A C T A U N I V E R S I T A T I S L O D Z I E N S I S
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DYNAMICS OF THE REAL ESTATE PRICES IN THE LIGHT OF THE CATASTROPHE THEORY

Abstract. The paper describes application of catastrophe theory for analysis of trends of real estate prices in Poznan. It turns out that the evolution of the real estate market is comprised of two main processes: long-term evolution in the area of a non-degenerate stability and discontinuous, rapid changes in the area of a degenerate stability. In the macro scale, the construction and developing branch contributes largely to the Gross Domestic Product affecting overall economic environment. In the micro scale, however, the knowledge about future price trends may help to decide whether or not to buy or sell the property.
Keywords: catastrophe theory, real estate market, dynamics.

I. INTRODUCTION

The paper deals with the real estate market seen as a dynamic system in perpetual seek for equilibrium in order to explain observed changes in housing prices. The system is immersed in a multidimensional phase space build up by several independent variables that contribute to an overall macroeconomic environment, and a dependent variable related directly to the housing price. A set of points within the state space visited by the system establishes the evolution path. Long-term changes in the real estate prices suggest that otherwise regular evolution of such a system could be rarely disturbed by violent instabilities of structural origin. This phenomenon is referred to as the catastrophe, according to the theory first described by R. Thom (1976), in which small changes in control parameters (independent variables) give rise to sudden, quasi-discrete change in the system state (dependent variables).

Catastrophe theory has been successfully applicable in various fields of contemporary science, including economics. For example, Zeeman (1974), and recently Barunik and Vosrvda (2009) used the stochastic cusp catastrophe model to explain the stock market crashes, Jakimowicz (2010) introduced the

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catastrophe into the theory of a business cycle, whereas Rosser (2007) made an extensive review of other applications in economics.

Catastrophe turns out to be highly non-stationary process occurring at very specific points onto the evolution path referred to as the critical points. In these points, previous evolution path of the system vanishes exhausting all possibilities of further development, and the system begins to follow a new path leading to a newly emerged stable state (Okninski 1990). What is also important, in the vicinity of critical points system becomes extremely sensitive to external stimuli, and hence even small fluctuations in control variables might eventually trigger large changes in the system state.

II. CATASTROPHE THEORY

As mentioned in previous paragraph, catastrophe theory deals with the systems immersed in a multidimensional phase space, in which a potential function exists:

$$V\left(\vec{y},\vec{x}\right)=V\left(\left(y_1,...y_n\right),\left(x_1,...x_k\right)\right) \tag{1}$$

where: $(y_1,\ldots y_n)$ – describes a set of state variables, while $(x_1,\ldots x_k)$ – a set of control parameters. The system evolves towards an equilibrium state driven by potential forces defined as:

$$f_i=-\frac{\partial V\left(\vec{y},\vec{x}\right)}{\partial y_i};i=1...n \tag{2}$$

The balancing process continues until a stationary state is reached, in which all potential forces become zero:

$$\frac{\partial V\left(\vec{y},\vec{x}\right)}{\partial y_i}=0;i=1...n \tag{3}$$

All the roots of equation (3) form an equilibrium surface R. However, the surface R could be divided into two subspaces taking into account the second derivative of the potential:

$$\Omega = \left\{ \left(\vec{y}_R, \overrightarrow{x_R} \right) : \frac{\partial V}{\partial y_i} = 0; \frac{\partial^2 V}{\partial y_i^2} \neq 0; i = 1...n \right\} \tag{4}$$

$$\Sigma = \left\{ \left(\vec{y}_R, \overrightarrow{x_R} \right) : \frac{\partial V}{\partial y_i} = 0; \frac{\partial^2 V}{\partial y_i^2} = 0; i = 1..n \right\} \tag{5}$$

Set Ω contains non-degenerate equilibrium states, while set Σ contains degenerate equilibrium states. Nonemptiness of Σ is a prerequisite for the catastrophe otherwise the system is always structurally stable. The border between those two sets projected onto the control space consists of critical points of the system evolution.

