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Theoretical analysis of the limiting rate of phreatic evaporation for aeolian sandy soil in Taklimakan Desert

HU ShunJun[1,2†], LEI JiaQiang[2], XU XinWen[2], SONG YuDong[2], TIAN ChangYan[1,2], CHEN XiaoBin[3] & LI XiuChang[2]

[1] Key Laboratory of Oasis Ecology and Desert Environmental, Xinjiang Institute of Ecology and Geography, Chinese Academy of Sciences, Urumqi 830011, China;
[2] Xinjiang Institute of Ecology and Geography, Chinese Academy of Sciences, Urumqi 830011, China;
[3] Yantai Institute of Coastal Zone Research for Sustainable Development, Chinese Academy of Sciences, Yantai 264000, China

Phreatic evaporation is a great lose for shallow groundwater in the Taklimakan Desert. Given soil type and groundwater table, the limiting rate of phreatic evaporation is defined as the maximum of water transferred from groundwater to soil surface per unit time, which is a key parameter and control condition for phreatic evaporation model developing. The soil water characteristic curve for the aeolian sandy soil in the Taklimakan Desert was fitted with the least square method based on the formula of soil moisture characteristics curve proposed by Van Genuchten, using observed soil moisture and soil water suction data. The unsaturated hydraulic conductivity was determined by the instantaneous profile method *in situ* and the calculation formula for unsaturated hydraulic conductivity was established. According to the steady flow theory, the quasi-analytical solution of limiting rate of phreatic evaporation was derived on the basis of generalization of the formula of unsaturated hydraulic conductivity. The results show that the soil moisture characteristics in the Taklimakan Desert can be well described by Van Genuchten's formula, and the limiting rate of phreatic evaporation declines by power function with the descending of groundwater table.

aeolian sandy soil, limiting rate of phreatic evaporation, steady flow, unsaturated hydraulic conductivity, Taklimakan Desert

Phreatic evaporation is a process that the water is transferred from phreatic zone to unsaturated zone and further to air through soil evaporation. Numerous studies have been done on the mechanism of phreatic evaporation in the past decades, concerning soil physics, hydrogeology, hydraulic engineering and water sources science, etc.[1−11]. Given the soil properties and groundwater table, the phreatic evaporation increases with atmospheric evaporation capacity. When the atmospheric evaporation capacity tends to the infinite, the phreatic evaporation approximates the limiting rate of phreatic evaporation which is defined as the maximum of water transferred from groundwater to soil surface per unit time at the given soil and groundwater conditions[12−15]. The limiting rate of phreatic evaporation is a key parameter and control condition for developing phreatic evaporation models[15]. Presently, this parameter is often determined by graphical solution, nonlinear regression iteration, theoretical analyses, etc. For example, Gardner[1], Bever et al.[16], Gao[17], Jury et al.[18], Zhang[19,20], Guo[21], Tang et al.[8], Ma et al.[22] explored the analytical solution of the limiting rate of phreatic evaporation, and Shi et al. [23] analyzed the effect of salinity on the phreatic evaporation. The analytical solution of the limiting rate of phreatic evaporation not only provides sound and reliable theoretical support for phreatic evaporation model, but also is of significance in water

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†Corresponding author (email: xjhushunjun@yahoo.com.cn)
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resources assessment, water balance analysis, saline-alkali soil amelioration, irrigation system improvement and the calculation of ecological water requirements[24−30].

1 Study site

The study site lies in the hinterland of Taklimakan Desert. The annual mean temperature is 12.4℃. The hottest month is July, with 28.2℃ in monthly mean temperature, and −8.1℃ for the coldest December. The precipitation averages 11.05 mm, but mainly concentrates between May and August, accounting for 92.3% of the annual total. The annual water surface evaporation is 3638.6 mm (obtained by evaporation pan 20 cm in diameter), with the monthly maximum mean, 563.2 mm, occurring in June, and the minimum, 34.4 mm, occurring in December. The annual sunshine duration is about 2571.3 h. The maximum monthly mean of sunshine hours is up to 8.3 h, occurring in October, but the minimum monthly mean, 5.2 h, occurring in January. The relative humidity annually averages 29.4%, and the maximum and minimum monthly mean appears in January and April, 46% and 15.5%, respectively. The wind is dominated by north-to-east, with an annual mean speed of 2.5 m/s and a maximum instantaneous value of 20 m/s[31]. There is a great change range of groundwater table, greater than 60 m at the top of the complex dune, 3—8 m at inter-dune, and about 1 m in the specific lowland The phreatic water is highly saline, with a mineralization degree of about 4—5 $g \cdot L^{-1}$. The soil, composed of thick sand layers, is classified as aeolian sandy soil, containing 0.05—0.25 mm particles greater than 70%—80%, which indicates that the soils are sufficiently sorted by wind. The soil bulk density ranges from 1.49 to 1.51 $g \cdot cm^{-3}$, and the saturated water content is 0.43 $m^3 \cdot m^{-3}$[31−33].

