Winter 12-2016

Essays on Artefactual and Virtual Field Experiments in Choice Under Uncertainty

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ABSTRACT

ESSAYS ON ARTEFACTUAL AND VIRTUAL FIELD EXPERIMENTS

IN CHOICE UNDER UNCERTAINTY

BY

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DECEMBER 2016

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In the area of transportation policy, congestion pricing has been used to alleviate traffic congestion in metropolitan areas. The focus of Chapter 1 is to examine drivers' perceived risk of traffic delay as one determinant of reactions to congestion pricing. The experiment reported in this essay recruits commuters from the Atlanta and Orlando metropolitan areas to participate in a naturalistic experiment where they are asked to make repeated route decisions in a driving simulator. Chapter 1 examines belief formation and adjustments under an endogenous information environment where information about a route can be obtained only conditional on taking the route. If the subjects arrive to the destination late, i.e. beyond an assigned time threshold, they are faced with a discrete (flat) penalty. In contrast, Chapter 2 examines subjective beliefs in a setting where the penalty for a late arrival is continuous, such that a longer delay incurs additional penalty on the driver. The primary research question is: does belief formation differ when the late penalty is induced as a continuous amount compared to when it is induced as a discrete amount? In particular, will we observe a difference in learning across the

range of congestion probabilities under different penalty settings? In the continuous penalty setting, we do *not* observe a difference in learning across the range of congestion probabilities. In contrast, in the discrete penalty setting we observe significant belief adjustments in the lowest congestion risk scenario.

In Chapter 3 the "source method" is used to examine how uncertainty aversion differs across events that have the same underlying objective probabilities but are presented under varying degrees of uncertainty. Subjects are presented with three lottery tasks that rank in order of increasing uncertainty. Given the choices observed in each task a source function is estimated jointly with risk attitudes under different probability weighting specifications of the source function. Results from the Prelec probability weighting suggest that, as the degree of uncertainty increases, subjects display increased pessimism; in contrast, the Tversky-Kahneman (1992) and the Power probability weightings detect no such difference. Thus, the conclusion regarding uncertainty aversion are contingent on which probability weighting specification is assumed for the source function.

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BY

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A Dissertation Submitted in Partial Fulfillment
of the requirements for the Degree
of
Doctor of Philosophy
in the
Andrew Young School of Policy Studies
of
Georgia State University

GEORGIA STATE UNIVERSITY
2016

ACCEPTANCE

This dissertation was prepared under the direction of the candidate's Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics in the Andrew Young School of Policy Studies of Georgia State University.

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December, 2016

ACKNOWLEDGEMENTS

I am grateful to have an excellent team of mentors and advisers, Dr. Elisabet Rutström and Dr. Glenn Harrison, whose guidance and support have been instrumental to my dissertation and to my career. Thank you for giving me the strict discipline that was much needed, and at times having a precious sense of humor! I cannot think of a better combination.

I would also like to thank Dr. Vjollca Sadiraj and Dr. Spencer Banzhaf for being members of my dissertation committee. Their insights and vigorous questioning have been an integral part in my development as a scholar. Special thanks to Dr. James Cox for the invaluable training in experimental designs and protocols, and to Dr. James Marton for the wonderful lessons in health economics which has fostered my research interest in behavioral health.

I want to thank the Center for the Economic Analysis of Risk, CEAR, for providing an excellent learning environment through its seminars and workshops that further my exposure to the field of risk, uncertainty and ambiguity.

I would like to thank my long-time mentor, Bryant Houston, who encouraged me to explore opportunities and introduced me to the field of behavioral economics.

Special thanks to my good old friends, Jean Yu and Ping Ma, who have stood by me through all these years, providing laughter and support.

Last and most importantly, I would like to thank my Mom and Dad, who teach me the values of education and character. To my Auntie, who grows the most beautiful roses in her garden and brings them to me at times when I needed "to smell the roses". Also, to my little brother, Dr. Chu Tsang, for sharing his research insights and providing technical support. He is *always* there for me.

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INTRODUCTION

The common theme across the three chapters in this dissertation is the study of decision making under uncertainty. The first two chapters investigate risk perception in the context of driving. In a driving simulator experiment, subjects are presented with an unknown probability of traffic delay and make route choices over multiple periods under a range of exogenous congestion probabilities. This allows us to compare if learning differs across the range of congestion probabilities. The third chapter examines if behavior differs across events that are presented under varying degrees of uncertainty in a context free task.

The goal of Chapters 1 and 2 is to examine drivers' perceived risk of travel delay in explaining their route choices. For any given trip, there are uncertainties about the amount of time that it takes to complete a trip as well as the level of congestion on the route. The importance of subjective beliefs in influencing drivers' behavior is well-stated in Hensher, Li and Ho (2014):

> Travelers need to assess the probability distribution of possible travel times for a future trip based on their experience, beliefs, etc.… Since travel time variability is best described under uncertainty rather than risk, respondents should be asked to provide their judged probabilities associated with different travel outcomes (i.e., subjective probabilities for uncertainty) in a choice study.

Consistent with this suggestion, the experiment described in Chapters 1 and 2 allow us to infer drivers' subjective beliefs over the uncertain risk of travel delay from their observed choices. The experiment is conducted using field drivers from the Atlanta and Orlando metropolitan areas, and the choice task that they participate in is designed with many features of a natural driving experience. These field subjects are asked to make route decisions as they are driving in a driving simulator. Using a driving simulator as an instrument to examine drivers' behavior is a relatively novel development in transportation experiments (Dixit, Harrison and Rutström (2014)). We apply this experimental approach to examine route choices in a repeated choice setting.

One design feature of the experiment is that subjective beliefs are elicited under an endogenous information environment: information about a route (such as its congestion level, travel time variability) can only be obtained if one drives on that route, thus information gathering is endogenous. This is an information environment that commonly occurs in practice, but has not received much attention in the literature on belief formation and learning.

In the experiment subjects are asked to make a binary choice between a route that has an uncertain level of congestion and an alternate route with no risk of congestion. For the route that has an uncertain level of congestion, four treatments are implemented that differ in terms of the range of congestion probabilities. The treatments range from a low risk of congestion to a high risk of congestion. Subjects are assigned monetary incentives for the value of making the drive, the penalty for arriving late to the destination, and the toll charged on the non-congested route. Apart from some prior information about frequency of congestion on the uncertain route, drivers only obtain additional information if they actually choose to drive on it. The research questions are: will the subjects be able to discern different levels of congestion risk (that are not told to them)? Furthermore, as the subjects gain experience driving, will we observe learning as well as differences in learning across the four levels of congestion risk? Our hypothesis is that, in this endogenous information environment, subjects who started with a prior belief of low congestion (i.e., those who are in the low-congestion risk treatment) are more likely to drive on the uncertain route and thus are able to obtain more information to revise their prior belief. Therefore, these subjects should display more learning than their counterparts who are in the high-congestion risk treatments.

Both Chapters 1 and 2 ask the same set of questions with respect to subjective beliefs. However, the two chapters differ in one crucial aspect. The difference is in the penalty that is associated with a late arrival. In Chapter 1 the penalty for a late arrival is fixed regardless of the extent of delay. In contrast, in Chapter 2 the penalty is variable and is contingent on the extent of delay. In other words, in Chapter 1 the penalty for a late arrival is discrete, whereas in Chapter 2 the penalty for a late arrival is continuous. While there is not yet a theoretical model that takes into account the possible behavioral difference under these two penalty settings, the empirical significance of this question is worth investigating. Since the delay penalty for each trip may differ depending on the purpose of the trip or the characteristics of the travelers, it is natural to investigate route behavior by delay penalties as it realistically reflects different types of trips and/or different groups of travelers.

Recall that in an endogenous information environment, we expect to observe more learning in the low-congestion risk treatments than in the high-congestion risk treatments. We can then ask if the same pattern of behavior will still take place when the penalty is induced in a discrete *vs*. a continuous manner. To examine behavior in a setting where the late penalty is continuous calls for an experimental design that has variability in arrival times so that the extent of delay varies. The advantage of using driving simulators in a choice task is that the amount of time it takes to complete the drive varies depending not only on route selection, and the congestion scenario on the uncertain route, but also on how the subjects drive on the simulator. In this way, the arrival times along with late penalties are naturalistically induced as continuous variables.

Chapter 3 builds on the previous two chapters, and the goal is to examine how this same group of field subjects perceive the unknown probabilities that are presented under varying degrees of uncertainty. Here the research question is: if an event with an unknown probability is presented to subjects under separate scenarios that vary in degrees of uncertainty, will it result in variations in behavior? In particular, does behavior vary in a systematic manner going from the least uncertain scenario to the most uncertain scenario? In the experiment, subjects are asked to complete three types of lottery tasks that are ranked in order of increasing uncertainty. Subjects' uncertainty attitudes are analyzed using the "source method" that is introduced by Abdellaoui, Baillon, Placido and Wakker (2011).

The source method assumes that different types of events imply potentially different sources of uncertainty, and that attitudes toward uncertainty and the perception of the likelihoods may be revealed by comparing decision weights inferred across different types of events. These decision weights are modeled using probability weighting functions (or source functions), and the parameters estimated from the source functions give rise to two indices of uncertainty aversion: pessimism and likelihood insensitivity. From here, behavior under uncertainty can be analyzed in a tractable manner using these indices. This allows one to pinpoint if the behavioral variation across different types of events is due to differences in pessimism and/or likelihood insensitivity. Chapter 3 asks if the behavioral variation going from the least uncertain scenario to the most uncertain scenario is due to an increased pessimism and/or likelihood insensitivity.

If the indices of uncertainty aversion are based on the estimates that are derived from a source function, one theoretical concern is whether different specifications of the source function imply different estimates of uncertainty aversion indices. In other words, is the analysis of behavior under uncertainty robust when we assume different specifications of the source function? Or will the observed behavior be better captured by one specification of the source function than another? The results show that the behavioral difference under uncertainty is better captured by the Prelec specification (Prelec (1998)) than the Tversky-Kahneman (1992) or Power specification. Thus, conclusions regarding uncertainty aversion are contingent on which specification is assumed for the source function.

CHAPTER 1

Estimating Subjective Beliefs in Naturalistic Tasks with Limited Information

1.1 Introduction

In the area of transportation policy, congestion pricing has been used to alleviate traffic congestion in metropolitan areas. The policy reduces traffic congestion by charging drivers for using congested routes. This gives them incentives to use an alternate route or mode of transportation. When drivers do not know the actual probability of delay and can only base their decisions on past experience on the routes or information from others, their expectations of delay as well as risk attitudes are crucial elements in determining their route choices. As pointed out by Savage (1971), to identify the model of decision making under risk one needs to understand the preference function of an agent *and* the way they perceive the probability of the unknown event (their subjective probabilities). This essay examines the perception of travel delay in explaining reactions to congestion pricing. Commuters from Atlanta and Orlando metropolitan areas are recruited to participate in an experiment that uses driving simulators and their subjective probabilities of the uncertain risk of delay are inferred through the route choices they make. The primary research question is whether the field subjects are able to form estimates of the risk of delay that vary with the underlying congestion probabilities in a simulator environment. Furthermore, do they *adjust* their beliefs in the direction of the objective congestion probability? Does the adjustment of beliefs differ depending on the underlying objective congestion probability?

An important and novel aspect of the design is information gathering is endogenous: the prior information of congestion on a route can only be updated if one decides to take that route, otherwise no new information is generated. Given some initial belief about the riskiness of a potentially congested route, those who start with a lower belief of congestion risk may be more inclined to take the risky route than those who start with a higher belief, thus leading to subjects with a lower belief of congestion risk obtaining more information than those with a higher belief of congestion risk, *ceteris paribus*. In a dynamic setting, the implication for belief adjustment is that subjects who start with a lower belief of congestion risk will experience faster belief adjustment than those who start with a high belief of congestion risk.

Another important design feature of this experiment is information about risk is presented using visual and immersive simulations. While much is known about beliefs in stylized experiments, such as those using urns of colored balls to model uncertain prospects, there is less known about risk perceptions in natural or naturalistic simulated environment. It has been suggested in the psychology literature that, depending on the framing of the experimental task, subjects may employ different decision heuristics, and thus there are reasons to believe that this may lead to different degrees of bias. For example, the dual-process theory suggests that some frames may induce slower cognitive modes that involves explicit deliberation, whereas others may induce faster cognitive modes that involves emotions and heuristics.[1] Applying the insights of dual process theories to decision making under risk and uncertainty, Mukherjee (2010) suggests that if people vary in their disposition to use either of these cognitive processes, then the task construction can directly affect the weight that either gets in the valuation of an uncertain

[1] See Chaiken and Trope (1999) for a discussion of dual-process theory.

prospect. Thus it is important that the experimental task imitates the real-life setting in which the agent would normally make the decision, so as to promote a more natural mode of decision making. Fiore, Harrison, Hughes and Rutström (2009) show that in visual and immersive simulations of risky environments, the estimated beliefs are closer to the actual underlying risk than in environments with some of the characteristics of standard survey instruments.

The purpose of this experiment is to elicit subjects' perceptions of the probability, *p*, of travel delay. This probability partly depends on the probability of congestion, which is given exogenously but varies across four scenarios. However, drivers also drive on non-congested parts of the route which contributes to their delay. Subjects' latent subjective probabilities are revealed through their binary choices over two routes: one has an uncertain level of congestion risk, the other has no congestion risk. One objective congestion probability is randomly assigned to each subject and stays constant through the session. This probability is known to the experimenter, but not to the subjects. Four levels of this probability are used: {0.2, 0.4, 0.6, 0.8}. The hypothesis is that, as subjects go through the ten driving periods of the experiment, their perception of the probability will change throughout, but only to the extent that they choose the relatively risky route to receive information feedback about its congestion conditions. When they choose the risk-free route they get no information feedback about the congestion conditions on the risky route.

The latent subjective beliefs of delay are estimated controlling for risk attitudes, and the task for eliciting risk attitudes (i.e., the preference functions) is implemented using stylized binary lottery choices. Andersen, Fountain, Harrison and Rutström (2014) and Manski (2014) emphasize that separate tasks are needed to identify both the preference function and the perceived probabilities, and this essay follows that advice. This experiment deviates from

Manski (2014) in that incentivized tasks are used to reveal both the risk perceptions and the risk attitudes of the subjects. There is much evidence that hypothetical methods can lead to biases in subject responses: for example, Harrison (2014) reports evidence of hypothetical bias in the estimation of risk perceptions, whereas Holt and Laury (2002) and Harrison (2005) report evidence of hypothetical bias in the estimation of risk attitudes.

The estimation results show that the subjective beliefs of delay, *p*, rank in the order of the objective probabilities of congestion. Across the driving periods only subjects in the lowest congestion risk treatment express significant adjustment in the belief of delay. In the higher risk treatments there is no belief adjustment. The result is consistent with the hypothesis that in an endogenous information environment collecting information about a route that has an uncertain level of congestion is perceived as riskier when the subjective probability of delay is higher. This leads to limited or no updating. The implication of this finding is that drivers in the field who habitually select an expressway over an alternative local route may do so because they persistently hold a high belief about the congestion level on the alternative route whether or not the objective congestion probability is high. This would imply that responses to congestion pricing could be limited since drivers would be reluctant to try the alternative route. In addition, if drivers are unable to adjust their beliefs about congestion after traffic events such as construction, lane closings, or lane conversions, it will result in suboptimal traffic allocation across alternate routes.

1.2 Literature Review

In experimental economics, different methods are used to elicit subjective beliefs in the laboratory. These methods include: survey questionnaires with hypothetical payoffs; the Becker-DeGroot-Marschak (BDM) method; choice tasks that are constructed in a Multiple Price List (MPL) format; or proper scoring rules such as the Quadratic Scoring Rule (QSR). For example, in hypothetical questionnaires, the experimenter may directly ask subjects what they think is the true probability of an event occurring, or what they think is the true state of nature out of the many possible states presented (Attneave (1953)). Regardless of which response subjects provide or which outcome will be played out, the payoffs that subjects "receive" are hypothetical (i.e., $0), which does not incentivize subjects to truthfully report their beliefs. Furthermore, it is well known in the valuation and the risk attitude elicitation literature that hypothetical bias exists (Harrison (2006, 2014)), therefore it is reasonable to suspect that belief elicitations that do not use monetary incentives may suffer from such hypothetical biases. Another approach is to use the BDM method to elicit subjective probabilities (rather than willingness to pay) (Holt and Smith (2009)). However, the instruction of the BDM may be difficult for subjects to understand thus potentially compromising its effectiveness. Choice tasks that are constructed in the MPL format may be used to elicit subjective probability intervals (rather than risk attitudes). The task may involve a series of lottery choice tasks (Moreno and Rosokha (2015)), or a series of betting tasks where subjects place bets with multiple bookies who offer different odds (Antoniou, Harrison, Lau and Read (2015, 2016)). Proper scoring rules such as the QSR can be implemented through a "slider task" that is constructed using the formula of the QSR. On a computer screen a number of possible events are shown to the subjects, and subjects are asked to allocate earnings or points across these events by adjusting the slider that represents each event

(Andersen, Fountain, Hole, and Rutström (2014); Harrison (2014); Harrison and Swarthout (2014)). The above mentioned elicitation methods are used in stylized or non-naturalistic settings with the exception of Fiore, Harrison, Hughes and Rutström (2009), who use visual and immersive simulations to present subjects with information about the unknown risk in a virtual reality setting. The study reports that the beliefs elicited in a virtual reality setting are closer to the objective risk than those elicited in a stylized setting with still images and/or textual descriptions.

Two classic experiments in the psychology literature: Preston and Baratta (1949) and Attneave (1953), exemplify the early experimental approach to belief elicitation. In both experiments, subjective beliefs are elicited and the elicitation procedure were not incentivized for the truthful reporting of subjective beliefs. Preston and Baratta (1949) present subjects with a series of gambles, and for each gamble the privilege to play the gamble is auctioned off to a number of subjects, who are bidders in the experiment. The highest bidder obtains the privilege to play the gamble. Probability theory would suggest that if a bidder on average pays in excess of the mathematical expectation for the privilege to play the gamble, then a long series of plays will result in systematic losses; *vice versa* if a bidder pays in less than the mathematical expectation. The study examines subjects' bidding prices of the gambles, or subjects' implied subjective probabilities. Under the range of probabilities studied, subjects tended to make high bets for events with objective probabilities that are below 0.2, and low bets for events with objective probabilities that are above 0.2. In other words, subjects overestimated low probabilities and underestimated high ones with an equality point at about 0.2.[2] The results of

[2] An equality point occurs when the subjective probability equals the objective probability. Results regarding equality points differ across experiments: some find an equality point around 02 or around 0.5, whereas others find no equality point, see Edwards (1954) for a detailed review.

the study may need to be taken with caution as it is well known in the auction literature that bidding price in a first-price sealed-bid auction is a function of the number of bidders present (Kagel and Levin (2011); Fullbrunn and Neugebauer (2013)), thus the number of bidders in the experiment may confound with subjects' subjective probabilities. Similar to the findings in Preston and Baratta (1949), Attneave (1953) reports that events with low frequencies of occurring are systematically overestimated and events with high frequencies of occurring are systematically underestimated. The Attneave (1953) experiment presents subjects with a newspaper clipping that has a thousand letters in it, and asks them to guess the occurrence of each of the 26 letters in the alphabet in the newspaper clipping.

To examine the importance of using incentivized methods in belief elicitation experiments, Harrison (2014) compares the subjective probabilities elicited using incentivized methods to the ones elicited using hypothetical methods. The study elicits the subjective belief *distribution* of subjects over various health risk and financial matters, using either an incentivized QSR or one with hypothetical payment. Subjects report their beliefs over possible events by allocating earnings across these events using a slider task shown on a computer screen. As subjects adjust the height of the slider for an event, the height on the slider corresponds to the earnings allocated to that event. Pooling across subject responses, the average belief differs significantly between the group who receive salient payment and the group for whom the payment is hypothetical. Furthermore, when controlling for demographic variations, hypothetical bias varies significantly across demographic sub-samples.

Subjective beliefs can be estimated as a discrete probability or as a probability distribution. It is important to control for subjects' risk attitudes when subjective belief is estimated as a discrete probability estimate.[3] For example, when subjects are asked to report beliefs in a QSR, risk averse subjects would be drawn toward a 50/50 report, thus they under-report high probabilities and over-report low probabilities (Offerman et al. (2009); Andersen, Fountain, Harrison and Rutström (2014)). To examine the importance of controlling for risk attitudes, Andersen, Fountain, Harrison and Rutström (2014) compare the subjective probabilities that are elicited with controlling for subjects' risk attitudes to those that are elicited without controlling for risk attitudes. Subjective probabilities are elicited over outcomes of the 2008 Presidential Election, and over the performance of a randomly chosen man and woman from the group of subjects in the experiment on a test in psychology known as the Eyes Test. Subjects report their beliefs over possible events by allocate earnings across the events using a slider task.[4] The payoff formula is designed using either the quadratic or the linear scoring rule. Beliefs are jointly estimated with risk attitudes assuming Subjective Expected Utility (SEU) and Rank Dependent Utility (RDU). The study reports that both the utility function and the probability weighting function are concave, and thus it is important to control for risk attitudes through utility curvature and probability curvature.

Many of the studies described above estimate a discrete probability estimate for an unknown event assuming a representative agent. Andersen, Fountain, Harrison, Hole and Rutström (2011) (hereafter, AFHHR) present subjects with a range of bookies offering odds on the outcome of some unknown event. As the subject allocates earnings over the range of offering odds, the individual's *probability distribution* over the possible probabilities for an

[3] When subjective probability is estimated as a probability *distribution*, Harrison, Martinez-Correa, Swarthout, and Ulm (2013) show that adjusting for risk attitudes is not needed if one is willing to assume that subjects behave consistently with EUT.

[4] Andersen, Fountain, Harrison and Rutström (2014) and Harrison (2014) both use a slider task to elicit subjective beliefs. The former uses a slider task with two bins to elicit a discrete probability, and the latter uses a slider task with ten bins to elicit a probability *distribution*.

unknown event is elicited. The study examines events that differ across a range of objective probabilities, and reports that in the low-probability treatment where the objective probability is 0.1 or 0.2, in each case the mode and the mean of the subjective distribution are significantly greater than its corresponding objective probability. As for the medium-probability and high-probability treatments, where the objective probabilities are 0.5 or 0.55, and 0.75 or 0.8, respectively, the mean of the subjective probability distributions are virtually the same as the objective probabilities. The experiment is conducted with stationary probabilities and real monetary incentives. Subjective beliefs are corrected for risk attitudes by including a lottery choice task and inferring beliefs with joint estimation methods.

Comparing the AFHHR (2011) study to the classic psychology studies of Preston and Baratta (1949) and Attneave (1953), a common finding is that subjects tend to overestimate low probabilities. This therefore seems to be true whether or not one uses monetary incentives or adjusts for risk attitudes. However, in the medium-probability and high-probability range, the perception of probabilities may differ depending on incentives and the task at hand.

Repeated Choice

The studies reviewed so far examine subjective probabilities in a one-task setting where beliefs are elicited only once. Gallistel et al. (2014) was interested in assessing beliefs in a dynamic setting with multiple periods. In each period, subjects are given signals and the (posterior) belief is elicited. The experimental task involves presenting subjects with “a box of circles” displayed on a computer screen that has unknown probabilities of green and red circles. In each period, the subject is allowed to sample one circle from the box and is asked to state his/her guess as to the proportion of green circles, *p*, by submitting the answer through a moving slider. Ten subjects are recruited for a flat fee and each go through 1,000 trials. An important feature in the design is that subjects are told that *p* changes randomly throughout the experiment (i.e., *p* is non-stationary), and thus the estimate of *p* needs to update with changes in the distribution. In each round, after the subjects submit their answers they receive no information feedback as to what the true *p* is. At any time during the experiment if the subjects think the proportion of circles in the box has changed they are told to click on the button that says "I think the box has changed." The study reports that "the mapping from the true probability to median report probability is the identity," which suggests that the *frequency* of overestimating the true probability is equal to the *frequency* of underestimating the true probability. Gallistel et al. (2014) do not compare the subjective probabilities to the true probabilities, therefore one cannot infer the degree to which subjects overestimate or underestimate the true probabilities.

To understand how subjects assess probabilities in a repeated choice setting, many studies have focused on belief updating. Conditional on a prior belief, if subjects update new information in a Bayesian manner, the posterior beliefs should converge on the objective probabilities over time as new information is acquired. Conversely, if subjects overweight or underweight new information, the posterior beliefs should deviate from objective probabilities. Kahneman and Tversky (1973), Grether (1980) and Grether (1992) report that subjects tend to make decisions based on how similar or representative the sample distribution is to the parent population (known as representativeness), disregarding any prior information they may have. Grether (1980) presents subjects with two urns with varying number of colored balls where the distribution of each urn is known to the subjects. Subjects are told that one of the urns will be randomly chosen and the (prior) probability of each being chosen is equal. After an urn is chosen, the experimenter draws six signals (with replacement) from the chosen urn, and subject's belief about the true urn is elicited. The study reports that subjects do not accurately weight the prior information in a Bayesian manner and that they tend to over-weight the new information. Building upon the design of Grether (1980, 1992), El-Gamal and Grether (1995) present subjects with two cages where the distribution of each cage is known to the subjects. Subjects are told that one of the cages will be randomly chosen and the (prior) probability of each cage being chosen may or may not be equal. Three (prior) probability treatments are examined. The study reports the three commonly-used updating rules used by subjects: (a) Bayes rule, (b) representativeness (over-weighting the new signal), and (c) conservatism (under-weighting the new signal). In these belief updating experiments, subjects are given extra monetary payment if their responses are correct.

Building upon past experiments on belief updating, Moreno and Rosokha (2015) examine belief updating between two environments. One environment is a *compound risk* environment where subjects are presented with a *compound* urn and are told of its composition "process" (i.e., its possible distributions). The second is an ambiguous environment where subjects are presented with an *ambiguous* urn in which they are not told of its composition process. Each treatment lasts for five rounds. In each round subjects are given three signals from the urn and are asked to state their choices in the MPL between a sure amount of money and a lottery (e.g. $X if black, $0 otherwise).[5] As subjects go through the rounds and gather more signals, one would expect their choices in the MPL to adjust and reflect learning. Subjective beliefs are

[5] A MPL is often used to elicit a subject's valuation of an event that has some subjective risk. The task involves presenting subjects with an array of ordered prices in a table. In each row, the subject is asked to choose between a fixed amount of money and taking a gamble. The gamble option stays the same throughout but the fixed amount of money changes across rows. The switching point between the two options would indicate subject's valuation for the event and hence their subjective probability interval if the subject is risk neutral.

estimated controlling for risk attitudes assume a generalized model of reinforcement learning and Bayesian updating. The study reports that subjects significantly underweight new information in an ambiguous environment compared to in a compound risk environment, and as a result the updating process is less volatile.

In a belief updating experiment, Antoniou, Harrison, Lau and Read (2015) use a revealed-preference approach to examine if subject's inferred posterior belief deviates from Bayes Rule across a range of low and high probabilities. Subjects are presented with a white box and a blue box, each containing 10-sided dice. The white box contains N 10-sided dice that each has 6 white and 4 blue sides, and the blue box contains similar dice that each has 6 blue and 4 white sides. Within each treatment, N is the same for both boxes; across treatments, N may take on the value of 3, 5, 9, or 17. Subjects are told that one of the boxes is randomly selected with 0.5 (prior) probability but they do not know which box. After a box is selected, a student-monitor randomly draws signals (or sample information) from the selected box. Next, the subjects are asked to place a bet in one of the available 19 betting houses offering different odds; this way, their choice of a betting house reveals their belief about the selected box. Each subject participates for 30 rounds of betting task, and at the end of the 30 rounds one of the bets is randomly selected for payment. In the experiment a separate task with known probabilities is implemented to elicit risk attitudes. Inferred beliefs are estimated assuming Subjective Expected Utility controlling for risk attitudes. The study reports that, in the low probability range subjects overestimate the (posterior) probability, and in the high probability range subjects underestimate the (posterior) probability. Furthermore, when one assumes (incorrectly) linear utility the deviation from Bayes Rule is higher.

What possibly explains the violation of Bayes Rule? Griffin and Tversky (1992) propose the strength-weight hypothesis as a plausible theory to explain violation of Bayes Rule. According to this hypothesis, decision makers who are fallible to the strength-weight bias tend to pay too much attention to the extremity (strength) of the information and too little attention to its predictive validity (weight). In an experiment that did not use incentive compatible elicitation methods, Griffin and Tversky (1992) report that the magnitude of bias is significant and in some cases probabilities diverge from Bayes Rule by 28%. Antoniou, Harrison, Lau and Read (2016) builds on the experimental design of Antoniou, Harrison, Lau and Read (2015) and test if the strength-weight bias is plausible when using an incentive compatible elicitation method and controlling for risk attitudes. The 2016 study reports an average bias of 6%, and after controlling for non-linear utility further reduces the bias.

In the studies with a single choice reviewed above, subjects consistently overestimate low probabilities. This result has been found in studies both with and without implementing incentivized elicitation methods or adjusting for risk attitudes. There is less consensus in the medium-probability and high-probability range where subjects underestimate the true probabilities in some settings but in others form an unbiased estimate. The consensus is that subjects do not typically form an unbiased estimate of the true probabilities over the range of objective probabilities. This generalization is further supported by the belief updating literature, which reports that subjects do not update probabilities in a Bayesian manner. If subjects do not properly weight the priors and update new information, then the elicited (posterior) probabilities will likely deviate from the objective probabilities.

Experimental Frame

The belief elicitation and updating experiments reviewed above are conducted in stylized environments. In contrast, Fiore, Harrison, Hughes and Rutström (2009) elicit beliefs using a virtual reality environment. To examine how the interactive and visual presentations of risk may contribute to differences in the perception of risk, Fiore, Harrison, Hughes and Rutström (2009) conduct an experiment that examines subjective beliefs under three settings that differ in the degree of interactiveness and immersiveness: 2-picture treatment, 52-picture treatment, and a virtual reality treatment. Subjects are presented with images or virtual simulations of a wild forest fire, and they have monetary interest in a property in the area where the fire can potentially be spreading. In a MPL subjects are asked for their willingness to pay for fire protective actions. Subjective beliefs are estimated controlling for risk attitudes. Given that the true probability of wild fire damage is 0.29, in the 2-picture treatment the estimated subjective probability was 0.45; in the 52-picture treatment it increases to 0.52; and in the virtual reality treatment it decreases to 0.25 which is quite close to the true probability. The study concludes that the immersive aspect of the virtual reality experiment has the effect of generating subjective beliefs that are closer to the objective risk than still images and/or textual descriptions.

It is well known in the psychology literature that the environment in which agents make decisions may affect how information is processed. The concept of "ecological rationality" claims that the rationality of a particular decision depends on the circumstances and environment in which it takes place (Gigerenzer and Todd (1999); Gigerenzer (2008)). Furthermore, when agents make decisions under risk and uncertainty, dual process theories suggest that two decision processes may be at work: the deliberative system and the affective system (Chaiken and Tope (1999); Stanovich and West (2000); Kahneman (2003); Evans and Frankish (2009); Mukherjee

(2010)). This suggests that an agent may value a gamble differently depending on which cognitive system is used, and which cognitive system is used depends on a number of attributes including the disposition of the agent, the framing of the task and the outcome of past gambles. Applying this insight to the construction of belief elicitation task, the framing of the experimental task may possibly lead to different degrees of bias in the perception of risk.

This essay employs virtual reality in order to elicit beliefs that are more relevant for discussions of beliefs as they apply in the field. This essay assumes that the immersive nature of the driving simulator has the effect of generating beliefs of delay that are closer to the actual risk of delay than if subjects are presented with still images and/or textual descriptions. The second purpose of this essay is to examine belief adjustment in an environment where information gathering is endogenous, such that subjects will acquire new information about a route only if they choose that route. This is an information condition that has not been studied in previous belief updating experiments. This information condition could mean that for sufficiently high risk cases belief adjustment will be very slow and possibly result in subjective risk deviating significantly from the objective risk.

1.3 Experimental Design

This experiment uses real money incentives. Each subject is presented with a driving simulator task with ten driving periods that elicit subjective beliefs and four binary lottery tasks that elicit risk attitudes. The experiment is not designed to elicit or infer the beliefs of individual subjects, but to do so using data pooled across subjects. This section describes the design of each task followed by the recruitment and experimental procedure.

1.3.1 Simulator Route Choice Task

The driving simulator task is designed to mimic a real-life commuting experience. The subjects drive in a simulator environment installed on a laptop that is equipped with a steering wheel, gas and brake pedals, and views everything from the perspective of sitting in the driver's seat. They drive from a simulated home origin to a simulated work destination as they make a binary choice between a route that has free-flow traffic and another route that could be congested with some probability. Each drive is referred to as a work day. If the subjects choose to take the free-flow route there is a toll charge that varies across subjects but is stationary across the drives. The drive takes approximately 2 to 4 minutes, depending on which route they take, which scenario they are in, and how they drive. To increase the realism of the setting, simulated vehicles are added to the road and subjects are required to follow general traffic rules, such as speed limits.

The number of variables that are assigned to the subjects include: a wage that serves as a monetary endowment for each drive, a time limit within which they have to arrive to work, a monetary penalty if they arrive to work late, a toll charge when taking the risk-free route, and an unknown probability of congestion on the risky road. These variables are adjusted on a between-subject basis. The wage can be a high wage of $5.00 or a low wage of $2.50. If travel time exceeds a certain time threshold, a discrete penalty amount will be subtracted from the wage. Table A1 shows the ranges of tolls, penalties and time thresholds. Tolls range from $0.50 to $2.00 if wage if $2.50, and from $0.50 to $4.00 if wage is $5.00. The range of toll is in 10-cent increments. Penalties range from $0.50 to $2.00 if the wage is $2.50, and from $0.50 to $4.00 if the wage is $5.00. The range of penalty is in 50-incent increments. Time thresholds range from

2 minutes and 10 seconds to 2 minutes and 45 seconds in 5-second increments. These assignments are constant across drives.

Each task is paid sequentially to avoid the issues that arise with random payment protocols.[6] Across the driving periods the cumulative earnings may present a wealth effect on risk aversion, such that an increase in earnings could reduce risk aversion in the following periods. However, an increase in earnings theoretically should not affect the belief of delay, and it should only affect the belief estimate indirectly through its effect on risk aversion. Cox, Sadiraj and Schmidt (2015) report that the PAS protocol did not induce a significant wealth effect; the same result is reported in Cox and Epstein (1989) and Cox and Grether (1996) who also use the PAS protocol. In contrast, Dixit, Harb, Martinez and Rutström (2015), who use the PAS protocol in a driving simulator task with exogenous delay probabilities, report that cumulative wealth significantly reduce risk aversion (*p*-value <1%). Here the cumulative wealth effect is assumed to be negligible on the belief estimate.

An aerial view of the simulated city where the subjects drive is shown in Figure A1. In the simulation, 7th Avenue is the express route that is risk free, and 9th Avenue is the alternate, local road that is congested with some probability. Before driving, subjects are shown a deck of

[6] The payment protocol that is used to elicit lottery choices must be compatible with the decision model in order to be incentive compatible. The pay-one-randomly (POR) protocol implicitly assumes that subjects view each outcome in each binary choice independently of each other, such that their behavior is in accordance with the Compound Independence Axiom (CIA). This payment protocol is incentive compatible under EUT (see Harrison and Swarthout (2014) and Cox, Sadiraj and Schmidt (2015)). However, it is incompatible with non-EUT models that are not based on the CIA, including Rank Dependent Utility (RDU).
This essay models choices over risky lotteries using both EUT and RDU, which necessitates the use of a payment protocol that is incentive compatible under both. The Pay-All-Sequentially (PAS) protocol does not rely on the CIA and is thus incentive compatible with both. However, PAS is not problem-free since it may induce a cumulative wealth effect. An alternative approach is to assume that there is one CIA that applies to the evaluation of a given lottery (in our case the evaluation of each route) and another CIA that applies to the payment protocol. One can then relax the former CIA and estimate the RDU model, while maintaining the assumption of the latter CIA. It is then possible to use the POR payment protocol and legitimately estimate the RDU model.

toll cards (face-down) and are asked to draw one card that will determine their toll fee if they were to take 7th Avenue. On 9th Avenue congestion is induced using a school bus that makes frequent stops on the road causing delay. The objective probability of congestion, or the probability of a school bus being present, takes four possible values, 0.2, 0.4, 0.6, or 0.8, and is varied across subjects but constant within subjects. Subjects are not told what probability treatment that they are assigned to nor are they told that these are the four possible congestion levels. Subjects are told that the congestion level stay the same across the ten drives.

To implement the random congestion process, at the start of each period subjects are presented with a deck of cards, where some of the cards have the word "bus" on them and others have the words "no bus" on them. They choose a card without seeing if the card says "bus" or "no bus". Next, the research assistant loads up the scenario stated on the chosen card. To ensure that subjects can trust that the research assistant actually loads the scenario indicated by the card drawn, the cards selected are saved in an envelope and revealed at the end of the ten drive tasks. Subjects do not know if a bus card is drawn unless they choose to drive on 9th Avenue in which case they will find out by experience. Thus, the information obtain on 9th Avenue will only be obtained if the route is selected. If the subjects drive very slowly, then late arrival is possible even when a bus does not come.[7] Prior to starting the drive task, subjects draw ten cards from the deck of bus cards with replacement, allowing them to form prior beliefs.

Earnings are recorded after each drive and tracked, along with cumulative earnings, throughout the drive periods in a transparent way.

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[7] These cases are not common. Pooling across all the drives and across all subjects, there are only 20 out of a total of 479 drives where the subjects take 9th Avenue, do not see a bus, and are still late (relative to the assigned time threshold). These are the subjects who are assigned the lowest time threshold and are late by less than 10 seconds; many of them are late by only 2 or 3 seconds.

1.3.2 Binary Lottery Task

Subjects are asked to complete four binary lottery tasks that elicit their risk attitudes. Figure A2 presents a screenshot from the lottery used in the practice task, and the set of prizes and probabilities used is listed in Table A2. In each task, a binary decision is made between a relatively safe lottery and a relatively risky one. After a decision is made, the outcome of the lottery is determined by the roll of a dice. Within each task the probability of getting the high prize is the same for the risky lottery and the safe lottery, but the probability varies across tasks. The tasks are randomly assigned to subjects. Each of the four lottery tasks is actualized sequentially, and the research assistants keep track of the task earnings along with the cumulative earnings in a way that was transparent to subjects.

1.3.3 Recruitment and Experimental Procedure

Subjects in this essay are selected from United States Postal Service (USPS) mailing lists and are recruited by invitation letters. The invitation letters direct them to a web page where they are instructed to create an anonymous Gmail account to use exclusively for this experiment to ensure strict privacy. Admission to participate in the experiment is contingent on being at least 18 years of age, holding a valid driver's license, and using a vehicle with a valid vehicle insurance.

The experimental tasks analyzed in this essay are part of a larger experiment described in Rutström et al. (2011). The larger experiment consists of four meetings separated by approximately two weeks each. The simulator driving task with uncertain congestion risk is only one of several tasks that subjects perform and is conducted during the second meeting. Two of the lottery tasks are conducted during the first meeting and the last two at the end of the second meeting. Subjects are paid for all tasks, and earnings for each task, along with cumulative earnings, are tracked in a clear and transparent manner. The subjects are commuters from the Atlanta and Orlando metropolitan areas and a total of 141 subjects are included for the purpose of this analysis.

1.4 Theory

The experiment described in this essay is designed with features of a theoretical model that is commonly used in transportation economics: the scheduling model. Below we provide a description of the scheduling model, and then followed by hypotheses and a description of the Subjective Expected Utility model that is assumed for structural analysis.

1.4.1 Scheduling Model in Transportation

One theoretical approach to modeling traveling decision is the scheduling model that was introduced by Small (1982). The difference between preferred arrival time (*PAT*) and actual arrival time is defined as schedule delay (*SD*). A late arrival relative to *PAT* is a schedule delay-late (*SDL*) and an early arrival relative to *PAT* is a schedule delay-early (*SDE*). There are two versions to this model. The first version specifies the arrival time as a discrete variable and hence assumes a discrete penalty (or fixed penalty) for any late arrival. The model is given in the following equation:

$$U = \alpha T + \beta(SDE) + \gamma(SDL) + \theta D_L$$

where utility *U* is a function of travel time *T*, schedule delay-early *SDE*, schedule delay-late *SDL*, and a fixed penalty for any late arrival D_L. D_L is a dummy variable equal to 1 when there is a delay and 0 otherwise. The estimated parameters (α, β, γ and θ) are assumed to be negative. The first version of the scheduling model does not include risk, hence there is not a probability attached to being early or late.

An example of a scenario that has a discrete arrival time that incurs a fixed late penalty is in airline travels, where the travelers' decision model considers only two possible *arrival outcomes*: arrive earlier than desired, or arrive late and miss the flight, thus the penalty is the same independent of how late the arrival is. Another example is for travelers who are motor-vehicle users and choose their departure time without having to adhere to a fixed timetable. They may choose their departure time away from the peak congestion hours and postpone traveling until the peak hours subside, thus departure times (as well as arrival times) may experience a "jump" before or after peak hours.[8]

Noland and Small (1995) relax the assumption that the arrival time is a discrete variable by extending it to include continuous arrival times by adding a probability distribution of travel times. When travel time *T* is assumed to be continuous and follows a probability distribution, the uncertainty about *T* propagates onto uncertainties about actual arrival times, also onto uncertainties about *SDE*, *SDL*, and late arrival. Thus, each of these variables also follows a probability distribution:

[8] Another example of a discrete arrival time is for travelers of public transport who plan their time of departure in accordance with a fixed timetable that is preset by a scheduler. Their time of departure may be, for example, every 15 minutes on the clock; it then follows that their time of arrival is also every 15 minutes.

$$E(U) = \alpha E(T) + \beta E(SDE) + \gamma E(SDL) + \theta P_L$$

where the expected utility $E(U)$ is dependent on expected (or mean) travel time *E(T)*, expected schedule delay-early *E(SDE)*, expected schedule delay-late *E(SDL)*, and the probability of arriving late P_L. Trips where there is no additional cost associated with the probability of arriving late would have $\theta = 0$.

This scenario is more representative of travelers who use private transport (i.e., motor-vehicles) and who choose departure times at any given moment without having to adhere to a fixed timetable. Hence their departure times as well as arrival times, are continuous variables.

In this continuous time setting the Noland and Small (1995) model can be generalized to model discrete penalty (i.e., fixed lump-sum amount), or continuous penalty (i.e., each minute of delay incurs an additional penalty).[9] In a setting where the late penalty is continuous, the traveler's decision model considers a *distribution* of arrival outcomes: arrive early, 1 minute of late penalty, 2 minutes of late penalty, …, etc. An example of this scenario is if the purpose of the trip is to attend an economics seminar, the longer the delay the more information is missed.

1.4.2 Hypotheses

This essay examines how field subjects perceive the risk of delay that is uncertain in a driving simulator. Specifically, are field subjects able to form estimates of the risk of delay that vary with the underlying objective congestion probability? Furthermore, under an endogenous

[9] Note that Chapters 1 and 2 both examine route choices in a setting where arrival time is continuous (following the model of Noland and Small (1995). Chapter 1 examines route choices where the late penalty is fixed, whereas Chapter 2 examines route choices where the late penalty is continuous.

information environment, do they adjust their beliefs in the direction of the objective congestion probability? Does the adjustment of beliefs differ depending on the underlying objective congestion probability?

The probability of delay depends partly on the probability of congestion, which is unknown to the subjects. When there is a bus, subjects could be late to work; but when there is not a bus, subjects could still be late to work. In other words, the *perceived risk of delay* depends on the following factors: (1) the probability of congestion, (2) the probability of delay conditional on the *presence* of congestion, and (3) the probability of delay conditional on the *absence* of congestion.

Once the subject selects 9th Avenue and finds out if a bus appears or not, the uncertainty about congestion is resolved, thus the conditional probabilities of delay in (2) and (3) is independent of the congestion probability in (1). To illustrate, supposed Subjects X and Y are two subjects from this essay. Subject X is assigned to a treatment where the probability of a bus is 0.2 on 9th Avenue (i.e., low congestion risk), whereas Subject Y is assigned to a treatment where the probability of a bus is 0.8 on 9th Avenue (i.e., high congestion risk), *ceteris paribus*. If Subject X decides to choose 9th Avenue *and* the bus appears, the chance of her arriving late is high. On a separate and independent choice task, if subject Y decides to choose 9th Avenue *and* the bus appears, the chance of him arriving late is high. In other words, any subject who chooses 9th Avenue and encounters a bus has a high chance of arriving late. This is true whether the subject is assigned to a low risk scenario or a high risk scenario. In other words, the probability of delay *conditional* on a bus is theoretically expected to be similar for all subjects; the same logic applies to the probability of delay *conditional* on no bus.

Recall that the *perceived risk of delay* depends on (1), (2) and (3). Since (2) and (3) are theoretically expected to be constant regardless of (1), it follows that the *perceived risk of delay* should follow the same rank-ordering as (1). In this essay, the perceived risk of delay is estimated without decomposing it into (1), (2) and (3).

The following two hypotheses are tested:

Hypothesis I – Subjects are able to form estimates of the risk of delay, and the perceived risk of delay will be ranked in the order of the congestion probabilities.

Hypothesis II – Subjects who start with a lower belief of delay will experience more belief adjustment than those who start with a higher belief. In an endogenous information environment, subjects who perceive that a route has a higher risk of delay also perceive collecting information to be riskier, therefore they are less like to drive on the route or to collect information. Since little or no information is gathered, it leads to limited or no belief adjustment.

Subjects are assumed to have a subjective belief of late arrival on each route. Conditional on their subjective beliefs and risk attitudes, they compare the utilities across routes and choose the one with a higher subjective expected utility.

1.4.3 Simulator Route Choice Task

Subjects are presented with a binary route choice: 7th Avenue is a risk-free route with no congestion and 9th Avenue is a risky route with an unknown probability of congestion. Subjects' route choices are modeled initially using Subjective Expected Utility (SEU) and Constant

Relative Risk Aversion (CRRA) utility function. The subjective expected utility of the risk-free route, *7th Avenue*, is:

$$SEU_7 = \left(\frac{m7^{(1-r)}}{(1-r)}\right) \tag{1}$$

where r is the coefficient of relative risk aversion,

$m_7 = w - t$ is money payoff,

w is wage, and

t is the toll charge on 7th Avenue.

Similarly, the subjective expected utility of the risky route, *9th Avenue*, is:

$$SEU_9 = p * \left(\frac{m9_{late}{}^{(1-r)}}{(1-r)}\right) + (1 - p) * \left(\frac{m9_{notlate}{}^{(1-r)}}{(1-r)}\right) \tag{2}$$

where p is the subjective probability of late arrival when taking 9th Avenue,

$m9_{late} = w - l$ is the money payoff when subject takes 9th Avenue and arrives late, where l is the late penalty for arriving late, and

$m9_{notlate} = w$ is the money payoff when subject takes 9th Avenue and arrives on time.

