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The Relationship of Mathematics Anxiety, Mathematical Beliefs, and Instructional Practices of Elementary School Teachers

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ACCEPTANCE

This dissertation, THE RELATIONSHIP OF MATHEMATICS ANXIETY, MATHEMATICAL BELIEFS, AND INSTRUCTIONAL PRACTICES OF ELEMENTARY SCHOOL TEACHERS, by PAMELA T. HUGHES, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree, Doctor of Philosophy, in the College of Education and Human Development, Georgia State University.

The Dissertation Advisory Committee and the student's Department Chairperson, as representatives of the faculty, certify that this dissertation has met all standards of excellence and scholarship as determined by the faculty.

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THE RELATIONSHIP OF MATHEMATICS ANXIETY, MATHEMATICAL BELIEFS, AND INSTRUCTIONAL PRACTICES OF ELEMENTARY SCHOOL TEACHERS

by

PAMELA T. HUGHES

Under the Direction of Dr. David W. Stinson

ABSTRACT

Since the early 1960s, mathematics education researchers have considered the affective domain (attitudes, beliefs, opinions, motivation) as an important aspect of teaching and learning mathematics (Goldin, 2002; Mcleod, 1992). It is suggested that the affective characteristics may be the missing variable that links teachers' instructional practices to students' learning (Ernest, 1989a). Two affective variables strongly related to teachers' instructional practices are mathematics anxiety and mathematical beliefs (see, e.g., Beswick, 2006; Jong & Hodges, 2013; Philipp, 2007; Wilkins, 2008).

The purpose of this quantitative survey study was to explore the relationships among mathematics anxiety, mathematical beliefs, and instructional practices of practicing elementary teachers as they relate to the mathematics reform efforts promoted by the National Council of

Teachers of Mathematics (see, e.g., 1989, 1991, 1995, 2000, 2014). The study was grounded, theoretically, in Ernest's social constructivism as a philosophy of mathematics and mathematics teaching and learning (1998) and in his model of relating teachers' content knowledge, attitudes, instructional beliefs, and instructional practice (1989). The study included 153 practicing elementary teachers who teach mathematics to students in Pre-K–5. These teachers completed the following online surveys: Mathematics Anxiety Scale, the Teaching Beliefs Survey and the Self-Report: Elementary Teachers Commitment to Mathematics Education Reform Survey. Quantitative data analysis methods included descriptive statistics, correlational analyses, and multiple regression analysis. Results indicated statistically significant correlational relationships between mathematics anxiety, mathematical beliefs, and instructional practices. Regression analyses were conducted to identify mathematics anxiety and mathematical beliefs as predictors of instructional practices. Results were significant for mathematical beliefs as a predictor, but not significant for mathematics anxiety as a predictor of instructional practices. Implications and recommendations for further study are discussed.

INDEX WORDS: Elementary teachers, Instructional practices, Mathematics anxiety, Mathematical beliefs, Practicing teachers, Prospective teachers

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TABLE OF CONTENTS

LIST OF TABLES

Page

LIST OF FIGURES

Page

CHAPTER 1

THE PROBLEM

A beginning third-grade teacher sits at her desk planning the mathematics lesson she will be teaching when her students return from lunch. She has had no difficulty planning her other lessons; but dreads the thought of teaching math, and frequently puts it off until the last possible moment. She had always struggled with mathematics and barely passed her college mathematics courses. At the time, however, she comforted herself that an elementary school teacher did not need to know a lot of mathematics. After all, how hard could it be to teach kids how to add, subtract, multiply, and divide numbers?

Notwithstanding the frustration and panic she usually experiences preceding her math lessons, the lesson that day appeared to be going well. The students gave every appearance of understanding the steps she wrote on the board. Fifteen minutes into the lesson, however, one student raised her hand and asked, "Isn't there another way to get the answer? My dad showed me an easier way." The young teacher's heartbeat began to accelerate rapidly, and she suddenly felt detached from her surroundings. Hot flashes swept over her, and she experienced an overwhelming urge to escape. She took a few long breaths, trying to ease her panic. "No," she finally responded. "This is the way they show you in the book, so this is the method you should use." Questions of this nature always made her very uncomfortable. After all, she was taught that there is normally one correct method for solving a math problem. Admittedly, her teacher training should motivate and direct her to engage the student and explore this question with the class, but she simply did not have the confidence or desire. She paused, and then said to the class, "Why don't we put up our math books for the day so we can have enough time to share our

Language Arts stories we created yesterday?" As the students cheerfully put up their math materials, the young teacher sighs with relief as her heart rate slowly begins to return to normal.

This fictional scenario, all too common in America's elementary classrooms, serves to illustrate the importance of the affective domain in facilitating an environment conducive to learning mathematics (Lim & Chapman, 2013; McLeod, 1992; Philipp, 2007). In recent decades research has begun to explore the importance of the affective domain and its role in teaching and learning mathematics. The existing research suggests that the affective domain, including beliefs and attitudes, significantly affects a teacher's choice of instructional practices (Goldin, 2002; McLeod, 1992; Philipp, 2007).

Since the early 1960's, researchers in mathematics education have considered the affective domain as an important aspect of teaching and learning mathematics (Ho, Senturk, Lam, Zimmer, Hong, & Okamoto, 2000; McLeod, 1992). The affective domain is comprised of emotions, attitudes, beliefs, opinions, interests, motivation, and values (Goldin, 2002). In the past few decades, research in mathematics education has begun to focus on the affect of mathematics teachers (Austin, Wadlington, & Bitner, 1992; Ernest, 1989a; McLeod, 1992; Philipp, 2007; Raymond, 1997). It is believed teacher affect may be the missing variable linking teachers' practices to students' learning (Ernest, 1989a). In addition to teachers' cognitive characteristics directly influencing student learning, it is also believed teachers' affect influences how teachers teach, which then influences student learning. In other words, instead of focusing on what the teacher *does* in the classroom, research has increasingly focused on *who* the teacher *is* in an affective sense (Hart, 2002). Two affective factors demonstrated by the teacher and believed to contribute to the instructional practices of elementary teachers are mathematics anxiety and mathematical beliefs. Both of these variables are evident in the beginning teacher's

thoughts and actions as she prepares and delivers the mathematics lesson. Exploring the relationships between these two constructs and how they relate to the instructional practices of elementary teachers is the focus of this research study.

Mathematics anxiety is a widespread phenomenon that has been linked to the implementation of effective instructional practices in mathematics education (Gresham, 2007; Malinsky, Ross, Pannells, & McJunkin, 2006; Trujillo & Hadfield, 1999). It has been described as a multidimensional construct with cognitive as well as affective roots (Bursal & Paznokas, 2006). Mathematics anxiety is characterized by feelings of tension, panic, and fear when confronted with mathematics. In addition to physical symptoms, individuals with mathematics anxiety frequently exhibit avoidance behaviors when faced with situations related to mathematics.

Another factor linked to the mathematical instructional practices of elementary teachers is their mathematical beliefs. Teachers enter the classroom with a set of predetermined beliefs about mathematics and the teaching and learning of mathematics (Raymond, 1997). It is likely, therefore, that these beliefs would provide a strong base for the methods of instruction they choose to use in the classroom (Wilkins, 2008). This is evident in the opening fictional scenario in which the teacher's beliefs contributed to the outcome of the lesson. The belief that there is only one way to solve a mathematics problem clearly influenced the way she handled the student's question.

My interest in this area developed during my graduate coursework. I have always taught middle or high school mathematics, and thought to widen my experience of elementary mathematics education during graduate school. I had the opportunity to enroll in an elementary mathematics education course for practicing elementary teachers working toward their masters'

degrees. What I witnessed in this class was perplexing. The teacher leader favored a constructivist, standards-based classroom, and planned activities that promoted cooperative learning and inquiry-based problem solving. Working in groups, I discovered that a number of the teachers were uncomfortable with many of the mathematical concepts, and were anxious and reluctant to present mathematical problems. When they did present solutions to the class, some of the teachers were seemingly nervous, relating their mathematical thinking with great difficulty. Furthermore, some of the teachers frequently demonstrated avoidance behaviors with some of the more difficult mathematical concepts, and often made negative remarks concerning their long-standing fear and struggles with mathematics. Interactions with these elementary teachers over the course of the semester inspired and led me to this research project.

Purpose of the Study

The purpose of this study was to explore the relationship between practicing elementary teachers' anxiety toward mathematics and the teachers' mathematical beliefs, and to examine how these affective characteristics were (or were not) related to the instructional practices of elementary teachers.

Research Questions and Hypotheses

Specifically, the research questions were:

1. What is the relationship between the mathematics anxiety and the instructional practices of elementary teachers?
2. What is the relationship between the mathematical beliefs and the instructional practices of elementary teachers?
3. What is the relationship between mathematics anxiety and the mathematical beliefs of elementary teachers?

The following null hypotheses were tested in this study:

H_0: There is no relationship between mathematics anxiety and the instructional practices of elementary teachers.

H_0: There is no relationship between the mathematical beliefs and the instructional practices of elementary teachers.

H_0: There is no relationship between mathematics anxiety and the mathematical beliefs of elementary teachers.

Operational Definitions

Mathematics Anxiety: a feeling of panic and tension that interferes with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations (Richardson & Suinn 1972). The construct also has been described as a state of discomfort that occurs in response to situations involving mathematical tasks that are perceived as threatening to self-esteem (Trujillo & Hadfield, 1990). Robertson and Claesgens (1983) define mathematics anxiety as an irrational fear of mathematics, which produces tension that interferes with the use of numbers and solving of mathematical problems. The following statements would characterize highly anxious teachers:

- I often experience anxiety when asked to add up a column of numbers.
- I often experience anxiety when figuring out a monthly budget.
- I often experience anxiety when trying to figure out the amount of tip to leave at a restaurant.
- I often experience anxiety when I open a math book to begin working on a homework assignment.

The level of mathematics anxiety in this study was operationally defined as the mean score

on the Mathematics Anxiety Rating Scale – Short Version (Suinn & Winston, 2003; see Appendix A).

Mathematical Beliefs: the beliefs teachers have about the nature of mathematics, and the teaching and learning of mathematics. These beliefs include teachers' opinions on the usefulness of mathematics, as well as their perceptions on how the subject should be presented (Beswick, 2006; Handal, 2003). The construct refers to the mathematical beliefs that are consistent with the mathematics education reform advocated by the National Council of Teachers of Mathematics (NCTM, 1989, 1991, 1995, 2000) grounded in a constructivist perspective. Teachers who hold constructivist-oriented beliefs toward the teaching and learning of mathematics would agree (most often) with the following statements (Beswick, 2005):

- Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics.
- Teachers can create for all students a nonthreatening, supportive learning environment.
- Effective mathematics teachers enjoy learning and doing mathematics themselves.

Teachers who hold traditional beliefs (most often) would agree with the following statements (Beswick, 2005):

- A mathematical mind is needed to be good at mathematics.
- There is usually one correct way to solve a mathematics problem.
- Males are better at mathematics than females.

The teachers' beliefs score in this study was operationally defined as the mean score on the Teacher Beliefs Survey (Beswick, 2005; see Appendix B).

Instructional Practices: the methods, activities, and strategies teachers' employ to teach mathematics. Traditional instructional practices are most often teacher-centered. They usually

involve the teacher presenting concepts by explanation and example, while students are expected to practice the procedures and memorize the facts until they are mastered. Reform-based instructional practices are most often student-centered. They are aligned with constructivist perspective that students must construct their own knowledge. The emphasis is on students "doing" mathematics by investigating problems and making conjectures, with the goal of developing conceptual as well as procedural understanding. For the purpose of this study, instructional practices refer to the degree to which elementary teachers engage in the reform practices consistent with those advocated by the NCTM *Principles and Standards for School Mathematics* (2000) and the Common Core State Standards Initiative for Mathematical Practice (CCSS; http://www.corestandards.org/Math/). The goals of CCSS are aligned with the constructivist perspective. These standards advocate instructional practices that emphasize constructivist learning through active inquiry-based activities. The following statements would characterize instructional practices aligned with a reform-based, constructivist ideology:

- Students invent their own methods to solve problems.
- I jump between topics as the need arises.
- Students learn through discussing their ideas.
- Students work collaboratively in pairs or small groups.

The instructional practices score was operationally defined as the mean score on the Self Report Survey: Elementary Teachers Commitment to Mathematics Education Reform (Ross, McDougall, Hogaboam-Gray, & LeSage, 2003; see Appendix C).

Practicing Teacher: a college graduate who holds a state-certified teaching certificate and is currently employed as a teacher. Practicing teachers are also referred to as inservice teachers.

Prospective Teacher: a college student who is currently enrolled in a teacher education program and involved in a school-based field experience. Prospective teachers are under the tutelage of supervisors from the education institution who direct and support the student in learning to prepare lessons, provide instruction, and measure student progress. Prospective teachers are also referred to as pre-service teachers.

Rationale for the Study

Although the role of mathematics in society has been of central importance for centuries, its role has never been more critical than it is currently in today's ever-changing world of rapid technological growth. Now more than ever, the mathematical skills of our citizens are crucial to the economic growth and the continued prosperity of our nation. With the demand for mathematical skills increasing, it has become increasingly urgent that we explore and identify those factors that promote the mathematical success of our students, as well as those factors that may prevent the development of these critical skills (Burton, 2012; Ewing, 2010). Many believe such answers lie with the development of standards and curriculum (Ewing, 2010; Hiebert, 2003; Heck, Weiss, & Pasley, 2011). Others propose that mathematical achievement lies not only with the knowledge and skills of the teacher but also with the instructional methods used in the classroom (Cross, 2009; Hadley & Dorward, 2011). Expectations at the national, state, and local level stress the need for the instructional practices of mathematics teachers to change. This change includes teaching methods undergirded by a constructivist paradigm, with student understanding of mathematical concepts paramount, and rote memorization of algorithms minimal. This change includes classroom learning environments that support problem solving, communication, and justification of mathematical ideas.

In recent decades research has begun to explore the importance of the affective domain and its role in teaching and learning mathematics. The existing research suggests that the affective domain, including beliefs and attitudes, significantly affects a teacher's choice of instructional practices (Goldin, 2002; McLeod, 1992; Philipp, 2007). Two constructs of the affective domain, mathematics anxiety and mathematical beliefs, have been investigated and are liberally represented in the research. However, much of this research focuses on students or prospective teachers. Although there have been significantly fewer studies on the mathematics anxiety of practicing elementary teachers, it has been reported that a disproportionately large percentage of this population experience significant levels of anxiety (Hadley & Dorward, 2011; Hembree, 1990; Wood, 1988). Furthermore, there is a definite gap in the literature with respect to the mathematics anxiety and beliefs of elementary teachers. There are limited studies that investigate two of the three constructs such as the mathematics anxiety and instructional methods of prospective teachers (see, e.g., Austin, Wadlington, & Bitner, 1992; Wilkins, 2008); however, no studies exist that investigate the three constructs simultaneously. That is, no research has attempted to model systematically the relationships among these variables with practicing elementary teachers. Furthermore, there has been little work that investigates these variables and their relationships by grade level within the elementary strand.

Much of the existing research in the fields of mathematics anxiety, mathematical beliefs, and instructional practices is qualitative in nature, involving only small groups of teachers (see, e.g., Brady & Bowd, 2005; Evans, 2012; Plaisance, 2008; Stuart & Thurlow, 2000; Trujillo & Hadfield, 1999). In order to advance understanding of the interaction of mathematics anxiety, mathematical beliefs, and instructional practices, it is also necessary to examine these constructs using large-scale quantitative studies that allow for the modeling of relationships among the

variables. Quantitative research has the ability to effectively translate data into easily quantifiable charts and graphs, as well as project that data to a larger population (Creswell, 2003).

This quantitative study used a survey design to explore this idea more specifically, expanding the knowledge base of mathematics education in the interrelated areas of mathematical beliefs, mathematics anxiety, and instructional practices of elementary mathematics teachers. More specifically, my goal was to target the mathematics anxiety, beliefs about the nature and teaching of mathematics, and instructional methods of practicing elementary teachers, as they are related to the mathematics reform movement promoted by NCTM. Furthermore, I illustrated how the study of mathematics anxiety and mathematical beliefs assists in revealing the extreme complexity of implementing constructivist-based reform, and might explain the failure of previous reform efforts (see, e.g., Yero, 2002). In making such a statement, it is necessary to provide a brief overview of the current mathematics reform movement.

Mathematics Reform

This section provides a brief historical overview of past and current reform efforts in mathematics education. In this attempt, my goal is to provide a context and rationale for my study, acknowledging that any brief account of this movement risks oversimplifying this important historical period. Although there have been other reform movements in mathematics education, I focus on the reform that originated in the 1970s and still exists today.

The National Council of Teachers of Mathematics (NCTM) is the world's largest mathematics education organization. Founded in 1920, NCTM has remained a public voice in aiming to provide vision and leadership to ensure high quality mathematics education for all

students. Its vision was built around the idea of classrooms where knowledgeable teachers create meaningful and challenging experiences for all students engaging in high-level mathematics. It fosters a curriculum that is "mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding" (NCTM, 2000, p. 3).

The Back to Basics education movement of the 70s saw decline in mathematics achievement (NCTM, 1980). Up to this point, NCTM had essentially maintained a passive role in curriculum policy. However, two publications opened the doors for NCTM to take a stronger position on mathematics curricula by calling for a collaborative approach to mathematics curriculum reform. The first publications by NCTM were *An Agenda for Action* (NCTM, 1980) and *Priorities in School Mathematics* (PRISM) (NCTM, 1981). The PRISM report was a summary of a survey study conducted by NCTM to assess the readiness of curriculum change. In *An Agenda for Action*, NCTM used the results of the study to make recommendations for the direction of mathematics education (Bullock, 2013). Kilpatrick (2009) states, "the *Agenda* provided direction for reform; it was essentially NCTM's first effort to influence national educational policy in a substantive way" (p. 110). This report made recommendations for instruction in mathematics to place a greater focus on problem solving rather than basic skills (Hekimoglu & Sloan, 2005). Although these brief reports did not receive widespread attention, NCTM had asserted its authority in the direction of mathematics education (Bullock, 2013). The Council began to hold meetings to discuss how mathematics education should change, and it formed committees to help teachers select appropriate textbooks and evaluate curricula programs (Kilpatrick, 2009).

In response to the low mathematics performance of students during the Back to Basics movement, the Department of Education charged the National Commission on Excellence in Education (NCEE) to report on the state of American education. In 1983 the NCEE published *A Nation at Risk: The Imperative for Educational Reform.* This strategically alarming report listed several "indicators of risk," including various literacy, mathematical, and technological deficiencies (NCEE, 1983). The document warned that American schools were ineffective in teaching mathematics and that educators should move to a more progressive approach for teaching and learning mathematics (Anderson, 2010).

A *Nation at Risk* was very influential in generating the proliferation of research during the 1980s, which brought about a significant change in the mathematics community. *Mathematics education* as a field of study began to gain importance, and, by the end of the decade, found itself accepted as a discipline in its own right (Hekimoglu & Sloan, 2005). The emergence of the constructivist theory also stimulated research in mathematics education. Research began to shift from the investigation of the teacher and student behaviors to an examination of cognition and context (Hekimoglu & Sloan, 2005). It was in this era that the NCTM Standards were born.

The NCTM Standards

As technology continued to advance, the pressure to prepare students for their role as global competitors did as well. Furthermore, the proliferation of research during the 1980s provided insight into how children best learn mathematics. In response, in 1989, NCTM developed the *Curriculum and Evaluation Standards for School Mathematics*, the first of four sets of standards to follow. This publication is often referred to as the "NCTM Standards," and includes 13 curriculum standards addressing both content and emphasis in mathematics

education. According to NCTM, "the study of mathematics should emphasize reasoning so that students can believe that mathematics makes sense" (NCTM, 1989, p. 29). The Standards overall emphasized conceptual understanding and problem solving based on a constructivist understanding of how children learn. In addition, they encouraged a more democratic vision of mathematics education by promoting equity and mathematical power as a goal for all students, including women and underrepresented minorities (Burrill, 1997).

The NCTM Standards soon became the foundation of many local and state curriculum frameworks, as well as the basis for several federally funded curricula. The initial reaction to the Standards were generally positive (Kilpatrick, 2009); however, critics of the Standards perceived the teaching of mathematics in the context of real life and the decrease of traditional rote learning as an elimination of basic skills and precise answers. Nontheless, critics, back then and still today, acknowledge that the "reform" curriculum is more successful than the traditional curriculum at reaching the lower 50% of mathematics students, all the while claiming that the serious and gifted students are being short-changed with such a "watered-down" curriculum (Wu, 1997).

Although the Standards required all students to pass high standards of performance in mathematics, many critics continued to oppose the suggested "radical" changes to mathematics instruction (Burrill, 1997). This conflict lead to the "math wars," a debate between the supporters of the constructivist paradigm proposed by NCTM and the supporters of a traditional curriculum emphasizing basic skills using standard algorithms.

Although the controversial Standards were accepted in many educational communities, some were critical of the radical changes expected in mathematics education (Burrill, 1997). In 2000, NCTM used a consensus process involving mathematicians, teachers, and educational

researchers to revise its original standards with the release of the *Principles and Standards for School Mathematics*. This was consistent with NCTM's initial plan to revise the Standards every decade and remain current with research and educational reform. This document was also published to address the sharp criticism against the 1989 Standards regarding decreased emphasis on the teaching of computation and algorithms learned in rote form (Kilpatrick, 2009). These standards sought more clarity and balance, and were organized around six principles (Equity, Curriculum, Teaching, Learning, Assessment, and Technology) and ten strands, which included five content areas (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) and five process standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation). Process standards specify the mathematical ways of thinking students should develop while learning mathematics content (NCTM, 2000). The new revised standards were not considered as radical as the 1989 standards and therefore did not provoke extensive criticism. These standards, still in place today, emphasize the importance of a constructivist, reform-based mathematics curriculum, and have been widely used to inform textbook creation, state and local curricula, and current trends in teaching (Hiebert, 2003). These standards have also been the inducement for creating the Common Core Mathematics Standards, which I discuss in a subsequent section.

In light of the movement to shift mathematics education into the realm of a constructivist paradigm, NCTM recommended instructional methods that foster learning mathematics with more emphasis on conceptual understanding. NCTM advocated instructional practices with emphasis on the processes, which provide students with a connected, coherent understanding of mathematics. In addition, NCTM emphasized that the development of mathematical

understanding is contingent on the ability of teachers to provide students with the opportunity to experience mathematics in real-life situations (NCTM, 2000).

According to NCTM, the (revised) Standards were written to describe a vision in which mathematics education would "promote mathematical thinking by creating an awareness of the nature of mathematics, its role in contemporary society, its cultural heritage, and the importance of mathematics as an instrument and tool of learning" (Hekimoglu & Sloan, 2005, p. 37). This vision spawned national discussions on the nature of mathematics and mathematical literacy. According to Hekimoglu and Sloan (2005), critics of the 2000 Standards continued to voice concerns over the recommendation for "reduced emphasis of computation and de-emphasis on the abstract in favor of the concrete (p. 38). A new criticism for the 2000 Standards included the suggested integration of technology in the mathematics classroom, which critics saw as yet another de-emphasis on basic skills. The basic skills issue continues today to be a major critique of the Standards, although that was never the intent of the developers (Hekimoglu & Sloan, 2005).

