A Model of the Determinants of Effort

Carl M. Campbell III*
Northern Illinois University

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Abstract

This study derives an expression for effort from utility-maximizing behavior on the part of workers, whose utility depends on consumption, effort, and the ratio between their wage and their perceived fair wage. Unlike many shirking models, this study treats effort as a continuous variable rather than as a dichotomous choice. Effort is shown to depend on wages at a worker's current firm, wages at other firms, the ratio between a worker's wage and perceived fair wage, unemployment benefits, the unemployment rate, and the firm's monitoring intensity.

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*Corresponding author: Carl M. Campbell III, Dept. of Economics, Northern Illinois University, DeKalb, IL 60115, U.S.A.; phone 815-753-6974; fax 815-752-1019; e-mail carlcamp@niu.edu.

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I. Introduction

Efficiency wage models are based on the premise that workers' productivity depends positively on their wages. One explanation for the positive relationship between wages and productivity is that higher wages induce more effort from workers. Their effort may depend on wages either because wages affect the cost of job loss, as in Shapiro and Stiglitz (1984), or because wages affect their morale, as in Akerlof (1982, 1984) and Akerlof and Yellen (1990). Given the importance of efficiency wage models, it is desirable to show how an equation describing effort can be derived from microeconomic principles.

This study develops a model of effort, in which effort is a continuous variable. Effort is determined from the utility-maximizing behavior of workers, whose utility depends on their consumption, their effort, and an interactive term between their effort and the ratio of their actual wage to their perceived fair wage. In addition, the probability that a worker is dismissed depends negatively on his or her effort. Workers choose the optimal level of effort in the current period to maximize their utility over an infinite horizon. The solution to the model shows that effort depends on wages at a worker's current firm, wages at others firms, the ratio between a worker's actual wage and perceived fair wage, unemployment benefits, the unemployment rate, and the firm's monitoring intensity.

Models of effort in which shirkers are dismissed have also been developed by Shapiro and Stiglitz (1984), Bowles (1985), and Sparks (1986). The most widely cited model in this literature is probably that of Shapiro and Stiglitz. In their model, effort can take on only two possible values, as workers provide either zero effort or a fixed positive level of effort. Workers who provide zero effort face a positive probability of dismissal, and workers who provide full effort are not dismissed. Since effort is a dichotomous variable, this model cannot be used to predict how effort is affected by other variables (e.g., wages, wages elsewhere, or the unemployment rate), as a change in one of these variables usually results in no change in effort. At some point, however, an infinitesimal change in these variables causes a change from zero effort to full effort, or vice-versa. In contrast, the present study treats effort as a continuous variable between 0 and 1, so that effort can take on an infinite number of possible values. As effort rises, the probability of dismissal (which is also continuous) falls. Because effort is a continuous variable, a small change in wages, wages elsewhere, unemployment benefits, the unemployment rate, or the firm's monitoring intensity will result in a small change in effort.

Two other models in which shirkers are dismissed are Bowles (1985) and Sparks (1986). Bowles analyzes effort in a two-period model and does not derive an explicit equation for effort. The present study, on the other hand, derives an explicit expression for effort in an infinite-period framework. The effort equation derived in Sparks has a positive second derivative of effort with respect to the wage and a negative cross derivative of effort with respect to the wage and the expected utility of an unemployed individual.[1] The former result means that the relationship between wages and effort is convex, and the latter result implies that wages affect effort more when unemployment is high.[2] In contrast, the present study derives an equation for effort with a negative second derivative of effort with respect to the wage and a negative cross derivative of effort with respect to the wage and the unemployment rate (so that wages affect effort less when unemployment is high). In addition, Sparks assumes that monitoring is fixed, while the present model allows firms to vary their monitoring.

Shirking models in which shirkers receive a monetary penalty instead of being dismissed have been developed by Alexopoulos (2002) and Felices (2002). In these models, the firm sets a required minimum effort level and pays part of the wage at the beginning of the period and part at the end. If a worker does not exert the minimum acceptable level of effort, there is a positive probability that he or she is caught, in which case the worker does not receive the payment at the end of the period. Workers can provide any level of effort, but, as a result of this payment scheme, they choose to provide either zero effort or the minimum acceptable level of effort. Thus, effort is effectively a dichotomous variable in these models. As in Shapiro and Stiglitz (1984), the fact that effort takes on only two values means that a change in other variables results in either no change in effort or a discrete change in effort. In addition, effort is not affected by labor market conditions in these models, since shirkers are not dismissed. As a result, effort does not depend on wages elsewhere, unemployment benefits, or the unemployment rate. In contrast, effort depends on all of these factors in the present model.

II. Previous Research on the Determinants of Effort

In several versions of efficiency wage theory, higher wages increase workers' effort. In Shapiro and Stiglitz (1984), a higher wage induces less shirking by raising the cost of job loss. Effort also depends positively on the unemployment rate in their model, since higher unemployment raises the cost of job loss.[3] In other studies, the relationship between wages and effort occurs for reasons that are more psychological in nature. In Akerlof (1982, 1984), a higher wage induces greater gratitude and loyalty from workers, who respond by working harder. Akerlof and Yellen (1990) present evidence that effort depends on the relationship between the wage and the wage a worker perceives as fair, particularly if the worker is paid less than what he or she feels is fair.

The hypothesis that effort depends on wages, unemployment, and the relationship between wages and perceived fair wages is supported by empirical work, survey evidence, and experimental studies.

While effort is difficult to observe, several studies provide evidence for an empirical relationship between wages and productivity or between wages and dismissals. Wadhwani and Wall (1991) and Levine (1992) find that a firm's relative wage has a positive and significant effect on, respectively, sales per worker and output per worker.[4] In addition, Cappelli and Chauvin (1991) find a negative relationship between a plant's relative wage and dismissal rate.

Empirical work also suggests that unemployment and productivity are positively related. Rebitzer (1987), Weisskopf (1987), and Green and Weisskopf (1990) show that labor productivity within disaggregated industries depends positively on the aggregate unemployment rate. In addition, Wadhwani and Wall (1991) find that the unemployment rate has a positive and significant effect on firms' sales (controlling for employment). The proposition that effort depends positively on the unemployment rate is also supported by survey evidence, as 72% of respondents in Blinder and Choi's (1990) survey and 90% of respondents in Agell and Lundborg's (1995) survey thought that higher unemployment would lead to higher effort.[5]

Survey evidence also suggests that there is a strong link between fairness and effort. Responses to the surveys of Kaufman (1984), Blinder and Choi (1990), Agell and Lundborg (1995), Campbell and Kamlani (1997), and Bewley (1999) show that managers believe that workers' perception of fairness has a large effect on their effort, particularly if workers feel they are paid less than what is fair. The responses to these surveys suggest that the fair wage depends on the wages of other workers, the firm's profits, and the workers' own past wage (which means that effort depends on wage changes, particularly in the downward direction).

The importance of fairness is also demonstrated by experimental studies. Fehr, Kirchsteiger, and Riedl (1993) and Fehr and Falk (1999) conducted experiments in which some participants acted as employers and others acted as workers.[6] They found that higher wages were correlated with higher effort, both when higher effort was more costly to the worker and when lower effort was more costly. The latter result suggests that workers are willing to "punish" firms paying low wages, even if it harms them. Thus, their results provide evidence for both positive and negative reciprocity.

III. Overview of the Model

Section II presents evidence that effort depends on wages, unemployment, and the relationship between wages and perceived fair wages. Sections III-IV demonstrate how these predictions can be obtained from a model based on utility maximization. This section presents an overview of the model's assumptions.