Next, subjects are assumed to behave as if they compare the two subjective expected utilities and choose the one with the higher SEU.[10]

[10] In the maximum likelihood estimation, the estimated belief of late arrival for 7th Avenue is zero and is implicit in (1).

This approach can easily be extended to Rank Dependent Utility (Quiggin (1982)). To illustrate, assume a simple power weighting function. The rank dependent utility of the risk-free route, *7th Avenue*, is:

$$RDU_7 = \left(\frac{m7^{(1-r)}}{(1-r)}\right) \tag{1'}$$

The rank dependent utility of the risky route, *9th Avenue*, is:

$$RDU_9 = p^{\gamma} * \left(\frac{m9_{late}{}^{(1-r)}}{(1-r)}\right) + (1 - p^{\gamma}) * \left(\frac{m9_{notlate}{}^{(1-r)}}{(1-r)}\right) \tag{2'}$$

where γ is the probability weighting parameter.

Next, subjects are assumed to behave as if they compare the two rank dependent utilities and choose the one with the higher RDU. This simple power weighting function can be given a nice behavioral interpretation. If $\gamma < 1$, then $p^{\gamma} > p$ and the function is everywhere concave. This means that the subjects puts more weight on the likelihood of late arrival than what is otherwise implied by p, and the subjective belief is effectively pessimistic. *Vice versa*, if $\gamma > 1$, then $p^{\gamma} < p$ and the function is everywhere convex. This means that the subjects puts less weight on the likelihood of late arrival than what is otherwise implied by p, and the subjective belief is effectively optimistic.

1.4.4 Binary Lottery Task

Subjects are presented with four binary lottery tasks with known probabilities that elicit their risk attitudes. In each task a decision is made between a relatively safe lottery and a relatively risky lottery. Risk attitudes are estimated assuming Expected Utility Theory (EUT) and a Constant Relative Risk Aversion (CRRA) utility function. The expected utility of the safe option (EU_S) is:

$$EU_S = p * \left(\frac{{x_L}^{(1-r)}}{(1-r)}\right) + (1 - p) * \left(\frac{{x_H}^{(1-r)}}{(1-r)}\right) \tag{3}$$

where p is the probability of a low prize, x_L,

(1-p) is the probability of a higher prize, x_H, and

r is the coefficient of relative risk aversion.

Similarly, the expected utility of the risky option is:

$$EU_R = p * \left(\frac{{y_L}^{(1-r)}}{(1-r)}\right) + (1 - p) * \left(\frac{{y_H}^{(1-r)}}{(1-r)}\right) \tag{4}$$

where p, is the probability of a low prize, y_L, and

(1-p) is the probability of a high prize, y_H.

This approach can be extended to RDU. If RDU is assumed for the route choice task, then the essentially same specification follows for the lottery task as with EUT. The rank dependent utility of the safe option is:

$$RDU_S = p^{\gamma} * \left(\frac{{x_L}^{(1-r)}}{(1-r)}\right) + (1 - p^{\gamma}) * \left(\frac{{x_H}^{(1-r)}}{(1-r)}\right) \tag{3'}$$

where p is the probability of a low prize, x_L,

(1-p) is the probability of a higher prize, x_H,

r is the coefficient of relative risk aversion, and

γ is the probability weighting parameter that weights the probability of the low prize.

Similarly, the rank dependent utility of the risky option is:

$$RDU_R = p^{\gamma} * \left(\frac{{y_L}^{(1-r)}}{(1-r)}\right) + (1 - p^{\gamma}) * \left(\frac{{y_H}^{(1-r)}}{(1-r)}\right) \quad (4')$$

with the same probability, p, for a low prizes, y_L, and

(1-p) for a high prize, y_H.

1.5 Empirical Analysis

The SEU estimation uses (1) – (4), and the RDU estimation uses (1') – (4'). Behavioral differences would be captured by subjective beliefs and risk attitudes, i.e., the curvature of the probability weighting function and the utility function, respectively. The full nonlinear estimation is performed using Maximum Likelihood techniques. Before estimating these non-linear models, a Probit model is estimated as a way of describing the data.

1.5.1 Estimation Approach

The estimation of beliefs uses data from the driving task pooling across subjects, and the estimation of risk attitudes uses data from the lottery task pooling across the same subjects. Subjective beliefs are estimated jointly with risk attitudes separately for each treatment, implying that any imprecision in the estimated risk attitudes are propagated into the estimation of beliefs. The joint estimation in the full structural SEU model below can be easily extended to RDU.

This joint estimation approach builds on previous work on structural estimation of risk attitudes by Andersen, Harrison, Lau and Rutström (2008) and Harrison and Rutström (2008b). A detailed description of the methodology can be found in Andersen, Fountain, Harrison and Rutström (2014).

Estimate Risk Attitudes from Lottery Tasks

Risk attitudes and subjective beliefs are estimated jointly using both the lottery data and the driving simulator data. For pedagogic reasons, the econometric model is shown separately for each model. We first describe the econometric model for estimating risk attitudes using only the lottery data.

Following (3) – (4), the index

$$\Delta EU = EU_R - EU_S \qquad (5)$$

is the difference in valuation between the risky lottery and the safe lottery.

The index (5) is then linked to observed choices by using a "logit" likelihood function:

$$prob\ (choose\ risky\ option) = \Lambda(\Delta EU) \qquad (6)$$

The risky option is chosen when $\Lambda(\Delta EU) > ½$.

Thus the likelihood of the observed responses, conditional on the EUT and CRRA specifications being true, depends on the estimated r given the above specification and the observed choices, c. The log-likelihood is then

$$\ln L\ (r; c) = \Sigma_i \big[\ \ln\Lambda(\nabla EU)\ \times \mathbf{I}(c_i = 1) + \ln\big(1 - \Lambda(\nabla EU)\big) \times\ \mathbf{I}(c_i = 0)\big] \qquad (7)$$

where $\mathbf{I}(\cdot)$ is the indicator function and c_i= l (0) denotes the choice of the lottery option R (S) in risk aversion task i.

An important extension of the core model is to allow for subjects to make some behavioral error. The latent index (5) then becomes

$$\Delta EU = [(EU_R - EU_S)/\nu]/\mu_{lottery} \tag{5'}$$

where $\mu_{lottery} > 0$ is a structural Fechner "noise parameter" used to allow some error when evaluating the difference in EU between the two lotteries. ν is a contextual normalizing term for each lottery pair R and S, which is defined as the difference between the maximum and the minimum utility in each lottery pair. This normalization is referred to as "contextual utility" and is due to Wilcox (2011).

One extends the likelihood specification to include the noise parameter $\mu_{lottery}$ and maximizes ln $L\ (r, \mu\ ; c)$ by estimating r and $\mu_{lottery}$, given observations on c.

Estimate Subjective Beliefs from Simulator Driving Tasks

Together with the estimation of risk attitudes described in the previous section, the estimation of beliefs follows (1) and (2), and the latent index is

$$\Delta SEU = (SEU_7 - SEU_9) \tag{8}$$

is the difference in valuation between 7th Avenue and 9th Avenue.

The estimation is performed using Maximum Likelihood and includes both a contextual utility normalization and a noise parameter. The noise parameter differs across the simulator task and the lottery task. Conditional on the SEU and the CRRA specifications being true, the maximized log-likelihood becomes,

$$\ln L\left(\hat{p}, r, \mu_{lottery}, \mu_{route}; c\right) = \Sigma_i\left[\ln \Lambda(\nabla SEU) \times \mathbf{I}(c_i = 0) + \ln\left(1 - \Lambda(\nabla SEU)\right) \times \mathbf{I}(c_i = 1)\right] \quad (9)$$

where $\mathbf{I}(\cdot)$ is the indicator function, $c_i = 0$ (1) denotes that the subject choose 7th Avenue (9th Avenue) in period i, and separate noise parameters are estimated for the lottery task ($\mu_{lottery}$) and the route choice task (μ_{route}). When Fechner errors are estimated separately by treatment, the results are essentially the same as when a common Fechner error is estimated across treatments. Thus to save on degrees of freedom, a common Fechner error is assumed across treatments.

Beliefs are estimated including fixed effects for the time periods, which is a non-parametric way of looking at belief formation.

1.5.2 Descriptive Statistics

The characteristics of the subject pool are described in Table A3. The proportion of commuters from Atlanta and Orlando are about equal. Each gender is evenly represented in the overall sample. About 44% have household income of above $100,000, and are labeled high income; the rest have household income of $100,000 or below, and are labeled low income. A significant majority hold a college education (78%). Within each risk treatment, the breakdown by demographics generally follows a similar trend as the overall sample distribution. An exception is in Treatment 0.8 where less than 10% are non-college graduates.

Travel Times and Frequency of Delay

The distribution of travel times is shown in Figure A3. On average, 7th Avenue takes the shortest time (115 seconds), next is 9th Avenue *without* a bus (134 seconds), and 9th Avenue *with* a bus takes the longest (201 seconds). The standard deviations are 3.7, 7.3, and 17.7, respectively. The increase in standard deviation is significantly different across the three scenarios (p-value < 0.001).[11] Thus the longer it takes to complete the drive, the higher is the variance of the distribution of travel times.

As expected, the average travel time is directly related to the frequency of delay. On average, the frequency of delay on 7th Avenue is 4%, on 9th Avenue *without* a bus it is 12%, and on 9th Avenue *with* a bus it is 97%.[12] Note that on 9th Avenue *without* a bus, the majority of delay happen to subjects who are assigned the lowest time thresholds (see Figure A4), and these subjects are late by 10 seconds or less, with many being late by only 2 or 3 seconds.

Pooling across time periods, the frequency of delay on 9th Avenue *with* a bus is not significantly different across the last three risk treatments. The same is true for the frequency of delay *without* a bus. This provides support for the claim that the conditional probability of delay *with* or *without* a bus is similar across congestion risks.[13] One exception is that the lowest risk treatment (i.e., Treatment 0.2) has significantly lower frequency of delay compare to other risk treatments. One possible explanation is that subjects in the lowest risk treatment drive more

[11] The test is performed using Levene's robust test for the equality of variances between the three groups.
[12] It is uncommon to see cases where subjects arrive late on 7th Avenue, arrive late on 9th Avenue *without* a bus, or arrive on-time on 9th Avenue *with* a bus. When the estimation is run dropping these uncommon cases, the main conclusion still holds.
[13] An estimation is performed dropping the cases where subjects are late on 9th Avenue *without* a bus, or are on time *with* a bus. This makes the frequency of delay on 9th Avenue *without* a bus zero, and the frequency of delay on 9th Avenue *with* a bus one. The results remain the same.

frequency on 9th Avenue and may therefore learn how to drive more efficiently, and this may help to shorten the travel time.[14]

Randomized Incentives across Risk Treatments

Recall that subjects are randomly assigned to a wage, and conditional on that wage they are randomly assigned to a late penalty and a toll. The assignment of time threshold is also random. The distribution of subjects who belong to each wage level as well as each level of penalties, tolls, and time thresholds are shown in Figures AA1, AA2, AA3, and AA4, respectively.

Even though the mean and standard deviation of these distributions are very similar, these distributions are not the same (i.e., the shapes of the distributions are not the same), as is revealed by the Kolmogorov-Smirnov test when comparing the distributions between any two given treatments. Given the small sample size in the experiment it is difficult to achieve perfect randomization that result in even representation of all possible values; the more values there are for a single parameter the more difficult it is to achieve an even representation for each value of the parameter. This is particularly the case for the toll assignment, where there are a total of 36 possible assignments. Comparing any two risk treatments, the distributions of tolls are significantly different from each other within a 10% significance level based on the Kolmogorov-Smirnov test. The same is for the penalty assignment where there are 8 possible values to be assigned, and the same is for the time threshold assignment where there are 8

[14] Here the possible implication is the perceived risk of travel delay on a particular route may be partly affected by how familiar the subjects are with driving on that route.

possible values to be assigned. For the wage variable, however, there are only 2 possible values to be assigned, the distribution of subjects who are assigned to each level of wage are not significantly different across the risk treatments except for Treatment 0.2.

Incentives and Choice of Route

Do the difference in incentives affect the choice of route? Since a higher toll may discourage driving on 7th Avenue, it is expected that the average subject who takes 9th Avenue has a higher toll than the average subject who takes 7th Avenue. The distribution of tolls for the drives on 7th Avenue and 9th Avenue is shown in Figure A5. The two distribution are significantly different based on the Kolmogorov-Smirnov[15] test, with the mean of the distribution being higher for 9th Avenue than 7th Avenue. This provides preliminary evidence that subjects with a high toll are more likely to drive on 9th Avenue.

Given that 9th Avenue on average takes longer to drive relative to 7th Avenue, subjects who are assigned a high penalty should be less likely to drive on 9th Avenue and more likely to drive on 7th Avenue. The distributions of assigned penalties for the drives on 7th Avenue and 9th Avenue are shown in Figure A6. The two distribution are significantly different, with the mean of the distribution being higher on 7th Avenue than on 9th Avenue. This provides preliminary evidence that subjects with a high penalty are more likely to take 7th Avenue since it has a shorter travel time.

[15] The Kolmogorov-Smirnov test if performed to examine the null hypothesis that two distributions are equal.

Wage is not expected to affect the choice of route. The distributions of assigned wages for the drives on 7th Avenue and 9th Avenue are not significantly different (p-value = 0.529).

Proportions of Route Choice

Prior to starting the driving task, the perception of congestion risk can be expected to reflect the prior bus card information. Figure A7 shows that the number of bus cards drawn increases with the objective risk. As the number of bus cards drawn increases with objective risk, the proportion of subjects who choose the risky route is expected to decrease. This is the case in our experiment as is shown in Figure A8, which describes the raw proportion of subjects who choose the risky route across the ten periods by treatment. It appears that subjects hold beliefs that are consistent with their prior bus card information.

Comparing the proportion of route choices across risk treatments, there is a larger proportion of subjects taking the risky route in the two treatments with objective congestion probabilities below 0.5 than in the two treatments with objective congestion probabilities above 0.5. Pooling across periods, the proportion of route choice ranks in the order of objective risks: Treatment 0.2 has the highest proportion of subjects taking the risky route (79%), followed by Treatments 0.4 (71%), 0.6 (53%), and 0.8 (47%). Most of the pairwise comparisons between treatments are significantly different, except between Treatments 0.2 and 0.4 (p-value 0.02) and between Treatments 0.6 and 0.8 (p-value 0.09). Thus, there is some preliminary evidence that subjects can perceive differences between high and low probabilities, lending partial support to Hypothesis I.

Across the ten periods, in each treatment there is some evidence of change in the proportion of risky route choices (see Figure A8). In particular there appears to be an increased proportion of risky choices following the first period, except in Treatment 0.4. This pattern of behavior is consistent with subjects initially overestimating the risk of delay and subsequently adjusting their beliefs. For example, in Treatment 0.2, the largest increase is 19% from period 1 to 8 (*p*-value 0.054). The next largest increase is 13% from period 1 to 5, but the change is not statistically significant. In Treatment 0.6 there is an increase of 25% from period 1 to 7 (*p*-value 0.045). In Treatment 0.8 the largest increase is from period 1 to 3 by 18% but this is not significant (*p*-value 0.105).

The number and proportion of subjects who switch routes between periods are shown in Table A4. In Treatment 0.2, subjects who take 9th Avenue are less likely to experience congestion than subjects in the higher risk treatments, so one would expect them to be less likely to switch away from using 9th Avenue. In fact, of the 22 subjects who initially selected 9th Avenue only 2 switched to 7th Avenue in period 2. That corresponds to only 9% of the sample. On the other hand, of those who took 7th Avenue, 56% switched to 9th Avenue in period 2. Pooling across periods, in Treatment 0.2 the proportion who switched from 9th Avenue to 7th Avenue (8%) is smaller than the proportion who switched from 7th Avenue to 9th Avenue (32%). This is consistent with frequent experiences of no congestion on 9th Avenue in Treatment 0.2. In Treatment 0.4 there is a similar but weaker pattern: 11% switched from 9th Avenue to 7th Avenue and 25% switched from 7th Avenue to 9th Avenue. In the two high risk treatments the average proportion of subjects switch from 9th Avenue to 7th Avenue and from 7th Avenue to 9th are similar: 19% and 14%, respectively, in Treatment 0.6; and 15% and 14%, respectively, in Treatment 0.8. However, in the later periods in Treatment 0.8, a higher proportion switched from 9th Avenue to 7th Avenue than in the opposite direction. This is a sign that, in Treatment 0.8, once subjects select 9th Avenue they experience on average more congestion than their counterparts in the lower risk treatments.

Based on the route switching behavior, Table A5 displays the conditions under which subjects switch away from 9th Avenue. It shows the proportion of subjects who switched from 9th Avenue to 7th Avenue conditional on encountering congestion or not. One would expect the proportion of subjects who switched in the former case to be at least as high as in the latter case. This is indeed the pattern observed in the three treatments with the highest congestion risk: Treatments 0.4, 0.6 and 0.8.

In summary, behavior appears consistent with subjects forming subjective beliefs that reflect the objective risks. Analysis of the raw data provides preliminary evidence that subjects can perceive the difference across high-probability and low-probability and that subjective beliefs may be ranked in the order of the objective risk. This lends partial support to Hypothesis I. Across periods there is some adjustments that indicate that subjects may come to believe they initially overestimate the risks, at least in Treatments 0.2 and 0.6. Given that the proportion of risky choices is higher in the low risk treatments than in the high risk treatments, subjects in the low risk treatments would obtain more information about the risky route than their counterparts in the high risk treatments. This result would imply that the estimated belief in the low risk treatments will be more likely to converge on the true probabilities than in the high risk treatments, which would lend support to Hypothesis II.

1.5.3 Propensity of Route Choice

In this section route choice is estimated controlling for variations in experimental parameters such as tolls and delay penalties, so to directly investigate whether changes in the tolls are less effective for subjects in the high risk treatments than in the low risk treatments, as suggested by the second hypothesis. Table A6 shows the result of a Probit model controlling for variations in payoff incentives and period fixed effects. The endogenous variable is the propensity to take the risky route and the independent variables are *Wage*, *Toll*, *Late Penalty*, the prior number of bus cards (*Prior*), and period fixed effects (i.e., *Period 2*, …, *Period 10*).

$$Propensity_{risky} = \beta_0 + \beta_1 \times Wage + \beta_2 \times Toll + \beta_3 \times Late\ penalty + \beta_4 \times Prior + \beta_5 \times Period\ 2 + \beta_6 \times Period\ 3 + \beta_7 \times Period\ 4 + \beta_8 \times Period\ 5 + \beta_9 \times Period\ 6 + \beta_{10} \times Period\ 7 + \beta_{11} \times Period\ 8 + \beta_{12} \times Period\ 9 + \beta_{13} \times Period\ 10$$

All coefficients are transformed to marginal probability effects computed using the delta method.[16]

Effects of Payoff Incentives

As expected from the descriptive data, *Wage* has no effect on the propensity to take the risky route. *Toll* has the theoretically expected positive effect on the propensity to take the risky

[16] The delta method takes a nonlinear transformation of an estimated parameter about its mean and its variance based on a Taylor approximation (Oehlert (1992)).

route, but the effect is only significant in the two low risk treatments. Comparing the coefficients across treatments reveals that the marginal effect of *Toll* is significantly higher in Treatment 0.2 than the other treatments, and the latter have coefficients that are not significantly different from each other. One explanation is that in the high risk treatments the alternate route has a higher risk of delay, and thus these subjects are reluctant to drive on the alternate route until a higher toll is set for the toll road. This finding suggests that changes in the *Toll* are less effective for subjects in the high risk treatments than in the low risk treatments. This is consistent with Hypothesis II: since subjects in the high risk treatments are more likely to start with a high belief of congestion for the risky route, they will be more likely to drive on the safe route, which means the effectiveness of *Toll* will be smaller for these subjects. Because information for the risky route can only be obtained if one drives on it, this would suggest that less information will be obtained about the risky route, resulting in asymmetric information across the two routes.

The variable *Late Penalty* has the theoretically expected negative sign but is only significant in Treatment 0.4. Within each treatment, the number of prior bus cards subjects draw does not have a significant effect on the propensity of route choice. Since the number of bus cards subjects draw does not vary much *within* each treatment, it is not surprising that the variable *Prior* is not significant *within* treatment. However, when pooling the data across all treatments, there is a significant decrease in the propensity to take the risky route as the number of prior bus cards increases (p-value < 0.001), which is expected, as is shown in the final column of Table A6.

Period Effects

Across treatments, in period 1 (captured by the coefficient on *Constant*) the propensity to choose the risky route is significantly higher in the low risk treatments than in the high risk treatments. However, the point estimates are not statistically significant except in Treatment 0.4. In all treatments, for subsequent periods the marginal propensity to take the risky route generally is higher, though the increase is not significant in most cases except in Treatment 0.2 in periods 5 and 8. Thus, results in the Probit model indicates that there is a higher propensity to take the risky route in the low risk treatments than in the high risk treatments, which suggests that subjects in the low risk treatments will obtain more information feedback about the risky route than their counterparts in the high risk treatments.

In summary, the conditional analysis of route choice in the Probit model tells a similar story to that of the unconditional descriptive analysis: the low risk treatments show a higher propensity to choose the risky route than the high risk treatments do. In addition, the responses to *Toll* variations is stronger in the two low risk treatments, and the only significant adjustment in route choice over time is found in the lowest risk treatment. The next sections analyze the subjective beliefs that are implied by this behavior, assuming SEU with a CRRA utility function. This allows for control for the influence of risk attitudes on inferred subjective beliefs.

1.5.4 Subjective Expected Utility

The estimation result of the SEU specification is shown in Table A7 assuming a CRRA utility function in equations (1) – (4). Risk attitudes and subjective probabilities are jointly estimated using both the lottery and the driving simulator data. The coefficients are the marginal probabilities computed using the delta method.[17] The top row of the table shows the estimated risk attitude for a representative agent. The middle of the table shows the estimated subjective probabilities, and the variable *Prior* captures the effect on route choice of the number of bus cards with the word "bus" that subjects drew before starting the drive task. The last part of the table shows the Fechner errors.

Risk Attitudes

Risk attitudes and subjective probabilities are estimated jointly. First the estimated risk attitudes are discussed, the next session discusses the estimated beliefs. The estimated distribution of the EUT CRRA risk attitudes are shown in Figure A9, by treatment. These CRRA estimates are from the model shown in Table A7 where we pool across all demographic subgroups. Thus, these estimated risk attitudes reflect the variations of demographics in the subject pool. Across the risk treatments there are considerable differences in risk attitudes. All estimated CRRA-values fall in the range 0 – 1, and are thus consistent with the literature (e.g., Harrison and Rutström (2008b)).

[17] The estimates for the subjective probability p are obtained as follows: first we estimate the parameter κ which can vary between $\pm\infty$, next κ is converted to p using $p = 1/(1+\exp(\kappa))$ and the resulting p is constrained to be in the unit interval. The non-linear transformation from κ to p uses the delta method that correctly calculate standard errors (Oehlert (1992)).

SEU Probabilities

The first period subjective delay probabilities (captured by the coefficient on *Constant*) are ranked in the order of congestion risk and are significant at the 1% level in all risk treatments. These joint ML results match those of the Probit model, showing how the propensity to choose the risky route decreases across risk treatments. The initial subjective delay probabilities are estimated to be .632, .688, 1 and 1 in Treatments 0.2, 0.4, 0.6 and 0.8, respectively.

Across the treatments, are subjects able to perceive that the treatment conditions are different? For example, between Treatments 0.8 and 0.2 the congestion risks differ objectively by 0.6 percentage points: are subjects able to perceive a difference? The same can be asked between Treatments 0.6 and 0.2 and between Treatments 0.8 and 0.4, which have congestion risks that differ objectively by 0.4 percentage points. To answer these questions, pairwise comparisons are made between the coefficients across treatments. In period 1, *only two* pairwise comparisons are significantly different: comparing Treatments 0.6 and 0.2 and Treatments 0.8 and 0.2. In subsequent periods, the difference between Treatments 0.6 and 0.2 are positive and significant, and the same is true between Treatments 0.8 and 0.2. Thus subjects are on average able to perceive differences in congestion risks when they objectively differ by 0.6 percentage points, and perhaps when they differ by 0.4 percentage points (such as when comparing 0.6 and 0.2). The difference in perceived probabilities lends support to Hypothesis I: the estimated probabilities are ranked in the order of congestion risk.

Belief Adjustment

There is evidence of significant belief adjustment across periods only in the lowest risk treatment, but even there it is limited. Although the marginal period effects are generally insignificant, the size of the marginal period effects ranks in the order of objective risk: Treatments 0.8 and 0.6 have the smallest marginal effect, next is Treatment 0.4, and treatment 0.2 has the largest marginal effect. This points toward there being less adjustments in the high risk treatments than in the low risk treatments.

In Treatment 0.2 the marginal effects in periods 5 and 8 are negative and significantly different from zero. They are -0.289 and -0.465 with *p*-values of 0.097 and 0.003, respectively. The marginal effects in the other periods are not small, although they are insignificant. These estimates have large 95% confidence intervals, and the large confidence intervals are likely a result of a great deal of heterogeneity in choices across subjects in this treatment. The subsequent section provides a discussion on how this can be captured to some degree by controlling for demographics.

In Treatment 0.4, the marginal effects are not significant in any periods. In Treatments 0.6 and 0.8 subjects start with a subjective probability of 1, which is an extreme belief of delay. There is no significant adjustment of belief across any periods, with point estimates that are also small.

In summary, subjects are able to perceive the difference in objective risk across treatments as shown by the rank ordering of estimated probabilities across treatments. In particular, the estimated probabilities reveal that they are able to distinguish probabilities that objectively differ by 0.6 percentage points, and sometimes when they differ by 0.4 percentage points, such as when comparing congestion risks of 0.6 and 0.2. Across periods, only subjects in the lowest risk treatment adjust their belief of delay, whereas in the high risk treatments they fail to adjust their beliefs. This is consistent with Hypothesis II under endogenous information feedback, showing that when the subjective belief of delay is higher, the riskier route is perceived as riskier and this leads to little or no belief adjustment.

Thus, results of the SEU model are similar to the unconditional descriptive analysis and the Probit model when comparing *across treatments*. Across periods, the SEU model agrees with the Probit model, but the unconditional descriptive data shows more adjustments.

Results in the SEU model for the lowest risk treatment are consistent with past experimental studies where subjective beliefs are elicited only once: Attneave (1953) reports that subjects overestimate low probabilities; Preston and Baratta (1949) and Andersen, Fountain, Harrison, Hole and Rutström (2011) report an equality point at 0.2 where subjects overestimate probabilities below 0.2. A crucial element to keep in mind when comparing past and present studies is that subjects in this essay make decisions under an endogenous information environment, i.e., information on a route can only be obtained if one takes that route, and this leads to asymmetric information across the two routes.

1.5.5 Demographics

Tables A8 shows the SEU probabilities estimated controlling for period fixed effects *and* demographic effects. The list of demographic variables include: *Female*, *College Education*, and *High Income*. Subjects are grouped into two education levels: those who hold a college degree are coded as *College Education* = 1 and those who do not are coded as *College Education* = 0. In terms of income, subjects with household income of above $100,000 are coded as *High Income* = 1 and those with household income of $100,000 or below are coded as *High Income* = 0.

The *Constant* is now referencing a particular demographic group, and none of the demographic variables is significant. The ideal model for studying demographics would include all possible demographic variables with a full set of interaction terms between demographic subgroups, but this would require a large data set.

Given the large confidence intervals in the main model it may be helpful to estimate subjective probabilities *by demographic subgroups*. This can reveal if a particular demographic subgroup contributes the most to the adjustment of belief in the overall data or to its imprecision.

Income

In Tables A9 and A10, subjective probabilities are shown separately for subjects with high income and low income, respectively. In Treatments 0.2 and 0.4, the high income subjects have initial beliefs of 0.766 and 0.751, respectively, and are not significantly different from each other. This shows that the high income subjects have a higher belief of delay than the average subject in Table A7. In Treatments 0.6 and 0.8, the high income subjects have initial beliefs of 1, which is the same estimate as the average subject. Treatment 0.2 is (again) the only treatment showing significant belief adjustment. Here, after dropping the low income subjects, significant adjustment is observed in periods 3, 6 and 10 in addition to periods 5 and 8. The marginal effects in the other periods are not small, but they are not statistically significant. In the higher risk treatments the marginal effects are (again) small and no adjustment is observed. Thus removing the low income subjects removes at least some of the noise in Table A7.

For the low income subjects, in Treatments 0.2 and 0.4 the initial beliefs are 0.278 and 0.528 respectively, but the estimates are not significant. Compared to the estimates shown in Table A7 that uses the aggregate data, the low income subjects have beliefs that are lower than the average subject. Across the periods, the low income subjects exhibit no significant belief adjustment, which also shows that the belief adjustments observed in the aggregate data are largely driven by high income subjects. Across high income and low income groups, the same pattern is observed in the high risk treatments: initial belief is 1 and subsequent periods show no belief adjustment.

Education

Next, subjective probabilities are estimated for college graduates and Table A11 shows the results. The initial beliefs are 0.718, 0.753, 0.999 and 1 across the risk treatments. After dropping the non-college subjects, Treatment 0.2 shows significant belief adjustment in periods 2, 3 and 6 in addition to periods 5 and 8; the subsequent beliefs are adjusted downward and none are significantly different from 0.2. In contrast, in Treatments 0.6 and 0.8 no belief adjustment is observed and subjective beliefs remain above the objective probabilities throughout. This shows that the belief adjustment in the aggregate data is largely driven by college graduates, implying that the non-college graduates may have contributed a considerable amount of noise to the aggregate data.[18]

1.5.6 Rank Dependent Utility - Robustness Check

The estimated probabilities for the RDU specification are shown in Table AA1. The RDU probabilities are jointly estimated with the utility curvature parameter and the probability-weighting parameter. The results are estimated assuming the CRRA utility function and the power weighting function of equations (1') – (4'). When the weighting parameter γ takes on the value of 1, the RDU probabilities are identical to the SEU probabilities. A value of γ above (below) 1 indicates an underweighting (overweighting) of probabilities, which means that the inferred probabilities are higher (lower) than under SEU. The estimated weighting parameter γ is not significantly different from 1 in any of the risk treatments, which means that subjects on average do not weigh probabilities in either direction. Thus it is not surprising that the estimated probabilities are not significantly different across the SEU and RDU specifications. The initial beliefs are 0.629, 0.630, 1 and 1 in Treatments 0.2, 0.4, 0.6 and 0.8, respectively. Across the periods, only in the lowest risk treatment is there significant belief adjustment, which occurs in period 8. In period 5 the marginal effects are virtually identical across the SEU and RDU specifications (0.39 and 0.343, respectively) but the effect is not significant in the latter specification.

[18] It is not possible to perform the estimation for the non-college subjects, since in treatment 0.8 only four subjects are non-college graduates.

The estimated RDU probabilities controlling for period fixed effects *and* demographic effects are shown in Table AA2. Similar to the results in the SEU specification, none of the demographic variables is significant in the RDU specification.

1.6 Conclusion

The goal of this essay is to examine drivers' perception of the risk of delay as one factor that influences route choice behavior in a simulated driving environment. This experiment recruits commuters from the Atlanta and Orlando metropolitan areas and presents them with a route choice task in a driving simulator. Subjects are required to make a binary choice between a route that has an uncertain level of congestion and an alternate route with no risk of congestion. Subjects are assigned monetary incentives for the value of making the drive, the discrete penalty for arriving late to the destination, and the toll charged on the non-congested route. Apart from some prior information about frequency of congestion on the risky route, drivers only obtain additional information if they actually choose to drive it. Information feedback is therefore endogenous and high risk scenarios can lead to less belief updating than low risk scenarios since drivers are more likely to avoid taking the risky route when it is riskier. The experiment implements four risk treatments that differ in the objective risk of congestion across a range of probabilities. This allows the examination of belief formation and adjustment across a range of probabilities.

Across risk treatments, the estimated *beliefs of delay* rank in the order of the *objective congestion probabilities*. In subsequent periods only subjects in the lowest risk treatment experience significant belief adjustments. In contrast, in the high risk treatments no belief adjustment is made. Behavior across treatments is as predicted under an endogenous information environment: subjects who start with a lower belief of delay are more inclined to take the route that has an uncertain level of congestion than those who start with a higher belief of delay, leading to subjects with a lower belief of delay obtaining more information than those with a higher belief of delay. Thus, subjects who start with a lower belief of delay experience more belief adjustment than those who start with a higher belief of delay. The results are consistent with past experimental findings in the low probabilities treatments but not in the high probabilities treatments.

Results of this essay show that when drivers hold a high initial belief of delay over a route they normally do not take, then they could be reluctant to try it out even when conditions on their usual route become less favorable. The policy implication is that for drivers to be more inclined to use unfamiliar routes as alternatives when their usual routes experience construction, maintenance or tolls, clear and credible information about the congestion situation on the alternate route is required. Furthermore, since the Probit model shows that the marginal effect of toll is lower for drivers in a high congestion risk scenario, this suggests that we will need to impose a higher toll on these subjects in order to incentivize them to switch to the alternate route.

Tables and Figures for Chapter 1

Table A1: Tolls and Wages in the Simulator Task

	Toll Range	**Late Penalty**	**Time Threshold**
Wage=$2.50	$0.50-$2.00	$0.50-$2.00	2min 10 secs to 2m 45 secs
Wage=$5.00	$0.50-$4.00	$1.00-$4.00	

The range of toll cards was in 10-cent increments.
The range of penalty was in 50-cent increments.
The range of time thresholds was in 5-second increments.

Table A2: Prizes and Probabilities in Lottery Task

Probability range	**Safe Lottery Low Prize**	**Safe Lottery High Prize**	**Risky Lottery Low Prize**	**Risky Lottery High Prize**
0.1 – 0.9	$2	$3	$0.25	$4
0.1 – 0.9	$2	$3	$0.25	$5
0.1 – 0.9	$2	$3	$0.25	$6
0.1 – 0.9	$4	$6	$0.50	$10

Table A3: Demographic Sub-groups in Each Treatment

	Treatment 0.2	**Treatment 0.4**	**Treatment 0.6**	**Treatment 0.8**	**All**
Number of subjects	31	40	32	38	141
Location					
Orlando	41.75%	50%	37.50%	47.49%	45.06%
Atlanta	58.25%	50%	62.50%	52.51%	54.94%
Gender					
Male	54.12%	52.88%	56.36%	54.91%	52.21%
Female	45.88%	47.12%	43.64%	45.09%	47.79%
Education					
College	68%	74.82%	72.27%	90.17%	78.09%
Non-college	32%	25.18%	27.73%	9.83%	21.91%
Income					
High: above $100K	49.18%	42.09%	34.09%	38.92%	44.09%
Low: $100K or below	50.82%	57.91%	65.91%	61.08%	55.91%

Table A4: Route Switches

From one period to the next	# of subj. who took 7th Ave. in the first of the two periods	# of subj. who took 9th Ave. in the first of the two periods	Proportion taking 9th	# of subj. switch from 7th to 9th	# of subj. switch from 9th to 7th	Conditional on taking 7th Ave., the proportion that switch to 9th	Conditional on taking 9th Ave., the proportion that switch to 7th	Difference
Treatment 0.2								
1 to 2	9	22	71%	5	2	56%	9%	47%
2 to 3	6	25	81%	1	2	17%	8%	9%
3 to 4	7	24	77%	3	2	43%	8%	35%
4 to 5	6	25	81%	3	2	50%	8%	42%
5 to 6	5	26	84%	0	2	0%	8%	-8%
6 to 7	7	24	77%	1	2	14%	8%	6%
7 to 8	8	23	74%	5	0	63%	0%	63%
8 to 9	3	28	90%	0	5	0%	18%	-18%
9 to 10	8	23	74%	4	2	50%	9%	41%
Average	-	-	79%	-	-	32%	8%	24%
Treatment 0.4								
1 to 2	12	28	70%	3	2	25%	7%	18%
2 to 3	11	29	73%	0	5	0%	17%	-17%
3 to 4	16	24	60%	7	1	44%	4%	40%
4 to 5	10	30	75%	1	3	10%	10%	0%
5 to 6	12	28	70%	3	2	25%	7%	18%
6 to 7	11	29	73%	3	4	27%	14%	13%
7 to 8	12	28	70%	6	3	50%	11%	39%
8 to 9	9	31	78%	3	3	33%	10%	23%
9 to 10	9	31	78%	1	4	11%	16%	-5%
Average	-	-	71%	-	-	25%	11%	14%
Treatment 0.6								
1 to 2	21	11	34%	5	2	24%	19%	5%
2 to 3	18	14	44%	5	1	28%	7%	21%
3 to 4	14	18	56%	3	3	21%	17%	4%
4 to 5	14	18	56%	2	2	14%	11%	3%
5 to 6	14	18	56%	3	2	21%	11%	10%
6 to 7	13	19	59%	3	3	23%	16%	7%
7 to 8	13	19	59%	0	2	0%	11%	-11%
8 to 9	15	17	53%	4	2	27%	12%	15%
9 to 10	13	19	59%	2	4	15%	21%	-6%
Average	-	-	53%	-	-	19%	14%	5%
Treatment 0.8								
1 to 2	25	13	34%	6	2	24%	15%	9%
2 to 3	21	17	45%	6	3	29%	18%	11%
3 to 4	18	20	52%	3	4	17%	20%	-3%
4 to 5	19	19	50%	2	1	11%	5%	6%
5 to 6	18	19	51%	3	3	17%	16%	1%
6 to 7	19	19	50%	2	4	11%	21%	-10%
7 to 8	21	17	45%	2	2	10%	12%	2%
8 to 9	21	17	45%	3	1	14%	6%	8%
9 to 10	19	19	50%	1	3	5%	16%	11%
Average	-	-	47%	-	-	15%	14%	1%

Table A5: Congestion Experiences

From one period to the next	**# of subjects who switch from 9th to 7th**	**Conditional on a bus card, # of subjects who switch from 9th to 7th**	**Conditional on *no* bus card, # of subjects who switch from 9th to 7th**	**Conditional on a bus card, % of subjects who switch from 9th to 7th**	**Conditional on no bus card, % of subjects who switch from 9th to 7th**
Treatment 0.2					
1 to 2	2	0	2	0%	100%
2 to 3	2	0	2	0%	100%
3 to 4	2	0	2	0%	100%
4 to 5	2	2	0	100%	0%
5 to 6	2	2	0	100%	0%
6 to 7	2	0	2	0%	100%
7 to 8	0	-	-	-	-
8 to 9	5	2	3	40%	60%
9 to 10	2	0	2	0%	100%
Average	--	--	--	30%	70%
Treatment 0.4					
1 to 2	2	1	1	50%	50%
2 to 3	5	3	2	60%	40%
3 to 4	1	1	0	100%	0%
4 to 5	3	0	3	0%	100%
5 to 6	2	2	0	100%	0%
6 to 7	4	2	2	50%	50%
7 to 8	3	2	1	67%	33%
8 to 9	3	2	1	67%	33%
9 to 10	4	2	2	50%	50%
Average	--	--	--	60%	40%
Treatment 0.6					
1 to 2	2	0	2	0%	100%
2 to 3	1	1	0	100%	0%
3 to 4	3	2	1	67%	33%
4 to 5	2	1	1	50%	50%
5 to 6	2	2	0	100%	0%
6 to 7	3	2	1	67%	33%
7 to 8	2	2	0	100%	0%
8 to 9	2	0	2	0%	100%
9 to 10	4	3	1	75%	25%
Average	--	--	--	62%	38%
Treatment 0.8					
1 to 2	2	1	1	50%	50%
2 to 3	3	2	1	67%	33%
3 to 4	4	4	0	100%	0%
4 to 5	1	0	1	0%	100%
5 to 6	3	2	1	67%	33%
6 to 7	4	4	0	100%	0%
7 to 8	2	2	0	100%	0%
8 to 9	1	1	0	100%	0%
9 to 10	3	3	0	100%	0%
Average	--	--	--	76%	24%

Table A6: Propensity of Route Choice Estimated with a Probit Model

	Treatment 0.2 N=31	**Treatment 0.4 N=40**	**Treatment 0.6 N=32**	**Treatment 0.8 N=38**	**Pooled Across Treatments N=141**
Constant	0.232 (0.192)	0.394*** (0.008)	0.011 (0.470)	0.003 (0.637)	0.511*** (<0.001)
Wage	-.059 (0.263)	0.022 (0.619)	-0.002 (0.697)	0.003 (0.569)	-.012 (0.550)
Toll	0.720*** (<0.001)	0.410*** (<0.001)	0.200 (0.157)	0.108 (0.381)	0., (<0.001)
Late Penalty	-.059 (0.451)	-0.243*** (<0.001)	-.011 (0.462)	-.003 (0.636)	-0.282*** (<0.001)
Period 2	0.235 (0.203)	0.031 (0.817)	0.016 (0.494)	0.007 (0.595)	0.114* (0.099)
Period 3	0.150 (0.394)	-0.144 (0.241)	0.066 (0.295)	0.023 (0.526)	0.127* (0.067)
Period 4	0.224 (0.211)	0.087 (0.535)	0.085 (0.257)	0.016 (0.553)	0.201*** (0.003)
Period 5	0.324* (0.080)	0.003 (0.984)	0.067 (0.289)	0.019 (0.545)	0.187*** (0.006)
Period 6	0.150 (0.394)	0.032 (0.814)	0.084 (0.261)	0.021 (0.531)	0.183*** (0.007)
Period 7	0.077 (0.639)	0.011 (0.936)	0.105 (0.230)	0.007 (0.604)	0.153** (0.026)
Period 8	0.498*** (0.004)	0.129 (0.365)	0.052 (0.322)	0.008 (0.600)	0.194*** (0.004)
Period 9	0.087 (0.593)	0.144 (0.316)	0.086 (0.260)	0.016 (0.552)	0.194*** (0.004)
Period 10	0.239 (0.190)	-0.045 (0.728)	0.051 (0.327)	0.008 (0.593)	0.130* (0.060)
Prior	-0.081* (0.100)	0.026 (0.206)	0.008 (0.283)	0.002 (0.476)	-0.044*** (<0.001)

p-values are in parentheses.
The coefficients are marginal propensities computed using the delta method that takes a nonlinear transformation of an estimated parameter about its mean and its variance based on a Taylor approximation.
*** means that the coefficient is significant at the 1% level.
** means that the coefficient is significant at the 5% level.
* means that the coefficient is significant at the 10% level.

Table A7: Subjective Expected Utility Estimates Across Periods

	Treatment 0.2 N=31	**Treatment 0.4 N=40**	**Treatment 0.6 N=32**	**Treatment 0.8 N=38**
Risk Aversion:				
r	.262* (0.074)	.424*** (0.002)	.554*** (0.001)	.525*** (<0.001)
Beliefs:				
Constant	.632*** (<0.001)	.688*** (<0.001)	1*** (<0.001)	1 (a)
Period 2	-.232 (0.178)	-.045 (0.661)	-.0001 (0.941)	<.001 (0.978)
Period 3	-.159 (0.330)	.154 (0.110)	-.005 (0.709)	<.001 (0.970)
Period 4	-.210 (0.165)	-.063 (0.440)	-.003 (0.738)	<.001 (0.974)
Period 5	-.289* (0.097)	.029 (0.526)	-.005 (0.727)	<.001 (0.977)
Period 6	-.159 (0.330)	.0005 (0.996)	-.006 (0.711)	<.001 (0.974)
Period 7	-.089 (0.545)	.035 (0.748)	-.005 (0.734)	<.001 (a)
Period 8	-.465*** (0.003)	-.092 (0.317)	-.001 (0.842)	<.001 (a)
Period 9	-.089 (0.618)	-.094 (0.283)	-.006 (0.720)	<.001 (a)
Period 10	-.198 (0.219)	.066 (0.574)	-.002 (0.754)	<.001 (a)
Prior	.009 (0.859)	-.026 (0.489)	-.0001 (0.944)	<.001 (0.826)
μRA	.177*** (<0.001)			
μBelief	.231*** (<0.001)			

The results are obtained using a joint estimation of risk attitudes and beliefs from the lottery data and the driving simulator data.
p-values are in parentheses. The coefficients are marginal effects computed using the delta method.
*** means that the coefficient is significant at the 1% level.
** means that the coefficient is significant at the 5% level.
* means that the coefficient is significant at the 10% level.
(a) implies that a standard error cannot be computed by the delta method due to numeric issues, because the estimated probabilities approach 0 or 1.
μRA is the Fechner error for the lottery data; μBelief is the Fechner error for the belief data.

Table A8: Subjective Expected Utility Estimates Across Periods and Demographic Effects

	Treatment 0.2 N=31	**Treatment 0.4 N=40**	**Treatment 0.6 N=32**	**Treatment 0.8 N=38**
Risk Aversion:				
r	.278* (0.053)	.423*** (0.004)	.474*** (0.003)	.549*** (0.001)
Beliefs:				
Constant	.844*** (<0.001)	.595*** (<0.001)	.976*** (<0.001)	.689** (0.021)
Period 2	-.266 (0.112)	-.050 (0.655)	-.038 (0.733)	.133 (0.701)
Period 3	-.176 (0.195)	.185* (0.096)	-.101 (0.497)	-.125 (0.781)
Period 4	-.233 (0.230)	-.070 (0.442)	-.140 (0.484)	-.224 (0.593)
Period 5	-.303* (0.049)	.031 (0.558)	-.128 (0.380)	-.389 (0.477)
Period 6	-.176 (0.195)	.001 (0.994)	-.047 (0.678)	.120 (0.774)
Period 7	-.107 (0.344)	.048 (0.694)	-.220 (0.344)	-.098 (0.788)
Period 8	-.555** (0.013)	-.102 (0.289)	-.083 (0.370)	.141 (0.692)
Period 9	-.110 (0.430)	-.102 (0.297)	-.047 (0.678)	-.224 (0.570)
Period 10	-.211 (0.130)	.088 (0.483)	-.108 (0.281)	.137 (0.710)
Female	.067 (0.321)	.004 (0.980)	.024 (0.808)	-.503 (0.654)
College Education	-.098 (0.477)	.031 (0.862)	-.059 (0.733)	.303 (0.362)
High income (above $100K)	-.220 (0.110)	-.076 (0.699)	-.041 (0.751)	.311 (a)
μRA	.177*** (<0.001)			
μBelief	.231*** (<0.001)			

p-values values are in parentheses. The coefficients are marginal effects.
*** means that the coefficient is significant at the 1% level.
** means that the coefficient is significant at the 5% level.
* means that the coefficient is significant at the 10% level.
(a) implies that a standard error cannot be computed by the delta method due to numeric issues, because the estimated probabilities approach 0 or 1.
μRA is the Fechner error for the lottery data; μBelief is the Fechner error for the belief data.