Adding It Up

As we entered the 21st century, NCTM was not the only organization calling for reform. In addition to NCTM, the National Research Council (NRC) has also played a significant role in the mathematics reform efforts. In response to the "math wars," the NRC formed a committee to conduct a mathematics learning study. In 2001, the NRC published a new report on Pre-K–8 mathematics education, *Adding It Up: Helping Children Learn Mathematics* (NRC, 2001). This publication addressed the "concerns expressed by many Americans, from prominent politicians to the people next door, that too few students in our elementary and middle schools are successfully acquiring the mathematical knowledge, the skill, and the confidence they need to

use the mathematics they have learned" (p. 1). A committee with diverse backgrounds reviewed and synthesized relevant research related to how children learn mathematics, and provided recommendations for changes in teaching, curricula, and teacher education. The goals of the committee were:

- To synthesize the rich and diverse research on pre-kindergarten through eighth-grade mathematics learning.
- To provide research-based recommendations for teaching, teacher education, and curriculum for improving student learning and to identify areas where research is needed.
- To give advice and guidance to educators, researchers, publishers, policy makers, and parents. (p. 3)

The committee used research findings to discuss the processes by which students develop mathematical proficiency, identified five interdependent components of mathematical proficiency, and described how students develop this proficiency. The five connected and equally important strands comprised the committee's definition of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. According to the NRC, these intertwined strands are critical to developing proficiency in mathematics; all students "must learn to think mathematically, and think mathematically to learn" (NRC, 2001, p. 17).

A shortened version of *Adding It Up* was sent to the superintendent of every school district in the country, which made the publication accessible. Kilpatrick (2009) acknowledges that it is generally regarded as "the Bible" of K□8 mathematics education in the United States, and Kilpatrick himself considers the publication a great resource for all mathematics educators.

To address the mathematics curriculum that has been characterized as "a mile wide and an inch deep," as well as the organization of the Standards' grade-level bands, the NCTM published *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006). The purpose of this report was to bring focus by identifying a small number of focal points for emphasis at each grade level. The report was generally well received by mathematics educators, and considered a positive contribution to the Standards debate (Kilpatrick, 2009). According to Kilpatrick, however, NCTM to propitiate the critics who supported a more traditional mathematics curriculum portrayed it in the media as a concession.

In 2006, a National Mathematics Advisory Panel was created to determine and recommend how scientific research could best be used to improve the teaching and learning of mathematics (Kilpatrick, 2009). Tasks groups were formed to analyze empirical research on teacher education, conceptual knowledge and skills, learning processes, instructional practices, and assessment. The panel's final report, *Foundations for Success: The Final Report of the National Mathematics Advisory Panel* (NMAP, 2008), was generally accepted as a positive document, especially due to its focus on algebra. There was some criticism on the standards of evidence, however. Many mathematics educators felt the "best scientific evidence" was limited and ruled out significant research conducted qualitatively (Kilpatrick, 2009). Critics also thought the report put too much emphasis on arithmetic and displayed a somewhat outdated view of algebra. The report was also criticized for its lack of statistics. Furthermore, geometry and measurement were not considered as stand-alone topics, but adjuncts to algebra (Kilpatrick, 2009).

Common Core Mathematics

The most recent reform in mathematics education relates to the development and implementation of the Common Core State Standards for Mathematics (CCSSM). The CCSSM is a part of the Common Core State Standards, and outlines the mathematical content all students should learn in Pre-K–12 (Heck, Weiss, & Pasley, 2011). The development of a common mathematics curriculum was launched in 2009 by state leaders, including governors and state commissioners of education. A team of experts in mathematics content, teaching, and research were directed to write standards based on three types of evidence: (a) evidence from high performing states and countries; (b) findings from cognitive science and mathematics education research, including student achievement studies; and (c) lessons learned from standards-based accountability systems. The CCSSM address both conceptual understanding of mathematics and procedural skills. The expectations of the CCSSM include:

- studying rigorous content benchmarked to international standards;
- focusing on fewer topics studied in greater depth;
- attending to coherence by connecting ideas within and across topics.

As in NCTM's *Principles and Standards for School Mathematics* (2000), the CCSSM contains both process and content standards. Process standards specify the mathematical ways of thinking students should develop while learning mathematics content. The CCSSM includes eight Standards for Mathematical Practice that are built upon the NCTM Process Standards and the National Research Council's five strands of mathematical proficiency. The Standards for Mathematical Practice describe fundamental approaches to, and dispositions toward, learning and doing mathematics.

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning (CCSSM, 2001).

The CCSSM is expected to have a significant impact on teachers' instructional practices. First, the implementation of the CCSSM requires a deeper understanding of mathematical content. In order to develop these student understandings, Ewing (2010) contends, "Teachers must have deep and appropriate content knowledge to reach that understanding; they must be adaptable, with enough mastery to teach students with a range of abilities; and they must have the ability to inspire at least some of their students to the highest levels of mathematical achievement" (para. 6). Second, teachers must master the instructional strategies needed to actively engage students in the study of mathematics. These effective strategies call for a significant change in the role of the teacher, however. Under the CCSSM, the teacher should be viewed as a facilitator rather than an authority of mathematical knowledge. The role of the teacher is not to dispense knowledge, but to guide and empower students to build on prior mathematics knowledge. The teacher is to create a rich learning environment that provides ample opportunities for problem solving, reasoning, and discussion of mathematics.

The current mathematics reform movement advocated by NCTM is intimidating for many prospective and practicing elementary teachers (Uusimaki & Nason, 2004). It is not surprising that many classroom teachers feel alienated from the reform process; for teaching

mathematics using constructivist-based methods can be intimidating and extremely difficult, even for those who have training and experience (Andrew, 2007; Dangel & Guyton, 2004; Ernest, 1991). Many teachers are asked to teach mathematics in a way that is completely different from the way in which they were taught mathematics (Bush, 1989; Marlow & Page, 2005). First-year teachers especially have difficulty and experience anxiety in teaching using constructivist-based instructional practices (Brooks & Brooks, 1999; Raymond, 1997; Thompson, 2013). Many prospective teachers enter a teacher education program believing the content in the elementary grades is simple, and that they already have the knowledge they need to teach. This is not the case with mathematics, however, for the level of instruction they have received is often not adequate for teaching mathematics (Hembree, 1990; Thompson, 2013).

For elementary teachers who experience mathematics anxiety, the rigor and depth of the content alone is sufficient to increase their level of frustration and anxiety. Added to this is the pressure to engage in instructional practices that often necessitate a radical change in their belief system. Although this study focuses on instructional practices in general, it must at the very least be acknowledged that any investigation into the instructional practices of elementary teachers is, in part, a reflection of their acceptance of the CCSSM standards.

NCTM has currently undertaken a major initiative to define and describe the principles and actions, including specific teaching practices that are essential for a high-quality mathematics education for all students. The most recent publication of NCTM is *Principles to Action: Ensuring Mathematical Success for All* (NCTM, 2014). The primary purpose of this publication is to fill the gap between the development and adoption of CCSSM and other standards and the enactment of practices, policies, programs, and actions required for their widespread and successful implementation.

Georgia adopted the Common Core Mathematics Standards in 2010. This adoption became a politically charged issue, for some in the public opposed the CCSSM. Georgia later withdrew from the associated national assessments so schools could administer a state-created assessment, due in part for financial reasons. In January 2015, Georgia formally renamed its standards the *Georgia Standards of Excellence (GSE)*. This change was made in part to preserve the Common Core Standards in Georgia, as well as to appease the critics. The GSE in Mathematics mirror the CCSSM except for a few minor additions, word changes, and rearranging of standards.

Conclusion

The platform of the mathematics reform movement is the NCTM Standards, while constructivism is the foundation on which the whole reform movement rests (Wu, 1997). Although the standards for what has been identified as effective mathematics instruction have been outlined by NCTM and more recently by the CCSSM, the implementation of these effective methods are still not consistently found among the instructional practices of elementary mathematics teachers. Why is this? Could the mathematics anxiety of elementary teachers hinder the implementation of these methods? What are elementary teachers' beliefs about the teaching and learning of mathematics? How do these beliefs align with their instructional methods?

My goal in this study was to seek answers to these questions, as well as advance the literature and expand the knowledge base of mathematics education in the interrelated areas of mathematical beliefs, mathematics anxiety, and instructional practices of elementary teachers. The idea that we are placing elementary teachers in classrooms with anxiety toward mathematics and negative beliefs about mathematics is troubling. The idea that we are placing teachers in the

classroom and doing little to understand and address the issue is even more disturbing. This study not only expands our knowledge of these variables but also extends our own understanding of the effects of these constructs on mathematics instruction. Through quantitative analysis, the extent to which mathematics anxiety and teachers' beliefs may influence their methods of instruction in the classroom was analyzed, thereby increasing the field of knowledge and level of understanding of these critical constructs.

In Chapter Two, I provide a thorough discussion of the theoretical framework that drives my thinking in regards to reform mathematics education; it draws primarily from the extensive scholarship of Paul Ernest, an internationally recognized authority on social constructivism in mathematics teaching and learning. Here I explore Ernest's philosophy of mathematics, education, and social constructivism, as well as provide evidence of how this perspective suitably frames my thinking and strengthens the overall purpose of the study.

In Chapter Three, I provide an in-depth review of the existing research in the areas of mathematics anxiety, mathematical beliefs, and instructional practices of elementary teachers, thus developing a case that supports and substantiates the findings of this study. In Chapter Four, I describe the methodology of the study, the instrumentation, the data collection, and the quantitative analysis that was used to measure the results of the teacher surveys. In Chapters Five and Six, I report the results, discuss the findings of the analysis, and summarize the project. I also discuss the limitations and implications of the project.

CHAPTER 2

THEORETICAL PERSPECTIVE

The Gods did not reveal, from the beginning,
All things to us; but in the course of time,
Through seeking, men find that which is the better.
But as for certain truth, no man has known it,
Nor will he know it; neither of the gods,
Nor yet of all the things of which I speak.
And even if by chance he were to utter
The final truth, he would himself not know it;
For all is but a woven web of guesses.
– Xenophanes

The above quote, translated and quoted by Karl Popper (1963, p. 34), clearly refers to the quest for truth and knowledge. According to the pre-Socratic philosopher, Xenophanes, all knowledge was not revealed to humanity by the Gods. Humanity was expected to seek and discover, not with the expectation of finding absolute truth, but with the sense that through the quest itself, humans would develop their own knowledge in making sense of the world. The old adage, "you wouldn't know the truth if it was staring you in the face" takes on new meaning under the light of Xenophanes' philosophical statement. The saying actually reflects a literal truth in some philosophies. For example, the philosophy of mathematics of Imre Lakatos's contends that we cannot establish foundations of mathematical knowledge truth or mathematical certainty. Instead, we can only make guesses and then critique and improve those guesses (Wilding-Martin, 2009). Lakatos's philosophy adheres to the views of fallibilism, and has been instrumental in shaping the works of other philosophies, most notably the works of Paul Ernest.

Ernest's research addresses fundamental questions about the nature of mathematics and how it relates to teaching, learning, and society (Sriraman, 2009). His philosophy is based on the fallibilist view that mathematical truth is never absolutely certain. To a fallibilist, no claim of knowledge, even one that is true, can ever be proven beyond a doubt. Fallibilism, as stated by

the philosopher Karl Popper (1945), refers to "the view, or acceptance of the fact, that we may err, and that the quest for certainty (or even the quest for high probability) is a mistaken quest" (p. 491). Fallibilism rejects the absolutists' view that mathematics is rigid, fixed, culture free, and inaccessible to all but an elite group blessed with mathematical minds. A fallibilist view of mathematics is, according to Ernest (1996), "eternally open to revision" (p. 2). Fallibilism perceives mathematics as the outcome of social processes (Ernest, 1994a). It is the fallibilist view that provides a springboard for Ernest's philosophy of social constructivism in mathematics. Many educational psychologists have studied social constructivism; however, the works of Ernest on the philosophy of social constructivism are unparalleled in the field of mathematics education (Thompson, 2003; Wilding-Martin, 2009).

Ernest is an internationally recognized authority on social constructivism in the field of mathematics education (Goodchild, 2010). He is, in fact, credited with introducing the term to mathematics education in the 1980s, thus distinguishing two forms of constructivism, the cognitive and social (Thompson, 2013). Ernest has published widely on the topic of social constructivism, as well as other theories of learning. His best-known works are perhaps *The Philosophy of Mathematics Education* (Ernest, 1991) and *Social Constructivism as a Philosophy of Mathematics* (Ernest, 1998). Before discussing the social constructivist philosophy of Ernest, it is necessary, however, to provide a brief account of the theories and philosophers who influenced his philosophy. I first begin by discussing constructivism, including the theories of Jean Piaget and Lev Vygotsky.

Social Constructivism

One cannot begin a proper discussion of social constructivism without first reviewing the theory of constructivism. Constructivism is the most important theoretical perspective to emerge

in the discipline of mathematics education in the past quarter of a century (Ernest, 1991; Thompson, 2013). What has been termed "simple constructivism" (Ernest, 2010, p. 40) is a philosophical view of learning based on the idea that individuals construct their own knowledge. That is, learners actively construct their own meanings through exploration, inquiry, interpretation, and analysis of prior experiences. Constructivism as a philosophy focuses on issues concerning the origins of human knowledge, as well as the development of individual understanding (Wilding-Martin, 2009). In reference to the latter, constructivism retains the view that students must actively construct meaning for themselves by using new information to build on prior knowledge.

There are four ways to characterize constructivist learning in contrast with traditional, mimetic learning. First, constructivist learning is based on constructing individual knowledge, not merely being told the information or receiving the knowledge. This constructing allows for assimilation of the information into existing schemata. Secondly, constructivist learning is not about recall, rather it is practiced understanding and application of knowledge and information. Thirdly, constructivist learning requires thinking and analyzing, not just memorizing and accumulating. It accentuates the thinking process rather than the quantity a learner memorizes. Fourthly, constructivist learning is considered active, not passive. Learners become more effective when they discover their own answers, concepts, solutions, and when they create interpretations and reflection about their own learning (Marlow & Page, 2005; Van de Walle, 2004).

Constructivism is based on the expectation that student learning is an interdependent process in which only the learner can actively construct personal meaning of the knowledge being acquired based on his or her cognitive developmental stages and his or her socio-cultural

experiences (NCTM, 2000; Piaget, 1971; Vygotsky, 1978). Naturally, constructivist pedagogy focuses on creating situations and activities in which students are encouraged and guided to construct meaning for themselves using such methods as exploration and inquiry (Van de Walle, 2004).

Constructivism draw upon the works of Jean Piaget (Piaget & Inhelder, 1969) and Lev Vygotsky (Vygotsky, 1978), who are often considered "master architects" (p. 76) in the design of human learning (Fogarty, 1999). Piaget's research on the epistemological stages of development, or cognitive constructivism, and Vygotsky's role of social interaction in the learning process, or social constructivism, are critical in understanding the human mind and the building process of cognitive knowledge (Fogarty, 1999).

No discussion of constructivism would be complete without acknowledging the influences of Piaget and Vygotsky. Indeed, much of Ernest's philosophical framework of social constructivism is based on their work, with Vygotsky's being the predominant influence. A proper discussion of the theories of Piaget and Vygotsky would be lengthy and out of the scope of this project. However, brief summaries, which highlight particularly relevant ideas, are necessary to establish the foundations that influenced the philosophy of Ernest.

Piaget

Jean Piaget (1896–1980), a Swiss scholar, was one of the most influential proponents of the constructivist theory of learning. In the late 20th century, Piaget explained the processes of learning in terms of assimilation, whereby learners add new knowledge to their existing framework and accommodation, where new information triggers cognitive conflict, which results in the reorganization of knowledge frameworks (Huitt & Hummel, 2003).

Piaget's research focused on cognitive development and the formation of knowledge. According to Huitt and Hummel (2003), his research led him to believe that progression of knowledge is the result of constructions formed individually by the learner. Through the process of conducting extensive clinical and case studies that emphasized the individual learner and the process of cognitive development, Piaget formulated his genetic epistemology of learning (Huitt & Hummel, 2003, Piaget & Inhelder, 1969). Through these observations and documentation, Piaget established structural changes that took place in the construction of knowledge and beliefs. Through this process, Piaget established four main stages of learning during a learner's development: (a) the sensori-motor stage in infancy, (b) the preoperational stage of toddlers and young children, (c) the concrete operational stage of elementary and preadolescent children, and (d) the formal operational period of adolescence students and adulthood (Piaget & Inhelder, 1969).

Through the development of schemata, or cognitive schemas, Piaget claimed that children could integrate new knowledge or accommodate new knowledge due to the action of cognitive conflict. Piaget defined this process as *assimilation* and *accommodation* (Piaget & Inhelder, 1969). Assimilation occurs when a learner perceives new objects or events in terms of existing schemes or operations. This information is then compared with existing cognitive structures. In other words, assimilation allows students the use of existing schema to give meaning to experiences. Accommodation occurs when existing schemes or operations must be modified to account for a new experience. Accommodation refers to the process of altering existing ways of viewing things or ideas that contradict or do not fit into their existing schema (Huitt & Hummel, 2003). According to Piaget, the process of assimilation and accommodation create equilibrium and a greater foundation for learning. Piaget believed that students should

play an active role in their learning processes and that cognitive growth is created when construction and reconstruction of knowledge related to previous experiences and environments has transpired (Piaget & Inhelder, 1969).

Among the integrated networks, or cognitive schemas, identified by Piaget, the construction of knowledge and the tools to construct new knowledge are created. As students learn, networks within the brain are rearranged, added to, changed, or modified through reflective, purposeful thought so that individuals can supplement their current understanding (Huitt & Hummel, 2003; Fogarty, 1999). Piaget's theories helped define the current constructivist view of learning through his view of cognitive constructivism by defining the thought processes that occur behind thinking, processing, and understanding (Fogarty, 1999; Thompson, 2013).

Vygotsky

Lev Vygotsky's (1896–1934) works began in the 1920's and contributed to and complemented the beliefs of Piaget (Fogarty, 1999). Although Piaget's works focused more directly on cognitive constructivism and suggested that teachers should play a limited role in students' learning, Vygotsky's works affirmed the significance of social interaction during the cognitive learning process. Vygotsky's theory, often called social constructivism or socio-cultural constructivism, suggests an active, involved teacher or peer during the learning process (Wilding-Martin, 2009).

Unlike Piaget and von Glasersfeld (subsequently discussed), who view the growth of knowledge as the mental organization of individual experience, Vygotsky regards the growth of knowledge as cultural. To Vygotsky, activities can only construct meaning within a system of

social behavior. There are two key features of Vygotsky's model that influences the work of Ernest: the role of language and the role of social interaction (Ernest, 1998).

Language plays an integral role in Vygotsky's social theory of cognitive development. According to Vygotsky, not only does language and communication provide the means for social interaction, but also it is a vehicle for learning. In other words, it teaches children (and adults) not only how they are supposed to act, but also provides them with the tools to formulate thought and construct their own conceptual understandings (Wilding-Martin, 2009).

Vygotsky's (1978) zone of proximal development (ZPD) was established on the belief learners maintain an area within their brain for future learning. According to Vygotsky, a child can become independent with a skill once she has been guided and instructed through the process prior to her independence. The ZPD theory emphasizes the need for a mentor (or "tools") during the learning process, especially as students learn a new process or concept. This mentor helps students advance to their personal zone of learning because they are challenged to think by a more advanced peer (Davydov, 1995). The social constructivist approach to Vygotsky's ZPD supports the foundational learning beliefs that students need social interaction, scaffolding instruction, and an opportunity to work with a more developed learner. Through this social constructivist approach to learning, educators could scaffold instruction and learning to promote collaborative processes that enhance and support students' cognitive development. Through collaborative efforts and communication, a wide range of useful mathematical connections are made so that students are capable of making profitable connections and constructions within their mathematical learning (Ernest, 1998). As students learn within their ZPD, the students create a process of cognitive, social, and emotional interchange because of the connections made through their cognitive assimilations and accommodations (Hausfather, 1996; Vygotsky, 1978).

When children are allowed to experience this social process in their mathematical learning, they experience a sense of justification and respect, and this promotes further learning (Davydov, 1995).

According to Vygotsky (1978), individual's higher functions originate through actual relationships between humans. As children grow and learn, their development appears twice. First, children's development occurs on a social level, between people; second, it occurs on an individual level. That is, individuals learn first through social interactions and then individually through an internalization process that leads to deep understanding. It is through social interactions and connections with other people that children (and humans in general) advance in their learning, owing to the connectivity they find with personal levels of development (Hausfather, 1996; Vygotsky, 1978).

According to Vygotsky (1978) "humans are active, vigorous participants in their own existence and that at each stage of development children acquire the means by which they can competently affect their world and themselves" (p.123). Vygotsky believed that children's play was a significant factor of conceptual knowledge and that, while playing with others, children emulated adult activities and roles that developed skills for future roles (Davydov, 1995). Vygotsky proposed that children's play in the educational setting did not disappear, rather surfaced during other learning, and created the foundation for the construction of future knowledge and beliefs (Vygotsky, 1978).

As part of this development, Vygotsky (1978) emphasized the importance of communication and speech. He claimed that speech not only facilitates the child's successful manipulation of objects but also controls the child's own actions. This development allows children the ability to form relationships through communication. According to Dangel and

Guyton (2004), schools must be models of interactive classrooms that encourage discourse and collaboration. Children's development depends upon the opportunities to interact, collaborate, and communicate; therefore, cooperative learning environments must encourage social discourse with others so that ideas and thoughts are shared, justified, and respected (Hausfather, 1996). It is essentially Vygotsky's views on the social aspect of learning, which Ernest incorporates, that distinguishes his model of social constructivism from the radical constructivism of von Glasersfeld.

Radical Constructivism

There are multiple strands of constructivism, most notably cognitive constructivism and social constructivism. They share common perspectives about teaching and learning. However, they differ in their emphasis on these perspectives. Constructivists such as Ernst von Glasersfeld view the construction of knowledge as an individual process, while others such as Ernest believe it requires social interaction and negotiation of meaning (Wilding-Martin, 2009).

Ernst von Glaserfeld introduced the term *radical constructivism* in the 1980s. His theory argues that knowledge cannot be passed directly from one learner to another; rather, it must be built up and developed by each individual learner. According to Thompson (2013), the radical form of constructivism "accounts for human interaction in terms of mutual interpretation and adaptation" (p. 4), whereas social constructivism introduces the idea of mathematical objectivity as a social construct.

