An advection-diffusion model to explain thermal surface anomalies off Cape Trafalgar

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ABSTRACT

The authors describe an almost permanent thermal anomaly, with low surface-temperature values, off Cape Trafalgar. The existence of strong tidal currents in the alongshore direction, and the local offshore orientation of isobaths at this point, support the hypothesis of vertical forcing by interaction between the barotropic tide and topography. This is simplified for modelisation as the pass of a tidal current over a ridge, which is considered uniform in the cross-shore direction. A bidimensional model in finite differences is developed to reproduce the main features observed experimentally. The combined effects of advection (both vertical and horizontal) and diffusion appear to be very important. The model is sensitive to the assumed values of thermal diffusion coefficients and their depth dependence, as well as to heat flux through the sea surface. To have realistic values for these parameters, a unidimensional diffusion model aimed at reproducing the mixed layer and thermocline observed in this area during summer has been developed. Heat flux and diffusion coefficients are adjustment parameters of the model, and, once determined, they are introduced in the 2-D advection-diffusion model. Results from simulation seem to be in good agreement with CTD observations, confirming our initial hypothesis.

Key words: Advection-diffusion, tidal currents, topography.

RESUMEN

Un modelo de advección-difusión para explicar anomalías térmicas en el cabo de Trafalgar

Se describe una anomalía térmica caracterizada por bajas temperaturas superficiales frente al cabo de Trafalgar. Debido a que la orientación de las isóbatas en esta área es casi perpendicular a la costa y a que existen fuertes corrientes de marea a lo largo de ella, se desarrolla la hipótesis de que la interacción entre las corrientes de marea y la topografía son los mecanismos responsables de esta anomalía. Para revisar esta hipótesis se desarrolla en el presente trabajo un modelo de advección-difusión que estudia el efecto del paso de una corriente de marea sobre un obstáculo. El modelo es sensible a ciertos parámetros, como los coeficientes de difusión térmica y el flujo de calor en la superficie del mar. Por ello desarrollamos un modelo monodimensional que reproduce la formación de la capa de mezcla y la termoclina estacional usando estos parámetros como parámetros de ajuste. Una vez encontrados los valores adecuados, son introducidos en el modelo de advección-difusión. Los resultados obtenidos parecen estar de acuerdo con los datos experimentales.

Palabras clave: Advección-difusión, corrientes de marea, topografía.

INTRODUCTION

Three multidisciplinary oceanographic surveys were conducted during consecutive years by the Instituto Español de Oceanografía, covering the western Alborán region and Gulf of Cadiz: Ictio.Alborán 0794, 0795, and 0796. During all of these surveys a thermal anomaly was observed off Cape Trafalgar (on the Atlantic coast of southern Spain, close to the Straits of Gibraltar, figure 1). It consists of a persistent pool of cold waters extending from the coast in the offshore direction, with surface temperatures 2-3 °C lower than surrounding waters. Using data from CTD trawlss repeated throughout the three surveys, we calculated the temperature distribution at a depth of 3 m corresponding to July 1994, 1995, and 1996 (figure 2). The pool of cold waters is evident in these three figures. This finding led us to do an analysis covering a longer period of time. A recompilation of thermal infrared images throughout 1988 (not included in the present paper) shows this feature to be almost present almost year-round, although it iss
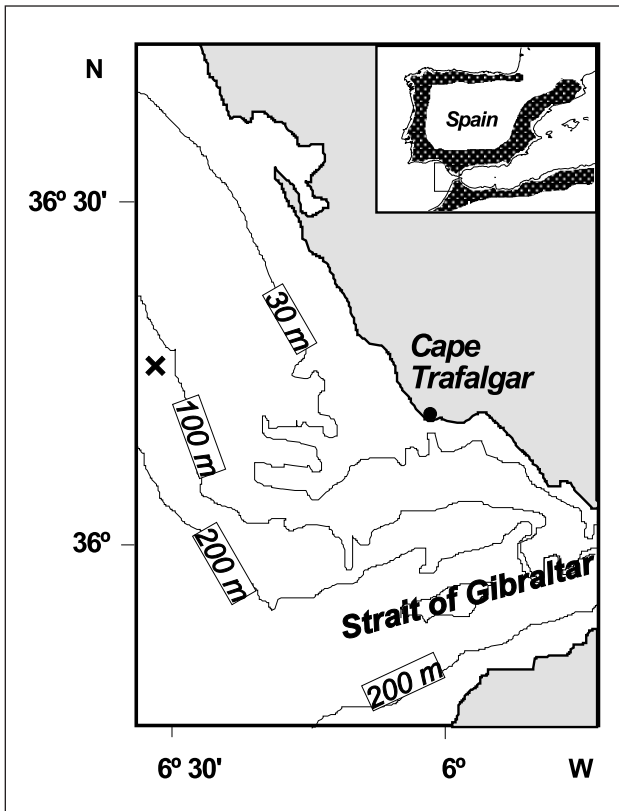

