4-3-2008

Forecasting Models for Economic and Environmental Applications

Shou Hsing Shih
University of South Florida

Forecasting Models for Economic and Environmental Applications

by

Shou Hsing Shih

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctoral of Philosophy
Department of Mathematics and Statistics
College of Arts and Science
University of South Florida

Major Professor: Chris P. Tsokos, Ph.D.
Gangaram Ladde, Ph.D.
Kandethody Ramachandran, Ph.D.
Wonkuk Kim, Ph.D.
Marcus McWaters, Ph.D.

Date of Approval:
April 3, 2008

Keywords: Time Series, Global Warming, Stock, S&P Price Index, Temperature, Carbon Dioxide

Acknowledgement

I would like to express deep appreciation to my major professor Dr. Chris P. Tsokos for his patient advice, generous support, friendship, and the many countless hours of personal time he has spent assisting me throughout my graduate career. With his guidance, it has fostered an environment that has helped me complete this work diligently and with greater ease than I would have thought possible otherwise.

I would like to thank Professor Ladde for his assistance and advisement on the theoretical aspects of this work.

I would also like to express my appreciation to Professors Ramachadran, McWaters, and Kim, for their support and guidance to complete this work.

Table of Contents

List of Tables

List of Figures

Forecasting Models for Economic and Environmental Applications

Shou Hsing Shih

ABSTRACT

The object of the present study is to introduce three analytical time series models for the purpose of developing more effective economic and environmental forecasting models, among others. Given a stochastic realization, stationary or nonstationary in nature, one can utilize exciting methodology to develop an autoregressive, moving average or a combination of both for short and long term forecasting. In the present study we analytically modify the stochastic realization utilizing (a) a k-th moving average, (b) a k-th weighted moving average and (c) a k-th exponential weighted moving average processes. Thus, we proceed in developing the appropriate forecasting models with the new (modified) time series using the more recent methodologies in the subject matter. Once the proposed statistical forecasting models have been developed, we proceed to modify the analytical process back into the original stochastic realization.

The proposed methods have been successfully applied to real stock data from a Fortune 500 company. A similar forecasting model was developed and evaluated for the daily closing price of S&P Price Index of the New York Stock Exchange.

The proposed forecasting model was developed along with the statistical model using classical and most recent methods. The effectiveness of the two models was compared

using various statistical criteria. The proposed models gave better results.

Atmospheric temperature and carbon dioxide, CO_2, are the two variables most attributable to GLOBAL WARMING. Using the proposed methods we have developed forecasting statistical models for the continental United States, for both the atmospheric temperature and carbon dioxide. We have developed forecasting models that performed much better than the models using the classical Box-Jenkins type of methodology.

Finally, we developed an effective statistical model that relates CO_2 and temperature; that is, knowing the atmospheric temperature we can at the specific location estimate the carbon dioxide and vice versa.

Chapter 1

Literature Review and Fundamental Concepts

1.0 Introduction

Time series analysis is one of the major areas in statistics that can be applied to many realistic problems. In the present chapter, we begin with summarizing the development of time series modeling and introduce some methodologies that have been developed recently. We then introduce some fundamental concepts that are essential for dealing with time series models. Most real-world time series are nonstationary in nature. Thus, before we deal with those nonstationary phenomena in the latter chapter, we shall first define stochastic process and white noise process, then survey some of the most popular stationary time series forecasting methodologies, finally give a brief outline of the basic structure of each model.

1.1 Literature Review

During the preparation of the present study, we have surveyed literature in the area of time series forecasting. Although the present study is concerned mainly with economic and environmental applications, our literature survey was extended to the forecasting in general.

In 1960, Muth presented a method for forecasting by an exponentially weighted moving average. Muth's study included the simple, seasonal effects and seasonal effects with linear trend. Actual sales data is used in the study to examine this proposed modeling scheme.

D. W. Trigg initiated a study of automatically monitoring a forecasting process to assure that the forecast remains in control. Trigg utilizes a first order exponential model with data that contained jumps.

A study for short term sales forecasting was carried out by Harrison in 1965. The study examines several methods for short term sales predictions; the method of Box and Jenkins in 1970 and Brown in 1965. It is argued that multi-parameter procedures for short term forecasting of sales do not give significantly better results. Harrison recommends the one parameter exponential procedure for sales data for non-seasonal forecasting. Some methods are recommended for short term sales process with seasonal effects.

A two stage exponential model with exponential smoothing and multiple regression was introduced by Crane and Eeatly in 1967. They first applied the exponential process forecasts along with other independent variables into developing a multiple regression model. This combined forecasting procedure was applied in modeling some economic data series related to bank deposits. This method produced good forecasting results of the economic series.

In 1971, Tsokos studied some forecasting models for short term forecasting of economic data. He studied the autoregressive, moving average and mixture of autoregressive moving average process as short term forecasting models for commodity contracts. Several models were developed for soybean, silver and New York Time Daily averages of fifty combined stocks for the first 100 business days. Their short term forecasts were evaluated using the minimum residual variance criteria.

The investigation of outliers in time series was the aim of Fox in 1972. Two types of outliers in the time series were considered. Gross error in the sample of observations and a single innovation are extreme, but they were included in the study. The second type of outlier does not only effect the present observations but also subsequent observations. A likelihood ratio criterion was developed to study the problem of the two outliers.

In 1994, Box and Jenkins introduced the multiplicative ARIMA model on airline data. This type of model addresses the issue of seasonal variation that was not identifiable by the classical ARIMA model. Thus, once we identify the period of a time series, we can use the multiplicative ARIMA model to forecast this series much better than the classical method. We shall investigate this model further in Chapter 4 and 5.

In 1982, Engle introduced the autoregressive conditional heteroskedasticity (ARCH) model that considers the variance of the current term to be a function of the variances of the previous time period’s error terms. ARCH relates the error variance to the square of a previous period’s error. In 1986, the generalized autoregressive conditional heteroskedasticity (GARCH) model was first proposed by Bollerslev, and it is employed commonly in modeling financial time series that exhibit time varying volatility clustering. For further discussion on GARCH modeling, see (Enders 1995, Engle 2001, Gujarati 2003, and Nelson 1991).

The state space representation of a system is related to the Kalman Filter and was originally developed by control engineers Kalman 1960, Kalman and Bucy 1961, and Kalman, Falb, and Arbib 1969. The system is also known as a state space model and is defined to be the minimum set of information from the present and past such that the future behavior of the system can be completely described by the knowledge of the present state and future input. For further discussion and development on the state space model, see (Chen 1999, Khalil, Nise 2004, Hinrichsen & Pritchard 2005, Sontag 1999, and Durbin & Koopman 2001)

1.2 Fundamental Concepts

A time series can be thought of as comprising of a sequence of measurements, almost certainly intercorrelated, representing some phenomena in different areas. Each of the measurements is associated with a moment of time, with some measurements incorporating other parameters. A time series can be classified as continuous or discrete depending upon whether the sequence is continuous or discrete. In the present study, we shall be only concerned with finite discrete time series which are measured at equal-distant time intervals or those continuous time series that have been digitized to a finite discrete time series.

1.2.1 Stochastic Process

A stochastic process is a family of time indexed random variables $X(\omega,t)$, where ω belongs to a sample space and t belongs to an index set. For a fixed t, $X(\omega,t)$ is a random variable. For a given ω, $X(\omega,t)$, as a function of t, is called a sample function or realization. The population that consists of all possible realizations is called ensemble in stochastic processes and time series analysis. Thus, a time series is a realization or sample function from a certain stochastic process.

Consider a finite set of random variables $\{X_{t_1}, X_{t_2}, ..., X_{t_n}\}$ from a stochastic process $\{X(\omega,t) : t = 0, \pm 1, \pm 2, ...\}$. The n-dimensional distribution function is defined by

$$F_{X_{t_1} X_{t_2}, \ldots, X_{t_n}}(x_1, x_2, ..., x_n) = P\{\omega : X_{t_1} \le x_1, ..., X_{t_n} \le x_n\} \qquad (1.2.1)$$

where $x_i, i = 1,2,...,n$ are any real numbers. A process is said to be first-order stationary if its one-dimensional distribution function is time invariant. i.e., if $F_{X_{t_1}}(x_1) = F_{X_{t_1+k}}(x_1)$ for any integers t_1, k and $t_1 + k$; second-order stationary if $F_{X_{t_1}, X_{t_2}}(x_1, x_2) = F_{X_{t_1+k}, X_{t_2+k}}(x_1, x_2)$ for any integer $t_1, t_2, k, t_1 + k$ and $t_2 + k$; and nth-order stationary if

$$F_{X_{t_1}, X_{t_2}, \ldots, X_{t_n}}(x_1, x_2, ..., x_n) = F_{X_{t_1+k}, X_{t_2+k}, \ldots, X_{t_n+k}}(x_1, x_2, ..., x_n) \qquad (1.2.2)$$

for any n-tuple $(t_1, t_2, ..., t_n)$ and k integers. A process is said to be strictly stationary if (1.1.2) is true for $n = 1,2,....$ The terms strongly stationary and completely stationary are also used to denote a strictly stationary process. Suppose (1.1.2) is true for some $n = m$, it would also be true for $n \le m$ because the *m*th-order distribution function determines all distribution functions of lower order. Therefore, a higher order of stationarity always implies a lower order stationarity.

For a real-value process $\{X_t : t = 0, \pm 1, \pm 2, ...\}$, we defined the mean function of the process

$$\mu_t = E(X_t) \tag{1.2.3}$$

the variance function of the process

$$\sigma_t^2 = E[(X_t - \mu_t)^2] \tag{1.2.4}$$

the covariance function

$$\gamma(t_1, t_2) = E(X_{t_1} - \mu_{t_1})(X_{t_2} - \mu_{t_2}) \tag{1.2.5}$$

and the correlation function

$$\rho(t_1, t_2) = \frac{\gamma(t_1, t_2)}{\sqrt{\sigma_{t_1}^2}\sqrt{\sigma_{t_2}^2}} \tag{1.2.6}$$

Since the distribution function for a strictly stationary process is the same for all *t*, the mean function $\mu_t = \mu$ is a constant, provided $E(|X_t|) < \infty$. This implies $E(X_t^2) < \infty$, and hence $\sigma_t^2 = \sigma^2$ for all t is also a constant. In addition, since $F_{X_{t_1}, X_{t_2}}(x_1, x_2) = F_{X_{t_1+k}, X_{t_2+k}}(x_1, x_2)$ for any integer t_1, t_2 and k, we have

$$\gamma(t_1, t_2) = \gamma(t_1 + k, t_2 + k)$$

and

$$\rho(t_1, t_2) = \rho(t_1 + k, t_2 + k)$$

Letting $t_1 = t - k$ and $t_2 = t$, we get

$$\gamma(t_1, t_2) = \gamma(t - k, t) = \gamma(t, t + k) = \gamma_k \tag{1.2.7}$$

and

$$\rho(t_1, t_2) = \rho(t - k, t) = \rho(t, t + k) = \rho_k \tag{1.2.8}$$

Thus, for a strictly stationary process with first two moments finite, the covariance and correlation between X_t and X_{t+k} depend only on the time difference *k*.

A process is said to be *n*th-order weakly stationary if all of its joint moments up to the *n*th-order exist and are time invariant, hence a second-order weakly stationary process will have constant mean and variance, and the covariance and correlation functions depend only on the time difference. Moreover, a strictly stationary process with the first two moments being finite is also a second-order weakly stationary or covariance stationary process. However, a strictly stationary process does not always have finite moments. When this occurs, it may not be covariance stationary. Therefore, it is possible for a strictly stationary process, with no joint moments existing, to not be weakly stationary of any order.

1.2.2 White Noise Process

A process is said to be a white noise process $\{W_t\}$ if it is a sequence of uncorrelated random variables from a fixed distribution with constant mean $E(W_t) = \mu_W$ and constant variance $Var(W_t) = \sigma_W^2$ and $\gamma_k = Cov(W_t, W_{t+k}) = 0$ for all $k \neq 0$. It is obvious to see that a white noise process $\{W_t\}$ is stationary with the autocovariance function $\gamma_k = \sigma_W^2$ when $k = 0$, $\gamma_k = 0$ when $k \neq 0$; the autocorrelation function $\rho_k = 1$ when $k = 0$, $\rho_k = 0$ when $k \neq 0$; and partial autocorrelation function $\phi_{kk} = 1$ when $k = 0$, $\phi_{kk} = 0$ when $k \neq 0$.

The basic phenomenon of the white noise is that both its Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) are equal to zero. In addition, a white noise is Gaussian if its joint distribution is normal. This concept plays an important role in constructing time series models in our later chapters.

1.2.3 Estimations on a Single Realization

A stationary time series is characterized by its mean μ, variance σ^2, autocorrelation ρ_k, and partial autocorrelation ϕ_{kk}. We can calculate the exact values of these parameters if the ensemble of all possible realizations is available. However, for most time series, only a single realization is available, thus making it impossible to calculate the ensemble average. In the following discussion, we discuss some statistical properties on these estimators by using time averages, which are useful when we deal with only a single realization. The mean μ can be estimated by its sample mean

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t \tag{1.2.9}$$

which is the time average of *n* observations. To see whether this is a valid or good estimator, we take the expectation of $\bar{x}$, we get

$$E(\bar{x}) = \frac{1}{n}\sum_{t=1}^{n} E(x_t) = \frac{1}{n} \bullet n\mu = \mu \tag{1.2.10}$$

which indicates that $\bar{x}$ is an unbiased estimator for μ. And since

$$\begin{aligned} Var(\bar{x}) &= \frac{1}{n^2}\sum_{t=1}^{n}\sum_{s=1}^{n} Cov(x_t, x_s) = \frac{\gamma_0}{n^2}\sum_{t=1}^{n}\sum_{s=1}^{n}\rho_{(t-s)} \\ &= \frac{\gamma_0}{n^2}\sum_{k=-(n-1)}^{n-1}(n-|k|)\rho_k \\ &= \frac{\gamma_0}{n}\sum_{k=-(n-1)}^{n-1}(1-\frac{|k|}{n})\rho_k \end{aligned} \tag{1.2.11}$$

Choose $k = (t-s)$, we know that if

$$\lim_{n\to\infty}\left[\sum_{k=-(n-1)}^{n-1}\left(1-\frac{|k|}{n}\right)\rho_k\right]$$

is finite, then $Var(\bar{x}) \to 0$ as $n \to \infty$, and $\bar{x}$ is a consistent estimator for μ. That is,

$$\lim_{n\to\infty} \frac{1}{n}\sum_{t=1}^{n} x_t = \mu \tag{1.2.12}$$

Similarly, the variance σ^2 can be estimated by its sample variance by using the time average as follows

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{t=1}^{n}(x_t - \bar{x})^2 \tag{1.2.13}$$

the sample autocovariance function $\hat{\gamma}_k$ is given by

$$\hat{\gamma}_k = \sum_{t=1}^{n-k}(x_t - \bar{x})(x_{t+k} - \bar{x}) \tag{1.2.14}$$

the sample ACF ρ_k can be estimated by

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} = \frac{\sum_{t=1}^{n-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{n}(x_t - \bar{x})^2},\ k = 0,1,2,... \tag{1.2.15}$$

and the sample PACF ϕ_{kk} can be estimated by using a recursive method starting with $\hat{\phi}_{11} = \hat{\rho}_1$ was published by Durbin(1960) as follows:

$$\hat{\phi}_{k+1,k+1} = \frac{\hat{\rho}_{k+1} - \sum_{j=1}^{k}\hat{\phi}_{kj}\,\hat{\rho}_{k+1-j}}{1 - \sum_{j=1}^{k}\hat{\phi}_{kj}\,\hat{\rho}_j} \tag{1.2.16}$$

1.3 The Stationary AR, MA, and ARMA Models

In time series analysis there are two very useful representations to express as a time series process, namely the autoregressive process and the moving average process. Before the discussion of these models, it is important to define several useful notations for simplifying purposes. The backshift operator is defined as

$$B^j x_t = x_{t-j} \tag{1.3.1}$$

and let

$$\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p) \quad (1.3.2)$$

and

$$\theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - ... - \theta_q B^q) \quad (1.3.3)$$

By defining (1.3.2) and (1.3.3), we can reduce models into compact forms in latter discussion.

1.3.1 The Autoregressive (AR) Models

The autoregressive process of order *p*, denoted as AR(*p*) was first introduced by Yule in 1926. It is useful in describing situations in which the present value of a time series is explained by its past values plus a random shock. The *p*th-order autoregressive process is defined as follows:

$$\dot{x}_t = \phi_1 \dot{x}_{t-1} + \phi_2 \dot{x}_{t-2} + ... + \phi_p \dot{x}_{t-p} + \varepsilon_t \quad (1.3.4)$$

or

$$\phi_p(B)\dot{x}_t = \varepsilon_t \quad (1.3.5)$$

where $\{\varepsilon_t\}$ is a zero mean white noise process, and $\dot{x}_t = x_t - \mu$. In addition, the AR(*p*) process contains $p+1$ parameters, which are $\phi_1, \phi_2, ..., \phi_p, \sigma_\varepsilon^2$. As $\sum_{j=1}^{p} |\phi_j| < \infty$, the process is always invertible. To be stationary, the roots of $\phi_p(B) = 0$ must lie outside of the unit circle.

1.3.2 The Moving Average (MA) Models

The moving average process of order *q*, denoted as MA(*q*) was first presented by Slutzky in 1927. It is useful in describing phenomena that produce an immediate effect that only lasts for short periods of time. The *q*th-order moving average process is defined as follows:

$$\dot{x}_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - ... - \theta_q \varepsilon_{t-q} \qquad (1.3.6)$$

or

$$\dot{x}_t = \theta_q(B)\varepsilon_t \qquad (1.3.7)$$

The MA(*q*) process contains $q+1$ parameters, which are $\theta_1, \theta_2, ..., \theta_q, \sigma_\varepsilon^2$. Moreover, a finite moving average process is always stationary as $1+\theta_1^2+\theta_2^2+...+\theta_q^2 < \infty$, and the process is invertible if the roots $\theta_q(B)=0$ of lie outside the unit circle.

1.3.3 The General Mixed ARMA Models

The mixture of autoregressive and moving average process, denoted as ARMA(*p*, *q*), can be produced if we combine the autoregressive and moving average process. The process was put together by Wold in 1938, and it is defined as follows:

$$\dot{x}_t = \phi_1 \dot{x}_{t-1} + \phi_2 \dot{x}_{t-2} + ... + \phi_p \dot{x}_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - ... - \theta_q \varepsilon_{t-q} \qquad (1.3.8)$$

or

$$\phi_p(B)\dot{x}_t = \theta_q(B)\varepsilon_t \qquad (1.3.9)$$

where

$$\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p)$$

and

$$\theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - ... - \theta_q B^q)$$

The general mixed ARMA(*p*, *q*) process has $p+q+1$ parameters, which are

$\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q, \sigma_\varepsilon^2$. In addition, the process is invertible if the roots of $\theta_q(B) = 0$ lie outside the unit circle and the process is stationary if the roots of $\phi_p(B) = 0$ lie outside of the unit circle and under the assumption that $\theta_q(B) = 0$ and $\phi_p(B) = 0$ share no common roots.

1.4 Aims of Present Study

The aim of the present study is to propose several new time series methodologies that can be applied to many economic and environmental problems. We begin with summarizing some fundamental concepts that are essential for dealing with time series modeling and introduce some preliminary processes that are necessary for model building in time series.

After we define all the necessities for stationary time series, we shall introduce the classical nonstationary time series modeling. Box and Jenkins methodology is one the most famous time series approach that is widely being used by many researchers working in many different areas. The idea of our proposed models is to filter a nonstationary time series and smooth the edges of that series so we can be better forecast the phenomena. The smoothing components that we implement into the time series shall be removed once we have obtained the model. Due to the complexity of the developmental processes, we summarize the methodologies for both the classical approach and our proposed models by providing step by step procedures in chapter 2.

In chapter 3, 4, 5 and 6, we introduce several economic and environmental applications by developing time series forecasting models using both the classical approach and our proposed methodologies. We shall show the quality and usefulness of our proposed models on those applications by comparing them with the classical methods. In each of the applications, we shall examine the proposed models by looking into some essential statistical properties numerically and identify the usefulness under circumstances. Finally, we shall evaluate the proposed models by ranking their efficiency under different circumstances according to their performance in forecasting the future phenomena.

1.5 Conclusion

In the present chapter, we began with surveying several major literatures in the area of time series forecasting and providing the references of the development of those methodologies in the subject area. We then defined some fundamental concepts such as stochastic process, white noise process and stationary ARMA process. Those processes are not only essential for our proposed model building procedures but also play a major role in time series forecasting. Finally, we briefly discuss the approach of our proposed methodologies and show the usefulness and effectiveness of our proposed models with the applications in the present study.

Chapter 2

Analytical Formulations of Modified Time Series

2.0 Introduction

In the present chapter, we shall summarize the updated step-by-step process on developing a forecasting model from a non-stationary stochastic realization. In addition, we introduce three additional models, namely simple moving average, weighted moving average, and the exponential moving average. The basic structure of these models begins with the actual non-stationary realizations and we formulate a new time series from the original data based on the models we mentioned above.

2.1 The General ARIMA Model

The time series processes we have discussed so far are all stationary processes, but most of the time series in reality are non-stationary. Therefore, the roots of their AR polynomials do not lie outside the unit circle. Hence, we will not be able to use the general mixed ARMA(p,q) as we discussed in the previous chapter on a non-stationary time series. In order for us to resolve this issue, we must first introduce the difference filter as follows:

$$(1-B)^d \tag{2.1.1}$$

where $B^j x_t = x_{t-j}$, and d is the degree of differencing of the series.

Due to the reason that the local behavior of such non-stationary time series is independent of its level, we let $\pi(B)$ be the autoregressive operator which describes the behavior of this homogeneous non-stationary time series, we have

$$\pi(B)(x_t + C) = \pi(B)x_t \tag{2.1.2}$$

for any constant *C*. This equation implies that $\pi(B)$ must be of the form

$$\pi(B) = \phi(B)(1-B)^d \tag{2.1.3}$$

for some $d > 0$, where $\phi(B)$ is a stationary autoregressive operator. Hence, we can reduce a non-stationary time series to a stationary time series after taking a proper degree of differencing of the series.

After we define the difference filter, we can now introduce the famous ARIMA(*p*,*d*,*q*) autoregressive integrated moving average model as follows:

$$\phi_p(B)(1-B)^d x_t = \theta_q(B)\varepsilon_t \tag{2.1.4}$$

where *d* is the degree of differencing,

$$\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)$$

and

$$\theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q)$$

Consider the simplest case, ARIMA(0,1,1), we have

$$(1-B)x_t = (1-\theta_1 B)\varepsilon_t$$

or we can expand the model as

$$x_t = x_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

which is a MA(1) model on the difference of the non-stationary time series.

Consider another simplest case, ARIMA(1,1,0), we have

$$(1-\phi_1 B)(1-B)x_t = \varepsilon_t$$

or we can expand the model as

$$x_t = (1+\phi_1)x_{t-1} - x_{t-2} + \varepsilon_t$$

which is a AR(1) model on the difference of the non-stationary time series.

In time series analysis, sometimes it is very difficult to make a decision in selecting the best order of the ARIMA(p,d,q) model when we have several models that all adequately represent a given set of time series. Hence, Akaike's information criterion (AIC), [1], plays a major role when it comes to model selection. AIC was first introduced by Akaike in 1973, and it is defined as follows:

$$\text{AIC}(M) = -2 \ln [\text{maximum likelihood}] + 2M, \tag{2.1.5}$$

where M is the number of parameters in the model and the unconditional log-likelihood function suggested by Box, Jenkins, and Reinsel in 1994, [4], is given by

$$\ln L(\phi,\mu,\theta,\sigma_\varepsilon^2) = -\frac{n}{2}\ln 2\pi\sigma_\varepsilon^2 - \frac{S(\phi,\mu,\theta)}{2\sigma_\varepsilon^2}, \tag{2.1.6}$$

where $S(\phi,\mu,\theta)$ is the unconditional sum of squares function given by

$$S(\phi,\mu,\theta) = \sum_{t=-\infty}^{n} [E(\varepsilon_t \mid \phi,\mu,\theta,x)]^2 \tag{2.1.7}$$

where $E(\varepsilon_t \mid \phi,\mu,\theta,x)$ is the conditional expectation of ε_t given ϕ,μ,θ,x.

The quantities $\hat{\phi}$, $\hat{\mu}$, and $\hat{\theta}$ that maximize (2.1.6) are called unconditional maximum likelihood estimators. Since $\ln L(\phi,\mu,\theta,\sigma_\varepsilon^2)$ involves the data only through $S(\phi,\mu,\theta)$, these unconditional maximum likelihood estimators are equivalent to the unconditional least squares estimators obtained by minimizing $S(\phi,\mu,\theta)$. In practice, the summation in (2.1.7) is approximated by a finite form

$$S(\phi,\mu,\theta)=\sum_{t=M}^{n}[E(\varepsilon_t|\phi,\mu,\theta,x)]^2 \qquad (2.1.8)$$

where M is a sufficiently large integer such that the backcast increment $|E(\varepsilon_t|\phi,\mu,\theta,x)-E(\varepsilon_{t-1}|\phi,\mu,\theta,x)|$ is less than any arbitrary predetermined small ε value for $t \le -(M+1)$. This expression implies that $E(\varepsilon_t|\phi,\mu,\theta,x)\cong\mu$; hence, $E(\varepsilon_t|\phi,\mu,\theta,x)$ is negligible for $t \le -(M+1)$.

After obtaining the parameter estimates $\hat{\phi}$, $\hat{\mu}$, and $\hat{\theta}$, the estimate $\hat{\sigma}_\varepsilon^2$ of σ_ε^2 can then be calculated from

$$\hat{\sigma}_\varepsilon^2=\frac{S(\hat{\phi},\hat{\mu},\hat{\theta})}{n}. \qquad (2.1.9)$$

For an ARMA(p,q) model based on n observations, the log-likelihood function is

$$\ln L=-\frac{n}{2}\ln 2\pi\sigma_\varepsilon^2-\frac{1}{2\sigma_\varepsilon^2}S(\phi,\mu,\theta). \qquad (2.1.10)$$

We proceed to maximize (2.1.10) with respect to the parameters $\phi,\mu,\theta,$ and σ_ε^2, from (2.1.9), we have

$$\ln\hat{L}=-\frac{n}{2}\ln\hat{\sigma}_\varepsilon^2-\frac{n}{2}(1+\ln 2\pi). \qquad (2.1.11)$$

Since the second term in expression (2.1.11) is a constant, we can reduce the AIC to the following expression

$$\text{AIC}(M)=n\ln\hat{\sigma}_\varepsilon^2+2M. \qquad (2.1.12)$$

Thus, we generate an appropriate time series model and select the statistical process with the smallest AIC. The model that we have identified will possess the smallest average mean square error. In addition, we summarize the development of the model as follows:

- Check for stationarity of the time series by determining the order of differencing *d*, where $d = 0,1,2,...$ according to KPSS test [12], until we achieve stationarity
- Deciding the order m of the process, for our case, we let $m = 5$ where $p + q = m$
- After (*d*, m) being selected, listing all possible set of (*p*, *q*) for $p + q \le m$
- For each set of (*p, q*), estimating the parameters of each model, that is, $\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q$
- Compute the AIC for each model, and choose the one with smallest AIC

According to the criterion that we mentioned above, we can obtain the ARIMA(*p*,*d*,*q*) model that best fit a given time series, where the coefficients are $\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q$.

2.2 The *k-th* Moving Average Time Series Model

In this section, we propose a model which is constructed based on the concept of the simple moving average. Before we actually discuss the proposed models, we shall first define the simple moving average. In time series analysis, the primary use for the *k-th* simple moving average is for smoothing a time series. It is very useful in discovering short-term, long-term trends and seasonal components of a time series. The *k-th* simple moving average process of a time series $\{x_t\}$ is defined as follows:

$$y_t = \frac{1}{k} \sum_{j=0}^{k-1} x_{t-k+1+j} \tag{2.2.1}$$

where $t = k, k+1, ..., n$.

It is obvious to see that as *k* increases, the number of observations of $\{y_t\}$ decreases, and the series $\{y_t\}$ gets closer and closer to the mean of the series $\{x_t\}$ as *k* increases. In addition, when $k = n$, the series $\{y_t\}$ reduces to a single observation, and equals to μ, because

$$y_t = \frac{1}{n}\sum_{j=1}^{n} x_j = \mu \tag{2.2.2}$$

On the other hand, if we choose a fairly small *k*, we can smooth the edges of the series without losing too much of the general information.

We proceed to develop our proposed model by transforming the original time series $\{x_t\}$ into $\{y_t\}$ by applying (2.2.1). After establishing the new time series, usually very nonstationary, we begin the process of reducing it to stationary time series by selecting the appropriate differencing order. We then proceed with the model building procedure of developing time series model and at each stage calculate the AIC. The best model will be the one with the smallest AIC.

Using the model that we developed for $\{y_t\}$ and subject to the AIC criteria, we forecast values of $\{y_t\}$ and proceed to apply the back-shift operator to obtain estimates of the original phenomenon $\{x_t\}$, that is,

$$\hat{x}_t = k\,\hat{y}_t - x_{t-1} - x_{t-2} - \ldots - x_{t-k+1}\,. \tag{2.2.3}$$

In addition, we summarize the development of the model as follows:

- Transforming the original time series $\{x_t\}$ into $\{y_t\}$ by applying (2.2.1)
- Check for stationarity of the series $\{y_t\}$ by determining the order of differencing *d*, where $d = 0,1,2,\ldots$ according to KPSS test, until we achieve stationarity
- Deciding the order m of the process, for our case, we let $m = 5$ where $p + q = m$
- After (d, m) being selected, listing all possible set of (*p*, *q*) for $p + q \le m$

- For each set of (p, q), estimating the parameters of each model, that is,

  $\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q$

- Compute the AIC for each model, and choose one with the smallest AIC
- Solve the estimates of the original time series by applying (2.2.3)

The proposed model and the corresponding procedure discussed in this section shall be illustrated with real economic application and the results will be compared with the classical time series model.

2.3 The *k-th* Weighted Moving Average Time Series Model

Compare to the model that we proposed in the previous section, the model we proposed in this section is more useful to those time series which behave more aggressively compared with some time series which do not. Before we actually present the model, we shall first introduce the weighted moving average process. The *k-th* weighted moving average process is also a very good smoothing tool. However, the structure of it is slightly different from the simple moving average process. It puts more weight on the most recent observation, and the weight consistently decreases up to the first observation. In addition, it captures the original time series better than the moving average process, which is suitable for those analysts who believe the recent observations should weight more than the old ones. The *k-th* weighted moving average process of a time series $\{x_t\}$ is defined as follows:

$$z_t = \frac{1}{(1+k)k/2} \sum_{j=0}^{k-1} (j+1) x_{t-k+1+j} \qquad (2.3.1)$$

where $t = k, k+1, ..., n$. Similar to the moving average process, as *k* increases, the number of observations of the series $\{z_t\}$ decreases, and as $k \to n$, from (2.3.1) the series $\{z_t\}$ becomes

$$z_t = \frac{1}{(1+n)n/2} \sum_{j=1}^{n} j x_j \qquad (2.3.2)$$

If we choose a fairly small *k*, we can smooth the edges of the series, and the series $\{z_t\}$ is closer to the series $\{x_t\}$ compared with the moving average method that we discussed in the last section.

Our first proposed model begins with transforming the original time series $\{x_t\}$ into $\{z_t\}$ by applying (2.3.1). After establishing the new time series, we begin the process of reducing it to stationary time series by selecting the appropriate differencing order. We then proceed with the model building procedure of developing time series model and at each stage calculate the AIC. The best model will be the one with the smallest AIC.

Using the model that we developed for $\{z_t\}$ and subject to the AIC criteria, we forecast values of $\{z_t\}$ and proceed to apply the back-shift operator to obtain estimates of the original phenomenon $\{x_t\}$, that is,

$$\hat{x}_t = \frac{[(1+k)k/2]\hat{z}_t - (k-1)x_{t-1} - (k-2)x_{t-2} - ... - x_{t-k+1}}{k} \qquad (2.3.3)$$

In addition, we summarize the development of the model as follows:

- Transforming the original time series $\{x_t\}$ into $\{z_t\}$ by applying (2.3.1)
- Check for stationarity of the series $\{z_t\}$ by determining the order of differencing *d*, where $d = 0,1,2,...$ according to KPSS test, until we achieve stationarity

- Deciding the order m of the process, for our case, we let $m = 5$ where $p + q = m$
- After (d, m) being selected, listing all possible set of (p, q) for $p + q \leq m$
- For each set of (p, q), estimating the parameters of each model, that is,

  $\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q$
- Compute the AIC for each model, and choose one with the smallest AIC
- Solve the estimates of the original time series by applying (2.3.3)

To conclude the goodness of fit of the proposed model, we will illustrate in a later chapter with numerical examples.

2.4 The *k-th* Exponential Weighted Moving Average Time Series Models

Similar to the last two sections, in order for us to present our proposed models, we must first introduce the exponential weighted moving average method. The *k*-days exponential weighted moving average serves pretty much the same purpose as those two average processes as we discussed in the last two sections. Instead of decreasing weight consistently as the weighted moving average method, it decreases weight exponentially. This fits perfect for those analysts who care most on the recent observations and not pay too much attention to the old ones. In addition, it is not difficult to see that the exponential weighted moving average catches the original series faster than both average processes as we discussed earlier. Hence, it is very useful in dealing with very aggressive time series. The *k-th* exponential weighted moving average process of a time series $\{x_t\}$ is defined as follows:

$$v_t = \frac{1}{\sum_{j=0}^{k-1}(1-\alpha)^j} \sum_{j=0}^{k-1} (1-\alpha)^{k-j-1} x_{t-k+1+j} \qquad (2.4.1)$$

where $t = k, k+1, ..., n$, and the smoothing factor α is defined as $\alpha = \frac{2}{k+1}$. If we let $k = n$, we have $\alpha = \frac{2}{n+1}$. Moreover, $\sum_{j=0}^{k-1}(1-\alpha)^j$ reaches its maximum when $k = 3$, and it gets closer and closer to 1 as *k* increases. As *k* increases, the number of observations the series $\{v_t\}$ decreases, and it eventually reduces to a single observation when $k = n$. As $k \to n$, the series $\{v_t\}$ becomes

$$v_t = \frac{1}{\sum_{j=0}^{n-1}(1-\alpha)^j} \sum_{j=0}^{n-1}(1-\alpha)^{n-j-1} x_{j+1} \tag{2.4.2}$$

From (2.4.2), it is obvious to see that the exponential weighted moving average process weighs heavily on the most recent observation and decreases the weight exponentially as time decreases. If we choose a fairly small *k*, we can smooth the edges of the series and the $\{v_t\}$ would be the closest to the series $\{x_t\}$ compared with both moving average processes that we discussed in the previous two sections.

We proceed to develop our proposed model by transforming the original time series $\{x_t\}$ into $\{v_t\}$ by applying (2.4.1). After obtaining the new time series, we select the appropriate differencing order to reduce the series $\{v_t\}$ into a stationary time series. We then proceed the model building procedure of developing time series model and at each stage calculate the AIC. The best model will be the one with the smallest AIC.

Using the model that we developed for $\{v_t\}$ and subject to the AIC criteria, we forecast values of $\{v_t\}$ and proceed to apply the back-shift operator to obtain estimates of the original phenomenon $\{x_t\}$, that is,

$$\hat{x}_t = \frac{\hat{v}_t}{\sum_{j=0}^{k-1}(1-\alpha)^j} - (1-\alpha)x_{t-1} - (1-\alpha)^2 x_{t-2} - \ldots - (1-\alpha)^{k-1} x_{t-k-1} \qquad (2.4.3)$$

In addition, we summarize the development of the model as follows:

- Transforming the original time series $\{x_t\}$ into $\{v_t\}$ by applying (2.4.1)
- Check for stationarity of the series $\{v_t\}$ by determining the order of differencing d, where $d = 0,1,2,...$ according to KPSS test, until we achieve stationarity
- Deciding the order m of the process, for our case, we let $m = 5$ where $p + q = m$
- After (d, m) being selected, listing all possible set of (p, q) for $p + q \leq m$
- For each set of (p, q), estimating the parameters of each model, that is, $\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q$
- Compute the AIC for each model, and choose one with the smallest AIC
- Solve the estimates of the original time series by applying (2.4.3)

The proposed model and the corresponding procedure discussed in this section shall be illustrated with real economic application and the results will be compared with the classical time series model.

2.5 The Multiplicative ARIMA Model

In time series analysis, seasonal variations sometimes dominate the variations of the original nonstationary time series. It occurs very commonly on the environmental applications, as most of the environmental forecasting problems, there exists some type of periodic trend that is obvious for us to recognize. We can treat the seasonal time series as a nonstationary time series that varies along some kind of seasonal periodic trend.

Hence, addressing the seasonal variations for the model becomes extremely useful when we deal with these types of problems.

The multiplicative seasonal autoregressive integrated moving average, ARIMA model is defined by

$$\Phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D x_t = \theta_q(B)\Gamma_Q(B^s)\varepsilon_t \tag{2.5.1}$$

where *p* is the order of the autoregressive process, *d* is the order of regular differencing, *q* is the order of the moving average process, *P* is the order of the seasonal autoregressive process, *D* is the order of the seasonal differencing, *Q* is the order of the seasonal moving average process, and the subindex *s* refers to the seasonal period. We shall denote the subject model by ARIMA$(p,d,q)\times(P,D,Q)_s$, and $\phi_p(B),\theta_q(B),\Phi_P(B^s),\Gamma_q(B^s)$ defined as follows:

$$\phi_p(B) = (1-\phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)$$

$$\theta_q(B) = (1-\theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q)$$

$$\Phi_P(B^s) = 1-\Phi_1 B^s - \Phi_2 B^{2s} - \ldots - \Phi_P B^{Ps}$$

and

$$\Gamma_Q(B^s) = 1-\Gamma_1 B^s - \Gamma_2 B^{2s} - \ldots - \Gamma_Q B^{Qs}.$$

The order of the multiplicative ARIMA model determines the structure of the model, and it is essential to have a good methodology in terms of developing the forecasting model.

Below we summarize the model identifying procedure:

- Determine the seasonal period *s*.

- Check for stationarity of the given time series $\{x_t\}$ by determining the order of differencing *d*, where $d = 0,1,2,...$ according to KPSS test, until we achieve stationarity.
- Deciding the order m of the process, for our case, we let $m = 5$ where $p + q + P + Q = m$.
- After (d, m) being selected, listing all possible configurations of (p, q, P, Q) for $p + q + P + Q \leq m$.
- For each set of (p, q, P, Q), estimates the parameters for each model, that is, $\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q, \Phi_1, \Phi_2, ..., \Phi_P, \Gamma_1, \Gamma_2, ..., \Gamma_Q$.
- Compute the AIC for each model, and choose the one with smallest AIC.
- After (p, d, q, P, Q) is selected, we determine the seasonal differencing filter by selecting the smaller AIC between the model with $D = 0$ and $D = 1$.
- Our final model will have identified the order of (p, d, q, P, D, Q).

To conclude the goodness of fit of the Multiplicative ARIMA model, we will illustrate in latter chapter with environmental applications.

2.6 Conclusion

In the present chapter, we began with developing a step-by-step forecasting procedure for the classical ARIMA model. We then introduced our three proposed models, namely the *k-th* Moving Average Time Series Model, the *k-th* Weighted Moving Average Time Series Model, and the *k-th* Exponential Weighted Moving Average Time Series Model, of which we believe perform better than the classical methodology. In addition, we have developed a step-by-step procedure for each of our proposed models. This step-by-step procedure of each model shall be very helpful when one needs to develop model following our methodologies. Finally, we introduce the multiplicative ARIMA model that is able to address the seasonal variation when we deal with environmental applications in later chapters and a step-by-step procedure for the subject model.

Chapter 3

Econometrics Forecasting Models

3.0 Introduction

The object of the present chapter is to illustrate the results of our proposed forecasting models for two nonstationary stochastic realizations. The subject models are based on modifying both given time series into a new *k-th* moving average, *k-th* weighted moving average, and *k-th* exponential weighted moving average time series to begin the development of the model. The study is based on the autoregressive integrated moving average process along with its analytical constrains. The analytical procedure of the proposed models is given in chapter two. The first series is a stock XYZ selected from the Fortune 500 companies and its daily closing price constitute the time series. The second series is the closing price of the S&P price index. Both the classical and proposed forecasting models were developed and a comparison of the accuracy of their responses is given.

