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VERYFICATION OF LOCATION PROBLEM IN ECONOMIC RESEARCH

Abstract. In economic research, problems with single-sample locations or estimation of differences between two-sample locations are commonly tested by experimental economists. The Wilcoxon test is usually used for single-sample locations and the Wilcoxon–Mann–Whitney test is often used when there are two-sample locations. Unfortunately those tests have some disadvantages, such as robustness against assumptions or weak efficiency. In this paper, some lesser-known procedures, which allow avoidance of those problems, will be presented. The methods considered will be illustrated by using data analysis from the real-estate market as an example.

Keywords: location problem, data analysis, real-estate market.

I. INTRODUCTION

Our research area involves, among other things, methods for both single-sample and two-sample location problems. We are particularly interested in nonparametric methods that are distribution-free. This is due to the fact that in practical analysis, it often occurs that sample sizes are too small for standard parametric methods (i.e. *t*-test) to be applicable, or that the data are measurable on an ordinal scale, or that other assumptions do not hold [Bandyopadhyay U. *et al.* 2007]. In such situations, nonparametric tests are useful for analysis.

In scientific literature, it is possible to find many nonparametric methods for testing location problems. But only a few procedures are usually used in practical research, including a sign test or Wilcoxon test for single-sample location problems and Wilcoxon–Mann–Whitney test or robust rank ordered test for two-sample location problems. These procedures present some disadvantages associated with robustness against assumptions or low efficiency.

For example, the Wilcoxon signed rank test (W) is an adequate test in situations where a distribution is symmetric. If the underlying distribution is

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asymmetric, then *W*-test is no longer distribution-free and may not maintain its nominal size. The problem with the *W*-test is that even a true null hypothesis may be rejected because of the skewness of the distribution. The sign test (*S*) is a valid test for situations with a skewness of distribution. However, if the assumed distribution is symmetric, then the *W*-test will be more efficient for testing the location than the *S*-test [Bandyopadhyay *et al.,* 2007].

The tests to compare two populations also have many drawbacks. A disadvantage of the Wilcoxon-Mann-Whitney test is that, on the one hand, it too frequently leads to rejection of a true null hypothesis of equal central tendencies. But on the other hand, it is excessively unlikely to reject the null hypothesis. This means that its performance varies greatly, depending on the characteristics of the underlying population distributions, which are typically not known to the researcher. The robust rank-order test is much less sensitive to the characteristics of distribution, and is better than the Wilcoxon-Mann-Whitney test when the sample size is small or very large. However, this test is too likely to give false positive results for medium-sized samples [Feltovich,2003].

The purpose of our research is to examine the robustness of procedures for both single-sample location problems and two-sample location problems. We are particularly interested in such methods which are appropriate for analysing the real estate market, because there is often a problem with outliers or lack of information about basic characteristics of distribution. In this paper we present only two less-known procedures for analysis of one sample location problem, which allows the avoidance of the difficulties described above.

These procedures are two adaptive tests. One of them is a probabilistic approach (*P*), which is a combination of the *S*-test and the *W*-test according to evidence of symmetry provided by the *p-value* from the triples test for symmetry [Bandyopadhyay *et al.,* 2007]. The second one is a deterministic approach (*D*) based on calculating a measure of symmetry and using this as a basis for choosing between the *S*-test and the *W*-test [Bandyopadhyay *et al*., 2007].

The presented methods will be illustrated through the analysis of real estate prices. The case of two-sample locations will be presented in a subsequent publication.

II. ADAPTIVE TESTS

Denote a random sample from a continuous population by $X_1, ..., X_n$. The cumulative distribution function is $F(x - \theta)$ where θ is an unknown median of population. A considered problem is to test a null hypothesis H_0: $\theta = \theta_0$ against some composite alternative hypothesis H_1. Furthermore, let $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(2)}$ be the order of statistics for this random sample $X_1, ..., X_n$.