In this model, workers choose a level of effort (e, with $0 \le e \le 1$) to maximize the present value of their utility over an infinite horizon. Mathematically, a worker seeks to maximize

$$\mathrm{E}[U^*] = E\left[\sum_{t=1}^{\infty} \frac{1}{(1+\delta)^{t-1}} U[c_t, e_t]\right] \tag{1}$$

$$\text{s.t.} \qquad \sum_{t=1}^{\infty} \frac{1}{\prod_{j=1}^{t-1}(1+r_j^e)} c_t = \sum_{t=1}^{\infty} \frac{1}{\prod_{j=1}^{t-1}(1+r_j^e)} \left[LI_t^e \Pr[Emp_t] + B(1-\Pr[Emp_t])\right] + A_1 ,$$

where c is consumption, δ is the discount rate, r^e is the expected interest rate, LI_t^e represents expected labor income in period t, B represents unemployment benefits, $\Pr[Emp_t]$ is the probability that the worker is employed in period t, and A_1 is the worker's initial wealth. In deriving a solution for effort, the following assumptions are made:

1. The utility of a worker in period t can be expressed as

$$U[c_t, e_t] = \ln[c_t] + \alpha e_t - \eta e_t^2 + f(W_t / W_t^F)e_t \qquad \text{with } \eta > 0,\ f > 0, \text{ and } f' > 0.$$

   In this equation, W represents the wage at a worker's current firm, and W^F represents the worker's perceived fair wage. The parameter α can be either positive or negative, depending on whether a worker receives satisfaction from work at lower levels of effort or finds any level of effort to be utility-reducing. However, the assumption that η>0 means that the marginal utility of effort becomes negative at higher levels of effort.

   The assumption that $f'(W/W^F)$ >0 implies that the disutility of effort rises as the ratio of a worker's actual wage to his or her perceived fair wage falls. In other words, the disutility of effort is higher for workers paid less than their perceived fair wage than for workers who believe they are being paid fairly. This relationship between the disutility of effort and the perceived fair wage is what is predicted by the literature on reciprocity. The exact form of the function $f(W/W^F)$ is left open to interpretation, since this relationship may be nonlinear.[7] It should be noted that while this study shows one possible way to integrate fairness into a model of effort, there may be other ways to incorporate fairness into the utility function of workers.

2. An employed worker provides one unit of labor per period.

3. The probability that a worker is dismissed in period t (PD_t) depends on the firm's monitoring intensity (m, with $0 \leq m \leq 1$) and on the worker's effort. Mathematically, this probability is expressed as

$$PD_t = m(1 - e_t)^2.$$

One way of viewing this equation is to think of m as the probability that the firm will monitor an individual worker and to think of e as the proportion of time that an employee actually works. If a firm observes a monitored worker twice and dismisses the worker if he or she is not working both times, the probability of dismissal will be given by this equation.[8] One of the appendices (discussed later) analyzes alternative forms for the dismissal function, and shows the advantages of the above dismissal equation over other specifications.

4. The probability of an exogenous separation in each period equals q.

5. The probability of rehire is the same for a previously dismissed worker and an exogenously separated worker.[9]

6. Hires occur at the beginning of a period, and separations occur at the end of a period.

7. The number of unemployed at the beginning of a period (NUB) equals the number unemployed in the previous period plus separations at the end of the previous period. Thus,

$$\begin{aligned} NUB_t &= u_{t-1}LF + (PD_{t-1} + q)L_{t-1} \\ &= u_{t-1}LF + (PD_{t-1} + q)(1 - u_{t-1})LF\ , \end{aligned}$$

where u is the unemployment rate, L is employment, and LF is the labor force.

8. Outflows from the pool of the unemployed (*OPU*) in period t equal separations in the previous period (since these workers need to be replaced) plus the change in employment between the previous period and the current period. Thus, outflows can be expressed as

$$\begin{aligned} OPU_t &= (PD_{t-1} + q)L_{t-1} + (L_t - L_{t-1}) \\ &= (PD_{t-1} + q)(1 - u_{t-1})LF - (u_t - u_{t-1})LF \, .^{10} \end{aligned}$$

9. There are perfect capital markets, so that workers can freely borrow and lend at the same interest rate. Note that a worker's assets will rise in periods after employment and will fall in periods after unemployment.[11]

10. In the steady-state, the interest rate is constant at r and equals the discount rate (δ).

The assumption that $r=\delta$ makes the model mathematically tractable. Given this assumption, Appendix A demonstrates that if utility is expanded around the point where consumption equals its average value, then the present value of utility can be approximated by the equation,

$$E[U^*] = E\left[\sum_{t=1}^{\infty} \frac{1}{(1+r)^{t-1}} \left[\ln[\rho\overline{W}] + \frac{Income_t}{\rho\overline{W}} - 1 + \alpha e_t - \eta e_t^2 + f(W_t / W_t^F)e_t\right]\right] + \frac{A_1}{\rho\overline{W}}, \quad (2)$$

where $\rho = \dfrac{(1-u)\overline{W} + uB + [r/(1+r)]A_1}{\overline{W}}$.

In the above equations, $\overline{W}$ represents the average wage, and $(1-u)\overline{W} + uB + [r/(1+r)]A_1$ represents the average value of consumption.[12] Thus, ρ is the ratio between average consumption and the average wage. It is likely that the average value of consumption will approximately equal $\overline{W}$, which means that ρ should approximately equal 1.[13]

IV. Model with a Constant Probability of Hire

In this section an expression for effort is derived for a situation in which the unemployment rate is constant at the natural rate (u^*), so that the probability of an unemployed individual finding work equals its steady-state value. This probability equals the equilibrium number of new hires divided by the pool of the unemployed at the beginning of the period. From assumptions 7 and 8, this probability is

$$h = \frac{(PD+q)(1-u^*)}{(PD+q)(1-u^*)+u^*}. \tag{3}$$

Let V_t^{EC} represent the expected utility of a worker employed at his or her current firm, V_t^{EO} represent the expected utility of a worker employed at another firm, and V_t^{UN} represent the expected utility of an unemployed individual. (These values represent the expected present value of utility at the beginning of period t.) It is assumed that a worker expects to be paid fairly if hired by another firm, which means that $\overline{W}=W^F$, and thus that $f(\overline{W}/W^F)=f(1)$.

The expected utility of a worker employed at another firm is

$$\begin{aligned} V_t^{EO} = \ln[\rho\overline{W}] + (\alpha + f(1))\bar{e}_t - \eta\bar{e}_t^2 + \frac{1}{1+r_t}(1-q-\overline{m}(1-\bar{e}_t)^2)E_t V_{t+1}^{EO} \\ + \frac{1}{1+r_t}(q+\overline{m}(1-\bar{e}_t)^2)E_t V_{t+1}^{UN}, \end{aligned} \tag{4}$$

where $\bar{e}$ represents the effort the worker would provide at another firm, $\overline{m}$ represents the level of monitoring at other firms, and $\overline{W}$ represents the wage offered by other firms.[14] It is assumed that the wage and the level of monitoring are the same at all other firms. In equilibrium, these should equal the wage and monitoring intensity at a worker's current firm.

The expected present value of an unemployed individual's utility is

$$V_t^{UN} = (1-h)\left(\ln[\rho\overline{W}] + \frac{B}{\rho\overline{W}} - 1 + \frac{1}{1+r_t} E_t V_{t+1}^{UN} \right) + hV_t^{EO} . \tag{5}$$

In the steady state, V^{UN} and V^{EO} are constant over time, so the subscripts on these variables and on the interest rate can be dropped. Solving (4) and (5) for V^{UN} yields

$$V^{UN} = \frac{1+r}{r}\left[\ln[\rho\overline{W}] + \frac{B}{\rho\overline{W}} - 1 + \frac{(1+r)h[(\alpha + f(1))\overline{e} - \eta\overline{e}^2 + 1 - (B/\rho\overline{W})]}{r + h + (1-h)(q + \overline{m}(1-\overline{e})^2)} \right]. \tag{6}$$

To determine the optimal amount of effort to provide at another firm, the derivative of (6) with respect to $\overline{e}$ is set equal to 0, and the resulting equation is solved for $\overline{e}$. Substituting this expression for $\overline{e}$ into (6) yields an expression for V^{UN} as a function of the exogenous variables. This equation for V^{UN} is quite complex and is not reported here.

