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In this paper we analyze the frequency and information content of small Nasdaq stock trades and their impacts on return volatility at the intraday interval. We employ an autoregressive conditional duration (ACD) model to estimate the intensity of the arrival and information content of trades by accounting for the deterministic nature of intraday periodicity and irregular trading intervals in transaction data. We estimate and compare the price duration of thinly and heavily traded stocks to assess the differential information content of stock trades. We find that the number of transactions is negatively correlated with price duration or positively correlated with return volatility. The impact of the number of transactions on price duration or volatility is higher for thinly traded stocks. On the other hand, the persistence of the impact on price duration adjusted for intradaily periodicity is about the same for thinly and heavily traded stocks on average.

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Introduction

The subject of price formation has always been intriguing to financial researchers. A vast financial literature has been devoted to the study of the pattern of information arrivals and how new information is incorporated into price. These studies range from simple event studies of the market response to news announcements, to more sophisticated information flow studies analyzing how information innovations are impounded into security prices. Interest in this issue has been fueled by recent advances in market microstructure theory and the availability of ultra-high-frequency data, thanks to modern technology.

The shift to high-frequency data analysis has posed significant challenges to empirical studies. A major difficulty faced in high-frequency data studies is that transactions arrive in irregular time intervals. Most empirical microstructure studies have employed data with a fixed time interval (e.g., hourly or half-hourly) to test the implications of market microstructure theory (see, for example, Foster and Viswanathan, 1995; Andersen and Bollerslev, 1997). This is because standard time-series econometric techniques build on the premise of fixed time intervals. The selection of the time interval is often arbitrary. Large heavily-traded stocks typically have transactions every few seconds whereas small thinly-traded stocks may not have transactions every hour or day. If a short time interval is chosen, there will be many intervals with no transactions for thinly traded stocks and heteroskedasticity of a particular form will be introduced. On the other hand, if a long interval is chosen, we may lose most of the microstructure features of the data. In particular, when transactions are averaged, the timing relation and characteristics of trades will be lost.

Empirical microstructure studies that examine transaction-by-transaction data (see, for example, Hasbrouck, 1991; Madhavan et al., 1997; Huang and Stoll, 1997) face a different estimation problem. Data points in these studies correspond to the transaction (event) time and so they are irregularly spaced. However, these studies have typically ignored the problem of irregular intervals when applying standard time-series econometric techniques. Assuming that data points are equally spaced, when in fact they are uneven, leaves out much of the important information about trade clustering, temporal order flow patterns and the information assimilation process.

Fortunately, new econometric methods have been developed recently to cope with the estimation problems of irregularly spaced data (see Engle, 2000; Dufour and Engle, 2000). Two time-series methods were developed to model irregularly spaced data: Time Deformation models (TD) and Autoregressive Conditional Duration (ACD) models. The TD approach uses auxiliary transformations to relate observational or economic time to calendar time. In contrast, the ACD approach directly models the time duration between events (e.g., trades). The ACD model typically adopts a dependent point process suitable for modeling characteristics of duration series such as clustering and overdispersion. In this paper, we employ the ACD model proposed by Engle and Russell (1998) to examine information clustering and trading responses to information at the intraday level. There are advantages of using this model. First, this model provides a framework for measuring and estimating the intensity of transaction arrivals that is particularly suited for the trading process. The model accounts for the irregular time interval, typically encountered in stock trading, by treating it as a random variable that follows a point process. This treatment resolves infrequent or nonsynchronous trading problems in empirical estimation using intraday data of thinly traded stocks. Second, the estimation procedure is relatively straightforward and the model can be easily adapted to test various microstructural hypotheses.

The primary objective of this paper is to examine the patterns of information arrival of small thinly-traded versus large heavily-traded stocks, and their impact on price movements. The trading pattern and the intensity of information-based trading of small thinly-traded stocks often deviates sharply from that of large heavily-traded stocks. Aside from the sheer difference in trading frequency, large stocks enjoy much higher liquidity than small stocks. Studies have shown that a good proportion of trades for large heavily-traded stocks is for liquidity purposes (see Easley et al., 1996). Thus, trades for large heavily-traded stocks may not always have high information content. Conversely, small stocks are not as liquid and are not traded as heavily as large stocks. Fewer analysts are interested in these stocks and so less information is available for investors. Due to lack of information and liquidity, there are typically no transactions for a good portion of the open market trading period. However, trading of these stocks often causes a significant price movement. Thinly traded stocks also have higher variations in order flow. Trades are clustered in that the occurrence of a trade induces another trade in a rather short time interval. Once these stocks are traded, there is a high probability that informed traders may trade to minimize price impacts (see Admati and Pfleiderer, 1988). As insiders' private information is impounded into price, return volatility increases. Since the number of trades is low for small inactive stocks, the information content per trade may be higher for these stocks.

In this paper we focus on the intensity of transaction arrivals and its effects on price movements of thinly-traded stocks in the Nasdaq market. Previous studies have shown that the price discovery process and bid-ask spread behavior of a dealer market such as Nasdaq differ from those of a auction market like the NYSE (see Hasbrouck, 1995; Huang and Stoll, 1996). Differences in market structures and trading mechanisms cause variations in trading costs, order flows and the speed of information transmission. Therefore, empirical findings of NYSE stocks do not necessarily characterize Nasdaq stocks. One distinct feature of Nasdaq is that the depth of the market often varies widely among stocks. This greater dispersion in trading activities provides an excellent opportunity for comparing the intensity of trade arrivals and the extent to which trades convey information for heavily- and thinly-traded stocks. Most empirical microstructure studies have not accounted for the uneven intervals in stock trades in examining the issues of order flow and information assimilation. An exception is Dufour and Engle (2000). However, their study covers only the most actively traded stocks at the NYSE. Unlike their study, we examine both the active and inactive stocks on Nasdaq.

The remainder of this paper is organized as follows. Section I presents the empirical model and methodology for estimating the intensity of trade arrivals and the effects of microstructure variables on the time duration of trades and price changes. Section II discusses data and empirical results. Finally, Section III summarizes the main findings of this paper.

I. The Model

Information arrivals induce trades and price changes (see Admati and Pfleiderer, 1988; Easley and O'Hara, 1992). To analyze information flow at irregular arrival times, we employ the autoregressive conditional duration (ACD) model proposed by Engle and Russell (1998).

Denote the interval between two arrival times, $x_i = t_i - t_{i-1}$, as duration. The expectation of the *i*th duration conditional on past x_i's is given by ψ_i, where

$$\psi_i \equiv E(x_i \mid x_{i-1}, x_{i-2}, ..., x_1) = \psi_i(x_{i-1}, x_{i-2}, ..., x_1; \Phi) \quad (1)$$

where Φ is the vector of the parameters of the duration process. Assuming that the stochastic process of the duration is

$$x_i = \psi_i \varepsilon_i \quad (2)$$

where ε_i is an i.i.d. error term with a distribution which must be specified. Following Engle and Russell (1998), we specify the conditional duration by a general model:

$$\psi_i = \omega + \sum_{j=1}^{m} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j}\,, \quad (3)$$

which follows an ACD (m, q) process with m and q referring to the orders of the lags, and $\Phi = (\omega, \alpha_j, \beta_k)$, *j = 1, 2,…, m* and *k = 1, 2,…, q*, are parameters to be estimated. This model has a close connection with GARCH models and shares many of their properties. The model is convenient because it can be easily estimated using a standard GARCH program by employing the square root of x_i as the dependent variable and setting the mean to zero (see Engle and Russell, 1998). In general, if durations are conditionally exponential, the conditional intensity is

$$\lambda(t \mid x_{N(t)}, ..., x_1) = \psi^{-1}_{N(t)+1} \quad (4)$$

It can be shown that the higher the conditional intensity, the higher the volatility of returns.

There are several ways to estimate the system of (2) and (3). The simplest way is to assume that the error term follows an exponential distribution and the lagged orders equal to one. This model is called the EACD(1,1) where E stands for the exponential distribution. Another way is to assume that the conditional distribution is Weibull, which is equivalent to assuming that x^{θ} is exponential where θ is the Weibull parameter. Similarly, we can estimate the Weibull model with the lagged orders equal to one, that is, WACD(1,1).

The Weibull distribution function can be written as

$$F(x_i) = (\theta / \psi_i^{\theta})\; x_i^{\theta-1} \exp[-(x_i / \psi_i)^{\theta}] \quad \text{for } \theta,\ \psi_i > 0 \quad (5)$$

When θ = 1, x_i / ψ_i follows an exponential distribution. The Weibull distribution is preferred if the data show an overdispersion with extreme values (very short or long durations) more likely than the exponential distribution would predict (see Dufour and Engle, 2000). Given the conditional density function, we can estimate the parameters of the ACD model by maximizing the following log likelihood function:

$$L(\eta) = \sum_{i=1}^{T} \ln(\theta / x_i) + \theta \ln[\ \Gamma(1+1/\theta)\ x_i/\psi_i\,] - [\ \Gamma(1+1/\theta)\ x_i/\psi_i\,]^{\theta} \qquad (6)$$

where $\Gamma(.)$ is the gamma function, and η is a column vector containing the parameters to be estimated. Engle and Russell (1998) commented that clever optimization can avoid repeated evaluation of the gamma function. This tactics is useful when the sample size is very large.

