Abstract

THE DESCRIPTION OF SCHUMANN ELECTROMAGNETIC RESONANCES BETWEEN EARTH AND ITS IONOSPHERE AS BOSE–EINSTEIN CONDENSATES OF EXTREMELY LOW FREQUENCY PHOTONS

by Davidson S. Wicker

April 2012

Director: Orville Day, PhD

Department of Physics

The purpose of this thesis is to show that Schumann resonances between Earth and its ionosphere are Bose–Einstein condensates of extremely low frequency photons. We will show that the photon densities of the first five modes of the Schumann resonances are each sufficient for the creation of a Bose–Einstein condensate. We will also study the coherence of the Schumann resonance electromagnetic waves, another necessary condition for the onset of a Bose–Einstein condensation.

THE DESCRIPTION OF SCHUMANN ELECTROMAGNETIC RESONANCES BETWEEN EARTH AND ITS IONOSPHERE AS BOSE–EINSTEIN CONDENSATES OF EXTREMELY LOW FREQUENCY PHOTONS

A Thesis

Presented to

The Faculty of the Department of Physics

East Carolina University

In Partial Fulfillment

Of the Requirements for the Degree

Master's of Science in Applied Physics

by

Davidson S. Wicker

April 2012

THE DESCRIPTION OF SCHUMANN ELECTROMAGNETIC RESONACES BETWEEN EARTH AND ITS IONOSPHERE AS BOSE-EINSTIEN CONDENSATES OF EXTREMELY LOW FREQUENCY PHOTONS

By

Davidson Sterling Wicker

APPROVED BY:

DIRECTOR OF DISSERTATION:

____

Orville Day, PhD

COMMITTEE MEMBER:

____

Michael Dingfelder, Ph.D

COMMITTEE MEMBER:

____

Edson Justiniano, Ph.D

COMMITTEE MEMBER:

____

Jason Yao, Ph.D

CHAIR OF THE DEPARTMENT OF PHYSICS:

____

John Sutherland, Ph.D

DEAN OF THE GRADUATE SCHOOL:

____

Paul J. Gemperline, Ph.D

Dedication

This thesis is dedicated to my lovely wife Fabrienne, and our daughter Mayla.

Acknowledgements

First, I would like to thank my thesis advisor, Dr. Orville Day, for allowing me to participate in this fascinating research. His guidance helped me through many of the challenges and obstacles I faced when pursuing the topic of the Schumann resonances. Our earlier work together on photon cavities surrounding black holes provided me with a great introduction to spherical photon cavities. Dr. Day realized that by considering the Schumann resonant cavity as a photon cavity, as opposed to simply an electromagnetic cavity, we may be able to show that the Schumann resonances are a Bose–Einstein condensate. As we investigated this possibility we came to realize there are likely several BEC's, composed of photons from each mode of the Schumann resonances.

I also want to thank the other members of my thesis committee: Dr. Michael Dingfelder, Dr. Edson Justiniano, and Dr. Jason Yao. The two semester course on quantum mechanics taught by Dr. Dingfelder was perhaps the most difficult, and interesting, course I have taken during my physics studies. The electrodynamics course taught by Dr. Justiniano was equally challenging, and helped prepare me well for much of the work related to the Schumann resonances. Dr. Yao (Department of Engineering) offered valuable insight and suggestions on successfully preparing this thesis. Although not on the advisory committee, Dr. John Kenney gave me some great suggestions and challenged me to study properties of the Schumann resonances that I had not considered.

Contents

1 Introduction

First predicted in 1952 by Winfried Otto Schumann[1], the Schumann resonances are a set of distinct peaks in the extremely low frequency (ELF) range[1] of the electromagnetic spectrum detected on Earth's surface. This electromagnetic radiation is confined in a natural spherical cavity formed between the Earth's surface and its lower ionosphere, as shown in Figure 1. Both the Earth and it's ionosphere are conductors, which act as a confining potential, allowing the Schumann resonance standing waves to exist[2]. The Schumann resonances propagate in the horizontal direction with a vertical electric field and horizontal magnetic field[3]. Vertical antennas and horizontally aligned magnetometers are used to detect the electric field and magnetic field components of the Schumann resonances[4]. The existence of the Schumann resonances has been well documented since the first experimental observations in the early 1960s and is now considered to be an empirical fact[5].

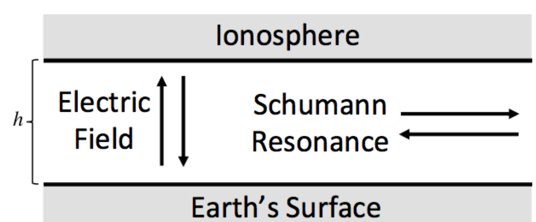

Figure 1: Diagram of the Earth-ionosphere cavity depicting the direction of propagation of the Schumann resonances, and the orientation of the electric field. Here, h is the height of the cavity (around 80 km).

[1]ELF radiation is typically defined to be from 3-300 Hz, although it can extend to 3 kHz as defined in some scientific fields.

Lightning strikes are responsible for excitations in the Earth-ionosphere cavity, as was pointed out by Schumann[1]. At any given time a total of about 2000 thunderstorms occur worldwide, covering around 10% of Earth's surface, with about 100 lightning strikes occurring every second worldwide[6]. This large occurrence of lightning strikes keeps the Schumann resonant cavity populated with extremely low frequency radiation that has been well documented[3, 4, 7, 8, 9, 10]. Although this radiation varies slightly in frequency and intensity over time, experimental data indicates that the resonant modes only vary by about ± 1 Hz[11]. The extremely low frequencies of the Schumann resonances involve huge wavelengths—about 40,000 km (Earth's circumference) for the fundamental mode.

We believe the Schumann resonances have quantum aspects that have never been identified. Most who have studied the Schumann resonances are geophysicists or electrical engineers with little or no understanding of quantum systems. All descriptions so far in the literature have been in terms of Maxwell's classical equations of electromagnetism. To the best of our knowledge there have been no studies of the Schumann resonances from a quantum point of view. Only the classical electrodynamic features of the Schumann resonances have been studied, and we believe there are interesting quantum aspects of the Schumann resonances that have never been studied before. We will show that by studying some of the quantum features of the Schumann resonances a remarkable property of this phenomenon will be realized; the Schumann resonances are Bose–Einstein condensates of extremely low frequency photons.

Before discussing the Schumann resonances as a quantum system, we will go over some of the necessary background information required for an in depth understanding of this global phenomenon. We will begin by studying some of the properties of the Earth-ionosphere cavity. Then, we will consider the theoretical foundations of the Schumann resonances. In the last section we will discuss Bose-Einstein condensates (BEC), and demonstrate that the Schumann resonances meet the necessary conditions for the onset of a BEC: coherence, containment in a confining potential, and densities above a critical density.

2 Theoretical Determination of the Schumann Resonance Frequencies

Resonant waves within the Earth-ionosphere cavity are governed by the wave equation

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0 \tag{1}$$

where c is the speed of the wave, and $\nabla^2\psi$ is given by Laplace's equation (in spherical coordinates):

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2} \tag{2}$$

The normal mode solutions to Equation 2 satisfy the equation

$$\frac{\partial}{\partial t}\Psi = i\omega\Psi \tag{3}$$

where

$$\Psi(r,\theta,\varphi,t) = \psi(r,\theta,\varphi)e^{i\omega t} \tag{4}$$

This equation tells us that the wavefunction Ψ has a spatial amplitude ψ, and evolves in time according to $e^{i\omega t}$. The function $\psi(r,\theta,\varphi)$ satisfies the Helmholtz equation given by

$$(\nabla^2 + k^2)\psi = 0 \tag{5}$$

where k is the wavenumber and $k^2 = \omega^2/c^2$. By substituting Equation 2 into Equation 5, we get (after some rearranging)

$$\left[\frac{1}{r}\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right] + k^2\right]\psi = 0 \tag{6}$$

The spatial part of Equation 4 can be separated into two components

$$\psi(r,\theta,t) = j(r)Y(\theta,\varphi) \tag{7}$$

where $j(r)$ describes the radial motion of the wave, and $Y(\theta, \varphi)$ is the spherical harmonic, which describes the angular motion of the wave. Now, we can substitute Equation 7 into Equation 6, separate out the r dependence from the (θ, φ) dependence and set them equal. This implies that they are equal to the same separation constant, λ. The angular portion of Equation 6 is written as

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial Y}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\varphi^2} = -\lambda Y \tag{8}$$

and the radial portion is written as

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + \left(k^2 - \frac{\lambda}{r^2}\right)\right] j(r) = 0 \tag{9}$$

where the eigenvalue λ is determined by the boundary conditions. Of course, the eigenvalues in this case correspond to the eigenfrequencies of the cavity.