Figure 1. Map of the area of study. Cape Trafalgar is on the western side of the Straits of Gibraltar. Note the offshore direction of 30 m and 100 m isobaths off Cape Trafalgar
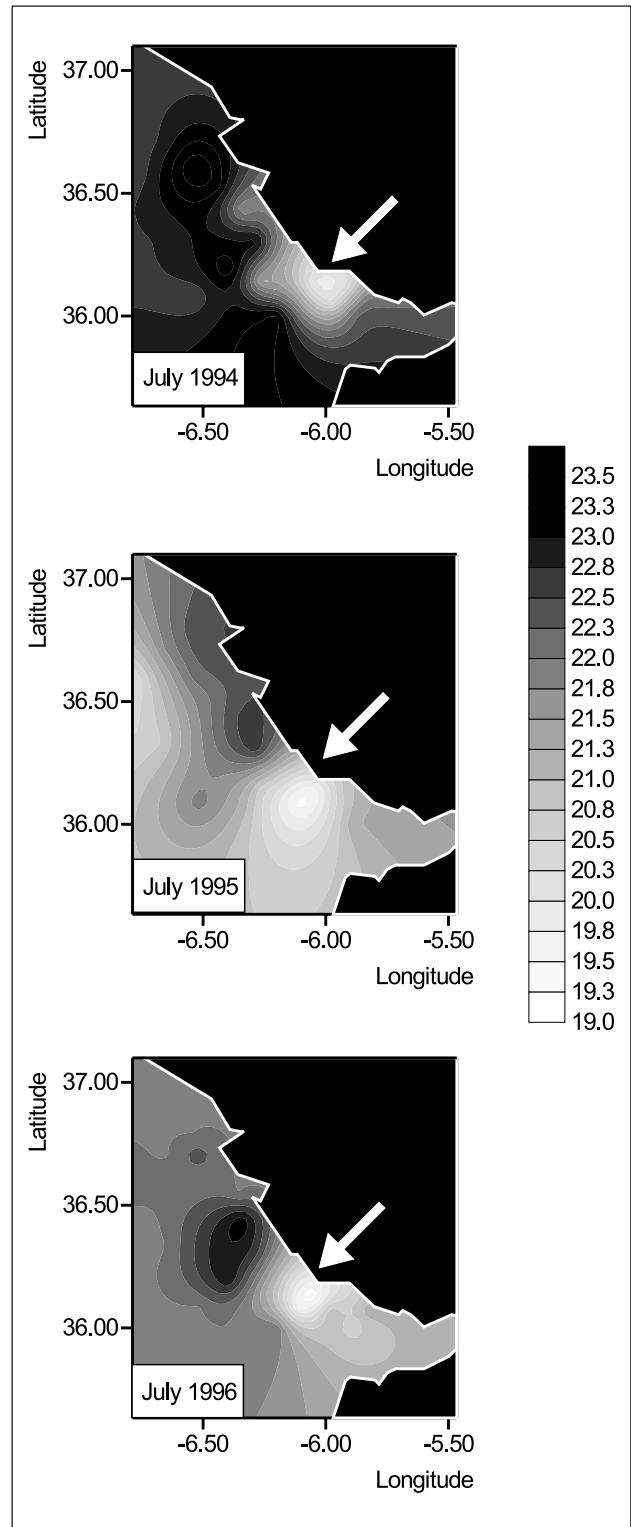

Figure 2. Temperature distribution at depth of 3 m from CTD trawls in July 1994, 1995 and 1996. Bright colours indicate cooler temperatures. A pool of cold surface waters can be seen in the three images

more evident in summer, when the surrounding waters are warmer and the contrast is more clear. It

is noteworthy that the anomaly extends in the off-shore direction, right in a place where isobath orientation is almost normal to the shoreline. Topography reproduces the cape's shape under the sea surface in this area. The existence of strong alongshore tidal currents in this region is also known; therefore, the first hypothesis to consider is the interaction between these currents and topography as the mechanism responsible. Similar effects have been reported by Tee, Smith and LeFaivre (1993), although in their paper a residual current forced by the pass of tidal currents over topography is responsible for advection-diffusion, whereas we shall consider the effect of tidal currents directly. Mazé (1983) explained the formation of a surface cold-water strip in the Bay of Biscay by the vertical excursions of the interface associated with internal tides generated by barotropic tide interaction with the continental slope: when the interface in a two-layered sea reaches its maximum elevation, the slight thickness of the upper layer favours mixing processes. This mechanism is quite similar to the one we propose. The aim of the present paper is to model the phenomenon described by an advection-diffusion model, where the forcing mechanism is the high vertical velocity forced by the pass of tidal currents over a ridge. To do this, a simplified topography is needed, so we shall consider our system to be homogeneous in the cross-shore direction. A description of topography and hydrodynamic fields calculation is offered in section 2. In section 3, we introduce these fields in the 2-D advection-diffusion model. We also reproduce the formation of a mixed layer over a thermocline using a unidimensional model for pure diffusion to determine thermal diffusion coefficients and heat fluxes as adjust parameters that will be introduced later in the two dimensional model.