2 Theoretical analysis

The distributions of volumetric water content and suction of homogenous soil under stable evaporation are presented in Figure 1. Given the groundwater table, the stable evaporation of homogenous soil is determined by[12]

$$\begin{cases} E = K(S)\dfrac{\mathrm{d}S}{\mathrm{d}Z} - K(S), \\ S(0) = 0, \end{cases} \quad (1)$$

where Z is vertical coordinate (m), K(s) is unsaturated hydraulic conductivity (mm/d), and S is soil water suction (m).

If K(s) is given by the general form

$$K(S) = \frac{K_s}{1 + fS^N}, \quad (2)$$

where K_s is the saturated conductivity (mm/d), f and N are pending parameters, then

$$E = \frac{K_s}{1 + fS^n}\left(\frac{\mathrm{d}S}{\mathrm{d}Z} - 1\right),$$

$$\mathrm{d}Z = \frac{1}{1 + \dfrac{E}{K_s} + \dfrac{E}{K_s} fS^N}\mathrm{d}S.$$

When Z=0, S=0; when Z=H (phreatic water depth) and $S\to\infty$, $E\to E_{\max}$. Therefore, the integral expression of the above-mentioned equation is

$$\int_0^H \mathrm{d}Z = \int_0^\infty \frac{1}{1 + \dfrac{E_{\max}}{K_s} + \dfrac{E_{\max}}{K_s} fS^N}\mathrm{d}S. \quad (3)$$

When $E_{\max} << K_s$, then

$$H = \int_0^\infty \frac{1}{1 + \dfrac{fE_{\max}}{K_s} S^N}\mathrm{d}S. \quad (4)$$

Supposing $\dfrac{fE_{\max}}{K_s} S^N = y^N$, then

$$S = \left(\frac{K_s}{fE_{\max}}\right)^{\frac{1}{N}} y,$$

$$\mathrm{d}S = \left(\frac{K_s}{fE_{\max}}\right)^{\frac{1}{N}} \mathrm{d}y.$$

If $S\to 0$, $y\to 0$; $S\to\infty$, $y\to\infty$, then H is determined by

$$H = \int_0^\infty \frac{1}{1+y^N}\left(\frac{K_s}{fE_{\max}}\right)^{\frac{1}{N}} \mathrm{d}y = \left(\frac{K_s}{fE_{\max}}\right)^{\frac{1}{N}} \int_0^\infty \frac{1}{1+y^N}\mathrm{d}y.$$

Because

$$\int_0^\infty \frac{1}{1+y^N}\mathrm{d}y = \frac{\pi}{N\sin\left(\dfrac{\pi}{N}\right)},$$

$$H = \left(\frac{K_s}{fE_{\max}}\right)^{\frac{1}{N}} \cdot \frac{\pi}{N\sin\left(\dfrac{\pi}{N}\right)},$$

and

$$E_{max} = \frac{K_s}{f}\left[\frac{\pi}{HN\sin\left(\frac{\pi}{N}\right)}\right]^N. \quad (5)$$

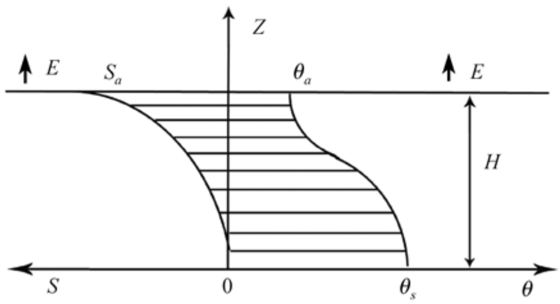

Figure 1 Distributions of soil moisture and suction under stable evaporation.