To find the eigenfrequencies associated with the Schumann resonances we must solve Maxwell's equations in spherical coordinates. Maxwell's equations are given by:

$$\nabla \times \mathbf{E} = \mu_0 \frac{\partial \mathbf{H}}{\partial t} \tag{10}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \tag{11}$$

$$\nabla \cdot \mathbf{D} = 0 \tag{12}$$

$$\nabla \cdot \mathbf{H} = 0 \tag{13}$$

where $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$ is the permeability of free space, $\varepsilon_0 = (36\pi \times 10^9)^{-1}\,\mathrm{F/m}$ is the permittivity of free space, $\mathbf{E}$ is the electric field, $\mathbf{D}$ is the electric induction, and $\mathbf{H}$ is the magnetic field. The more commonly used magnetic $\mathbf{B}$-field is related to $\mathbf{H}$ by $\mathbf{B} = \mu_0\mathbf{H}$ (valid in Earth's atmosphere—explained in more detail later), and $\mathbf{D}$ is related to $\mathbf{E}$ by $\mathbf{D} = \varepsilon_0\varepsilon\mathbf{E}$, where ε is the permittivity (its value being material dependent).

Equation 11 can also be written as[12]

$$\nabla \times \mathbf{H} = i\omega\varepsilon_0\mathbf{E} + \mathbf{J} + \mathbf{J}_{\text{ext}} \tag{14}$$

where $\mathbf{J}$ is the conduction current, and $\mathbf{J}_{\text{ext}}$ is the external source current. The conduction current is driven by the electric field and can be written as $\mathbf{J}(\mathbf{r}) = \sigma(\mathbf{r})\mathbf{E}(\mathbf{r})$, if the electrical conductivity $\sigma(\mathbf{r})$ is taken to be a scalar function of position[12]. For the Schumann resonant cavity the external source current $\mathbf{J}_{\text{ext}}$ comes from lightning, which excites the Earth-ionosphere cavity, and can be treated as a radial electric dipole. The rate at which energy from lightning is pumped into the nth mode is given by[12]

$$J_n = \frac{1}{K}\int_A \mathbf{E}_n^* \cdot \mathbf{J}_{\text{ext}}\, dA \tag{15}$$

where K is a normalization constant, and $\mathbf{E}_n^*$ is the complex conjugate of the nth mode electric field.

The field components of the Schumann resonances have a time dependence in the form $\exp(i\omega t)$. Therefore, we can convert Equations 10–13 from the time domain to the frequency domain by application of a Fourier transform:

$$\mathbf{E}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r},t)\exp(-i\omega t)\, dt \tag{16}$$

and

$$\mathbf{H}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} \mathbf{H}(\mathbf{r},t)\exp(-i\omega t)\, dt \tag{17}$$

where $\mathbf{r} = r\hat{r} + \theta\hat{\theta} + \varphi\hat{\varphi} = (r,\theta,\varphi)$ is the position vector in spherical coordinates.

Schumann considered an idealized concentric spherical cavity with perfectly conducting boundaries. Using these assumptions, the r, θ, and φ components of Equations 10 and 11 can be written as[2]

$$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial r}r\sin\theta E_\varphi - \frac{\partial E_r}{\partial\varphi}\right) = i\omega\mu_0 H_\theta \tag{18}$$

$$\frac{1}{r}\left(\frac{\partial}{\partial r}rE_\theta - \frac{\partial E_r}{\partial\theta}\right) = -i\omega\mu_0 H_\varphi \tag{19}$$

$$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial r}r\sin\theta H_\varphi - \frac{\partial H_r}{\partial\varphi}\right) = i\omega\varepsilon_0\varepsilon E_\theta \tag{20}$$

$$\frac{1}{r}\left(\frac{\partial}{\partial r}rH_\theta - \frac{\partial H_r}{\partial\theta}\right) = i\omega\varepsilon_0\varepsilon E_\varphi \tag{21}$$

$$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta E_\varphi - \frac{\partial E_\theta}{\partial\varphi}\right) = -i\omega\mu_0 H_r \tag{22}$$

$$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta H_\varphi - \frac{\partial H_\theta}{\partial\varphi}\right) = i\omega\varepsilon_0\varepsilon E_r \tag{23}$$

By treating the Earth-ionosphere cavity as being filled with a homogeneous dielectric with $\varepsilon \approx \varepsilon_0$, Equations 18–23 can be reduced to just three field equations at Earth's surface[13]:

$$E_\theta = -Z_g H_\varphi \tag{24}$$

$$E_r = -i\frac{I\,dl f(f+1)}{4\pi a^2\varepsilon_0\omega h}\sum_{m=0}^{\infty}\frac{(2m+1)P_m(\cos\theta)}{m(m+1)-f(f+1)} \tag{25}$$

$$H_\varphi = -\frac{I\,dl}{4\pi ah}\sum_{m=0}^{\infty}\frac{2m+1}{m(m+1)-f(f+1)}\frac{dP_m(\cos\theta)}{d\theta} \tag{26}$$

where a is Earth's radius[2], h is the height of the lower-ionosphere, Z_g is the surface impedance of the ionosphere, ε_0 is the permittivity of free space, f is the eigenfrequency, $P_m(\cos\theta)$ is the Legendre polynomial, θ is the angle between the source and observer, and $I\,dl$ is the current moment of the radial electric dipole.

The vertical electric field given by Equation 25 is called a TM (transverse magnetic) wave since it is perpendicular to the horizontal magnetic field. Likewise, Equation 26 is

[2]Earth's equatorial radius is 6378.1370 km and the polar radius is 6356.7523 km, with a mean radius of 6371 km.

called a TE (transverse electric) wave since it propagates in a direction perpendicular to the electric field. Electromagnetic fields such as the Schumann resonances are called TEM waves, or transverse electromagnetic waves, since both fields (E and H) are perpendicular and the direction of propagation is parallel to the cavity boundaries. The actual direction of propagation for the Schumann resonance TEM waves is given below.

Equation 24 is found experimentally to be effectively zero[2]. By taking the cross product of the remaining **E** and **H** fields we get

$$\mathbf{S} = \mathbf{E} \times \mathbf{H} = E_r H_\varphi \sin\alpha\, \hat{\theta} \tag{27}$$

where α is the angle between the radial E-field and horizontal (relative to Earth's surface) H-field, and clearly points in the $\hat{\theta}$ direction. Equation 27 is called the Poynting vector, and specifies the power flow along the direction of propagation. This theoretical result also fits with experimental data[4, 7].

Although a spherical cavity formed from two concentric spheres has no preferred direction, models that take into account the variations in the Earth's ionosphere place the polar axis $\theta = 0$ either at one of Earth's poles or at the center of the day (or night) hemisphere[2].

With perfectly conducting cavity boundaries the eigenfrequencies are given by

$$f_m = \frac{c}{2\pi a}\sqrt{m(m+1)} \tag{28}$$

and produces the 10.5, 18.2, 25.7, 33.2, and 40.7 Hz as the first five eigenfrequencies (if we take Earth's mean radius to be 6371 km). To reproduce the eigenfrequencies that match values obtained experimentally we can multiply Equation 28 by a correction factor which takes in to account the variations in the conductivity of the Earth's ionosphere[13]

$$f'_m \approx f_m \left(1 - \frac{c\sqrt{\varepsilon_0}}{4h\sqrt{\pi f_m \sigma_i}}\right) \tag{29}$$

where σ_i is the conductivity of the lower ionosphere (between a height of 60-100 km), and c is the speed of light. Variations in the conductivity of the ionosphere are due to Earth's geomagnetic field and solar radiation (as mentioned above).

The eigenfrequencies can also be found from the following dispersion relation [14]

$$k_n R_E = \sqrt{n(n+1)\left(1 - \frac{h}{R_E}\right) - \left(\frac{R_E Z}{2h}\right)^2} + i\frac{R_E}{2h}Z \tag{30}$$

where Z is the effective dimensionless surface impedance, and arises when taking into account the non-sharp boundary of the lower-ionosphere and effects from Earth's geomagnetic field. Z becomes a tensor whose components depend on uniquely defined coordinates. We must define two sets of coordinates, one with respect to the geographic pole, and the other with respect to the geomagnetic pole[3]. The geographic spherical coordinates (r, θ, φ) place $\theta = 0$ at Earth's rotational axis, and the geomagnetic spherical coordinates (R, θ', φ') place $\theta' = 0$ aligned with Earth's geomagnetic pole (tilted at 11° from Earth's rotational axis). The two coordinate systems are related by [2]

$$\begin{aligned} r\sin\theta\cos\varphi &= R\sin\theta'\cos\varphi' + \delta \\ r\sin\theta\sin\varphi &= R\sin\theta'\sin\varphi' \\ r\cos\theta &= R\cos\theta' \end{aligned} \tag{31}$$

where δ is the displacement from the geomagnetic axis to the geomagnetic pole, which is taken to be *parallel to the geographic axis.* This odd choice for the geomagnetic axis is

[3]Earth's outer magnetic field is treated as the field due to a dipole source shifted by δ from Earth's rotational axis

made so the Sun is always normal to both the geomagnetic and geographic axis. Earth's geomagnetic field components, in terms of the geomagnetic coordinates are [2]

$$
\begin{aligned}
H_R^0 &= -\frac{2M_0}{4\pi R^3}\cos\theta' \\
H_{\theta'}^0 &= \frac{M_0}{4\pi R^3}\sin\theta' \\
H_{\varphi'}^0 &= 0
\end{aligned}
\tag{32}
$$

and in terms of the geographical coordinates are written as

$$
\begin{aligned}
H_r^0 &\approx -\frac{2M}{4\pi a^3}\cos\theta\left[1+\frac{7}{2}\frac{\delta}{a}\sin\theta\sin\Psi\right] \\
H_\theta^0 &\approx \frac{M_0}{4\pi a^3}\sin\theta\left[1+\frac{\delta}{a}\frac{\sin\Psi}{\sin\theta}(1+2\sin^2\theta)\right] \\
H_\varphi^0 &\approx \frac{\delta}{a}\frac{M_0}{4\pi a^3}\cos\theta\cos\Psi
\end{aligned}
\tag{33}
$$

where $\Psi = \frac{\pi}{12}(t_U - 6) + \lambda_d$ is the angle between the day-night interface (called the terminator) and the normal to the Sun, t_U Greenwich mean time (GMT) in hours, and λ_d is the angle between the Greenwich meridian and the geomagnetic pole. The angle Ψ is used in place of φ when treating the Schumann resonant cavity as anisotropic (see Figure 2).