Dynamics of the real estate market is studied using the cusp catastrophe model. The model relies on a single state variable y, and two control variables: α and β (asymmetry and bifurcation parameters, respectively) expressed explicitly in the form:

$$V(y, \alpha, \beta) = \frac{1}{4} y^4 + \frac{1}{2} \alpha y^2 + \beta y \tag{6}$$

Each control variable is canonical one, which means that it can be actually composed as a linear combination of various observables (time-series of economic data):

$$\begin{cases} \alpha = \alpha_0 + \sum_{i=1}^{n} \alpha_i x_i \\ \beta = \beta_0 + \sum_{i=1}^{n} \beta_i x_i \end{cases} \tag{7}$$

where: x_i – the i-th observable (independent variable), while α_0, and β_0 – free terms.

Equilibrium surface R contains roots of the equation:

$$R = \left\{ (y, \alpha, \beta); \frac{\partial V}{\partial y} = y^3 + \alpha y + \beta = 0 \right\} \tag{8}$$

Subset of degenerate equilibrium states is given by:

$$\Sigma = R \cap \left\{ (y, \alpha, \beta); \frac{\partial^2 V}{\partial y^2} = 3y^2 + \alpha = 0 \right\} \tag{9}$$

Finally, critical points are those which meet the equation:

$$B = \left\{ (\alpha, \beta); 4\alpha^3 + 27\beta^2 = 0 \right\} \tag{10}$$

On the whole, the evolution path can be thought of as a curve within the state space, driven straight onto the equilibrium surface or in the nearby space. As long as it follows non-degenerate states, the system evolves in a regular manner, otherwise alternative paths could appear suddenly.

III. DATA DESCRIPTION

In general, the economic data used throughout this study can be thought of as samples of processes which are random to a certain degree. However, catastrophe theory is based on deterministic equations, and hence it is necessary to get rid of that randomness. To this end, various methods of averaging were used: arithmetic mean, median, and others, which are described further in the text.

Changes in real estate prices were studied by fitting the cusp catastrophe model to data coming from 11 794 sale prices of residential apartments settled in Poznan (Poland) in the period between January 2001 and September 2011, maintained by the City Administration Office in the Register of Prices and Values. Fig. 1 shows the plots of the quarterly-averaged (per square meter) prices: transaction price, real transaction price, and the median price as a function of time.

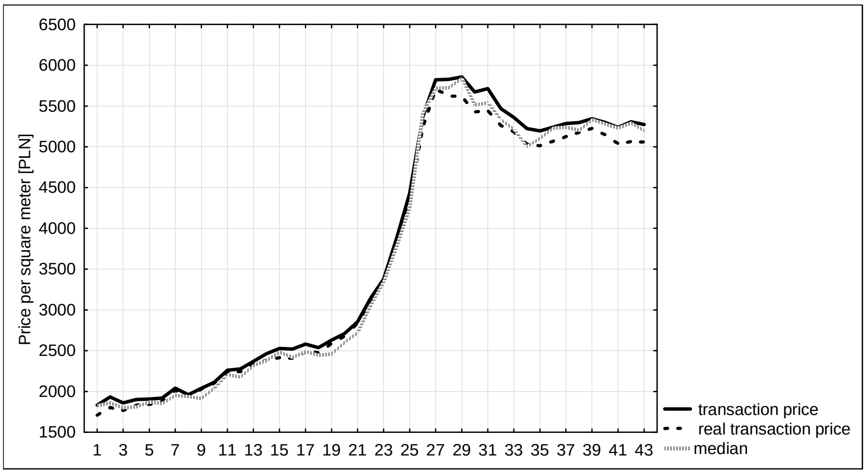

Figure 1. Plots of the quarterly-averaged (per square meter) prices: transaction price, real transaction price, and the median price as a function of time

Source: own study.

The most important results of preliminary linear fit, and the rates of price changes are summarized in Table 1.

Table 1. Results of linear fit, and the rates of price changes from data seen in Fig. 1.

Time interval	Simple regression equation	R^2	Averaged transaction price [$1m^2$]	Price changes [month]	Price changes [year]
Whole time interwal (Q1 2001 – Q3 2011)	$y = 109.28\,x + 1202.8$	0.84	3606.86 zł/m^2	3.03 %	36.36 %
Subinterval I (Q1 2001 – Q3 2006)	$y = 61.49\,x + 1543.8$	0.92	2281.65 zł/m^2	2.69 %	32.28 %
Subinterval II (Q4 2006 – Q4 2007)	$y = 495.77\,x + 34595$	0.90	4946.85 zł/m^2	10.02 %	120.24 %
Subinterval III (Q1 2008 – Q3 2011)	$y = -28.833\,x + 5422.9$	0.52	3606.86 zł/m^2	–0.56 %	6.72 %

Source: own study.

