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TRANSITION RADIATION EMITTED FROM A SURFACE IRREGULARITY SHAPED AS RECTANGULAR STEP AND RECTANGULAR GROOVE

JERZY H. RUTKOWSKI

University of Łódź, Institute of Physics
ul. Nowotki 149/153, 90-236 Łódź, Poland

ABSTRACT. Transition radiation emitted when a nonrelativistic particle ($\beta=0.2$–0.5) enters a surface irregularity shaped as a rectangular step and a rectangular groove and having the dielectric constant $\varepsilon=1.4$–3.8 has been investigated. The angular distribution of the radiation intensity as well as its dependence on the height of the irregularities, the width of the groove and the distance between the irregularity edge and the entry point of the particle have been calculated on the basis of the theory of Bagiyan and Ter-Mikaelyan.

Transition radiation (TR) emitted from a rough surface has not been examined thoroughly yet. Most of the empirical facts follow from the experiments (e.g. [1–4]) performed on smooth surface being, however, rough to some extent due to the preparation technology. The degree of this roughness as well as the optical properties of the materials used as targets were not well determined. Besides, the results of these experiments were not always reproducible and the conclusions following could only be of a qualitative character. In particular, these experiments showed that the polarization degree of the radiation detected decreased as the roughness of the surface increased. This occurred as a result of the appearance of radiation in the plane perpendicular to the emission plane*, which does not exist on a perfectly smooth surface.

The TR on surfaces with a well known degree of roughness has been investigated by Harutunian *et al.* [5]. It is the only experiment of a quantitative character.

The theory of the TR emitted from a rough surface has been proposed by Bagiyan and Ter-Mikaelyan [6–8]. In general, the theory describes the radiation from a surface with irregularities of arbitrary shape. Besides, the authors give

* An emission plane is a plane defined by the normal to the surface and the direction of the emitted photon.

analytical expressions for several particular shapes of these irregularities, such as: isolated irregularities (e.g. rectangular groove), irregularities with periodical distribution on the surface (e.g. a set of parallel grooves shaped rectangularly or sinusoidally), a set of irregularities with statistical distribution of their heights and widths randomly distributed on the surface, and others. By analyzing them the authors determine the properties of the TR and their connection with the parameters describing both the physical and geometrical properties of the irregularities.

In the above theory, two parameters are introduced. One is the so-called coherency length l_c, introduced earlier by Frank [9]. It is the distance between two points of a particle trajectory in which the emitted radiation with wavelength λ has phases differing by π. The coherency length may be expressed by the formula:

$$l_c = \frac{\bar{\lambda}\beta\sqrt{\varepsilon_0}}{1-\beta\sqrt{\varepsilon_0}\cos\theta} \tag{1}$$

where $\bar{\lambda}=\lambda/2\pi$, $\beta=v/c$, v – particle velocity, c – light velocity in vacuum, $\varepsilon_0=0.5(\varepsilon_1+\varepsilon_2)$, ε_1, ε_2 – dielectric constants, θ – emission angle of the photon measured from the z-axis ($0\leqslant\theta\leqslant\pi$) (see also Fig. 1). The parameter l_c describes the influence of irregularities occurring along the particle trajectory on the emission process. When investigating the influence of an oxide layer on the TR emitted from a smooth metallic boundary, Pafomov and Frank [10] showed that when the thickness of the layer is considerably smaller than the coherence length, the layer does not affect the TR (the particle does not "feel" the irregularity). This conclusion has been confirmed by Ter-Mikaelyan [11] when examining various processes of high energy physics.

The influence on the radiation of the irregularities distributed perpendicularly to the particle trajectory is described by the other parameter of the theory, ρ. It defines the radius of the electromagnetic field of frequency ω accompanying the particle and is expressed by the formula

$$\rho = \frac{\bar{\lambda}\beta\sqrt{\varepsilon_0}}{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}}, \tag{2}$$

where φ is an azimuthal angle of the emitted photon measured from the x-axis ($0\leqslant\varphi\leqslant 2\pi$) (Fig. 1).