3.1 Forecasting Models on Stock XYZ

In this section, we first begin with collecting 500 observations for a stock from the fortune 500 companies and try to forecast its daily closing value by using the traditional approach versus different forecasting models that we proposed in chapter two. We then examine each model by looking into the properties of its plots and residuals and rank each model based on the results of our findings. Finally, we hide the last 25 observations and forecast the next 25 observations without using the future information. Thus, the coefficients of the models are updated every time when we get new information. We then examine the last 25 residuals so we can determine the goodness of fit of those models and draw conclusions based on these results.

3.1.1 Data Preparation

In order for us to properly illustrate different types of moving average ARIMA models, we need to first create another three time series, namely 3 days moving average time series (MA3), 3 days weighted moving average time series (WMA3), and 3 days exponential weighted moving average time series (EWMA3) by using the methodology that we discussed in chapter two.

Suppose we let $\{x_t\}$ (see Appendix A1) be the daily closing values of stock XYZ that we collect from the fortune 500 companies that we mentioned earlier. A plot of the actual information is given by Figure 3.1

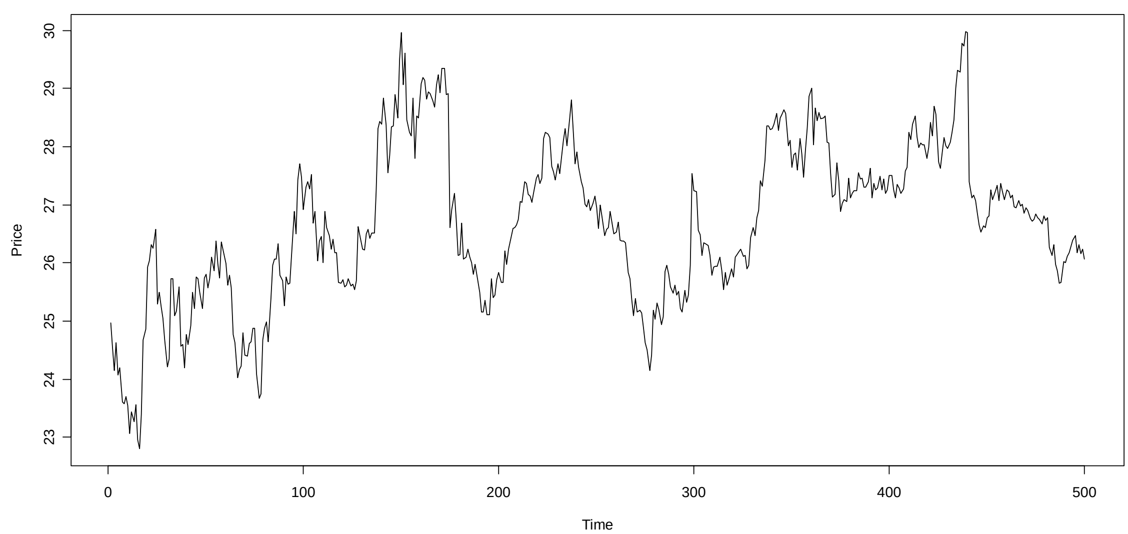

Figure 3.1 Daily Closing Price for Stock XYZ

In order for us to apply the fitting procedure for our first proposed model, we must transform $\{x_t\}$ into a 3 days moving average series $\{y_t\}$ (see Appendix A2) by referring to (2.2.1). Hence, we have $y_1 = y_2$ are not available, and

$$y_3 = \frac{x_1 + x_2 + x_3}{3}$$

$$y_4 = \frac{x_2 + x_3 + x_4}{3}$$

$$\vdots$$

$$y_t = \frac{x_{t-2} + x_{t-1} + x_t}{3} \tag{3.1.1}$$

Similar to the last transformation, we need to create another series by referring to (2.3.1), which is transferring $\{x_t\}$ into a 3 days weighted moving average series $\{z_t\}$ (see Appendix A3). Hence, we have $z_1 = z_2$ are not available, and

$$z_3 = \frac{x_1 + 2x_2 + 3x_3}{6}$$

$$z_4 = \frac{x_2 + 2x_3 + 3x_4}{6}$$

$$\vdots$$

$$z_t = \frac{x_{t-2} + 2x_{t-1} + 3x_t}{6} \tag{3.1.2}$$

The last transformation we need to make is turning the original time series $\{x_t\}$ into a 3 days exponential weighted moving average series $\{v_t\}$ (see Appendix A4) by referring to (2.4.1). that is $v_1 = v_2$ are not available, and

$$v_3 = \frac{.25x_1 + .5x_2 + x_3}{1.75}$$

$$v_4 = \frac{.25x_2 +,5x_3 + x_4}{1.75}$$

$$\vdots$$

$$v_t = \frac{.25x_{t-2} + .5x_{t-1} + x_t}{1.75} \tag{3.1.3}$$

After finishing all of the above transformations, we end up with 4 set of different time series observations, which are $\{x_t\}$, $\{y_t\}$, $\{z_t\}$, and $\{v_t\}$. Now we are ready for the model fitting procedures in the next section.

In the following sub-sections, we shall follow those step by step procedures as we discussed in chapter two for both the classical time series approach and our proposed methods.

3.1.2 The General ARIMA Model

Following the step-by-step procedure that we introduced in Section 2.1, the classical forecasting model with the best AIC score is the ARIMA(1,1,2). That is, a combination of first order autoregressive (AR) and a second order moving average (MA) with a first difference filter. Thus, we can write it as

$$(1-.9631B)(1-B)x_t = (1-1.0531B+.0581B^2)\varepsilon_t\,. \tag{3.1.4}$$

After expanding the autoregressive operator and the difference filter, we have

$$(1-1.9631B+.9631B^2)x_t = (1-1.0531B+.0581B^2)\varepsilon_t$$

and the model can be written as

$$x_t = 1.9631x_{t-1} - .9631x_{t-2} + \varepsilon_t - 1.0531\varepsilon_{t-1} + .0581\varepsilon_{t-2}$$

by letting $\varepsilon_t = 0$, we have the one day ahead forecasting time series of the closing price of stock XYZ as

$$\hat{x}_t = 1.9631x_{t-1} - .9631x_{t-2} - 1.0531\varepsilon_{t-1} + .0581\varepsilon_{t-2}. \qquad (3.1.5)$$

Using the above equation, we graph the forecasting values obtained by using the classical approach on top of the original time series, shown as Figure 3.2

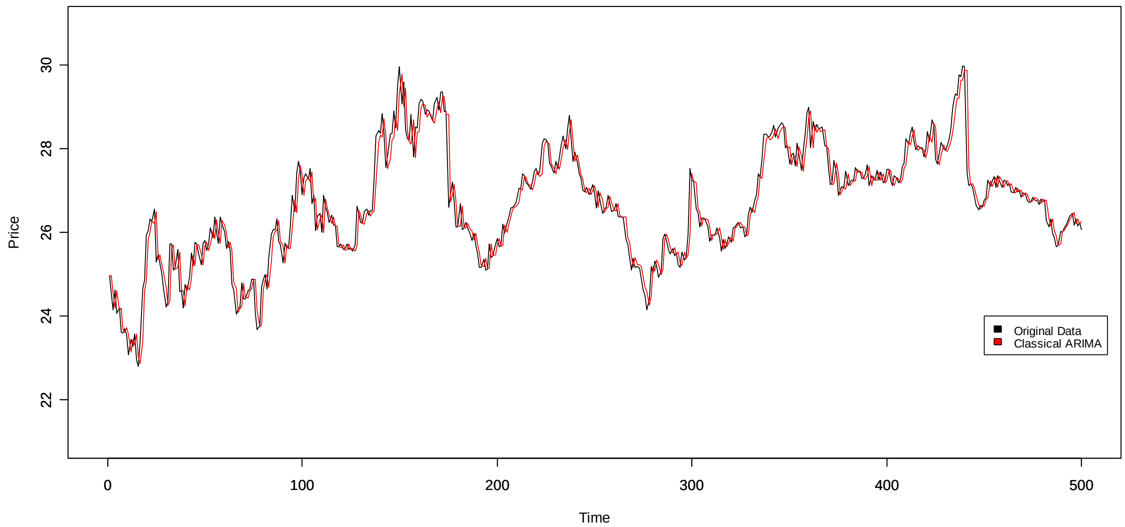

Figure 3.2 Comparisons on Classical ARIMA Model VS. Original Time Series

The basic statistics that reflect the accuracy of model (3.1.5) are the mean $\bar{r}$, variance S_r^2, standard deviation S_r and standard error $S_r/\sqrt{n}$ of the residuals. Figure 3.3 gives a time series plot of the residual and table 3.1 provides the basic statistics.

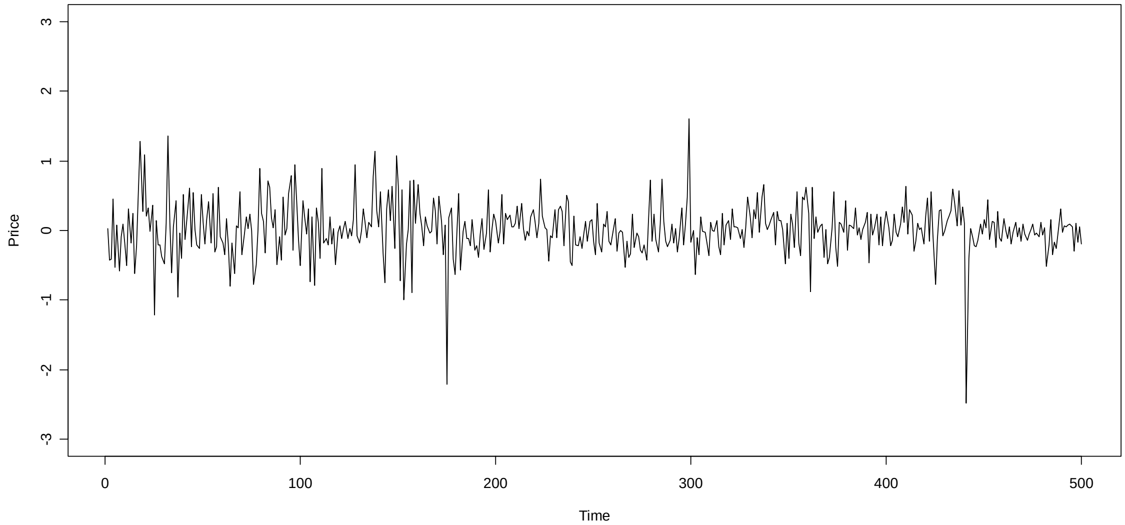

Figure 3.3 Time Series Plot of the Residuals for Classical Model

Table 3.1 Basic Evaluation Statistics for Classical ARIMA

$\bar{r}$	S_r^2	S_r	$S_r / \sqrt{n}$
0.02209169	0.1445187	0.3801562	0.0170011

Furthermore, we restructure the model (3.1.5) with $n = 475$ data points to forecast the last 25 observations only using the previous information. The purpose is to see how accurate our forecast prices are with respect to the actual 25 values that have not been used. Table 3.2 gives the actual price, predicted price, and residuals between the forecasts and the 25 hidden values.

Table 3.2 Actual and Predicted Price for Classical ARIMA

N	Actual Price	Predicted Price	Residuals
476	26.78	26.8473	-0.0673
477	26.75	26.7976	-0.0476
478	26.67	26.7673	-0.0972
479	26.8	26.6922	0.1078
480	26.73	26.8064	-0.0764
481	26.78	26.7490	0.0310
482	26.27	26.7911	-0.5211
483	26.12	26.3277	-0.2077
484	26.32	26.1631	0.1569
485	25.98	26.3364	-0.3564
486	25.86	26.0349	-0.1749
487	25.65	25.9068	-0.2568
488	25.67	25.6670	0.0031
489	26.02	25.7119	0.3081
490	26.01	26.0335	-0.0235
491	26.11	26.0427	0.0674
492	26.18	26.1343	0.0457
493	26.28	26.2032	0.0768
494	26.39	26.2986	0.0914
495	26.46	26.4043	0.0557
496	26.18	26.4743	-0.2943
497	26.32	26.2219	0.0981
498	26.16	26.3354	-0.1754
499	26.24	26.1953	0.0447
500	26.07	26.2602	-0.1902

The average of these residuals is $\bar{r} = -0.05608$, and the Figure 3.4 is a graph presentation of the forecasting result.

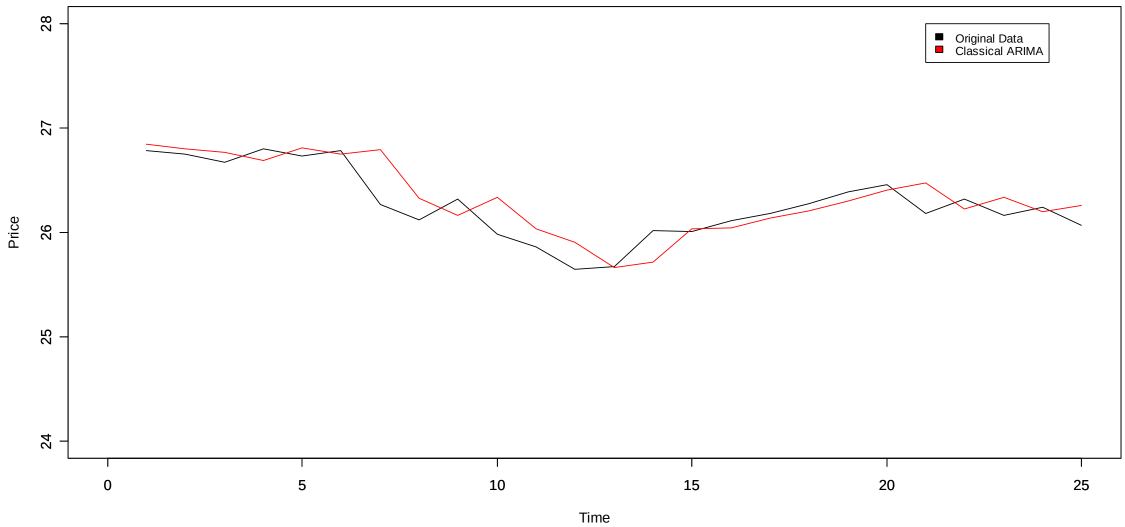

Figure 3.4 Classical ARIMA Forecasting on the Last 25 Observations

3.1.3 The 3 Days Moving Average ARIMA Models

Figure 3.5 shows the new time series $\{y_t\}$ along with the original time series $\{x_t\}$, that we shall use to develop the proposed forecasting model.

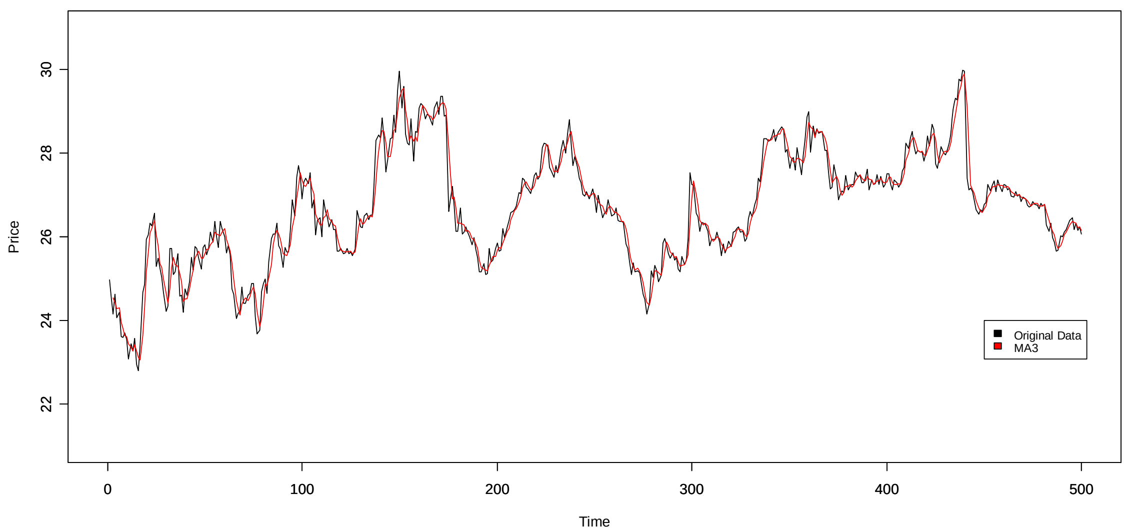

Figure 3.5 MA3 Series VS. The Original Time Series

Following the procedure we discussed in section 2.2, we found the best model that characterizes the behavior of $\{y_t\}$ to be ARIMA(2,1,3). That is,

$$(1-.8961B-.0605B^2)(1-B)y_t = (1+.0056B-.0056B^2-B^3)\varepsilon_t. \qquad (3.1.6)$$

Expanding the autoregressive operator and the first difference filter, we have

$$(1-1.8961B+.8356B^2+.0605B^3)y_t = (1+.0056B-.0056B^2-B^3)\varepsilon_t.$$

and the model can be written as

$$y_t = 1.8961y_{t-1} - .8356y_{t-2} - .0605y_{t-3} + \varepsilon_t + .0056\varepsilon_{t-1} - .0056\varepsilon_{t-2} - \varepsilon_{t-3}.$$

The final analytical form of the proposed forecasting model can be written as

$$\hat{y}_t = 1.8961y_{t-1} - .8356y_{t-2} - .0605y_{t-3} + .0056\varepsilon_{t-1} - .0056\varepsilon_{t-2} - \varepsilon_{t-3}. \qquad (3.1.7)$$

Using the above equation, we graph the forecasting values obtained by using the MA3 ARIMA approach on top of the original time series, shown as Figure 3.6

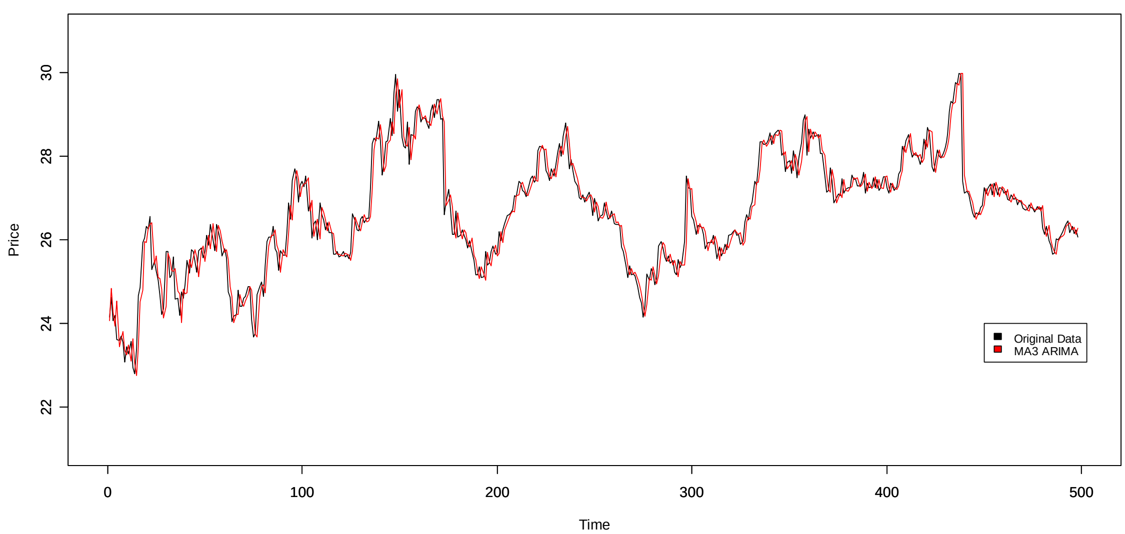

Figure 3.6 Comparisons on MA3 ARIMA Model VS. Original Time Series

Note the very closeness of the two plots that reflect the quality of the proposed model.

Similar to the classical model approach that we discussed earlier, we shall use the first 475 observations $\{y_1, y_2, ..., y_{475}\}$ to forecast $\hat{y}_{476}$. Then we use the observations $\{y_1, y_2, ..., y_{476}\}$ to forecast $\hat{y}_{477}$, and continue this process until we obtain forecasts all the observations, that is, $\{\hat{y}_{476}, \hat{y}_{477}, ..., \hat{y}_{500}\}$. From equation (2.2.3), we can see the relationship between the forecasting values of the original series $\{x_t\}$ and the forecasting values of 3 days moving average series $\{y_t\}$,that is,

$$\hat{x}_t = 3\hat{y}_t - x_{t-1} - x_{t-2}. \tag{3.1.8}$$

Hence, after we estimated $\{\hat{y}_{476}, \hat{y}_{477}, ..., \hat{y}_{500}\}$, we can use the above equation, (3.1.8), to solve the forecasting values for $\{x_t\}$. Figure 3.7 is the residual plot generated by the MA3 ARIMA model, and followed by Table 3.3, that includes the basic evaluation statistics.

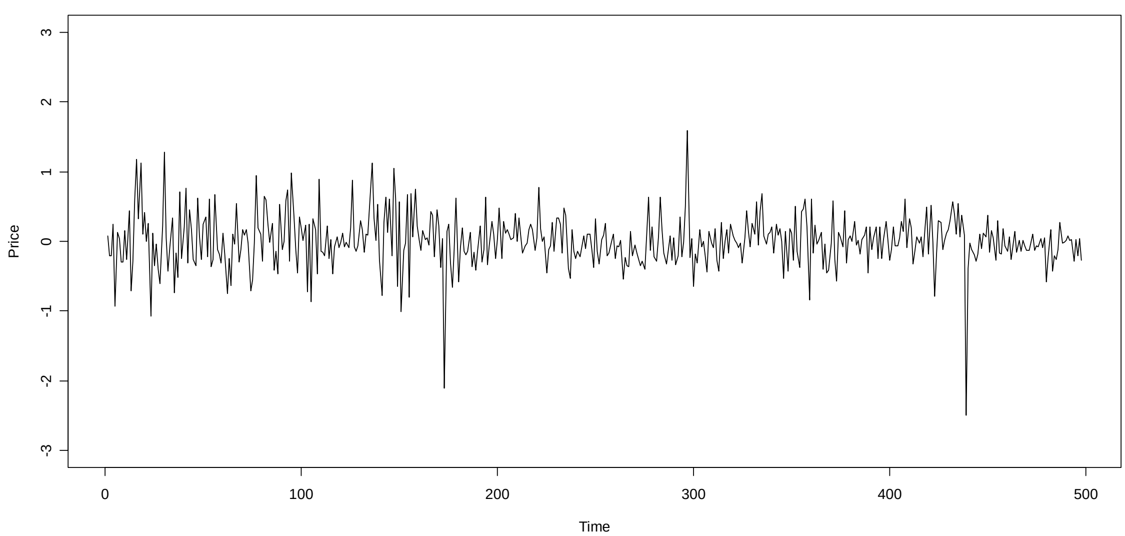

Figure 3.7 Time Series Plot of the Residuals for MA3 Model

Table 3.3 Basic Evaluation Statistics for MA3 ARIMA

$\bar{r}$	S_r^2	S_r	$S_r/\sqrt{n}$
0.01016814	0.1437259	0.3791119	0.01698841

Both of the above displayed evaluations reflect on accuracy of the proposed model. The actual daily closing prices of stock XYZ from the 476th day along with the forecasted prices and residuals are provided in Table 3.4.

Table 3.4 Actual and Predicted Price for MA3 ARIMA

N	Actual Price	Predicted Price	Residuals
476	26.78	26.8931	-0.1131
477	26.75	26.7715	-0.0215
478	26.67	26.7121	-0.0421
479	26.8	26.7239	0.0761
480	26.73	26.7854	-0.0554
481	26.78	26.6892	0.0908
482	26.27	26.8292	-0.5592
483	26.12	26.3027	-0.1827
484	26.32	26.0808	0.2392
485	25.98	26.3603	-0.3803
486	25.86	25.9868	-0.1268
487	25.65	25.8443	-0.1943
488	25.67	25.7115	-0.0414
489	26.02	25.6499	0.3701
490	26.01	25.9650	0.0450
491	26.11	26.0526	0.0574
492	26.18	26.0912	0.0888
493	26.28	26.1449	0.1351
494	26.39	26.3090	0.0810
495	26.46	26.3752	0.0848
496	26.18	26.4223	-0.2423
497	26.32	26.2461	0.0739
498	26.16	26.2964	-0.1364
499	26.24	26.1437	0.0963
500	26.07	26.2678	-0.1978

The Results given above attest to the good forecasting estimates for the hidden data. The average of these residuals is $\bar{r} = -0.0342$, and the Figure 3.8 is a graph presentation of the forecasting result.

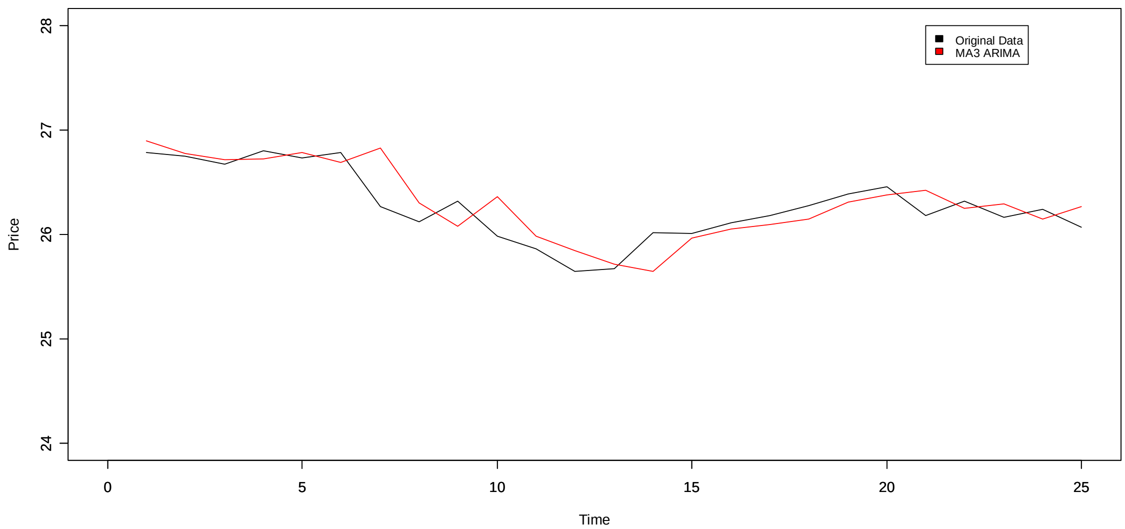

Figure 3.8 MA3 ARIMA Forecasting on the Last 25 Observations

3.1.4 The 3 Days Weighted Moving Average ARIMA Models

Figure 3.9 shows the new time series $\{y_t\}$ along with the original time series $\{x_t\}$, that we shall use to develop the proposed forecasting model.

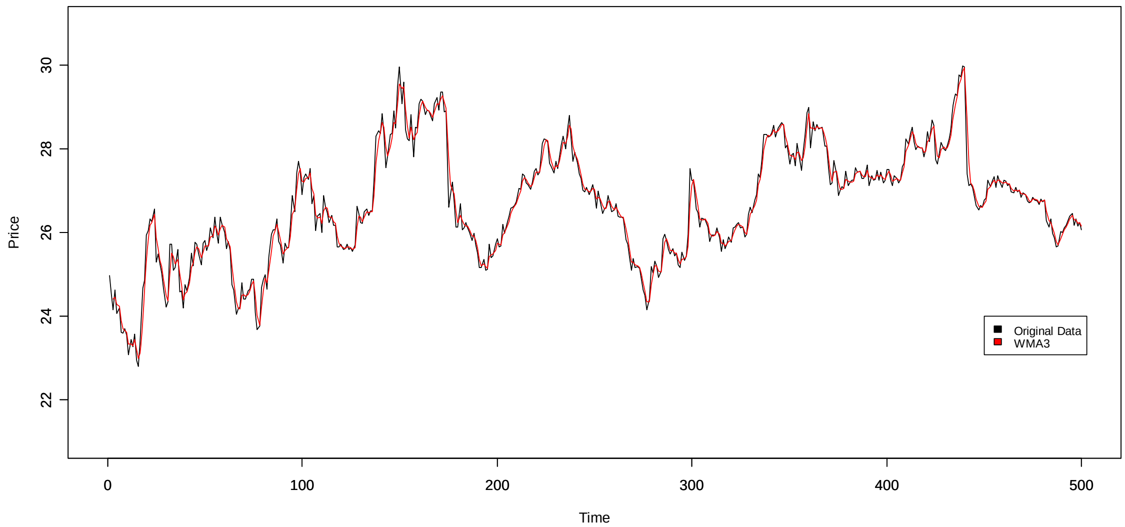

Figure 3.9 WMA3 Series VS. The Original Time Series

Following the procedure we have stated in section 2.3, we found the best model that characterizes the behavior of $\{z_t\}$ to be ARIMA(1,1,3). That is,

$$(1+.9073\,B)(1-B)z_t = (1+1.5084\,B+.8348\,B^2+.2456\,B^3)\varepsilon_t \qquad (3.1.9)$$

expand the autoregressive operator and the difference filter, we have

$$(1-.0927B-.9073B^2)z_t = (1+1.5084B+.8348B^2+.2456B^3)\varepsilon_t$$

and the model can be written as

$$z_t = .0927z_{t-1} + .9073z_{t-2} + \varepsilon_t + 1.5084\varepsilon_{t-1} + .8348\varepsilon_{t-2} + .2456\varepsilon_{t-3}$$

by letting $\varepsilon_t = 0$, we have the one day ahead forecasting series as

$$\hat{z}_t = .0927z_{t-1} + .9073z_{t-2} + 1.5084\varepsilon_{t-1} + .8348\varepsilon_{t-2} + .2456\varepsilon_{t-3} \qquad (3.1.10)$$

Using the above equation, we graph the forecasting values obtained by using the WMA3 ARIMA approach on top of the original time series, shown as Figure 3.10

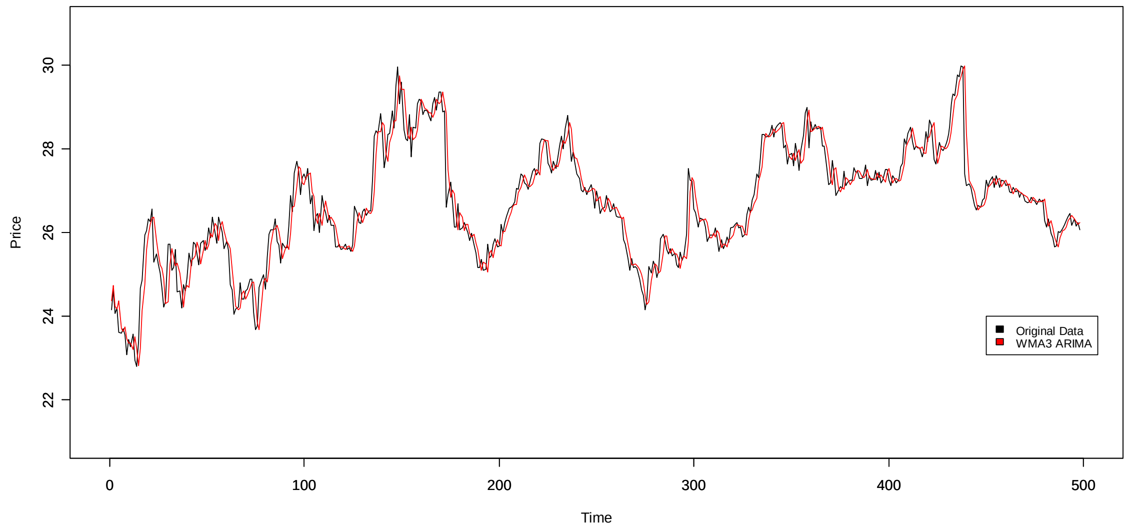

Figure 3.10 Comparisons on WMA3 ARIMA Model VS. Original Time Series

Similarly, we shall use the first 475 observations $\{z_1, z_2, ..., z_{475}\}$ to forecast $\hat{z}_{476}$.

Then we use the observations $\{z_1, z_2, ..., z_{476}\}$ to forecast $\hat{z}_{477}$, and continue this process until we obtain forecasts all the observations, that is, $\{\hat{z}_{476}, \hat{z}_{477}, ..., \hat{z}_{500}\}$. From equation (2.3.3), we can see the relationship between the forecasting values of the original series $\{x_t\}$ and the forecasting values of 3 days moving average series $\{z_t\}$,that is,

$$\hat{x}_t = \frac{6\hat{z}_t - 2x_{t-1} - x_{t-2}}{3}. \qquad (3.1.11)$$

Hence, after we estimated $\{\hat{z}_{476}, \hat{z}_{477}, ..., \hat{z}_{500}\}$, we can use the above equation, (3.1.11), to solve the forecasting values for $\{x_t\}$. Figure 3.11 is the residual plot generated by our proposed model, and followed by Table 3.5, that includes the basic evaluation statistics.

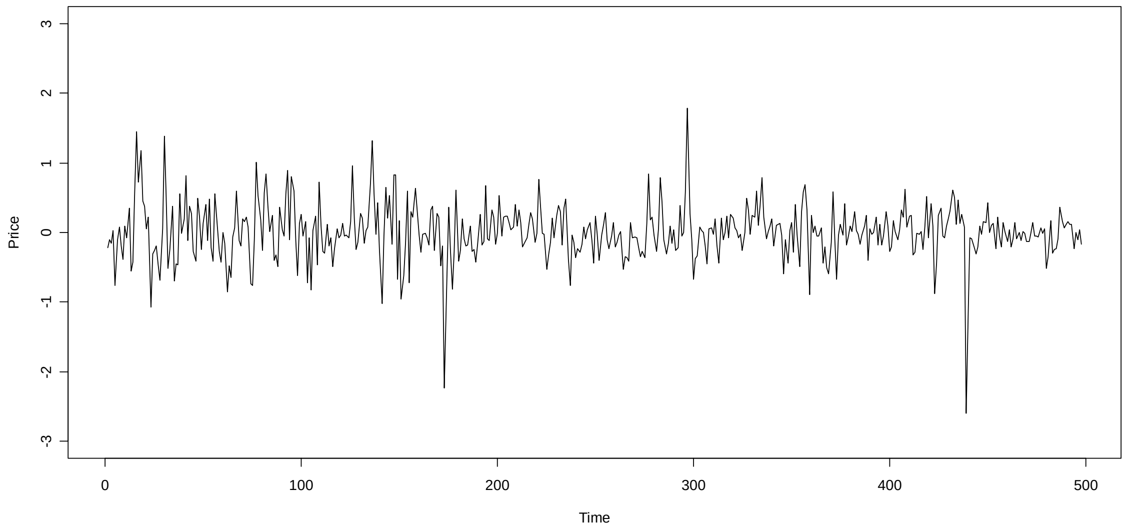

Figure 3.11 Time Series Plot of the Residuals for WMA3 Model

Table 3.5 Basic Evaluation Statistics for WMA3 ARIMA

$\bar{r}$	S_r^2	S_r	$S_r / \sqrt{n}$
0.00866631	0.1578446	0.3972966	0.01776764

A detail ranking comparison between models will be illustrated in latter section, and Table 3.6 shows a head to head comparison between the actual and predicted price for WMA3 ARIMA model.

Table 3.6 Actual and Predicted Price for WMA3 ARIMA

N	Actual Price	Predicted Price	Residuals
476	26.78	26.8435	-0.0635
477	26.75	26.77416	-0.0242
478	26.67	26.76265	-0.0926
479	26.8	26.6686	0.1314
480	26.73	26.79599	-0.0660
481	26.78	26.72857	0.0514
482	26.27	26.7817	-0.5117
483	26.12	26.30523	-0.1852
484	26.32	26.13652	0.1835
485	25.98	26.30601	-0.3260
486	25.86	25.98931	-0.1293
487	25.65	25.89196	-0.2420
488	25.67	25.65289	0.0171
489	26.02	25.67122	0.3488
490	26.01	25.9955	0.0145
491	26.11	26.00049	0.1095
492	26.18	26.11326	0.0667
493	26.28	26.1717	0.1083
494	26.39	26.26891	0.1211
495	26.46	26.38754	0.0725
496	26.18	26.44883	-0.2688
497	26.32	26.20532	0.1147
498	26.16	26.31132	-0.1513
499	26.24	26.16914	0.0709
500	26.07	26.23146	-0.1615

The average of these residuals is $\bar{r} = -0.0325$, and the Figure 3.12 is a graph presentation of the forecasting result.

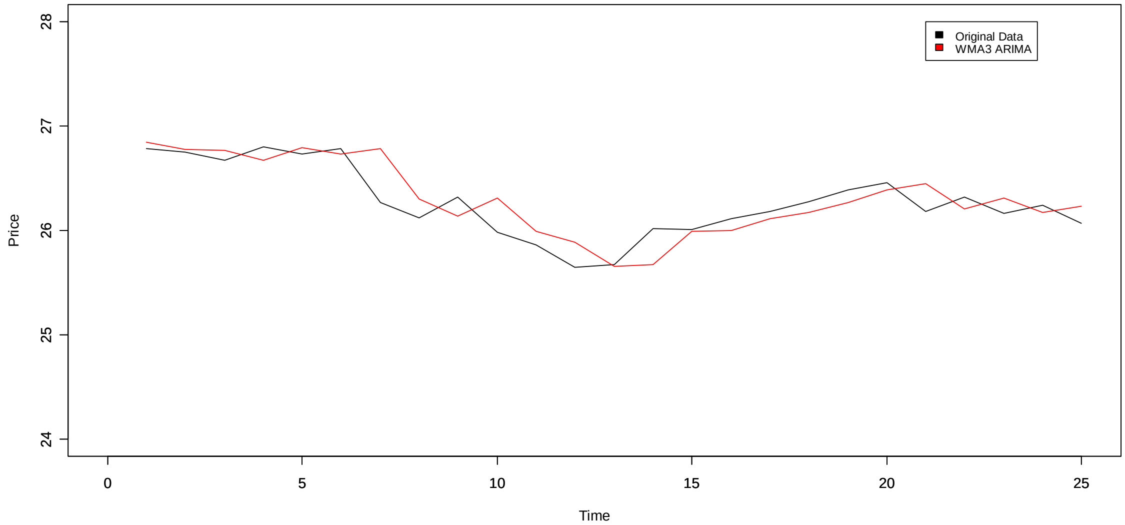

Figure 3.12 WMA3 ARIMA Forecasting on the Last 25 Observations

3.1.5 The 3 Days Exponential Weighted Moving Average ARIMA Models

Figure 3.13 shows the new time series $\{y_t\}$ along with the original time series $\{x_t\}$, that we shall use to develop the proposed forecasting model.

