Lower-Bound on Blocking Probability of A Class of Crosstalk-free Optical Cross-connects (OXCs)

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Abstract - A combination of horizontal expansion and vertical stacking of optical Banyan (HVOB) is the general architecture for building Banyan-based optical cross-connects (OXCs), and the intrinsic crosstalk problem of optical signals is a major constraint in designing OXCs. In this paper, we analyze the blocking behavior of HVOB networks and develop the lower-bound on blocking probability of a HVOB network that is free of first-order crosstalk in switching elements (SEs). The proposed lower-bound is significant because it provides network designers an effective tool to estimate the minimum blocking probability they can expect from a HVOB architecture regardless what kind of routing strategy to be adopted. Our lower-bound can accurately depict the overall blocking behavior in terms of the minimum blocking probability in a HVOB network, as verified by extensive simulation based on a network simulator with both random routing and packing routing strategies. Surprisingly, the simulated and theoretical results show that our lower-bound can be used to efficiently estimate the blocking probability of HVOB networks applying packing strategy. Thus, our analytical model can guide network designers to find the tradeoff among the number of planes (stacked copies), the number of SEs, the number of stages and blocking probability in a HVOB network applying packing strategy.

Index Terms –OXCs, banyan networks, blocking probability, horizontal expansion and vertical stacking, crosstalk.

I. INTRODUCTION

Optical network technology is expected to form the base infrastructure for the next-generation data-centric Internet. The recent development of such technology has already increased network capacities significantly, providing scalable bandwidth levels for rising traffic demands. The commercially available wavelength-division multiplexing (WDM) transmission systems can yield massive capacity relief between adjacent network elements (e.g., routers, switches), where each wavelength channel essentially provides a high bandwidth "virtual fiber" and recent advances in commercial products indicate that the number of wavelengths can be as large as 100 per fiber [1].

With the sprouting deployment of WDM transmission facilities in the telecommunications field, the next logical step is to deploy light-path switches to reconfigure the WDM facilities. This requires devices that can switch light signals with multiple wavelengths. Directional coupler (DC) is such a device, and DCs can handle signals with the speed of some terabits per second and with multiple wavelengths [2],[3]. A DC is an electro-optical switching device that is created by putting two channel waveguides close to each other. It has a switch function similar to a 2×2 switching element (SE) with both the cross and bar states. DCs are best used in a circuit switching environment because once the state of the coupler is set up electronically, the couplers become transparent and optical signals can pass through the coupler with the same speed as they travel in optical fibers.

To build a large-scale optical switch with good scalability, numerous basic 2×2 switching elements (SEs) are usually grouped in multiple stages with a specified interconnection topology between adjacent stages. The basic SEs and the interconnecting optical links will form a switching network such that the optical flows arriving at inputs can be switched appropriately to outputs as requested. Banyan networks [4],[5],[6],[7]are a class of attractive interconnection topologies for constructing DC-based optical switching networks because they have a simple switch setting ability (self-routing), as well as a smaller and identical number of SEs along any path between an input-output pair; therefore, absolute loss uniformity and smaller attenuation of optical signals are guaranteed in such networks. However, with a banyan topology, only a unique path can be found from each network input to each network output, which degrades the network to a blocking one. A general approach to building banyan-based nonblocking optical switching networks is to jointly perform horizontal expansion and vertical stacking [8],[9], in which a regular banyan network is first horizontally expanded by adding some extra stages to the back of the network, and then multiple copies of the horizontally expanded banyan network are vertically stacked as illustrated in Fig.1. We use HVOB to denote a DC-based optical switching network built on a combination of horizontal expansion and vertical stacking of optical banyan networks.

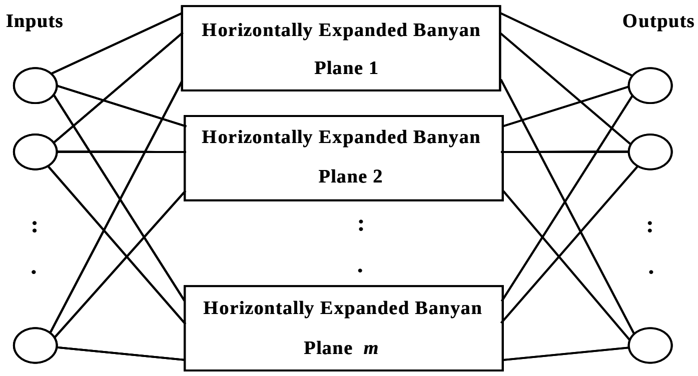


Fig.1 A combination of horizontal expansion and vertical stacking of banyan networks.

A major shortcoming of a directional coupler is *crosstalk* [10]. When two optical signals traverse through a DC at the same time, a small portion of optical power in one waveguide will be coupled into the other unintended waveguide. This undesirable coupling is called the *first-order crosstalk*. This first-order crosstalk will propagate downstream stage by stage, leading to a higher order crosstalk in each downstream stage with a decreasing magnitude. A cost-effective solution to the crosstalk problem is to guarantee that only one signal passes through a DC at a time, thus eliminating the first-order crosstalk. Although signal attenuation is another shortcoming of a DC-based photonic switching system, it can be resolved by optical amplifiers due to the passive nature of the device. Unfortunately, optical amplifiers are usually linear in the sense that they amplify signals as well as crosstalk. Due to the stringent bit-error rate requirement of fiber optics, crosstalk elimination has become an important issue for improving the signal-to-noise ratio of the optical flow transmission.

