Development of a Finite Difference Time Domain (FDTD) Model for Propagation of Transient Sounds in Very Shallow Water

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Introduction

Underwater sounds include steady-state vessel noise, transient animal calls, and impulsive pile driving sounds. We developed a finite difference time domain (FDTD) model (Yee, 1966; Botteldooren, 1994; Sakamoto *et al.*, 2002) using sample grids of sound pressure and velocity in alternating time steps to model sound propagation in very shallow water (depth $\leq$ 10 m). Since this is a time domain method it is particularly useful for modeling the propagation of transient and impulsive sounds. In this presentation, we compare propagation predicted by our model for single-frequency sources to the RAM model (Collins, 1995) and discuss sound propagation calculations for various transient sounds.

The Finite Difference Time Domain (FDTD) Method

We approximate the linearized acoustic differential equations as finite difference equations. Spatial coordinates are computed on a grid (e.g., x_1, x_2, x_3, ...), and time t is taken in discrete steps (e.g., t_1, t_2, t_3, ...). Time derivatives are approximated as finite differences of time

$$\frac{\partial p(x,t)}{\partial t} \rightarrow \frac{\Delta p(x,t)}{\Delta t} = \frac{p(x,t_2) - p(x,t_1)}{t_2 - t_1}, \qquad (1)$$

and spatial derivatives are approximated as finite differences of spatial coordinates

$$\frac{\partial p(x,t)}{\partial x} \rightarrow \frac{\Delta p(x,t)}{\Delta x} = \frac{p(x_2,t) - p(x_1,t)}{x_2 - x_1}. \qquad (2)$$

The resulting finite difference propagation equations are solved for the time evolution of the acoustic parameters pressure and particle velocity, each of which depends on the spatial variations of the other parameter.

In an approach known as leapfrogging, spatial variations of pressure are used to calculate changes to the particle velocity, and spatial variations in particle velocity are used to calculate changes to the pressure. In the leapfrogging scheme, the particle velocity spatial grid points are halfway between the pressure grid points (see Figure 1), and the pressure and particle velocity values are computed $1/2$ time-step apart. The calculation alternates between particle velocity and pressure changes in each $1/2$ time-step. We assume the seafloor to be an equivalent fluid and use its sound speed and density in the time-increment equations.

We use perfectly matched layers (PMLs; Teixeira and Chew, 1997) to eliminate numerical reflections from the ends of the grid space. We assume a pressure-release surface and terminate the grid with a PML in the other directions.

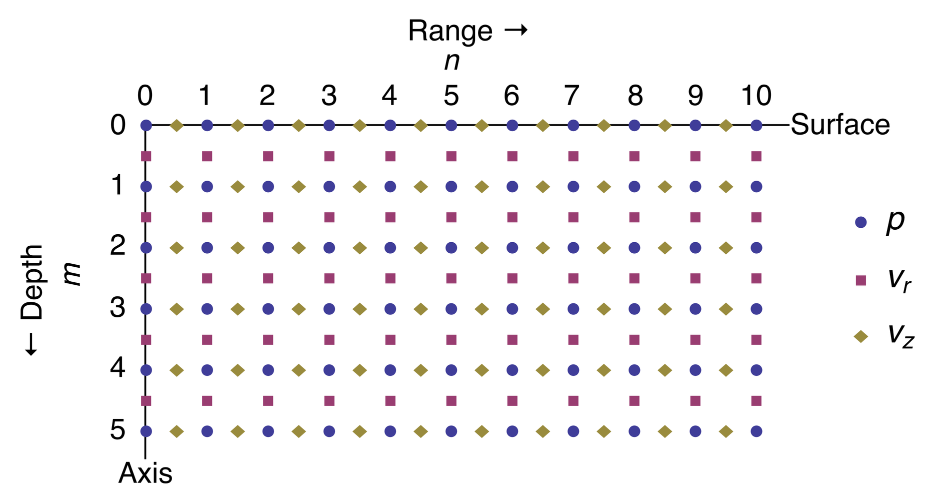

Figure 1: The grid used for FDTD calculations. Pressure p and the velocity component v_r and v_z values are separated by a half grid-space to simplify finite difference calculations involving each variable. In the leapfrogging technique spatial differences in particle velocities result changes to the pressure values in between them, and spatial differences in pressures result in changes to the particle velocity values between them.