The ACD model is essentially a model for intertemporally correlated transaction (event) arrival times. The arrival times are treated as random variables following a point process. In the context of security trading, associated with each arrival time are random variables such as volume, price or bid-ask spread. These variables are defined as "marks". Finance researchers are often interested in modeling these marks associated with the arrival times. For example, not all transactions occur because of the arrival of new information. Instead, some are triggered by pure liquidity or portfolio adjustment reasons, which are not related to changes in the expected (fundamental) value of stock. On the other hand, there are times when transactions occur as a result of new information arrival that is not publicly observable. Market microstructure theory suggests that traders possessing private information will trade as long as their information has value. This results in clustering of transactions following an information event. To examine this hypothesis, we can define the events as a subset of the transaction arrival times with specific "marks". For example, to examine the effect of information events, we can select data points for which price has moved beyond the bid-ask bound. This process is called dependent thinning.

To distinguish informed from uninformed trades, we modify transaction arrival times into price arrival times. The basic idea is to leave out those transactions that do not significantly alter price. The price movements can be classified either as transitory or permanent movements. Define the midpoint of the bid-ask spread or "midprice" to be the current price. Following Engle and Russell (1998), we define a permanent price movement as any movement in the midprice (midquote) greater than or equal to \$0.25 or 2 ticks.[1] Once we define the price arrival times, we can apply the ACD model to these new event arrival times. In this case, we are modeling how quickly the price is changing rather than the arrival rate of transactions. The intensity function is now called price intensity, which measures the instantaneous probability of a permanent price change.

The basic formulation of the ACD model parameterizes the conditional intensity of event arrivals as a function of the time between past events. It can be easily extended to include other effects such as characteristics associated with past transactions or other outside influences. For example, previous studies have shown that important information is contained in the number of trades, and the trade size which is the average volume per transaction. To examine this hypothesis, we can modify the ACD model to include these two variables:

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[1] A tick is 1/8 dollar.

$$\psi_i = \omega + \sum_{j=1}^{p} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j} + \gamma \text{ \#Trans} + \lambda \text{ Volume/Trans} \quad (7)$$

where the duration is now between two consecutive prices with a movement greater than or equal to two ticks, and the number of transactions per duration and trade size per transaction are added as determinants of duration. Market microstructure theory contends that trades contain information that affects price movements (or volatility). Including the number of transactions and trade size allows us to test this important hypothesis. In addition, dividing the accumulated volume by the number of transactions yields the average volume per transaction (or trade size) at the interval *x*. Previous studies have indicated that trade size may contain information. The ACD model in (7) now describes how quickly the price changes, by taking into consideration the effects of transaction rate and trade size. The intensity function becomes a measurement of the instantaneous probability of a price movement called "price intensity." It can be shown that price duration is inversely related to the volatility of price changes.

In addition to transaction frequency and trade size, we also test the ACD model with the bid-ask spread variable. Microstructure theory suggests that the specialist's (or dealer's) bid-ask spread reflects the intensity of informed trading. It will be interesting to see whether this variable will increase the explanatory power of the model. Thus, we also estimate the following extended model:

$$\psi_i = \omega + \sum_{j=1}^{p} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \psi_{i-j} + \gamma \text{ \#Trans} + \lambda \text{ Volume/Trans} + \delta \text{ Spread} \quad (8)$$

where Spread is the bid-ask spread divided by mid-quote.

It is widely known that intraday return volatility exhibits significant deterministic (periodic) patterns. Since price duration is the inverse of volatility, the duration measure is expected to contain a deterministic component. This deterministic component needs to be separated from the stochastic component in empirical estimation. The strategy followed here to eliminate the intraday pattern is a simple seasonal adjustment approach. The time span within a trading day is divided into non-overlapping time intervals of 15 minutes each. The mean of price durations within each interval is computed over the entire sample period. The adjusted price duration is then computed as the price duration divided by the average price duration within that interval. The adjusted price duration series now has a mean approximately equal to one. If the adjusted duration is greater (less) than one, the duration is greater (less) than the average duration in that time interval. We estimate the ACD model using these adjusted price durations, as well as the raw (unadjusted) durations.[2]

II. Data and Empirical Estimation

Data on price, size, and trading time for Nasdaq stocks are obtained from the TAQ database over the period of July 1 to September 30, 1997. Trades and quotes are selected

[2] We have also tried the spline method to filter the deterministic intraday components. The results using this method are quite similar.

strictly for Nasdaq-listed firms, thus excluding NYSE stocks traded on Nasdaq and stocks listed on regional exchanges. We also exclude all preferred stocks, stock funds, stock rights, warrants and ADRs.[3]

Previous studies (see, for example, Easley et al., 1996; Wu and Xu, 2000; Wu, 2003) have used trading volume as a measure for defining the activeness of stocks. Following the influential paper by Easley et al. (1996), we use trading volume to classify the activeness of stocks for the purpose of comparing with their results. Trading volume is a preferred measure for this classification because it contains the information of frequency and size of trades, both of which are important indicators of the activeness or depth of stocks. We rank all Nasdaq common stocks by the average daily trading volume over the sample period, and then divide the sample into volume deciles. The first volume decile includes the highest-volume stocks and the tenth decile contains the lowest-volume stocks. To insure enough trading activities for purposes of empirical estimation, we choose stocks from the first, fifth and eighth volume deciles. To control for the price effect, we construct a matched sample of stocks having transaction prices close to each other at the beginning of the sample period (July 1), but at different levels of trading volume. Stocks from the three selected deciles are ranked in order of initial price and adjacent triplets of stocks are matched. We randomly choose five matched stocks from each of the three volume deciles to perform empirical estimation. We choose only five stocks from each volume decile for empirical estimation to alleviate the computation burden.

Transaction duration can be easily computed as the time difference between consecutive trades. Consecutive trades with same time stamp and price are aggregated and treated as one trade. We then "thin" the transaction data by constructing price duration with price changes greater than or equal to two ticks. Volume is expressed in terms of the number of shares traded at each interval.

Table 1A shows the summary statistics after dependent thinning where any midquote movements less than two ticks are ignored. More heavily traded stocks have lower spreads, more transactions (or shorter trade durations) and higher volume. Note that the daily number of transactions (or trading frequency) in the high-volume group is higher than those in the medium- and low-volume groups for all stocks except DURA. Similarly, the daily number of transactions (or trading frequency) in the medium-volume group is higher than that of low-volume group for all stocks except PSUN. In the analysis to follow, we compute the parameter estimates with and without these two stocks. The averages without these two stocks represent the average parameter estimates of high and medium trading frequency groups. After the data are "thinned" by price, the price duration still tends to be lower for more actively traded stocks. On the other hand, trade size or the average volume per transaction is about the same for both active and inactive stock groups. Table 1B lists the names of sample stocks.

Figure 1 shows the average price duration throughout a typical trading day for three selected stocks. The vertical axis indicates the price duration in seconds, and the horizontal axis indicates the intraday intervals. We divide each trading day into 25 intervals of 15

[3] To avoid the problem at the market open (e.g., stale quotes, and delay of the open), data for the first fifteen minutes are dropped as suggested by Miller et al. (1994). This avoids serious stale quote problems, especially for thinly traded stocks.

minutes each. The average price duration within each interval is computed over the entire sample period. As shown, price duration exhibits an inverted U-shape pattern. This is not a surprise since price duration is an inverse of price volatility, and intraday price volatility exhibits a pronounced U shape. Price duration is negatively related to trading frequency or number of transactions. As indicated, the price duration (in seconds) of ASND is much shorter than that of WIND because the former has a much greater number of daily transactions (see Table 1A).

2.1 Model Estimation Using Unadjusted Data

We first estimate the baseline ACD models with no microstructure variables. We use the Polak-Ribiere Conjugate Gradient (PRCG) to obtain the MLE estimates of the ACD parameters. The model is first estimated using the unadjusted price duration and then the adjusted duration. The adjusted duration is the price duration adjusted for the intraday deterministic pattern.

Table 2 reports the empirical estimates for the EACD(1,1) model using unadjusted data. As shown, most parameter estimates are statistically significant. The ARCH and GARCH parameters, α and β, are positive in most cases, consistent with the prediction and their values fall in the theoretical range. The results indicate that a short price duration is likely to be followed by another short price duration. Or equivalently, high price volatility in the current trading interval is likely to bring high price volatility at the next trading interval. The sum of α and β represents the persistence of price duration. The results do not show a material difference in persistence for high and low trading volume groups.

Table 3 reports the estimates of the WACD(1,1) model. Again, the estimates of α and β are positive in most cases and most of them are significant. The Weibull parameter θ is highly significant. The values of the Weibull parameter are all less than one and tend to be smaller for less heavily traded stocks. The results suggest that the EACD model is not suitable because the error term does not follow exactly the exponential distribution. The persistence of price duration is measured by the sum of α and β. Ignoring CBSS, the result again does not show a material difference in persistence for high and low trading volume groups.[4]

We next test the implications of market microstructure theories. On theoretical grounds, Easley and O'Hara (1992) predict that the number of transactions would influence the price process through the information-based clustering of transactions. Admati and Pfleiderer (1988, 1989) predict that the number of transactions will have no impact on price intensity. Glosten and Milgrom (1985) and Kyle (1985) predict that volume tends to be higher as the probability of informed trading increases. Most empirical studies have documented a positive relationship between volatility and volume for both individual securities and portfolios. Schwert (1989) and Gallant, Rossi, and Tauchen (1992) find a positive correlation between volatility and trading volume. Jones, Kaul, and Lipson (1994) show that the positive volatility-volume relationship actually reflects the positive relationship between volatility and the number of transactions. Based on this finding, they conclude that trade size carries no information beyond that contained in the frequency of transactions. None

[4] Note that although the estimate of α for CBSS is significantly negative, this estimate improves when adjusted price duration is used as dependent variable as shown in Table 6 below.

of these studies has addressed the issue of uneven trading intervals or infrequent trading. In the following, we re-examine this issue using the ACD model at the intraday level.