From January 2001 to September 2006 residential apartments prices increased slowly and steadily, somewhere 2.7 percent by month. However, the prices increased from October 2006 to December 2007 the most in the studied decade rising monthly by 10 percent. This behavior could be indicative of structural instability in the market. Later on, the prices asymptotically decreased following a downward trend, – 0.6 percent by month, observed from January 2008 to September 2011. As such, the steep increase can be thought of as a critical point in the evolution of the studied market.

Fig. 2. presents histograms of residential apartments prices computed for the whole time interval (Fig. 2A) and the three above-mentioned subintervals (Fig. 2B-D). Note that the first histogram has a bimodal character corresponding to at least two evolution paths of the system in its phase space. Such a possibility is supported by the fact that sub-interval histograms are unimodal as if current evolution path was gradually replaced by another. In that sense, Fig. 2A exhibits three equilibrium states of the system: two stable equilibria (attractors) at the edges and one unstable equilibrium (repeller) in the middle.

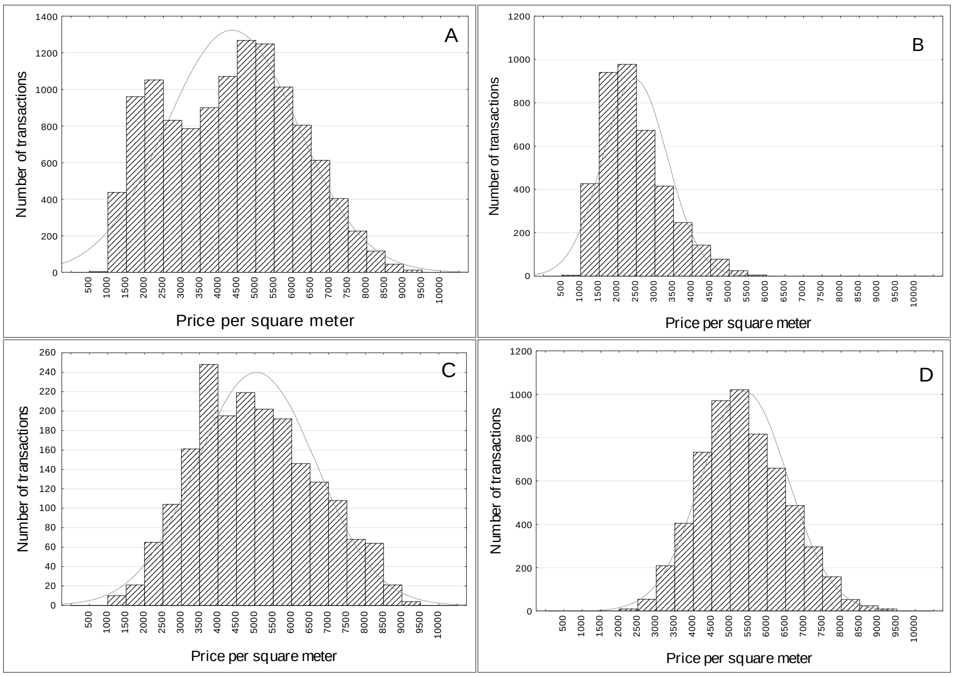

Figure 2. Histograms of residential apartments prices computed for the whole time interval (Figure 2A) and the three subintervals of regular and instable evolution (Figure 2B-D).

Source: own study.

IV. RESULTS

Macroeconomic data used for construction of canonical variables in the cusp catastrophe model are described in detail in Table 2. Given Eq. (8), numerical fit procedure iteratively steps to find out the best relation between control variables α and β, and a state variable y in the sense of the minimum least square deviations. Hence, the algorithm tunes the control variables by scaling the contribution of each observable, but unfortunately fails to report those results as output data.