Numerous results are available in studies of HVOB networks, such as [8],[9],[11],[12], and their main focus has been on determining the minimum number of stacked copies (planes) required for a nonblocking HVOB network. These results indicate that the HVOB structure, although is attractive, usually requires either a high hardware cost or a large network depth to guarantee the nonblocking property. Blocking behavior analysis of a network is an effective approach to the study of network performance and to finding a desirable trade-off between hardware cost and blocking probability. Lee [13] and Jacobaeus [14] have developed two well-known probabilistic models for analyzing the blocking behavior of Clos networks [15]. A number of studies with approaches similar to those proposed by Lee and Jacobaeus have been conducted to analyze the performance of banyan networks [5],[6],[16],[17]; however, they present probabilistic results only for electronic networks. In other words, these studies only addressed link-blocking. Some analytical models have been developed to understand the blocking behaviors of vertically stacked optical banyan networks (without horizontal expansion) that do not meet the nonblocking condition (i.e., with fewer stacked copies than required by the nonblocking condition) [18],[19],[20]. To our best knowledge, however, no research has been reported for modeling and evaluating the performance behavior of general HVOB networks, in which not only the number of planes (network hardware cost) but also the number of stages (network depth) are incorporated in the performance analysis.

In this paper, we focus on the HVOB networks that are free of first-order crosstalk in each SE (we refer to this quality as '*crosstalk-free*' hereafter) and derive the lower bound on the blocking probability of a HVOB network with respect to the number of planes and number of stages in the network. The lower-bound indicates the inherent minimum blocking probability a HVOB architecture can provide no matter what kind of routing strategy to be adopted. In particular, the proposed lower-bound can be used to efficiently estimate the blocking probability of HVOB networks adopting packing routing strategy [21], in which a graceful compromise among blocking probability, network hardware cost, and network depth can be explored.

The rest of the paper is organized as follows. Section 2 provides preliminaries that will facilitate the discussion. Section 3 introduces the proposed lower-bound for a HVOB network. Section 4 presents the simulation results, which are compared with the theoretical ones estimated by our analytical model. Section 5 concludes the paper.

II. PRELIMINARIES

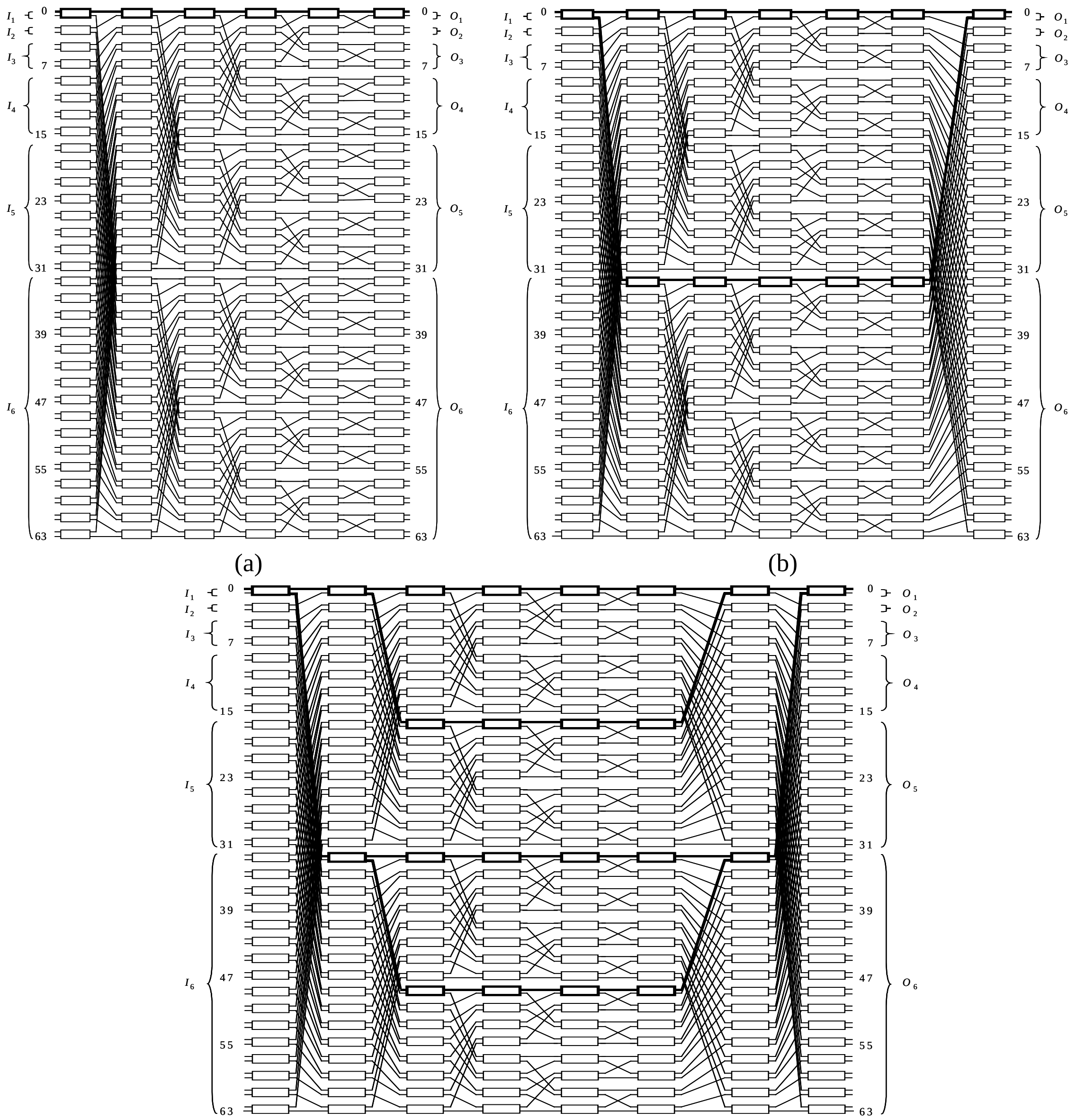
A typical $N\times N$ banyan network has $\log N$ stages * and one unique path between any input-output pair. One basic technique for creating multiple paths between an input-output pair is horizontal expansion, in which the reverse of the first x ($1\leq x\leq \log N-1$) stages of a regular $N\times N$ banyan network is appended to the back of the network such that 2^x paths are created between the input-output pair, as illustrated in Fig.2 for a 64×64 banyan network. Another technique for generating multiple paths between an input-output pair is the vertical stacking of multiple banyan networks [22]. The general scheme for building banyan-based optical switching networks is a combination of the horizontal expansion and vertical stacking of an optical banyan network [8],[9], as illustrated in Fig.1. For simplicity, we use HVOB(N,m,x) to denote an $N\times N$ HVOB network that has m stacked planes of an $N\times N$ optical banyan network with x extra stages.