We estimate the ACD model with two additional explanatory variables: the number of transactions per duration and average trade size. Table 4 reports the results of estimation. The coefficients of the number of transactions are mostly negative. The results suggest that the expected price duration tends to be shorter, or equivalently the volatility is higher, following an interval of high transaction rates. This relationship is much stronger for less-heavily traded stocks. This conclusion holds regardless of whether DURA and PSUN are included or not. On the other hand, the effect of trade size is less conclusive. The sign of the coefficients of average volume per transaction (or trade size) is negative for more-heavily traded stocks but positive for less-heavily traded stocks.

Table 5 reports the estimates of the ACD model when the bid-ask spread is added as an additional explanatory variable. The coefficients of the number of transactions continue to be quite significant with a predicted negative sign. The coefficients of trade size again have mixed signs. The coefficients of spreads are generally negative, suggesting that higher spreads generally lead to shorter price duration (or higher volatility). Excluding PSUN in the middle group does not change the conclusion.[5]

2.2 Model Estimation Using Adjusted Data

We next turn to the estimation of the ACD model using the adjusted data where price duration is adjusted for the intradaily periodicity. Table 6 reports the estimates of the baseline WACD(1,1) model where no microstructure variables are included. As shown, after removing the intraday deterministic effect to retain the stochastic component of price duration, parameter estimates of the WACD(1,1) model become much more stable. The parameters α and β are now all within the theoretical range with a sum less than one. The results suggest that it is necessary to account for the intraday periodic pattern in empirical estimation. Again, the results show little difference in the persistence of price duration between the high- and low-volume stocks. On average, the sum of α and β is quite close for the three groups.

Table 7 reports the results of the WACD model with microstructure variables. The coefficients of the number of transactions are all negative, indicating that the higher the number of transactions, the shorter the price duration. The size of the coefficients (in absolute value) is much larger for less-heavily traded stocks. The average value of the coefficient for the number of transactions is –0.51 for the lowest-volume group compared to -0.09 for the highest-volume group. Excluding DURA and PSUN does not affect the results materially (-0.06 for the high volume group). Thus, the impact of the number of transactions (#Trans) is higher not only for low volume group but also for low trade frequency group. Another interesting finding is that the coefficients of trade size have mixed signs. The sign tends to be negative for most-heavily traded stocks. As the trading volume decreases, the sign becomes positive. Thus, trade size does not necessarily decrease price duration (or increase price volatility).

[5] Note that DURA in the first group does not converge. Therefore, the results for the high-volume group also represent the results for stocks with high-trade frequency.

Table 8 reports the results when bid-ask spread is added as an additional variable. Results show that the effect of spread is much higher for less-heavily traded stocks. The coefficients of spread are mostly negative and significant for thinly traded stocks. On average, the absolute value of the spread coefficient for the thinly traded group (-0.77) is much higher than that of the heavily traded group, which is close to zero. Furthermore, the absolute value of the coefficient of the number of transactions is again much higher for the thinly traded group. The average value of this coefficient is -0.55 for the thinly traded group compared to -0.08 for the heavily traded group. Excluding DURA and PSUN does not change the results materially (-0.06 for the heavily traded group). Moreover, the coefficient of trade size (average volume per transaction) is positive, which contrasts with the negative sign of the trade size coefficient for the heavily traded group. Thus, the price intensity appears to be quite different between active and inactive stocks. The response of price intensity to the arrival of information, as captured by the spread and transactions, is much stronger for the thinly traded stocks than for heavily traded stocks. The results support the contention that trades of thinly traded stocks have a larger impact on their duration (or volatility); that is, a trade of thinly traded stock tends to trigger another trade much faster. This result also suggests a greater trade clustering for thinly traded stocks.

2.3 One-Step-Ahead Forecast of Price Durations

Figure 2 shows the one-step-ahead forecast of price durations in a trading day for three selected stocks ASND (July 2), SEBL (July 18) and WIND (July 14), respectively. These dates are chosen such that they are among the days with more transactions after the “thinning” process. The one-step-ahead forecasts are obtained as follows. Consider the ACD(1,1) model for duration in (3). Define $\eta_i = x_i - \psi_i$, which is a martingale difference sequence with mean 0.

Rewrite the ACD (1,1) equation in (3) as

$$x_i = \omega + (\alpha + \beta)x_{i-1} + \eta_i - \beta\eta_{i-1} \qquad \text{(3a)}$$

The price duration x_i therefore has an ARMA(1,1) representation, and its forecast can be obtained using the usual ARMA method.

We employ the adjusted duration and compute its one-step-ahead forecast, using the WACD(1,1) estimates in Table 6. These forecasts are shown in the left-hand panel of Figure 2 for three selected stocks. They are then multiplied back by the intraday periodic pattern (i.e., average duration with a typical shape as shown in Figure 1) according to the time of day the transaction occurs. The resulting forecast of the unadjusted duration is shown in the right-hand panel of Figure 2. The horizontal axis indicates the sequence of transactions for this particular trading day and the vertical axis indicates the duration.

Figure 2 shows that actual durations are in general subject to higher fluctuation than the forecasted durations. As shown, the one-step-ahead forecast obtained by multiplying the adjusted duration forecast and the intraday periodic component performs reasonably well for the less-heavily traded stocks. The forecasts for stock ASND, which is in the heavily traded group, is comparatively more stable at the level of around 1000 (seconds). Incorporating the deterministic intraday periodic component into the forecasting enhances the forecasting precision. The results suggest that one needs to consider the intraday deterministic components to provide a good forecast for price duration.

Engle and Russell (1997) establish a relationship between price durations and volatility. In brief, they assume the underlying price process is a binomial process with increments of $\pm c$ which takes expected time Ψ. They show (see eq. (22) of their paper) that the expected variance per unit of time is inversely related to expected duration; in particular, $\hat{\sigma}_i^2 = c^2 / \psi_i$. Using this relationship, we can transform our estimates of price duration to volatility.

2.4 Impulse Response Function

We next employ the concept of impulse response function to examine the impact of information shock on price duration. Consider the ACD(1,1) model with a microstructure variable z,

$$\psi_i = \omega + \alpha\ x_{i-1} + \beta\psi_{i-1} + \gamma\ z_i \tag{9}$$

Let $\eta_i = x_i - \psi_i$, and rewrite the ACD equation (9) as

$$x_i = \omega + (\alpha\ + \beta)x_{i-1} + \eta_i - \beta\eta_{i-1} + \gamma\ z_i$$

Denote $\mu = \omega/(1-\alpha-\beta)$ as the mean, and B the back-shift operator such that $By_i = y_{i-1}$. It follows that

$$x_i - \mu = \frac{1-\beta\ B}{1-(\alpha+\beta)B}\eta_i + \frac{\gamma}{1-(\alpha+\beta)B} z_i$$

Let $\phi = \alpha\ + \beta$ be the sum of the ARCH and GARCH parameters. The expansion

$$\frac{\gamma}{1-\phi B} = \gamma + \gamma\ \phi\ B + \gamma\ \phi^2 B^2 + ... + \gamma\ \phi^k B^k + ...$$

is useful for studying the lasting effect of the microstructure variable z on duration. Furthermore, the lagged *k* term is $\gamma\phi^k$, which measures the impact of one unit increase in the microstructure variable z on price duration x (mean adjusted) *k*-lags (or *k*-transactions) later. We refer to this term as the impulse response function at lag k. It is clear that for ϕ less than 1 in magnitude, the impact goes down to zero eventually. But the impact decreases with a 'slower' rate or is more persistent, when ϕ is closer to 1. Also, we refer to

$$\gamma + \gamma\ \phi\ + \gamma\ \phi^2 + ... + \gamma\ \phi^k = \gamma(1 - \phi^{k+1})/(1-\phi) \qquad (10)$$

as the *k*-cumulative impulse response function of a unit increase in z on price duration x (mean adjusted) after *k* transactions. It goes to a limiting value of $\gamma/(1-\phi)$. Note that the (first) difference in the k-th and (k-1)th cumulative impulse response function gives $\gamma\phi^k$.

Table 9 reports the k-cumulative impulse response functions of two microstructure variables, number of transactions and average volume per transaction, based on the parameter estimates for adjusted duration in Table 7.[6] For each stock, the first row reports the estimates of ω, α, β, $\alpha+\beta$, γ, and λ. The second row reports the k-cumulative impulse response function of one unit increase in #Trans (the number of transactions per duration) for *k* = 0, 1, ….., 9 lags later, and with the limiting value (when k is infinite) given as the last value. Similarly, the third row is the cumulative response functions that correspond to Volume/Trans (or trade size). The results show that an increase of one unit in #Trans has a higher (negative) impact on price duration for stocks that are less active. On the other hand, the impact of a unit increase in Volume/Trans on price duration is mixed and has no clear pattern across stock groups.