MATERIALS AND METHODS

Hydrodynamic description of vertical forcing

In the case of a barotropic tide propagating in a flat-bottomed sea, we can consider that the main horizontal pressure gradients are due to the free-surface elevation, ξ. In the shallow water inviscid theory, the vertical velocity field has its maximum at the sea surface, and will decrease linearly to the bottom according to the expression:

$$W(z) = -(z + H)\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) \qquad [1]$$

where H is the sea depth, W is the vertical velocity, and U and V are the horizontal components. The z-axis is considered positive upward, and its origin at the free surface at rest. The vertical velocity of the surface will be given by:

$$\frac{\partial}{\partial t}\xi = -H\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) \qquad [2]$$

A simple calculation shows that this quantity is very low when considering tidal frequencies, and compared with those generated by steep topographies (Baines, 1973).

If we now consider the presence of topographic variations along the propagation direction, higher vertical speeds are forced. Taking the x-axis along this direction, and considering no topographic variations along the y-axis, the problem can be reduced to a 2-D one, as is usually done in analytical models for internal tide generation, which is closely related to the present problem (cf. Baines, 1973, 1982). For the ridge outlined in figure 3, the zero flux condition through the sea bottom imposes a non-zero vertical velocity at those points where the bottom slope is different from zero, and will satisfy the condition:

$$U + \alpha W = 0 \qquad [3]$$

α being the bottom slope. Following Baines (1973), vertical velocity will be caused by the gradient of water flux along the propagation direction, which is responsible for free surface elevation, and by bottom slope. The former causes very low vertical velocities when compared with topographic effects, as we noted above, and we can take into account only the latter. Baines (1973) gives the following expression for topographically forced vertical velocity:

$$W = -Qz\,(1/H)_x\,sin\,(\omega t) \qquad [4]$$

where Q is the water flux along the propagation direction, H the water column thickness, the x-subindex stands for derivatives along the x-axis, and ω is the tidal frequency, M_2 in this paper. Q, as discussed above, can be taken as constant, and will be Q = H(x) U(x). Therefore, for a given topography, e.g. the one in the figure 3, and fixing the flux,

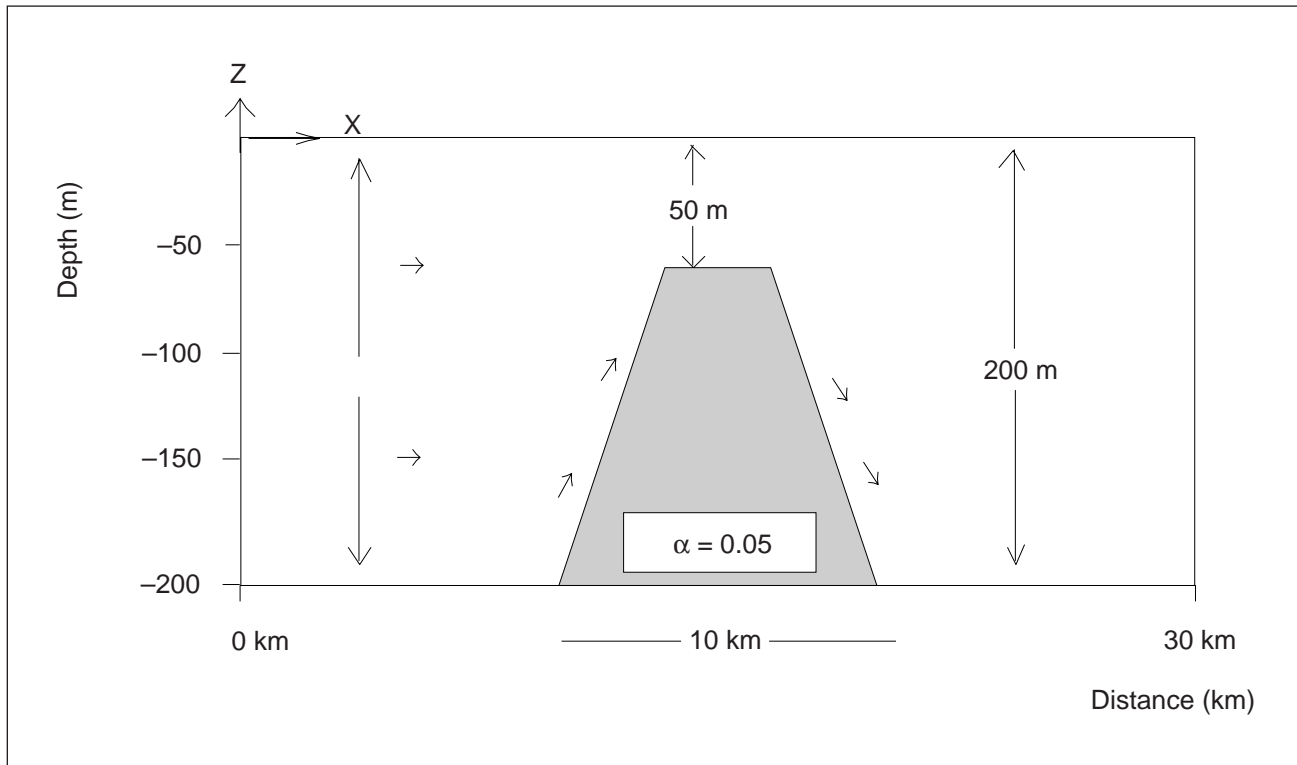Figure 3. Diagram of the model domain and parameters used in the 2-D advection-diffusion model