The consideration of the crosstalk-free constraint distinguishes the analysis of optical switching networks from that of electronic ones. In electronic switching networks, blocking occurs when two connections intend to use the same link, which is referred to as *link-blocking*. Obviously, all signals passing through a network should follow link-disjoint paths in transmission to avoid link-blocking. In HVOB networks, however, we need to address another type of blocking. If adding the connection causes some paths, including the new one, to violate the crosstalk-free constraint, the connection cannot be added even if the path is available. We refer to this second type of blocking as *crosstalk-blocking*. Since the crosstalk-free constraint requires that no two optical signals ever share an SE in transmission (i.e. they should be node-disjoint in transmission), we need to consider only the crosstalk-blocking in HVOB networks. Obviously, the consideration of crosstalk-blocking will increase the overall blocking probability than considering only the link-blocking.

Due to their symmetric structures, all paths in banyan networks have the same property in terms of blocking. We define the blocking probability as the probability that a feasible connection request is blocked, where a feasible connection request is a connection request between an idle input port and an idle output port of a network. Without loss of generality, we choose the path between the first input port and the first output port (which is termed the *tagged path* in the following context) for the blocking analysis. All the SEs and links on the tagged path are called *tagged SEs* and *tagged links*, respectively. For a banyan network with x extra stages, we number the stages of SEs from left (stage 1) to right (stage $\log N+x$). We define the *input intersecting set* $I_i=\{2^{i-1}, 2^{i-1}+1,\ldots, 2^i-1\}$ associated with stage i as the set of all inputs that intersect a tagged SE at stage i and define an *output intersecting set* $O_i=\{2^{i-1}, 2^{i-1}+1,\ldots, 2^i-1\}$ associated with stage i is the set of all outputs that intersect a tagged SE at stage $\log N+x-i+1$, as illustrated in Fig.2.

To simplify the analysis, we make the same assumption held in [13],[14] for multistage interconnection networks: the correlation between signals arriving (or leaving) at different input (or output) ports will be neglected. This leads to a fact that the status (either busy or idle) of each individual input (output) port in the network is independent. This assumption matches the practical situation, since optical switching networks are becoming larger in size, with increasingly complex interconnections, so as to transport a huge amount of data at once. In such circumstances, instead of being fixed with a certain extent of mutual correlation, the communication patterns of the input (or output) signals to an optical switch are becoming statistically random such that the correlation between signals at input (or output) ports becomes approximately negligible.

* In this paper log means the logarithm to the base 2.



(c)

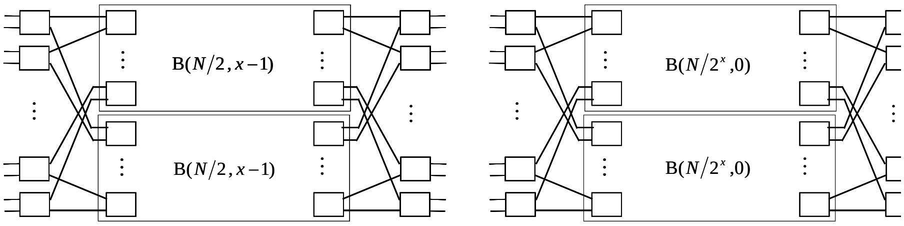
Fig.2 64×64 banyan network with: (a) zero, (b) one, and (c) two extra stages. The tagged paths between the input 0 and output 0 are illustrated.

III. LOWER BOUND ON BLOCKING PROBABILITY

The lower-bound on the blocking probability of a HVOB(N,m,0) network has been developed in [18]. In this paper, we focus on the lower-bound on the blocking probability of general HVOB(N,m,x) networks with x≥1. To establish the lower-bound on the blocking probability of a HVOB network with crosstalk-free constraint, we adopt a "aggressive" routing control strategy in bound derivation, in which we only guarantee that each of those connection requests that blocks a same tagged SE falls in a distinct plane to meet the crosstalk-free constraint, such that the minimum number of blocked planes can be achieved based on the routing strategy. Here, we define a plane as a blocked plane if all its tagged paths are blocked. Thus, the connection request of tagged path in a HVOB network will be blocked if all the planes of the network are blocked planes.