The response function for #Trans is most interesting and is plotted in Figure 3 for each stock group of high, medium and low trading activities. The graph on the top left-hand corner shows the k-cumulative impulse response function (k = 0, 1, …, 99) for adjusted price duration after a unit increase in #Trans. These response functions are obtained based on the average WACD(1,1) and γ estimates for each stock group as reported in Table 7. As shown, the effect of a unit increase in #Trans has a higher (negative) impact on price duration for stocks that are less active.

The two graphs on the bottom show the impulse response function $\gamma\phi^k$. The graph on the lower right-hand corner is similar but with the γ value set equal to –1 for all three groups. This setting aims at singling out the effect caused by the sum of the ARCH and GARCH parameters by standardizing the impact of #Trans. For illustration, consider the thinly traded group in the lower left-hand corner of Figure 3. If #Trans increases by one unit, after one transaction (k = 1), the change in adjusted duration is -0.51*(0.64+0.23)= -0.44 (see Table 7), which is the second point on the impulse response function for this group. We need to divide this number by 100 to obtain the actual amount of reduction because the unit of adjusted duration was multiplied by 100 in Table 7. In other words, the reduction in adjusted duration due to a trade innovation is 0.44% of the average price duration (depends on the time of day) for this group after one transaction. From Table 1A, we can compute the average duration for this group, which is 2,308 seconds. Therefore, the reduction amounts to about 10 seconds per #Trans. Since the average #Trans is 19 (from Table 1A) for this group, the overall reduction in price duration is about 190 seconds. Similar calculation shows the overall reduction is about 56 seconds after k = 10 transactions.

From the lower left-hand corner of Figure 3, we again see that the effect of a unit increase in #Trans has a higher (negative) impact for stocks that are less active, which eventually goes down to zero. After standardizing the impact of number of trades by keeping

[6] Results are qualitatively the same if we add the spread variable.

γ the same (= -1), the graph on the lower right-hand corner shows more clearly that the persistence of impact on the adjusted price duration for the heavily and thinly traded stock groups is about the same. Thus, while trades of thinly traded stocks have a larger impact (or a greater information effect), the duration of the impact is close to that of heavily traded stocks on average.

III. Summary

In this paper, we examine the frequency of information arrivals of small thinly-traded stocks and its impacts on price duration or return volatility at the intraday level. We employ the autoregressive conditional duration (ACD) model to estimate the intensity of information arrivals and information content of trades. The unique feature of this model is its ability to handle high-frequency transaction data recorded at irregular time intervals.

We find that intraday periodicity must be considered in the transaction data analysis. Our results show that the data adjusted for the intraday deterministic pattern produce much more stable parameter estimates. In addition, the accuracy of forecasts is enhanced when the intraday pattern is accounted for in the one-step-ahead forecasting.

Our results show that there are differences in transaction and price durations between heavily and thinly traded stocks. The impact of the number of transactions on adjusted price duration is much larger for thinly traded stocks than for heavily traded stocks. On the other hand, the persistence of the impact on adjusted price duration is about the same between heavily and thinly traded stocks on average. The results show that the number of transactions has higher explanatory power than average trade size. We also examine the impact of spread on price duration. The results show a consistent significantly negative (positive) relationship between spreads and price duration (volatility) only for thinly traded stocks. In addition, the effect of spreads is much stronger for thinly traded stocks, suggesting a larger impact of asymmetric information for these stocks. Overall, we find that the number of trades contains most of the relevant information affecting price duration or volatility, and the trades of thinly traded stocks have greater impact on price duration. The results suggest that the trades of thinly traded stocks contain more private information than the trades of heavily traded stocks.

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Table 1A
Summary Statistics

Stock Symbol	No. of Durations	Ave. Price	Ave. Spread /Duration	Ave. # Tran /Duration	Ave. Daily # Trans	Ave. Vol/Trans	Ave. Price Duration	Ave. Daily Volume (shares)
ASND	1,030	45.55	0.09	333.39	5,723.20	1,497.70	1,083.55	8,431,497
ORCL	612	47.24	0.09	304.34	3,104.27	1,335.79	1,724.23	4,131,497
NSCP	734	40.86	0.13	97.86	1,197.15	1,288.94	1,494.21	1,551,877
SBUX	395	39.97	0.13	82.49	543.06	1,304.65	2,313.90	686,025
DURA	403	39.14	0.22	45.72	307.09	1,687.19	2,446.58	577,185
CLST	519	35.88	0.27	27.62	238.91	1,626.74	1,866.45	435,837
ADTN	664	35.92	0.22	34.17	378.15	1,173.34	1,423.13	439,343
IRIDF	479	34.51	0.31	53.63	428.15	749.06	1,745.48	334,698
PSUN	463	36.72	0.46	14.34	110.66	1,737.35	1,807.68	225,440
SEBL	698	36.19	0.34	15.81	183.92	1,250.37	1,535.55	246,112
SDTI	487	38.20	0.27	18.44	149.67	1,344.45	1,952.97	223,817
APOL	463	37.31	0.29	17.18	132.57	1,428.63	1,965.89	211,208
LHSPF	311	36.16	0.31	25.31	131.19	1,285.17	2,031.17	173,928
WIND	610	41.93	0.35	15.84	161.04	1,322.36	1,704.94	220,510
CBSS	86	36.36	0.28	18.49	26.50	1,500.64	3,886.57	41,073

This table provides summary statistics for stocks in three groups classified based on trading volume. The first group is the high-volume or heavily traded group and the third group is the low-volume or thinly traded group. The medium volume group is in between these two groups. The first group can be classified as the frequently traded group, if DURA is excluded. The second group can be classified as the medium-frequency group if PSUN is excluded while the third group can be classified as the infrequently traded group. The data are “thinned” by ignoring price movements less than two ticks ($0.25). Duration is the time interval between two trades. The duration calculated after thinning is called price duration. Price duration is measured in seconds. Volume is measured in number of shares. The number of durations is the number of observations for the duration variable; average price and spread are expressed in dollars; average #Trans/Duration is the number of transaction per duration; average daily #Trans is the mean transaction number per day; and average Vol./Trans is the average volume (in shares), or trade size per transaction.

Table 1B
Company Names

High-Volume Group	ASND	ASCEND COMMUNICATIONS
	ORCL	ORACLE CORP
	NSCP	NETSCAPE COMMUNICATIONS CORP
	SBUX	STARBUCKS CORP
	DURA	DURA PHARMACEUTICALS, INC.
Medium-Volume Group	CLST	CELLSTAR CORP
	ADTN	ADTRAN INC
	IRIDF	IRIDIUM LLC
	PSUN	PAC SUNWEAR CA
	SEBL	SIEBLE SYSTEMS
Low-Volume Group	SDTI	SECURITY DYNAMICS
	APOL	APOLLO GROUP
	LHSPF	LERNOUT & HAUSPIE SPEECH PRODUCTS
	WIND	WIND RIVER SYSTEMS
	CBSS	COMPASS BNCSHRS

Table 2
Estimates of the EACD(1,1) model for unadjusted price duration

Stock	ω	α	β
ASND	8.34(4.01)	0.26(5.13)	0.41(4.08)
ORCL	7.63(3.54)	0.31(4.81)	0.46(4.64)
NSCP	6.66(3.09)	0.20(4.29)	0.54(4.95)
SBUX	7.45(1.79)	-0.03(-0.93)	0.83(8.72)
DURA	0.70(1.09)	0.01(0.84)	0.97(37.07)
Average		0.15	0.64
Average (without DURA)		0.19	0.56
CLST	1.97(3.39)	0.37(5.64)	0.63(12.68)
ADTN	8.34(4.01)	0.26(5.13)	0.41(4.08)
IRIDF	3.10(4.44)	0.25(5.26)	0.67(15.65)
PSUN	6.27(3.48)	0.16(3.89)	0.64(8.16)
SEBL	3.80(2.31)	0.06(2.82)	0.79(11.11)
Average		0.22	0.63
Average (without PSUN)		0.24	0.63
SDTI	14.78(1.38)	0.02(0.84)	0.51(1.52)
APOL	5.56(2.30)	0.07(2.28)	0.76(8.32)
LHSPF	2.63(2.95)	0.59(5.41)	0.46(7.66)
WIND	7.81(5.82)	0.45(6.69)	0.35(5.52)
CBSS	--	--	--
Average		0.28	0.52

This table reports the parameter estimates of the EACD(1,1) model and the t-values (in parentheses) for three stock groups described in Table 1. The EACD(1,1) model for duration is

$$x_i = \psi_i \ \varepsilon_i$$

$$\psi_i = \omega + \alpha \ x_{i-1} + \beta \psi_{i-1}$$

where x_i is the duration and ψ_i is the conditional mean of the duration between two arrival times t_i and t_{i-1}. The price duration was divided by 60 in estimation. The parameter estimates for CBSS did not converge. Average parameter estimates are mean estimates for each group. Averages without DURA or PSUN are mean parameter estimates excluding each of these two stocks which are removed because their trading frequencies are too low to be qualified in the high and medium frequency groups. The mean estimates excluding these stocks represent the group average using the measure of trade frequency to define the activeness of stocks.

