Market Constraints as a Rationale for the Friedman-Savage Utility Function

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I. Introduction

Since the classic paper by Friedman and Savage (1948), it has generally been accepted that the observed fact that individuals or firms[1] participate in unfair lotteries and other forms of unfair risk taking[2] may be explained by a section in the individual's utility function in which the individual shows risk preference rather than risk aversion. In their model, Friedman and Savage specify a utility function which is, in turn, concave, convex, and concave, thus allowing for simultaneous purchase of insurance (risk aversion) and participation in lotteries (risk preference).

However, while their specified utility function is indeed capable of explaining observed behavior, it does seem to be rather unsatisfactory in that it is an ad hoc specification. It is the purpose of this paper to suggest a set of circumstances which give rise to a Friedman-Savage–type utility function.

In particular, it is shown that when certain capital market imperfections exist the utility function defined over intermediate wealth should be distinguished from and may have different properties than the one defined on final wealth. Then, even if we accept the common assumption made in the literature that individuals are risk averse, that is, that their utility function is concave over final wealth, it is still possible that they participate in unfair gambling (and, of course, may also purchase insurance).

[1] The word "individual" is, throughout this paper, used to denote economic units, including both individuals and firms.

[2] The term "unfair lottery" in the paper encompasses all investment opportunities offering actuarially unfair risks.

The motivation behind our endeavor to provide an institutional explanation for gambling is a reluctance (shown, e.g., by Stigler and Becker [1977]) to use non-well-behaved utility functions as explanations for economic phenomena without some a priori reasons for assuming that the utility function is of an unusual shape. Clearly all phenomena in economics can be technically explained by recourse to tastes. Thus, for example, individuals will necessarily show risk preference over a section of their utility function which is assumed convex. However, is there any a priori reason to assume that there may be such a section? It is a positive answer to this question which this paper attempts to provide.

In Section II we discuss certain capital market imperfections and their implications about the nature of rates of return on investment. In Section III we show how the Friedman-Savage utility function may emerge when utility is defined on intermediate wealth and there exist capital market imperfections. The possibility of gambling with risk aversion and of simultaneous gambling and insurance therefore immediately arises.

II. Rates of Return and Capital Market Imperfections

A number of recent studies discuss the effects of uncertainty, imperfect information, and various transaction costs on capital markets and show that the result may be that capital markets are characterized by certain imperfections. These imperfections are usually in terms of the nonexistence of certain markets and the fact that there may not be free and equal access to other markets. Uncertainty and the possibility of costly default may lead lenders to introduce collateral requirements or take default costs into account in their loan rates. Consequently, it can be said that one has to have certain assets (providing collateral services) in order to have easier or cheaper access to capital markets.

Barro (1976) and Benjamin (1978) derive these conclusions explicitly and show that market imperfections will lead to loan rates being functions of loan sizes and available collateral. In particular, they derive a loan-supply function which is constant for some initial range and then becomes an increasing and convex function of loan sizes.

In the same vein, Jaffee and Modigliani (1969) show that, as a result of uncertainty and imperfect and costly information, capital markets may be characterized by credit rationing. They show that beyond a certain point loan rates will generally depend on loan sizes, with a possible upper bound on loan sizes. Furthermore, they provide empirical evidence supporting these types of imperfections.

Empirical evidence indicating capital market imperfections is also

given by Eckstein (1961) and Nerlove (1968), who find capital markets
to be characterized by differential rates of return on given invest-
ments.

The important implication of the capital market imperfections is
that individuals face a variety of capital market constraints and, there-
fore, do not have free and equal access to the market. Consequently,
capital provides additional services by either weakening the accessi-
bility constraints or by reducing the cost of the acquisition of capital.
By providing collateral services, capital, therefore, increases the set of
"feasible activities" or reduces their cost, so that its "full rate of
return" should take this additional role into account.