the 2-D velocity field U,W can be calculated according to the expressions:

$$U = [Q/H(x)]\ sin\ (\omega t) \qquad [5]$$

$$W = Q\ z\ (1/H^2)H_x\ sin\ (\omega t) \qquad [6]$$

For a given water flux Q, a decrease in the thickness of the water column will be followed by an increase of the absolute value of horizontal velocity. When this component is positive, as z always takes negative values, W will be positive (upward) when H decreases, i.e., upstream, and will be negative (downward) when H increases, i.e. downstream.

Using [5] and [6] and a value of 100 m²/s for Q (corresponding to U = 0.5 m/s for the undisturbed horizontal field), we calculated amplitudes for U and W for a 200-m deep sea, with a ridge where the sea depth was 50 m, the slope on both sides of the ridge was 0.05°, and the total length of the area considered 30 km. These values are taken as an idealisation of those found in Cape Trafalgar. Results can be seen in figure 4. The figure shows amplitudes; the instantaneous values must be obtained by multiplying the velocity vector times the function *sin* ωt.

An important point to be noted here is that, if the sea does not have a constant density, the vertical motions caused by the mechanism described above will generate internal horizontal pressure gradients, which will propagate to both sides of the ridge as internal tides (Rattray, 1960; Prinsenberg and Rattray, 1975; Baines, 1982; Mazé, 1983). The description of the system's dynamics will no longer be as simple as we have depicted it till now. A commonly-used technique to study this kind of phenomena is to consider the actual motion of the fluid as the superposition of the barotropic part that we have already dealt with, and introducing it as a forcing term in the internal part of the motion, which accounts for the density stratification. In the present paper we merely take the barotropic tide and introduce it as a forcing mechanism for advecting low temperature waters from the bottom sea to subsurface areas where turbulent diffusion is more efficient, and we shall not address the propagation of pressure gradients produced by vertical excursions of material surfaces. A complete study of both phenomena will be left for future projects.
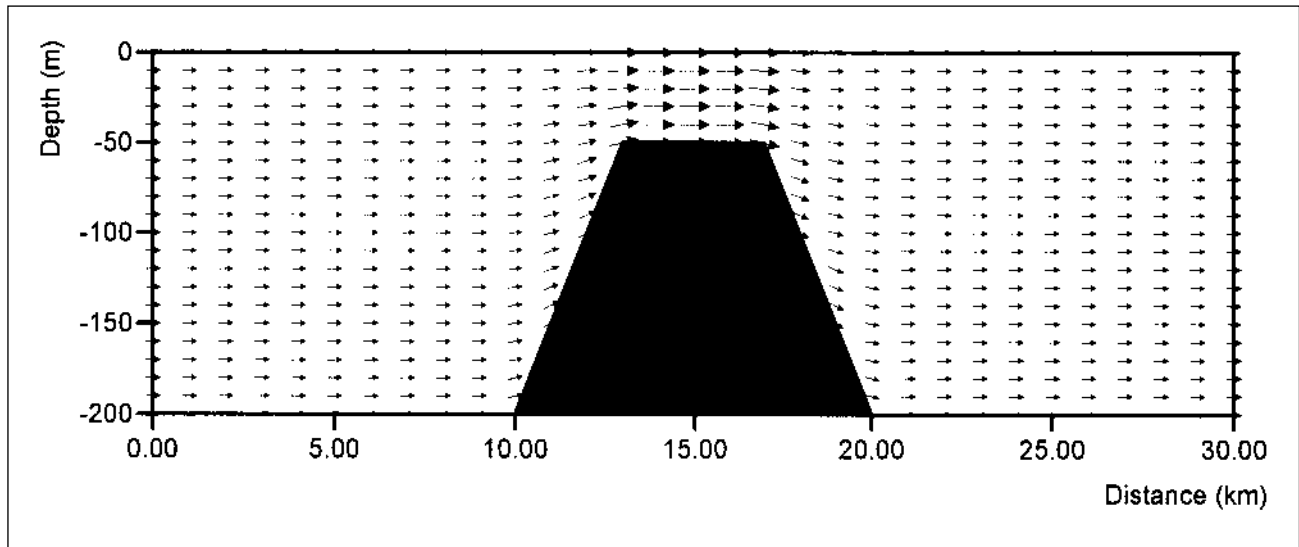
Figure 4. Velocity amplitude vectors. As a reference, consider that the vectors out of the ridge are 0.5 m s⁻¹ and on the top of the ridge 2 m s⁻¹. Instantaneous values are obtained by multiplying these vectors times a sine or cosine function