Table 3
Estimates of the WACD(1,1) model for unadjusted price duration

Stock	ω	α	β	θ
ASND	7.07(5.19)	0.40(7.58)	0.23(2.34)	0.91(47.99)
ORCL	7.71(3.36)	0.30(3.88)	0.45(4.29)	0.94(33.37)
NSCP	0.67(1.66)	0.06(3.28)	0.91(30.81)	0.81(36.10)
SBUX	7.84(1.57)	-0.03(-0.89)	0.83(7.49)	0.78(25.51)
DURA	0.53(0.64)	0.01(0.47)	0.98(26.01)	0.71(23.58)
Average		0.15	0.68	
Average (without DURA)		0.18	0.61	
CLST	1.79(2.51)	0.40(4.33)	0.61(9.13)	0.68(35.59)
ADTN	8.62(2.72)	0.22(3.33)	0.42(2.81)	0.72(33.64)
IRIDF	2.96(3.05)	0.29(4.02)	0.64(10.65)	0.68(31.04)
PSUN	5.91(2.36)	0.21(2.71)	0.61(5.16)	0.57(29.19)
SEBL	4.82(1.93)	0.12(2.38)	0.70(5.70)	0.61(37.17)
Average		0.25	0.60	
Average (without PSUN)		0.26	0.59	
SDTI	16.46(1.09)	0.04(0.75)	0.45(0.96)	0.68(29.54)
APOL	5.11(1.60)	0.07(1.58)	0.78(6.66)	0.65(29.48)
LHSPF	2.53(2.30)	0.61(4.49)	0.44(5.82)	0.77(25.05)
WIND	7.80(4.19)	0.46(4.86)	0.33(3.83)	0.67(34.95)
CBSS	41.10(1.85)	-0.24(-4.09)	0.60(2.13)	0.75(11.52)
Average		0.19	0.52	

This table reports the parameter estimates of the WACD(1,1) model and the t-values (in parentheses) for three stock groups described in Table 1. θ is the parameter of the Weibull distribution. Estimation is based on the likelihood function in equation (6). The WACD(1,1) model for duration is

$$x_i = \psi_i \ \varepsilon_i$$

$$\psi_i = \omega + \alpha \ x_{i-1} + \beta \psi_{i-1}$$

where x_i is the duration and ψ_i is the conditional mean of the duration between two arrival times t_i and t_{i-1}. The price duration was divided by 60 in estimation. Average parameter estimates are mean estimates for each group. Averages without DURA or PSUN are mean parameter estimates excluding each of these two stocks which are removed because their trading frequencies are too low to be qualified in the high and medium frequency groups. The mean estimates excluding these stocks represent the group average using the measure of trade frequency to define the activeness of stocks.

Table 4
Estimates of the WACD(1,1) model for unadjusted price duration with number of transactions and trade size

Stock	ω	α	β	θ	#Trans	Ave Vol/ #Trans
ASND	9.18 (5.21)	0.39 (6.67)	0.21 (2.43)	0.91 47.28)	0.02 (0.13)	-1.11(-1.81)
ORCL	5.74 (8.08)	0.32 (2.51)	0.44 (4.31)	0.94 (33.97)	-0.01 (-0.01)	1.55 (1.51)
NSCP	15.07(4.45)	0.39 (3.94)	0.35 (2.91)	0.81 (36.97)	-4.44 (-2.91)	-2.81 (-1.58)
SUBX	15.46(2.73)	-0.07(-1.17)	0.77 (7.87)	0.78 (28.33)	1.91 (0.75)	-4.37 (-2.26)
DURA	11.44(1.37)	0.05 (0.96)	0.74 (3.91)	0.71 (28.00)	-5.55 (-1.08)	-0.48 (-0.31)
Average	11.38	0.22	0.50	0.83	-1.61	-1.44
Average (without DURA)	11.36	0.26	0.44	0.86	-0.63	-1.69
CLST	4.90 (3.33)	0.51 (4.60)	0.57 (8.43)	0.68 (31.45)	-10.33 (-1.87)	-1.07 (-1.74)
ADTN	4.40 (2.38)	0.03 (0.89)	0.89 (15.93)	0.73 (35.15)	-4.52 (-3.55)	-0.91 (-0.95)
IRIDF	2.16 (1.19)	0.35 (3.62)	0.65 (9.94)	0.68 (28.09)	-4.58 (-1.72)	2.00 (0.79)
PSUN	5.11 (1.69)	0.24 (2.61)	0.56 (4.50)	0.57 (28.79)	-7.16 (-0.64)	1.48 (1.10)
SEBL	3.73 (1.24)	0.16 (2.57)	0.64 (4.27)	0.62 (36.75)	-11.11 (-1.47)	2.54 (1.95)
Average	4.06	0.26	0.66	0.66	-7.54	0.81
Average (without PSUN)	3.80	0.26	0.69	0.68	-7.64	0.64
SDTI	6.72(9.89)	0.05 (0.90)	0.77 (7.97)	0.68 (30.02)	-21.11 (-1.50)	2.25 (1.24)
APOL	2.47 (0.99)	0.08 (1.83)	0.80 (10.19)	0.65 (29.00)	-18.23 (-1.30)	3.35 (1.86)
LHSPF	-2.07(-2.28)	0.72 (5.11)	0.44 (6.26)	0.79 (25.52)	-12.67 (-2.22)	4.72 (3.48)
WIND	6.76 (2.79)	0.55 (4.32)	0.33 (3.88)	0.67 (33.46)	-17.40 (-1.30)	1.38 (1.28)
CBSS	41.40(5.23)	-0.27(-4.15)	0.61 (6.19)	0.78 (11.39)	-15.54 (-0.76)	4.82 (1.17)
Average	21.06	0.23	0.59	0.714	-16.99	3.30

This table reports the parameter estimates of the WACD(1,1) model with the number of transaction and trade size, and the t-values (in parentheses) for three stock groups described in Table 1. θ is the parameter of the Weibull distribution. Estimation is based on the model

$$\psi_i = \omega + \alpha\ x_{i-1} + \beta\psi_{i-1} + \gamma \#Trans + \lambda\ Volume/Trans$$

where x_i is the duration and ψ_i is the conditional mean of the duration between two arrival times t_i and t_{i-1}; #Trans is the number of transactions per duration and Volume/Trans is trade size or average volume per transaction. The unadjusted duration was divided by 60, the number of transactions was divided by 100, and the average volume over number of transactions was divided by 1000. Average parameter estimates are mean estimates for each group. Averages without DURA or PSUN are mean parameter estimates excluding each of these two stocks which are removed because their trading frequencies are too low to be qualified in the high and medium frequency groups. The mean estimates excluding these stocks represent the group average using the measure of trade frequency to define the activeness of stocks.

Table 5
Estimates of the WACD(1,1) model for unadjusted price duration with spread, number of transactions and average trade size

Stock	ω	α	β	θ	Spread	#Trans	Ave Vol/ #Trans
ASND	3.80 (1.72)	0.02 (0.86)	0.91 (17.28)	0.73 (34.35)	0.01 (0.40)	-4.22 (-3.03)	-0.84 (-0.93)
ORCL	9.52 (2.81)	0.31 (2.96)	0.48 (5.12)	0.94 (34.42)	-0.42 (-3.28)	-0.10 (-0.17)	0.98 (0.52)
NSCP	16.99 (6.67)	0.34 (4.22)	0.44 (4.96)	0.81 (37.93)	-0.38 (-6.85)	-4.26 (-4.38)	-1.31 (-0.96)
SUBX	50.60 (4.43)	0.33 (2.98)	-0.40 (-2.67)	0.79 (26.65)	0.69 (1.43)	-13.09 (-2.09)	-5.52 (-1.52)
DURA	--	--	--	--	--	--	--
Average	21.91	0.34	0.18	0.87	-0.07	-4.37	-1.71
CLST	7.91 (3.27)	0.54 (4.88)	0.55 (8.23)	0.68 (31.91)	-0.12 (-2.23)	-9.94 (-2.16)	-0.98 (-0.76)
ADTN	3.80 (1.72)	0.02 (0.86)	0.91 (17.28)	0.73 (34.35)	0.01 (0.40)	-4.22 (-3.03)	-0.84 (-0.93)
IRIDF	4.13 (1.40)	0.34 (3.83)	0.66 (10.58)	0.68 (33.69)	-0.05 (-0.92)	-4.49 (-1.87)	1.26 (0.52)
PSUN	3.73 (1.06)	0.24 (2.52)	0.55 (4.77)	0.57 (28.01)	0.04 (0.56)	-8.36 (-0.67)	1.51 (1.30)
SEBL	6.01 (1.61)	0.17 (2.77)	0.62 (4.72)	0.62 (36.87)	-0.05 (-0.80)	-13.14 (-2.01)	2.64 (2.16)
Average	5.12	0.26	0.66	0.66	-0.03	-8.03	0.72
Average (without PSUN)	5.46	0.27	0.69	0.68	-0.05	-7.95	0.52
SDTI	27.91 (1.80)	0.10 (1.28)	0.29 (0.66)	0.68 (30.91)	-0.13 (-1.31)	-25.21 (-1.49)	0.06 (0.05)
APOL	10.99 (1.10)	0.10 (1.68)	0.72 (3.73)	0.65 (29.09)	-0.23 (-1.49)	-20.44 (-1.14)	3.49 (1.53)
LHSPF	2.62 (0.62)	0.53 (1.84)	0.52 (2.91)	0.80 (25.34)	-0.09 (-1.56)	-16.63 (-4.12)	4.21 (1.84)
WIND	13.07 (3.82)	0.54 (4.49)	0.34 (4.23)	0.68 (32.45)	-0.15 (-2.50)	-21.71 (-1.66)	1.15 (1.17)
CBSS	25.91 (1.56)	-0.24 (-5.18)	0.21 (1.86)	0.81 (11.98)	-1.24 (-2.24)	18.01 (0.41)	4.59 (0.56)
Average	16.10	0.21	0.42	0.72	-0.37	-13.20	2.70

This table reports the parameter estimates of the WACD(1,1) model with spread, the number of transactions, and trade size, and the t-values (in parentheses) for three stock groups described in Table 1. θ is the parameter of the Weibull distribution. Estimation is based on the model

$$\psi_i = \omega + \alpha \ x_{i-1} + \beta \psi_{i-1} + \delta \ Spread + \gamma \#Trans + \lambda \ Volume / Trans$$

Spread is the percentage bid-ask spread and the remaining variables are as defined in Table 4. The unadjusted duration was divided by 60, the number of transactions was divided by 100, the average volume over number of transactions was divided by 1000 and the spread was multiplied by 100. The estimates for DURA did not converge. Average parameter estimates are mean estimates for each group. Averages without DURA or PSUN are mean parameter estimates excluding each of these two stocks which are removed because their trading frequencies are too low to be qualified in the high and medium frequency groups. The mean estimates excluding these stocks represent the group average using the measure of trade frequency to define the activeness of stocks.