3D and 2D Implementations

We have developed implementations of our FDTD model in both two dimensional (2D) cylindrical coordinates and three dimensional (3D) Cartesian coordinates. The 2D implementation assumes axial symmetry to reduce the number of grid points resulting in faster computation times and smaller data sets. The 3D implementation allows asymmetrical geometries at the cost of a geometric increase in computation time and data storage requirements.

Impulse Propagation

We use the FDTD impulse propagation method (Sakamoto *et al.*, 2002) to propagate a pressure impulse (see Figure 2) from the source position(s) throughout the grid to the receiver positions. This propagated impulse response signal contains all geometrical, reflective, and diffractive effects on the signal for all frequencies below the Nyquist frequency associated with the time step Δt. We then convolve the impulse response signal at the desired receiver position with the source signal function to obtain the propagated signal at the receiver position. This technique works for both steady-state and transient signals.

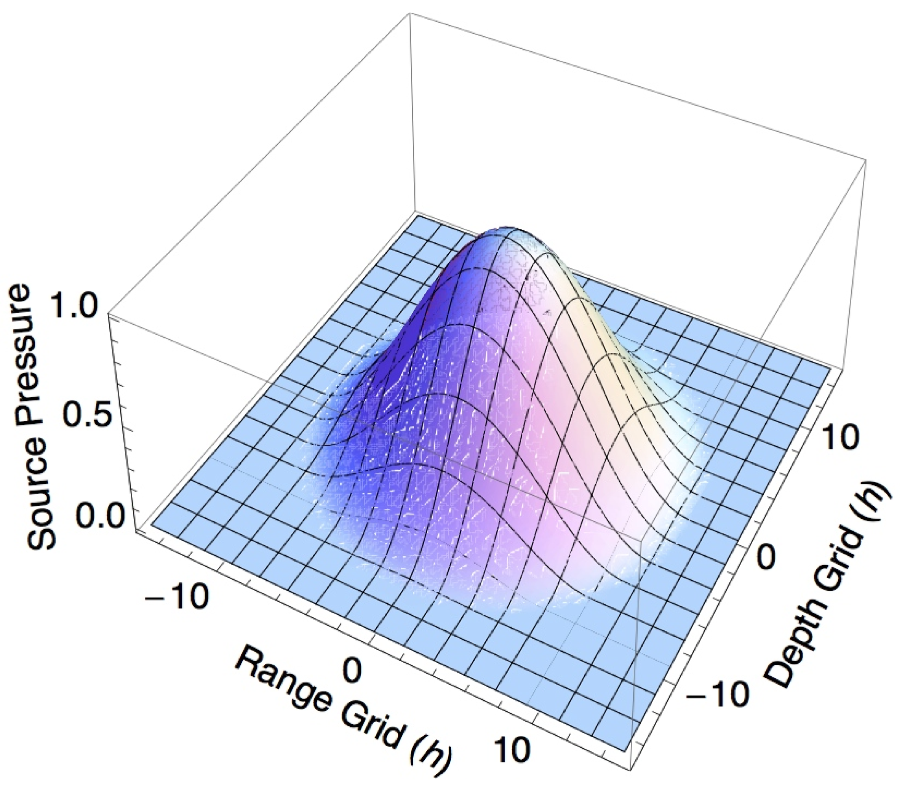

Figure 2: The impulsive function (Sakamoto *et al.*, 2002) used for the source in our FDTD propagation calculations. The pressure increases from zero to the maximum over 12 grid spaces in a sinusoidal function.