When considering a sharply bounded magnetized plasma[4], the dimensionless surface impedance is found by solving

$$
\sum_\theta Z_{r\theta} Z_{\theta\varphi}\eta_{r\varphi} = \hat{\varepsilon}^{-1}
\tag{34}
$$

where $\hat{\varepsilon}$ is given by Equation 50. The surface impedance (Equation 34) has generalized

[4]Equations 34–38 are also valid for spherical boundaries whenever $\kappa \gg \delta_S$, where κ is the radius of curvature, and δ_S is the skin depth of the wave in the atmosphere. This also tells us that the electromagnetic wave in the plasma always propagates along the normal to the boundary, regardless of the incident angle, and is why we only have ω_r in Equations 38.

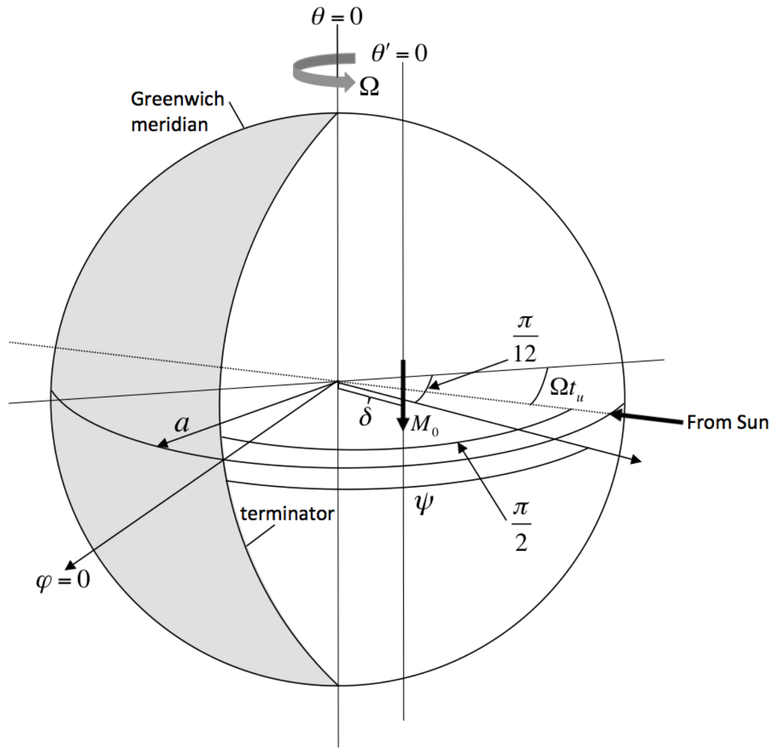

Figure 2: Diagram of the spherical coordinate system for a non-uniform Earth-ionosphere cavity, where M_0 is the magnetic moment (see Equation 49).

components equal to [14]

$$\begin{aligned}
Z_{\theta\theta} &= \frac{1}{\chi}\left[\eta_{\theta\theta} + \sqrt{\eta_{\theta\theta}\eta_{\varphi\varphi} - \eta_{\theta\varphi}\eta_{\varphi\theta}}\right] \\
Z_{\varphi\varphi} &= \frac{1}{\chi}\left[\eta_{\varphi\varphi} + \sqrt{\eta_{\theta\theta}\eta_{\varphi\varphi} - \eta_{\theta\varphi}\eta_{\varphi\theta}}\right] \\
Z_{\varphi\theta} &= \frac{\eta_{\varphi\theta}}{\chi} \\
Z_{\theta\varphi} &= \frac{\eta_{\theta\varphi}}{\chi}
\end{aligned} \tag{35}$$

where $\chi^2 = \eta_{\theta\theta} + \eta_{\varphi\varphi} + 2\sqrt{\eta_{\theta\theta}\eta_{\varphi\varphi} - \eta_{\theta\varphi}\eta_{\varphi\theta}}$. The surface impedance tensor is given by

$$Z_{\theta\varphi} = \hat{Z} = \begin{pmatrix} Z_1 & -Z_2 \\ Z_2 & Z_1 \end{pmatrix} \tag{36}$$

If we assume that the conductivity of the ionospheric plasma is high, then we have

$$\frac{\omega v_{\text{eff}}}{\omega_0^2} \ll 1 \tag{37}$$

where ω_0 is the plasma frequency of the ionosphere, and v_{eff} is the effective collision frequency. By using this assumption we find that each component in Equation 36 is given by

$$\begin{aligned} Z_1 = Z_{\theta\theta} = Z_{\varphi\varphi} &= \sqrt{i\frac{\omega v_{\text{eff}}}{2\omega_0^2}}\sqrt{1+\sqrt{1+\frac{\omega_r^2}{v_{\text{eff}}^2}}} \\ Z_2 = Z_{\varphi\theta} = -Z_{\theta\varphi} &= \sqrt{i\frac{\omega v_{\text{eff}}}{2\omega_0^2}}\frac{1}{\sqrt{1+\sqrt{1+\frac{\omega_r^2}{v_{\text{eff}}^2}}}}\frac{\omega_r}{v_{\text{eff}}} \end{aligned} \tag{38}$$

These components depend only on the radial geomagnetic field at the specific frequencies of the Schumann resonances, and the positive sign implies wave absorption in the ionosphere[2]. For small electron gyrofrequency (i.e., a negligible geomagnetic field), we have the surface impedance of an isotropic ionosphere[2]

$$Z_{\theta\theta} = Z_{\varphi\varphi} = Z_0 = \sqrt{i\frac{\omega v_{\text{eff}}}{\omega_0^2}}. \tag{39}$$

By studying the surface impedance of the ionosphere we now have a better understanding of why the Schumann resonances are reflectedt.

3 Properties of the Schumann Resonant Cavity

Schumann developed the theoretical underpinnings of electromagnetic resonances confined between Earth's surface and its ionosphere by considering an idealized cavity. When the wave equation (Equation 1) is applied to a perfectly spherical resonant cavity formed by two concentric shells we get a set of eigenfrequencies given by[4]

$$f_\ell = \frac{c}{2\pi(a+h/2)}\sqrt{\ell(\ell+1)}, \tag{40}$$

where c is the speed of light, a is the radius of the inner spherical surface, h is the height of the cavity, and $\ell = \{1, 2, \dots\}$ is the mode number (see Figure 3). The square root factor in Equation 40 is present due to the spherical symmetry of the resonant cavity.[5]. The factor in front of the square root specifies the separation frequency of resonant waves. The separation frequency $\frac{c}{2\pi(a+h/2)}$ is about 7.4 Hz, but is found experimentally to be closer to 6 Hz[3, 4, 7, 8, 9, 10].

When Equation 40 is applied to the Earth-ionosphere cavity the first five modes are given

[5] If an experimenter had no prior knowledge of Earth's spherical geometry, then the presence of this factor could be used as experimental proof that the Earth-ionosphere resonant cavity possesses spherical symmetry[2]

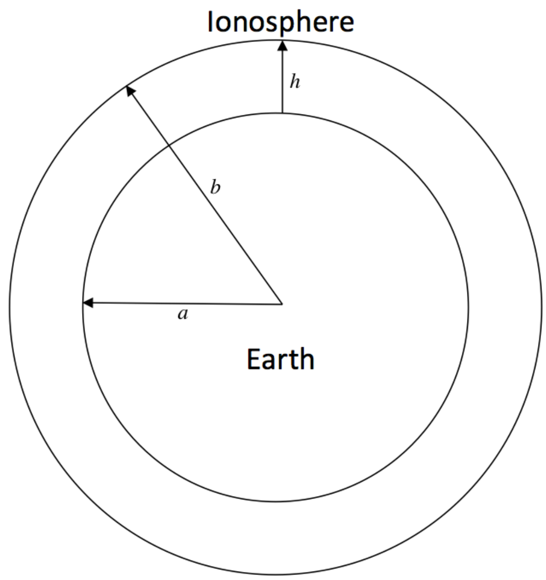

Figure 3: Diagram of the Earth-ionosphere cavity (not to scale, $\frac{b-a}{a} \ll 1$), where a is Earth's mean radius (6371 km), b is the radius of the lower ionosphere, and h is the height of the cavity (around 80 km).

as

$$\begin{aligned} f_1 &= 10.5 \text{ Hz} \\ f_2 &= 18.2 \text{ Hz} \\ f_3 &= 25.7 \text{ Hz} \\ f_4 &= 33.2 \text{ Hz} \\ f_5 &= 40.7 \text{ Hz} \end{aligned} \tag{41}$$

where we use $a = 6371$ km (Earth's mean radius), and $h = 80$ km (the distance between Earth's surface and the average height of the lower ionosphere). However, the experimental values are found to be around[5]

$$
\begin{aligned}
f_{1,\text{exp}} &= 8\ \text{Hz} \\
f_{2,\text{exp}} &= 14\ \text{Hz} \\
f_{3,\text{exp}} &= 21\ \text{Hz} \\
f_{4,\text{exp}} &= 27\ \text{Hz} \\
f_{5,\text{exp}} &= 34\ \text{Hz}
\end{aligned}
\tag{42}
$$

Higher frequency modes are often seen (up to about 7) in the experimental spectra. The discrepancy between the theoretical values and the values obtained by experiment is due to variations in the height at which the standing waves reflect off the lower ionosphere. Details of the variation in the height of the lower ionosphere will be discussed later.