Numerical model

The velocity field calculated in the previous section was used to consider the effect of advection and diffusion in a sea whose temperature distribution will initially depend only on the vertical co-ordinate. Obviously, after some computation time, this distribution will depend on the horizontal one, also. The equation to be solved is the 2-D advection-diffusion equation:

$$\frac{\partial T}{\partial t} + U\,\frac{\partial T}{\partial x} + W\,\frac{\partial T}{\partial z} =$$

$$= K_x\,\partial^2\,\frac{T}{\partial x^2} + \frac{\partial}{\partial z}\left(K_z\,\frac{\partial T}{\partial z}\right) \qquad [7]$$

where K_x, and K_z are the horizontal and vertical thermal diffusion coefficients, and U, and W the components of the 2-D velocity field obtained in section 2. The problem is solved using a mesh with $\Delta x = 500$ m, $\Delta z = 5$ m and $\Delta t = 1$ s. The finite differences scheme used was forward in time and centered in space. This integration scheme has very severe stability conditions which can be expressed as (Fletcher, 1987):

$$S_x + S_z = 0.5 \qquad \left(\frac{C_x^2}{S_x}\right) + \left(\frac{C_z^2}{S_z}\right) \le 2 \qquad [8]$$

$$\text{where} \quad S_x = K_x\,\frac{\Delta t}{(\Delta x)^2} \quad , \quad C_x = U\,\frac{\Delta t}{\Delta x}$$

$$\text{and} \quad S_z = K_z\,\frac{\Delta t}{(\Delta z)^2} \quad , \quad C_z = W\,\frac{\Delta t}{\Delta z}.$$

These conditions do not consider the spatial variation of eddy diffusion coefficients. To avoid stability problems we have reduced considerably the Δt value. We imposed the boundary condition of zero heat flux through the bottom, but a realistic boundary condition for the sea surface has yet to be established.

This point presents some difficulties. One possibility is to estimate the heat flux through the sea surface, Q_H as $Q_H = \rho c K_z \left(\frac{\partial T}{\partial z}\right)_0$ where ρ is the water density, c the specific heat capacity for sea water, K_z the eddy diffusion coefficient, and $(\partial T/\partial z)_0$ is the vertical gradient of temperature at the sea surface. Haney (1971) points out the weakness of this formulation, since we usually lack sufficient knowledge of K_z, and offers an alternative formulation for the net heat exchange between ocean and atmosphere; however, atmosphere temperature measurements are required. Since these data were not available in the present work, we imposed the boundary condition:

$$q_H = K_z\,\frac{\partial T}{\partial z}\bigg|_0 \quad \text{at } z = 0 \qquad [9]$$

$$\text{where } q_H = \frac{Q_H}{\rho c}.$$

However, q_H and K_z are still parameters to determine.

Estimation of K_z and q_H

Temperature profiles, obtained from different oceanographic surveys in our study area, show temperature variations throughout the year, so that the water column is quite homogenous during the end of winter, while at the end of summer or beginning of autumn, a well-mixed warm layer is developed over a sharp thermocline.

Furthermore, temperature profiles are assumed to remain basically unchanged over a period 1-2 days.

Values chosen for q_H and K_z should reproduce these features. On the other hand, the determination of eddy diffusion coefficients does not reduce to finding a single value, but to finding its dependence with depth. K_z variability has been parameterised in different ways. In Pacanowsky and Philander (1981), the expression $K_z = K_b + \left[\dfrac{\nu}{(1 + \gamma Ri)} \right]$ is used, where K_b is a background dissipation parameter, ν is a eddy diffusion coefficient for momentum, Ri is the Richardson number, and γ is a numerical adjust parameter. We see that K_z depends on the vertical coordinate through variations in the stratification of water and vertical momentum shear. In a dynamic model, these parameters can also depend on time. We shall not consider this time dependence in the present paper. K_z will depend on the vertical coordinate to allow for the existence of a turbulent mixed layer over a water column where turbulence is supposed to decrease with depth, but this depth dependence will not change with time.

Regarding the estimation of q_H, let us consider a time dependence

$$q_H = q_0 \, sin \, (\omega t) \qquad [10]$$

where $\omega = 2\pi/T$, and the period T a year.

The problem is now to estimate q_0, which will be considered a negative magnitude indicating heat flux into the ocean. This assumption implies taking the time origin at the end of winter, when the periodic heat flux postulated in [10] would start going into the sea. Note that [10] implies zero net flux when averaged over an entire year, which is not generally true. When considering net heating as:

$$Q_N = Q_s - Q_I - Q_T - Q_L \qquad [11]$$

where Q represents heat flux, and the subindices N, S, I, T, and L stand for net heating, solar radiation absorption, net loss of infrared radiative energy, loss at the surface through turbulent fluxes, and loss of latent flux through evaporation, respectively, this amount averaged over one year can be positive or negative, depending on the latitude considered. Horizontal advection accounts for this locally unbalanced amount of heat in order to keep the year-averaged temperature of the mixed layer constant, i.e. regions that receive heat from the atmosphere when averaging over a year must export heat to those areas where losses exceed heat gain. Behringer and Stommel (1981) use this balance as an alternative method to estimate Q_N. Therefore, expression [10] should not be taken as equivalent to [11], as the sinusoidal boundary condition used in our model averages to zero; i.e. heat export to adjacent regions has been taken into account implicitly.