Let $NBP(N,x)$ denotes the minimum number of blocked planes in a HVOB(N,m,x) network under the "aggressive" routing control strategy, and B(N,x) denotes an N×N banyan network with x(x≥1) extra stages. A

B(N,x) network can be defined in a recursive way and this recursive definition will end at the central column of 2^x banyan network B($N/(2^x)$,0), as shown in Fig.3. Note that the B(N,x) is just a plane of a HVOB(N,m,x) network, and this plane is blocked if both its upper and lower tagged paths are blocked.



(a) B(N,x) (b) B($N/2^{x-1}$,1)

Fig.3 Recursive definition of B(N,x) network ($x \geq 1$). (a) The first step of the recursive definition. (b) The last step of the recursive definition.

We use Pr(A) to denote the probability that event A happens and use $\Pr^-(A)$ to denote the lower-bound of Pr(A). For a HVOB(N,m,x) network, a lower-bound of its blocking probability is thus

$$\Pr^-(blocking) = 1 - \Pr\big(NBP(N,x) < m\big) \tag{1}$$

Equation 1 indicates that we need to evaluate the probability Pr($NPB(N,x)<m$) for a HVOB(N,m,x) network to get the lower-bound on its blocking probability. Let $n_1(N,x)$ and $n_{\log N+x}(N,x)$ denote the number of connections passing through the first and last tagged SEs of a B(N,x) network, respectively. We shall establish the follow theorem concerning the evaluation of Pr($NPB(N,x)<m$).

Theorem 1: For a HVOB(N,m,x) network,

$$\begin{aligned}&\Pr\big(NBP(N,x)<m\big) = \Pr\big(n_1(N,x)<m\big)\cdot\Pr\big(n_{\log N+x}(N,x)<m\big)\\ &\times\Big\{1-\big[1-\Pr\big(NBP\big(\tfrac{N}{2},x-1\big)<m\big)\big]^2\Big\}\end{aligned} \tag{2}$$

Proof: We use $NBP_1(N/2,x\text{-}1)$ and $NBP_2(N/2,x\text{-}1)$ to denote the minimum numbers of planes blocked in the upper B($N/2$,x-1) and lower B($N/2$,x-1) of the B(N,x) network, respectively, and thus $\min\{NBP_1(N/2,x\text{-}1), NBP_2(N/2,x\text{-}1)\}$ is the maximum number of planes that the upper B($N/2$,x-1) and lower B($N/2$,x-1) combined can block. Since $NBP(N,x)=\max\{n_1(N,x),\min\{NBP_1(N/2,x\text{-}1),NBP_2(N/2,x\text{-}1)\},n_{\log N+x}(N,x)\}$, then we have the following formula based on the recursive definition of B(N,x) network shown in Fig.3.

$$\begin{aligned}&\Pr\big(NBP(N,x)<m\big)\\ &=\Pr\big(n_1(N,x)<m\big)\cdot\Pr\Big(\min\big\{NBP_1\big(\tfrac{N}{2},x-1\big),NBP_2\big(\tfrac{N}{2},x-1\big)\big\}<m\Big)\cdot\Pr\big(n_{\log N+x}(N,x)<m\big)\end{aligned} \tag{3}$$

Note that

$$\begin{aligned}&\Pr\Big(\min\big\{NBP_1(\tfrac{N}{2},x-1),NBP_2(\tfrac{N}{2},x-1)\big\}<m\Big)\\ &=1-\Pr\Big(\min\big\{NBP_1(\tfrac{N}{2},x-1),NBP_2(\tfrac{N}{2},x-1)\big\}\geq m\Big)\\ &=1-\Pr\big(NBP_1(\tfrac{N}{2},x-1)\geq m, NBP_2(\tfrac{N}{2},x-1)\geq m\big)\\ &=1-\Pr\big(NBP_1(\tfrac{N}{2},x-1)\geq m\big)\cdot\Pr\big(NBP_2(\tfrac{N}{2},x-1)\geq m\big)\\ &=1-\big[1-\Pr\big(NBP_1(\tfrac{N}{2},x-1)<m\big)\big]\cdot\big[1-\Pr\big(NBP_2(\tfrac{N}{2},x-1)<m\big)\big]\end{aligned} \tag{4}$$

Based on the symmetric structure of a B(N,x) network, we have:

$$\Pr\big(NBP_1(\tfrac{N}{2},x-1)<m\big)=\Pr\big(NBP_2(\tfrac{N}{2},x-1)<m\big)=\Pr\big(NBP(\tfrac{N}{2},x-1)<m\big) \tag{5}$$

Thus, we can prove that Pr($NBP(N,x)<m$) is given by (2) based on Eqs. (3)-(5). □

Theorem 1 clearly shows a recursive relationship between Pr($NPB(N,x)<m$) and Pr($NPB(N/2,x\text{-}1)<m$), and we will be able to use (2) to calculate the probability Pr($NPB(N,x)<m$) recursively if we can get the results for probabilities Pr($n_1(N)<m$),Pr($n_{\log N+x}(N)<m$) and Pr($NPB(N,0)<m$). For the evaluation of Pr($n_1(N)<m$) and Pr($n_{\log N+x}(N)<m$), we have the following lemma 1.

