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Essays on Insurance Economics

BY

Jinjing Wang

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree

Of

Doctor of Philosophy

In the Robinson College of Business

Of

Georgia State University

GEORGIA STATE UNIVERSITY

ROBINSON COLLEGE OF BUSINESS

2015

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ACCEPTANCE

This dissertation was prepared under the direction of the Jinjing Wang Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctoral of Philosophy in Business Administration in the J. Mack Robinson College of Business of Georgia State University.

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ABSTRACT

Essays on Insurance Economics

BY

Jinjing Wang

July 11, 2015

Committee Chair: Ajay Subramanian

Major Academic Unit: Department of Risk Management and Insurance

This dissertation thesis consists of four chapters to address how aggregate shocks affect insurance firms' risk management and asset investment decisions as well as the impact of these decisions on insurance prices and regulation. The first chapter focuses on the transfer of aggregate risk by insurance firms. Specifically, this chapter develops a signaling model to examine how insurance firms choose among retention, reinsurance and securitization especially for catastrophe risks. The second chapter examines the determination of insurance prices in an integrated equilibrium framework where insurance firms' assets may be subject to both idiosyncratic and aggregate shocks. The third chapter presents an empirical analysis of the hypothesized impacts of internal capital and asset risk on insurance prices as predicted by the results of the second chapter. The last chapter investigates the optimal design of insurance regulation to achieve the Pareto optimal asset and liquidity management by insurers as well as risk sharing between insurers and insurees.

Chapter 1 provides a novel explanation for the predominance of retention and reinsurance relative to securitization in catastrophe risk transfer using a signaling model. An insurer's

risk transfer choice trades off the lower signaling costs of reinsurance against the additional costs of reinsurance stemming from sources such as their market power, higher cost of capital relative to capital markets, and compensation for their monitoring costs. In equilibrium, the lowest risk insurers choose reinsurance, while intermediate and high risk insurers choose partial and full securitization, respectively. An increase in the loss size increases the average risk of insurers who choose securitization. Consequently, catastrophe risks, which are characterized by low frequency-high severity losses, are only securitized by very high risk insurers. Chapter 2 develops a unified equilibrium model of competitive insurance markets where insurers' assets may be exposed to idiosyncratic and aggregate shocks. We endogenize the asset and liability sides of insurance firms' balance sheets. We obtain new insights into the relationship between insurance prices and insurers' internal capital that potentially reconcile the conflicting predictions of previous theories that investigate the relation using partial equilibrium frameworks. Equilibrium effects lead to a non-monotonic U-shaped relation between insurance price and internal capital. Specifically, the equilibrium insurance price first decreases with a positive shock to the internal capital when it is below certain threshold level, and then increases with a positive shock to the internal capital when it is above the threshold level. Further, we also derive another testable implication that an increase in the asset default risk increases the insurance price and decrease the insurance coverage. Chapter 3 studies the property and casualty insurance industry in periods from 1992 to 2012 based on the aggregate level of NAIC data. We show that the insurance price decreases with an increase in the surplus of insurance firms at the end of the previous year when the surplus is lower than 8.5 billion, and then increase when the surplus is higher than 8.5 billion. Our results provide support for the hypothesis of a U-shaped relationship between internal capital and insurance price. Our results also provide evidence for the positive relationship between asset portfolio risk and insurance price. Chapter 4 studies the effects of aggregate risk on the Pareto optimal asset and liquidity management by insurers as well as risk-sharing between insurers and insuees. When aggregate risk is low, both insurees and insures hold no liquidity reserves, insurees are fully insured, and insurers bear all aggregate risk. When aggregate risk takes intermediate values, both insurees and insurers still hold no

liquidity reserves, but insurees partially share aggregate risk with insurers. When aggregate risk is high, however, it is optimal to hold nonzero liquidity reserves, and insurees partially share aggregate risk with insurers. The efficient asset and liquidity management policies as well as the aggregate risk allocation can be implemented through a regulatory intervention policy that combines a minimum liquidity requirement when aggregate risk is high, "*ex post*" contingent on the aggregate state, comprehensive insurance policies, and reinsurance.

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Chapter 1

Catastrophe Risk Transfer

1.1 Introduction

Insurers with limited capital to completely cover the risks in their portfolios often exploit external risk transfer mechanisms such as reinsurance and securitization. Although these risk-sharing mechanisms are used for all types of insurable risks, they are especially important in the case of catastrophe risks because of the large magnitudes of the potential losses involved. A strand of literature argues that securitization has a significant advantage over reinsurance because of the substantially higher available capital and risk-bearing capacity of capital markets (Durbin 2001). Nevertheless, an enduring puzzle is that reinsurance is still the dominant risk transfer mechanism for catastrophe risks. By the end of 2011, the outstanding risk capital of asset-backed-security catastrophe (CAT) bonds amounted to $12 billion, while the reinsurance capacity was $470 billion. CAT bonds are often issued to provide "high layers of protection" that are not covered by reinsurance. It is often argued that CAT bonds are too expensive even though CAT risks are uncorrelated with market risks suggesting that they are somehow "mispriced" relative to their payoffs. Further, many CAT bonds receive ratings that are below investment grade (see Cummins (2008, 2012)).

We provide a novel explanation for the above stylized facts using a signaling model to analyze an insurer's risk transfer choice. When an insurer with private information about its portfolio faces a choice between reinsurance and securitization, its choice represents a signal

of the nature of risks in its portfolio. The insurer's choice trades off the lower adverse selection or information costs associated with reinsurance (because of the superior monitoring abilities of reinsurers) against the higher costs of reinsurance arising from various sources such as reinsurers' market power, higher cost of capital relative to capital markets, and compensation for their costs of monitoring (Froot (2001)). We show that Perfect Bayesian Equilibria (PBE) of the signaling game have a partition form where an insurer chooses reinsurance if its risk is below a low threshold, partial securitization if its risk lies in an intermediate interval, and full securitization if its risk is above a high threshold. The threshold risk level above which the insurer chooses securitization increases with the magnitude of potential losses in its portfolio. Given that catastrophe risks are usually characterized by "low frequency–high severity" losses, our results imply that an insurer is more likely to choose retention or reinsurance to transfer catastrophe risk. Further, because an insurer only opts for securitization if its risk of potential losses is high, securitization typically provides high layers of protection, catastrophe bonds have high premia (relative to the ex ante expected losses) and often have ratings below investment grade.[1] Importantly, our results suggest that the high costs of catastrophe securities reflect the rational incorporation of their inherent risks by capital markets based on the information they glean from insurers' risk transfer choices.

In our signaling model, a representative insurer with a limited amount of capital holds a portfolio of insurable risks. The insurer incurs significant bankruptcy costs if it is unable to meet its liabilities, which provides incentives for it to transfer its risks. The insurer can choose to retain its risks or transfer them either partially or wholly through reinsurance or securitization. The insurer has private information about its risks so that there is adverse selection regarding its "type." Reinsurers have a significant information advantage over capital markets because they possess the resources to more effectively monitor insurers. For simplicity, we assume that reinsurers know an insurer's risk type and, therefore, do not face any adverse selection. (Our results are robust to allowing for adverse selection in reinsurance as long as its degree is less than that in securitization.) On the flip side, however, reinsurers

[1]Because CAT bonds are fully collateralized, CAT bond ratings are determined by the probability that the bond principal will be hit by a triggering event. Thus, the bond ratings indicate the layer of catastrophic-risk coverage that is provided by the bonds.

charge a markup over the actuarially fair premium that could arise through various channels. Consistent with Froot (2001), reinsurers have significant market power that allows them to extract additional rents relative to competitive capital markets. (The market power of reinsurers is analogous to the market power of informed lenders in Rajan (1992).) The reinsurance markup could also arise as compensation for reinsurers' monitoring costs and the higher cost of capital of reinsurers relative to capital markets that have higher risk-bearing capacity. The insurer's choice among retention, reinsurance and securitization reflects the tradeoff between the lower adverse selection costs associated with reinsurance and the costs stemming from the reinsurance markup.

For robustness, we analyze two versions of framework. In the first version, the insurer incurs *fixed* bankruptcy costs if it is unable to meet its liabilities. In the second version, it incurs *variable* bankruptcy costs that are proportional to the magnitude of its losses. In both versions, the insurer's "risk" is determined by its probability of incurring a loss that exceeds its capital level so that it is unable to meet its liabilities.

In the model with fixed bankruptcy costs, we show that Perfect Bayesian equilibria (PBE) of the signaling game (under stability restrictions on off-equilibrium beliefs along the lines of the D1 refinement) have a "partition form" that is characterized by two thresholds. The insurer chooses reinsurance if its risk is below the low threshold, self-insurance if its risk lies between the thresholds, and securitization if its risk is above the high threshold. The intuition for the equilibria is as follows. With fixed bankruptcy costs, the costs the insurer incurs are independent of the magnitude of its shortfall in meeting its liabilities. Consequently, it is never optimal for the insurer to partially retain its risks, that is, it either chooses to retain all its risks or completely transfer them. Because of the reinsurance markup, the cost of reinsurance is increasing and convex in the insurer's risk, while the cost of retention is increasing and linear. The costs of securitization, which stem from the cross-subsidization of higher risk types are, however, *decreasing* and convex in the insurer's risk. Consequently, if the insurer's risk is below a low threshold, it prefers reinsurance to retention as well as securitization. If the insurer's risk lies in an intermediate interval, it prefers retention to reinsurance because the increasing and convex costs of reinsurance dominate those of

retention for intermediate risks. If the insurer's risk is above a high threshold, the fact that the cost of securitization is decreasing in the insurer's risk type implies that securitization dominates retention and reinsurance.

An increase in the size of potential losses increases the marginal cost of subsidizing higher risk insurers, thereby increasing the trigger risk level above which insurers choose securitization. In the context of catastrophe risk, which is characterized by low frequencies and large magnitudes of potential losses, our results imply that an insurer chooses securitization if and only if its risk of potential losses is high, that is, reinsurance is more likely to be chosen as a risk transfer mechanism. Further, the prediction that only very high-risk insurers choose securitization explains why catastrophe bonds have high premia relative to their expected losses, and ratings of catastrophe-linked securities are often below investment grade.

In the model with proportional bankruptcy costs, an insurer's bankruptcy costs vary with the magnitude of its shortfall in meeting its liabilities. Consequently, it is always optimal for the insurer to transfer at least some portion of its risk either through reinsurance or securitization by choosing a retention level. The PBE of the risk transfer signaling game again have a partition structure, which depends on the reinsurance markup. If the reinsurance markup is below a threshold, then the lowest risk insurers choose full reinsurance, the intermediate risk insurers choose separating securitization contracts with retention levels that decrease with their risk, while the highest risk insurers choose full pooling securitization. If the reinsurance markup is above the threshold, however, the equilibria are characterized by two intervals where the lower risk insurers choose separating partial securitization contracts, while the high risk insurers choose full securitization.

When the reinsurance markup is sufficiently low, the costs of reinsurance are lower than the signaling costs associated with (partial or full) securitization. To avoid the costs associated with the reinsurance markup, and the costs of subsidizing high-risk insurers, intermediate risk insurers signal their types by choosing separating securitization contracts that are characterized by retention levels that decline with their risk. For high-risk insurers, the costs of signaling are too high so that they choose to pool by offering full securitization contracts. When the reinsurance markup is high, however, the lowest risk insurers too prefer separating

partial securitization to reinsurance.

The implication that only high risks are securitized is consistent with a noticeable increase in catastrophe securitization after Hurricane Katrina. Anecdotal evidence suggests that actuaries significantly increased their estimates of catastrophe risks following Katrina (see Ahrens et al. (2009)). The spike in securitization transactions is, therefore, consistent with the higher perceived levels of risk. In recent years, more sophisticated investors such as dedicated hedge funds have entered the catastrophe securization market and this has been followed by an increase in the volume of securitization. This observation is also consistent with our basic story. The entry of more sophisticated investors has likely reduced the level of adverse selection in securitization markets, thereby lowering securitization costs.

To highlight our results as crisply as possible, we assume that an insurer chooses one of three possible risk transfer mechanisms, namely, retention, reinsurance, and securitization. Our results can, however, be naturally extended to the scenario in which an insurer is exposed to multiple risks. In this context, our analysis suggests that the lowest risks are reinsured, the intermediate risks are either retained or partially securitized, and the highest risks are fully securitized. Consequently, our results are also consistent with the observation that insurers often choose both reinsurance and securitization to transfer their *portfolios* of risks. In particular, the results comport with evidence that catastrophe bonds are typically issued to provide high layers of protection that are not reinsured.

Our study relates to two branches of the literature that investigate insurers' choice between reinsurance and securitization, especially in the context of catastrophe risk transfer. The first branch examines the factors that affect the *demand* for insurance-linked securities such as ambiguity and loss aversion (Bantwal and Kunreuther (2000)) as well as aversion to downside risk and parameter uncertainty (Barrieu and Louberge (2009)). The second branch examines the factors that affect the *supply* of insurance-linked securities. Cummins and Trainar (2009) argue that the benefits of securitization relative to reinsurance increase when the magnitude of potential losses and the correlation of risks increase. Finken and Laux (2009) argue that, given low basis risk, catastrophe bonds with parametric triggers are insensitive to adverse selection, and can serve as an alternative risk transfer mechanism that

is more attractive to low risk insurers who suffer from adverse selection with reinsurance contracts. Lakdawalla and Zanjani (2012) argue that catastrophe bonds can improve the welfare of insureds when reinsurers face contracting constraints on the distribution of assets in bankruptcy, and when they must insure a heterogeneous group of risks. Gibson *et al.* (2014) analyze the tradeoff between the costs and benefits of loss information aggregation procedures to determine the prevalent risk transfer form. They argue that traders in capital markets may produce too much information, thereby making securitization prohibitively costly. Hagendorff et al. (2014) empirically show that reinsurance dominates securitization when loss volatility is above a threshold. We complement the above literature by providing a novel explanation based on signaling considerations for the dominance of retention and reinsurance in the market for catastrophe risk transfer. Insurers' risk transfer choice reflects the tradeoff between the lower adverse selection costs associated with reinsurance and the additional costs stemming from reinsurance markup.

It is often argued that a significant deterrent to the growth in the market for insurance-linked securities is the presence of *basis risk*, which is present when securities have parametric triggers where payouts are based on an index not directly tied to the sponsoring insurer's losses. It is, however, unclear what the quantitative impact of basis risk is on the securitization decision given that insurers can choose the volume of securities to issue to hedge their exposure to the catastrophe underlying the index. Indeed, Cummins, Lalonde and Phillips (2004) empirically show that insurers, except perhaps for the smallest ones, can hedge their exposures almost as effectively using contracts with index triggers as they can using contracts that settle on their own losses. Further, basis risk can be often be reduced substantially by appropriately defining the location where the event severity is measured (Cummins (2008)). Moreover, a substantial percentage of CAT bonds also have indemnity-based triggers that are tied to the insurer's losses, and CAT bonds with indemnity triggers have significantly larger issue volumes than those with parametric triggers (Braun (2014)).

Another related argument that is proffered for the low volume of securitization is the presence of capital market transaction costs. A major component of these costs are *endogenous* costs due to adverse selection that play a central role in our analysis. Further,

CAT bond issuers annualize the fixed costs over multiple periods, thereby reducing annual transaction costs. In addition, the favorable tax treatment of CAT bonds allow insurers to reduce tax costs associated with equity financing (Niehaus (2002), Harrington and Niehaus (2003)). Moreover, CAT bond interest paid offshore is also deducted for tax purposes in the same way as reinsurance premia (see Cummins (2008)). Consequently, it is not clear that transaction costs associated with securitization, *apart from adverse selection costs* that we already incorporate, are high enough to significantly deter securitization. Further, even if transaction costs were significant, it is not clear whether they explain why securitization is typically used to provide high layers of protection.

Although we focus on catastrophe risks for concreteness, our framework and results can be more broadly applied to analyze the sharing of all insurable risks, and the transfer of other types of risk such as credit risk by firms (e.g., see Gorton and Pennacchi (1995), Duffee and Zhou (2001), Parlour and Plantin (2008), Parlour and Winton (2013), Thompson (2014)). In the context of credit risk transfer, our results suggest that only high credit risks are optimally securitized that offers a potential explanation for why securities such as credit default swaps were actually very risky and triggered huge losses during the financial crisis. Indeed, Drucker and Puri (2009) examine the secondary market for loan sales and find that sold loans are riskier than average.

More broadly, our paper fits into the literature on the analysis of information revelation through the choice of the risk sharing arrangement (e.g., see Leland and Pyle (1977), Nachman and Noe (1994), DeMarzo and Duffie (1999)). We contribute to this literature by comparing information generation channels associated with differing risk transfer mechanisms. We examine two channels through which information is revealed: one is through costly monitoring performed by informed counterparties, and the other one is through signaling to competitive counterparties. Our results imply that information about low risk types is monitored by the risk bearer, information about intermediate risk types is signaled by the risk transferrers, and no information about high risk types is revealed in equilibrium.

1.2 The Model

The economy consists of a continuum of insurers. The representative insurer has a limited amount of capital W and a risky portfolio of insurable risks. The insurer is faced with the choice between retaining the risk (that is, self-insuring) or transferring the risk through reinsurance or securitization. The insurer's portfolio has two possible realizations. In the "good" state, which occurs with probability $1 - p$, the portfolio suffers no loss and the insurer earns the premium A. However in the "bad" state, which occurs with probability p, the portfolio suffers a loss and the insurer has to make the net payment B (total indemnity net of the premium). We assume that $W - B < 0$ so that the insurer's capital is not enough to cover the net loss payment in the bad state.

The insurer has *private information* about the probability p so that there is adverse selection regarding the type p of the insurer. The loss probability p is drawn from the cumulative distribution F with support in $[0, 1]$. The insurer incurs an *additional* deadweight bankruptcy cost C in the bad state if it is unable to fully cover the loss. Note that the bankruptcy cost is in addition to the loss $B - W$. The bankruptcy cost could comprise of non-pecuniary as well as pecuniary costs that arise from a loss of reputation, the loss of future business opportunities, etc.

We assume a fixed bankruptcy cost C in this section. In Section 1.3, we alter the model to consider variable bankruptcy costs that increase with the magnitude of the insurer's shortfall in meeting its liabilities. Hoerger *et al.*(1990) show that the demand for reinsurance might be created by the existence of bankruptcy costs even if the insurer is risk neutral. If the magnitude of underwriting losses and the correlations of risks are large, the risk warehousing function of insurers may collapse. The presence of bankruptcy costs could motivate the insurer to hedge its underwriting losses through reinsurance or securitization.

Given its linear objective function, it is optimal for the insurer to choose either reinsurance or securitization for its entire portfolio provided it chooses to transfer its risk. As we discuss later, however, our results extend naturally to the scenario in which an insurer is exposed to differing risks and chooses different risk transfer mechanisms for different types of risks. We

first derive the reinsurance and securitization contracts separately assuming that insurers only have access to one of the two risk transfer mechanisms. We then analyze the insurer's choice among retention, reinsurance and securitization.

1.2.1 Reinsurance

Reinsurers have an information advantage over investors in capital markets due to their specialized expertise and ability to monitor/screen insurers (e.g. Jean-Baptise *et al.*(2000)). To simplify matters, and to focus attention on the information advantage of reinsurers relative to capital markets, we assume that reinsurers have the monitoring technology to know the risk type of the insurer perfectly so that they do not face any adverse selection. (Our results are robust to allowing for adverse selection in reinsurance as long as its degree is less than that in securitization.)

On the flip side, reinsurers charge a proportional markup $\delta > 0$ over the actuarially fair insurance premium that could arise from multiple sources.

First, as argued by Froot (2001), reinsurers have significant market power relative to competitive capital market investors. The presence of market power for reinsurers is analogous to the market power of informed lenders in Rajan (1992). As in Rajan (1992), we can endogenize reinsurers' market power stemming from their informational advantage by incorporating competition between informed reinsurers and uninformed capital market investors. Adapting his results to our setting, reinsurers' excess rents increase with the expected loss payment (under full information), which is the actuarially fair insurance premium. Apart from (or in addition to) arising from reinsurers' bargaining power, the markup could also emerge through various other channels.

Second, as argued by the literature, capital markets have higher risk-bearing capacity than reinsurers (see Cummins (2008, 2012)). In this context, the markup arises from the higher cost of capital of reinsurers relative to capital markets. More specifically, if L is the total payment made by a reinsurer to the insurer if the latter incurs a loss with probability p, then the present value of this payment from the reinsurer's standpoint is $\frac{pL}{1+\beta}$, where β is the reinsurer's cost of capital. Consequently, the reinsurance premium is $(1 + \beta)$ times the

actuarially fair premium, pL.

Third, reinsurers' monitoring technology is costly and the markup compensates them for their monitoring costs. It is straightforward to endogenize reinsurers' incentives for monitoring and the resulting markup stemming from compensation for monitoring costs. For example, we can formalize the arguments as follows. If the reinsurer monitors the insurer, it learns the insurer's type, but its monitoring costs are κpL, that is, the monitoring costs are a proportion κ of the expected indemnity. If the reinsurer does not monitor, it remains uninformed about the insurer's type as with other competitive capital market investors. Competition among investors in capital markets then ensures that the reinsurer receive zero expected rents from its contract with the insurer. The reinsurer, therefore, chooses to monitor the insurer if the reinsurance premium is at least $(1 + \kappa)pL$, that is, the premium compensates the reinsurer for its expected payment to the insurer if the latter incurs a loss and its monitoring costs. If we incorporate competition among informed reinsurers and uninformed capital market investors, then the reinsurance premium is exactly $(1 + \kappa)pL$, that is, reinsurers are indifferent between between monitoring (and becoming informed) and not monitoring (and remaining uninformed).

In reality, of course, all these forces—market power, costs of capital and monitoring costs—are simultaneously present so the reinsurance markup in the model represents their cumulative effect. We, therefore, remain agnostic about the specific channel through which the markup arises and simply refer to it as the reinsurance markup throughout the paper.

For simplicity, we assume that reinsurers have sufficient capital to fully insure the insurance company so that they do not face default risk.[2] Reinsurers usually have better diversification opportunities that may lower their default risks (*e.g.* Jean-Baptise *et al.*(2000)). The main objective of our study is to compare the trade-off between the information advantage of reinsurers against the lower costs of risk-sharing with capital markets. Consequently, we avoid further complicating the analysis and the intuition for our results by also introducing default risk for reinsurers.

[2] According to the Guy Carpenter report, the total losses of the global property/casualty sector in 2011 exceeded $ 100 billion, but shareholder funds exceeded $ 160 billion. Consequently, the reinsurance sector continued to function normally despite the heavy losses in 2011.