The expected utility of a worker employed at his or her current firm is

$$\begin{aligned} V_t^{EC} = {} & \ln[\rho\overline{W}] + \frac{W_t}{\rho\overline{W}} - 1 + \alpha e_t - \eta e_t^2 + f(W_t / W_t^F)e_t \\ & + \frac{1}{1+r_t}(1 - q - m(1-e_t)^2)E_t V_{t+1}^{EC} + \frac{1}{1+r_t}(q + m(1-e_t)^2)E_t V_{t+1}^{UN} . \end{aligned} \tag{7}$$

In a steady-state equilibrium, V^{EC} can be expressed as

$$V^{EC} = \frac{(1+r)[\ln[\rho\overline{W}] + (W/\rho\overline{W}) - 1 + \alpha e - \eta e^2 + f(W/W^F)e]}{r + q + m(1-e)^2} + \frac{q + m(1-e)^2}{r + q + m(1-e)^2} V^{UN} . \tag{8}$$

If the derivative of V^{EC} with respect to e is set equal to 0 and the resulting equation is solved for e, the following expression for effort is obtained:

$$e = \frac{X - \sqrt{(X - mY)^2 + (r+q)mY^2}}{mY}, \tag{9}$$

where

$$X = \eta(r+q+m) + m(\ln[\rho\overline{W}] + (W/\rho\overline{W}) - 1) - m\frac{r}{1+r}V^{UN}[\overline{W}, B, h, \overline{m}, \rho],$$

and

$$Y = 2\eta - \alpha - f(W/W^F).$$

Appendix B demonstrates that

$$\left.\frac{\partial e}{\partial W}\right|_{\frac{W}{W^F}} > 0; \quad \frac{\partial e}{\partial \overline{W}} < 0; \quad \frac{\partial e}{\partial B} < 0; \quad \frac{\partial e}{\partial (W/W^F)} > 0; \quad \frac{\partial e}{\partial u*} > 0; \quad \frac{\partial e}{\partial m} > 0;$$

$$\left.\frac{\partial^2 e}{\partial W^2}\right|_{\frac{W}{W^F}} < 0; \text{ and } \frac{\partial^2 e}{\partial W \partial u*} < 0.$$

Thus, effort depends positively on a worker's own wage, the ratio between the wage and the perceived fair wage, the unemployment rate, and the firm's monitoring intensity. In addition, effort depends negatively on wages at other firms and on unemployment benefits.

The second derivatives and cross derivatives are also important. The second derivative of effort with respect to the wage is negative, which means that the relationship between wages and effort is concave. The fact that this relationship is concave means that an interior maximum exists under reasonable assumptions about the profit function. Suppose that a firm's profit function can be expressed as $\pi = PF[L, K, e(W,u)] - WL - rK$, where P is the price of output, F is the production function (with $F_{ee} \leq 0$), L is labor input, and K is the capital stock. Then, profit maximization requires that $\partial^2\pi / \partial W^2 < 0$. Since $\partial^2\pi / \partial W^2 = PF_{ee}e_W^2 + PF_e e_{WW}$, the fact that e_{WW} is negative means that an interior maximum exists.

The cross derivative of effort with respect to the wage and the unemployment rate is also negative, which implies that wages have a smaller effect on effort at higher levels of unemployment. This negative cross derivative can explain why Blanchflower and Oswald (1994) found evidence for a negatively-sloped wage curve.[15]

In addition, Appendix B demonstrates that the cross derivative of effort with respect to wages and monitoring can be either positive or negative, which means that they can be either complements or substitutes.[16] It should be noted that efficiency wage models generally treat wages and monitoring as substitutes. However, monitoring has two opposing effects on wages. On one hand, higher monitoring increases effort, lessening the need to pay high wages. On the other hand, higher monitoring increases the chances of a shirking worker being dismissed, and workers earning higher wages should be more concerned about the threat of dismissal.

Simulations were run to calculate first, second, and cross derivatives for effort. In calibrating the model, a period is assumed to be equivalent to a quarter. In these simulations, the interest rate equals .01 (implying an annual interest rate of approximately 4%), W and $\overline{W}$ are set at \$15, the level of unemployment benefits is set at \$7.50 (half the value of W and $\overline{W}$), the value of ρ is set equal to 1, and the natural rate of unemployment rate is set equal to 5.5%. In addition, labor turnover data from *Employment and Earnings* and work by Bansak and Raphael (1998) and Blanchard and Diamond (1990) suggest that a reasonable value for the probability of an exogenous separation (q) is 0.0375.[17] The form of $f(W/W^F)$ is difficult to pin down, since fairness has many dimensions, and workers may respond differently to each. For simplicity, $f(W/W^F)$ is expressed as a linear function of the form $f(W/W^F) = W/W^F$. It would be reasonable to view the fair wage as given by the expression $W^F = \overline{W}^{1-u} B^u$, a specification that has been suggested by Danthine and Donaldson (1990).

Values for α, η, and m are chosen to satisfy three conditions. First, Campbell (1994) finds that probability of dismissal (or induced resignation) averaged .015 per quarter during the first two years of employment. Since workers with longer tenures tend to have lower dismissal rates, the dismissal rate for the average worker is probably lower than this value. Thus, parameters are chosen so that the probability of dismissal (*PD*) equals 0.012.[18] Second, the average level of effort is assumed to equal 0.8. Since this value for average effort is chosen arbitrarily, simulations were also run under the assumptions that this value equals 0.7 and 0.85. Third, according to the Solow (1979) condition, the elasticity of efficiency with respect to the wage equals 1 for firms that pay efficiency wages. Accordingly, this elasticity is assumed to equal 1 in most simulations. However, some firms may not pay efficiency wages,[19] and firms that pay efficiency wages may have additional motives (e.g., turnover or adverse selection) besides inducing effort. Thus, this elasticity is assumed to equal 0.50 in one of the simulations.

The results of the simulations are presented in Table 1. Reported in this table are elasticities, second derivatives, and cross derivatives of effort with respect to the model's independent variables. Also reported are the values of α, η, and m obtained from the three conditions discussed above. In column 1 the equilibrium level of effort equals 0.8, and the elasticity of effort with respect to the wage equals 1.0. In columns 2 and 3, the wage elasticity of effort remains at 1.0, but the equilibrium effort level equals 0.7 and 0.85 respectively. Then, in column 4, the equilibrium level of effort is 0.8 and the wage elasticity of effort is 0.5. In Figures 1a and 1b, the relationship between wages and effort is graphed using the parameters from columns 1 and 4 of Table 1. Both graphs are smooth and well-behaved.

Several important results are obtained from these simulations. First, all of the second derivatives in Table 1 are negative, including those that cannot be definitively signed using comparative statics ($\partial^2 e/\partial u^2$, $\partial^2 e/\partial \overline{W}^2$, $\partial^2 e/\partial B^2$, and $\partial^2 e/\partial m^2$). Second, the cross derivative of effort with respect to wages and monitoring is always positive. Thus, given the model's parameters, wages and monitoring are complements. (Recall that the sign of this cross derivative is theoretically ambiguous.) Third, the elasticity of effort with respect to the unemployment rate lies between 0.0516 and 0.0672. Wadhwani and Wall (1991) found an elasticity of productivity with respect to the unemployment rate equal to 0.05 for British manufacturing firms, and Weisskopf (1987) estimated that a one percentage point rise in unemployment raised labor productivity by approximately 0.01% in the United States and the United Kingdom. Given the assumption that u*=5.5%, the latter result implies an elasticity of productivity with respect to the unemployment rate of 0.055. Thus, the simulations predict elasticities of effort with respect to unemployment that are close to empirically estimated elasticities of productivity with respect to unemployment. Fourth, footnote 15 describes how a wage curve elasticity can be calculated. If it is assumed that $F(L, K, e(W, u))=A(eL)^{\phi}K^{1-\phi}$ and that ϕ=0.7, the simulations predict wage curve elasticities between –0.0550 and –0.103. Using data from many countries, Blanchflower and Oswald (1994) estimated wage curve elasticities that generally lie between –0.05 and –0.15 and that are centered around –0.10. Thus, the simulated elasticities in this study are close to the elasticities estimated by Blanchflower and Oswald, even though the model was calibrated with parameters that have no relationship to their work.

