insignificant.

We have also considered the influence of industry on the impact that FDI has on firm-level productivity, as seen in Table 13.

Table 13. Sectoral differences in averages of the productivity gains within the first three years of FDI

	Avg. productivity (h_t) gain after		
Sector	**1 year**	**2 years**	**3 years**
p_{t1}	0.023	0.070	-0.009
p_{t2}	-0.000	-0.019	-0.017
p_{t3}	0.015	-0.007	-0.026
p_{t4}	0.018	0.000	0.018
p_{t5}	0.013	0.013	0.025
p_{t6}	0.008	-0.018	-0.025
p_{t7}	-0.063	-0.038	--
p_{t8}	-0.004	-0.016	-0.016
p_{t9}	0.002	0.050	0.067

Source: own calculations

It seems that the impact of conducting FDI on firm-level efficiency in the case of an average investor was not significant in any industry.

5. Conclusions

Our research focused on three key aspects: differences in productivity levels, the presence of self-selection and the existence of learning effects. We investigated three kinds of internationalized economic subjects from the Lodz Voivodeship: exporters, importers (of intermediate and capital goods) and direct investors. We compared them among each other and with companies that refrained from any international activities.

As for productivity distribution, firms engaged in all kinds of international business were more productive than purely domestic enterprises. The average productivity of exporters and importers was similar, with a minor surplus to the advantage of importers. The productivity gap between FDI investors and other subjects was almost twice the size and increased over time. What is more, with the exception of just a few companies, FDI investors were also engaged in both exports and imports.

Using logit estimations of the exporter's, importer's and investor's status, we found evidence of self-selection among importers and investors. However, we found no proof for such a phenomenon among exporters. Thus, these results are not entirely consistent with the NNTT models which assume a productivity threshold for exporters. It is of course possible that such a threshold does exist but is low enough to be insignificant. In such a case, the decision whether to export or not would not be productivity driven, as non-exporters also meet the necessary condition of exporting.

In terms of learning effects, we examined average productivity shifts among new exporters, importers and investors in the first three years of their international operations. We found positive increments, however, they turned out to be statistically insignificant in all cases. Two caveats arise immediately. Firstly, our data came from the years 2005-2011, with most of that period being highly influenced by the global financial crisis. This might have had a strong influence on firms' capability to develop, especially to adopt new technologies and successfully meet higher standards, which means gaining experience and learning. Secondly, because of the short time horizon of available data, we could not focus on longer

time-lags than three years. It seems reasonable, however, to believe that especially benefitting from FDI could be more of a long distance goal.

In spite of those problems we think that there are industries which may act as growth engines in the Lodz Voivodeship due to the internationalization of firms (an increase in efficiency may be the result of intra-industry reallocations and learning). Specifically, we point to vehicles, transport equipment, and the chemical and pharmaceutical industries. However, support from the government (central or local) and business support institutions is needed.

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Appendix 1. TFP distribution and different modes of internationalization

Table A.1 summarizes the distribution of TFP across different kinds of firms when multiple forms of internationalization are taken into account.

Table A.1. TFP estimates for firms engaged in different forms of internationalization

		2005	2008	2011
Exporters, importers, investors	avg. TFP	–	6.87	7.40
	st. dev.	–	0.76	0.98
	no. of firms	2	17	26
Exporters,importers	avg. TFP	6.56	6.64	6.65
	st. dev.	0.68	0.74	0.76
	no. of firms	408	362	379
Exporters	avg. TFP	6.12	6.22	6.27
	st. dev.	0.48	0.55	0.64
	no. of firms	140	132	130
Importers	avg. TFP	6.34	6.46	6.50
	st. dev.	0.56	0.70	0.74
	no. of firms	143	165	141
No intern. activities	avg. TFP	6.00	6.04	6.02
	st. dev.	0.46	0.48	0.52
	no. of firms	353	336	299

Source: own calculations

Firstly, the general conclusion is that the more forms of internationalization there are, the higher the productivity. In all years of the analysis, exporters have higher TFP than non-internationalized firms, simultaneous exporters and importers have higher TFP than firms limited to just one mode of international expansion and companies engaged in all considered international activities have the highest TFP. What is more, the subset of firms that are exporters and importers at the same time is the most numerous. When referring to all kinds of exporters, one can notice consistent progress of TFP in time. This cannot be stated about non-internationalized firms.