The influence of von Glasersfeld's radical constructivism can be seen in Ernest's view of mathematical learning (Wilding-Martin, 2009). Both see knowledge as an active endeavor that must be constructed. However, Ernest and von Glasersfeld deviate from each other in their views of how this knowledge is constructed. For von Glasersfeld, the active construction of

knowledge is viewed as an individual process. Although von Glasersfeld acknowledges that much knowledge is socially induced, he argues it can only be formed through the cognitive efforts of the individual (von Glasersfeld, 1999). He states in his review of Ernest's *Social Constructivism as a Philosophy of Mathematics,*

> From my perspective, the active constructing of knowledge is under all circumstances an individual's enterprise–hemmed in, constrained, and guided, if you will, by interactions with others, but having access to no other raw material than the 'stuff' of the individual's own experience. (von Glasersfeld, 1999, p. 4)

He furthermore questions Ernest's shift away from a Piagetian/constructivist viewpoint. He points out that Ernest made this shift but did not suggest any other "conceptual tools that would enable individuals to profit from social transmission" (p. 4). In von Glasersfeld's opinion, Piaget's assimilation, accommodation, and reflective abstraction are compatible with the ideas of social constructivism (Wilding-Martin, 2009).

Similar to Ernest, Stephen Lerman (1996) believes in a constructivist view based on Vygotsky's social theory of learning rather than Piaget's theory of cognitive construction. In "Intersubjectivity in Mathematics Learning: A Challenge to the Radical Constructivist Paradigm?" Lerman challenges the idea of simply adding on a sociocultural view of learning to the radical constructivist philosophy and calling it social constructivism. He argues that a sociocultural theory is much more than a greater emphasis on social interaction. Lerman, like Ernest, believes the two are fundamentally different. To Lerman, radical constructivism views the individual as the creator of meaning; whereas social constructivism views meaning as "first sociocultural, to be internalized by the subject's regulation within discursive practices" (p. 147).

Radical constructivism and social constructivism share many characteristics in the learning environment, however. Jonassen (1994) provides a succinct summary of eight characteristics that underline constructivist learning environments and are applicable to both perspectives:

1. Constructivist learning environments provide multiple representations of reality.
2. Multiple representations avoid oversimplification and represent the complexity of the real world.
3. Constructivist learning environments emphasize knowledge construction inserted of knowledge reproduction.
4. Constructivist learning environments emphasize authentic tasks in a meaningful context rather than abstract instruction out of context.
5. Constructivist learning environments provide learning environments such as real-world settings or case-based learning instead of predetermined sequences of instruction.
6. Constructivist learning environments encourage thoughtful reflection on experience.
7. Constructivist learning environments "enable context- and content-dependent knowledge construction."
8. Constructivist learning environments support collaborative construction of knowledge through social negotiation, not competition among learners for recognition.

Davydov (1995) identifies four principles attached to concept of teaching developmentally as they relate to the constructivist theory. In order for teachers to use a developmental approach in their instruction that aligns to the theoretical beliefs of Piaget, Vygotsky, and von Glasersfeld, the following beliefs must be honored: (a) children construct

their own knowledge and understanding; we cannot transmit ideas to passive learners; (b) knowledge and understanding are unique for each learner; (c) reflective thinking is the single most important ingredient for effective learning; and (d) effective teaching is a child-centered activity (Van de Walle, 2004).

Ernest' Philosophical Influences

Whereas von Glasersfeld bases his theory of cognitive development on the work of Piaget, Ernest instead draws from Vygotsky's social theory of learning. In addition to Vygotsky, significant portions of Ernest philosophy of mathematics and philosophy of mathematics education are drawn from the works of Wittgenstein and Lakatos (Ernest, 1994; Wilding-Martin, 2009). Wittgenstein's philosophy relates language and mathematics. Whereas Vygotsky links language to the formation of the mind, Wittgenstein links language to the formation of a body of mathematical knowledge (Wilding-Martin, 2009). For Wittgenstein, language is based on social agreement. Through social interaction we agree on the meanings of words to communicate. According to Wilding-Martin, Wittgenstein uses the term *language games*, which refer to the linguistic patterns we follow and the rules that govern them, and *forms of life*, which refer to the rules, behaviors, and language games that provide the necessary context for meaning to develop. The development of knowledge and meaning through language games applies to mathematics terms and concepts as well. According to Wilding-Martin, for Wittgenstein, mathematics itself is merely a language game. Wittgenstein posits that language is learned through practice and that the meaning of a word is defined by its use. Therefore, terms for mathematical meaning are gained through patterns of social use. The two aspects of Wittgenstein's philosophy that are central to Ernest's own philosophy are: (a) mathematics is a language, and (b) mathematics is socially constructed (Wilding-Martin, 2009). Ernest (1998) holds that Wittgenstein's philosophy

of mathematics as language can be applied to the construction of individual knowledge, which can lead to a social view of mathematics learning as well.

Ernest further draws on the ideas of Lakatos in the formation of his philosophy of social constructivism. Lakatos's philosophy of mathematics is often referred to as "the logic of mathematical discovery" (Ernest, 1994; Wilding-Martin, 2009). His view of the formation of mathematical knowledge involves a cycle of conjecture, refutation, and new conjecture. According to Wilding-Martin, Lakatos refers to his philosophy as quasi-empirical because "counterexamples are generated abstractly, not observed in the spatio-temporal world" (p. 36). Lakatos believes that conversation, conflict, and argument should be present during the mathematical process. His logic of mathematical discovery involves doing mathematics through mental experimentation and interaction with others in order to discover new mathematical relationships. Lakatos argues that mathematical knowledge develops through quasi-empirical speculation and criticism rather than through the development of formal theorems (Wilding-Martin, 2009). He depicts the study of mathematics as interactive and lively. Lakatos holds, as Wilding-Martin states, "mathematical knowledge is generated through a creative process of speculating, finding counterexamples, and making appropriate adjustments to one's reasoning" (p. 38). He further argues that we cannot establish foundations of mathematical certainty; that we should make conjectures and then critique those conjectures, in order to correct or improve them. Ernest (1998) incorporated these ideas into his philosophy of mathematics. Ernest's views of the logic of mathematical discovery are based on Wittgenstein's concepts of language games and forms of life, and described by him as the "dialectical logic of human conversation and interaction" (p. 135).

Ernest's view of social constructivism actually consists of two philosophies: a philosophy of mathematics and a philosophy of mathematics education. As previously stated, Ernest's social constructivist philosophy draws on von Glasersfeld's radical constructivism, which posits that mathematics is constructed, not discovered. However, Ernest's view of social constructivism deviates from the views of radical constructivists by his emphasis on interpersonal communication and the use of conversation. According to Ernest (1998),

> Conversation includes any sequence of linguistic utterances or texts in a common language (or languages) made by a number of speakers or authors, who take it in turn to 'speak' (contribute) and who respond with further relevant contributions to the conversation. (p. 163)

Conversation in this context is a metaphor that refers to an interchange between people, both verbal and written (Wilding-Martin, 2009). To Ernest, the use of conversation is critical to the creation and justification of mathematical knowledge. The ideas of both Wittgenstein and Lakatos influenced Ernest to adopt the view that social and linguistic dimensions have a critical role in the genesis of mathematical knowledge, in addition to its justification (Ernest, 1998).

At this point, we have discussed the following influences that have shaped Ernest's philosophy of mathematics. First and foremost, his philosophy is based on the fallibilist assumption that mathematical truth is never certain. Ernest contends, like Vygotsky, that knowledge is actively constructed in social situations. He builds upon von Glasersfeld's view that subjective, independent knowledge results from the construction of meaning through experience. Drawing from Wittgenstein, Ernest believes that mathematics is a set of language games based on shared forms of life. And finally, Ernest uses the ideas of Lakatos to describe

how mathematical knowledge is formed. Next I conduct a closer examination of Ernest's views of mathematical knowledge.

Objective and Subjective Mathematical Knowledge

Although Ernest rejects the idea that mathematics has no claim to absolute truth, he does acknowledge the existence of *objective* mathematical knowledge. This objectivity refers to mathematical knowledge that is intersubjective and shared among the mathematical community. This intersubjective knowledge includes mathematical theorems, proofs, and conjectures, as well as the shared conventional symbols and use of language. Although inherently arbitrary, this objective knowledge, with its own set of rules and standards, can be passed down in history. According to Ernest (1994a), this body of knowledge "has an existence of its own that is separate from those who have contributed to it" (p. 67). This body of socially accepted knowledge cannot be altered except by widespread agreement within the mathematical community.

As previously stated, Ernest draws upon the works of Lakatos's logic of mathematical discovery. However, he extends and generalizes Lakatos's view of the generation of mathematical knowledge into a "generalized logic of mathematical discovery," which views the generation of objective mathematical knowledge as cyclic. A new proposal of mathematical context is presented publicly and subjected to further critique. The most important factor in the acceptance of a proposal is its proof. The claims of the proof are examined by the mathematical community, and then either rejected or accepted. Furthermore, the cultural values, preferences, and interests play a role in the objective knowledge formation. According to Wilding-Martin (2009), it is this generalized logic of mathematical discovery that allows for major shifts in conceptual frameworks of mathematical knowledge.

Ernest (1998) contends, however, that there are mathematical ideas that are *not* acknowledged by the mathematical community. The personal values and preferences of the mathematical community often influence what kind of mathematical knowledge is accepted, and the process by which it is validated (Wilding-Martin, 2009). Ernest sees this as social injustice. He states:

> In absolutist terms, there is no basis for asserting that the system of values of one culture or society is superior to all others. It cannot be asserted, therefore, that Western mathematics is superior to any other forms because of its greater power over nature. This would be to commit the fallacy of assuming that the values of Western culture and mathematics are universal. (p. 72)

Ernest contends that mathematical systems developed by different cultures may have different uses for mathematics, and all cultural mathematical systems should be acknowledge and valued by the mathematical community. This view is shared by others and is the basis for the development of ethnomathematics, a philosophy that "rejects inequity, arrogance, and bigotry while challenging the Eurocentric bias that denies the mathematical contributions and rigor of other cultures" (Arismendi-Pardi, 1999, p. 1).

Wilding-Martin (2009) points out that individual, subjective knowledge is typically not linked to a discussion of social constructivism. However, Ernest places it in his philosophy of mathematics because of its importance to the account of mathematics as it is passed from one generation to the next, and in allowing for the creativity of individual mathematicians. According to Ernest (1998), individuals actively construct their own meaning based on experiences, through a process of internalizing objective knowledge. He bases his view of subjective knowledge on von Glasersfeld's radical constructivist theory that learning is the active

construction of knowledge through experience. However, Ernest contends the role of communication and agreement between individuals is missing in von Glasersfeld's theory. As stated earlier, von Glasersfeld bases his theory of development on Piaget's cognitive view of learning; whereas, Ernest models his on Vygotsky's social theory of learning. According to Wilding-Martin (2009), "For Ernest, socially situated conversation is instrumental in the formation of the mind, and thus also in its use" (p. 76). For von Glasersfeld, learning is an individual process that is informed by social interaction with others. An individual first constructs meaning individually, and then collaborates with others, which may cause perturbations in the constructions. In reaction, the individual then revises the constructions to accommodate the experience. Although the conversation may have contributed to further understanding, the construction itself is formed individually. For Ernest, learning is a social process that happens through the interaction with others. The construction of meaning is thoroughly social, not individual. To Ernest, thought itself is based in language, and thus social experiences shape the very process of thinking and learning (Wilding-Martin, 2009).

According to Ernest (1998), mathematical learning begins very early, and is developed through participation in language games embedded in forms of life. Therefore, as Vygotsky contends, children learn by engaging in socially situated conversation through play. Through play, Ernest posits that children learn two concepts central to mathematical thought: the use of signal or signifier, and the creation of imaginary realities (Wilding-Martin, 2009). Then Lakatos's logic of mathematical discovery allows for the construction of subjective mathematical knowledge. As Ernest describes this process, "Learners push the boundaries of concepts, and through a cycle of conjectures and refutations learn more about those concepts and refine their personal constructions to achieve consistency and compatibility with others" (p. 219). Ernest's

view of the construction of subjective knowledge delineates the social nature of the mind and the central role of conversation in learning mathematics, which provides the basis for his theories of teaching and learning mathematics (Wilding-Martin, 2009).

Philosophy of Mathematics Education

We will now look to Ernest's philosophy of mathematics education. Wilding-Martin (2009) posits that although there may appear to be a natural association between Ernest's social constructivist philosophy of mathematics and his social constructivist philosophy of mathematical education, the latter can be considered on its own merits. In other words, one can follow his philosophy of mathematics without necessarily following his philosophy of mathematics education, and vice versa.

According to Ernest, social constructivism by itself, does not represent an approach to education, but must be combined with a set of values and an educational ideology. Ernest identifies five ideologies pertaining to mathematics education (in the United Kingdom): Industrial Trainer, Technological Pragmatist, Old Humanist, Progressive Educator, and Public Educator.

Ernest incorporates the "public educator ideology" and builds on it to form his philosophy of mathematics education. This ideology is based on the goals of democracy and social equity (Wilding-Martin, 2009). Ernest (1991, 1998) goes into to great detail in describing these ideologies, which I do not attempt here as it deviates from the scope of this project. As Ernest developed these ideologies in a British culture, there is some argument as to whether they can be applied in other countries (Wilding-Martin, 1999).

Aims

Ernest (1998) addresses three main areas of interest in his philosophy of mathematics education: aims and values for mathematics education, a theory of learning, and a theory of teaching. As Ernest emphasizes the importance of social responsibility to the discipline of mathematics, he also stresses the importance of an ethical responsibility in mathematics education. According to Wilding-Martin (2009), Ernest believes the aims of mathematics education should reflect the social nature of the discipline and include the promotion of social justice. In Ernest's opinion, the portrayal of mathematics as objective and disconnected from other disciplines also serves to dehumanize it. He therefore argues that mathematics education should recognize and work against the stereotypes that undermine the abilities of females and minorities. Ernest further posits that mathematics education should value discovery and creativity as much as justification, as well as respect the mathematical contributions of non-Western, non-traditional cultures (Wilding-Martin, 2009).

According to Ernest (1998), another aim of mathematics education is the idea of student empowerment. Ernest argues that mathematics education should empower students to "take control of their life, and to participate fully and critically in a democratic society" (p. 84). According to Ernest, this type of empowerment has three dimensions. Empowered students have mathematic ability, the proficiency to use mathematics in their lives, and confidence in their mathematical abilities.

The idea of humanizing mathematics is also advocated by Anna Sfard. In "Balancing the Unbalanceable: The NCTM Standards in Light of Theories of Learning Mathematics," Sfard (2003) examines the NCTM Standards in the light of theories of learning mathematics. Like Ernest, Sfard is a social constructivist who views mathematics as a social discipline. In her

assessment of the Standards, she gives credit for what she views is a comprehensive attempt to teach mathematics with a human face. She states, "the Standards must be applauded for the values they promote…the norms of the mathematics classroom seem more in tune with the norms of a democratic society than they have ever been" (p. 387). Sfard, like Ernest, believes the aims of mathematics education should reflect the social nature of the discipline and include the promotion of social justice and a democratic society to empower students.

Theory of Learning

Ernest's (1998) theory of learning centers around his view of subjective knowledge, which is formed by the social construction of meaning through conversation, and Lakatos's logic of mathematical discovery. He emphasizes that school mathematics is different from the mathematics learned in early childhood. Ernest states, "school mathematics involves enculturation into a new form of life and language games, which will form the basis of the learning that takes place there" (p. 85). Therefore, the social context of the classroom will shape how students think about themselves and mathematics.

According to Ernest (1998), conversation and dialogue are necessary in the construction of personal, subjective knowledge. Therefore, students need experiences that allow them to construct and refine concepts by discovering connections and testing their ideas in new contexts. They should be actively engaged in mathematical discussions, encountering different perspectives, and critiquing the arguments of others. Ernest's (1991) implications for school mathematics include:

- School mathematics for all should be centrally concerned with human mathematical problem posing and solving.

- Inquiry and investigation should occupy a central place in the school mathematics curriculum.
- The fact that mathematics is a fallible and changing human construction should be explicitly admitted and embodied in the school mathematics curriculum.
- The pedagogy employed should be process and inquiry focused, or else the previous implications are contradicted. (p. 283)

Differentiated Curriculum

Perhaps the most controversial facet of Ernest's philosophy of mathematics education is his view of a differentiated curriculum for future mathematicians. Ernest (1998) questions the practice of sending all students through the traditional advanced mathematics sequence in secondary (high) school. He feels that all students should share the same curriculum while in elementary school, but once they have acquired the basic mathematical competency, they should be allowed to select (or not select) mathematics courses based on their future plans. Ernest calls for a general mathematics curriculum for those students who do not plan to be further trained in mathematical fields after secondary school.

A general curriculum would focus on mathematics as an "intrinsically valuable and interesting part of human culture" (Wilding-Martin, 1998, p. 115), and would address big ideas of mathematics instead of detailed procedures. Ernest (1998) believes that there are many mathematical ideas that can be explored at this level without advanced mathematical knowledge, such as randomness, infinity, and symmetry. In referring back to his philosophy of mathematics, Ernest believes mathematics should be presented as an integral part of human culture, with the goal that students learn to appreciate its role in areas such as philosophy, art, science, and technology. This appreciation should extend to social justice as well. Ernest states that students

should be able to "identify, interpret, evaluate, and critique the mathematics embedded in social and political systems and claims, from advertisements to government and interest-group pronouncements" (p. 116).

Ernest also proposes a curriculum for future mathematicians. In this curriculum, all students share the same mathematics curriculum through elementary school. Then the future mathematicians will need to be enculturated into the mathematical community. The predominant areas of preparation for future mathematicians would be mathematical content knowledge, mathematical forms of life, and mathematical practice (Wilding-Martin, 2009). This curriculum should include content knowledge that includes the standards methods and procedures used to solve problems, as well include representative content such as statements, proofs, and procedures from various areas of mathematics. Furthermore, a curriculum for future mathematicians should address the history and philosophy of mathematics in order for future mathematicians to be able to "recognize the role of humanity in its development through history and the philosophical debates over its foundations" (Wilding-Martin, 2009, p. 128).

Ernest's vision of the curriculum for future mathematicians provides students in secondary school with opportunities to deepen and refine their knowledge of mathematical language games, learning to use the mathematical language in a more sophisticated way. They will expand their mathematical vocabulary and study new sets of symbols to accompany the new mathematical knowledge attained through conversation, problem posing, proofs, and refutations. But how does Ernest envision the *teaching* of mathematics?

Theory of Teaching

Ernest's theory of teaching also reflects the importance of the social context in teaching subjective knowledge. According to Ernest (1998), it is the responsibility of the mathematics

teacher to teach mathematics and promote social justice. The teacher should design activities that facilitate the construction of subjective knowledge through conversation and encourage democratic, critical thinking. To provide opportunities for the social construction of mathematical knowledge in a democratic context, Ernest proposes an investigatory approach to teaching mathematics. An investigatory approach is not synonymous with a problem-solving approach, however. Investigatory activities are more open-ended; the students investigate issues and choose the problems they will study. According to Wilding-Martin (2009), Ernest borrows from Freire the idea of *problem posing* pedagogy, which "encourages empowerment and social engagement by allowing students to question the curriculum and pedagogy in the classroom" (p. 90). Inquiry and problem solving are central to mathematics education, and should be used as a pedagogical approach in the entire mathematic curriculum. Students should be given opportunities to work in groups, choosing problems and topics from socially relevant contexts through engagement and conversation with others. Ernest believes that students should work alone at times as well to develop and explore creativity and self-direction (Ernest, 1998).

Ernest's social constructivist classroom is not a simple one, nor is it easy to attain. The teacher must have a deep understanding of mathematics to be able to guide the in-depth investigations. The expertise of the teacher is critical in this endeavor. The teacher must have sufficient mathematical knowledge to recognize interesting, relevant mathematical questions, as well as the skills to evaluate multiple approaches to solving the problems that students pose (Wilding-Martin, 2009). Indeed, if the teacher lacks either the necessary epistemological or pedagogical understanding of teaching mathematics, such a learning environment can quickly lead to chaos.

The mathematics curriculum Ernest proposes requires teachers who are knowledgeable and well trained in social constructivist pedagogy. How does Ernest account for teachers acquiring the knowledge and skills needed to implement instructional practices concurrent with social constructivism? According to Ernest (1991), mathematical pedagogical knowledge is only one variable that affects teachers' instructional practices. He proposes that the structures of knowledge, beliefs, and attitudes all interact in elucidating the overall understanding of mathematics teaching (Ernest, 1989a).

Ernest (1989) believes that teachers' mathematical content knowledge, beliefs, and attitudes toward mathematics play an important role in teachers' effectiveness and their decisions concerning instructional methodology. He proposes a model that describes how these three variables, which are stored in the mind as schemas, relate to teachers' instructional practices. Teacher knowledge represents the cognitive component of the model, and according to Ernest, includes the knowledge of mathematics, other subject matter, pedagogy and curriculum, and classroom management (Wilkins, 2008). The affective components of the model are teachers' beliefs and attitudes. Beliefs include conceptions of the nature of mathematics, models of teaching and learning mathematics, and principles of education. Attitudes include attitudes toward mathematics and attitudes towards teaching mathematics.

Ernest (1989b) posits that knowledge provides an essential foundation for the teaching of mathematics. Teachers need a substantial knowledge base in mathematics in order to plan for instruction and to understand and guide the learner's responses. This knowledge will facilitate teachers' explanations, demonstrations, diagnosis of misconceptions, acceptance of students' individual methods, and curriculum decisions. According to Ernest, knowledge of teaching mathematics can be divided into two areas, curriculum knowledge and pedagogical knowledge

of mathematics. Curriculum knowledge refers to knowledge of the materials and media through which mathematics instruction is carried out and assessed, such as school produced curricular materials and other resources. Pedagogical knowledge includes knowledge of how to represent mathematical topics and ideas in a way that children can understand; this includes knowledge of children's methods, concepts, difficulties, and common errors. It consists of those mathematical tasks, activities, and explanations that a teacher uses to transform and represent mathematical knowledge. Knowledge of organizational skills for teaching mathematics, knowledge of the school, and knowledge of educational theories also serve to impact teachers' instructional practices.

According to Ernest's (1989b) conceptual model, teachers' instructional practices begin with a personal philosophy of mathematics and what it means to do mathematics, which in turn influences their conceptions, or beliefs about teaching and learning mathematics. Ernest holds that these beliefs about mathematics and teaching mathematics also influence teachers' instructional practices. Ernest posits that a mathematics teacher's belief system has three parts: the teacher's ideas of mathematics as a subject for study, the teacher's idea of the nature of mathematics teaching, and the teacher's idea of the learning of mathematics. According to Ernest, the beliefs teachers hold about the nature of mathematics affects how teachers present the discipline to their students as well as the assumptions they hold about learning (Ernest, 1991).