The equation for effort obtained in this study is derived under the assumption that the probability of dismissal is given by the equation $PD = m(1-e)^2$. However, there are other possible forms for the dismissal function in which effort could still be treated as a continuous variable. For example, it could be assumed that $PD = m(1-e)$. In addition, it could be assumed that firms set a minimum target level of effort and that workers' effort is measured with error. In this case, a worker whose actual effort exceeds the target level could still be dismissed if his or her measured effort lies below this target value. As a worker's effort increases, the probability that measured effort exceeds the target level increases in a continuous manner, so that the probability of dismissal is a continuous function of effort. Appendix C discusses the implications for the effort equation under three alternative specifications for the dismissal function: a dismissal function in which PD=m(1-e), a dismissal function in which firms set a target level of effort and measure effort with an error that is uniformly distributed, and a dismissal function in which firms set a target level of effort and measure effort with an error that is normally distributed. With the first two specifications, $\partial^2 e / \partial W^2 > 0$, which means that the relationship between wages and effort is convex. With the third specification, it is not possible to obtain a closed-form solution for effort. While it is possible to obtain an approximate solution for effort with the third specification, the relationship between wages and effort is not necessarily concave. In contrast, the specification for effort used in this study yields a closed-form solution for effort in which the relationship between wages and effort is unambiguously concave.

V. Conclusion

This study derives an effort equation from microeconomic principles. Given the model's assumptions, effort depends positively on a worker's own wage, the ratio between the worker's wage and perceived fair wage, the firm's monitoring intensity, and the unemployment rate. In addition, it depends negatively on wages at other firms and the level of unemployment benefits. The model also makes predictions concerning some of the second and cross derivatives of effort. The second derivative of effort with respect to the wage is negative. Thus this study provides support for the common assumption that the relationship between wages and effort is concave. The cross derivative of effort with respect to wages and unemployment is negative, so this model is able to explain the wage curve. In addition, the cross derivative of effort with respect to wages and monitoring can be either positive or negative, which means that wages and monitoring can be either complements or substitutes.

The equations for effort derived in this study can be used in efficiency wage work. The effort function derived here is particularly suited for efficiency wage models, since it is derived from microeconomic principles, treats effort as a continuous variable, and has first and second derivatives of reasonable signs. The effort equation derived in this study is static, as it assumes that unemployment is constant. However, Campbell (2003a) shows how the effort equation derived in the present study can be extended into a situation in which unemployment varies over time. The effort equation derived in Campbell (2003a) expresses effort in the current period as a function of current and expected future unemployment rates. Simulations of this model indicate that effort depends positively on current and future unemployment rates, with declining effects as the time horizon lengthens.[20] An effort equation that allows unemployment to differ across time periods can be incorporated into models of the business cycle in which shocks cause unemployment to vary over time.[21] Thus, while the present model develops an expression for effort in a steady-state equilibrium, this expression can be modified for use in dynamic models of the economy.

Appendix A

This appendix demonstrates that the assumption that $r=\delta$ allows us to treat utility in a period as a function of income in that period. Suppose the utility derived from consumption is expanded around its average value, which equals $(1-u)\overline{W}+uB+[r/(1+r)]A_1$. Then the utility derived from consumption in period t can be expressed as

$$U[c_t]=\ln[c_t]\approx \ln[(1-u)\overline{W}+uB+\frac{r}{1+r}A_1]+\frac{c_t-\{(1-u)\overline{W}+uB+[r/(1+r)]A_1\}}{(1-u)\overline{W}+uB+[r/(1+r)]A_1}.$$

Suppose we define

$$\rho=\frac{(1-u)\overline{W}+uB+[r/(1+r)]A_1}{\overline{W}}.$$

Then, the utility from consumption in period t can be approximated by the equation

$$U[c_t]\approx \ln[\rho\overline{W}]+\frac{c_t}{\rho\overline{W}}-1,$$

and the present value of the utility from consumption (E[$U^{*,c}$]) can be approximated by

$$E[U^{*,c}]\approx E\left[\sum_{t=1}^{\infty}\frac{1}{(1+\delta)^{t-1}}\left[\ln[\rho\overline{W}]+\frac{c_t}{\rho\overline{W}}-1\right]\right].$$

Since the budget constraint requires that

$$\sum_{t=1}^{\infty}\frac{1}{(1+r)^{t-1}}c_t=\sum_{t=1}^{\infty}\frac{1}{(1+r)^{t-1}}Income_t+A_1=\sum_{t=1}^{\infty}\frac{1}{(1+r)^{t-1}}\left[Income_t+\frac{r}{1+r}A_1\right],$$

the assumption that δ=r means that the present value of the utility from consumption can be expressed as

$$E[U^{*,c}] \approx E\left[\sum_{t=1}^{\infty}\frac{1}{(1+r)^{t-1}}\left[\ln[\rho\overline{W}]+\frac{Income_t}{\rho\overline{\overline{W}}}-1+\frac{r}{1+r}\frac{A_1}{\rho\overline{\overline{W}}}\right]\right],$$

and the present value of total utility can be expressed as

$$E[U^*] \approx E\left[\sum_{t=1}^{\infty}\frac{1}{(1+r)^{t-1}}\left[\ln[\rho\overline{W}]+\frac{Income_t}{\rho\overline{\overline{W}}}-1+\alpha e_t-\eta e_t^2+f(W_t/W_t^F)e_t\right]\right]+\frac{A_1}{\rho\overline{\overline{W}}}.$$

Appendix B

This appendix shows how signs can be determined for derivatives of effort with respect to the model's independent variables. (For simplicity, ρ is set equal to 1 in these derivations.) Implicit differentiation can be used to calculate the signs of the first derivatives. Note that

$$\frac{\partial V^{EC}}{\partial e} = (1+r)\frac{\begin{array}{l}(r+q)(\alpha - 2\eta e + f(W/W^F)) + m(\alpha + f(W/W^F))(1-e^2) - 2\eta me(1-e)\\ + 2m[\ln[\overline{W}] + (W/\overline{W}) - 1](1-e) - 2m(1-e)\dfrac{r}{1+r}V^{UN}\end{array}}{[r+q+m(1-e)^2]^2},$$

where profit maximization requires that

$$H = \frac{\partial^2 V^{EC}}{\partial e^2} < 0.$$

Therefore,

$$\left.\frac{\partial e}{\partial W}\right|_{\frac{W}{W^F}} = -\frac{\left.\dfrac{\partial^2 V^{EC}}{\partial e \partial W}\right|_{\frac{W}{W^F}}}{H} = -\frac{(1+r)\dfrac{2m(1/\overline{W})(1-e)}{[r+q+m(1-e)^2]^2}}{H} > 0,$$

$$\frac{\partial e}{\partial (W/W^F)} = -\frac{\dfrac{\partial^2 V^{EC}}{\partial e \partial (W/W^F)}}{H} = -\frac{(1+r)\dfrac{f'(W/W^F)(r+q+m-me^2)}{[r+q+m(1-e)^2]^2}}{H} > 0,$$