As for importers, internationalization is associated with higher productivity. Firms that do not expand abroad were the least efficient enterprises in each year. Secondly, the degree of internationalization (the number of different forms of international activities) mattered; the most productive firms were those which combined import with FDI and export. Slightly less efficient were enterprises engaged in two-way trade, etc. Thirdly, the number of firms engaged in both import and FDI was negligible (hence we decided to exclude presenting statistics for that subset of firms). It means that when a company decides to invest abroad, it actively engages in international production networks and prefers to add both exporting and importing activities instead of only one of them. Finally, firms sourcing and operating domestically were the only group with a rather stagnant average productivity (and the only one experiencing a decline in efficiency – between 2008 and 2011 their productivity decreased slightly).

The striking feature of firms from the Lodz Voivodeship was that when a firm decided to invest abroad, it combined that activity with international trade. When a firm engages in vertical FDI, it usually imports

intermediate goods from foreign affiliate. In the context of horizontal FDI, a firm may export when foreign local demand is lower than the affiliate's capacity. If the third-country effect is included (as it is common in analysis of the complex FDI), a firm may be the owner of a foreign affiliate but also export (import) to (from) that third country. Combining FDI with trade, as is prevalent in the case of international expansion of firms from the Lodz Voivodeship, seems to be reasonable

Appendix 2. The Olley-Pakes Algorythm

The Olley-Pakes Algorythm (OPA) is a semiparametric method of calculating productivity presented in detail by Olley and Pakes (1996). The purpose of this appendix is to briefly summarize the idea of this method and give practical guidelines on how to use it.

1. Theoretical background

Olley and Pakes begin with stating the basic conditions for a firm's operations. They define capital accumulation and ageing of companies:

$$(1) \qquad k_{t+1} = (1-\delta)k_t + i_t \qquad \text{and} \qquad a_{t+1} = a_t + 1$$

where k denotes capital, i denotes investment, a denotes age, δ is the depreciation parameter and t denotes time.

Then, they assume that productivity in period t+1 (ω_{t+1}) of each firm is determined by a Markov process conditioned on all the information known in period t. It is sampled from a certain distribution, and the family F_ω of such distributions is defined as:

$$(2) \qquad F_\omega = \{F(\cdot\,|\omega), \omega \in \Omega\}$$

It is assumed that the company's operational decisions depend on the fact that it maximizes the expected discounted value (V) of future cash flows. This results in Bellman's optimization problem:

$$(3) \qquad V_t(\omega_t, a_t, k_t) = \max\{\Phi, sup_{i_t \geq o}\Pi_t(\omega_t, a_t, k_t) - c(i_t) + \beta\mathbb{E}[V_{t+1}(\omega_{t+1}, a_{t+1}, k_{t+1})|J_t\}$$

where Φ is the company plant's value should it be sold, Π_t is the current profit, conditional on the costs of investment $c(i_t)$, and β is the firm's discount factor for the expected future value, which is conditional on the information held by the company in current period J_t.

In other words, Olley and Pakes claim that a company's decisions about staying on the market and investment depend on its perception of the future based on current information. If the company's productivity is above a certain threshold $\tilde{\omega}_t$, which is dependent on the firm's age and capital, then the company continues its operations. Otherwise it has to shut down. This is known as the exit rule:

$$(4) \qquad \mathrm{X}_t = \begin{cases} 1 \; if \; \omega_t \geq \tilde{\omega}_t(a_t, k_t) \\ 0 \; otherwise \end{cases}$$

The firm's productivity and age (which is a proxy for experience) are also crucial for its investment:

$$(5) \qquad i_t = i_t(\omega_t, a_t, k_t)$$

2. Estimation

Firstly, we assume that the production function is a Cobb-Douglas type, and after logging it is:

$$(6) \qquad y_{it} = \beta_0 + \beta_a a_{it} + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \eta_{it}$$

Note that the symbols now stand for logged: age, capital, labour and productivity. The last variable η could be interpreted simply as the error term, but Oley and Pakes prefer to view it as a productivity shock unforecastable in period t. The estimation is on panel data, thus the variables are now indexed by companies (i) and time (t).