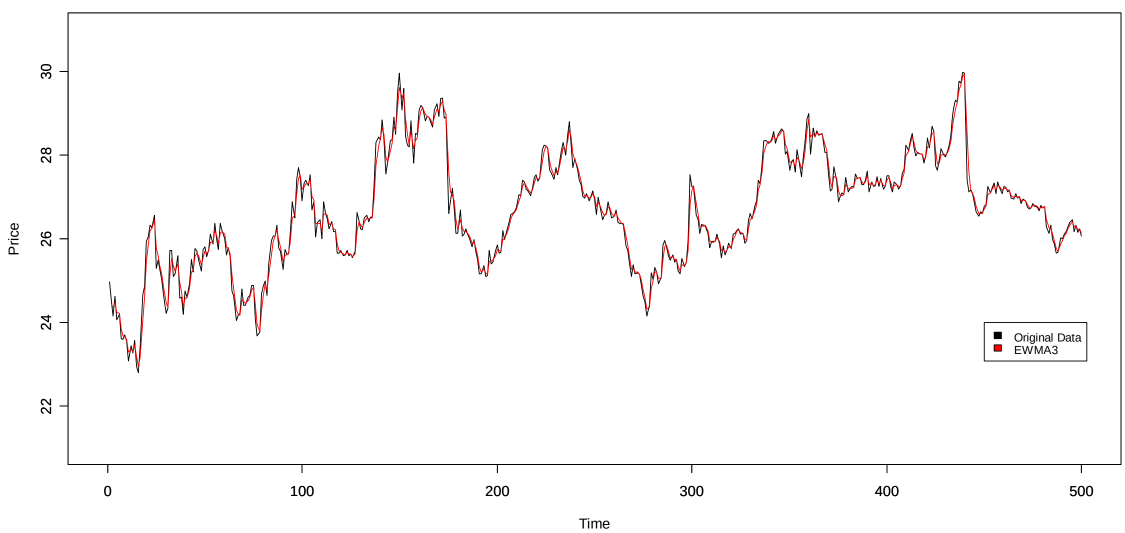

Figure 3.13 WMA3 Series VS. The Original Time Series

Following the procedure we have stated in section 2.4, we found the best model that characterizes the behavior of $\{v_t\}$ to be ARIMA(3,1,2), that is,

$$(1-.4766B-.9045B^2)(1-B)v_t=(1-.4362B-.0728B^2-.9071B^3)\varepsilon_t \qquad (3.1.12)$$

expand the autoregressive operator and the difference filter, we have

$$(1-1.4766B-.4279B^2+.9045B^3)v_t=(1-.4362B-.0728B^2-.9071B^3)\varepsilon_t$$

and rewrite the model as

$$v_t=1.4766v_{t-1}+.4279v_{t-2}-.9045v_{t-3}+\varepsilon_t-.4362\varepsilon_{t-1}-.0728\varepsilon_{t-2}-.9071\varepsilon_{t-3}$$

by letting $\varepsilon_t=0$, we have the one day ahead forecasting series as

$$\hat{v}_t=1.4766v_{t-1}+.4279v_{t-2}-.9045v_{t-3}-.4362\varepsilon_{t-1}-.0728\varepsilon_{t-2}-.9071\varepsilon_{t-3} \quad (3.1.13)$$

Using the above equation, we graph the forecasting values obtained by using the EWMA3 ARIMA approach on top of the original time series, shown as Figure 3.14

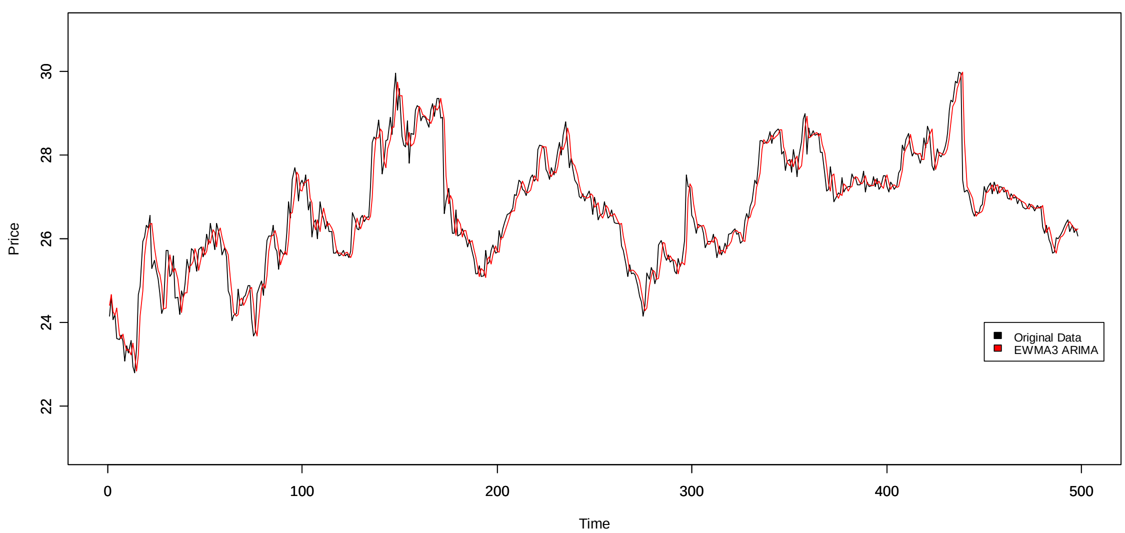

Figure 3.14 Comparisons on EWMA3 ARIMA Model VS. Original Time Series

Similarly, we shall use the first 475 observations $\{v_1,v_2,...,v_{475}\}$ to forecast $\hat{v}_{476}$.

Then we use the observations $\{v_1,v_2,...,v_{476}\}$ to forecast $\hat{v}_{477}$, and continue this process until we obtain forecasts all the observations, that is, $\{\hat{v}_{476}, \hat{v}_{477}, ..., \hat{v}_{500}\}$. From equation (2.4.3), the relationship between the forecasting values of the original series $\{x_t\}$ and the forecasting values of 3 days moving average series $\{v_t\}$ can be derived as

$$\hat{x}_t = 1.75\hat{v}_t - .5x_{t-1} - .25x_{t-2}. \tag{3.1.14}$$

Hence, after we estimated $\{\hat{v}_{476}, \hat{v}_{477}, ..., \hat{v}_{500}\}$, we can use the above equation, (3.1.14), to solve the forecasting values for $\{x_t\}$. Figure 3.15 is the residual plot generated by our proposed model, and followed by Table 3.7, that includes the basic evaluation statistics.

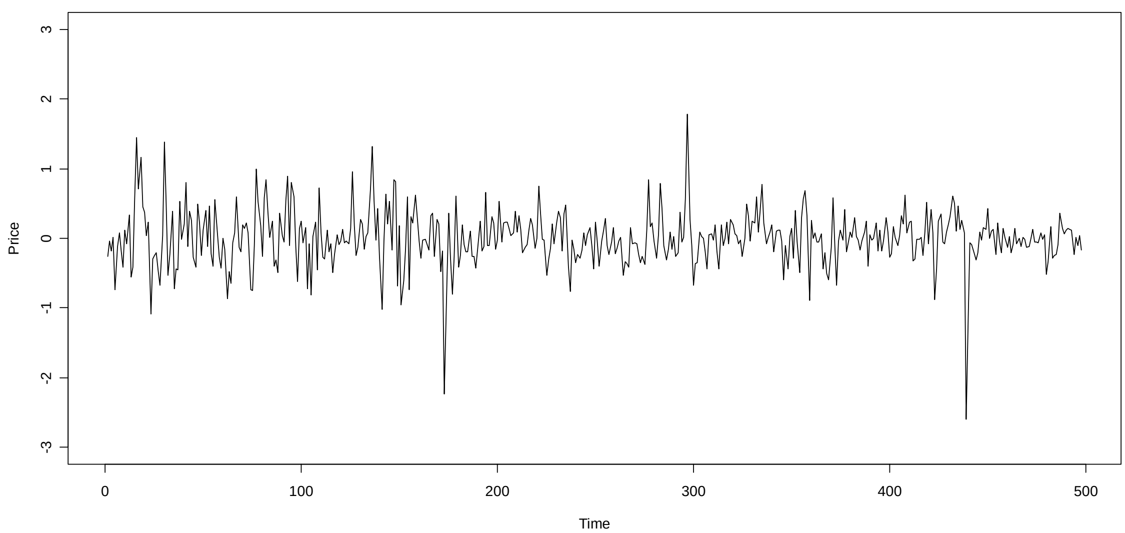

Figure 3.15 Time Series Plot of the Residuals for EWMA3 Model

Table 3.7 Basic Evaluation Statistics for EWMA3 ARIMA

$\bar{r}$	S_r^2	S_r	$S_r/\sqrt{n}$
0.008076663	0.1573456	0.3966682	0.01773954

A detail ranking comparison between models will be illustrated in latter section, and Table 3.8 shows a head to head comparison between the actual and predicted price for WMA3 ARIMA model.

Table 3.8 Actual and Predicted Price for EWMA3 ARIMA

N	Actual Price	Predicted Price	Residuals
476	26.78	26.86504	-0.0850
477	26.75	26.80201	-0.0520
478	26.67	26.79553	-0.1255
479	26.8	26.69121	0.1088
480	26.73	26.81608	-0.0861
481	26.78	26.75107	0.0289
482	26.27	26.81218	-0.5422
483	26.12	26.3397	-0.2197
484	26.32	26.16653	0.1535
485	25.98	26.30794	-0.3279
486	25.86	26.02086	-0.1609
487	25.65	25.94825	-0.2983
488	25.67	25.68467	-0.0147
489	26.02	25.71181	0.3082
490	26.01	26.03114	-0.0211
491	26.11	26.05992	0.0501
492	26.18	26.18416	-0.0042
493	26.28	26.22193	0.0581
494	26.39	26.31487	0.0751
495	26.46	26.43036	0.0296
496	26.18	26.48979	-0.3098
497	26.32	26.25195	0.0680
498	26.16	26.34178	-0.1818
499	26.24	26.19074	0.0493
500	26.07	26.2655	-0.1955

The average of these residuals is $\bar{r} = -0.0678$, and the Figure 3.16 is a graph presentation of the forecasting result.

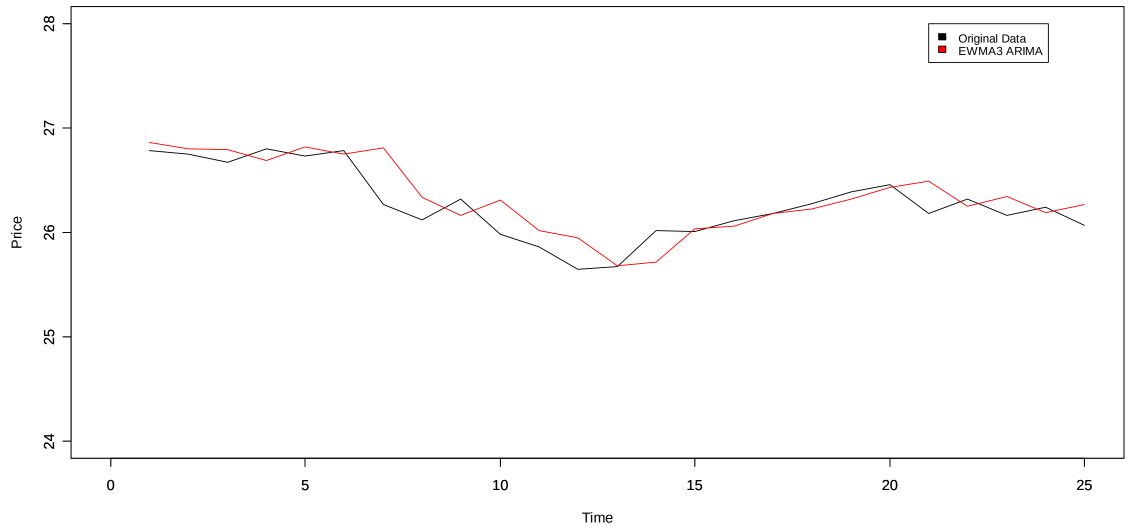

Figure 3.16 EWMA3 ARIMA Forecasting on the Last 25 Observations

3.2 Forecasting Models on S&P Price Index

In the following application, we shall use the S&P Price Index, and consider its daily closing price for 500 days to constitute the time series $\{x_t\}$. A plot of the actual information is given by Figure 3.17.

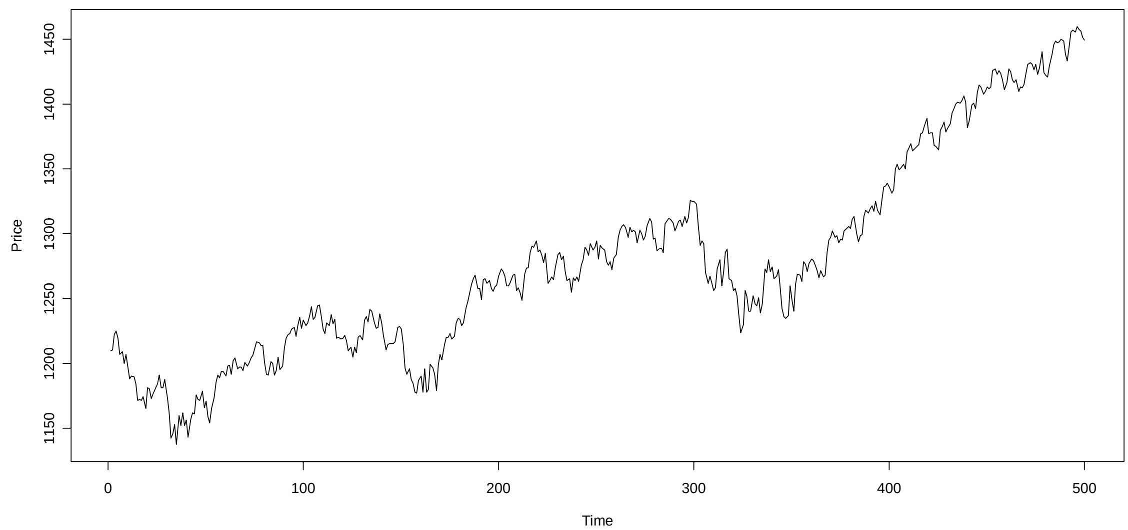

Figure 3.17 Daily Closing Price for S&P Price Index

3.2.1 Data Preparation

To proceed with the model building processes, we shall first create another three time series, namely 3 days moving average time series (MA3), 3 days weighted moving average time series (WMA3), and 3 days exponential weighted moving average time series (EWMA3) by using the methodologies that we discussed in chapter two.

Suppose we let $\{x_t\}$ (see Appendix B1) be the daily closing values of the S&P Price Index that we mentioned earlier. In order for us to proceed the fitting procedure for our first proposed model, we must transform $\{x_t\}$ into a 3 days moving average series $\{y_t\}$ (see Appendix B2), a 3 days weighted moving average series $\{z_t\}$ (see Appendix B3), and a 3 days exponential weighted moving average series $\{v_t\}$ (see Appendix B4) by referring to (3.1.1), (3.1.2), and (3.1.3) respectively.

After finishing all of the above transformations, we end up with 4 sets of different time series observations, which are $\{x_t\}$, $\{y_t\}$, $\{z_t\}$, and $\{v_t\}$. Hence, we are now ready to proceed with the model fitting procedures.

In the following sub-sections, we shall follow those step by step procedures as we discussed in chapter two for both the classical time series approach and our proposed methods.

3.2.2 The General ARIMA Model

Following the step-by-step procedure that we introduced in Section 2.1, the classical forecasting model with the best AIC score is the ARIMA(0,1,2). That is, a second order moving average (MA) with a first difference filter. Thus, we can write it as

$$(1-B)x_t = (1-.0331B-.1104B^2)\varepsilon_t\,. \tag{3.2.1}$$

After expanding the autoregressive operator and the difference filter, we have

$$x_t - x_{t-1} = (1 - .0331B - .1104B^2)\varepsilon_t$$

and the model can be written as

$$x_t = x_{t-1} + \varepsilon_t - .0331\varepsilon_{t-1} - .1104\varepsilon_{t-2}$$

by letting $\varepsilon_t = 0$, we have the one day ahead forecasting time series of the closing price of stock XYZ as

$$x_t = x_{t-1} - .0331\varepsilon_{t-1} - .1104\varepsilon_{t-2}. \qquad (3.2.2)$$

Using the above equation, we graph the forecasting values obtained by using the classical approach on top of the original time series, shown as Figure 3.2

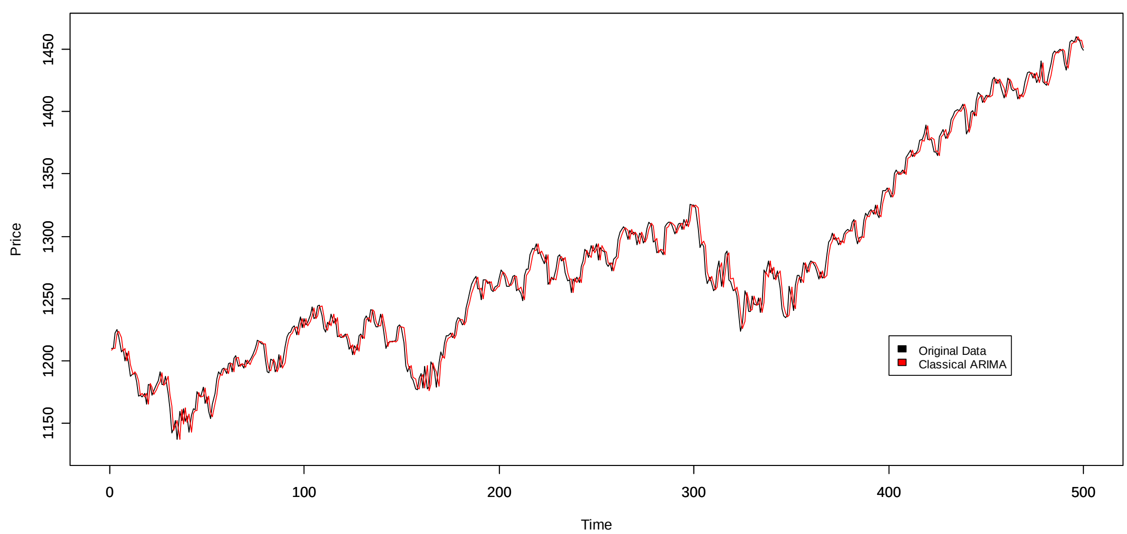

Figure 3.18 Comparisons on Classical ARIMA Model VS. Original Time Series

The basic statistics that reflect the accuracy of model (3.1.5) are the mean $\bar{r}$, variance S_r^2, standard deviation S_r and standard error $S_r/\sqrt{n}$ of the residuals. Figure 3.19 gives a time series plot of the residual and table 3.9 provides the basic statistics.

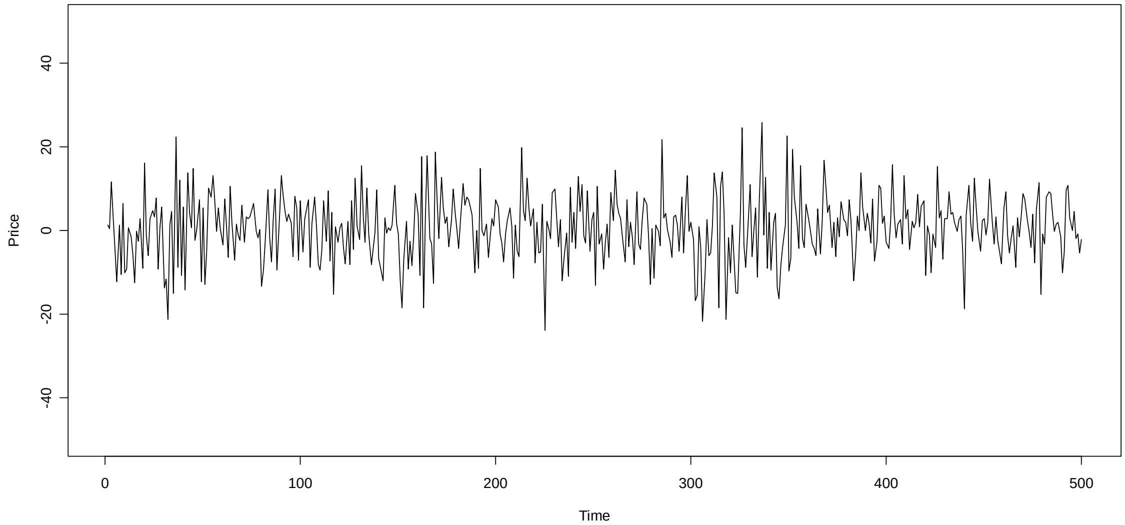

Figure 3.19 Time Series Plot of the Residuals for Classical Model

Table 3.9 Basic Evaluation Statistics for Classical ARIMA

$\bar{r}$	S_r^2	S_r	$S_r / \sqrt{n}$
0.5621225	60.76364	7.795104	0.3493070

Furthermore, we restructure the model (3.1.5) with $n = 475$ data points to forecast the last 25 observations only using the previous information. The purpose is to see how accurate our forecast prices are with respect to the actual 25 values that have not been used. Table 3.10 gives the actual price, predicted price and residuals between the forecasts and the 25 hidden values.

Table 3.10 Actual and Predicted Price for Classical ARIMA

N	Actual Price	Predicted Price	Residuals
476	1422.95	1430.7690	-7.8190
477	1427.99	1422.8510	5.1390
478	1440.13	1428.5800	11.5500
479	1423.9	1439.1420	-15.2420
480	1422.18	1423.3170	-1.1370
481	1420.62	1423.8770	-3.2570
482	1428.82	1420.8820	7.9380
483	1438.24	1428.8140	9.4260
484	1445.94	1436.9820	8.9580
485	1448.39	1444.5870	3.8030
486	1446.99	1447.3060	-0.3160
487	1448	1446.6020	1.3980
488	1450.02	1447.9800	2.0400
489	1448.31	1449.7950	-1.4850
490	1438.06	1448.1510	-10.0910
491	1433.37	1438.5980	-5.2280
492	1444.26	1434.6300	9.6300
493	1455.3	1444.4730	10.8270
494	1456.81	1453.8570	2.9530
495	1455.54	1455.5140	0.0260
496	1459.68	1455.2130	4.4670
497	1457.63	1459.5290	-1.8990
498	1456.38	1457.2000	-0.8200
499	1451.19	1456.6180	-5.4280
500	1449.37	1451.4620	-2.0920

The average of these residuals is $\bar{r} = 0.9336$, and the Figure 3.20 is a graph presentation of the forecasting result.

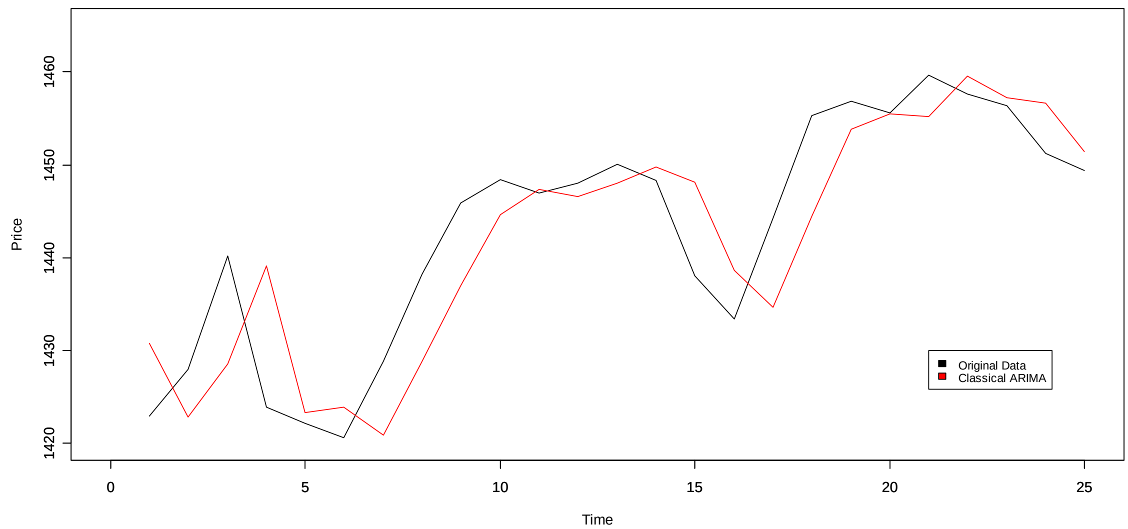

Figure 3.20 Classical ARIMA Forecasting on the Last 25 Observations

3.2.3 The 3 Days Moving Average ARIMA Models

Figure 3.21 shows the new time series $\{y_t\}$ along with the original time series $\{x_t\}$, that we shall use to develop the proposed forecasting model.

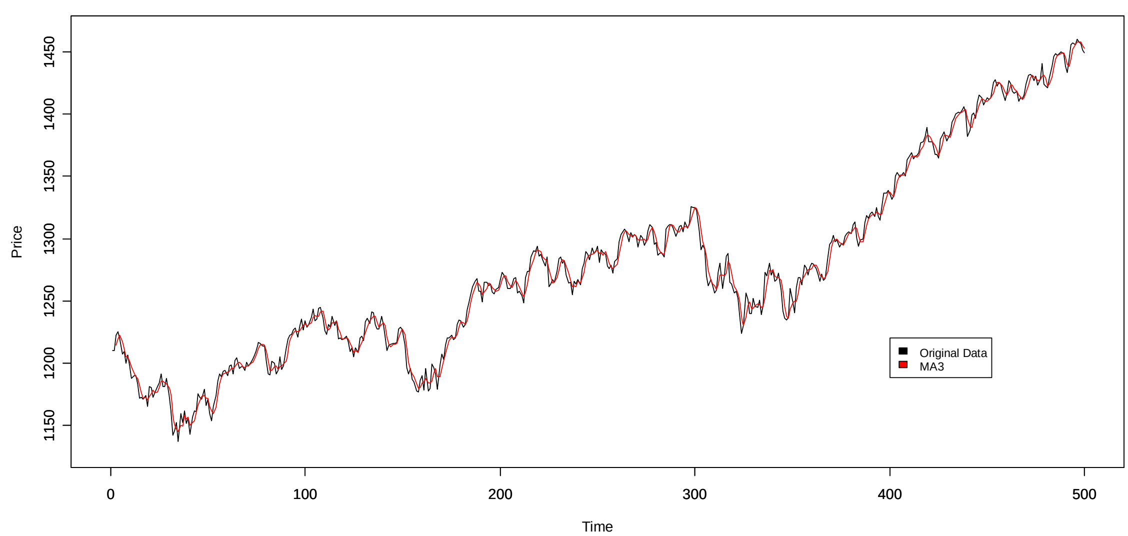

Figure 3.21 MA3 Series VS. The Original Time Series

Following the procedure we discussed in section 2.2, we found the best model that characterizes the behavior of $\{y_t\}$ to be ARIMA(0,1,4). That is,

$$(1-B)y_t = (1+.9608B+.8441B^2-.1562B^3-.1132B^4)\varepsilon_t . \qquad (3.1.3)$$

Expanding the autoregressive operator and the first difference filter, we have

$$y_t - y_{t-1} = (1+.9608B+.8441B^2-.1562B^3-.1132B^4)\varepsilon_{tt} .$$

and the model can be written as

$$y_t = y_{t-1} + \varepsilon_t + .9608\varepsilon_{t-1} + .8441\varepsilon_{t-2} - .1562\varepsilon_{t-3} - .1132\varepsilon_{t-4} .$$

The final analytical form of the proposed forecasting model can be written as

$$y_t = y_{t-1} + .9608\varepsilon_{t-1} + .8441\varepsilon_{t-2} - .1562\varepsilon_{t-3} - .1132\varepsilon_{t-4} . \qquad (3.1.4)$$

Using the above equation, we graph the forecasting values obtained by using the MA3 ARIMA approach on top of the original time series, shown as Figure 3.22

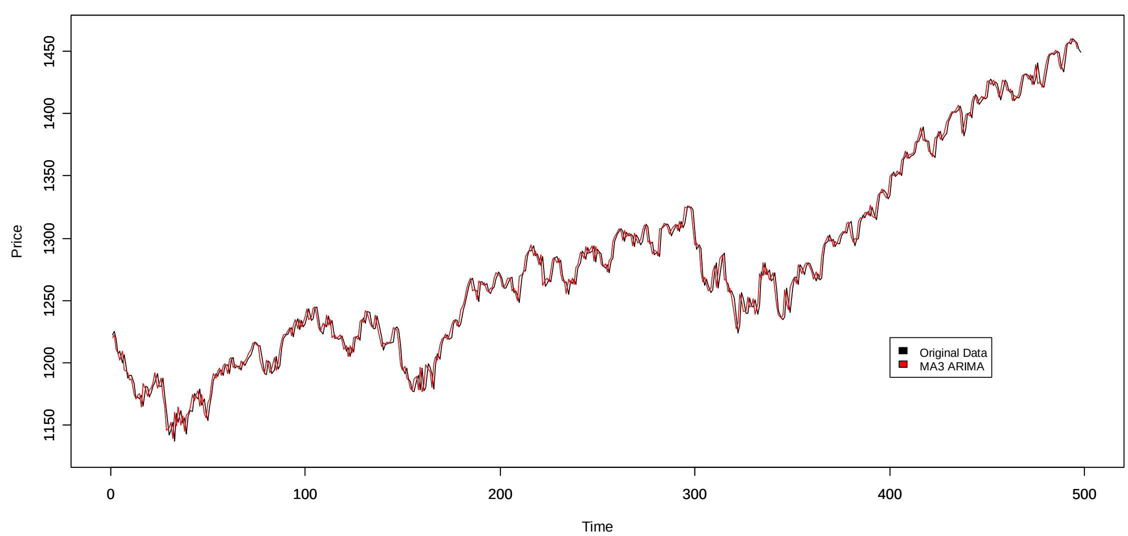

Figure 3.22 Comparisons on MA3 ARIMA Model VS. Original Time Series

Note the very closeness of the two plots that reflect the quality of the proposed model.

Similar to the classical model approach that we discussed earlier, we shall use the first 475 observations $\{y_1, y_2, ..., y_{475}\}$ to forecast $\hat{y}_{476}$. Then we use the observations $\{y_1, y_2, ..., y_{476}\}$ to forecast $\hat{y}_{477}$, and continue this process until we obtain forecasts all the observations, that is, $\{\hat{y}_{476}, \hat{y}_{477}, ..., \hat{y}_{500}\}$. From equation (2.2.3), we can see the relationship between the forecasting values of the original series $\{x_t\}$ and the forecasting values of 3 days moving average series $\{y_t\}$,that is,

$$\hat{x}_t = 3\hat{y}_t - x_{t-1} - x_{t-2}. \quad (3.1.8)$$

Hence, after we estimated $\{\hat{y}_{476}, \hat{y}_{477}, ..., \hat{y}_{500}\}$, we can use the above equation, (3.1.8), to solve the forecasting values for $\{x_t\}$. Figure 3.23 is the residual plot generated by the MA3 ARIMA model, and followed by Table 3.11, that includes the basic evaluation statistics.

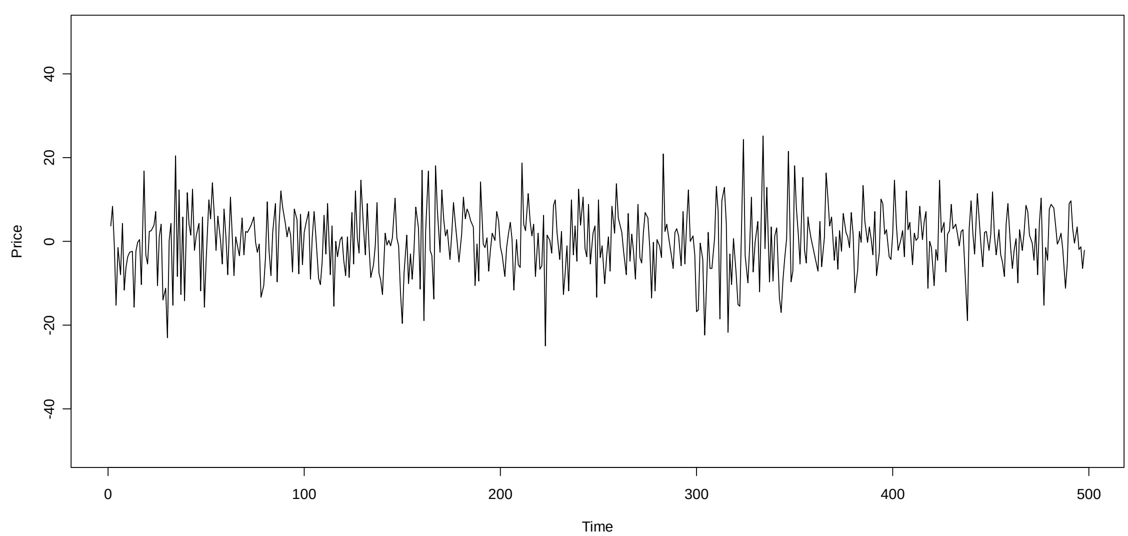

Figure 3.23 Time Series Plot of the Residuals for MA3 Model

Table 3.11 Basic Evaluation Statistics for MA3 ARIMA

$\bar{r}$	S_r^2	S_r	$S_r / \sqrt{n}$
-0.0006352779	60.60183	7.784718	0.3488415

Both of the above displayed evaluations reflect on accuracy of the proposed model. The actual daily closing prices of stock XYZ from the 476th day along with the forecasted prices and residuals are provided in Table 3.12.

Table 3.12 Actual and Predicted Price for MA3 ARIMA

N	Actual Price	Predicted Price	Residuals
476	1422.95	1430.3760	-7.4260
477	1427.99	1422.7950	5.1950
478	1440.13	1429.0290	11.1010
479	1423.9	1438.7110	-14.8110
480	1422.18	1423.2030	-1.0230
481	1420.62	1424.5170	-3.8970
482	1428.82	1420.4510	8.3690
483	1438.24	1428.5580	9.6820
484	1445.94	1437.5200	8.4200
485	1448.39	1444.1100	4.2800
486	1446.99	1447.1560	-0.1660
487	1448	1447.2460	0.7540
488	1450.02	1447.5280	2.4920
489	1448.31	1449.6110	-1.3010
490	1438.06	1448.7910	-10.7310
491	1433.37	1438.1860	-4.8160
492	1444.26	1434.5520	9.7080
493	1455.3	1444.9280	10.3720
494	1456.81	1453.3290	3.4810
495	1455.54	1455.4640	0.0760
496	1459.68	1455.7900	3.8900
497	1457.63	1459.0210	-1.3910
498	1456.38	1457.0970	-0.7170
499	1451.19	1457.2440	-6.0540
500	1449.37	1450.9800	-1.6100

The Results given above attest to the good forecasting estimates for the hidden data. The average of these residuals is $\bar{r} = 0.9551$, and the Figure 3.24 is a graph presentation of the forecasting result.

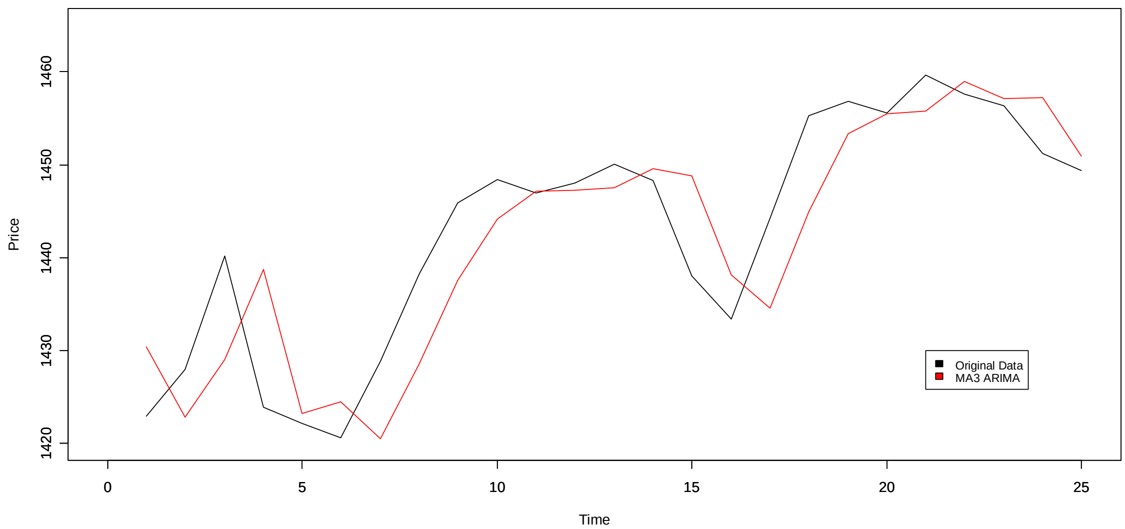

Figure 3.24 MA3 ARIMA Forecasting on the Last 25 Observations

3.2.4 The 3 Days Weighted Moving Average ARIMA Models

Figure 3.25 shows the new time series $\{y_t\}$ along with the original time series $\{x_t\}$, that we shall use to develop the proposed forecasting model.

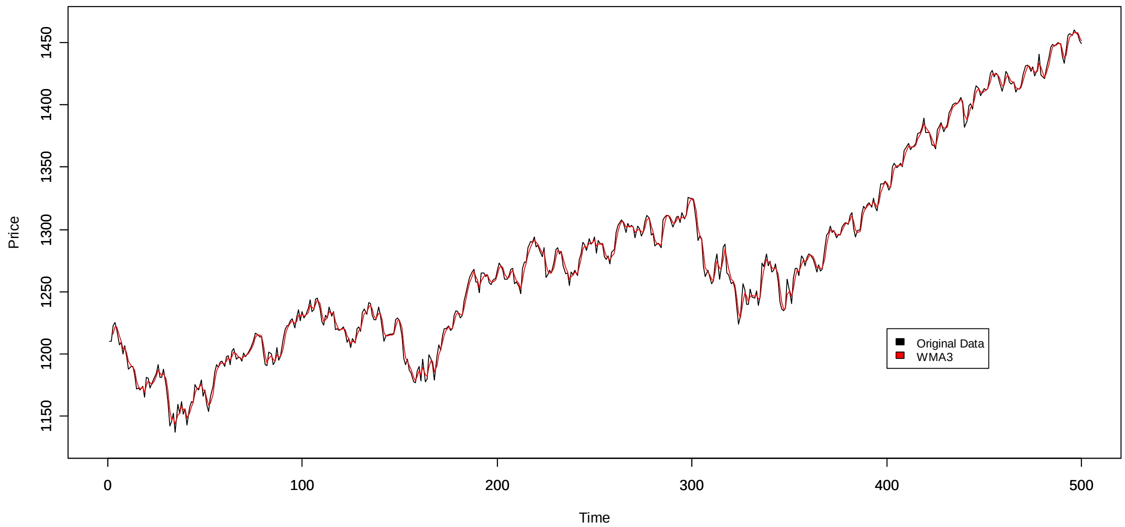

Figure 3.25 WMA3 Series VS. The Original Time Series

Following the procedure we have stated in section 2.3, we found the best model that characterizes the behavior of $\{z_t\}$ to be ARIMA(0,1,2). That is,

$$(1-B)z_t = (1+.6339B+.2249B^2)\varepsilon_t \tag{3.1.9}$$

expand the autoregressive operator and the difference filter, we have

$$z_t - z_{t-1} = (1+.6339B+.2249B^2)\varepsilon_t$$

and the model can be written as

$$z_t = z_{t-1} + \varepsilon_t + .6339\varepsilon_{t-1} + .2249\varepsilon_{t-2}$$

by letting $\varepsilon_t = 0$, we have the one day ahead forecasting series as

$$z_t = z_{t-1} + .6339\varepsilon_{t-1} + .2249\varepsilon_{t-2} \tag{3.1.10}$$

Using the above equation, we graph the forecasting values obtained by using the WMA3 ARIMA approach on top of the original time series, shown as Figure 3.26

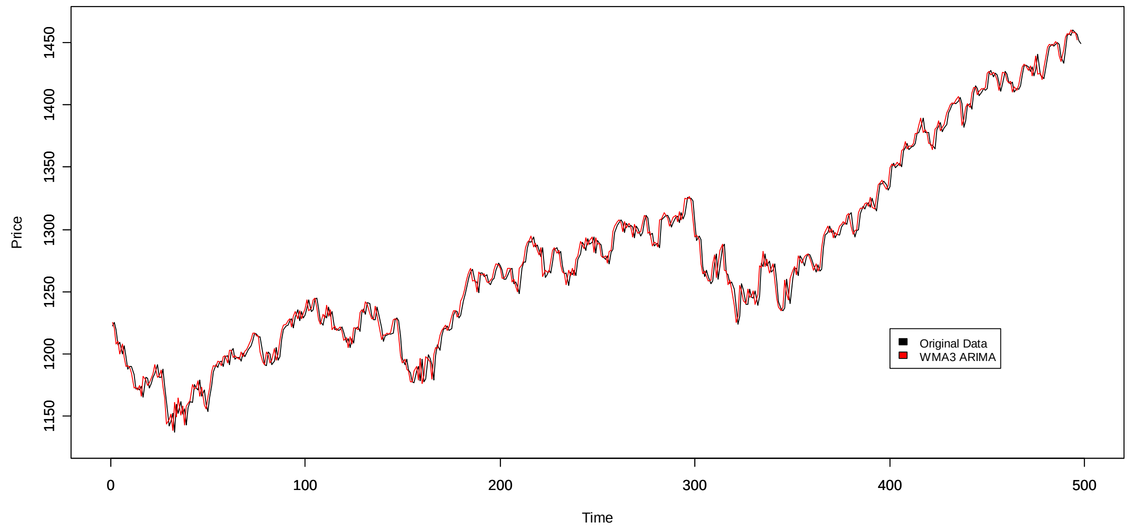

Figure 3.26 Comparisons on WMA3 ARIMA Model VS. Original Time Series

Similarly, we shall use the first 475 observations $\{z_1, z_2, ..., z_{475}\}$ to forecast $\hat{z}_{476}$. Then we use the observations $\{z_1, z_2, ..., z_{476}\}$ to forecast $\hat{z}_{477}$, and continue this process until we obtain forecasts all the observations, that is, $\{\hat{z}_{476}, \hat{z}_{477}, ..., \hat{z}_{500}\}$. From equation (2.3.3), we can see the relationship between the forecasting values of the original series $\{x_t\}$ and the forecasting values of 3 days moving average series $\{z_t\}$,that is,

$$\hat{x}_t = \frac{6\hat{z}_t - 2x_{t-1} - x_{t-2}}{3}. \tag{3.1.11}$$

Hence, after we estimated $\{\hat{z}_{476}, \hat{z}_{477}, ..., \hat{z}_{500}\}$, we can use the above equation, (3.1.11), to solve the forecasting values for $\{x_t\}$. Figure 3.27 is the residual plot generated by our proposed model, and followed by Table 3.13, that includes the basic evaluation statistics.