Property price best reflects various processes on the market and in surrounding environment, hence monthly-averaged real transaction price per square meter of residential apartment is to be chosen as a dependent variable in the cusp catastrophe model. Other parameters can be used as well, for example: property value, rent prices, return rates, house price index etc., but most surveys rely on transaction prices solely (see: Dittman (2013), Foryś (2011), Krajewska (2013), Kuryj-Wysocka and Wisniewski (2013), Renigier-Biłozor and Wisniewski (2012), Tanaś (2013)).

Table 2. Macroeconomic data used for construction of canonical variables in the cusp catastrophe model

Symbol	Variable	Description
CTR	Real transaction price	Average apartment price per 1 m^2, given in quarterly intervals and adjusted for inflation
GDP	Gross Domestic Product	Fixed prices are average annual prices for the previous year. The corresponding period in the previous year = 100.
IR	Interest rates	Average interest rates quoted by the National Bank of Poland; rediscount rate, lombard rate, reference rate (at the end of period)
MOD	Residential construction sector	Number of new dwellings, in '000
UN	Total registered unemployment	At the end of period
ILS	Economic welfare	Gross nominal monthly salary. The corresponding period in the previous year = 100.
USPL	USD/PLN	Relations between USD and PLN
WBD	Total gross value added	Fixed prices are average annual prices for the previous year. The corresponding period in the previous year = 100.

Source: own study.

The development of the housing market is correlated with economic growth. For this reason, control variables α and β were built based on the key measure of national income - Gross Domestic Product. On the other hand, investors need ample access to financing sources. Changes in mortgage rates reflect the country's macroeconomic situation being affected by the interest rates taken by the National Bank of Poland. Therefore, interest rates of the central bank (calculated as an average of the reference rate, the lombard rate and the rediscount rate) are added to control parameters together with the number of new dwellings and the unemployment rate. Apart from that, however, such macroeconomic factors as: gross nominal monthly salary, USD/PLN exchange rate, and total gross value added are also suggested as control parameters. Fig. 3 plots time-dependent changes in the above macroeconomic data in order to compare them with average transaction prices per square meter, and to illustrate the correlations between the analyzed variables. Each independent variable is found to oscillate with specific repetition period.

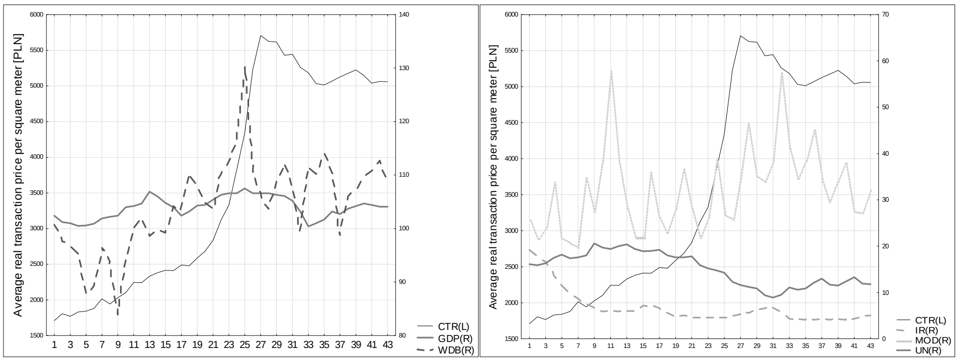

Figure 3. Changes in macroeconomic data used for construction of canonical variables in the cusp model plotted together with changes in real transaction prices

Source: own study

The usability of the cusp model for analysis of real estate market was assessed by comparison of its results with those obtained for more popular models: linear and logistics. The following metrics were used to evaluate the fit quality: pseudo-R^2 (cusp model (Cobb 1998)), coefficient of determination R^2 (linear and logistic models), likelihood ratio test (logLik), Akaike information criterion (AIC) and Bayesian information criterion (BIC). Unlike the R^2 which still holds positive, pseudo-R^2 can take on negative values as well. In turn, the better fit corresponds to higher value of logLik, and lower values of AIC and BIC.

The results presented in Table 3 confirm that the cusp model is superior mostly for its logLik, AIC, and BIC values. Comparison of pseudo-R^2 value for the cusp model (0.987) with R^2 values for logistic (0.9819) and linear (0.9617) models reveals an insignificant advantage of the cusp model.