Estimating (6) with OLS is biased, because of two problems. First of all, the expected value of current productivity is conditional on past values of productivity, but also on current inputs, which results in endogeneity problem. What is more, since profits (Π) are a growing function of inputs, then companies with e.g. higher capital require lower productivity in order to maintain their operations (see (3)). In other words, the exit rule (4) is also a source of the so called selection bias for the estimation.

Olley and Pakes try to deal with that problem. First, they suggest inverting (5) to obtain a productivity function:

$$(7) \qquad \omega_{it} = h_{it}(i_{it}, a_{it}, k_{it})$$

Now, the unobservable current productivity is a function h of observable current inputs and characteristics.

Incorporating (7) into (6) makes it impossible to estimate the β_a and β_k parameters, but we can still estimate β_l from the partially linear semiparametric model:

$$(8) \qquad y_{it} = \beta_l l_{it} + \phi_{it}(i_{it}, a_{it}, k_{it}) + \eta_{it}$$

where

$$(9) \qquad \phi_{it}(i_{it}, a_{it}, k_{it}) = \beta_0 + \beta_a a_{it} + \beta_k k_{it} + h_{it}(i_{it}, a_{it}, k_{it})$$

Let us now consider the survival probability. Surviving through to the forthcoming period t+1 is conditional on the future productivity threshold, which depends on future capital and age levels (see (4)). Although Olley and Pakes are not clear about it in their paper, the construction of Bellman's problem for the company (see (3)) and the selection bias they describe indicate that every company should have its own productivity threshold, which is a decreasing function of age and capital. Let us now notice that future capital depends on current capital and investment, while future age depends on current age (see (1)). Productivity changes are a Markov process, so they depend on past productivity and other past information (e.g. data about inputs) as well. Thus we have:

$$(10) \qquad P\{\mathrm{X_{i,t+1}} = 1 | \widetilde{\omega}_{i,t+1}, J_{it}\} = p_{it}(i_{it}, a_{it}, k_{it}) \equiv P_{it}$$

From (6) we know, that output is conditional on age, inputs and the very fact the company has survived. That is why, when we consider (8), we get:

$$(11) \qquad \mathbb{E}\{y_{i,t+1} - \beta_l l_{i,t+1} | a_{i,t+1}, k_{i,t+1}, \mathrm{X_{i,t+1}} = 1\}$$
$$= \beta_0 + \beta_a a_{\mathrm{i,t+1}} + \beta_k k_{\mathrm{i,t+1}} + \mathbb{E}\{\omega_{i,t+1} | \omega_{it}, \mathrm{X_{i,t+1}} = 1\}$$
$$\equiv \beta_a a_{\mathrm{i,t+1}} + \beta_k k_{\mathrm{i,t+1}} + g_{i,t+1}(\widetilde{\omega}_{i,t+1}, \omega_{it})$$

where parameter β_0 has been incorporated into the g function, which is:

$$(12) \qquad g_{i,t+1}(\widetilde{\omega}_{t+1}, \omega_{it}) = \beta_0 + \int_{\widetilde{\omega}_{i,t+1}} \omega_{i,t+1} \frac{F(d\omega_{i,t+1}|\omega_{it})}{\int_{\widetilde{\omega}_{i,t+1}} F(d\omega_{i,t+1}|\omega_{it})}$$