We may expect that when the distance between the irregularity and the particle trajectory is considerably larger than ρ or the irregularity dimension is much bigger then ρ, then the irregularity will not affect the radiation. Moreover, when the irregularity dimension is considerably smaller than ρ, the irregularity would not affect the TR.

The role of the parameters l_e and ρ as well as the irregularity dimensions in the radiation process are presented in more detail by Bagiyan in [8].

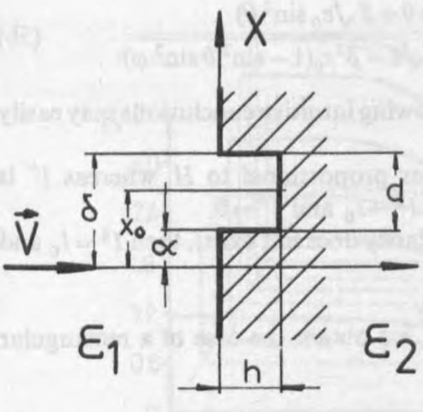

Fig. 1. Geometry in the case of irregularity shaped as rectangular groove. When $d \to \infty$, the case of irregularity shaped as rectangular step is obtained.

Here, we consider the TR emitted from a surface shaped as a rectangular groove and a rectangular step. The choice of irregularities so simple in shape is due to methodical reasons because in these cases all quantities of the theory have clear physical or geometrical interpretation. All the quantities having length in their dimensions are expressed in the units of l_c or ρ, which give the results obtained a deeper physical sense than in the case of the previous calculations [12].

It is convenient to write the theoretical equations for a groove in the form

$$W^{\parallel}=\frac{I^{\parallel}}{I_0}=1+(V_R-V_{R+D})\sin H+[G_{R,R+D}+(V_R-V_{R+D})]\sin^2\tfrac{1}{2}H \qquad \text{(3a)}$$

$$W^{\perp}=\frac{I^{\perp}}{I_0}=Ae^{-2|R|}\{1+e^{-D}[e^{-D}-2\cos(\gamma D)]\}\sin^2\tfrac{1}{2}H \qquad \text{(3b)}$$

where $I^{\parallel}, I^{\perp}, I_0$ – radiation energy emitted in the frequency interval $d\omega$ and into the solid angle interval $d\Omega$ in the emission plane, in the plane perpendicular to the emission plane, and from the smooth surface, respectively; $H=h/l_c$, $R=\alpha/\rho$, $D=d/\rho$, h – groove height, d – groove width, α – distance between the particle trajectory and the groove edge,

$$V_{R,R+D}=[F\cos(|R,R+D|\gamma)+\sin(|R,R+D|\gamma)]\,e^{-|R,R+D|}, \qquad \text{(4a)}$$

$$G_{R,R+D}=(\mu_R\mp\mu_{R+D})(2+\mu_R\mu_{R+D}), \qquad \text{(4b)}$$

the signs: "+" and "−" refer to the cases when the particle enters and does not enter the groove, respectively, and

$$\gamma=\frac{\beta\sqrt{\varepsilon_0}\cos\varphi\sin\theta}{\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}}, \qquad \text{(4c)}$$

$$A=\frac{\sin^2\varphi(1-\beta^2\varepsilon_0\cos^2\theta)^2}{\sin^2\theta(1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta)^2[1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)]}. \qquad \text{(4d)}$$

In the above equations we have

$$\mu_{R,R+D}=[F\sin(|R,R+D|\gamma)-\cos(|R,R+D|\gamma)]\,e^{-|R,R+D|}, \qquad \text{(5a)}$$

$$F=\frac{\cos\varphi(1-\beta^2\varepsilon_0)(\cos\theta+\beta\sqrt{\varepsilon_0}\sin^2\theta)}{\sin\theta(1-\beta^2\varepsilon_0-\beta\sqrt{\varepsilon_0}\cos\theta)^2\sqrt{1-\beta^2\varepsilon_0(1-\sin^2\theta\sin^2\varphi)}}\,. \tag{5b}$$