Because reinsurance companies know the insurer's type, they offer distinguishing contracts $(A_r(p), B_r(p))$ that are contingent on the insurer's type, where $A_r(p)$ is the reinsurance premium and $B_r(p)$ is the *net payment*—the indemnities less the premium—to the insurer in the bad state. The optimal contract for each insurer type, p, maximizes its expected utility subject to the reinsurance premium being at least a proportion δ above the actuarially fair premium. Given the fixed bankruptcy cost C, it is easy to show that no insurer type chooses reinsurance if $\delta \geq \frac{C}{\tilde{B}}$ because it is too expensive. Consequently, we consider the case where $\delta < \frac{C}{\tilde{B}}$. If an insurer chooses reinsurance, the optimal reinsurance contract solves

$$\max_{(A_r(p),B_r(p))} (W + A - A_r(p))(1-p) + (W - B + B_r(p))p - Cp \cdot \mathbf{1}_{\{Br(p)<B-W\}}$$

such that

$$\begin{aligned} A_r(p) &\geq p(1+\delta)(B_r(p) + A_r(p)) \\ 0 \leq B_r(p) &\leq B - W \end{aligned} \tag{1.1}$$

Proposition 1 (Reinsurance Contract) *Define*

$$\dot{p} = \frac{C - \tilde{B}\delta}{C(1+\delta)} < 1. \tag{1.2}$$

If $p > \dot{p}$, the insurer chooses retention. If $p < \dot{p}$, the insurer chooses reinsurance. The optimal reinsurance contract, $(A_r^*(p), B_r^*(p))$, is

$$A_r^*(p) = \frac{\tilde{B}p(1+\delta)}{1 - p(1+\delta)}, \qquad B_r^*(p) = B - W = \tilde{B}$$

As one would expect, a higher loss probability raises the reinsurance premium. If the insurer's risk is higher than $\dot{p}$, the expected bankruptcy cost is lower than the cost of reinsurance so that the insurer retains its risk. Because the bankruptcy cost is fixed, the insurer chooses full reinsurance if it opts to transfer its risks.

1.2.2 Securitization

We now examine the case where insurers only have access to capital markets. An insurer's cost of transferring its risks is potentially reduced by the fact that capital markets are competitive. On the flip side, however, capital markets are marred by adverse selection since they cannot obtain the information about an insurer's risk type *ex ante,* that is, before it issues securities. We model the securitization game as a signaling game whose timing is as follows. An insurer offers a contract, (A_s, B_s), where A_s is the premium received by the investors if there is no loss, and B_s is the net payment made by investors if a loss occurs. We restrict consideration to equilibria in pure strategies for the insurer. Investors update their prior beliefs based on the offered contract and then either accept or reject it. In all our subsequent results, we employ reasonable stability restrictions on off-equilibrium beliefs along the lines of Banks and Sobel's (1987) D1 refinement for signaling games to address the potential multiplicity of Perfect Bayesian Equilibria (PBE).

Because the bankruptcy cost is fixed and does not depend on the magnitude of the insurer's shortfall in the bad state, it is easy to see that separating securitization contracts are not incentive compatible. In other words, it is better for an insurer to self-insure rather than choose a securitization contract with a nonzero retention level that reveals its type because it incurs the same bankruptcy cost in either case so that its expected payoff is the same. (Recall that the bankruptcy cost is in addition to the loss payment.)

We conjecture that there exists a trigger level such that insurers with types above the trigger choose full securitization, while those with types below the trigger choose full retention. Consider a candidate equilibrium defined by a trigger level, p. Let $\mu(.)$ denote the posterior beliefs of capital markets regarding an insurer's type given that it has chosen securitization. Given that insurers with types above p choose full securitization in the conjectured equilibrium, investors' posterior beliefs about the insurer's type are given by

$$d\mu(p') = \frac{dF(p')}{1 - F(p)} \tag{1.3}$$

The equilibrium is determined by a function, $R(.)$—the *subsidization ratio function*—that

is defined as follows:

$$R(p) = \frac{\int_p^1 t d\mu(t) - p}{1 - \int_p^1 t d\mu(t)} \tag{1.4}$$

The function, which depends on the distribution of insurers' risk types and the threshold level p that defines the conjectured equilibrium, determines the costs incurred by an insurer with risk type p from pooling with higher risk insurers and, thereby, subsidizing them. If p is the *equilibrium* threshold, then the insurer with risk type p should be indifferent between full retention and full securitization. In other words, the expected bankruptcy cost associated with full retention should be the same as the cross-subsidization cost associated with full pooling securitization for an insurer of type p. We now characterize the equilibrium choice between retention and securitization and the optimal securitization contracts.

Proposition 2 (Securitization Contract) *Suppose there is a unique $\ddot{p}$ satisfying the following equation:*

$$C\ddot{p} = \tilde{B}R(\ddot{p}). \tag{1.5}$$

In the unique PBE of the securitization game (under the D1 refinement), insurers with types p in the interval $[\ddot{p}, 1]$ fully transfer their risks and offer the same contract (A_s^*, B_s^*), where

$$B_s^* = \tilde{B}, \qquad A_s^* = \frac{\tilde{B}\int_{\ddot{p}}^1 t d\mu(t)}{1 - \int_{\ddot{p}}^1 t d\mu(t)}.$$

Insurers with types p below $\ddot{p}$ choose full self-insurance.

The threshold, $\ddot{p}$, is the point of indifference between the cross-subsidization costs from pooling with higher types and the expected bankruptcy costs from retaining risk. Consider a candidate equilibrium where insurers with types greater than or equal to p offer pooling securitization contracts, while those with types less than p retain their risk. By the definition of the subsidization ratio function, the subsidization costs incurred by the insurer with type p from pooling with higher types are given by $\tilde{B}R(p)$. The expected bankruptcy cost incurred by the insurer of type p if it retains its risks is given by Cp. Consequently, the indifference

point, $\ddot{p}$, is determined by (1.5).

In general, (1.5) could have multiple solutions so that there could be multiple PBEs each determined by the threshold risk type that is indifferent between retention and pooling securitization. As is common in the signaling literature, we add a "single crossing" assumption, which ensures that the above equation has a unique solution, that is, the curves Cp and $R(p)$ intersect at exactly one point $\ddot{p}$. A sufficient condition that ensures this is

$$R'(p) < \frac{C}{\tilde{B}}^{3} \tag{1.6}$$

Because the subsidization costs incurred by insurer types greater than $\ddot{p}$ decline with the type, it is optimal for all such insurers to pool by offering full securitization contracts. Given that $\ddot{p}$ satisfies (1.5), the expected bankruptcy cost incurred by an insurer with type less than $\ddot{p}$ is less than the subsidization costs incurred by choosing securitization so that $\ddot{p}$ determines the unique equilibrium.

1.2.3 Risk Transfer Equilibria

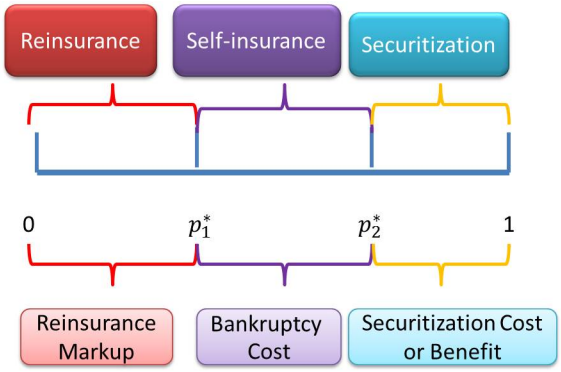

Figure 1.1: Conjecture of "Partition" Form

We now show that the PBE of the risk transfer game have the conjectured "partition" form as shown in Figure 1.1.

Proposition 3 (Partition Equilibria) *Suppose that condition (1.6) holds.*

[3]Let the function $g(p) = Cp - \tilde{B}R(p)$. Since $g(0) = -\tilde{B}R(0) < 0$, and $g(1) = C > 0$ It is easy to show that $g(p) = 0$ has a unique solution $\ddot{p}$ as long as $g(p)$ is increasing over the interval $[0, 1]$; that is,$R'(p) < \frac{C}{\tilde{B}}$.

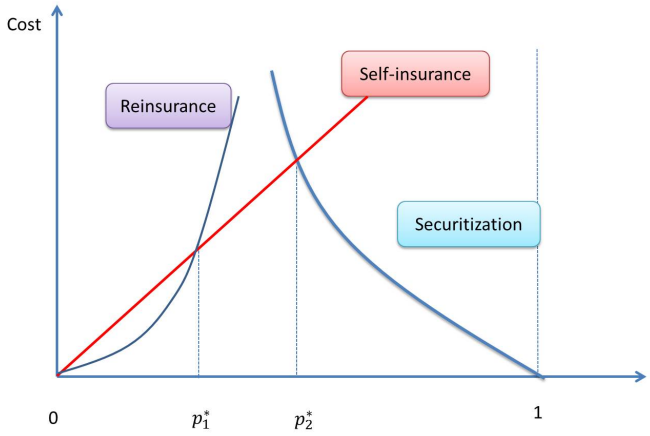

Figure 1.2: The cost of different risk transfer mechanisms

1. *If $C < \tilde{B}\delta$, there exists a unique PBE (under the D1 refinement) with two partitions determined by the threshold $\ddot{p}$ that solves (1.5). Insurers with risk type below $\ddot{p}$ choose full retention, while those with risk type above $\ddot{p}$ choose full pooling securitization.*

2. *If $\tilde{B}\delta < C \leq \frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$, the unique equilibrium (under the D1 refinement) is characterized as follows. Insurers with types in the interval $[0, \dot{p}]$ choose full reinsurance, insurers with types in the interval $[\dot{p}, \ddot{p}]$ choose full self-insurance, and insurers with types in the interval $[\ddot{p}, 1]$ choose full pooling securitization where $\dot{p}$ is defined by (1.2) and $\ddot{p}$ is defined by (1.5).*

3. *If $C > \frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$ and $R'(p) < \frac{\delta}{(1-p(1+\delta))^2}$, there exists a unique trigger p_3^* that solves*

$$\frac{\delta p_3^*}{1 - p_3^*(1+\delta)} = R(p_3^*) \tag{1.7}$$

such that insurers with types in the interval $[0, p_3^*]$ choose full reinsurance, while insurers with types in the interval $[p_3^*, 1]$ choose full pooling securitization.

Figure 1.2 shows the cost function of each risk transfer choice faced by insurers. For all types, the chosen form of risk transfer is the one that has the lowest expected cost. As illustrated in the figure, the expected bankruptcy cost is increasing and linear in an insurer's type, the expected cost of reinsurance is increasing and convex in an insurer's type, and the expected cost of securitization decreases with an insurer's type. Consequently, in general,

the equilibrium takes a partition form with three subintervals of insurer types. Insurers with sufficiently low risk in the interval $[0, \dot{p}]$ choose full reinsurance, intermediate-risk insurers with types in interval $[\dot{p}, \ddot{p}]$ choose full retention, and high-risk insurers with types in the interval $[\ddot{p}, 1]$ choose full securitization. The thresholds that determine the various subintervals are the "indifference" points. Depending on the relative magnitudes of the bankruptcy cost C, the reinsurance markup δ and the loss payment B, however, one of the subintervals may be empty so that the partition equilibrium is characterized by two subintervals.

Part 1 of the above proposition shows that reinsurance is dominated by retention if the fixed bankruptcy cost is lower than the threshold $\tilde{B}\delta$ so that all insurer types choose between retention and securitization. The lower risk insurers choose retention by avoiding the subsidization cost due to information asymmetry in capital markets, while higher risk insurers choose securitization due to the relatively lower cost of risk sharing. When the fixed bankruptcy cost is between the thresholds $\tilde{B}\delta$ and $\frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$, the risk transfer choices of intermediate insurer types reflect the tradeoff between the additional costs stemming from reinsurance markup and the fixed bankruptcy cost. Consequently, as described by part 2 of the proposition, the equilibrium has a partition form with three subintervals. When the fixed bankruptcy cost is high enough, retention is dominated by either full reinsurance or full securitization. Consequently, the equilibrium has a partition form with only two subintervals as described by part 3 of the proposition.

From (1.5) and the implicit function theorem, we get

$$\frac{d\ddot{p}}{dB} = \frac{R(\ddot{p})}{C - \tilde{B}R'(\ddot{p})} \tag{1.8}$$

The numerator of (1.8) is positive. Because $\ddot{p}$ is the unique solution of (1.5), we can show that the denominator of the R.H.S. of (1.8) is positive. Thus $d\ddot{p}/dB > 0$. In other words, $\ddot{p}$ is an increasing function of B. When the bankruptcy cost $C \leq \frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$, it follows from parts 1 and 2 of the proposition that $\ddot{p}$ is the threshold risk level above which insurers choose securitization. If $C > \frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$, it follows from condition (1.7) that the trigger level above which insurers choose securitization does not depend on the loss payment B. Taken together,

the above discussion shows that an increase in the magnitude of the insurer's aggregate losses weakly *increases* the threshold risk level above which insurers choose securitization so that the securitization subinterval *shrinks*.

Corollary 1 (Effects of Loss Size) *An increase in the size of the net loss payments B reduces the size of the subinterval of insurer risk types that choose securitization.*

The intuition for the effect of the loss size is that an increase in the loss size increases the marginal cost borne by an insurer of subsidizing higher risk types through securitization. Consequently, as the loss size increases, the marginal insurer who is indifferent between retention and securitization has higher risk.

Catastrophe risks are characterized by low probabilities and large magnitudes of potential losses. The fact that an increase in the magnitude of potential losses increases the trigger risk level above which securitization is chosen suggests that catastrophe risks are more likely to be retained by insurers or reinsured rather than securitized. Further, because only high-risk insurers choose securitization, they pay high premia in securities markets (relative to the ex ante expected loss determined by the average probability $\int_0^1 p dF(p)$), which could also explain why catastrophe-linked securities are usually expensive and credit ratings of many catastrophe bonds are below investment grade. As discussed at the end of Section 1.1, our results are also directly applicable to the more general scenario in which insurers hold portfolios of risks. In this context, our results explain why catastrophe bonds are typically issued to provide high layers of protection, that is, they cover high risks (Cummins (2008)).

We can also investigate the effects of changes in the distribution of insurer losses on risk transfer equilibria. A "first order stochastic dominance" shift in the distribution of insurer types $F(.)$ pushes up the subsidization cost function $R(.)$, thereby causing the securitization subinterval in the PBE to shrink. We, therefore, have the following corollary.

Corollary 2 (Effects of FOSD Shift in Loss Distribution) *A "first order stochastic dominance"shift in the distribution of insurer types $F(.)$ increases the threshold risk level above which insurers choose securitization.*

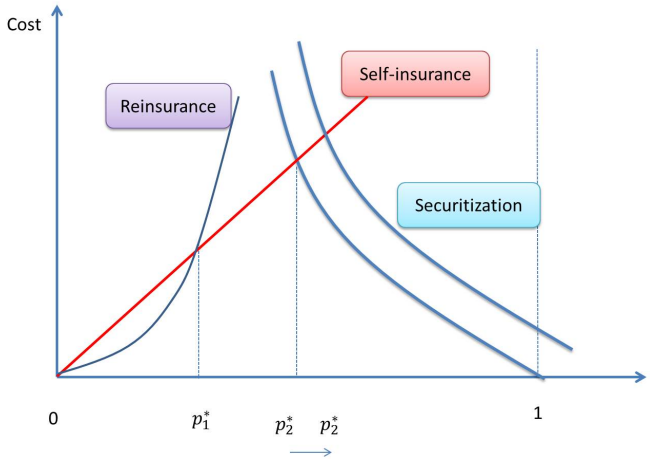

Figure 1.3: Loss payment shift or FOSD shift of the insurer types

Figure 1.3 illustrates the effects of an increase in the amount of net loss payment and a "first order stochastic dominance" shift in the distribution of insurer types $F(p)$. As discussed above, both shift up the expected cost of securitization since the cross-subsidization on securitization market is more severe. Consequently, the upper threshold level of risk that determines the level at which insurers choose securitization increases since the relatively lower risk insurers find retention or reinsurance less costly relative to securitization.

Although we focus on insurance risks for concreteness, our framework is also applicable to the transfer of other types of risk such as credit risk. In the context of credit risk transfer, our results suggest that only high credit risks are optimally transferred through securitization. Our analysis, therefore, offers a potential explanation for why securities such as credit default swaps were actually very risky and triggered huge losses for providers of default protection. Consistent with our prediction that only high credit risks are securitized, Drucker and Puri (2009) examine the secondary market for loan sales and find that sold loans are riskier that average.

1.3 Variable Bankruptcy Costs

We now modify the model to allow for variable bankruptcy costs that are proportional to the insurer's shortfall in the bad state. More precisely, if the insurer chooses to transfer some or all of its risk through reinsurance or securitization, and receives a payment $\overline{B}$ in the

bad state, then the bankruptcy cost is $c \cdot (\tilde{B} - \overline{B})$, where c is a constant. The *maximum* bankruptcy cost, which occurs when the insurer retains all its risk, is $c \cdot \tilde{B}$. We set $c\tilde{B} = C$ to compare our results in this section with those in the previous sections. All other assumptions in the previous section remain the same. As we alluded to in the previous sections, in the presence of variable bankruptcy costs, separating partial securitization contracts may be the optimal choice for some insurer types in the equilibrium since they benefit from sharing risk with investors in capital markets at the cost of retaining some risk to signal their type.

1.3.1 Reinsurance

We first consider the case where insurers only have access to reinsurance. Because of the presence of the reinsurance markup, it is easy to see that it is either optimal for an insurer to choose full reinsurance or no reinsurance at all. Consequently, the insurer's optimal choice between retention and reinsurance, and the optimal reinsurance contract if it chooses reinsurance, are given by Proposition 1. The risk transfer choice and the reinsurance contract are, therefore, the same as in the model with fixed bankruptcy costs.

1.3.2 Securitization

Suppose now that insurers only have access to capital markets. The proportional bankruptcy cost provides low risk insurers the room to bear some risk by choosing partial securitization. The insurer's choice of risk retention level serves as a signal of its type and, thereby, reduces the adverse selection cost due to information asymmetry. An insurer's optimal choice of securitization coverage reflects the tradeoff between the adverse selection/cross-subsidization cost and the expected bankruptcy cost.

We conjecture that a candidate PBE is characterized by a threshold risk type p such that insurers with risk types below the threshold partially transfer their risk through separating contracts, while insurers with risk types above the threshold fully transfer their risk through pooling contracts. Insurers who partially transfer their risk through separating securitization contracts reveal their types and, therefore, incur no adverse selection costs, but nonzero expected bankruptcy costs arising from partial retention. In contrast, the high risk insurers

who fully transfer their risks through the pooling securitization contract incurs zero expected bankruptcy costs, but nonzero cross-subsidization costs. The equilibrium threshold p^* is determined by three conditions.

First, for insurers with risk types below the threshold, each type chooses an incentive compatible risk retention level. The incentive compatibility condition implies that the loss amount transferred through separating securitization satisfies the following ordinary differential equation (please see the Appendix for the proof)

$$\frac{dB_s^{sep}(p)}{dp} = \frac{B_s^{sep}(p)}{cp(1-p)} \tag{1.9}$$

The general solution to the above ODE is

$$B_s^{sep}(p) = \exp(\lambda)\left(\frac{p}{1-p}\right)^{\frac{1}{c}} \tag{1.10}$$

where the constant λ is determined endogenously along with the equilibrium threshold p^*.

Second, an insurer with the threshold risk, p^*, is indifferent between the pooling and separating securitization contracts. It incurs nonzero expected bankruptcy costs associated with the retention level if it chooses to signal its type, while it bears subsidization costs associated with the full risk transfer if it pools with higher risk insurers. The equilibrium threshold, p^*, should therefore satisfy the following condition:

$$\underbrace{c\left(\tilde{B} - B_s^{sep}(p^*)\right)p^*}_{\text{expected bankruptcy costs from separating contracts}} = \underbrace{\tilde{B}R(p^*)}_{\text{subsidization costs from pooling contracts}}, \tag{1.11}$$

where $R(.)$ is the subsidization ratio function defined in (1.4). Rearranging the above equation and using (1.10), we obtain

$$\exp(\lambda) = \tilde{B}\left(1 - \frac{R(p^*)}{cp^*}\right)\left(\frac{1-p^*}{p^*}\right)^{\frac{1}{c}} \tag{1.12}$$

Third, for p^* to be the equilibrium threshold, it should be sub-optimal for the insurers in the two subintervals to deviate from their securitization choices. For insurers with risk

types below p^*, the marginal subsidization costs must exceed the marginal bankruptcy costs, thereby motivating the insurers to signal their types by retaining some risk. On the other hand, for insurers with risk types above p^*, the expected bankruptcy costs must exceed the cross-subsidization costs. As we show in the Appendix, the equilibrium trigger, p^*, satisfies the following condition:

$$c - \left(c + \frac{1}{1-p^*}\right)\left(1 - \frac{R(p^*)}{cp^*}\right) + \frac{1}{1 - \int_{p^*}^{1} t d\mu(t)} \geq 0 \tag{1.13}$$

In general, there is a continuum of threshold levels, p^*, satisfying the above inequality. For each such p^*, there exists a corresponding λ satisfying (1.12) so that each p^* determines a PBE of the securitization game. More formally, we define the set $\mathcal{P}$ satisfying (1.13), that is,

$$\mathcal{P} = \{p^* : c - \left(c + \frac{1}{1-p^*}\right)\left(1 - \frac{R(p^*)}{cp^*}\right) + \frac{1}{1 - \int_{p^*}^{1} t d\mu(t)} \geq 0\}. \tag{1.14}$$

The following proposition characterizes the multiple PBEs of the securitization game.