It is possible to invert (10) in a way that enables making the future productivity threshold a function of the probability of surviving and present productivity. This allows us to transform function g into function $\gamma_{i,t+1}(P_{it},\omega_{it})$. Considering this and (7)-(12) we get:

$$(13) \qquad y_{i,t+1} - \beta_l l_{i,t+1} = \beta_a a_{\mathrm{i,t+1}} + \beta_k k_{\mathrm{i,t+1}} + \gamma_{i,t+1}(P_{it}, \omega_{it})$$
$$= \beta_a a_{\mathrm{i,t+1}} + \beta_k k_{\mathrm{i,t+1}} + \gamma_{i,t+1}(P_{it}, h_{it})$$
$$= \beta_a a_{\mathrm{i,t+1}} + \beta_k k_{\mathrm{i,t+1}} + \gamma_{i,t+1}(P_{it}, \phi_{it} - \beta_a a_{\mathrm{it}} - \beta_k k_{\mathrm{it}}) + \eta_{i,t+1} + \xi_{i,t+1}$$

where ξ could be interpreted as innovation. Of course, in practice both ξ and η are unobservable and combine to an error term. The fact that (13) is a two-period expression constitutes the need for the initial estimation of (8).

3. Practical guideline

Parts 1 and 2 of this appendix present the theory of estimating productivity with OPA. It might seem complicated because of the numerous confounded functions of unknown form. However, every function can be approximated by a polynomial, which is a key to facilitating the OPA procedure (as it transforms the estimated problems into a quasi-linear one and enables OLS regressions[4]). Olley and Pakes used fourth order polynomials, but they later admitted that in fact using only third order polynomials where possible does not change the results significantly. In practice, OPA requires three steps of estimation.

In the first step we estimate the production function:

$$y_{it} = \hat{\beta}_l l_{it} + \hat{\phi}_{it}(i_{it}, a_{it}, k_{it}) + \varepsilon_{it} \quad (14)$$

where dashes indicate estimated parameters and ε denotes the error term. Function φ is a third (or fourth) order polynomial with full interaction of variables i_{it}, a_{it} and k_{it}. The obvious effect of this step is obtaining an estimate for the β_l parameter, but receiving the theoretical values of φ_{it} is in fact just as important.

In the second step we estimate survival probability:

$$\mathrm{X_{i,t+1}} = \hat{P}_{it}(i_{it}, a_{it}, k_{it}) + \varepsilon_{i,t+1} \quad (15)$$

where ε again denotes the error term and P is a third (or fourth) order polynomial with full interaction of variables i_{it}, a_{it} and k_{it}. X is a binary dependent variable, whichis why Olley and Pakes used probit estimation for this step. In our calculations we have also utilized a probit model, but logit would also be suitable for this purpose. The estimates of survival probability are necessary for conducting the third and final step of the OPA procedure.

In the last step we carry out an estimation of a model

$$(y_{i,t+1} - \hat{\beta}_l l_{i.t+1}) = \hat{\beta}_a a_{i.t+1} + \hat{\beta}_k k_{i.t+1} + \hat{\gamma}_{i.t+1}\left(\hat{P}_{it}, \hat{\phi}_{it}, a_{\mathrm{it}}, k_{\mathrm{it}}\right) + \varepsilon_{i,t+1} \quad (16)$$

where ε again denotes the error term and γ is a fourth order polynomial with full interaction of variables a_{it} and k_{it} and estimates (theoretical values based on polynomial approximation) of functions P_{it} and φ_{it}, which we know from the first two steps of the estimation. Since variables a and k are in two periods, we get the estimates of β_a and β_k even though they are confounded in function γ. However, this also results in the collinearity problem, which is why Olley and Pakes used kernel estimators to get theoretical values of γ. Nevertheless, since biased estimation of parameters of this function is of minor consequence, we argue that OLS is sufficient, as it results in consistent (although not efficient in general) estimates of parameters β_a and β_k.

Finally, we can use our results to calculate estimated productivity using the expression:

$$\hat{h}_{it} = \hat{\phi}_{it} - \hat{\beta}_a a_{\mathrm{it}} - \hat{\beta}_k k_{\mathrm{it}} \quad (17)$$

where function h_{it} is in fact a polynomial approximation of ω_{it}, which stands for productivity.

[4] Another possibility (tried by Olley and Pakes), instead of approximating the unknown functions with polynomials, is to use kernel density estimators to obtain their expected values. However, the results are comparable, while OLS is much easier in practice. Using KDE results in the unknown functions having forms of mathematical expectations of certain distributions, which can be observed in the notation for function g (see (12)), which is in fact an expected value of a continuous distribution of ω (corrected with β_0 parameter).