Lemma 1: For a HVOB(N,m,x) network, the probabilities Pr($n_1(N,x)<m$) and Pr($n_{\log N+x}(N,x)<m$) are given by:

$$\Pr\left(n_1(N,x)<m\right)=\Pr\left(n_{\log N+x}(N,x)<m\right)=\sum_{k=0}^{\min\{m-1,1\}}\binom{1}{k}\cdot r^k\cdot(1-r)^{1-k} \tag{6}$$

where r is the occupancy probability of an input (output) port.

Proof: Note that $n_1(N,x)$ ($n_{\log N+x}(N,x)$) is the number of connections passing through the first (last) tagged SE of a B(N,x) network, and we have at most 1 such kind of connections, i.e., the connection passing through the second input port (the connection passing through the second output port of the network), therefore we have the following formula based on the symmetric structure of a B(N,x) network:

$$\Pr\left(n_1(N,x)<m\right)=\Pr\left(n_{\log N+x}(N,x)<m\right)=\sum_{k=0}^{\min\{m-1,1\}}\Pr\left(n_1(N,x)=k\right)$$

Let r be the occupancy probability of an input (output) port of a B(N,x) network, then it is easy to see that the probability $\Pr(n_1(N,x)=k)$ is given by:

$$\Pr\left(n_1(N,x)=k\right)=\binom{1}{k}\cdot r^k\cdot(1-r)^{1-k}$$

This finishes our proof. □

Hereafter, we will calculate the probability $\Pr(NPB(N,0)<m)$ for the cases in which $\log N$ is even and $\log N$ is odd, respectively.

A. Calculation of $\Pr(NPB(N,0)<m)$ *When logN is Even*

For a HVOB(N,m,0) network where $\log N$ is even, the probability $\Pr(NPB(N,0)<m)$ is given by [18]:

$$\begin{aligned}&\Pr\left(NBP(N,0)<m\right)\\&=\left(\Pr\left(n_1(N,0)<m\right)\cdot\prod_{i=1}^{(1/2)\log N-1}\frac{\Pr\left(n_i(N,0)<m,n_{i+1}(N,0)<m\right)}{\Pr\left(n_i(N,0)<m\right)}\right)^2\times\frac{\Pr\left(n_{\frac{1}{2}\log N}(N,0)<m,n_{\frac{1}{2}\log N+1}(N,0)<m\right)}{\left(\Pr\left(n_{\frac{1}{2}\log N}(N,0)<m\right)\right)^2}\end{aligned} \tag{7}$$

where $n_i(N,0)$ is the number of connections passing through the i-th tagged SE in the HVOB(N,x,0) network. The $\Pr(n_i(N,0)<m)$ (including $\Pr(n_1(N,0)<m)$ and $\Pr(n_{(1/2)\log N}(N,0)<m)$) can be evaluated using the following formula:

$$\Pr(n_i(N,0)<m)=\sum_{k=0}^{\min\left\{m-1,\sum_{j=1}^{i}|I_j|\right\}}\binom{\sum_{j=1}^{i}|I_j|}{k}(\eta_i+\xi_i)^k(1-\eta_i-\xi_i)^{\sum_{j=1}^{i}|I_j|-k} \tag{8}$$

where η_i and ξ_i are given by:

$$\eta_i=r\cdot\left(N/2^i-1\right)/(N-1),\ \xi_i=r\cdot\left(N/2^i\right)/(N-1)$$

The probability $\Pr(n_i(N,0)<m,n_{i+1}(N,0)<m)$ (including $\Pr(n_{(1/2)\log N}(N,0)<m,n_{(1/2)\log N+1}(N,0)<m)$) are determined by:

$$\begin{aligned}&\Pr(n_i(N,0)<m,n_{i+1}(N,0)<m)\\&=\sum_{k=0}^{\min\left\{m-1,\sum_{j=1}^{i}|I_j|,\sum_{j=1}^{\log N-i+1}|O_j|\right\}}\ \sum_{l=0}^{\min\left\{m-1,|I_{i+1}|+k,\sum_{j=1}^{\log N-i}|O_j|\right\}}\ \sum_{c=0}^{\min\{k,l\}}\Pr(n_i(N,0)=k,t_{i,i+1}=c)\times\Pr(n_{i+1}(N,0)=l\,|\,t_{i,i+1}=c)\end{aligned} \tag{9}$$

where $t_{i,i+1}$ is the number of connections passing through both the i-th and the (i+1)-th tagged SEs in the HVOB(N,m,0) network. For i=1,2,…,(1/2)log$N$, $\Pr(n_i(N,0)=k,t_{i,i+1}=c)$ is given by

$$\Pr(n_i(N,0)=k,t_{i,i+1}=c)=\binom{\sum_{j=1}^{i}|I_i|}{c}\binom{\sum_{j=1}^{i}|I_i|-c}{k-c}\cdot\eta_i^{\,c}\cdot\xi_i^{\,k-c}\cdot(1-\eta_i-\xi_i)^{\sum_{j=1}^{i}|I_i|-k} \tag{10}$$