A consequence of these capital market imperfections is that for
some levels of asset holdings the rates of return on assets may depend
on the levels of the asset holdings, even from an individual's (rather
than an aggregate) point of view. The range within which this is likely
to happen is when wealth levels are low, but not below some minimum
level. When an individual's wealth is below some minimum level the
capital market constraints may be so effective that except for the
possibility of obtaining small loans the market is in effect inaccessible.
Since small loans can usually be obtained at constant loan rates (as is
shown in Jaffee and Modigliani [1969], Barro [1976], and Benjamin
[1978]), the additional role of asset holding is ineffective within this
initial range, and consequently the rates of return will not depend on
asset holdings. However, as an individual's asset holdings increase
above some minimum level, his higher level of wealth will provide him
with the additional "collateral services," and rates of return within this
range will depend on the levels of wealth.

Since rates of return are usually not known with certainty, the
dependence of rates of return on asset holdings is in a probabilistic
sense. In other words, the rates-of-return probability distribution
function changes with asset holdings, that is, the distribution is condi-
tional on asset holdings.[3]

In addition to the above considerations, rates of return may be
increasing functions of wealth (over some initial range) for other
reasons as well. First, a large number of fixed costs may be incurred in
effecting profitable investment. Information costs incurred in locat-
ing high-return investments may be considerable, and a large element
of these costs may be relatively fixed. Transaction costs, where again
the large part is fixed, are also likely to comprise a high percentage of
small investments. In the presence of these costs the rate of return will
be an increasing function of the investment. Second, there may exist

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[3] Thus, e.g., an increase in asset holdings will over some range make the probability
distribution more stochastically dominant.

significant indivisibilities which (especially in view of the capital market constraints) will imply increasing returns to scale.

In view of these considerations, we conclude that individuals, especially those with small or moderate levels of wealth, may very often face various capital market constraints which lead to rates of return being functions of wealth. Furthermore, the relationship between wealth and the rates of return is such that rates of return are (at least over some range) increasing functions of wealth.

III. Gambling with Risk Aversion

Having discussed some of the capital market imperfections and the constraints they impose on individuals, we now consider their effects on the individual's attitudes toward risk taking.

In order to focus on the gambling problem and to separate it from the investment problem, we assume that the rate of return is nonrandom. This assumption is not crucial and does not change the results. For the sake of simplicity, we also assume that there is only one asset. A discussion of the case with many assets and random rates of return can be found in Appelbaum and Katz (1979), where the same results are derived.

In line with the discussion in the previous section, it will be assumed that the individual faces a rate of return R on his investment, A, such that

$$R = R(A) \quad \begin{cases} R' = 0 & 0 \leq A \leq A^* \\ R' > 0 & \text{for } A^* \leq A \leq A^{**} \\ R' = 0 & \text{for } \quad A > A^{**}, \end{cases} \quad (1)$$

where R' is the partial derivative of R. In other words, the rate of return is constant over some initial range, increasing over some subsequent range,[4] and then constant again.

An individual investing an amount A will end up with final wealth W, such that

$$W = A[1 + R(A)]. \quad (2)$$

Clearly, convexity of $R(A)$ over some range is sufficient, but not necessary, for the convexity of $W(A)$ over the same range; both convex and concave rate-of-return functions may lead to a convex $W(A)$.[5]

[4] This type of return function is discussed also in Blinder (1974) and Appelbaum and Harris (1978).

[5] It does, however, seem reasonable that at least over some initial range (when R is not constant) $R'' > 0$, i.e., the function is convex. The convexity of $R(A)$ at the initial range would, e.g., follow from the convexity of the loan-supply function derived by Jaffee and Modigliani (1969), Barro (1976), and Benjamin (1978).

Let us now consider the individual's decision problem. Following the literature,[6] we assume the individual has a utility function defined on final wealth, $U(W)$, and that

$$U' > 0, \quad U'' < 0; \tag{3}$$

that is, the individual is risk averse.

Now, consider the individual's utility function when it is defined over A—his intermediate wealth. This is given by

$$V(A) = U\{A[1 + R(A)]\} \tag{4}$$

and

$$V'(A) = U' \cdot (1 + R + AR'), \tag{5}$$

$$V''(A) = U'' \cdot (1 + R + AR')^2 + U' \cdot (2R' + AR''). \tag{6}$$

Therefore, in accordance with our above discussion, $V' > 0$ for all values of A. For $A < A^*$ and $A > A^{**}$, V'' is clearly negative. Within the range $A^* < A \leq A^{**}$ the second term on the right-hand side of (6) may be positive and, hence, V'' may be positive. If this occurs there will be a section of $V(A)$ that will be convex and, hence, V will show the Friedman-Savage shape of being in turn concave, convex, and concave.