From the expressions (3) and (4) the following intuitive conclusions may easily be obtained:

a) When $H \ll 1$ then $I^{\|} > I_0$ by a factor proportional to H, whereas $I^{\perp}$ is proportional to $(\frac{1}{2}H)^2$. When $H \to 0$, then $I^{\|} \to I_0$ and $I^{\perp} \to 0$.

b) If $d=0$ or $d=\infty$ (i.e. when the irregularity does not exist), then $I^{\|}=I_0$ and $I^{\perp}=0$.

c) When $R \gg 1$, then $I^{\|} \simeq I_0$ and $I^{\perp} \simeq 0$.

Putting $d=\infty$ in the equations (3)–(5), we obtain the case of a rectangular step. Then we have

$$W=1+V_R\sin H+[2\mu_R+(1+F^2)e^{-2|R|}]\sin^2\tfrac{1}{2}H \tag{6a}$$

$$W=Ae^{-2|R|}\sin^2\tfrac{1}{2}H\,. \tag{6b}$$

Calculations based on the equations (3)–(6) have been performed for nonrelativistic particles with velocities $\beta=0.2$–0.5, entering the medium with dielectric constant $\varepsilon_2=1.4$–3.8 from vacuum ($\varepsilon_1=1$). The weak dependence of the obtained results on the particle velocity and the dielectric constant of the medium has been observed.

The intensity of TR for the irregularity shaped as a step is greater than for the irregularity having the form of a groove under the same physical conditions. This is due to the destructive interference of the radiation emitted from both groove edges. If it is not specially emphasized below, the results to be presented will refer to the irregularity in the form of a step.

The dependence of TR intensity on the magnitude of the irregularities. If the angles θ, φ and the parameter R are constant and the parameter H changes, then both components $W^{\|}$ and $W^{\perp}$ have their maxima in the neighbourhood of $H=1$. If H is constant and R changes, then the intensity of the perpendicular component $W^{\perp}$ decreases when R increases. The parallel component $W^{\|}$ shows no such regularity of its run. There may exist some range of the parameter H values such that the component $W^{\|}$ runs similarly to $W^{\perp}$, when R changes. Outside this range the intensity of the parallel component can reach its minimum. The quickest changes of both TR components occur when the parameter $H \simeq 1$ and/or $R \simeq 1$. The component $W^{\|}$ attains values both greater and smaller than 1, i.e. $I^{\|}$ reaches values both greater and smaller than the intensity of TR emitted from the smooth surface. For some set of values of the parameters considered and for some angles θ and φ, the component $W^{\perp}$ is greater than the component $W^{\|}$, i.e. the polarization degree of the radiation becomes negative. In Fig. 2 we present the numerical calculations concerning the behaviour of the components described.