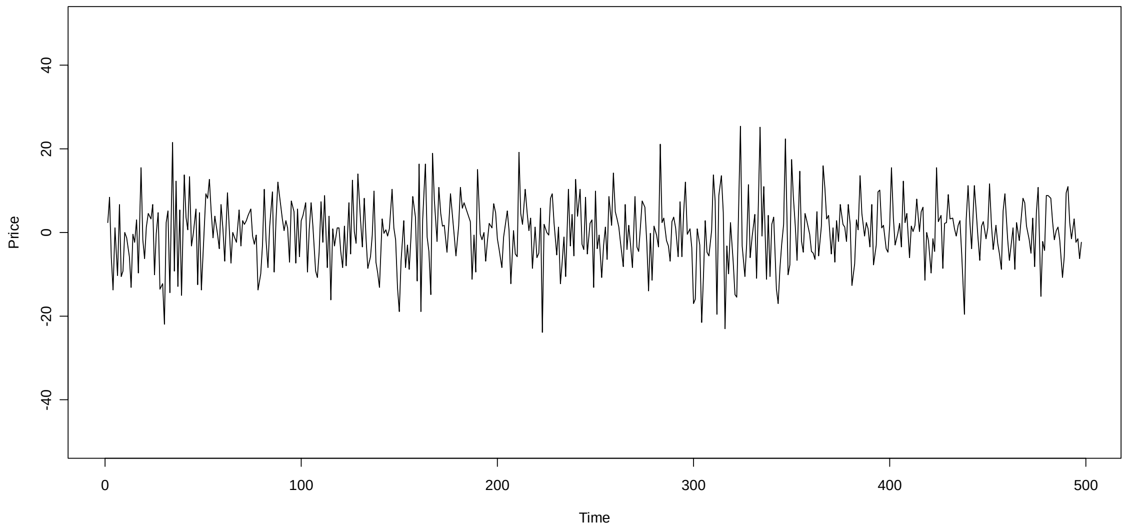

Figure 3.27 Time Series Plot of the Residuals for WMA3 Model

Table 3.13 Basic Evaluation Statistics for WMA3 ARIMA

$\bar{r}$	S_r^2	S_r	$S_r/\sqrt{n}$
-0.005440737	60.87297	7.802113	0.3496211

A detail ranking comparison between models will be illustrated in a later section, and Table 3.14 shows a head to head comparison between the actual and predicted price for WMA3 ARIMA model.

Table 3.14 Actual and Predicted Price for WMA3 ARIMA

N	Actual Price	Predicted Price	Residuals
476	1422.95	1430.643	-7.6930
477	1427.99	1422.655	5.3350
478	1440.13	1428.753	11.3770
479	1423.9	1438.699	-14.7990
480	1422.18	1423.973	-1.7930
481	1420.62	1424.451	-3.8310
482	1428.82	1419.478	9.3420
483	1438.24	1428.872	9.3680
484	1445.94	1437.154	8.7860
485	1448.39	1445.274	3.1160
486	1446.99	1448.124	-1.1340
487	1448	1447.094	0.9060
488	1450.02	1448.109	1.9110
489	1448.31	1449.747	-1.4370
490	1438.06	1448.302	-10.2420
491	1433.37	1438.673	-5.3030
492	1444.26	1434.242	10.0180
493	1455.3	1443.687	11.6130
494	1456.81	1453.938	2.8720
495	1455.54	1456.578	-1.0380
496	1459.68	1455.873	3.8070
497	1457.63	1459.531	-1.9010
498	1456.38	1457.274	-0.8940
499	1451.19	1456.895	-5.7050
500	1449.37	1451.229	-1.8590

The average of these residuals is $\bar{r} = 0.8329$, and the Figure 3.28 is a graph presentation of the forecasting result.

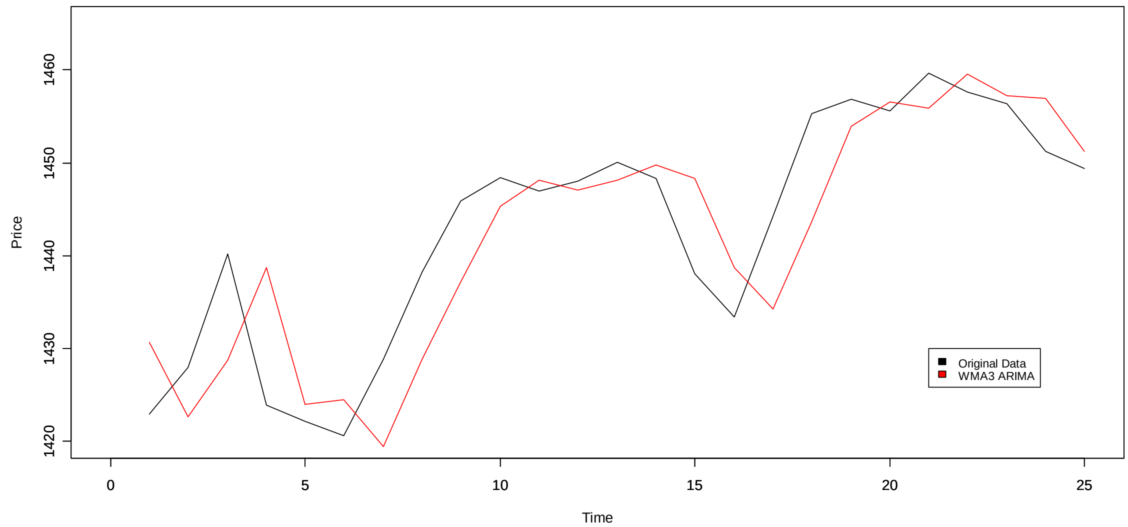

Figure 3.28 WMA3 ARIMA Forecasting on the Last 25 Observations

3.2.5 The 3 Days Exponential Weighted Moving Average ARIMA Models

Figure 3.29 shows the new time series $\{y_t\}$ along with the original time series $\{x_t\}$, that we shall use to develop the proposed forecasting model.

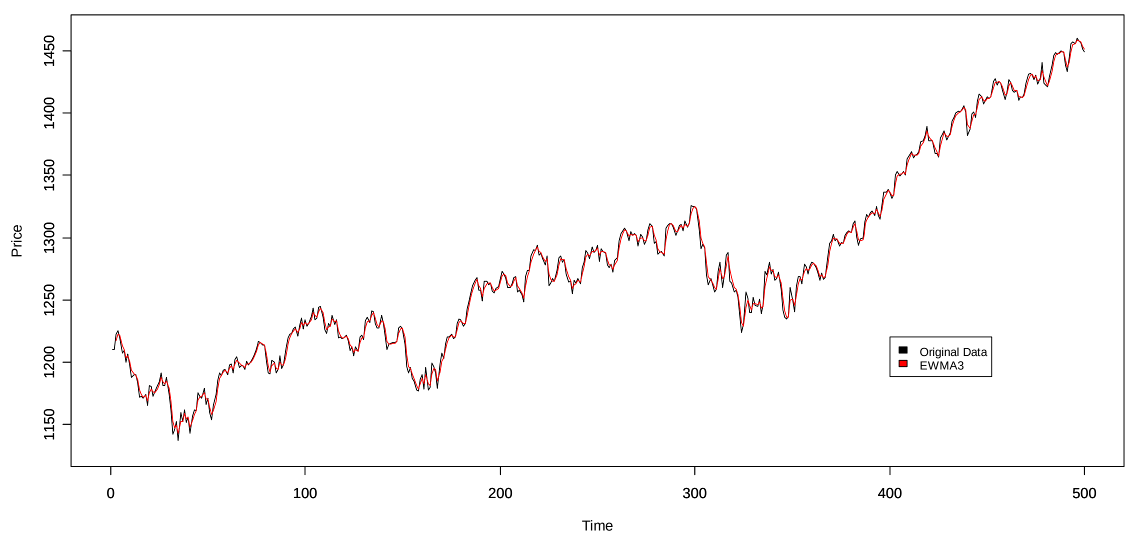

Figure 3.29 EWMA3 Series VS. The Original Time Series

Following the procedure we have stated in section 2.4, we found the best model that characterizes the behavior of $\{v_t\}$ to be ARIMA(3,1,2), that is,

$$(1-.9617B+.1296B^2+.0671B^3)(1-B)v_t=(1-.5144B-.1946B^{2^3})\varepsilon_t \quad (3.1.12)$$

expand the autoregressive operator and the difference filter, we have

$$(1-1.9617B+1.0913B^2+.1967B^3-.0671B^4)v_t=(1-.5144B-.1946B^{2^3})\varepsilon_t$$

and rewrite the model as

$$v_t=1.9617v_{t-1}-1.0913v_{t-2}-.1967v_{t-3}+.0671v_{t-4}+\varepsilon_t-.5144\varepsilon_{t-1}-.1946\varepsilon_{t-2}$$

by letting $\varepsilon_t=0$, we have the one day ahead forecasting series as

$$v_t=1.9617v_{t-1}-1.0913v_{t-2}-.1967v_{t-3}+.0671v_{t-4}-.5144\varepsilon_{t-1}-.1946\varepsilon_{t-2} \quad (3.1.13)$$

Using the above equation, we graph the forecasting values obtained by using the EWMA3 ARIMA approach on top of the original time series, shown as Figure 3.30

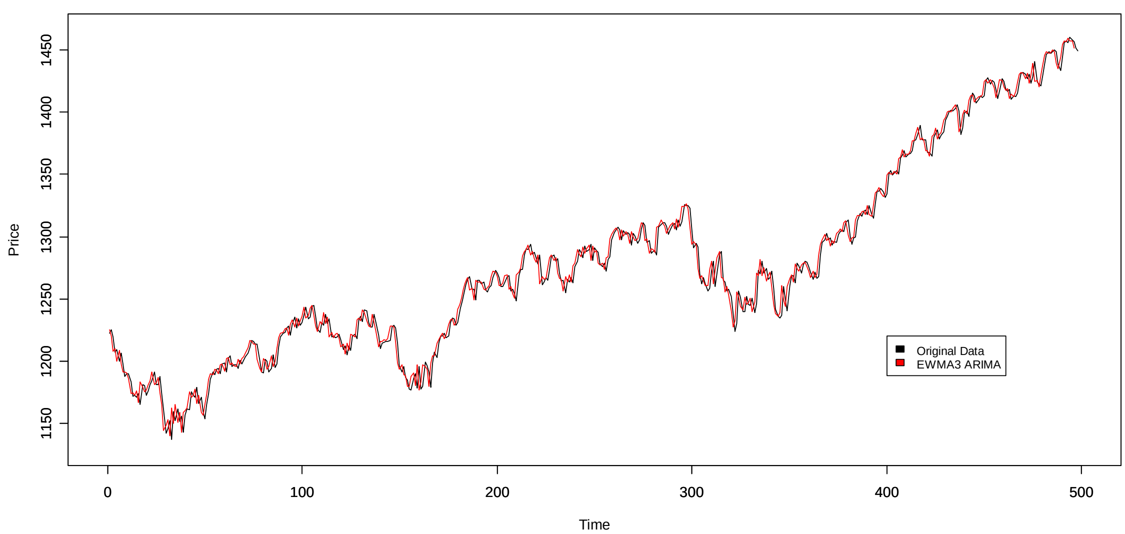

Figure 3.30 Comparisons on EWMA3 ARIMA Model VS. Original Time Series

Similarly, we shall use the first 475 observations $\{v_1,v_2,...,v_{475}\}$ to forecast $\hat{v}_{476}$.

Then we use the observations $\{v_1,v_2,...,v_{476}\}$ to forecast $\hat{v}_{477}$, and continue this process until we obtain forecasts all the observations, that is, $\{\hat{v}_{476}, \hat{v}_{477}, ..., \hat{v}_{500}\}$. From equation (2.4.3), the relationship between the forecasting values of the original series $\{x_t\}$ and the forecasting values of 3 days moving average series $\{v_t\}$ can be derived as

$$\hat{x}_t = 1.75\hat{v}_t - .5x_{t-1} - .25x_{t-2}\ . \qquad (3.1.14)$$

Hence, after we estimated $\{\hat{v}_{476}, \hat{v}_{477}, ..., \hat{v}_{500}\}$, we can use the above equation, (3.1.14), to solve the forecasting values for $\{x_t\}$. Figure 3.31 is the residual plot generated by our proposed model, and followed by Table 3.15, that includes the basic evaluation statistics.

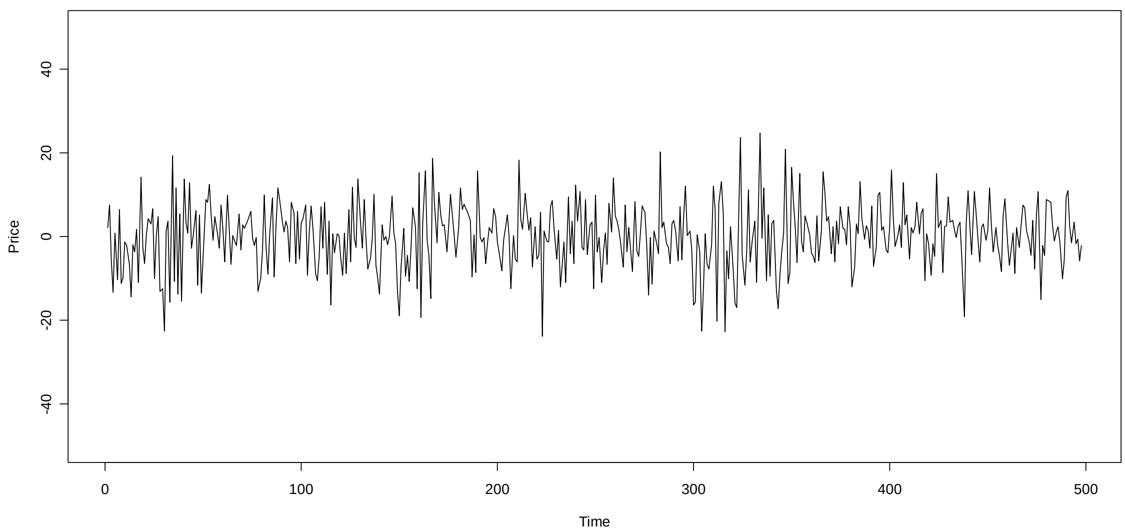

Figure 3.31 Time Series Plot of the Residuals for EWMA3 Model

Table 3.15 Basic Evaluation Statistics for EWMA3 ARIMA

$\bar{r}$	S_r^2	S_r	$S_r / \sqrt{n}$
0.01651362	60.42038	7.773055	0.3483189

A detail ranking comparison between models will be illustrated in latter section, and Table 3.16 shows a head to head comparison between the actual and predicted price for WMA3 ARIMA model.

Table 3.16 Actual and Predicted Price for EWMA3 ARIMA

N	Actual Price	Predicted Price	Residuals
476	1422.95	1430.186	-7.2360
477	1427.99	1422.34	5.6500
478	1440.13	1428.754	11.3760
479	1423.9	1438.368	-14.4680
480	1422.18	1423.878	-1.6980
481	1420.62	1424.578	-3.9580
482	1428.82	1419.391	9.4290
483	1438.24	1428.912	9.3280
484	1445.94	1437.024	8.9160
485	1448.39	1445.242	3.1480
486	1446.99	1447.595	-0.6050
487	1448	1446.488	1.5120
488	1450.02	1447.386	2.6340
489	1448.31	1449.043	-0.7330
490	1438.06	1447.769	-9.7090
491	1433.37	1438.381	-5.0110
492	1444.26	1434.177	10.0830
493	1455.3	1443.617	11.6830
494	1456.81	1453.801	3.0090
495	1455.54	1456.459	-0.9190
496	1459.68	1455.642	4.0380
497	1457.63	1458.982	-1.3520
498	1456.38	1456.648	-0.2680
499	1451.19	1456.533	-5.3430
500	1449.37	1450.962	-1.5920

The average of these residuals is $\bar{r} = 1.1166$, and the Figure 3.32 is a graph presentation of the forecasting result.

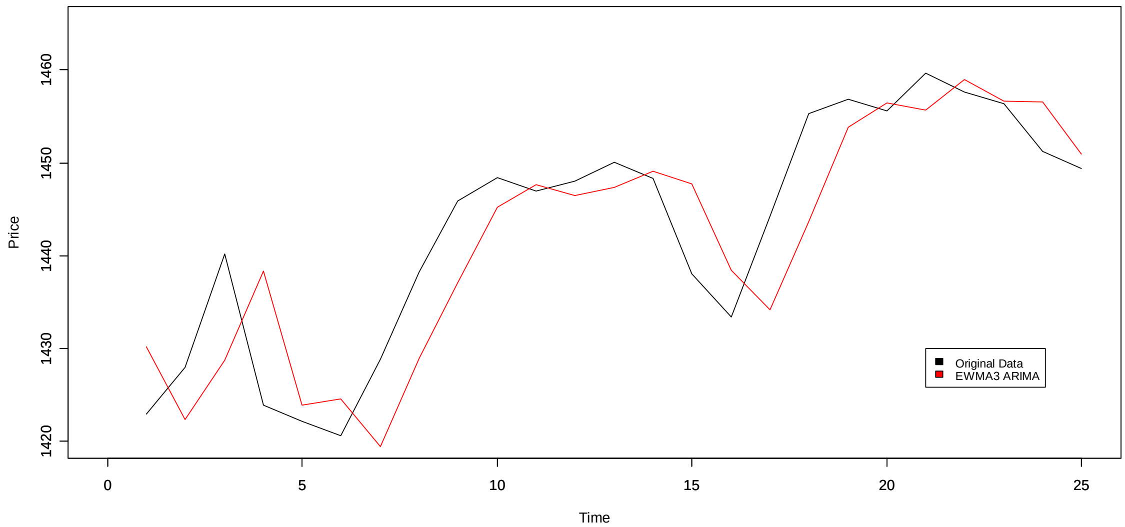

Figure 3.32 EWMA3 ARIMA Forecasting on the Last 25 Observations

3.3 Evaluations on Proposed Models VS. Classical Method

In the previous sections, we have discussed the classical ARIMA, MA3 ARIMA, WMA3 ARIMA, and EWMA3 ARIMA models on both stock XYZ and S&P Price Index. We shall now compare the performance of each model through examining their basic statistical properties. The Following Table 3.17 shows the comparison between models for stock XYZ.

Table 3.17 Ranking Comparison on stock XYZ

	$\bar{r}$	S_r^2	S_r	$S_r/\sqrt{n}$	Order
Classical ARIMA	0.02209169	0.1445187	0.3801562	0.0170011	1,1,2
MA3 ARIMA	0.01016814	0.1437259	0.3791119	0.01698841	2,1,3
WMA3 ARIMA	0.00866631	0.1578446	0.3972966	0.01776764	1,1,3
EWMA3 ARIMA	0.008076663	0.1573456	0.3966682	0.01773954	3,1,2

According to Table 3.17 the MA3 ARIMA model performs best on forecasting stock XYZ compare to other models.

Table 3.18 Ranking Comparison on S&P Price Index

	$\bar{r}$	S_r^2	S_r	$S_r / \sqrt{n}$	RANK
Classical ARIMA	0.5621225	60.76364	7.795104	0.3493070	0,1,2
MA3 ARIMA	-0.0006352779	60.60183	7.784718	0.3488415	0,1,4
WMA3 ARIMA	-0.005440737	60.87297	7.802113	0.3496211	0,1,2
EWMA3 ARIMA	0.01651362	60.42038	7.773055	0.3483189	3,1,2

According to Table 3.18 the EWMA3 ARIMA model performs best on forecasting S&P Price Index among all models. In addition, the MA3 ARIMA model also performs better than the classical ARIMA model; it speaks out the quality of our proposed models.

3.4 Conclusion

In the present chapter, we try to show the usefulness and effectiveness of our proposed models by applying them on two different economic time series, namely the daily closing price of Stock XYZ and the daily closing price of S&P Price Index. In both cases, once we obtained our proposed models we compare them with the classical approach and rank their efficiency by examining some basic statistical criteria of the residuals. In addition, by hiding the last 25 observations and trying to predict the future information, we are able to show that our proposed models perform well without knowing the future information. The encouraging results speak out the quality of the models.

Chapter 4

Global Warming: Atmospheric Temperature Forecasting Model

4.0 Introduction

Temperature plays a very important role in Global Warming and its relation with Carbon Dioxide. The aim of the present chapter is to develop a statistical forecasting model for the temperature in the Continental United States. There are two methods being used in recording temperatures and we shall refer them as Version 1 (see Appendix C1) and Version 2 (see Appendix C2) data sets. Thus, an additional aim in the present study is to determine if the two methods of recording temperatures are indeed different. Version 1 data was collected by the United States Climate Division, USCD, and Version 2 data by the United States Historical Climatology Network, USHCN.

The Version 1 dataset consists of monthly mean temperature and precipitation for all 344 climate divisions in the contiguous U. S. from January 1895 to June 2007. The data is adjusted for time of observation bias. However, no other adjustments are made for inhomogeneities. These inhomogeneities include changes in instrumentation, observer, and observation practices, station and instrumentation moves, and changes in station composition resulting from stations closing and opening over time within a division. For additional information concerning Version 1 of the data, see (Easterling & Peterson, 1995; Karl et al., 1986; Karl & Williams, 1987; Karl et al., 1988; Karl et al., 1990; Peterson & Easterling, 1994; Quayle et al., 1991). A graphical presentation of the Version 1 dataset is given by Figure 4.1.

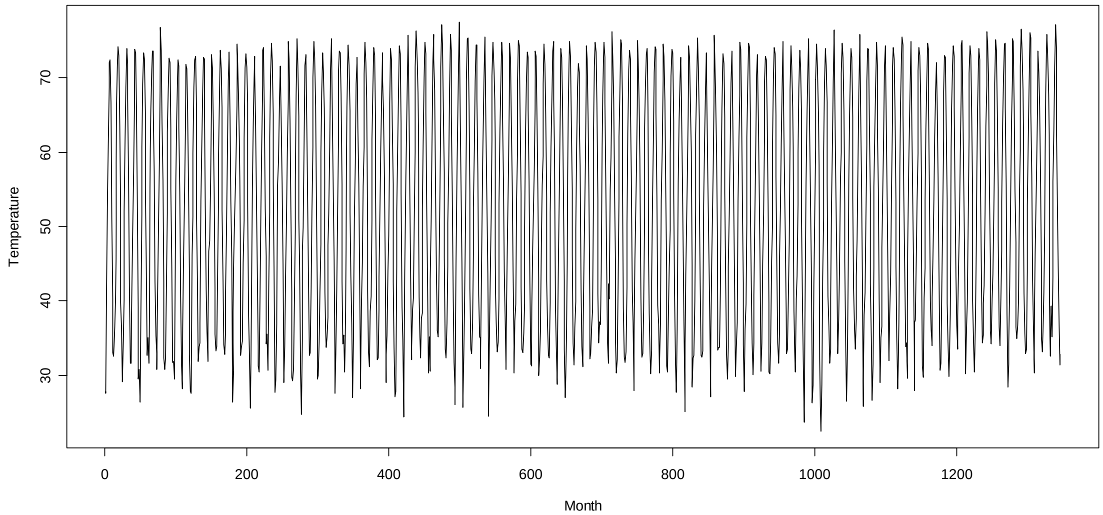

Figure 4.1 Time Series Plot for Monthly Temperature for 1895-2007 (Version 1)

The Version 2 dataset was first become available in July 2007, and it consists of data from a network of 1219 stations in the contiguous United States that were defined by scientists at the Global Change Research Program of the U. S. Department of Energy at National Climate Data Center. A methodology was developed and applied to test known station changes for their impact on the homogeneity, and necessary adjustments were made if the changes caused a statistically significant response in the time series. They claim that the data set is a consistent network through time, which minimizes any biasing due to network changes through time. For information on Version 2 of the time series, see (Alexandersson & Moberg, 1997; Baker, 1975; Easterling et al., 1996; Easterling et al., 1999; Hughes et al., 1992; Karl et al., 1990; Karl et al., 1988; Karl et al., 1986; Karl & Williams, 1987; Lund & Reeves, 2002; Menne & Williams, 2005; Quinlan et al., 1987; Vose et al., 2003; Wang, 2003). A graphical presentation of the Version 2 dataset is given by Figure 4.2.

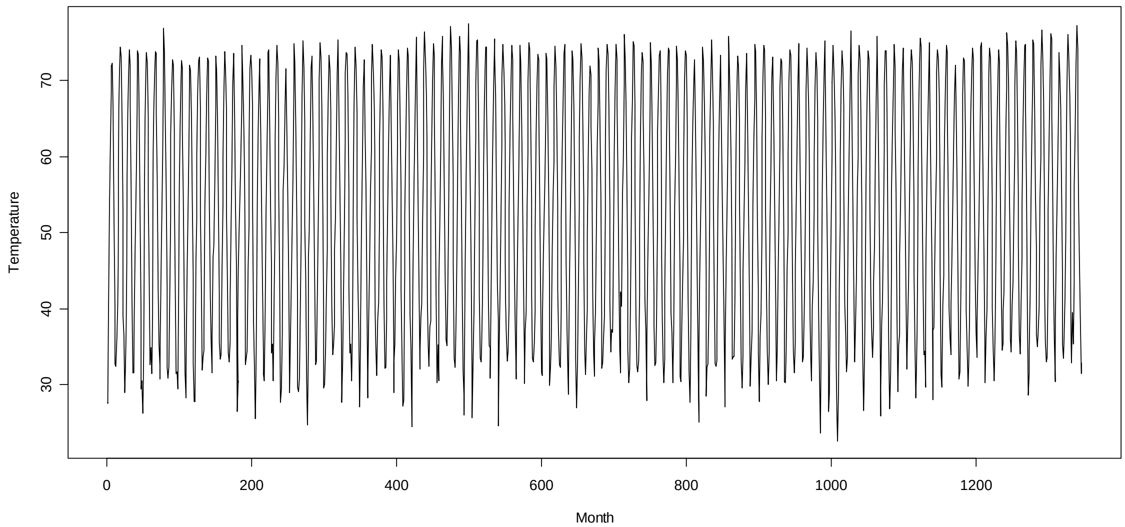

Figure 4.2 Time Series Plot for Monthly Temperature for 1895-2007 (Version 2)

4.1 Analytical Procedure

The multiplicative seasonal autoregressive integrated moving average, ARIMA model is defined by

$$\Phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D x_t = \theta_q(B)\Gamma_Q(B^s)\varepsilon_t \tag{4.1.1}$$

where p is the order of the autoregressive process, d is the order of regular differencing, q is the order of the moving average process, P is the order of the seasonal autoregressive process, D is the order of the seasonal differencing, Q is the order of the seasonal moving average process, and the subindex s refers to the seasonal period. We shall denote the subject model by ARIMA$(p,d,q)\times(P,D,Q)_s$, and $\phi_p(B),\theta_q(B),\Phi_P(B^s),\Gamma_q(B^s)$ defined as follows:

$$\phi_p(B) = (1-\phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)$$

$$\theta_q(B) = (1-\theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q)$$

$$\Phi_P(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \ldots - \Phi_P B^{Ps}$$

and

$$\Gamma_Q(B^s) = 1 - \Gamma_1 B^s - \Gamma_2 B^{2s} - \ldots - \Gamma_Q B^{Qs}.$$

The order of the multiplicative ARIMA model determines the structure of the model and it is essential to have a good methodology in terms of developing the forecasting model. In the present study, we start with addressing the issue of the seasonal subindex *s*. After we examine the original data, shown by Figure 4.1 and 4.2, we have reason to believe the monthly temperature of the Continental United States behaves as a periodic function with a cycle of 12 months. Hence, we let the seasonal subindex $s = 12$. In time series analysis, one cannot proceed with a model building procedure without confirming the stationarity of a given stochastic realization, thus, we test the overall stationarity of the series by using the method introduced by Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., and Shin, Y in 1992, (Kwiatkowski et al., 1992).

Once the order of the differencing is identified, it is common for one ARIMA$(p,d,q)\times(P,D,Q)_s$ model that we have several sets of (p,q,P,Q) that are all adequately representing a given set of time series. Akaike's information criterion, AIC, (Akaike, 1974), was first introduced by Akaike in 1974 plays a major role in our model selecting process. We shall choose the set of (p,q,P,Q) that produces the smallest AIC. Another important aspect in our model selection process is to determine the seasonal differencing, *D*, the goal is to select a smaller AIC without complicating the selected model. Hence, we only compute the AIC for both $D = 0$ and $D = 1$ based on our previous selection of the orders (p,d,q,P,Q), and choose the model with smaller AIC to be our final model.

Below we summarize the model identifying procedure:

- Determine the seasonal period *s*.
- Check for stationarity of the given time series $\{x_t\}$ by determining the order of differencing *d*, where $d = 0,1,2,...$ according to KPSS test, until we achieve stationarity.
- Deciding the order m of the process, for our case, we let $m = 5$ where $p + q + P + Q = m$.
- After (d, m) being selected, listing all possible configurations of (p, q, P, Q) for $p + q + P + Q \leq m$.
- For each set of (p, q, P, Q), estimates the parameters for each model, that is, $\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q, \Phi_1, \Phi_2, ..., \Phi_P, \Gamma_1, \Gamma_2, ..., \Gamma_Q$.
- Compute the AIC for each model, and choose the one with smallest AIC.
- After (p, d, q, P, Q) is selected, we determine the seasonal differencing filter by selecting the smaller AIC between the model with $D = 0$ and $D = 1$.
- Our final model will have identified the order of (p, d, q, P, D, Q).

4.2. Development of Forecasting Models

The historical temperature data for the continental United States that we shall use are shown by Figure 4.1 and 4.2. A visual inspection does not show any obvious trends being present. Thus, we let the seasonal period $s = 12$. Following the step-by-step procedure we described above, we found that the model best characterizes the average monthly temperature of the Continental United States for both Version 1 and 2 is a $\text{ARIMA}(2,1,1) \times (1,1,1)_{12}$ process, analytical given by

$$(1-\Phi_1 B^{12})(1-\phi_1 B-\phi_2 B^2)(1-B)(1-B^{12})x_t = (1-\theta_1 B)(1-\Gamma_1 B^{12})\varepsilon_t \qquad (4.2.1)$$

Expanding both sides of the above ARIMA, we have

$$\begin{aligned}&[1-(1+\phi_1)B+(\phi_1-\phi_2)B^2+\phi_2 B^3-(1+\Phi_1)B^{12}+(1+\phi_1+\Phi_1+\phi_1\Phi_1)B^{13}\\&+(\phi_2+\phi_2\Phi_1-\phi_1-\phi_1\Phi_1)B^{14}-(\phi_2+\phi_2\Phi_1)B^{15}+\Phi_1 B^{24}-(\phi_1+\Phi_1)B^{25}\\&+(\phi_1\Phi_1-\phi_2\Phi_1)B^{26}+\phi_2\Phi_1 B^{27}]x_t = (1-\theta_1 B-\Gamma_1 B^{12}+\theta_1\Gamma_1 B^{13})\varepsilon_t\end{aligned}$$

Simplify it, we get

$$\begin{aligned}&x_t-(1+\phi_1)x_{t-1}+(\phi_1-\phi_2)x_{t-2}+\phi_2 x_{t-3}-(1+\Phi_1)x_{t-12}+(1+\phi_1+\Phi_1+\phi_1\Phi_1)x_{t-13}\\&+(\phi_2+\phi_2\Phi_1-\phi_1-\phi_1\Phi_1)x_{t-14}-(\phi_2+\phi_2\Phi_1)x_{t-15}+\Phi_1 x_{t-24}-(\phi_1+\Phi_1)x_{t-25}\\&+(\phi_1\Phi_1-\phi_2\Phi_1)x_{t-26}+\phi_2\Phi_1 x_{t-27} = \varepsilon_t-\theta_1\varepsilon_{t-1}-\Gamma_1\varepsilon_{t-12}+\theta_1\Gamma_1\varepsilon_{t-13}\end{aligned}$$

Thus, the one-step ahead forecasting model for Version 1 data is given by

$$\begin{aligned}\hat{x}_t &= 1.0941x_{t-1}-0.057x_{t-2}-0.0371x_{t-3}+0.9954x_{t-12}-1.0891x_{t-13}+\\&0.0567x_{t-14}+0.0369x_{t-15}+0.0046x_{t-24}+0.0895x_{t-25}-0.0004x_{t-26}+\\&0.00017x_{t-27}-0.9861\varepsilon_{t-1}-0.9742\Gamma_1\varepsilon_{t-12}+0.9607\varepsilon_{t-13}\end{aligned} \qquad (4.2.2)$$

and the one-step ahead forecasting model for Version 2 data is given by

$$\begin{aligned}\hat{x}_t &= 1.0952x_{t-1}-0.0556x_{t-2}-0.0396x_{t-3}+0.9964x_{t-12}-0.9009x_{t-13}+\\&0.0554x_{t-14}+0.0395x_{t-15}+0.0036\Phi_1 x_{t-24}+0.0916x_{t-25}+0.0002x_{t-26}+\\&0.00014x_{t-27}-0.9855\varepsilon_{t-1}-0.9741\varepsilon_{t-12}+0.9599\varepsilon_{t-13}\end{aligned} \qquad (4.2.3)$$

Note the closeness of the two forecasting models.

4.3 Evaluation of the Proposed Models

We begin by forecasting for the last one hundred observations the monthly average temperature in the Continental United States for both Version 1 and 2, using the models given by expression 4.2.2 and 4.2.3. A graphical presentation of the results is presented below by Figure 4.3 and 4.4.

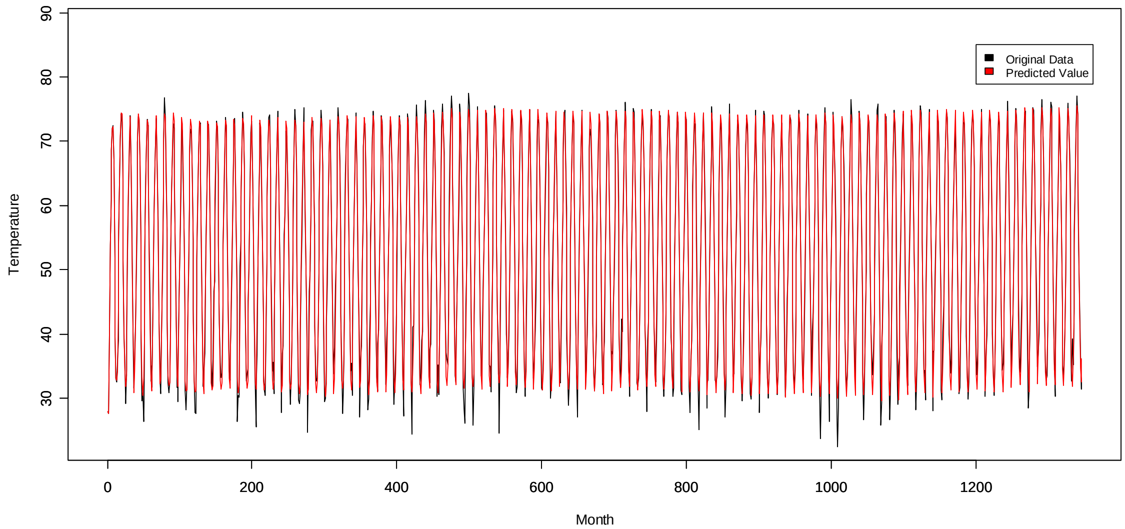

Figure 4.3 Actual VS. Predicted Values for Version 1 Dataset

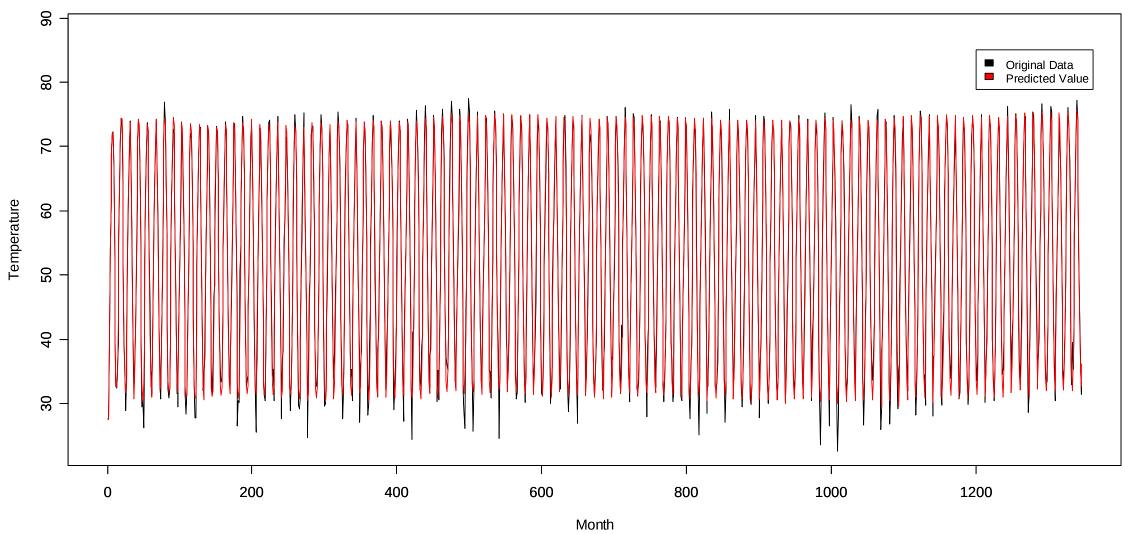

Figure 4.4 Actual VS. Predicted Values for Version 2 Dataset

As can be observed that both models are similar and the one-step ahead forecasting is quite good, except the temperature of January 2006 took an unexpected turn. We identify this inconsistency as a possible outlier.

We proceed to calculate the residuals estimates, $r_t = x_t - \hat{x}_t$, for both forecasting process given by (4.2.2) and (4.2.3). The results are graphically presented below by Figure 4.5 and 4.6.

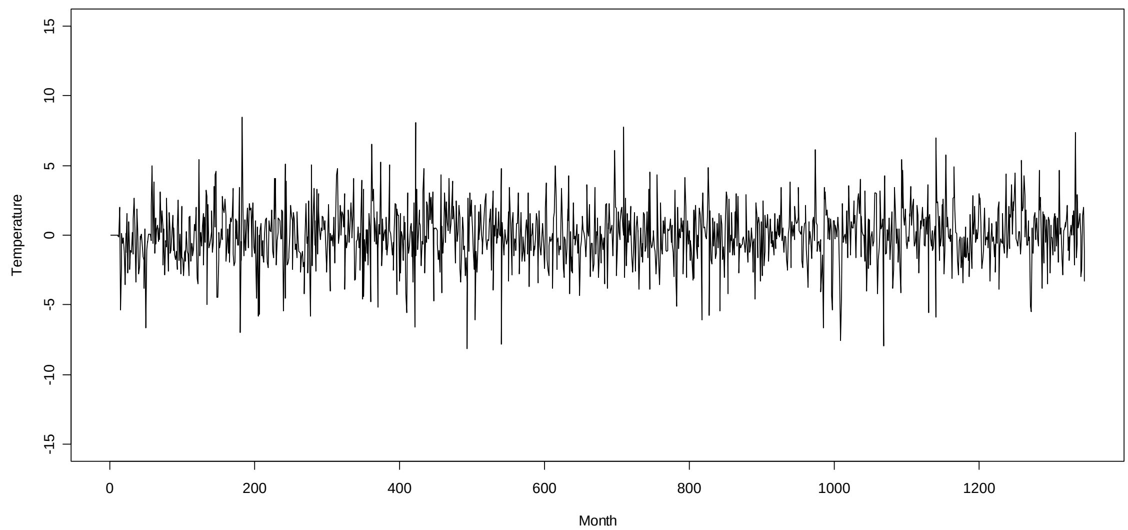

Figure 4.5 Residual Plot for Monthly Temperature (Version 1 Dataset)

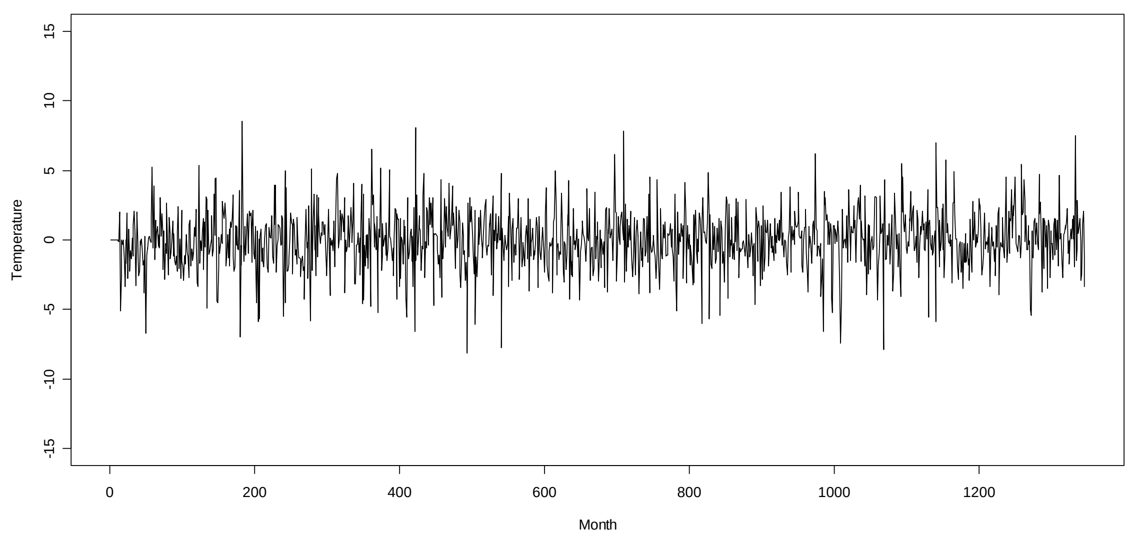

Figure 4.6 Residual Plot for Monthly Temperature (Version 2 Dataset)

We observe that the residuals are quite small and isolating around the zero axis as expected. It indicates that both models are good models in predicting the Version 1 and Version 2 of the time series.