Table A9: High Income Subject' Subjective Expected Utility Estimates Across Periods

	Treatment 0.2 N=18	**Treatment 0.4 N=20**	**Treatment 0.6 N=19**	**Treatment 0.8 N=25**
Risk Aversion:				
r	.171 (0.201)	.533* (0.065)	.550 (0.605)	.991 (0.124)
Beliefs:				
Period 1	.766*** (<0.001)	.751*** (0.002)	1 (a)	1 (a)
Period 2	-.205 (0.172)	<.001 (1.000)	<.001 (a)	<.001 (a)
Period 3	-.206* (0.063)	.123 (0.339)	<.001 (a)	<.001 (a)
Period 4	-.205 (0.172)	<.001 (1.000)	<.001 (a)	<.001 (a)
Period 5	-.353** (0.029)	.070 (0.423)	<.001 (a)	<.001 (a)
Period 6	-.206* (0.063)	.031 (0.719)	<.001 (a)	<.001 (a)
Period 7	-.205 (0.172)	.026 (0.736)	<.001 (a)	<.001 (a)
Period 8	-.502*** (<0.001)	-.049 (0.730)	<.001 (a)	<.001 (a)
Period 9	-.206 (0.203)	-.051 (0.550)	<.001 (a)	<.001 (a)
Period 10	-.501*** (<0.001)	.026 (0.736)	<.001 (a)	<.001 (a)
Prior	-.005 (0.895)	-.047 (0.223)	<.001 (a)	<.001 (a)
***μ*RA**	.093*** (<0.001)	.183*** (0.006)	.729 (0.387)	.181** (0.024)
***μ*Belief**	.100*** (<0.001)	.204* (0.067)	.383*** (0.009)	.287*** (0.010)

p-values are in parentheses. The coefficients are marginal effects.
*** means that the coefficient is significant at the 1% level.
** means that the coefficient is significant at the 5% level.
* means that the coefficient is significant at the 10% level.
(a) implies that a standard error cannot be computed by the delta method due to numeric issues, because the estimated probabilities approach 0 or 1.
μRA is the Fechner error for the lottery data; μBelief is the Fechner error for the belief data.

Table A10: Low Income Subjects' Subjective Expected Utility Estimates Across Periods

	Treatment 0.2 N=13	**Treatment 0.4 N=20**	**Treatment 0.6 N=13**	**Treatment 0.8 N=13**
Risk Aversion:				
r	.302 (0.196)	.354** (0.011)	.696*** (<0.001)	.340* (0.098)
Beliefs:				
Constant	.278 (0.239)	.528 (0.134)	1 (a)	1 (a)
Period 2	-.177 (0.492)	-.105 (0.560)	<.001 (a)	-.035 (0.776)
Period 3	-.043 (0.900)	.232 (0.234)	-.014 (0.715)	-.105 (0.806)
Period 4	-.133 (0.533)	-.142 (0.459)	-.004 (0.745)	-.074 (0.782)
Period 5	-.148 (0.511)	-.043 (0.367)	-.007 (0.746)	-.084 (0.783)
Period 6	-.043 (0.900)	-.034 (0.883)	-.011 (0.708)	-.067 (0.825)
Period 7	.141 (0.673)	.090 (0.771)	-.008 (0.750)	-.038 (0.819)
Period 8	-.220 (0.210)	-.142 (0.210)	-.008 (0.750)	-.010 (0.854)
Period 9	.141 (0.735)	-.147 (0.438)	-.022 (0.703)	-.065 (0.829)
Period 10	.210 (0.244)	.232 (0.624)	-.006 (0.773)	-.014 (0.878)
Prior	.114 (0.205)	.030 (0.683)	<.001 (a)	<.001 (a)
μRA	.188*** (0.001)	.144*** (<0.001)	.141** (0.031)	.201*** (0.001)
μBelief	.305*** (0.001)	.267*** (<0.001)	.195*** (<0.001)	.191*** (<0.001)

p-values are in parentheses. The coefficients are marginal effects.
*** means that the coefficient is significant at the 1% level.
** means that the coefficient is significant at the 5% level.
* means that the coefficient is significant at the 10% level.
(a) implies that a standard error cannot be computed by the delta method due to numeric issues, because the estimated probabilities approach 0 or 1.
μRA is the Fechner error for the lottery data; μBelief is the Fechner error for the belief data.

Table A11: College Graduates' Subjective Expected Utility Estimates Across Periods

	Treatment 0.2 N=21	**Treatment 0.4 N=30**	**Treatment 0.6 N=23**	**Treatment 0.8 N=34**
Risk Aversion:				
r	.169 (0.231)	.289* (0.092)	.647*** (0.002)	.494** (0.030)
Beliefs:				
Period 1	.718*** (0.004)	.753*** (<0.001)	.999*** (<0.001)	1 (a)
Period 2	-.319* (0.089)	<.001 (1.000)	.001 (a)	<.001 (0.921)
Period 3	-.316* (0.091)	.113 (0.172)	-.006 (0.708)	<.001 (0.923)
Period 4	-.094 (0.602)	-.046 (0.336)	-.006 (0.720)	<.001 (0.902)
Period 5	-.452* (0.054)	.043 (0.335)	-.011 (0.709)	<.001 (0.908)
Period 6	-.316* (0.091)	.040 (0.598)	-.014 (0.680)	<.001 (0.921)
Period 7	-.094 (0.384)	.031 (0.667)	-.011 (0.697)	<.001 (a)
Period 8	-.452* (0.054)	-.046 (0.567)	-.005 (0.746)	<.001 (a)
Period 9	-.199 (0.361)	-.007 (0.910)	-.003 (0.752)	<.001 (a)
Period 10	-.180 (0.259)	.056 (0.582)	-.004 (0.729)	<.001 (a)
Prior	-.009 (0.909)	-.037 (0.162)	-.001 (0.761)	<.001 (a)
μRA	.158*** (<0.001)	.158*** (<0.001)	.164*** (0.001)	.189*** (<0.001)
μBelief	.201*** (<0.001)	.187*** (<0.001)	.250*** (0.0001)	.220*** (<0.001)

p-values are in parentheses. The coefficients are marginal effects.
*** means that the coefficient is significant at the 1% level.
** means that the coefficient is significant at the 5% level.
* means that the coefficient is significant at the 10% level.
(a) implies that a standard error cannot be computed by the delta method due to numeric issues, because the estimated probabilities approach 0 or 1.
μRA is the Fechner error for the lottery data; μBelief is the Fechner error for the belief data.

Figure A1: Downtown Network With Bus on 9th Avenue

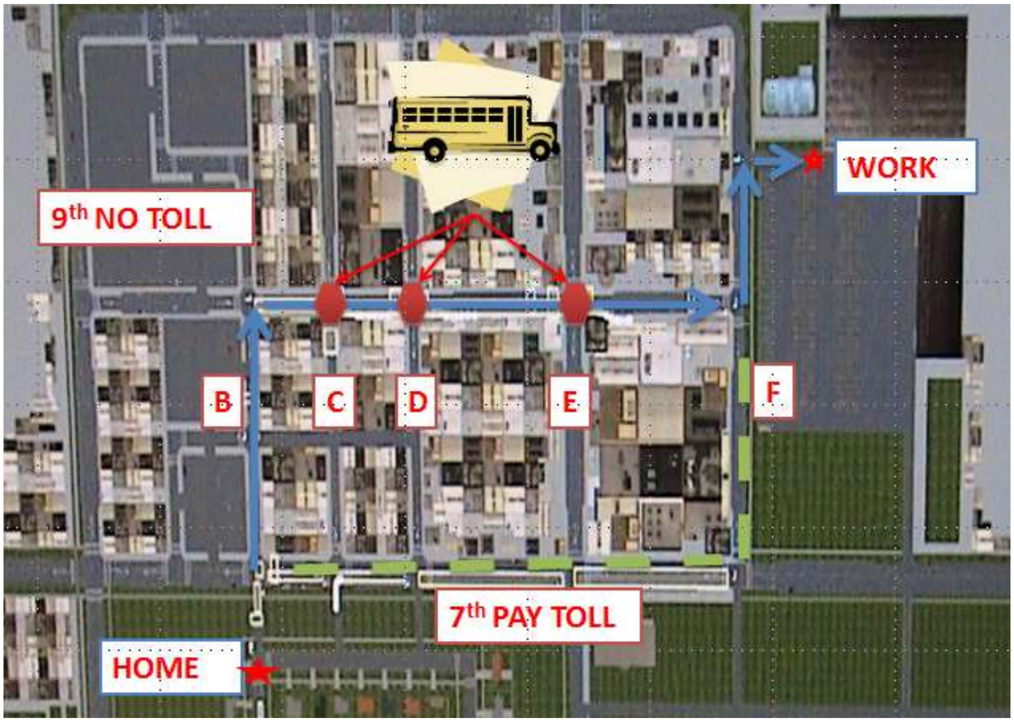


Figure A2: Screen Shot For Lottery Practice Task

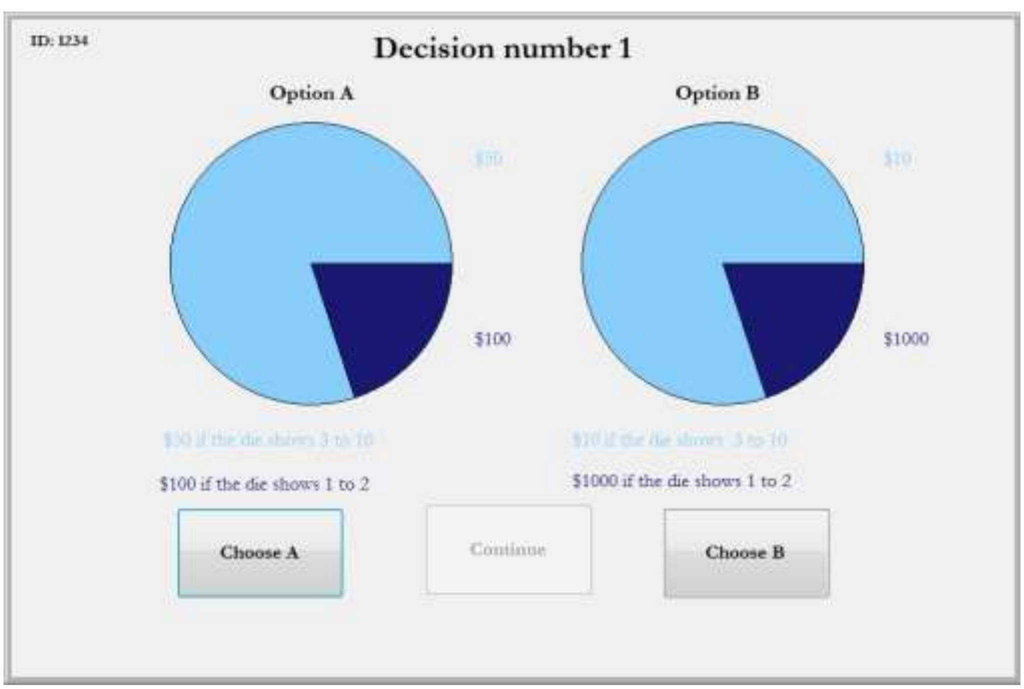
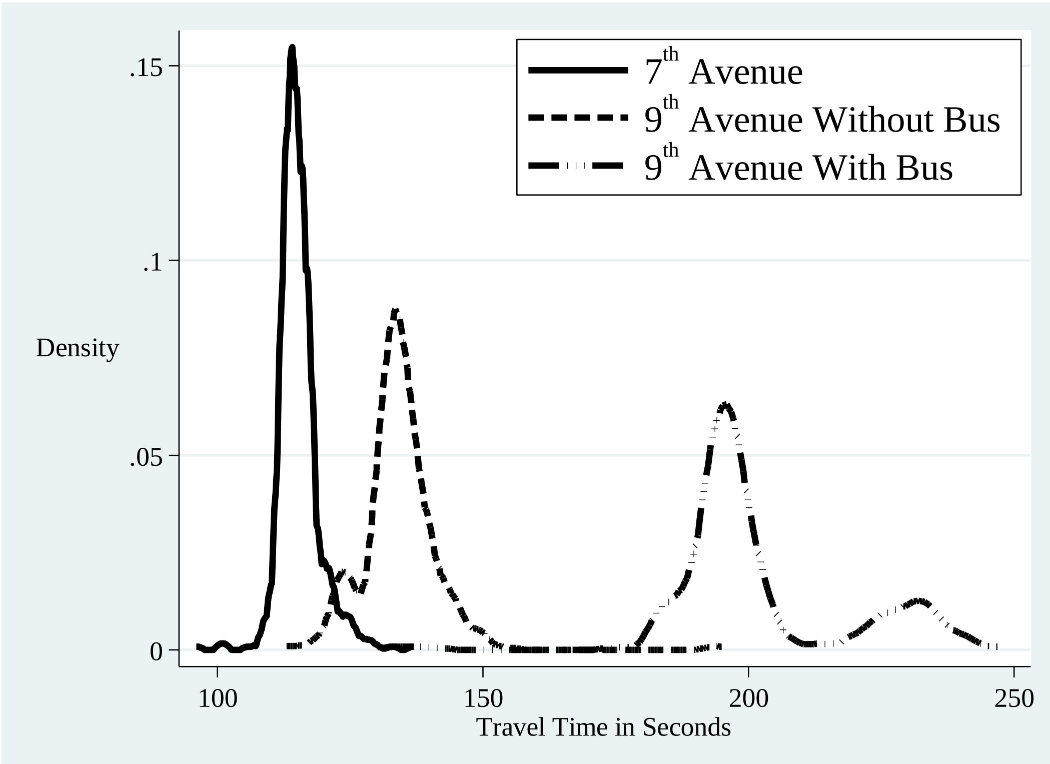


Figure A3: Distribution of Observed Travel Times



Travel time on 7th Avenue has a mean of 115 and standard deviation of 3.7.
Travel time on 9th Avenue *without* a bus has a mean of 134 and standard deviation of 7.3.
Travel time on 9th Avenue *with* a bus has a mean of 201 and standard deviation of 17.7.

Figure A4: Frequency of Delay by Time Threshold

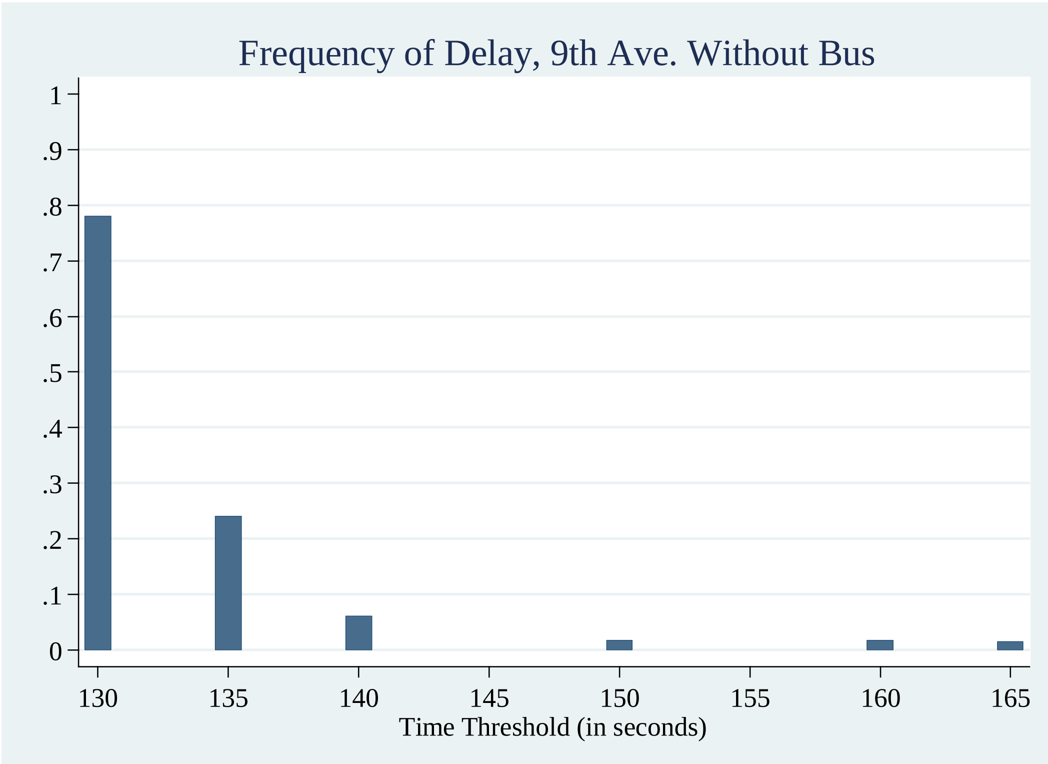


Figure A5: Distribution of Toll by Route

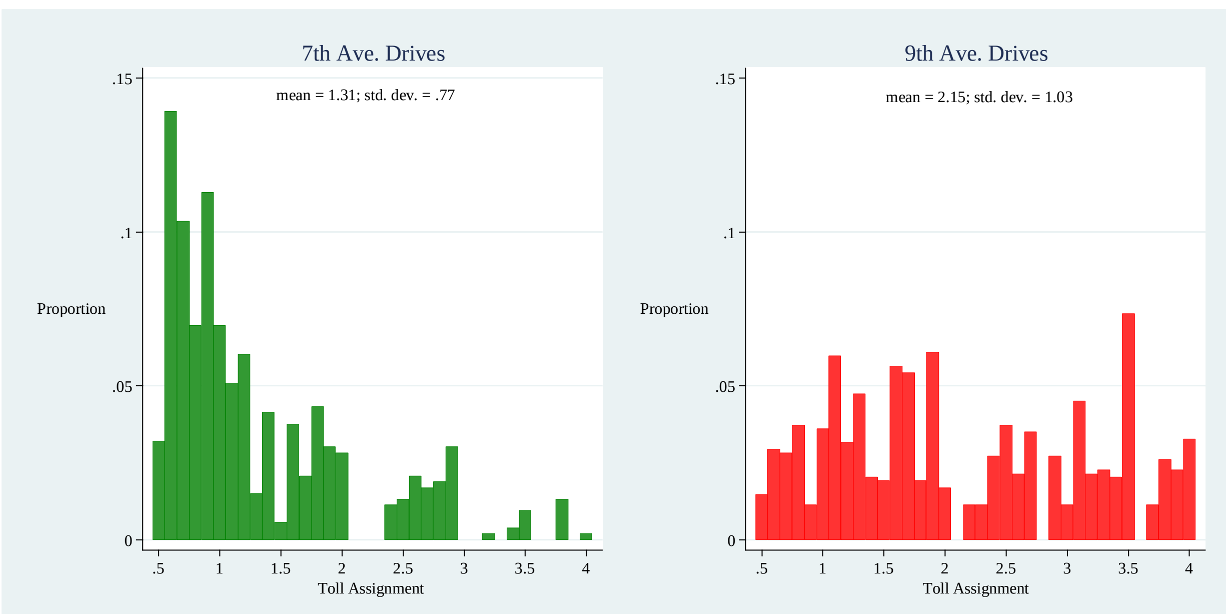


Figure A6: Distribution of Penalty by Route

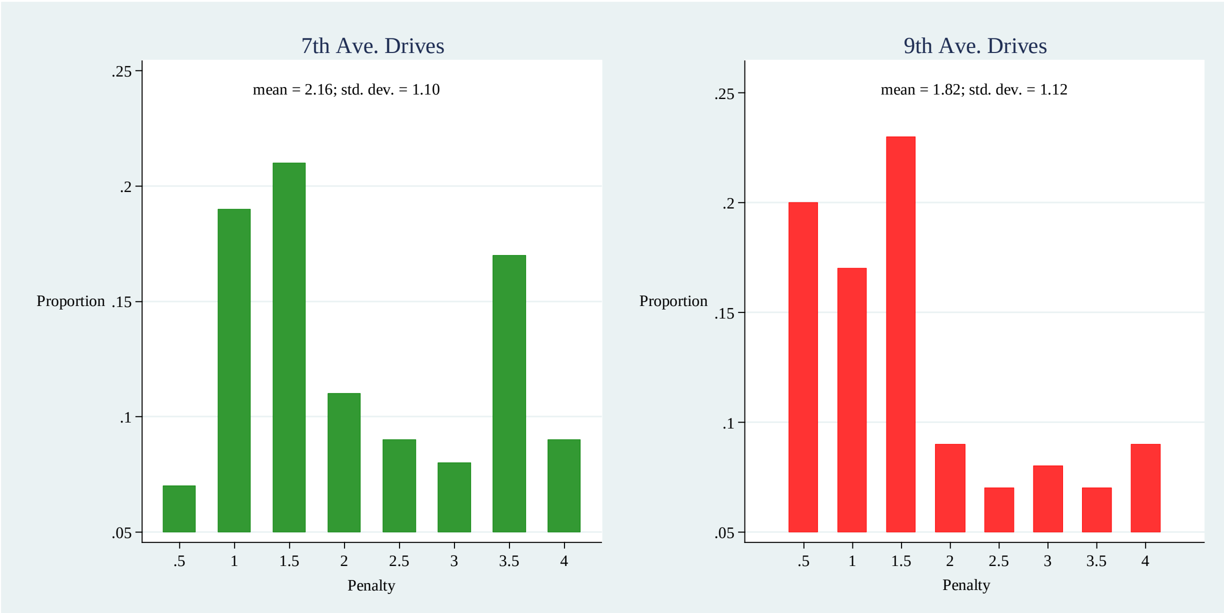


Figure A7: Number of Bus Cards Subjects See Before First Period

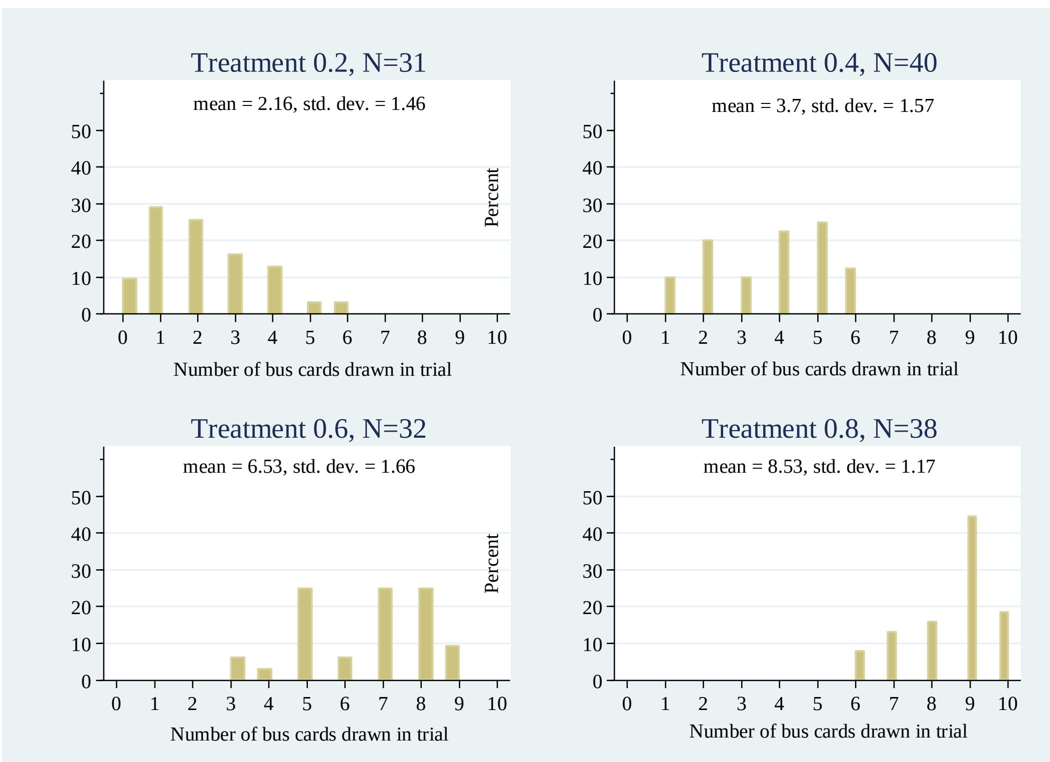


Figure A8: Proportion of Subjects who take 9th Avenue Across Periods

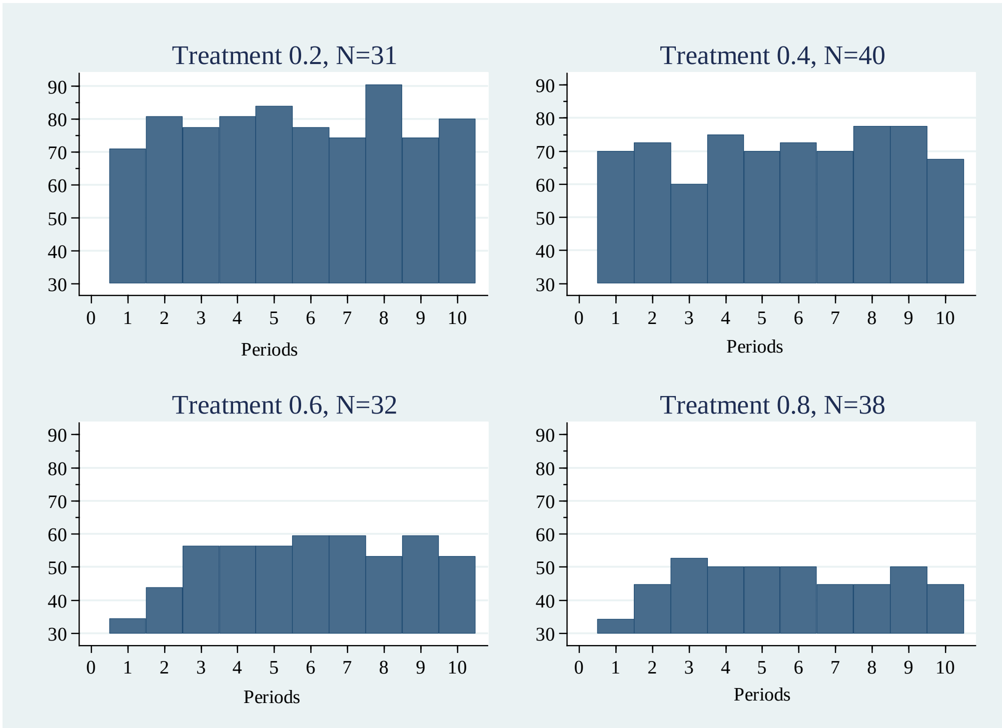
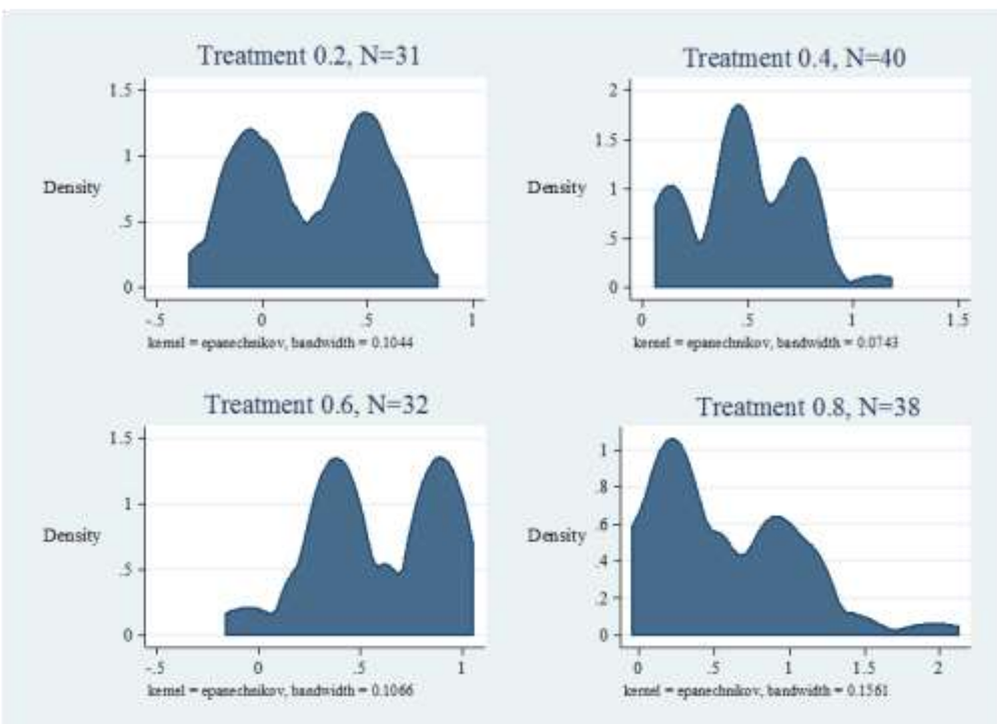


Figure A9: Estimated Distributions of Risk Aversion Coefficients for the EUT CRRA Models



Note: These are kernel densities representing the heterogeneity due to demographics.

CHAPTER 2

Estimating Subjective Beliefs in Naturalistic Tasks with Limited Information Under Variable Delay Penalty

2.1. Introduction

This essay examines drivers' subjective beliefs of congestion as a way of explaining their route choices. We examine subjective belief in a setting where the penalty for a late arrival is variable and is contingent on the *extent* of delay, such that a longer delay incurs additional penalty on the driver. This continuous penalty setting complements the discrete penalty setting that is examined in Chapter 1. This is consistent with route choice models that simply subtract the value of the travel time from the value of the trip (Small (1982); Jackson and Jucker (1982)).

The primary research question in this essay is: if the penalty for a late arrival is a varying amount, does belief formation differ compared to when the penalty is a fixed amount? Recall from Chapter 1, where the penalty is a fixed amount, that we observe belief adjustment only when the underlying congestion risk is low. This behavior is said to be expected under an endogenous information environment, as it is in the context of driving, where information about a route can only be obtained if one drives on that route. Thus, in a scenario where the underlying congestion risk is low and subjects start with a prior belief of low congestion, they are more likely to drive on the route and are able to obtain more information and result in more belief adjustment. Here in the continuous penalty setting, will we observe the same pattern of behavior?

There are reasons to believe that the *consequences* of delay (here referred to as *late penalties*) affect which route an individual may select. For example, an individual whose purpose of the trip is to attend a conference meeting faces a different delay consequence than another individual whose purpose of the trip is to catch a flight. For the first individual, the consequence of delay is missing part of the meeting, where the longer the delay the more information is missed; for the second individual, the consequence of delay is missing the flight and the loss of the entire value of the trip. The first scenario exemplifies a penalty that is continuous with longer delay incurring additional penalty, whereas the second scenario exemplifies a penalty that is discrete with a fixed amount. Even if no appointment is being missed, fully or partially, the fact that more of the individual's valuable time is wasted sitting in traffic reduces the utility of the trip.

To examine behavior in a setting where the late penalty is *continuous* calls for an experimental design that has variability in arrival times so that the extent of delay varies. In the experiment subjects are asked to make route choices using a driving simulator, and the amount of time it takes to complete the drive varies depending on route selection, the congestion scenario on the uncertain route, and how the subjects drive on the simulator. In this way, the arrival times along with late penalties are induced as continuous variables.

Commuters from the Atlanta and Orlando metropolitan areas are recruited to participate in this experiment. The field subjects are asked to make binary choices over two routes: one has an uncertain level of congestion risk, the other has no congestion risk. We elicit subjects' perceptions of the probability, *p*, that there is congestion on the uncertain route. This probability is known to the experimenter, but not to the subjects. Four levels of this probability are used: {0.2, 0.4, 0.6, 0.8}. One of the congestion probabilities is randomly assigned to each subject and stays constant throughout the session for that subject. We also elicit each subject's perceptions of the amount of time it takes to travel on the route that has an uncertain congestion risk when there is actually congestion *vs*. no congestion, and the amount of time it takes to travel on the route that has no congestion risk. The route choices are made using driving simulators, and subject's subjective probabilities of the uncertain risk of congestion, as well as their subjective probabilities of the travel time distributions, are inferred through the route choices they make. The latent subjective probabilities are estimated controlling for risk attitudes which are estimated from separate tasks with binary lottery choices.

In this continuous penalty setting, the results indicate that subjects are able to discern the difference between low-congestion and high-congestion risks, which is the same result as reported in a discrete penalty setting. In terms of learning (or belief adjustment), however, we draw different conclusions from those in Chapter 1. In the discrete penalty setting, we saw adjustments in beliefs over time in the lowest risk scenario. In this essay we compare the standard deviations of the estimated travel time belief distributions as an indication of whether more is learnt in the low risk scenario than the high risk scenario. We do not see a significant difference across treatments in these standard deviations.

2.2 Experimental Design

The field subjects are asked to make route choices over two routes: 9th Avenue which has an uncertain level of congestion risk, and 7th Avenue which has no congestion risk. Subjects are assigned: a wage that serves as a monetary endowment for each drive, a time threshold after which the variable penalty kicks in,[19] a monetary penalty per-second beyond the threshold that they arrive, a toll charge when taking the congestion-free route (7th Avenue), and an unknown probability of congestion on the risky road (9th Avenue). Table B1 shows the ranges of wage, toll, penalties and time thresholds. These assignments are constant across the drives for a given subject. On 9th Avenue congestion is induced using a school bus that makes frequent stops on the road, causing delay.

Subjects make route choices in a setting where the late penalty is continuous, and presented as a per-second amount. This is different from Chapter 1 where the late penalty is discrete, and presented as a fixed lump-sum. All other aspects of the experimental design are the same as for the discrete penalty case. The penalty is $0.03 per second for some subjects or $0.05 per second for other subjects if the wage is $2.50, and is $0.05 per second for some subjects or $0.10 per second for other subjects if the wage is $5.00, thus there are only three possible assignments. The time thresholds are lower than in Chapter 1 and range from 1 minute and 50 seconds to 2 minutes and 30 seconds in 5-second increments.[20] The purpose of assigning a time threshold for each trip is to induce a large value of time use at the margin so that any "extra time" that is above an assigned threshold incurs a larger marginal cost to the subject. This is indeed the case.

The rest of the design is a replica of the experiment described in Chapter 1. We employ the same joint-task design: the driving simulator task is used to elicit subjective belief, and the lottery choice task is used to elicit risk attitudes. The lottery choice task is exactly the same; see Table B2 for the range of prizes and probabilities that are used in the experiment. We also

[19] The time threshold is induced to make sure that the incentives at the margin (per second) are salient in relation to the overall payments for the drive.
[20] This is in contrast to Chapter 1 where the time thresholds range from 2 minutes and 10 seconds to 2 minutes and 45 seconds in 5-second increments.

employ the same payment protocol. The subjects are recruited in the same manner as the subjects described in Chapter 1.

2.3 Theory

This essay examines how field subjects perceive the risk of congestion in a setting where the penalty of delay is continuous. To test if the same behavioral pattern can be observed in both continuous and discrete penalty settings, here we ask a similar set of questions as in Chapter 1. Are the field subjects able to form estimates of the risk of congestion that vary with the underlying objective probability? Furthermore, under this endogenous information environment, do they adjust their beliefs in the direction of the underlying congestion probability? Does learning differ depending on the underlying congestion probability?

Since the penalty increases for each second that the subject is late relative to a time threshold, it is important to consider the beliefs over the distribution of travel times on each route as well as the belief of congestion on 9th Avenue. Thus, the subjects are assumed to hold a belief distribution of possible travel times in addition to the belief of congestion.

In this continuous penalty setting we test two hypotheses:

Hypothesis I – Subjects are able to form estimates of the risk of congestion, and the perceived risk of congestion ranks in the order of the underlying congestion probabilities.

Hypothesis II – Subjects who are in the low risk treatments are more likely to try out 9th Avenue than those who are in the high risk treatments. With more experience driving on 9th Avenue, these subjects should be able to learn about its congestion condition and to form more accurate estimates of the underlying objective probability. Hence, we should see a *difference in learning* when compared across the risk treatments, such that there should be evidence of more learning in the low risk treatment than in the high risk treatment.

Across the treatments the *difference in learning* is measured in two ways. First, it would be reflected by a more precise estimate of the subjective probabilities of *congestion*, thus we would expect a smaller standard error for the estimated subjective probability of congestion in the low risk treatments than in the high risk treatments.[21] Second, with more experience driving on 9th Avenue, these subjects should be able to form a more precise estimate of the time it takes to drive on each route. A more precise belief estimate over travel times would be reflected by a smaller standard deviation in the subjective probability distribution of travel times, as a sign of learning. Thus, we would expect the standard deviation of the estimated travel time distribution for 9th Avenue to be smaller in the low risk treatments than the high risk treatments. Thus, the *difference in learning* across the treatments are measured in terms of the estimated subjective probabilities of *congestion* and/or the estimated subjective probability distributions of *travel times*.

Lastly, we will compare the results obtained from the continuous penalty setting to the results from the discrete penalty setting in Chapter 1.

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[21] This implication, and the next implication, tacitly assumes an underlying Bayesian model of learning in which the individual starts off with a relatively diffuse prior. If the individual started off with a relatively precise prior that was biased, then this implication does not follow. We see no reason *a priori* to expect the prior belief to differ across discrete environment of Chapter 1 and the continuous environment examined here.

2.3.1 Simulator Route Choice Task

Recall that 7th Avenue is a risk-free route with no congestion and 9th Avenue is a risky route with an unknown probability of congestion. Subjects' route choices are modeled using Subjective Expected Utility (SEU) and a Constant Relative Risk Aversion (CRRA) utility function. We view each subject as making the decision to take 7th Avenue or 9th Avenue by evaluating the SEU of each route.[22]

For 7th Avenue the subject holds a subjective belief $p7_t$ about the time, t, that a trip will take. We assume that this distribution is normally distributed with mean μ_7 and standard deviation σ_7, and that it is a distribution over n discrete values of time t defined as seconds. For each time taken, t, we know the earnings that the subject would make, which we denote $m7_t$. Specifically,

$$m7_t = w - f \tag{1}$$

if the time t taken is *less* than the time threshold t^* allowed, where w is an exogenous wage and f is the toll fee on 7th Avenue, and

$$m7_t = w - f - l_t \times (t - t^*) \tag{2}$$

if the time taken t *exceeds* the time threshold t^*, where l_t is the penalty for late arrival associated with time t. If $m7_t$ is negative, it is set to zero.

We can then define the lottery entailed by taking 7th Avenue in terms of the combination of probability $p7_t$ and payoffs $m7_t$, using the CRRA utility function

[22] In principle this approach could be extended to consider alternatives to SEU.

$$u(m7_t) = \left(\frac{m7_t{}^{(1-r)}}{(1-r)}\right)$$

to evaluate the utility of payoffs. Assuming a lower and upper time to the distribution of travel times on 7th Avenue, the SEU for 7th Avenue is then

$$SEU7 = \sum_{\underline{t}=50}^{\overline{t}=300} p7_t \times u7_t \tag{3}$$

The lower and upper bounds of the distribution are selected based on the range of travel times we have observed in the experiment (shown in Figure B1). We choose increments between $\underline{t}$ and $\overline{t}$ to obtain good convergence properties; the later empirical analysis uses increments of 1 second. As the parameters μ_7 and σ_7 vary, the value of $p7_t$ changes for each t, and hence the SEU of 7th Avenue changes.

The evaluation for 9th Avenue follows essentially the same logic, apart from the fact that the distribution of travel times on 9th Avenue is conditional on whether or not there is a bus on that route. Hence we view the subject as having some probability, π, that there will be a bus (on 9th Avenue) and as having two conditional distributions of travel times for the trips on 9th Avenue. One conditional distribution assumes no bus, and is again assumed to be normally distributed with mean $\mu_9{}^{no\ bus}$ and standard deviation $\sigma_9{}^{no\ bus}$. Another conditional distribution assumes there is a bus, and is assumed to be normally distributed with mean $\mu_9{}^{bus}$ and standard deviation $\sigma_9{}^{bus}$. Given values of $\mu_9{}^{no\ bus}$ and $\sigma_9{}^{no\ bus}$ we can generate n discrete probabilities $p9^{no\ bus}{}_t$, and given values of $\mu_9{}^{bus}$ and $\sigma_9{}^{bus}$ we can generate n discrete probabilities $p9^{bus}{}_t$. We know that payoffs on 9th Avenue are given by

$$m9_t = w \tag{4}$$

if the time t taken is *less* than the time threshold t^* allowed, and

$$m9_t = w - l_t \times (t - t^*) \quad (5)$$

if the time taken t *exceeds* the time threshold t^*. Again, if $m9_t$ is negative, it is set to zero.

We can then define the SEU for 9th Avenue respecting the fact that there is a compound risk of there being a bus and a distribution of times conditional on the presence of the bus. Hence we have

$$SEU9 = \sum_{\underline{t}=50}^{\bar{t}=300} \{\pi\left(p9^{bus}{}_t \times u9^{bus}{}_t\right) + (1-\pi)\left(p9^{no\ bus}{}_t \times u9^{no\ bus}{}_t\right)\} \quad (6)$$

We use the same lower and upper bounds as in (3). Subjects are assumed to behave as if they compare the two SEUs and choose the one with the higher SEU.

2.3.2 Binary Lottery Task

Subjects are asked to complete four binary lottery tasks with known probabilities that allow us to elicit their risk attitudes. In each lottery task a decision is made between a relatively safe lottery and a relatively risky lottery. Risk attitudes are estimated assuming Expected Utility Theory (EUT) and a CRRA utility function. The expected utility of the safe option (EU_S) is:

$$EU_S = p \times \left(\frac{x_L{}^{(1-r)}}{(1-r)}\right) + (1-p) \times \left(\frac{x_H{}^{(1-r)}}{(1-r)}\right) \quad (7)$$

where p is the probability of a low prize, x_L, (1-p) is the probability of a higher prize, x_H, and r is the coefficient of relative risk aversion. Similarly, the expected utility of the risky option is:

$$EU_R = p \times \left(\frac{y_L{}^{(1-r)}}{(1-r)}\right) + (1-p) \times \left(\frac{y_H{}^{(1-r)}}{(1-r)}\right) \quad (8)$$

where *p* is the probability of a low prize, y_L, and (1-*p*) is the probability of a high prize, y_H.

2.4 Empirical Analysis

The non-linear structural estimation of the SEU model is performed using Maximum Likelihood techniques. Before estimating the SEU structural model, a Probit model is estimated as a way of describing the data.

2.4.1 Estimation Approach

Following the same estimation procedure as in Chapter 1 we jointly estimate risk attitudes and subjective beliefs. Instead of estimating the probability of delay, here we estimate the joint probability of congestion and travel time. We estimate subjective beliefs using the data from the driving task pooling across subjects, and we estimate risk attitudes using the data from the lottery tasks pooling across the same subjects. Subjective beliefs about travel times are estimated conditional on risk treatment dummies, and jointly with risk attitudes.[23]

In a setting where the late penalty is continuous and where the *extent* of delay matters, subjects hold a belief as to how long the trip will take, i.e., a belief of all the possible travel times. Subjects also hold a belief as to the probability of congestion (or a bus) on the risky route. Therefore, in the estimation we derive separate estimates for each belief.

[23] This joint estimation approach builds on previous work on structural estimation of risk attitudes by Andersen, Harrison, Lau and Rutström (2008) and Harrison and Rutström (2008b). A detailed description of the methodology can be found in Andersen, Fountain, Harrison and Rutström (2014).

Estimate Risk Attitudes from Lottery Tasks

First we describe the estimation of risk attitudes. The estimation of risk attitudes uses only the lottery data, and the equations shown here are the same as equations (5) – (7) in Chapter 1. For purpose of completeness we present a similar set of equations so that one can see how the econometrics follow from the theories stated in Section 3.

Following (7) – (8), the index

$$\nabla EU = EU_R - EU_S \tag{9}$$

is the difference in valuation between the risky lottery and the safe lottery.

The index in (9) is then linked to observed choices by using a "logit" likelihood function:

$$prob\ (choose\ risky\ option) = \Lambda(\nabla EU) \tag{10}$$

The risky option is assumed to be chosen when $\Lambda(\nabla EU) > ½$.

Thus the likelihood of the observed responses, conditional on the EUT and CRRA specifications being true, depends on the estimated *r* given the above specification and the observed choices, *c*. The log-likelihood is then

$$\ln L\ (r; c) = \Sigma_i \left[\ln\Lambda(\nabla EU)\ \times \mathbf{I}(c_i = 1) + \ln\left(1 - \Lambda(\nabla EU)\right) \times\ \mathbf{I}(c_i = 0)\right] \tag{11}$$

where $\mathbf{I}(\cdot)$ is the indicator function and c_i= 1 (0) denotes the choice of the lottery option *R* (*S*) in lottery task *i*.

We allow for behavioral errors in the core model, and the latent index (9) then becomes

$$\nabla EU = [(EU_R - EU_S)\ /\nu]\ /\mu_{lottery} \tag{9'}$$

where $\mu_{lottery} > 0$ is a structural Fechner "noise parameter" used to allow some error when evaluating the difference in EU between the two lotteries. The constant ν is a contextual normalizing term for each lottery pair *R* and *S*, and is defined as the difference between the maximum and the minimum utility in each lottery pair. This normalization is referred to as "contextual utility" and is due to Wilcox (2011).

One can extend the likelihood specification in (11) to include the noise parameter $\mu_{lottery}$ and maximize ln$L$ $(r, \mu_{lottery}\,; c)$ by estimating *r* and $\mu_{lottery}$, given observations on *c*.

Estimate Subjective Beliefs from Simulator Driving Tasks

Recall that for 7th Avenue, the subject holds a distribution of subjective beliefs $p7_t$ about the time t that a trip will take. We assume that this distribution is normally distributed with mean μ_7 and standard deviation σ_7, and that it is a distribution over time *t* defined as seconds. Given this distribution, we model the subject as evaluating a discrete lottery that evaluates this continuous distribution at τ equally-spaced intervals of t between some lower time $\underline{t}$ and upper time $\overline{t}$. We set $\underline{t}$ = 50 and $\overline{t}$ = 300 at τ = 250 equally-spaced intervals.[24] We select $\underline{t}$, $\overline{t}$ and τ solely for numerical purposes, and as we increase τ for a given $\underline{t}$ and $\overline{t}$ we can obtain a better discrete approximation of the underlying continuous distribution. Once these τ evaluations are taken, the τ densities are normalized to sum to 1. The approach for 9th Avenue follows the same logic for the distribution of travel time when there is a bus (with mean $\mu_9{}^{bus}$ and standard deviation $\sigma_9{}^{bus}$) and when there is no bus (with mean $\mu_9{}^{no\ bus}$ and standard deviation $\sigma_9{}^{no\ bus}$).

[24] Even though we evaluate these lotteries with τ equally-spaced intervals of time t, the density of each interval is generally a different value.

For numerical reasons we initially constrain the numerical values of the parameters of the travel time distributions (i.e., μ_7, σ_7, $\mu_9{}^{bus}$, $\sigma_9{}^{bus}$, $\mu_9{}^{no\ bus}$ and $\sigma_9{}^{no\ bus}$) to be equal to the distributions of travel times shown in Figure B1, and to estimate only the subjective probability of a bus, π. We do allow π to vary with the exogenous treatments in which there is a true probability of the bus of 20%, 40%, and 60%.[25] Normalizing on Treatment 0.2 (where the true probability of a bus is 20%), and defining binary variables $T40$ and $T60$ for those treatments, we estimate

$$\pi = \pi_{20} + \pi_{40} \times T40 + \pi_{60} \times T60 \tag{12}$$

and constrain π to lie in the unit interval. Once we obtain estimates of π_{20}, π_{40}, and π_{60}, we selectively relax the rational expectation constraints for the parameters of the travel time distributions (i.e., μ_7, σ_7, $\mu_9{}^{bus}$, $\sigma_9{}^{bus}$, $\mu_9{}^{no\ bus}$ and $\sigma_9{}^{no\ bus}$).