Table 6
Estimates of the WACD(1,1) model for adjusted price duration

Stock	ω	α	β	θ
ASND	7.35(2.21)	0.15(4.25)	0.78(12.43)	1.05(41.77)
ORCL	7.40(2.64)	0.13(4.73)	0.80(20.90)	1.16(32.20)
NSCP	3.70(1.98)	0.11(3.66)	0.85(21.34)	0.96(35.32)
SUBX	31.91(1.75)	0.12(2.00)	0.56(2.72)	1.02(25.99)
DURA	7.51(0.92)	0.05(1.49)	0.87(8.11)	0.82(23.13)
Average		0.11	0.77	
Average (without DURA)		0.13	0.75	
CLST	6.09(2.19)	0.31(4.16)	0.66(8.83)	0.81(28.70)
ADTN	16.17(2.41)	0.12(3.17)	0.72(8.21)	0.83(35.44)
IRIDF	11.12(3.04)	0.22(3.91)	0.68(11.56)	0.84(31.13)
PSUN	31.99(2.21)	0.22(2.73)	0.48(2.78)	0.66(27.75)
SEBL	12.63(2.62)	0.12(3.57)	0.76(12.56)	0.71(33.31)
Average		0.20	0.66	
Average (without DURA)		0.19	0.71	
SDTI	15.30(1.22)	0.06(1.55)	0.79(5.72)	0.76(27.21)
APOL	16.15(0.93)	0.02(0.61)	0.82(4.52)	0.74(26.60)
LHSPF	7.19(2.38)	0.44(4.47)	0.54(7.16)	0.89(23.76)
WIND	20.99(2.75)	0.33(4.30)	0.49(4.35)	0.75(32.29)
CBSS	13.50(1.14)	0.14(1.13)	0.73(3.54)	1.07(12.58)
Average		0.20	0.67	

This table reports the parameter estimates of the WACD(1,1) model, and the t-value (in parentheses) for three stock groups described in Table 1. θ is the parameter of the Weibull distribution. Estimation is based on the likelihood function in equation (6). The WACD(1,1) model for duration is

$$x_i = \psi_i \ \varepsilon_i$$

$$\psi_i = \omega + \alpha \ x_{i-1} + \beta \psi_{i-1}$$

where x_i is the duration and ψ_i is the conditional mean of the duration between two arrival times t_i and t_{i-1}. The adjusted price duration was multiplied by 100 in estimation. Average parameter estimates are mean estimates for each group. Averages without DURA or PSUN are mean parameter estimates excluding each of these two stocks which are removed because their trading frequencies are too low to be qualified in the high and medium frequency groups. The mean estimates excluding these stocks represent the group average using the measure of trade frequency to define the activeness of stocks.

Table 7
Estimates of the WACD(1,1) model for adjusted price duration with number of transactions and average volume/number of transactions

Stock	ω	α	β	θ	#Trans	Ave Vol/ #Trans
ASND	10.50 (4.98)	0.19 (1.78)	0.74 (24.79)	1.05 (34.56)	-0.01 (-1.36)	-0.68 (-1.96)
ORCL	3.18 (1.04)	0.19 (4.18)	0.73 (11.19)	1.16 (30.43)	-0.01 (-1.56)	6.35 (1.21)
NSCP	5.45 (4.25)	0.13 (3.22)	0.84 (15.12)	0.96 (34.41)	-0.05 (-2.08)	1.18 (0.61)
SBUX	62.96 (3.04)	0.19 (2.56)	0.44 (2.33)	1.03 (24.32)	-0.16 (-2.62)	-9.69 (-1.61)
DURA	28.98 (1.97)	0.12 (2.39)	0.71 (5.47)	0.83 (22.29)	-0.20 (-1.73)	-1.93 (-0.54)
Average	22.21	0.16	0.69	1.01	-0.09	-0.95
Average (without DURA)	20.52	0.18	0.69	1.05	-0.06	-0.71
CLST	15.35 (5.71)	0.37 (5.07)	0.72 (12.49)	0.83 (30.69)	-0.66 (-6.03)	-1.41 (-1.68)
ADTN	21.08 (4.98)	0.04 (1.78)	0.87 (24.79)	0.85 (34.56)	-0.23 (-4.46)	-4.39 (-1.96)
IRIDF	9.55 (1.45)	0.23 (4.79)	0.69 (13.43)	0.84 (28.68)	-0.05 (-1.05)	2.98 (0.42)
PSUN	31.50 (2.26)	0.29 (3.04)	0.41 (13.00)	0.66 (27.35)	-0.68 (-1.73)	6.68 (1.32)
SEBL	11.23 (8.85)	0.13 (3.23)	0.74 (14.11)	0.72 (32.34)	-0.11 (-0.58)	3.29 (1.14)
Average	17.74	0.21	0.69	0.78	-0.35	1.43
Average (without PSUN)	14.30	0.19	0.76	0.81	-0.26	0.12
SDTI	60.27 (2.16)	0.15 (2.47)	0.44 (1.84)	0.77 (28.10)	-0.97 (-4.76)	-0.93 (-0.14)
APOL	9.92 (48.06)	0.03 (1.04)	0.88 (13.67)	0.74 (25.79)	-0.29 (-0.95)	2.79 (0.76)
LHSPF	-0.02 (-0.03)	0.43 (4.69)	0.56 (8.37)	0.89 (24.70)	-0.17 (-1.85)	8.00 (3.06)
WIND	20.89 (2.46)	0.39 (3.84)	0.49 (4.39)	0.76 (31.52)	-0.58 (-1.67)	2.55 (0.88)
CBSS	6.45 (129.69)	0.13 (1.71)	0.85 (10.55)	1.09 (22.61)	-0.52 (-1.37)	3.66 (47.63)
Average	19.50	0.23	0.64	0.85	-0.51	3.21

The table reports the parameter estimates of the WACD(1,1) model with the number of transactions and average volume, and the t-value (in parentheses) for three stock groups described in Table 1. θ is the parameter of the Weibull distribution. Estimation is based on the model

$$\psi_i = \omega + \alpha\, x_{i-1} + \beta\psi_{i-1} + \gamma \#Trans + \lambda\, Volume / Trans$$

where the variables are as defined in Table 4. The adjusted duration was multiplied by 100, and the average volume over number of transactions was divided by 1000. Average parameter estimates are mean estimates for each group. Averages without DURA or PSUN are mean parameter estimates excluding each of these two stocks which are removed because their trading frequencies are too low to be qualified in the high and medium frequency groups. The mean estimates excluding these stocks represent the group average using the measure of trade frequency to define the activeness of stocks.

Table 8
Estimates of the WACD(1,1) model for adjusted price duration with spread, number of transactions and average trade size