Next, we evaluate the mean of the residuals, $\bar{r}$, the variance, S_r^2, the standard deviation, S_r, standard error, *SE*, and the mean square error, *MSE*. The results are presented below by Table 4.1 and 4.2, for Version 1 and Version 2 data, respectively.

Table 4.1 Basic Evaluation Statistics (Version 1 Dataset)

$\bar{r}$	S_r^2	S_r	*SE*	*MSE*
-0.008512476	4.331902	2.081322	0.05673052	4.328756

Table 4.2 Basic Evaluation Statistics (Version 2 Dataset)

$\bar{r}$	S_r^2	S_r	*SE*	*MSE*
-0.01310953	4.323726	2.079357	0.05667696	4.320685

We observe that all evaluation criteria support the quality of the proposed forecasting model. We can also conclude the similarity of the two models. Thus, it raises the question is the effort to collect two data sets implement two different procedures by two agencies necessary?

We have demonstrated that our proposed models are capable of representing the past monthly average temperature of the Continental United States, it is also essential to show that these models are also capable of forecasting the future values of the temperature. Therefore, we hide the last 12 months of the temperature, restructure the models (4.2.2) and (4.2.3) and try to predict the following months only using the previous information. For example, we used the first 1334 observations $\{x_1, x_2, ..., x_{1334}\}$ to forecast $\hat{x}_{1335}$. Then we use the observations $\{x_1, x_2, ..., x_{1335}\}$ to forecast $\hat{x}_{1336}$, and continue this process until we obtain the forecasting values of the last 12 observations, that is, $\{\hat{x}_{1335}, \hat{x}_{1336}, ..., \hat{x}_{1346}\}$. Table 4.3, gives the actual, forecasting and residual data for the subject 12 months.

Table 4.3 Original VS. Forecast Values (Version 1 Dataset)

	Original Values	Forecast Values	Residuals
March 2006	43.31	44.0291	-0.7191
April 2006	56.03	53.1361	2.89395
May 2006	63.06	62.5318	0.52821
June 2006	71.44	70.6153	0.82467
July 2006	77.1	75.5855	1.51453
August 2006	74.1	74.2054	-0.1054
September 2006	63.69	66.6904	-3.0004
October 2006	52.97	55.4991	-2.5291
November 2006	44.68	43.2673	1.41275
December 2006	36.64	34.6357	2.00433
January 2007	31.39	32.58	-1.19
February 2007	32.86	36.2024	-3.3424

Figure 4.7 below gives a graphical presentation of the information presented in Table 4.3 for Version 1 observed time series.

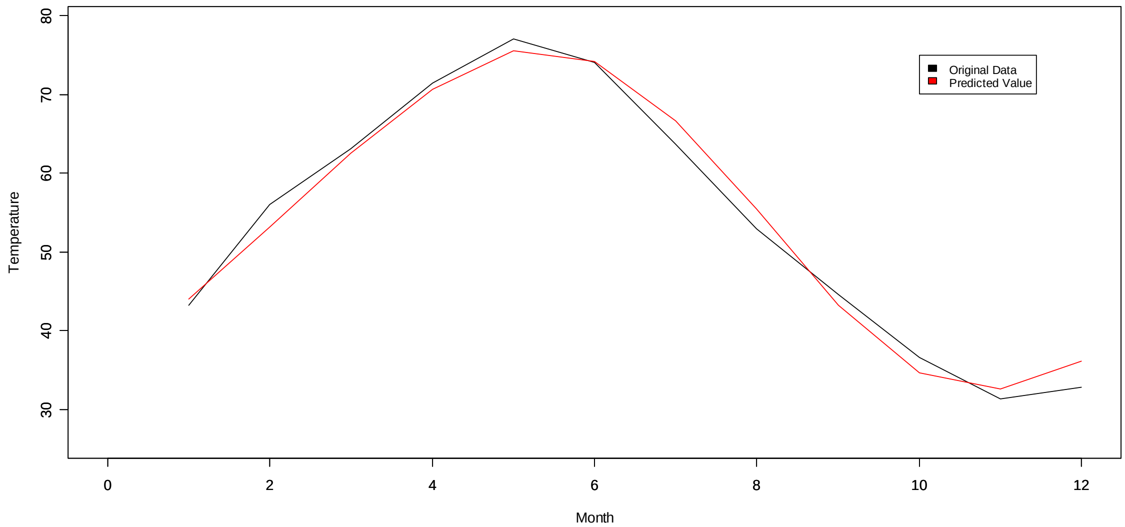

Figure 4.7. Actual VS. Predicted Values for the Last 12 Observations (Version 1)

Similarly, for Version 2 of the data set, we have calculated the estimates presented by Table 4.4.

Table 4.4 Original VS. Forecast Values (Version 2 Dataset)

	Original Values	Forecast Values	Residuals
March 2006	43.45	44.1812	-0.7312
April 2006	56.12	53.2506	2.86942
May 2006	63.12	62.6351	0.48486
June 2006	71.55	70.7152	0.83478
July 2006	77.22	75.6947	1.52532
August 2006	74.19	74.3167	-0.1267
September 2006	63.86	66.8069	-2.9469
October 2006	53.13	55.6137	-2.4837
November 2006	44.58	43.3947	1.18529
December 2006	36.79	34.7224	2.06761
January 2007	31.46	32.6854	-1.2254
February 2007	32.86	36.3025	-3.4425

A graphical presentation of the results given in Table 4.4 are given below by Figure 4.8.

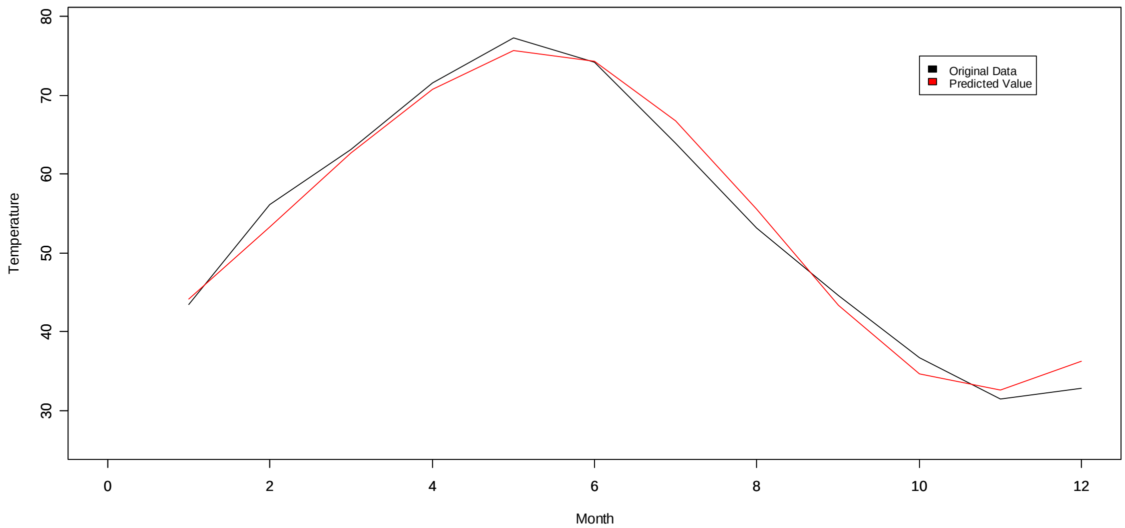

Figure 4.8 Actual VS. Predicted Values for the Last 12 Observations (Version 2)

We remark the similarity of the results of both models and the good forecast values.

4.4 Conclusion

In the present study, we have developed two seasonal autoregressive integrated moving average models to forecast the monthly average temperature in degrees Fahrenheit in the Continental United States using historical monthly data from 1895-2007. The two statistical models are based on two different methods of recording and weighting the subject temperature namely, USCD (Version 1) and USHCN (Version 2). Although the two different sets of data are somewhat similar, we believe from a statistical perspective that the one from USHCN is more appropriate to use. The two developed statistical models were evaluated using various statistical criteria and it was shown that both forecasting processes produced good estimates of the subject matter.

Chapter 5

Global Warming: Carbon Dioxide Proposed Forecasting Model

5.0 Introduction

Global Warming is one of the most compelling and difficult problems facing our society. It is well understood that carbon dioxide (CO_2), along with temperature are the primary causes of global warming. The present study is concerned with developing analytical statistical models to predict CO_2. Jim Verhulst, Perspective Editor, St. Petersburg Times, writes, “Carbon dioxide is invisible- no color, no odor, no taste. It puts out fires, puts the fizz in seltzer, and is to plants what oxygen is to us. It is hard to think of it as a poison.” (Verhulst 2007). The United States is emitting approximately 5.91221 billion metric tons of carbon dioxide in the atmosphere, which makes us one of the World leaders. In addition to CO_2 in the atmosphere, we have CO_2 emissions that are related to gas, liquid, and solid fuels along with gas flares and cement production.

The aim of the present chapter is to develop two different statistical models for the carbon emissions and atmospheric carbon dioxide in the United States using historical data from the subject matter.

5.1 Carbon Dioxide Emission Modeling

The CO_2 emissions data set that we used to develop the proposed model contains the monthly emissions data from 1981 to 2003 (see Appendix D). It was published by Carbon Dioxide Information Analysis Center (CDIAC), which is supported by the United

States Department of Energy. The CDIAC is a well known organization, which responds to data and information requests from users worldwide investigating the greenhouse effect and global climate change. For detailed information, see (United States Environmental Protection Agency (EPA), 2004; Marland et al., 2003). A graphical presentation of the emissions data is given by Figure 5.1.

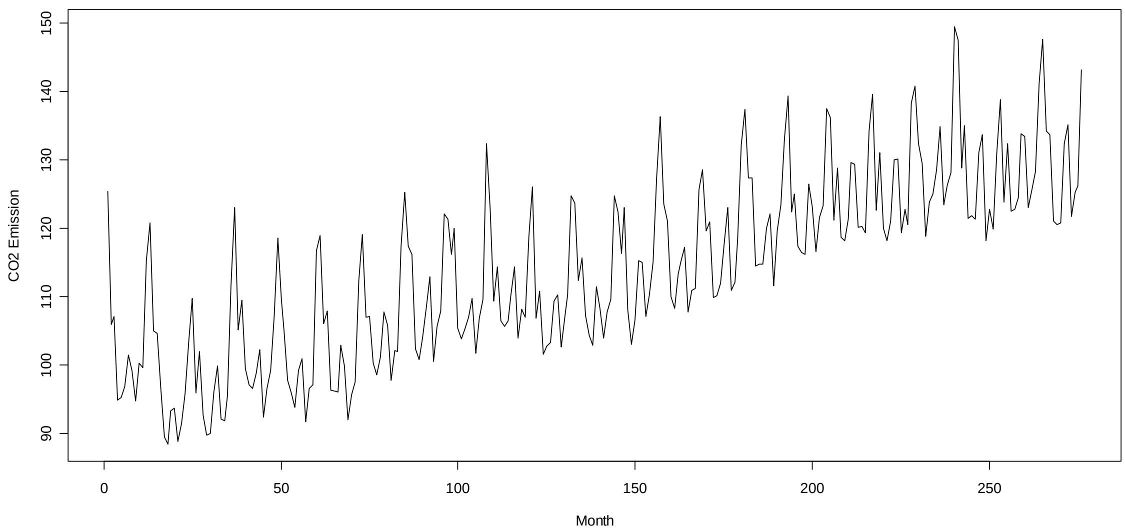

Figure 5.1 Time Series Plot on CO_2 Emission 1981-2003

In forecasting the CO_2 emission, we start with addressing the issue of the seasonal subindex *s*. After we examine the original dataset, shown by Figure 5.1, we note that the monthly CO_2 emission behaves as a periodic function with a cycle of 12 months and contains a small upward trend. Thus, we let the seasonal subindex $S = 12$. Follow by the step-by-step procedure as we described in section 4.2, we found the model that best characterizes the monthly emissions of the United States is an ARIMA$(1,1,2) \times (1,1,1)_{12}$ process, analytically given by

$$(1 - \Phi_1 B^{12})(1 - \phi_1 B)(1 - B)(1 - B^{12}) x_t = (1 - \theta_1 B - \theta_2 B^2)(1 - \Gamma_1 B^{12}) \varepsilon_t \qquad (5.1.1)$$

After expanding both sides of model (5.1) and estimate its coefficients. The final statistical model for CO_2 emission, $\hat{CO}_{2_E}$, with the appropriate estimate of the weights are given by

$$\begin{aligned}\hat{CO}_{2_E} &= 1.5203x_{t-1} - 0.5203x_{t-2} + 1.0049x_{t-12} - 1.527749x_{t-13} + 0.5228495x_{t-14} \\ &- 0.0049x_{t-24} + 0.007449x_{t-25} - 0.002549x_{t-26} - 0.9988\varepsilon_{t-1} + 0.1234\varepsilon_{t-2} \\ &- 0.8523\varepsilon_{t-12} + 0.8512772\varepsilon_{t-13} - 0.10517\varepsilon_{t-14}\end{aligned} \quad (5.1.2)$$

Once we obtained proper coefficients, we shall proceed to evaluate the proposed model and illustrate the quality of the model.

The forecasting values that obtained from the proposed statistical model, (5.1.2), for CO_2 emissions in the United States is graphically presented Figure 4.2.

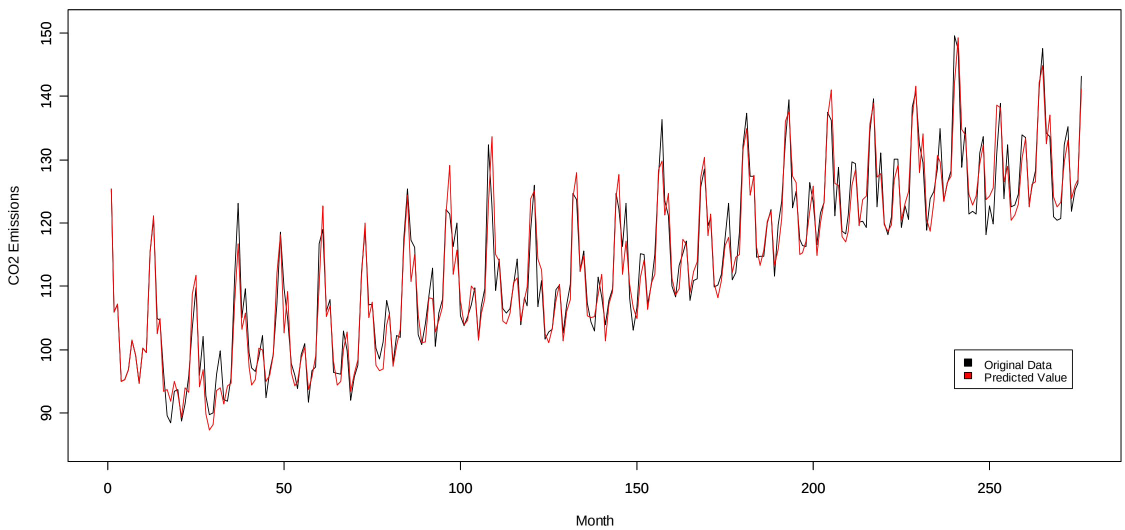

Figure 5.2 Actual VS. Predicted Values for CO_2 Emission 1981-2003

As can be observed, the predicted values follow the actual values of CO_2 Emission closely. It indicates that the overall quality of the model is good. We shall proceed to calculate the residuals estimates, $r_t = x_t - \hat{x}_t$, and the results are graphically presented below by Figure 5.3

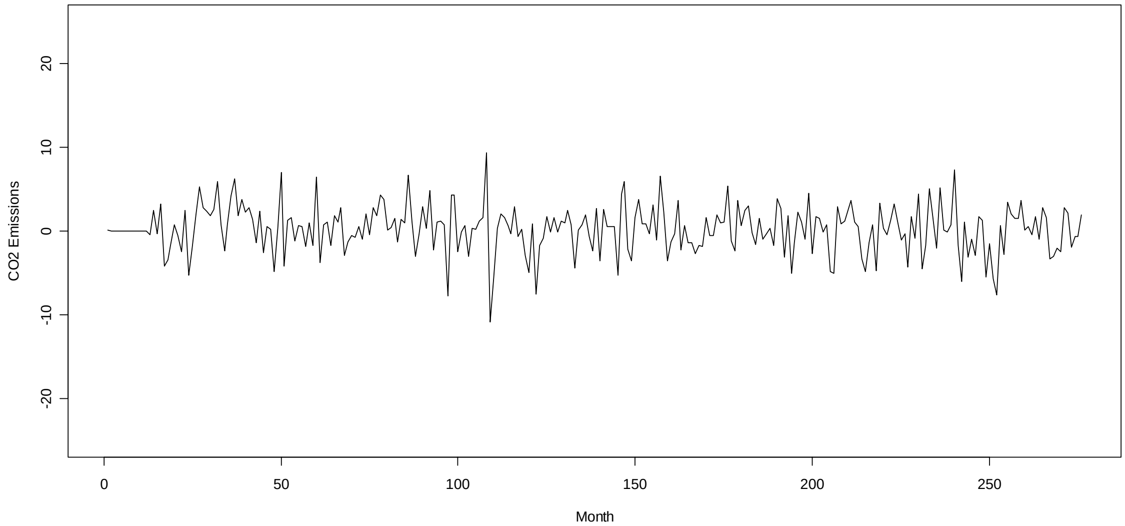

Figure 5.3 Residuals Plot for CO_2 Emissions

The residuals are quite small and isolating around the zero axis as expected. It indicates that the proposed model forecasts the CO_2 emissions closely in the United States.

The mean of the residuals, $\bar{r}$, the variance, $S_r{}^2$, the standard deviation, S_r , standard error, *SE*, and the mean square error, *MSE*, are presented below by Table 5.1.

Table 5.1 Basic Evaluation on CO_2 Emissions Model

$\bar{r}$	$S_r{}^2$	S_r	*SE*	*MSE*
0.2339641	8.055668	2.838251	0.1708426	8.08122

We observe that all evaluation criteria support the quality of the proposed forecasting model for CO_2 emissions.

We now proceed to further evaluate model (5.1.1) hiding the last 12 months of the CO_2 recordings and re-estimating the coefficients of the model (5.1.1). Having restructured the model (5.1.1) we proceed to estimate the hidden recordings. For example, we used the first 264 observations $\{x_1, x_2, ..., x_{264}\}$ to forecast $\hat{x}_{265}$. Then we use the observations $\{x_1, x_2, ..., x_{265}\}$ to forecast $\hat{x}_{266}$, and continue this process until we obtain the forecasting values of the last 12 observations, that is, $\{\hat{x}_{265}, \hat{x}_{266}, ..., \hat{x}_{276}\}$. Table 5.2, gives the actual, forecasting and residual data for the subject 12 months.

Table 5.2 Original VS. Forecasting Values on CO_2 Emissions Model

	Original Values	Forecast Values	Residuals
January 2003	147.6298	145.2361	2.3937
February 2003	134.1716	132.6554	1.5162
March 2003	133.6979	137.3912	-3.6933
April 2003	121.0047	124.5518	-3.5471
May 2003	120.4789	122.4091	-1.9302
June 2003	120.7394	123.101	-2.3616
July 2003	132.4187	129.3481	3.0706
August 2003	135.1314	132.787	2.3444
September 2003	121.7753	123.8295	-2.0542
October 2003	125.2487	125.9811	-0.7324
November 2003	126.2127	126.812	-0.5993
December 2003	143.1509	141.1834	1.9675

Note the closeness between the original and forecast values. A graphical presentation of the results given in Table 4.2 is given below by Figure 5.4.

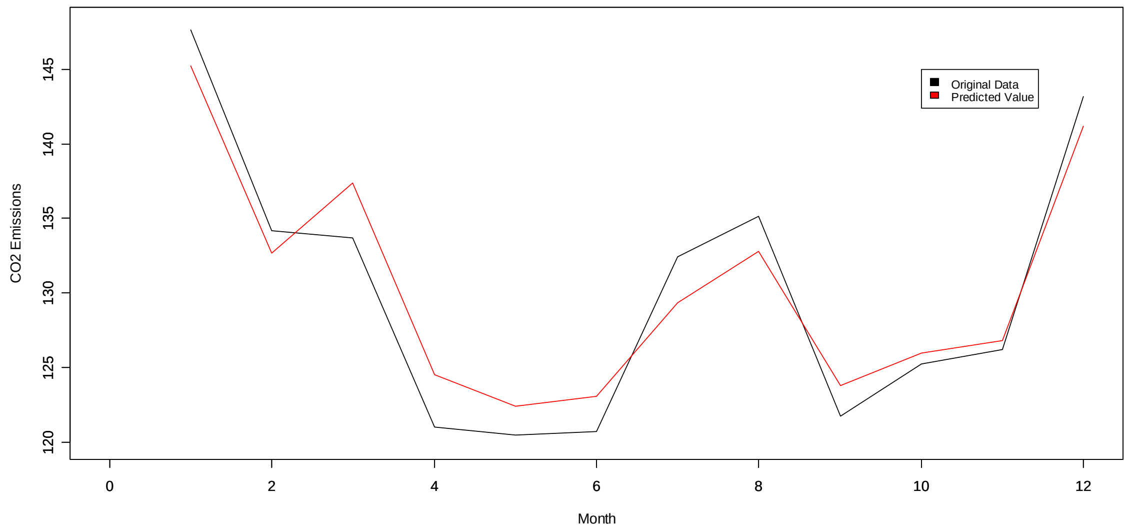

Figure 5.4 Monthly CO_2 Emission VS. Forecast Values for the Last 12 Observations

It can be seen that the predicted values produced by our proposed model follow the actual values of CO_2 Emissions closely. It not only shows that our proposed model is capable of forecasting CO_2 Emissions without using any future information but also speaks out the usefulness of the model.

5.2 Atmospheric Carbon Dioxide Modeling

The data set that we used to develop our second proposed model consists of monthly CO_2 concentrations in the atmosphere from 1958 to 2004 (see Appendix E). The data was collected in Mauna Loa by Carbon Dioxide Research Group, Scripps Institution of Oceanography, University of California. A map of geographical location of Mauna Loa is provided by Figure 5.5.

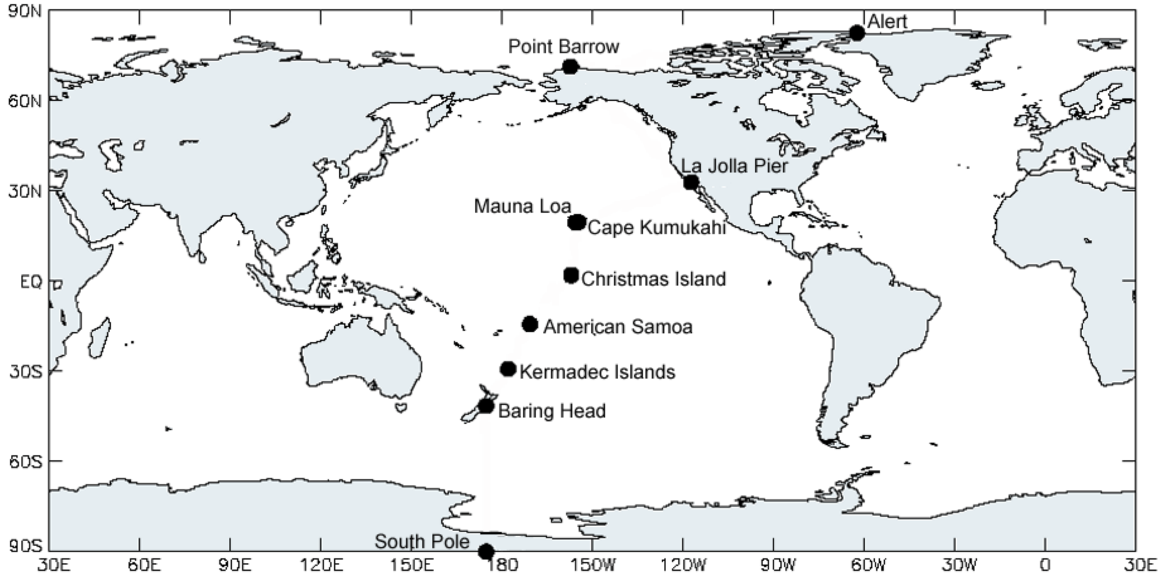

Figure 5.5 Geographical Location of Mauna Loa

At the earlier stage of our model building process, we spot several missing values in the early 1960s. To address this problem, we decided to use the data from 1965 to 2004, which is a period which contains no missing values. For additional information concerning the data set on CO_2 concentrations in the atmosphere, see (Bacastow, 1979; Bacastow & Keeling, 1981; Bacastow et al., 1980; Bacastow et al., 1985; Keeling, 1960; Keeling, 1984; Keeling, 1998; Keeling et al., 1976; Keeling et al., 1982; Keeling et al., 1989; Keeling et al., 1996; Keeling et al., 1995; Pales & Keeling, 1965; Keeling et al., 2002; Whorf & Keeling, 1998).

A plot of the actual CO_2 concentration in the atmosphere is given by Figure 5.6. It provides a visual presentation of the time series plot of CO_2 concentrations in the atmosphere.

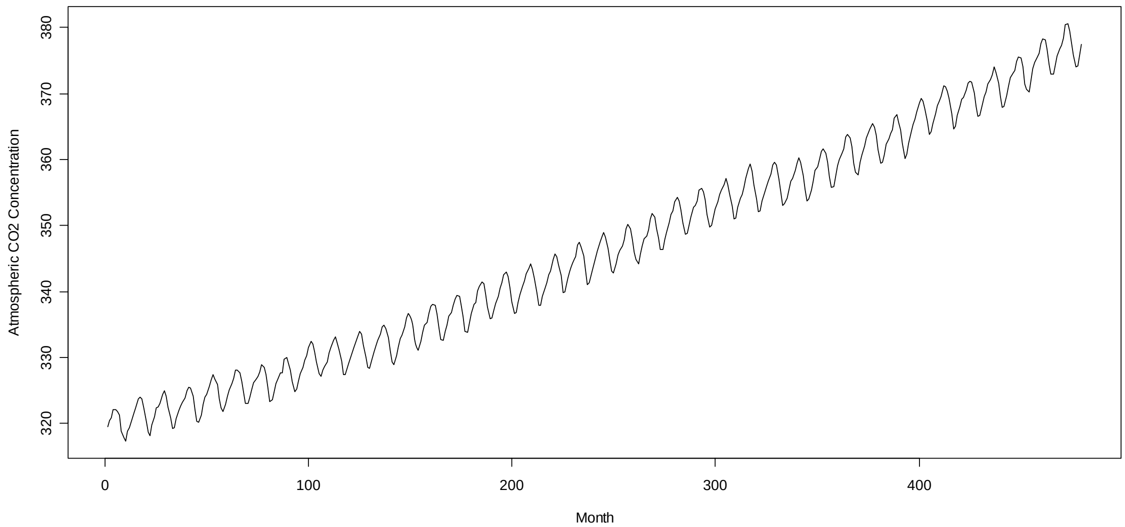

Figure 5.6 Time Series Plot for Monthly CO_2 in the Atmosphere 1965-2004

In forecasting the atmospheric CO_2, we begin by addressing the issue of the seasonal subindex *s*. After we examine the original data sets, shown by Figure 5.6, we note that the CO_2 in the atmosphere data has a more obvious upward trend compare to the CO_2 emission and the shape of its pattern is almost identical every year, as shown by Figure 5.6. Thus, we set the seasonal period $S = 12$. We have identified that the model that best described the monthly CO_2 concentrations in the atmosphere according to the procedure that we discussed in section 4.2 is an ARIMA$(2,1,0)\times(2,1,1)_{12}$ process, analytically given by

$$(1-\Phi_1 B^{12} - \Phi_2 B^{24})(1-\phi_1 B - \phi_2 B^2)(1-B)(1-B^{12})x_t = (1-\Gamma_1 B^{12})\varepsilon_t \quad (5.2.1)$$

After expanding both sides of model (5.2.1) and estimate its coefficients. The final statistical model for atmospheric CO_2, $\hat{CO}_{2_A}$, with the appropriate estimate of the weights are given by

$$\hat{CO}_{2_A} = 0.6887x_{t-1} + 0.1989x_{t-2} + 0.1124x_{t-3} + 1.0759x_{t-12} - 0.74097x_{t-13} - 0.213997x_{t-14} - 0.12093x_{t-15} - 0.0683x_{t-24} + 0.047038x_{t-25} + 0.013585x_{t-26} + 0.00768x_{t-27} - 0.0076x_{t-36} + 0.005234x_{t-37} + 0.0015116x_{t-38} + 0.00085x_{t-39} - 0.8787\varepsilon_{t-12} \quad (5.2.2)$$

We shall proceed to evaluate these models and illustrate the quality of both models in the next section.

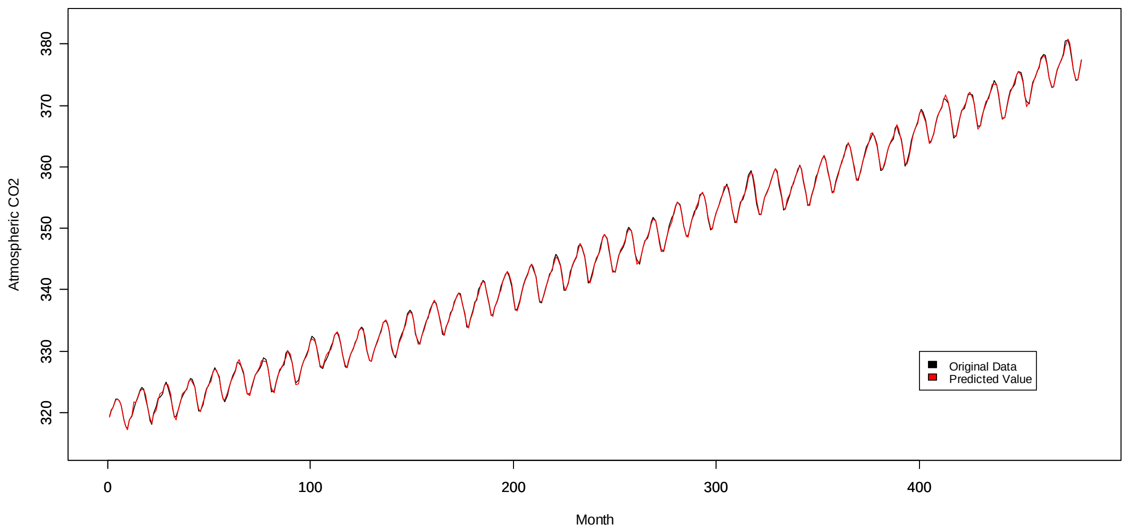

Figure 5.7 Actual VS. Predicted Values for Atmospheric CO_2 1965-2004

Obviously, this graphical presentation attests to show the quality of the proposed model. A plot of the residuals is given by Figure 5.8 below.

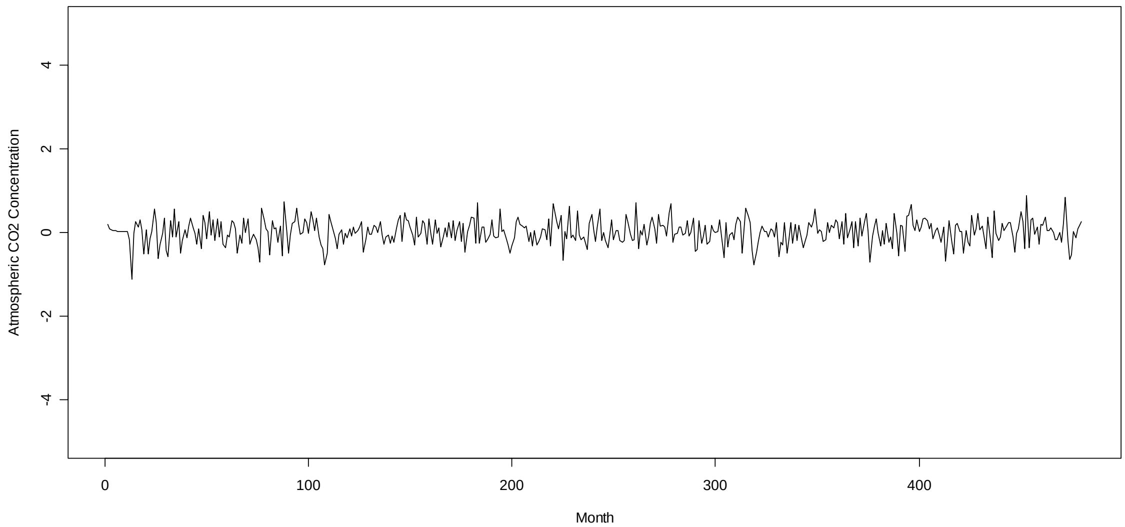

Figure 5.8 Residuals Plot for Atmospheric CO_2

The residuals of our proposed model are very small and isolating around the zero axes. It illustrates the quality of the model. The following Table 5.3 gives a basic evaluation statistics of the proposed model.

Table 5.3 Basic Evaluation on Atmospheric CO_2 Model

$\bar{r}$	$S_r^{\ 2}$	S_r	SE	MSE
0.01140137	0.08460756	0.2908738	0.01327651	0.08456128

These results also confirm the effectiveness of the proposed model for forecasting CO_2 in the atmosphere.

We shall use the same technique as we used in the previous application to illustrate the quality of our proposed model in terms of forecasting values in the future. Again, we hide the last 12 months of atmospheric CO_2 recordings and try to predict them only using the information from the past. Table 5.4 gives the numerical comparison between the original series and the forecasting.

Table 5.4 Original VS. Forecasting Values on Atmospheric CO_2 Model

	Original Values	Forecast Values	Residuals
January 2004	376.79	376.7963	-0.0063
February 2004	377.37	377.609	-0.239
March 2004	378.41	378.1837	0.2263
April 2004	380.52	379.6653	0.8547
May 2004	380.63	380.8268	-0.1968
June 2004	379.57	380.2339	-0.6639
July 2004	377.79	378.3489	-0.5589
August 2004	375.86	375.837	0.023
September 2004	374.06	374.1871	-0.1271
October 2004	374.24	374.1482	0.0918
November 2004	375.86	375.6897	0.1703
December 2004	377.48	377.2186	0.2614

The residuals that were calculated are shown by Table 5.4 are all very small, and a graphical presentation of the results is given below by Figure 5.9.

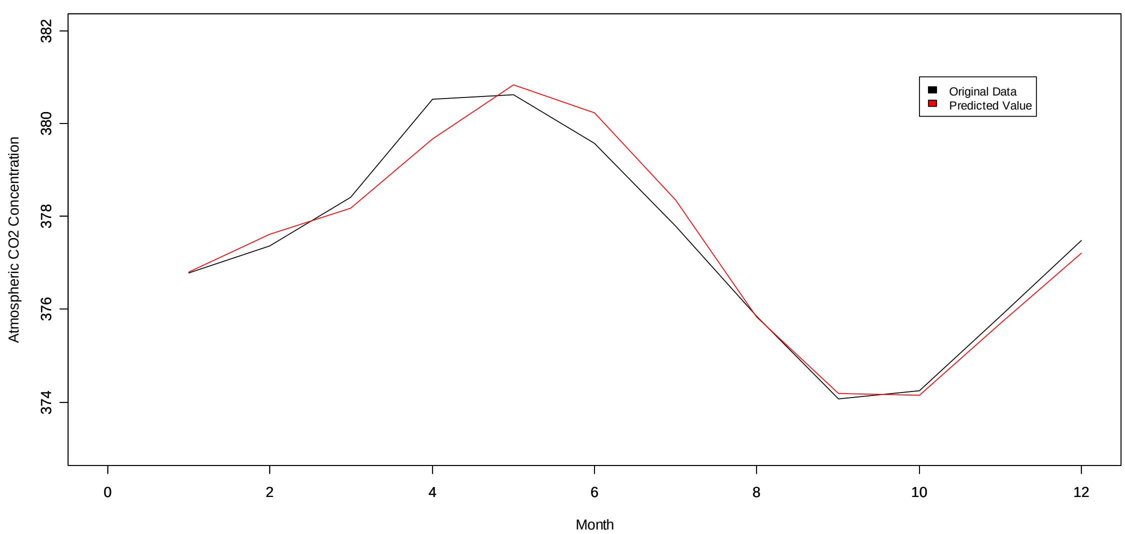

Figure 5.9 Actual VS. Predicted Values for CO_2 in the Atmosphere

Thus, we can conclude that the proposed model (5.2.1), forecasts very well on the future behavior of CO_2 in the atmosphere.

5.3 Conclusion

We have developed two non-stationary time series statistical models with trend and seasonal effects to predict future estimates of carbon dioxide emissions and that in the atmosphere. We use actual CO_2 recordings in both situations to develop the subject statistical models. The developed processes were evaluated to attest the degree of quality by using various statistical criteria. Finally, we tested the accuracy of the proposed models by predicting and analyzing the CO_2 emission and atmosphere for 12 months. The results are very encouraging.

Chapter 6

Global Warming: Temperature & Carbon Dioxide Prediction Modeling

6.0 Introduction

The object of the present chapter is to propose forecasting models for the monthly Carbon Dioxide in the atmosphere and monthly temperature of the Continental United States. The approach of the subject model is to use regression analysis on the monthly temperature of Continental United States to explain the difference of the monthly Carbon Dioxide and vice versa. Therefore, the final form of the subject models is a combination of regression model based on monthly temperature predicting the difference of the monthly Carbon Dioxide adding a time series term of the previous month atmospheric Carbon Dioxide.

6.1 Relationship between Carbon Dioxide & Temperature

Many studies have been done in the subject of Global Warming. In fact, it is common to use time series analysis to form a forecasting model when historical information is available. Shih & Tsokos introduces the time series approach on forecasting both temperature of the Continental United States and Carbon Dioxide in the atmosphere. In the present study, we take the monthly temperature of the Continental United States from 1965 to 2004 along with the monthly atmospheric CO_2 from 1965 to 2004 and try to explore the relationship between those two. Figure 6.1 is the illustration of the time series plot on monthly temperature of the Continental United States from 1965 to 2004.

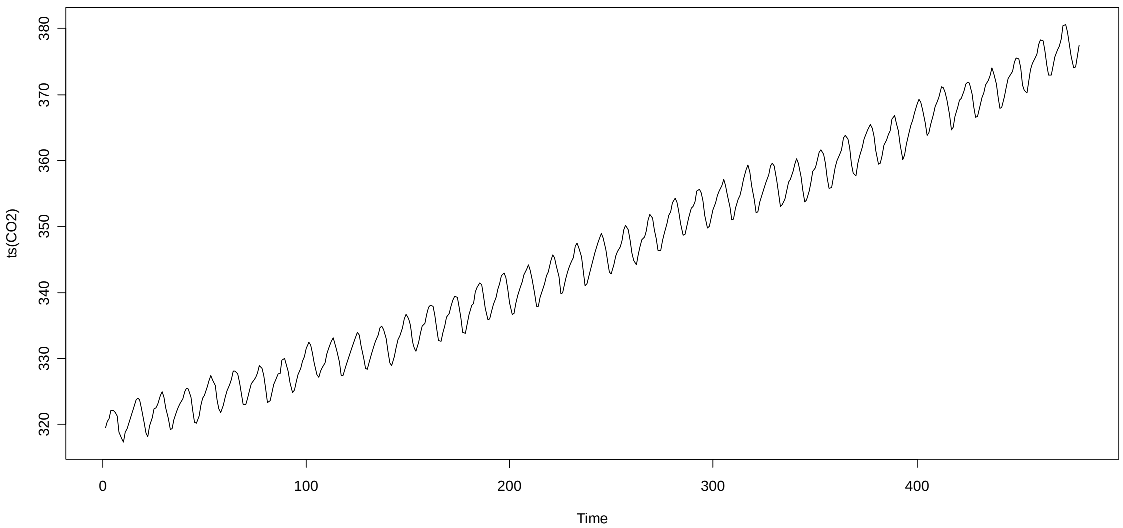

Figure 6.1 Monthly Atmospheric Carbon Dioxide from 1965 to 2004

Obviously, the monthly atmospheric CO_2 behave as a periodic function and has a stable upward trend present at all time period. Since there are total of 480 observations, we denote them as $x_1, x_2, ..., x_{480}$. Figure 6.2 is the time series plot of the mean temperature of the Continental United States from 1965 to 2004.