Together with the estimation of risk attitudes, the estimation of beliefs follows (3) and (6), and the latent index is

$$\nabla SEU = (SEU7 - SEU9) \tag{13}$$

where ∇SEU is the difference in valuation between 7th Avenue and 9th Avenue.

The estimation is performed using Maximum Likelihood. One parameter was estimated using the "profile likelihood" method,[26] due to local numerical flatness of the likelihood

[25] The likelihood function was flat with respect to the data from Treatment 0.8, so data from that sub-sample was dropped. The local flatness of the likelihood with respect to these data is not surprising, given the discussion from Chapter 1 about this treatment and the lack of behavioral variation it generated in comparison to the other treatments.

[26] The profile likelihood method assumes a grid of values for one of the model parameters, and solves for the conditional maximum likelihood, allowing all other parameters to vary as the constrained parameter is varied over that grid. The conditional ML estimates are then used as constrained estimates. The parameter evaluated in this manner is the mean of the distribution of subjective beliefs of travel times on the 9th Avenue with a bus, $\mu_9{}^{bus}$.

function. Conditional on the SEU and the CRRA specifications being true, the maximized log-likelihood becomes

$$\ln L\left(\pi_{20}, \pi_{40}, \pi_{60},\ \mu_9{}^{bus}, \sigma_9{}^{bus}, \mu_9{}^{no\ bus}, \sigma_9{}^{no\ bus}, \mu_7, \sigma_7, r, \mu_{route}, \mu_{lottery}\,; c\ \right) =$$

$$\Sigma_i\left[\ln \Lambda(\nabla SEU)\ \times \mathbf{I}(c_i = 0) + \ln\left(1 - \Lambda(\nabla SEU)\right) \times\ \mathbf{I}(c_i = 1)\right] \qquad (14)$$

where $\mathbf{I}(\cdot)$ is the indicator function, c_i = 0 (1) denotes that the subject choose 7th Avenue (9th Avenue) in drive task *i*, and separate noise parameters are estimated for the simulator driving task (μ_{route}) and the lottery task ($\mu_{lottery}$). A common noise parameter is assumed across risk treatments.[27]

Estimate Unconditional Travel Time Distribution for 9th Avenue

We also estimate an *unconditional* travel time distribution for 9th Avenue by taking the weighted average of the two conditional distributions, and using the estimated bus probability as the weight.

We allow the two parameters μ_9 and σ_9 to vary with the exogenous probability of a bus being 20%, 40%, and 60%. Normalizing on Treatment 0.2, and defining binary variables $T40$ and $T60$ for those treatments, we estimate

$$\mu_9 =\ \mu_{9\_T20} +\ \mu_{9\_T40}\ \times T40 +\ \mu_{9\_T60} \times\ T60 \text{ , and} \qquad (15)$$

$$\sigma_9 =\ \sigma_{9\_T20} +\ \sigma_{9\_T40}\ \times T40 +\ \sigma_{9\_T60} \times\ T60 \qquad (16)$$

[27] Note that (13) and (14) in this essay is the same as equations (8) and (9) in Chapter 1.

Following the same approach as in the estimation of (14), the maximized log-likelihood now becomes

$$\ln L\,(\mu_9, \sigma_9, \mu_7, \sigma_7, r, \mu_{route}, \mu_{lotter}; c\,) = \Sigma_i\big[\ln \Lambda(\nabla SEU)\ \times \mathbf{I}(c_i = 0) +$$

$$\ln\big(1 - \Lambda(\nabla SEU)\big) \times\ \mathbf{I}(c_i = 1)\big] \qquad (17)$$

Note that the *unconditional* travel time distribution for 9th Avenue does not distinguish between a bus or no bus, thus we do *not* estimate the subjective probability of a bus, i.e., π is not in the model.

2.4.2 Descriptive Statistics

The characteristics of the subject pool are described in Table B3.[28] The proportion of commuters from Atlanta and Orlando are about the same. Each gender is evenly represented in the overall sample. About 49% have household income of above $100,000. A significant majority hold a college education (82%). Within each risk treatment, the breakdown by demographics generally follows a similar trend as the overall sample distribution.

In addition to being randomly assigned to a congestion risk on 9th Avenue, subjects are randomly assigned to a toll, late penalty, and a wage. Since all the experimental parameters are randomly assigned to subjects and stay constant throughout the drives, the distribution of subjects who belong to each level of toll (or each level of late penalty, or wage) should be similar across the four congestion risk treatments. However, given the relatively small sample size in the experiment it is difficult to achieve perfect randomization that result in even representation of

[28] These are not the same subjects as those we observed in Chapter 1, although they are recruited from the same population. Here the sample size is 141 subjects.

all possible values. Because of this it is important to control for all of the parameters in the analysis. This is particularly the case for the toll assignment, where there are a total of 36 possible assignments. Comparing any two risk treatments, the distributions of tolls are significantly different from each other within a 10% significance level based on the Kolmogorov-Smirnov[29] test.[30] In contrast, for the penalty assignment, where there are only 3 possible values to be assigned, the distribution of subjects who are assigned to each level of penalty are not significantly different from each other in half of the pairwise treatment comparisons (within a 10% significance level). For the wage variable, where there are only 2 possible values to be assigned, the distribution of subjects who are assigned to each level of wage are not significantly different across the four risk treatments, with the exception of only two pairwise comparisons (i.e., when comparing Treatments 0.4 and 0.6 with p-value = 0.001, and when comparing Treatments 0.6 and 0.8 with p-value = 0.010).

Travel Times and Realized Penalties

The distribution of travel times pooling over all subjects and treatments is shown in Figure B1. On average 7th Avenue takes the shortest time (114 seconds), next is 9th Avenue *without* a bus (133 seconds), and 9th Avenue *with* a bus takes the longest (195 seconds). The standard deviations are 5.5, 10.1 and 17.4 seconds, respectively. The increase in standard deviation across the three scenarios is significant (p-value < 0.001).[31] This shows that the longer it takes to complete the drive, the higher is the variance of the distribution of travel times.

[29] The Kolmogorov-Smirnov test whether two distributions are equal.
[30] On average, Treatments 0.4 and 0.6 have the larger tolls ($1.84 and $1.80, respectively). However, the averages in these two treatments are not much different than the averages in Treatments 0.2 and 0.8 ($1.73 and $1.68, respectively). Other features of the distributions differ across these treatments.
[31] This test is performed using Levene's robust test for the equality of variances between the groups.

Overall, the descriptive data in Figure B1 shows that subjects perceive a travel time difference across routes and congestion scenarios.

On average as travel time increases, we should expect late penalties to increase. The distribution of the resulting penalties (or realized penalties) pooling over all subjects and treatments is shown in Table B4. On average 7th Avenue incurs the lowest penalty ($0.05), next is 9th Avenue *without* a bus ($0.36), and 9th Avenue *with* a bus incurs the highest penalty ($3.06). The standard deviations are $0.15, $0.62 and $1.45, respectively. The increase in standard deviation across the three scenarios is significant (p-value < 0.001). Thus, the distributions of realized penalties follow the ranking of the distributions of travel times: the longer it takes to complete the trip, the higher is the realized penalty.

Proportions of Route Choice

Given that subjects are randomly assigned to a toll, a late penalty and a wage, route choice is predicted to depend on many of these parameters, in addition to the main congestion treatments. We expect that subjects who are assigned a higher toll are more likely to drive on 9th Avenue than subjects who are assigned a lower toll. Figure B2 shows the distribution of toll across drivers who chose to drive on 7th and 9th Avenue, respectively. The two distributions are significantly different at the 1% level. The drivers who chose to drive on 9th Avenue have on average a toll assignment of $2.10, whereas those who chose to drive on 7th Avenue have on average a toll assignment of $1.50. This provides preliminary evidence that subjects with a high toll are more likely to drive on 9th Avenue.

We expect that subjects who are assigned a higher late penalty are more likely to select the faster route. There are only three assignments for penalties: $0.03, $0.05 and $0.10 per second. The distributions of late penalties do not differ significantly across subjects who took 7th and 9th Avenue (p-value = 0.109). However, this test is unconditional, not controlling for variations in any of the other variables. We will look at a conditional test below in a Probit model.

Proportion of Route Choice by Risk Treatment

Prior to starting the driving task, the only information that subjects have for forming a prior belief about congestion are the drawings of the ten bus cards before starting the drives. Figure B3 shows that the number of bus cards drawn increases with the objective risk. As the number of bus cards drawn increases with objective risk, in the first period the proportion of subjects who chose the risky route decreases. This is shown in Figure B4 (in the first period). It appears that subjects hold initial beliefs of congestion that are consistent with their prior bus card information.

Comparing the proportion of route choice across risk treatments, we see that a larger proportion of subjects taking the risky route in the two treatments with objective congestion probabilities below 0.5 than in the two treatments with objective congestion probabilities above 0.5. Pooling across periods, the proportion of route choice rank in the order of objective risks: Treatment 0.2 has the highest proportion of subjects taking the risky route (62%), follow by Treatment 0.4 (52%), 0.6 (38%), and 0.8 (33%). Most of the pairwise comparisons between treatments are significantly different (at the 1% level), except between Treatments 0.6 and 0.8 (p-value = 0.18). Thus, there is some preliminary evidence that subjects on average can perceive differences between high and low probabilities.

Across the periods, there is no significant change in the proportion of route choice following the first period. This is true for all risk treatments. Even though Figure B4 shows some differences in the proportion of route choice across the periods, none of the between-period pairwise comparisons are significant.

The number and proportions of subjects who switched routes between periods are shown in Table B5. Recall that 9th Avenue is the risky route with an uncertain risk of congestion and 7th Avenue is the safe route with no risk of congestion. In Treatment 0.2, subjects who take 9th Avenue are less likely to experience congestion than subjects in the higher risk treatments, so one would expect them to be less likely to switch away from using 9th Avenue. In fact, of the 25 subjects who initially selected 9th Avenue only 7 (OR 28%) switched to 7th Avenue in period 2. On the other hand, of those who took 7th Avenue, 47% switched to 9th in period 2. Pooling across periods, in Treatment 0.2 the proportion who switched from 9th Avenue to 7th Avenue (16%) is smaller than the proportion who switched from 7th Avenue to 9th Avenue (24%). This is consistent with frequent experiences of no congestion on 9th Avenue in Treatment 0.2. In Treatment 0.4, there is a similar but weaker pattern: 19% switched from 9th Avenue to 7th Avenue and 21% switched from 7th Avenue to 9th Avenue.

In the two high risk treatments, the pattern is reversed: there is a higher proportion of subjects that switched from 9th Avenue to 7th Avenue than from 7th Avenue to 9th Avenue. In Treatment 0.6, 39% switched from 9th Avenue to 7th Avenue and 22% switched from 7th Avenue to 9th Avenue; in Treatment 0.8, 29% switched from 9th Avenue to 7th Avenue and 14% switched from 7th Avenue to 9th Avenue. This is a sign that in the high risk treatments, once subjects selected 9th Avenue, they experienced on average more congestion than their counterparts did in the lower risk treatments.

Based on the route switching behavior, Table B6 displays the conditions under which subjects switched out of 9th Avenue. It shows the proportion of subjects who switched from 9th Avenue to 7th Avenue conditional on encountering congestion or not. One would expect the proportion of subjects who switched in the former case to be at least as high as in the latter case. This is indeed the pattern observed in the three treatments with the highest congestion risk: Treatments 0.4, 0.6 and 0.8.

In summary, behavior appear consistent with subjects forming subjective beliefs of congestion that reflect its underlying objective probability. Analysis of the raw data provides preliminary evidence that subjects can perceive the difference across high-probabilities and low-probabilities and that subjective beliefs of congestion rank in the order of the objective risk of congestion. This lends partial support to Hypothesis I and confirms the results obtained in Chapter 1.

Given that the proportion of risky choices is higher in the low risk treatments than in the high risk treatments, subjects in the low risk treatments would obtain more information about the risky route and display more adjustments in route choices compared to their counterparts in the high risk treatments. However, the route proportions data revealed that across the periods the proportion of risky choices stay virtually the same with no significant adjustments. This is true for all risk treatments.

2.4.3 Propensity of Route Choice

Before discussing the results of the full structural model estimation, in this section route choice is estimated controlling for variations in experimental parameters such as tolls and late penalties, so to directly investigate whether, under this endogenous information environment, the changes in the tolls are less effective for subjects in the high risk treatments than in the low risk treatments, as suggested by the second hypothesis. Table B7 shows the result of a Probit model controlling for variations in payoff incentives and period fixed effects. The endogenous variable is the propensity to take the risky route and the independent variables are *Toll*, *Late Penalty Per Second*, *Wage*, the prior number of bus cards (*Prior*), and period fixed effects (i.e., *Period 2*, …, *Period 10*)..

$$\begin{aligned} Propensity_{risky} &= \beta_0 + \beta_1 \times Toll + \beta_2 \times Late\ Penalty\ Per\ Second\ + \beta_3 \times Wage + \beta_4 \\ &\times Prior + \beta_5 \times Period\ 2 + \beta_6 \times Period\ 3\ + \beta_7 \times Period\ 4 + \beta_8 \\ &\times Period\ 5 + \beta_9 \times Period\ 6 + \beta_{10} \times Period\ 7\ + \beta_{11} \times Period\ 8 + \beta_{12} \\ &\times Period\ 9\ + \beta_{13} \times Period\ 10 \end{aligned}$$

As in Chapter 1, all coefficients are transformed to marginal probability effects computed using the delta method.

Effects of Payoff Incentives

Toll has the expected positive effect on the propensity to take the risky route. The coefficients are significantly different from zero in all risk treatments. Comparing any two treatments the coefficients are significantly different from each other except between Treatments 0.2 and 0.6 (p-value = 0.145). The influence of *Toll* increases across Treatments 0.4, 0.6 and 0.8.

Late Penalty Per Second has the theoretically expected negative effect on the propensity to take the risky route in the last three treatments, but only in Treatments 0.4 and 0.8 where the coefficients are significantly different from zero. *Wage* is predicted to have no effect on the propensity of route choice. This is true for all risk treatments except the lowest risk treatment, and even there the coefficient is very small, at -0.080 (p-value = 0.026).

Within each treatment (except in Treatment 0.8) the number of prior bus cards subjects draw does not have a significant effect on the propensity of route choice. Since the number of bus cards subjects drew does not vary much *within* each treatment, it is not surprising that the variable *Prior* is not significant *within* treatment. However, when pooling the data across all treatments, there is a significant decrease in the propensity to take the risky route as the number of prior bus cards increases (p-value < 0.001), which is expected and shown in the final column of Table B7.

Period Effects

In period 1 (captured by the coefficient on *Constant*) the propensity to take the risky route is not rank-ordered across the four treatments. The pairwise comparisons across treatments do not show significant differences except between Treatments 0.2 and 0.4 and between Treatments 0.4 and 0.6, and the latter is in the opposite direction from what we expect. In subsequent periods, there is no significant change in the propensity to take the risky route, and this is true for all four treatments.

In summary, the conditional analysis of route choice using a Probit model tells a slightly different story compared to the unconditional descriptive analysis: the low risk treatments generally do not show a significantly higher propensity to choose the risky route than the high risk treatments. Across the periods the proportion of risky choices (or the propensity for the risky route) stay virtually the same with no significant adjustments, and this is true for all risk treatments. The next section estimates the subjective beliefs that are implied by this behavior, assuming SEU with a CRRA utility function.

2.4.4 Subjective Expected Utility

We estimate the conditional travel time model in (14) using maximum likelihood. The estimation results of the SEU model are shown in Table B8. The subjective belief distributions over *travel times* are estimated for 7th Avenue, 9th Avenue without a bus, and 9th Avenue with a bus. The subjective beliefs of *congestion* are estimated conditional on treatment dummies for the objective risk of congestion. The likelihood function was flat with respect to the data from Treatment 0.8, so data from that sub-sample was dropped.

Figure B5 displays the estimated distribution of actual travel times, assuming that the actual data are normally distributed. This is similar to Figure B1, but imposes the parametric assumption that each distribution is Gaussian. This is the assumption under which the model is estimated, so Figure B5 is easier to compare to the estimated subjective belief distributions.

Figure B6 displays the estimated subjective belief distributions for the travel times on 7th Avenue, with the actual distribution (from Figure B5) shown in a dashed line. We see that the estimated subjective belief distribution for 7th Avenue has virtually no dispersion compared to the actual distribution. Although less extreme, we will see this pattern with respect to beliefs about travel times on 9th Avenue as well.

Figure B7 displays the conditional distributions of subjective beliefs about travel times on 9th Avenue, depending on whether there is a bus or not. As expected *a priori*, but without any constraints on the estimates, we see that the bus does lead to subjective beliefs of time delay.

Moreover, the dispersion of the subjective distribution with a bus is significantly larger than the dispersion of the subjective distribution with no bus. Figure B8 compares these estimated subjective belief distributions with the actual distributions, and again we see a tendency for the beliefs to be less dispersed than the actual.

In general we see that the averages of the subjective beliefs about travel times are close to the true average. But we see that these subjective beliefs are much more precise in terms of their standard deviation, with the subjects behaving as if they had more confidence in these estimated average travel times than the data would justify. Of course, the subjects are actively learning about these distributions in real-time, so these implied subjective beliefs may reflect that partial adaptation to the data they are seeing.

By sharp contrast, the estimated subjective probabilities of a bus in Table B8 are apparently very imprecise. The estimated probability of the bus in Treatment 0.2 is 13.3% but it is not significantly different from zero given its large standard error (p-value = 0.627). In terms of the 95% confidence interval, Treatment 0.2 has a wide 95% confidence interval between -40% and 67%. The estimated probability of the bus in Treatment 0.4 is 51.9%, again with a wide 95% confidence interval between 16% and 87%. The estimated probability of the bus in Treatment 0.6 is 58.0%, also with a wide 95% confidence interval between 24% and 92%.

One implication of these results is to suggest, for future research, an identifying restriction in which the probability of a bus is set to 0 for Treatment 0.2 and set to 1 for Treatment 0.8. This does not reduce the agent's decision problem here to one of certainty, because there are still conditional distributions of travel times for 9th Avenue whether or not a bus is assumed to be present or not. In other words, if the subjective probability of a bus is 0, then the agent still has a risky choice between 7th Avenue (which is never affected by the bus) and 9th Avenue with no bus. The latter route still has a subjective distribution of travel times. Similarly, if the subjective probability of a bus is 1, then the agent still has a risky choice between 7th Avenue and 9th Avenue with a bus; again, the latter route still has a subjective distribution of travel times. In this compound risk setting, it could be that the imprecision in the estimated first-stage risks (the risk of a bus) in Treatments 0.4 and 0.6 lead to an "overcompensation" by reducing the uncertainty in the estimated second-stage risks (the distribution of travel times, conditional on a route and/or bus). On the other hand, given the uncertainty over the probabilities of a bus in Treatments 0.4 and 0.6, it may be cognitively easier for the subjects to behave as if the probabilities are at their extremes of 0 and 1 in Treatments 0.2 and 0.8, respectively.

From these estimates we can infer some estimates for 9th Avenue that might be more natural to interpret in terms of behavior and the hypotheses of interest. Consider the *unconditional* travel time distribution for 9th Avenue. In this case we need to take a weighted average of the distribution conditional on there being a bus and the distribution conditional on there being no bus, where the weights are the (estimated) probability of a bus and the (estimated) probability of no bus. The weighted average of the mean of the two conditional distributions is just the weighted average of the means. But the weighted average of the variance of the two conditional distributions is not just the weighted average of the variances unless the covariance is zero. In fact, there is a non-zero covariance, since these reflect estimated subjective belief distributions. A subject who holds beliefs that travel times without a bus are high compared to the beliefs of other subjects is also likely to hold beliefs that travel times with a bus are high compared to other subjects. Hence we expect there to be a positive correlation (and hence covariance) between the two estimated distributions, and the parameters characterizing them. In turn, this correlation might be naturally generated by subjects that have different efficiencies of driving, or have different degrees of optimism or pessimism with respect to travel time. When we allow for this non-zero covariance, the weighted variance is the weighted average of the two variances plus a term that is 2 times the two weights times the correlation.[32] Once we know the weighted variance we immediately have the weighted standard deviation as the square root of the weighted variance. Since we have estimated the model using Full Information Maximum Likelihood we can recover all of these terms correctly accounting for the non-linearity between them and inferring their correct standard errors. Table B9 shows the estimates that result for 9th Avenue.

We see that the unconditional travel time distributions for 9th Avenue have means (151, 170 and 173) that are higher than for 7th Avenue (116). We also see that the ranking of the mean travel times matches the ranking of the objective bus probability across treatments, although the differences are small apart from Treatment 0.2. It is easy to check with a *t*-test that Treatment 0.2's distribution is significantly different from either of the other two, at a 1% level. We do not see a lower standard deviation for the lower risk treatments than for the higher risk treatments.

Hypotheses

What do these estimates of the structural model tell us about the hypotheses?

Hypothesis I states that subjects are able to discern the differences in congestion risk across treatments. The evidence from estimates of the subjective probabilities of a bus does not provide support for this hypothesis, even though the point estimates are in the predicted direction. The reason is that they are just so imprecisely estimated. However, when we combine

[32] This is a basic property of statistics when evaluating the weighted sum of variances.

these estimates with the conditional distributions of travel time, we can discern a clear difference between Treatment 0.2 and Treatments 0.4 and 0.6, even though Treatments 0.4 and 0.6 are not distinguishable (Figure B9). We are able to make such a claim because the subjective beliefs about the conditional travel time distributions are estimated so precisely (Figure B8), offsetting the imprecision of the subjective probabilities that are used to condition those distributions, to arrive at the unconditional distributions for each treatment.

Hypothesis II states that under an endogenous information environment, we should see a *difference in learning* comparing across the risk treatments, such that there should be evidence of more learning in the low congestion risk treatments than in the high congestion risk treatments . Across the treatments, the *difference in learning* is measured by comparing the estimated subjective probabilities of *congestion* or the subjective probability distributions of *travel times*. The estimates of the subjective probability of congestion do not provide any support for the hypothesis, since the standard error is larger, not smaller, in Treatment 0.2 compared to the higher risk treatments. The estimates of the subjective probability distribution of travel times also do not provide support for the hypothesis, since the standard deviations for the three distributions are *not* significantly different from each other: they are 10.992, 10.298, and 10.524, respectively (Figure B9). Thus, neither the subjective probability of *congestion* nor the subjective probability distribution of *travel times* provide support for Hypothesis II. The results of the structural estimation are consistent with the results of the descriptive data and Probit model in showing a lack of support for Hypothesis II: in the descriptive data, there is no significant changes in the proportion of risky choices in any of the treatments; in the Probit model, there is no significant changes in the propensity to take the risky route in any of the treatments.

Subjects have an expectation as to how long it will take to complete the drive on 9th Avenue, but how accurate is their expectation? In other words, does their expectation match the actual travel time that it took them to complete the drive? Here, another way to examine learning is perhaps by comparing the *expected* outcome to the *realized* outcome. If subjects hold a fairly accurate expectation of the travel times on 9th Avenue, then the estimated belief of travel time distribution should be close to the actual realized travel time distribution. Hence we compare the mean and standard deviation of the estimated subjective distribution to the mean and standard deviation of the actual distribution. In that case there is some slight evidence that the estimated distribution in Treatment 0.2 is closer to its corresponding actual distribution than the other treatments. However, this "evidence" is not statistically significant. Overall, we do not have significant results to support Hypothesis II regardless of what we describe as learning.[33]

Overall, results of the structural model are consistent with results of the Probit model, showing support for Hypothesis I but not Hypothesis II. Both models are consistent in showing that subjects are able to discern the difference between low congestion risk and high congestion risk. In terms of learning, however, we do not observe a difference in learning across the treatments from either model. In the Probit model, there is no significant changes in the propensity of route choice in any of the treatments, thus we do not observe a *difference in learning* across the treatments. In the structural model, there is also no evidence of a *difference in learning* across the risk treatments, as it is measured by the estimated probabilities of

[33] The usual Bayesian approach might expect that subjects start with a diffuse prior, and refine that as sample data are observed. This would suggest that the posterior distribution (i.e., the estimated travel time distribution) would initially be more diffuse than the sample (i.e., the actual distribution), and converge towards the sample standard deviation over time, which would seem to be inconsistent with what we infer (Figure B8). However, the Bayesian logic does not require that one start with a diffuse prior, and the evidence that these subjects do not have a subjective belief distribution that is more diffuse than the data does not violate Bayesian logic. To test the Bayesian version of Hypothesis II we would need to have independent estimates of the prior distributions that subjects had at the beginning of the experiment with respect to the travel times on 9th Avenue conditional on a bus or no bus.

congestion and the estimated travel time distributions. The standard error of the estimated probability of congestion is not smaller (in fact, it is larger) in the lowest risk treatment than the high risk treatments, and the standard deviation of the estimated travel time distribution is also not smaller (or significantly different) in the lowest risk treatment than the high risk treatments.

2.5 Conclusion

The goal of this essay is to examine belief formation and learning under a continuous penalty setting, so that we can compare it to results from Chapter 1 that examines behavior under a discrete penalty setting. Under an endogenous information environment, we examine if learning differs depending on the underlying congestion probability under different penalty settings. Recall in the discrete penalty setting, the results show there is more learning in the low risk treatment than in the higher risk treatments. The difference in learning across the risk treatments is expected under an endogenous information environment, since in a low risk treatment subjects are more likely to start with a prior belief of low congestion, and are therefore more likely to drive on the risky route, and subsequently obtain more information to revise their prior belief compared to their counterparts who starts with a prior belief of high congestion. This same pattern of behavior would still be expected to be taking place when the penalty is induced in a continuous rather than discrete manner; however, here we do not report evidence that learning differs across the risk treatments. What could explain this difference?

Recall that in the Probit model, in the continuous penalty setting the marginal effect of Toll is significant in the propensity to choose the risky route and this is true in all risk treatments. In contrast, in the discrete penalty setting the marginal effect of Toll is only significant in the low risk treatments but not in the high risk treatments. Thus, depending on the penalty setting the marginal effect of toll may or may not be significant on the propensity to choose the risky route. Further, given that information about the risky route can only be obtained if one chooses to drive on it, the difference in the marginal effect of Toll on route propensities may contribute to the difference in learning across the penalty settings.

Tables and Figures for Chapter 2

Table B1: Tolls and Wages in the Simulator Task

	Toll Range	**Late Penalty**	**Time Threshold**
Wage=$2.50	$0.50-$2.00	$0.03 or $0.05 per sec	1min 50 secs to 2m 30 secs
Wage=$5.00	$0.50-$4.00	$0.05 or $0.10 per sec	

The range of toll cards was in 10-cent increment.
The range of time thresholds was in 5-second increment.

Table B2: Prizes and Probabilities in Lottery Task

Probability range	**Safe Lottery Low Prize**	**Safe Lottery High Prize**	**Risky Lottery Low Prize**	**Risky Lottery High Prize**
0.1 – 0.9	$2	$3	$0.25	$4
0.1 – 0.9	$2	$3	$0.25	$5
0.1 – 0.9	$2	$3	$0.25	$6
0.1 – 0.9	$4	$6	$0.50	$10

Table B3: Demographic Sub-groups in Each Treatment

	Treatment 0.2	**Treatment 0.4**	**Treatment 0.6**	**Treatment 0.8**	**All**
Number of subjects	40	33	30	36	139
Location					
Orlando	30%	39%	50%	56%	56%
Atlanta	70%	61%	50%	44%	44%
Gender					
Male	55%	52%	54%	54%	53%
Female	45%	48%	46%	46%	47%
Education					
College	85%	82%	81%	78%	82%
Non-college	15%	18%	19%	22%	18%
Income					
High: above $100K	42%	57%	47%	50%	49%
Low: $100K or below	58%	43%	53%	50%	51%

Table B4: Distribution of Realized Penalties

Realized Penalties	**7th Avenue**	**9th Avenue without bus**	**9th Avenue with bus**
0	90.46%	61.52%	1.57%
0.5	4.90%	20.76%	0.39%
1	0.68%	6.33%	1.18%
1.5	0%	4.81%	11.37%
2	0.14%	2.78%	16.08%
2.5	1.50%	3.29%	30.98%
3	0%	0%	3.53%
3.5	0%	0%	7.06%
4	0%	0.25%	6.67%
4.5	0%	0.00%	1.96%
5	2.32%	0.25%	19.22%
Total	100%	100%	100%

Pooling over all subjects and treatments. Number of observations is 1,347.
Realized penalty on 7th Avenue has a mean of $0.05 and standard deviation 0.15.
Realized penalty on 9th Avenue *without* a bus has a mean of $0.36 and standard deviation 0.62.
Realized penalty on 9th Avenue *with* a bus has a mean of $3.06 and standard deviation 1.45.

Table B5: Route Switches

From one period to the next	**# of subjects who took 7th Ave. in the first of the two periods**	**# of subjects who took 9th Ave. in the first of the two periods**	**Proportion taking 9th Avenue**	**# of subjects who switch from 7th to 9th**	**# of subjects who switch from 9th to 7th**	**For those who take 7th Ave., proportion that switch to 9th**	**For those who take 9th Ave., proportion that switch to 7th**	**Difference**
Treatment 0.2								
1 to 2	15	25	62.5%	7	7	47%	28%	19%
2 to 3	15	25	62.5%	4	3	27%	12%	15%
3 to 4	14	26	65%	3	4	21%	15%	6%
4 to 5	15	25	62.5%	4	3	27%	12%	15%
5 to 6	14	26	65%	2	4	14%	15%	-1%
6 to 7	16	24	60%	4	2	25%	8%	17%
7 to 8	14	26	65%	4	5	29%	19%	9%
8 to 9	15	25	62.5%	0	3	0%	12%	-12%
9 to 10	19	21	52.5%	5	3	26%	14%	12%
Average	-	-	61.94%	-	-	24%	16%	8%
Treatment 0.4								
1 to 2	16	17	51.52%	5	4	31%	24%	8%
2 to 3	15	18	54.55%	2	1	13%	6%	8%
3 to 4	14	19	57.58%	3	5	21%	26%	-5%
4 to 5	16	17	51.52%	4	4	25%	24%	1%
5 to 6	16	17	51.52%	3	3	19%	18%	1%
6 to 7	16	17	51.52%	7	3	44%	18%	26%
7 to 8	12	21	63.64%	1	6	8%	29%	-20%
8 to 9	17	16	48.48%	4	2	24%	13%	11%
9 to 10	15	18	54.55%	1	3	7%	17%	-10%
Average	-	-	53.88%	-		21%	19%	2%
Treatment 0.6								
1 to 2	16	14	46.67%	4	3	25%	21%	4%
2 to 3	15	15	50%	4	8	27%	53%	-27%
3 to 4	19	11	36.67%	4	5	21%	45%	-24%
4 to 5	20	10	33.33%	7	4	35%	40%	-5%
5 to 6	17	13	43.33%	4	3	24%	23%	0%
6 to 7	16	14	46.67%	5	7	31%	50%	-19%
7 to 8	18	12	40%	1	4	6%	33%	-28%
8 to 9	21	9	30%	4	4	19%	44%	-25%
9 to 10	21	8	27.59%	3	3	14%	38%	-23%
Average	-	-	39.36%	-		22%	39%	-16%
Treatment 0.8								
1 to 2	24	12	33.33%	4	1	17%	8%	8%
2 to 3	21	15	41.67%	1	5	5%	33%	-29%
3 to 4	25	11	30.56%	7	4	28%	36%	-8%
4 to 5	22	14	38.89%	3	4	14%	29%	-15%
5 to 6	23	13	36.11%	3	5	13%	38%	-25%
6 to 7	26	10	27.78%	5	5	19%	50%	-31%
7 to 8	26	10	27.78%	3	3	12%	30%	-18%
8 to 9	26	10	27.78%	3	2	12%	20%	-8%
9 to 10	25	11	30.56%	3	1	12%	9%	3%
Average	-	-	32.72%	-		14%	29%	-15%

Table B6: Congestion Experiences

From one period to the next	# of subjects who switch from 9th to 7th	Conditional on a bus card, # of subjects who switch from 9th to 7th	Conditional on *no* bus card, # of subjects who switch from 9th to 7th	Conditional on a bus card, % of subjects who switch from 9th to 7th	Conditional on no bus card, % of subjects who switch from 9th to 7th
Treatment 0.2					
1 to 2	7	2	5	29%	71%
2 to 3	3	0	3	0%	100%
3 to 4	4	0	4	0%	100%
4 to 5	3	1	2	33%	67%
5 to 6	4	0	4	0%	100%
6 to 7	2	2	0	100%	0%
7 to 8	5	3	2	60%	40%
8 to 9	3	1	2	33%	67%
9 to 10	3	1	2	33%	67%
Average	--	--	--	32%	68%
Treatment 0.4					
1 to 2	4	4	0	100%	0%
2 to 3	1	0	1	0%	100%
3 to 4	5	2	3	40%	60%
4 to 5	4	2	2	50%	50%
5 to 6	3	3	0	100%	0%
6 to 7	3	2	1	67%	33%
7 to 8	6	4	2	67%	33%
8 to 9	2	1	1	50%	50%
9 to 10	3	3	0	100%	0%
Average	--	--	--	64%	36%
Treatment 0.6					
1 to 2	3	3	0	100%	0%
2 to 3	8	8	0	100%	0%
3 to 4	5	1	4	20%	80%
4 to 5	4	3	1	75%	25%
5 to 6	3	1	2	33%	67%
6 to 7	7	5	2	71%	29%
7 to 8	4	3	1	75%	25%
8 to 9	4	3	1	75%	25%
9 to 10	3	2	1	67%	33%
Average	--	--	--	68%	32%
Treatment 0.8					
1 to 2	1	1	0	100%	0%
2 to 3	5	3	2	60%	40%
3 to 4	4	4	0	100%	0%
4 to 5	4	4	0	100%	0%
5 to 6	5	3	2	60%	40%
6 to 7	5	5	0	100%	0%
7 to 8	3	3	0	100%	0%
8 to 9	2	1	1	50%	50%
9 to 10	1	1	0	100%	0%
Average	--	--	--	86%	14%

Table B7: Propensity of Route Choice Estimated with a Probit Model

	Treatment 0.2 N=40	**Treatment 0.4 N=33**	**Treatment 0.6 N=30**	**Treatment 0.8 N=36**	**Pooled Across Treatments N=139**
Constant	.397*** (0.001)	.791*** (<0.001)	.232 (0.159)	.596** (0.037)	.619*** (<0.001)
Toll	.301*** (<0.001)	.069** (0.040)	.200*** (0.002)	.351* (0.087)	.194*** (<0.001)
Late Penalty Per Second	.453 (0.599)	-.791*** (<0.001)	-.232 (0.159)	-.596** (0.037)	-.619*** (<0.001)
Wage	-.080** (0.026)	.027 (0.312)	.028 (0.305)	.057 (0.190)	-.013 (0.407)
Period 2	-.001 (0.997)	.030 (0.741)	.025 (0.819)	.114 (0.385)	.030 (0.622)
Period 3	.031 (0.795)	.052 (0.561)	-.080 (0.424)	-.047 (0.741)	-.009 (0.882)
Period 4	.002 (0.984)	-.001 (0.991)	-.103 (0.308)	.088 (0.505)	-.015 (0.803)
Period 5	.029 (0.809)	.005 (0.958)	-.029 (0.776)	.0382 (0.778)	.009 (0.883)
Period 6	-.029 (0.800)	-.003 (0.978)	-.005 (0.963)	-.104 (0.471)	-.025 (0.682)
Period 7	.028 (0.810)	.094 (0.282)	-.057 (0.567)	-.140 (0.331)	.006 (0.920)
Period 8	.0002 (0.999)	-.028 (0.777)	-.122 (0.242)	-.096 (0.501)	-.066 (0.293)
Period 9	-.106 (0.337)	.030 (0.745)	-.145 (0.191)	-.059 (0.678)	-.081 (0.202)
Period 10	-.053 (0.638)	-.031 (0.762)	-.115 (0.262)	.032 (0.814)	-.057 (0.366)
Prior	.008 (0.738)	-.018 (0.159)	.018 (0.200)	-.069*** (0.001)	-.051*** (<0.001)
The *p*-values are in parentheses, testing if the coefficient is different from zero. The coefficients are marginal effects computed using the delta method. *** means the coefficient is significant at the 1% level. ** means the coefficient is significant at the 5% level. * means the coefficient is significant at the 10% level.					

Table B8: Subjective Expected Utility Beliefs

Risk attitudes:	
$\boldsymbol{r}$	.367*** (<0.001) {0.203, 0.532}
Belief of travel time on 9th Avenue *with* bus:	
$\boldsymbol{\mu_9}^{\boldsymbol{bus}}$	194.952 (a) {a}
$\boldsymbol{\sigma_9}^{\boldsymbol{bus}}$	5.385 (0.111) {-1.241, 12.020}
Belief of travel time on 9th Avenue *without* bus:	
$\boldsymbol{\mu_9}^{\boldsymbol{no\ bus}}$	143.966*** (<0.001) {140.946, 146.99}
$\boldsymbol{\sigma_9}^{\boldsymbol{no\ bus}}$	1.034 (0.407) {-1.410, 3.478}
Belief of travel time on 7th Avenue:	
$\boldsymbol{\mu_7}$	115.642*** (<0.001) {115.499, 115.785}
$\boldsymbol{\sigma_7}$	.365* (0.083) {-0.048, 0.778}
Belief of congestion (or bus) on 9th Avenue:	
$\boldsymbol{\pi_{20}}$	.133 (0.627) {-0.405, 0.672}
$\boldsymbol{\pi_{40}}$	.519*** (0.004) {0.164, 0.874}
$\boldsymbol{\pi_{60}}$	.580*** (0.001) {0237, 0.923}

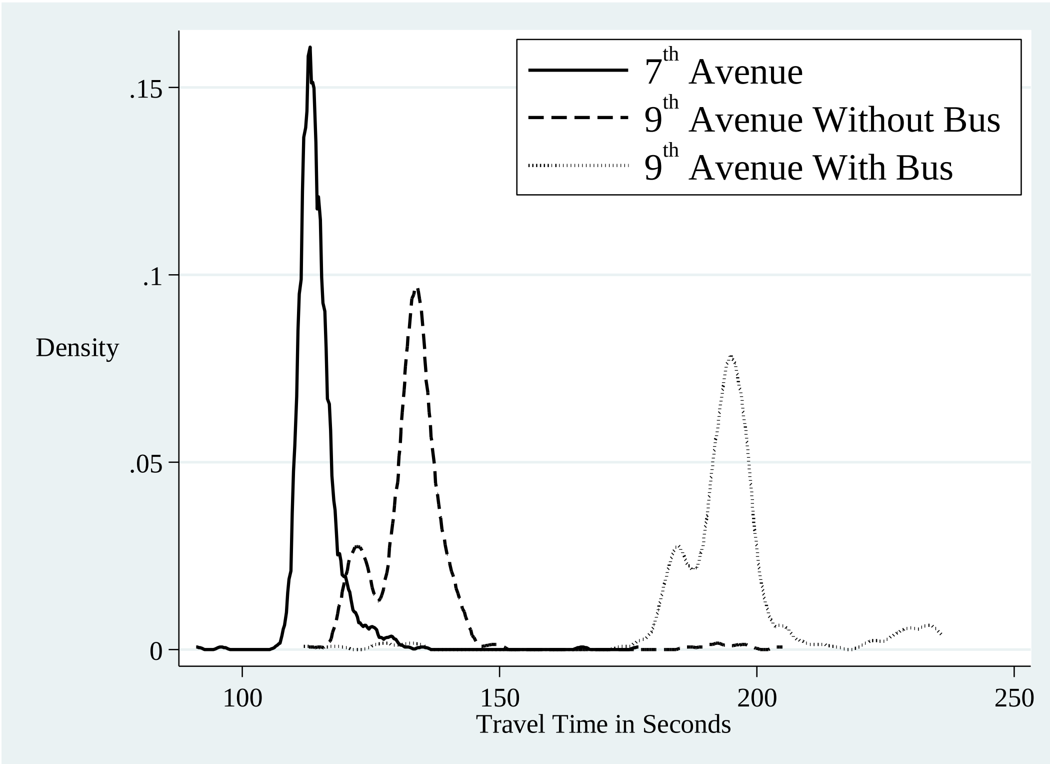
The *p*-values are in parentheses, testing if the coefficient is different from zero.
The 95% confidence intervals are in brackets.
*** means the coefficient is significant at the 1% level.
** means the coefficient is significant at the 5% level.
* means the coefficient is significant at the 10% level.
(a) implies that a standard error or confidence interval cannot be computed by the delta method due to numeric issues, because the estimated probabilities approach 0 or 1.

Table B9: Unconditional Estimated Travel Time Distributions

	Treatment 0.2	**Treatment 0.4**	**Treatment 0.6**
Belief of travel time on 9th Avenue:			
$\boldsymbol{\mu_9}$	150.781*** (<0.001) {125.400, 176.161}	170.437*** (<0.001) {153.109, 187.765}	173.560*** (<0.001) {156.718, 190.402}
$\boldsymbol{\sigma_9}$	10.996*** (<0.001) {7.532, 14.460}	10.298*** (0<0.001) {9.268, 11.329}	10.524*** (<0.001) {9.030, 12.017}
Belief of travel time on 7th Avenue:			
$\boldsymbol{\mu_7}$	115.642*** (<0.001) {115.499, 115.785}		
$\boldsymbol{\sigma_7}$	.365* (0.083) {-0.048, 0.778}		

The *p*-values are in parentheses, testing if the coefficient is different from zero.
The 95% confidence intervals are in brackets
*** means the coefficient is significant at the 1% level.
** means the coefficient is significant at the 5% level.
* means the coefficient is significant at the 10% level.

Figure B1: Distribution of Observed Travel Times



Pooling over all subjects and treatments.
Travel time on 7th Avenue has a mean of 114 and standard deviation of 5.5.
Travel time on 9th Avenue *without* a bus has a mean of 133 and standard deviation of 10.1.
Travel time on 9th Avenue *with* a bus has a mean of 195 and standard deviation of 17.4.