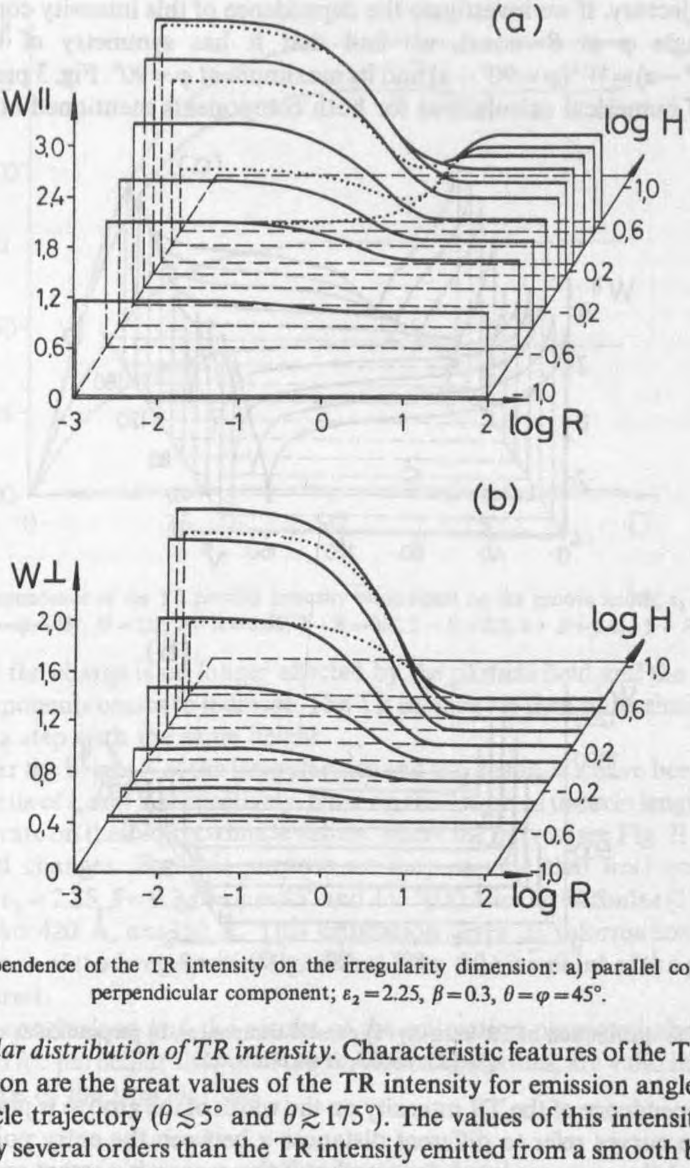

Fig. 2. Dependence of the TR intensity on the irregularity dimension: a) parallel component, b) perpendicular component; $\varepsilon_2 = 2.25$, $\beta = 0.3$, $\theta = \varphi = 45°$.

Angular distribution of TR intensity. Characteristic features of the TR angular distribution are the great values of the TR intensity for emission angles close to the particle trajectory ($\theta \lesssim 5°$ and $\theta \gtrsim 175°$). The values of this intensity may be greater by several orders than the TR intensity emitted from a smooth boundary. This refers to both the components and takes place for all azimuthal angles φ. If $\varphi \lesssim 90°$ and the angle θ increases from 0° to 180°, $W^{\parallel}$ decreases reaching its deep minimum for $\theta \lesssim 20°$ and then increases up to considerable values for angles θ close to 180°. The TR intensity may be both greater and smaller than the TR intensity emitted from the smooth boundary. As for the component $W^{\perp}$, its intensity depends slightly on the angle θ, except for emission angles close to the particle trajectory. If we investigate the dependence of this intensity component on the angle φ at $\theta=\text{const}$, we find that it has symmetry of the type $W^{\perp}(\varphi=90°-\alpha)=W^{\perp}(\varphi=90°+\alpha)$ and its maximum at $\varphi=90°$. Fig. 3 presents an example of numerical calculations for both components mentioned above.