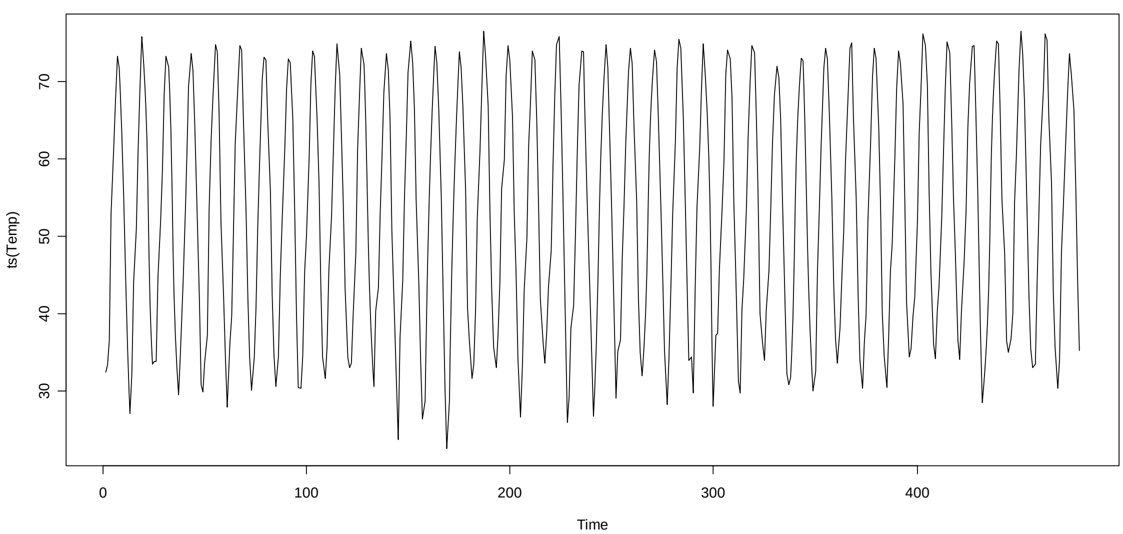

Figure 6.2 Monthly Temperature of the Continental United States from 1965 to 2004

It is expected that the average monthly temperature of the Continental United States behave as a periodic function with a clear seasonal variation that is obvious to identify. Similar to the CO_2 data, we denote the 480 temperature observations as $y_1, y_2, ..., y_{480}$. The correlation coefficient is defined as

$$r = \frac{1}{n-1} \sum \frac{(x - \bar{x})}{s_x} \bullet \frac{(y - \bar{y})}{s_y} \qquad (6.1.1)$$

By using (6.1.1), we calculated the correlation coefficient between the temperature and CO_2 and found $r = 0.04704972$, which does not show much relationship between those two variables.

In order for our proposed model to be accurate, it is important to have high correlation coefficient to drive the regression part of the subject model. Hence, a filtering process becomes necessary during our model building procedure. Consider the following difference filter

$$(1-B)x_t = x_t - x_{t-1} \qquad (6.1.2)$$

It is obvious that the atmospheric CO_2 series contains an upward trend, but the temperature series doesn't. Hence, removing the upward trend from the atmospheric CO_2 series is the first step of our model building procedure. By applying (6.1.2), we can produce the differencing series of the atmospheric CO_2, denote as $z_1, z_2, ..., z_{479}$. Where

$$z_t = (1-B)x_t \qquad (6.1.3)$$

Figure 6.3 gives the time series plot of the first order differencing atmospheric CO_2 series.

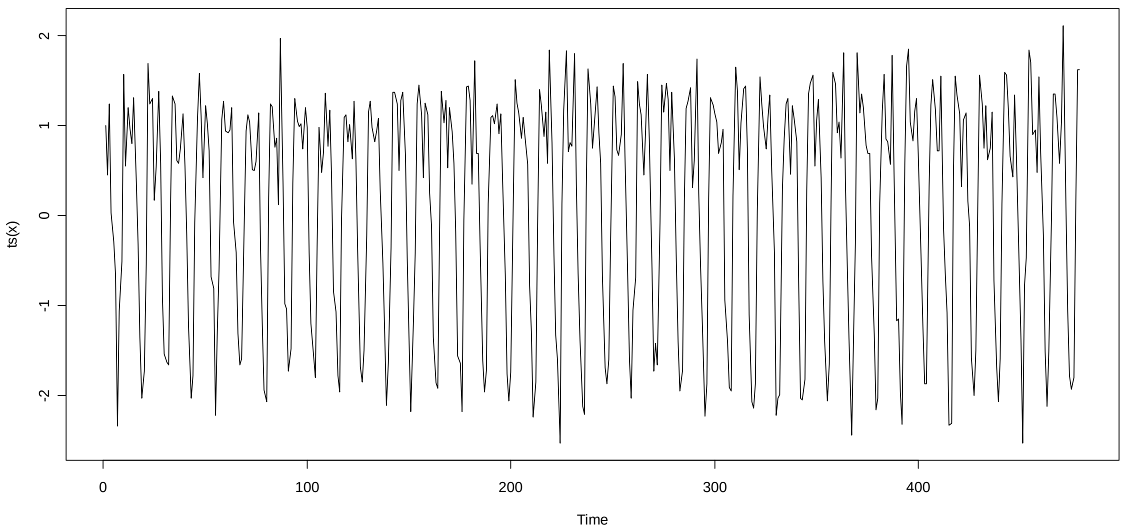

Figure 6.3 First Order Differencing Monthly Atmospheric CO_2 Series

It can be seen that the differencing filter has remove the upward trend from the atmospheric CO_2 series, and turn it into a periodic function similar to the temperature series. After the examination of the temperature and the differencing series, we want to compare both series by graphing both series on a time series plot. Figure 6.4 shows the illustration of that subject matter.

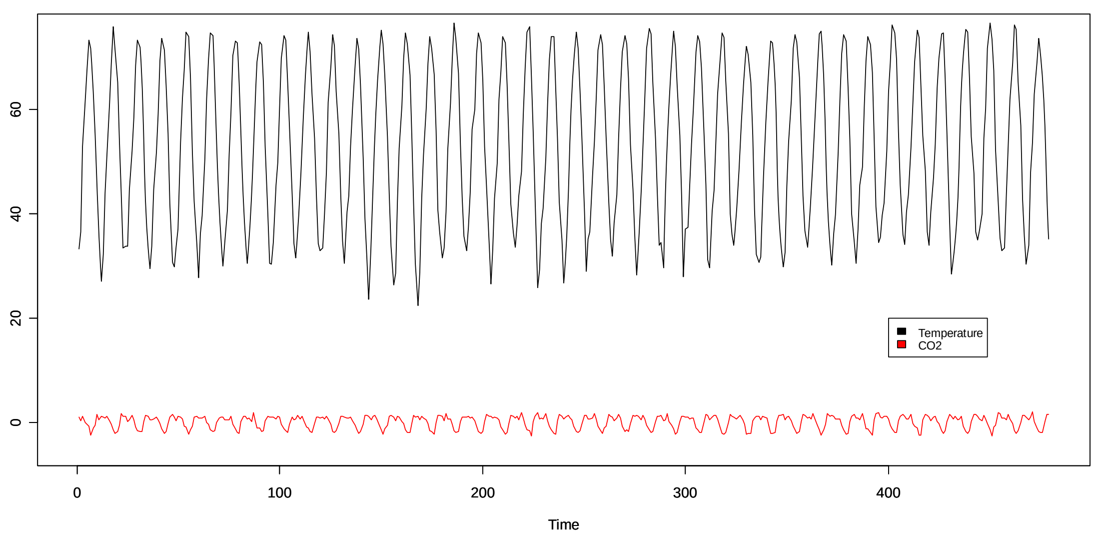

Figure 6.4 The Temperature Series VS. The Differencing CO_2 Series

The purpose of our next several transformations is to bring the temperature series to the same level as the CO_2 series, so we can force a correlation between two series. We begin with computing the mean of both series using (6.1.4).

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{6.1.4}$$

We found the mean of the temperature series is 53.03569, and the mean of the differencing CO_2 series is 0.1211691. Hence, the first transformation is to subtract $d = 52.91452$ from the temperature series. That is

$$u_t = y_t - d \tag{6.1.5}$$

Therefore, after the transformation (6.1.5), both series have the same mean as a periodic function. We then examine the minimum and maximum of both series. The following Table 6.1 gives a comparison on both series.

Table 6.1 Comparison between Two Series

Series	Minimum	Maximum	Difference
Temperature	-30.5357	23.51431	54.05
CO_2	-2.53	2.11	4.64
		Ratio	11.64871

After the examination of both series, it is important to determine the ratio between both series, because it plays a major role on our next transformation.

$$v_t = \frac{u_t}{11.64871} \tag{6.1.6}$$

After the 2nd transformation being made, we examine the time series plot on both series, shown as Figure 6.5.

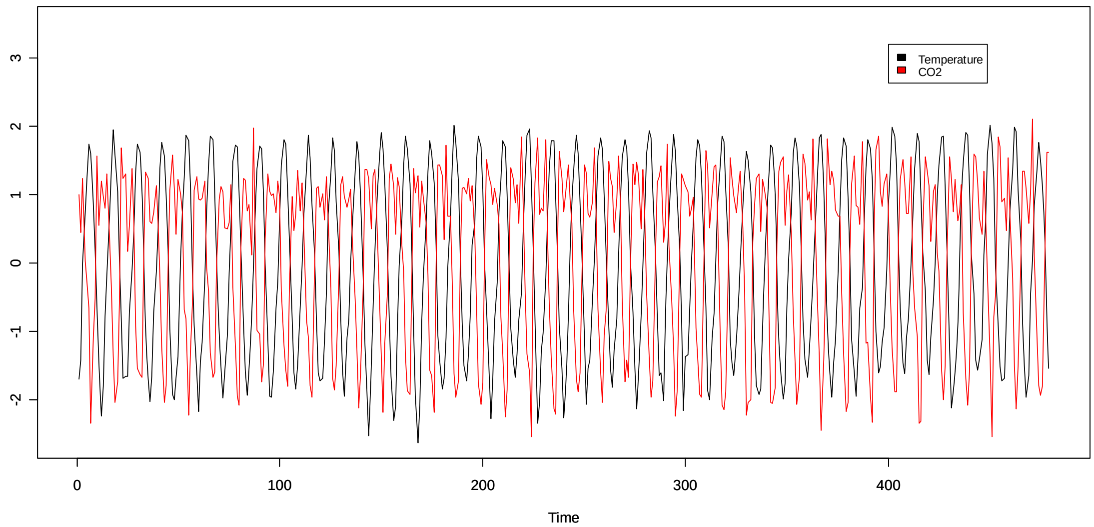

Figure 6.5 Comparison between Both Series After Transformations

It is obvious that both series behave as periodic functions at the same level. By examine the cross correlation function of the two, we can find the lag that shall give us the highest correlation. Consider two series x_i and y_i, where $i = 0,1,2,...,t-1$. The cross correlation *r* at delay *d* is defined as

$$r = \frac{\sum_i [(x_i - \bar{x})(y_{i-d} - \bar{y})]}{\sqrt{\sum_i (x_i - \bar{x})^2} \sqrt{\sum_i (y_i - \bar{y})^2}} \qquad (6.1.7)$$

We then plot the cross correlation at different lag between the difference atmospheric CO_2 and temperature on Figure 6.6.

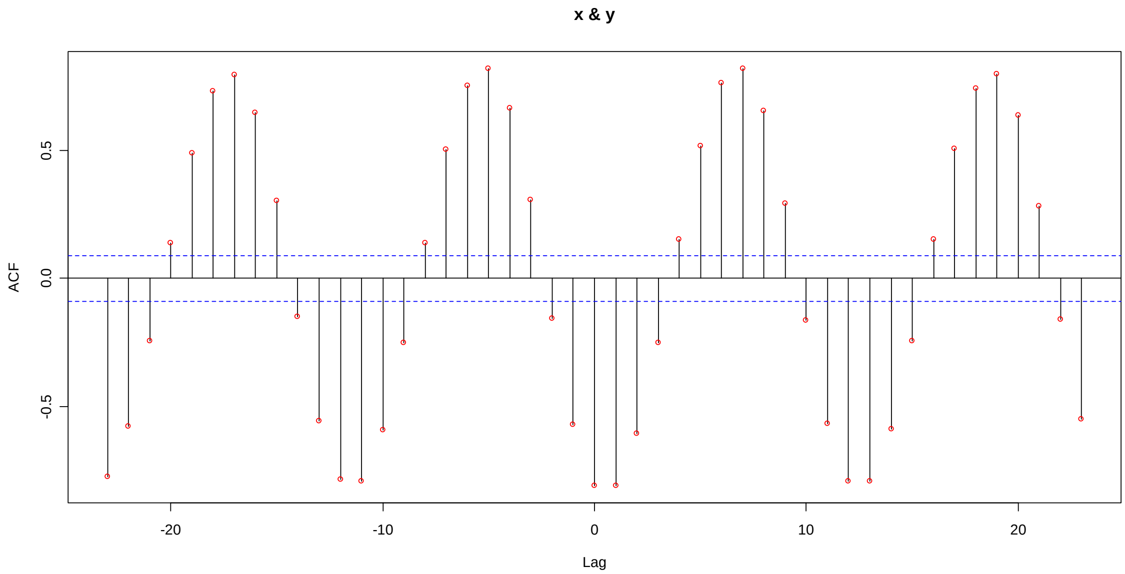

Figure 6.6 Cross Correlation at Different Lag

After reviewing the cross correlation function shown as Figure 6, it is clear to see that there exists a negative correlation between the difference of atmospheric CO_2 and the temperature at lag 0, because $|r|$ is the maximum at lag 0. Hence we can now calculate the correlation coefficient between these two series by using (6.1.1), and we have r = -0.8109993, which indicate a negative linear relationship between two series.

6.2 Carbon Dioxide & Temperature Model-01

The aim of this section is to develop a model for the atmospheric CO_2 knowing the monthly temperature. Since all transformations being made are invertible, we can go ahead build the regression model first, and then use backward filter to solve back the original series. Let the first 479 observations of the temperature series after transformations be $v_1, v_2, ..., v_{479}$. The simple regression model formulated between series z_t and v_t is

$$\hat{z}_t = 0.1319 - 0.7640 v_t \tag{6.2.1}$$

The simple regression model given by equation (6.2.1) indicates the difference of the atmospheric CO_2 is explained by the transforming temperature series. In addition, we can solve back the original atmospheric CO_2 and temperature series by using equation (6.1.3), (6.1.5), and (6.1.6). Therefore, the final analytical form of the proposed model is

$$\hat{x}_t = 0.1319 - 0.7620\left(\frac{y_t - 52.91452}{11.64871}\right) + x_{t-1}$$

And we can simplify the model as

$$\hat{x}_t = 3.5933 - 0.06541 y_t + x_{t-1} \tag{6.2.2}$$

Table 6.2 illustrates the residual analysis of (6.2.2)

Table 6.2 Residual Analysis for Combine Model-1

$\bar{r}$	S_r^2	S_r	$S_r / \sqrt{n}$
2.650722e-17	0.5242516	0.7240522	0.03280994

After we develop our proposed model, it is essential to evaluate the performance of the model. In this section, we shall illustrate the evaluation and usefulness of our proposed model by using one-step ahead forecasting technique. That is we hide the last 12 month of atmospheric CO_2 data, and try to forecast them using only the previous information. In example, we forecast $\hat{x}_{469}$ using only $x_1, x_2, ..., x_{468}$ and $y_1, y_2, ..., y_{469}$; forecast $\hat{x}_{470}$ using only $x_1, x_2, ..., x_{469}$ and $y_1, y_2, ..., y_{470}$;…; and forecast $\hat{x}_{480}$ using only $x_1, x_2, ..., x_{479}$ and $y_1, y_2, ..., y_{480}$. In addition, we update the coefficient of the model once we obtain new information. Table 2 gives the monthly comparison between the actual and the forecasts that produces by our proposed model.

Table 6.3 Comparison between Actual and Forecast for Model-1

Month (2004)	Temperature	CO_2 (Actual)	CO_2 (Forecast)	Residuals
January	30.34	376.79	376.9535	-0.1635
February	33.91	377.37	378.3881	-1.0181
March	47.94	378.41	378.7315	-0.3215
April	53.53	380.52	378.8619	1.6581
May	62.87	380.63	380.6131	0.0169
June	68.87	379.57	380.1183	-0.5483
July	73.59	377.79	378.6683	-0.8783
August	70.8	375.86	376.5785	-0.7185
September	66.26	374.06	374.8254	-0.7654
October	55.84	374.24	373.3174	0.9226
November	44.34	375.86	374.1783	1.6817
December	35.15	377.48	376.5517	0.9283

From Table 6.3, it can be seen that the forecasts fit closely to the actual CO2, it speaks out the quality of the model.

6.3 Carbon Dioxide & Temperature Model-02

The aim of this section is to develop a model for the monthly average temperature knowing the CO_2 in the atmosphere. We shall use those transformations that we discussed in section 6.1 to build the regression model first, and then use backward filter to solve back the original series. Let the first 479 observations of the temperature series after transformations be $v_1, v_2, ..., v_{479}$. The simple regression model formulated between series v_t and z_t is

$$\hat{v}_t = 0.1180 - 0.8578 z_t \qquad (6.3.1)$$

The simple regression model given by equation (6.3.1) indicates the transforming temperature series is explained by the difference of the atmospheric CO_2. In addition, we can solve back the original temperature and atmospheric CO_2 series by using equation (6.1.3), (6.1.5), and (6.1.6). Therefore, the final analytical form of the proposed model is

$$\frac{\hat{y}_t - 52.91452}{11.64871} = 0.1180 - 0.8578(x_t - x_{t-1})$$

And we can simplify the model as

$$\hat{y}_t = 54.2891 - 9.99226 x_t + 9.99226 x_{t-1} \qquad (6.3.2)$$

Table 6.2 illustrates the residual analysis of (6.2.2)

Table 6.4 Residual Analysis for Combine Model-2

$\bar{r}$	S_r^2	S_r	$S_r / \sqrt{n}$
-1.646039e-06	80.08299	8.94891	0.4088861

After we develop our proposed model, it is essential to evaluate the performance of the model. In this section, we shall illustrate the evaluation and usefulness of our proposed model by using one-step ahead forecasting technique. That is we hide the last 12 month of the temperature data and try to forecast them using only the existing information. In example, we forecast $\hat{y}_{469}$ using only $x_1, x_2, ..., x_{469}$ and $y_1, y_2, ..., y_{468}$; forecast $\hat{y}_{470}$ using only $x_1, x_2, ..., x_{470}$ and $y_1, y_2, ..., y_{469}$;…; and forecast $\hat{y}_{480}$ using only

$x_1, x_2, ..., x_{480}$ and $y_1, y_2, ..., y_{479}$. In addition, we update the coefficient of the model once we obtain new information. Table 2 gives the monthly comparison between the actual and the forecasts that produces by our proposed model.

Table 6.5 Comparison between Actual and Forecast for Model-2

Month (2004)	CO2	Temperature	Temperature (Forecast)	Residuals
January	376.79	30.34	40.70947	-10.36947
February	377.37	33.91	43.27709	-9.36709
March	378.41	47.94	48.37591	-0.43591
April	380.52	53.53	43.7548	9.7752
May	380.63	62.87	33.13691	29.73309
June	379.57	68.87	53.18363	15.68637
July	377.79	73.59	64.91555	8.67445
August	375.86	70.8	72.14047	-1.34047
September	374.06	66.26	73.62139	-7.36139
October	374.24	55.84	72.27571	-16.43571
November	375.86	44.34	52.4835	-8.1435
December	377.48	35.15	38.11712	-2.96712

From Table 2, it can be seen that the forecasts fit closely to the actual CO_2, it speaks out the quality of the model.

6.4 Temperature Model

It is obvious that the average monthly temperature of the Continental United States contains a seasonal pattern. We consider the temperature series $x_1, x_2, ..., x_{480}$, and take the average of each month as $m_1, m_2, ..., m_{12}$ by using the following transformation.

$$m_1 = \frac{x_1 + x_{13} + ... + x_{469}}{40}$$

$$m_2 = \frac{x_2 + x_{14} + ... + x_{470}}{40}$$

$$\vdots$$

$$m_{12} = \frac{x_{12} + x_{24} + ... + x_{480}}{40} \qquad (6.4.1)$$

We then create a new series $\{\gamma_t\}$ simply by repeating the series $m_1, m_2, ..., m_{12}$ shown as (6.4.2).

$$\{\gamma_1, \gamma_2, ..., \gamma_{12}\} = \{m_1, m_2, ..., m_{12}\}$$

$$\{\gamma_{13}, \gamma_{14}, ..., \gamma_{24}\} = \{m_1, m_2, ..., m_{12}\}$$

$$\vdots$$

$$\{\gamma_{469}, \gamma_{470}, ..., \gamma_{480}\} = \{m_1, m_2, ..., m_{12}\} \qquad (6.4.2)$$

Let $\{\lambda_t\}$ be the difference between the temperature series $\{x_t\}$ and the new series $\{\gamma_t\}$, shown as (6.4.3).

$$\lambda_t = x_t - \gamma_t \qquad (6.4.3)$$

By using the methodology that we discussed in section 2.1, we found the best ARIMA model on the series $\{\lambda_t\}$ is a ARIMA(2,1,2), that is

$$(1 - .7755B + .1938B^2)(1 - B)\lambda_t = (1 + .0181B - .9819B^2)\varepsilon_t . \qquad (6.4.4)$$

Expanding the autoregressive operator and the first difference filter, we have

$$(1 - 1.7755B + .9693B^2 + .1938B^3)\lambda_t = (1 + .0181B - .9819B^2)\varepsilon_t .$$

and the model can be written as

$$\lambda_t = 1.7755\lambda_{t-1} - .9693\lambda_{t-2} - .1938\lambda_{t-3} + \varepsilon_t + .0181\varepsilon_{t-1} - .9819\varepsilon_{t-2} .$$

The final analytical form of the proposed forecasting model can be written as

$$\hat{\lambda}_t = 1.7755\lambda_{t-1} - .9693\lambda_{t-2} - 0.1938\lambda_{t-3} + .0181\varepsilon_{t-1} - .9819\varepsilon_{t-2}. \qquad (6.4.5)$$

We can solve back the original temperature series by combining (6.4.2) and (6.4.3), we have

$$\hat{x}_t = 1.7755\lambda_{t-1} - .9693\lambda_{t-2} - 0.1938\lambda_{t-3} + .0181\varepsilon_{t-1} - .9819\varepsilon_{t-2} + \gamma_t \qquad (6.4.6)$$

After we obtained the predicted values for the temperature series, we proceed to evaluate the residuals of the model by calculating $\bar{r}$, S_r^2, S_r, and $S_r/\sqrt{n}$, shown as Table 6.6.

Table 6.6 Basic Evaluation Statistics for the Temperature Model

$\bar{r}$	S_r^2	S_r	$S_r/\sqrt{n}$
0.1290907	3.75889	1.938786	0.08849305

It can be seen that the evaluation of Table 6.6 supports the quality of the proposed temperature model.

6.5 Conclusion

In the present chapter, we look into the trend and seasonal patterns of the CO_2 and temperature time series and examine the relationship between the two series. We discover that there exists a strong linear relationship between two series after we applied several transformations into both series. Thus, the first two proposed statistical models that relates CO_2 to temperature are semi-regression based models; that is, knowing the atmospheric temperature we can at the specific location estimate the carbon dioxide and vice versa. Our last proposed model begins with removing the seasonal variation of the temperature series by subtracting the mean function from the series. We found the new series become much easier to predict than the original series. Since all transformations that applied to the series are invertible, we can predict the original series by applying a back shift operator. The comparisons between the actual and forecasts of each model were provided and they attest the quality of the proposed models.

Chapter 7

Future Research

As a result of the present study, we will continue the research on the subject area by studying the following problems.

Investigate the selection of the best ARIMA model utilizing AIC versus BIC with respect to small, medium and large sample sizes.

In the proposed forecasting models with *k*-th moving average, *k*-th weighted moving average, and *k*-th exponential weighted moving average processes, we want to be able to determine the optimal *k* that will produce the smallest residuals. We will also like to study the robustness and sensitivity of the selected models when *k* changes.

Once a particular model has been identified with an actual sample size, we want to study the consistency of the orders of the model when the sample size changes. In addition, we will study the Fourier transform of the developed models so that we can investigate the behavior of the variance as a function of time for a given set of data. This information will assist us in improving the selected model.

Finally, we will restructure the proposed models so that would be instantly updated as new information becomes available. We will obtain confidence limits for short term and long term forecasting and compare the confidence range with other acceptable and useful models.

Reference

Akaike, H. (1974). A New Look at the Statistical Model Identification, IEEE Transactions on Automatic Control, AC-19, 716-723.

Alexandersson, H. & A. Moberg, (1997). Homogenization of Swedish temperature data. Part I: Homogeneity test for linear trends. Int. J. Climatol., 17, 25-34.

Bacastow, R.B. (1979). Dip in the atmospheric CO2 level during the mid-1960s. Journal of Geophysical Research 80:3109-14.

Bacastow, R.B., & C.D. Keeling. (1981). Atmospheric carbon dioxide concentration and the observed airborne fraction. In B. Bolin (ed.), Carbon Cycle Modelling, SCOPE 16. John Wiley and Sons, New York.

Bacastow, R.B., J.A. Adams, Jr., C.D. Keeling, D.J. Moss, T.P. Whorf, & C.S. Wong. (1980). Atmospheric carbon dioxide, the Southern Oscillation, and the weak 1975 El Niño. Science 210:66-68.

Bacastow, R.B., C.D. Keeling, & T.P. Whorf. (1985). Seasonal amplitude increase in atmospheric CO2 concentration at Mauna Loa, Hawaii, 1959-1982. Journal of Geophysical Research 90(D6):10529-40.

Baker, D. G., (1975). Effect of observation time on mean temperature estimation. J. Appl. Meteor., 14, 471-476.

Banerjee, A., J. J. Dolado, J. W. Galbraith, & D. F. Hendry (1993), Cointegration, Error Correction, and the Econometric Analysis of Non-Stationary Data, Oxford University Press, Oxford.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity, Journal of Econometrics, 31, 307-327.

Box, G. E. P. and G. M. Jenkins, (1970). Time Series Analysis Forecasting and Control. Holden-Day, San Franscisco.

Box, G. E. P., G. M. Jenkins, & G. C. Reinsel, (1994). Time Series Analysis: Forecasting and Control, 3rd ed., Prentice Hall, Englewood Cliffs, NJ., 89-99.

Box, G. E. P., G. M. Jenkins, & G. C. Reinsel, (1994). Time Series Analysis: Forecasting and Control, 3rd ed., Prentice Hall, Englewood Cliffs, NJ., 224-247.

Brockwell, P. J., & R. A. Davis, (1996). Introduction to Time Series and Forecasting., Springer, New York., Sections 3.3 and 8.3.

Brown, R. G. (1962). Smoothing, Forecasting and Prediction of Discrete Time Series, Prentice-Hall, New Jersey.

Chen, C. T. (1999). Linear System Theory and Design, 3rd. ed., Oxford University Press.

Crane, D. B. and J. R. Eratty, (1967). A two stage forecasting model: exponential smoothing and multiple regression, Management Science, Vol. 13, No. 8.

Dickey, D. A., & W. A. Fuller, (1979). "Distribution and the Estimators for Autoregressive Time Series With a Unit Root"., Journal of the American Statistical Association, Vol. 74, No. 366, 427-431

Dickey, D. A., D. P. Hasza, & W. A. Fuller, (1984). "Testing for Unit Roots in Seasonal Time Series"., Journal of the American Statistical Association, Vol. 79, No. 386, 355-367.

Durbin, J., & S. J. Koopman, (2001). Time Series Analysis by State Space Methods., Oxford University Press.

Easterling, D.R., & T.C. Peterson, (1995). A new method of detecting undocumented discontinuities in climatological time series, Int. J. of Climatol., 15, 369-377.

Easterling, D. R., T. R. Karl, E.H. Mason, P. Y. Hughes, & D. P. Bowman. (1996). United States Historical Climatology Network (U.S. HCN) Monthly Temperature and Precipitation Data. ORNL/CDIAC-87, NDP-019/R3. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tennessee.

Easterling, D. R., T. R. Karl, J. H. Lawrimore, & S. A. Del Greco. (1999). United States Historical Climatology Network Daily Temperature, Precipitation, and Snow Data for 1871-1997. ORNL/CDIAC-118, NDP-070. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tennessee.

Enders, W. (1995). Applied Econometrics Time Series, John-Wiley & Sons, 139-149.

Engle, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of variance of United Kingdom Inflation, Econometrica, 50, 987-1007.

Engle, R. F. (2001). "GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics", Journal of Economic Perspectives 15(4):157-168.

EPA (U.S. Environmental Protection Agency) (2004). Inventory or U.S. Greenhouse Gas Emissions and Sinks: 1990-2002, EPA 430-R-04-003, U.S. Environmental Protection Agency, Washington, D.C., 308 pp. plus annexes (291 pp.). Available electronically from: http://yosemite.epa.gov/oar/globalwarming.nsf/content/ResourceCenterPublicationsGHGEmissionsUSEmissionsInventory2003.html

Fox, A. J. (1972). Outliers in time series, Journal of the Royal Stat. Society, Series B, Vol. 34, No. 3.

Gardner, G., A. C. Harvey, & G. D. A. Phillips, (1980). “Algorithm AS154. An algorithm for exact maximum likelihood estimation of autoregressive-moving average models by means of Kalman filtering”., Applied Statistics, 29, 311-322.

Gujarati, D. N. (2003). Basic Econometrics, 856-862.

Harrison, P. J. (1965). Short-term sales forecasting. Applied Statistics. 14, 102-139.

Harvey, A. C., (1993). Time Series Models, 2nd Edition, Harvester Wheatsheaf., sections 3.3 and 4.4.

Hinrichsen, D. and Pritchard, A. J. (2005). Mathematical Systems Theory I, Modelling, State Space Analysis, Stability and Robustness. Springer.

Hughes, P. Y., E. H. Mason, T. R. Karl, & W. A. Brower. (1992). United States Historical Climatology Network Daily Temperature and Precipitation Data. ORNL/CDIAC-50, NDP-042. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tennessee.

Jones, R. H., 1980). “Maximum likelihood fitting of ARMA models to time series with missing observations”., Technometrics, 20, 389-395.

Karl, T. R., C. N. Williams, Jr., & F. T. Quinlan. (1990). United States Historical Climatology Network (HCN) Serial Temperature and Precipitation Data. ORNL/CDIAC-30, NDP-019/R1. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tennessee.

Karl, T.R., C.N. Williams, Jr., P.J. Young, & W.M. Wendl&, (1986). A model to estimate the time of observation bias associated with monthly mean maximum, minimum, and mean temperature for the United States, J. Climate Appl. Meteor., 25, 145-160.

Karl, T.R., & C.W. Williams, Jr., (1987). An approach to adjusting climatological time series for discontinuous inhomogeneities, J. Climate Appl. Meteor., 26, 1744-1763.

Karl, T.R., H.F. Diaz, & G. Kukla, (1988). Urbanization: its detection and effect in the United States climate record, J. Climate, 1, 1099-1123.

Karl, T.R., C.N. Williams, Jr., F.T. Quinlan, & T.A. Boden, (1990). United States Historical Climatology Network (HCN) Serial Temperature and Precipitation Data, Environmental Science Division, Publication No. 3404, Carbon Dioxide Information and Analysis Center, Oak Ridge National Laboratory, Oak Ridge, TN, 389 pp.

Keeling, C.D. (1960). The concentration and isotopic abundance of carbon dioxide in the atmosphere. Tellus 12:200-203.

Keeling, C.D. (1984). Atmospheric and oceanographic measurements needed for establishment of a data base for carbon dioxide from fossil fuels. In The Potential Effects of Carbon Dioxide-Induced Climatic Changes in Alaska. (Miscellaneous, etc.). The Proceedings of a Conference. Fairbanks, Alaska, April 7-8, 1982. School of Agriculture and Land Resources Management, University of Alaska, Fairbanks.

Keeling, C.D. (1998). Rewards and penalties of monitoring the earth. Annual Review of Energy and the Environment 23:25-82. Annual Reviews Inc., Palo Alto.

Keeling, C.D., R.B. Bacastow, A.E. Bainbridge, C.A. Ekdahl, Jr., P.R. Guenther, L.S. Waterman, & J.F.S. Chin. (1976). Atmospheric carbon dioxide variations at Mauna Loa Observatory, Hawaii. Tellus 28(6):538-51.

Keeling, C.D., R.B. Bacastow, & T.P. Whorf. (1982). Measurements of the concentration of carbon dioxide at Mauna Loa Observatory, Hawaii. In W.C. Clark (ed.), Carbon Dioxide Review: 1982. Oxford University Press, New York.

Keeling, C.D., R.B. Bacastow, A.F. Carter, S.C. Piper, T.P. Whorf, M. Heimann, W.G. Mook, & H. Roeloffzen. (1989). A three-dimensional model of atmospheric CO2 transport based on observed winds: 1. Analysis of observational data. In D.H. Peterson (ed.), Aspects of Climate Variability in the Pacific and the Western Americas. Geophysical Monograph 55:165-235.

Keeling, C.D., J.F.S. Chin, & T.P. Whorf. (1996). Increased activity of northern vegetation inferred from atmospheric CO2 measurements. Nature 382: (6587) 146-49. MacMillan Magazines Ltd., London.

Keeling, C.D., T.P. Whorf, M. Wahlen, & J. van der Plicht. (1995). Interannual extremes in the rate of rise of atmospheric carbon dioxide since 1980. Nature 375:666-670.

Keeling, C.D., P.R. Guenther, G. Emanuele III, A. Bollenbacher, & D.J. Moss. (2002). Scripps Reference Gas Calibration System for Carbon Dioxide-in-Nitrogen and Carbon Dioxide-in-Air Standards: Revision of 1999 (with Addendum). SIO Reference Series No. 01-11.

Khalil, H. K. Nonlinear Systems, 3rd. ed., Prentice Hall.

Kwiatkowski, D., P. C. B. Phillips, P. Schmidt, & Y. Shin. (1992). Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root., Journal of Econometrics, 54, 159-178.

Lund, R., & J. Reeves, (2002). Detection of undocumented changepoints: a revision of the two-phase regression model. J. Climate, 15, 2547-2554.

Marland, G., T.A. Boden, & R. J. Andres (2003). Global, Regional, and National CO2 Emissions. In Trends: A Compendium of Data on Global Change. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, TN, USA.

Menne, M.J., & C.N. Williams, Jr., (2005). Detection of undocumented changepoints using multiple test statistics and composite reference series. J. Climate, 18, 4271-4286.

Muth, J. (1960). Forecasting sales by exponentially weighted moving averages. Journal American Statistical Assoc., 55, 299.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach, Econometrica 59: 347-370.

Nise, N. S. (2004). Control Systems Engineering, 4th ed., John Wiley & Sons, Inc.

Pales, J.C., & C.D. Keeling. (1965). The concentration of atmospheric carbon dioxide in Hawaii. Journal of Geophysical Research 24:6053-76.

Peterson, T.C., & D.R. Easterling, (1994). Creation of homogeneous composite climatological reference series, Int. J. Climatol., 14, 671-680.

Quayle, R.G., D.R. Easterling, T.R. Karl, & P.Y. Hughes, (1991). Effects of recent thermometer changes in the cooperative station network, Bull. Am. Meteorol. Soc., 72, 1718-1724.

Quinlan, F. T., T. R. Karl, & C. N. Williams, Jr. (1987). United States Historical Climatology Network (HCN) Serial Temperature and Precipitation Data. NDP-019. Carbon Dioxide Information Analysis Center. Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tennessee.

Rogers, A. J., (1986). “Modified Lagrange Multiplier Tests for Problems with One-Sided Alternatives”, Journal of Econometrics, North-Holland., 31, 341-361

Said, S. E., & D. A. Dickey, (1984). “Testing for Unit Roots in Autoregressive-Moving Average Models of Unknown Order”., Biometrika, 71, 599-607.

Sakamoto, Y., M. Ishiguro, & G. Kitagawa, (1986). Akaike Information Criterion Statistics., D. Reidel Publishing Company.

Shih, S. H., & Tsokos, C. P., (2008). A Weighted Moving Average Process for Forecasting, To Appear, Journal of Modern Applied Statistical Methods.

Shih, S. H., & Koutras, D., (2007). Analytical Model for Economic Forecasting, To Appear, Proceedings of The 5th International Conference on Dynamic Systems and Applications.

Shih, S. H., & Tsokos, C. P., (2007). New Nonstationary Time Series Models with Economic Applications, To Appear, Proceedings of The 5th International Conference on Dynamic Systems and Applications.

Shih, S. H., & Tsokos, C. P., (2008). A Temperature Forecasting Model for the Continental United States, To Appear, The International Journal Neural, Parallel & Scientific Computations, Volume 16.

Shih, S. H., & Tsokos, C. P., (2008). Prediction Models for Carbon Dioxide Emissions and the Atmosphere, To Appear, The International Journal Neural, Parallel & Scientific Computations, Volume 16.

Shumway, R. H., & D. S. Stoffer, (2006). Time Series Analysis and Its Applications: with R Examples, 2nd ed., Springer, New York.

Slutzky, E. (1927). The summation of random causes as the source of cyclical processes, Econometria, 5, 105-146 (1937). Translated from the earlier paper of the same title in Problems of Economic Conditions (Ed. Conjuncture Institute Moscow).

Sontag, E. D. (1999). Mathematical Control Theory: Deterministic Finite Dimensional Systems. Second Edition. Springer.

Trigg, D. W. and A. G. Leach, (1967). Exponential smoothing with an adaptive response rate. Operations Research Quarterly, Vol. 18, No. 1.

Tsokos, C. P., (1973). "Forecasting Models from Non-Stationary Time Series-Short Term Predictability of Stocks"., Mathematical Methods in Investment and Finance., North Holland Publishing Co., 520-63.

Verhulst, J. (April 22, 2007). "Feeling The Heat". St. Petersburg Times. St. Petersburg, Florida.

Vose, R.S., C.N. Williams, T.C. Peterson, T.R. Karl, & D.R. Easterling, (2003). An evaluation of the time of observation bias adjustment in the US Historical Climatology Network. Geophysical research letters, 30 (20), 2046, clim3-1--3-4 doi:10.1029/2003GL018111.

Wang, X.L., (2003). Comments on "Detection of undocumented changepoints: A revision of the two-phase model". J. Climate, 16, 3383-3385.

Whorf, T.P., & C.D. Keeling. (1998). Rising carbon. New Scientist 157:(2124) 54-54. New Scientist Publ Expediting Inc., Elmont.

Wei, W. W. S., (2006). Time Series Analysis: Univariate and Multivariate Methods, 2nd ed., Pearson Education, Inc.

Woldm H. O. A. (1938). A study in the analysis of stationary time series, Almqvist and Wiksell, Uppsala.