If we assume that the distribution is symmetric, then the *W*-test is the most appropriate test for the above null hypothesis. On the other hand, if we assume that the underlying distribution is asymmetric, then the *S*-test is more efficient for testing the location than the *W*-test. In practical analysis it often appears that there is no information about the skewness of the distribution. Some mixed procedures of both mentioned tests are presented in the literature for such situations. These combined methods are proposed for obtaining a reasonable power while maintaining the nominal significance level. Each adaptive procedure uses in its first stage some measures or test of symmetry as a preliminary test, and uses that as a basis for choosing between the *W*-test and the *S*-test [Bandyopadhyay *et al.,* 2007, Baklizi 2005].

The main aim of this paper is to present two adaptive procedures for testing the null hypothesis that is formulated above.

The first one, the procedure (*P*), has a probabilistic approach. It is a combination of the *S*-test or the *W*-test with a triples test for symmetry. The choice of test (*W*-test or *S*-test) depends on the *p–value* from the triples test for symmetry. The *p-value,* also known as observed significance level or critical significance level, is a probability of getting a value of the test statistic that is as extreme or more extreme than observed value of the test statistic. This probability is computed on the basis that the null hypothesis is correct. Moreover, all tests are performed at significance level α, which is pre-assumed probability of the rejection of a true hypothesis [Maddala 2001].

The triples test for symmetry is an asymptotically distribution-free procedure to verify the null hypothesis of the symmetry of the underlying population of θ, against the alternative hypothesis that it is asymmetry. This procedure is based on the *U*-statistic estimator $\hat{\eta}$ [Bandyopadhyay*et al.,* 2007, Baklizi 2005, Kochar, 1992]. Assume that

$$h(x_1, x_2, x_3) = \frac{1}{3[sign(x_1 + x_2 - 2x_3) + sign(x_1 + x_3 - 2x_2) + sign(x_2 + x_3 - 2x_1)]'}$$

where $sign(x) = -1, 0, 1$ according as $z <, =, > 0$. Furthermore, assume that

$$\hat{\eta} = \frac{1}{\binom{n}{3}} \sum_{i<j<k} h(X_i, X_j, X_k).$$

The triples test statistic is of the form

$$V = \frac{\sqrt{n}\hat{\eta}}{\hat{\sigma}},$$

where

$$\hat{\sigma}^2 = n\binom{n}{3}^{-1}\sum_{c=1}^{3}\binom{3}{c}\binom{n-3}{3-c}\hat{\zeta}_c,$$

$$\hat{\zeta}_1 = \frac{1}{n}\sum_{i=1}^{n}(\hat{h}_1(X_1) - \hat{\eta}^2),$$

$$\hat{\zeta}_2 = \frac{1}{\binom{n}{2}}\sum_{j<k}^{n}(\hat{h}_2(X_j, X_k) - \hat{\eta})^2,$$

$$\hat{\zeta}_3 = \frac{1}{9} - \hat{\eta}^2,$$

$$\hat{h}_1(X_i) = \frac{1}{\binom{n-1}{2}}\sum_{j<k,\, j\neq i,\, i\neq k} h(X_i, X_j, X_k),$$

$$\hat{h}_2(X_j, X_k) = \frac{1}{n-2}\sum_{j\neq i,\, i\neq k} h(X_i, X_j, X_k).$$

If then reject the null hypothesis of symmetry. $\frac{U_\alpha}{2}$ is the upper quantile of the standard normal distribution. Proceedings in the procedure (P) are as follows: Let p represent the *p*-value associated with the observed $\hat{\eta}$. The the *p-value* can be considered as the amount of evidence against symmetry of the distribution present in the data. Small values of *p* are evidence of asymmetry [Bandyopadhyay *et al.,* 2007, Baklizi 2005]. If $W > w_\alpha^+$ then reject the null hypothesis H_0 with the probability *p* and if $S > s_\alpha^+$ then reject the null hypothesis with the probability (1 – *p*). Equivalently, do not reject the null hypothesis with the probability *p* if $W \leq w_\alpha^+$ and with probability (1 – *p*) if $S \leq s_\alpha^+$, where w_α^+ and s_α^+ are the upper *α* – critical values from the *W*-test and the *S*-test respectively. This procedure is robust and has high power compared to the deterministic approach (*D*) [Bandyopadhyay *et al.,* 2007].

