
Hubbert, S. and Levesley, J., (2001). An open problem concerning Fourier transforms of radial functions in Euclidean space and on spheres. *In* W. Haussmann; K. Jetter and M. Reimer, eds. *Recent Progress in Multivariate Approximation*. 4th International Conference, September 2000, Witten-Bommerholz. International Series of Numerical Mathematics, 137. Basel: Birkhäuser, pp. 225-226.

Open Problem

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Let $P_n^(\lambda)$ be the Gegenbauer polynomials, orthogoanl on $[-1,1]$ with respect to the weight $(1-t^2)^{\lambda-1/2}$, normalised by

$$P_n^{(\lambda)}(1) = \frac{\Gamma(n+2\lambda)}{(\Gamma(2\lambda)\Gamma(n+1)}$$

Let ϕ be continuous and $\phi^{(m)}$ be completely monotone on $(0,\infty)$ for some $m \in \mathbb{N}$. Let $\Phi(x) = \phi(\|x\|)$ have a (generalised) Fourier transform with polynomial decay, that is

$$\hat{\Phi}(\xi) = \mathcal{O}(\|\xi\|^{-d-\alpha},$$

for some $\alpha > 0$ and $d = 2\lambda + 1$. Then, the restriction of Φ to the sphere,

$$\Psi(x,y) = \phi\left(\sqrt{2-2x^Ty}\right)$$

has a representation as a spherical Fourier series

$$\Psi(x,y) = \sum_{k=0}^{\infty}\sum_{l=1}^{d_k} c_k Y_{kl}(x)Y_{kl}(y),$$

whose spherical Fourier coefficients $\{c_k\}$ have the analagous decay rate

$$c_k = \mathcal{O}(k^{-d+1-\alpha}).$$

Here $\{Y_{kl}\}$, $l = 1,\ldots,d_k$, $k = 0,1,\ldots$, form an orthonormal basis for the spherical harmonics of degree k, which has dimension d_k.