The result may also be derived diagrammatically, as is shown in figure 1. The curve $V(A)$ drawn in the first quadrant is clearly a Friedman-Savage type of utility function which allows both gambling and insurance in intermediate wealth to take place simultaneously.

Even if $V(A)$ is concave within the range $A^* < A < A^{**}$, we can still get a convex region, as in the Friedman-Savage utility function, since $V(A)$ will, in general, have a kink at A^*. Thus, although in this case $V(A)$ is made of concave segments only, it will nevertheless not be globally concave. This case is shown in figure 2, where again it is possible to observe gambling and insurance simultaneously.

Hence, we have provided an explanation of the shape of the Friedman-Savage utility function without suggesting that an individual is anything but everywhere risk averse. Rather than focus on the tastes of the individual, we have focused on his institutional constraints.[7]

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[6] See, e.g., Arrow 1970.

[7] An alternative way of obtaining these results is to employ the state preference approach and consider the effects of capital market imperfections on the set of acceptable gambles. It can be easily shown (see Appelbaum and Katz 1979) that the isoutility curves defined over two random states are not necessarily convex, and thus both gambling and insurance buying may occur simultaneously.
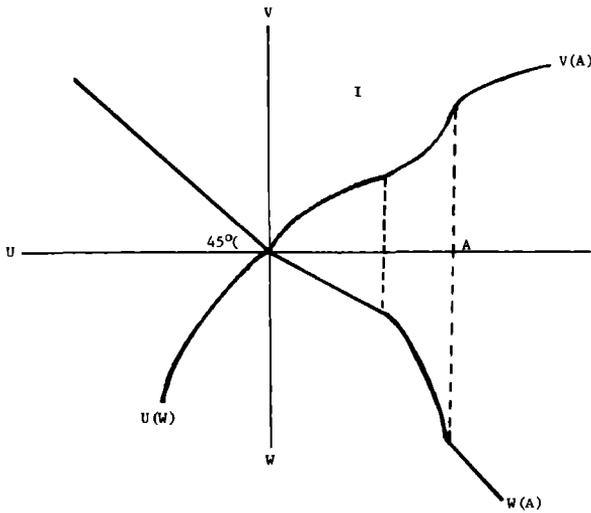
FIG. 1
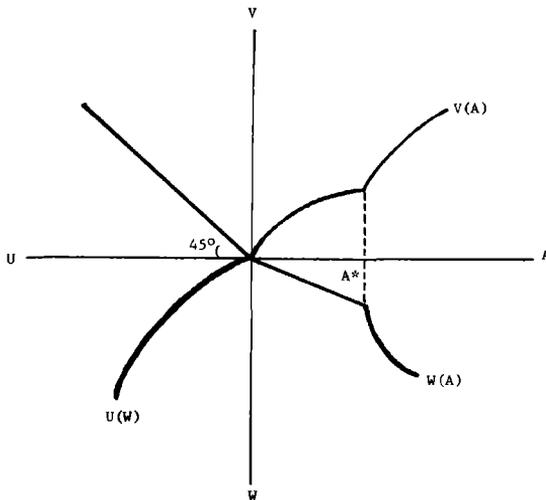
FIG. 2

IV. Conclusion

This paper provides a possible rationale for the Friedman-Savage utility function. It suggests that the existence of market imperfections, in particular in the capital markets, may impose various constraints on individuals and thus affect their behavior. The existence of these constraints could lead individuals to participate in unfair gambling, since this may reduce the implicit costs of the constraints.

Our explanation of gambling focuses on the constraints facing an individual rather than on his preferences, and consequently it does not require a modification of the standard assumption that individuals are risk averse. Of course, it may be that individuals show risk preference over a section of their utility function. Such a section is not, however, necessary in order to derive a Friedman-Savage type of utility function.

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