Measurements of the Schumann resonances show wide peaks, that have been attributed to the finite conductivity of the lower-ionosphere[2]. The width of resonant peaks can be quantified by considering the Q-factor. The Q-factor of a resonant cavity is given as [15]

$$Q = \frac{\omega_0}{\delta\omega} = \frac{\omega_0}{\Gamma} \tag{43}$$

where ω_0 is the resonant frequency and Γ is the width of the peak at half-maximum. Schumann predicted a cavity Q of about 12 in his theoretical work[1], but experimental data showed a cavity $Q_1 = 4$ for the fundamental mode, and $Q_5 = 6$ for the fifth resonant mode[2, 5]. Since we have $h/R_E \ll 1$, where R_E equals Earth's radius, and h is the height of the lower-ionosphere, Equation 43 reduces to[2]

$$Q = \frac{h}{\delta_S} \tag{44}$$

where

$$\delta_S = \frac{\lambda}{\sqrt{\varepsilon}} = \lambda Z \tag{45}$$

Equation 45 is called the skin depth of the cavity walls, and describes the distance that electromagnetic waves penetrate beyond the cavity boundaries. Here, λ is the resonance wavelength, ε is the permittivity of the cavity walls, and Z is the effective dimensionless surface impedance of the ionosphere (Equation 36). With an average cavity height of 80 km and a Q-factor equal to 4, we find that $\delta_S = 20$ km. In other words, the electromagnetic field of the fundamental Schumann resonance penetrates 20 km into the ionosphere, to an average maximum height of 100 km, representing just 1/2000th of the fundamental mode wavelength. The skin depth of the Schumann resonances partially explains why experimental data[4, 7, 8, 9, 10] of their resonant frequencies does not show sharp peaks.[6]

We believe the peak widths are also partially a result of incomplete resonant waves being generated from lightning. Accelerated free electrons from a typical lightning event do not always have lifetimes long enough for them to form a complete resonant wave. Single stroke lightning has an average duration of 30-40 ms, but multiple stroke lightning, which accounts for 80-85% of all lightning events can last several times longer[6]. The period of oscillation for the fundamental mode of the Schumann resonances is $T_1 = \dfrac{2\pi R_E}{c} = 134$ ms, where R_E is Earth's radius, and c is the speed of light. Therefore, we would expect the measured

[6]Although the Schumann resonant frequencies vary by $\sim \pm 1$ Hz, it is interesting to note that, on average, the attenuation rate of Schumann resonant waves appears to be equal to their rate of production by lightning. Otherwise, the Schumann resonances would continue to build up, or eventually die off from attenuation. This implies a connection between the frequency of occurrence of lightning strikes and the conductivity of the ionosphere.

resonant peaks to be wide since, for at least 15-20% of all lightning events, there isn't enough time to generate a complete Schumann resonant wave. The free electrons created by shorter duration lightning strikes will bind to molecules in the atmosphere too quickly.

3.1 Cavity Boundaries

3.1.1 Earth's Ionosphere

The Schumann resonant cavity is bounded by the confining potentials of Earth's surface and its lower ionosphere. The cavity boundaries are "confining" due to their conductivity. The conductivity σ of a material is a proportionality constant that relates the current density $\mathbf{J}$ to the force per unit charge[16]:

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{46}$$

where $\mathbf{E}$ is the electric field, $\mathbf{v}$ is the velocity of the charged particles, and $\mathbf{B}$ is the magnetic field through which the particles move. When the velocity of the charged particles is small, Equation 46 reduces to

$$\mathbf{J} = \sigma\mathbf{E} \tag{47}$$

and is known as Ohm's law.

The conductivity of Earth's ionosphere varies with time and increases rapidly with elevation[2]. The height of the lower ionosphere varies throughout the day and time of year, with daily variations due to a decrease in ion production at night from the solar wind. Seasonal variations are more complex, but are also due to the Sun's influence on Earth's atmosphere. The presence of free electrons at altitudes above 60 km cause the conductivity of the atmosphere to increase to a level that supports the reflection of electromagnetic excitations in the Earth-ionosphere cavity. At 100 km in altitude the atmospheric conductivity is similar to the average conductivity of Earth's surface[17]. Although the lower ionosphere is not a perfect conductor, the Schumann resonances have a low occurrence of penetration through this boundary due to their very low frequencies. The conductivity of the lower-ionosphere at an altitude of 75 km has been measured to be $\sigma = 3.7 \times 10^{-6}$ S·m^{-1}, and electron density at this altitude is found to be $N_e = 400$ electrons/cm^3 [2]. Electromagnetic waves with frequencies greater than around 10 MHz are generally not reflected by the ionosphere since its conductivity is not high enough to support wave reflection at these high frequencies[2].

3.1.2 Earth's Surface

Earth's surface has an average electrical conductivity of about 10^{-3} S·m^{-1} [17], and carries a net negative charge of about 5×10^5 C[17]. The electrical conductivity σ is very sensitive to temperature and composition[18]. Most dry rocks are actually insulators at temperatures below a few hundred degrees Celsius, but fluids found in the rocks pores can substantially increase its conductivity[18]. The average conductivity of crystalline rocks in Earth's crust is in the range of 10^{-6} to 10^{-2} S·m^{-1} [19]. This empirical observation is contained in the following relation (known as Archie's Law)

$$\sigma_{\text{bulk}} = \sigma_{\text{fluid}} P^2 \tag{48}$$

where σ_{bulk} is the conductivity of the rock, σ_{fluid} is the conductivity of the fluid within the rocks pores, and P is the "fractional porosity" of the rock[18]. Geological data suggests that Earth's conductivity at a depth of 20 km (the same as the skin depth δ_S of the lower-ionosphere) varies from 5-15 mS·m^{-1}. The conductivity of Earth's oceans, which accounts for most of the Earth's surface area, is between 2.5-6 S·m^{-1}[19].

An understanding of the conductive properties of Earth's surface helps us understand why we can consider it to be a near-perfect conductor. The extremely low frequencies of the Schumann resonances ensures they will be reflected from Earth's surface.

3.2 Earth's Geomagnetic Field

Earth's rotation provides it with a natural magnetic field. This field can be modeled as being due to a dipole with a magnetic moment[2]

$$M_0 = 8.07 \times 10^{22} \mathrm{A} \times \mathrm{m}^2. \tag{49}$$

Earth's geomagnetic field causes an anisotropy in the conductivity of the ionosphere at altitudes above about 80 km[3, 17]. The geomagnetic field (discussed in more detail later) influences plasma in the ionosphere, thus affecting its relative permittivity. As a result of this influence, the permittivity of the ionosphere is described by the tensor[2]

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{rr} & \varepsilon_{r\theta} & \varepsilon_{r\varphi} \\ \varepsilon_{\theta r} & \varepsilon_{\theta\theta} & \varepsilon_{\theta\varphi} \\ \varepsilon_{\varphi r} & \varepsilon_{\varphi\theta} & \varepsilon_{\varphi\varphi} \end{pmatrix} \tag{50}$$

whose components can be simplified with the condition $\omega \ll v_{\text{eff}}, \omega_H, \omega_0$:

$$\varepsilon_{rr,\theta\theta,\varphi\varphi} = 1 - i\frac{\omega_0^2}{\omega v_{\text{eff}}} + \frac{v_{\text{eff}}^2 + \omega_{r,\theta,\varphi}^2}{\omega_H^2 + v_{\text{eff}}^2}$$
$$\varepsilon_{r\theta,\theta r} = i\frac{\omega_0^2}{\omega v_{\text{eff}}} \pm \frac{v_{\text{eff}}\omega_\varphi + \omega_r\omega_\theta}{\omega_H^2 + v_{\text{eff}}^2}$$
$$\varepsilon_{\theta\varphi,\varphi\theta} = i\frac{\omega_0^2}{\omega v_{\text{eff}}} \pm \frac{v_{\text{eff}}\omega_r + \omega_\theta\omega_\varphi}{\omega_H^2 + v_{\text{eff}}^2}$$
$$\varepsilon_{\varphi r,r,\varphi} = i\frac{\omega_0^2}{\omega v_{\text{eff}}} \pm \frac{v_{\text{eff}}\omega_\theta + \omega_r\omega_\varphi}{\omega_H^2 + v_{\text{eff}}^2} \tag{51}$$

where ω_0 is the frequency of the ionospheric plasma. The vector electron gyrofrequency[7] is given by

$$\vec{\omega_H} = \frac{e\vec{B}_0}{m_e}, \tag{52}$$

where $\omega_H^2 = \omega_r^2 + \omega_\theta^2 + \omega_\varphi^2$, $e = 1.6022 \times 10^{-19}$ C is the charge of an electron, $m_e = 9.11 \times 10^{-31}$ kg is the mass of an electron, and $\vec{B}_0$ is Earth's geomagnetic field (taken to be static). Equation 50 reduces to $\varepsilon_{\theta\varphi}$ when the electron gyrofrequency is small.