Stock	ω	α	β	θ	Spread	#Trans	Ave Vol/ #Trans
ASND	15.95 (3.87)	0.21 (5.23)	0.72 (13.08)	1.05 (43.52)	-0.43 (-1.72)	-0.01 (-2.09)	-0.75 (-0.98)
ORCL	10.01 (2.40)	0.16 (4.44)	0.79 (18.94)	1.16 (33.77)	-0.76 (-3.19)	-0.01 (-2.15)	4.59 (1.80)
NSCP	8.43 (1.09)	0.14 (3.30)	0.83 (13.36)	0.96 (35.18)	-0.16 (-1.06)	-0.05 (-3.06)	1.65 (0.50)
SBUX	60.68 (2.91)	0.20 (2.66)	0.41 (2.32)	1.03 (25.76)	0.42 (0.59)	-0.17 (-2.42)	-10.38 (-1.71)
DURA	0.67 (0.12)	0.10 (1.08)	0.79 (5.01)	0.83 (20.78)	1.05 (1.67)	-0.16 (-1.17)	-2.83 (-0.79)
Average	19.15	0.16	0.71	1.01	0.02	-0.08	-1.54
Average (without DURA)	23.77	0.18	0.69	1.05	-0.23	-0.06	-1.22
CLST	22.45 (3.68)	0.34 (5.26)	0.74 (13.33)	0.83 (31.11)	-0.22 (-1.36)	-0.63 (-7.80)	-2.57 (-2.23)
ADTN	20.15 (2.61)	0.04 (1.33)	0.87 (15.51)	0.85 (35.85)	0.04 (0.27)	-0.22 (-3.33)	-4.49 (-1.35)
IRIDF	12.40 (1.26)	0.24 (3.37)	0.68 (8.28)	0.84 (28.61)	-0.07 (-0.43)	-0.06 (-0.92)	2.74 (0.40)
PSUN	19.27 (1.21)	0.30 (3.15)	0.41 (2.97)	0.67 (27.39)	0.27 (1.22)	-0.75 (-1.97)	6.29 (1.34)
SEBL	14.51 (1.22)	0.13 (3.01)	0.74 (9.12)	0.72 (33.11)	-0.09 (-0.40)	-0.14 (-0.74)	3.55 (0.75)
Average	17.76	0.21	0.69	0.78	-0.01	-0.36	1.10
Average (without PSUN)	17.38	0.19	0.76	0.81	-0.09	-0.26	-0.19
SDTI	64.12 (2.46)	0.14 (2.80)	0.47 (2.06)	0.77 (26.75)	-0.23 (-0.74)	-0.93 (-4.90)	-1.07 (-0.19)
APOL	111.88 (2.84)	0.01 (0.19)	0.25 (0.89)	0.75 (27.58)	-1.41 (-3.93)	-0.27 (-0.55)	5.93 (0.84)
LHSPF	14.30 (1.83)	0.39 (4.75)	0.59 (8.62)	0.91 (22.70)	-0.35 (-3.17)	-0.34 (-2.46)	8.28 (2.13)
WIND	35.76 (3.22)	0.41 (4.48)	0.48 (4.68)	0.76 (31.51)	-0.33 (-1.82)	-0.76 (-2.28)	1.80 (0.59)
CBSS	57.54 (3.92)	0.62 (5.23)	0.20 (1.66)	1.12 (11.82)	-1.53 (-5.44)	-0.44 (-0.89)	16.36 (2.19)
Average	56.72	0.31	0.40	0.86	-0.77	-0.55	6.26

This table reports the parameter estimates of the WACD(1,1) model with spreads, the number of transactions, and trade size for three stock groups described in Table 1. t-values are in parentheses. θ is the parameter of the Weibull distribution. Estimation is based on the model

$$\psi_i = \omega + \alpha\ x_{i-1} + \beta\psi_{i-1} + \delta\ Spread + \gamma\#Trans + \lambda\ Volume/Trans$$

where the variables are as defined in Table 5. The adjusted duration was multiplied by 100, the average volume over number of transactions was divided by 1000, and the spread was multiplied by 100. Average parameter estimates are mean estimates for each group. Averages without DURA or PSUN are mean parameter estimates excluding each of these two stocks which are removed because their trading frequencies are too low to be qualified in the high and medium frequency groups. The mean estimates excluding these stocks represent the group average using the measure of trade frequency to define the activeness of stocks.

Table 9

Cumulative impulse response function of the microstructure variables (number of transactions, and average volume/number of transactions) for adjusted duration

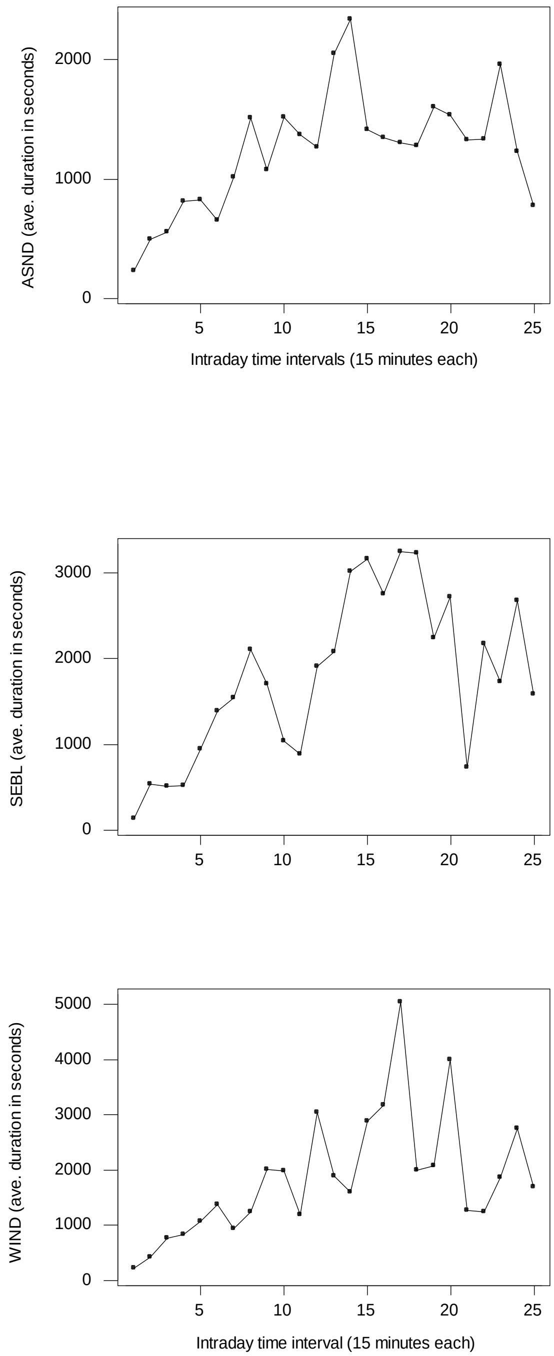
Stock	First row: $\omega, \alpha, \beta, \alpha+\beta$, and γ, λ. Second and third row: cumulative impulse response function of # trans and volume/trans									
ASND	10.50	0.19	0.74	0.93	-0.01	-0.68				
	-0.0	-0.0	-0.0	-0.0	-0.0	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
	-0.7	-1.3	-1.9	-2.4	-3.0	-3.4	-3.9	-4.3	-4.7	-5.0	-9.7
ORCL	3.18	0.19	0.73	0.92	-0.01	6.35				
	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0	-0.1	-0.1	-0.1	-0.1	-0.1
	6.3	12.2	17.6	22.5	27.1	31.2	35.1	38.6	41.9	44.9	79.4
NSCP	5.45	0.13	0.84	0.97	-0.05	1.18				
	-0.1	-0.1	-0.1	-0.2	-0.2	-0.3	-0.3	-0.4	-0.4	-0.4	-1.7
	1.2	2.3	3.4	4.5	5.6	6.6	7.6	8.5	9.4	10.3	39.3
SBUX	62.96	0.19	0.44	0.63	-0.16	-9.69				
	-0.2	-0.3	-0.3	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4	-0.4
	-9.7	-15.8	-19.6	-22.1	-23.6	-24.6	-25.2	-25.5	-25.8	-25.9	-26.2
DURA	28.98	0.12	0.71	0.83	-0.20	-1.93				
	-0.2	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	-0.9	-1.0	-1.0	-1.2
	-1.9	-3.5	-4.9	-6.0	-6.9	-7.6	-8.3	-8.8	-9.2	-9.6	-11.4
CLST	15.35	0.37	0.72	1.09	-0.66	-1.41				
	$\alpha+\beta > 1$										
ADTN	21.08	0.04	0.87	0.91	-0.23	-4.39				
	-0.2	-0.4	-0.6	-0.8	-1.0	-1.1	-1.2	-1.4	-1.5	-1.6	-2.6
	-4.4	-8.4	-12.0	-15.3	-18.3	-21.1	-23.6	-25.8	-27.9	-29.8	-48.8
IRIDF	9.55	0.23	0.69	0.92	-0.05	2.98				
	-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.3	-0.3	-0.3	-0.4	-0.6
	3.0	5.7	8.2	10.6	12.7	14.7	16.5	18.1	19.7	21.1	37.3
PSUN	31.50	0.29	0.41	0.70	-0.68	6.68				
	-0.7	-1.2	-1.5	-1.7	-1.9	-2.0	-2.1	-2.1	-2.2	-2.2	-2.3
	6.7	11.4	14.6	16.9	18.5	19.6	20.4	21.0	21.4	21.6	22.3
SEBL	11.23	0.13	0.74	0.87	-0.11	3.29				
	-0.1	-0.2	-0.3	-0.4	-0.4	-0.5	-0.5	-0.6	-0.6	-0.6	-0.8
	3.3	6.2	8.6	10.8	12.7	14.3	15.8	17.0	18.1	19.0	25.3

Table 9
Continued

SDTI	60.27	0.15	0.44	0.59	-0.97	-0.93					
	-1.0	-1.5	-1.9	-2.1	-2.2	-2.3	-2.3	-2.3	-2.3	-2.4	-2.4
	-0.9	-1.5	-1.8	-2.0	-2.1	-2.2	-2.2	-2.2	-2.2	-2.3	-2.3
APOL	9.92	0.03	0.88	0.91	-0.29	2.79					
	-0.3	-0.6	-0.8	-1.0	-1.2	-1.4	-1.6	-1.7	-1.8	-2.0	-3.2
	2.8	5.3	7.6	9.7	11.7	13.4	15.0	16.4	17.7	18.9	31.0
LHSPF	-0.02	0.43	0.56	0.99	-0.17	8.00					
	-0.2	-0.3	-0.5	-0.7	-0.8	-1.0	-1.2	-1.3	-1.5	-1.6	-17.0
	8.0	15.9	23.8	31.5	39.2	46.8	54.3	61.8	69.2	76.5	800.0
WIND	20.89	0.39	0.49	0.88	-0.58	2.55					
	-0.6	-1.1	-1.5	-1.9	-2.3	-2.6	-2.9	-3.1	-3.3	-3.5	-4.8
	2.5	4.8	6.8	8.5	10.0	11.4	12.6	13.6	14.5	15.3	21.2
CBSS	6.45	0.13	0.85	0.98	-0.52	3.66					
	-0.5	-1.0	-1.5	-2.0	-2.5	-3.0	-3.4	-3.9	-4.3	-4.8	-26.0
	3.7	7.2	10.8	14.2	17.6	20.9	24.1	27.3	30.4	33.5	183.0