Then we developed a 1-D model in finite differences to solve:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(K_z \frac{\partial T}{\partial z} \right) \qquad [12]$$

and to model the temporal evolution of the water column temperature. A scheme that was forward in time and centred in space was used, with $\Delta z = 5$ m, and $\Delta t = 15$ s.

The following step was to run the model corresponding to equation [12], with different values for the parameters to be established: q_0, K_z. Comparison between model outputs and experimental data will determine if values used are the right ones. Therefore, the two kinds of simulations conducted were the following:

1. From a temperature profile taken from experimental data collected in this area during summer, e.g. the one in figure 5, we ran the model for a period of 1-2 days under different values of q_0, K_z. A choice in these parameters was considered correct if the temperature profile did not change appreciably in such a brief period of time.

2. We took a uniform distribution of temperature along the water column (13 °C), similar to the one found at the end of winter. From this initial condition, we ran the model for a period of several motns. Parameters chosen were now considered acceptable if the mixed layer and thermocline were developed correctly.
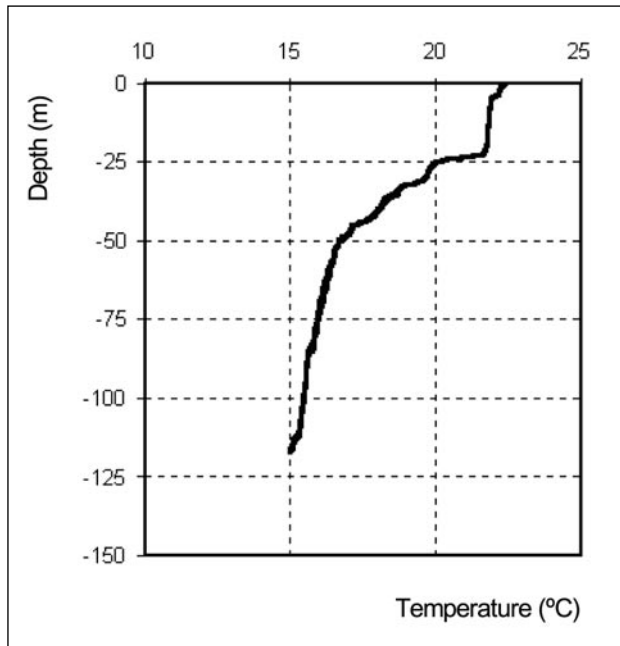
Figure 5. A typical temperature profile from Ictio.Alborán surveys in the Gulf of Cadiz in an area not affected by the thermal anomaly, when a warm mixed layer is well developed. Its position is marked on figure 1, and can be used for imposing initial conditions to the advection-diffusion model

Figure 6 shows results from simulations over a 3-month period with different values of K_z. Heat flux corresponds to $q_0 = -0.000072$ °C km/s (equivalent to 300 W m^{-2}), and three different choices for K_z (marked on figure 6 with different letters). In figure 6A, this coefficient changes linearly from 0.1 m^2 s^{-1} at the surface to 0.001 m^2 s^{-1} at 30 m. For the rest of the water column this value remains constant. In figures 6B and 6C, we have results corresponding to constant values of K_z along the water column. In 6B, we have taken what we consider a turbulent one, 0.1 m^2 s^{-1}, and in 6c a much lower value, 0.001 m^2 s^{-1}. Simulations with different heat fluxes had worse results, so, finally, parameters from figure 6A were considered the right ones. Note the importance of the change of K_z with depth. High values in the upper layer (due to turbulent motion) favour mixing processes. The change in these coefficients with depth is responsible for heat being trapped mainly in this mixed layer, forming the seasonal thermocline. On the other hand, constant values of K_z did not give satisfactory results. When this value is a turbulent one, heat can transmit into the deep sea easily, and distributes uniformly along the water column. In this case, high temperature values cannot be reached. If the coefficient is low, heat coming

from the atmosphere cannot propagate into the sea, and surface temperature values rise too much. On the other hand, when the model was run with parameters chosen from an initial condition similar to figure 5, for a period of 1-2 days, the situation does not change appreciably.

Once we had determined the heat flux value and the K_z dependence with depth, (the one used in figure 6A), we ran the model corresponding to equation [7] for a real time of 12 tidal cycles. The initial temperature distribution only depends on z. It is 22 °C on a 30-m thick mixed layer, and then a linear variation to 13 °C at a depth of 100 m. The rest of the water column has a constant temperature of 13 °C to the 200 m bottom in those places where depth reaches this value. Figure 7 shows the temperature distribution after the simulation and averaged over a tidal period to avoid advection effects, which would depend on the moment of the tidal cycle when we stopped the simulation.