For i=1,2,…,(1/2)log$N$ –1, $\Pr(n_{i+1}(N,0)=l|t_{i,i+1}=c)$ is evaluated by

$$\Pr(n_{i+1}(N,0)=l\,|\,t_{i,i+1}=c)=\binom{|I_{i+1}|}{l-c}\cdot\mu_{i+1}^{\,l-c}\cdot(1-\mu_{i+1})^{|I_{i+1}|-l+c} \tag{11}$$

where μ_{i+1} is given by

$$\mu_{i+1} = r \cdot \left(N/2^i - 1 - c\right) / \left(N - 1 - c\right)$$

When i=(1/2)log$N$, the probability Pr($n_{i+1}(N,0)=l|t_{i,i+1}=c$) is determined by

$$\Pr(n_{\frac{1}{2}\log N+1}(N,0) = l \big| t_{\frac{1}{2}\log N, \frac{1}{2}\log N+1} = c) = \binom{\sum_{j=1}^{\frac{1}{2}\log N} |O_j| - c}{l - c} \cdot \lambda^{l-c} \cdot (1-\lambda)^{\sum_{j=1}^{\frac{1}{2}\log N} |O_j| - l}$$

where λ is evaluated as

$$\lambda = r \cdot \left(\left|I_{\frac{1}{2}\log N+1}\right|\right) \Big/ \left(N - 1 - \sum_{j=1}^{\frac{1}{2}\log N} |I_j|\right)$$

B. Calculation of Pr($NPB(N,0)<m$) *When logN is Odd*

When log$N$ is odd, the Pr($NPB(N,0)<m$) can also be evaluated based on the model proposed in [18], as summarized in the following lemma 2.

Lemma 2: For a HVOB(N,m,0) network, when log$N$ is odd, the probability Pr($NPB(N,0)<m$) is given by

$$\Pr\left(NBP(N,0) < m\right) = \frac{\left(\Pr\left(n_1(N,0) < m\right) \cdot \prod_{i=1}^{(1/2)(\log N - 1)} \frac{\Pr\left(n_i(N,0) < m, n_{i+1}(N,0) < m\right)}{\Pr\left(n_i(N,0) < m\right)}\right)^2}{\Pr\left(n_{\frac{1}{2}(\log N+1)}(N,0) < m\right)} \tag{12}$$

where Pr($n_i(N,0)<m$) and Pr($n_i(N,0)<m, n_{i+1}(N,0)<m$) can be evaluated using (8) and (9), respectively.

IV. EXERIMENTAL RESULTS

An experimental study was performed to verify our lower-bound on the blocking probability (also denoted by *BP* hereafter) of a HVOB network. In this section, we first introduce the network simulator we developed, then we present the simulation results based on the simulator.

A. The Network Simulator

We developed a network simulator, which consists of two modules: the request pattern generator and request router. The request pattern generator randomly generates a set of connection request patterns for a HVOB network based on the occupancy probability *r*. To verify the lower-bound on *BP*, the "aggressive" routing strategy, random routing strategy, and packing strategy [21] are used to route the connection requests in a connection pattern through the network. In the "aggressive" routing strategy, each connection request has the probability of 0.5 to go through either the upper or the lower part of the network recursively, and we only guarantee that all the requests that block a same tagged SE will fall within distinct plans. To establish the connection request in random routing, the request router randomly chooses one of the planes that can be used by a request to establish the connection. Under the packing strategy, a connection is realized on a path found by trying the most used plane of the network first and least used plane last. For a connection pattern, if no plane can satisfy the request of the tagged path using a routing strategy, the connection pattern is recorded as a blocked connection pattern corresponding to the routing strategy. The blocking probability is then estimated by the ratio of the number of blocked connection patterns to the total number of connection patterns generated.

B. Theoretical Versus Simulated Lower-Bound on BP

We have examined two networks, HVOB(512,m,x) and HVOB(1024,m,x) with x={1,2}, to verify the derived lower-bound. For each network configuration, blocking probability is examined by using both the theoretical bound and the simulator for r=0.8. The corresponding results are summarized in Tables 1 and 2.

The results in Table 1 and Table 2 show clearly that our theoretical model can correctly estimates the lower-bound on the blocking probability of general HVOB networks, and the results from the random routing and packing strategy are all nicely bounded by the derived lower bound. We also observed in our simulation and theoretical study that the proposed lower-bound follows closely the nonblocking condition of a rearrangeable HVOB network [9]. For the network with N=512, the lower bound goes to zero at m=16 when the network has 1 extra stage and goes to zero at m=16 when the network has 2 extra stages. For HVOB(1024,m,1) network and HVOB(1024,m,2) network, the lower bound of blocking probability becomes zero at m=32 and m=16, respectively. The results Table 1 and Table 2 further indicate surprisingly that our lower-bound can be used to efficiently estimate the blocking probability of a HVOB network applying packing strategy. Thus, our lower-bound is significant because it not only provides network designers an effective tool to estimate the minimum blocking probability they can expect from a HVOB architecture but also can reveal the inherent relationship among blocking probability, network depth and network hardware cost in a general HVOB network applying packing strategy. Hereafter, our bound will be used to illustrate the tradeoff among the number of planes (stacked copies), the number of SEs, the number of stages and blocking probability in a HVOB network applying packing strategy.

Table 1 Blocking probability of HVOB(512,m,x) networks with x={1,2}and r=0.8.