Ernest (1991) posits that, as educators, our practices are informed by the beliefs and personal theories that we hold about mathematics, learning, and teaching. Beliefs about our own personal philosophies are formed by how we come to know and how students learn, which are reflected in and guide our daily teaching practices. These beliefs may not be explicit; in fact, they may have never been actually articulated. Nevertheless, if we pause to reflect on our

teaching practices, or especially if we try to explain our actions and behaviors, our beliefs surface and we become more aware of them. When our system of beliefs are acknowledged at a conscious level, we can begin to question, challenge, compare, and communicate them or even replace them by new beliefs that we might further decide to embrace. Epistemological beliefs or beliefs about what constitutes knowledge and how we come to know are fundamental because they influence or provide a basis for our beliefs about learning and teaching. As Ernest (1999) argues, "all practice and theories of learning and teaching rest on an epistemology, whether articulated or not" (p. 1).

In addition, teacher attitudes about mathematics have the potential to impact student attitudes and subsequent achievement in mathematics (Ernest, 1991). According to Ernest, attitudes towards mathematics and attitudes toward teaching mathematics also have a strong influence on teachers' instructional practices. Teachers' attitudes toward the discipline of mathematics include liking, enjoyment, interest, and confidence in mathematics. Mathematics anxiety and low self-efficacy are two constructs that may be embedded in attitudes towards mathematics for situations in which negative attitudes are prevalent.

Teachers' attitudes toward teaching mathematics also include liking, enjoyment, and enthusiasm for the teaching of mathematics, and confidence in the teacher's own mathematics teaching ability (Ernest, 1989). According to Ernest, attitudes toward the teaching of mathematics are especially important because of the effect they can have on the atmosphere of the classroom, as well as students' attitudes toward mathematics and achievement in mathematics.

This relationship is represented in the model (see Figure 1) with a single-sided arrow from each of these variables directed toward instructional practice. Teachers' instructional

beliefs are hypothesized to mediate the effect of teachers' subject-matter knowledge and mathematical attitudes, which in this study was replaced with the more specific attitude, mathematics anxiety. In addition to their direct influence on instructional practices, teachers' subject-matter knowledge and mathematical attitudes (mathematics anxiety) are hypothesized to indirectly influence instructional practice through their influence on teachers' instructional beliefs. In this case, beliefs would be considered a mediating variable in that, for example, subject-matter knowledge influences beliefs which in turn influences instructional practice. This relationship is represented in the model by the direct paths leading from subject-matter knowledge and mathematical attitudes (mathematics anxiety) to instructional beliefs to instructional practices. Finally, teachers' subject-matter knowledge and mathematics attitude (mathematics anxiety) are posited to have a reciprocal relationship. Whereas a teacher's level of knowledge in mathematics would likely influence her mathematical attitude (mathematics anxiety), it is also likely that a teacher's mathematical attitude (mathematics anxiety) could influence her attainment of content knowledge. This part of the model represents the hypothesized interrelationship among teachers' knowledge, beliefs, attitudes, and practice.

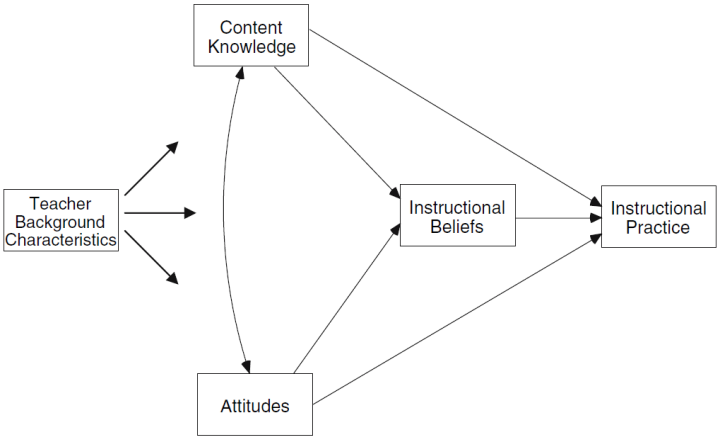

Figure 1. Theoretical model relating teachers' content knowledge, attitudes, instructional beliefs, and instructional practice.

Ernest's (1989) conceptual model illustrates how teachers' instructional practices are affected by teachers' subject-matter knowledge, instructional beliefs, and mathematical attitudes (mathematics anxiety). The model suggests that teacher's instructional practice originates with a personal philosophy of mathematics and what it means to do mathematics, which in turn influences their conceptions about teaching and learning mathematics. In considering Ernest's model for teacher education, the cognitive outcome of knowledge can be addressed directly in teacher education programs as the content of instructional and learning experiences. However, according to Ernest, the affective goals of the model cannot be addressed in this same way. Unlike content knowledge, beliefs and attitudes are formed by the teacher's personal reactions to experiences. Teacher education programs should take this into account when addressing the affective components of mathematics teaching. Therefore, Ernest's model provided a beneficial lens for examining and interpreting the interactions between the mathematics anxiety, mathematical beliefs, and instructional practices.

It should be noted, however, that research also suggests the relationships between attitudes, beliefs and instructional practices are reciprocal (Beswick, 2005). That is, attitudes and beliefs determine instructional practices, but change in teachers' beliefs may also be a consequence of change in their practices. Although the model (see Figure 1) appears causal due to the one-directional arrows, the relationship is actually didactical. According to Beswick (2005), attitudes, beliefs, and instructional practices develop together and influence each other. Hoyles, cited in Beswick (2005), describes beliefs as contextual. That is, "all of a teacher's beliefs are constructed as a result of experiences that necessarily occur in contexts" (p. 41). She argues that it is not contextual factors that prevent teachers from enacting certain beliefs, rather the contextual factors can elicit a set of beliefs that are in fact enacted. Simply stated, different

contexts give rise to different beliefs. Furthermore, according to Beswick, "it is unreasonable to expect consistency between broad collections of beliefs that are not closely linked with a specific context, and practice that is not described in equally broad, contextually independent terms" (p. 42). Therefore, Beswick suggests that teachers' beliefs and practices should be considered in broad terms.

Conclusion

While Ernest's work is based on constructivist principles, his social constructivism incorporates a theory of knowledge construction that is based on "socially situated conversation" (Ernest, 1999). Ernest social constructivist philosophy is influenced by Vygotsky's social learning theory, Wittgenstein's philosophy of language, and Lakatos's quasi-empiricist philosophy of mathematics. Ernest draws on Vygotsky's theory that learning is a social process, occurring when individuals are engaged in social activities. People create meaning through their interactions with each other and the objects in the environment. Ernest posits that learning is an active process of creating knowledge, often with others in a social context, so that it becomes personally meaningful.

Ernest draws upon von Glasersfeld's constructivist theory that subjective, independent knowledge results from the construction of meaning through experience. However, he deviates from von Glasersfeld's radical constructivist views on individual construction of knowledge without reference to the social process. Drawing from Wittgenstein, Ernest believes that mathematics is a set of language games based on shared forms of life. In addition, Ernest uses the ideas of the mathematics philosopher Lakatos to describe how mathematical knowledge is formed through his logic of mathematical discovery.

According to Ernest (1998), the theory of social constructivism encourages students to master goals more thoroughly than other instructional practices. The use of communication and discourse in the classroom promotes higher order thinking skills and focuses on the depth of knowledge required for mastering mathematics. In addition, NCTM strongly encourages the use of conversation and interaction in the mathematics classroom (NCTM, 2000). Ernest's social constructivist vision of mathematics education is ambitious. Furthermore, his vision of the mathematics classroom is a tall order for teachers, as well as teacher educators. Teachers must be trained to implement a social constructivist classroom, which presents many difficulties. One of the difficulties is that teachers often teach mathematics the way they were taught. If the teacher has mathematics anxiety, then this hurdle becomes a mountain.

Ernest provides a conceptual model that illustrates how teachers' instructional practices are a function of their subject-matter knowledge, instructional beliefs, and mathematical attitudes. This model served as the overarching theme in my study due to its relevance, context, and flexibility. It, I believe, expands the knowledge base of mathematics education in the interrelated areas of mathematical beliefs, mathematics anxiety, and instructional practices of elementary mathematics teachers. For only through recognizing the factors that negatively influence teacher' instructional practices in mathematics, can efforts be extended toward alleviating and eliminating such influences.

CHAPTER 3

LITERATURE REVIEW

To perceive the world differently, we must be willing to change our belief system, let the past slip away, expand our sense of now, and dissolve the fear in our minds.
—William James

It is a truth universally acknowledged in research communities that a comprehensive and methodical review of the literature is required in the research process. What is not universally acknowledged (and should be) is that this review should set the context for defining how the research will be an original contribution to the overall literature in the field (Garson, 2012). The purpose of this chapter is to illustrate, through the literature, the relationships between the mathematics anxiety, mathematical beliefs, and instructional practices of elementary teachers. This review is not simply an encyclopedia of all previous knowledge of these constructs; rather, it is meant to serve as a work in itself, which hopefully serves to provide the reader with a road map of the existing literature in the field related to the hypotheses. With that being said, however, it is also necessary to provide the reader with individual background information on these constructs. Therefore, the following questions are explored in this chapter:

- What is the affective domain, particularly with reference to mathematics anxiety and mathematical beliefs?
- What do we know about mathematics anxiety, it causes, and how it affects elementary mathematics teachers?
- What do we know about the mathematical beliefs concerning the nature of mathematics, and the teaching and learning of mathematics?
- How do these beliefs relate to elementary mathematics teachers?
- How are instructional practices defined for this study?

- How do these instructional practices relate to the mathematics anxiety and/or the mathematical beliefs of elementary school teachers?

According to Boote and Beile (2005), a quality literature review should be "a thorough, critical examination of the state of the field that sets the stage for the authors' substantive research projects" (p. 9). With that goal in mind, a comprehensive and systematic literature review was conducted in the spring and fall of 2014, bearing directly on the mathematics anxiety, mathematical beliefs, and instructional practices of elementary teachers. A keyword-based computerized search was conducted. All years were searched with no publication date limit. A search with combinations of the key-words "mathematics anxiety," "mathematical beliefs," "instructional practices," "elementary teachers," "inservice teachers," "pre-service teachers," "prospective teachers," and "practicing teachers" yielded a vast amount of literature. For example, a search of the keywords "mathematics anxiety" yielded a massive amount of literature, most of which did not focus on elementary teachers. Therefore, the additional keywords "elementary teachers" were added.

It is significant to note that the majority of research studies investigating mathematics anxiety and/or mathematical beliefs focuses on prospective (i.e., pre-service) elementary teachers currently enrolled in colleges and universities (Bekdemir, 2010; Bursal & Paznokas, 2006; Burton, 2012; Swars, Daane, & Giesen, 2006). This is understandable; the subjects are more accessible to the researchers who are often members of the faculty for these institutions. It is necessary to consider, however, that prospective teachers will be practicing (i.e., inservice) teachers in a few short years, taking their anxieties and beliefs toward mathematics into the classroom. Thus, I believe the literature focusing on prospective teachers can be extremely valuable and enlightening; and I deemed it important to include this literature in my review as

well, to gain an understanding of the disposition of elementary teachers as they enter the classroom.

The literature was sifted through, and narrowed to notable research journals, books, and dissertations. I further used citations in many of these works to lead to further related works, careful to focus on works found in notable research journals and books that relate to the stated hypotheses of this project. Through this iterative process, the results were narrowed to what I deem if not an exhaustive certainly a comprehensive inventory of literature relating to the mathematics anxiety, mathematical beliefs, and instructional practices of elementary teachers.

This literature review is divided into five sections: (a) the affective domain, (b) mathematics anxiety, (c) mathematics beliefs, (d) relationships between mathematics anxiety and mathematical beliefs, (e) and instructional practices. In the first part of this chapter, I provide a brief description of the affective domain, outlining its importance and relevance to the constructs in this study. I then define mathematics anxiety and mathematical beliefs (as they pertain to this study), and explore probable causes of these constructs. These definitions are followed by a synthesis of the literature relating to the mathematics anxiety and mathematical beliefs of prospective and practicing elementary teachers. The final part of the chapter is dedicated to reviewing the literature related to instructional practices of elementary teachers, specifically focusing on those scholarly works that demonstrate how instructional practices might (or might not) align with mathematics anxiety and mathematical beliefs.

The Affective Domain

Since the early 1960s, researchers in mathematics education have considered the affective domain as an important aspect of teaching and learning mathematics (McLeod, 1992). Although the learning of mathematics is primarily a cognitive endeavor, affective variables also play an

important role (McLeod, 1992; Reyes, 1984). In fact, McLeod (1992) claims that it has long been clear that affective issues play a central role in the learning of mathematics. The affective domain is comprised of emotions, attitudes, beliefs, opinions, interests, motivation, moods, and values (Goldin, 2002; McLeod, 1992). According to Reyes (1984), positive attitudes, in addition to knowledge, are essential to the learning of mathematics—one without the other is not sufficient.

In the past few decades, research in mathematics education has begun to focus on the affective characteristics of mathematics teachers (Ernest, 1989; Hart, 2002; Ho, 2000; Lim & Chapman, 2013; McLeod, 1992; Philipp, 2007; Raymond, 1997). It is believed that the affective characteristics may be the missing variable that links teachers' practices to students' learning (Ernest, 1989a). According to McLeod (1992), "all research in mathematics education can be strengthened if researchers will integrate affective issues into studies of cognition and instruction" (p. 575). In addition to teachers' cognitive characteristics directly influencing student learning, it is also believed teachers' characteristics influence how teachers teach, which then influences student learning. In other words, instead of focusing on what the teacher *does* in the classroom, research has increasingly focused on *who* the teacher *is* in an affective sense (Hart, 2002). Philipp (2007) states,

> Although few researchers have examined the relationship between mathematics teachers' affect and their instruction, the existing research shows that the feelings teachers experienced as learners carry forward to their adult lives, and these feelings are important factors in the ways teachers interpret their mathematical worlds. (p. 258)

According to Reyes (1984), confidence is one of the most important affective variables in the learning of mathematics. He states, "Confident students tend to learn more, feel better about themselves, and be more interested in pursuing mathematical ideas than students who lack confidence" (p. 560). Two affective variables strongly related to confidence are mathematics anxiety and mathematical beliefs. Although beliefs are more cognitive in nature, they are central in the development of attitudinal and emotional responses to mathematics. Therefore, beliefs can be considered part of the affective domain (McLeod, 1992). McLeod states, "Because beliefs provide an important part of the context within which attitudinal and emotional responses to mathematics develop, we need to establish stronger connections between research on beliefs and research on other aspects of the affective domain" (p. 248).

Since the 1980s, there has been an explosion of research investigating both student-related and teacher-related factors contributing to mathematics anxiety and mathematical beliefs (e.g., Beilock, Guderson, Ramirez, & Levine, 2010; Jackson & Leffingwell, 1999). Much of this research focuses on student-related factors (Philipp, 2007). However, there is ample literature on teacher-related factors as well. It is in this area that one finds evidence that mathematics anxiety and mathematical beliefs may influence the preferred instructional practices of elementary school teachers (Gresham, 2007; Wilkins, 2008). As previously stated, there have been significantly fewer studies on the mathematics anxiety and mathematical beliefs of practicing elementary teachers; however, it has been reported that a disproportionately large percentage of this population experience significant levels of mathematics anxiety and hold traditional beliefs about mathematics (Hembree, 1990; Hadley & Dorward, 2011; Wilkins, 2008; Wood, 1988).

Mathematics Anxiety

Of the wide range of affective variables related to teaching and learning mathematics, mathematics anxiety has been the most actively researched. Mathematics anxiety is a complex construct and has been defined in numerous ways. Mathematics anxiety has been defined as both a cognitive dread of mathematics, and a learned emotional feeling of intense frustration or helplessness about one's ability to complete mathematical tasks (Gresham, 2007). Robertson and Claesgens (1983) define mathematics anxiety as an irrational fear of mathematics, which produces tension that interferes with the use of numbers and solving of mathematical problems. Trujillo and Hadfield (1990) describe mathematics anxiety as a state of discomfort that occurs in response to situations involving mathematical tasks that are perceived as threatening to self-esteem. While there are many definitions of mathematics anxiety, the most frequently quoted is probably that of Richardson and Suinn (1972) who described mathematics anxiety as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (p. 551).

The research on the classification of mathematics anxiety is conflicting and ambiguous, however. Some researchers consider mathematics anxiety as an affective variable in addition to the broad constructs of motivation and attitudes. Lim and Chapman (2013) investigated the relationship between variables of the affective domain and mathematics achievement. In their study, anxiety, motivation, and attitudes were classified as three distinct but highly correlated affective variables. Other researchers represent mathematics anxiety as a sub-construct of attitudes toward mathematics (Jong & Hodges, 2013). In Aslan's (2013) study of the mathematics anxiety and beliefs of prospective teachers, he referred to attitudes toward mathematics as a multi-dimensional construct, which encompasses mathematics anxiety. In fact,

in educational research, mathematics anxiety is frequently consolidated with the broader issue, attitudes toward mathematics (Bessant, 1995; Kolstad & Hughes, 1992; Wilkins, 2008).

Mathematics anxiety became a prominent topic of research in the 1970s, although the existence of the problem was acknowledged much earlier (Dreger & Aiken, 1957; Gough, 1954). In 1957, Dreger and Aiken first used the term mathematics anxiety in describing students' negative attitudes toward mathematics. In the 1950s, mathematics anxiety initially paralleled test anxiety, and this pairing is often formed in the present. In 1972, Richardson and Suinn brought the construct of mathematics anxiety to the research forefront with the development of the Mathematics Anxiety Rating Scale (MARS). In 1978, Tobias, considered a pioneer in the field of mathematics anxiety, popularized mathematics anxiety with the publication of her book, *Overcoming Math Anxiety*, which significantly assists in demystifying the subject of mathematics.

Sub-constructs of Mathematics Anxiety

Hembree (1990) conducted a meta-analysis that included 151 studies involving mathematics anxiety. As an initial emphasis, Hembree focused on the sub-constructs of mathematics anxiety that included test anxiety and mathematics anxiety. Hembree's goal was to determine the relationship between mathematics anxiety and mathematics performance. He found that mathematics anxiety appears to be a learned condition, more behavioral than cognitive in nature. He also found that mathematics anxiety and test anxiety were positively related. Furthermore, it was concluded that mathematics anxiety is not restricted to text anxiety, but appears to comprise of a general distress from any contact with mathematics.

Due to the high public profile of mathematics achievement in our nation, elementary schools place a great emphasis on test scores in this subject (Vinson, 2001). Teachers are under

constant pressure to ensure their students attain high achievement scores, especially in mathematics. Such pressures may create mathematics anxiety among elementary teachers, which paradoxically may result in lower mathematics achievement scores for students on statewide tests (Shaw, 1990). Studies have emerged that differentiate between mathematics anxiety and mathematics test anxiety (Bush, 1989; Kelly &Tomhave, 1985). Nevertheless, despite the overlap among mathematics anxiety and mathematics test anxiety, the evidence is convincing that mathematics anxiety is a distinct and separate phenomenon (Ashcraft, 2002).

Not only do elementary teachers experience mathematics anxiety, some suffer from mathematics teaching anxiety. According to Peker (2009), mathematics teaching anxiety is based on an individual's ability to teach mathematics. Whereas mathematics anxiety tends to be internally focused and reflects an individual's lack of knowledge or confidence in mathematics, mathematics teaching anxiety is externally focused and reflects on how well an individual engages students in the process of learning mathematics (Brown, Westenskow, & Moyer-Packenham, 2011).

In addition, Brown, Westenskow, and Moyer-Packenham (2011) found that anxiety of teaching mathematics is not always related to previous mathematics anxiety. Their study examined the relationship between mathematics anxiety and mathematics teaching anxiety of elementary prospective teachers. It was supposed that there would be a positive correlation between mathematics anxiety from prior experiences and mathematics teaching anxiety. That is, prospective teachers having high mathematics anxiety from prior experiences would also exhibit anxiety in teaching the subject. Likewise, it was hypothesized that prospective teachers who had little or no prior mathematics anxiety experiences also would have little or no teaching anxiety. The results showed that one-third of the group either had mathematics anxiety but no

mathematics teaching anxiety, or did not have mathematics anxiety but had mathematics teaching anxiety. Therefore, the relationship between the two constructs is not consistent. Nevertheless, these results have important implications for teacher training programs. Not only should teacher educators focus on prospective teachers who experience mathematics anxiety, but also provide support to those who experience mathematics teaching anxiety (Brown, et. al, 2011).

Causes of Mathematics Anxiety

To develop a thorough understanding of elementary teachers' mathematics anxiety, it is necessary to explore the research that examines the causes to which they attribute their anxieties. Various precursors of mathematics anxiety have been identified in the literature (Battista, 1986; Gresham, 2008; Harper & Daane, 1999; Hembree, 1990; Jackson & Leffingwell, 1999); however, an explicit, definitive cause of mathematics anxiety has not been established (Ashcraft & Kirk, 2001). Although the specific causes of mathematics anxiety are not known, research does shed light on possible sources.

In the early 1970s, gender differences were thought to be the reason for mathematics anxiety. Many believed that men had stronger backgrounds in math or that math was a male's subject (Tobias, 1976). Therefore, women taking mathematics courses were considered to be weaker than male students. This debility was thought to cause mathematics anxiety among girls and women. According to Tobias (1976), women may often change their educational career choices if mathematics plays a major role in course selection.

According to the 2012 National Survey of Science and Mathematics Education, over 90% of elementary teachers are women (Banilower et al., 2013). According to Hembree (1990), there are studies that have found a higher degree of mathematics anxiety in women majoring in

elementary education than in the men. Although gender is not examined as a variable in the current study, it is worthwhile to also note that several studies report there is no correlation between gender and anxiety in mathematics (Ho, 2000; Lindberg, Hyde, Petersen, & Linn, 2010; Ma, 1999). However, elementary majors in colleges were identified as a largely female population, and this population was found to have the highest level of mathematics anxiety and mathematics avoidance behaviors of any college major (Hembree, 1990; Beilock et al., 2010).

Trujillo and Hadfield (1999) hypothesized the factors that might cause mathematics anxiety, and grouped them into three areas: environmental, intellectual, and personality factors. The environmental factor includes classroom experiences, insensitive teachers, and parental pressure. The intellectual factor includes teaching mathematics with mismatched learning styles, lack of confidence in mathematical ability, and lack of perceived usefulness of mathematics. The personality factors include reluctance to ask questions due to low self-efficacy, and viewing mathematics as a male domain.

Mathematics anxiety is considered by many to stem from negative prior experiences in the mathematics classroom (Battista, 1986; Harper & Daane, 1999; Hembree, 1990; Gresham, 2008; Jackson & Leffingwell, 1999). A study by Uusimaki and Nason (2004) investigated the causes underlying a sample of primary prospective teachers' negative attitudes and anxiety about mathematics. Through in-depth interviews, it was discovered that the prospective teachers often felt anxiety in mathematical content involving space, as well as algebra. It was also discovered that prospective teachers often felt highly anxious when teaching mathematics in practicum situations. It was found that the origin for mathematics anxiety for most of the participants could be attributed to prior school experiences involving their primary school teachers.