DISCUSSION

We can observe the decrease of surface temperature and the steeping of isotherms on both sides of the ridge. The time averaging suppresses the effect of advection, so the results are due to the heat diffusion. According to [6], any fluid element which is on the slope of the ridge will have a vertical motion. This will be upward for a half period and downward for the second half period. Let's consider the rest position Z_0 as that one when the velocity field is zero. After T/2 (being T the tidal period), this fluid element reaches its highest position that can be estimated using [6]:

$$W = \frac{dz}{dt} = Qz \left(\frac{1}{H^2} \right) \left(\frac{dH}{dx} \right) sin (\omega t)$$

$$\frac{1}{z} \left(\frac{dz}{dt} \right) = Q \left(\frac{1}{H^2} \right) \left(\frac{dH}{dx} \right) sin (\omega t) \qquad [13]$$

Integrating [13] between t = 0 and t = $\frac{T}{2}$ we get:

$$ln \, Z \left(\frac{T}{2} \right) = ln \, Z_0 + \left(\frac{Q}{H^2} \right) \left(\frac{dH}{dx} \right) \left(\frac{-1}{\omega} \right) cos (\omega t) \Big|_0^{T/2}$$

Finally, if we call α to the slope of the ridge we get the expression:

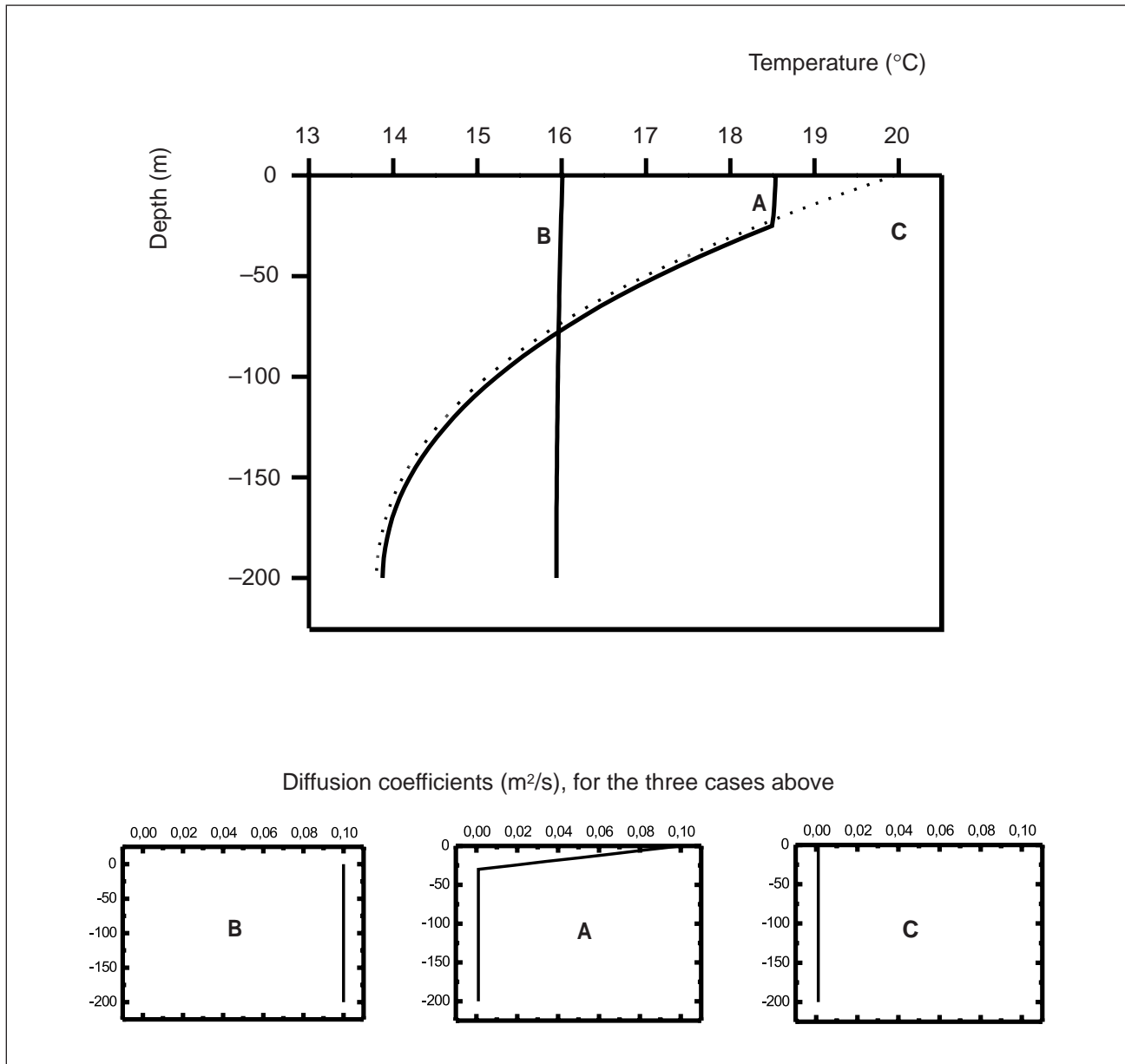$$Z \left(\frac{T}{2} \right) = Z_0 \exp \left(\frac{2Q\alpha}{H^2 \omega} \right) \qquad [14]$$