The complexities of the ionospheres permittivity arise largely from the fact that the poles of Earth's geomagnetic field are not aligned with Earth's rotational axis. By considering the Earth's geomagnetic field, we now have a better understanding of the reflective properties of the ionosphere.

[7] Gyrofrequency, or Larmor frequency, is the frequency at which a charged particle rotates when moving across a magnetic field

3.3 Atmospheric Conditions within the Schumann Resonant Cavity

Earth's lower atmosphere has a net positive charge of about 5×10^5 C, and balances out the net negative charge distributed over Earth's surface (as mentioned above). More than 90% of the atmospheric charge is located between Earth's surface and a height of 5 km.[6] The region of the atmosphere within the Schumann resonance cavity has a particle density that varies from $2.7 \times 10^{19}\,\mathrm{cm}^{-3}$ at ground level to $3 \times 10^{14}\,\mathrm{cm}^{-3}$ at an altitude of 80 km[2]. The average energy of a Schumann resonant wave in the fundamental mode is $hf_1 = 330 \times 10^{-12}$ eV, far too low to be absorbed by molecules in the Earth's atmosphere. Atmospheric densities at a height of 35 km (close to the average mid-point height of the Schumann resonant cavity) are about 1% of their value at sea level. At 35 km in altitude the atmospheric conductivity increases to 10^{-11} S·m^{-1} [6].

Cosmic rays and Earth's natural radioactivity are responsible for the presence of ions in the atmosphere at heights below 50 km. These ions make the lower atmosphere slightly conductive, but their short lifetimes (on the order of microseconds) means that atmospheric conductivity[8] below about 50 km can usually be neglected[6]. Ion production over land is about twice a high as it is over oceans since bodies of water with large surface area do not have significant radioactivity[6]. Cosmic rays are primarily responsible for atmospheric ions at heights greater than 1 km, where the number of ions produced is a function of solar activity and geomagnetic latitude (the latitude as measured from Earth's geomagnetic pole)[6].

[8]Atmospheric conductivity just above Earth's surface is about 10^{-14} S·m^{-1}, increasing rapidly with altitude[6]

An important property of the Earth-ionosphere cavity is that it is approximately in thermal equilibrium, due to the size of the cavity. Average temperatures are generally between 270-300 K. The condition of thermal equilibrium will be important later when we discuss the Schumann resonances as Bose–Einstein condensates.

4 Source of the Schumann Resonance Electromagnetic Waves

Schumann suggested that lightning strikes on Earth excite the Earth-ionosphere cavity and generate low frequency standing waves[2]. This idea has been supported by experimental data[7, 8, 10, 17, 20]. It is useful to understand some of the properties of lightning, since this is the source of the Schumann resonances.

There are three primary thunderstorm centers responsible for excitations in the Earth-ionosphere cavity; Middle and South America, equatorial Africa, and Southeast Asia, with most activity occurring in the local late afternoon[3, 11]. The electromagnetic radiation emitted from the typical lightning strike is peaked in the 5-10 kHz range with a large low frequency band, and temperatures typically reaching 30,000 K[6, 2]. Lightning events always involve clouds, which can generally be modeled as dipoles, with the base of the cloud being negatively charged and the cloud top positively charged[2].

There are four different types of lightning discharges: upward positive, downward positive, upward negative, and downward negative, with downward negative lightning accounting for around 90% of cloud-to-ground lightning[6]. It is believed that 3/4 of all lightning discharges are cloud-to-cloud discharges[6].

Cloud to ground (or ground to cloud) lightning is rather complicated. The process begins when a leader forms after the cloud's electrostatic charge builds to a certain threshold, dependent on the size of the cloud. The leader is a conducting column that moves in a zig-zag motion from the cloud to the ground at a speed of about 1.5×10^5 m/s and carries a negative charge of a few Coulombs[2]. Before the leader reaches the ground a column of current called a return stroke moves from the ground upward toward the cloud. This return stroke is what is commonly referred to as the “lightning bolt”, or “lightning flash”. The entire process lasts about 1/3 of a second[2]. The vertical region surrounding a lightning strike contains a large number of free electrons, which are stripped from molecules in the atmosphere, due to the large potential difference between the Earth’s surface and clouds. Typically, free electrons in the return stroke move only a few meters before binding with molecules in the atmosphere[6].

Charge that builds up in clouds can be transferred to Earth in several different ways. Dart-leader-return-stroke sequences, continuing currents, and M-components represent the three possible modes of charge transfer to Earth’s surface during lightning strikes. All of these methods have channel conductivities on the order of 10^4 S·m^{-1} [6].

During a leader-return-stroke sequence a conductive path is created between the ground and cloud, which is the charge source, and deposits negative charge along its path[6]. This is followed by a return stroke, following the same path but in the opposite direction (ground-to-cloud), thus neutralizing the negative charge from the leader[6]. Both the leader and return stroke transport negative charge from the cloud to Earth’s surface[6]. “Continuing current” lightning strikes are arcs between the cloud and Earth’s surface, where the arc current is between tens and hundreds of amperes, lasting upwards of hundreds of milliseconds[6]. M-component lightning “requires the existence of a grounded channel carrying a continuing current that acts as a wave-guiding structure.”[6]

5 Energy Density of the Schumann Resonance Electromagnetic Waves

We can determine the photon density of the Schumann resonances by knowing the energy density of the Schumann resonances. Equations 25 and 26 tell us that the modes of the Schumann resonances are mutually orthogonal. Therefore, we can write

$$\int_V \varepsilon_0 \mathbf{E}_n^* \cdot \mathbf{E}_m \, dV = u_E \delta_{n,m} \tag{53}$$

and

$$\int_V \mu_0 \mathbf{H}_n^* \cdot \mathbf{H}_m \, dV = u_H \delta_{n,m} \tag{54}$$

where u_E and u_H are normalized energies, and

$$\delta_{n,m} = \begin{cases} 0 & \text{if } n \neq m \\ 1 & \text{if } n = m \end{cases} \tag{55}$$

is the Kronecker delta function. The energy density (energy per unit volume) of the electric field is then given by

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\frac{D^2}{\epsilon_0} \tag{56}$$

where ϵ_0 is the permittivity of free space, and $D = \epsilon_0 E$ is the electric displacement field. In general, we have $D = \varepsilon_0 E + P$, where P is the electric polarization, which quantifies the induced field due to dipoles in the material. In other words, it is a measure of induced (or permanent) dipole moments in a dielectric material. For our purposes we can neglect P since the electric polarization of Earth's atmosphere is negligible in response to the Schumann resonance waves due to low frequency.

The energy density stored in the magnetic field follows from Equation 54, and is given by

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\mu_0 H^2 \tag{57}$$

where μ_0 is the permeability of free space, and $B = \mu_0 H$. In general, we have $B = \mu_0(H+M)$, where M is the magnetization of the material. Magnetization is a measure of the response of a material to a magnetic field. Since the magnetization of Earth's atmosphere is negligible in response to the Schumann resonance waves, we can neglect M and use $B = \mu_0 H$. The B-field was originally called "magnetic induction"[21], but today it is common to simply call it the magnetic field.

The total energy stored in the electromagnetic field is the sum of Equations 56 and 57

$$u_{\text{total}} = u_E + u_B \tag{58}$$

and represents the energy per unit volume of an electromagnetic field.

By dividing Equation 58 by the energy per photon[9], hf, we can obtain the photon density of the Schumann resonances at the frequency f. Since the Earth-ionosphere cavity satisfies the condition $\dfrac{b-a}{a} \ll 1$ (see Figure 3) the Schumann resonant waves change very slowly in the radial direction and approximately linearly[2]. Therefore, we can treat the Schumann resonances as effectively two dimensional and obtain a photon surface density σ_n for each of the n modes (the case where $n = 1$ is shown below). Due to the horizontal propagation direction of the Schumann resonances we take the surface area of the two dimensional

[9]Here we have switched to the quantum description of an electromagnetic wave in anticipation of our discussion of the Schumann resonances as Bose–Einstein condensates.