Comparison to RAM

To validate our model, we compared FDTD propagation calculations to those made using a split-step parabolic equation calculation with the freely available Range-dependent Acoustic Model (RAM) program (Collins, 1995). Figures 3–5 are examples of these calculations for a 5 m deep uniform ocean (sound speed 1536 m/s, density 1700 kg/m^3) over a sandy bottom (sound speed 1024 m/s, density 2035 kg/m^3). There is very good agreement between the FDTD calculations and the RAM calculations for all geometries.

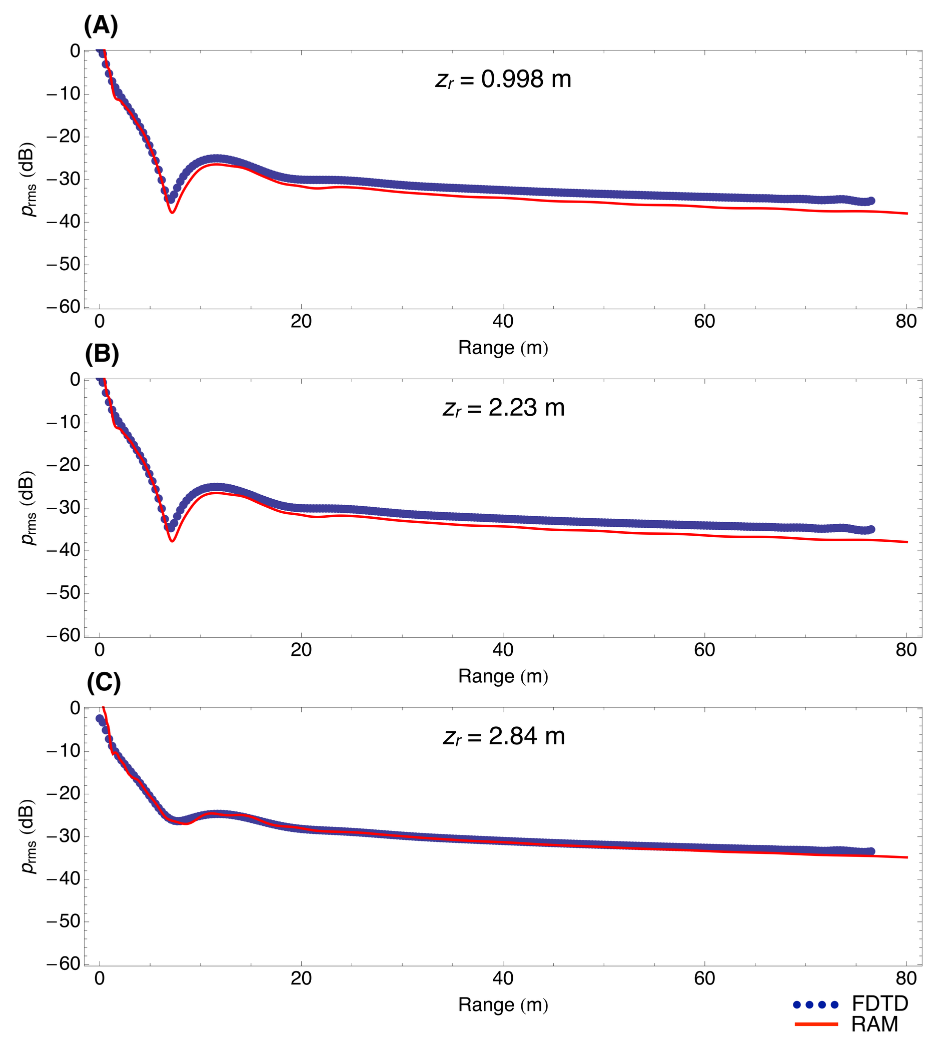

Figure 3: Comparison of sound propagation calculations by our FDTD and the RAM. All calculations are for a 250 Hz constant frequency source at depth 2.38 m in a flat 5.00 m deep ocean. (A) Receiver at depth z_r = 0.998 m. (B) Receiver at depth z_r = 2.23 m. (C) Receiver at depth z_r = 2.84 m.