Proposition 4 (Securitization Contracts) *Define the set $\mathcal{P}$ as in (1.14). For any $p^* \in \mathcal{P}$, the optimal securitization contract, $(A_s^*(p), B_s^*(p))$,is characterized as follows.*

- *For an insurer of type $p < p^*$*

$$B_s^*(p) = \tilde{B}\left(1 - \frac{R(p^*)}{cp^*}\right)\left(\frac{1-p^*}{p^*}\right)^{\frac{1}{c}}\left(\frac{p}{1-p}\right)^{\frac{1}{c}},$$
$$A_s^*(p) = \tilde{B}\left(1 - \frac{R(p^*)}{cp^*}\right)\left(\frac{1-p^*}{p^*}\right)^{\frac{1}{c}}\left(\frac{p}{1-p}\right)^{\frac{1}{c}+1}$$

- *For an insurer of type $p > p^*$*

$$B_s^*(p) = B_s^* = \tilde{B}, \qquad A_s^*(p) = A_s^* = \frac{\tilde{B}\int_{p^*}^{1} t d\mu(t)}{1 - \int_{p^*}^{1} t d\mu(t)}$$

where

$$\mu(p) = \frac{F(p) - F(p^*)}{1 - F(p^*)}$$

Among the set of PBEs described in the proposition, the most efficient one minimizes the expected deadweight bankruptcy costs incurred by insurers. Consequently, the most efficient PBE is the one defined by the threshold $\mathbf{p}$ where

$$\mathbf{p} = \arg\min_{p \in \mathcal{P}} \int_0^p c(\tilde{B} - B_s^{sep}(t,p)) t f(t) dt$$

1.3.3 Risk Transfer Equilibria

We now consider the scenario where insurers have access to both reinsurance and securitization. In this general scenario, there exist a variety of candidates for PBEs. The reinsurance markup plays a key role in determining the properties of the PBEs. Intuitively, when the reinsurance markup is below a low threshold, reinsurance dominates (partial or full) securitization for low and intermediate risk insurers because the costs due to the reinsurance markup for such insurers are low relative to the expected bankruptcy costs from partial securitization or the cross-subsidization from full pooling securitization. High risk insurers choose full pooling securitization. If the reinsurance markup is in an intermediate region, partial securitization becomes attractive to intermediate risk insurers, while low risk insurers choose reinsurance and high risk insurers choose full pooling securitization. If the reinsurance markup exceeds a high threshold, partial securitization dominates reinsurance even for low risk insurers.

To formalize the above intuition, we begin by noting that the expected cost of an insurer with risk type p if it chooses full reinsurance is $\frac{\tilde{B}\delta p}{1-p(1+\delta)}$. The expected cost from choosing a separating partial securitization contract with retention level $\tilde{B} - B_s^{sep}(p)$ is $pc\left(\tilde{B} - B_s^{sep}(p)\right)$. By the arguments used to derive (1.10), incentive compatibility of the securitization contracts implies that

$$B_s^{sep}(p) = \exp(\lambda)\left(\frac{p}{1-p}\right)^{\frac{1}{c}}. \tag{1.15}$$

In the above, the constant λ is endogenously determined along with the trigger p_1 representing the point of indifference between full reinsurance and partial securitization, and the trigger p_2 representing the point of indifference between partial securitization and full

securitization. The trigger, p_1, must therefore satisfy

$$\begin{aligned}\frac{\tilde{B}\delta p_1}{1-p_1(1+\delta)} &= c(\tilde{B} - B_s^{sep}(p_1))p_1 \\ &= c(\tilde{B} - \exp(\lambda)\left(\frac{p_1}{1-p_1}\right)^{\frac{1}{c}})p_1\end{aligned}$$

Rearranging the above equation, we have

$$\exp(\lambda) = \tilde{B}\left(1 - \frac{\delta}{(1-p_1(1+\delta))\,c}\right)\left(\frac{1-p_1}{p_1}\right)^{\frac{1}{c}} \tag{1.16}$$

For any p_1 satisfying $p_1 < \frac{c-\delta}{c(1+\delta)}$, a corresponding λ exists satisfying the above equation so that any such p_1 is a candidate indifference point between reinsurance and partial securitization. Accordingly, we define the set $\mathcal{U}$ as

$$\mathcal{U} = \{p_1 : p_1 < \frac{c-\delta}{c(1+\delta)}\} \tag{1.17}$$

Given any $p_1 \in \mathcal{U}$, the threshold, p_2, which represents the point of indifference between partial and full securitization, must satisfy

$$c(\tilde{B} - B_s^{sep}(p_2))p_2 = \tilde{B}R(p_2). \tag{1.18}$$

By (1.16),

$$B_s^{sep}(p_2) = \tilde{B}\left(1 - \frac{\delta}{(1-p_1(1+\delta))\,c}\right)\left(\frac{1-p_1}{p_1}\right)^{\frac{1}{c}}\left(\frac{p_2}{1-p_2}\right)^{\frac{1}{c}}.$$

Accordingly, we define the set $\mathcal{L}$ that consists of the possible equilibrium indifference thresholds, p_2, as follows.

$$\mathcal{L} = \{p_2 : 1 - \left(1 - \frac{\delta}{(1-p_1(1+\delta))\,c}\right)\left(\frac{1-p_1}{p_1}\right)^{\frac{1}{c}}\left(\frac{p_2}{1-p_2}\right)^{\frac{1}{c}} = \frac{R(p_2)}{cp_2}; \forall p_1 \in \mathcal{U}\} \tag{1.19}$$

If $p_1 < p_2$, there is a nontrivial intermediate interval of insurer types who choose partial

securitization. If $p_1 > p_2$, however, insurers who choose securitization choose full pooling securitization. Accordingly, the set of candidate equilibrium triggers, p_1 and p_2, can be divided into two subsets.

For p_2 to be an equilibrium threshold, however, it must be sub-optimal for insurers choosing partial or full securitization to deviate from their respective choices. As we show in the Appendix, p_2 must satisfy the following inequality for any given $p_1 \in \mathcal{U}$

$$c - \left(c + \frac{1}{1-p_2}\right)\left(1 - \frac{\delta}{c(1-p_1(1+\delta))}\right)\left(\frac{(1-p_1)p_2}{p_1(1-p_2)}\right)^{\frac{1}{c}} + \frac{1}{1-\int_{p_2}^{1} t d\mu(t)} \geq 0. \qquad (1.20)$$

Accordingly, we first define the set $\mathcal{G}$ as

$$\mathcal{G} = \{p_1, p_2 : p_1 < p_2,$$
$$c - \left(c + \frac{1}{1-p_2}\right)\left(1 - \frac{\delta}{c(1-p_1(1+\delta))}\right)\left(\frac{(1-p_1)p_2}{p_1(1-p_2)}\right)^{\frac{1}{c}} + \frac{1}{1-\int_{p_2}^{1} t d\mu(t)} \geq 0,$$
$$\forall p_1 \in \mathcal{U}, p_2 \in \mathcal{L}\}.$$

Next, we define the set $\mathcal{F}$ as

$$\mathcal{F} = \{p_1, p_2 : p_2 < p_1, \forall p_1 \in \mathcal{U}, p_2 \in \mathcal{L}\}$$

We now have the requisite definitions in place to characterize the risk transfer equilibria.

Proposition 5 (Partition Equilibrium)

1. *Suppose $\delta < c$. Risk transfer equilibria are characterized as follows.*

   a. *For all pairs of p_1^*, p_2^* such that $\{p_1^*, p_2^*\} \in \mathcal{G}$, insurers with types in the interval $[0, p_1^*]$ choose full reinsurance, insurers with types in the interval $[p_1^*, p_2^*]$ choose separating partial securitization, and insurers with types in the interval $[p_2^*, 1]$ choose pooling full securitization.*

   b. *For all pairs of p_1^*, p_2^* such that $\{p_1^*, p_2^*\} \in \mathcal{F}$, there exists $p_3^* \in [0,1]$ with $0 < p_2^* \leq p_3^* \leq p_1^* < 1$ such that insurers with types in the interval $[0, p_3^*]$ choose*

full reinsurance and insurers with types in the interval $[p_3^*, 1]$ *choose pooling full securitization if condition* $R'(p) < \frac{\delta}{(1-p(1+\delta))^2}$ *holds, where* $\frac{\delta p_3^*}{1-p_3^*(1+\delta)} = R(p_3^*)$.

2. Suppose $\delta > c$. *Equilibria are characterized by two subintervals as in Proposition 4.*

The above proposition suggests that full reinsurance dominates partial risk sharing for low risk insurers if the reinsurance markup is lower than the bankruptcy coefficient c. Intermediate risk insurers choose partial securitization provided the proportional bankruptcy cost is below a threshold. If the bankruptcy cost exceeds the threshold, however, partial securitization is sub-optimal for all insurers, that is, high risk insurers chooses full securitization, while low risk insurers choose full reinsurance. When the reinsurance markup is above the bankruptcy cost coefficient, however, insurers choose partial or full securitization.

1.4 Discussion and Conclusions

When an insurer has private information about its portfolio of risks, its risk transfer choice serves as a signal of the quality of risks in its portfolio. The insurer's choice reflects the tradeoff between the lower adverse selection costs associated with reinsurance against the additional costs of reinsurance stemming from a number of sources such as reinsurers' market power relative to that of competitive capital market investors, compensation for reinsurers' costly monitoring, and the higher cost of capital of reinsurers relative to capital markets. PBE of the signaling game have a partition form where the lowest risk insurers choose reinsurance, intermediate risk insurers choose partial securitization, and highest risk insurers choose full securitization. An increase in the magnitude of potential losses in the portfolio increases the threshold level of risk above which insurers choose securitization. Consequently, catastrophe risk, which is characterized by "low frequency–high severity" losses is only securitized by very high risk insurers. Further, because only the highest risk insurers choose securitization, they pay high premia in securities markets, which could explain why catastrophe-linked securities are usually expensive, and why catastrophe securities often receive "below investment grade" ratings. Our results, therefore, provide an novel alternate explanation for the

relative predominance of reinsurance in the market for catastrophe risk transfer and the high cost of catastrophe bonds.

The prediction that only risks above a threshold are securitized is also consistent with the observed spike in securitization transactions following major catastrophes such as Hurricane Katrina following which actuaries' assessments of future catastrophic events were revised upward. Our story also suggests that, as more sophisticated investors such as hedge funds enter the market for catastrophe-linked securities, the adverse selection costs associated with securitization would be expected to decline, thereby encouraging securitization transactions. An increase in the degree of competitiveness of reinsurance markets would also provide a fillip to securitization by lowering reinsurers' market power relative to capital markets.

Our framework can be used to analyze the transfer of all types of risks, and not just insurance risks. If our model were adapted to analyze credit risk transfer in the context of the recent financial crisis, our results suggest that only high credit risks are optimally transferred through securitization, thereby suggesting that instruments such as credit default swaps were, indeed, very risky as was borne out by the large losses suffered by providers of default protection. Indeed, consistent with this prediction, Drucker and Puri (2009) examine the secondary market for loan sales and find that sold loans are riskier that average. Our model could also be potentially adapted to the study of firms' choices between alternate modes of financing such as private versus public financing, and "informed " versus "arms length "financing (e.g., see Rajan (1992) , Chemmanur and Fulghieri (1994), Winton and Yerramilli (2008). We leave the analysis of these extensions to future research.

Appendix: Proofs

Proof of Proposition 1

Proof. Suppose $\delta < \frac{C}{\tilde{B}}$.

Given the presence of fixed bankruptcy costs, it is easy to see that it is sub-optimal for an insurer to choose partial reinsurance, that is, if an insurer chooses reinsurance, it chooses full reinsurance. Consequently, the second constraint in (1.1) must be binding for an insurer of type p that chooses reinsurance. Hence, the net reinsurance payment $B_r^*(p)$ for it in the bad state is $B_r^*(p) = B - W = \tilde{B}$. The insurer's maximization problem is equivalent to minimizing $A_r(p)(1-p) - B_r p$ which implies that the first constraint in (1.1) is also binding. Hence, the premium is given by $A_r^*(p) = \frac{\tilde{B}p(1+\delta)}{1-p(1+\delta)}$.

The expected payoff of reinsurance for the insurer with type p is $EU_r(p) = W + \big(A(1-p) - Bp\big) - \frac{\tilde{B}\delta p}{1-p(1+\delta)}$. The expected payoff of full self-insurance for the insurer with type p is $EU_{self}(p) = W + \big(A(1-p) - Bp\big) - Cp$. Thus, $EU_r(p) > EU_{self}(p)$ for all $p < \frac{C-\tilde{B}\delta}{C(1+\delta)} = \dot{p}$, where $\dot{p}$ is defined in (1.2). Accordingly, reinsurance is sub-optimal for insurers with types $p > \dot{p}$, but optimal for insurers with types $p < \dot{p}$. ■

Proof of Proposition 2

Proof. Consider first a candidate fully separating equilibrium $(A_s^*(p), B_s^*(p))$, where $(A_s^*(p), B_s^*(p))$ is the securitization contract offered by the insurer with type p. The capital market investors break even, thereby leading the investors' participation condition to be binding. Hence, the premium is $A_s^*(p) = \frac{pB_s^*(p)}{1-p}$. However, $(A_s^*(p), B_s^*(p))$ is not incentive compatible because the higher risk insurers are strictly better off by deviating and offering the lower risk insurers' contract. Consequently, we cannot have a fully separating equilibrium. Hence, any equilibrium must necessarily involve some pooling.

Next, we observe that there cannot be an equilibrium in which there exists a quadruple, $\{p_1, p_2, p_3, p_4\}$ with $p_1 \leq p_2 < p_3 \leq p_4$ such that insurers with types in $[p_1, p_2]$ pool together and choose a single full securitization contract, and insurers with types in $[p_3, p_4]$ pool together and choose a single full securitization contract, but the two intervals of insurers choose different contracts. This assertion follows easily from the observation that insurers with types in $[p_3, p_4]$ would prefer the contract offered by the insurers with types in $[p_1, p_2]$.

It follows from the above arguments that it suffices to consider candidate equilibria in which insurers with types below a threshold choose self-insurance, while insurers with types above the threshold choose full pooling securitization. Accordingly, consider a candidate equilibrium defined by a trigger level p. We now examine the conditions for p to be an *equilibrium* threshold. An insurers with type $k \geq p$ chooses full pooling securitization, $B_s^*(k) = B^* = \tilde{B}$. The break-even condition of investors requires that the premium be given

by

$$A_s^*(k) = A^* = \tilde{B}\frac{\int_p^1 t d\mu(t)}{1 - \int_p^1 t d\mu(t)},$$

where $\mu(p)$ is the investors' posterior beliefs about insurer's types. Therefore, the insurer's expected payoff from securitization is

$$EU_s^{pooling}(k) = W + \big(A(1-k) - Bk\big) - \tilde{B}\frac{\int_p^1 t d\mu(t) - k}{1 - \int_p^1 t d\mu(t)}. \tag{1.21}$$

The insurer, whose type k is less than or equal to p, chooses full retention. Its expected payoff is, therefore,

$$EU_{self}(k) = W + \big(A(1-k) - Bk\big) - Ck.$$

It is easy to see that, if $p = \ddot{p}$ satisfying (1.5), then $EU_s^{pooling}(p) = EU_{self}(p)$. Hence, the insurer with risk type $\ddot{p}$ is indifferent between pooling with higher types and self-insurance.

Next, we check that $\ddot{p}$ is, indeed, the equilibrium threshold. First, we establish incentive compatibility of the set of contracts defined by $\ddot{p}$. If an insurer with type $k < \ddot{p}$ deviates to choose the pooling contract (A^*, B^*), its expected payoff is

$$EU_s^{deviate}(k) = W + \big(A(1-k) - Bk\big) - \tilde{B}\frac{\int_{\ddot{p}}^1 t d\mu(t) - k}{1 - \int_{\ddot{p}}^1 t d\mu(t)}.$$

It is easy to show that, if $k < \ddot{p}$, then

$$\tilde{B}\frac{\int_{\ddot{p}}^1 t d\mu(t) - k}{1 - \int_{\ddot{p}}^1 t d\mu(t)} > \tilde{B}\frac{\int_{\ddot{p}}^1 t d\mu(t) - \ddot{p}}{1 - \int_{\ddot{p}}^1 t d\mu(t)} = \tilde{B}R(\ddot{p}) = C\ddot{p} > Ck.$$

Thus, $EU_{self}(k) > EU_s^{deviate}(k)$. As a result, the insurer with type $k < \ddot{p}$ will not choose the pooling contract (A^*, B^*). If an insurer with type $k > \ddot{p}$ deviates to choose full self-insurance, the expected payoff is

$$EU_{self}^{deviate}(k) = W + \big(A(1-k) - Bk\big) - Ck.$$

It is easy to see that, if $k > \ddot{p}$, then

$$\tilde{B}\frac{\int_{\ddot{p}}^1 t d\mu(t) - k}{1 - \int_{\ddot{p}}^1 t d\mu(t)} < \tilde{B}\frac{\int_{\ddot{p}}^1 t d\mu(t) - \ddot{p}}{1 - \int_{\ddot{p}}^1 t d\mu(t)} = \tilde{B}R(\ddot{p}) = C\ddot{p} < Ck.$$

Thus, $EU_s^{pooling}(k) > EU_{self}^{deviate}(k)$. Consequently, the insurer whose type is greater than $\ddot{p}$

will not choose retention.

Now suppose that an insurer with type $k > \ddot{p}$ finds it profitable to deviate to some other securitization contract (A'_s, B'_s). Suppose first that the contract involves a full transfer of risk. The deviation is profitable for the insurer iff $A'_s < A^*$. In this case, however, the deviation is also profitable for insurers with higher risk types. Consequently, reasonable off-equilibrium beliefs of investors must necessarily pool insurers with types greater than or equal to k, which makes the hypothesized deviation unprofitable for insurer k. Alternately, applying the D1 refinement, the sets of investor beliefs under which a deviation to the full risk transfer contract (A'_s, B'_s) is profitable increases with the insurer risk type. Iteratively applying the D1 refinement, therefore, implies that, on observing such a deviation, investors' beliefs assign probability one that the insurer has the highest risk type, which makes it unprofitable for all lower risk insurers to deviate.

Suppose that the deviating contract (A'_s, B'_s) does not involve a full transfer of risk so that $B'_s < B^*$ and the insurer bears the additional bankruptcy cost C in the bad state. Because the insurer's expected cost under the pooling contract given by (1.21) is decreasing and linear in its type k, in this case too, the sets of investor beliefs under which the deviation is profitable are increasing in the insurer type. Iteratively applying the D1 refinement, investors' beliefs assign probability one that the insurer has the highest risk type on observing such a deviation, thereby making it unprofitable for lower risk types.

Similarly, suppose that an insurer with type $k < \ddot{p}$ finds it profitable to deviate to a securitization contract $(A^{"}_s, B^{"}_s)$. If the contract involves a full transfer of risk, it must also be profitable for insurers with types in $[k, \ddot{p}]$. Consequently, reasonable off-equilibrium beliefs must pool together such insurers, which makes the hypothesized deviation unprofitable. Alternately, iteratively applying the D1 refinement, off-equilibrium beliefs following such a deviation assign probability one that the insurer is of type $\ddot{p}$, thereby making the deviation unprofitable for all lower risk insurers. If the contract does not involve a full transfer of risk, then the insurer necessarily bears the bankruptcy cost C in the bad state. In this case too, if such a deviation is profitable for the insurer, it must also be profitable for insurers with types in $[k, \ddot{p}]$. We can again argue as above that reasonable off-equilibrium beliefs following such a deviation make it unprofitable for the insurer.

Hence, the threshold $\ddot{p}$ satisfying (1.5) defines an equilibrium. Moreover, if (1.5) has a unique solution, then it determines the unique PBE of the risk transfer game. ■

Proof of Proposition 3.

Proof.

1. If $C < \tilde{B}\delta$, it is sub-optimal for an insurer to choose reinsurance. We are, thus, in the scenario described in Proposition 2.

2. Suppose $\tilde{B}\delta < C < \frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$.

It follows from Proposition 1 that insurers with types in the interval $[0, \dot{p}]$ prefer full reinsurance to full self-insurance. By Proposition 2, insurers with types in the interval $[\ddot{p}, 1]$ prefer full pooling securitization to full self-insurance. By condition (1.6), there is a unique $\ddot{p}$ satisfying (1.5).

Since $C < \frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$, $\frac{\tilde{B}\delta\dot{p}}{1-\ddot{p}(1+\delta)} > C\dot{p} = \frac{\tilde{B}\delta\dot{p}}{1-\dot{p}-\delta}$. Thus, $\dot{p} < \ddot{p}$.

It follows from the results of Propositions 1 and 2 that $\dot{p}$ and $\ddot{p}$ are two indifference points. Now check whether they are, indeed, the equilibrium thresholds. If an insurer with type in the interval $[0, \dot{p}]$, deviates to choose the pooling securitization contract given by Proposition 2, the expected payoff is

$$EU_s^{deviate}(p) = W + (A(1-p) - Bp) - \tilde{B}\frac{\int_{\ddot{p}}^1 td\mu(t) - p}{1 - \int_{\ddot{p}}^1 td\mu(t)}.$$

It is easy to see that, since

$$\tilde{B}\frac{\int_{\ddot{p}}^1 td\mu(t) - p}{1 - \int_{\ddot{p}}^1 td\mu(t)} \geq \tilde{B}\frac{\int_{\ddot{p}}^1 td\mu(t) - \ddot{p}}{1 - \int_{\ddot{p}}^1 td\mu(t)} = \tilde{B}R(\ddot{p}) = C\ddot{p} > C\dot{p} > Cp,$$

it will not deviate to choose full pooling securitization. Consequently, the insurers with types in the interval $[0, \dot{p}]$ will not deviate to choose full pooling securitization. Under restrictions on reasonable off-equilibrium beliefs along the lines of the D1 refinement as in the proof of Proposition 2, an insurer with type in the interval $[0, \dot{p}]$ will also not deviate to choose any other securitization contract.

For an insurer with type in the interval $[\dot{p}, \ddot{p}]$, Proposition 1 implies that it will not choose full reinsurance. Proposition 2 implies that it will not choose full pooling securitization or any other securitization contract. As a result, it is optimal for it to choose full self-insurance.

For an insurer with type in the interval $[\ddot{p}, 1]$, Proposition 2 shows that it will not choose full self-insurance. If it deviates to choose full reinsurance, it pays the additional rents due to reinsurance markup arising from a variety of sources. Thus, the expected payoff is $EU_r^{deviate}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1-p(1+\delta)}$. It is easy to show that

$$\frac{\tilde{B}\delta p}{1 - p(1+\delta)} > Cp$$

since the function $C - \frac{\tilde{B}\delta}{1-p(1+\delta)}$ decreases with p and equals zero at $\dot{p}$. Also,

$$Cp > C\ddot{p} = \tilde{B}\frac{\int_{\ddot{p}}^1 td\mu(t) - \ddot{p}}{1 - \int_{\ddot{p}}^1 td\mu(t)} > \tilde{B}\frac{\int_{\ddot{p}}^1 td\mu(t) - p}{1 - \int_{\ddot{p}}^1 td\mu(t)}.$$

As a result, $EU_r^{deviate}(p) < EU_s^{pooling}(p)$ if $p > \ddot{p}$. By arguments similar to those used in the proof of Proposition 2, which plays restrictions on reasonable off-equilibrium beliefs, it is also sub-optimal for an insurer with type in the interval $[\ddot{p}, 1]$ to deviate to any other securitization contract. Consequently, it is optimal for insurers with types greater than $\ddot{p}$ to choose pooling securitization. Further, the conjectured PBE is the unique equilibrium since the values of $\dot{p}$ and $\ddot{p}$ are unique under condition (1.6) and $\tilde{B}\delta < C < \frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$.