Figure B2: Distribution of Toll by Route

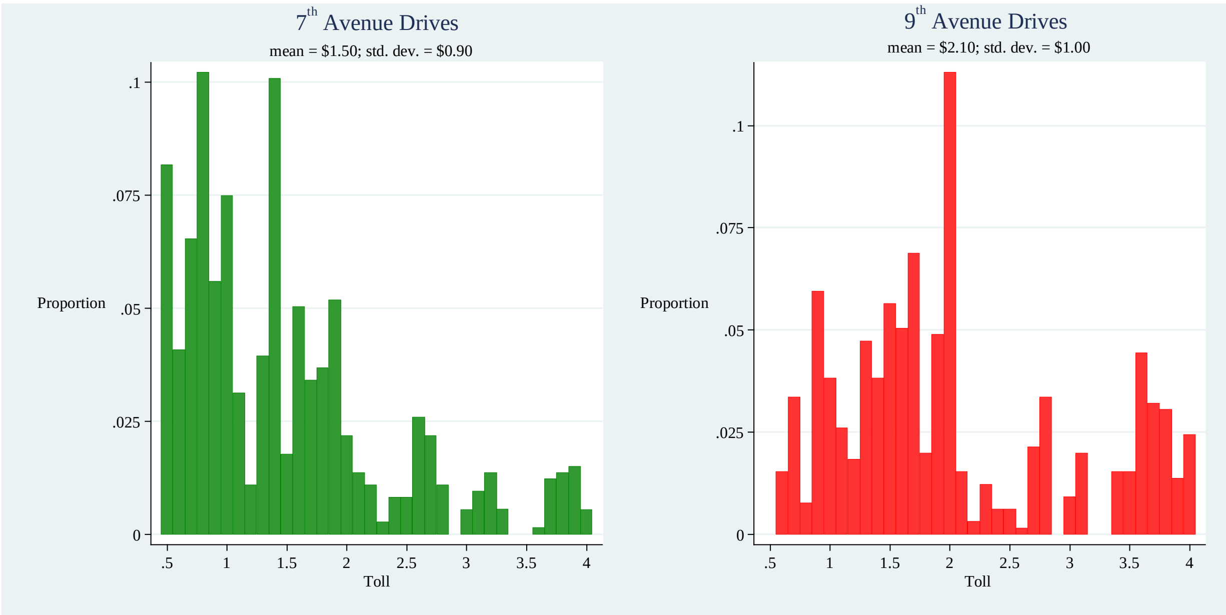


Figure B3: Number of Bus Cards Subjects See Before First Period

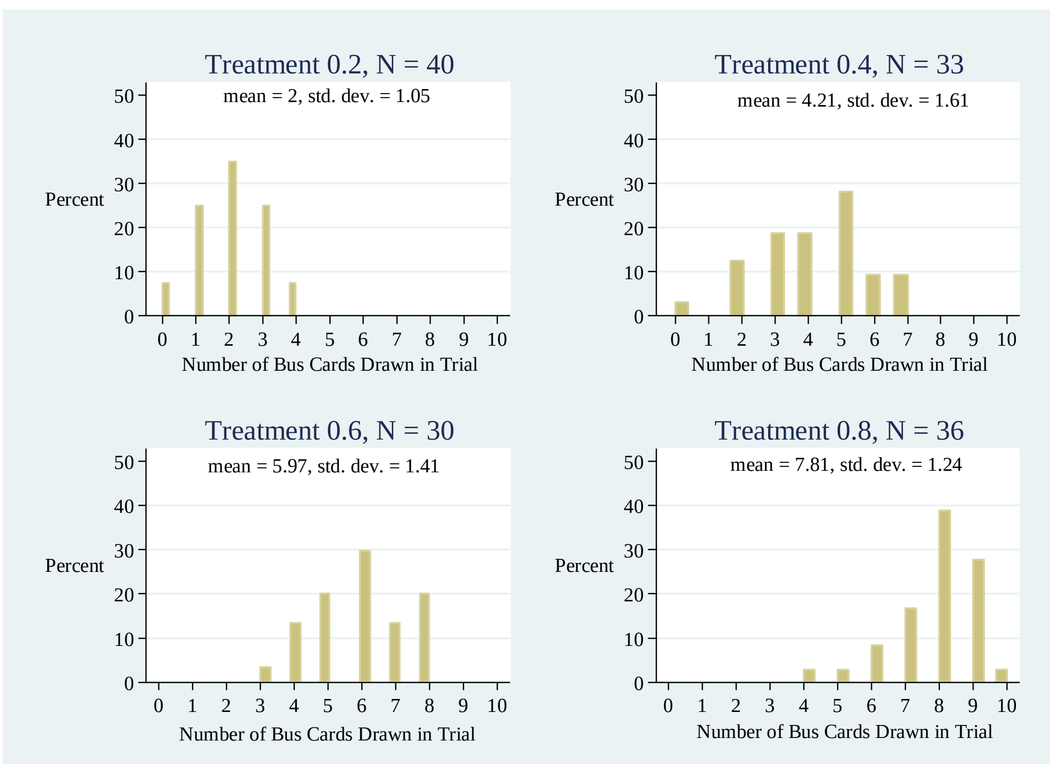


Figure B4: Proportion of Subjects who Take 9th Avenue Across Periods

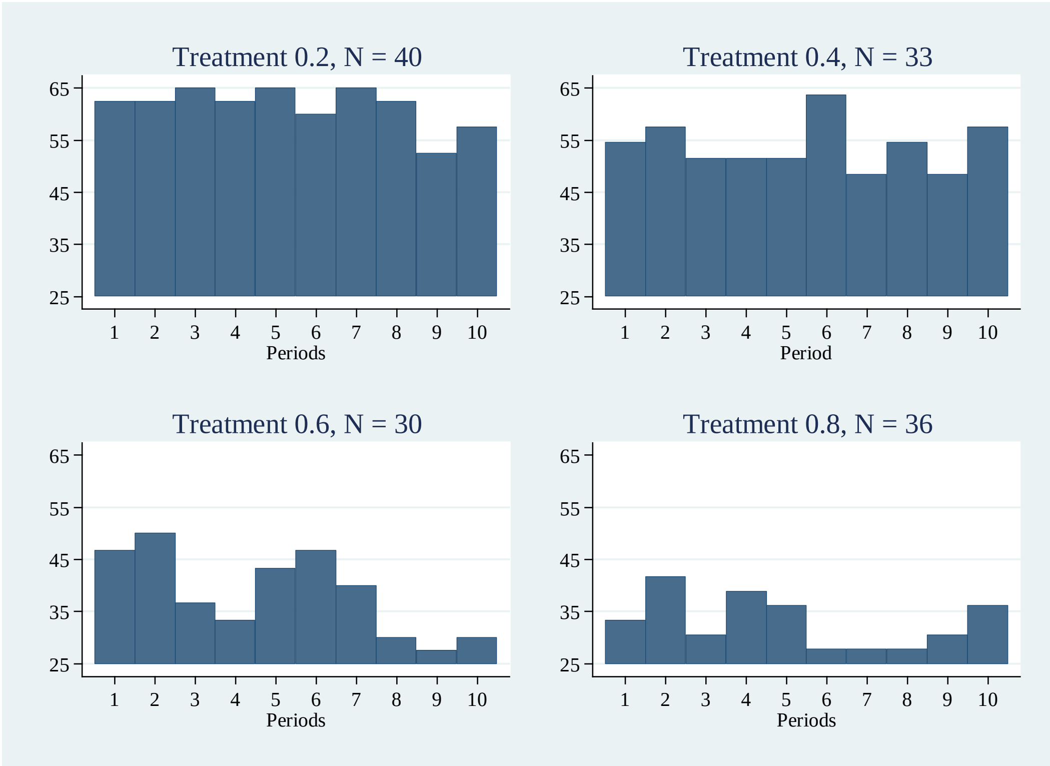


Figure B5: Distribution of Observed Travel Times, Assume Normal Distribution

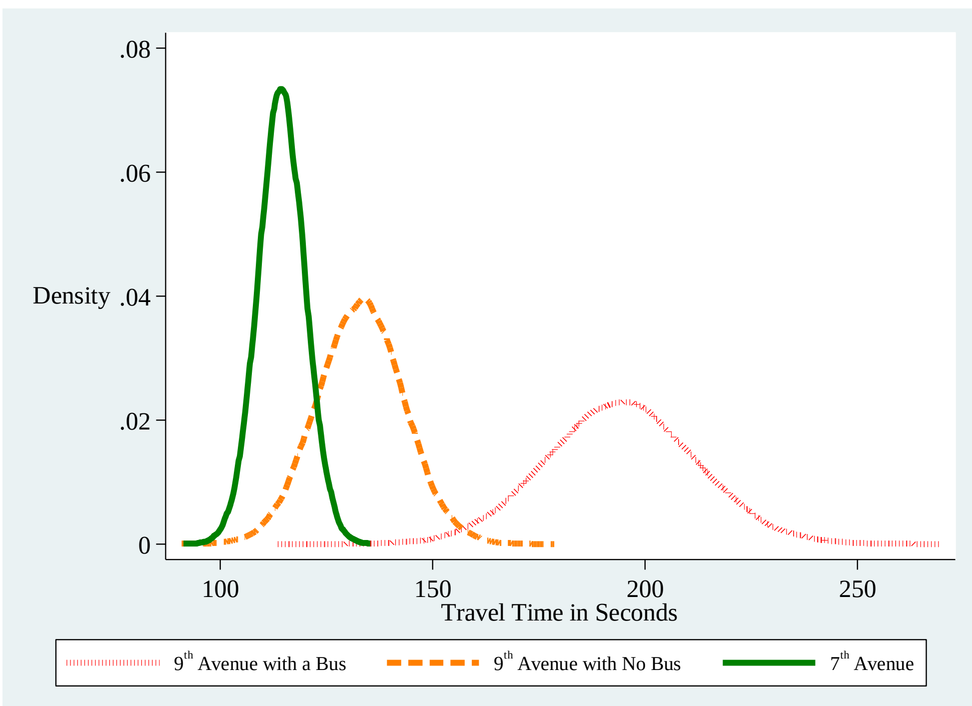


Figure B6: Subjective Travel Time Beliefs for 7th Avenue

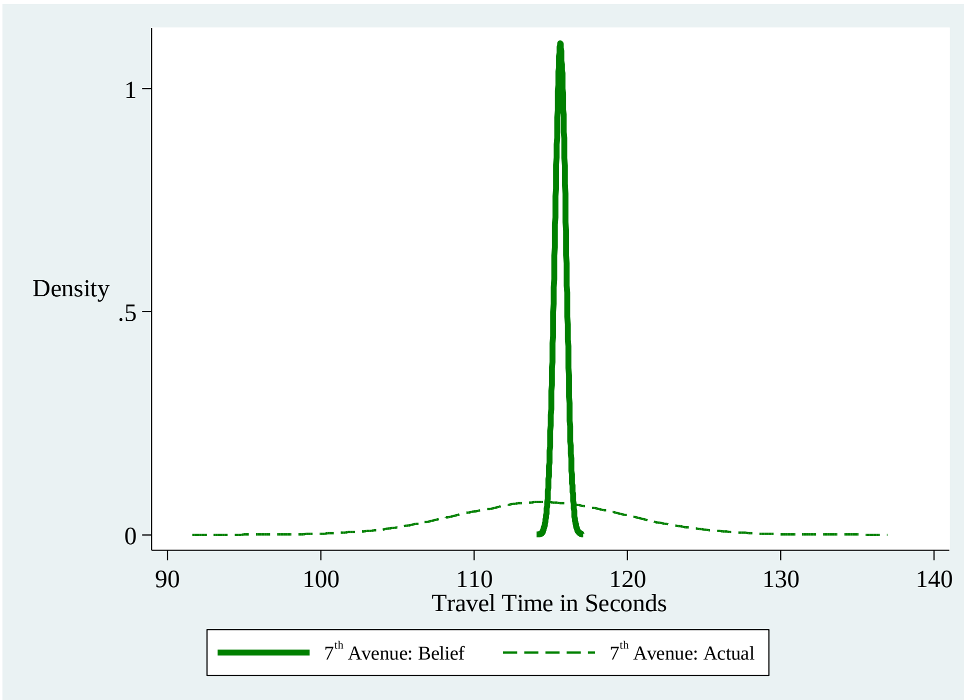


Figure B7: Subjective Travel Time Beliefs for 9th Avenue

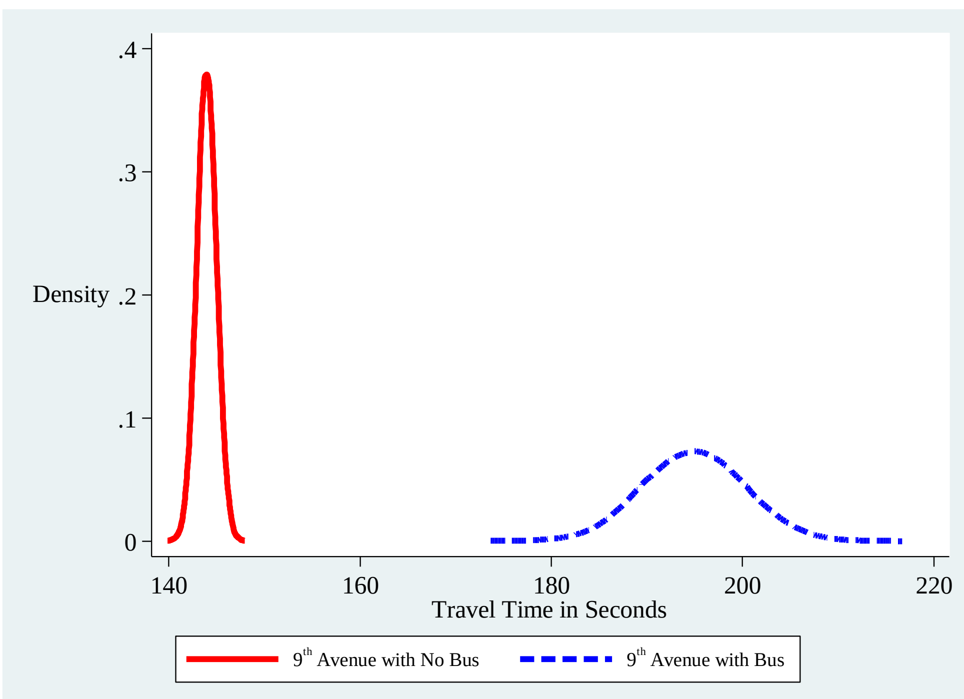


Figure B8: Subjective Travel Time Beliefs for 9th Avenue Compared to Actual Distributions of Travel Time

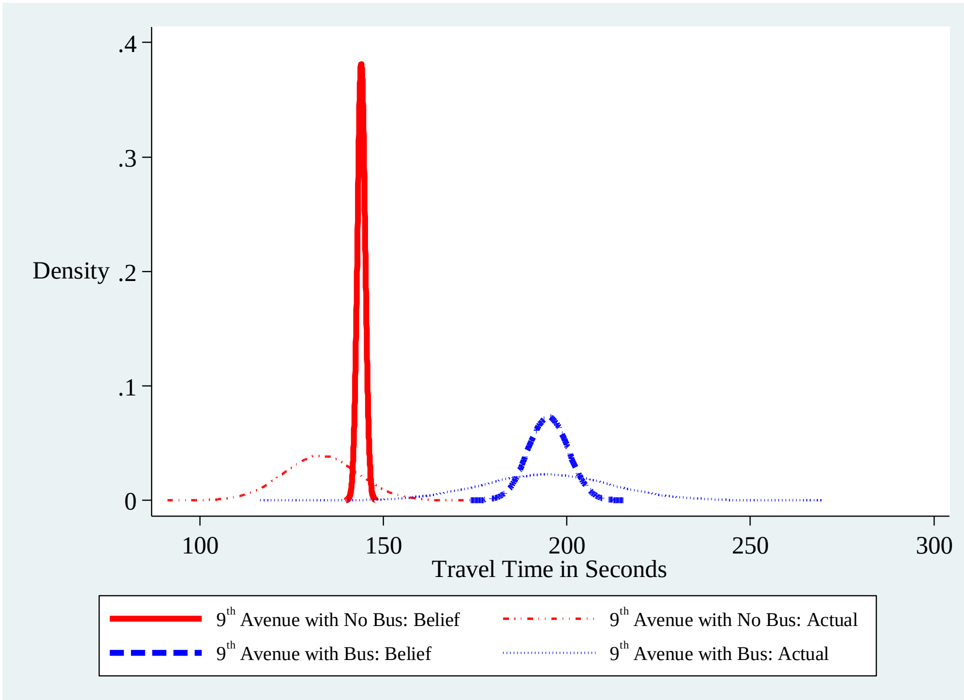
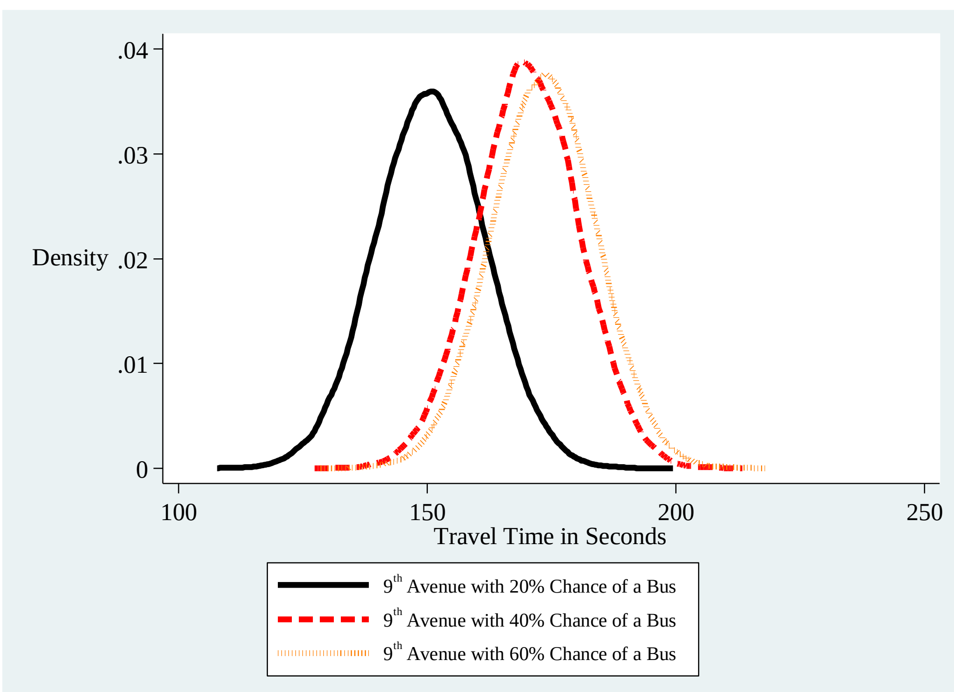


Figure B9: Subjective Travel Time Beliefs for 9th Avenue Depending on the Exogenous Chance of a Bus



CHAPTER 3

Estimating Uncertainty Aversion Using the Source Method in Stylized Tasks With Varying Degrees of Uncertainty

3.1 Introduction

This essay examines uncertainty aversion[34] using the source method introduced in Abdellaoui, Baillon, Placido and Wakker (2011), hereafter ABPW. The source method assumes that different types of events imply potentially different sources of uncertainty; for example, an event with an unknown probability is a different source of uncertainty from an event with a known probability. For each type of event a probability weighting function can be estimated in a rank-dependent model. The probability weighting function estimated from each source of uncertainty is referred to as the *source function*. The source function transforms the probabilities into decision weights and the transformation partially reflects preferences and partially perceptions. Attitudes toward uncertainty can be examined by *comparing* source functions. Note that a source function is essentially a probability weighting function, and since our analysis is performed using the source method, we use the term source function (instead of probability weighting function) hereafter.

[34] The terms *uncertainty* and *ambiguity* both refer to events that have unknown probabilities, but they differ in that *uncertainty* refers to events that the individual has *some* information about the probability distribution, whereas *ambiguity* refers to a complete lack of information about the probabilities. To put it differently, both uncertainty and ambiguity refer to an underlying nondegenerate distribution (i.e., not assuming the Reduction of Compound Lotteries), however, ambiguity differs from uncertainty in that in the former the individual does not even have enough information to form any subjective belief distribution, degenerate or non-degenerate.
In our lottery task experiment, subjects are provided with partial information about the unknown probabilities instead of a complete lack of information. Thus in the analysis we use the term uncertainty aversion instead of ambiguity aversion.

The primary research question in this essay is: if an event with an *unknown* probability is presented to subjects under two scenarios that vary in degrees of uncertainty (i.e., in one scenario the event is presented with little uncertainty about its probabilities, whereas in another scenario the same event is presented with more uncertainty about its probabilities), will this result in variations in behavior? If we estimate a source function for each scenario, will the estimated source functions shift in a way that is consistent with these variations in uncertainty? In other words, does behavior vary in a systematic manner going from the least uncertain scenario to the most uncertain scenario? The experiment described in this essay uses a within-subjects design where each subject completes three types of lottery tasks that are ranked in order of increasing uncertainty. Each task involves making a pairwise comparison between a relatively safe lottery and a relatively risky lottery. In the first task the probabilities of the outcomes are known, in the second and third tasks the probabilities are unknown and are presented under varying degrees of uncertainty.

Using a revealed preference approach, a source function is estimated for each of the three types of tasks based on the choices observed. The estimation is performed using maximum likelihood assuming the two-parameter Prelec (Prelec (1998)), Tversky-Kahneman (Tversky and Kahneman (1992)), hereafter TK, and Power probability weighting specifications assuming a rank ordering of outcomes consistent with Rank Dependent Utility Theory (Quggin (1982)). Next, the resulting source functions are compared using the two indices of uncertainty aversion from the source method: the index of *pessimism* and the index of *likelihood insensitivity*. Pessimism is a tendency to place a lower decision weight for the best outcome relative its underlying objective probability (i.e., $w(p) < p$). It reflects a source function that is convex, globally or locally, such that on the unit interval a subject displays pessimism only for a particular range of probabilities. A concave region would similarly be interpreted as optimism. Likelihood insensitivity is a tendency to overweigh low probabilities and underweigh high probabilities: for an event that has a low probability of occurring, subjects weigh the event higher than its underlying objective probability (i.e., $w(p) > p$), and for an event that has a high probability of occurring, subjects weigh the event lower than its underlying objective probability (i.e., $w(p) < p$). Likelihood insensitivity reflects an inverse-S shaped source function, but notice that likelihood insensitivity could also reflect a change in subjective probabilities in the direction of uniformity, as posited under the principle of insufficient reason.[35]

The index of pessimism and the index of likelihood insensitivity can be estimated, for example, using the two-parameter Prelec probability weighting specification.[36] Given the choices that subjects make in the three types of lottery tasks, we estimate a source function for each type of task assuming the Prelec specification. Next, across the three estimated source functions we compare the pessimism indices, and the *difference* of the pessimism indices is interpreted as uncertainty aversion, as is defined by the source method. Likewise, we compare the likelihood insensitivity indices across the three source functions, and the *difference* of the likelihood insensitivity indices is interpreted as another characteristic property of uncertainty aversion. We undertake a similar estimation for the TK and Power probability weighting specifications.

[35] The "principle of insufficient reason" or the "principle of indifference" states that if one is ignorant of the ways an event can occur, the event will occur equally likely in any way (first enunciated by Jakob Bernoulli).

[36] The Rieger and Wang weighting function is another relatively flexible probability weighting specification with two parameters (Rieger and Wang (2006)). Harrison, Humphrey and Verschoor (2009) estimate probability weighting using both Prelec (1998) and Rieger and Wang (2006) specifications and report only small differences in results between the two specifications.

Based on an experiment that presents simply stylized lottery tasks to adult non-student participants in Atlanta and Orlando, we find that the results obtained using the source method are consistent with past studies that do not use the source method in showing that behavior differs under events with known *vs.* unknown probabilities (i.e., Cubitt, Kuilen and Mukerji (2012) and Attanasi, Gollier, Montesano and Pace (2014)). Furthermore, the estimated source functions show that, compare to events with known probabilities subjects display more likelihood insensitivity for events with *unknown* probabilities, which is consistent with the findings by ABPW. We also find that the behavioral difference under *unknown* probabilities is better captured by the Prelec specification than the TK or Power specification. Results from the Prelec specification suggest that, as the degree of uncertainty increases, subjects display increased *pessimism*; in contrast, the TK and the Power specifications detect no such difference. Thus, the conclusion regarding uncertainty aversion are contingent on which probability weighting specification is assumed for the source function.

3.2 Literature Review

Models of uncertainty or ambiguity can be categorized into two general types.[37] One type is models with multiple-priors. Multiple-priors models consider a set of *priors* or a set of probability distributions over the outcomes, *not* just one distribution. Examples are the smooth model (Klibanoff, Marinacci and Mukerji (2005)) and the α-MEU model (Ghirardato et al.

[37] Models of uncertainty or ambiguity describe events that have probabilities *not* known to the subject. The parameter p refers to subject's subjective probability and is an *unknown* parameter in the decision model. In contrast, models of risk describe events that have known probabilities, thus p, the objective probability, is a known parameter in the decision model.

(2004)). The smooth model considers a set of priors for the possible outcomes,[38] plus separate utility parameters to capture attitudes toward risk and attitudes toward uncertainty.[39] In evaluating any two-stage lotteries, the smooth model takes the Expected Utility (EU) of each one-stage lottery within each prior and then takes the Expected Value of all EUs across the priors. The α-MEU model also considers a set of priors or scenarios. The parameter, α, weighs the worst scenario, and (1- α) weighs the best scenario, thus α serves as the index of uncertainty aversion. Another type is rank-dependent models such as Choquet Expected Utility (CEU) (Gilboa (1987) and Schmeidler (1989)) and Cumulative Prospect Theory (CPT) (Tversky and Kahneman (1992)). These models use decision weights to model preferences toward risk and uncertainty; they rank outcomes in the order of attractiveness and attach decision weights to each ranked outcome. When testing which model has better descriptive and predictive power for the observed choices, Savage's Subjective Expected Utility (SEU) model (Savage (1971)) typically serves as a baseline comparison to the more complex uncertainty models.[40]

This literature review focuses on studies that model preferences toward uncertainty using decision weights. Decision weights have been elicited using methods such as the indifference approach (Mangelsdorff and Webber (1994)), the Quadratic Scoring Rule (Offerman, Sonneman, Kuilen and Wakker (2009), Andersen, Fountain, Hole, and Rutström (2014) and Harrison (2014)), or lottery pairs in a list format (ABPW (2011), Abdellaoui, Vossman and Weber

[38] Events that have a set of priors may be viewed as having elements of a *compound* risk. One should note that the smooth model assumes that compound risk is not reducible to a single probability, i.e., the model does *not* assume the axiom of Reduction of Compound Lotteries (ROCL).

[39] For example, the Constant Relative Ambiguity Aversion (CRAA) parameter serving as the index of uncertainty aversion under uncertainty is analogues to the Constant Relative Risk Aversion (CRRA) parameter serving as the index of risk aversion under risk.

[40] One important distinction between Savage's subjective risk model and a more complex uncertainty model is that the former assumes ROCL and the latter doesn't. In a subjective risk model, any probability distribution is reducible to a single (degenerate) probability, i.e., to the mean of its distribution. In contrast, in an uncertainty model a probability distribution is not reducible to a single probability estimate.

(2005)). The common finding across these methods is that, under uncertainty subjects tend to overweigh low probabilities and underweigh high probabilities, displaying a tendency to place equal decision weights on all possible outcomes.

The rank-dependent models CEU and CPT can both be generalized to non-additive decision weights. CPT is different from CEU in that the former estimates separate weighting functions W^+ and W^- for gains and losses, respectively. The experimental literature on uncertainty aversion considered CEU to be rank-dependent and similar to Rank Dependent Utility (RDU) under risk. As Hey and Pace (2014) puts it: “CEU is the same as RDU (which is not regarded by all as a theory of behavior under uncertainty because it uses objective probabilities but also uses a weighting function, mapping objective probabilities into subjective probabilities) under an appropriate interpretation of that latter theory.” In the estimation of CEU, Hey and Pace (2014) rank-order the outcomes and assume non-additive capacities. The capacities are estimated *non-parametrically*, such that each of the capacities is a parameter to be estimated, as are the pairwise unions. The same estimation approach is used in Conte and Hey (2013) and Hey, Lotito and Maffioletti (2010). A recent approach is to assume that the capacities add up to 1 and estimate them *parametrically* assuming a probability weighting function, as it is shown in Kothiyal, Spinu and Wakker (2014). In this way, the CEU capacities are essentially RDU decision weights.

The following review first discusses studies that examine uncertainty aversion using the source method. In these studies decision weights are elicited over the full range of objective probabilities, and then these elicited values are compared to their respective underlying objective probabilities using two indices of uncertainty aversion defined below. The next set of studies discusses different approaches to elicit decision weights or to correct the reported probabilities for risk and/or uncertainty premium.

3.2.1 Abdellaoui, Baillon, Placido and Wakker (2011), ABPW

In this study different types of events are presumed to constitute different sources of uncertainty, and that the decisions made can be examined using the *source method*. The source method employs a class of uncertainty models that are rank-dependent: RDU/CEU or CPT.[41] These models rank the possible outcomes according to the level of attractiveness, then estimate decision weights assuming a probability weighting function. The probability weighting function estimated from each source of uncertainty is referred to as the *source function* that maps the probabilities, *p*, into decision weights, *w(p)*. A source function can be estimated from events with known or unknown probabilities. Each source function reflects interactions between beliefs and preferences, and by comparing two source functions attitudes toward uncertainty that reflect a *combination* of beliefs and preferences are revealed. Note that a source function is presumed to reflect a combination of beliefs and preferences; it does *not* separate the two. It is the difference in two source functions that is said to reveal uncertainty aversion. Here, the term *uncertainty aversion* reflects the differences of beliefs across events *as well as* differences of preferences across events.[42]

[41] ABPW (2011) perform the estimation under the name Rank Dependent Utility, whereas Kothiyal, Spinu and Wakker (2014) perform the same estimation under the name Choquet Expected Utility. Both studies use the source method.

[42] In the context of the source method, the term "uncertainty aversion" reflects beliefs *as well as* preferences. We are aware that this way of defining uncertainty aversion may cause confusion with many uncertainty models that separate beliefs from preferences. Nevertheless, we use the source method for our analysis for the purpose of examining uncertainty aversion using *decision weights*.

Attitudes toward uncertainty and the degree of perceived uncertainty can be defined in a tractable manner using two indices of uncertainty: pessimism and likelihood insensitivity. As mentioned previously, the index of pessimism reflects the concavity or convexity of a source function, and the difference in concavity or convexity across the graphs of the source functions is interpreted as uncertainty aversion by ABPW. Likelihood insensitivity reflects a source function that is inverse-S shaped, i.e., for an event that has a low probability of occurring, subjects weigh the event higher than its underlying objective probability (i.e., $w(p) > p$), and for an event that has a high probability of occurring, subjects weigh the event lower than its underlying objective probability (i.e., $w(p) < p$), displaying a tendency to place equal decision weights on all possible outcomes. The difference in insensitivity across the graphs of the source functions is interpreted as another characteristic property of uncertainty aversion by ABPW.[43] In the study these two indices are captured using the two-parameter Prelec weighting function (Prelec (1998)).

The source method does not propose a new theoretical model of uncertainty. Instead, it proposes a way to analyze behavior under uncertainty using a class of theoretical models that already exist in the literature: rank-dependent models. The novelty here is to *define* uncertainty preferences by means of two indices based on the decision weights.

The ABPW study is based on two experimental tasks. The first task is the classic Ellsberg urn experiment and the second involves natural uncertainties such as the weather and a stock index in an obscure country. In the Ellsberg task subjects are presented with two urns each containing eight balls. The known urn *K* contains eight balls of different colors: red, blue, yellow, black, green, purple, brown, and cyan. The unknown urn *U* contains eight balls with the

[43] One should note that *likelihood insensitivity*, by itself, may solely reflect diffuse perceptions and may not reflect any preferences toward uncertainty at all. It is the *difference* in insensitivities between two source functions that is considered a characteristic property of uncertainty preferences in the source method.

same set of colors but the composition is unknown to the subjects in the sense that some colors might appear several times and others might be absent. Using a list format, subjects are presented with a series of choices, each between a prospect and an ascending range of a sure payment, with the switching point taken as the certainty equivalent. One of the choices on the list is selected for payment. The second task involves natural uncertainties, such as the French stock index CAC40, temperature in Paris (the home city where the experiment is conducted), and temperature in a foreign city. The elicitation method is again a list varying the amount of the certain option. One of the choices is randomly selected for payment.[44]

To control for risk preferences, utilities are elicited using lotteries with known probabilities that are presented in a list format similar to that used for the uncertainty tasks, with the switching values taken as the certainty equivalent. The lotteries always have an objective probability of 0.5. Utilities are estimated assuming a power utility function using nonlinear least-squares methods.

In the uncertainty task ABPW calculate, rather than estimate, decision weights, after which they fit these weights to a two-parameter Prelec function by minimizing quadratic distance. The Ellsberg task compares the certainty equivalents for risk and for uncertainty. The source functions for urns K and U significantly deviate from linearity (i.e., decision weights deviate from the underlying objective probabilities), and display significant likelihood insensitivity, with significantly more insensitivity in the urn U than in the urn K. In particular,

[44] In the Ellsberg task, ABPW (2011) implement a random incentive payment procedure where one of the choices is randomly selected for real payment. In the task that involves natural uncertainties, ABPW implement two treatments of payment procedures. In one treatment subjects receive a flat payment and the choices are hypothetical, thus the choices are not incentivized. In the other treatment one of the choices is randomly selected for real payment. The money that the subjects earned is collected about three months from the date of the experiment, after the uncertainty is resolved.

for large probabilities ($p > 0.5$) there is more underweighting of probabilities for urn U than for urn K; for small probabilities ($p \leq 0.5$) there is no significant difference. The pessimism index is not significantly different from zero in either urn. Pessimism in urn U, however, significantly exceeds that in urn K. The ABPW interpret the underweighting or overweighting of the subjective probabilities as indicative of willingness to bet,[45] and report that there is more willingness to bet for risk than is for uncertainty in the high probabilities ($p > 0.5$), and that the willingness to bet is the same for both in the low probabilities ($p \leq 0.5$).

The second task makes observations on the certainty equivalents for natural uncertainties. All source functions display a common inverse-S shaped with low probabilities overweighted and high probabilities underweighted. The insensitivity and pessimism indices are significantly different from zero but are not significantly different across the sources of natural uncertainties. Furthermore, the source functions for natural uncertainties are not significantly different from the one for the uncertain urn in the Ellsberg task. These findings suggest that events with unknown probabilities are perceived to be similar, but they are perceived differently from events with known probabilities.

3.2.2 Kothiyal, Spinu and Wakker (2014), KSW

This study compares the predictive power of the source method to that of popular alternatives. The list of models examined include CEU using the source method, CPT using the source method, and multiple-priors models such as maxmin EU and maxmax EU.

[45] ABPW elicit subject's certainty equivalent for each event. It appears that ABPW equate the difference in certainty equivalents across events to difference in willingness to bet across events.

In the experiment uncertainty is implemented using a bingo blower that contains balls of three different colors in 0.2, 0.3, 0.5 proportions. There are three treatments with the total number of balls being 10, 20, to 40 balls. They find that the more balls there are in the bingo blower, the harder it is for subjects to guess the proportion of each color, i.e., the harder it is for them to figure out the underlying objective probability of each color. In this way, the treatments are ranked in order of increasing uncertainty, with the 10-ball bingo blower being the least uncertain treatment and the 40-ball bingo blower being the most uncertain treatment. The outcomes include both gains and losses, and one of the choices is randomly selected for payment.[46]

Some of the observed choices are used for model fitting, and the remaining choices are used as a prediction set. The estimation uses maximum likelihood and the comparison between models is based on the predicted log-likelihood of the test set. The study reports that CPT using the source method outperforms alternative theories for predicting decisions under uncertainty. Furthermore, when estimating CPT using different specifications of the source function (i.e., Prelec (1998), Tversky-Kahneman (1992), Goldstein and Einhorn (1987), and Neo-additive (2007)), CPT still outperforms alternative theories. Hence, the conclusion that CPT best predicts choices under uncertainty is not sensitive to the particular parameterization chosen. The study does not report the shape of the estimated source functions.[47] [48]

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[46] KSW (2014) do not mention how they would handle a situation where the subject's realized outcome turns out to be a loss.

[47] KSW (2014) estimate separate source function parameters for gains and losses and report that the weighting parameters for gains are significantly different than for losses.

[48] Recall that some events involve possible gains and losses, to avoid taking the utility of a negative payment, KSW "normalize" the utilities such that the utility of the best outcome is 1 and the utility of the worst outcome is 0.

3.2.3 Abdellaoui, Vossmann and Weber (2005), AVW

AVW uses an approach that is essentially the source method without referring to it as such. The experiment aims to decompose decision weights into a decision attitude component and a belief component. To do so they use three different types of tasks to elicit certainty equivalents for utility, choice-based probabilities (beliefs) and decision weights, respectively. In each type of task subjects go through a series of binary choice questions in a list format. In the "utility" task subjects choose between two risky prospects; in the "choice-based probability" task they choose between a risky prospect and an uncertain prospect; in the "decision weight" task they choose between an uncertain prospect and a sure amount. Subjects are paid a flat fee for participating in the experiment and the lottery outcomes are hypothetical, thus subjects' choices are not incentivized in a salient manner.

Decision weights are estimated from the inferred choice-based probabilities using a linear-in-log-odds function. The linear-in-log-odds function has two parameters that can be interpreted along the lines as the two indices of uncertainty described in ABPW (2011): one parameter controls the concavity or convexity of the source function and the other parameter reflects a source function that is inverse-S shaped. AVW report that SEU is violated, since the estimated source function is non-linear. This is consistent with the findings reported in ABPW (2011). However, the AVW study does not report the shape of the estimated weighting functions.[49]

[49] The AVW (2005) experiment has outcomes for both gains and losses, and behavior is modeled using CPT. They report violation of SEU in both gains and losses domains.

3.2.4 Dimmock, Kouwenberg and Wakker (2015), DKW

Based on the source method, this study measures uncertainty attitudes using matching probabilities, which involves eliciting a probability that would make subjects indifferent between choosing a risky option and an uncertain option. The matching probabilities approach is claimed to directly capture uncertainty attitudes without the need to measure utility or probability weighting.

In the experiment, subjects go through a series of choice tasks that are designed in a list format, and each task is shown separately one at a time on the computer screen. Subjects are asked to choose between Choice *K* that could result in a gain of €15 or €0 with known probabilities, and Choice *U* that could result in a gain of €15 or €0 with unknown probabilities. If subjects are indifferent between the choices, then they could select *Indifferent*. If the subject selects Choice *K* in the current task, then Choice *K* is made less attractive in the next task by lowering the probability of the winning prize. If, instead, the subject selects Choice *U*, then Choice *K* is made more attractive in the next task by increasing the probability of the winning prize. This iterative process continues until the subject selects *Indifferent*, when this happens a *matching probability* is found.[50] Alternatively, the subject could reach the maximum number of six iterations without selecting *Indifferent*, in which case the experimenter infers the average of the remaining upper and lower bound. One of the decision tasks is randomly selected for payment.

[50] This iterative process for eliciting preferences, i.e., chaining "old responses" into new lotteries, may not be incentive compatible since any strategic misrepresentation of preferences in the current round (or any decision error for that matter) have consequences into the following rounds, producing a different final outcome than otherwise. See Harrison and Rutström (2008b) for a discussion of the Trade-Off elicitation procedure.

For each probability on the unit interval, the authors derive the local uncertainty attitude by mapping the underlying objective probability, *p*, to the elicited matching probability, *m(p)*. The function that maps *p* to *m(p)* has two parameters that have similar interpretations as the pessimism and likelihood sensitivity indices in ABPW (2011). DKW report that behavior display uncertainty-generated likelihood insensitivity, which is a tendency to treat subjective likelihood as a 50-50 probability. This is consistent with an inverse-S weighting function reported in ABPW (2011).

3.2.5 Mangelsdorff and Webber (1994), MW

Mangelsdoff and Webber (1994) introduce the indifference approach to eliciting preferences under uncertainty. The approach is based on asking what change that is needed in one lottery in order to make subjects indifferent between it and another lottery. There are two different aspects of the lottery that could be changed: the money outcome, or the probability. The first indifference approach is demonstrated in ABPW (2011) through eliciting certainty equivalents; the latter is demonstrated in DKW (2016) through eliciting matching probabilities.

MW use both of these approaches to elicit Choquet capacities.[51] MW did not estimate a (parametric) function that maps probabilities into decision weights. In the experiment, subjects are asked to choose between two lottery options: an ambiguous lottery that has two possible outcomes with unknown probabilities, and a risky lottery that has two possible outcomes with known probabilities. Subjects are asked to specify what changes had to be made to the lotteries

[51] Studies that use the source method treat RDU the same as CEU, thus for purpose of discussing these studies the two terms are used interchangeably.

in order for them to be indifferent between the lotteries, thus the name of the elicitation method: *indifference approach*. The first approach to elicitation is by changing the amount to be won. To implement the version where the money outcome is changed, subjects are asked to specify what changes in the amount of winning have to be made in order for them to be indifferent between the lotteries. For example, if the subjects prefer the risky lottery (over the uncertainty lottery), then how much does the winning amount in the risky lottery have to be reduced (increased) in order for them to be indifferent between the two lotteries? The second approach to elicitation is by changing the probability of winning. After each lottery comparison subjects are asked what changes to the probabilities have to be made in order for them to be indifferent between the lotteries. For example, if the subjects prefer the risky lottery (over the uncertain lottery), then what changes have to be made to the probabilities in the risky lottery in order for them to be indifferent between the two lotteries. Subjects are paid a flat fee for participating in the experiment and the money in the lotteries is not paid out, thus subjects' choices are not incentivized.

MW use a simple approach to categorize behavior based on whether the subjects select the option with known probabilities or the option with unknown probabilities. MW report that in the group of subjects who are categorized as having non-neutral attitudes toward uncertainty, CEU predicts behavior better than EU when EU is assumed the "principle of insufficient reason".[52] For the uncertain option, the underlying objective probabilities do not vary across the full range of probabilities, thus no probability function could be estimated and no conclusion is drawn regarding the shape of the function.

[52] EU plus the assumption of the "principle of insufficient reason" is essentially SEU that assumes subjective probabilities for all possible outcomes are equal.

3.2.6 Offerman, Sonneman, Kuilen and Wakker (2009), OSKW

This study introduces a calibration method in which elicited probabilities (from the Quadratic Scoring Rule) are calibrated *to correct* for possible effects of risk premium (for events with known probabilities) or uncertainty premium (for events with unknown probabilities). For choices made under unknown probabilities, the corrected probabilities may be viewed as nonparametric decision weights.

Subjects are presented with statements of events that have known probabilities. They are asked to choose a probability (i.e., 1%, 2%, …, 100%) that the statement is true or false. Depending on the probability they choose, referred to as the *reported* probability, the subject receives one score if the statement is true and another score if the statement is false; the scores are determined by the Quadratic Scoring Rule (QSR). The reported probabilities and their corresponding scores are presented on a list shown on a computer screen. After subjects select a reported probability, any awarded points are converted to money using an exchange rate. OSKW obtain measurements for the reported probabilities over the full range of objective probabilities. Subjects are assigned to one of the two payment treatments: pay all tasks,[53] or pay one task randomly.[54]

The probability that is reported by the subject is confounded by their utility and probability weighting curvatures. The goal is to estimate the reported probability as a function of

[53] In the pay-all task treatment OSKW actualize the earnings for all tasks at the end of the experiment; note that the earnings are not actualized *after each* task.

[54] Many of the experiments reviewed above employ a pay-one-randomly payment protocol *and* model behavior using non-EU models such as RDU and CPT. Theoretically this causes incentive compatibility issues. The pay-one-randomly (POR) protocol implicitly assumes that subjects view each outcome in each binary choice independently of each other, such that their behavior is in accordance with the Compound Independence Axiom (CIA). This payment protocol is incentive compatible under EUT (see Harrison and Swarthout (2014) and Cox, Sadiraj and Schmidt (2015)). However, it is incompatible with non-EUT models that are not based on the CIA, including RDU and CPT.

utility and probability weighting, so that this function can serve as a *correction function* to correct the reported probability for curvatures of the utility and probability weighting. Based on the choices subjects make, OSKW calibrate the parameters of the utility and probability weighting functions assuming power utility and Prelec probability weighting. Estimation is performed using maximum likelihood.

The results show that correcting for utility curvature significantly increases the likelihood compared to the model without correction, and correcting for probability weighting curvature also significantly increases the likelihood compared to the model without correction, though less so than correcting for utility curvature does. Next, the *reported* probability can be compared to the *corrected* probability, and the *difference* between the two can be interpreted as a risk premium.

This *correction* method can be extended to events with unknown probabilities, and the *difference* between the reported and the corrected probabilities can be interpreted as uncertainty premium. While OSKW refer to the *corrected* probability as reflecting "beliefs," referring to them as nonparametric decision weights would also be valid.[55]

3.3 Experimental Design

The experiment reported here uses real money incentives. Each subject is presented with four lotteries that have *known* probabilities, hereafter pure risk lotteries (Figure C1), and two

[55] OSKW implement a separate task that uses similar events but with unknown probabilities, and they use the data from that task to test if the *corrected* probabilities (which they refer to as "subjective beliefs") are additive. OSKW report that the correction method reduces the violations of additivity in subjective beliefs. For this task with unknown probabilities there was no variation over the full range of objective probabilities and OSKW did not estimate the uncertainty premium.

lotteries that have *unknown* probabilities, hereafter uncertainty lotteries. The two uncertainty lotteries are presented in varying degrees of uncertainty: the uncertainty lottery that is less uncertain is referred to as the scrambled lottery (Figures C2.1 and C2.2), and the uncertainty lottery that is the most uncertain is referred to as the blackened lottery (Figures C3.1 and C3.2). Thus three types of lottery tasks are administered: pure risk, scrambled, and blackened. This section describes the design of each lottery task followed by the recruitment and experimental procedures.

3.3.1 Binary Lottery Tasks

In each lottery task subjects choose between a relatively safe option and a relatively risky option. Each option has two possible prizes. The set of prizes and probabilities used for the pure risk lotteries is listed in Table C1. Each subject completes four pure risk lottery tasks. In each task, the probability of obtaining the higher prize is the same in each option, thus the only difference between the two options are the prizes. The set of prizes and probabilities used for the uncertainty lotteries is listed in Table C2. Each subject completes one scrambled lottery task and one blackened lottery task. The *set* of possible parameter values is the same for both types of uncertainty lotteries, but a subject may be assigned a scrambled lottery that has different underlying probabilities than the blackened lottery.

The lotteries are presented using pie chart images. Each pie has two colors (dark blue and light blue) and each color represents a prize. The proportion of the pie that is dark blue represents the probability of getting the high prize, and the proportion that is light blue represents the probability of getting the low prize.[56]

In the pure risk lottery task subjects know the probabilities of the outcomes. Figure C1 shows a screenshot of the practice round before starting the round for real payment. In the pie chart the colors that represent the probabilities are divided into two distinct sections, making it easy to see the proportions. In addition the subjects are explicitly told which numbers on a ten-sided die correspond to which prize. It is safe to assume that there is no uncertainty, only risk, in this task.

In the scrambled lottery task (Figures C2.1 and C2.2), subjects are *not* told the probabilities, and the colors that represent the probabilities are divided into small segments that scrambled across the pie, thus making it difficult to see the proportion that each color occupies. This likely generates some degree of uncertainty on the probabilities.[57]

The most uncertain task is likely the blackened lottery task (Figures C3.1 and C3.2). It builds on the scrambled lottery and adds an additional layer of uncertainty by hiding parts of the pie with a black field so that subjects are not able to see the colors. For both the scrambled and the blackened lotteries there is a time limit (of 15 seconds) on how long subjects can view the pie chart images.

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[56] In the black-and-white version of this text, the dark blue appears as dark gray and the light blue appears as light gray.

[57] We acknowledge that the uncertainty surrounding how the colors are scrambled across the pie chart images may be perceived as an additional risk by the subjects. In other words, the uncertainty lotteries could be perceived as having a compound risk. In that case the analysis may be viewed as examining the difference in behavior under one compound risk (the scrambled lottery) *vs*. another compound risk (the blackened lottery).

3.3.2 Recruitment and Experimental Procedure

The experimental tasks analyzed here are part of a larger experiment described in Rutström et al. (2011). The subjects who are described here are selected from United States Postal Service (USPS) mailing lists and are recruited by invitation letters. The invitation letters direct them to a web page where they are instructed to create an anonymous Gmail account to use exclusively for the experiment to ensure strict privacy. Admission to participate in the experiment is contingent on being at least 18 years of age, holding a valid driver's license, and using a vehicle with a valid vehicle insurance.

The larger experiment consists of four meetings separated by approximately two weeks each. Subjects participate in an experiment that takes place over four sessions with many other tasks than those analyzed here. They complete two pure risk tasks in session 1, another two in session 2, and one scrambled and one blackened task in session 3. Subjects are paid for all tasks, and earnings are actualized immediately following each task, so to avoid issues that arise with the random payment protocols.[58] Earnings in each task, along with cumulative earnings, are tracked in a clear and transparent manner. The subjects are commuters from the Atlanta and

[58] As mentioned previously, the pay-one-randomly (POR) protocol implicitly assumes that subjects view each outcome in each binary choice independently of each other, such that their behavior is in accordance with the Compound Independence Axiom (CIA). This payment protocol is incentive compatible under EUT (see Harrison and Swarthout (2014) and Cox, Sadiraj and Schmidt (2015)). However, it is incompatible with non-EUT models that are not based on the CIA, including Rank Dependent Utility (RDU).
This essay models choices over risky lotteries using RDU, which necessitates the use of a payment protocol that is incentive compatible with RDU. The Pay-All-Sequentially (PAS) protocol does not rely on the CIA and is thus incentive compatible with RDU. However, PAS is not problem-free since it may induce a cumulative wealth effect. Cox, Sadiraj and Schmidt (2015) report that the PAS protocol did not induce a significant wealth effect; the same result is reported in Cox and Epstein (1989) and Cox and Grether (1996) who also use the PAS protocol. In contrast, Dixit, Harb, Martinez and Rutström (2015), who use the PAS protocol in a driving simulator task with exogenous delay probabilities, report that cumulative wealth significantly reduces risk aversion (p-value $<1\%$). Here the cumulative wealth effect is assumed to be negligible on the probability weighting estimates.

Orlando metropolitan areas, and a total of 270 subjects are included for the purpose of this analysis.

3.4 Theory

We assume that all uncertainty preferences are captured by probability weighting, and the perceptions of the likelihoods are measured using two indices: the index of pessimism and the index of likelihood insensitivity. Each of the three types of lotteries are presumed to constitute a different source of uncertainty, and we assume that utility is constant across uncertainty sources. The latter assumption is supported by the findings of ABPW (2011) and Abdellaoui, L'Haridon and Paraschiv (2009); both studies report no difference in utility estimates when measuring utilities for risk and uncertainty tasks.[59] [60] Risk attitudes are controlled through curvature of the utility function and the estimation is performed assuming RDU.[61]

This essay examines decision weights using three probability weighting specifications: a two-parameter Prelec function (Prelec (1998)), a one-parameter Tversky-Kahneman function (Tversky and Kahneman (1992)), and a one-parameter Power function. The first two can measure both of the desired indices, but the last can only measure pessimism, not likelihood

[59] Despite the findings of ABPW (2011) and Abdellaoui, L'Haridon and Paraschiv (2009), we acknowledge the limitations of this assumption. The assumption that utility curvature is constant across sources implicitly assumes that utility curvature may be constant across risk domains, which may not be consistent with empirical findings.
[60] As a response to ABPW (2011)'s findings, Harrison (2011) undertake a maximum likelihood evaluation of ABPW's data and report no evidence of source dependence in either the utility function or the power weighting function.
[61] Using the source method, ABPW (2011) estimate a probability weighting function under the name RDU, whereas KSW (2014) perform the same estimation but under the name CEU. In contrast to KSW (2014), Hey and Pace (2014) and Conte and Hey (2013) use a different approach to estimate CEU (i.e., they estimate CEU capacities non-parametrically and do not assume a probability weighting function). Since CEU has been estimated under different approaches, to avoid confusion the name RDU is used here.

insensitivity. It is included here only because it is a popular function in the literature on choice under risk.