Schumann cavity to be

$$A_{\mathrm{SR}} = 4\pi(R_E + h/2)^2 = 5.2 \times 10^{18}\,\mathrm{cm}^2 \tag{59}$$

where $R_E = 6400 \times 10^5$ cm is Earth's radius and $h = 80 \times 10^5$ cm is the height above Earth's surface at which the Schumann resonances reflect off the lower ionosphere. Nickolaenko and Hayakawa developed a heuristic model of the Schumann resonance fields that we use to find the energy densities of the Schumann resonance waves for the first five modes[2]. The first five resonant modes have electric fields peaked at

$$\begin{aligned} E_1 &= 130 \text{ mV/m} \\ E_2 &= 60 \text{ mV/m} \\ E_3 &= 80 \text{ mV/m} \\ E_4 &= 75 \text{ mV/m} \\ E_5 &= 60 \text{ mV/m} \end{aligned} \tag{60}$$

and magnetic fields peaked at

$$\begin{aligned} H_1 &= 195\,\mu\text{A/m} \\ H_2 &= 230\,\mu\text{A/m} \\ H_3 &= 180\,\mu\text{A/m} \\ H_4 &= 130\,\mu\text{A/m} \\ H_5 &= 150\,\mu\text{A/m} \end{aligned} \tag{61}$$

These values correspond well with experimental measurements[4, 7, 8, 9, 20]. After dividing Equation 58 by the height of the cavity, we obtain the energy surface densities for the first five Schumann resonance modes:

$$
\begin{aligned}
u_{\sigma,1} &= 7.9 \times 10^{-13}\ \mathrm{J/cm^2} \\
u_{\sigma,2} &= 3.9 \times 10^{-13}\ \mathrm{J/cm^2} \\
u_{\sigma,3} &= 3.9 \times 10^{-13}\ \mathrm{J/cm^2} \\
u_{\sigma,4} &= 2.8 \times 10^{-13}\ \mathrm{J/cm^2} \\
u_{\sigma,5} &= 2.4 \times 10^{-13}\ \mathrm{J/cm^2}
\end{aligned}
\tag{62}
$$

We recognize the energy of each wave to be

$$E_n = hf_n \tag{63}$$

where $h = 6.63 \times 10^{-34} \mathrm{J} \cdot \mathrm{s}$. Finally, we divide Equations 62 by Equation 63 (using the corresponding frequency) to obtain the photon surface densities for the first five modes of the Schumann resonances

$$
\begin{aligned}
\sigma_1 &= 1.5 \times 10^{20}\,\mathrm{cm^{-2}} \\
\sigma_2 &= 4.2 \times 10^{19}\,\mathrm{cm^{-2}} \\
\sigma_3 &= 2.8 \times 10^{19}\,\mathrm{cm^{-2}} \\
\sigma_4 &= 1.6 \times 10^{19}\,\mathrm{cm^{-2}} \\
\sigma_5 &= 1.1 \times 10^{19}\,\mathrm{cm^{-2}}
\end{aligned}
\tag{64}
$$

6 Coherence of the Schumann Resonances

Lasers provide the most well known example of coherent radiation. However, the Schumann resonances may be the most common form of coherent radiation on Earth. Radiation inside a cavity can be said to be coherent if the phase relationship of all the waves is constant in space and time. For measurements of frequency f taken at two spatially separated locations, L_1 and L_2, the coherence γ is given by[10]

$$\gamma(f) = \sqrt{\frac{|L_1(f)^* L_2(f)|^2}{|L_1(f)|^2 |L_2(f)|^2}} \tag{65}$$

where L_1^* is the complex conjugate of L_1. We would have perfect coherence if $\gamma^2 = 1$.

The coherence of the the Schumann resonances has been found to be around 94–97% for the fundamental mode ($\sim$ 8 Hz), and 82–90% for the second mode ($\sim$ 13 Hz), where measurements of the horizontal magnetic field were made at two locations separated by 1100 km[10]. The large separation distance of these two measurement locations emphasizes in a very convincing way that the Schumann resonances are indeed coherent. These experiments involved placing large magnetic coils underground at each location with axes aligned within 0.5° of the geographic North-South and East-West directions. Other measurements of the Schumann resonances found the coherence to be 90% and higher for the first two modes[22].

Even though the level of coherence of the Schumann resonances has been measured experimentally, what has not been studied is the mechanism behind this coherence. The free electrons generated during a lightning event are acted on by the Schumann resonance electromagnetic waves that already exist and forced into coherence. This effect is similar to the generation of laser light. Each electron has a component of acceleration in phase with the Schumann resonances that surround Earth. As a result, these accelerating electrons emit radiation in phase with the Schumann resonances. Experiments have shown that the Earth-ionosphere cavity is filled with long-lasting ions ($\sim$100 s) created from the Earth's natural radioactivity and cosmic rays[6], but these ions are too heavy to be acted on to produce Schumann resonant waves.

The period of oscillation for the fundamental mode of the Schumann resonance is about 134×10^{-3} s. This is found by simply dividing the circumference of the Earth by the speed of light. So, we can see that this is less than the time (about 1/3 second) it takes for a complete single stroke lightning strike. However, multiple stroke lightning lasts much longer[6]. It is multiple stroke lightning that produces free electrons with lifetimes long enough for complete Schumann resonance waves to be created. Perfectly coherent Schumann resonance waves inside a perfectly symmetrical cavity with perfectly conducting boundaries would show sharp peaks at the eigenfrequencies. However, the data shows wide peaks[2]. We believe this is due to accelerated free electrons that bind to molecules faster than the period of oscillation of the Schumann resonance wave that acts on it.

7 The Schumann Resonances as Bose–Einstein Condensates

First, we will give a brief overview of Bose–Einstein condensates. Then, we will discuss the necessary conditions for the onset of a BEC and show that the Schumann resonances meet these conditions.

7.1 Bose–Einstein Condensates

A Bose–Einstein condensate (BEC) is a condensed gas of particles in a confining potential, all of which exist in their ground state and can be described by a single collective wavefunction[23]. The particles that form a BEC act collectively as a coherent wave, and manifest themselves as a macro quantum state that is distinct from any other form of matter or radiation. A BEC can form when the thermal de Broglie wavelength of its constituent particles is on the order of their separation distance. This phenomenon was predicted by Satyendra Bose and Albert Einstein in 1925 and first observed experimentally in 1995[24]. The experimental observation of a BEC in 1995 involved a condensed gas of trapped rubidium atoms cooled to 170 nK. Since that time, several other experimental demonstrations have successfully created BEC's[25, 26, 27, 28, 29]. There are only a small number of massive bosons that can become a BEC. These include helium-4, rubidium, and the carbon-12 nucleus. A Bose–Einstein condensate has some similarities with lasers; both are coherent sources of particles, and require a cavity to form. The main difference is that a Bose–Einstein condensate is in thermal equilibrium[30].

A Bose–Einstein condensate of massless bosons are fundamentally different from a massive BEC. The main reason is that massless particles do not interact significantly with each other, whereas massive particles do. In 2010 a 2-dimensional photonic BEC was observed for the first time[30]. This was a 2-dimensional BEC because of the very close boundary walls (small third dimension) compared to the other two dimensions. This was accomplished in a dye-filled optical microcavity bounded by mirrors and pumped with a laser. The cavity boundaries acted as a confining potential. The ground state mode occurred even when the laser pump spot was spatially displaced within the cavity. We can see some similarities between this experiment and the Schumann resonances. First, the Earth-ionoshere cavity is an effective 2-dimensional cavity filled with an approximately homogeneous dielectric. Second, the Earth's surface and the lower ionosphere act as the cavity's confining potential. Lastly, the Earth-ionosphere cavity is constantly "pumped" with lightning strikes that are spatially (and temporally) displaced. As mentioned above, although lightning strikes are not a coherent source of light, many of the electromagnetic waves that are generated from lightning may become coherent due to the immersion of the free electrons in a very dense BEC (see Equation 64).

It is important to point out some of the distinctions between the two types of particles, bosons and fermions. Bosons are particles that obey Bose–Einstein statistics and Fermions are particles that obey Fermi–Dirac statistics. All bosons have integer spin, whereas fermions have half-integer spin. More than one boson can occupy the same quantum state, but the Pauli exclusion principle limits only one fermion from occupying the same quantum state. The wavefunction for a collection of bosons is symmetric and remains unchanged upon exchange of two particles. For example, a two particle wavefunction consisting of bosons, with one having a probability of being located a position $\mathbf{r_1}$ and the other having a probability of being located a position $\mathbf{r_2}$ is written as

$$\psi(\mathbf{r_1}, \mathbf{r_2}) = \psi(\mathbf{r_2}, \mathbf{r_1}) \qquad \text{(bosons)} \tag{66}$$

A collection of fermions, on the other hand, will have an anti-symmetric wavefunction under particle exchange. The exchange of two fermions is characterized by the property

$$\psi(\mathbf{r_1}, \mathbf{r_2}) = -\psi(\mathbf{r_2}, \mathbf{r_1}) \qquad \text{(fermions)} \tag{67}$$

Combinations of an even number of fermions, called mesons, are composite bosons, including Cooper pairs (coupled pairs of electrons), which give rise to the phenomenon of superconductivity. Both massive and massless bosons[10] can theoretically form a BEC if the necessary conditions of coherence and critical density/temperature are met.