3. Suppose $C > \frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$, that is $\ddot{p} < \dot{p}$. It follows that it is sub-optimal for an insurer to choose self-insurance, thereby leading the equilibria to have a partition form with two subintervals.

First solve for the point of indifference between choosing full reinsurance and pooling with higher risk insurers through securitization. The optimal reinsurance contracts are given by Proposition 1, and the corresponding expected payoff is $EU_r(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1-p(1+\delta)}$. The optimal pooling securitization coverage is $B_s^* = \tilde{B}$. The indifference point, p_3, between securitization and reinsurance must solve

$$\frac{\tilde{B}p_3\delta}{1-p_3(1+\delta)} = \tilde{B}R(p_3) \tag{1.22}$$

Condition $R'(p) < \frac{\delta}{(1-p(1+\delta))^2}$ ensures that there is a unique solution p_3^* to (1.22)

Next, we check whether the unique solution p_3^* is the equilibrium threshold. For insurers with types in the interval $[0, p_3^*]$, the expected payoff of full reinsurance is

$$EU_r(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1-p(1+\delta)},$$

while the expected payoff of full pooling securitization is

$$EU_s^{deviate}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p)}{1-\int_{p_3^*}^1 td\mu(t)}.$$

For any $p \in [0, p_3^*]$,

$$\frac{\tilde{B}p\delta}{1-p(1+\delta)} < \frac{\tilde{B}p_3^*\delta}{1-p_3^*(1+\delta)} = \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p_3^*)}{1-\int_{p_3^*}^1 td\mu(t)} < \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p)}{1-\int_{p_3^*}^1 td\mu(t)}$$

Then, $EU_r(p) > EU_s^{deviate}(p)$. The insurer types in the interval $[0, p_3^*]$, therefore, will not deviate to choose full securitization. By arguments similar to those used in the earlier proofs, an insurer with type in the interval $[0, p_3^*]$ will also not deviate to choose any other securitization contract.

Similarly, the expected payoff of insurers with types in the interval $[p_3^*, 1]$ from choosing securitization is

$$EU_s(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p)}{1 - \int_{p_3^*}^1 td\mu(t)},$$

while the expected payoff of choosing full reinsurance is

$$EU_r^{deviate}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}p\delta}{1 - p(1+\delta)}.$$

For any $p \in [p_3^*, 1]$, we have

$$\frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p)}{1 - \int_{p_3^*}^1 td\mu(t)} < \frac{\tilde{B}(\int_{p_3^*}^1 td\mu(t) - p_3^*)}{1 - \int_{p_3^*}^1 td\mu(t)} = \frac{\tilde{B}p_3^*\delta}{1 - p_3^*(1+\delta)} < \frac{\tilde{B}p\delta}{1 - p(1+\delta)}$$

Thus, $EU_s(p) > EU_r^{deviate}(p)$. Therefore,insurers with types in the interval $[p_3^*, 1]$ will not deviate to choose reinsurance. By arguments similar to those used in earlier proofs, they will also not deviate to choose any other securitization contract.

Consequently, the conjectured equilibrium is, indeed, the unique PBE of the signaling game if condition (1.22) and $C > \frac{\tilde{B}\delta}{1-\ddot{p}(1+\delta)}$ hold. ■

Proof of Proposition 4

Proof. We first show that PBEs cannot be fully pooling or fully separating.

Consider first a candidate pooling equilibrium where all insurers offer the same contract $(A_s^*(p), B_s^*(p))$ given by Proposition 2. Because bankruptcy costs now depend on an insurer's shortfall in meeting its liabilities, the lower risk insurers have incentives to retain some risk to signal their types, thereby reducing the subsidization costs from pooling securitization contracts.

Now consider a candidate fully separating equilibrium where each insurer type chooses corresponding securitization contracts at fair price since its risk type is perfectly revealed in the capital markets. Thus, the optimal risk retention level $\tilde{B} - B_s^{sep}(p)$ (or the optimal risk coverage $B_s^{sep}(p)$) solves

$$\max_{\tilde{p}} W + (A(1-p) - Bp) - (A_s(\tilde{p})(1-p) - B_s^{sep}(\tilde{p})p) - c\left(\tilde{B} - B_s^{sep}(\tilde{p})\right)p$$

such that

$$A_s(\tilde{p})(1-\tilde{p}) - B_s^{sep}(\tilde{p})\tilde{p} \geq 0 \tag{1.23}$$

The break-even condition (1.23) for capital markets is binding. The above is, therefore,

equivalent to

$$\min_{\tilde{p}} \frac{B_s^{sep}(\tilde{p}-p)}{1-\tilde{p}} + c\left(\tilde{B} - B_s^{sep}(p)\right)p$$

The first order condition is

$$\frac{\left(B_s^{sep\prime}(\tilde{p})(\tilde{p}-p) + B_s^{sep}(\tilde{p})\right)(1-\tilde{p}) + B_s^{sep}(\tilde{p})(\tilde{p}-p)}{(1-\tilde{p})^2} - cB_s^{sep\prime}(\tilde{p})p = 0.$$

Setting $\tilde{p} = p$, we obtain

$$B_s^{sep}(p) = \frac{dB_s^{sep}(p)}{dp} cp(1-p) \tag{1.24}$$

The general solution of the above ordinary differential equation is given by (1.9), that is

$$B_s^{sep}(p) = \exp(\lambda)\left(\frac{p}{1-p}\right)^{\frac{1}{c}},$$

where λ is the constant of integration. It is easy to show that, for any λ, there is a $\tilde{p}$ where $0 < \tilde{p} < 1$, such that $B_s^{sep}(\tilde{p}) = \tilde{B}$. It follows that the pure separating equilibrium is also violated since not all insurers are able to signal their types.

Using arguments similar to those used in the proof of Proposition 2, we can show that it suffices to consider candidate *semi pooling* equilibria characterized by a threshold risk type p^* such that insurers with types below it partially transfer their risks through separating contracts, while insurers with risk types above it fully transfer their risks through pooling contracts. Insurers who choose separating contracts reveal their types and, therefore, incur no adverse selection costs, but nonzero expected bankruptcy costs from the partial retention. The insurer of type p^* should be indifferent between a separating and pooling contract.

The expected cost to an insurer of type p from choosing a separating contract that reveals its type is

$$C_s^{sep}(p)p = c\left(\tilde{B} - \exp(\lambda)\left(\frac{p}{1-p}\right)^{\frac{1}{c}}\right)p$$

The expected cost to the insurer with type p from choosing a pooling contract is $\tilde{B}R(p)$, where $R(p)$ is defined by equation (1.4).

Thus, an indifference threshold p^* is determined by

$$c\left(\tilde{B} - \exp(\lambda)\left(\frac{p^*}{1-p^*}\right)^{\frac{1}{c}}\right)p^* = \tilde{B}R(p^*).$$

Any p^* satisfying the above equation is a candidate for the threshold that supports the

conjectured semi pooling PBE. The indifference point p^* also determines the incentive compatible pooling and separating contracts in terms of the value of λ. Rearranging (1.11) and using (1.10), we obtain (1.12), that is

$$\exp(\lambda) = \tilde{B}\left(1 - \frac{R(p^*)}{cp^*}\right)\left(\frac{1-p^*}{p^*}\right)^{\frac{1}{c}}.$$

Clearly, $\forall p^* \in [0,1]$, there exists a corresponding λ such that p^* is the point of indifference between pooling and separating contracts.

For the given indifference point p^*, plugging (1.12) into (1.10), we obtain the corresponding separating contracts for the insurer with type $p < p^*$.

$$B_s^{sep*}(p) = \tilde{B}\left(1 - \frac{R(p^*)}{cp^*}\right)\left(\frac{1-p^*}{p^*}\right)^{\frac{1}{c}}\left(\frac{p}{1-p}\right)^{\frac{1}{c}} \tag{1.25}$$

$$A_s^{sep*}(p) = \tilde{B}\left(1 - \frac{R(p^*)}{cp^*}\right)\left(\frac{1-p^*}{p^*}\right)^{\frac{1}{c}}\left(\frac{p}{1-p}\right)^{\frac{1}{c}+1} \tag{1.26}$$

The break-even condition for capital markets requires the pooling contract premium to be

$$A_s^* = \frac{\tilde{B}\int_{p^*}^1 t d\mu(t)}{1 - \int_{p^*}^1 t d\mu(t)}$$

We now show that $p^* \in [0,1]$ is an *equilibrium* indifference threshold if it satisfies condition (1.13).

For an insurer with type $p \in [0, p^*]$, the expected payoff of choosing partial securitization is

$$EU_s^{sep}(p) = W + (A(1-p) - Bp) - c\left(\tilde{B} - B_s^{sep}(p)\right)p$$

If it deviates to the pooling contract, the expected payoff is

$$EU_s^{deviatepool}(p) = W + (A(1-p) - Bp) - \tilde{B}\frac{\int_{p^*}^1 t d\mu(t) - p}{1 - \int_{p^*}^1 t d\mu(t)}$$

We now show that $EU_s^{deviatepool}(p) \leq EU_s^{sep}(p)$ if condition (1.13) holds. Define

$$G_1(p) = cp\left(\tilde{B} - B_s^{sep}(p)\right) - \tilde{B}\frac{\int_{p^*}^1 t d\mu(t) - p}{1 - \int_{p^*}^1 t d\mu(t)}$$

where $B_s^{sep}(p)$ is given by (1.25). We have

$$\begin{aligned} G_1'(p) &= c\left(\tilde{B} - B_s^{sep}(p)\right) - cpB_s^{sep\prime}(p) + \frac{\tilde{B}}{1 - \int_{p^*}^1 td\mu(t)} \\ &= c\left(\tilde{B} - B_s^{sep}(p)\right) - \frac{B_s^{sep}(p)}{1-p} + \frac{\tilde{B}}{1 - \int_{p^*}^1 td\mu(t)} \end{aligned}$$

$$G_1''(p) = -cB_s^{sep\prime}(p) - \frac{B_s^{sep\prime}(p)(1-p) + B_s^{sep}(p)}{(1-p)^2} \leq 0.$$

Thus, $G_1(p)$ is a concave function of p. So we have

$$\frac{\partial G_1(p)}{\partial p}|_{p<p^*} \geq \frac{\partial G_1(p)}{\partial p}|_{p=p^*}$$

Next, note that

$$\begin{aligned} \frac{\partial G_1(p)}{\partial p}|_{p=p^*} &= c\left(\tilde{B} - \tilde{B}\left(1 - \frac{R(p^*)}{cp^*}\right)\right) - \frac{\tilde{B}\left(1 - \frac{R(p^*)}{cp^*}\right)}{1-p^*} + \frac{\tilde{B}}{1 - \int_{p^*}^1 td\mu(t)} \\ &= \tilde{B}\left(c - \left(c + \frac{1}{1-p^*}\right)\left(1 - \frac{R(p*)}{cp^*}\right) + \frac{1}{1 - \int_{p^*}^1 td\mu(t)}\right). \end{aligned}$$

Under condition (1.13), $\frac{\partial G_1(p)}{\partial p}|_{p<p^*} \geq 0$, that is $G_1(p)$ is an increasing function of p for $p < p^*$ so that $G_1(p) < G_1(p^*) = 0$. Consequently,

$$cp\left(\tilde{B} - B_s^{sep}(p)\right) < \tilde{B}\frac{\int_{p^*}^1 td\mu(t) - p}{1 - \int_{p^*}^1 td\mu(t)},$$

and $EU_s^{sep}(p) > EU_s^{deviatepool}$. Hence, the insurers with risk types below p^* will not deviate to pooling securitization by (1.13). Because the Spence-Mirrlees single-crossing condition holds (due to the linear objective function of insurers), the "local" incentive compatibility condition (1.24) ensures that an insurer with risk type $p \leq p^*$ will also not deviate to choose the partial securitization contract of some other type $p' \leq p^*$. Finally, as in the proof of Proposition 2, we can show that, under reasonable off-equilibrium beliefs, it is sub-optimal for an insurer with risk type $p \leq p^*$ to deviate to some other arbitrary securitization contract (A_s, B_s) that is not chosen by another risk type $p' \leq p*$. If such a deviation were profitable for the insurer of type $p < p^*$, it would also be profitable for types $p' \in (p, p^*]$. Consequently, on observing such an off-equilibrium deviation, the beliefs of capital market investors would pool

the insurer of type p with the insurers of types $p' \in (p, p^*]$, thereby making the deviation unprofitable. Alternatively, iteratively applying the D1 refinement, investors believe that the deviating insurer is of the risk type p^* with probability one, which makes the deviation unprofitable for all lower risk types.

For insurers with types $p \in [p^*, 1]$, the expected payoff of choosing full pooling securitization is

$$EU_s^{pool}(p) = W + (A(1-p) - Bp) - \tilde{B}\frac{\int_{p^*}^1 td\mu(t) - p}{1 - \int_{p^*}^1 td\mu(t)}$$

The expected payoff of mimicking an arbitrary lower-risk insurer of type $\hat{p} < p^*$ is

$$EU_s^{deviatesep}(p) = W + (A(1-p) - Bp) - \underbrace{c\left(\tilde{B} - B_s^{sep}(\hat{p})\right)p}_{\text{expected bankruptcy costs of mimicking}} \underbrace{-\frac{B_s^{sep}(\hat{p})(\hat{p}-p)}{1-\hat{p}}}_{\text{benefits from mimicking}}$$

Define

$$G_2(p) = c\left(\tilde{B} - B_s^{sep}(\hat{p})\right)p + \frac{B_s^{sep}(\hat{p})(\hat{p}-p)}{1-\hat{p}} - \tilde{B}\frac{\int_{p^*}^1 td\mu(t) - p}{1 - \int_{p^*}^1 td\mu(t)}$$

The first derivative is

$$\begin{aligned} G_2'(p) &= c\left(\tilde{B} - B_s^{sep}(\hat{p})\right) - \frac{B_s^{sep}(\hat{p})}{1-\hat{p}} + \frac{\tilde{B}}{1 - \int_{p^*}^1 td\mu(t)} \\ &= c\left(\tilde{B} - B_s^{sep}(\hat{p})\right) + \left(\frac{\tilde{B}}{1 - \int_{p^*}^1 td\mu(t)} - \frac{B_s^{sep}(\hat{p})}{1-\hat{p}}\right) > 0 \end{aligned}$$

It is obvious that $G_2(p)$ is an increasing function of $p \in [p^*, 1]$. Thus, $G_2(p) \geq G_2(p^*) = 0 \quad \forall p \geq p^*$. That is,

$$c\left(\tilde{B} - B_s^{sep}(\hat{p})\right)p + \frac{B_s^{sep}(\hat{p})(\hat{p}-p)}{1-\hat{p}} > \tilde{B}\frac{\int_{p^*}^1 td\mu(t) - p}{1 - \int_{p^*}^1 td\mu(t)}.$$

Hence, it is easy to show that $EU_s^{pool}(p) > EU_s^{deviate}(p) \qquad \forall p > p^*$. As a result, insurers with risk types greater than p^* will not deviate to choose separating contracts. As earlier, we can iteratively apply the D1 refinement to show that an insurer with risk type $p > p^*$ will also not deviate to choose some other arbitrary securitization contract.

By the above arguments, each candidate threshold $p^* \in \mathcal{P}$ defined in (1.14) defines a semi-pooling PBE. ■

Proof of Proposition 5

Proof.

1. Suppose first that

$$\delta < c \tag{1.27}$$

It then follows from Proposition 1 that the insurer with risk type below a threshold chooses full reinsurance. By Proposition 4, higher risk insurers prefer pooling securitization, while lower risk insurers prefer separating securitization. Therefore, we conjecture that there are two types of PBEs under condition (1.27). The differences between the two types of PBEs lie in the intermediate risk insurers' choice between full reinsurance and partial securitization.

First considers the candidates for a pair of triggers (p_1, p_2), where p_1 is the point of indifference between full reinsurance and partial securitization, and p_2 is the point of indifference between partial securitization and full securitization. The intermediate interval is nonempty iff $p_1 < p_2$. By our earlier arguments, p_1 must satisfy

$$\frac{\tilde{B}\delta p_1}{1 - p_1(1+\delta)} = c(\tilde{B} - \exp(\lambda)\left(\frac{p_1}{1-p_1}\right)^{\frac{1}{c}})p_1 \tag{1.28}$$

where the constant of integration, λ, is determined by p_1 if it is the equilibrium threshold.

Rearranging the above equation, we obtain (1.16), where

$$\exp(\lambda) = \tilde{B}\left(1 - \frac{\delta}{(1 - p_1(1+\delta))\, c}\right)\left(\frac{1-p_1}{p_1}\right)^{\frac{1}{c}} \tag{1.29}$$

The trigger, p_2, must satisfy equation (1.18), that is, $c(\tilde{B} - B_s^{sep}(p_2))p_2 = \tilde{B}R(p_2)$, where it follows from (1.16) that

$$B_s^{sep}(p_2) = \tilde{B}\left(1 - \frac{\delta}{(1 - p_1(1+\delta))\, c}\right)\left(\frac{1-p_1}{p_1}\right)^{\frac{1}{c}}\left(\frac{p_2}{1-p_2}\right)^{\frac{1}{c}}$$

The above two equations lead to the following relationship between p_1 and p_2:

$$1 - \left(1 - \frac{\delta}{(1 - p_1(1+\delta))\, c}\right)\left(\frac{1-p_1}{p_1}\right)^{\frac{1}{c}}\left(\frac{p_2}{1-p_2}\right)^{\frac{1}{c}} = \frac{R(p_2)}{cp_2}$$

We define the set $\mathcal{U}$ by (1.17), which comprises of all possible indifference points p_1^*. We define the set $\mathcal{L}$ by (1.19), which contains all possible indifference points p_2^*.

Suppose that $p_1^* < p_2^*$. Conjecture a partition equilibrium where insurers with types in the range $[0, p_1^*]$ choose full reinsurance, insurers with types in the range $[p_1^*, p_2^*]$ choose

separating partial securitization, and insurers with types in the range $[p_2^*, 1]$ choose pooling full securitization. We now show that the pair of indifference points are, indeed, equilibrium thresholds.

For insurers with types in the range $[0, p_1^*]$, their expected payoff of full reinsurance is

$$EU_r(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1 - p(1+\delta)}.$$

If they deviate to choose partial securitization by choosing the corresponding coverage, where

$$B_s^{sep}(p) = \tilde{B}\left(1 - \frac{\delta}{(1 - p_1^*(1+\delta))c}\right)\left(\frac{1-p_1^*}{p_1^*}\right)^{\frac{1}{c}}\left(\frac{p}{1-p}\right)^{\frac{1}{c}} \tag{1.30}$$

their expected payoff is

$$EU_{deviatesep}(p) = W + (A(1-p) - Bp) - c(\tilde{B} - B_s^{sep}(p))p.$$

If they deviate to choose full pooling securitization, where

$$B_s^* = \tilde{B}; \qquad A_s^* = \frac{\tilde{B}\int_{p_2^*}^1 t d\mu(t)}{1 - \int_{p_2^*}^1 t d\mu(t)}, \tag{1.31}$$

the expected payoff is

$$EU_{deviatepool}(p) = W + (A(1-p) - Bp) - \tilde{B}\frac{\int_{p_2^*}^1 t d\mu(t) - p}{1 - \int_{p_2^*}^1 t d\mu(t)}.$$

Define

$$\Phi(p) = \frac{\tilde{B}\delta p}{1 - p(1+\delta)} - cp\left(\tilde{B} - B_s^{sep}(p)\right).$$

It is easy to show that $\Phi(p)$ is a convex function. Since $\Phi(0) = \Phi(p_1^*) = 0$, then, $\Phi(p) \leq 0 \forall p \in [0, p_1^*]$. That is

$$\frac{\tilde{B}\delta p}{1 - p(1+\delta)} < c\left(\tilde{B} - B_s^{sep}(p)\right)p$$

Define

$$\Psi(p) = cp\left(\tilde{B} - B_s^{sep}(p)\right) - \tilde{B}\frac{\int_{p_2^*}^1 t d\mu(t) - p}{1 - \int_{p_2^*}^1 t d\mu(t)}.$$

It is easy to see that function $\Psi(p)$ is a concave function of p. Then $\frac{\partial\Psi(p)}{p}|_{p<p_2^*} > \frac{\partial\Psi(p)}{p}|_{p=p_2^*}$ and

$$\frac{\partial\Psi(p)}{p}|_{p=p_2^*} = \tilde{B}\Bigg(c-\left(c+\frac{1}{1-p_2^*}\right)\left(1-\frac{\delta}{c(1-p_1^*(1+\delta))}\right)\left(\frac{(1-p_1^*)p_2^*}{p_1^*(1-p_2^*)}\right)^{\frac{1}{c}}+\frac{1}{1-\int_{p_2^*}^1 td\mu(t)}\Bigg).$$

For any $p_1^* \in \mathcal{L}$, it follows that $\frac{\partial\Psi(p)}{p} > 0$ for all $p < p_2*$ if condition (1.20) holds. Therefore, $\Psi(p)$ is an increasing function of p. So $\Psi(p) \leq \Psi(p_2^*) = 0$ for all $p < P_2^*$; that is

$$cp\left(\tilde{B} - B_s^{sep}(p)\right) < \tilde{B}\frac{\int_{p_2^*}^1 td\mu(t) - p}{1-\int_{p_2^*}^1 td\mu(t)}$$

.

Since

$$\frac{\tilde{B}\delta p}{1-p(1+\delta)} < c\left(\tilde{B} - B_s^{sep}(p)\right)p < \tilde{B}\frac{\int_{p_2^*}^1 td\mu(t) - p}{1-\int_{p_2^*}^1 td\mu(t)},$$

$$EU_r(p) > EU_{deviatesep}(p), \qquad EU_r(p) > EU_{deviatepool}(p)$$

Therefore, insurers with types in the range $[0, p_1^*]$ will not deviate to choose either separating partial securitization or pooling full securitization. Iteratively applying the D1 refinement, they will also not deviate to choose some other arbitrary securitization contract.

Now consider insurers with types in the range $[p_1^*, p_2^*]$. If they choose partial securitization contracts given by (1.30), the expected payoff is $EU_s^{sep}(p) = W + (A(1-p) - Bp) - c(\tilde{B} - B_s^{sep}(p))p$.