3 Parameters of soil water transportation

3.1 Soil water characteristic curve

The soil water characteristics formula developed by Van Genuchten[34,35] is defined as

$$\theta(S) = \theta_r + \frac{\theta_s - \theta_r}{[1+(\alpha S)^n]^m}, m = 1 - \frac{1}{n}, \quad (6)$$

where θ is volumetric soil moisture (m^3/m^3), θ_r is residual soil water content (m^3/m^3), and m, n and α are parameters. According to the relationship between soil water content and soil suction presented in Table 1[1)] [32], θ is fitted as

$$\theta = \frac{0.43}{[1+(1.8947S)^{2.8997}]^{0.6551}}. \quad (7)$$

3.2 Infiltration properties

The relationships between infiltration rate and time and between cumulative infiltration water and time determined by double loop infiltration experiment are shown in Figures 2 and 3, respectively. The former follows the Horton formula fitt as:

$$i = 3.3015 + (6.5446 - 3.3015)\exp(-0.0517t), \quad (8)$$

where t is infiltration time (min), i is infiltration rate (mm/min). The infiltration rate peaks at the beginning, then declines with time. It declines dramatically at the beginning, then slowly, until the infiltration rate close to stable infiltration rate. The infiltration capacity of the aeolian sandy soil is quite high, with the stable infiltration rate $i_c \approx 3.3015$ mm/min=3.3015×24×60=4754.16 mm/d.

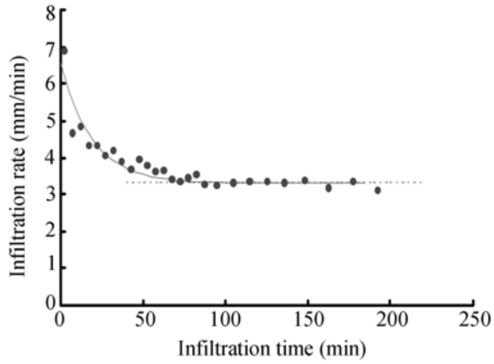

Figure 2 The relationship between infiltration rate and time.

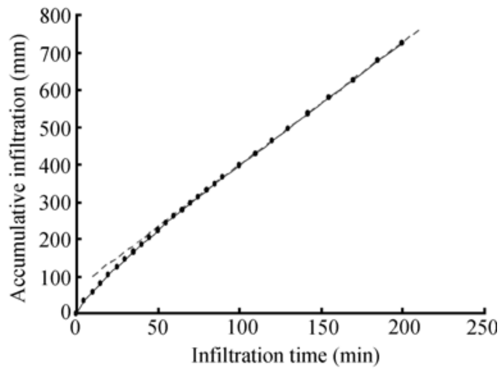

Figure 3 The relationship between accumulative infiltration and time.

3.3 Saturated hydraulic conductivity

The soil saturated hydraulic conductivity is determined by[36]

$$K_S = 2d_{10}^2 e^2 \times 10 \times 24 \times 3600, \quad (9)$$

where d_{10} is the effective particle size (mm); e is the soil porosity rate. Because $e/(1+e) = \theta_S$ where θ_S, the saturated conductivity equals 0.43, e can be obtained as 0.75. In addition, the effective particle size, d_{10}, is determined as 0.07 mm through particle grading curve based on the Soil mechanical composition (Table 2). Therefore, the soil saturated hydraulic conductivity is determined as 4762.8 mm/d according to formula (9).

Given enough time, the stable infiltration rate ap-

Table 1 The corresponding soil water suction of soil water content for aeolian sandy soil

Moisture content (m^3/m^3)	0.23	0.24	0.22	0.14	0.13	0.12	0.10	0.07	0.06	0.06
Soil water suction (m)	0.58	0.58	0.70	0.84	1.00	1.04	1.11	1.21	1.37	1.40

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Table 2 Soil mechanical composition

	Coarse sand	Fine sand	Coarse sand	Coarse sand	Coarse sand	Clay	Physical clay
Particle diameter (mm)	1－0.25	0.25－0.05	0.05－0.01	0.01－0.005	0.005－0.001	<0.001	<0.001
Content (%)	23.47	71.62	1.94	0.07	0.39	2.52	2.98

proaches to the saturated infiltration rate when the soil wsater is saturated[37,38]. Since K_s is 4754.16 mm/d obtained by double loop infiltration experiment, it averages 4758.5 mm/d over the calculated and observed results.