Similarly, Harper and Daane (1999) investigated the mathematics anxiety of 53 prospective teachers before and after they completed an undergraduate mathematics methods course. Through qualitative methods, they found that mathematics anxiety often began in elementary school, possibly instilled by teachers who experience mathematics anxiety themselves. Harper and Daane hypothesized that "many of the causes of mathematics anxiety have stemmed from rigid and structured classroom instructional practices" (p. 35). Students in this type of classroom may feel pressure to perform mathematics within a certain time limit, or solve math problems using only the one "right" way. Some students report being made to feel "stupid" in front of the math class by asking a "dumb" question. According to Harper and Daane, these embarrassing incidences are likely to cause students to lose confidence in as well as develop a negative attitude toward mathematics. They further posit that undue pressure on test scores and grades can produce anxiety in students (Harper & Daane, 1999).

In addition, Jackson and Leffingwell (1999) studied the mathematics anxiety of 157 prospective teachers. The prospective teachers were asked, "Describe your worst or most challenging mathematics classroom experience from kindergarten through college" (1999, p. 583). Jackson and Leffingwell analyzed and categorized the responses of the 27% whose anxiety developed in their freshman year as follows: (a) communication and language barriers, (b) insensitive and uncaring attitude of instructors, (c) quality of instruction, (d) evaluation of instruction, (e) manipulative instructor dislike for level of class, (f) gender bias, (g) and age discrimination. They also classified the instructor behaviors as covert and overt behaviors, noting that both have a detrimental effect on mathematics anxiety (Jackson & Leffingwell, 1999). Overall, the literature clearly identifies negative classroom experiences as the leading

cause of mathematics anxiety in elementary teachers (Battista, 2006; Bekdemir, 2010, Hembree, 1990; Swars, Daane, & Giesen, 2006).

Cognitive Aspects

Although mathematics anxiety is believed to be caused by negative prior experiences, it also has origins in the cognitive domain. Sloan, Daane, and Giesen (2002) investigated the relationship between prospective teachers' mathematics anxiety and their learning styles. The findings indicated that a global (right-brain dominant) learning style was related to levels of mathematics anxiety. There was a low but significant positive correlation between the global learning style and mathematics anxiety, which suggest learning styles could be a contributing factor of mathematics anxiety. As global orientation scores increased, levels of mathematics anxiety increased as well, indicating that global learners tend to exhibit higher levels of mathematics anxiety. Sloan and colleagues acknowledged that the positive correlation was low, and hypothesized that other variables such as instructional methods, mathematics achievement levels, and mathematical confidence may account for more of the variance.

In addition, Ashcraft (2002) contends mathematics anxiety lowers ability level in mathematics because "paying attention to these intrusive thoughts acts like a secondary task, distracting attention from the math task" (p. 184). In other words, the mathematics ability of a highly anxious individual may be masked by the anxiety, whereby the mathematics anxiety takes precedence over the mathematics ability, preventing it from emerging. Ashcraft further argues that mathematics anxiety disrupts cognitive processing by compromising current activity in working memory. Generally speaking, researchers agree that more research is needed on the origins of mathematics anxiety and on its "signature" in cognitive activity in order to analyze both its affective and cognitive components (Ashcraft, 2002; Peker, 2009; Stodolsky, 1985).

Avoidance Tendencies

The most pervasive tendency of mathematics anxiety is avoidance. Individuals with mathematics anxiety take fewer mathematics courses in high school and college (Hembree, 1990; Stodolsky, 1985). Avoidance compounds the problem, however. Avoidance prevents mastery of the content skills and makes individuals with mathematics anxiety less competent in mathematics (Ashcraft & Kirk, 2001). Students with mathematics anxiety also tend to avoid educational paths and careers that require mathematical courses. The anxiety can be so severe for some individuals that they actively and purposefully avoid mathematics at all costs (Hembree, 1990).

Research has shown that elementary teachers with mathematics anxiety may also avoid mathematics in the classroom. Studies have shown that teachers with high mathematics anxiety spend less time teaching mathematical concepts in the classroom (Aslan, 2013; Austin, Wadlinton, & Bitner, 1992; Hadley & Dorward, 2011). Swetman, Munday, and Windham (1993) discovered that elementary school teachers with high levels of mathematics anxiety spend less time planning mathematics lessons and use mathematics instructional time for activities unrelated to mathematics.

Similarly, Trice and Ogden (1986) investigated the instructional practices of first-year elementary teachers. Forty first-year elementary school teachers were observed once a week for three weeks and asked to submit lesson plans for analysis. The Revised Mathematics Anxiety Rating Scale was also administered to the teachers to determine the levels of mathematics anxiety. The results suggested mathematics was not a major focus in the classroom for teachers with mathematics anxiety. In fact, the teachers who were found to have the highest levels of

anxiety avoided teaching mathematics altogether. Through interview analysis, these teachers also revealed a mild dislike for mathematics (Trice & Ogden, 1986).

Robertson (1991) described a cycle of mathematics avoidance that leads to a series of phases of mathematics anxiety. In phase one, a person experiences negative reactions to mathematical situations perhaps resulting from past negative experiences with mathematics, and subsequently leading to phase two in which a person avoids mathematical situations. This avoidance leads to phase three, poor mathematics preparation, later resulting in phase four, poor mathematics performance. This poor performance generates more negative experiences with mathematics that subsequently cycles back to phase one. This cycle can be repeated, resulting in a mathematics anxious person becoming increasingly convinced that he or she cannot do mathematics. Research by Robertson suggests that individuals are rarely able to break the cycle.

The Cyclic Nature of Mathematics Anxiety

Learned behaviors, as previously noted, can extend to create further mathematics anxiety. The literature reviewed has strongly suggested that mathematics anxiety is a learned behavior, which is contagious (Austin, Wadlinton, & Bitner, 1992; Brady & Bowd, 2005; Burns, 1998). Burns (1998) explains the phenomenon of the repetitive nature of mathematics anxiety in her book *Math: Facing an American Phobia*. Burns states, "The way we've traditionally been taught mathematics has created a recurring cycle of *math phobia*, generation to generation, that has been difficult to break" (p. x).

For example, Brady and Bowd (2005) conducted a study among 238 prospective teachers in Canada, and the findings were aligned with those of Austin, Wadlinton, and Bitner (1992). After administering the Mathematics Anxiety Rating Scale (MARS), the researchers concluded that the participants' mathematics anxiety stemmed from previous formal instruction in the

subject area; however, although the participants identified the source of their anxiety, the study indicated that the cycle of mathematics anxiety in teachers and students would be continued through the future teachers' instructional practices (Brady & Bowd, 2005). The research clearly suggests that teachers who exhibit a sense of mathematics anxiety convey that anxiety to students through their instructional practices, and the cycle of mathematics anxiety continues (Austin, Wadlinton, & Bitner, 1992; Beilock et al., 2010; Brady & Bowd, 2005).

Reducing Mathematics Anxiety

Educational researchers are in agreement that reducing mathematics anxiety is an important component in the preparation of elementary teachers. The literature reviewed indicates that prospective elementary teachers often enter education programs with mathematics anxiety related to prior experiences (Battista, 1986; Hembree, 1990; Kelly & Tomhave, 1985; Levine, 1993). It is critical, therefore, to provide support for prospective teachers in their teacher education programs; early mathematics teaching experiences affect future mathematics teaching experiences. Uusimaki and Nason (2004) recommend that facilitators of teacher training courses in elementary mathematics be non-intimidating and supportive in nature. They also emphasize that learning environments should allow prospective teachers to freely explore and communicate about mathematics in a supportive group environment.

In order to alleviate mathematics anxiety, teachers need to understand their feelings toward mathematics (Austin, Wadlinton, & Bitner, 1992; Battista, 1986; Gresham, 2007). At the elementary level, educators who are not comfortable with mathematics owe it to themselves and to their students to undergo a process of confronting the source of their discomfort and continuing their own education in mathematics (Harper & Daane, 1998). Teachers must counteract myths and negative beliefs about mathematics developed from prior experiences with

new positive experiences (Beswick, 2006). Professional development can help teachers accomplish these goals (Vinson, 2001).

Several studies have demonstrated the effectiveness of a mathematics methods course in working with prospective teachers who exhibit mathematics anxiety (Battista, 1986, Harper & Daane, 1998; Swars, 2007; Vinson, 2001). These studies suggest that teacher education programs have the ability to influence and reduce the mathematics anxieties of prospective teachers. Creating a stress-free learning environment is essential in accomplishing this goal (Battista, 1986, Harper & Daane, 1998; Kelly & Tomhave, 1985). Researchers agree that highly anxious prospective teachers require teacher educators who are supportive and nonthreatening to ensure their success in overcoming their mathematics anxieties (Battista, 1986; Malinsky et al., 2006; Swars, Daane, & Giesen, 2007).

Mathematical Beliefs

In addition to mathematics anxiety, the literature suggests that teachers' beliefs about the nature of mathematics and the teaching and learning of mathematics may also have a significant impact on the instructional practices of elementary teachers (Akinsola, 2008; Beswick, 2006; Ernest, 2000; Uusimaki & Nason, 2004; Wilkins & Brand, 2004). Research findings suggest that beliefs about the nature of mathematics influence teachers' perceptions of how mathematics should be presented (Beswick, 2006; Handal, 2003). Elementary teachers were once students, and many of their mathematical beliefs were developed while they were in the classroom. For many prospective teachers, their early classroom experiences, too often saturated with inadequate mathematics instruction and ineffective teaching practices, have contributed to limited content knowledge and a lack of confidence in mathematics (Hembree, 1990). It is not surprising that these negative classroom experiences may have affected their attitudes toward

mathematics, and would likely impede their effectiveness in teaching mathematics. Therefore, the methods of mathematics instruction that teachers use in the classroom may very well be products of their beliefs.

Beliefs Defined

Various definitions or descriptions of the term beliefs have been proposed in the educational and psychological literature over the past two decades, which illustrates the difficulty of establishing a clear definition of the term (Barkatsas & Malone, 2005). It is suggested by some that a consensus on a definition is not necessary. Leder and Forgasz (2002), for example, argue that useful work can be done without full agreement about the precise definition. Törner (2002), however, argues that the functional role of a definition helps to define areas of research and pose relevant research questions (as cited in McLeod & McLeod, 2002). Nevertheless, there are some prevalent definitions or descriptions of the term beliefs that have been proposed in the educational and psychological literature (Barkatsas & Malone, 2005).

Philipp (2007) defines beliefs as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (p. 259). According to Thompson (1992), beliefs are more cognitive, are felt less intensely, and are harder to change than attitudes. Furthermore, beliefs might be thought of as lenses through which one's view of some aspect of the world is affected (Philipp, 2007). Thompson identifies conceptions as beliefs, views, and preferences. Although she identifies beliefs as a subset of conceptions, she frequently uses the terms interchangeably. Hart's (2002) interpretation classifies beliefs as part of our subjective knowledge, with a strong affective component. Raymond (1997) defines mathematical beliefs as "personal judgments about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics" (p.

552). In Goldin's (2002) view, beliefs are "multiply-encoded, internal cognitive/affective configurations, to which the holder attributes truth value of some kind" (p. 59).

Ernest (1991) contends that a mathematics teacher's belief system has three parts: the teacher's ideas of mathematics as a subject for study, the teacher's idea of the nature of mathematics teaching, and the teacher's idea of the learning of mathematics. Askew, Brown, Rhodes, Johnson, and William (1997) characterized the orientations of teachers toward each of these components as transmission, discovery, or connectionist. According to Swan (2006), transmission-oriented teachers believe that mathematics is a set of factual information that must be conveyed or presented to students, and typically enact didactic, teacher-centered methods. Discovery-oriented teachers view mathematics as a set of knowledge best learned through student-guided exploration, and frequently tend to focus on designing effective classroom experiences that are appropriately sequenced. Lastly, connectionist-oriented teachers view mathematics as an intertwined set of concepts, and they rely heavily on experiences to help students learn about the connections between mathematical topics (Swan, 2006).

According to McLeod (1992), the terms beliefs, attitudes, and emotions are terms that express the range of affect involved in mathematical learning. Although he considers beliefs to be an affective construct, he also notes that beliefs are "largely cognitive in nature, and are developed over a relatively long period of time" (p. 579). He further describes beliefs as relatively stable and resistant to change.

According to McLeod (1989), there are two categories of beliefs which influence mathematics teaching and learning: beliefs about mathematics, and beliefs about self. First, students develop a system of beliefs about mathematics as a discipline. McLeod claims, "These beliefs generally involve very little affect, but they form an important part of the context in

which affect develops" (p. 246). A second category of beliefs deals with students' (and teachers') beliefs about themselves and their relationship to mathematics. This category has a stronger affective component and includes beliefs that are related to confidence, self-concept, and causal attributions of success or failure. In this study, I considered both categories as described by McLeod.

Classification of Beliefs

Pajares (1992) proposed, "All teachers hold beliefs, however defined and labeled, about their work, their students, their subject matter, and their roles and responsibilities" (p. 314). Because "humans have beliefs about everything" (p. 315), Pajares recommended that researchers make a distinction between teachers' broader, general belief systems and their educational beliefs. In addition, he recommended that educational beliefs be narrowed further to specify the focus of those beliefs, for example, educational beliefs concerning the nature of knowledge. Following Pajares' recommendation, this study focused specifically on teachers' beliefs about the nature of mathematics, and the teaching and learning of mathematics (Ernest, 1989a). Henceforth, in this study, the term *beliefs* refers to beliefs about the nature of mathematics, and the teaching and learning of mathematics.

Mathematical beliefs can be further narrowed and classified as constructivist-oriented beliefs and traditional beliefs. Teachers who hold constructivist-oriented beliefs perceive learning as an active construction and reconstruction of knowledge, and teaching as a process of guiding and facilitating learners in the process of knowledge construction. This belief is contrasted with teachers who hold traditional beliefs and tend to perceive learning as a passive activity, with students holding little responsibility for their own learning, and view teaching as

merely dispensing knowledge to students. Teachers who hold constructivist-oriented beliefs (most often) would agree with the following statements (Beswick, 2005):

- Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics.
- Teachers can create for all students a nonthreatening, supportive learning environment.
- I would be comfortable with a child suggesting a solution to a mathematics problem I had not thought of previously.
- Effective mathematics teachers enjoy learning and doing mathematics themselves.
- Mathematics is useful.

Teachers who hold traditional beliefs (most often) would agree with the following statements (Beswick, 2005):

- A mathematical mind is needed to be good at mathematics.
- There is usually one correct way to solve a mathematics problem.
- Males are better at mathematics than females.
- It is important to cover all the topics in the mathematics curriculum in the textbook sequence.
- Mathematics is not useful.

The Constraining Nature of Educational Environments

In accordance with mathematics anxiety, the literature suggests that teachers' mathematical beliefs may be formulated from prior experiences in mathematics (Hembree, 1990). In addition, the surrounding culture and society can influence the formation and development of mathematical beliefs (McLeod, 1992). According to Handal (2003), in traditional environments, even teachers with progressive educational beliefs are forced to

compromise and conform to traditional instructional styles. Teachers generate their own beliefs about how to teach during their school years and these beliefs are aligned with their teaching practice. Thus, their educational beliefs are passed on to the students. By the time candidates enroll in a teacher education program, these ideas are so solidified and entrenched in their personal philosophy that they are most often passed on to their students once the candidates begin their teaching careers, thus perpetuating a cycle similar to one identified with mathematics anxiety (Handal, 2003).

Shaw (1990) conducted a qualitative study with three middle school teachers that determined the differences between teachers' ideal beliefs and actual beliefs about understanding and how these factors influenced teachers' instructional practices. In his study, ideal beliefs represented what teachers preferred to teach in order for students to learn; actual beliefs represented how the teachers actually taught based on contextual factors. The results indicated that teachers held a system of beliefs about mathematics teaching and learning that were different from their actual teaching beliefs and implemented classroom practices. Teachers were led by their actual beliefs, rather than their ideal beliefs. Shaw concluded that several contextual factors attribute to the way teachers define their beliefs, such as how they learn mathematics, how they teach mathematics, students' backgrounds, students' goals for learning mathematics, standardized tests, administrative demands, textbooks, and time.

Similarly, according to Ernest (1999), there is a great disparity between espoused and enacted models of teaching and learning mathematics. Although they may have been taught to adopt a reformed practice during their teacher education program, practicing teachers are subject to the constraints and contingencies of the school context once they enter the classroom. They may be influenced by the expectations of others, especially other teachers and superiors. This

influence also results from the institutionalized curriculum represented by adopted curriculum materials and assessment methods. Ernest points out that "the socialization effect of the context is so powerful that teachers in the same school, despite having differing beliefs about mathematics and its teaching, are often observed to adopt similar classroom practices" (p. 27).

Furthermore, Richardson (1996) argues that in some cases a new belief does not promote a change in practices because they may be unfamiliar with a specific educational innovation. According to Richardson:

> It cannot be assumed that all changes in beliefs translate into changes in practices, certainly not practices that may be considered worthwhile. In fact, a given teacher's belief or conception could support many different practices or no practices at all if the teacher does not know how to develop or enact a practice that meshes with a new belief. (p. 114)

As previously stated, beliefs are structured, stable, and develop over a long period of time. Ernest (1989) contends, "Teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change" (p. 99). Can beliefs be changed? According to Pajares (1992), beliefs about mathematics are unlikely to change unless individuals are dissatisfied with their existing beliefs. And they are unlikely to be dissatisfied unless they are challenged and one is able to assimilate them into existing conceptions. Pajares states, "new beliefs must be intelligible and appear plausible before most accommodation can take place" (p. 320). Part of the process of changing mathematical beliefs must be creating a context in which it is emotionally safe to do so. Affecting change in mathematical beliefs is critical for teachers. For, as Lloyd (2002) claims, "the success of current mathematics education

initiatives depends on our identification of viable ways to encourage and enable teachers to make significant shifts in their beliefs" (p. 150).

Instructional Practices

Research clearly indicates that teachers can influence students' learning experiences through instructional practices (Austin 1992; Brady & Bowd, 2005; Burns, 1998; Hembree, 1990). Mathematics anxious teachers often exhibit characteristics and behaviors in their instructional practices similar to the ones that caused their own mathematics anxiety (Beilock et al., 2010; Brady & Bowd, 2005; Burns, 1998). Research also shows that the relationship between teachers' mathematical beliefs and their instructional practice is mediated by many conflicting factors.

Instructional Practices Related to Anxiety

While the teaching of mathematics using constructivist-based practices remains a principal aim of the current mathematics reform movement, it is nevertheless viewed as threatening for many prospective and practicing elementary teachers (Uusimaki, 2001). It is not surprising that many classroom teachers feel alienated from the reform process; for teaching mathematics using constructivist-based methods can be intimidating and extremely difficult, even for those who have training and experience (Ernest, 1991). Many teachers are asked to teach mathematics in a way that is completely different from the way in which they were taught mathematics. First-year teachers especially have difficulty and experience anxiety in teaching using constructivist-based instructional practices (Raymond, 1997). Furthermore, many prospective teachers enter education programs believing the content in the elementary grades is simple, and that they already have the knowledge they need to teach. This belief is not the case

with mathematics, however, for the level of content knowledge they have received is often not adequate for teaching mathematics (Hembree, 1990).

There is a plethora of research suggesting that mathematics anxiety has a direct relationship with teachers' instructional practices (Bush, 1989; Furner & Berman, 2003; Hadley, 2011; Jackson & Leffingwell, 1999; Karp, 1991). Many researchers posit that teachers who exhibit a sense of mathematics anxiety convey that anxiety to students through their instructional practices. In this section, a few of these studies have been synthesized to illustrate this relationship.

Numerous studies have shown that teachers who experience mathematics anxiety tend to use more traditional methods of instruction to teach mathematics (Brush, 1981; Bush, 1989; Hiebert, 2003; Karp, 1991). A study conducted by Brush (1981) investigated the mathematics anxiety levels of 31 upper-level elementary teachers and their selected teaching practices. Teachers were administered the MARS, and were required to audio-record typical mathematics lessons. The investigator also visited and observed each classroom. The findings indicated that teachers with high levels of mathematics anxiety tend to teach mathematics using more traditional methods, while those with lower anxiety levels used more games and activities in their mathematics lessons (Brush, 1981).

Bush (1989) conducted a study regarding mathematics anxiety of upper elementary teachers. Bush focused on how teachers' mathematics anxiety related to student anxiety and achievement, teaching exercises, and teacher characteristics. The results of the study indicated that mathematics anxious teachers tended to teach more traditionally, meaning their instruction included the following practices:

- taught a great number of skills but addressed fewer concepts;

- gave more seatwork and whole group instruction;
- gave less time to homework correction;
- conducted less small group instruction sessions;
- involved students less in problem solving, and;
- used less interactive game activities while teaching

According to Bush, the teachers were insecure and failed to venture into activities that allowed students to take more mathematical risks.

Karp (1991) studied the mathematical attitudes and instructional behaviors of upper-elementary teachers (Grades 4–6). The upper grades were selected because research suggests that this time period is critical to the development of attitudes toward mathematics and confidence in mathematics ability. Teachers demonstrated only one correct way to solve problems and students were not allowed much time to interact throughout the lesson. According to Karp, the teachers indicated that the mathematics instructor was the primary mathematical authority, and this left the students dependent on the teacher for acquiring information about the subject. Overall, teachers with negative attitudes employed methods that typically fostered a dependent atmosphere in the mathematics classroom, whereas teachers with positive attitudes encouraged student initiative and independence (Karp, 1991).

The studies by Bush (1989) and Karp (1991) both illustrate the highly anxious teachers' tendency to use traditional instruction in the mathematics classroom. Students more times than not received direct instruction and individualized seat work with little peer interaction. Additionally, students engaged in limited mathematical discussions and became dependent on the classroom teacher as the mathematical authority (Bush, 1989; Karp, 1991). These teaching practices are in direct conflict with the recommendations of NCTM that advocates a student-

centered classroom, which fosters social collaboration and peer support through cooperative learning.

Teaching strategies, techniques, and policies throughout an individual's educational career can have a tremendous impact on developing and increasing mathematics anxiety. Furner and Berman (2003) explain that "one size fits all" instruction, rote instruction, and assigning mathematics homework as punishment all contribute to creating mathematics anxiety. Furthermore, generalizing instruction with no differentiation, assigning mathematics problems that require computation in isolation, and focusing on one correct method for solving a problem also cause feelings of anxiety (Bush, 1989; Jackson & Leffingwell, 1999).

The literature reviewed suggests that for many prospective and practicing elementary teachers who experience mathematics anxiety, their early-classroom experiences were most often saturated with inadequate mathematics instruction and ineffective teaching practices (Battista, 2006; Bekdemir, 2010; Gresham, 2008; Harper & Daane, 1999). It is reasonable to hypothesize that these negative experiences contributed to their condition due to limited content knowledge and a lack of confidence in mathematics (Hembree, 1990). Furthermore, it is not surprising that these negative classroom experiences have affected their attitudes toward mathematics, and would likely impede their effectiveness in teaching mathematics.