If they deviate to choose full reinsurance contracts given by Proposition 1 , the expected payoff is

$$EU_{deviatere}(p) = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1-p(1+\delta)}.$$

If they deviate to choose pooling securitization given by (1.31), the expected payoff is

$$EU_{deviatepool}(p) = W + (A(1-p) - Bp) - \tilde{B}\frac{\int_{p_2^*}^1 td\mu(t) - p}{1-\int_{p_2^*}^1 td\mu(t)}.$$

Since $\Phi(p)$ is a convex function with $\Phi(0) = \Phi(p_1^*) = 0$, $\Phi(p) > 0$ for $p > p_1^*$, that is, $\frac{\tilde{B}\delta p}{1-p(1+\delta)} > cp\left(\tilde{B} - B_s^{sep}(p)\right)$. Thus $EU_{deviatere}(p) < EU_s^{sep}(p)$.

Also, when $p < p_2^*$, it follows that $c(\tilde{B} - B_s^{sep}(p))p < \tilde{B}\frac{\int_{p_2}^1 td\mu(t) - p}{1 - \int_{p_2}^1 td\mu(t)}$. Hence, $EU_s^{sep}(p) > EU_{deviatepool}(p)$. Therefore, insurers with types in the range $[p_1^*, p_2^*]$ will choose neither full reinsurance nor full pooling securitization. Because the Spence-Mirrlees single-crossing condition holds, the "local" incentive compatibility condition (1.24) for the partial securitization contracts ensures that an insurer with risk type $p \in [p_1^*, p_2^*]$ will also not deviate to choose some other type's partial securitization contract. Finally, iteratively applying the D1 refinement, they will also not deviate to choose some other arbitrary securitization contract that is not chosen by another type.

We now consider the insurers with types in the range $[p_2^*, 1]$.

If they choose pooling securitization given by (1.31), the expected payoff is

$$EU_s^{pool}(p) = W + (A(1-p) - Bp) - \tilde{B}\frac{\int_{p_2^*}^1 td\mu(t) - p}{1 - \int_{p_2^*}^1 td\mu(t)}.$$

If they deviate to choose full reinsurance, the expected payoff is

$$EU_{deviatere} = W + (A(1-p) - Bp) - \frac{\tilde{B}\delta p}{1 - p(1+\delta)}.$$

By the property of the function $\Phi(p)$, it is easy to show that, when $p > p_2^* > p_1^*$, $\Phi(p) > 0$. Thus,

$$\frac{\tilde{B}\delta p}{1 - p(1+\delta)} > cp\left(\tilde{B} - B_s^{sep}(p)\right).$$

By the property of function $\Psi(p)$, it is easy to show that, when $p > p_2^* > p_1^*$,

$$cp\left(\tilde{B} - B_s^{sep}(p)\right) > \tilde{B}\frac{\int_{p_2^*}^1 td\mu(t) - p}{1 - \int_{p_2^*}^1 td\mu(t)}.$$

As a result, $EU_s^{pooling}(p) > EU_{deviatere}(p)$. The insurers, therefore, will not deviate to choose full reinsurance. It follows from the results of Proposition 4 that the insurers on this interval would not choose partial securitization. There are multiple possible PBEs, where the thresholds $\{p_1^*, p_2^*\} \in \mathcal{G}$ and $p_1^* < p_2^*$.

Now consider the case where $p_1^* > p_2^*$ so that partial securitization is sub-optimal for insurers. In this case, we conjecture a PBE with two partitions, where insurers with type in the range $[0, p_3^*]$ choose full reinsurance, while insurers with types in the range $[p_3^*, 1]$ choose pooling full securitization. We are, therefore, in the scenario as characterized by Part 3 of Proposition 3.

2. Suppose $\delta > c$. It follows that full reinsurance is the sub-optimal choice for all insurers.

Consequently, we are in the scenario as characterized by Proposition 4. ■

Chapter 2

Capital, Risk and Insurance Prices

2.1 Introduction

Financial institutions such as insurers and banks are usually required to hold sufficient equity capital on the liability side of their balance sheets and liquid reserves on the asset side as a buffer against the risk of insolvency, especially when their loss portfolios are imperfectly diversified and/or returns on their assets shrink dramatically. The financial crisis of 2007-2008 was precipitated by the presence of insufficient liquidity buffers and excessive debt levels in the financial system that made banks vulnerable to large aggregate negative shocks. In the context of insurers, the imperfect incorporation of the externality created by aggregate risk on their investment decisions when markets are incomplete may lead them to hold insufficient liquidity buffers to meet insurance liabilities. The resulting increase in insurer insolvency risk has an impact on the amount of insurance they can supply to insurees and, therefore, the degree of risk-sharing in the insurance market. Indeed, empirical evidence shows that, in response to Risk Based Capital (RBC) requirements, under-capitalized insurers not only increase their capital holdings to meet minimum capital requirements, but also take more risks to reach higher returns (Cummins and Sommer, 1996; Shim, 2010; Sager, 2002). Insurers' propensity to "reach for yield" contributes to their overall insolvency risk.[1] Aggregate risk may, therefore, lead to misallocation of capital and suboptimal risk sharing among insurees and insurers when markets are incomplete. To the best of our knowledge, however,

[1] Cox(1967) describes bank's tendency to invest in high risk loans with higher returns. Becker and Ivashina (2013)) support insurers' reaching for yield behavior by examining insurers' bond investment decisions

the above arguments have yet to be theoretically formalized in an equilibrium framework that endogenizes the demand and supply of insurance as well as insurers' asset and liability risks. Such a framework could potentially shed light on the optimal regulation of insurance firms taking into account both the asset and liability sides of insurers' balance sheets.

We contribute to the literature by developing an equilibrium model of competitive insurance markets where insurers' assets may be exposed to idiosyncratic and aggregate shocks. In the unregulated economy, we show that the equilibrium insurance price varies non-monotonically in a U-shaped manner with the level of internal capital held by insurers. In other words, the insurance price decreases with a positive shock to internal capital when the internal capital is below a threshold, but increases when the internal capital is above the threshold. We thereby reconcile conflicting predictions in previous literature on the relation between insurance premia and internal capital that are obtained in partial equilibrium frameworks that focus on either demand-side or supply-side forces. We also obtain the additional testable implications that an increase in insurers' asset risk, which raises the default probability, raises insurance premia and reduces coverage. We then proceed to derive insights into the solvency regulation of insurers by studying the benchmark "first best" economy in which there is perfect risk-sharing among insurers and insurees (so that they are only exposed to aggregate risk) and the effects of aggregate risk are fully internalized. We analyze the effects of aggregate risk on the Pareto optimal allocation of insurer capital to liquidity reserves and risky assets as well as risk sharing among insurees and insurers. We show that, when aggregate risk is below a threshold, it is Pareto optimal for insurers and insurees to hold zero liquidity reserves, insurees are fully insured, and insurers bear all aggregate risk. When aggregate risk takes intermediate values, both insurees and insurers still hold no liquidity reserves, but insurees partially share aggregate risk with insurers. When the aggregate risk is high, however, both insurees and insurers hold nonzero liquidity reserves, and insurees partially share aggregate risk with insurers. We demonstrate that the efficient allocation can be implemented through regulatory intervention that comprises of comprehensive insurance policies that combine insurance and investment, reinsurance, a minimum liquidity requirement when aggregate risk is high, and ex post budget-neutral taxation and subsidies

contingent on the realized aggregate state.

Our model features two types of agents: a continuum of ex ante identical, risk averse insurees each facing a risk of incurring a loss in their endowment of capital, and a continuum of ex ante identical risk neutral insurers each endowed with a certain amount of internal "equity" capital. There is a storage technology/safe asset that provides a constant risk free return and a continuum of risky assets that generate higher expected returns than the risk free asset. Although both insurees and insurers can directly invest in the safe asset, only insurers have access to the risky assets. In addition to their risk-sharing function, insurance firms, therefore, also serve as intermediaries to channel individual capital into productive risky assets. Insuree losses are independently and identically distributed, but insurers' assets are exposed to aggregate risk. Specifically, a certain proportion of insurers is exposed to a common asset shock, while the remaining insurers' asset risks are idiosyncratic. A priori, it is unkown whether a particular insurer is exposed to the common or idiosyncratic shock. The proportion of insurers who are exposed to the common shock is, therefore, the natural measure of the aggregate risk in the economy. Insurees invest a portion of their capital in the risk-free asset and use the remaining capital to purchase insurance. Insurers invest their internal capital and the external capital raised from selling insurance claims in a portfolio of risk-free and risky assets.

We first derive the market equilibrium of the unregulated economy. In the unregulated economy, asset markets are incomplete because there are no traded securities contingent on the asset realizations of individual insurers or the aggregate state. Insurees make their insurance purchase decisions rationally anticipating insurers' investment strategy and default risk given their observations of insurers' internal capital, the size of the insurance pool, and the menu of traded insurance contracts that comprise of the insurance price (the premium per unit of insurance) and the face value of coverage. *Ceteris paribus*, an increase in insurers' internal capital or a decrease in asset risk increases the demand for insurance due to the lower likelihood of insurer insolvency. An increase in the risk of insuree losses leads to a decrease in insurance demand because it increases the proportion of insurees who suffer losses and, therefore, decreases the amount that each insuree recovers if he incurs a loss, but

the insurance company is insolvent. Insurers, in turn, take the menu of traded insurance contracts as given and choose how many units of each contract to sell. There is free entry in that any contract that is expected to make positive expected profits for insurers is supplied. Competition among insurers then ensures that, in equilibrium, each insurer earns zero expected economic profits that incorporate the opportunity costs of internal capital that is used to make loss payments when insurers are insolvent. An increase in the insurance price, therefore, lowers the amount of insurance that each insurer sells in equilibrium leading to a downward sloping "zero economic profit" or "competitive" supply curve for insurance. An increase in the internal capital or an increase in asset risk, *ceteris paribus*, increases the opportunity costs of providing insurance, thereby increasing the amount of insurance that provides zero economic profits to insurers. An increase in the loss proportion increases the cost of claims, thereby pushing up the competitive supply level.

In competitive equilibrium, the insurance price is determined by market clearing—the demand for insurance must equal the supply—and zero economic profits for insurers. The insurance demand curve and the "zero economic profit" or "competitive" supply curve are both downward sloping with the demand curve being steeper due to the risk aversion of insurees. Consequently, any factor that increases the insurance demand curve, *ceteris paribus*, decreases the equilibrium price, while a factor that increases the competitive supply curve has a positive effect. We analytically characterize the competitive equilibrium of the economy and explore its comparative statics.

We demonstrate that there is a U-shaped relation between the insurance price and insurers' internal capital. Specifically, the insurance price decreases with a positive shock to internal capital when the internal capital is below a threshold, but increases when the internal capital is above the threshold. The intuition for the non-monotonic U-shaped relation hinges on the influence of both demand-side and supply-side factors. An increase in insurers' internal capital increases the competitive supply of insurance coverage because of the increased opportunity costs of internal capital. Because insurers are risk-neutral, however, the change in the competitive supply of insurance coverage is linear in the internal capital. On the demand side, an increase in insurers' internal capital increases insurers' insolvency

buffer, thereby increasing the demand for insurance coverage. An increase in internal capital also increases the funds available for investment that further has a positive impact on the demand for insurance. The demand, however, is concave in the internal capital due to insurees' risk aversion. Because the competitive insurance supply varies linearly with capital, while the insurance demand is concave, there exists a threshold level of capital at which the demand effect equals the supply effect. Consequently, the demand effect dominates the supply effect so that the equilibrium insurance price goes down when the internal capital level is lower than the threshold. When the capital is above the threshold, the supply effect dominates so that the insurance price increases.

As suggested by the above discussion, equilibrium effects that integrate both demand side and supply side forces play a central role in driving the U-shaped relation between the insurance price and insurer capital. Our results, therefore, reconcile and further refine the opposing predictions for the relation in the literature that stem from a focus on only demand or only supply effects in partial equilibrium frameworks. Specifically, the "capacity constraints" theory, which focuses on the supply of insurance, predicts a negative relationship between insurance price and capital by assuming that insurers are free of insolvency risk (Gron, 1994; Winter, 1994). In contrast, the "risky debt" theory incorporates the default risk of insurers, but predicts a positive relationship between insurance price and capital (Doherty and Garven, 1986; Cummins, 1988, Cummins and Danzon, 1997). Empirical evidence on the relationship is also mixed. We make the simple, but fundamental point that the insurance price reflects the effects of capital on both the demand for insurance and the supply of insurance in equilibrium. We show that the relative dominance of demand-side and supply-side forces depends on the level of internal capital, thereby generating a U-shaped relation between price and internal capital.

Next, we show that an increase in insurers' asset risk, which increases their insolvency probability, increases the insurance price and reduces the insurance coverage in equilibrium. The intuition for the results again hinges on a subtle interplay between the effects of an increase in asset risk on insurance supply and demand. A positive shock to insurers' asset risk, *ceteris paribus*, has the *direct* effect of increasing the competitive supply of insurance

coverage, that is, the level of insurance supply at which insurers earn zero economic profits. Consequently, the amount of funds available to pay loss claims in distress increases, thereby having the *indirect* effect of increasing the demand for insurance. On the other hand, an increase in the asset risk increases the insurers' insolvency probability that has a negative effect on the demand for insurance. We show that, under reasonable conditions, the direct effect outweighs the indirect effect. Consequently, an increase in asset risk reduces insurance demand, but increases the competitive supply level, thereby increasing the insurance price and decreasing the coverage level in equilibrium. Our results imply that the response to the increased asset risk of insurance firms is the shift of insuree's capital accumulation from indirect investment in risky assets to direct storage in safe assets.

2.2 Related Literature

Two streams of the literature investigate the relation between insurer capital and insurance premia. The first branch proposes the "capacity constraint"theory, which assumes that insurers are free from insolvency risk. The prediction of an inverse relation between insurance price and capitalization crucially hinges on the assumption that insurers are limited by regulations or by infinitely risk averse policyholders so that they can only sell an amount of insurance that is consistent with zero insolvency risk (e.g.,Gron, 1994; Winter, 1994). Winter(1994) explains the variation in insurance premia over the "insurance cycle"using a dynamic model. Empirical tests using industry-level data prior to 1980 support the predicted inverse relation between insurance capital and price, but data from the 1980s do not support the prediction. Gron (1994) finds support for the result using data on short-tail lines of business. Cagle and Harrington (1995) predict that the insurance price increases by less than the amount needed to shift the cost of the shock to capital given inelastic industry demand with respect to price and capital.

Another significant stream of literature—the "risky corporate debt" theory—incorporates the possibility of insurer insolvency and predicts a positive relation between insurance price and capitalization (e.g., Doherty and Garven, 1986; Cummins, 1988). The studies in this

strand of the literature emphasize that, because insurers are not free of insolvency risk in reality, the pricing of insurance should incorporate the possibility of insurers' financial distress. Higher capitalization levels reduce the chance of insurer default, thereby leading to a higher price of insurance associated with a higher amount of capital. Cummins and Danzon (1997) show evidence that the insurance price declines in response to the loss shocks in the mid-1980s that depleted insurer's capital using data from 1976 to 1987. While the "capacity constraint"theory concentrates on the supply of insurance, "the pricing of risky debt"theory focuses on capital's influence on the quality of insurance firms and, therefore, the demand for insurance. The empirical studies support the mixed results for different periods and business lines.

We complement the above streams of the literature by integrating demand-side and supply-side forces in an equilibrium framework. We show that there is a U-shaped relation between price and internal capital. In contrast with the literature on "risk debt pricing", which assumes an exogenous process for the asset value, we endogenize the asset value which depends on the total invested capital including both internal capital and capital raised through the selling of insurance policies. Insurers' assets and total liabilities are, therefore, simultaneously determined in equilibrium in our analysis.

Our paper is also related to the studies that examine the relation between capital holdings and risk taking of insurance companies. Cummins and Sommer (1996) empirically show that insurers hold more capital and choose higher portfolio risks to achieve their desired overall insolvency risk using data from 1979 to 1990. It is argued that insurers response to the adoption of RBC requirements in both property-liability and life insurance industry by increasing capital holdings to avoid regulation costs, and by investing in riskier assets to obtain high yields (e.g.,Baranoff and Sager, 2002; Shim, 2010). Insurers are hypothesized to choose risk levels and capitalization to achieve target solvency levels in response to buyers' demand for safety. Filipovic, Kremslehner and Muermann (2015) show that limited liability creates an incentive for insurers to engage in risk-shifting, thereby transferring wealth from policy holders, and that solvency capital requirements that restrict investment and premium policies can improve efficiency. Our paper fits into this literature by studying the response of

the market price to shocks to insurers' internal capital as well as aggregate and idiosyncratic shocks to insurers' assets in an equilibrium framework. Our results shed light on the solvency regulation of insurance firms by incorporating the liability and asset sides of insurers' balance sheets. We show that efficient allocations can be implemented through comprehensive insurance policies sold by insurers that combine insurance with investment, reinsurance, a minimum liquidity requirement, and ex post budget-neutral taxation contingent on the aggregate state. The tradeoff between holding sufficient capital to meet insurance liabilities and diverting capital to the most productive assets implies that a liquidity requirement should be imposed only when aggregate risk is sufficiently high.

Our paper also contributes to the literature on capital allocation and insurance pricing. Zanjani (2002) argues that price differences across markets are driven by different capital requirements to maintain solvency assuming that capital is costly to hold. Our paper endogenizes the cost of capital in terms of the opportunity cost of holding internal capital, which is used to pay for loss claims when insurers default. We highight insurees' and insurers' responses to internal capital shocks. Consequently, insurance prices reflect insurees' demand for safety and insurers' abilities to provide insurance with imperfect protection.

2.3 The Model

We consider a single-period economy with two dates 0 and 1. There is a single consumption/capital good. There are two types of agents: a continuum of measure 1 of risk-averse *insurees* or *policy holders* and a continuum of measure 1 of risk-neutral *insurance firms*. Each insuree is endowed with 1 unit of capital at date 0 and has a logarithmic utility function. Each insurance firm is endowed with K units of "internal" capital. There is a storage technology/safe asset that is in sufficiently large supply that it provides a constant return of R_f per unit of capital invested.

At date 1, an insuree i can incur a loss $l \leq 1$ so that a portion of each insuree's endowment is at risk. Losses are independently and identically distributed across insurees. Each insuree's loss probability is p. At date 0, each insuree invests a portion of her capital in the safe

asset and the remainder in buying an insurance contract, (κ, C), where κ is the premium per unit of insurance coverage and C is the face value of insurance coverage. Similar to Rothschild and Stiglitz (1976), we consider an insurance market in which insurance contracts, $\Phi \equiv \{(\kappa, C); \kappa > 0, C > 0\}$ that combine the "price" of insurance and the "quantity" of insurance are traded. Each insuree chooses a single contract from the set of traded contracts. Insurers have internal capital K and raise external capital by selling insurance contracts. Insurers and insurees take the set of insurance contracts Φ as given in making their supply and demand decisions, respectively. The set Φ is such that any contract that is demanded and expected to be profitable for an insurance company is supplied.

Each insurance firm j has access to a risky technology that generates a return of R_H per unit of invested capital with probability $1 - q$ when it "succeeds" but $R_L < R_H$ with probability q when it "fails." Insurance firms first raise capital in insurance markets and then invest it. Further, insurance firms *cannot commit* to their investment policy when they raise capital. A proportion $1 - \tau$ of insurance firms are exposed to idiosyncratic technology shocks, that is, the technology shocks are independently and identically distributed for this group of insurance firms. The remaining proportion τ of insurers are, however, exposed to a *common* shock, that is, the technology shock described above is the *same* for these insurers. Although insurers know that a proportion τ of them is exposed to a common shock, an individual insurer does not know whether it is exposed to an idiosyncratic or common shock a priori. τ is a measure of the aggregate *risk* in the economy.

We assume that

$$(1 - q)R_H + qR_L \geq R_f. \tag{2.1}$$

The above condition ensures that the expected return on the risky project is at least as great as the risk-free rate. While policy holders can directly invest in the safe asset, only insurance firms have access to the production technology. Consequently, in addition to the provision of insurance to policy holders, insurance firms also play important roles as financial intermediaries who channel the capital supplied by policy holders to productive assets. In addition to the fact that insurees do not have direct access to asset markets in the unregulated

economy, asset markets are incomplete because there are no traded securities contingent on the asset realizations of individual insures or the realization of the aggregate shock.

Let C_j be the *total* face value of insurance contracts sold by insurer j and K_j be the external capital it raises. The insurer can invest its total capital, $K + K_j$ in a portfolio comprising of the risk-free storage technology and the risky project. In an autarkic economy with no regulation, it follows from condition (2.1), and the fact that insurance firms cannot commit to their investment policy when they raise capital by selling insurance contracts, that it is optimal for risk-neutral insurance firms to invest their entire capital in the risky technology.

By our earlier discussion, the total liability of the insurer j is pC_j because a proportion p of its pool of insurees incur losses. Insurers default if their total liability cannot be covered by the total investment returns when the risky technology fails, that is when

$$pC_j > (K + K_j)R_L. \tag{2.2}$$

In the event of default, the total available capital of an insurer is split up among insurees in proportion to their respective indemnities. The internal capital plays the role of a buffer that increases an insurer's capacity to meet its liabilities and, thereby, the amount of insurance it can sell. The cost of holding internal capital in our model is an opportunity cost, which refers to the returns from the invested internal capital that are depleted to pay out liabilities when insurers default.

Each individual insuree observes the total capital, $K + K_j$, held by each insurer j in marking her insurance purchase decision. In making the decision on the level of insurance coverage to purchase, insurees rationally anticipate the possibility of default, and the amount they will be paid for a loss when insurers' asset returns are insufficient to pay out the aggregate loss claims as shown by (2.2).

2.3.1 The Equilibrium of the Unregulated Economy

We now derive the equilibrium of the unregulated economy by analyzing the demand and supply of insurance by insurees and insurers, respectively. In equilibrium, the demand for insurance equals the supply.

Insurance Demand

Each insuree chooses its portfolio, which comprises of his investment in the safe asset (self-insurance) and his choice of insurance contract, to maximize his expected utility. Without insurance, each insuree i's expected utility is given by the autarkic utility level,

$$\text{Autarkic Utility } = p\ln(R_f - l) + (1-p)\ln(R_f). \tag{2.3}$$

Insurees take the set, Φ, of traded insurance contracts as given in making their purchase decisions. Each insuree observes the total capital held by each insurer and, therefore, rationally anticipates the possibility that she may not be fully indemnified in the scenario where the insuree incurs losses, but the insurer is insolvent. Insurees also rationally incorporate insurers' investment portfolio choices in making their insurance demand decisions. As previously stated, insurers invest all their capital in the risky technology, thereby causing insurers to be likely to default in the "bad" state where the technology fails. The likelihood that insurees' loss claims may not be fully indemnified is then affected by the risk in the investment portfolio of insurance firms and the total liabilities insured by them. In general, the loss payment obtained by each insuree is determined by three factors: the proportion of insurees in the insurer's pool who incur losses, the total amount of capital held by the insurer, and the return of the insurer's investment project.