3.4.1 Prelec Weighting

The Prelec weighting function (Prelec (1998)) has parameters η and φ:

$$w(p) = exp(-\eta(-lnp)^{\varphi}) \tag{1}$$

where p is the objective probability of the event, and η and φ are the parameters that weigh the probability. This function is defined for $0 \leq p \leq 1$, $\eta > 0$ and $\varphi > 0$.[62] [63] In Appendix D, Figures BB1.1 – BB1.5 provide examples of how an agent with a Prelec weighting function weighs the probabilities under different values of η and φ. These two parameters represent the two indices of uncertainty: pessimism and likelihood insensitivity. The index of pessimism is captured by η which controls the concavity or convexity of the function. The difference in pessimism across the lotteries reflects uncertainty aversion under the maintained assumption of ABPW. The index of likelihood insensitivity is captured by φ which give an S-shaped or inverse-S shaped to the function. The inverse-S shaped in probability weighting suggests a tendency to overweigh low probabilities and underweigh high probabilities (i.e., in the direction of 50-50) and a lack of

[62] Holding η =1 and letting ϕ vary, the function takes on an S-shaped or inverse-S shaped. Holding ϕ =1 and letting η vary, the function takes on a convex or concave shape.

[63] In our estimation ϕ is constrained to be nonnegative, but *not* to the unit interval. Constraining ϕ to the unit interval implies that subjects exhibit an inverse-S weighting function, which would be contrary to many received evidence that shows otherwise. In particular, Andersen, Harrison, Lau and Rutström (2015) report that constraining ϕ to the unit interval incorrectly leads to evidence of no probability weighting for their average subjects. Thus, we do not impose such assumption and let the data speak for itself.

sensitivity to variation in the objective probabilities. If η and φ are both equal to one in the pure risk lottery the subject is an EU maximizer.

Four hypotheses are tested with respect to the Prelec function:

Hypothesis I – For both uncertainty lotteries, the index of pessimism, η, differs significantly from 1, controlling for utility curvature.

$$\eta_{scrambled} \neq 1; \eta_{blackened} \neq 1$$

Hypothesis II – For both uncertainty lotteries, the index of likelihood insensitivity, φ, differs significantly from 1, controlling for utility curvature.

$$\varphi_{scrambled} \neq 1; \varphi_{blackened} \neq 1$$

Hypothesis III – As the level of uncertainty increases, going from the pure risky lottery to the uncertainty lotteries, the *index* of pessimism decreases. This suggests that pessimism for the uncertainty lotteries exceeds that for the pure risk lottery, such that the weighting functions for the uncertainty lotteries are more convex than the one for the pure risk lottery.

$$\eta_{pure\ risk} > \eta_{scrambled}; \ \eta_{pure\ risk} > \eta_{blackened}$$

Hypothesis IV – As the level of uncertainty increases going from the pure risky lottery to the uncertainty lotteries, the *index* of likelihood insensitivity decreases. This suggests that subjects are increasingly more likelihood insensitive: the weighting functions for the uncertainty lotteries are “flatter” than the one for the pure risk lottery.

$$\varphi_{pure\ risk} > \varphi_{scrambled}; \ \varphi_{pure\ risk} > \varphi_{blackened}$$

3.4.2 Tversky – Kahneman Weighting

The TK weighting function (Tversky and Kahneman (1992)) has only one parameter, γ:

$$w(p) = \frac{p^{\gamma}}{[p^{\gamma}+(1-p)^{\gamma}]^{\frac{1}{\gamma}}} \quad (2)$$

where $0 \leq p \leq 1$. This function exhibits inverse-S probability weighting (i.e., optimism for small *p*, and pessimism for large *p*) for $0 < \gamma < 1$, and S-shaped probability weighting (i.e., pessimism for small *p*, and optimism for large *p*) for $1 < \gamma < 2$. A review by Gonzalez and Wu (1999) indicates that the commonly reported weighting function is an inverse-S function; in contrast, Wilcox (2015) reports that a concave weighting function (i.e., optimism for the best outcome) is the most prevalent weighting function in his subjects.

Within the range $0 < \gamma < 1$, as γ moves closer to 1 the crossover point where $w(p) = p$ moves toward $p = 0.5$ such that the concave and convex regions are about equal; as γ moves closer to 0 the crossover point moves toward $p = 0$ such that the convex region becomes larger than the concave region. See Figures BB2.5 – BB2.7 for comparisons. Within the range $1 < \gamma < 2$, as γ moves closer to 1 the crossover point moves toward $p = 0.5$ such that the concave and convex regions are about equal; as γ moves closer to 2 the crossover point moves toward $p = 1$ such that the convex region is larger than the concave region. See Figures BB2.2 – BB2.4 for comparisons. At $\gamma > 2$, the function is everywhere convex, and as γ increases the convexity increases. This function is undefined at $\gamma = 0$ and non-monotonic for really small γ. For more examples of how an agent would weigh the probabilities under different values of γ, see Figures BB2.1 – BB2.8 in the Appendix.

The TK specification does not allow independent specification of location and curvature, therefore it is less flexible than the two-parameter Prelec. For $\gamma = 1$ in the pure risk lottery the subject is an EU maximizer. Comparing γ between the pure risk and uncertainty lotteries reveals possible uncertainty aversion.

Two hypotheses are tested with respect to the TK function:

Hypothesis I – For both uncertainty lotteries, γ differs significantly from 1, controlling for utility curvature.

$$\gamma_{scrambled} \neq 1; \gamma_{blackened} \neq 1$$

Hypothesis II – Subjects display uncertainty aversion for the uncertainty lotteries relative to the pure risk lotteries.

$$\gamma_{pure\ risk} > \gamma_{scrambled}; \gamma_{pure\ risk} > \gamma_{blackened}$$

3.4.3 Power Weighting

The Power weighting function has parameter γ:

$$w(p) = p^{\gamma} \qquad (3)$$

where $0 \leq p \leq 1$. For $\gamma > 1$ this function is everywhere convex and subjects underweigh the probabilities; for $\gamma < 1$ the function is everywhere concave and subjects overweigh the probabilities.[64] For $\gamma = 1$ in the pure risk lottery, the subject is an EU maximizer. In Appendix D

[64] In the Power weighting function $\gamma = 0$ raises issues because it would imply that the agent places a probability of 1 for an event regardless of its objective probability. We do not encounter this issue in the estimation, though one could still constrain $\gamma > 0$.

Figures BB3.1 – BB3.3 provide examples of what the weighting function would look like under different values of γ and its implied decision weights.

The Power function has the least flexible functional form compared to the Prelec or TK function, given that the former can only accommodate the index of pessimism, *not* likelihood insensitivity. Comparing γ between the pure risk and uncertainty lotteries reveals possible uncertainty aversion that is due to difference in pessimism.

Two hypotheses are tested with respect to the Power function:

Hypothesis I – For the uncertainty lotteries, γ differs significantly from 1, controlling for utility curvature.

$$\gamma_{scrambled} \neq 1; \gamma_{blackened} \neq 1$$

Hypothesis II – Pessimism for the uncertainty lottery exceeds that for the pure risk lotteries, such that the weighting function for the uncertainty lotteries is more convex than the one for the pure risk lottery.

$$\gamma_{pure\ risk} < \gamma_{scrambled}; \gamma_{pure\ risk} < \gamma_{blackened}$$

3.5 Empirical Analysis

The weighting functions are jointly estimated with risk attitudes assuming RDU with CRRA utility functions. This joint estimation approach builds on previous work on structural estimation of risk attitudes by Andersen, Harrison, Lau and Rutström (2008), Harrison and Rutström (2008b) and Andersen, Fountain, Harrison and Rutström (2014).

3.5.1 Estimation Approach

Data are pooled across the three types of lotteries. The main assumption is: different sources of uncertainty generate different source functions but not different utility functions. Therefore, we estimate a common CRRA utility function but allow the weighting parameters to vary with type of lotteries. The reference lottery is the pure risk lottery, and the covariates are the scrambled and blackened lotteries. The econometric approach is illustrated using the Prelec weighting function shown in (1). This approach can be easily extended to the TK and Power weighting functions.

Recall the Prelec weighting function in (1):

$$w(p) = exp(-\eta(-lnp)^{\varphi})$$

where $w(.)$ is the function that transforms p into decision weight, $w(p)$, and $w(.)$ weighs the best option and $(1 - w(.))$ weighs the worst option.

The parameters, η and φ, are allowed to vary with the exogenous treatments for type of lotteries. With the pure risk lottery as the reference, both η and φ are estimated conditional on dummy covariates *Scrambled* and *Blackened*:

$$\eta = \eta^{pure\ risk} + \eta^{scramabled} \times Scrambled + \eta^{blackened} \times Blackened \tag{4}$$

$$\varphi = \varphi^{pure\ risk} + \varphi^{scramabled} \times Scrambled + \varphi^{blackened} \times Blackened \tag{5}$$

with η and φ each constrained to be non-negative.

In the full structural model, the evaluation of the risky option is:

$$RDU_{risky} = exp(-\eta(-lnp)^{\varphi}) * \left(\frac{{x_H}^{risky^{(1-r)}}}{(1-r)}\right) + (1 - exp(-\eta(-lnp)^{\varphi})) * \left(\frac{{x_L}^{risky^{(1-r)}}}{(1-r)}\right) \quad (6)$$

where r is the coefficient of relative risk aversion, x_H is the high prize, x_L is the low prize, and p is the objective probability of the high prize, x_H. Similarly, the evaluation for the safe option is:

$$RDU_{safe} = exp(-\eta(-lnp)^{\varphi}) * \left(\frac{{x_H}^{safe^{(1-r)}}}{(1-r)}\right) + (1 - exp(-\eta(-lnp)^{\varphi})) * \left(\frac{{x_L}^{safe^{(1-r)}}}{(1-r)}\right) \quad (7)$$

The latent preferences for evaluating each option, or the RDU for each lottery pair, is calculated for the candidate estimates of r, η and φ. Following (6) and (7), the index

$$\Delta RDU = RDU_{risky} - RDU_{safe} \quad (8)$$

is the difference in valuation between the risky option and the safe option. The index is then linked to observed choices by using a "logit" likelihood function that is denoted as $\Lambda(\Delta E)$:

$$prob\ (choose\ risky\ option) = \Lambda(\Delta RDU) \quad (7)$$

The risky option is assumed to be chosen when $\Lambda(\Delta RDU) > ½$.

Thus the likelihood of the observed responses, conditional on the RDU specification, CRRA utility function and the Prelec probability weighting function specifications being true, depends on the estimated parameters r, η and φ given the above stochastic specification and the observed choices, y. The log-likelihood is then

$$\ln L\ (r, \eta, \varphi; y) = \Sigma\big[\ \ln\Lambda(\nabla RDU)\ \times \mathbf{I}(y = 1) + \ln\big(1 - \Lambda(\nabla RDU)\big) \times\ \mathbf{I}(y = 0)\big] \quad (9)$$

where $\mathbf{I}(\cdot)$ is the indicator function and y = l (0) denotes the choice of the risky (safe) option.

An important extension of the core model is to apply contextual utility, due to Wilcox (2011), and to allow for subjects to make some behavioral errors. The latent index in (8) then becomes:

$$\Delta RDU = [(RDU_{risky} - RDU_{safe}) / \text{v}] / \mu \qquad (8\text{-})$$

where ν is a normalizing term defined as the difference between the maximum and the minimum utility in each lottery pair. The parameter $\mu > 0$ is a structural Fechner "noise parameter" used to allow some error when evaluating the difference in RDU between the two options. A common Fechner error is assumed for the three types of lotteries.

We extend the likelihood specification to include the noise parameter, μ, and maximize $\ln L\ (r, \eta, \varphi, \mu\ ; y)$ by estimating r, η, φ and μ, given observations on y. The estimation is performed using maximum likelihood.

3.5.2 Descriptive Statistics

The characteristics of the subject pool are described in Table C3. Each gender is about evenly represented in the overall sample. Income is divided into two groups: 40% have household income above $100,000 and the rest have household income of $100,000 or below. Age is divided into two groups: 56% are between the ages of 18 and 40 and the rest between the ages of 41 and 75. A majority hold a college education (80%).

The underlying objective probabilities (i.e., 0.1, …, 0.9) are randomly assigned to subjects. If subjects are evenly distributed across the 9 objective probabilities, then the proportion of subjects in each probability should make up around 11% of the sample size. This is true for the pure risk lottery tasks where each subject completes four tasks; Table C4 shows that the proportion of subjects in each objective probability is about 11%. This is also true for many of the probabilities in the uncertainty lottery tasks. However, since each subject only completes one scrambled and one blackened lottery task, the sample set there is smaller, and thus, it is harder to achieve an even representation of subjects for all probability assignments. A few of the probability assignments have proportions that are as low as 6% or as high as 19% of the sample size (relative to the ideal 11%).

The proportion of safe choices made by subjects is illustrated in Figure C4.1; the solid, dash and dotted curves represent the pure risk, scrambled and blackened lotteries, respectively. For the pure risk lotteries, as the probability of getting the high prize increases, the proportion of safe choices decreases, as shown by the downward-sloping solid curve. This behavior is expected since subjects know the probabilities of all possible outcomes. In contrast, the curves for the uncertainty lotteries are flatter, which suggests that behavior is similar across the range of underlying probabilities. The same general pattern is observed across demographic sub-samples. Thus, there is preliminary evidence that in the uncertainty lotteries subjects tend to overweigh low likelihood events and underweigh high likelihood events, displaying likelihood insensitivity.

Comparing across the three curves, in the low probabilities the pure risk curve is below the uncertainty curves, whereas in the high probabilities the pure risk curve is above the uncertainty curves. To examine if the three curves are significantly different from one another, a *proportions* test is performed to examine if the proportions of safe choices are significantly different across the three types of lotteries.[65] When comparing between the uncertainty lotteries,

[65] The *proportions* test, by the Stata command *prtesti*, tests whether the same proportion from two classes are significantly different.

behavior is *not* significantly different. When comparing between the uncertainty and the pure risk lotteries, behavior is significantly different. This is shown in Figure C4.2, where the "black dot" denotes if a given uncertainty lottery has proportion of safe choices that is significantly different from the pure risk lotteries. Overall there is preliminary evidence showing that subjects behave differently in scenarios with known *vs.* unknown risk.

3.5.3 Results of Structural Estimation

Recall the assumption from ABPW that different sources of uncertainty generate different source functions but not different utility functions, hence the structural estimation pools across the three types of lotteries to estimate one common CRRA utility function. The weighting parameters are estimated conditional on the type of lotteries. The reference lottery is the pure risk lottery and the covariates are the scrambled and blackened lotteries. The dummy variable *Scrambled* takes the value of 1 for the scrambled lottery, and the dummy *Blackened* takes the value of 1 for the blackened lottery. The results are presented in Table C5 for each probability weighting specification.

A. Constant Relative Risk Aversion

The estimated CRRA coefficients are significantly different from zero; they are 0.508 (*p*-value < 0.001), 0.205 (*p*-value = 0.034), and 0.636 (*p*-value = 0.008), respectively for the three probability weighting specifications. Being in the range of 0 – 1, they are consistent with the experimental literature reviewed by Harrison and Rutström (2008b).

B. RDU - Prelec Probability Weighting

Results from the Prelec specification are shown in column *(b)* in Table C5. In the pure risk lottery η nor φ differ significantly from 1; EU is therefore not rejected in the pure risk lottery. Hypotheses I states that the index of pessimism, η, should differ significantly from 1 for both uncertainty lotteries; however, a χ^2 test shows that η differs significantly from 1 for the blackened lottery (*p*-value = 0.036) but *not* for the scrambled lottery (*p*-value = 0.727), thus lacking full support for Hypothesis I.

Hypothesis II states that the index of likelihood insensitivity, φ, should differ significantly from 1 for both uncertainty lotteries. This is confirmed by a χ^2 test with a *p*-value <0.001 for both uncertainty lotteries, thus providing support for Hypothesis II.

According to assumptions of the source method, the three types of lotteries each constitute a different source of uncertainty. One should then expect behavior under the uncertainty lotteries to differ from the pure risk lotteries, and the index of pessimism and the index of likelihood insensitivity should capture this behavioral difference, as is suggested in Hypotheses III and IV.

As uncertainty increases, going from pure risk to scrambled to blackened lotteries, the estimate of η decreases (see Table C6), showing more *pessimism*. Comparing each of the estimates for the uncertainty lotteries to the estimates for the pure risk lotteries, *pessimism* is significantly higher for the former compared to the latter, by 0.457 (*p*-value = 0.020) and 0.227 (*p*-value = 0.228), respectively. Furthermore, comparing the uncertainty lotteries, *pessimism* is higher for the blackened lottery than the scrambled lottery (*p*-value = 0.071), showing that for events with *unknown* probabilities that vary in the degrees of uncertainty subjects display different degrees of *pessimism*.

As for the index of likelihood insensitivity, *likelihood insensitivity* is significantly higher when comparing each of the uncertainty lottery to the pure risk lottery, by 1.418 (p-value < 0.001) and 1.297 (p-value = 0.001), respectively. However, when comparing the uncertainty lotteries, there is no significant difference in *likelihood insensitivity* (p-value = 0.322). This shows that for events with *unknown* probabilities that vary in the degrees of uncertainty subjects display a similar degree of *likelihood insensitivity*.

For each of the three types of lotteries a graph of the weighting function is constructed based on its estimates of η and φ; see Figures C5.1, C5.2 and C5.3 for the three respective lotteries. The graphs are constructed based on a lottery with two outcomes. The left panel shows how the subjects weigh the probability of the best outcome given a range of objective probabilities; the right panel shows a specific example where the probabilities of the worst and best outcomes are equal (p = ½) and their respective decision weights. A pattern emerges going from the pure risk to the scrambled to the blackened lotteries: the weighting function becomes increasingly flatter, showing that subjects display increasing tendency to place equal weights for all underlying probabilities.[66]

Results from the Prelec specification are consistent with results from the descriptive analysis. Subjects display increasing pessimism and likelihood insensitivity going from events

[66] Comparing the uncertainty lotteries, the blackened lottery has a crossover point (i.e., $w(p) = p$) at 0.5 and the scrambled lottery has a crossover point at *below* 0.5 (see Figures C5.2 and C5.3). The weighting function for the blackened lottery has roughly the same area for pessimism and optimism, whereas for the scrambled lottery there is disproportionately a bigger area for pessimism than optimism. Note that this difference in crossover points is consistent with the difference in estimates for the index of pessimism, η.

with *known* probabilities to events with *unknown* probabilities. This result is consistent with past studies that examine uncertainty aversion with or without using the source method (ABPW (2011), DKW (2015), Hey and Pace (2014), Attanasi, Gollier, Montesano and Pace (2014)).

Furthermore, when comparing behavior under events with *unknown* probabilities that vary in the degree of uncertainty, results from the Prelec specification suggest that this behavioral difference is attributed to difference in *pessimism*, not likelihood insensitivity.

C. RDU – Tversky – Kahneman Probability Weighting

Results from the TK specification are shown in column *(c)* in Table C5. The estimated γ for the pure risk lottery is 1.846 and is significantly different from 1 (*p*-value < 0.001), thus EU is rejected. Figure C6.1 shows that in the pure risk lottery subjects significantly underweigh the low probabilities (i.e., $w(p) < p$), but not the high probabilities, where $w(p) = p$. This is in contrast to the results from the Prelec specification where the decisions weights are not significantly different from the underlying objective probabilities over the full range of probaiblities.

For the uncertainty lotteries the γ estimates are 0.672 (p-value < 0.001) and 0.683 (p-value < 0.001), respectively, and they are significantly different from 1, which lend support to Hypothesis I. Comparing each of the estimates for the uncertainty lotteries to the estimate for the pure risk lottery, γ in the former is significantly lower than the latter, by 1.174 (*p*-value < 0.001) and 1.163 (*p*-value < 0.001), respectively. This suggests that subjects behave differently under events with known *vs*. unknown probabilities. Furthermore, this behavioral difference is in the direction of likelihood insensitivity: Figures C6.2 and C6.3 show that behavior displays likelihood insensitivity for the uncertainty lotteries, providing support for Hypothesis II.

However, when comparing the two uncertainties lotteries there is no significant difference (p-value = 0.592). This is in contrast to the results from the Prelec specification, which show that the blackened lottery displays significantly more *pessimism* than the scrambled lottery. These results suggest that when comparing behavior under events that have *unknown* probabilities, the flexible Prelec specification is better at capturing the behavioral difference due to varying degrees of uncertainty.

D. RDU - Power Probability Weighting

The Power weighting function is the least flexible specification compared to the Prelec or TK specification, and can only accommodate the index of pessimism. Perhaps due to its inability to accommodate data that is of an inverse-S or S shape, when the data of the blackened lottery is included the estimation experiences numerical problems. Thus we undertake the estimation without the blackened lottery data, and the results that are shown in column *(d)* of Table C5 are only based on the data from the pure risk and scrambled lotteries. Here the hypothesis testing is performed only for the pure risk and scrambled lotteries.

For the pure risk lottery γ is *not* significantly different from 1 with coefficient 0.811 (p-value = 0.484); Figure C7.1 shows the weighting function for the pure risk lottery. EU is therefore not rejected for the pure risk lottery. For the scrambled lottery γ is significantly different from 1 with coefficient 0.789 (p-value = 0.088), which provides support for Hypothesis

I. However, subjects display *optimism* for the best outcome (as is shown in Figure C7.2), instead of pessimism that the index of pessimism would predict for behavior under uncertainty.

Comparing the scrambled lottery to the pure risk lottery, the γ estimates are not significantly different from each other (p-value = 0.891): 0.811 *vs.* 0.789 (see Figure C7.1 *vs*. C7.2), hence lending no support for Hypothesis II. This is in contrast to the results from the Prelec and TK specifications, where both specifications detect a behavioral difference between events with known *vs*. unknown probabilities. Overall, results from using the Power specification do not display behavior that is consistent with results from the Prelec and TK specifications, or from past studies at large.

In summary, for both the Prelec and TK specifications, the results suggest that subjects behave differently under events with known *vs.* unknown probabilities. Relative to events whose probabilities are known, subjects behave in the direction of likelihood insensitivity when probabilities are *not* known, such that they overweigh low probabilities and underweigh high probabilities. One should note that likelihood insensitivity, by itself, may be solely a reflection of diffuse perceptions and may not reflect any preferences toward uncertainty at all. In the source method, it is the *difference* in insensitivities between two source functions that is considered a characteristic property of uncertainty preferences.

3.6. Conclusion

The goal of this essay is to examine uncertainty aversion between events that have the same underlying objective probability but are presented differently under varying degrees of uncertainty. Using a within-subject design subjects are asked to complete three lottery tasks that are ranked in order of increasing uncertainty. Two presentations of uncertainty are used, one presumably more uncertain for the decision maker than the other.

Based on the choices subjects make, a source function is estimated for each lottery task using three different specifications of the source function. The source functions are compared using two uncertainty indices: pessimism and likelihood insensitivity. Overall, the results are consistent with past studies that do not use the source method, in showing that behavior differs under events with known *vs.* unknown probabilities. We report that the source function for events with *known* probabilities differ significantly from the source function for events with *unknown* probabilities. In particular, when probabilities are *not* known subjects behave in the direction of likelihood insensitivity, such that they overweigh low probabilities and underweigh high probabilities.

However, when comparing the difference in behavior between events that both have *unknown* probabilities but vary in the degree of uncertainty, the behavioral difference is better captured by the Prelec specification than the TK or Power specification. Results from the Prelec specification suggest that as the degree of uncertainty increases, subjects display increased *pessimism*, whereas the TK and the Power specifications show no such difference. Thus, the conclusion regarding uncertainty aversion are contingent on which specification is assumed for the source function.

Tables and Figures for Chapter 3

Table C1: Prizes and Probabilities for Pure Risk Lottery

Probability range	**Safe Lottery Low Prize**	**Safe Lottery High Prize**	**Risky Lottery Low Prize**	**Risky Lottery High Prize**
0.1 – 0.9	$2	$3	$0.25	$4
0.1 – 0.9	$2	$3	$0.25	$5
0.1 – 0.9	$2	$3	$0.25	$6
0.1 – 0.9	$4	$6	$0.50	$10

Table C2: Prizes and Probabilities for the Uncertainty Lotteries

Probability range	**Safe Lottery Low Prize**	**Safe Lottery High Prize**	**Risky Lottery Low Prize**	**Risky Lottery High Prize**
0.1 – 0.9	$2	$3	$0.25	$5

Table C3: Demographic Subsample by Lottery Type

N = 270	**Pure Risk**	**Scrambled**	**Blackened**	**Pool all lotteries**
Female	47%	46%	46%	47%
Male	53%	54%	54%	53%
College	80%	80%	80%	80%
Non-college	20%	20%	20%	20%
Income > $100K	40%	41%	40%	40%
Income ≤ $100K	60%	60%	60%	60%
Ages 18-40	55%	56%	56%	56%
Ages 41-75	45%	44%	44%	44%
Each subject completes 6 lottery tasks: 4 pure risk, 1 scrambled and 1 blackened.				

Table C4: Proportion of Subjects Assigned to Each Objective Probability

Objective probability of the higher prize	***p* = 0.1**	***p* = 0.2**	***p* = 0.3**	***p* = 0.4**	***p* = 0.5**	***p* = 0.6**	***p* = 0.7**	***p* = 0.8**	***p* = 0.9**
Pure Risk Lottery	11%	13%	12%	12%	11%	11%	10%	11%	12%
Scrambled Lottery	12%	17%	10%	13%	8%	7%	9%	9%	15%
Blackened Lottery	13%	13%	12%	6%	7%	8%	8%	14%	19%

Table C5: Estimate Rank Dependent Utility Probability Weighting

Column (a)	*Column (b)*	*Column (c)*	*Column (d)*
	RDU Prelec	**RDU TK**	**RDU Power**
r	.508*** (<0.001)	.205** (0.034)	.636*** (0.008)
γ			
_cons	---	1.846*** (<0.001)	.811*** (0.003)
Scrambled	---	-1.174*** (<0.001)	-.021 (0.891)
Blackened	---	-1.163*** (<0.001)	---
η			
_cons	1.171*** (<0.001)	---	---
Scrambled	-.227 (0.228)	---	---
Blackened	-.457** (0.020)	---	---
φ			
_cons	1.585*** (<0.001)	---	---
Scrambled	-1.297*** (0.002)	---	---
Blackened	-1.418*** (0.001)	---	---
μ	.180*** (<0.001)	.208*** (<0.001)	.136** (0.041)

Pure Risk Lottery is the reference lottery; *Scrambled* = 1 for the scrambled lottery; *Blackened* =1 for the blackened lottery.

Perhaps due to the Power function's inability to accommodate data that is of an inverse-S shaped, when the data of the blackened lottery is included the estimation experiences numerical issues. Thus we perform the estimation without the blackened lottery data, and the results that are shown in *column (d)* of Table 5 is based on only the data from the pure risk and scrambled lotteries.

The coefficients are marginal effects, computed using the delta method that takes a nonlinear transformation of an estimated parameter about its mean and its variance based on a Taylor approximation. The *p*-values are in parentheses.
*** means that the coefficient is significantly different from zero at the 1% level.
** means that the coefficient is significantly different from zero at the 5% level.
* means that the coefficient is significantly different from zero at the 10% level.

Table C6: Rank Dependent Utility Weighting Parameters in Total Effects

	Pure Risk	**Scrambled**	**Blackened**
Prelec			
$\boldsymbol{\eta}$	1.171 (0.978)	0.943 (0.727)	0.713** (0.036)
$\boldsymbol{\varphi}$	1.585 (0.346)	0.288* (<0.001)	0.167*** (<0.001)
TK			
$\boldsymbol{\gamma}$	1.846*** (<0.001)	0.672*** (<0.001)	0.683*** (<0.001)
Power			
$\boldsymbol{\gamma}$	0.811 (0.484)	0.79* (0.088)	--

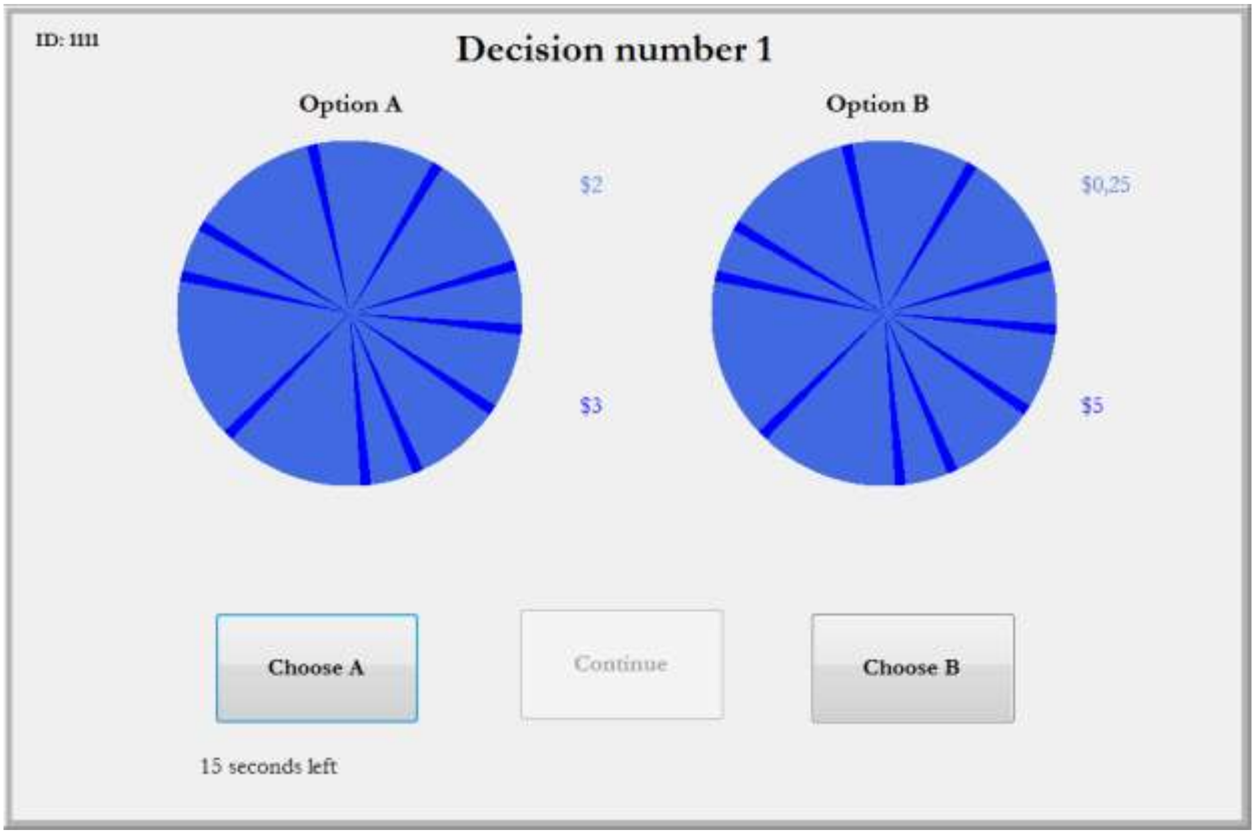
The values are in *total effects*.

*** means that the coefficient is significantly different from 1 at the 1% level.
** means that the coefficient is significantly different from 1 at the 5% level.
* means that the coefficient is significantly different from 1 at the 10% level.
The *p*-values are in parentheses.

Perhaps due to the Power function's inability to accommodate data that is of an inverse-S shaped, when the data of the blackened lottery is included the estimation experiences numerical issues. Thus we perform the estimation without the blackened lottery data, and the results that are shown in *column (d)* of Table 5 is based on only the data from the pure risk and scrambled lotteries.

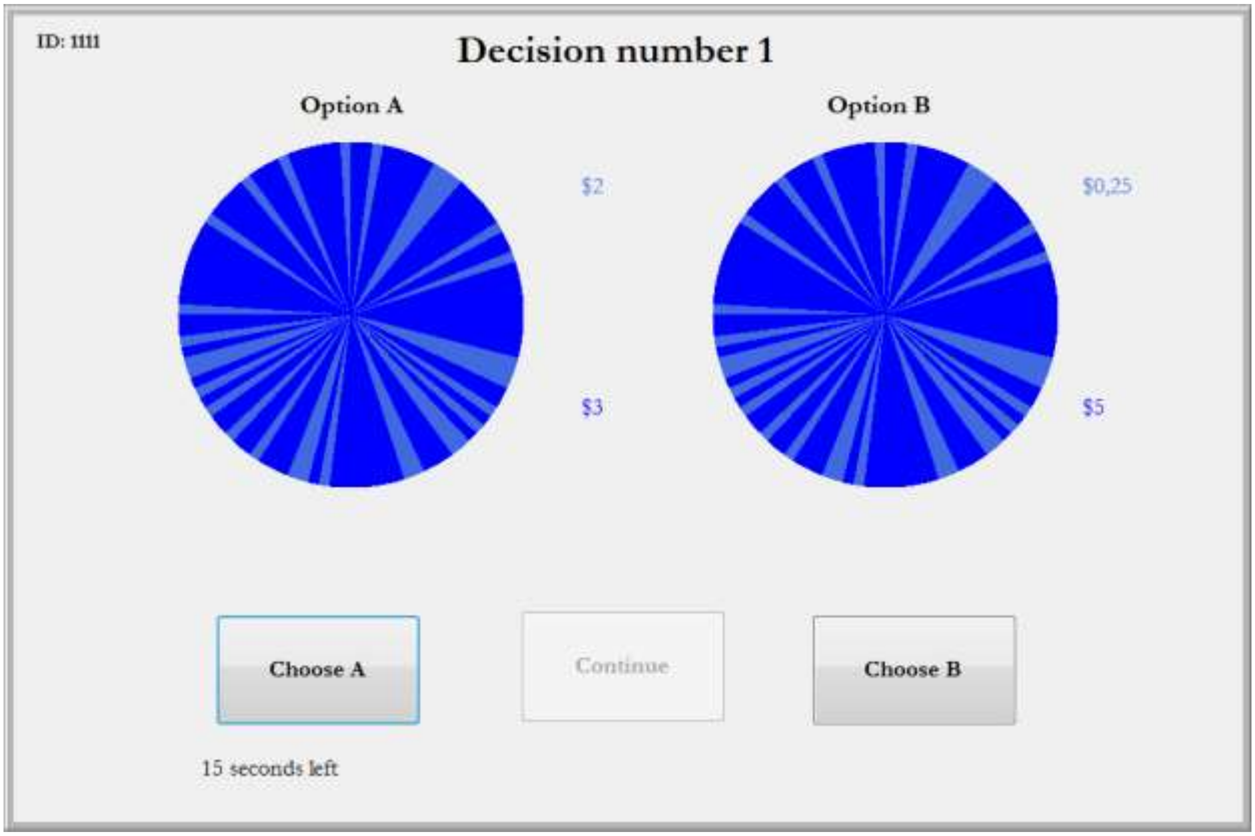
Figure C1: Screen Shot For Pure Risk Lottery Practice Task

Figure C2.1: Screen Shot For Scrambled Lottery, $p = 0.1$



In the above figure, the objective probability of the higher prize (in dark blue) is 0.1.

Figure C2.2: Screen Shot For Scrambled Lottery, p = 0.7



In the above figure, the objective probability of the higher prize (in dark blue) is 0.7.

Figure C3.1: Screen Shot For Blackened Lottery Practice Task

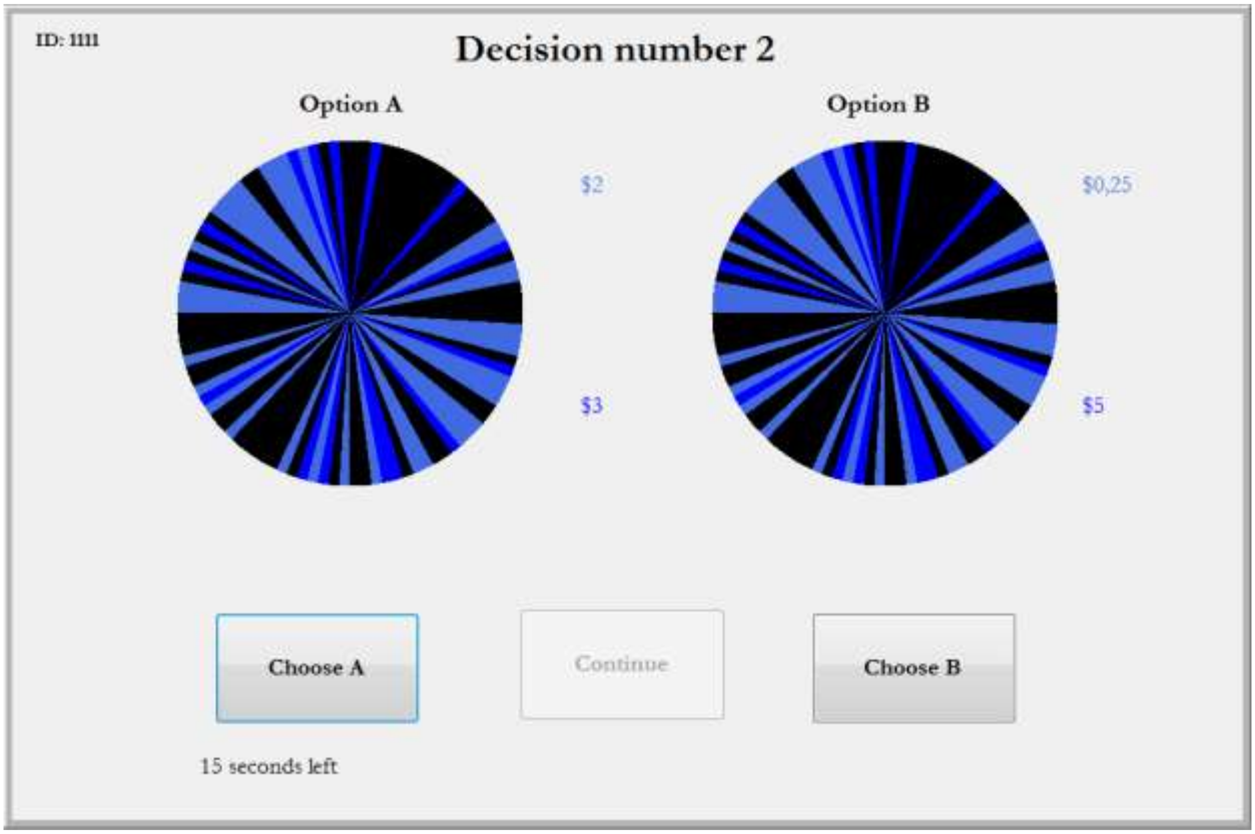


Figure C3.2: Screen Shot For Blackened Lottery Practice Task

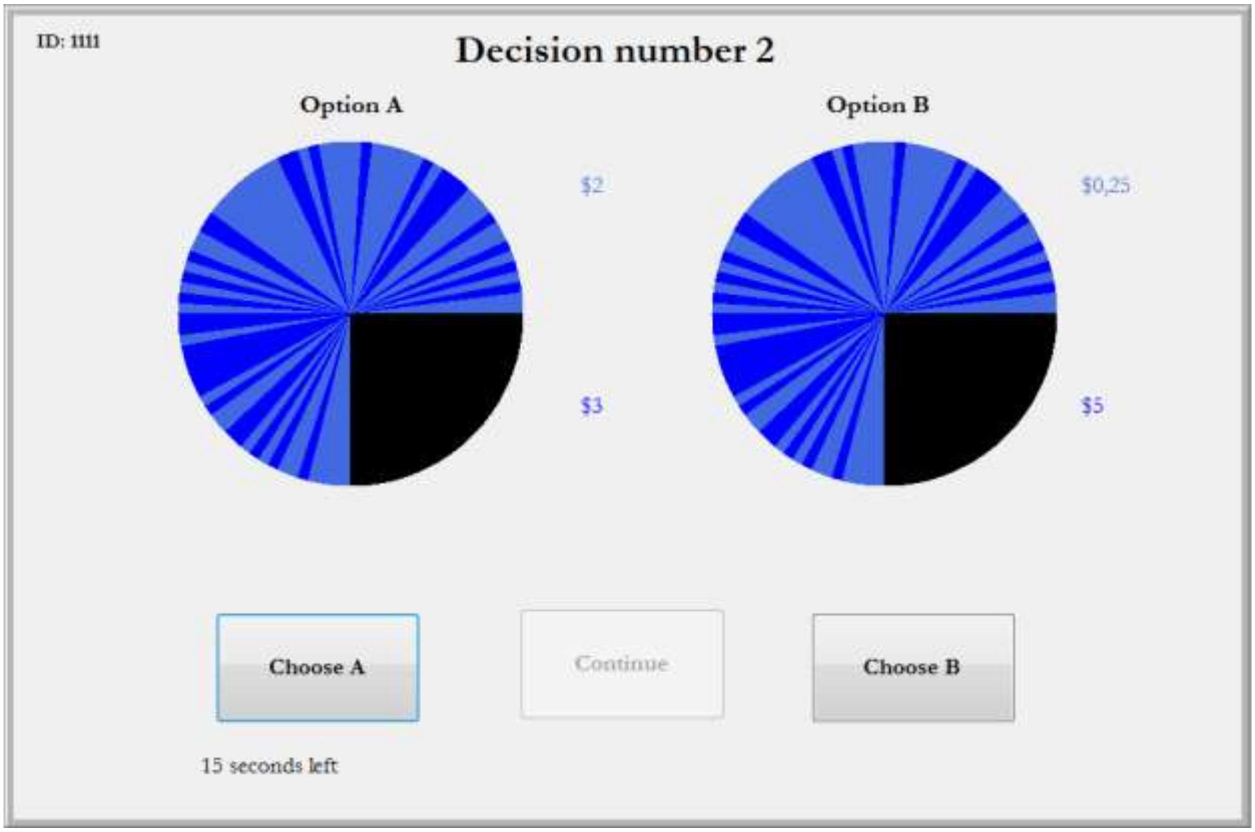


Figure C4.1: Proportion of Safe Choices

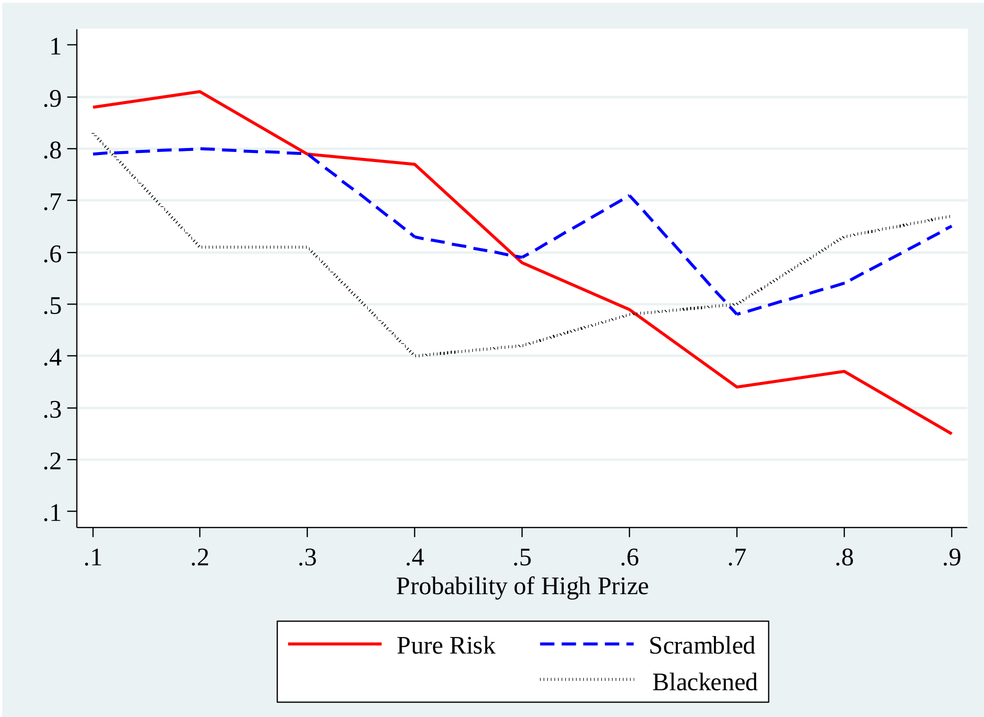


Figure C4.2: Difference in the Proportion of Safe Choices

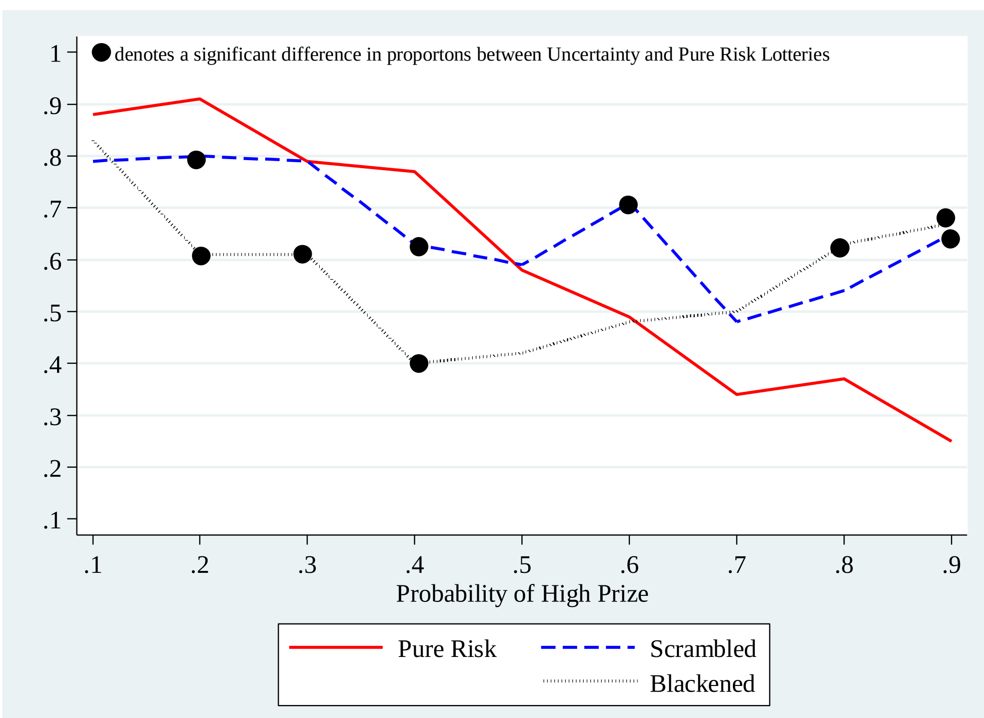


Figure C5.1: Estimate Rank Dependent Utility Prelec Weighting for Pure Risk Lottery