3.4 Unsaturated hydraulic conductivity

The unsaturated hydraulic conductivity is determined by the soil moisture redistribution under vertical infiltration, also called instantaneous profile method[12,39,40].

(i) Measurement principle. Assuming the downward direction is positive, the soil water flux $q(Z)$ for unsaturated one-dimensional flow, according to Darcy's law, may be written as

$$q(Z) = -K(\theta)\frac{\partial(\varphi_m - Z)}{\partial Z} = -K(\theta)\frac{\partial(-S - Z)}{\partial Z} = K(\theta)\left(\frac{\partial S}{\partial Z} + 1\right). \quad (10)$$

Thus

$$K(\theta) = \frac{q(Z)}{\frac{\partial S}{\partial Z} + 1} \approx \frac{q(Z)}{\frac{\Delta S}{\Delta Z} + 1}. \quad (11)$$

According to the principle of mass conservation, we can obtain the equation:

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial Z}. \quad (12)$$

Therefore, integrating eq. (12) from Z_0 to Z, soil moisture flux at any depth Z, $q(Z)$, and unsaturated hydraulic conductivity (mm/d), $K(\theta)$, can be given in the form

$$q(Z) = q(Z_0) - \frac{1}{\Delta t}\left[\int_{Z_0}^{Z} \theta(t_{i+1})\mathrm{d}Z - \int_{Z_0}^{Z} \theta(t_i)\mathrm{d}Z\right], \quad (13)$$

$$K(\theta) = \frac{q(Z_0) - \frac{1}{\Delta t}\left[\int_{Z_0}^{Z} \theta(t_{i+1})\mathrm{d}Z - \int_{Z_0}^{Z} \theta(t_i)\mathrm{d}Z\right]}{\frac{\Delta S}{\Delta Z} + 1}, \quad (14)$$

where θ is volumetric soil water content (m^3/m^3) and t is time (d).

(ii) Measurement method. The measurements were carried out at a 1 m×1 m plot without vegetation during September 25 to October 8 and October 14 to October 20, 2007. A neutron tube was buried in the plot center. To set the one-dimensional vertical infiltration condition, the plot was irrigated overland flow. When the soil was highly wetted, the irrigation was terminated, and the plot was covered with plastic film to prevent evaporation and keep the soil moisture flux at zero in the soil surface. Whereafter, soil water content was measured with neutron probe at various time. Soil water suction was transferred from the soil water content observed through soil water characteristic curve.

(iii) Results. Figure 4 shows the relationship between unsaturated hydraulic conductivity and soil water suction of the aeolian sandy soil, which may be well fitted in the form

$$K(S) = \frac{4758.5}{1 + 1186.10386 S^{3.8937}}. \quad (15)$$

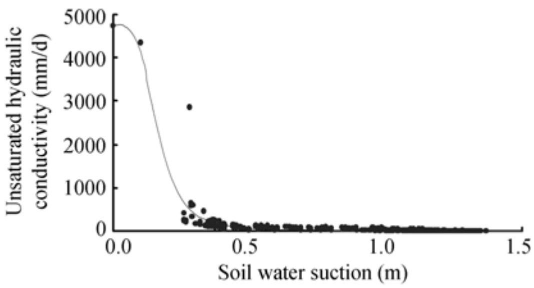

Figure 4 The relationship between soil water suction and unsaturated hydraulic conductivity.

4 The relationship between phreatic evaporation and phreatic depth

Substituting K_s=4762.8, f=1186.1038 and N=3.8937 into eq. (5), we obtain

$$E_{\max} = 6.1797 H^{-3.8937}. \quad (16)$$

5 Conclusions and discussion

(1) The soil water characteristic curve proposed by Van Genuchten can well describe the soil moisture characteristics in the Taklimakan Desert in which the aeolian sandy soils are composed of coarse and uniform particles.

(2) The limiting rate of phreatic evaporation only depends on soil properties and phreatic water depth. It is declined by power function with the descending of phreatic water depth. The exponent m is equal to 3.8937,

which agrees with the previous result about *m* ranging between 1—4 and generally large for sandy soil[1)].

(3) The formula of the limiting rate of phreatic evaporation can be used to estimate phreatic evaporation in the non-freezing period, because of the low precipitation, high evaporation, extreme dry soil surface, high soil water suction approaching the infinite and stable groundwater table in the Taklimakan Desert[41,42], which satisfy the condition of stable evaporation of homogeneous soils at the stable groundwater table.

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