Given that insurees and insurers are ex ante identical, we focus on *symmetric equilibria* where insurees make identical portfolio choices and insurers have ex ante identical pools of insurees. Without loss of generality, therefore, we focus on a representative insurer and a representative insuree. Suppose that the representative insuree chooses the contract (κ, C_d). If C_s is the *total face value* of the insurance contracts sold by the insurer, its total capital

is $K + \kappa C_s$. The insurer's available capital if its project fails is, therefore, $(K + \kappa C_s)R_L$. Consequently, the payment received by each insuree who incurs a loss when the insurer's project fails is $\min(C_d, \frac{(K+\kappa C_s)R_L}{p})$. It is clear from our subsequent results that it is suboptimal for the insurer to sell so much coverage that it is unable to meet losses in the "good" state where its project succeeds. In the following, therefore, we assume this result to avoid unnecessarily complicating the exposition.

Among all the available contracts, (κ, C_d), where the premium per unit of coverage is κ, the representative insuree chooses the contract that maximizes its expected utility, that is, the insuree's choice of coverage solves

$$\max_{C_d} \overbrace{p(1-q)\ln\left[(1-\kappa C_d)R_f - l + C_d\right]}^{\text{insuree incurs loss in insurer's "good" state}} + \overbrace{pq\ln\left[(1-\kappa C_d)R_f - l + \min(C_d, \frac{(K+\kappa C_s)R_L}{p})\right]}^{\text{insuree incurs loss in insurer's "bad" state}} + \overbrace{(1-p)\ln\left[(1-\kappa C_d)R_f\right]}^{\text{insuree does not incur loss}} \tag{2.4}$$

such that

$$\kappa C_d \leq 1 \tag{2.5}$$

As is clear from the above, an atomistic insuree makes her insurance purchase decision based on her probability of a loss and the probability that the insurer's assets fail. Because she observes the insurer's total capital when she makes her decision, the insuree's decision rationally incorporates the proportion of the population of insurees that will incur losses.

The properties of the logarithmic utility function guarantee that it is suboptimal for insurees to invest all their capital in risky insurance so that the budget constraint, (2.5) is not binding. The necessary and sufficient first order condition for the insuree's optimal

choice of insurance coverage, C_d^*, is

$$\left\{ \frac{p(1-q)(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{pq\kappa R_f}{(1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s)R_L}{p}} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} \right\} \cdot \mathbf{1}_{\{pC_d^* \geq (K+\kappa C_s)R_L\}}$$
$$+ \left\{ \frac{p(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} \right\} \cdot \mathbf{1}_{\{pC_d^* < (K+\kappa C_s)R_L\}} = 0 \tag{2.6}$$

The solution to the above equation can be expressed as a function, $C_d^*(K, C_s, \kappa)$, where we suppress the dependence of the optimal demand on the liability and asset risk parameters, p and q, and the safe asset return, R_f, to simplify the notation.

The following lemma characterize the insuree's optimal demand for insurance coverage for a given insurance price, κ. The optimal demand depends on whether or not the representative insurer defaults in the bad state where its assets fail.

Lemma 1 • *If the representative insurer defaults in the "bad" state where its assets fail, the optimal insurance demand C_d^* is given by*

$$C_d^* = C_d^*(K, C_s, \kappa), \tag{2.7}$$

where $C_d^*(K, C_s, \kappa)$ satisfies equation

$$\frac{p(1-q)(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{pq\kappa R_f}{(1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s)R_L}{p}} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} = 0 \tag{2.8}$$

- *If the representative insurer does not default in the "bad" state where its assets fail, the optimal insurance demand $C_d^*$is given by*

$$C_d^* = C_d^*(\kappa), \tag{2.9}$$

where $C_d^*(\kappa)$ satisfies

$$\frac{p(1-\kappa R_f)}{(1-\kappa C_d^*)R_f - l + C_d^*} - \frac{(1-p)\kappa R_f}{(1-\kappa C_d^*)R_f} = 0 \tag{2.10}$$

By (2.8) and (2.10), we note that the insurer's internal capital, K, total supply, C_s, and

asset risk parameter, q, influence the optimal demand for insurance coverage only when insurees foresee insurer insolvency in the "bad" state, where its assets fail. For generality, we allow for the case that the market insurance price might lead to over insurance, i.e., $C_d > l$.

The following lemma shows how the optimal demand for insurance coverage varies with the fundamental parameters of the model that will be useful when we derive the equilibrium of the economy.

Lemma 2 (Variation of Insurance Demand) *The optimal demand for insurance, C_d^*, (i) decreases with the insurance price, κ; (ii) decreases with the return, R_f, on the safe asset; (iii) increases with insurers' internal capital, K; (iv) increases with the total face value of policies sold by the insurer, C_s; increases with the insurer's asset return in the low state, R_L; and (v) decreases with the insurer's expected probability of failure; q.*

The optimal demand for insurance claims reflects the tradeoff between self-insurance through investments in the safe asset and the purchase of insurance coverage with potential default risk for the insurer and, therefore, imperfect insurance for the insuree. Capital allocated in safe assets plays an alternative role in buffering the losses that cannot be indemnified by insurers when their assets fail. The insurance demand decreases with the insurance price, that is, the demand curve is downward-sloping, since the utility function of insurees satisfies the properties highlighted by Hoy and Robson (1981) for insurance to be a normal good. An increase in the risk-free return raises the autarkic utility level, thereby diminishing the demand for insurance coverage.

In addition to functioning as a risk warehouse, which absorbs and diversifies each insuree's idiosyncratic loss, insurance firms also serve as financial intermediaries who channel external capital supplied by policyholders to productive assets. In our model, the overall insolvency risk faced by insurance firms are simultaneously determined by the asset and liability sides of insurer's balance sheets. An increase in the aggregate loss proportion of the insuree pool; a decrease in the internal capital held by insurers; a decrease in the amount of external capital raised by the insurer from selling insurance; and a decrease in the asset return in the low state all lower the insurance coverage of an insuree when the insurer is insolvent so that the

optimal insurance demand declines.

Insurance Supply

Each insurer chooses which contracts from the set, Φ, to supply and the number of units of each contract to maximize its total net expected payoffs from providing insurance for insurees and investing the capital it raises. As discussed earlier, in the absence of regulatory intervention, it is optimal for each insurer to invest its entire capital in the risky project due to its risk neutrality and the asset return condition (2.1). Recall that an insurer cannot commit to its investment policy when it raises external capital by selling insurance contracts. An insurer chooses to supply insurance if and only if its expected net profits are at least as great as its autarkic expected payoff, that is, its expected payoff from not selling insurance and investing its internal capital. An insurer's autarkic expected payoff is

$$\text{Autarkic Expected Payoff} = K\left(\left(1-q\right)R_H + qR_L\right). \tag{2.11}$$

Each insurer makes its supply decision knowing the proportion, p, of its pool of insurees who will incur losses. In the bad state where its technology fails, if its available capital is lower than the total loss payments to insurees, then the capital is divided equally among the insurees. If the premium per unit of coverage is κ, the optimal supply of insurance coverage, therefore, solves

$$\max_{C_s}\left\{\left(1-q\right)\left(\left(K+\kappa C_s\right)R_H - pC_s\right)\right\} + \left\{q\left(\left(K+\kappa C_s\right)R_L - pC_s\right)\right\}\cdot \mathbf{1}_{\{pC_s \leq (K+\kappa C_s)R_L\}} \tag{2.12}$$

such that

$$\begin{aligned}&\left\{\left(1-q\right)\left(\left(K+\kappa C_s\right)R_H - pC_s\right)\right\} + \left\{q\left(\left(K+\kappa C_s\right)R_L - pC_s\right)\right\}\cdot \mathbf{1}_{\{pC_s \leq (K+\kappa C_s)R_L\}} \\ &\geq K\left(\left(1-q\right)R_H + qR_L\right) \qquad (P.C)\end{aligned} \tag{2.13}$$

The participation constraint, (2.13), ensures that the insurer chooses to sell a nonzero amount of coverage if and only if its expected net profit exceeds its expected payoff in

autarky; that is, its expected *economic profit* (profit in excess of the autarkic level) is nonnegative. From (2.12) and (2.13), it is clear that it is optimal for the insurer to supply no coverage if the premium rate, $\kappa < \frac{p}{R_H}$ and infinite coverage if $\kappa > \frac{p}{R_L}$. In equilibrium, therefore, we must have $\kappa \in [\frac{p}{R_H}, \frac{p}{R_L}]$. It also follows from the linearity of the objective function and the fact that any insurance contract that makes nonnegative expected economic profit for an insurer is supplied that the participation constraint, (2.13), must bind in equilibrium, that is, insurers make zero expected *economic profits*. Consequently, if the insurer will not default in the "bad" state where its asset fails, the *zero economic profit* supply of insurance coverage will completely hinge on the demand for insurance coverages because the insurers is always solvency and its opportunity cost of holding internal capital is zero in this scenario. Nevertheless, if the insurer will default in the "bad" state where its asset fails, the *zero economic profit supply* of insurance coverage for any insurance price $\kappa \in [\frac{p}{R_H}, \frac{p}{R_L}]$, which we hereafter refer to as the *competitive insurance supply* for expositional convenience, is

$$C_s^*(K, \kappa) = \frac{qKR_L}{(1-q)(\kappa R_H - p)} \tag{2.14}$$

Lemma 3 (Competitive Insurance Supply) *For $\kappa \in (\frac{p}{R_H}, \frac{p}{R_L})$, the competitive insurance supply level, $C_s^*(K, \kappa)$, (i) decreases with the insurance price, κ; (ii) increases with insurers' internal capital, K; (iii) increases with insurers' expected default probability, q; (iv) increases with the asset return, R_L, in the bad state; and (v) increases with the loss probability of insurees, p.*

An increase in the insurance price increases the expected return from supplying insurance and, therefore, decreases the coverage level at which each insurer's participation constraint, (2.13), is binding. For given $\kappa \in (\frac{p}{R_H}, \frac{p}{R_L})$, an increase in the insurer's internal capital, asset risk, or the aggregate risk of the pool of insurees lowers the expected returns from providing insurance and, therefore, increases the competitive insurance supply level.

Insurance Market Equilibria

We now derive the insurance market equilibrium that is characterized by the insurance price (per unit of coverage) κ^*. The equilibrium satisfies the following conditions.

1. Given the equilibrium price κ^*, the face value of coverage supplied by each insurer is $C_s^*(K, \kappa^*)$ given by (2.14) and insurers make zero expected economic profits.

2. Given the equilibrium price κ^* and the supply level $C_s^*(K, \kappa^*)$, the coverage purchased by each insuree is $C_d^*(K, C_s^*(\kappa^*), \kappa^*)$ given by (2.7) and (2.9).

3. The equilibrium price κ^* clears the market, that is, $C_d^*(K, C_s^*(\kappa^*), \kappa^*) = C_s^*(K, \kappa^*) = C^*$.

The following proposition characterizes the equilibria of the insurance market. We begin with some necessary definitions. Define the expected return from the insurer's risky technology,

$$ER = (1 - q)R_H + qR_L. \tag{2.15}$$

Define the excess demand function

$$F(K, \kappa) = C_d^*(K, C_s^*(\kappa), \kappa) - C_s^*(K, \kappa), \tag{2.16}$$

where $C_d^*(K, C_s^*(\kappa), \kappa)$ is the demand function described by Lemma (1) and $C_s^*(K, \kappa)$ is given by ((2.14)).

Proposition 6 (Insurance Market Equilibria)

- *Suppose* $K \leq \overline{K}_1$, *where* $\overline{K}_1$*is given by*

$$F(K = \overline{K}_1, \kappa = \frac{p}{ER}) = 0 \tag{2.17}$$

In equilibrium, insurers default in the "bad" state when their assets fail. The equilibrium price, κ^*, *satisfies:*

$$F(K, \kappa^*) = 0. \tag{2.18}$$

- *Suppose $K > \overline{K}_1$. In equilibrium, insurers do not default in the "bad" state where their assets fail. The equilibrium insurance price is $\kappa^* = \frac{p}{ER}$ and the equilibrium coverage level, C^*, is given by*

$$C^* = \frac{p}{\kappa^*} - \frac{(1-p)(R_f - l)}{1 - \kappa^* R_f} > l.$$

The above proposition shows that there are two possible equilibria that are determined by the internal capital of insurers. When the internal capital is lower than the threshold level $\overline{K}_1$, the representative insurer defaults in the "bad" state that is rationally foreseen by all agents. When the internal capital is higher than the threshold $\overline{K}_1$, the insurer faces no insolvency risk and this is rationally anticipated by all agents. The equilibrium insurance price is simply determined by the aggregate loss proportion of insurees adjusted by the expected return from the risky technology, $\frac{p}{ER}$, at which the insurer's participation constraint (2.13) is always binding. The equilibrium insurance coverage is determined by the probability and degree of individual loss of the insuree, expected returns to risky technology as well as the returns to risk free asset.

We now focus on the more interesting first scenario in which the representative insurer with insufficient internal capital may default after its technology fails. Many fundamental factors; such as the internal capital endowed by the representative insurer, the risk of the insurer's investment portfolio and individual losses, will play significant roles in jointly the determination of market equilibrium. Figure 2.1 shows this equilibrium determination. As analyzed earlier, both the demand curve for insurance and the competitive insurance supply curve are downward slopping, and the demand curve is stepper than the competitive supply curve due to the risk aversion of the insuree and risk neutrality of the insurer. The crossing point of the two curves represents the insurance contracts traded in the equilibrium. In other words, the equilibrium price, κ^*, and coverage level, C^*, satisfy the implicit equation (2.24). In addition, the condition (2.25) ensures that the insurer, indeed, defaults in the bad state. It also implies the equilibrium insurance price must be less than $\frac{p}{ER}$;[2] otherwise, the

[2]In general, the equilibrium insurance price can not exceed $\frac{p}{ER}$. The intuition is that,if the insurer's internal capital is level greater than $\overline{K}_1$, price higher than $\frac{p}{ER}$ will make the insurer positive economic profit; if insurer's internal capital is less than $\overline{K}_1$, price higher than $\frac{p}{ER}$ will make the insurer still solvent in the "bad" state where its asset fails.

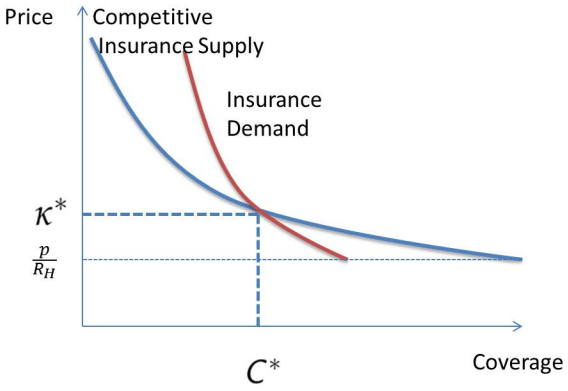

Figure 2.1: Insurance Market Equilibrium

conjecture of insurer insolvency in its "bad" state will be violated. The condition for the existence of the conjectured equilibrium is:

$$F(\kappa)|_{\kappa\to\frac{p}{ER}} = \lim_{\kappa\to\frac{p}{ER}} C_d^*(\kappa, C_s^*(\kappa)) - \lim_{\kappa\to\frac{p}{ER}} C_s^*(\kappa) > 0 \tag{2.19}$$

As shown in the proof of Proposition 6 in Appendix, condition (2.19) are satisfied when internal capital, K, is less than $\overline{K}_1$. Since an individual insurer can deviate and supply contracts with lower premia that still ensure nonnegative economic profits, the equilibrium price must be the smallest κ^* at which $\frac{\partial F(\kappa)}{\partial \kappa}|_{\kappa^*} > 0$.

We next identify the effects of shocks in the economy on the equilibrium insurance price, coverage level and social welfare.

2.3.2 The Effects of Capital and Risk

Internal Capital

Internal capital influences the equilibrium insurance price through the demand for and supply of insurance. By (2.14), an increase in internal capital increases the competitive insurance supply level. There are both direct and indirect effects of an increase in internal capital on the demand for insurance. An increase in internal capital has the direct effect of increasing the demand for insurance because of the higher available capital to meet insurance claims. The demand for insurance coverage is further enlarged by the insurees' anticipation of the increase

in the competitive supply of insurance with the increase in internal capital. Consequently, the overall effect of internal capital on the demand for insurance is also positive. The net effects of an increase in internal capital on the equilibrium insurance price depend on the relative dominance of demand-side and supply-side effects.

The equilibrium price κ^* satisfies $\frac{\partial F(K,\kappa)}{\partial \kappa}|_{\kappa=\kappa^*} > 0$. The marginal effects of internal capital on the insurance price can be understood through its effects on the excess demand function.

$$\frac{\partial F(K,\kappa^*)}{\partial K} = \underbrace{\frac{\partial C_d^*(K,C_s^*,\kappa^*)}{\partial K}}_{\text{direct effect on demand}} + \underbrace{\frac{\partial C_d(K,C_s^*,\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*(K,\kappa^*)}{\partial K}}_{\text{indirect effect on demand}} - \underbrace{\frac{\partial C_s^*(K,\kappa^*)}{\partial K}}_{\text{direct effect on competitive supply}}.$$

The following proposition describes the effects of internal capital on the equilibrium insurance price.

Proposition 7 (The Effects of Internal Capital)

- *Suppose* $\frac{\partial F(K,\kappa^*)}{\partial K}|_{K\to 0} > 0$. *There exist a threshold* $\widetilde{K}$ *such that the equilibrium insurance price* κ^* *decreases with internal capital when* $K < \widetilde{K}$, *and increases when* $K > \widetilde{K}$.
- *Suppose* $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} < 0$, *the equilibrium insurance price increases with the amount of internal capital.*

where the threshold level of internal capital, $\widetilde{K}$, *and its associated equilibrium* $\widetilde{\kappa}$ *are jointly determined by the following two equations:*

$$\frac{\partial F(K,\kappa^*)}{\partial K}|_{K=\widetilde{K},\kappa^*=\widetilde{\kappa};} = 0 \tag{2.20}$$

$$C_d^*\left(\widetilde{K}, C_s^*(\widetilde{\kappa},\widetilde{K}),\widetilde{\kappa},\right) = C_s^*(\widetilde{K},\widetilde{\kappa}). \tag{2.21}$$

The above proposition shows that the insurance premium decreases with insurers' internal capital when the internal capital level is relatively low, while it increases with insurers' internal capital when its level is relatively high. This result reconciles the conflicting predictions on the relation between insurance price and capital in previous literature. The "capacity constraint"theory relies on the assumption that insurance firms are free of insolvency. Winter (1990) argues that insurance firms can only write the volume of business consistent with

zero insolvency due to regulation. The total capital amount determines the capacity of the insurance market. A significant negative shock to insurer capital shrinks the supply of insurance in imperfect capital markets. It follows that the insurance price increases and insurance coverage declines while the demand for insurance is not affected in the absence of insurer insolvency. The "pricing of risky debt"theory incorporates the insolvency risk of insurance firms. Cummins and Sommer (1996) theoretically show both a postive and negative relation between price and a retroactive loss shock based on an optimal endogenous capitalization structure of insurance firms.

As mentioned earlier, an increase in internal capital increases the insurance demand and supply so the net impact depends on which of the two effects is dominant. By (2.14), the competitive supply of insurance is linear in the internal capital level. Because insurees are risk-averse, the demand for insurance is concave in the insurer's internal capital. Consequently, the excess demand function, $F(K, \kappa^*)$, is concave in the internal capital, that is, if $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} > 0$, then there exists, in general, a threshold level of internal capital, $\widetilde{K}$, at which the marginal effect of internal capital on the excess demand is zero. It follows from the concavity of the excess demand that the marginal effect of internal capital on the excess demand is positive for $K < \widetilde{K}$ and negative for $K > \widetilde{K}$. In other words, the risk aversion of insurees causes the "demand effect" of an increase in internal capital on the insurance price to dominate the "supply effect" for $K < \widetilde{K}$ and vice versa for $K > \widetilde{K}$. Hence, the equilibrium insurance premium varies in a U-shaped manner with the level of internal capital. If $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} \leq 0$, then the marginal effect of internal capital on the excess demand is always non-positive so that the equilibrium insurance premium increases with internal capital.

The Effects of Asset Risk

We now address the impacts of the representative insurer's asset risk on the equilibrium insurance price and coverage. The presence of asset induced insolvency complicates the decisions on both the demand and supply sides. The impact of asset risk on insurance supply indirectly influences insurance demand by affecting the total capital available to the insurer to meet liabilities in insolvency. Specifically, it follows from (2.14) that an increase

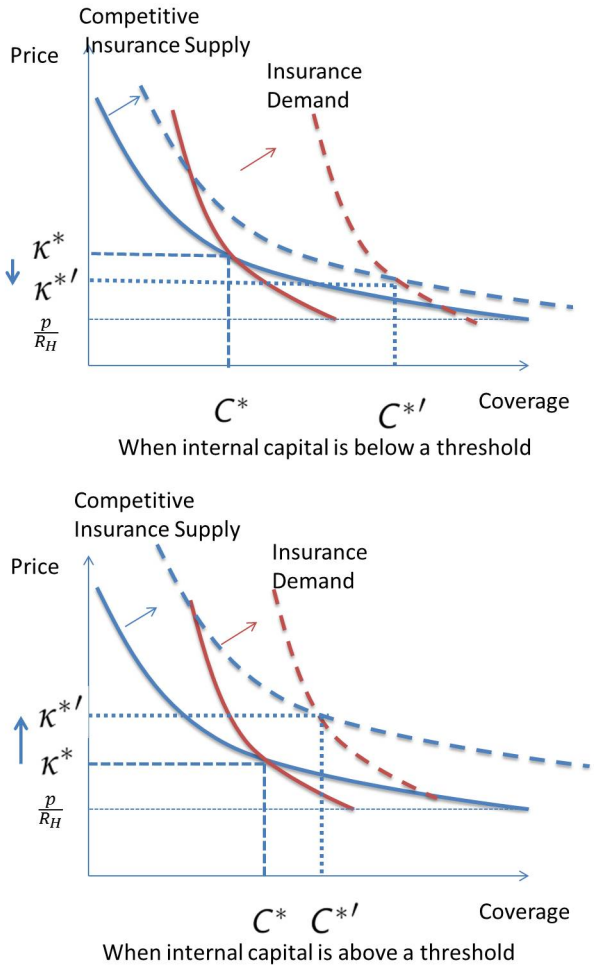

Figure 2.2: Effects of Internal Capital

in asset risk increases the competitive insurance supply level. Because insurees rationally foresee the likelihood that their losses will not be fully indemnified by insurers, the direct effect of an increase in asset risk on insurance demand is negative. The increase in the competitive supply level with asset risk, however, increases the amount each insuree is able to recover if it incurs a loss, but the insurer is insolvent. The indirect impact of an increase in asset risk on insurancey demand is, therefore, positive. The net impact of asset risk on

the equilibrium insurance price is determined via its effect on the excess demand function,

$$\frac{\partial F(\kappa^*, q)}{\partial q} = \underbrace{\frac{\partial C_d(\kappa^*, q)}{\partial q}}_{\text{direct effect on demand}<0} + \underbrace{\frac{\partial C_d(\kappa^*, q)}{\partial C_s^*}\frac{\partial C_s^*}{\partial q}}_{\text{indirect effect on supply}>0} - \underbrace{\frac{\partial C_s^*(\kappa^*, q)}{\partial q}}_{\text{direct effect on zero-economic-profit supply}>0}, \tag{2.22}$$

where we explicitly indicate the dependence of the demand and supply functions on the asset risk parameter, q. The following proposition characterizes the effect of asset risk on the equilibrium insurance price and coverage.