The results of other studies concluded that teachers were an important factor in impacting students' attitudes toward mathematics (Jackson & Leffingwell, 1999; Nathan & Koedinger, 2000). However, Jackson and Leffingwell contend that teachers' behaviors were detrimental to students' attitudes, while Nathan and Koedinger expressed that it was teachers' beliefs that were more important. Conclusively, the teacher, in some manner, was considered the extending cause of mathematics anxiety. Therefore, it is clear that teachers who exhibit a sense of mathematics

anxiety convey that anxiety to students through their instructional practices, and the cycle of mathematics anxiety continues (Austin, Wadlington, & Bitner, 1992; Beilock et al., 2010; Brady & Bowd, 2005).

Instructional Practices Related to Beliefs

Likewise, the literature suggests a relationship between teachers' mathematical beliefs and their instructional practices. It would seem logical that the instructional decisions made by a teacher who believes mathematics to be a set of rules and procedures to follow would look different from those of a teacher who views mathematics as a social construction that encourages the active process of solving meaningful, real-world problems. Research supports this logic. Numerous studies have shown that mathematics teachers tend to shape their classroom practice based upon their beliefs (Austin, Wadlington, & Bitner, 1992; Beswick, 2006; Goldin, 2002; Hart, 2002). Additionally, beliefs that are consistent with the constructivist mathematics education reform advocated by NCTM have been measured in several studies (Barkatsas & Malone, 2005; Beswick, 2005; Dede & Uysal, 2012; Polly et al., 2013; Wilkins, 2008). The following section briefly outlines and synthesizes studies that have investigated these relationships.

Thompson (1992) stresses that any attempt to improve mathematics education must begin with an understanding of the conceptions (beliefs) held by the teachers and how these are related to their instructional practices. Through qualitative methods, Thompson examined the beliefs and conceptions of mathematics teachers and found that the teachers held rigid beliefs about mathematics. Through observation, Thompson discovered they played a significant role in shaping their instructional behavior, noting, "the observed consistency between the teachers' professed conceptions of mathematics and the manner in which they typically presented the

content strongly suggests that the teachers' views, beliefs, and preferences about mathematics do influence their instructional practice (p. 125).

In addition, Polly and colleagues (2013) examined the relationships between mathematical beliefs, teachers' instructional practices, and student achievement for 35 prospective teachers and 494 practicing teachers. The findings indicated a significant relationship between teacher beliefs and instructional practices, but not between teacher beliefs or instructional practice when related to student achievement.

Raymond (1997) found that beginning elementary school teachers, who enter the teaching profession with constructivist beliefs regarding mathematical instructional practices, might not necessarily implement those beliefs once they are in the classroom. According to Raymond, when the beginning teachers are faced with the limitations and constraints of teaching, they tend to implement more traditional classroom practices. For example, in Raymond's study, one teacher viewed cooperative learning as an effective way to teach mathematics, but did not implement it due to her strong concern for classroom management. As stated earlier, the teacher fell into the pattern of allowing the expectations of the social and school context to overshadow her constructivist beliefs about the teaching and learning of mathematics. The results of this study suggest that deeply held traditional beliefs about the nature of mathematics have the potential to promote traditional instructional practices, even when teachers hold nontraditional beliefs about mathematics pedagogy (Raymond, 1997).

Whereas mathematics anxiety in elementary teachers has been found to correlate to reduced confidence to teach mathematics (Bursal & Paznokas, 2006), research examining the relationship between elementary teachers' mathematics anxiety and their beliefs about mathematics is limited. However, in a study of prospective elementary teachers, Swars, Daane,

and Giesen (2006) found that the participants with low anxiety expressed different perceptions of the nature of mathematics than participants with high anxiety. Participants with low anxiety viewed mathematics as problem solving and play, whereas participants with high anxiety described mathematics as procedural knowledge and rules requiring memorization.

Another study by Aslan (2013) investigated the mathematics anxiety and mathematical beliefs of 100 prospective and 50 practicing teachers using quantitative methods. The mathematics anxiety of the practicing teachers was found to be significantly higher than the anxiety of the prospective teachers. Furthermore, beliefs about mathematics were more positive for the practicing teachers. Again, it is not surprising that the powerful influences of the school context may increase practicing teachers' mathematics anxiety and affect their instructional practices.

Perhaps most aligned with the present project is a study by Wilkins (2008), which examined the content knowledge, attitudes, and beliefs of elementary teachers. The relationships among these variables were also investigated by grade level within the elementary strand. Wilkin's study suggests that beliefs may serve as a mediator between teachers' attitudes toward mathematics and instructional practice. Content knowledge, attitudes, and beliefs were all found to be related to teachers' instructional practice. Furthermore, beliefs were found to partially mediate the effects of content knowledge and attitudes, and were found to have the strongest effect on teachers' instructional practices. Teachers with more positive attitudes toward mathematics were more likely to believe in the effectiveness of inquiry-based instruction and use it more frequently in their classrooms. As previously stated, mathematics anxiety may be considered a subconstruct of attitudes in research communities (Aslan, 2013; Jong & Hodges, 2013; Lange, 1992). Likewise, inquiry-based instruction can be aligned with the reform-based

teaching strategies promoted by NCTM (Richardson & Liang, 2008). With these related variables in mind, the results of Wilkin's study were useful in formulating the hypotheses of the current project.

Conclusion

According to the research reviewed, mathematics anxiety is a widespread phenomenon for many prospective and practicing elementary teachers. Mathematics anxiety has been identified as an influential factor in elementary prospective teachers' beliefs and behavior, both of which influence teachers' instructional practices (Brady & Bowd, 2005, Swetman et al., 1993). A large number of elementary prospective teachers have been identified as having a high level of mathematics anxiety, which follows them to the classroom (Hembree, 1990; Levine, 1993; Swars et al., 2006).

Another affective variable thought to influence elementary teachers' instructional practices is their beliefs about the nature of mathematics and the teaching and learning of mathematics (Beswick, 2006). Prospective teachers who experience anxiety with mathematics often have negative beliefs about mathematics (Swars, Daane, & Giesen, 2006). These negative beliefs can lead teachers to develop traditional instructional practices, thus increasing their mathematics anxiety and decreasing student achievement (Kolstad & Hughes, 1994). The literature reviewed shows that the relationship between teachers' mathematical beliefs and their instructional practice is dialectical in nature and is mediated by many conflicting factors. Researchers agree that these beliefs should be identified and addressed in teacher education programs, as well as prospective and practicing teachers, in order to reform the teaching of mathematics toward congruence with a constructivist paradigm (Haciomeroglu, 2013; Handal, 2003; Philipp, 2007; Stipek, Givvin, Salmon, & MacGyvers, 2001).

Despite the depth of research in the areas of mathematics anxiety, mathematical beliefs, and instructional practices of prospective teachers, there is a lack of research that examines the relationships between these constructs and practicing elementary teachers. Furthermore, there are few studies that link both teachers' beliefs about mathematics teaching and learning and their instructional practices (Polly et al., 2013). The findings of this study will add to the knowledge gap in this area.

Collectively, the studies reviewed suggest that students' mathematics anxiety is cultivated and fostered, at least in part, through classroom teachers' behaviors and instructional practices (Brady & Bowd, 2005; Hadley & Dorward, 2011; Hembree, 1990; Jackson, 2008). The literature suggests that mathematics anxiety is learned and contagious, and highly anxious elementary teachers can spread mathematics anxiety to their students (Austin, Wadlington, & Bitner, 1992). The research also suggests that environmental factors rather than innate ability are often at the root of mathematics anxiety (Hembree, 1990). Furthermore, the literature reviewed establishes that mathematics anxiety generally evolves during the elementary years of schooling and is influenced by prior negative experiences (Bush, 1989; Bekdemir, 2010). Often, it is these prior negative experiences with mathematics that contribute to teachers' anxiety and negative beliefs toward mathematics, which would likely impede their effectiveness in teaching mathematics (Hembree, 1990).

Meta-analysis studies found that the negative association between mathematics anxiety and mathematics achievement establishes a need for cognitively based treatments to help students overcome their mathematics anxiety (Hembree, 1990; Ma, 1999). Researchers agree that action must be taken to prevent the anxiety from affecting students' attitudes and achievement in mathematics. Many mathematics educators have concluded that changing

instructional practices in mathematics classrooms should be not only a matter of new curricula or resources, but also a matter of challenging traditional personal philosophies of teachers (see, e.g., Ernest, 1991).

Educational researchers agree that it is crucial for teacher training programs to assist prospective and practicing elementary teachers in acknowledging their mathematics anxiety and beliefs about teaching and learning mathematics (Harper & Daane, 1998; Swars, 2007; Trujillo & Hadfield, 1999). Furthermore, the development of safe and non-threatening learning environments is crucial to ensure that highly anxious prospective student teachers can feel safe to explore and communicate about mathematics in a supportive group environment (Sloan, 2010). An overarching theme found in the literature reviewed is the importance of determining and developing methods to alleviate mathematics anxiety and shift beliefs in elementary teachers; this is critical to the mathematical success of future generations of students.

The literature reviewed for this study clearly suggests that the understanding of teachers' use of particular instructional practices is a complex undertaking and depends on many factors including mathematics anxiety and mathematical beliefs. It is my hope that this review not only provides support for establishing a foundation for this project, but also assists further research in exploring the relationships between and among mathematics anxiety, mathematical beliefs, and instructional practices of elementary school teachers.

Collectively, the studies reviewed suggest that mathematics anxiety and mathematical beliefs are two constructs that have an impact on the instructional practices of elementary teachers. The hypotheses of my study are concurrent with the literature reviewed: the mathematics anxiety and beliefs of elementary teachers are related to their instructional practices. It is important to consider, however, that the mathematics anxiety and mathematical

beliefs of the teachers may not align with their instructional practices. The powerful influences of the school context may increase inservice teachers' mathematics anxiety and affect their instructional practices. According to Ernest (1999), there is a great disparity between espoused and enacted models of teaching and learning mathematics. Although prospective teachers may have been taught to adopt a reformed practice during their teacher-training years, the practicing teachers are subject to the constraints and contingencies of the school context once they enter the classroom (Ernest, 1999). Practicing teachers are influenced by the expectations of others, especially other teachers and superiors. This influence also results from the institutionalized curriculum represented by adopted curriculum materials and assessment methods. As noted previously, Ernest points out that "the socialization effect of the context is so powerful that teachers in the same school, despite having differing beliefs about mathematics and its teaching, are often observed to adopt similar classroom practices" (p. 27).

The results of this study expand upon the vast amount of mathematics anxiety research, as well as the research on mathematical beliefs, to illuminate connections among teachers' instructional practices in the elementary classroom. Current research is deficient in directly aligning the relationship of mathematics anxiety and mathematical beliefs with practicing elementary teachers' instructional practices. Therefore, the results of this study help illuminate the relationships that exist among these constructs, thus better informing teacher preparation and enriching the mathematics instruction of elementary teachers.

CHAPTER 4

METHODOLOGY

Purpose of the Study

The purpose of this study was to explore the relationship between practicing elementary teachers' anxiety toward mathematics and the teachers' mathematical beliefs, and to examine how these affective characteristics are related (or not) to the instructional practices of elementary teachers. This chapter provides a description of the research design, participants, instrumentation, procedures, and treatment of the data for the study.

Research Questions and Hypotheses

Specifically, the research questions are:

1. What is the relationship between the mathematics anxiety and the instructional practices of elementary teachers?
2. What is the relationship between the mathematical beliefs and the instructional practices of elementary teachers?
3. What is the relationship between mathematics anxiety and the mathematical beliefs of elementary teachers?

The following null hypotheses were tested in this study:

H_0: There is no relationship between mathematics anxiety and the instructional practices of elementary teachers.

H_0: There is no relationship between the mathematical beliefs and the instructional practices of elementary teachers.

H_0: There is no relationship between mathematics anxiety and the mathematical beliefs of elementary teachers.

This quantitative study used an online survey design to examine the relationship of mathematics anxiety, mathematical beliefs, and instructional practices of elementary teachers. A survey design was selected as the research method for several reasons. Previous research suggests that teacher self-report surveys provide a relatively accurately picture of classroom practice (Ross et al., 2003). The purpose of survey research is to be able to generalize results from a sample to a population so that inferences can be made about characteristics, attitudes, or behaviors of the population (Creswell, 2003). The purpose of this study was to explore the relationship between practicing elementary teachers' anxiety toward mathematics and the teachers' mathematical beliefs, and to examine how these affective characteristics are related (or not) to the instructional practices of elementary teachers. Therefore, the purpose of survey research closely aligned with the stated purpose of my study. Second, a survey design was selected due to the economy of the method and rapid turnaround in data collection (Creswell, 2003). Third, the instruments available to measure the constructs of this study had high levels of reliability and validity.

To answer the research questions and to test the hypotheses, correlational analyses were conducted to determine significant (or not) relationships between the constructs. Multiple regression was also used to explore the relationships among the variables in the study: mathematics anxiety and mathematical beliefs (as the independent variables), and instructional practices (as the dependent variable). This design can establish that a set of independent variables explains a proportion of the variance in a dependent variable at a significant level, as well as establish the relative predictive importance of the independent variables (Creswell, 2003; Hoy, 2010). Multiple regression shares all the assumptions of correlation such as the linearity of relationships, homoscedasticity (or the same level of relationship throughout the range of the

independent variable), interval data, absence of outliers, and data whose range is not truncated (Tolmie, Muijs, & McAteer, 2011). The specification of the model being tested is critical, and the exclusion of important causal variables or the inclusion of extraneous variables can considerably change the beta weights (Creswell, 2003). The change in beta weights can considerably influence and change the interpretation of the importance of the independent variables (Connolly, 2007), so anticipation of causal and extraneous variables were accounted for in the design of the study. The self-report survey design of the study provided relevant and insightful information about elementary mathematics teachers within a reasonable timeframe, which proved to be beneficial in the research design (Connolly, 2007; Creswell, 2003).

Instrumentation

Demographic data were collected and incorporated into this online survey design. In survey research, demographic data are critical in making comparisons across groups and generalizing findings (Connolly, 2007). Once survey data are collected, it can be divided into various data groups based on demographic information gathered from the survey, and differentiation between different sub-groups can be made and analyzed (Tolmie, Muijs, & McAteer (2011). At the beginning of this survey, participants were asked to disclose the following information:

1. Number of years of teaching experience.
2. Number of years of experience teaching mathematics.
3. Highest college degree level obtained.
4. The number of mathematics courses taken in college.

The instruments used in this study were The Mathematics Anxiety Rating Scale: Short Version (Suinn & Winston, 2003), the Teacher Beliefs Survey (Beswick, 2005), and the Self-

Report Survey: Elementary Teachers Commitment to Mathematics Education Reform (Ross et al., 2003). The three instruments were chosen based on their wide use and acceptance in the field of educational research, high levels of reliability and validity, and relevance to this particular study.

The Mathematics Anxiety Rating Scale: Short Version (MARS-SV)

The most frequently used instrument to measure mathematics anxiety is the Mathematics Anxiety Rating Scale (Capraro, Capraro, & Henson, 2001). Initially developed in 1972 as a self-report Likert scale survey, it contains 98 items designed to measure the respondent's level of anxiety related to mathematics tasks (Richardson & Suinn, 1972). This study utilized a shortened version of the original survey, the Mathematics Anxiety Rating Scale: Short Version (MARS-SV). The MARS-SV is a 30-item survey developed by Suinn and Winston (2003) to reduce the length of the 98-item survey. This instrument is based on a 5-point Likert 80 scale, where 1 represents "not at all" and 5 represents "very much."

Cronbach's alpha was used to test the reliability of the instrument. Cronbach's alpha is a reliability coefficient that "indicates the degree of homogeneity in the items; a high coefficient tells us that the items tend to be measuring the same characteristic of the respondents, while a low coefficient means that the items are disparate in what they are measuring (Goodwin & Goodwin, 1996, p. 79). Cronbach alpha was found to be 0.96, an indication of high internal consistency. In addition, a test-re-test reliability of 0.91 was found (Suinn & Winston, 2003). Concurrent validity of the MARS-SV with the MARS was conducted using a Pearson correlation with $r = 0.92$ indicating a high correlation (Suinn & Winston, 2003).

The Teacher Beliefs Survey

The Teacher Beliefs Survey, Beswick (2005), consists of 26 items with which teachers were asked to indicate the extent of their agreement concerning beliefs about teaching and learning mathematics. Like the MARS-SV, the Teacher Beliefs Survey is based on a 5-point Likert scale, where 1 represents "Strongly Disagree", and 5 represents "Strongly Agree." Higher scores indicate greater consistency with a constructivist, reform view of teaching and learning mathematics. The Teacher Beliefs Survey items can be divided to determine two subscale scores: problem-solving (social constructivist) and instrumentalist (traditional) views of mathematics. The problem-solving and instrumentalist views of mathematics are two of the three categorizations used by Ernest (1989b) to categorize teacher beliefs. Fourteen items measure the level of agreement with the problem-solving view of mathematics. The remaining 12 items measure the level of agreement with an instrumentalist view of mathematics. For the purpose of this study, the mean scores for the survey subscale totals were used to determine the teacher's orientation towards problem-solving and instrumentalist views of mathematics. Therefore, a higher problem-solving subscale score indicates that a teacher views mathematics as a dynamic subject involving inquiry and discovery, which is consistent with the constructivist view advocated by NCTM. The problem-solving view of mathematics includes student-centered approaches to learning mathematics. A higher instrumentalist subscale score indicates that a teacher views mathematics as an accumulation of facts, rules, and skills and tends to utilize teacher-directed methods. Beswick (2005) found the Teachers Belief Survey measured two factors, essentially corresponding with the respective views of mathematics teaching and learning that were identified as theoretically consistent with instrumentalist and problem-solving views of mathematics. The Cronbach's alpha reliability coefficient associated with an

instrumentalist view of mathematics factor is 0.77 and the alpha reliability coefficient associated with a problem-solving view of mathematics factor is 0.78.

It is plausible, however, that the elementary teachers' mathematical beliefs will not be aligned with their instructional practices. That is, their enacted instructional practices may not correspond to their stated beliefs about teaching and learning mathematics. Previous research has shown that elementary teachers who espoused beliefs were related to a student-centered constructivist approach to teaching still relied heavily on district-mandated curricula and assessments for classroom instruction. Mandatory assessment of students is a factor that needs to be considered when referencing teachers' instructional practices. Many teachers implement traditional, performance-driven instruction in their classrooms because of the pressures caused by state mandates even though the teachers may express constructivist beliefs (Raymond, 1997; Shaw, 1990).

Self-Report Survey: Elementary Teachers Commitment to Mathematics Education Reform

The instrument used to measure the instructional practices of elementary teachers was the Self-Report Survey: Elementary Teachers Commitment to Mathematics Education Reform (Ross et al., 2003). It measures the extent to which elementary teachers' implement mathematics education reform in their teaching practices. The developers created a blueprint for standard-based teaching based on a review of key NCTM documents and 153 empirical studies. The resulting blueprint contained the nine dimensions of reform-based mathematics teaching practices, including the ability to develop complex, authentic learning tasks for students, facilitate student-to-student interaction, and implement appropriate assessment strategies. The standards-based survey was developed from this blueprint and contains 20 Likert items with a 5-point response scale ranging from "Strongly Agree" to "Strongly Disagree." To guard against

response bias, seven of the items are negatively worded for which the coding is reversed, so that in the analysis all the items run in the same direction. These items are marked with an asterisk in Appendix C. Using Cronbach's alpha, a reliability coefficient of 0.88 was obtained in two independent studies (Ross, et. al, 2003). Elementary teachers established content and face validity through review.

Setting and Participants

The setting for this study is a public school district in a suburban county in Georgia with a population 27,736 students. There are 10,849 students enrolled in the 19 elementary schools in the district. The racial demographics of the student population is 64% Caucasian, 21% African American, 8% Hispanic, 2% Asian, and 5% Multi-racial. Approximately 13% of the students in the district are economically disadvantaged and 11% of the students are served under Individual Education Plans (Georgia Department of Education, 2014).

The overall teacher retention rate for the school district is 95.2%, and the average number of years of teaching experience is 13.7 years. In fall of 2014, only 3% of the teacher population was first-year beginning teachers (Georgia Department of Education, 2014). A Bachelor's Degree is the highest level of education for 34% of the teacher population. Approximately 46% of the teachers have a Master's Degree, 19% have earned an Education Specialist Degree, and 2% have a Doctorate Degree (Office of Student Achievement, 2014).

There are approximately 505 elementary teachers in the district. The sample identified for this study consists of 153 elementary mathematics teachers from the 19 elementary schools in the district. All teachers were notified by email and asked to participate in the study (see Appendix E). Each participant had the option of refusing to participate in the study.

Procedures

I received approval to conduct the study from the local school district and Georgia State University's Institutional Review Board. In order to conduct research within the local school system, I submitted a completed research request application and a detailed proposal package to the school system's Department of Research office. There are specific guidelines set to standardize research activities effectively within the district in order to protect individual rights of students and staff in the school system, and to avoid interference with ongoing instructional programs in the schools. I also received consent from the developers of the survey instruments that were utilized in the study.

The research coordinator of the county then emailed the consent letter and survey link to all 505 elementary teachers in the county in August 2015. The consent letter (see Appendix D) included the purpose of the research, risks and benefits, confidentiality involved, institutional affiliation of the researcher, and contact information for the researcher. A link was provided to direct participants to the online survey on www.surveymonkey.com if they agreed to participate.

Data Collection

Participation in this study consisted of completing the following self-reporting instruments: The Mathematics Anxiety Rating Scale: Short Version (Suinn & Winston, 2003), the Teacher Beliefs Survey (Beswick, 2005), and the Self-Report Survey: Elementary Teachers Commitment to Mathematics Education Reform (Ross et al., 2003). Software from the website www.surveymonkey.com was used to administer the survey instruments and gather the responses. A total of 153 participants completed the online survey, yielding a return rate of approximately 30%.

The online survey data was exported to the Statistical Package for Social Sciences (SPSS) for data analysis. The data includes:

- Demographic data
- Mean scores from The Mathematics Anxiety Rating Scale
- Mean scores from The Teacher Beliefs Instrument
- Mean scores from the Self-Report Survey: Commitment to Mathematics Education Reform

Data Analyses

Data from the survey were analyzed using SPSS, a program that organizes data, conducts statistical analyses, and generates tables and graphs that summarize data. Descriptive and inferential statistics were used, including correlational and regression procedures. The Pearson product-moment correlation was used to analyze the following relationships:

1. To determine whether a significant correlation exists between the mathematics anxiety levels and instructional practices. The findings were expected to be concurrent with the literature (Hadley & Dorward, 2011; Jackson, 2008; Uusimaki & Nason, 2004), showing a negative correlation between the two constructs, with higher levels of mathematics anxiety corresponding to lower scores on the standards-based survey, indicating more traditional, teacher-centered instructional practices.
2. To determine whether a significant correlation exists between teacher beliefs scores and the instructional practices scores. Several studies reviewed found a positive correlation between mathematical beliefs and instructional practices (Handal, 2003; Polly et al., 2013; Raymond, 2007; Thompson, 1984; Wilkins, 2008). Therefore, the expectation here was that higher beliefs scores, which suggest reform-based

constructivist beliefs, would be positively correlated with higher scores on the standards-based survey, indicating a student-centered and constructivist teaching style.