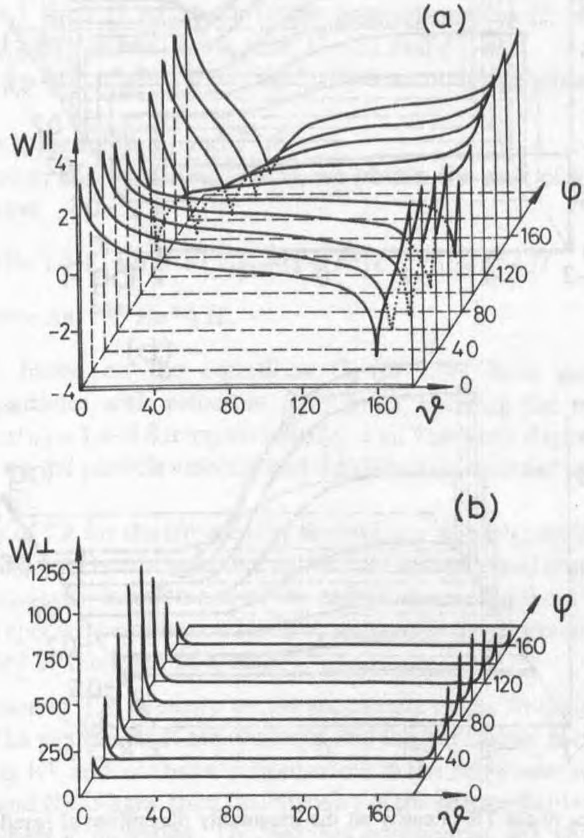

Fig. 3. Angular distribution of TR intensity: a) parallel component, b) perpendicular component; $\varepsilon_2=2.25$, $\beta=0.3$, $H=1.0$, $R=0.1$.

The dependence of the TR intensity on the width of the groove is presented in Fig. 4. The curves refer to different distances α between the entry point of the particle and the groove edge. It is seen that if the groove is narrow enough, the intensity of the parallel component $I^{\parallel}$ differs a little from the TR intensity emitted from the smooth surface, whereas the perpendicular component of the TR intensity (not shown in Fig. 4) almost vanishes ($I^{\perp}\simeq 0$). Along with the increase in groove width, the intensity of both components increases. When the groove width is large enough ($\alpha+d>\rho$), the edge more distant from the entry

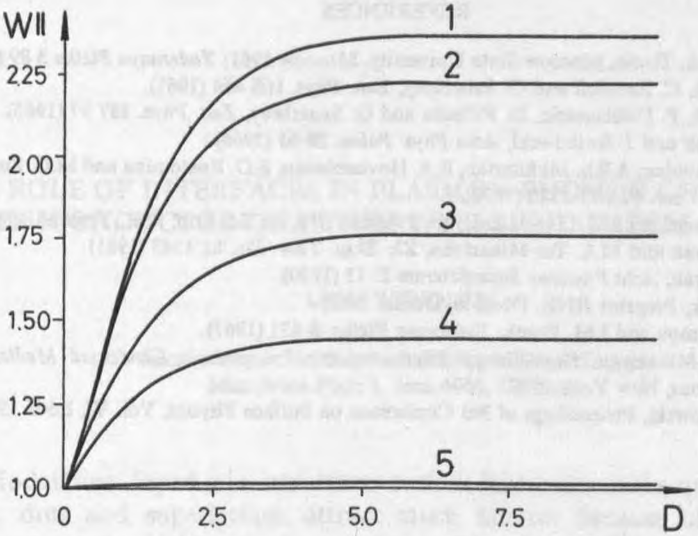

Fig. 4. Dependence of the TR parallel intensity component on the groove width; $\varepsilon_2=2.25$, $\beta=0.3$, $\theta=\varphi=45°$, $H=1.0$; 1 – $R=0.01$, 2 – $R=0.1$, 3 – $R=0.5$, 4 – $R=1.0$, , 5 – $R=5.0$.

point of the charge is no longer affected by the particle field and the intensity of the components ceases to increase. The TR intensity is then quite similar as in the case of a step with the same height.

So far the height h of the irregularities and the distance α have been expressed in the units of l_e and ρ, respectively. Now we shall express them in length units and concentrate on their approximate values, where the curves (see Fig. 2) are affected by rapid changes. For this purpose we may assume that $h \simeq l$ and $\alpha \simeq d \simeq \rho$. Putting $\varepsilon_2=2.25$, $\beta=0.3$, $\theta=\varphi=45°$ and $\lambda=5000$ Å in the formulae (1) and (2), we obtain $h \simeq 420$ Å, $\alpha \simeq 330$ Å. This estimation gives us information about the dimensions of the irregularities significant from the viewpoint of the processes of our interest.

The conclusions and the results of the estimation presented above, though obtained for particular shapes of the surface irregularities, are valid in general for any shape of the irregularities.

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