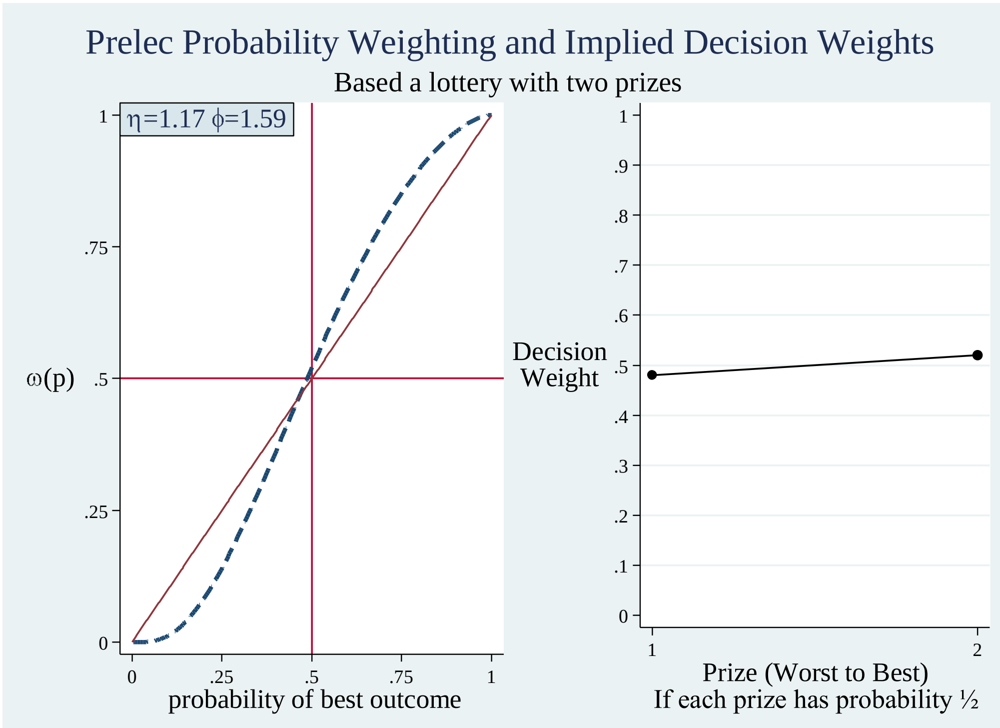


Figure C5.2: Estimate Rank Dependent Utility Prelec Weighting for Scrambled Lottery

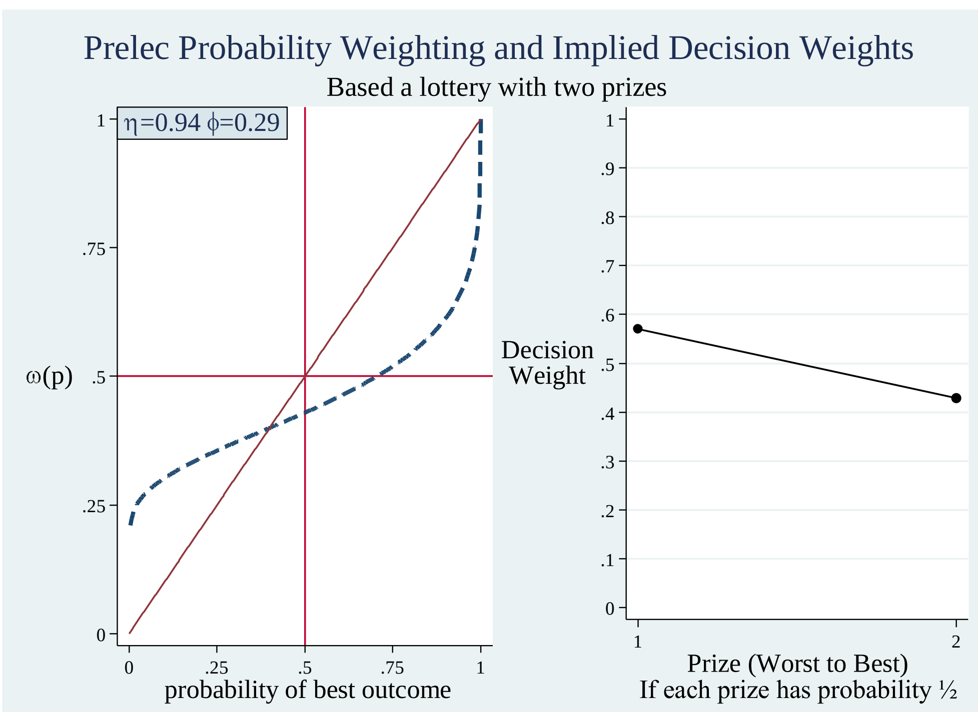


Figure C5.3: Estimate Rank Dependent Utility Prelec Weighting for Blackened Lottery

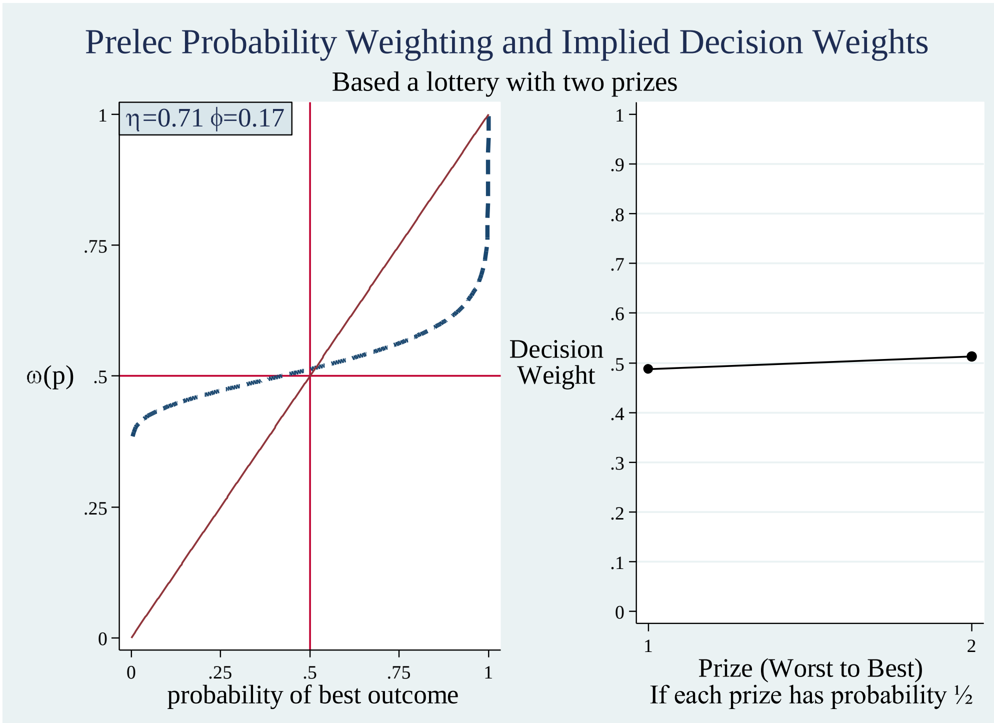


Figure C6.1: Estimate Rank Dependent Utility TK Weighting for Pure Risk Lottery

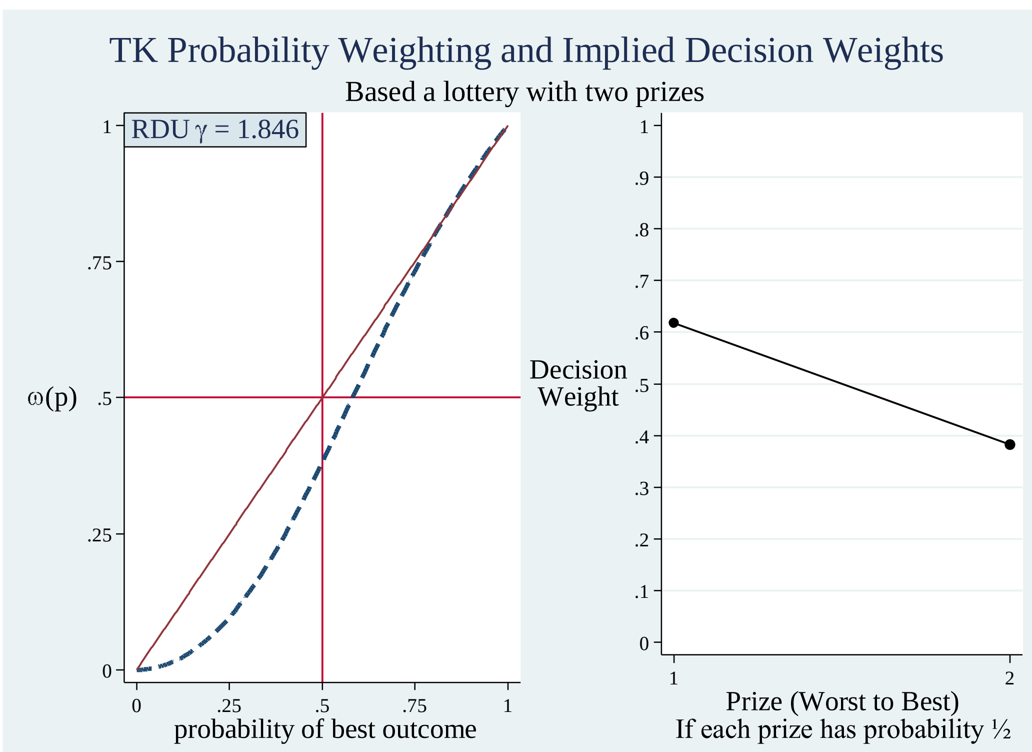


Figure C6.2: Estimate Rank Dependent Utility TK Weighting for Scrambled Lottery

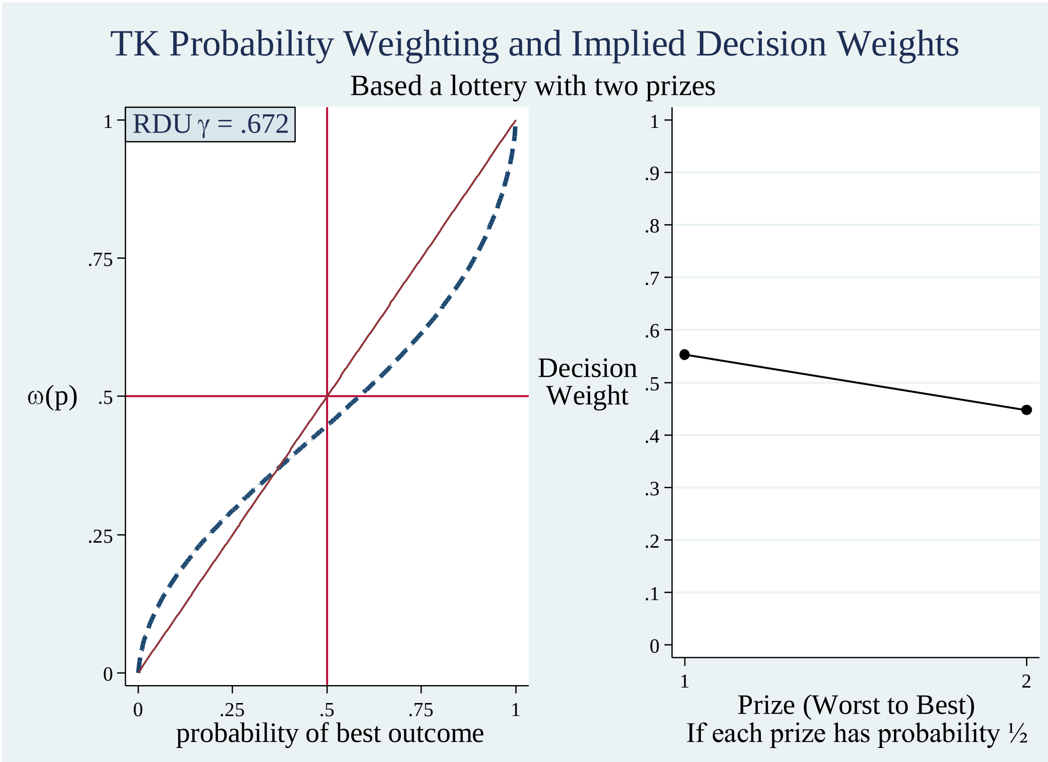


Figure C6.3: Estimate Rank Dependent Utility TK Weighting for Blackened Lottery

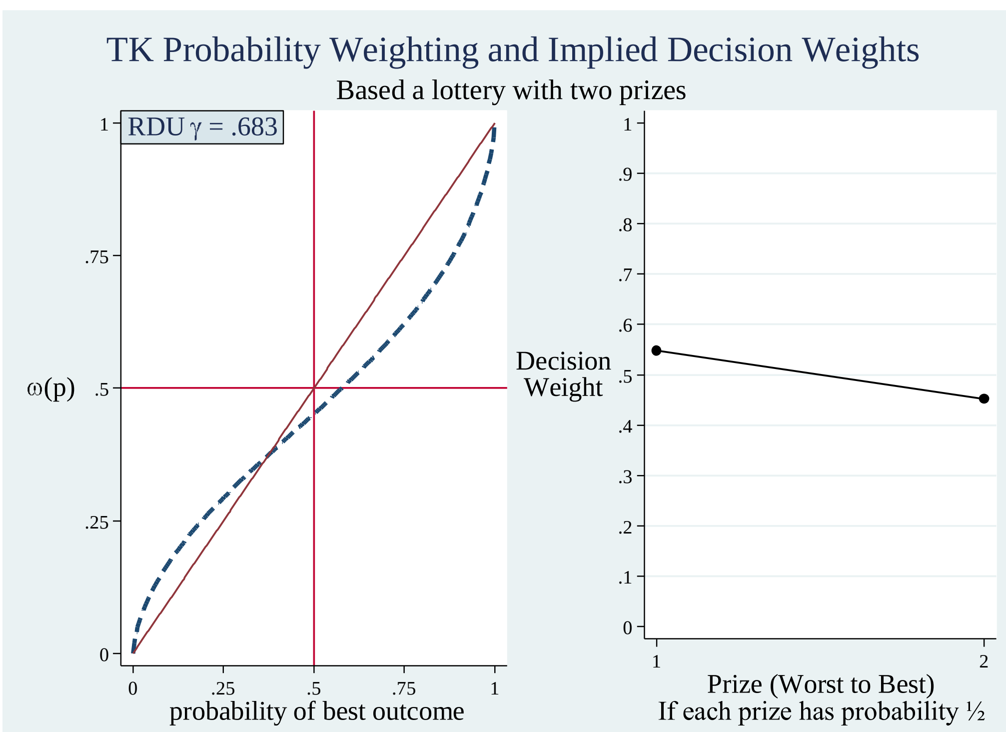


Figure C7.1: Estimate Rank Dependent Utility Power Weighting for Pure Risk Lottery

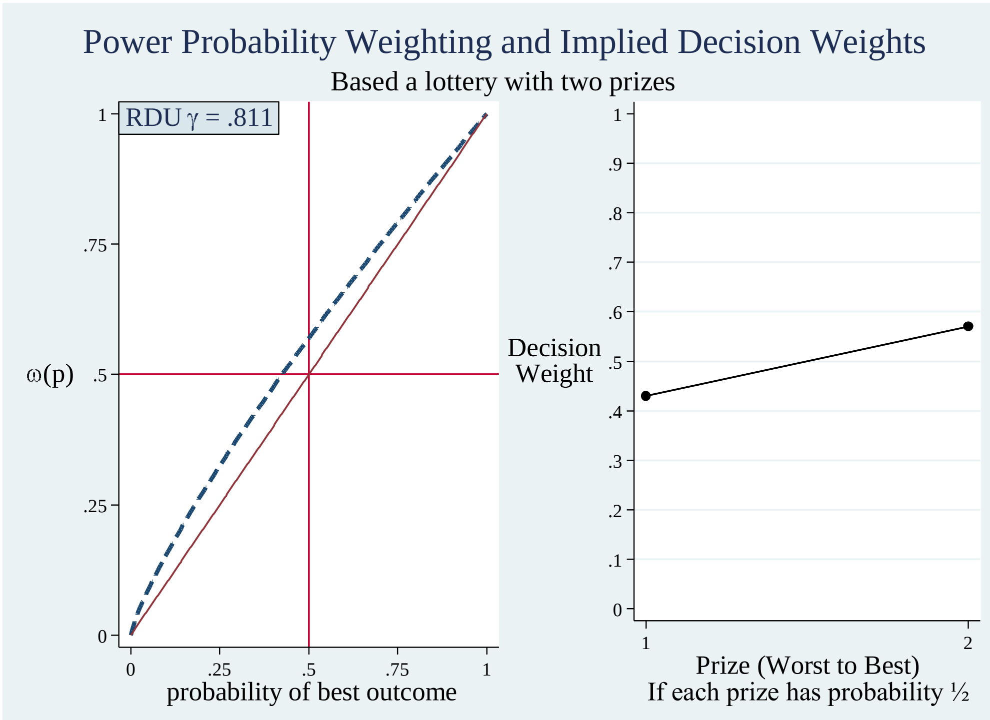
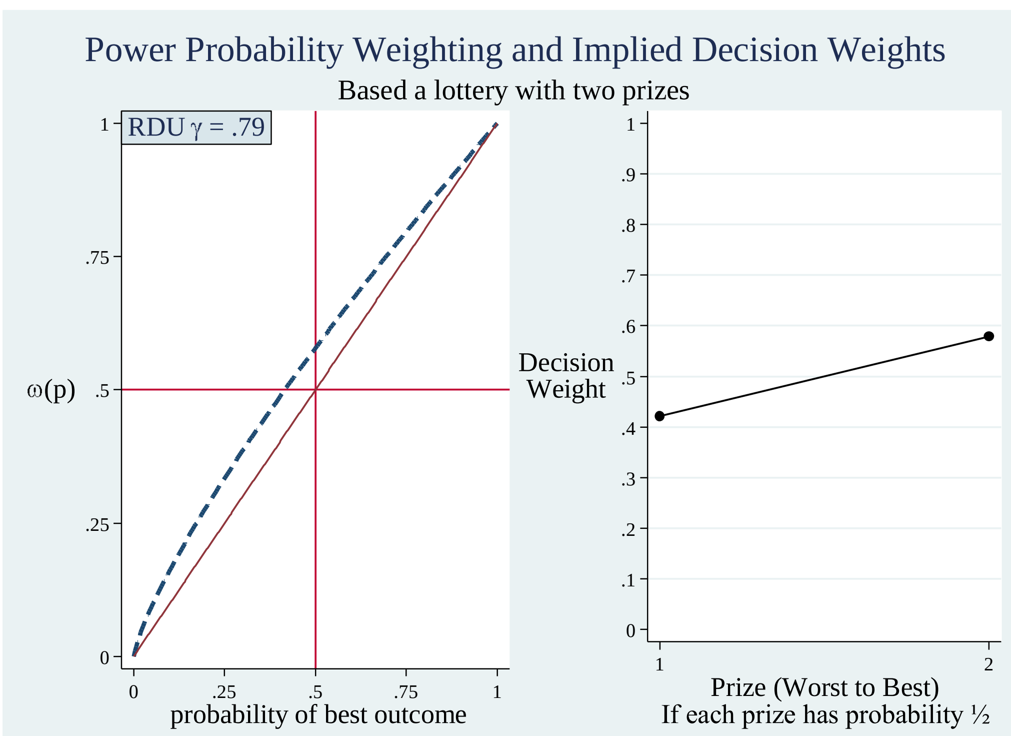


Figure C7.2: Estimate Rank Dependent Utility Power Weighting for Scrambled Lottery



CONCLUSION

The goal of Chapters 1 and 2 was to examine drivers' perception of the risk of delay as one factor that influences route choice behavior in a simulated driving environment. Both experiments recruited commuters from the Atlanta and Orlando metropolitan areas and present them with a route choice task in a driving simulator. Subjects were required to make a binary choice between a route that has an uncertain level of congestion and an alternate route with no risk of congestion. The task is repeated over ten periods. Apart from some prior information about the frequency of congestion on the uncertain route, drivers only obtained additional information if they actually chose to drive on it. Information feedback is therefore endogenous and high risk scenarios can lead to less belief updating than low risk scenarios, since drivers are more likely to avoid taking the uncertain route when it is riskier. The experiment implements four risk treatments that differ in the objective risk of congestion across a range of probabilities. This allows the examination of belief formation and adjustment across these treatments.

Chapter 1 examines belief formation under a discrete penalty setting, whereas Chapter 2 examines belief formation under a continuous penalty setting. In Chapter 1 the belief estimate refers to the belief of delay, and does not distinguish between the belief of congestion and the belief of delay with and without congestion. Assuming the Subjective Expected Utility model, this belief of delay is estimated conditional on period fixed effects for the ten driving periods in the experiment. In Chapter 2 separate belief estimates are derived for congestion and the distribution of travel times for each route and congestion scenario. The estimation is performed by pooling across all driving periods.

Across the risk treatments, subjects appear to be able to discern the difference between low-congestion risk *vs*. high-congestion risk. This is true for both continuous and discrete penalties. For Chapter 1, subjects in the lowest risk treatment experience significant belief adjustments in later periods only, whereas in the high risk treatments no belief adjustment is observed. This behavior is expected: in a low risk treatment subjects are more likely to start with a prior belief of low congestion, and are therefore more likely to drive on the uncertain route. This should allow them to obtain more information about the uncertain route so to revise their prior belief.

In Chapter 2, where data is pooled across periods, differences in learning is inferred by comparing the standard deviation of the inferred travel time distributions across risk treatments. We find no significant difference and conclude that there is no evidence for differences in learning.

Do subjects react to changes in the toll differently across the two penalty settings? In Chapter 1 in the Probit regression, the coefficient Toll has the theoretically expected positive effect but is only significant in the two low risk treatments, which suggests subjects in the high risk treatments is less responsiveness to the variations in Toll than subjects in the lowest risk treatments. This difference in the responsiveness to Toll across the risk treatments is consistent with behavior under an endogenous information environment: subjects in the high risk treatments are more likely to start with a high belief of congestion for the uncertain route, so they will be more likely to drive on the safe route (i.e., the toll road), which means that the effectiveness of Toll will be smaller for these subjects. In contrast, in Chapter 2 the effect of Toll is significant in all risk treatments.

Given that subjects behave differently under these two penalty settings, what is implied for transportation policies? The discrete and continuous penalty settings naturally apply to different types of travelers who face different penalties for late arrivals, and each type is reported to have different responsiveness to variations in the toll. This suggests that if one is able to identify travelers by the type of penalties they face, then one may be able to better evaluate how different types of travelers would react to the change in a congestion pricing policy.

Congestion pricing policies typically employ variations in the toll as a way to redirect and optimize traffic flow. When travelers have limited response to changes in the toll, it would render the policy ineffective in redirecting traffic flow. In addition, if travelers are able to *learn* the underlying objective probability of different traveling outcomes and make decisions optimally, then it would help optimize traffic flow and improve welfare (even in the event that they are not responsive to changes in the toll by a congestion pricing policy). The most problematic case that is identified by our experiments is when the travelers are not responsive to changes in the toll *nor* are they able to learn the underlying probabilities of different traveling outcomes, which would imply that policy makers will need to employ other measures to redirect traffic flow (e.g., by actively providing credible traffic information). This is especially the case for the group of subjects who face a discrete late penalty (Chapter 1) and who are in a scenario where the underlying objective probability of congestion is high. For this group of subjects, we observe a lack of learning across driving periods as well as a lack of responsiveness to variations in the toll. On the other hand, for the group of subjects who face a continuous penalty (Chapter 2) whichever risk scenario they are in, even though these subjects do not display learning, they do display a significant response to variations in the toll. This suggests that congestion pricing policy will be more effective in redirecting traffic flow for this latter group of subjects than for the previous group.

Following a similar line of research as in Chapters 1 and 2, Chapter 3 also examines risk perception for events with unknown probabilities. Furthermore, it examines how subjects perceive uncertainty between events that have the same underlying objective probability but are presented differently under varying degrees of uncertainty. Using a within-subject design subjects are asked to complete three lottery tasks that are ranked in order of increasing uncertainty. Their choices are analyzed using the "source method". Based on the choices subjects make, a source function is estimated for each lottery task using three different specifications of the source function. Overall, the results are consistent with past studies that do not use the source method, in showing that behavior differs under events with known vs. unknown probabilities. In particular, when probabilities are not known subjects behave in the direction of likelihood insensitivity, in the sense that they overweigh low probabilities and underweigh high probabilities. The source method assumes that all behavioral differences for alternative sources is characterized by differences in probability weighting, and hence in the form and parametric values of different probability weighting functions.

However, when comparing the difference in behavior between events that both have unknown probabilities but vary in the degree of uncertainty, the behavioral difference is better captured by a Prelec specification of the probability weighting function than the Tversky-Kahneman (1992) or Power probability weighting function. Results from the Prelec specification suggest that as the degree of uncertainty increases, subjects display increased pessimism, whereas the Tversky-Kahneman (1992) and the Power specifications show no such difference. The implication of this result is that conclusions regarding uncertainty aversion are contingent on which specification is assumed for the source function (i.e., the probability weighting function).

One general conclusion regarding behavior under uncertainty is that subjects are not very good at learning the true probability of the uncertain events, and that under uncertainty they have a tendency to place equal decision weights on all possible outcomes.

Appendix AA: More Tables for Chapter 1

Table AA1: Rank Dependent Utility Estimates Across Periods

	Treatment 0.2 N=31	**Treatment 0.4 N=40**	**Treatment 0.6 N=32**	**Treatment 0.8 N=38**
Risk Aversion:				
r	.265 (0.333)	.241 (0.574)	.975* (0.078)	.643 (0.469)
γ	1.003*** (<0.001)	.767** (0.040)	1.72* (0.068)	1.173 (0.354)
Beliefs:				
Constant	.629*** (<0.001)	.630*** (<0.001)	1*** (<0.001)	1 (a)
Period 2	-.233 (0.181)	-.052 (0.651)	-.00005 (0.949)	<.001 (0.979)
Period 3	-.160 (0.331)	.184 (0.107)	-.003 (0.706)	<.001 (0.971)
Period 4	-.211 (0.169)	-.073 (0.418)	-.002 (0.738)	<.001 (0.975)
Period 5	-.290 (0.102)	.033 (0.540)	-.003 (0.728)	<.001 (0.977)
Period 6	-.160 (0.331)	-.0004 (0.997)	-.004 (0.714)	<.001 (0.975)
Period 7	-.090 (0.541)	.040 (0.750)	-.003 (0.734)	<.001 (a)
Period 8	-.467*** (0.000)	-.107 (0.291)	-.0007 (0.842)	<.001 (a)
Period 9	-.089 (0.618)	-.109 (0.265)	-.004 (0.719)	<.001 (a)
Period 10	-.199 (0.220)	.078 (0.569)	-.001 (0.752)	<.001 (a)
Prior	.010 (0.848)	-.029 (0.549)	-.0002 (0.818)	<.001 (a)
***μ*RA**	.175*** (<0.001)			
***μ*Belief**	.234*** (<0.001)			

The results are obtained using a joint estimation of risk attitudes and beliefs from the lottery data and the driving simulator data. *p*-values are in parentheses. The coefficients are marginal effects computed using the delta method.
*** means that the coefficient is significant at the 1% level.
** means that the coefficient is significant at the 5% level.
* means that the coefficient is significant at the 10% level.
(a) implies that a standard error cannot be computed by the delta method due to numeric issues, because the estimated probabilities approach 0 or 1.
μRA is the Fechner error for the lottery data; μBelief is the Fechner error for the belief data.

Table AA2: Rank Dependent Utility Estimates Across Periods and Demographic Effects

	Treatment 0.2 N=31	**Treatment 0.4 N=40**	**Treatment 0.6 N=32**	**Treatment 0.8 N=38**
Risk Aversion:				
r	.309 (0.265)	.149 (0.762)	.198 (0.850)	.769 (0.481)
γ	1.053*** (<0.001)	.693* (0.073)	.691 (0.441)	1.366 (0.436)
Beliefs:				
Constant	.850*** (<0.001)	.543*** (0.007)	.971*** (<0.001)	.758** (0.028)
Period 2	-.255 (0.127)	-.054 (0.659)	-.044 (0.762)	.092 (0.762)
Period 3	-.169 (0.200)	.208 (0.113)	-.125 (0.567)	-.119 (0.754)
Period 4	-.221 (0.230)	-.077 (0.427)	-.175 (0.561)	-.202 (0.582)
Period 5	-.290* (0.064)	.035 (0.562)	-.162 (0.474)	-.355 (0.517)
Period 6	-.169 (0.200)	.001 (0.996)	-.055 (0.713)	.078 (0.829)
Period 7	-.102 (0.329)	.052 (0.698)	-.281 (0.463)	-.089 (0.770)
Period 8	-.536** (0.018)	-.113 (0.290)	-.104 (0.456)	.100 (0.745)
Period 9	-.106 (0.428)	-.113 (0.298)	-.055 (0.713)	-.202 (0.561)
Period 10	-.202 (0.132)	.097 (0.485	-.137 (0.425)	.096 (0.764)
Female	.065 (0.295)	.012 (0.951)	.029 (0.829)	-.475 (0.719)
College Education	-.093 (0.470)	-.014 (0.948)	-.087 (0.779)	.234 (0.518)
High income (>$100K)	-.212 (0.131)	-.078 (0.703)	-.053 (0.770)	.242 (0.483)
μRA	.173*** (<0.001)			
μBelief	.225*** (<0.001)			

p-values are in parentheses. The coefficients are marginal effects.
*** means that the coefficient is significant at the 1% level.
** means that the coefficient is significant at the 5% level.
* means that the coefficient is significant at the 10% level.
(a) implies that a standard error cannot be computed by the delta method due to numeric issues, because the estimated probabilities approach 0 or 1.
μRA is the Fechner error for the lottery data; μBelief is the Fechner error for the belief data.

Figure AA1: Distribution of Wage by Congestion Risk

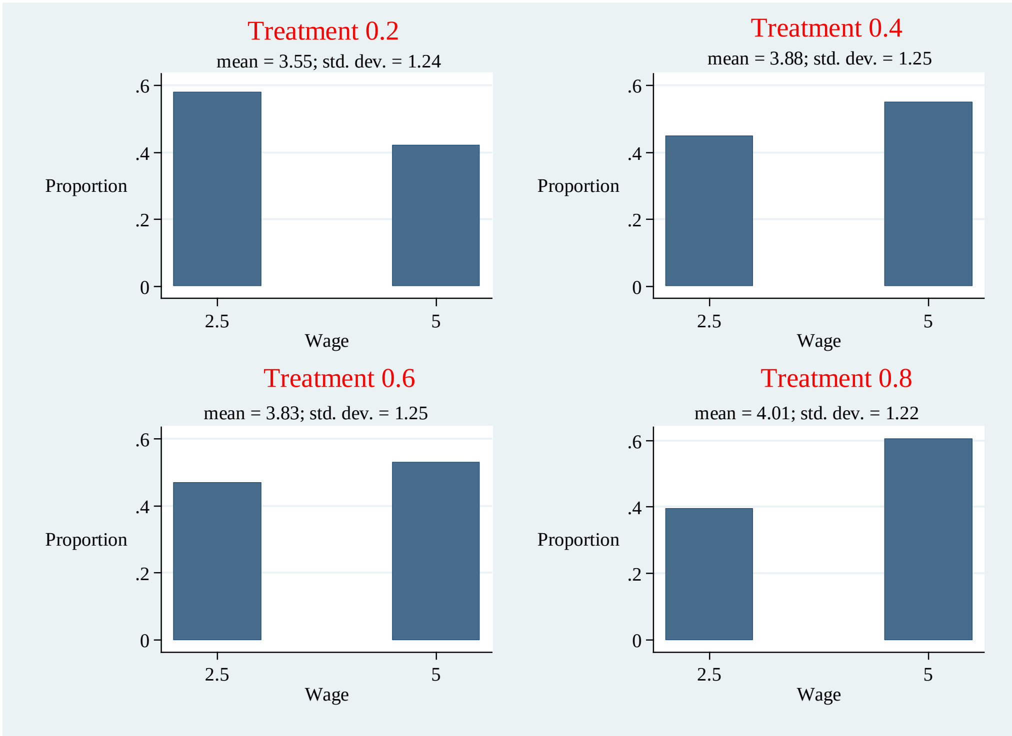


Figure AA2: Distribution of Penalty by Congestion Risk

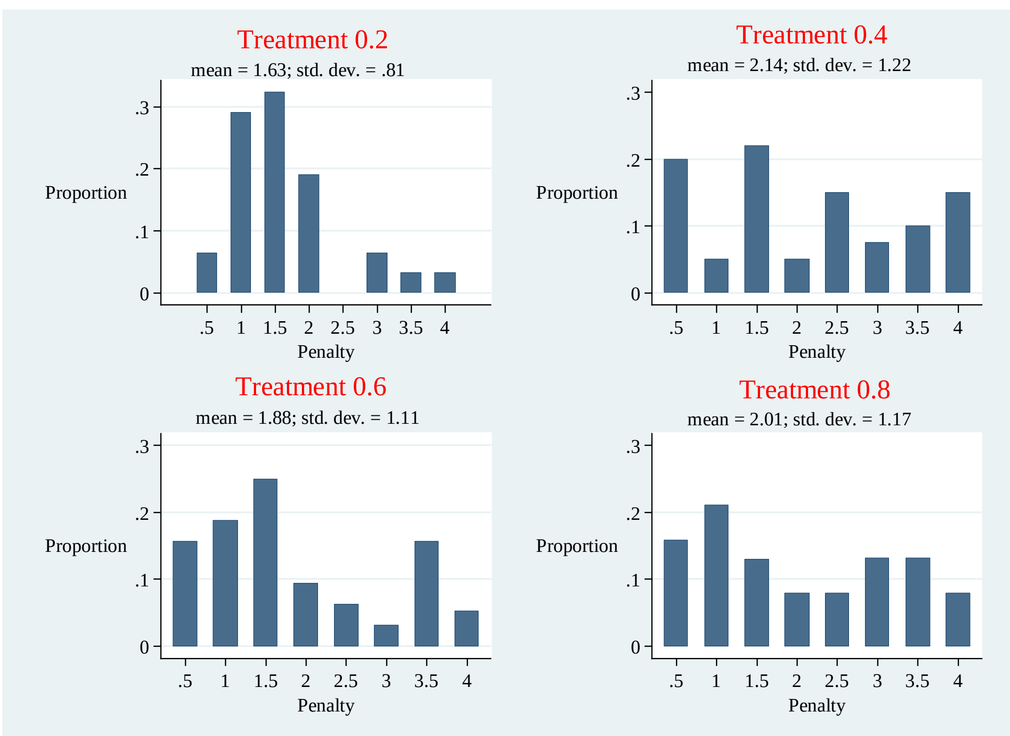


Figure AA3: Distribution of Toll by Congestion Risk

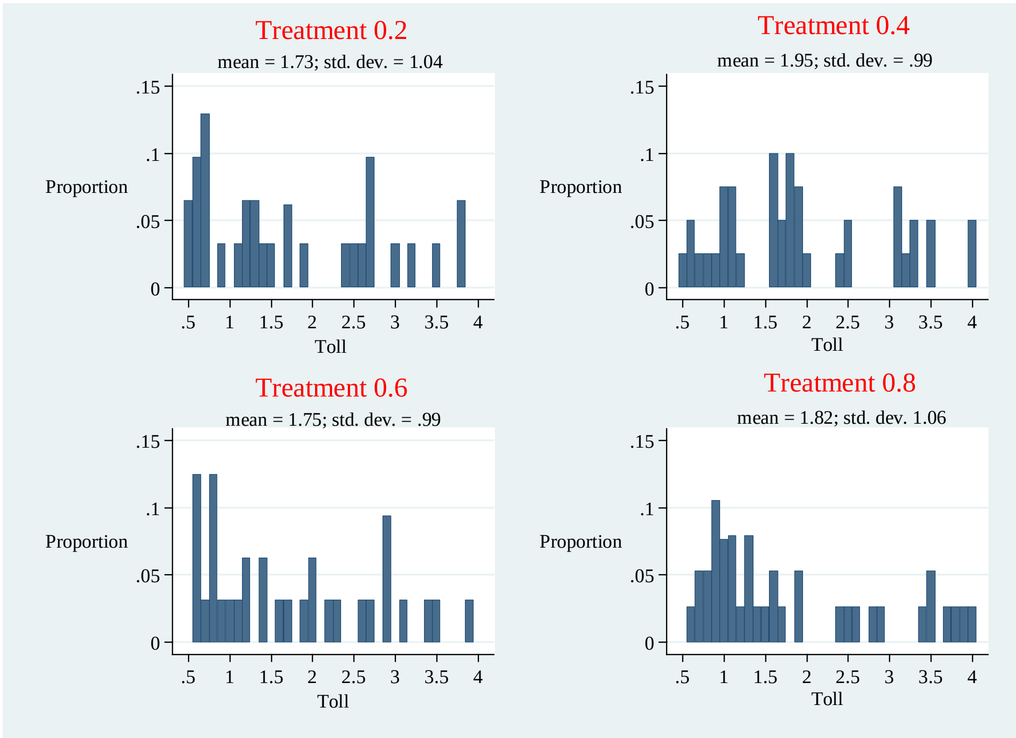
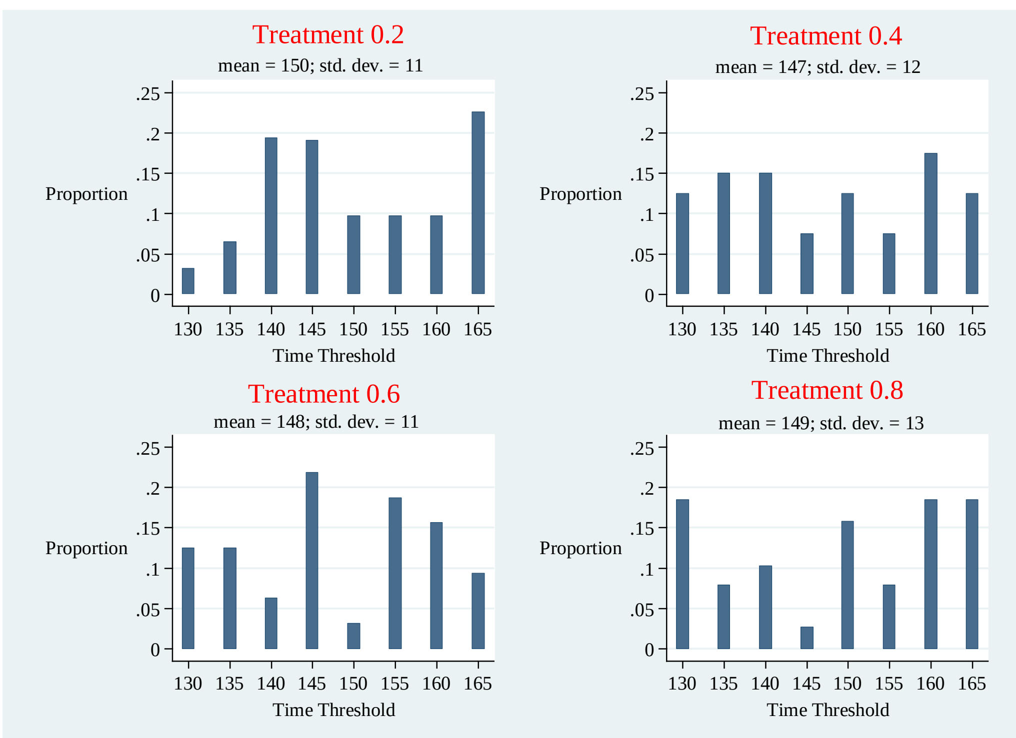


Figure AA4: Distribution of Time Threshold by Congestion Risk



Appendix BB: More Figures for Chapter 3

Figure BB1.1: Rank Dependent Utility Prelec Weighting, η =1 and φ =1

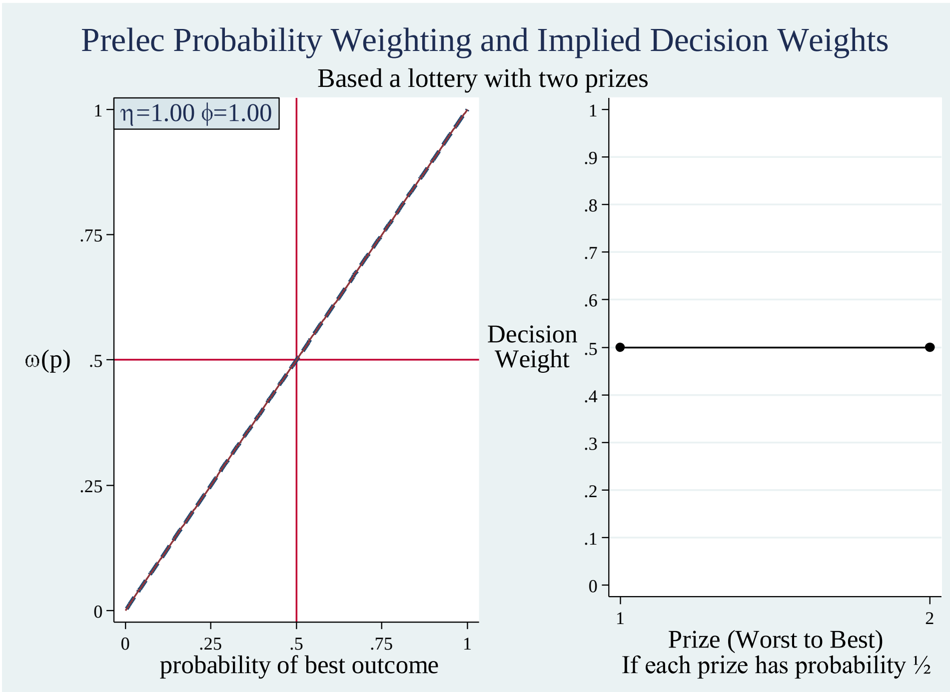


Figure BB1.2: Rank Dependent Utility Prelec Weighting, η =1 and φ =0.50

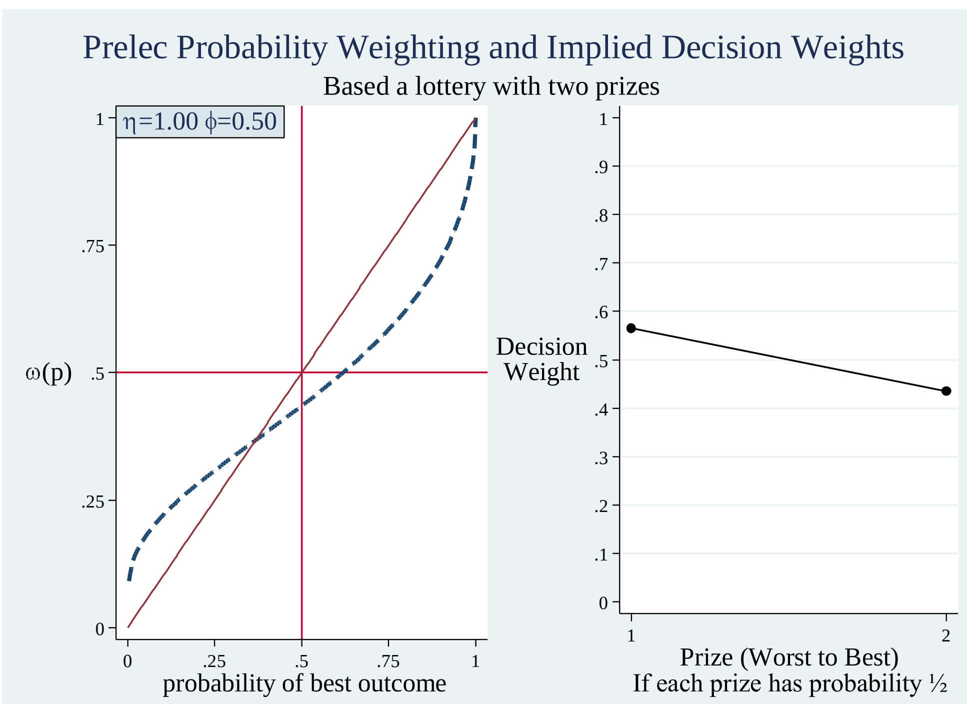


Figure BB1.3: Rank Dependent Utility Prelec Weighting, $\boldsymbol{\eta}$ =1 and $\boldsymbol{\varphi}$ =1.50

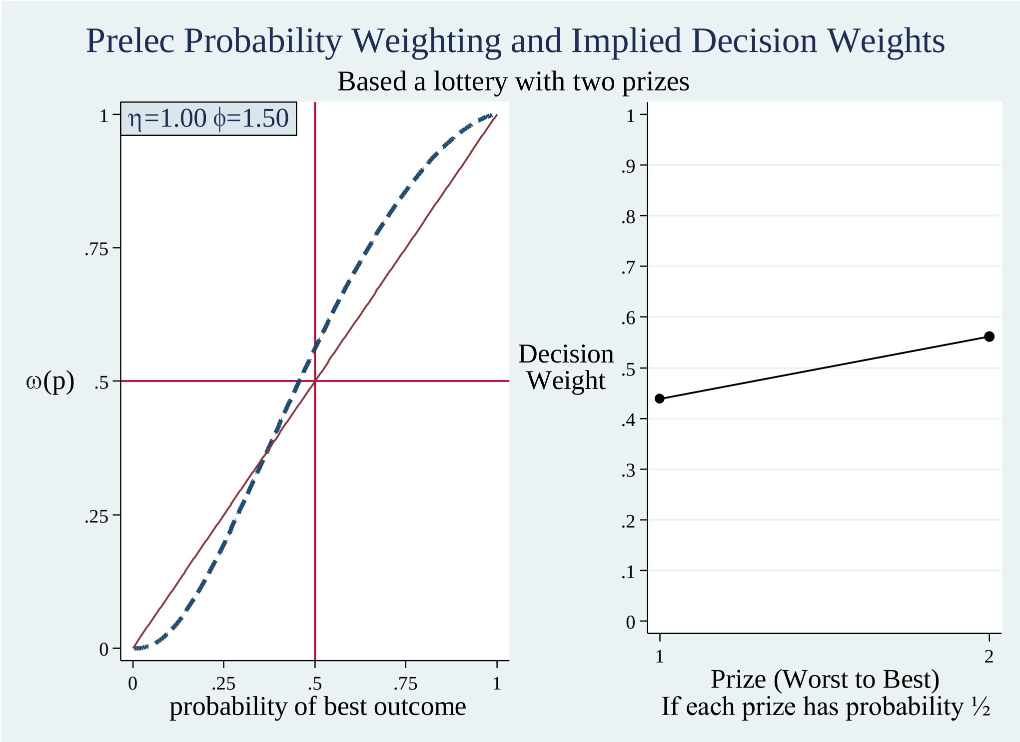


Figure BB1.4: Rank Dependent Utility Prelec Weighting, $\boldsymbol{\eta}$ =0.50 and $\boldsymbol{\varphi}$ =1