Table 3. Results of the numerical fit procedure

Model	R^2	Cobb's pseudo R^2	logLik	AIC	BIC
Linear model	0.9617	x	–304.7882	627.5364	643.3972
Logistic model	0.9819	x	–288.7225	597.4451	615.0571
Cusp model	x	0.9870	41.8092	–61.6184	–42.2452

Source: own study.

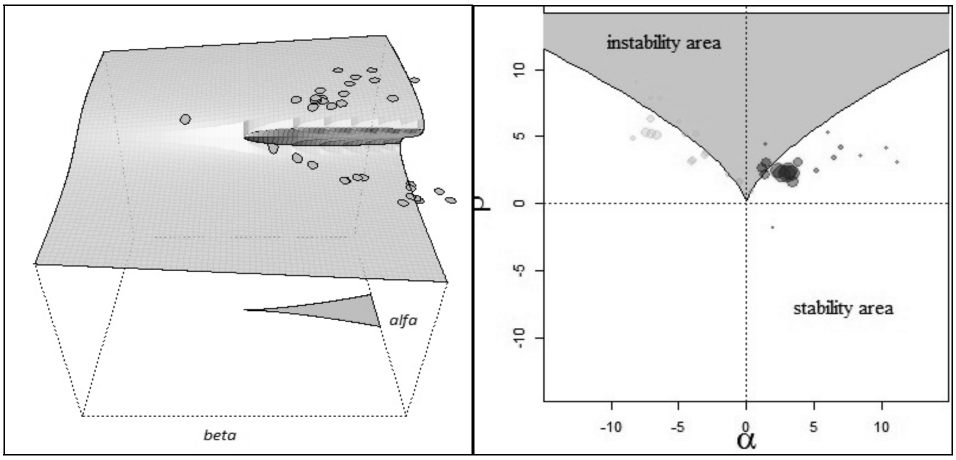

Figure 4. Evolution paths of the studied system in the phase space: (left) 3-dimensionl view, (right) 2-dimensional projection onto the control plane.

Source: own study.

As mentioned previously, time-series of macroeconomic data are blended together, and contribute to control variables α and β, which in turn best correlate with the state variable y and assumed equilibrium plane. Now, it is possible to draw a two-dimensional curve within three-dimensional phase space by combining the single values of α, β and y for each point in time, which is shown in Fig. 4. It can be seen that the evolution of real estate market combines two intermittent processes: long-term evolution, which occurs mostly in the non-degenerated equilibrium states (white area in Fig. 4B), and short-term, sudden changes triggered on the verge of instability zone (shaded area in Fig. 4B).

V. CONCLUSIONS

The obtained results confirm in general that the real estate market can be treated as a dynamic system. The results also confirm the presence of instability periods in its history. Unlike regular, long-term evolution over the equilibrium surface, the short-term catastrophic influence of control parameters on the system gives rise to discontinuous transition, i.e. sudden changes between different equilibrium states of the system.

Although the actual point at which the catastrophic change occurs can be hardly predicted, changes can be forecasted in a wider perspective. In such a case, this theory can be a useful tool for developing robust indicators of a catastrophe. Graphic representation of the evolution path casts light on possible evolution scenarios.

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DYNAMIKA CEN NIERUCHOMOŚCI BADANA NA GRUNCIE TEORII KATASTROF

W pracy przedstawiono dane weryfikujące praktyczną stosowalność teorii katastrof do analizy przebiegów czasowych cen nieruchomości na przykładzie danych zebranych na lokalnym rynku nieruchomości w Poznaniu. Wyniki pokazują, że ewolucja rynku nieruchomości jest procesem wieloskalowym, na który składają się: długofalowa ewolucja w obszarze niezdegenerowanej stabilności strukturalnej układu przerywana gwałtownymi zmianami wywołanymi przechodzeniem przez obszary stabilności zdegenerowanej. Zagadnienie to ma fundamentalne znaczenie zarówno w skali makro, gdyż budownictwo mieszkaniowe generuje istotny wkład do PKB, jak też w skali mikro, gdyż pozwala podejmować decyzje o kupnie/sprzedaży nieruchomości w sposób bardziej świadomy.