The quantum field operators $\hat{\psi}(\mathbf{r})$ and $\hat{\psi}^\dagger(\mathbf{r'})$, also called the "creation" and "annihilation" operators, satisfy the the Bose commutation relation

$$\left[\hat{\psi}(\mathbf{r}), \hat{\psi}^\dagger(\mathbf{r'})\right] = \delta(\mathbf{r} - \mathbf{r'}) \tag{68}$$

where the r.h.s. is the Dirac delta function, and $\hat{\psi}^\dagger$ is the Hermitian conjugate of $\hat{\psi}$. These operators can be used to describe all observables, including the density operator

$$\hat{n}(\mathbf{r}) = \hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r}). \tag{69}$$

To describe a Bose–Einstein condensate in terms of these operators, we write the second factor on the r.h.s. of Equation 69 as[31]

$$\hat{\psi}(\mathbf{r}) = \langle\hat{\psi}(\mathbf{r})\rangle + \tilde{\psi}(\mathbf{r}) \tag{70}$$

[10] Massless bosons include the photon, graviton, and gluon.

The first term on the r.h.s. is the Bose macroscopic wavefunction, used as the "order parameter" for Bose superfluid phase transitions and is defined as

$$\langle\hat{\psi}(\mathbf{r})\rangle \equiv \Phi(\mathbf{r}) = \begin{cases} 0, & \text{if } T > T_{\mathrm{BEC}}, \\ \neq 0, & \text{if } T < T_{\mathrm{BEC}}. \end{cases} \tag{71}$$

where T_{BEC} is the critical temperature at which we see the onset of a Bose–Einstein condensate. In order for Equation 71 to be finite a small symmetry-breaking perturbation must be introduced[31]

$$\hat{H}_{\mathrm{SB}} = \lim_{\eta\to 0}\int d\mathbf{r}\left[\eta(\mathbf{r})\hat{\psi}^{\dagger}(\mathbf{r}) + \eta^{*}(\mathbf{r})\hat{\psi}(\mathbf{r})\right]. \tag{72}$$

In other words, a Bose–Einstein condensate is defined as a broken-symmetry order parameter.[11]. In contrast to a Fock state, where N (the number of particles) is fixed, Equation 71 is a coherent state with a "clamped" value of the phase[31].

The evolution of an atomic Bose–Einstein condensate is described by the Gross-Pitaevskii (GP) equation[32]

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + (V_0 + V_{NL})\psi \tag{73}$$

where ψ is the order parameter (Bose wavefunction) from Equation 71, $V_0 = V(\mathbf{r})$ is the external confining potential, and V_{NL} is the interaction strength of the constituent bosons (the subscript NL refers to this term being non-linear). For a photonic BEC we can neglect the V_{NL} term in Equation 73, since photons are very weakly interacting, and write

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V_0\psi \tag{74}$$

which is the familiar form of the Schrödinger equation, where m is the effective photon mass given in Equation 75 below.

[11] In fact, all conservation laws can be defined using broken-symmetry order parameter theories[31]

From a quantum perspective, the Schumann resonances are a high density collection of bosons (photons) existing in the ground-state. For the Schumann resonances (massless photons/bosons) to become a Bose–Einstein condensate we must satisfy the conditions of coherence and critical density. An additional constraint placed on the Schumann cavity (which we treat as effectively 2-dimensional), is that a 2-dimensional BEC can only exist within a confining potential, unless the temperature is absolute zero[33]. Earth's surface and the lower ionosphere act as the confining potential for the Schumann photons, but these two boundaries also provide an effective photon mass[34]. The Earth-ionosphere cavity can be considered as being filled with an ideal gas of photons with an effective mass

$$m_{\text{ph}} = \frac{hf}{c^2} \approx 5.78 \times 10^{-50} \text{ kg}, \tag{75}$$

where h is Planck's constant ($h = 6.626 \times 10^{-34}\,\text{J}\cdot\text{s}$), and c is the speed of light ($c = 2.99 \times 10^8$ m/s).

Einstein showed that an atomic BEC will form when the de Broglie wavelength $\lambda = h/p$ of atoms is comparable to the interatomic spacing, where p is the momentum of the particle[23]. The corresponding equation for photons is $\lambda = hc/E$, where the photons momentum is given by $p = E/c$. Since the fundamental mode of the Schumann resonances has a wavelength equal to the "length" of the cavity ($\sim$ 40,000 km) we see that each of these Schumann photons necessarily overlaps. A three dimensional atomic gas with density n will begin to condense into a single quantum state when we have the phase space density $n\lambda^3$[35]. But, it was recently shown[30] that a 2-dimensional collection of photons will condense into a BEC when the phase space density is greater than unity

$$n\lambda_{\text{th}}^2 > 1, \tag{76}$$

where n is the surface density of photons confined to the Schumann resonance cavity, and λ_{th} is the de Broglie wavelength that corresponds with thermal motion in the resonator surface[30]. The thermal wavelength λ_{th} is given by

$$\lambda_{\text{th}} = \frac{h}{\sqrt{2\pi m_{\text{ph}} k_{\text{B}} T}} \tag{77}$$

where k_{B} is Boltzmann's constant, T is the absolute temperature, and m_{ph} is the effective photon mass given in Equation 75. For the Earth-ionosphere cavity we get $\lambda_{\text{th}} = 17.1$ m. Using the results from Equation 64 we find that (for the fundamental mode) $n\lambda_{\text{th}}^2 = 4.4 \times 10^{22} \gg 1$. This result is not too surprising when we consider that Equation 76 makes no assumptions about the wavelength to cavity length ratio, λ/ℓ. For the Earth-ionosphere cavity this ratio is 1, and thermal motion in the resonant plane is not important. Equation 77 can therefore be considered relevant for resonant cavity's where $\lambda/\ell \ll 1$. Once a Bose–Einstein condensate forms, the energy of its constituent particles is defined in terms of the Heisenberg uncertainty principle $\Delta x \Delta p \geq \hbar/2$ [35].

7.2 Critical Temperature and Critical Density Requirements

Massive bosons must meet a critical temperate requirement to become a BEC. The critical temperature for a 2-dimensional Bose–Einstein condensate is given by[33]

$$T_C = \frac{hf}{k_B}\left(\frac{N}{\zeta(2)}\right)^{1/2} \tag{78}$$

where k_B is Boltzmann's constant, h is Planck's constant, f is the fundamental frequency, N is the total particle number, and $\zeta(s)$ is the Riemann zeta-function ($\zeta(2) = \frac{\pi^2}{6}$), defined as $\zeta(s) = \sum_{n=1}^{\infty} 1/n^s$. The corresponding equation for the critical temperature of a 3-

dimensional BEC is

$$T_C = \frac{hf}{k_B}\left(\frac{N}{\zeta(3)}\right)^{1/3} \tag{79}$$

If we use the number density of the fundamental mode as given by Equation 86 below, then Equation 78 gives us $T_C = 8.4 \times 10^9$ K. So, it is clear why temperature is not one of the defining parameters for the onset of a photonic BEC in most situations.

To connect the critical temperature to the critical density we use[33]

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^2 \tag{80}$$

for the 2-dimensional case. In 3 dimensions we would use

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^3 \tag{81}$$

If we solve Equation 80 for N_0 we get $N_1 = 7.8 \times 10^{38}$ photons in the fundamental mode of the Schumann resonances. This value is exactly the same as that given by Equation 86, and is not surprising since $\frac{T}{T_c} \ll 1$.

Whereas the critical temperature is the defining parameter for the onset of a massive BEC, it is the density that is the defining parameter for the onset of a massless 2-dimensional BEC. The equation that determines the critical number density for a 2-dimensional BEC is given by[30]

$$N_c = \frac{\pi^2}{3}\left(\frac{k_B T}{\hbar\Omega}\right)^2 \tag{82}$$

where T is the absolute temperature of the cavity, $\hbar = \frac{h}{2\pi}$ is Planck's reduced constant, and Ω is the trapping frequency given by

$$\Omega = \frac{c}{\sqrt{DR/2}} \tag{83}$$

Here, D is the height of the cavity, c is the speed of light, and R is the radius of curvature of the spherical cavity. Using $D = 80$ km, and $R = 6400$ km we get $\Omega = 591$ Hz. If we substitute this value into Equation 82 (with $T = 300$ K) we get

$$N_c = 1.45 \times 10^{22} \tag{84}$$

The total number of photons in the Schumann resonant cavity at the nth modal frequency if given by

$$N_n = \sigma_n 4\pi (R_E + h/2)^2 \tag{85}$$

where σ_n is the photon surface density of the nth mode (Equation 64), and the remaining factors on the r.h.s. are the total surface area of the 2-dimensional Schumann resonant cavity. By taking the results from Equation 64 and plugging in to Equation 85 we obtain the total number of photons that surround Earth for the first five Schumann modes:

$$\begin{aligned} N_1 &= 7.8 \times 10^{38} \\ N_2 &= 2.2 \times 10^{38} \\ N_3 &= 1.4 \times 10^{38} \\ N_4 &= 8.2 \times 10^{37} \\ N_5 &= 5.5 \times 10^{37} \end{aligned} \tag{86}$$

When compared to Equation 84 we see that we clearly meet the critical density requirement for the onset of a BEC by approximately 15–16 orders of magnitude.