3. To determine whether a significant correlation exists between mathematics anxiety levels and teacher beliefs scores. The findings are expected to be concurrent with the literature (Akinsola, 2008; Aslan, 2013; Austin, Wadlington, & Bitner, 1992; Haciomeroglu, 2013), which suggests a negative correlation between the two constructs, with higher anxiety scores corresponding with lower beliefs scores, which suggests traditional, instrumentalist beliefs about teaching and learning mathematics.

Multiple regression analyses were conducted to determine if teachers' instructional practices were impacted by their mathematics anxiety and mathematical beliefs. Through the regression analysis, the relationships, if any, among the identified constructs of mathematics anxiety, mathematical beliefs, and instructional practices of elementary teachers were identified. The expectation here was that the instructional practices would be impacted by the mathematics anxiety and mathematical beliefs, as suggested in the literature (Barkatsas & Malone, 2005; Beswick, 2005; Bush, 1989; Hembree 1990). A significance level of .05 was used to determine statistical significance on all tests.

The teachers in the sample were separated into three groups according to years of teaching experience to determine any differences in the three constructs by teaching longevity. The following groups were defined: 0□5 years (beginning teachers), □□□□ years (middle teachers), and 16+ years (veteran teachers). There is a deficiency of research studies comparing teaching longevity with mathematics anxiety, mathematical beliefs, and instructional practices. Raymond (1997) studied beginning elementary teachers' beliefs and instructional practices, and

found that beginning teachers' instructional practices are impacted by their beliefs. Wilkins (2008) examined the mathematical beliefs and instructional practices of elementary teachers with respect to the number of years of teaching experience. However, no significant relationship was found between the number of years of teaching experience and beliefs or instructional practices. Limited studies were found that compared levels of mathematics anxiety of beginning teachers with veteran teachers (Hadley & Dorward, 2011). The expectation here was that teachers who are new to teaching would have significantly higher mathematics anxiety levels, lower beliefs scores, and lower instructional practices scores. These findings are concurrent with the literature reviewed for prospective teachers (Aslan, 2013; Brady & Bowd, 2005; Battista, 1986; Bekdemir, 2010; Burton, 2012; Jackson, 2008; Philippou & Christou, 1998).

The teachers in the sample were also divided into three groups according to their highest degree level: bachelor's degree, master's degree, and specialist or doctoral degree. No research studies were found that investigated degree level with mathematics anxiety, mathematical beliefs, or instructional practices of elementary school teachers.

Limitations

All research methods involving measurement have limits (Creswell, 2003), and the potential limitations should be considered at the data analysis and interpretation stage. Although every attempt was made to conduct a thorough, comprehensive research project, there are several limitations in this study that should be acknowledged.

1. The participants are all elementary teachers in one school system. Therefore, the results of this study may not be generalizable to other school systems due to differences in teacher demographics.

2. Because this study relies on accessible and willing participants, a random sample was not possible. Therefore, the voluntary, non-random nature of participant recruitment may limit the sample to participants who have a lower level of mathematics anxiety and higher degree of alignment of beliefs with reform based approaches to mathematics. The research suggests that elementary teachers with mathematics anxiety tend to avoid mathematics (Hembree, 1990). This avoidance behavior could very well extend to any activity that addresses mathematics, including a survey that asks them to identify their anxieties and beliefs about mathematics.
3. The inclusion of instructional practices as a variable to be surveyed may "cue" the participants to select answers that theoretically sound more appropriate rather than select answers corresponding to their actual, enacted instructional practices.
4. The surveys were taken online. Although this method increased the speed at which the data was gathered, there may have been periods of time in which the teachers could not access the Internet. Although this does not happen frequently, it does occur. A teacher who elected to participate, and then could not access the survey, may not have done so later when Internet access was resumed.

Strengths

1. The instruments used in this study have been well reviewed, and have substantial validity and reliability.
2. The sample size (N = 153) is sizeable, which provides a more specialized, identifiable profile of the constructs of the study.
3. The data collection was cost effective with a rapid turnaround rate.

4. Because of the nature of the research, little to no risk occurred with participation in this study. Participants were assured of confidentiality, and were able to answer survey questions on their own computers at a time convenient to them.
5. The information will be used for proactive purposes and will not be associated with evaluations and/or plans of improvement for any teacher.

Summary

This chapter outlined the research methodology used to collect and analyze the data for this study, beginning with a restating of the problem being investigated, the purpose of the study, and research questions. The methodology of the study is designed to provide necessary data to determine the relationship among mathematics anxiety, mathematical beliefs, and the instructional practices of elementary teachers represented in the selected school district. Detailed descriptions of the research design and self-reporting instruments were also provided. The sample and procedures were then described, followed by an outline of the data collection and analysis used in the study. The chapter was concluded by a specification of the strengths and limitations of the study. The next chapter focuses on the survey results that examined elementary teachers' mathematics anxiety, mathematical beliefs, and instructional practices. Chapter Six discusses the results and implications of the study.

CHAPTER 5

RESULTS

The purpose of this quantitative, correlational study was to determine if a relationship existed among mathematics anxiety, mathematical beliefs, and the instructional practices of elementary school teachers. The research questions and hypotheses explored in the study were designed to help identify the specific relationships, if any, that existed among the named constructs to determine their influences on teachers' classroom instructional practices. Although research about mathematics anxiety, mathematical beliefs have all been explored as separate factors in their relationships to teachers' instructional practices in mathematics (see, e.g., Barkatsas & Malone, 2005; Brady & Bowd, 2005; Cross, 2009; Gresham, 2008; Hadley & Dorward, 2011; Lake & Kelly, 2014; McLeod & McLeod, 2002; Mujis & Reynolds, 2002), the lack of studies incorporating all of the components established the basis for the study.

Research Questions and Associated Hypotheses

The research questions and hypotheses for this study are restated below. The results and analyses from testing these questions and hypotheses are discussed and presented in the next section. Specifically, the research questions are:

1. What is the relationship between the mathematics anxiety and the instructional practices of elementary teachers?
2. What is the relationship between the mathematical beliefs and the instructional practices of elementary teachers?
3. What is the relationship between mathematics anxiety and the mathematical beliefs of elementary teachers?

The following null hypotheses were tested in this study:

H_0: There is no relationship between mathematics anxiety and the instructional practices of elementary teachers.

H_0: There is no relationship between the mathematical beliefs and the instructional practices of elementary teachers.

H_0: There is no relationship between mathematics anxiety and the mathematical beliefs of elementary teachers.

Analysis of Data

The data presented in this chapter describe the relationships found among mathematics anxiety, mathematical beliefs, and the instructional practices of elementary school teachers participating in the study. Demographic data were collected and analyzed. Following the demographic information, charts showing the means and standard deviations of scores from each instrument are presented along with the relevant research question. The research design also included correlational analyses and multiple regression, showing the relationships found among the constructs studied. The chapter continues with the results of the statistical analysis of the study's research questions and hypotheses regarding teachers' mathematical anxiety, mathematical beliefs, and instructional practices. The relationships found between the constructs are then explained through the interpretations and analyses of the findings.

Demographic Characteristics

Participants in the study were 153 Pre-K–5 teachers from 19 elementary schools from a suburban school district in the state of Georgia. Demographic characteristics of participants were collected in the online survey, providing the number of years of teaching experience, the number of years of teaching mathematics, highest degree level, and number of college

mathematics courses taken. For the years of teaching experience, the answer choices were given in intervals: 0–5 years, 6–15 years, and 16+ years. These groupings make comparisons of teaching longevity more logical and comprehensible. This information is presented in Table 1.

Table 1

Number of Years of Teaching Experience

Years	*n*	Percentage
0–5	12	7.8%
6–15	67	43.8%
16+	74	48.4%

Most participants reported their highest degree as a master's degree ($n = 65$), and the number of participants who hold a bachelor's degree ($n = 43$) is the same as the number of educational specialists. Only 2 of the practicing classroom teachers hold doctoral degrees, therefore they were combined with the educational specialists. This information is presented in Table 2.

Table 2

Highest Degree Level

Years	*n*	Percentage
Bachelor's Degree	43	28.1%
Master's Degree	65	42.5%
Specialist/Doctoral Degree	45	29.4%

For the number of years of teaching mathematics, the following intervals were used: Never taught mathematics, 0–5 years, 6–15 years, and 16+ years. The results indicate that all but 1 participant has experience teaching mathematics. There was also an option for those who have never taught mathematics. This information is presented in Table 3.

Table 3

Number of Years of Teaching Mathematics

Years	*n*	Percentage
Never taught mathematics	1	0.7%
0–5	23	15.0%
6–15	74	48.4%
16+	55	35.9%

For the number of college mathematics courses in college, the options were: 0, 1, 2, or 3+. The number of elementary teachers who were enrolled in 3 or more college mathematics courses (n = 105) is considerably larger than the other groups. This information is presented in Table 4. This finding is encouraging, for the mathematics requirements (outside the field of education) for students majoring in elementary education in many U. S. colleges and universities are minimal (Beilock et al, 2010).

Table 4

Number of Mathematics Courses Taken in College

Number of courses	*n*	Percentage
0	2	1.3%
1	11	7.2%
2	35	22.9%
3+	105	68.6%

The Mathematics Anxiety Rating Scale: Short Version (MARS-SV)

The MARS-SV (Suinn & Winston, 2003) was used to measure this construct based on a 5-point Likert 80 scale, with response options ranging from 1–5, with 1 representing low anxiety, 3 representing neutral anxiety, and 5 representing high anxiety. The mean and standard deviation of the sample on the MARS-SV was $M = 2.35$ and $SD = 0.90$, which indicates that teachers had a lower to neutral sense of mathematics anxiety. The four items with the highest means are provided to give more information on which statements prompted the highest sense of anxiety among the participating teachers (see Table 5). The highest mean found, 3.32, was in response to a question about being given a homework assignment of difficult problems. The lowest mean, 1.45, references a question concerning the reading of a cash register receipt after a purchase. It is important to note that the highest scores were from statements showing mathematics anxiety regarding assessments, indicating that testing in itself could cause more anxiety than the subject of mathematics.

Table 5

Mathematics Anxiety Results

	M	*SD*
MARS-SV (total)	2.35	0.90
14. Being given a homework assignment of many difficult problems due the next class period	3.32	1.25
11. Taking the mathematics section of a college entrance exam	3.31	1.28
13. Being given a "pop quiz" in class	3.30	1.17
5. Thinking about an upcoming math test 5 minutes before.	3.28	1.26

The Teacher Beliefs Survey (TBS)

The TBS (Beswick, 2005) was used to measure teacher beliefs. It is also based on a 5-point Likert scale, where 1 represents "Strongly Disagree", and 5 represents "Strongly Agree." a A higher score indicates greater consistency with a problem-solving view of mathematics. This view supports a constructivist, reform view of teaching and learning mathematics. Lower scores indicate beliefs that align with an instrumentalist view of mathematics. This view supports a traditional, content-based view of teaching and learning mathematics. The mean and standard deviation of the sample on the TBS was $M = 3.54$ and $SD = 0.39$, which indicates that the teachers leaned toward a problem-solving view of mathematics. The four items with the highest means are provided to give additional information on which belief statements were most aligned with a reform view of teaching and learning mathematics (see Table 6).

Table 6

Mathematical Beliefs Results

	M	*SD*
TBS (total)	3.54	0.39
23. Teachers can create for all students a nonthreatening environment for learning mathematics.	4.25	0.67
7. It is important teachers to understand the structured way in which mathematics concepts and skills relate to each other.	4.14	0.72
1. A vital task for the teacher is motivating children to solve their own mathematical problems.	4.11	0.85
5. It is important for children to be given opportunities to reflect on and evaluate their learning.	4.09	0.92

Self-Report Survey: Elementary Teachers Commitment to Mathematics Education Reform

The Self-Report Survey: Elementary Teachers Commitment to Mathematics Education Reform (ETCMER) (Ross et al., 2003) was used to measure instructional practices. This standards-based survey measures the extent to which elementary teachers' implement mathematics education reform in their teaching practices. It contains 20 Likert items with a 5-point response scale ranging from "Strongly Agree" to "Strongly Disagree." Higher scores indicating practices aligned with a reformed, constructivist view of instruction, including the ability to develop complex, authentic learning tasks for students and facilitate student-to-student interaction. The mean and standard deviation of the sample on the standards implementation survey ETCHMER was $M = 3.12$ and $SD = 0.45$ (see Table 7).

Table 7

Instructional Practices Results

	M	*SD*
ETCHMER (total)	3.12	0.45
7. Every child in my room should feel that mathematics is something he/she can do.	4.38	0.80
11. When students are working on math problems, I put more emphasis on getting the correct answer than on the process followed.	3.94	0.92
6. It is not very productive for students to work together during math time.	3.76	1.00
17. I teach students how to explain their mathematical ideas.	3.61	0.93

The four items with the highest means are also provided in Table 7 to give additional information on which practices are most commonly used and whether they are aligned with reform view of teaching and learning mathematics. It is important to note that of the four practices, two indicate a low reform, traditional view of teaching mathematics, and two indicate a high reform, constructivist view of teaching mathematics. This range of the means is 2.25 to 4.38, which also indicates an even split between low reform and high reform practices.

Correlations between Mathematics Anxiety, Mathematical Beliefs, and Instructional Practices

After determining the descriptive statistics for mathematics anxiety, mathematical beliefs, and instructional practices of participating teachers, correlational analyses of these data were conducted. A Pearson product-moment coefficient was computed to assess the relationship between the mathematics anxiety and instructional practices of the participating teachers. The results indicate a significant (albeit moderate) inverse relationship between the two variables,

r = -0.35, p < .05. These results suggest that lower anxiety corresponds to higher scores for instructional practices, and high anxiety corresponds to lower scores for instructional practices (see Figure 2). Therefore, we can reject the null hypothesis stating there is no relationship between mathematics anxiety and instructional practices for elementary teachers.

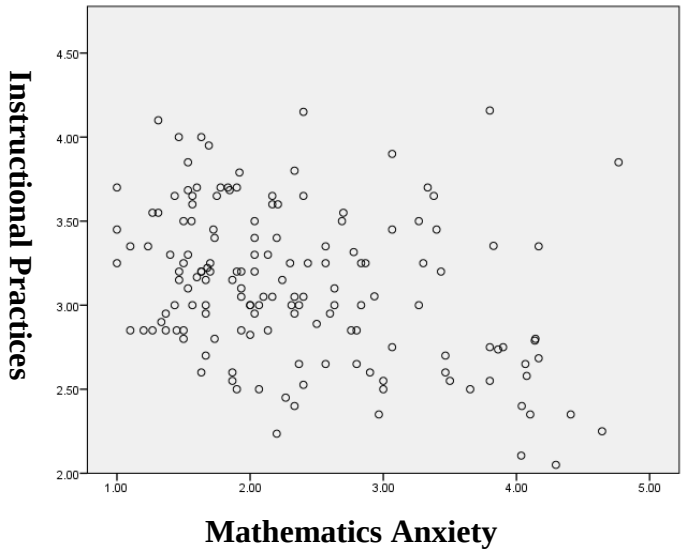

Figure 2: Relationship between mathematics anxiety and instructional practices.

Correlational analyses were also used to examine the relationship between the mathematical beliefs and instructional practices of the participating teachers. The results indicate a significant positive relationship between the two variables, r = 0.61, p < .05. These results suggest that beliefs indicative of a problem-solving, constructivist view correlate with higher scores for reform practices, and lower scores for beliefs, which are indicative of an instrumentalist view, correlate with lower scores for reform practices (see Figure 3). Therefore, we can also reject the null hypothesis stating there is no relationship between mathematical beliefs and instructional practices for elementary teachers.

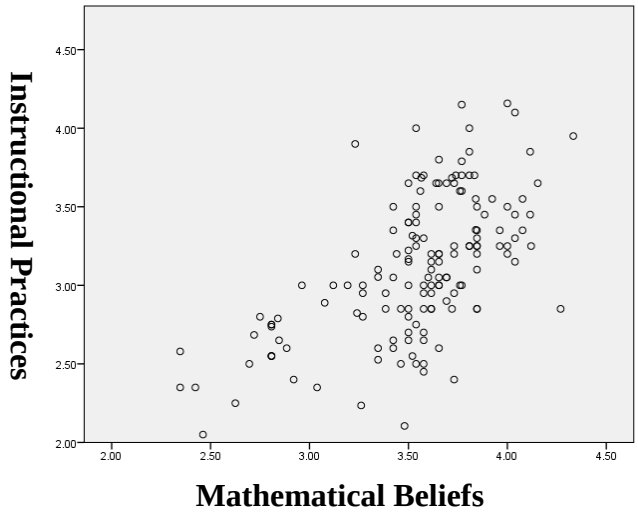

Figure 3: Relationship between mathematical beliefs and instructional practices.

The third correlational analysis was computed to investigate the relationship between mathematics anxiety and mathematical beliefs of elementary teachers. The results indicate a significant inverse relationship between the two variables, $r = -0.56$, $p < .05$. These results suggest that lower anxiety corresponds to beliefs aligned with a problem solving, constructivist view, and high anxiety corresponds to beliefs aligned with an instrumentalist, content-oriented view (see Figure 4). Therefore, we can reject the null hypothesis stating there is no relationship between mathematics anxiety and mathematical beliefs for elementary teachers.

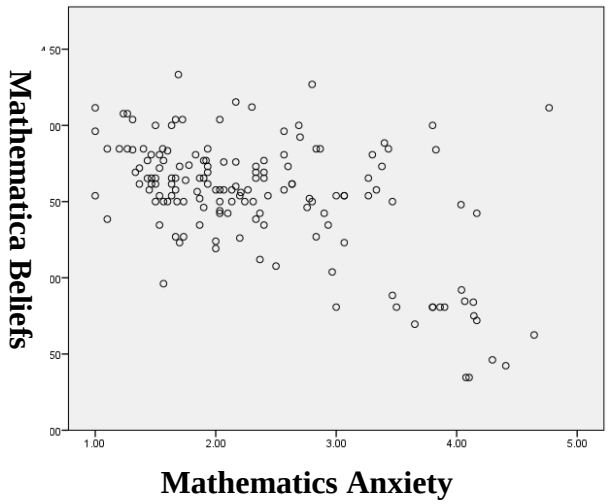

Figure 4: Relationship between mathematics anxiety and mathematical beliefs.

The data for each correlational analysis is summarized in Table 8.

Table 8

Pearson Product-moment Correlations

Variable 1	*Variable 2*	*r*	*p*
Mathematics Anxiety	Instructional Practices	-.35	.000*
Mathematical Beliefs	Instructional Practices	.61	.000*
Mathematics Anxiety	Mathematical Beliefs	-.56	.000*

*denotes significance at the .05 level

Multiple Regressions

Both predictor variables were statistically correlated with instructional practices, which indicate that the data were suitably correlated with the dependent variable for examination through multiple linear regressions to be reliably undertaken. A multiple linear regression was first computed with mathematics anxiety as the independent variables and instructional practices as the dependent variable (see Table 9). This test yielded significant results for mathematics anxiety as a predictor of instructional practices.

Table 9

Model 1: Coefficients of Predictor Variables Regarding Instructional Practices

Predictor Variable	*B*	*t*	*p*
Constant	3.53	36.84	.000
Mathematics Anxiety	-.18	-4.60	.000*

A second model incorporated mathematical beliefs into the regression (see Table 10). With the inclusion of mathematical beliefs, the results for mathematical anxiety were not statistically significant. This suggests a spurious relationship between mathematics anxiety and instructional practices.

Table 10

Model 2: Coefficients of Predictor Variables Including Mathematical Beliefs

Predictor Variable	*B*	*t*	*p*
Constant	.71	1.87	.064
Mathematics Anxiety	-.01	-.25	.803
Mathematical Beliefs	.69	7.65	.000*

Frequently in research, one may statistically detect a correlation between two variables that does not result from any direct relation between them but from their relation to other variables. This detection may imply a spurious relationship between the variables. A spurious relationship occurs when a third variable creates the appearance of relationship between two other variables, but this relationship disappears when that third variable is included in the analysis (Walker & Maddan, 2013). In this case, mathematical beliefs is the third variable due to the fact that the relationship between mathematics anxiety and instructional practices disappears when mathematical beliefs were included in the regression analysis.

A third regressional analysis was computed with instructional practices as the dependent variable and the following independent variables: mathematics anxiety, teaching longevity, and educational degree status. The teachers were divided into three groups according to their years of experience: beginning (0–5 years), middle (6–15 years), and veteran (16+ years). Three

groups were also formed according to educational degree: bachelor's degree, master's degree, and specialists/doctoral degree. There were only two teachers with doctoral degrees; therefore, I deemed it pragmatic to group the doctoral degrees with the specialist degrees. This test yielded significant results for mathematics anxiety controlling for teaching longevity and educational degree level (see Table 11).

Table 11

Model 3: Coefficients of Predictor Variables Regarding Instructional Practices

Predictor Variable	*B*	*t*	*p*
Constant	3.73	21.20	.000
Mathematics Anxiety	-.19	-4.74	.000
Teaching Longevity (Middle)	-.22	-1.44	.151
Teaching Longevity (Veteran)	-.30	-1.93	.056
Degree Level (Master's)	.06	.69	.489
Degree Level (Specialist/Doctorate)	.12	1.23	.222

A fourth regression model incorporated mathematical beliefs as an additional independent variable and yielded different results (see Table 12). As in Models 1 and 2, the results for mathematics anxiety were not significant when mathematical beliefs were incorporated into the regression model. However, the results for mathematical beliefs were significant when controlling for mathematics anxiety, teaching longevity, and educational degree. In Model 2, without the control variables, the coefficient for mathematical beliefs was $B = .69$. In Model 4, the coefficient was very similar, $B = .72$, when controlling for teaching longevity and educational degree level. Therefore, a one point increase in the mathematical

beliefs scale is associated with an expected 0.72 point increase in the instructional practice scale. This suggests that teachers with constructivist beliefs tend to use more reform practices in their classrooms.

Table 12

Model 4: Coefficients of Predictor Variables with Mathematical Beliefs

Predictor Variable	*B*	*t*	*p*
Constant	.86	2.23	.027
Mathematics Anxiety	-.02	-.46	.647
Teaching Longevity (Middle)	-.23	-1.81	.072
Teaching Longevity (Veteran)	-.36	-2.73	.007
Degree Level (Master's)	.04	.48	.631
Degree Level (Specialist/Doctorate)	.07	.84	.400
Mathematical Beliefs	.72	8.00	.000*

As we saw in the first two models, Models 3 and 4 also suggests a spurious relationship between mathematics anxiety and mathematical beliefs. According to Walker and Maddan (2013), spurious variables show a relationship because of a similar trend in both variables over time: "They influence both the independent and dependent variables such that the relationship between the independent and dependent variables is inflated" (p. 46). These results suggest that it is actually the mathematical beliefs that directs and drives the relationship between mathematics anxiety and instructional practices. This is consistent with Wilkins (2008) who found beliefs to be a mediating variable between mathematics anxiety and instructional practices.