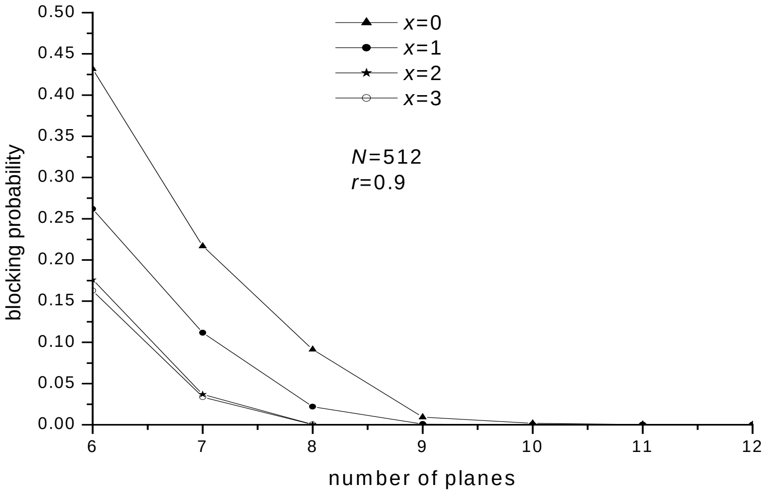
N=512, r=0.8								
m	Random routing		Packing strategy		Simulated lower bound		Theoretical lower bound	
	x=1	x=2	x=1	x=2	x=1	x=2	x=1	x=2
1	0.99998	0.99993	0.98512	0.96511	0.97536	0.95555	0.97440	0.95460
2	0.99953	0.99882	0.96228	0.92897	0.95275	0.91977	0.95181	0.91886
3	0.99518	0.99004	0.88537	0.81562	0.87660	0.80754	0.87573	0.80674
4	0.97104	0.95088	0.76531	0.67395	0.75773	0.66727	0.75698	0.66661
5	0.88688	0.84249	0.54973	0.44571	0.54428	0.48052	0.54374	0.48004
6	0.69512	0.64124	0.26184	0.21586	0.25924	0.21372	0.25899	0.21351
7	0.41475	0.38907	0.11597	0.10784	0.11482	0.10677	0.11470	0.10666
8	0.17670	0.16541	0.08263	0.02598	0.08181	0.02572	0.08173	0.02569
9	0.05771	0.03835	4.73E-5	4.95E-6	4.683e-5	4.90E-6	4.67E-5	4.89E-6
10	0.01296	0.00466	9.51E-7	2.1E-8	9.415E-7	2.0E-8	9.40E-7	2.07E-8
11	0.00200	0.00026	1.2E-8	1E-10	1.171E-8	0.96E-10	1.18E-8	0.9E-10

Table 2 Blocking probability of HVOB(1024,m,x) networks with x={1,2}and r=0.8.

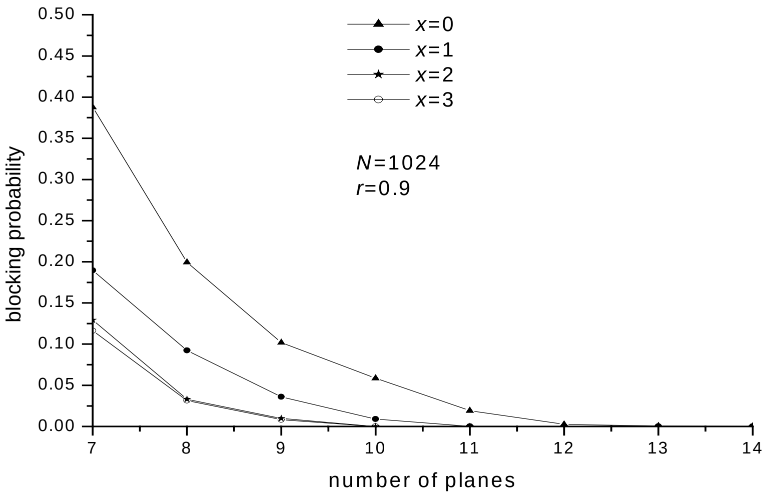
N=1024, r=0.8								
m	Random routing		Packing strategy		Simulated lower bound		Theoretical lower bound	
	x=1	x=2	x=1	x=2	x=1	x=2	x=1	x=2
1	0.999993	0.999971	0.99025	0.98474	0.98926	0.98375	0.98916	0.98365
2	0.999795	0.999504	0.91740	0.89121	0.91648	0.89031	0.91639	0.89023
3	0.997616	0.995322	0.78452	0.71208	0.78373	0.71136	0.78365	0.71129
4	0.984656	0.975557	0.69442	0.62530	0.69372	0.62467	0.69365	0.62461
5	0.935214	0.912479	0.61133	0.49332	0.61071	0.49282	0.61065	0.49277
6	0.805000	0.776470	0.47120	0.28840	0.47072	0.28811	0.47068	0.28808
7	0.575995	0.567048	0.15826	0.11734	0.15810	0.11722	0.15808	0.11721
8	0.336548	0.307700	0.08454	0.06586	0.08445	0.06579	0.08444	0.06578
9	0.156134	0.109983	6.19E-4	4.51E-4	6.18E-4	4.5E-4	6.1E-4	4.5E-4
10	0.054834	0.023836	7.59E-6	6.50E-6	7.58E-6	6.4E-6	7.5E-6	6.5E-6
11	0.014314	0.002802	5.0E-7	0.98E-8	4.99E-7	0.88E-8	4.9E-7	0.9E-8
12	0.002812	0.000180	1.01E-10	1.0E-12	0.89E-10	1.1E-12	0.92E-10	0.998E-12
13	0.000402	0.000015	0.99E-11	1.2E-13	0.96E-11	0.9E-13	0.90E-11	0.991E-13
14	0.000047	0.000006	1.1E-12	0.9E-14	0.88E-12	0.91E-14	0.96E-12	0.990E-14