Figure 6. Numerical solution for a 1-D model and three different values for eddy diffusion coefficients. The simulation is stopped after a 3-month period. Figure 6A reproduces a reasonable situation for springtime. Further simulations confirmed these parameters as the best choice for the advection-diffusion model

If we consider a moment when Q is positive, i.e. U is to the right in our model, the sign in the argument of the exponential in [10] will depend on the sign of α. For a fluid element on the left side of the ridge, $\alpha = -0.05$. If initially it is at the bottom of the sea ($Z_0 = -200$ m), and right in the place where the slope becomes non-zero, after T/2 will be at Z (T/2) = −33 m.

This means that material surfaces undergo strong vertical excursions, and so, depending of

the moment of the tidal period we consider, this will cause a strong slope of isotherms. On the other hand, we see how a fluid element with 13 °C, which is immersed in a homogenous layer where thermal diffusion is very unlikely (because of the low K_z value), is able to almost reach the turbulent layer and to absorb heat from it (higher value of K_z), thus cooling surface waters and warming deep waters. Therefore, waters on the slope of the ridge become more uniform with lower tempera-
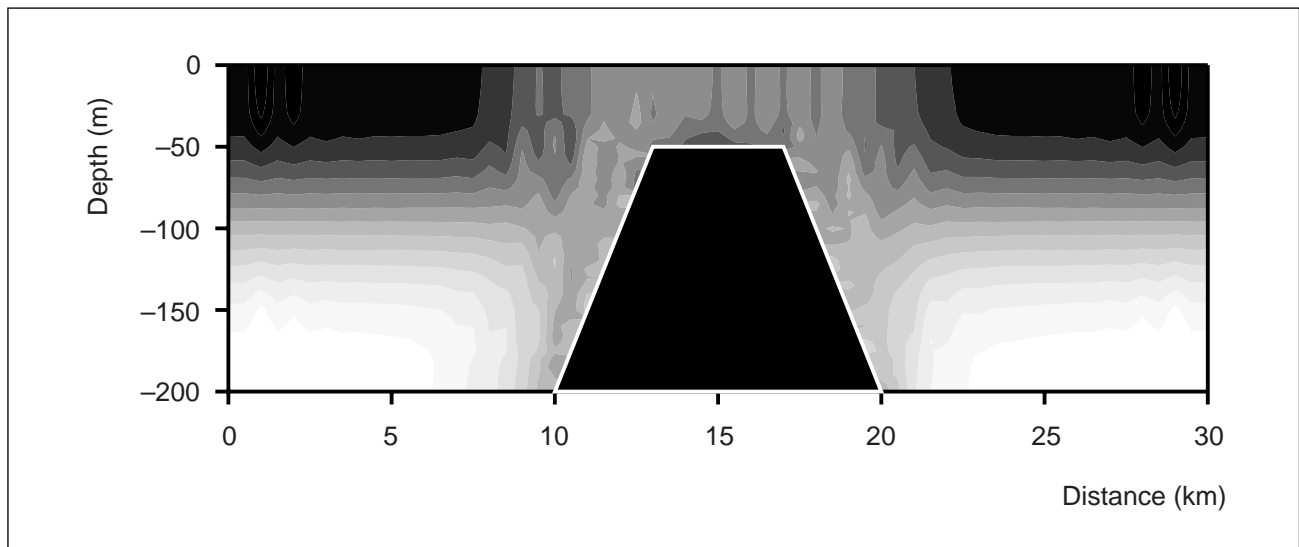
Figure 7. Results from the 2-D advection-diffusion numerical model after 12 tidal cycles of simulation. Bright colours indicate cooler waters. Different results could be expected depending on the moment of the tidal cycle when we stop the simulation because of the effect of advection. To avoid this effect results have been averaged over a tidal cycle

ture values at the surface. This cannot happen on the top of the ridge, where the slope is zero and so are vertical excursions. In this case we have to consider the noticeable increase of horizontal velocity as we impose the constant flux condition: Q = HU. For a depth change as used in our model from 200 m to 50 m, this implies that U will be four times higher on the top of the ridge than off it. Thus, horizontal advection is able to transport cold water from the sides of the ridge to the top, forcing horizontal diffusion. The result is as shown in figure 7: a cooling and uniformisation of waters over the whole area of the ridge. This coincides quite well with the thermal anomaly described in section 1, and so it is the mechanism that we propose to explain it. We are also convinced that this advection diffusion mechanism, as well as the important role of the mixed turbulent layer, can be extrapolated to other properties, e.g. nutrient concentration. In this case, new terms to express source-or-sink terms by biological processes must be included in equation [7], and this will be a subject for future research.

ACKNOWLEDGEMENTS

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