and

$$\frac{\partial e}{\partial \overline{W}} = -\frac{\dfrac{\partial^2 V^{EC}}{\partial e \partial \overline{W}}}{H} = -\frac{(1+r)\dfrac{2m\left[\dfrac{1}{\overline{W}} - \dfrac{W}{\overline{W}^2}\right](1-e) - 2m(1-e)\dfrac{r}{1+r}\dfrac{\partial V^{UN}}{\partial \overline{W}}}{[r+q+m(1-e)^2]^2}}{H}$$

$$= -\frac{(1+r)\dfrac{2m\left[\dfrac{1}{\overline{W}} - \dfrac{W}{\overline{W}^2}\right](1-e) - 2m(1-e)\left[\dfrac{1}{\overline{W}} - \dfrac{B}{\overline{W}^2} + \dfrac{(1+r)h(B/\overline{W}^2)}{r+h+(1-h)(q+\overline{m}(1-\overline{e})^2)}\right]}{[r+q+m(1-e)^2]^2}}{H}$$

$$= -\frac{(1+r)\dfrac{-2m(1-e)\dfrac{W-B}{\overline{W}^2} - 2m(1-e)\dfrac{(1+r)h(B/\overline{W}^2)}{r+h+(1-h)(q+\overline{m}(1-\overline{e})^2)}}{[r+q+m(1-e)^2]^2}}{H} < 0\,.$$

To calculate $\partial e/\partial B$, note that

$$\frac{\partial e}{\partial V^{UN}} = -\frac{\dfrac{\partial^2 V^{EC}}{\partial e \partial V^{UN}}}{H} = -\frac{-\dfrac{2m(1-e)r}{[r+q+m(1-e)^2]^2}}{H} < 0\,,$$

and

$$\frac{\partial V^{UN}}{\partial B} = \frac{1+r}{r}\frac{1}{\overline{W}}\left[1 - \frac{(1+r)h}{r+h+(1-h)(q+\overline{m}(1-\overline{e})^2)}\right] > 0\,.$$

Therefore,

$$\frac{\partial e}{\partial B} = \frac{\partial e}{\partial V^{UN}}\frac{\partial V^{UN}}{\partial B} < 0.$$

In calculating $\partial e/\partial u^*$, we make use of the fact that $\partial e/\partial V^{UN}<0$. In addition, we know that

$$\frac{\partial h}{\partial u^*} = -\frac{PD+q}{[(PD+q)(1-u^*)+u^*]^2} < 0\,,$$

and

$$\frac{\partial V^{UN}}{\partial h} = \frac{(1+r)^2}{r}\frac{r+q+\overline{m}(1-\overline{e})^2}{[r+h+(1-h)(q+\overline{m}(1-\overline{e})^2)]^2}\left[(\alpha + f(1))\overline{e} - \eta\overline{e}^2 + 1 - \frac{B}{\overline{W}}\right] > 0\,.$$

The reason why $\partial V^{UN}/\partial h$ must be positive is that the expected present value of an unemployed worker's utility (V^{UN}) must be at least as high as the present value of unemployment benefits, which equals $((1+r)/r)[\ln[\overline{W}]+(B/\overline{W})-1]$. Thus, the last term in equation (6) is positive, which means that $(\alpha + f(1))\overline{e} - \eta\overline{e}^2 + 1 - (B/\overline{W})$ is positive.

From the signs of $\partial e/\partial V^{UN}$, $\partial V^{UN}/\partial h$, and $\partial h/\partial u^*$, it follows that

$$\frac{\partial e}{\partial u^*} = \frac{\partial e}{\partial V^{UN}} \frac{\partial V^{UN}}{\partial h} \frac{\partial h}{\partial u^*} > 0 .$$

Calculating $\partial e/\partial m$ is somewhat more difficult. To calculate this value, note that

$$\frac{\partial e}{\partial m} = -\frac{\dfrac{\partial^2 V^{EC}}{\partial e \partial m}}{H} .$$

Suppose we define

$$A = (\alpha + f(W/W^F))(1-e^2) - 2\eta e(1-e) + 2(1-e)[\ln[\overline{W}] + (W/\overline{W}) - 1] - 2(1-e)\frac{r}{1+r}V^{UN} ,$$

and

$$B = (r+q)(\alpha - 2\eta e + f(W/W^F)) .$$

Then,

$$\frac{\partial V^{EC}}{\partial e} = (1+r)\frac{Am+B}{[r+q+m(1-e)^2]^2} ,$$

and

$$\frac{\partial^2 V^{EC}}{\partial e \partial m} = (1+r)\frac{[r+q+m(1-e)^2]^2 A - [Am+B]\times 2[r+q+m(1-e)^2](1-e)^2}{[r+q+m(1-e)^2]^4}$$

$$= (1+r)\frac{[r+q+m(1-e)^2]A - 2[Am+B](1-e)^2}{[r+q+m(1-e)^2]^3}$$

$$= (1+r)\frac{(r+q)A - m(1-e)^2 A - 2B(1-e)^2}{[r+q+m(1-e)^2]^3}\,.$$

Since utility-maximization requires that *Am*+*B*=0, the above equation can be expressed as

$$\frac{\partial^2 V^{EC}}{\partial e \partial m} = (1+r)\frac{(r+q)A - B(1-e)^2}{[r+q+m(1-e)^2]^3}$$

$$= (1+r)\frac{\begin{array}{l}(r+q)\{(\alpha + f(W/W^F))(1-e^2) - 2\eta e(1-e) + 2(1-e)[\ln[\overline{W}] + (W/\overline{W}) - 1] \\ -2(1-e)\frac{r}{1+r}V^{UN}\} - (r+q)(\alpha - 2\eta e + f(W/W^F))(1-e)^2\end{array}}{[r+q+m(1-e)^2]^3}$$

$$= (1+r)\frac{\begin{array}{l}(r+q)\{2(\alpha + f(W/W^F))e(1-e) - 2\eta e^2(1-e) \\ +2[\ln[\overline{W}] + (W/\overline{W}) - 1](1-e) - 2(1-e)\frac{r}{1+r}V^{UN}\}\end{array}}{[r+q+m(1-e)^2]^3}$$

$$= (1+r)\frac{2(r+q)(1-e)\{\alpha e + f(W/W^F)e - \eta e^2 + \ln[\overline{W}] + (W/\overline{W}) - 1 - \frac{r}{1+r}V^{UN}\}}{[r+q+m(1-e)^2]^3} > 0\,.$$

The above equation must be positive, since the term in brackets equals the current utility of a worker employed at his or her present firm minus $r/(1+r)$ times the discounted value of the lifetime utility of an unemployed worker. Since $\partial^2 V^{EC}/\partial e \partial m$ is positive, $\partial e/\partial m$ must be positive.