C. Blocking Probability, Network Depth and Hardware Cost

The Table 1 and Table 2 indicate that for a HVOB network with a given number of planes, we can reduce the blocking probability by appending more extra stages to the network. To show further the impact of increasing network depth upon blocking probability, we illustrate in Fig.4 the lower-bound on blocking probability of different HVOB(N,m,x) configurations with N={512,1024},x={0,1,2,3},and r=0.9. Fig.4 shows that for a HVOB(N,m,x) network with a given number of planes and a given constant network utilization r, the blocking probability decreases sharply as the number of extra stages increases from 0 to 2, but this decrease in blocking probability becomes insignificant if we increase number of extra stages further from 2 to 3. Thus, our results indicate that we can achieve a good tradeoff between network depth and *BP* by appending only two extra stages in a HVOB network.



(a)



(b)

Fig.4. Blocking probability of different HVOB(N,m,x) networks with r=0.9 and x={0,1,2,3}. (a) N=512. (b) N=1024.

To show the significant hardware saving raised by using our lower bound in designing a HVOB network, we focus on the HVOB(N,m,2) architecture and summarize in Table 3 the minimum numbers of planes required by the HVOB(N,m,2) networks with r=0.9 and different requirements on *BP* and summarize in Table 4 the minimum numbers of planes required by the HVOB(N,m,2) networks with *BP*<1% and r={0.5,0.75,1.0}. For comparison, we also show in both Table 3 and Table 4 the minimum numbers of planes corresponding to the condition of strictly nonblocking [8] (*BP*=0) and the minimum number of planes corresponding to random routing. The results in Table 3 and Table 4 indicate that for large-scale HVOB networks, the hardware cost required by the nonblocking condition is considerably high, and this hardware cost can be significantly reduced by allowing an almost negligible blocking probability and adopting random routing and can be further reduced by adopting our lower bound (or packing strategy). For example, in HVOB(1024,m,2) with workload r =0.9 and an upper limit of blocking probability 0.1% (i.e., *BP* < 0.1%), the minimum number of planes required to achieve the nonblocking condition is 35 while the minimum number of required planes is 12 for random routing and 10 for our lower-bound. Therefore, (35-12)/35≅65% and (35-10)/35≅71% of the hardware cost can be saved by using random routing and packing strategy, respectively, while a very low blocking probability (*BP*<0.1%) is guaranteed.

Table 3 Minimum numbers of planes for HVOB(*N*,*m*,2) networks with *r*=0.9 and different requirements of blocking probability.

N			32	64	128	256	512	1024
r=0.9		*BP*=0	9	11	15	19	27	35
	Random routing	*BP*<0.1%	6	7	9	9	11	12
		BP<1%	4	6	8	9	11	11
		BP<5%	4	5	7	8	10	10
	Lower bound	*BP*<0.1%	3	4	6	7	8	10
		BP<1%	3	4	5	6	8	9
		BP<5%	2	3	4	5	7	8

Table 4 Minimum numbers of planes for HVOB(*N*,*m*,2) networks with *BP*<1% and different workloads.

N			32	64	128	256	512	1024
BP=0	*r*=1.0		9	11	15	19	27	35
BP<1 %	Random routing	*r*=0.5	2	4	4	6	7	9
		r=0.75	3	5	6	7	9	10
		r=1.0	4	5	8	9	11	12
	Lower bound	*r*=0.5	2	2	3	4	5	6
		r=0.75	2	3	4	5	7	8
		r=1.0	3	4	5	6	8	9

We can also observe from Table 3 that for a given workload, the hardware cost estimated by our bound is not sensitive to the requirements of blocking probability. For the network with *N*=1024 and *r*=0.9, the minimum number of planes estimated by our bound is 9 for the requirement of *BP*<1% and is 10 for the requirement of *BP*<0.1%, both of which are much less than the number of planes (i.e.,35) required by the nonblocking condition. The results in Table 4 further indicate that for a given requirement on *BP*, the hardware cost estimated by our lower-bound is also not sensitive to variation of workload. For the HVOB(512,*m*,2) network with the requirement of *BP*<1%, the minimum number of planes estimated by the bound is 7 for *r*=0.75 and 8 for *r*=1.0, both of which are much less than the 27 planes required by the nonblocking condition.

V. CONCLUSIONS

In this paper, we have developed an analytical model for evaluating the lower-bound on the blocking probability of general HVOB networks built on a combination of horizontal expansion and vertical stacking of optical banyan networks. Extensive simulation results based on both random routing and packaging strategy indicate that our bound can efficiently estimate the minimum blocking probability we can expect from a HVOB network and we can actually achieve the lower-bound by applying packaging strategy in the network. The model provides network developers with guidance for quantitatively determining the impact of appending extra stages and reducing the number of planes on the minimum possible blocking probability of a HVOB network, by which a graceful compromise among the hardware cost, network depth and the blocking probability can be initialized. Our model reveals an unobvious overall behavior of HVOB networks, that the hardware cost of a HVOB network can be reduced dramatically while a small network depth and a negligible small blocking probability are guaranteed. We expect that the modeling method employed in this paper will help in deriving the lower bound on the blocking probabilities of other types of optical switching networks as well.

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