8 Conclusion

The first Bose–Einstein condensate composed of photons was reported in 2010[30]. We are the first to propose that the Schumann resonances are Bose–Einstein condensates of extremely low frequency photons. These resonances are confined within a cavity bordered by Earth's surface and its lower ionosphere. All other studies of the Schumann resonances have used Maxwell's equations, without consideration of quantum mechanical effects.

It is remarkable that the quantum aspects of the Schumann resonances have never been studied before. Perhaps this is due to the belief that quantum mechanics only applies to small scale phenomenon. Wave-particle duality forces us to accept that all electromagnetic waves can also be described as particles—photons. The extremely large size of the Schumann resonant waves ($\sim$ 40,000 km for the fundamental mode) may have caused other researchers to subconsciously suppress any consideration of them from a quantum perspective.

In our research, each of the two main requirements for the onset of a BEC were discussed: coherence, and critical density/temperature. We also pointed out that two dimensional cavities, which effectively describes the Earth-ionosphere cavity, are further bound by the condition that a confining potential must exist between cavity boundaries if a Bose–Einstein condensation is to be achieved. This condition was shown to be met by Earth's surface, and the lower-ionosphere.

Although the coherence of the Schumann resonances has already been studied, we are the first to make the connection with this being one of the necessary conditions for the onset of a BEC. We showed that the first five modes of the Schumann resonances meet the condition for coherence, confirming measurements of coherence with measurements made at two locations separated by over 1000 kilometers.

Lastly, we calculated the number densities for the first five modes of the Schumann resonances and showed that they far exceed the necessary densities needed to form a BEC. Based on the high photon densities for each of the first five modes, it is likely that higher modes of the Schumann resonances may also meet the density requirements for Bose–Einstein condensation, although we did not study these higher modes.

During our research we kept coming back to the question of when the Schumann resonances first began. Clearly, at some point during Earth's evolution lightning strikes produced radiation at just the right frequencies to excite the Earth-ionosphere cavity at the Schumann resonant frequencies. Then, the electric field of these Schumann resonant waves accelerated free electrons in the atmosphere, producing waves in phase with each other. Eventually, the photon density became high enough that Bose–Einstein condensation was achieved. The entire process has probably sustained itself ever since, although it would be interesting to consider the effects of Earth's recurring geomagnetic pole reversals on the Schumann resonances. Another area that could be explored is determining whether the Schumann resonances have an effect, or cause, multiple stroke lightning.

This research represents a critical step forward in understanding the Schumann resonances at a new level. Future studies on treating a Schumann resonance as a Bose–Einstein condensate may benefit from this research.

References

[1] W. O. Schumann, “Über die strahlungslosen Eigenschwingungen einer leitenden Kugel, die von einer Luftschicht und einer Ionosphärenhülle umgeben ist,” Zeitschrift für Naturforsch, vol. 7a, pp. 149–154, 1952.

[2] A. Nicholaenko and M. Hayakawa, Resonances in the Earth-Ionosphere Cavity, vol. 19 of Modern Approaches In Geophysics. London: Kluwer Academic Publishers, 2002.

[3] D. Labendz, “Investigation of Schumann resonance polarization parameters,” J. Atmos. Terr. Phys., vol. 60, pp. 1779–1789, 1998.

[4] D. Beamish and A. Tzanis, “High resolution spectral characteristics of the Earth–ionosphere cavity resonances,” J. Atmos. Terr. Phys., vol. 48, no. 2, pp. 187–203, 1986.

[5] M. Balser and C. A. Wagner, “Observations of Earth-ionosphere cavity resonances,” Nature, vol. 188, pp. 638–641, November 1960.

[6] V. A. Rakov and M. A. Uman, Lightning: Physics and Effects. Cambridge University Press, 2003.

[7] K. Sao, M. Yamashita, S. Tanahashi, H. Jindoh, and K. Ohta, “Experimental investigations of Schumann resonance frequencies,” J. Atmos. Terr. Phys., vol. 35, pp. 2047–2053, 1973.

[8] G. Satori, J. Szendroi, and J. Vero, “Monitoring Schumann resonances - I. methodology,” J. Atmos. Terr. Phys., vol. 58, no. 13, pp. 1475–1481, 1996.

[9] G. G. Belyaev, A. Y. Schekotov, A. V. Shvets, and A. P. Nickolaenko, "Schumann resonances observed using Poynting vector spectra," J. Atmos. Terr. Phys., vol. 61, pp. 751–763, 1999.

[10] P. M. Holtham and B. J. McAskill, "The spatial coherence of Schumann activity in the polar cap," J. Atmos. Terr. Phys., vol. 50, no. 2, pp. 83–92, 1988.

[11] H. Volland, ed., CRC Handbook of Atmospherics, vol. 1. CRC Press, 1982.

[12] D. D. Sentman, "Schumann resonance spectra in a two-scale-height Earth-ionosphere cavity," Journal of Geophysical Research, vol. 101, pp. 9479–9487, April 1996.

[13] J. R. Wait, Electromagnetic Waves in Stratified Media. IEEE/OUP Series on Electromagnetic Wave Theory, Oxford University Press, 1962.

[14] P. V. Bliokh, A. P. Nickolaenko, and Y. F. Filippov, Schumann resonances in the Earth-ionosphere cavity. Oxford, New York, Paris, 1980.

[15] J. D. Jackson, Classical Electrodynamics. Wiley, 1999.

[16] D. J. Griffiths, Introduction to Electrodynamics. Prentice Hall, 1999.

[17] M. J. Rycroft, "Some effects in the middle atmosphere due to lightning," J. Atmos. Terr. Phys., vol. 56, pp. 343–348, 1994.

[18] J. F. Hermance, "Electrical conductivity models of the crust and mantle," American Geophysical Union, 1995.

[19] S. Maus, "Electromagnetic ocean effects," tech. rep., National Geophysical Data Center, NOAA E/GC1, 325 Broadway, Boulder, CO 80305-3328, 2003.

[20] A. P. Nickolaenko, M. Hayakawa, and Y. Hobara, "Temporal variations of the global lightning activity deduced from the Schumann resonance data," J. Atmos. Terr. Phys., vol. 58, no. 15, pp. 1699–1709, 1996.

[21] J. C. Maxwell, "A dynamical theory of the electromagnetic field," Phil. Trans. R. Soc. Lond., vol. 155, pp. 459–512, 1865.

[22] R. J. Dinger and J. A. Goldstein, "Spatial coherence measurements and evaluation of a noise reduction technique for ambient noise from 0.3 to 40 [h]z," tech. rep., Naval Research Laboratory, Washington, D.C., October 1980.

[23] A. Einstein, "Quantentheorie des einatomigen idealen Gases," Zweite Abhandlung. Sitz.ber. Preuss. Akad. Wiss., vol. 1, pp. 3–14, 1925.

[24] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, "Observation of Bose–Einstein condensation in a dilute atomic vapor," Science, vol. 269, no. 5221, pp. 198–201, 1995.

[25] K. B. Davis, M. . Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, and W. Ketterle, "Bose–Einstein condensation in a gas of sodium atoms," Phys. Rev. Lett., vol. 75, pp. 3969–3973, 1995.

[26] C. C. Bradley, C. A. Sackett, and R. G. Hulet, "Bose–Einstein condensation of lithium: Observation of limited condensate number," Phys. Rev. Lett., vol. 78, pp. 985–989, 1997.

[27] S. Jochim, M. Bartenstein, A. Altmeyer, G. Hendl, S. Riedl, C. Chin, J. H. Denschlag, and R. Grimm, "Bose–Einstein condensation of molecules," Science, vol. 302, pp. 2101–2103, 2003.

[28] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, and et. al., "Bose–Einstein condensation of exciton polaritons," Nature, vol. 443, pp. 409–414, 2006.

[29] R. Balili, V. Hartwell, D. Snoke, L. Pfeiffer, and K. West, "Bose–Einstein condensation of microcavity polaritons in a trap," Science, vol. 316, pp. 1007–1010, 2007.

[30] J. Klaers, J. Schmitt, F. Vewinger, and M. Weitz, "Bose–Einstein condensation of photons in an optical microcavity," Nature, vol. 468, pp. 545–548, November 2010.

[31] A. Griffin, T. Nikuni, and E. Zaremba, Bose-condensed gases at finite temperatures. Cambridge University Press, 2009.

[32] J. M. P. Carmelo, P. D. Sacramento, J. M. B. L. dos Santos, and V. R. Vieira, eds., Strongly correlated systems, coherence and entanglement. World Scientific, 2007.

[33] S. Burger, F. S. Cataliotti, C. Fort, P. Maddaloni, F. Minardi, and M. Inguscio, "Quasi-2D Bose–Einstein condensation in an optical lattice," Europhys. Lett., vol. 57, pp. 1–6, January 2002.

[34] J. Klaers, F. Vewinger, and M. Weitz, "Thermalization of a two-dimensional photonic gas in a 'white wall' photon box," Nature Physics, vol. 6, pp. 512–515, July 2010.

[35] R. A. Serway, C. J. Moses, and C. A. Moyer, Modern Physics. Brooks/Cole, 3 ed., 2005.