To calculate $\partial^2 e/\partial W^2$ and $\partial^2 e/\partial W\partial u^*$, recall that

$$e = \frac{X - \sqrt{(X - mY)^2 + (r+q)mY^2}}{mY},$$

where

$$X = \eta(r+q+m) + m(\ln[\overline{W}] + (W/\overline{W}) - 1) - m\frac{r}{1+r}V^{UN}[\overline{W}, B, h, \overline{m}],$$

and

$$Y = 2\eta - \alpha - f(W/W^F).$$

Thus,

$$\frac{\partial e}{\partial X} = \frac{1}{mY}\left[1 - \left((X - mY)^2 + (r+q)mY^2\right)^{-1/2}(X - mY)\right] > 0,$$

$$\frac{\partial^2 e}{\partial X^2} = -\frac{1}{mY}\left((X - mY)^2 + (r+q)mY^2\right)^{-3/2}(r+q)mY^2 < 0,$$

and

$$\frac{\partial X}{\partial V^{UN}} = -m\frac{r}{1+r} < 0.$$

As a result,

$$\left.\frac{\partial^2 e}{\partial W^2}\right|_{\frac{W}{W^F}} = \frac{\partial}{\partial W}\left(\left.\frac{\partial e}{\partial W}\right|_{\frac{W}{W^F}}\right) = \frac{\partial}{\partial W}\left(\frac{\partial e}{\partial X}\frac{\partial X}{\partial W} + \frac{\partial e}{\partial Y}\left.\frac{\partial Y}{\partial W}\right|_{\frac{W}{W^F}}\right) = \frac{\partial e}{\partial X}\frac{\partial^2 X}{\partial W^2} + \frac{\partial^2 e}{\partial X^2}\left(\frac{\partial X}{\partial W}\right)^2$$

$$= \frac{\partial^2 e}{\partial X^2}\left(\frac{\partial X}{\partial W}\right)^2 < 0,$$

and

$$\frac{\partial^2 e}{\partial W du*} = \frac{\partial}{\partial u*}\left(\frac{\partial e}{\partial W}\right) = \frac{\partial}{\partial u*}\left(\frac{\partial e}{\partial X}\frac{\partial X}{\partial W} + \frac{\partial e}{\partial Y}\frac{\partial Y}{\partial W}\right)$$

$$= \left[\frac{\partial e}{\partial X}\frac{\partial^2 X}{\partial W \partial V^{UN}} + \frac{\partial^2 e}{\partial X^2}\frac{\partial X}{\partial W}\frac{\partial X}{\partial V^{UN}} + \frac{\partial e}{\partial Y}\frac{\partial^2 Y}{\partial W \partial V^{UN}} + \frac{\partial^2 e}{\partial Y^2}\frac{\partial Y}{\partial W}\frac{\partial Y}{\partial V^{UN}}\right]\frac{\partial V^{UN}}{\partial h}\frac{\partial h}{\partial u*}$$

$$= \underset{-}{\frac{\partial^2 e}{\partial X^2}}\,\underset{+}{\frac{\partial X}{\partial W}}\,\underset{-}{\frac{\partial X}{\partial V^{UN}}}\,\underset{+}{\frac{\partial V^{UN}}{\partial h}}\,\underset{-}{\frac{\partial h}{\partial u*}} < 0.$$

To calculate $\partial^2 e / \partial W \partial m$, define

$$X_1 = X / m = \frac{\eta(r+q+m)}{m} + \ln[\overline{W}] + (W/\overline{W}) - 1 - \frac{r}{1+r} V^{UN}[\overline{W}, B, h, \overline{m}],$$

so that

$$e = \frac{X_1 - \sqrt{(X_1 - Y)^2 + \frac{r+q}{m} Y^2}}{Y}.$$

Then,

$$\frac{\partial e}{\partial W} = \frac{1}{Y}\left\{\frac{\partial X_1}{\partial W} - \left[(X_1 - Y)^2 + \frac{r+q}{m}Y^2\right]^{-1/2}(X_1 - Y)\frac{\partial X_1}{\partial W}\right\},$$

and

$$\frac{\partial^2 e}{\partial W \partial m} = \frac{1}{Y}\left\{\frac{\partial^2 X_1}{\partial W \partial m} - \left[(X_1 - Y)^2 + \frac{r+q}{m}Y^2\right]^{-1/2}\left[(X_1 - Y)\frac{\partial^2 X_1}{\partial W \partial m} + \frac{\partial X_1}{\partial W}\frac{\partial X_1}{\partial m}\right]\right.$$

$$\left. + \frac{1}{2}\left[(X_1 - Y)^2 + \frac{r+q}{m}Y^2\right]^{-3/2}\left[2(X_1 - Y)\frac{\partial X_1}{\partial m} - \frac{r+q}{m^2}Y^2\right](X_1 - Y)\frac{\partial X_1}{\partial W}\right\}$$

$$= \frac{1}{Y}\left[(X_1 - Y)^2 + \frac{r+q}{m}Y^2\right]^{-3/2} \frac{\partial X_1}{\partial W}\left\{-\left[(X_1 - Y)^2 + \frac{r+q}{m}Y^2\right]\frac{\partial X_1}{\partial m} + (X_1 - Y)^2 \frac{\partial X_1}{\partial m} - \frac{1}{2}\frac{r+q}{m^2}(X_1 - Y)Y^2\right\}$$

$$= \frac{1}{Y}\left[(X_1 - Y)^2 + \frac{r+q}{m}Y^2\right]^{-3/2} \frac{\partial X_1}{\partial W}\left\{-\frac{r+q}{m}Y^2 \frac{\partial X_1}{\partial m} - \frac{1}{2}\frac{r+q}{m^2}(X_1 - Y)Y^2\right\}$$

$$= \frac{1}{Y}\left[(X_1 - Y)^2 + \frac{r+q}{m}Y^2\right]^{-3/2} \frac{Y^2}{\overline{W}}\frac{r+q}{m^2}\left\{\frac{r+q}{m}\eta - \frac{1}{2}(X_1 - Y)\right\}.$$

The sign of this last expression is ambiguous.

Appendix C
Effort Equation with Other Dismissal Functions

In the model developed in this study, it is assumed that the probability of dismissal is described by the equation, $PD = m(1-e)^2$. This appendix analyses the effort equations that would be obtained under three other assumptions concerning the probability of dismissal. In the first case, it is assumed that PD=m(1-e). In the second and third cases, it is assumed that firms set a target level of effort and dismiss those whose measured effort falls below this target level. However, it is assumed that workers' effort is measured with error. This means that as effort rises, the probability of dismissal falls in a continuous manner. In the second case it is assumed that the measurement error is uniformly distributed, and in the third case it is assumed that the measurement error is normally distributed. (For simplicity, ρ is set equal to 1 in these derivations.)

Case 1: PD=m(1-e)

Under this assumption concerning the probability of dismissal, the expected utility of a worker employed at his or her current firm is

$$V_t^{EC} = \ln[\overline{W}] + \frac{W_t}{\overline{\overline{W}}} - 1 + \alpha e_t - \eta e_t^2 + f(W_t / W_t^F)e_t + \frac{1}{1+r_t}(1-q-m(1-e_t))E_t V_{t+1}^{EC} + \frac{1}{1+r_t}(q+m(1-e_t))E_t V_{t+1}^{UN}.$$

In a steady-state equilibrium, V^{EC} can be expressed as

$$V^{EC} = \frac{(1+r)[\ln[\overline{W}] + (W/\overline{W}) - 1 + \alpha e - \eta e^2 + f(W/W^F)e]}{r+q+m(1-e)} + \frac{q+m(1-e)}{r+q+m(1-e)}V^{UN}.$$

If the derivative of V^{EC} with respect to e is set equal to 0 and the resulting equation is solved for e, the following expression for effort is obtained:

$$e = \frac{(r+q+m)\eta - \sqrt{(r+q+m)^2\eta^2 - m\eta Z}}{m\eta},$$

where $Z = (r+q+m)(\alpha + f(1)) + m[\ln \overline{W} + (W/\overline{W}) - 1] - \frac{r}{1+r} mV^{UN}$.

In this case, the first and second derivatives of the effort equation are

$$\frac{\partial e}{\partial W} = (1/2)[(r+q+m)^2\eta^2 - m\eta Z]^{-\frac{1}{2}} m(1/\overline{W}) > 0, \qquad \text{and}$$

$$\frac{\partial^2 e}{\partial W^2} = (1/4)[(r+q+m)^2\eta^2 - m\eta Z]^{-\frac{3}{2}} m^3\eta(1/\overline{W})^2 > 0.$$

The positive sign of $\partial^2 e/\partial W^2$ means that, given the assumption that PD=m(1-e), the relationship between wages and effort is convex. As stated previously, if the profit function is expressed as $\pi = PF[L, K, e(W,u)] - WL - rK$, then the condition for there to be an interior maximum is that $PF_{ee}e_W^2 + PF_e e_{WW} < 0$. Thus, a positive value for e_{WW} means that an interior maximum may not exist.