The second procedure (*D*) has a deterministic approach. It is a combination of a preliminary test based on a simple measure of symmetry with the *S*-test and the *W*-test. The preliminary test is of the form

$$Q = \frac{X_{(n)} - 2\overline{X} + X_{(1)}}{X_{(n)} - X_{(1)}},$$

where $\widetilde{X}$ denotes the median of distribution. The median is equidistant from both the extremes for the symmetric distribution. It will be closer to one of the extremes for skewed distribution [Bandyopadhyay *et al.,* 2007]. The proposed measure is from the interval (–1, 1). The adaptive test statistic is given by the formula

$$D = S \cdot I(|Q| > c) + W \cdot I(|Q| \le c),$$

where $I(x)$ is an indicator function assuming the values 0 or 1 depending on *x*. If *x* is true, then $I(x) = 1$ and if *x* is false, then $I(x) = 0.$ The value $c = 0.075$ in terms of robustness of the test.This procedure maintains the designated α levels fairly accurately while displaying power. It performs satisfactorily unless there is an outlier in the data [Bandyopadhyay*et al.,* 2007].

III. APPLICATION

The presented methods will be illustrated through the analysis of real estate prices. The data used in the analysis come from the secondary real estate market in the Mokotow district in Warsaw. We use data only from one district because the real estate market is local, that is, both the price and a set of characteristics of the property are local. We used 50 prices per property square meter (m^2), which were similar in terms of features considered as important in this local real estate market.

The problem was to verify null hypothesis H_0: θ = 7 500 PLN/m^2 vs. H_1: θ >7 500 PLN/m^2. No assumption was made regarding the symmetry of the distribution.

The use of P approach required the calculation of p-value for the triples test. The calculated *p*-value was 0,96. Then we performed the Bernoulli trial with probability of success equal 0,96. Supposing success occurred, then we had to use the *W*-test. The observed value of the *W*-test statistic is 51 while $w^{+}_{0.05} = 61..$ Therefore at significance level equal 0,05 we do not reject the null hypothesis.

The use of D approach required the calculation of the measure $Q = 0.008.$ The observed *Q* is less than *c* = 0.075 and we use also *W*-test. The results of this test are described above.

IV. CONCLUSION

In this paper we have presented two adaptive procedures for testing the hypothesis about the median of distribution. The first one is the probabilistic approach (*P*), which is based on the triples test of symmetry as the preliminary test. The second one is the deterministic approach (*D*), which is based on a very simple measure of symmetry as the preliminary test. Both of them contained the *W*-test or *S*-test depending on the results of the preliminary test. The proposed methods are developed to be used without any assumptions of distribution, especially without the assumption of symmetry. So, as it was described at the beginning, they are a solution to the problems of testing the hypothesis about the median of the distribution.

Although both methods are described as reasonably robust, it is necessary to be aware of the dangers associated with the use of such adaptive procedures. It is important to remember that the tests which are the most appropriate in their own rights are not necessarily the most appropriate ones to use as the preliminary test in the adaptive procedure [Bandyopadhyay *et al.,* 2007].

In the practical analysis of real estate prices the use of probabilistic approach (*P*) is rather complicated and requires good knowledge of mathematical statistics. The triples test tends to reach calculations, but the bigger the sample then the more calculations we need. For example, if there are thirty-five observations, then more than six thousand calculations are needed, and for fifty observations there are almost twenty thousand calculations.

The deterministic approach (*D*) is much easier in practical applications, though it has less power compared to the probabilistic approach (*P*).

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PROBLEM WERYFIKACJI POŁOŻENIA W BADANIACH EKONOMICZNYCH

W badaniach ekonomicznych często rozważanym problem jest ocena położenia w pojedynczej próbie lub ocena różnic w położeniu dwóch prób. Zazwyczaj wykorzystywanymi wówczas testami są test znaków lub test Wilcoxona dla jednej próby oraz test Wilcoxona – Manna – Whitney'a w przypadku porównywania dwóch prób. Testy te jednakże obarczone są pewnymi wadami związanymi z brakiem odporności na niespełnienie wymaganych założeń lub niską efektywnością. W pracy zostaną przedstawione mniej znane procedury, dzięki którym można uniknąć powyższych problemów. Przedstawione metody zostaną zilustrowane na przykładzie analizy danych pochodzących z rynku nieruchomości.