Discussion and Conclusion

Our FDTD approach compares well with the RAM calculations. The FDTD holds promise for modeling the propagation of transient sounds in very shallow estuaries and rivers. We used an earlier version of this model to calculate the propagation of transient *Cynoscion regalis* (weakfish) sounds in very shallow water with both level and sloped seabeds (Sprague and Luczkovich, 2012b) and to calculate the propagation of weakfish sounds in order to estimate numbers of calling fish in aggregations (Sprague and Luczkovich, 2012a). Another application of our model is to use motion of a vibrating pile as the source function to calculate the propagation of the pressure and particle velocity produced during pile driving. In the future, we would like to implement a poroelastic substrate model for more realistic investigations of the water-substrate interactions for transient sounds.

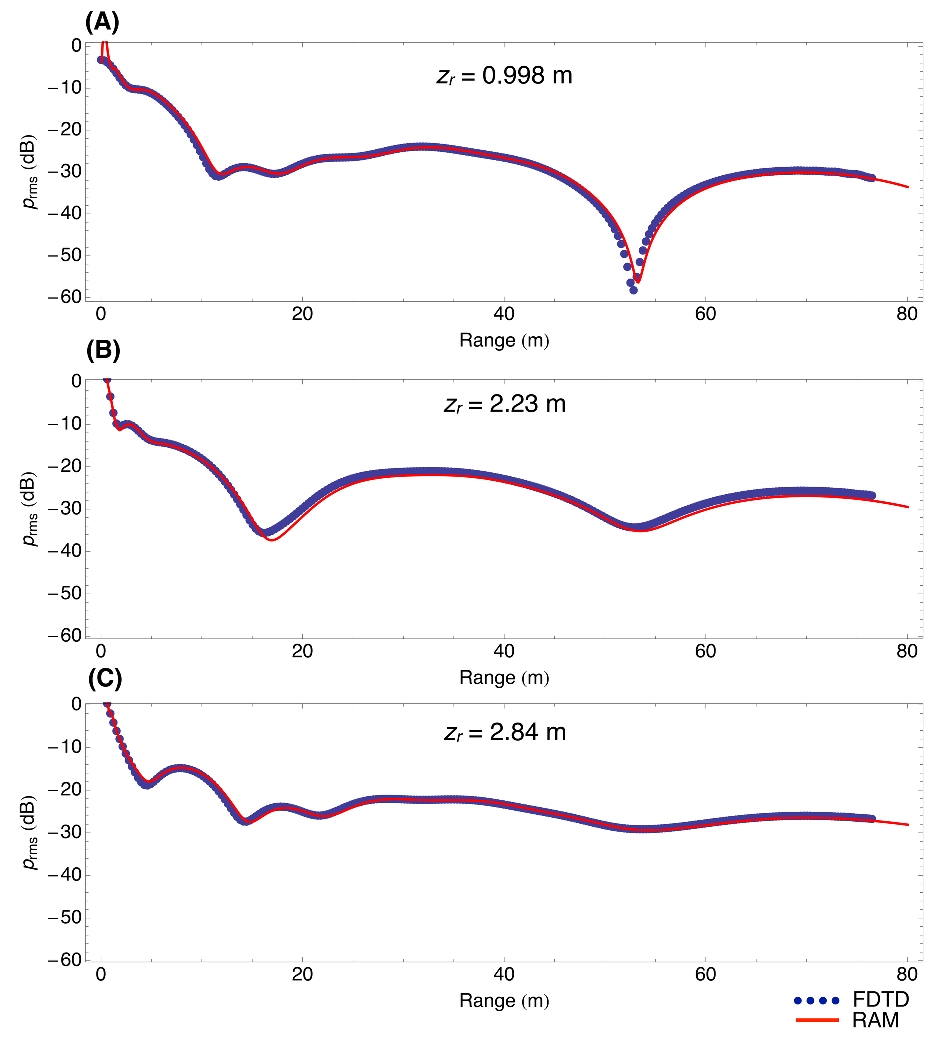

Figure 4: Comparison of sound propagation calculations by our FDTD and the RAM. All calculations are for a 500 Hz constant frequency source at depth 2.38 m in a flat 5.00 m deep ocean. (A) Receiver at depth z_r = 0.998 m. (B) Receiver at depth z_r = 2.23 m. (C) Receiver at depth z_r = 2.84 m.