In addition, it can be shown that this specification for the dismissal function results in a positive cross derivative of effort with respect to the wage and the unemployment rate. To see this, note that

$$\frac{\partial^2 e}{\partial W \partial V^{UN}} = -(1/4)[(r+q+m)^2\eta^2 - m\eta Z]^{-\frac{3}{2}} m^3\eta \frac{r}{1+r}(1/\overline{W}) < 0,$$

and that

$$\frac{\partial^2 e}{\partial W \partial u^*} = \frac{\partial}{\partial W}\left(\frac{\partial e}{\partial V^{UN}}\frac{\partial V^{UN}}{\partial u^*}\right) = \frac{\partial e}{\partial V^{UN}}\frac{\partial^2 V^{UN}}{\partial W \partial u^*} + \frac{\partial^2 e}{\partial W \partial V^{UN}}\frac{\partial V^{UN}}{\partial u^*} = \frac{\partial^2 e}{\partial W \partial V^{UN}}\frac{\partial V^{UN}}{\partial u^*}$$

$$= \frac{\partial^2 e}{\partial W \partial V^{UN}}\frac{\partial V^{UN}}{\partial h}\frac{\partial h}{\partial u^*}.$$

In Appendix B it was shown that $\partial V^{UN}/\partial h>0$ and $\partial h/\partial u^*<0$. Thus, $\partial^2 e/\partial W\partial u^*>0$, which means that wages have a greater effect on effort as the unemployment rate rises. This positive cross derivative means that higher unemployment may raise wages. Footnote 15 demonstrates that $\partial W/\partial u = -(\partial^2\pi/\partial W\partial u)/(\partial^2\pi/\partial W^2)$, where $\partial^2\pi/\partial W\partial u = PF_e e_{Wu} + PF_{ee}e_W e_u$. Thus, even if $\partial^2\pi/\partial W^2 < 0$, so that an interior maximum exists, the fact that $e_{Wu}>0$ means that $\partial W/\partial u$ may be positive.

<u>Case 2: Effort Measured with Error that is Uniformly Distributed</u>

Suppose we let e^T represent the target level of effort, so that workers whose measured effort falls below this level are dismissed. Suppose also that workers' effort is measured with error. Let e represent the actual effort of a worker and ε represent the error in measuring effort. In Case 2, this error is assumed to be uniformly distributed with a mean of 0 and a range of d, which means that measured effort is uniformly distributed with a mean of e and a range of d. The probability that a worker is dismissed is thus equal to

$$PD_t = \Pr\left[e_t + \varepsilon_t < e^T\right] = \frac{e^T - e_t + (d/2)}{d} = \frac{1}{2} + \frac{1}{d}(e^T - e_t).$$

Thus, the expected utility of a worker employed at his or her current firm is

$$V_t^{EC} = \ln[\overline{W}] + \frac{W_t}{\overline{\overline{W}}} - 1 + \alpha e_t - \eta e_t^2 + f(W_t / W_t^F)e_t + \frac{1}{1+r_t}(1 - q - .5 - d^{-1}(e^T - e_t))E_t V_{t+1}^{EC}$$
$$+ \frac{1}{1+r_t}(q + .5 + d^{-1}(e^T - e_t))E_t V_{t+1}^{UN}.$$

In a steady-state equilibrium, V^{EC} can be expressed as

$$V^{EC} = \frac{(1+r)[\ln[\overline{W}] + (W/\overline{W}) - 1 + \alpha e - \eta e^2 + f(W/W^F)e]}{r + q + .5 + d^{-1}e^T - d^{-1}e} + \frac{q + .5 + d^{-1}e^T - d^{-1}e}{r + q + .5 + d^{-1}e^T - d^{-1}e} V^{UN}.$$

Differentiating this equation with respect to *e*, and setting the derivative equal to 0 yields the following equation for effort:

$$e = \frac{(r + q + .5 + d^{-1}e^T)\eta - \sqrt{(r + q + .5 + d^{-1}e^T)^2 \eta^2 - d^{-1}\eta Z}}{\eta d^{-1}},$$

where

$$Z = (r + q + .5 + d^{-1}e^T)(\alpha + f(1)) + d^{-1}[\ln \overline{W} + (W/\overline{W}) - 1] - \frac{r}{1+r} d^{-1} V^{UN}.$$

This equation has the same form as the equation derived in Case 1. Thus, this specification yields a convex effort function in which wages affect effort more as unemployment rises.

Case 3: Effort Measured with Error that is Normally Distributed

Suppose that effort is measured with error, as in Case 2, but now suppose that the error is normally distributed with a mean of 0 and a standard deviation of σ. Then, the probability of dismissal equals

$$PD_t = \Pr[e_t + \varepsilon_t < e^T] = \int_{-\infty}^{e^T - e_t} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{\varepsilon_t}{\sigma}\right)^2\right] d\varepsilon_t .$$

In this case it is impossible to obtain a closed-form solution for effort. However, it is possible to obtain an approximation to the probability of dismissal and thus to effort. From Liebnitz's rule,

$$\frac{\partial PD}{\partial e_t} = -\frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{e^T - e_t}{\sigma}\right)^2\right], \text{ and}$$

$$\frac{\partial^2 PD}{\partial e_t^2} = -\frac{1}{\sqrt{2\pi}\sigma}\left(\frac{e^T - e_t}{\sigma}\right) \exp\left[-\frac{1}{2}\left(\frac{e^T - e_t}{\sigma}\right)^2\right].$$

If we expand effort around the point *e**, the probability of dismissal can be approximated by the Taylor series expansion,

$$PD \approx \int_{0}^{e^T - e*} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{\varepsilon}{\sigma}\right)^2\right] d\varepsilon - \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{e^T - e*}{\sigma}\right)^2\right](e_t - e*)$$
$$- \frac{1}{2\sqrt{2\pi}\sigma}\left(\frac{e^T - e*}{\sigma}\right) \exp\left[-\frac{1}{2}\left(\frac{e^T - e*}{\sigma}\right)^2\right](e_t - e*)^2 .$$

The effort function obtained with this specification for *PD* is quite complex, and is not reported here. However, it can be demonstrated that the resulting effort function is not necessarily concave.

Thus, in Cases 1 and 2, the effort function is convex. In Case 3, a closed-form solution for effort cannot be obtained, and an approximation of the effort function is not necessarily concave. In contrast, under the assumption that $PD = m(1-e)^2$, a closed-form solution for effort can be obtained, and the relationship between wages and effort is unambiguously concave.

Footnotes

[1]The equation for effort derived in Sparks is $e = \min\{(1+r)\bar{e} - [\bar{e}^2(1+r)^2 - (w - rV^U)]^{1/2}, \bar{e}\}$, where $\bar{e}$ represents the minimum effort standard and V^U represents the expected utility of an unemployed individual. In equilibrium, a corner solution for effort is obtained in which effort equals $\bar{e}$.

[2]The negative cross derivative of effort with respect to the wage and the expected utility of an unemployed individual implies that the cross derivative of effort with respect to the wage and the unemployment rate is positive, since higher unemployment reduces the probability of an unemployed person being rehired and thus lowers the expected utility of an unemployed individual. A positive cross derivative of effort with respect to the wage and the unemployment rate means that a rise in the unemployment rate may raise wages.

[3]It has been suggested that involuntary unemployment can be eliminated if workers posted bonds, which would be forfeited if caught shirking. (See, for example, Carmichael (1985).) However, Shapiro and Stiglitz (1984, 1985) and Dickens, Katz, Lang, and Summers (1989) argue that firms may have good reasons for not requiring workers to post bonds.