Appendices

Appendix A1

Daily Closing Price of Stock XYZ

[1] 24.96 24.53 24.14 24.63 24.07 24.19 23.61 23.58 23.70 23.54 23.07 23.44 23.27 23.56 22.95 22.80 23.39 24.67 24.86 25.93 26.04 26.32 26.25 26.57 25.29

[26] 25.49 25.25 25.04 24.67 24.21 24.35 25.72 25.73 25.09 25.17 25.58 24.57 24.59 24.20 24.76 24.60 24.92 25.50 25.21 25.76 25.72 25.49 25.22 25.74 25.80

[51] 25.57 25.72 26.10 25.86 26.37 25.99 25.74 26.36 26.20 25.99 25.62 25.79 25.57 24.76 24.63 24.03 24.16 24.22 24.79 24.41 24.40 24.61 24.64 24.88 24.87

[76] 24.09 23.67 23.75 24.68 24.88 24.99 24.65 25.39 25.96 26.07 26.07 26.33 25.78 25.70 25.26 25.75 25.63 25.64 26.15 26.88 26.50 27.42 27.70 27.47 26.91

[101] 27.31 27.40 27.27 27.52 26.69 26.89 26.04 26.38 26.45 26.00 26.89 26.61 26.47 26.23 26.41 26.17 26.18 25.66 25.65 25.71 25.58 25.61 25.73 25.60 25.63

[126] 25.54 25.70 26.62 26.45 26.24 26.22 26.50 26.57 26.42 26.51 26.52 27.26 28.30 28.43 28.38 28.84 28.37 27.55 27.84 28.34 28.36 28.90 28.50 29.50 29.96

[151] 29.07 29.60 28.46 28.24 28.19 28.83 27.80 28.52 28.50 29.08 29.19 29.14 28.82 28.94 28.91 28.78 28.68 29.07 29.23 28.93 29.35 29.35 28.89 28.91 26.61

[176] 26.91 27.20 26.74 26.12 26.14 26.68 26.07 26.10 26.23 26.10 26.00 25.80 25.98 25.69 25.50 25.15 25.15 25.35 25.10 25.11 25.73 25.40 25.45 25.71 25.84

[201] 25.66 25.67 26.20 25.98 26.24 26.38 26.59 26.61 26.65 26.74 27.06 27.04 27.40 27.36 27.18 27.15 27.04 27.21 27.46 27.52 27.37 27.45 28.14 28.24 28.21

[226] 28.16 27.66 27.57 27.43 27.70 27.54 27.81 28.10 28.30 28.01 28.48 28.80 28.25 27.71 27.91 27.65 27.40 27.29 27.01 26.96 27.08 26.90 27.02 27.15 26.95

[251] 26.59 26.99 26.77 26.46 26.57 26.61 26.88 26.70 26.50 26.53 26.70 26.39 26.37 26.37 26.35 25.83 25.72 25.37 25.09 25.38 25.16 25.18 25.13 24.89 24.63

[276] 24.50 24.15 24.41 25.19 25.03 25.31 25.20 24.93 25.08 25.85 25.95 25.80 25.59 25.48 25.61 25.45 25.51 25.22 25.16 25.53 25.33 25.45 25.95 27.54 27.24

[301] 27.22 26.56 26.48 26.13 26.35 26.33 26.30 26.12 25.78 25.93 25.94 25.94 26.10 25.86 25.54 25.83 25.62 25.73 25.89 25.76 26.10 26.14 26.19 26.23 26.11

[326] 26.13 25.89 25.95 26.43 26.60 26.47 26.77 26.90 27.41 27.32 27.77 28.35 28.35 28.29 28.30 28.39 28.57 28.28 28.50 28.56 28.63 28.57 28.02 28.10 27.64

[351] 27.86 27.89 27.60 28.13 27.87 27.48 27.95 28.32 28.86 29.00 28.03 28.66 28.44 28.58 28.48 28.49 28.52 28.07 28.06 27.53 27.14 27.18 27.72 27.41 26.88

Appendix A1: (Continued)

[376] 27.02 27.09 27.05 27.46 27.12 27.20 27.24 27.24 27.55 27.44 27.46 27.30 27.30 27.39 27.62 27.11 27.36 27.26 27.28 27.49 27.25 27.44 27.19 27.26 27.51

[401] 27.51 27.26 27.12 27.35 27.29 27.19 27.27 27.58 27.65 28.25 28.12 28.38 28.53 28.17 27.99 28.06 28.03 28.03 27.80 27.99 28.41 28.18 28.70 28.56 27.74

[426] 27.63 27.90 28.15 28.01 27.97 28.08 28.24 28.47 29.00 29.31 29.28 29.77 29.73 29.98 29.97 27.39 27.12 27.17 27.07 26.86 26.65 26.53 26.64 26.60 26.77

[451] 26.81 27.25 27.09 27.23 27.33 27.07 27.36 27.23 27.08 27.25 27.23 27.11 27.16 26.96 26.95 27.07 26.97 27.01 26.85 26.95 26.90 26.76 26.72 26.74 26.84

[476] 26.78 26.75 26.67 26.80 26.73 26.78 26.27 26.12 26.32 25.98 25.86 25.65 25.67 26.02 26.01 26.11 26.18 26.28 26.39 26.46 26.18 26.32 26.16 26.24 26.07

Appendix A2: MA3 Series on Daily Closing Price of Stock XYZ

[1]	na	na	24.54	24.43	24.28	24.30	23.96	23.79	23.63	23.61	23.44	23.35	23.26	23.42	23.26	23.10	23.05	23.62	24.31	25.15	25.61	26.10	26.20	26.38	26.04
[26]	25.78	25.34	25.26	24.99	24.64	24.41	24.76	25.27	25.51	25.33	25.28	25.11	24.91	24.45	24.52	24.52	24.76	25.01	25.21	25.49	25.56	25.66	25.48	25.48	25.59
[51]	25.70	25.70	25.80	25.89	26.11	26.07	26.03	26.03	26.10	26.18	25.94	25.80	25.66	25.37	24.99	24.47	24.27	24.14	24.39	24.47	24.53	24.47	24.55	24.71	24.80
[76]	24.61	24.21	23.84	24.03	24.44	24.85	24.84	25.01	25.33	25.81	26.03	26.16	26.06	25.94	25.58	25.57	25.55	25.67	25.81	26.22	26.51	26.93	27.21	27.53	27.36
[101]	27.23	27.21	27.33	27.40	27.16	27.03	26.54	26.44	26.29	26.28	26.45	26.50	26.66	26.44	26.37	26.27	26.25	26.00	25.83	25.67	25.65	25.63	25.64	25.65	25.65
[126]	25.59	25.62	25.95	26.26	26.44	26.30	26.32	26.43	26.50	26.50	26.48	26.76	27.36	28.00	28.37	28.55	28.53	28.25	27.92	27.91	28.18	28.53	28.59	28.97	29.32
[151]	29.51	29.54	29.04	28.77	28.30	28.42	28.27	28.38	28.27	28.70	28.92	29.14	29.05	28.97	28.89	28.88	28.79	28.84	28.99	29.08	29.17	29.21	29.20	29.05	28.14
[176]	27.48	26.91	26.95	26.69	26.33	26.31	26.30	26.28	26.13	26.14	26.11	25.97	25.93	25.82	25.72	25.45	25.27	25.22	25.20	25.19	25.31	25.41	25.53	25.52	25.67
[201]	25.74	25.72	25.84	25.95	26.14	26.20	26.40	26.53	26.62	26.67	26.82	26.95	27.17	27.27	27.31	27.23	27.12	27.13	27.24	27.40	27.45	27.45	27.65	27.94	28.20
[226]	28.20	28.01	27.80	27.55	27.57	27.56	27.68	27.82	28.07	28.14	28.26	28.43	28.51	28.25	27.96	27.76	27.65	27.45	27.23	27.09	27.02	26.98	27.00	27.02	27.04
[251]	26.90	26.84	26.78	26.74	26.60	26.55	26.69	26.73	26.69	26.58	26.58	26.54	26.49	26.38	26.36	26.18	25.97	25.64	25.39	25.28	25.21	25.24	25.16	25.07	24.88
[276]	24.67	24.43	24.35	24.58	24.88	25.18	25.18	25.15	25.07	25.29	25.63	25.87	25.78	25.62	25.56	25.51	25.52	25.39	25.30	25.30	25.34	25.44	25.58	26.31	26.91
[301]	27.33	27.01	26.75	26.39	26.32	26.27	26.33	26.25	26.07	25.94	25.88	25.94	25.99	25.97	25.83	25.74	25.66	25.73	25.75	25.79	25.92	26.00	26.14	26.19	26.18
[326]	26.16	26.04	25.99	26.09	26.33	26.50	26.61	26.71	27.03	27.21	27.50	27.81	28.16	28.33	28.31	28.33	28.42	28.41	28.45	28.45	28.56	28.59	28.41	28.23	27.92
[351]	27.87	27.80	27.78	27.87	27.87	27.83	27.77	27.92	28.38	28.73	28.63	28.56	28.38	28.56	28.50	28.52	28.50	28.36	28.22	27.89	27.58	27.28	27.35	27.44	27.34
[376]	27.10	27.00	27.05	27.20	27.21	27.26	27.19	27.23	27.34	27.41	27.48	27.40	27.35	27.33	27.44	27.37	27.36	27.24	27.30	27.34	27.34	27.39	27.29	27.30	27.32

Appendix A2: (Continued)

[401] 27.43 27.43 27.30 27.24 27.25 27.28 27.25 27.35 27.50 27.83 28.01 28.25 28.34 28.36 28.23 28.07 28.03 28.04 27.95 27.94 28.07 28.19 28.43 28.48 28.33

[426] 27.98 27.76 27.89 28.02 28.04 28.02 28.10 28.26 28.57 28.93 29.20 29.45 29.59 29.83 29.89 29.11 28.16 27.23 27.12 27.03 26.86 26.68 26.61 26.59 26.67

[451] 26.73 26.94 27.05 27.19 27.22 27.21 27.25 27.22 27.22 27.19 27.19 27.20 27.17 27.08 27.02 26.99 27.00 27.02 26.94 26.94 26.90 26.87 26.79 26.74 26.77

[476] 26.79 26.79 26.73 26.74 26.73 26.77 26.59 26.39 26.24 26.14 26.05 25.83 25.73 25.78 25.90 26.05 26.10 26.19 26.28 26.38 26.34 26.32 26.22 26.24 26.16

Appendix A3: WMA3 Series on Daily Closing Price of Stock XYZ

[1] na na 24.41 24.45 24.27 24.22 23.88 23.69 23.64 23.60 23.33 23.33 23.29 23.44 23.21 22.98 23.12 23.93 24.55 25.36 25.81 26.16 26.24 26.42 25.88

[26] 25.60 25.34 25.18 24.89 24.50 24.36 25.01 25.50 25.41 25.24 25.36 25.01 24.75 24.39 24.54 24.59 24.79 25.16 25.26 25.53 25.65 25.61 25.39 25.52 25.68

[51] 25.68 25.68 25.89 25.92 26.16 26.09 25.93 26.09 26.18 26.12 25.84 25.77 25.65 25.20 24.83 24.35 24.20 24.17 24.50 24.50 24.47 24.51 24.59 24.75 24.83

[76] 24.48 24.01 23.78 24.20 24.62 24.90 24.80 25.08 25.55 25.92 26.05 26.20 26.01 25.83 25.49 25.58 25.61 25.66 25.89 26.43 26.57 27.02 27.41 27.54 27.23

[101] 27.20 27.29 27.32 27.42 27.06 26.93 26.43 26.35 26.36 26.21 26.52 26.60 26.59 26.37 26.36 26.26 26.21 25.92 25.74 25.68 25.63 25.62 25.66 25.65 25.64

[126] 25.58 25.63 26.13 26.38 26.37 26.26 26.36 26.49 26.48 26.49 26.50 26.89 27.66 28.19 28.38 28.62 28.53 28.04 27.83 28.04 28.27 28.63 28.61 29.07 29.56

[151] 29.44 29.48 28.94 28.54 28.25 28.52 28.21 28.33 28.39 28.79 29.04 29.15 28.99 28.93 28.91 28.85 28.75 28.89 29.09 29.05 29.19 29.28 29.12 28.98 27.76

[176] 27.14 27.00 26.92 26.51 26.23 26.41 26.29 26.19 26.16 26.14 26.07 25.92 25.92 25.80 25.64 25.36 25.21 25.25 25.19 25.15 25.42 25.46 25.48 25.57 25.73

[201] 25.73 25.70 25.93 26.00 26.15 26.27 26.46 26.56 26.63 26.69 26.88 27.00 27.22 27.32 27.28 27.19 27.10 27.14 27.31 27.45 27.44 27.43 27.78 28.07 28.21

[226] 28.19 27.92 27.70 27.52 27.59 27.57 27.70 27.91 28.15 28.12 28.29 28.56 28.47 28.07 27.90 27.75 27.57 27.39 27.17 27.03 27.03 26.97 26.99 27.07 27.03

[251] 26.80 26.85 26.81 26.65 26.57 26.57 26.74 26.74 26.63 26.55 26.61 26.52 26.43 26.37 26.36 26.09 25.86 25.56 25.29 25.28 25.22 25.21 25.15 25.02 24.80

[276] 24.61 24.35 24.34 24.76 24.98 25.20 25.21 25.08 25.05 25.44 25.77 25.86 25.72 25.57 25.56 25.51 25.51 25.36 25.24 25.36 25.37 25.42 25.68 26.66 27.12

[301] 27.28 26.89 26.63 26.32 26.30 26.30 26.32 26.21 25.98 25.91 25.91 25.94 26.02 25.95 25.74 25.74 25.68 25.71 25.79 25.80 25.95 26.06 26.16 26.20 26.16

[326] 26.14 26.01 25.96 26.18 26.43 26.51 26.64 26.78 27.13 27.28 27.56 27.99 28.25 28.32 28.30 28.34 28.46 28.39 28.44 28.49 28.58 28.59 28.30 28.15 27.86

[351] 27.83 27.84 27.74 27.91 27.91 27.72 27.78 28.06 28.53 28.84 28.49 28.51 28.45 28.55 28.51 28.50 28.50 28.29 28.14 27.80 27.42 27.23 27.44 27.48 27.20

[376] 27.04 27.03 27.06 27.26 27.22 27.22 27.21 27.23 27.39 27.44 27.47 27.38 27.33 27.35 27.49 27.33 27.32 27.27 27.29 27.38 27.34 27.39 27.28 27.27 27.37

Appendix A3: (Continued)

[401] 27.47 27.39 27.23 27.26 27.28 27.25 27.25 27.41 27.56 27.94 28.09 28.27 28.41 28.33 28.14 28.05 28.03 28.04 27.91 27.93 28.17 28.22 28.48 28.54 28.17

[426] 27.82 27.78 27.98 28.04 28.01 28.03 28.14 28.33 28.70 29.07 29.24 29.53 29.67 29.86 29.93 28.68 27.69 27.19 27.11 26.98 26.79 26.62 26.61 26.60 26.69

[451] 26.76 27.02 27.10 27.19 27.26 27.18 27.26 27.25 27.18 27.19 27.21 27.17 27.16 27.05 26.99 27.01 27.00 27.01 26.92 26.93 26.91 26.84 26.76 26.74 26.79

[476] 26.79 26.77 26.72 26.75 26.74 26.77 26.52 26.28 26.25 26.12 25.98 25.77 25.70 25.84 25.96 26.06 26.13 26.22 26.32 26.41 26.31 26.30 26.22 26.23 26.14

Appendix A4: EWMA3 Series on Daily Closing Price of Stock XYZ

```
[1]   na    na   24.37 24.48 24.24 24.22 23.84 23.68 23.65 23.59 23.29 23.35 23.29 23.46 23.17 22.95 23.16 24.04 24.60 25.44 25.84 26.18 26.24 26.44 25.79
[26] 25.59 25.32 25.16 24.86 24.46 24.36 25.11 25.53 25.36 25.23 25.39 24.94 24.73 24.36 24.58 24.59 24.81 25.21 25.25 25.57 25.66 25.59 25.37 25.56 25.70
[51] 25.66 25.69 25.92 25.91 26.19 26.08 25.90 26.13 26.18 26.10 25.81 25.77 25.64 25.14 24.80 24.31 24.19 24.18 24.54 24.49 24.46 24.52 24.60 24.77 24.84
[76] 24.43 23.96 23.78 24.27 24.66 24.91 24.78 25.12 25.61 25.94 26.05 26.22 25.98 25.81 25.46 25.60 25.61 25.65 25.93 26.49 26.56 27.08 27.45 27.53 27.18
[101] 27.22 27.30 27.31 27.43 27.01 26.92 26.38 26.36 26.37 26.18 26.57 26.60 26.57 26.35 26.37 26.25 26.21 25.88 25.73 25.69 25.63 25.62 25.67 25.64 25.64
[126] 25.57 25.64 26.20 26.39 26.35 26.26 26.38 26.50 26.47 26.49 26.50 26.94 27.75 28.23 28.38 28.65 28.51 27.97 27.83 28.08 28.28 28.67 28.59 29.13 29.62
[151] 29.39 29.50 28.87 28.50 28.24 28.56 28.15 28.36 28.41 28.83 29.06 29.15 28.96 28.93 28.91 28.84 28.74 28.92 29.11 29.04 29.21 29.29 29.09 28.97 27.59
[176] 27.11 27.03 26.90 26.45 26.22 26.45 26.25 26.17 26.17 26.14 26.06 25.90 25.93 25.79 25.62 25.33 25.20 25.26 25.18 25.14 25.46 25.45 25.48 25.59 25.75
[201] 25.72 25.69 25.97 26.00 26.16 26.28 26.48 26.57 26.63 26.70 26.91 27.00 27.25 27.33 27.26 27.19 27.09 27.15 27.33 27.46 27.43 27.44 27.83 28.10 28.21
[226] 28.19 27.88 27.68 27.50 27.60 27.57 27.72 27.94 28.17 28.11 28.32 28.60 28.44 28.02 27.90 27.73 27.54 27.37 27.15 27.02 27.04 26.96 26.99 27.08 27.02
[251] 26.77 26.87 26.81 26.62 26.57 26.58 26.76 26.74 26.61 26.55 26.62 26.50 26.42 26.37 26.36 26.06 25.84 25.54 25.26 25.30 25.21 25.20 25.15 25.00 24.78
[276] 24.59 24.32 24.35 24.82 24.99 25.21 25.21 25.06 25.05 25.50 25.80 25.85 25.70 25.56 25.57 25.50 25.51 25.34 25.23 25.38 25.36 25.43 25.72 26.79 27.14
[301] 27.27 26.85 26.61 26.29 26.31 26.31 26.32 26.20 25.95 25.91 25.91 25.94 26.03 25.94 25.71 25.75 25.67 25.71 25.81 25.79 25.97 26.07 26.16 26.21 26.16
[326] 26.14 25.99 25.96 26.22 26.46 26.50 26.66 26.80 27.17 27.29 27.59 28.04 28.27 28.32 28.30 28.35 28.48 28.38 28.45 28.50 28.59 28.59 28.26 28.14 27.83
[351] 27.83 27.85 27.72 27.94 27.91 27.68 27.80 28.09 28.58 28.86 28.43 28.53 28.44 28.55 28.50 28.50 28.51 28.26 28.13 27.76 27.38 27.22 27.48 27.47 27.15
[376] 27.04 27.04 27.06 27.29 27.21 27.21 27.21 27.23 27.42 27.44 27.47 27.37 27.32 27.35 27.51 27.30 27.33 27.27 27.29 27.40 27.32 27.39 27.27 27.27 27.39
```

Appendix A4: (Continued)

[401] 27.47 27.37 27.22 27.27 27.28 27.24 27.25 27.44 27.58 27.98 28.09 28.29 28.43 28.30 28.12 28.06 28.03 28.03 27.90 27.94 28.20 28.22 28.51 28.55 28.11

[426] 27.79 27.80 28.00 28.03 28.01 28.04 28.16 28.35 28.74 29.10 29.25 29.56 29.68 29.88 29.94 28.50 27.60 27.19 27.11 26.96 26.77 26.61 26.61 26.60 26.70

[451] 26.77 27.06 27.10 27.19 27.27 27.17 27.27 27.24 27.16 27.20 27.21 27.16 27.16 27.04 26.98 27.02 27.00 27.01 26.91 26.93 26.91 26.83 26.76 26.74 26.79

[476] 26.79 26.77 26.71 26.76 26.74 26.77 26.48 26.26 26.26 26.10 25.96 25.76 25.69 25.87 25.96 26.07 26.14 26.23 26.33 26.41 26.29 26.30 26.21 26.23 26.13

Appendix B1: Daily Closing Price of S&P Price Index

[1] 1210.08 1210.47 1222.12 1225.31 1219.43 1207.01 1209.25 1200.08 1206.83 1197.75 1188.07 1190.21 1189.65 1183.78 1171.71 1172.53 1171.42

[18] 1174.28 1165.36 1181.41 1180.59 1172.92 1176.12 1181.39 1184.07 1191.14 1181.20 1181.21 1187.76 1173.79 1162.05 1142.62 1145.98 1152.78

[35] 1137.50 1159.95 1152.12 1162.10 1151.83 1156.38 1143.22 1156.85 1162.16 1161.17 1175.65 1172.63 1171.35 1178.84 1166.22 1171.11 1159.36

[52] 1154.05 1165.69 1173.80 1185.56 1191.08 1189.28 1193.86 1194.07 1190.01 1197.62 1198.78 1191.50 1202.22 1204.29 1196.02 1197.51 1197.26

[69] 1194.67 1200.93 1198.11 1200.82 1203.91 1206.58 1210.96 1216.96 1216.10 1213.61 1213.88 1200.73 1191.57 1190.69 1201.57 1199.85 1191.33

[86] 1194.44 1204.99 1194.94 1197.87 1211.86 1219.44 1222.21 1223.29 1226.50 1227.92 1221.13 1229.35 1235.20 1227.04 1233.68 1229.03 1231.16

[103] 1236.79 1243.72 1234.18 1235.35 1244.12 1245.04 1235.86 1226.42 1223.13 1231.38 1229.13 1237.81 1230.39 1233.87 1219.34 1220.24 1219.02

[120] 1219.71 1221.73 1217.59 1209.59 1212.37 1205.10 1212.28 1208.41 1220.33 1221.59 1218.02 1233.39 1236.36 1231.67 1241.48 1240.56 1231.20

[137] 1227.16 1227.73 1237.91 1231.02 1221.34 1210.20 1214.62 1215.29 1215.63 1215.66 1216.89 1227.68 1228.81 1226.70 1214.47 1196.39 1191.49

[154] 1195.90 1187.33 1184.87 1177.68 1176.84 1186.57 1190.10 1178.14 1195.76 1177.80 1179.59 1199.38 1196.54 1191.38 1178.90 1198.41 1207.01

[171] 1202.76 1214.76 1219.94 1220.14 1222.81 1218.59 1220.65 1230.96 1234.72 1233.76 1229.01 1231.21 1242.80 1248.27 1254.85 1261.23 1265.61

[188] 1268.25 1257.46 1257.48 1249.48 1264.67 1265.08 1262.09 1263.70 1257.37 1255.84 1259.37 1260.43 1267.43 1272.74 1270.94 1267.32 1259.92

[205] 1259.62 1262.79 1268.12 1268.66 1256.54 1258.17 1254.42 1248.29 1268.80 1273.46 1273.48 1285.45 1290.15 1289.69 1294.18 1286.06 1287.61

[222] 1283.03 1277.93 1285.04 1261.49 1263.82 1266.86 1264.68 1273.83 1283.72 1285.19 1280.08 1282.46 1270.84 1264.03 1265.02 1254.78 1265.65

[239] 1263.78 1266.99 1262.86 1275.53 1280.00 1289.38 1287.24 1283.03 1292.67 1287.79 1289.43 1294.12 1280.66 1291.24 1289.14 1287.23 1278.26

[256] 1275.88 1278.47 1272.23 1281.42 1284.13 1297.48 1303.02 1305.33 1307.25 1305.08 1297.23 1305.04 1301.67 1302.95 1301.61 1293.23 1302.89

Appendix B1: (Continued)

[273] 1300.25 1294.87 1297.81 1305.93 1311.56 1309.04 1295.50 1296.62 1286.57 1288.12 1289.12 1285.33 1307.28 1309.93 1311.46 1311.28 1308.11

[290] 1301.74 1305.41 1309.72 1310.61 1305.19 1313.21 1308.12 1312.25 1325.76 1324.66 1325.14 1322.85 1305.92 1291.24 1294.50 1292.08 1270.32

[307] 1261.81 1267.03 1262.07 1256.58 1258.57 1272.88 1280.16 1259.87 1270.09 1285.71 1288.22 1265.29 1263.85 1256.15 1257.93 1252.30 1237.44

[324] 1223.69 1230.04 1256.16 1251.54 1240.13 1240.12 1252.20 1245.60 1244.50 1250.56 1239.20 1246.00 1272.87 1270.20 1280.19 1270.91 1274.08

[341] 1265.48 1267.34 1272.43 1258.60 1242.28 1236.20 1234.49 1236.86 1259.81 1249.13 1240.29 1260.91 1268.88 1268.40 1263.20 1278.55 1276.66

[358] 1270.92 1277.41 1280.27 1279.36 1275.77 1271.48 1265.95 1271.81 1266.74 1268.21 1285.58 1295.43 1297.48 1302.30 1297.52 1298.82 1292.99

[375] 1296.06 1295.09 1301.78 1304.28 1305.37 1303.82 1311.01 1313.25 1300.26 1294.02 1298.92 1299.54 1313.00 1318.07 1316.28 1319.66 1321.18

[392] 1317.64 1325.18 1318.03 1314.78 1326.37 1336.35 1336.59 1338.88 1335.85 1331.32 1334.11 1350.20 1353.22 1349.59 1350.66 1353.42 1349.95

[409] 1362.83 1365.62 1369.06 1364.05 1365.80 1366.96 1368.60 1377.02 1377.38 1382.22 1389.08 1377.34 1377.93 1377.94 1367.81 1367.34 1364.30

[426] 1379.78 1382.84 1385.72 1378.33 1380.90 1384.42 1393.22 1396.57 1399.76 1401.20 1400.50 1402.81 1406.09 1400.95 1381.96 1386.72 1399.48

[443] 1400.63 1396.71 1409.12 1414.76 1412.90 1407.29 1409.84 1413.04 1411.56 1413.21 1425.49 1427.09 1422.48 1425.55 1423.53 1418.30 1410.76

[460] 1416.90 1426.84 1424.73 1418.30 1416.60 1418.34 1409.71 1412.84 1412.11 1414.85 1423.82 1430.73 1431.90 1430.62 1426.37 1430.50 1422.95

[477] 1427.99 1440.13 1423.90 1422.18 1420.62 1428.82 1438.24 1445.94 1448.39 1446.99 1448.00 1450.02 1448.31 1438.06 1433.37 1444.26 1455.30

[494] 1456.81 1455.54 1459.68 1457.63 1456.38 1451.19 1449.37

Appendix B2: MA3 Series on Daily Closing Price of S&P Price Index

```
  [1]       NA       NA 1214.223 1219.300 1222.287 1217.250 1211.897 1205.447 1205.387 1201.553 1197.550 1192.010 1189.310 1187.880 1181.713
 [16] 1176.007 1171.887 1172.743 1170.353 1173.683 1175.787 1178.307 1176.543 1176.810 1180.527 1185.533 1185.470 1184.517 1183.390 1180.920
 [31] 1174.533 1159.487 1150.217 1147.127 1145.420 1150.077 1149.857 1158.057 1155.350 1156.770 1150.477 1152.150 1154.077 1160.060 1166.327
 [46] 1169.817 1173.210 1174.273 1172.137 1172.057 1165.563 1161.507 1159.700 1164.513 1175.017 1183.480 1188.640 1191.407 1192.403 1192.647
 [61] 1193.900 1195.470 1195.967 1197.500 1199.337 1200.843 1199.273 1196.930 1196.480 1197.620 1197.903 1199.953 1200.947 1203.770 1207.150
 [76] 1211.500 1214.673 1215.557 1214.530 1209.407 1202.060 1194.330 1194.610 1197.370 1197.583 1195.207 1196.920 1198.123 1199.267 1201.557
 [91] 1209.723 1217.837 1221.647 1224.000 1225.903 1225.183 1226.133 1228.560 1230.530 1231.973 1229.917 1231.290 1232.327 1237.223 1238.230
[106] 1237.750 1237.883 1241.503 1241.673 1235.773 1228.470 1226.977 1227.880 1232.773 1232.443 1234.023 1227.867 1224.483 1219.533 1219.657
[121] 1220.153 1219.677 1216.303 1213.183 1209.020 1209.917 1208.597 1213.673 1216.777 1219.980 1224.333 1229.257 1233.807 1236.503 1237.903
[136] 1237.747 1232.973 1228.697 1230.933 1232.220 1230.090 1220.853 1215.387 1213.370 1215.180 1215.527 1216.060 1220.077 1224.460 1227.730
[151] 1223.327 1212.520 1200.783 1194.593 1191.573 1189.367 1183.293 1179.797 1180.363 1184.503 1184.937 1188.000 1183.900 1184.383 1185.590
[166] 1191.837 1195.767 1188.940 1189.563 1194.773 1202.727 1208.177 1212.487 1218.280 1220.963 1220.513 1220.683 1223.400 1228.777 1233.147
[181] 1232.497 1231.327 1234.340 1240.760 1248.640 1254.783 1260.563 1265.030 1263.773 1261.063 1254.807 1257.210 1259.743 1263.947 1263.623
[196] 1261.053 1258.970 1257.527 1258.547 1262.410 1266.867 1270.370 1270.333 1266.060 1262.287 1260.777 1263.510 1266.523 1264.440 1261.123
[211] 1256.377 1253.627 1257.170 1263.517 1271.913 1277.463 1283.027 1288.430 1291.340 1289.977 1289.283 1285.567 1282.857 1282.000 1274.820
[226] 1270.117 1264.057 1265.120 1268.457 1274.077 1280.913 1282.997 1282.577 1277.793 1272.443 1266.630 1261.277 1261.817 1261.403 1265.473
```

Appendix B2: (Continued)

[241] 1264.543 1268.460 1272.797 1281.637 1285.540 1286.550 1287.647 1287.830 1289.963 1290.447 1288.070 1288.673 1287.013 1289.203 1284.877

[256] 1280.457 1277.537 1275.527 1277.373 1279.260 1287.677 1294.877 1301.943 1305.200 1305.887 1303.187 1302.450 1301.313 1303.220 1302.077

[271] 1299.263 1299.243 1298.790 1299.337 1297.643 1299.537 1305.100 1308.843 1305.367 1300.387 1292.897 1290.437 1287.937 1287.523 1293.910

[286] 1300.847 1309.557 1310.890 1310.283 1307.043 1305.087 1305.623 1308.580 1308.507 1309.670 1308.840 1311.193 1315.377 1320.890 1325.187

[301] 1324.217 1317.970 1306.670 1297.220 1292.607 1285.633 1274.737 1266.387 1263.637 1261.893 1259.073 1262.677 1270.537 1270.970 1270.040

[316] 1271.890 1281.340 1279.740 1272.453 1261.763 1259.310 1255.460 1249.223 1237.810 1230.390 1236.630 1245.913 1249.277 1243.930 1244.150

[331] 1245.973 1247.433 1246.887 1244.753 1245.253 1252.690 1263.023 1274.420 1273.767 1275.060 1270.157 1268.967 1268.417 1266.123 1257.770

[346] 1245.693 1237.657 1235.850 1243.720 1248.600 1249.743 1250.110 1256.693 1266.063 1266.827 1270.050 1272.803 1275.377 1274.997 1276.200

[361] 1279.013 1278.467 1275.537 1271.067 1269.747 1268.167 1268.920 1273.510 1283.073 1292.830 1298.403 1299.100 1299.547 1296.443 1295.957

[376] 1294.713 1297.643 1300.383 1303.810 1304.490 1306.733 1309.360 1308.173 1302.510 1297.733 1297.493 1303.820 1310.203 1315.783 1318.003

[391] 1319.040 1319.493 1321.333 1320.283 1319.330 1319.727 1325.833 1333.103 1337.273 1337.107 1335.350 1333.760 1338.543 1345.843 1351.003

[406] 1351.157 1351.223 1351.343 1355.400 1359.467 1365.837 1366.243 1366.303 1365.603 1367.120 1370.860 1374.333 1378.873 1382.893 1382.880

[421] 1381.450 1377.737 1374.560 1371.030 1366.483 1370.473 1375.640 1382.780 1382.297 1381.650 1381.217 1386.180 1391.403 1396.517 1399.177

[436] 1400.487 1401.503 1403.133 1403.283 1396.333 1389.877 1389.387 1395.610 1398.940 1402.153 1406.863 1412.260 1411.650 1410.010 1410.057

[451] 1411.480 1412.603 1416.753 1421.930 1425.020 1425.040 1423.853 1422.460 1417.530 1415.320 1418.167 1422.823 1423.290 1419.877 1417.747

[466] 1414.883 1413.630 1411.553 1413.267 1416.927 1423.133 1428.817 1431.083 1429.630 1429.163 1426.607 1427.147 1430.357 1430.673 1428.737

[481] 1422.233 1423.873 1429.227 1437.667 1444.190 1447.107 1447.793 1448.337 1448.777 1445.463 1439.913 1438.563 1444.310 1452.123 1455.883

[496] 1457.343 1457.617 1457.897 1455.067 1452.313

Appendix B3: WMA3 Series on Daily Closing Price of S&P Price Index

```
  [1]       NA       NA 1216.230 1221.773 1221.838 1214.200 1210.200 1204.292 1204.983 1201.165 1194.423 1190.753 1189.573 1186.808 1178.723
 [16] 1174.132 1171.838 1173.035 1169.343 1174.872 1178.325 1176.892 1175.798 1178.222 1181.852 1187.158 1184.992 1182.862 1184.483 1179.683
 [31] 1170.248 1154.292 1147.538 1148.820 1144.007 1151.272 1152.293 1158.415 1155.302 1155.817 1149.042 1152.228 1157.233 1160.780 1168.575
 [46] 1171.727 1172.493 1175.308 1171.282 1170.768 1164.420 1158.663 1160.755 1167.805 1178.328 1186.360 1189.260 1191.870 1193.202 1192.005
 [61] 1194.492 1196.932 1194.947 1198.073 1201.468 1199.810 1198.143 1197.137 1196.007 1198.232 1198.477 1199.935 1201.913 1204.730 1208.325
 [76] 1213.230 1215.530 1214.998 1214.160 1207.260 1198.342 1192.657 1196.277 1198.897 1195.877 1194.305 1199.197 1198.207 1198.080 1204.377
 [91] 1213.318 1219.562 1222.288 1224.715 1226.675 1224.288 1226.372 1230.905 1230.145 1231.720 1230.248 1230.870 1233.620 1239.317 1237.795
[106] 1236.355 1239.540 1243.118 1240.297 1232.670 1226.348 1227.803 1228.880 1233.845 1232.653 1233.367 1226.025 1222.212 1219.480 1219.568
[121] 1220.605 1219.323 1214.280 1212.313 1208.272 1209.902 1209.148 1215.015 1218.973 1219.595 1226.300 1232.313 1233.520 1237.357 1239.385
[136] 1236.033 1230.740 1228.118 1232.725 1232.768 1227.328 1217.383 1214.267 1214.218 1215.348 1215.588 1216.270 1222.080 1226.447 1227.567
[151] 1220.937 1207.468 1196.953 1194.512 1190.880 1187.528 1181.685 1178.458 1181.845 1186.713 1183.532 1188.943 1183.843 1181.688 1189.187
[166] 1194.662 1194.433 1186.000 1190.735 1199.458 1203.452 1209.468 1215.350 1219.177 1221.442 1220.255 1220.323 1225.462 1231.122 1233.613
[181] 1231.545 1230.902 1236.638 1243.603 1250.648 1256.943 1262.357 1266.200 1262.415 1259.268 1253.477 1258.408 1262.343 1263.517 1263.393
[196] 1260.267 1257.660 1257.860 1259.312 1263.753 1268.918 1270.955 1269.430 1264.223 1261.003 1261.255 1264.927 1267.502 1262.510 1259.375
[211] 1256.023 1251.980 1259.567 1267.712 1272.693 1279.462 1285.805 1289.137 1292.012 1289.372 1288.188 1285.062 1281.243 1282.335 1272.080
[226] 1266.580 1264.952 1265.263 1269.618 1277.250 1282.807 1282.390 1282.122 1276.253 1269.372 1265.660 1259.735 1261.922 1262.903 1265.697
```

Appendix B3: (Continued)

[241] 1264.390 1269.883 1275.653 1283.945 1286.747 1285.492 1288.552 1288.623 1289.423 1291.502 1286.608 1288.193 1288.427 1288.535 1283.063

[256] 1278.565 1277.572 1274.918 1277.865 1281.243 1290.353 1298.025 1303.252 1305.905 1305.845 1301.517 1302.443 1302.053 1302.872 1302.067

[271] 1297.643 1299.457 1299.960 1298.000 1297.237 1301.380 1307.392 1309.362 1302.690 1298.317 1291.408 1289.020 1288.362 1287.058 1296.937

[286] 1304.947 1310.253 1311.115 1309.725 1305.453 1304.637 1306.953 1309.447 1307.752 1310.103 1309.328 1311.033 1318.317 1322.958 1325.083

[301] 1323.915 1314.767 1301.402 1295.317 1292.747 1281.603 1269.692 1265.838 1263.680 1260.152 1258.490 1265.393 1274.135 1268.802 1268.362

[316] 1276.197 1284.362 1276.337 1268.392 1260.240 1258.323 1254.818 1245.808 1233.042 1229.157 1242.042 1249.497 1246.605 1242.027 1246.162

[331] 1246.887 1246.150 1247.713 1243.870 1244.493 1258.302 1267.057 1275.640 1273.885 1274.042 1269.252 1267.843 1269.575 1264.667 1252.745

[346] 1241.960 1236.358 1235.960 1247.940 1250.645 1246.490 1252.073 1261.458 1267.312 1265.880 1271.742 1275.047 1274.105 1275.122 1277.758

[361] 1279.338 1277.717 1274.223 1269.430 1269.802 1268.298 1268.320 1276.650 1287.610 1294.813 1299.548 1299.107 1298.967 1295.688 1295.497

[376] 1295.063 1298.597 1301.915 1304.408 1304.413 1307.673 1310.932 1306.382 1299.305 1297.510 1298.413 1306.167 1313.292 1316.330 1318.268

[391] 1319.857 1319.157 1322.000 1320.348 1317.597 1321.117 1329.428 1334.807 1337.695 1336.983 1334.090 1333.470 1341.690 1349.028 1350.902

[406] 1350.730 1351.862 1351.225 1356.968 1362.078 1366.875 1365.982 1365.760 1366.088 1367.587 1372.537 1375.797 1379.740 1384.843 1382.067

[421] 1379.592 1377.837 1372.873 1369.263 1365.898 1372.547 1378.730 1383.770 1381.545 1380.847 1382.232 1388.233 1393.428 1397.607 1399.948

[436] 1400.610 1401.772 1404.065 1402.973 1392.312 1387.505 1392.307 1397.928 1398.478 1403.568 1409.872 1412.890 1410.405 1409.500 1411.015

[451] 1411.767 1412.632 1419.075 1424.243 1424.518 1424.783 1424.028 1421.252 1415.402 1415.087 1420.847 1424.128 1421.867 1418.522 1417.753

[466] 1413.735 1412.713 1411.953 1413.602 1418.878 1425.780 1430.163 1431.065 1428.708 1429.143 1426.037 1426.728 1433.220 1429.992 1425.745

[481] 1421.687 1424.980 1432.163 1440.520 1445.882 1447.282 1447.728 1448.842 1448.828 1443.470 1437.423 1439.597 1447.965 1454.215 1455.923

[496] 1457.822 1457.965 1457.347 1453.993 1451.145

Appendix B4: EWMA3 Series on Daily Closing Price of S&P Price Index

```
  [1]       NA       NA 1217.071 1222.279 1221.494 1213.173 1210.064 1203.690 1205.247 1200.677 1193.516 1190.676 1189.584 1186.376 1177.721
 [16] 1173.903 1171.779 1173.213 1168.774 1175.806 1178.649 1176.324 1175.844 1178.674 1182.169 1187.727 1184.450 1182.626 1184.951 1178.841
 [31] 1169.077 1152.624 1147.316 1149.386 1143.077 1152.511 1152.269 1158.941 1154.806 1155.897 1148.210 1152.889 1157.937 1160.836 1169.586
 [46] 1171.856 1172.330 1175.813 1170.559 1170.817 1163.697 1158.004 1161.460 1168.661 1179.361 1187.034 1189.263 1192.154 1193.326 1191.720
 [61] 1194.939 1197.196 1194.454 1198.666 1201.871 1199.269 1198.053 1197.154 1195.816 1198.617 1198.424 1200.061 1202.199 1204.994 1208.701
 [76] 1213.763 1215.611 1214.800 1214.120 1206.327 1197.374 1192.376 1197.033 1199.033 1195.227 1194.324 1200.024 1197.740 1198.050 1205.446
 [91] 1214.193 1219.940 1222.431 1224.970 1226.853 1223.837 1226.797 1231.519 1229.701 1232.000 1230.074 1230.911 1234.073 1239.946 1237.279
[106] 1236.211 1240.194 1243.393 1239.663 1231.777 1225.889 1228.314 1228.916 1234.411 1232.330 1233.439 1225.070 1221.930 1219.414 1219.589
[121] 1220.766 1219.076 1213.610 1212.321 1207.819 1210.241 1209.043 1215.774 1219.347 1219.370 1227.313 1232.891 1233.256 1237.946 1239.553
[136] 1235.343 1230.229 1228.063 1233.466 1232.519 1226.473 1216.357 1214.317 1214.371 1215.389 1215.599 1216.359 1222.880 1226.784 1227.443
[151] 1220.013 1205.886 1196.173 1194.710 1190.373 1187.149 1181.113 1178.227 1182.520 1187.197 1182.761 1189.917 1182.980 1181.389 1190.643
[166] 1194.930 1193.997 1184.986 1191.831 1200.537 1203.353 1210.224 1216.006 1219.314 1221.637 1220.017 1220.370 1226.247 1231.636 1233.634
[181] 1231.183 1230.946 1237.519 1244.270 1251.249 1257.556 1262.821 1266.493 1261.707 1259.013 1252.906 1259.303 1262.734 1263.313 1263.437
[196] 1259.853 1257.400 1258.076 1259.471 1264.279 1269.464 1270.953 1269.129 1263.609 1260.806 1261.474 1265.383 1267.667 1261.657 1259.203
[211] 1255.794 1251.453 1260.886 1268.533 1272.806 1280.317 1286.426 1289.216 1292.321 1288.899 1288.106 1284.771 1280.770 1282.721 1270.567
[226] 1266.186 1265.224 1265.180 1270.220 1278.174 1283.147 1282.060 1282.170 1275.480 1268.609 1265.569 1259.027 1262.454 1263.029 1265.881
```