Proposition 8 (The Effects of Asset Risk) *Suppose $\frac{R_L}{p} < R_f$. The equilibrium insurance price increases with the asset risk, q, while the coverage level declines. If $\frac{R_L}{p} \geq R_f$, then the effect of asset risk on the insurance price is ambiguous.*

The intuition for the condition $\frac{R_L}{p} < R_f$ is as follows. $\frac{R_L}{p}$ captures the marginal contribution of an increase in the supply of insurance claims to the marginal utility of each insuree in the default state, while R_f measures the marginal contribution of an increase in insurance demand to marginal utility of each insuree in the default state. Consequently, the condition $\frac{R_L}{p} < R_f$ implies that the marginal contribution of insurance supply to the marginal utility is less than that of insurance demand. In other words, one unit increase in insurance supply will induce less than one unit increase in insurance demand. It follows that the indirect effect of an increase in asset risk on insurance demand through the increase in the competitive insurance supply level is less than the direct effect on the competitive supply level. Consequently, the excess demand decreases with asset risk so that the equilibrium price increases.

2.4 Conclusions

We develop an equilibrium model of competitive insurance markets where insurers' assets may expose to both idiosyncratic shocks and aggregate shocks. We reconcile the conflicting predictions in previous literature and provide new insights into the relationship between

insurance premia and internal capital that stem from the influence of both demand and supply side forces. The insurance price varies non-monotonically in a U-shaped manner with the level of internal capital held by insurers. We also obtain additional testable implications for the effects of insurers' asset risks on premia and the level of insurance coverage. We then empirically test these results in next chapter.

Appendix: Proofs

Proof of Lemma 1

Proof. We first consider the case where the representative insurer defaults in the "bad "state where its assets fail; that is, $C_d p \leq (K + \kappa C_s)R_L$. It follows that $\min\left(C_d, \frac{(K+\kappa C_s)R_L}{p}\right) = \frac{(K+\kappa C_s)R_L}{p}$. The necessary and sufficient first order condition for insuree's optimal choice of coverage, C_d^*, is simplified as equation (2.8). We next consider the other case where the representative insurer does not default in the "bad "where its assets fail; that is, $C_d p > (K + \kappa C_s)R_L$. It then follows that $\min\left(C_d, \frac{(K+\kappa C_s)R_L}{p}\right) = C_d$. The optimal choice of insurance coverage, therefore, has to satisfy equation (2.10). ■

Proof of Lemma 2

Proof. We consider the case where the representative insurer defaults in the "bad"state where its assets fail; that is, $C_d p \leq (K + \kappa C_s)R_L$. From the first section in Lemma 1, we first define the implicit function for the optimal demand for insurance coverage $\mathcal{G}(C_d^*, \kappa, p, q, R_f, R_L, K, C_s)$ as

$$\mathcal{G}(C_d^*, \kappa, p, q, R_f, R_L, K, C_s) = \frac{p(1-q)(1-\kappa R_f)}{W_1} - \frac{pq\kappa R_f}{W_2} - \frac{(1-p)\kappa R_f}{W_3} \tag{2.23}$$

where $W_1 = (1-\kappa C_d^*)R_f - l + C_d^*$, $W_2 = (1-\kappa C_d^*)R_f - l + \frac{(K+\kappa C_s^*)R_L}{p}$, $W_3 = (1-\kappa C_d^*)R_f$. We then show how the optimal demand for insurance coverage varies with the fundamental parameters of the model. It is easy to derive the signs for the following two equation:

$$\frac{\partial \mathcal{G}(C_d)}{\partial C_d} = -\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} - \frac{pq\kappa^2 R_f^2}{W_2^2} - \frac{(1-p)\kappa^2 R_f^2}{W_3^2} < 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial \kappa} = -\frac{p(1-q)R_f(R_f-l)}{W_1^2} - \frac{pqR_f(R_f - l + \frac{KR_L}{p})}{W_2^2} - \frac{(1-p)R_f^2}{W_3^2} < 0$$

It then follows that $\frac{\partial C_d^*}{\partial \kappa} < 0$. The optimal demand for insurance C_d^*, therefore, decreases

with the insurance price. Similarly, it is easy to show the following:

$$\frac{\partial \mathcal{G}(C_d)}{\partial R_f} = -\frac{p(1-q)\kappa}{W_1} - \frac{p(1-q)(1-\kappa R_f)(1-\kappa C_d)}{W_1^2} - \frac{pq\kappa(l-\frac{(K+\kappa C_s)R_L}{p})}{W_2^2} - \frac{(1-p)\kappa R_f(R_f-C_d)}{W_3^2} < 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial K} = \frac{pq\kappa R_f \frac{R_L}{p}}{W_2^2} > 0 \qquad \frac{\partial \mathcal{G}(C_d)}{\partial C_s} = \frac{pq\kappa^2 R_f \frac{R_L}{p}}{W_2^2} > 0$$

$$\frac{\partial \mathcal{G}(C_d)}{\partial R_L} = \frac{q\kappa R_f(K+\kappa C_s)}{W_2^2} > 0 \qquad \frac{\partial \mathcal{G}(C_d)}{\partial q} = -\frac{p(1-\kappa R_f)}{W_1} - \frac{p\kappa R_f}{W_2} < 0$$

Consequently, the optimal demand for insurance coverage, C_d, decreases with the return, R_f, on the safe asset and the default probability of insurer's risk assets, q, while increases with insurer's internal capital, K, the total face value of insurance contracts sold by the insurer, C_s, and the insurer's asset return in the low state, R_L. ■

Proof of Lemma 3

Proof. We show the effects of fundamental parameters on the competitive insurance supply by checking the signs for the following equations based on the competitive insurance supply in the case where insurer's asset may fail in "bad" state, C_s^*, given by equation (2.14). It is obvious that

$$\frac{\partial C_s^*}{\partial \kappa} = -\frac{qKR_LR_H}{(1-q)(\kappa R_H - p)^2} < 0 \qquad \frac{\partial C_s^*}{\partial K} = \frac{qR_L}{(1-q)(\kappa R_H - p)} > 0$$

$$\frac{\partial C_s^*}{\partial q} = \frac{KR_L}{(1-q)^2(\kappa R_H - p)} > 0 \qquad \frac{\partial C_s^*}{\partial R_L} = \frac{qK}{(1-q)(\kappa R_H - p)} > 0$$

$$\frac{\partial C_s^*}{\partial p} = \frac{qKR_L}{(1-q)(\kappa R_H - p)^2} > 0$$

It follows that the competitive insurance supply level, C_s^*, decreases with the insurance price, κ, while increases with insurers' internal capital, K, the default probability of insurer's risky assets, q, the risky asset return, R_L, in the bad state, and the loss probability of insurees, p.

■

Proof of Proposition 6

Proof. The insurance market equilibria depends on the internal capital level, K, held by insurance companies. We first conjecture that the representative insuree rationally foresees that the representative insurer will default in the "bad" state if the insurer's internal capital level is below $\overline{K}_1$ (where $\overline{K}_1$ satisfies equation (2.17)), whereas the representative insuree will anticipate that the representative insurer will still be solvent in the "bad" state if its internal capital level is above $\overline{K}_1$. We then derive the equilibrium insurance contracts, which consists of insurance price κ^* and the face value of insurance coverage C^* for each case, and later validate that the equilibrium where insurers defaults in "bad" state cannot exist given

any level of the internal capital level above the threshold level, $\overline{K}_1$.

1. Suppose $K \leq \overline{K}_1$, we conjecture that the insurer is expected to default in the "bad" state. It follows that the optimal demand for insurance coverage, C_d^*, has to satisfy equation (2.8), and the competitive insurance supply level, C_s^*, has to satisfies (2.14). The equilibrium insurance price, therefore, have to satisfy the following equation:

$$F(K, \kappa) = 0 \tag{2.24}$$

where $F(K, \kappa)$ is the excess demand function defined as (2.16), and C_s^* and C_d^* have to satisfy (2.8) and (2.14) separately.

In addition, to ensure the solution, κ, to equation (2.24) to be the equilibrium insurance price, it also needs to satisfy

$$pC^* \geq (K + \kappa^* C^*) R_L \tag{2.25}$$

where C^* is the face value of equilibrium insurance coverage such that $C_s^* = C_d^* = C^*$.

We next show that, given any K less or equal to $\overline{K}_1$, there exists an equilibrium insurance contract which includes the equilibrium insurance price κ^* and equilibrium face value of insurance coverage C^*.

(2.14) implies that the equilibrium insurance price, κ^*, need to lie in the interval $\left(\frac{p}{R_H}, \frac{p}{R_L}\right)$ because the insurer would like to supply either zero or infinite amount of insurance coverage for any price outside this region. Further, from (2.25) and (2.14), it is easy to show that the equilibrium insurance price κ^* also has to satisfy $\kappa^* \leq \frac{p}{ER}$, where $ER = (1-q)R_H + qR_L$; otherwise, the conjecture will be violated due to the violation of (2.14).

To show the existence of κ^* that satisfies (2.24), we check the boundary conditions for $\kappa \in \left(\frac{p}{R_H}, \frac{p}{ER}\right]$.

The derivative of $F(K, \kappa)$ with respect to κ for any K less or equal to $\overline{K}_1$; that is,

$$\frac{\partial F(\kappa^*|K)}{\partial \kappa^*} = \frac{\partial C_d^*(\kappa^*, C_s^*(\kappa^*)|K)}{\partial \kappa^*} + \frac{\partial C_d^*(\kappa^*, C_s^*(\kappa^*)|K)}{\partial C_s^*} \frac{\partial C_s^*(\kappa^*|K)}{\partial \kappa^*} - \frac{\partial C_s^*(\kappa^*|K)}{\partial \kappa^*} \tag{2.26}$$

According to the proof of Lemma 2, it is easy to show

$$\frac{\partial F(\kappa)}{\partial \kappa} = \underbrace{\frac{\partial C_d^*(\kappa, C_s^*(\kappa))}{\partial \kappa}}_{<0} + \underbrace{\frac{\partial C_d^*(\kappa, C_s^*(\kappa))}{\partial C_s^*}}_{>0} \underbrace{\frac{\partial C_s^*(\kappa)}{\partial \kappa}}_{<0} - \underbrace{\frac{\partial C_s^*(\kappa)}{\partial \kappa}}_{<0} \tag{2.27}$$

Howver, the sign of $\frac{\partial F(\kappa)}{\partial \kappa}$ is indeterminate.

We then check the sign of $F(\kappa)$ at the lower boundary of κ.

$$\lim_{\kappa\to\frac{p}{R_H}} F(\kappa|K) = \lim_{\kappa\to\frac{p}{R_H}} C_d^*(\kappa, C_s^*(\kappa)|K) - \lim_{\kappa\to\frac{p}{R_H}} C_s^*(\kappa|K)$$

It is obvious that $\lim_{\kappa\to\frac{p}{R_H}} C_s^*(\kappa|K) \to +\infty$ for any K such that $0 < K \leq \overline{K}_1$ because insurers have to sell a very large finite amount so that condition (2.13)is binding. In addition, $C_d^*|_{\kappa\to\frac{p}{R_H}} = C_d^*(\kappa \to \frac{p}{R_H}|K) < \frac{R_H}{p} < +\infty$. It follows that $\lim_{\kappa\to\frac{p}{R_H}} F(\kappa|K) < 0$

To ensure the existence of equilibrium insurance price κ^*, a necessary condition is that

$$F(\kappa|K)|_{\kappa=\frac{p}{ER}} = \lim_{\kappa=\frac{p}{ER}} C_d^*(\kappa, C_s^*(\kappa)|K) - \lim_{\kappa=\frac{p}{ER}} C_s^*(\kappa|K) \geq 0 \tag{2.28}$$

We now examine that condition (2.28) is satisfied for any K, such that $0 < K \leq \overline{K}_1$. In other words, we have to show that $F(K|\kappa = \frac{p}{ER})$ is a decreasing function and $\overline{K}_1$ is the solution to (2.17).

It is obvious that

$$F(K \to 0|\kappa = \frac{p}{ER}) = \underbrace{C_d^*(K \to 0|\kappa = \frac{p}{ER})}_{>0} - \underbrace{C_s^*(K \to 0|\kappa = \frac{p}{ER})}_{\to 0} > 0 \tag{2.29}$$

When $\kappa = \frac{p}{ER}$, the insurance claims received by each insuree who incurs losses are equal to the insurance claims seld by each insurer. Thus, under the reasonable condition$R_f > \frac{R_L}{p}$

$$\frac{\partial F(K|\kappa = \frac{p}{ER})}{\partial K} = \frac{\partial C_d^*(K|\kappa = \frac{p}{ER})}{\partial C_s^*}\frac{\partial C_s^*}{\partial K} - \frac{\partial C_s^*}{\partial K} < 0 \tag{2.30}$$

Conditions (2.29) and (2.30) imply that there exists a solution $\overline{K}_1$ to the equation (2.17). Condition (2.30) also implies for any $K \leq \overline{K}_1$,$F(K|\kappa = \frac{p}{ER}) \geq 0$. Thus when $K < \overline{K}_1$,there exists at least one equilibrium insurance price. However, since $\frac{\partial EU(\kappa^*)}{\kappa^*} < 0$, we focus on the equilibrium with the smallest price κ; that is at which $\frac{\partial F(\kappa)}{\partial \kappa}|_{\kappa^*} > 0$ and social welfare are maximized.

2. Suppose $K > \overline{K}_1$. As we shown in previous case, when $K > \overline{K}_1$, (2.28) will be violated. It follows that the solution to equation (2.24) will be greater than $\frac{p}{ER}$. Consequently, the conjecture that equilibrium where the insurer will default in its "bad" state cannot be maintained. We now conjecture that in equilibrium insurers will not default in its "bad" state. According to previous argument, insuree's demand for insurance coverage is not binding. In this case, the insurers face no opportunity cost and earns zero profit at the actuarially fair price $\frac{p}{ER}$, at which the insurer is indifferent between selling insurance and no insurance. The equilibrium insurance coverage is, then determined by insurance demands, which satisfies

(2.10), where $\kappa = \frac{p}{ER}$. It is easy to see the solution to (2.10) is $C^* = \left(p-(1-p)\frac{R_f-l}{\frac{ER}{p}-R_f}\right)\frac{ER}{p}$.

We next check that the insurance contracts $\kappa^* = \frac{p}{ER}$ and C^* are the equilibrium contracts. In other words, we have to check whether (2.25) holds. (2.25) implies that the insurer's internal capital has to satisfy $K > C^*(\frac{p}{R_L} - \frac{p}{ER})$; that is,

$$K > \underbrace{\left(p-(1-p)\frac{R_f-l}{\frac{ER}{p}-R_f}\right)\frac{ER}{p}(\frac{p}{R_L}-\frac{p}{ER})}_{let=\overline{K}_2}$$

We next show that $\overline{K}_1 = \overline{K}_2$. According to condition(2.17), we have $C_d^*(\overline{K}_1|\kappa \to \frac{p}{ER}) - \frac{q\overline{K}_1 R_L}{(1-q)(\frac{p}{ER}R_H - p)} = 0$. Thus $\overline{K}_1 = C_d^*(\overline{K}_1|\kappa \to \frac{p}{ER})\frac{(1-q)(\frac{p}{ER}R_H-p)}{qR_L}$. Since the equilibrium insurance demand $C_d^*(K = \overline{K}_1)$ is equal to $C^*(K > \overline{K}_2)$. In other words, $C_d^*(K = \overline{K}_1) = C^* = \left(p-(1-p)\frac{R_f-l}{\frac{ER}{p}-R_f}\right)\frac{ER}{p}$. Now we have $\overline{K}_1 = \left(p-(1-p)\frac{R_f-l}{\frac{ER}{p}-R_f}\right)\frac{ER}{p}\frac{(1-q)(\frac{p}{ER}R_H-p)}{qR_L}$. It is easy to show that $\frac{(1-q)(\frac{p}{ER}R_H-p)}{qR_L} = \frac{p}{R_L} - \frac{p}{ER}$; therefore, $\overline{K}_1 = \overline{K}_2$. Q.E.D. ■

Proof of Proposition 7

Proof. To examine the effects of internal capital of the insurer on the insurance price, we integrate its effects on both the competitive supply of insurance coverage and the demand for insurance coverage. In other word, we need to determine the sign of $\frac{\partial \kappa^*}{\partial K} = -\frac{\frac{\partial F(\kappa^*,K)}{\partial K}}{\frac{F(\kappa^*,K)}{\partial \kappa^*}}$. From (2.24), we have

$$\frac{\partial F(\kappa^*)}{\partial K} = \underbrace{\frac{\partial C_d(\kappa^*)}{\partial K}}_{\text{direct effect on demands}} + \underbrace{\frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*(\kappa^*)}{\partial K}}_{\text{indirect effect on demands}} - \underbrace{\frac{\partial C_s(\kappa^*)}{\partial K}}_{\text{direct effect on zero-expected-profit supply}}$$

Following the results of Lemma 2 and 3, it is easy to show the following

$$\frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*}{\partial K} = \frac{pq\kappa^2 R_f \frac{R_L}{p}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}\frac{qR_L}{(1-q)(\kappa^* R_H - p)} > 0$$

$$\frac{\partial C_s(\kappa^*)}{\partial K} = \frac{qR_L}{(1-q)(\kappa^* R_H - p)} > 0$$

$$\frac{\partial C_d^*}{\partial K} = -\frac{\frac{\partial \mathcal{G}(C_d^*)}{\partial K}}{\frac{\partial \mathcal{G}(C_d^*)}{\partial C_d}} = \frac{\frac{pq\kappa R_f \frac{R_L}{p}}{W_2^2}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} + \frac{pq\kappa^2 R_f^2}{W_2^2} + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}} > 0$$

Thus we have

$$\frac{\partial F(\kappa^*,K)}{\partial K} = \underbrace{\frac{\partial C_d(\kappa^*,K)}{\partial K}}_{>0} + \underbrace{\frac{\partial C_d(\kappa^*,K)}{\partial C_s^*}\frac{\partial C_s^*(\kappa^*,K)}{\partial K}}_{>0} - \underbrace{\frac{\partial C_s(\kappa^*,K)}{\partial K}}_{>0}$$

Further, we know $\frac{\partial F(\kappa^*,K)}{\partial \kappa^*} > 0$. Thus the sign of $\frac{\partial F(\kappa^*,K)}{\partial K}$ is indeterminate, and the effects of internal capital on the equilibrium price is non-monotonic. However, the excess insurance demand function is concave because

$$\begin{aligned}\frac{\partial^2 F(\kappa^*,K)}{\partial K^2} =& \frac{\partial^2 C_d(\kappa^*,K)}{\partial K^2} + \frac{\partial\left(\frac{\partial C_d(\kappa^*,K)}{\partial C_s^*}\frac{\partial C_s^*}{\partial K}\right)}{\partial K} \\ =& \underbrace{-\frac{2W_2 pq\kappa^* R_f \frac{R_L}{p}\left[\left(\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}+\frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}\right)\frac{\partial W_2}{\partial K}\right]}{\left[\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}W_2^2 + pq\kappa^{*2}R_f^2 + \frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}W_2^2\right]^2}}_{\text{direct effect part}<0} \\ & \underbrace{-\frac{2W_2 pq\kappa^{*2} R_f \frac{R_L}{p}\left[\left(\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}+\frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}\right)\frac{\partial W_2}{\partial K}\right]\frac{\partial C_s^*}{\partial K}}{\left[\frac{p(1-q)(1-\kappa^* R_f)^2}{W_1^2}W_2^2 + pq\kappa^{*2}R_f^2 + \frac{(1-p)\kappa^{*2}R_f^2}{W_3^2}W_2^2\right]^2}}_{\text{indirect effect part}<0} < 0\end{aligned}$$

Thus the marginal effect of internal capital K on insurance demand C_d is decreasing, while the marginal effect on competitive insurance supply is constant. Consequently, the overall effects are decreasing.