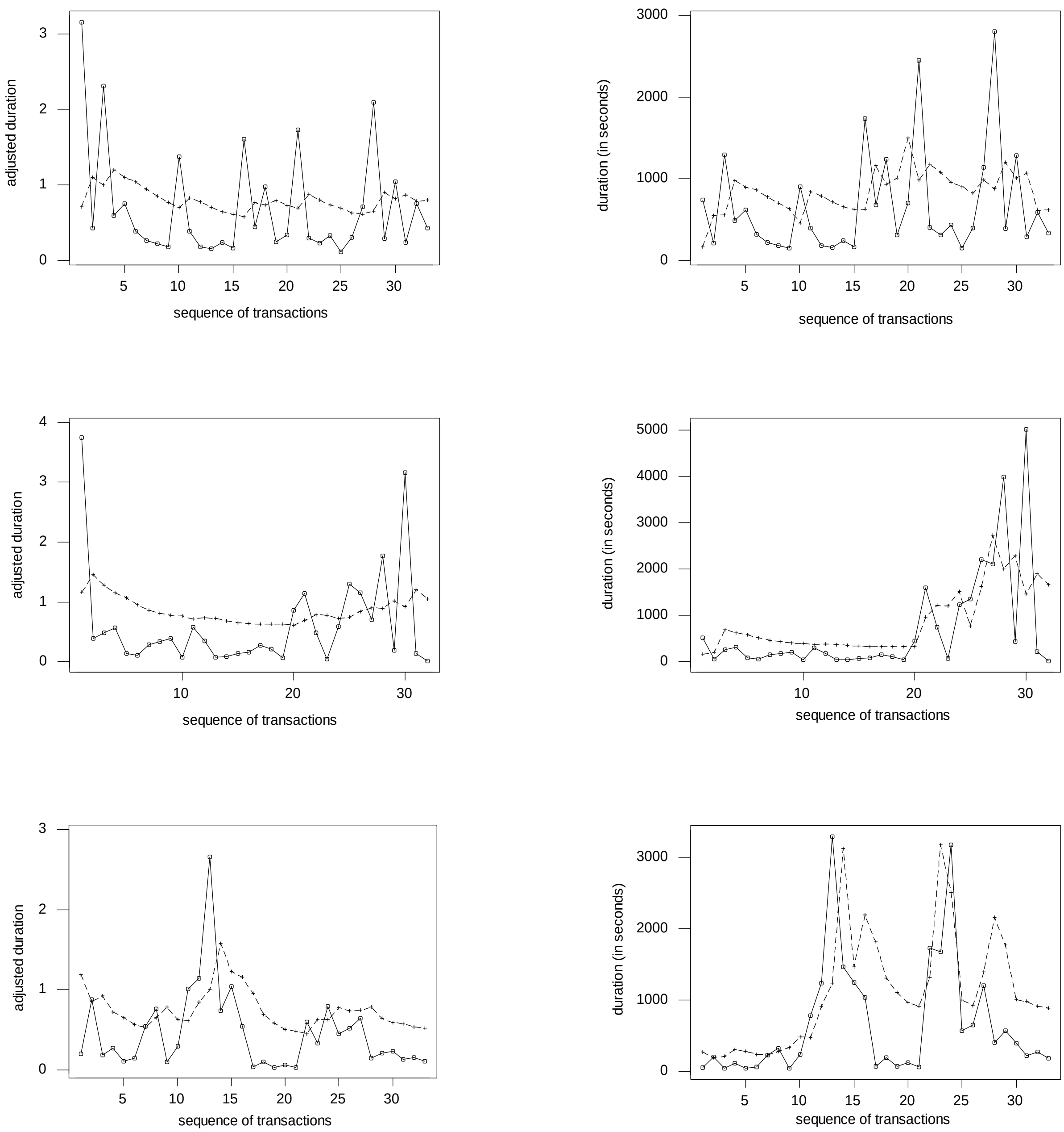
For each stock, the first row reports the estimates of the WACD parameters ω, α, β, $\alpha+\beta$, γ, and λ in Table 7. The units used in this table follow from Table 7. The second row reports the k-cumulative impulse response function of one unit increase in #Trans (the number of transactions) for k = 0, 1, ….., 9 lags later, and with the limiting value (when k is infinite) given as the last value. The value indicates the cumulative response in term of percentage change in average price duration. The third row reports the response functions corresponding to Volume/trans (or trade size). The value (times 0.001) indicates the cumulative response in term of percentage change in average price duration. The response function is not reported if $\alpha+\beta$ is negative or $\geq$ 1.

Figure 1
Average Price Duration for A Typical Trading Day



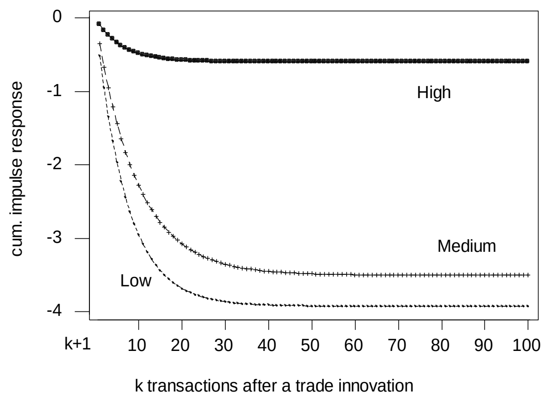
The time span from 9:45:00 to 16:00:00 in a trading day is divided into 25 intraday time intervals of 15 minutes each. The average price duration within each interval is computed over the entire sample period. Plots of the average price duration (in seconds) over the 25 intervals for stocks ASND, SEBL and WIND are shown.

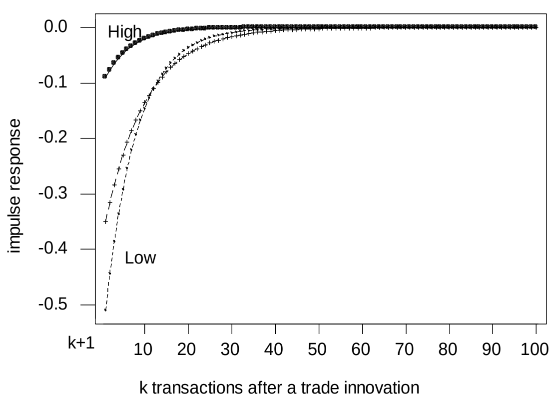
Figure 2
One-Step-Ahead Forecast of Price Duration

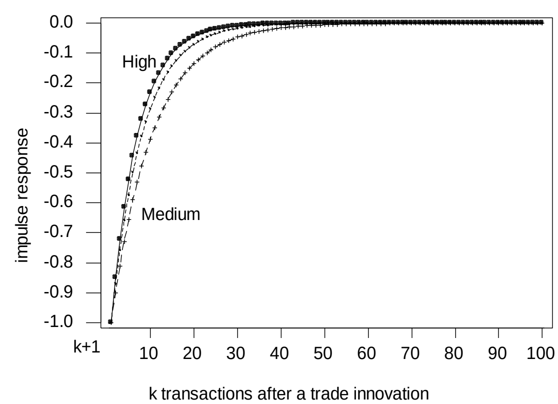


One-step-ahead forecast of price duration for a selected day with more transactions for three selected stocks: ASND (July 2), SEBL (July 18), and WIND (July 14). The graphs on the left show the forecast of adjusted durations using the WACD(1,1) model for adjusted durations in Table 6; it is then multiplied by the intraday periodic pattern to obtain the forecasts for unadjusted durations (graphs on the right). The adjusted duration on the vertical axis in the left-side panel is expressed in terms of ratios (the series has a mean approximately equal to 1) and the unadjusted duration on the vertical axis of the right panel is expressed in seconds. In all graphs, the horizontal axis indicates the sequence of transactions for each trading day. The dashed line represents forecast values while the solid line represents actual values.

Figure 3
Impulse Response for Adjusted Duration to Trade Innovations







The vertical axis of the graphs indicates the response in terms of percentage change in average price duration, and the horizontal axis indicates the number of transactions (k = 0, 1, …, 99) taking place after one unit increase (innovation) in the variable #Trans. The graph on the top left-hand corner shows the k-cumulative impulse response function for adjusted price duration after a trade innovation (i.e., a unit increase in #Trans, the number of transactions). These response functions are obtained based on the average WACD(1,1) and γ estimates for each stock group of high, medium and low trading activities reported in Table 7. The bottom graphs are the impulse response function $\gamma\phi^k$. The graph on the lower right-hand side is the response function with the γ value sets equal to –1 (standardized) for all three stock groups. For illustration, in the lower left-hand graph, when there is a trade innovation, the effect of that trade on the adjusted duration will go down by 0.44% (0.13%) after k = 1 (10) transactions for the thinly traded group.