Appendix B4: (Continued)

[241] 1264.171 1270.690 1276.274 1284.721 1286.817 1285.140 1289.140 1288.504 1289.424 1291.876 1285.759 1288.629 1288.529 1288.349 1282.377

[256] 1278.181 1277.700 1274.534 1278.373 1281.656 1291.371 1298.739 1303.549 1306.097 1305.736 1300.904 1302.814 1301.999 1302.883 1302.001

[271] 1297.013 1299.947 1300.001 1297.553 1297.319 1302.030 1307.987 1309.316 1301.663 1298.074 1290.717 1288.891 1288.470 1286.811 1298.414

[286] 1305.659 1310.426 1311.139 1309.494 1304.923 1304.747 1307.349 1309.613 1307.386 1310.547 1309.156 1311.207 1319.380 1323.201 1325.091

[301] 1323.763 1313.503 1299.950 1295.200 1292.651 1279.991 1268.566 1266.009 1263.450 1259.641 1258.501 1266.463 1274.996 1267.526 1268.609

[316] 1277.556 1284.913 1274.759 1267.743 1259.656 1258.267 1254.459 1244.613 1231.706 1229.283 1244.059 1249.789 1245.680 1241.754 1247.024

[331] 1246.703 1245.914 1248.120 1243.203 1244.709 1260.383 1267.506 1276.290 1273.460 1274.047 1268.713 1267.771 1269.983 1263.800 1251.250

[346] 1241.137 1236.091 1236.089 1249.636 1250.429 1245.604 1253.336 1262.519 1267.467 1265.497 1272.714 1275.277 1273.650 1275.449 1278.117

[361] 1279.341 1277.439 1273.831 1268.933 1270.089 1268.076 1268.304 1277.926 1288.727 1295.194 1299.941 1298.880 1298.946 1295.303 1295.577

[376] 1295.067 1299.051 1302.253 1304.546 1304.329 1308.150 1311.263 1305.507 1298.550 1297.711 1298.574 1307.143 1313.974 1316.323 1318.467

[391] 1320.046 1318.940 1322.454 1320.017 1317.194 1321.867 1330.417 1335.061 1337.864 1336.821 1333.694 1333.561 1342.906 1349.627 1350.714

[406] 1350.720 1352.084 1351.043 1357.806 1362.584 1367.187 1365.706 1365.766 1366.213 1367.731 1373.177 1376.023 1380.094 1385.449 1381.391

[421] 1379.354 1377.851 1372.150 1368.989 1365.670 1373.580 1379.317 1384.049 1381.086 1380.854 1382.544 1388.946 1393.877 1397.914 1400.127

[436] 1400.594 1401.920 1404.354 1402.684 1390.833 1387.393 1393.331 1398.314 1398.226 1404.361 1410.570 1412.891 1409.960 1409.549 1411.304

[451] 1411.737 1412.714 1419.991 1424.650 1424.227 1424.893 1423.957 1420.830 1414.739 1415.346 1421.703 1424.214 1421.357 1418.247 1417.837

[466] 1413.160 1412.731 1411.976 1413.780 1419.584 1426.487 1430.411 1431.001 1428.374 1429.337 1425.596 1426.909 1434.207 1429.121 1425.236

[481] 1421.534 1425.529 1433.031 1441.294 1446.240 1447.240 1447.767 1449.010 1448.754 1442.697 1436.844 1440.263 1449.013 1454.586 1455.869

[496] 1458.087 1457.917 1457.209 1453.593 1450.891

Appendix C1: Monthly Temperature for 1895-2007 (Version 1 Dataset)

[1] 27.88 27.63 41.14 53.89 61.00 68.68 72.08 72.38 66.75 51.23 40.29 33.09 32.60 36.07 38.97 53.51 63.38 70.30 74.24 73.06 63.60 52.55 39.79

[24] 36.31 29.11 34.56 40.20 52.06 61.69 68.73 74.00 71.82 67.21 56.05 41.91 31.65 31.62 36.15 42.32 51.56 60.36 69.79 73.87 73.39 66.07 52.32

[47] 39.09 29.54 30.83 26.43 38.74 51.67 60.71 69.15 73.40 72.49 65.27 55.13 45.89 32.65 35.02 31.63 42.09 52.40 62.27 70.60 73.60 73.63 65.91

[70] 57.34 42.51 35.28 32.90 30.77 41.67 50.19 61.54 69.08 76.84 73.73 63.82 56.16 42.52 32.33 30.83 32.47 42.85 51.81 62.73 68.62 72.66 72.00

[93] 63.22 55.08 44.70 31.73 31.85 29.47 43.34 51.05 60.22 66.43 72.44 71.56 63.24 54.59 40.85 31.28 28.20 32.60 42.60 50.19 60.73 67.84 71.86

[116] 71.20 65.46 54.64 43.91 32.86 27.82 27.60 46.24 51.37 59.86 68.76 72.44 72.91 66.48 52.28 43.08 31.90 33.89 34.42 37.00 53.55 60.23 68.00

[139] 72.82 72.56 66.78 53.05 41.45 35.91 31.85 36.24 46.72 48.15 56.53 66.41 73.09 71.40 64.62 54.42 41.69 35.57 33.25 33.93 44.80 53.46 59.35

[162] 67.55 73.67 71.21 65.99 52.31 43.24 34.15 32.88 35.90 41.28 49.70 58.64 69.51 73.18 73.53 64.80 53.40 45.73 26.42 30.41 30.21 50.39 54.45

[185] 59.78 68.85 74.59 71.63 66.16 56.25 41.93 32.73 33.77 34.59 45.10 50.87 61.87 71.37 73.18 71.47 66.58 53.31 38.02 33.03 25.59 32.00 36.75

[208] 51.52 61.22 67.35 72.87 70.77 63.23 53.88 43.09 33.95 31.32 30.45 39.74 52.33 60.87 68.94 73.66 74.07 64.24 52.33 46.02 34.22 35.56 30.66

[231] 41.91 51.92 61.94 70.08 74.69 72.32 65.11 55.95 44.42 27.72 29.43 37.02 38.10 55.53 58.89 66.78 71.58 70.71 65.00 56.15 44.07 33.94 29.02

[254] 33.90 43.31 51.18 59.99 67.02 74.93 72.48 64.17 52.87 41.10 29.76 29.23 30.93 39.46 49.54 55.88 67.68 75.20 71.58 64.78 51.11 44.47 30.35

[277] 24.76 34.80 46.74 49.88 61.53 71.75 72.97 73.16 62.39 56.87 41.25 35.18 32.65 33.12 41.82 52.12 60.68 70.00 74.89 72.78 66.14 53.30 40.07

[300] 29.49 29.95 34.45 41.25 47.59 59.97 68.03 73.31 71.27 65.85 55.26 40.43 33.78 35.10 37.82 47.53 51.74 60.71 71.00 75.23 72.60 67.02 55.95

[323] 43.35 35.80 27.65 32.79 41.39 51.15 61.83 70.91 73.54 73.38 67.79 55.65 42.65 34.24 35.42 30.41 39.31 50.77 60.07 68.73 74.37 71.70 65.56

[346] 51.91 43.54 36.86 27.06 36.03 38.16 51.10 58.55 69.22 72.45 72.71 63.26 55.66 43.27 28.21 29.84 39.20 45.17 55.50 60.66 70.53 74.73 72.05

Appendix C1: (Continued)

[369] 67.17 49.45 41.33 33.32 31.15 38.59 40.68 50.89 61.70 68.91 74.02 72.95 64.54 55.53 41.37 32.16 32.33 38.73 43.25 52.40 60.38 67.87 73.31

[392] 69.98 65.53 56.57 45.14 29.02 32.79 35.26 44.10 48.88 62.11 66.55 74.00 72.40 63.74 55.66 42.52 33.77 27.15 27.84 44.65 51.99 59.52 68.17

[415] 74.28 73.27 63.93 54.53 39.21 34.67 24.43 41.02 41.40 55.23 59.92 69.15 75.67 73.57 66.28 52.43 42.01 32.08 33.88 38.91 40.43 51.90 60.13

[438] 71.45 76.36 72.67 68.70 57.27 44.28 36.02 32.41 37.63 38.37 52.42 61.28 69.98 74.83 73.22 65.25 53.04 41.64 30.32 35.25 30.56 42.93 50.78

[461] 60.62 72.34 75.85 72.55 68.25 55.89 43.46 37.18 36.00 35.18 43.79 54.63 65.38 71.48 77.09 74.25 64.43 57.22 45.91 33.55 32.33 36.94 44.62

[484] 50.54 58.02 68.37 75.86 73.66 65.32 54.42 40.39 31.71 28.20 26.08 44.52 51.04 64.15 71.35 77.52 75.32 66.89 54.11 40.69 35.37 25.77 32.72

[507] 39.90 51.03 62.67 69.56 75.28 75.36 66.26 54.58 41.69 33.52 32.98 36.95 46.16 52.60 60.56 69.54 74.47 74.46 67.24 57.15 40.89 35.14 34.91

[530] 30.98 43.56 52.37 63.43 69.58 75.51 73.30 67.95 55.47 43.20 38.55 24.56 34.85 42.57 51.07 61.57 70.29 74.82 72.79 65.94 56.88 39.98 37.01

[553] 33.20 34.53 40.35 53.07 63.15 68.71 74.74 72.82 65.16 56.02 43.97 37.13 30.79 31.80 42.51 53.99 59.88 68.72 74.68 72.51 64.30 55.36 44.06

[576] 33.84 30.28 37.28 39.61 53.86 60.48 69.70 75.01 74.25 64.84 54.42 41.90 33.55 33.04 35.62 39.69 49.97 62.82 69.16 73.49 72.82 65.81 56.04

[599] 42.56 31.57 31.34 36.26 46.70 51.17 59.00 66.63 73.65 72.89 65.50 54.86 43.01 30.01 32.34 35.93 47.93 55.56 59.46 69.16 74.54 71.83 65.13

[622] 54.03 42.73 37.08 32.72 32.38 39.80 51.94 60.83 67.54 73.54 74.84 67.12 59.58 39.20 33.90 28.83 32.59 39.83 54.31 61.04 69.56 73.97 72.81

[645] 66.67 53.97 42.70 33.55 27.07 32.95 41.93 53.03 62.54 70.05 74.88 72.82 64.33 54.84 46.22 34.31 31.41 36.51 39.95 49.65 60.24 68.74 71.89

[668] 70.72 64.14 58.17 41.58 33.88 31.16 35.43 39.62 50.69 61.50 67.35 74.29 72.56 64.61 54.75 39.07 32.18 32.96 36.50 38.84 52.61 61.34 71.55

[691] 74.74 73.50 66.47 53.66 41.00 34.36 37.26 36.90 44.79 49.66 59.94 70.92 74.78 72.78 66.69 56.95 44.85 34.66 31.64 42.30 40.32 55.04 59.31

[714] 69.61 76.15 72.87 67.07 55.43 44.98 34.86 30.36 32.07 41.10 53.80 62.09 67.26 75.12 74.54 66.46 55.77 39.10 32.45 31.78 33.16 41.80 50.34

[737] 62.58 71.09 73.77 72.36 65.67 56.90 41.31 37.05 27.95 38.93 42.37 51.64 60.50 69.52 74.99 72.41 64.77 52.43 41.26 37.95 32.51 32.89 38.75

[760] 51.31 63.28 68.74 73.26 73.98 66.08 55.48 43.96 32.98 30.26 33.40 42.58 52.56 61.70 70.72 74.24 73.89 65.35 54.02 39.52 36.82 30.30 31.55

Appendix C1: (Continued)

[783] 36.84 53.54 60.10 69.84 74.52 72.86 66.98 55.49 43.62 31.24 30.51 38.38 44.56 49.57 59.68 70.26 73.89 73.37 64.06 54.57 41.45 31.86 27.75

[806] 35.86 38.95 52.77 63.13 68.70 72.75 72.78 64.76 56.83 44.03 34.17 25.14 35.03 44.84 52.88 61.83 69.49 74.34 72.69 66.94 60.03 44.83 28.50

[829] 32.34 32.76 39.89 52.26 62.29 68.78 75.35 71.37 64.31 54.43 43.19 32.76 32.43 33.35 36.49 52.85 61.68 67.65 73.33 71.74 62.38 55.61 45.40

[852] 36.51 27.11 32.17 44.00 50.93 61.02 68.76 75.78 71.50 65.15 53.53 43.96 33.41 33.81 33.81 44.66 52.03 58.50 68.38 73.22 71.92 64.68 54.60

[875] 42.30 32.93 29.55 33.81 44.66 51.63 59.05 69.30 73.60 71.36 64.50 55.31 41.93 30.74 29.86 33.67 37.13 53.62 62.27 67.65 74.81 73.87 66.16

[898] 51.74 42.44 34.17 27.83 36.23 39.59 50.52 61.97 69.59 74.69 74.07 65.21 52.99 42.11 34.35 30.07 34.33 40.76 51.33 59.27 70.34 73.16 72.78

[921] 65.00 55.82 42.41 34.76 30.56 34.37 45.07 51.82 61.44 69.11 72.91 72.46 65.10 52.98 39.74 30.46 30.27 34.52 45.44 49.86 60.16 69.80 74.03

[944] 73.29 64.99 56.74 43.58 34.36 31.61 35.63 45.44 52.60 61.31 69.29 74.87 70.99 62.90 54.55 43.11 34.33 32.95 33.52 39.76 47.88 61.34 68.34

[967] 74.30 72.26 63.51 55.29 43.34 34.77 30.49 40.35 43.39 52.93 59.97 68.64 73.67 71.46 64.85 50.70 39.16 31.52 23.66 36.86 44.32 55.12 62.64

[990] 71.17 75.21 72.51 66.51 54.57 43.54 33.81 26.36 28.69 42.22 52.62 60.64 69.81 74.59 72.51 66.52 55.05 42.13 30.46 22.50 28.74 43.16 51.17

[1013] 60.13 68.73 73.91 71.72 66.72 55.91 40.70 36.60 31.62 33.50 40.37 52.46 61.21 69.55 76.49 73.53 66.82 53.27 43.03 35.66 32.96 37.43 43.91

[1036] 56.19 59.92 70.67 74.66 72.78 65.50 52.95 45.41 34.14 26.60 33.19 43.01 49.71 61.80 67.14 73.96 72.80 64.65 53.89 41.86 36.41 33.59 37.68

[1059] 43.43 48.09 58.91 68.00 74.81 75.85 66.21 55.63 43.78 25.85 29.28 38.17 41.08 50.65 61.08 69.51 73.93 73.89 63.76 54.31 42.29 35.07 26.72

[1082] 31.56 44.53 55.38 63.03 68.95 74.81 71.71 63.38 54.94 40.13 29.05 35.11 36.59 47.17 53.91 62.08 71.34 74.33 72.39 64.44 54.62 42.01 34.93

[1105] 31.95 38.23 43.58 54.51 64.03 71.08 74.04 72.54 65.76 53.34 44.65 35.23 28.21 34.26 43.47 52.84 62.23 71.71 75.51 74.38 65.13 54.29 44.00

[1128] 33.94 34.40 29.68 43.49 53.76 61.42 68.99 74.91 72.02 64.59 54.92 43.45 27.99 37.15 37.46 45.63 53.47 59.88 70.97 74.05 72.97 67.90 54.59

[1151] 45.54 31.28 29.74 40.44 44.43 53.61 63.00 69.95 74.70 73.73 65.48 55.25 40.02 36.12 33.96 40.43 45.62 53.48 62.07 68.29 72.02 70.46 65.17

[1174] 55.00 40.75 32.22 30.75 31.69 42.52 50.95 61.95 68.19 73.05 72.71 63.88 53.60 39.69 34.77 29.91 32.55 45.42 53.73 61.73 71.77 74.32 72.90

Appendix C1: (Continued)

[1197] 66.06 54.86 43.37 36.73 33.59 38.06 44.36 50.52 59.69 68.54 74.38 74.99 65.27 55.05 42.50 34.01 30.26 36.25 39.85 51.70 61.94 70.60 74.30

[1220] 73.02 64.13 54.31 40.05 34.45 30.46 36.93 45.49 48.97 60.33 69.24 73.98 72.44 67.07 54.50 41.42 34.40 35.45 39.68 42.09 51.99 63.51 69.01

[1243] 76.18 74.62 69.59 55.66 45.45 36.03 34.20 40.30 43.64 52.55 61.06 69.18 75.13 73.73 64.80 54.94 48.35 36.48 34.04 40.36 46.84 53.35 64.05

[1266] 69.81 74.53 74.70 66.06 55.55 38.32 28.48 31.50 34.40 42.04 53.92 63.62 70.00 75.26 74.86 66.07 54.76 47.83 36.53 34.92 36.74 40.02 54.39

[1289] 60.19 71.63 76.55 73.49 67.44 52.23 42.14 35.43 33.00 33.48 44.02 52.85 61.90 68.84 76.13 75.38 65.41 56.87 42.76 35.68 30.34 33.91 47.94

[1312] 53.53 62.87 68.87 73.59 70.80 66.26 55.84 44.34 35.15 33.23 38.22 42.90 52.98 60.34 70.01 75.89 73.88 67.84 55.75 45.28 32.64 39.29 35.16

[1335] 43.31 56.03 63.06 71.44 77.10 74.10 63.69 52.97 44.68 36.64 31.39 32.86

Appendix C2: Monthly Temperature for 1895-2007 (Version 2 Dataset)

[1] 27.63 27.54 40.95 54.05 61.11 68.61 71.97 72.24 66.43 51.57 40.25 32.73 32.38 36.04 39.21 53.27 63.23 70.57 74.44 73.05 63.63 52.79 39.17

[24] 36.44 28.92 34.29 39.90 52.24 62.03 68.75 74.03 72.03 67.04 55.69 41.72 31.62 31.59 36.21 42.03 51.80 60.51 69.93 73.92 73.45 66.01 52.13

[47] 39.25 29.49 30.55 26.27 38.57 51.60 60.62 69.28 73.65 72.42 65.33 54.68 45.98 32.63 34.93 31.42 42.18 52.56 62.52 71.01 73.76 73.55 65.62

[70] 57.17 42.44 35.37 32.54 30.71 41.81 50.31 61.83 69.11 76.94 73.87 63.80 56.16 42.66 32.32 30.85 32.37 42.78 51.93 62.88 68.74 72.74 72.09

[93] 63.36 55.13 44.58 31.51 31.72 29.43 43.40 51.20 60.30 66.58 72.61 71.64 63.32 54.71 41.13 31.44 28.29 32.40 42.70 50.35 60.88 67.99 72.02

[116] 71.36 65.56 54.67 44.18 32.83 27.83 27.76 46.24 51.53 59.95 68.71 72.57 73.04 66.52 52.36 43.17 31.96 33.76 34.59 37.20 53.74 60.37 68.06

[139] 73.00 72.48 66.87 53.20 41.45 35.74 31.55 36.34 46.68 48.43 56.65 66.46 73.21 71.47 64.79 54.46 41.86 35.59 33.34 33.94 44.96 53.73 59.59

[162] 67.65 73.83 71.34 66.08 52.41 43.41 34.15 33.02 35.86 41.50 49.91 58.86 69.62 73.25 73.58 64.93 53.55 45.96 26.52 30.47 30.22 50.61 54.67

[185] 60.02 68.98 74.69 71.73 66.20 56.29 42.04 32.68 33.61 34.56 45.21 51.04 62.01 71.41 73.29 71.56 66.63 53.41 38.18 33.13 25.55 32.06 36.90

[208] 51.67 61.39 67.44 72.90 70.80 63.32 53.90 43.17 33.95 31.32 30.47 39.86 52.42 60.93 68.97 73.67 74.06 64.22 52.36 46.01 34.20 35.39 30.48

[231] 41.93 51.98 62.01 70.11 74.67 72.30 65.09 55.90 44.33 27.68 29.33 36.92 38.18 55.60 58.94 66.71 71.53 70.70 64.97 56.03 44.01 33.96 28.94

[254] 33.80 43.24 51.25 60.07 66.96 74.92 72.44 64.06 52.81 41.06 29.73 29.13 30.89 39.54 49.64 55.92 67.65 75.22 71.63 64.71 51.05 44.44 30.33

[277] 24.69 34.74 46.84 50.03 61.62 71.80 73.02 73.21 62.41 56.86 41.37 35.26 32.62 33.10 41.94 52.28 60.80 70.04 74.97 72.86 66.19 53.27 40.15

[300] 29.55 30.00 34.46 41.38 47.73 60.12 68.07 73.35 71.31 65.90 55.32 40.54 33.93 35.12 37.81 47.64 51.95 60.87 71.08 75.34 72.68 67.20 56.07

[323] 43.45 35.88 27.70 32.84 41.55 51.33 62.02 70.94 73.64 73.45 67.83 55.70 42.78 34.23 35.40 30.48 39.44 50.92 60.27 68.82 74.44 71.79 65.63

[346] 51.97 43.66 36.92 27.06 36.08 38.33 51.25 58.69 69.29 72.51 72.75 63.34 55.69 43.34 28.25 29.82 39.19 45.35 55.67 60.80 70.57 74.76 72.05

Appendix C2: (Continued)

[369] 67.19 49.46 41.36 33.33 31.20 38.59 40.82 50.99 61.84 68.92 74.03 72.98 64.56 55.55 41.40 32.14 32.33 38.71 43.36 52.53 60.47 67.92 73.33

[392] 70.01 65.54 56.58 45.15 29.03 32.77 35.29 44.25 49.02 62.24 66.60 74.03 72.46 63.81 55.73 42.57 33.82 27.17 27.88 44.80 52.15 59.61 68.20

[415] 74.28 73.26 63.99 54.58 39.31 34.69 24.42 41.02 41.52 55.33 60.01 69.14 75.68 73.56 66.31 52.42 42.07 32.09 33.90 38.94 40.55 52.00 60.23

[438] 71.44 76.38 72.65 68.74 57.31 44.36 36.03 32.44 37.58 38.48 52.51 61.34 70.00 74.82 73.25 65.29 53.06 41.67 30.33 35.20 30.56 43.02 50.87

[461] 60.70 72.35 75.80 72.60 68.27 55.91 43.47 37.11 35.97 35.15 43.86 54.71 65.47 71.45 77.08 74.25 64.40 57.22 45.92 33.48 32.27 36.91 44.71

[484] 50.65 58.10 68.36 75.85 73.67 65.34 54.41 40.40 31.66 28.14 26.06 44.58 51.12 64.25 71.34 77.49 75.28 66.88 54.10 40.72 35.33 25.70 32.64

[507] 39.96 51.08 62.74 69.57 75.27 75.33 66.26 54.53 41.73 33.49 32.97 36.91 46.22 52.67 60.62 69.51 74.43 74.44 67.25 57.17 40.88 35.13 34.87

[530] 30.91 43.62 52.45 63.50 69.56 75.50 73.31 67.96 55.45 43.27 38.56 24.55 34.80 42.61 51.13 61.62 70.28 74.78 72.75 65.93 56.84 39.97 36.95

[553] 33.14 34.47 40.38 53.10 63.17 68.62 74.69 72.77 65.16 55.99 43.92 37.05 30.71 31.70 42.55 54.04 59.91 68.62 74.62 72.47 64.27 55.33 44.03

[576] 33.73 30.20 37.18 39.63 53.87 60.48 69.58 74.94 74.18 64.85 54.42 41.84 33.48 32.97 35.49 39.68 49.99 62.82 69.08 73.44 72.79 65.82 56.03

[599] 42.58 31.55 31.28 36.17 46.71 51.17 59.01 66.54 73.59 72.83 65.48 54.83 42.99 29.95 32.28 35.80 47.96 55.59 59.49 69.04 74.46 71.79 65.05

[622] 53.97 42.67 36.99 32.63 32.29 39.83 51.94 60.85 67.43 73.44 74.77 67.09 59.55 39.16 33.80 28.77 32.47 39.87 54.30 61.05 69.49 73.92 72.73

[645] 66.64 53.97 42.68 33.45 26.96 32.84 41.98 53.08 62.59 70.02 74.87 72.81 64.32 54.81 46.23 34.20 31.30 36.45 40.02 49.71 60.29 68.70 71.89

[668] 70.68 64.13 58.15 41.59 33.83 31.11 35.40 39.67 50.72 61.53 67.32 74.27 72.50 64.60 54.74 39.06 32.12 32.85 36.46 38.89 52.65 61.38 71.50

[691] 74.70 73.45 66.45 53.61 40.97 34.30 37.20 36.84 44.85 49.69 59.97 70.87 74.74 72.76 66.67 56.94 44.83 34.58 31.56 42.26 40.38 55.08 59.33

[714] 69.57 76.11 72.83 67.02 55.39 44.96 34.81 30.28 32.00 41.13 53.83 62.12 67.26 75.09 74.53 66.44 55.74 39.09 32.38 31.75 33.08 41.81 50.37

[737] 62.56 71.07 73.73 72.35 65.63 56.89 41.28 36.96 27.90 38.86 42.41 51.68 60.53 69.48 74.98 72.39 64.77 52.42 41.27 37.88 32.47 32.86 38.81

[760] 51.33 63.29 68.72 73.29 73.98 66.07 55.47 43.94 32.91 30.23 33.36 42.61 52.59 61.70 70.71 74.25 73.88 65.30 53.98 39.50 36.77 30.24 31.51

Appendix C2: (Continued)

[783] 36.88 53.57 60.12 69.83 74.52 72.86 66.94 55.45 43.59 31.16 30.39 38.30 44.58 49.59 59.71 70.25 73.88 73.36 64.01 54.53 41.41 31.80 27.65

[806] 35.77 38.96 52.78 63.11 68.69 72.73 72.78 64.71 56.77 44.01 34.12 25.11 34.97 44.86 52.89 61.85 69.49 74.34 72.67 66.91 60.00 44.81 28.46

[829] 32.27 32.70 39.91 52.29 62.33 68.79 75.34 71.37 64.30 54.42 43.20 32.71 32.37 33.29 36.52 52.86 61.70 67.66 73.31 71.75 62.38 55.60 45.39

[852] 36.49 27.07 32.11 44.03 50.96 61.04 68.77 75.76 71.51 65.14 53.52 43.95 33.37 33.74 33.75 44.65 52.02 58.51 68.36 73.25 71.93 64.67 54.58

[875] 42.30 32.87 29.52 33.77 44.66 51.63 59.06 69.28 73.59 71.33 64.49 55.29 41.92 30.71 29.81 33.61 37.11 53.64 62.28 67.64 74.81 73.87 66.13

[898] 51.71 42.41 34.15 27.79 36.18 39.56 50.52 61.96 69.58 74.68 74.04 65.19 52.95 42.06 34.27 30.02 34.30 40.73 51.31 59.25 70.32 73.14 72.77

[921] 64.98 55.80 42.38 34.70 30.53 34.31 45.03 51.80 61.46 69.11 72.92 72.47 65.08 52.97 39.73 30.44 30.26 34.47 45.47 49.87 60.17 69.78 74.02

[944] 73.29 64.99 56.71 43.55 34.33 31.61 35.60 45.43 52.62 61.30 69.32 74.86 70.99 62.88 54.53 43.10 34.32 32.96 33.54 39.79 47.89 61.35 68.36

[967] 74.31 72.26 63.50 55.30 43.35 34.77 30.49 40.36 43.37 52.93 59.98 68.67 73.67 71.46 64.85 50.69 39.14 31.50 23.64 36.86 44.32 55.12 62.65

[990] 71.17 75.19 72.48 66.52 54.58 43.54 33.86 26.45 28.75 42.24 52.62 60.68 69.82 74.58 72.50 66.52 55.05 42.15 30.46 22.57 28.82 43.15 51.17

[1013] 60.11 68.74 73.93 71.72 66.72 55.91 40.70 36.62 31.67 33.58 40.38 52.49 61.24 69.58 76.51 73.54 66.83 53.26 43.03 35.71 33.01 37.49 43.93

[1036] 56.19 59.94 70.68 74.68 72.77 65.52 52.97 45.43 34.20 26.65 33.29 42.98 49.69 61.78 67.12 73.95 72.78 64.66 53.94 41.86 36.47 33.64 37.75

[1059] 43.45 48.08 58.90 67.99 74.79 75.84 66.22 55.63 43.78 25.95 29.35 38.24 41.08 50.66 61.03 69.46 73.91 73.89 63.75 54.33 42.31 35.16 26.80

[1082] 31.65 44.54 55.36 63.01 68.92 74.78 71.70 63.39 54.94 40.14 29.11 35.17 36.64 47.15 53.88 62.05 71.30 74.33 72.38 64.42 54.63 42.03 34.93

[1105] 31.99 38.26 43.56 54.47 63.97 71.09 74.07 72.55 65.80 53.43 44.69 35.24 28.26 34.27 43.43 52.80 62.19 71.73 75.57 74.42 65.21 54.34 44.05

[1128] 33.97 34.43 29.69 43.48 53.79 61.42 69.03 74.94 72.07 64.66 54.96 43.47 28.00 37.17 37.46 45.64 53.47 59.86 71.03 74.06 73.02 67.96 54.65

[1151] 45.59 31.31 29.74 40.41 44.41 53.59 62.95 69.98 74.69 73.73 65.51 55.24 40.05 36.15 34.00 40.44 45.60 53.45 62.05 68.29 72.03 70.47 65.22

[1174] 55.01 40.79 32.26 30.76 31.67 42.48 50.89 61.86 68.16 73.02 72.68 63.86 53.57 39.74 34.76 29.84 32.56 45.38 53.65 61.67 71.70 74.29 72.88

Appendix C2: (Continued)

[1197] 66.07 54.87 43.42 36.74 33.58 37.95 44.33 50.45 59.62 68.50 74.36 74.98 65.28 55.04 42.52 33.99 30.22 36.21 39.83 51.63 61.87 70.62 74.28

[1220] 73.03 64.13 54.28 40.11 34.49 30.48 36.91 45.48 48.96 60.28 69.27 74.02 72.48 67.13 54.55 41.46 34.51 35.53 39.75 42.13 52.03 63.51 69.03

[1243] 76.25 74.70 69.63 55.76 45.58 36.10 34.29 40.36 43.68 52.55 61.07 69.23 75.17 73.75 64.88 55.06 48.46 36.59 34.11 40.47 46.89 53.41 64.03

[1266] 69.83 74.62 74.75 66.18 55.66 38.49 28.59 31.63 34.49 42.15 53.98 63.63 70.05 75.32 74.93 66.17 54.89 47.98 36.67 35.03 36.95 40.20 54.33

[1289] 60.13 71.67 76.62 73.54 67.49 52.31 42.33 35.65 33.01 33.53 44.05 52.84 61.88 68.89 76.21 75.44 65.51 57.03 42.92 35.91 30.45 34.07 48.05

[1312] 53.57 62.88 68.93 73.69 70.95 66.40 56.02 44.54 35.42 33.45 38.45 43.10 53.18 60.47 70.13 76.02 73.98 67.98 55.93 45.46 32.91 39.53 35.35

[1335] 43.45 56.12 63.12 71.55 77.22 74.19 63.86 53.13 44.58 36.79 31.46 32.86

Appendix D: Monthly CO_2 Emission 1981-2003

```
  [1] 125.4146 105.9197 107.1396  94.9415  95.2932  96.8175 101.4611  99.1624  94.7263 100.2223  99.5731 115.2323 120.7643 104.9733 104.5716
 [16]  96.7353  89.5450  88.4151  93.3538  93.7069  88.7784  91.4506  95.8438 103.4164 109.8079  95.9954 102.0473  92.6503  89.7150  90.0224
 [31]  96.1035  99.8732  92.1287  91.9029  95.5590 111.6509 123.0677 105.0796 109.5487  99.4655  97.1149  96.6004  98.8165 102.1989  92.3669
 [46]  96.5541  99.2773 107.1774 118.5610 109.6339 104.9004  97.7826  95.9263  93.8377  99.2186 100.9150  91.7477  96.6222  97.1792 116.6590
 [61] 119.0228 106.0296 107.9398  96.3769  96.1893  96.1216 102.9323  99.8575  92.0380  95.7214  97.5536 112.3446 119.0369 107.0488 107.1302
 [76] 100.2641  98.5374 101.2423 107.7090 105.7530  97.7900 102.1821 101.9277 117.4361 125.3272 117.3372 116.2031 102.3907 100.7750 104.1288
 [91] 108.4163 112.9515 100.5518 105.7330 107.9122 122.1634 121.3611 116.2543 120.0471 105.3687 103.7629 105.2683 107.0433 109.7045 101.6754
[106] 106.8046 109.6547 132.4097 122.7457 109.3402 114.3499 106.5081 105.7040 106.4926 110.1887 114.3188 103.9005 108.1243 106.9206 118.8626
[121] 126.0021 106.8170 110.8317 101.5886 102.7917 103.2419 109.4145 110.2427 102.5933 107.0639 110.2982 124.6845 123.6433 112.3913 115.6407
[136] 107.3133 104.4047 102.8995 111.4495 108.3450 103.9191 107.7787 109.6073 124.7419 122.3291 116.2767 123.0597 107.9980 103.0110 106.5694
[151] 115.2173 115.0036 107.1554 110.2313 114.9989 127.2304 136.3117 123.5147 121.0627 110.0240 108.2796 113.2922 115.1996 117.1827 107.7320
[166] 110.8785 111.2215 125.6154 128.5581 119.5553 120.9017 109.9400 110.1609 111.9193 117.5974 123.0613 110.9679 112.1697 118.6156 132.1565
[181] 137.3295 127.4181 127.3773 114.5245 114.7983 114.7599 120.0355 122.1473 111.5481 119.6606 123.3725 132.9810 139.4169 122.3559 124.9457
[196] 117.3911 116.4253 116.2448 126.3987 123.1898 116.6263 121.5586 123.3120 137.4636 136.2017 121.1348 128.8153 118.6826 118.2228 121.3047
[211] 129.6144 129.3553 120.1381 120.3204 119.3293 134.2790 139.6680 122.5791 131.1086 120.0453 118.1919 121.0179 130.0581 130.0953 119.2973
[226] 122.7435 120.5918 138.3181 140.7672 132.3686 129.4892 118.8712 123.8231 125.0370 128.5651 134.8708 123.4820 126.2876 128.2345 149.5021
[241] 147.4769 128.7643 135.0432 121.4761 121.8473 121.3621 131.0966 133.6699 118.2129 122.7140 119.8921 130.9794 138.8554 123.7673 132.4239
```

Appendix D: (Continued)

[256] 122.5456 122.7795 124.5207 133.8627 133.4808 123.0272 125.6926 128.2429 141.2141 147.6298 134.1716 133.6979 121.0047 120.4789 120.7394

[271] 132.4187 135.1314 121.7753 125.2487 126.2127 143.1509

Appendix E: Monthly CO_2 in the Atmosphere 1965-2004

```
  [1] 319.44 320.44 320.89 322.13 322.16 321.87 321.21 318.87 317.81 317.30 318.87 319.42 320.62 321.59 322.39 323.70 324.07 323.75 322.40 320.37 318.64
 [22] 318.10 319.79 321.03 322.33 322.50 323.04 324.42 325.00 324.09 322.55 320.92 319.26 319.39 320.72 321.96 322.57 323.15 323.89 325.02 325.57 325.36
 [43] 324.14 322.11 320.33 320.25 321.32 322.90 324.00 324.42 325.64 326.66 327.38 326.70 325.89 323.67 322.38 321.78 322.85 324.12 325.06 325.98 326.93
 [64] 328.13 328.07 327.66 326.35 324.69 323.10 323.07 324.01 325.13 326.17 326.68 327.18 327.78 328.92 328.57 327.37 325.43 323.36 323.56 324.80 326.01
 [85] 326.77 327.63 327.75 329.72 330.07 329.09 328.05 326.32 324.84 325.20 326.50 327.55 328.54 329.56 330.30 331.50 332.48 332.07 330.87 329.31 327.51
[106] 327.18 328.16 328.64 329.35 330.71 331.48 332.65 333.09 332.25 331.18 329.40 327.44 327.37 328.46 329.58 330.40 331.41 332.04 333.31 333.96 333.59
[127] 331.91 330.06 328.56 328.34 329.49 330.76 331.74 332.56 333.50 334.58 334.87 334.34 333.05 330.94 329.30 328.94 330.31 331.68 332.92 333.42 334.70
[148] 336.07 336.74 336.27 334.93 332.75 331.58 331.16 332.40 333.85 334.97 335.39 336.64 337.76 338.01 337.89 336.54 334.68 332.76 332.54 333.92 334.95
[169] 336.23 336.76 337.96 338.89 339.47 339.29 337.73 336.09 333.91 333.86 335.29 336.73 338.01 338.36 340.08 340.77 341.46 341.17 339.56 337.60 335.88
[190] 336.01 337.10 338.21 339.23 340.47 341.38 342.51 342.91 342.25 340.49 338.43 336.69 336.85 338.36 339.61 340.75 341.61 342.70 343.56 344.13 343.35
[211] 342.06 339.82 337.97 337.86 339.26 340.49 341.37 342.52 343.10 344.94 345.75 345.32 343.99 342.39 339.86 339.99 341.16 342.99 343.70 344.51 345.28
[232] 347.08 347.43 346.79 345.40 343.28 341.07 341.35 342.98 344.22 344.97 346.00 347.43 348.35 348.93 348.25 346.56 344.69 343.09 342.80 344.24 345.56
[253] 346.29 346.96 347.86 349.55 350.21 349.54 347.94 345.91 344.86 344.17 345.66 346.90 348.02 348.47 349.42 350.99 351.84 351.25 349.52 348.10 346.44
[274] 346.36 347.81 348.96 350.43 351.72 352.22 353.59 354.22 353.79 352.39 350.44 348.72 348.88 350.07 351.34 352.76 353.07 353.68 355.42 355.67 355.13
[295] 353.90 351.67 349.80 349.99 351.30 352.53 353.66 354.70 355.39 356.20 357.16 356.22 354.82 352.91 350.96 351.18 352.83 354.21 354.72 355.75 357.16
[316] 358.60 359.34 358.24 356.17 354.03 352.16 352.21 353.75 354.99 355.98 356.72 357.81 359.15 359.66 359.25 357.03 355.00 353.01 353.31 354.16 355.40
[337] 356.70 357.16 358.38 359.46 360.28 359.60 357.57 355.52 353.70 353.98 355.33 356.80 358.36 358.91 359.97 361.26 361.68 360.95 359.55 357.49 355.84
```

Appendix E: (Continued)

[358] 355.99 357.58 359.04 359.96 361.00 361.64 363.45 363.79 363.26 361.90 359.46 358.06 357.75 359.56 360.70 362.05 363.25 364.03 364.72 365.41 364.97

[379] 363.65 361.49 359.46 359.60 360.76 362.33 363.18 364.00 364.57 366.35 366.79 365.62 364.47 362.51 360.19 360.77 362.43 364.28 365.32 366.15 367.31

[400] 368.61 369.29 368.87 367.64 365.77 363.90 364.23 365.46 366.97 368.15 368.87 369.59 371.14 371.00 370.35 369.27 366.94 364.63 365.12 366.67 368.01

[421] 369.14 369.46 370.52 371.66 371.82 371.70 370.12 368.12 366.62 366.73 368.29 369.53 370.28 371.50 372.12 372.87 374.02 373.30 371.62 369.55 367.96

[442] 368.09 369.68 371.24 372.43 373.09 373.52 374.86 375.55 375.40 374.02 371.49 370.71 370.24 372.08 373.78 374.68 375.63 376.11 377.65 378.35 378.13

[463] 376.62 374.50 372.99 373.00 374.35 375.70 376.79 377.37 378.41 380.52 380.63 379.57 377.79 375.86 374.06 374.24 375.86 377.48