Summary

In this chapter, data analysis was performed to examine the relationships among mathematics anxiety, mathematical beliefs, and instructional practices of the teachers in the sample (N = 153). Data was first exported from www.surveymonkey.com into SPSS and Excel. Descriptive statistics were used to examine the results of each survey. To test the hypotheses, a Pearson correlation-moment was then calculated for each pairing of the three variables in the study (see Table 8). Each correlational analysis was significant, enabling a rejection of each of the null hypotheses.

To determine whether teachers' instructional practices were affected by mathematics anxiety and/or mathematical beliefs, multiple regressions were computed. The results initially produced significant results for mathematics anxiety and instructional practices, but when mathematical beliefs were incorporated into the design, the relationship no longer existed. Regressional analysis was then used to test these variables controlling for teaching longevity and educational degree level. As before, the relationship between mathematics anxiety, controlling for the aforementioned demographic factors, disappeared when mathematical beliefs were incorporated into the model. The only significant result obtained from the regression analysis was teaching longevity for veteran teachers. The results indicate that there is a significant relationship between teaching longevity and instructional practices for veteran teachers. This relationship will be discussed further in the next chapter.

Chapter 6 provides further insight into the findings and offers conclusions. This next chapter also discusses implications for improving teaching education based on the study and provides suggestions for future research.

CHAPTER 6

DISCUSSION

In this chapter, I address the research findings posed in this study by summarizing and discussing the findings presented in Chapter 5. This closing chapter includes the following sections: summary of the study, discussion of findings, implications, recommendations for future research, and closing thoughts.

Summary of the Study

Mathematics anxiety, mathematical beliefs, and the instructional practices of elementary school teachers are all topics that have been explored as individual constructs in the professional research literature (see, e.g., Barkatsas & Malone, 2005; Brady & Bowd, 2005; Cross, 2009; Gresham, 2008; Hadley & Dorward, 2011; Lake & Kelly, 2014; McLeod & McLeod, 2002; Mujis & Reynolds, 2002). Because of the varied information found among the separate studies addressing the isolated constructs, the intention of this study was to determine the interrelatedness of all of the components to affirm or counter the previous research.

A thorough review of the literature revealed no study that addressed the constructs of mathematics anxiety, mathematical beliefs, and the instructional practices of elementary school teachers collectively. Informed by the theory of social constructivism, this study was designed to determine the relationships among all of the constructs, and to establish if mathematics anxiety and mathematical beliefs influence teachers' instructional practices in mathematics.

This study included 153 participants who teach Pre-K–5 in a suburban county in Georgia. An online survey (www.surveymonkey.com) was used to collect data over a 2-week period in August 2015. Data were collected for the study using three instruments: The Mathematics Anxiety Rating Scale: Short Version (Suinn & Winston, 2003), The Teacher Beliefs Instrument

(Beswick, 2005), and the Self-Report Survey: Elementary Teachers Commitment to Mathematics Education Reform (Ross et al., 2003).

Findings

The findings that address the three research questions and hypotheses were presented in detail in Chapter 5 and are summarized in this section.

Demographics

The survey collected demographic information from the participants, including number of years of teaching experience, highest degree level, number of years of teaching mathematics, and number of mathematics courses taken in college. There are several important factors to note about this information. First, approximately 93% of the participants have at least six years of teaching experience. Very few beginning teachers participated in the survey. It may be that this percentage is indicative of the population, in which only 7% of the teachers in the district have less than five years teaching experience. It may be that novice teachers are less inclined to participate because of the demands and time constraints associated with just learning to manage the role of classroom teacher.

The percentage of participants who have taught mathematics for at least six years is approximately 85%. Only one participant had never taught mathematics. This may be due to the perception that teachers having never taught mathematics were less inclined to participate in a survey related to mathematics. It is also noteworthy that 92% of the participants had more than one mathematics course in college, and almost 70% had at least three mathematics courses. In addition, 73% of the participants had a master's degree or higher degree. This is reasonable, as one would expect a positive correlation between the degree and the number of mathematics courses taken. Because the study focused on elementary teachers, it was not anticipated that

many of the teachers would have an exhaustive number of mathematics courses because the majority of colleges and universities in the United States require little mathematics for prospective teachers outside the field of elementary education.

Correlation between Mathematics Anxiety and Instructional Practices

After determining the descriptive statistics for the participating teachers, correlational analyses of the survey data were conducted. The data indicate that overall teachers had a lower to neutral sense of mathematics anxiety. The correlational analyses showed a significant, negative relationship between mathematics anxiety and instructional practices, thus the null hypothesis was rejected. This finding indicates that lower anxiety corresponds to high reform instructional practices, and high anxiety corresponds to low reform instructional practices. This finding is consistent with the literature (Bush, 1989; Hadley & Dorward, 2011; Jackson, 2008; Uusimaki & Nason, 2004), which also found the relationship between mathematics anxiety and instructional practices to be negatively correlated.

Correlation between Mathematical Beliefs and Instructional Practices

For mathematical beliefs, the overall data suggest the participants have a neutral view, with beliefs leaning slightly toward a constructivist, problem-solving view of teaching and learning mathematics. The correlational analyses showed a significant, positive relationship between mathematical beliefs and instructional practices, thus the null hypothesis was rejected. This finding indicates that beliefs aligned with the traditional, instrumentalist view corresponds to low reform practices, and beliefs aligned with the constructivist, problem-solving view corresponds to high reform practices. This finding is also consistent with the literature (Barkatsas & Malone, 2005; Beswick, 2005; Golafshani, 2000; Handal, 2003; Polly et al., 2013;

Wilkins, 2008), which found the relationship between mathematical beliefs and instructional practices to be positively correlated.

Multiple Regressions

Multiple linear regressions were conducted to determine whether mathematics anxiety and mathematical beliefs could be considered predictors of teachers' instructional practices. For mathematical beliefs, this finding was significant; beliefs teachers hold toward the teaching and learning of mathematics may affect their methods of instruction. This was not the case for mathematics anxiety, however. The findings showed the relationship between mathematics anxiety and instructional practices was not significant. This finding is consistent with Hadley and Dorward (2011), who also found mathematics anxiety not significantly related to instructional practices. Nevertheless, this result is surprising and contradicts other literature (Beilock et al., 2010; Bush, 1989; Jackson, 2008; Uusimaki & Nason, 2004; Vinson, 2001), which suggests teachers' instructional practices may be influenced by mathematics anxiety. Previous research, however, suggests that individuals who have had mathematics anxiety may be able to overcome it through communication and collaboration with peers (Liu, 2007). By discussing their personal anxiety toward teaching mathematics, the teachers' sense of mathematics anxiety decreased considerably, and they were able to share strategies for mathematics instruction.

It was expected that mathematics anxiety would have a significant relationship with elementary teachers' instructional practices; however, because no significance was found, other explanations must account for its insignificance. The failure to find a causal relationship between the two constructs could possibly be explained by the security teachers find in utilizing their textbook series. Because teachers may have been responsible for allowing textbooks to be

the driving force behind their mathematics instruction, it is feasible that a reliance on these resources helps to mask, or possibly eliminate, the factor of mathematics anxiety. Supporting resources, including teacher manuals that frequently provide instructional ideas and the scope and sequence of mathematics lessons, often provide enough stability to relieve teachers' anxieties due to the constant reassurance provided through their resources. Furthermore, by utilizing these resources teachers do not have to rely heavily on their own independent thought or skills in regard to instructional planning, and they may develop more confidence in their instructional lessons. The literature suggests that teachers who have more confidence in their classroom practices through presentation of information and questioning strategies (which often accompany textbook series) develop better instructional practices (Brady & Bowd, 2005). By considering this rationale, failure to find a significant causal effect between mathematics anxiety and instructional practices can be justified.

Another reason mathematics anxiety may have been found to be insignificant in relation to teachers' instructional practices in mathematics could also be contributed to the direction of their anxiety. As previously noted, the participating teachers in this study were found to have a higher sense of mathematics test anxiety (as reflected by the higher mean on the MARS-SV questions related to assessment) rather than an anxiety about mathematics in general. While this finding suggests teachers' instructional practices were not directly influenced by their mathematics anxieties, it is important to recognize that elementary teachers are not expected to perform on mathematics assessments in their daily practices (to show personal mastery of the content on a mathematics test). Rather, teachers are required to teach the mathematics concepts on an elementary level. Because the MARS-SV was possibly not directly aligned to a factor that causes an insecurity or threat among teachers in their daily instructional practices, it is a possible

reason why mathematics anxiety was not found to have a significant relationship with teachers' instructional practices. Although the teachers' mathematics anxiety may collectively be strongly influenced by their mathematics test anxiety, it was insignificant in their daily instructional practices as a classroom teacher. By putting elementary teachers in another setting, possibly where a mathematics assessment would be given to hold them accountable for the content, the results may be different. However, this is not the case included in the daily responsibilities of school teachers, so it could account for the insignificance of the construct in the study.

Teaching Longevity

Although not directly related to the hypotheses of the study, I deemed it important to explore teaching longevity as it relates to mathematics anxiety, mathematical beliefs, and instructional practices. In reviewing the literature for this project, I found that beginning teachers often enter the profession with specific intentions of teaching using constructivist methods, but find themselves facing barriers that often curtail their efforts (Ernest, 1989b; Wilcox, Schram, Lappan, & Lanier, 1991). Furthermore, the literature indicates that mathematics anxiety of prospective teachers is prevalent in early childhood teaching programs (Hembree, 1990; Brady & Bowd, 2005). Do prospective teachers carry their anxiety with them into the classroom? Do they overcome or adjust to that anxiety as they gain more experience over the years? What kind of instructional practices do beginning teachers use to teach mathematics? Although these questions could very likely be used as a premise for a research project in and of itself, I thought it noteworthy to investigate teaching longevity as it relates to the constructs for the participating teachers in this study.

The teachers were divided into three groups according to the number of years of teaching experience. Multiple regressional analysis yielded significant results for the veteran teachers but

not for the beginning or middle teachers. When controlling for just mathematics anxiety and degree status, the veteran teachers were almost significantly related to instructional practices, but when mathematical beliefs was incorporated into the model, the veteran teachers were significantly related to instructional practices. The results imply that for veteran teachers, their instructional practices tend to be more traditional. This may be due to what Ernest (2004) classifies as "the socialization effect" of the school. Ernest contends that although beginning teachers may have been taught to adopt a reformed practice during their teacher education program, practicing teachers are subject to the constraints and contingencies of the school context once they enter the classroom. This may be due to the expectations of others, especially other teachers and superiors.

Implications

School districts, teachers, administrators, and professional development organizations can benefit from an increased awareness concerning the impact teachers' mathematics anxiety and mathematical beliefs have on the instructional practices of elementary teachers. Expectations at the national, state, and local level stress the need for the instructional practices of mathematics teachers to change. This change includes teaching methods that foster a constructivist paradigm, with student understanding of mathematical concepts paramount and rote memorization of algorithms minimal. This change includes classroom learning environments that support problem solving, communication, and justification of mathematical ideas. Also requiring change, the role of the teacher is to create meaningful tasks that engage pupils with mathematical ideas and encourage pupils to explain their solutions strategies so that they may internalize the concepts.

Strategies should be put in place to support teacher growth as they learn to establish a consistent foundation for effective, constructivist, reform-based instructional practices. It is imperative that educational stakeholders provide teachers with long-term, consistent professional development and support that guides them to better understand not only the mathematics curriculum, but also their own anxieties and beliefs exposed as they teach that curriculum.

The aforementioned expectations are the result of a mathematics reform movement that began with the publication of the NCTM Standards in 1989, and continues to the present with the adoption of Georgia's new mathematics curriculum, the *Georgia Standards of Excellence* (GSE; see https://www.georgiastandards.org/Georgia-Standards/Pages/Math.aspx).

The teacher's role in the mathematics classroom is described in the following quote:

> Teachers establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create. Teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions. The teacher is responsible for creating an intellectual environment where serious mathematical thinking is the norm. More than just a physical setting with desks, bulletin boards, and posters, the classroom environment communicates subtle messages about what is valued in learning and doing mathematics (NCTM, 2000, p. 18).

Teacher educators need to focus on teaching sound pedagogy and fundamental mathematics content; however, they also need to focus on the affective behaviors to instill confidence in and positive attitudes toward mathematics. As the literature suggests, mathematics anxiety is considered by many to stem from negative prior experiences in the mathematics classroom (Battista, 1986; Harper & Daane, 1998; Hembree, 1990; Gresham, 2008; Jackson &

Leffingwell, 1999). Therefore, teacher educators should provide opportunities for prospective teachers to reflect on their prior experiences and acknowledge the origins of their anxiety. Prospective teachers (and beginning practicing teachers) should also be provided with opportunities to reflect on and analyze their teaching methods. Through awareness of their teaching methods, teachers may gain a better understanding of themselves, which may improve their instructional delivery. It is imperative that teachers also understand the usefulness and relevance of the mathematics they are teaching.

Recommendations for Future Research

In reviewing the literature for this project, I found it dominated by small-scale qualitative studies, or mixed-methods designs. Conducting more large-scale, quantitative studies with participants from a wider population range can extend this research. Incorporating observation methods into the design can also strengthen studies that measure the instructional practices of teachers. It is beneficial for classroom observations to be integrated into studies that examine the influence of teachers' instructional practices; however, time constraints and added costs often prevent such endeavors. In addition, research can be improved by designing longitudinal studies that extend beyond the prospective teachers' training or first year of teaching. Whether data collection procedures include surveys, interviews, or observations, research studies could often be extended and repeated to capture long-term effects and change over time (Creswell, 2003).

Mathematics anxiety and mathematical beliefs are thought to be rooted in biographical events in a person's prior experiences (Hembree, 1990; McLeod, 1992). These biographical events are thought to influence instructional practices (Ernest, 1989a; Stipek et al., 2001), and therefore should be further explored in future studies. Understanding how teachers conceptualize their learning experiences and generalize them to form beliefs about teaching and

learning mathematics may contribute to improvements in professional developments and teacher education programs.

Future studies should also incorporate teachers' content knowledge when examining instructional practices. A certain level of mathematical knowledge and expertise is needed to teach mathematics effectively at the elementary school level. How this knowledge (or lack thereof) relates to mathematics anxiety and/or mathematical beliefs would be an illuminating project.

Closing Thoughts

The purpose of this study was to investigate the mathematics instructional practices of elementary school teachers with respect to two contributing factors: mathematics anxiety and mathematical beliefs. More specifically, it explored any possible relationships among the constructs of mathematics anxiety and mathematical beliefs to determine if these independent variables impact (or not) elementary teachers' mathematical instructional practices. Generally, the findings of this study support the literature reviewed. The findings indicated a significant relationship between mathematics anxiety and instructional practices, between mathematical beliefs and instructional practices, and between mathematics anxiety and mathematical beliefs.

Only by identifying the relationship between or among any of the given constructs, can the most effective mathematics practices among elementary school teachers be promoted. As educators, if we assist teachers in alleviating their mathematics anxieties and redirect their mathematical beliefs, we might prevent further influences of mathematics anxiety and negative beliefs among students. I have three hopes for this project. First, it is my hope that this study expands the knowledge base of mathematics education in the interrelated areas of mathematical beliefs, mathematics anxiety, and instructional practices of elementary mathematics teachers.

For only through recognizing the factors that negatively influence teacher' instructional practices in mathematics, can efforts be extended toward alleviating and eliminating such influences. Secondly, it is my hope that the results of this project assists further research in identifying the relationships between and among mathematics anxiety, mathematical beliefs, and instructional practices of elementary school teachers. And finally, it is my hope this project may create a sense of urgency for all educational stakeholders to find appropriate ways to help elementary teachers' acknowledge their mathematics anxieties and beliefs about mathematics.

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APPENDIX A

The Mathematics Anxiety Rating Scale, Short Version (MARS-SV)

The items in the questionnaire refer to things that may cause fear of apprehension. For each item decide which of the ratings best describes how much you are frightened by it nowadays –

1 Not at all 2 A little 3 A fair amount 4 Much 5 Very much

	1	2	3	4	5
1. Taking an examination (final) in a math course.					
2. Thinking about an upcoming math test one week before.					
3. Thinking about an upcoming math test one day before.					
4. Thinking about an upcoming math test one hour before.					
5. Thinking about an upcoming math test five minutes before.					
6. Waiting to get a math test grade returned in which you expect to do well.					
7. Receiving your final math grade in the mail.					
8. Realizing that you have to take a certain number of math classes to fulfill the requirements in your major.					
9. Being given a "pop" quiz in a math class.					
10. Studying for a math test.					

11. Taking the math section of a college entrance exam.					
12. Taking an examination (quiz) in a math course.					
13. Picking up the math textbook to begin a homework assignment.					
14. Being given a homework assignment of many difficult problems which is due the next class period.					
15. Getting ready to study for a math test.					
16. Dividing a five digit number by a two digit number in private with pencil and paper.					
17. Adding up 976 + 777 on paper.					
18. Reading a cash register receipt after your purchase.					
19. Figuring the sales tax on a purchase that cost more than $1.00.					
20. Figuring out your monthly budget.					
21. Being given a set of numerical problems involving addition to solve on paper.					
22. Having someone watch you as you add up a column of numbers.					

23. Totaling up a dinner bill you think overcharged you					
24. Being responsible for collecting dues for an organization and keeping track of the amount.					
25. Studying for a driver's license test and memorizing the figure involved, such as the distance it takes to stop a car going at different speeds.					
26. Totaling up the dues received and the expenses of a club you belong to.					
27. Watching someone work with a calculator.					
28. Being given a set of division problems to solve.					
29. Being given a set of subtraction problems to solve.					
30. Being given a set of multiplication problems to solve. 					

APPENDIX B

THE TEACHER BELIEFS SURVEY

Place a check in the box that describes your level of agreement with each statement.

Beliefs About Mathematics, Its Teaching, and Its Learning					
	Strongly Agree (5)	Agree (4)	Not Decided (3)	Disagree (2)	Strongly Disagree (1)
1. A vital task for the teacher is motivating children to solve their own mathematical problems.					
*2. Mathematics is computation.					
3. Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.					
4. Children always benefit by discussing their solutions to mathematical problems with each other.					
5. It is important for children to be given opportunities to reflect on and evaluate their learning.					
6. Allowing a child to struggle with a mathematical problem, even a little tension, can be necessary for learning to occur.					
7. It is important for teachers to understand the structured way in which mathematics concepts and skills relate to each other.					

8. Mathematics is a beautiful, creative, and useful human endeavor that is both a way of knowing and a way of thinking.					
9. Effective mathematics teachers enjoy learning and doing mathematics themselves.					
10. Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics.					
11. Knowing how to solve a mathematics problem is as important as getting the right solution.					
12. Teachers of mathematics should be fascinated with how children think and intrigued by alternative ideas.					
13. Persistent questioning has a significant effect on children's mathematical learning.					
*14. If a child's explanation of a mathematical solution doesn't make sense to the teacher it is best to ignore it.					
*15. Telling the children the answer is an efficient way of facilitating their mathematics learning.					

*16. It is important that mathematics content be presented to children in the correct sequence.					
17. Justifying the mathematical statements that a person makes is an extremely important part of mathematics.					
*18. It is important to cover all the topics mathematics curriculum in the textbook sequence.					
*19. I would feel uncomfortable if a child suggested a solution to a mathematical problem that I hadn't thought of previously.					
20. As a result of my experience in mathematics classes, I have developed an attitude of inquiry.					
*21. There is an established amount of mathematical content that should be covered at each grade level.					
22. Mathematical material is best presented in an expository style: demonstrating, explaining, and describing concepts and skills.					
23. Teachers can create, for all children, a nonthreatening environment for learning mathematics.					

*24. It is not necessary for teachers to understand the source of children's errors; follow–up instruction will correct their difficulties.					
*25. Listening carefully to the teacher explaining a mathematics lesson is the most effective way to learn mathematics.					
*26. It is the teacher's responsibility to provide children with clear and concise solution methods for mathematical problems.					

*denotes negatively worded item and are reverse coded.

APPENDIX C

SELF REPORT SURVEY: ELEMENTARY TEACHERS COMMITMENT
TO MATHEMATICS EDUCATION REFORM

	Strongly Disagree 1	Disagree 2	Undecided 3	Agree 4	Strongly Agree 5
1. I like to use math problems that can be solved in many different ways.					
2. I regularly have my students work through real–life math problems that are of interest to them.					
3. When two students solve the same math problem correctly using two different strategies I have them share the steps they went through with each other.					
4. I tend to integrate multiple strands of mathematics within a single unit.					
5. I often learn from my students during math time because my students come up with ingenious ways of solving problems that I have never thought of.					
*6. It is not very productive for students to work together during math time.					
7. Every child in my room should feel that mathematics is something he/she can do.					
8. I integrate math assessment into most math activities.					
9. In my classes, students learn math best when they can work together to discover mathematical ideas.					

10. I encourage students to use manipulatives to explain their mathematical ideas to other students.					
*11. When students are working on math problems, I put more emphasis on getting the correct answer than on the process followed.					
12. Creating rubrics for math is a worthwhile assessment strategy.					
13. In my class it is just as important for students to learn data management and probability as it is to learn multiplication facts.					
14. I don't necessarily answer students' math questions but rather let them puzzle things out for themselves.					
*15. A lot of things in math must simply be accepted as true and remembered.					
*16. I like my students to master basic mathematical operations before they tackle complex problems.					
17. I teach students how to explain their mathematical ideas.					
*18. Using computers to solve math problems distracts students from learning basic math skills.					

19. If students use calculators they won't master the basic skills they need to know.					
20. You have to study math for a long time before you see how useful it is.					

*denotes negatively worded item and are reverse coded

APPENDIX D

Certificate of Informed Consent

Overview and Procedure: The purpose of this research is to examine the relationships between and among mathematics anxiety, mathematical beliefs, and instructional practices of elementary school teachers. By granting consent to be part of this research, you agree to voluntarily participate in taking 3 online surveys: The Mathematics Anxiety Rating Scale: Short Version, The Teacher Beliefs Survey, and a Commitment to Mathematics Reform Survey. The total time to complete these surveys is approximately 15–20 minutes.

Risks and Benefits: There are no risks greater than the risks of everyday living. There are no direct benefits for participating in the study.

Confidentiality: Your privacy will be protected. Any information obtained during the course of your participation will remain confidential and will be used solely for research purposes. The surveys will be completed anonymously. Your name will only be used to track completion of the surveys. It will never be shared with the public. The data will remain confidential and be stored on a password–encrypted website www.surveymonkey.com. Results of the study will be made available to you upon request.

Your Rights: As with any research project, your participation is voluntarily. You may withdraw from the survey at any time, or decline to answer any questions without penalty.

Contact Information: If you have questions or concerns about your rights as a participant, please contact, Pamela T. Hughes, or the Faculty Advisor at Georgia State University.

Investigator	**Faculty Advisor**
Pamela T. Hughes	Dr. David W. Stinson
pam.hughes@cowetaschools.org	dstinson@gsu.edu

By clicking AGREE below, you are agreeing 1) to participate in this study, and 2) that you have read and understand all of the information provided on this form.

AGREE