[4]Levine examined data from the PIMS line-of-business data set, and Wadhwani and Wall used data from British manufacturing firms.

[5]In 1998, Agell and Lundborg (1999) surveyed most of the same firms that were included in their earlier survey (which was conducted in 1991). The Swedish unemployment rate was 3.1% in 1991 and 8.4% in 1998. Managers reported less shirking in 1998 than in 1991, and Agell and Lundborg attribute this decreased shirking to the higher unemployment rate.

[6]These researchers did not use terms such as employer, worker, or effort. Instead, they framed their experiments in terms of the goods market, since they viewed fairness considerations as less important in the goods market than in the labor market. Thus their results are biased against finding fair wage effects.

[7]For example, Akerlof and Yellen (1990) assume that effort equals the ratio between a worker's wage and fair wage if the wage is less than the fair wage, but equals 1 if the wage is greater than the fair wage.

[8]Firms may find it advantageous to observe workers twice if they fear that a single observation may result in dismissing hard-working employees who are unlucky about the timing of the firm's observation.

[9]The assumption that the probability of rehire is the same for both groups of workers is made to simplify the model, but this assumption is not necessary to obtain a solution for effort. For example, we could assume that $h_t^D = ah_t^{ES}$ (with $a < 1$), where h_t^D is the probability of hire for a previously dismissed worker, h_t^{ES} is the probability of hire for an exogenously separated worker, and a represents the degree to which the probability of rehire is lower for previously dismissed workers than for exogenously separated workers. (See Parsons (1990) and Gibbons and Katz (1991) for evidence that labor market outcomes may be worse for dismissed workers than for workers who had not been dismissed.) Under the assumption that the probability of rehire is lower for dismissed workers, an equation for effort can be derived, but it is considerably more complex than the equation derived in this study. (A derivation of the model under this assumption is available from the author upon request.)

[10]If employment falls by more than the number of separations at the end of the previous period, then layoffs occur. In this case, $NUB_t = u_{t-1}LF + L_t - L_{t-1}$, and $OPU_t = 0$.

[11]In analyzing workers' effort choice, the studies of Collard and de la Croix (2000), Alexopoulos (2002), and Felices (2002) assume that workers cannot borrow or lend between periods, but instead assume that they have access to income insurance. Alexopoulos and Felices assume that individuals are part of households that provide income insurance, and Collard and de la Croix assume that workers receive perfect income insurance from private firms. The model in the present study could be altered to assume that workers do not have access to capital markets but do have access to perfect income insurance. Under these assumptions, the equation for effort would be similar to the equation derived in the present study. (The only ways in which the effort function would differ under these alternate assumptions are that the interest rate (r) would not appear in the equation for effort and that we would not be able to separate out the effects of W and $\overline{W}$ on effort.) Thus, the assumption that capital markets are perfect is not critical for the results of this study.

[12]While $\overline{W}$ represents the average wage, it should be noted that (as discussed below) all firms pay the same wage in equilibrium.

[13]To see that average consumption approximately equals $\overline{W}$, suppose that unemployment benefits (B) equal $b\overline{W}$ (where b can be viewed as the replacement ratio). Then, average consumption will approximately equal $(1-u)\overline{W} + ub\overline{W} + [r/(1+r)]A_1$, which can be expressed as $\overline{W}[1-(1-b)u] + [r/(1+r)]A_1$. If the unemployment rate

is 5.5% and the replacement ratio is 0.5, then the difference between c and $\overline{W}$ that is cause by the fact that people occasionally experience unemployment is –2.75%. In addition, figures from Wolff (2000) indicate that median household wealth in 1998 was \$60,700, and figures from the Census Bureau indicate that median household income in 1998 was \$38,885. Given the real interest rate in 1998 (on a three-month Treasury Bill), $[r/(1+r)]A_1$ is 4.8% of household income. For the median household, the vast majority of household income probably represents labor income, as figures from Wolff indicate that over 70% of wealth for the middle three quintiles is in non-interest bearing form (real estate and pension plans). Accordingly, the difference between c and $\overline{W}$ that is caused by the fact that consumption depends on asset holdings is approximately +4.8%. Thus, the overall difference between c and $\overline{W}$ is probably no more than about 2%.

[14]In this expression for V_t^{EO} (as well as in the expressions for V_t^{UN} and V_t^{EC}), we can ignore the term for an individual's wealth, because $A_1/\overline{W}$ does not affect the optimal level of effort in equation (2). However, an individual's initial wealth will affect his or her effort through the effect of A_1 on the value of ρ.

[15]Suppose, as before, that profits are expressed as $\pi = PF[L, K, e(W,u)] - WL - rK$. Then, $\partial W/\partial u = -(\partial^2\pi/\partial W\partial u)/(\partial^2\pi/\partial W^2)$, where it was previously demonstrated that $\partial^2\pi/\partial W^2 < 0$. Since $\partial^2\pi/\partial W\partial u = PF_e e_{Wu} + PF_{ee} e_W e_u < 0$, the fact that $e_{Wu}<0$ means that $\partial W/\partial u$ must be negative.

[16]In addition, $\partial^2 e/\partial u^2$, $\partial^2 e/\partial \overline{W}^2$, $\partial^2 e/\partial B^2$, and $\partial^2 e/\partial m^2$ cannot be definitively signed.

[17]We can think of an exogenous separation as resulting from a retirement, a death, a plant closing, a permanent layoff not related to the worker's effort, or a quit into unemployment. There are several sources from which we can estimate the probability of an exogenous separation. Labor turnover data is reported in *Employment and Earnings*. According to figures between 1960 and 1981 (these data were discontinued in 1981), the monthly quit rate was 1.73 per 100 workers, the layoff rate was 1.60, and the rate of other separations was 0.84. (Other separations were defined as those resulting from a discharge, a permanent disability, a death, a retirement, a transfer to another establishment of the company, or entrance into the armed forces.) In addition, data on layoffs and recalls suggest that permanent layoffs are approximately 30% of total layoffs. If we assume that 40% of quits, 30% of layoffs, and 50% of other separations represent exogenous separations, we get a figure for exogenous separations of 0.016 per month, or 0.048 per quarter. (The assumption that 40% of quits are exogenous separations is based on the findings of Blanchard and Diamond (1990) that 60% of quits are into another job.)

Bansak and Raphael (1998) find that the annual probability of a permanent layoff (not including dismissals), a retirement, and a quit (without the worker having lined up another job) are, respectively, 4.05%, 1.12%, and 4.90. These figures imply an annual exogenous separation rate of 0.101. Since many workers experience multiple separation within a year, dividing the annual separation rate by four would probably understate the quarterly separation rate. If the quarterly separation rate is 20-50% higher than what would be obtained by dividing the annual separation rate by four, an appropriate figure for quarterly separations would be 0.030 to 0.038.

Blanchard and Diamond (1990) estimate that monthly transitions between employment and unemployment average 1.5% of employment. On a quarterly basis, this figure represents a quarterly separation rate of .045. If, as discussed later, the dismissal rate is assumed to equal .012, this implies an exogenous separation rate of .033.

Given the results of these studies, a value of q=.0375 appears to be reasonable. This value for q implies that an individual who works 40 years (160 quarters) experiences six exogenous separations over his or her life.

[18]This value implies that someone working 160 quarters experiences slightly less than two lifetime dismissals.

[19]For example, the work of Kaufman (1984) suggests that about half of firms pay efficiency wages and the other half pay market-clearing wages. An unpublished question in Campbell and Kamlani's (1997) survey also suggests that approximately 50% of firms pay efficiency wages.

[20]In addition, results from the simulations indicate that the second derivatives of effort with respect to unemployment in each period are negative and that the cross derivatives of effort with respect to unemployment in two different periods are positive.

[21]Campbell (2003b, 2003c) shows examples of how these equations can be embedded into general equilibrium models.