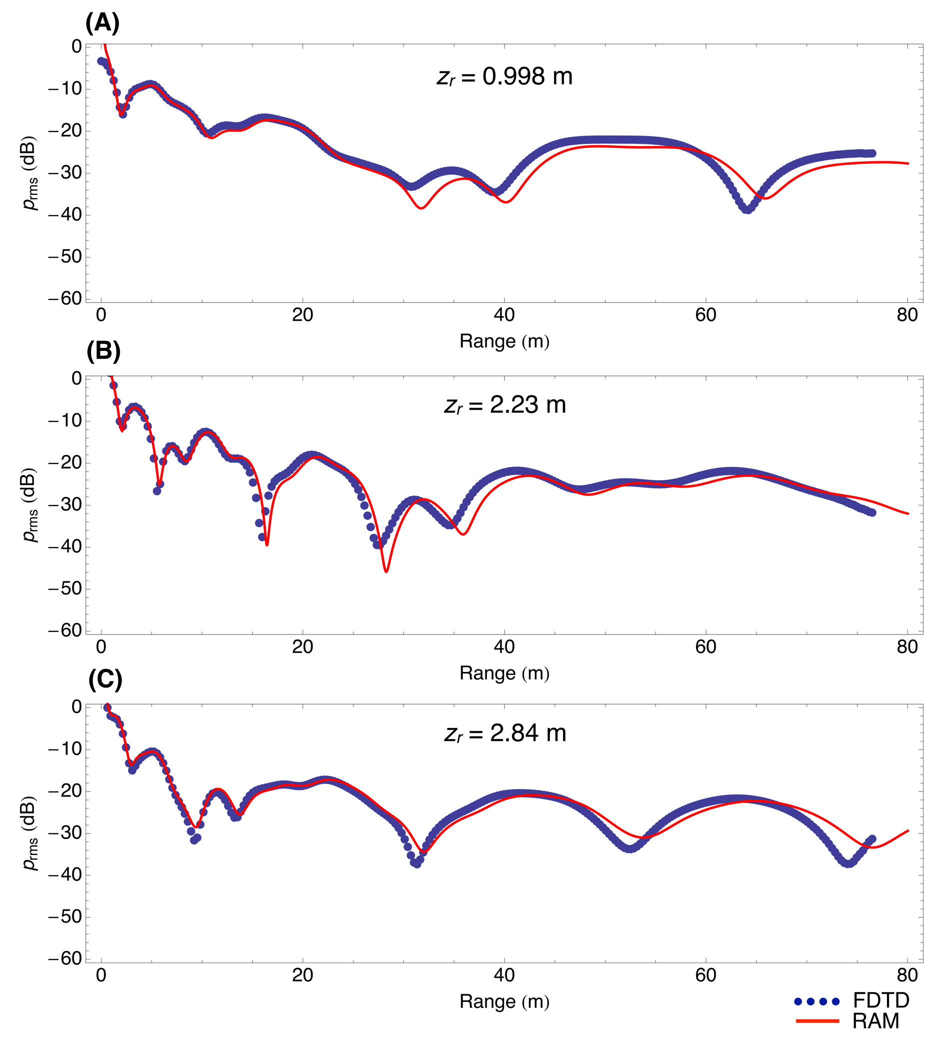

Figure 5: Comparison of sound propagation calculations by our FDTD and the RAM. All calculations are for a 1000 Hz constant frequency source at depth 2.38 m in a flat 5.00 m deep ocean. (A) Receiver at depth z_r = 0.998 m. (B) Receiver at depth z_r = 2.23 m. (C) Receiver at depth z_r = 2.84 m.

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