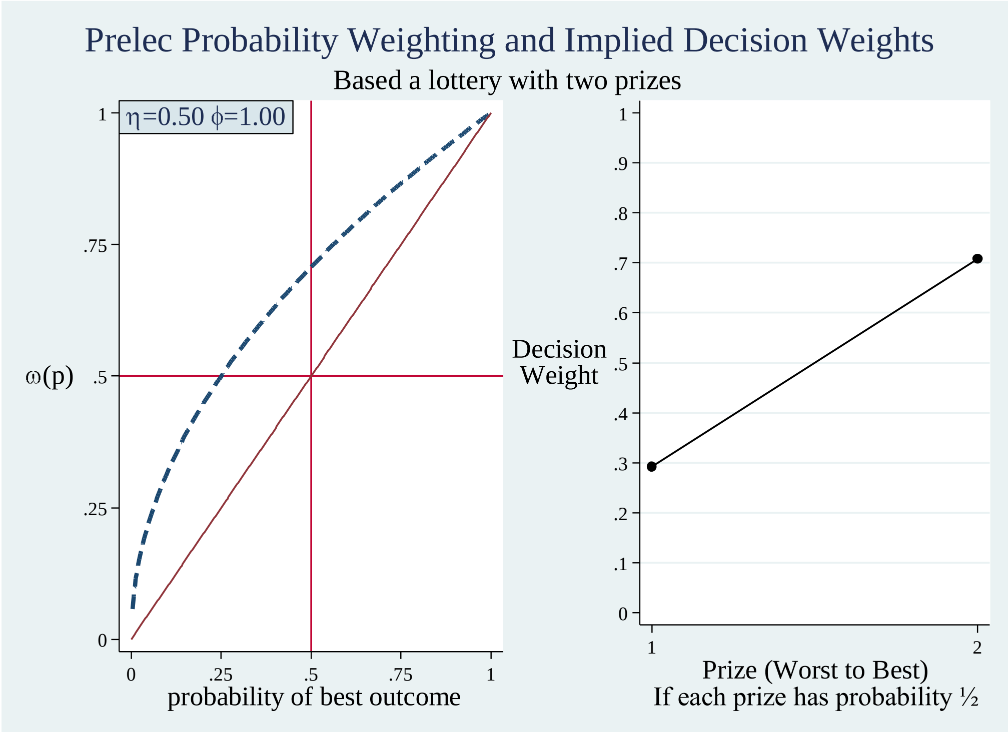


Figure BB1.5: Rank Dependent Utility Prelec Weighting, $\boldsymbol{\eta}$ =1.50 and $\boldsymbol{\varphi}$ =1

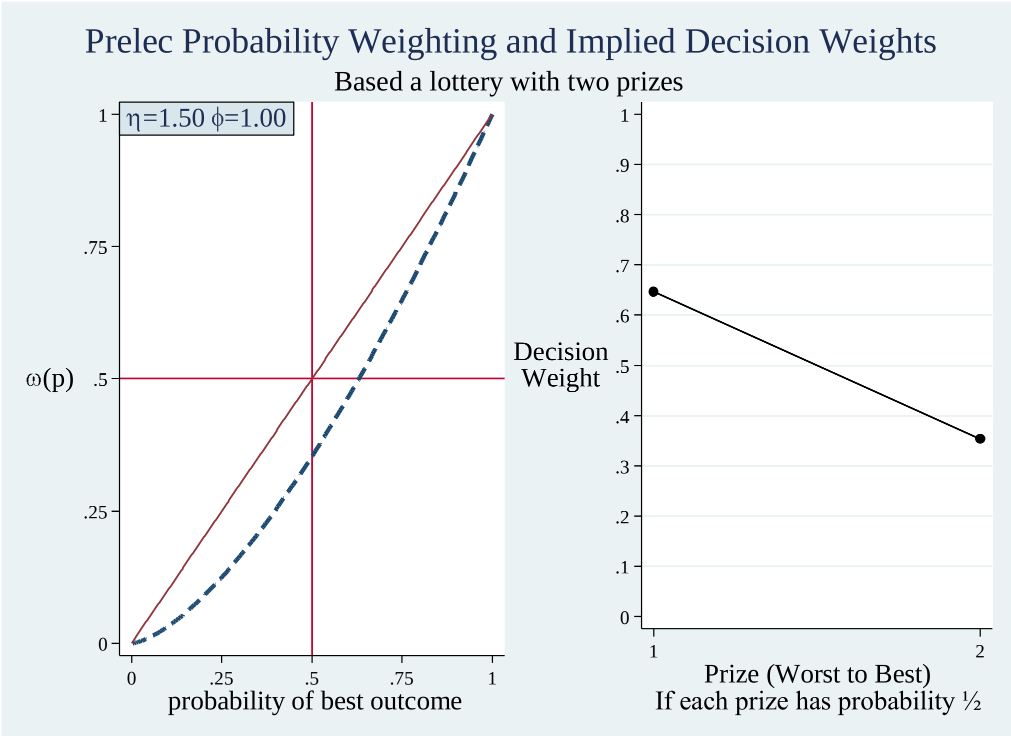


Figure BB2.1: Rank Dependent Utility TK Weighting, γ = 1

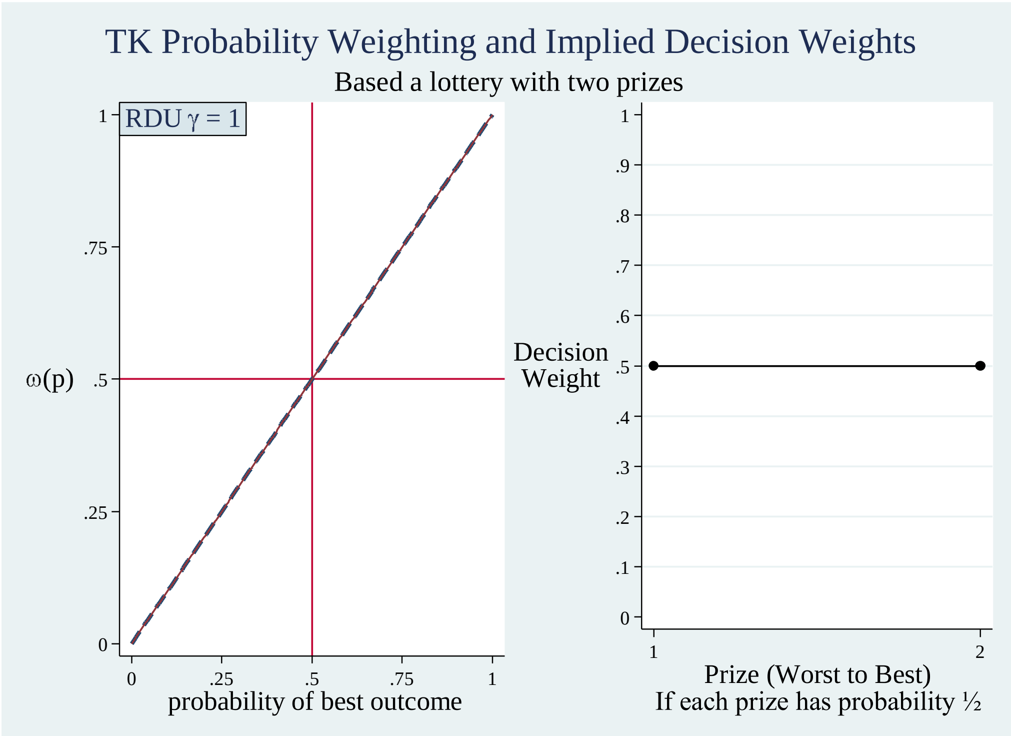


Figure BB2.2: Rank Dependent Utility TK Weighting, γ = 1.8

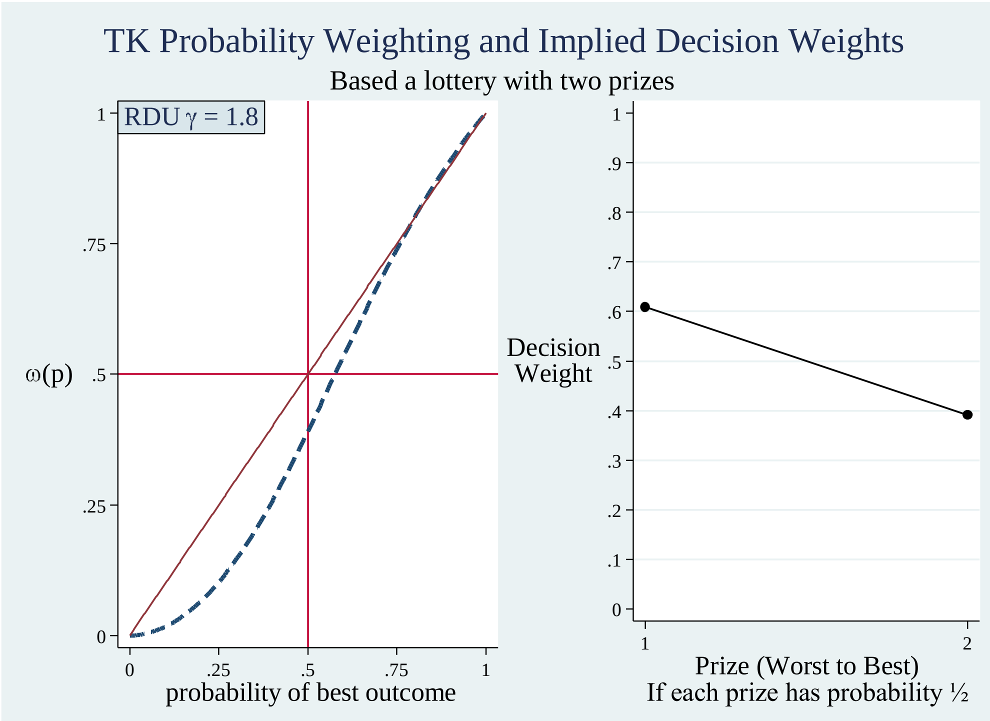


Figure BB2.3: Rank Dependent Utility TK Weighting, γ = 1.5

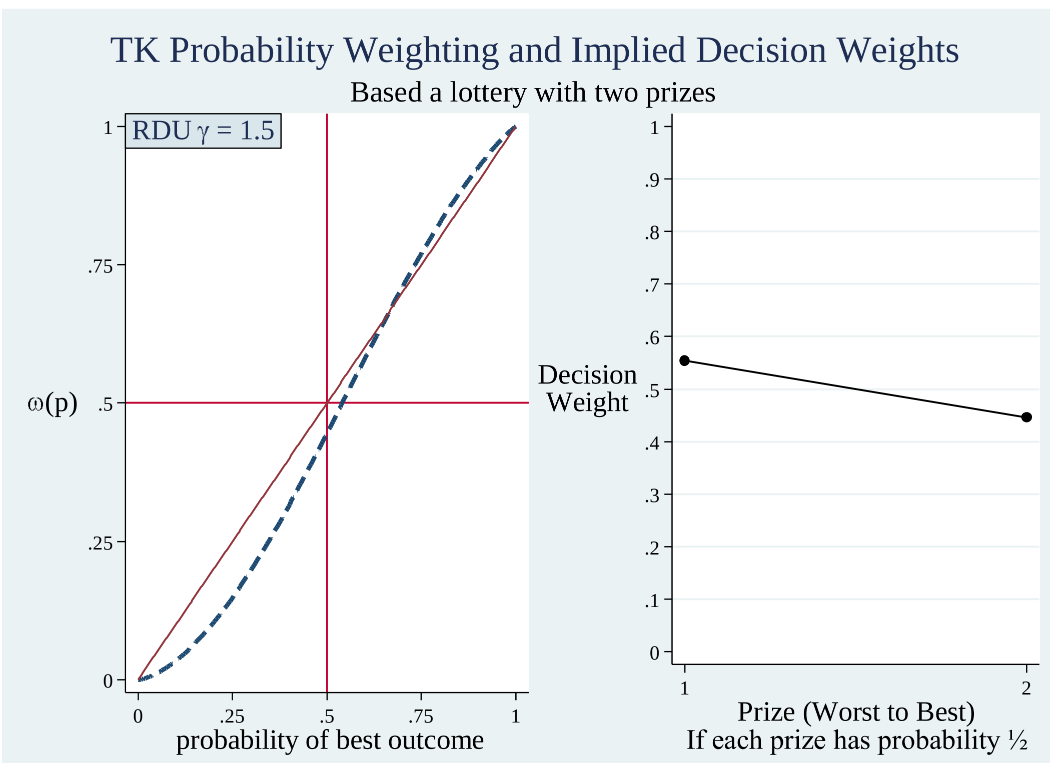


Figure BB2.4: Rank Dependent Utility TK Weighting, γ = 1.2

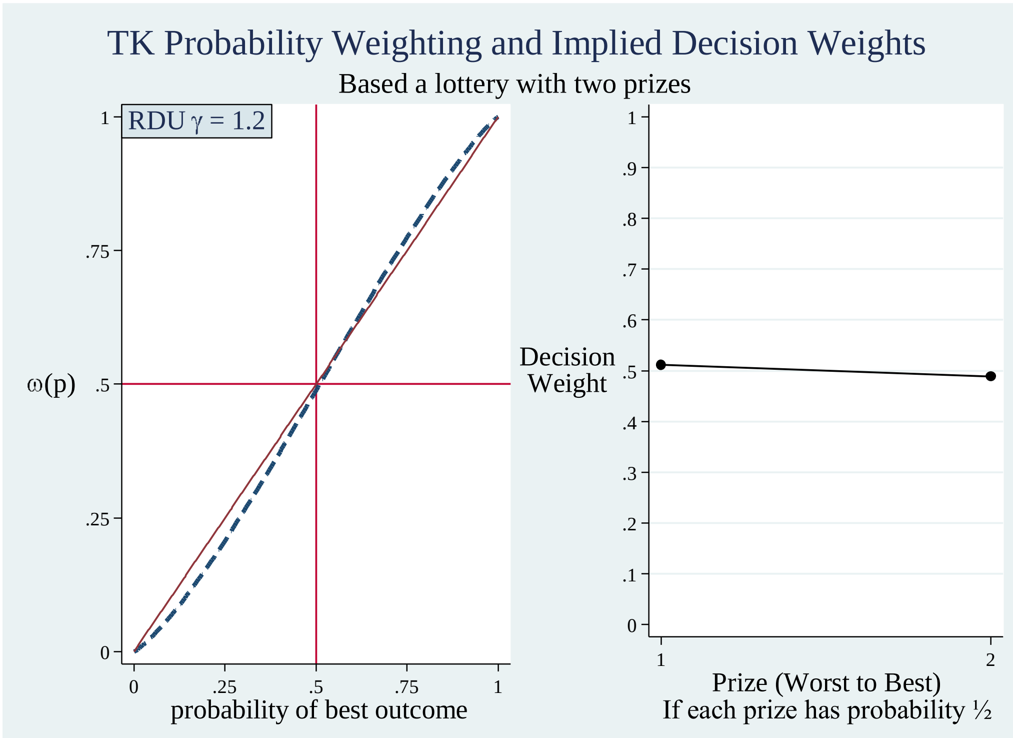


Figure BB2.5: Rank Dependent Utility TK Weighting, γ = 0.8

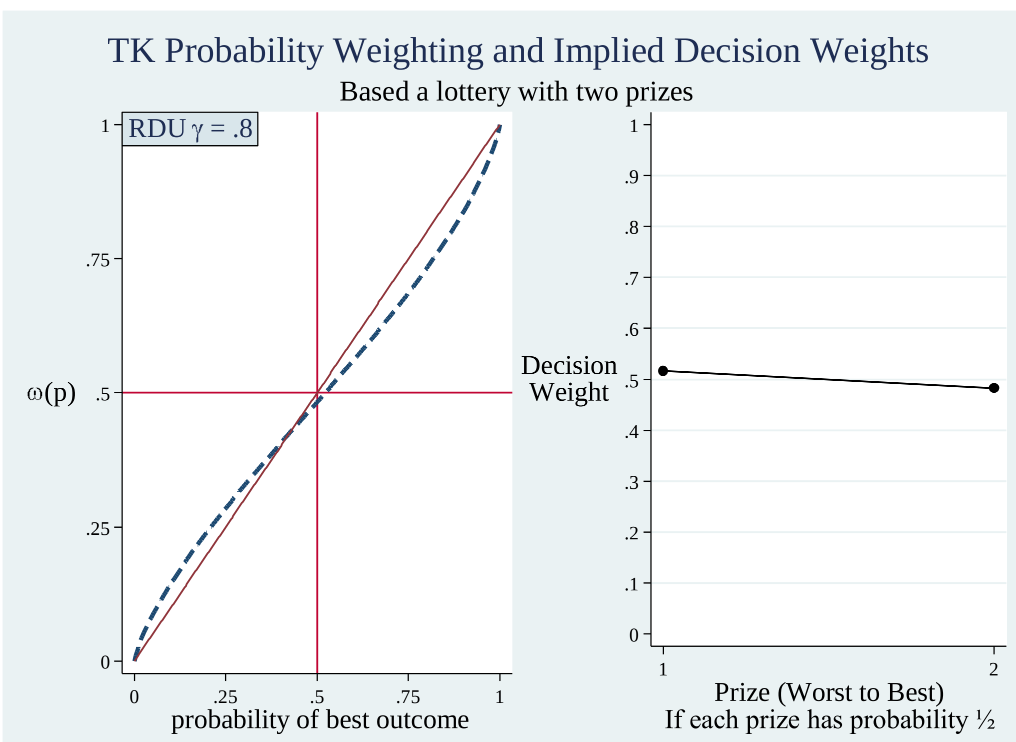


Figure BB2.6: Rank Dependent Utility TK Weighting, γ = 0.5

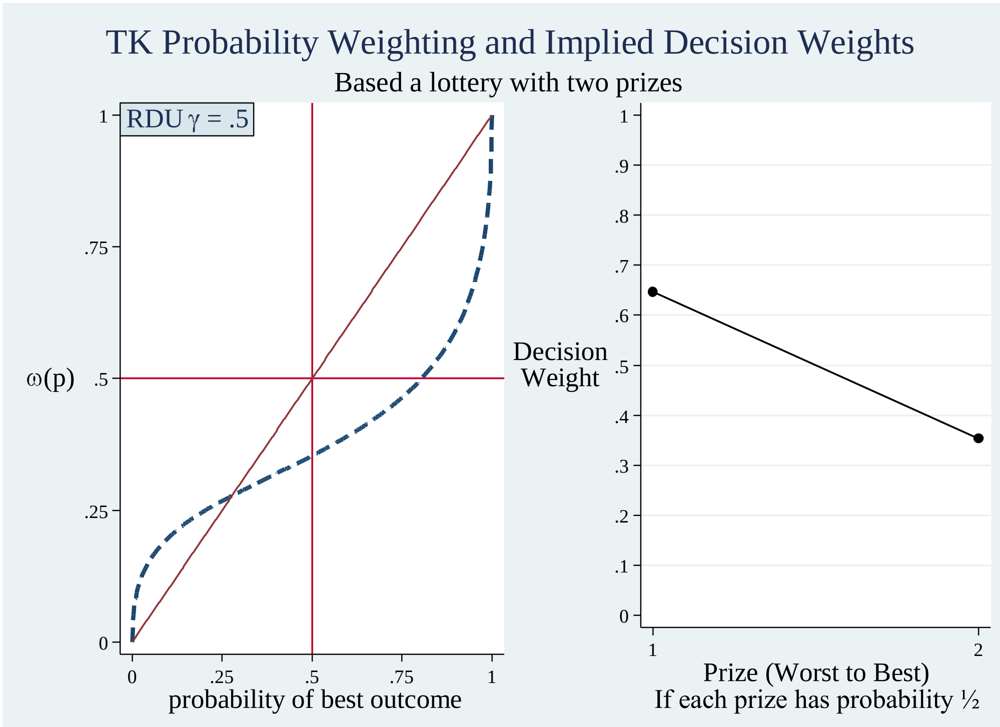


Figure BB2.7: Rank Dependent Utility TK Weighting, γ = 0.2

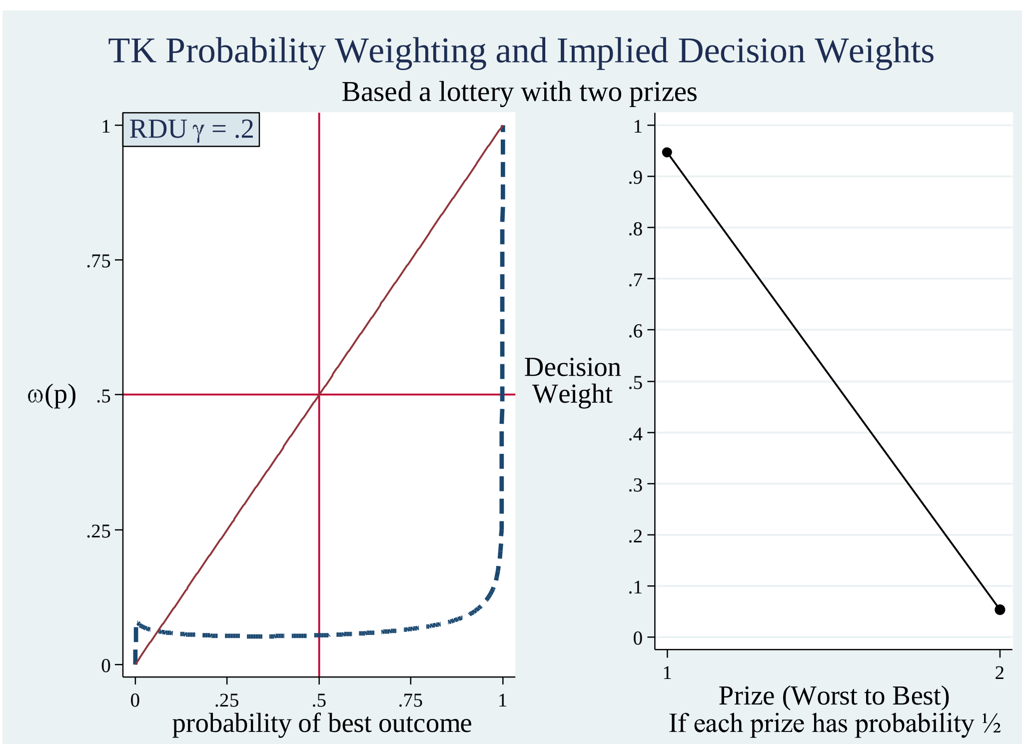


Figure BB2.8: Rank Dependent Utility TK Weighting, γ = 2.5

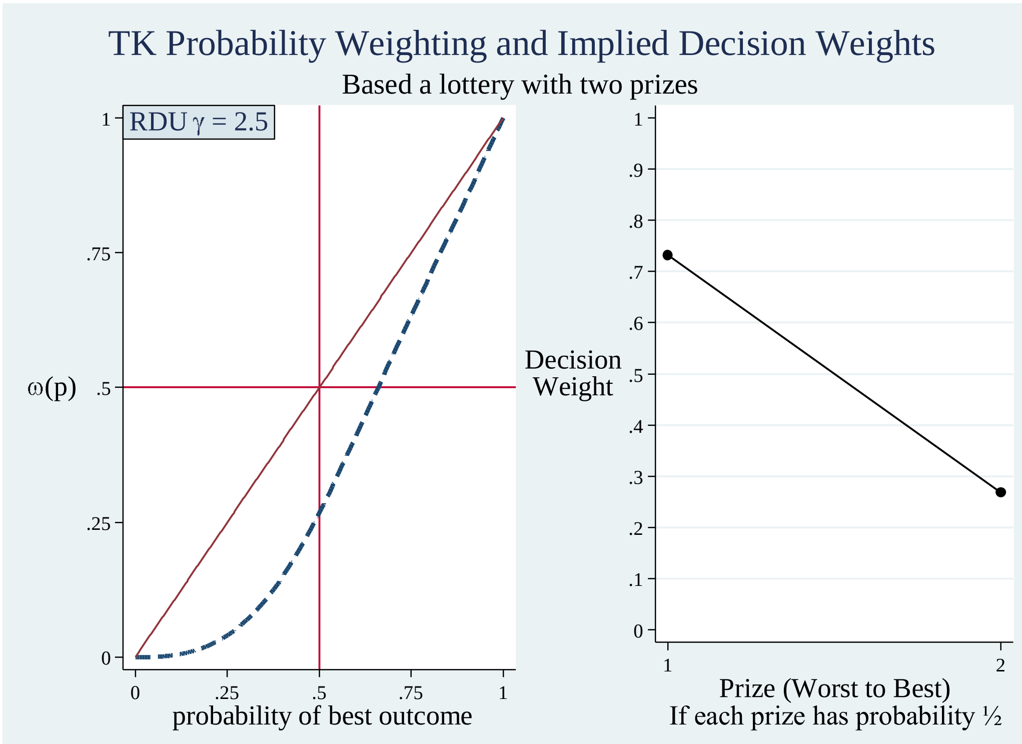


Figure BB3.1: Rank Dependent Utility Power Weighting, γ = 1

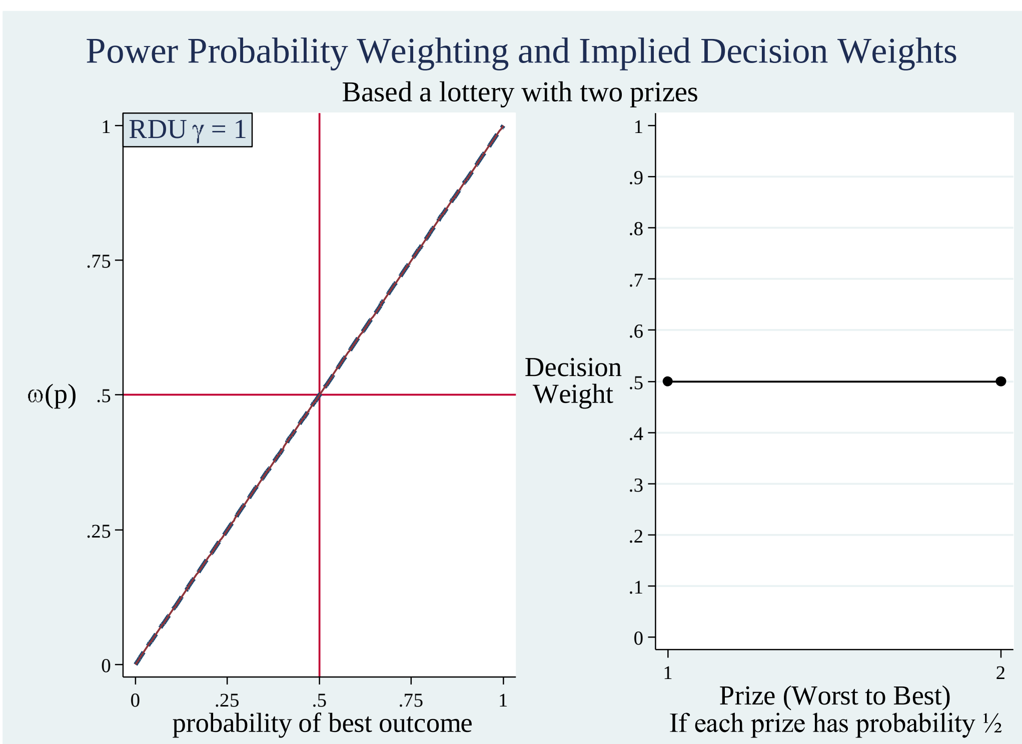


Figure BB3.2: Rank Dependent Utility Power Weighting, $\gamma = 1.5$

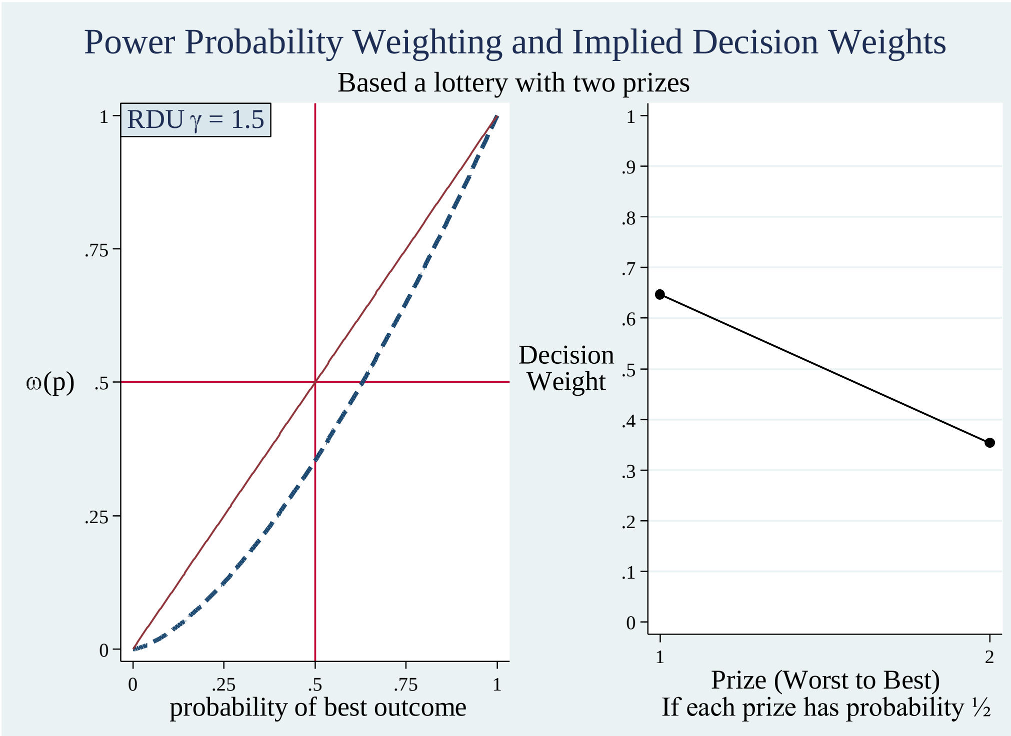
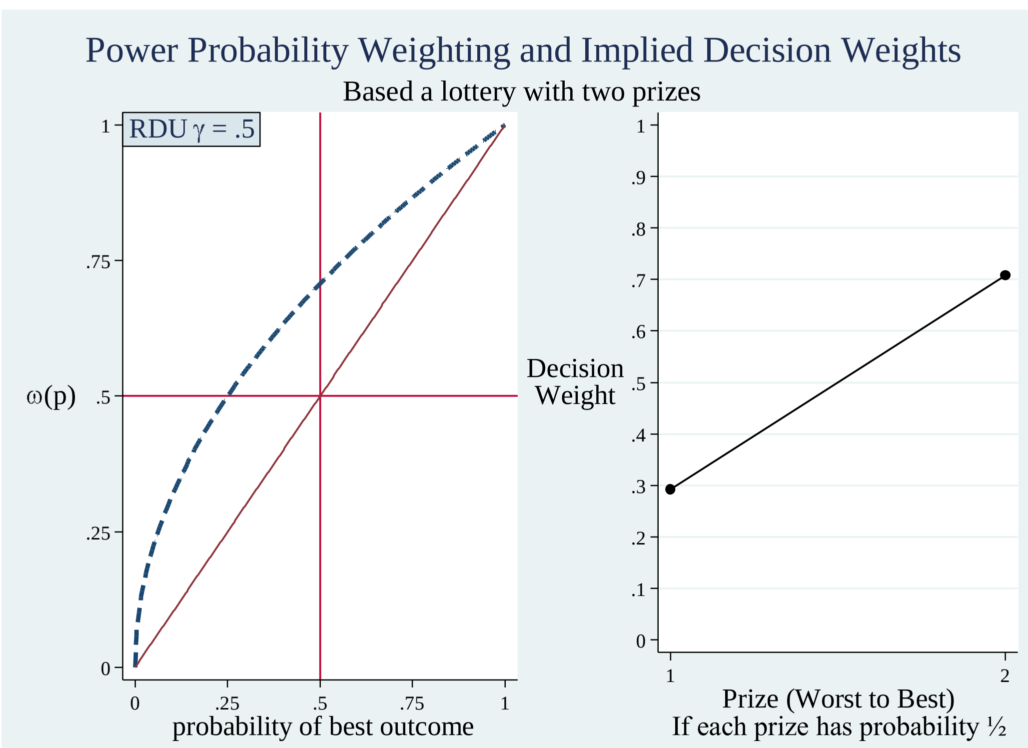


Figure BB3.3: Rank Dependent Utility Power Weighting, $\gamma = 0.5$



Appendix CC: More Literature Review for Chapter 3

This appendix provides a review of a number of recent ambiguity experiments.[67] The implementation of ambiguity varies, such as the use of a bingo blower or an urn containing balls with unknown proportions. The choice tasks may include allocating tokens across events with known or unknown probabilities, stating a reservation price for a lottery using the Becker–DeGroot–Marschak method, or choosing between prospects in a list format to elicit certainty equivalents. An overview is provided with respect to experimental design, model specifications and findings, and followed by a more detailed description of each study.

For the purpose of this review, models of uncertainty or ambiguity are categorized into two general specifications: the smooth specification (Klibanoff, Marinacci and Mukerji (2005)) and the kinked specification. The smooth specification (or the smooth model) considers a set of priors for the possible outcomes, plus separate utility parameters to capture attitudes toward risk and attitudes toward uncertainty. In evaluating any two-stage lotteries, the smooth model takes the EU of each one-stage lottery within each prior and then takes the Expected Value of all EUs across the possible priors. The kinked specification also considers a set of priors but the unknown probabilities are skewed by decision weights. The kinked specification includes models such as Choquet EU (Gilboa (1987) and Schmeidler (1989)), Rank Dependent Utility (Quiggin (1982)), α-MEU (Ghirardato et al. (2004)),[68] Vector EU (Siniscalchi (2009)), and Contraction EU (Gajdos et al. (2008)).

[67] For the experiments that are described in this appendix, subjects are provided with some information about the probability distribution on the unknown events, instead of a complete lack of information. By definition, these experiments should be categorized as uncertainty experiments (instead of ambiguity experiments). However, since these authors refer to their experiments as testing ambiguity instead of uncertainty, for purpose of discussing their studies we use the term ambiguity so to be consistent with the labeling.
[68] An alternative name is the Alpha-Maxmin EU model or Alpha EU (Ghirardato et al. (2004)). We use the name α-MEU throughout this essay.

The descriptive power of the specifications is contingent on the data set that is used to fit the models and the types of questions that are asked in the experiment. For example, in experiments that involve one-stage probabilities (or simple lotteries), the α-MEU model performs better relative to the smooth model; in experiments that involve two-stage lotteries (or compound lotteries) where the second-order probability distribution may or may not be known, the smooth model performs better. For these findings, see Ahn et al. (2010), Bossaert et al. (2010), Cubitt et al. (2012) and Attanasi et al. (2014).

A number of studies report that SEU outperforms many ambiguity models in explaining choices. For example, Hey and Pace (2014) report that SEU is just as good a predictor of observed choices as the more complicated ambiguity models (such as Choquet EU, α-MEU, Vector EU, and Contraction EU); Ahn et al. (2010) report that SEU explains the majority of the observed choices better than the kinked or smooth specification; Mangelsdorff and Weber (1994) report that SEU, assuming the principle of insufficient reason,[69] is a better predictor of the observed choices than Choquet EU.

The proportion of subjects who are ambiguity averse varies widely across experiments, relative to the proportion of subjects who are ambiguity neutral or ambiguity seeking. This may not come as a surprise given that experiments vary with respect to the implementation of ambiguity, elicitation approach, and econometric specifications. Furthermore, when examining the correlation between risk premium and ambiguity premium (i.e., when comparing the risk aversion and ambiguity aversion parameters), some studies report a positive correlation whereas

[69] "Principle of insufficient reason" or the "principle of indifference" states that if one is ignorant of the ways an event can occur, the event will be assumed to occur equally likely in any way (first enunciated by Jakob Bernoulli). Mangelsdorff and Weber (1994) use the term "principle of insufficient reason" when referring to the assumption of equal probabilities for all outcomes.

others report a negative correlation. For instance, see Bossaert et al. (2010), Cubitt et al. (2012), and Attanasi et al. (2014).

The rest of the appendix describes in detail several selected studies. The first group of studies consists of experiments with one-stage probability (Ahn et al. (2010) and Bossaert et al. (2010)), and the second group consists of experiments with two-stage probabilities (Conte and Hey (2013), Cubitt et al. (2012) and Attanasi et al. (2014)).[70]

C1 Ahn, Choi, Gale and Kariv (2010)

In this study subjects are presented with a portfolio choice task where they are asked to allocate tokens across three accounts given a fixed number of tokens, and each account corresponds to a state of nature that is to be selected at random. The first account has 1/3 probability of success, and the other two have probability of success that sum up to 2/3. The allocation across the three account is made in one choice. Subjects are asked to complete 50 allocation choice tasks. One of the choices is randomly selected for payment, and tokens are converted to real money. The data are fit using three models: SEU, a kinked specification using the α-MEU model, and the smooth model. For each subject and for each model specification, the parameters are estimated using nonlinear least squares. SEU explains a majority of the observed choices, next is the α-MEU model, and last is the smooth model.

[70] For more review of ambiguity experiments, see Hey (2016) and Etner, Jeleva and Tallon (2010).

C2 Bossaerts, Ghirardato, Guarnaschelli and Zame (2010)

In a competitive market setting, subjects are presented with an opportunity to buy or sell securities. They may choose from two types of securities. One type is a bond that pays a fixed dividend. Another type is stocks, three of them (x, y, and z), that pay dividends randomly but that are negatively correlated, such that in the experiment if state x is realized, stock x pays $0.50 and stocks y and z pay nothing.[71] The payout is determined by a draw from an urn that contains eighteen balls of red, green and blue. The composition of the urn may or may not be known to the subjects depending on which treatment they are assigned to. If they are assigned to the risky treatment, they are told the composition of the urn; if they are assigned to the ambiguous treatment, they are told only the proportion of red balls and the number of balls in total. The experiment consists of eight trading periods, and in each period a ball is drawn without replacement to determine the payout, thus the total number of balls as well as the composition of balls in the urn changes throughout the course of the experiment. Subjects are paid their cumulative earnings.

An α-MEU model is used for the observed choices, and the population appears to be heterogeneous with some being quite ambiguity averse. Furthermore, a positive correlation between risk and ambiguity premiums is reported; i.e., a positive correlation between the risk aversion and ambiguity aversion parameters.

[71] If two stocks are negatively correlated, when the earnings of one increases, the other is likely to decrease, thus the gain in one stock is likely to offset the loss in the other. If the stocks are positively correlated, on the other hand, they tend to rise and fall together.

C3 Conte and Hey (2013)

In this study subjects are presented with two *compound* lotteries. Each compound lottery is made up of a number of *simple* lotteries with binary outcomes (red or blue), and the compositions of the simple lotteries are known to the subjects. The subject's first decision is to choose whether they want to bet on red or blue. The next decision is to choose which of the two compound lotteries they prefer to play out the color they choose. For example, supposed in the first decision a subject choose the color 'red' to bet on, and supposed that the left-side compound lottery has 5 simple lotteries, and each simple lottery consists of majority red, whereas the right-side compound lottery has 3 simple lotteries, and each simple lottery consists of majority blue. In this scenario, the subject would be more inclined to choose the left-side compound lottery to play out his or her chosen color (red), because the left-side compound lottery will have a higher chance of drawing a red relative to the right-side compound lottery.

An interesting design feature in this study is that one compound lottery is designated as the "changing lottery" whereas the other is the "unchanged lottery". After the subject makes a pairwise decision, one of the simple lotteries inside the "changing lottery" disappears. Then in the next round the subject is asked again to make a pairwise decision, except this time the choice is between the *updated* "changing lottery" and the "unchanged lottery". This iterative process continues until the "changing lottery" is left with only one simple lottery. In this way, as the distribution in the "changing lottery" becomes narrower, one can study how the narrowing of possible states affects subject's pairwise decisions.

The experiment consists of 49 tasks with a total of 256 pairwise decisions. To determine the payout, one of the 49 tasks is randomly selected, then conditional on that task a pairwise decision is chosen. If the subject has chosen the left (right) compound lottery, then within that left (right) compound lottery one of the simple lotteries is played out. Observed choices are used to estimate four models (i.e., EU,[72] the smooth model, RDU, and α-MEU). The smooth model fares the best and α-MEU the least.

C4 Cubitt, Kuilen and Mukerji (2012)

In this study subjects are presented with a number of gambles, and the gambles are played out by a deck that has 10 playing cards. An example of a gamble is: if a spade is drawn from a particular deck then the subject wins $20, otherwise nothing. The drawing is performed using three decks of cards that are ranked by degree of ambiguity: *deck #1* has 7 spades and 3 hearts; *deck #2* takes on two possible compositions, and *deck #3* takes on four possible compositions.

For each gamble the subjects are asked if they want to keep the gamble or choose a sure amount of money instead; if they keep the gamble then it is played out by *deck #1*. Subjects go through a series of these choices in a list format and their certainty equivalent for *deck #1* is elicited in this way. In the next part of the experiment, *deck #2* is used to play out the gambles. In contrast to *deck #1* (whose composition is known), subjects are told that *deck #2* can take on two possible compositions but the probability of each composition is unknown. For each gamble the subjects face they are asked if they want to keep the gamble or to choose a sure amount of money instead; if they keep the gamble then it is played out by *deck #2*. Using the same elicitation procedure as before, subjects go through a series of these choices in a list format that elicits their certainty equivalent for *deck #2*. In the next part of the experiment, a different deck

[72] Conte and Hey (2013) use the term EU instead of Subjective EU.

of 10 cards (*decks #3*) is used to play out the gambles. *Deck #3* can take on four possible compositions but subjects are not told the probability of each composition. The subjects, again, go through a series of choices in a list format that elicit their certainty equivalent for *deck #3*.

At the end of the experiment, one of the choice tasks is randomly selected for payment. The data are used to estimate the smooth model and the α-MEU model, with stronger support for the former. Each subject is categorized into ambiguity-seeking, ambiguity-neutral, or ambiguity-averse, and ambiguity-neutral preference holds the largest group of subjects. Pooling across subjects, the estimation results show some ambiguity-aversion for the average subject.

C5 Attanasi, Gollier, Montesano and Pace (2014)

This study involves two-stage lotteries, which is the feature of the smooth model (Klibanoff, Marinacci and Mukerji (2005)). Three types of lotteries are implemented: a lottery with a known one-stage probability, a lottery that is partially-ambiguous, and a lottery with a second-stage probability that is either known or unknown. Certainty equivalents are elicited for each type of lotteries and comparisons made between these elicited certainty equivalents.

The experiment consists of ten tasks. In the first part of the experiment (tasks 1 – 4), each subject is presented with four simple lotteries with known probabilities. Each lottery has binary outcomes and the outcome is determined by an urn that has 5 white balls and 5 orange balls (thus a 50/50 chance). These four lottery tasks are used to elicit risk preferences. Next, the subjects is asked to choose *one* out of the four lotteries. In task 5, the subject states a reservation price for the simple lottery that she chooses among tasks 1 – 4. This reservation price is elicited using the BDM procedure. In a latter part of the experiment, the elicited certainty equivalent for this risky lottery is compared to the certainty equivalents for the ambiguous lotteries in tasks 6, 7, 8, 9 and 10.

In task 6, the chosen lottery from tasks 1 – 4 is changed from a risky distribution (of 50/50) to an ambiguous distribution described as follows: instead of the chosen lottery (from tasks 1 – 4) being determined by a one-stage 50/50 distribution, the lottery is now determined by a distribution that has a second-order probability. If a subject is assigned to the *binomial treatment*, the lottery is played out by an urn that has 10 balls, and these 10 balls are assembled by randomly selecting balls from a "construction" urn that has 50 white balls and 50 orange balls. If the subject is assigned to the *uniform treatment*, there are 11 urns each containing 10 balls that encompass all possible combinations of whites and oranges, and one of the 11 urns is chosen to play out the lottery (with equal probability). If the subject is assigned to the *unknown treatment*, the lottery is played out by an urn containing 10 balls, and these 10 balls are assembled by randomly selecting balls from a construction urn that has 100 balls of unknown proportions of whites and orangs. After a subject is assigned to one of the three treatments, she is asked to state a reservation price for this *revised* lottery that originally is chosen from tasks 1 – 4, since now the original chosen lottery has become two-stage instead of one-stage.

Next, the urn constructed in task 6 is used in tasks 7, 8 and 9. As a test for partial ambiguity, the proportions for white and orange balls are narrowed down. For example, in task 7 the subject is told that there are between 3 to 7 white balls in the urn; in task 8 the subject is told that there are between 3 to 10 white balls in the urn; in task 9 there are between 3 to 10 orange balls in the urn. Conditional on this new information, the subject is asked to state a reservation price for each *revised* lottery. The certainty equivalents elicited for these (partially) ambiguous lotteries is compared to the risky lottery in task 5. At the end of the experiment, one of the ten tasks is randomly selected for payment. Subjects are paid a fixed show-up fee.

Roughly 90% of the subjects can be classified as averse, neutral or loving according to their operational definition of coherent-ambiguity attitudes, whereas the rest cannot be classified because they are incoherent. Furthermore, when the distribution is unknown, subjects behave as if the probabilities are uniformly distributed, in line with the principle of insufficient reason.[73] This result is consistent with Hey and Pace (2014),[74] who report that the estimated beliefs are closer to equal probabilities in the more ambiguous treatment than in the least ambiguous treatment.

C6 Andersen, Fountain, Harrison, Hole and Rutström (2011)

Recall that SEU assumes the axiom of ROCL such that any prior probability distribution for an event is reduced to a single (degenerate) probability estimate. In contrast, uncertainty models do not assume ROCL and thus a probability distribution is not reducible to a single probability estimate. In theory an uncertainty model preserves the characteristics of the subjective probability distribution whereas the SEU model doesn't. Keeping to the SEU framework, Andersen, Fountain, Harrison, Hole and Rutström (2011) perform an estimation procedure that allows one to preserve some distributional characteristics of the estimated belief. To do so, they estimate a *distribution* of beliefs, instead of a single probability estimate, by

[73] Attanasi, Gollier, Montesano and Pace (2014) use the term "principle of insufficient reason" when discussing the assumption that subjects behave as if the probabilities are uniformly distributed when the distribution is unknown.
[74] Hey and Pace (2014) do *not* use the term "principle of insufficient reason" to describe the estimated beliefs being closer to equal probabilities in their most ambiguous treatment.

estimating the parameters that give rise to the shape of the belief distribution. In this way, one is able to obtain more information about the underlying belief.

Andersen, Fountain, Harrison, Hole and Rutström (2011) present each subject with a range of bookies offering odds on the outcome of some unknown event. As the subject allocates earnings over the range of offering odds, the individual probability *distribution* of the unknown event is elicited. The study is conducted with stationary probabilities, and subjective probabilities are corrected for risk attitudes by including a lottery choice task and inferring probabilities with joint estimation methods. Each subjects completes 9 betting tasks in total and one of them is randomly selected for real payment.

The study examines events that have underlying objective probabilities that differ across a range of probabilities, and reports that in the low-probability treatment where the objective probability is 0.1 or 0.2, in each case the mode and the mean of the subjective distribution are significantly greater than the objective probability. As for the medium- and high- probability treatments, where the objective probabilities are 0.5 or 0.55, and 0.75 or 0.8, respectively, the mean of the subjective probability distributions are virtually the same as the objective probabilities. Thus, results for the low probability range appears to be consistent with past studies (ABPW (2011); KSW (2014)) that report a concave probability region, but not for the high probability range.

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