



## Birkbeck ePrints: an open access repository of the research output of Birkbeck College

<http://eprints.bbk.ac.uk>

---

Hubbert, S. and Levesley, J., (2001). An open problem concerning Fourier transforms of radial functions in Euclidean space and on spheres. In W. Haussmann; K. Jetter and M. Reimer, eds. *Recent Progress in Multivariate Approximation*. 4th International Conference, September 2000, Witten-Bommerholz. International Series of Numerical Mathematics, 137. Basel: Birkhäuser, pp. 225-226.

---

This is an author-produced version of a paper published in *Recent Progress in Multivariate Approximation* (ISBN 3764365056). This version has been peer-reviewed but does not include the final publisher proof corrections, published layout or pagination. All articles available through Birkbeck ePrints are protected by intellectual property law, including copyright law. Any use made of the contents should comply with the relevant law.

### Citation for this version:

Hubbert, S. and Levesley, J., (2001). An open problem concerning Fourier transforms of radial functions in Euclidean space and on spheres. London: Birkbeck ePrints. Available at: <http://eprints.bbk.ac.uk/archive/00000397>

### Citation for the publisher's version:

Hubbert, S. and Levesley, J., (2001). An open problem concerning Fourier transforms of radial functions in Euclidean space and on spheres. In W. Haussmann; K. Jetter and M. Reimer, eds. *Recent Progress in Multivariate Approximation*. 4th International Conference, September 2000, Witten-Bommerholz. International Series of Numerical Mathematics, 137. Basel: Birkhäuser, pp. 225-226.

---

<http://eprints.bbk.ac.uk>

Contact Birkbeck ePrints at [lib-eprints@bbk.ac.uk](mailto:lib-eprints@bbk.ac.uk)

# Open Problem

J. Levesley and S. Hubbert

Let  $P_n^{(\lambda)}$  be the Gegenbauer polynomials, orthogonal on  $[-1, 1]$  with respect to the weight  $(1 - t^2)^{\lambda-1/2}$ , normalised by

$$P_n^{(\lambda)}(1) = \frac{\Gamma(n + 2\lambda)}{(\Gamma(2\lambda)\Gamma(n + 1))}$$

Let  $\phi$  be continuous and  $\phi^{(m)}$  be completely monotone on  $(0, \infty)$  for some  $m \in \mathbb{N}$ . Let  $\Phi(x) = \phi(\|x\|)$  have a (generalised) Fourier transform with polynomial decay, that is

$$\hat{\Phi}(\xi) = \mathcal{O}(\|\xi\|^{-d-\alpha}),$$

for some  $\alpha > 0$  and  $d = 2\lambda + 1$ . Then, the restriction of  $\Phi$  to the sphere,

$$\Psi(x, y) = \phi\left(\sqrt{2 - 2x^T y}\right)$$

has a representation as a spherical Fourier series

$$\Psi(x, y) = \sum_{k=0}^{\infty} \sum_{l=1}^{d_k} c_k Y_{kl}(x) Y_{kl}(y),$$

whose spherical Fourier coefficients  $\{c_k\}$  have the analogous decay rate

$$c_k = \mathcal{O}(k^{-d+1-\alpha}).$$

Here  $\{Y_{kl}\}$ ,  $l = 1, \dots, d_k$ ,  $k = 0, 1, \dots$ , form an orthonormal basis for the spherical harmonics of degree  $k$ , which has dimension  $d_k$ .