Therefore, suppose $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} > 0$, there may exist a threshold level of $\widetilde{K}$ and the corresponding insurance price $\widetilde{\kappa}$ such that the equilibrium price κ^* decrease with the amount of internal capital when $K < \widetilde{K}$, while increase with an increase in the amount of internal capital when $K > \widetilde{K}$; suppose $\frac{\partial F(\kappa^*,K)}{\partial K}|_{K\to 0} < 0$, the equilibrium price increases with the amount of internal capital, where $\widetilde{K}$ and $\widetilde{\kappa}$ are jointly determined by the following two equations

$$\begin{aligned}&\frac{\partial F(\kappa^*,K)}{\partial K}\left(\kappa^* = \widetilde{\kappa}; K = \widetilde{K}\right) = 0 \\ &C_d^*\left(\widetilde{\kappa},\widetilde{K}, C_s^*(\widetilde{\kappa},\widetilde{K})\right) = C_s^*(\widetilde{\kappa},\widetilde{K})\end{aligned}$$

■

Proof of Proposition 8

Proof. To examine the effects of default risk of insurer's risky assets on the equilibrium insurance price, we integrate its effects on both the competitive supply of insurance coverage and the demand for insurance coverage. In other word, we need to determine the sign of We check the sign of $\frac{\partial \kappa^*}{\partial q} = -\frac{\frac{\partial F(\kappa^*)}{\partial q}}{\frac{F(\kappa^*)}{\partial \kappa^*}}$. According to the proof of Lemma 2 and 3, it is easy to show the following:

$$
\begin{aligned}
\frac{\partial F(\kappa^*)}{\partial q} =& \frac{\partial C_d(\kappa^*)}{\partial q} + \frac{\partial C_d(\kappa^*)}{\partial C_s^*}\frac{\partial C_s^*}{\partial q} - \frac{\partial C_s^*(\kappa^*)}{\partial q} \\
=& -\frac{\frac{p(1-\kappa R_f)}{W_1} + \frac{p\kappa R_f}{W_2}}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2} + \frac{pq\kappa^2 R_f^2}{W_2^2} + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}} \\
&+ \underbrace{\frac{pq\kappa^2 R_f \frac{R_L}{p} - pq\kappa^2 R_f^2}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}}_{<0 \quad \text{if} \quad \frac{R_L}{p} < R_f} \frac{JR_L}{(1-q)^2(\kappa R_H - p)} \\
&- \frac{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2}{\frac{p(1-q)(1-\kappa R_f)^2}{W_1^2}W_2^2 + pq\kappa^2 R_f^2 + \frac{(1-p)\kappa^2 R_f^2}{W_3^2}W_2^2} \frac{JR_L}{(1-q)^2(\kappa R_H - p)}
\end{aligned}
$$

Given condition that $\frac{R_L}{p} < R_f$, we have $\frac{\partial F(\kappa^*)}{\partial q} < 0$. It follows that $\frac{\partial \kappa^*}{\partial q} > 0$. The equilibrium insurance price, thus, increases with an increase in the asset risk. In other words, when $\frac{R_L}{p} < R_f$, the indirect effects of asset risk on insurance demand is offset by the direct effects on competitive insurance supply. Consequently, the demand effects dominates so that the equilibrium price goes up and the equilibrium coverage shrinks. ■

Chapter 3

Empirical Evidence of Internal Capital, Asset Risk and Insurance Prices

3.1 Introduction

The traditional theories of the determination of insurance prices (Myers and Cohn, 1986) suggest that insurance premia are given by the discounted value of the expected costs of providing coverage given perfect capital markets. However, insurance prices fluctuate in different phases of the insurance underwriting cycle, which suggests that capital market imperfections may make it difficult for insurance firms to adjust their capital holdings freely and immediately after a large negative shock that depletes their total capital. Financial capital is the major determinant of insurance output capacity. Moreover, insurers are required by regulation, such as "Risk Based Capital" or "Solvency II" regime, to hold sufficient equity capital. Equity capital can serve as a buffer against the risk of insurer insolvency, especially when their loss portfolios are imperfectly diversified and/or returns on their assets shrink dramatically. The amount of equity capital held by insurers, thus, crucially affects insurance prices and reflects insurers' ability of meeting its loss payments.

A significant stream of the literature examines how insurer capital affects insurance prices, but generates contrasting predictions regarding the relationship both theoretically and empirically. The "capacity constraints" theory focuses on the supply of insurance and predicts

a negative relationship between insurance price and capital(Gron, 1994; Winter, 1994). The "capacity constraints" theory assume insurers are free of insolvency risk because insurance firms are constrained by either infinite risk averse policy holders or regulator. Insurance firms, thus can only supply the amount of insurance that is consistent with zero insolvency risk. Equity capital, therefore, plays a significant role in determining the insurance capacity. Large negative shocks significantly reduce insurer capital, thereby pushing up the insurance price and reducing the insurance coverage, and vice versa. The empirical studies supporting this negative relation use pre 1980s industry data or short-tail insurance line data. The "capacity constraint" theories can explain the underwriting cycle, where "hard market" periods following portfolio losses that are characterized by rising prices and reduced coverage alternate with "soft market" periods where there is excess capital that results in falling insurance prices and increased availability of insurance.

However, individual insurance firms are exposed to significant insolvency risk. Insurance prices should, therefore, also reflect the financial quality of insurers that is significantly affected by available equity capital held by insurance firms. The "risky debt" theories incorporate the default risk of insurers, but predict a positive relationship between insurance price and capital (Doherty and Garven, 1986; Cummins, 1998; Cummins and Danzon, 1997). Insurers with sufficiently higher amount of capital may be less likely to default after loss portfolio shocks or asset shocks, thereby leading to a relatively higher insurance price. The positive relation is also supported by long-tail lines of insurance data (Gron, 1994).

Extant studies primarily examine the relation between capital and price by focusing on either the supply or demand side. Insurance prices, are, however, endogenously determined in the equilibrium that reflects both the demand and supply of insurance. Recall that in Chapter 2, we derive a unified equilibrium model of competitive insurance markets incorporating both demand side and supply side factors affecting insurance market, as well as the asset and liability sides of insurance firm's balance sheets. We predict a non monotonic U-shaped relationship between insurance prices and the level of internal capital held by insurance firms. Specifically, the equilibrium insurance price decreases with a positive shock to internal capital when the internal capital is below a threshold, but increases when the

internal capital is above the threshold. The results are driven by equilibrium effects, and could potentially reconcile the conflicting results predicted by the previous studies. The insurance demand is concave in the internal capital due to the risk aversion of insurees, while the insurance supply is linear in the level of internal capital due to the risk neutrality of insurance firms. Therefore, there exists a threshold level of internal capital, at which the effects of internal capital on insurance demand is equal to that on insurance supply. When the internal capital is below the threshold level, the demand effects dominate the supply effects, thereby leading to a negative relationship between insurance price and internal capital. When the internal capital is above the threshold level, the supply effects dominate the demand effects, thereby causing a positive relationship between insurance price and internal capital.

The U-shaped relation between the insurance price and internal capital could potentially reconcile the conflicting results predicted by previous theories. In this chapter, our empirical analysis, using industry-level data including all lines of property and casualty insurance for the period 1992-2012, supports the hypothesis that the relationship between insurance price and internal capital is U-shaped. The results in this chapter are consistent with the theoretical predictions of the equilibrium model in Chapter 2.

The results of Chapter 2 also predict that an increase in the asset investment risk increases the insurance price. An increase in the asset default risk increases the opportunity costs of insurance firms' internal capital, and also increases the chance that the policyholders do not receive full insurance protection. The equilibrium price is expected to rise by integrating the effects of asset risk on both insurers and policyholders.

Section 2 discusses the related literature. Section 3 introduces the data we use in this study, explains the main variables estimation, and discuss the main testable hypotheses and regression specification. Section 4 shows the results that support the hypothesis of the relation between insurance price and internal capital as well as the relation between insurance price and asset risk, using aggregate level data for all lines of property and casualty insurance during the period 1992-2012. Section 5 concludes this chapter.

3.2 Related Literature

Our paper is related to two lines of literature that investigate the relation between capital and price. The first branch proposes the "capacity constraint"theory, which assumes that insurers are free from insolvency risk. The prediction of an inverse relation between insurance price and capitalization crucially hinges on the assumption that insurers are limited by regulations or by infinite risk averse policyholders so that they can only sell an amount of insurance that is consistent with zero insolvency risk (e.g.,Gron, 1994; Winter, 1994). Winter(1994) explains the variation in insurance premia over the "insurance cycle"using a dynamic model. Empirical tests using industry-level data prior to 1980 support the predicted inverse relation between insurance capital and price, but data from the 1980s do not support the prediction. Gron (1994) finds support for the result using data on short-tail lines of business. Cagle and Harrington (1995) predict that the insurance price increases by less than the amount needed to shift the cost of the shock to capital given inelastic industry demand with respect to price and capital.

Another significant stream of literature—the "risky corporate debt" theory—incorporates the possibility of insurer insolvency and predicts a positive relation between insurance price and capitalization (e.g., Doherty and Garven, 1986; Cummins, 1988). The studies in this strand of the literature emphasize that, because insurers are not free of insolvency risk in reality, the pricing of insurance should incorporate the possibility of insurers' financial distress. Higher capitalization levels reduce the chance of insurer default, thereby leading to a higher price of insurance associated with a higher amount of capital. Cummins and Danzon (1997) show evidence that the insurance price declines in response to the loss shocks in the mid-1980s that depleted insurer's capital using data from 1976 to 1987. While the "capacity constraint" theory concentrates on the supply of insurance, "the pricing of risky debt" theory focuses on capital's influence on the quality of insurance firms and, therefore, the demand for insurance. The empirical studies support the mixed results for different periods and business lines.

We complement the above streams of the literature by showing that the insurance price

is negatively related to internal capital when internal capital is relatively low, but positively related with internal capital is relatively high.

Our paper is also related to the studies that examine the relation between capital holdings and risk taking of insurance companies. Cummins and Sommer (1996) empirically show that insurers hold more capital and choose higher portfolio risks to achieve their desired overall insolvency risk using data from 1979 to 1990. It is argued that insurers respond to the adoption of RBC requirements in both property-liability and life insurance industry by increasing capital holdings to avoid regulation costs, and by investing in riskier assets to obtain high yields (e.g.,Baranoff and Sager, 2002; Shim, 2010). Insurers are hypothesized to choose risk levels and capitalization to achieve target solvency levels in response to buyers' demands for safety. Our paper fits into the literature by studying the relationship between assets risk and insurance price. Higher default risk assets may potentially increase insurance price driven by the effects on both competitive insurance supply and policy holder's demand decisions. Our empirical results support this positive relation.

3.3 Data and Variable Construction

3.3.1 Data

The primary data source for the study is taken from the regulatory annual statements filed by property and casualty insurers with the National Association of Insurance Commissioners (NAIC) from 1992 to 2012. The main analysis is based on the aggregate level data for insurance lines. The NAIC data includes detailed information on the net premium written, net losses incurred and expenses for each line of insurance. We can simple add up those variables across all individual insurance firms (including stock, mutual and other types of firms) to get the aggregated market level data for these variables. Other aggregated market level variables, such as dividends paid to the policy holder, assets and so on, however, are calculated in two steps since NAIC data provide no information on those variables for each insurance line. In general, we first divide the value of those variables into each insurance line for each insurer relying on the corresponding weights, which will be discussed later in detail.

We then generate the aggregated market level data for each line by integrating all insurance firms for that line, respectively. The market level variables generated by the above two steps include the premiums, losses, expenses, surplus, dividends paid to policy holders and each type of assets.

To calculate the variance covariance matrix of insurer's asset portfolios, we use the index data including S&P 500, Moody's corporate bond total return, National Association of Real Estate Investment Trusts total return(NAREIT), the Merrill Lynch mortgage backed securities total return, and 30 days US Treasury bill rate from bloomberg terminal database.

The key variables we need to construct for this analysis are the price of insurance, capital and asset allocated into each line, and the measure of asset risk.

3.3.2 Estimating the Price of Insurance

The standard price measure in the insurance literature (e.g., Winter, 1994; Cummins and Danzon, 1997; Cummins, Lin and Phillips, 2006) is the *economic premium ratio* (EPR). The EPR for a line of insurance is defined as the ratio of the premiums for each line to the expected losses discounted at risk-free rate associated for that line, that is:

$$EPR_{it} = \frac{NPW_{it} - DIV_{it} - E_{it}}{\sum_{t=1}^{T}(NLI_{it} + LAE_{it})/(1 + r_t)^t} \tag{3.1}$$

where

EPR_{it} =the economic premium-to-liability ratio for line i at time t,

NPW_{it} =net premiums written for line i at time t,

DIV_{it} =dividends paid to policyholders for line i at time t,

E_{it} =underwriting expenses incurred for line i at time t,

NLI_{it} =net loss cash flow for line i at time t after the policy is issued,

LAE_{it} =net loss adjustment expense cash flow for line i at time t,

r_t =US Treasury spot-rate of interest for maturity of t,

T =the number of periods in the loss cash flow stream.

In our analysis, we assume the loss cash flow tail and the loss adjustment expense cash

flow tail are constantly distributed over the sample period. We, then use the incurred loss for line i at year t to measure the expected losses for the policy issued at year t. Thus the ERL_i is calculated separately for each line and for year over the sample period. The net premium written NPW_{it} and net loss incurred NLI_{it} for line i of insurance at time t are calculated by summing NPW_{ijt} and NLI_{ijt} across all the insurers j, respectively. However, as mentioned earlier, the NAIC annual statements do not have detailed information of dividends paid to policy holders, underwriting expenses incurred and net loss adjustment expense for each line. We adopt the two steps to generate the market level data for these variables. First, for each insurance company each year, the dividends paid to policy holders, underwriting expenses incurred and net loss adjustment expense incurred are divided into each insurance line based on the corresponding allocation weight, that is, the proportion of premiums written for each line over the total premium written by that company. We then aggregate each of those variables over all insurance firms each year. We apply all the aggregated market level variables into equation (3.1); and therefore construct the measure of insurance price for each insurance line at each year over the sample period.

3.3.3 Estimating the Capital Allocations by Line

We measure the amount of internal capital held by insurers using the amount of surplus from the annual statement page of "liabilities, surplus and other funds" at the end of previous filing year. We need to calculate capital allocations by lines of business for each insurance firm since we only have the information of total firm surplus. There are several capital allocation methodology.

We first use "the weighted liability" method. We divide the total firm surplus into different business lines weighted by the ratio of the net losses incurred of each line to the total net losses incurred of the firm. Specifically, the capital held for line i of insurance firm j at the statement filing year t is

$$C_{ijt} = C_{jt} \frac{NLI_{ijt}}{\sum_i^I NLI_{ijt}}$$

Figure 3.1: Asset Returns Index

Asset Class	Indices
Stocks	S&P 500 Total Return Index
Bonds	Moody's Corporate Bond Total Return Index
Real Estate	National Association of Real Estate Investment Trusts Total Return(NAREIT)
Mortgage	Merrill Lynch Mortgage Backed Securities Total Return
Cash or Equivalence	30 days US Treasury Bill Rate

where

C_{jt} =total surplus of insurance firm j at firm at filing year t

NLI_{ijt} =net loss incurred of line i of insurance firm j at filing year t

We then add up the capital of each line i across all the insurance firms who provide the line i; that is, $C_{it} = \sum_j^J C_{ijt}$

3.3.4 Estimating the Asset Risk

Insurance companies invest their funds including internal capital and collected premiums to a variety of asset classes. An insurance firm's asset portfolio can be well captured by the combination of bonds, stocks, real estates, mortgages, cash and other cash equivalent investments. The existing literature has two alternative proxies for asset risk of insurance firms (Baranoff e.t. al, 2007; Eling and Marek, 2013).

One measure is *opportunity asset risk* (OAR), which is based on the volatility of asset returns to calibrate investment risk in portfolio theory. OAR measures the gains or losses presented by the insurer's allocation choices among different asset categories in its investment portfolios. We assume each insurance firm could invest its actual investment portfolio in the corresponding investment indices as summertime in Table 3.1. We calculate the variance-covriance matrix Σ_t for the hypothetical index investment portfolio using the monthly returns of each investment index during the period from 1992 to 2012.

We then assume each line of insurance is operated as a single representative firm, and derive the asset portfolios held by the single representative firm. Similar to the calculation of capital allocation by line of insurance business, we also apply the two steps to the asset

allocation for each of the five major types of assets. We first divide the total asset values into different lines of insurance for each insurance company. This allocation is weighted by the ratio of the net loss incurred of each insurance line to the total net incurred losses of that company. For each line of insurance, we next add up assets allocated to that line over all the insurance firms that supply that line. We, therefore, have constructed the asset portfolios for each representative line of insurance.

For each insurance line i, portfolio weights are assumed to be constant throughout the year. We can calculate the assets portfolio weight vector in year t, $(\alpha_{\text{stocks}}, \alpha_{\text{bonds}}, \alpha_{\text{real estates}}, \alpha_{\text{mortgages}}, \alpha_{\text{cash}})_{it}$, where each component represents the ratio of each type of asset value over the total portfolio value. The volatility of returns to the asset portfolios for each line of insurance, then can be calculated by:

$$(\alpha_{\text{stocks}}, \alpha_{\text{bonds}}, \alpha_{\text{real estates}}, \alpha_{\text{mortgages}}, \alpha_{\text{cash}})_{it} * \Sigma * (\alpha_{\text{stocks}}, \alpha_{\text{bonds}}, \alpha_{\text{real estates}}, \alpha_{\text{mortgages}}, \alpha_{\text{cash}})'_{it}$$

The OAR is then calculated as the logarithm of the annualized standard deviation for each insurance line at each year over the sample period.

Another measure is *regulatory asset risk* (RAR), which is related to the C-1 component of risk-based capital from the regulatory tradition of concern with solvency, minimize the risk of failure or ruin from investment activities. Specifically,

$$RAR = \log(\frac{\text{C-1 measure of risk-based capital}}{\text{total invested assets}})$$

where

$$\begin{aligned} & \text{C-1 measure of risk-based capital} \\ = \; & bond * 0.065 + stocks * 0.3 + realestate * 0.1 + mortgage * 0.03 + cash * 0.003 \end{aligned}$$

3.3.5 Regression Analysis

We test the following two hypotheses in this chapter.

Hypothesis 1 *Insurance price is positively related to the level of internal capital when internal capital is relatively low, while insurance price is negatively related to the level of internal capital when internal capital is relatively high.*

Hypothesis 2 *Insurance price is positively related to the asset risk of insurance firms' investment portfolios.*

To test the above two hypotheses, the basic regression specification is as follows:

$$P_{it} = \beta_0 + \beta_1 C_{it} + \beta_2 C_{it}^2 + \beta_3 R_{it} + \gamma' X_i + \nu_t + \eta_i + \epsilon_{it}$$

where P_{it} = proxy for insurance price for line i of business in year t

C_{it} = proxy for the internal capital which is the surplus allocated in line i in year $t-1$

C_{it}^2 = square term of the proxy for internal capital which is the surplus allocated in line i in year $t-1$

R_{it} = proxy for the asset risk allocated in line i in year t

X_i = vector of control variables for the line i

ν_t = line fixed effect for year t over the sample periods.

η_i = year fixed effect for line i

We control line fixed effects and time fixed effects in our regression analysis. C_{it} is calculated as the logarithm of insurance surplus allocated in line i at the end of year $t-1$. C_{it}^2 is the square term of C_{it}. We thus expect that the coefficient of C_{it} is negative while the coefficient of C_{it}^2 is positive. Moreover, we anticipate the coefficient of R_{it} is positive.

For the robustness check of Hypothesis 1, we divide our sample into two groups: one group where the capital allocated to lines of insurance is above the threshold level 85 trillion dollars[1], and the other group where capital allocated to lines of insurance is below the threshold. The regression specification for both groups is as follows:

$$P_{it} = \beta_0 + \beta_1 C_{it} + \beta_3 R_{it} + \gamma' X_i + \nu_t + \eta_i + \epsilon_{it}$$

[1]The threshold level is determined by the coefficient of the first regression equation

We expect the sign of the β_1 is positive for the first group with higher level of internal capital, while negative for the second group with lower level of internal capital.

3.4 Empirical Results

Summary statistics for the variables included in the regression analysis are shown in Table 3.1 based on the aggregated line level data.

The regression results based on the lines of property and liability insurance are presented in table 2. Several specifications are presented. The results in table 2 provide strong support for both hypotheses. Without adding both the proxy for internal capital and the square term, the coefficients are not significant. Both the simple OLS and Fixed Effects models predict significant negative coefficient of the proxy for internal capital and positive coefficient of the square term. It supports the non-monotonic relationship between insurance price and internal capital. It shows that the insurance price is negatively correlated with internal capital at first, and then positively correlated with internal capital as the level of internal capital is above certain threshold.

Besides, the results presented in table 2 also support the Hypothesis 2. The coefficient of asset investment portfolio risk is significantly positive, which suggests that the insurance price is positively related with asset investment portfolio risk. It supports our theoretical predictions in Charter 2

The results based on subgroup robustness test for hypothesis 1 are presented in table 3. We divide the total sample into two subgroups: one with internal capital level below 8.5 billion, and the other one with capital level above 8.5 billion. For the low internal capital group, the sign of the coefficient of internal capital is significantly negative. In contrast, for the high internal capital group, the sign of the coefficient of internal capital is significantly positive. The results in table 3 show further support for the hypothesis about the relationship between insurance price and internal capital in the U-shaped manner. It could potential reconcile the conflicting results predicted by the previous empirical studies either focusing on line level or firm level analysis.

3.5 Conclusion

The unsolved "puzzle" of the relationship between insurance price and internal capital are supported by either supply driven theory or demand driven theory. The existing empirical studies also show mixed support. The non monotonic relationship predicted by the equilibrium effects, however could potentially reconcile the controversial results theoretically. We study all the property and casualty industry lines in periods from 1992 to 2012. We show that the internal capital decrease with an increase in the surplus of insurance firms at the end of the previous year when the surplus is lower than 8.5 billion, and then increase when the surplus is higher than 8.5 billion. Our results provide support for the hypothesis of a U-shaped relationship between internal capital and insurance price. Our results also provide evidence for the positive relationship between asset portfolio risk and insurance price.

3.6 Appendix

Table 3.1: Summary Statistics for Line Level Data: 1992-2012

Statistic	N	Mean	St. Dev.	Min	Max
Net Premium Written	588	24,186,545,526	43,054,623,809	89,298,769	236,284,657,645
Loss Adjustment Expenses	588	4,218,219,734	7,838,413,599	−3,978,015	71,068,656,869
Underwriting Expenses	588	5,238,684,419	10,292,170,313	−3,331,025	62,159,308,812
Asset Risk	588	−1.630	0.454	−2.787	−0.669
Insurance Price	588	1.406	1.714	0.045	21.857
Internal Capital	560	22.706	1.773	17.706	27.431
Assets	588	7.776	14.277	0.018	207.864

Table 3.2: Regression Results at Aggregate Line Level: 1993 - 2012

	Dependent variable:			
	Insurance Price			
	OLS	*Fixed Effect*	*Panel Fixed Effect*	*Fixed Effect*
	(1)	(2)	(3)	(4)
Internal Capital	−3.899***	−0.202		−3.619**
	(0.906)	(0.147)		(1.467)
Internal Capital Square	0.082***		−0.004	0.078**
	(0.021)		(0.003)	(0.033)
Asset Size	−0.000	−0.000	−0.000	−0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Asset Risk (OAR)	1.354***	1.296**	1.300**	1.374**
	(0.479)	(0.536)	(0.536)	(0.535)
Constant	49.939***			
	(10.047)			
Observations	560	560	560	560
R^2	0.139	0.057	0.055	0.067
Adjusted R^2	0.102	0.051	0.050	0.061
Residual Std. Error	1.641 (df = 536)			
F Statistic	3.758*** (df = 23; 536)	1.389 (df = 22; 510)	1.360 (df = 22; 510)	1.579** (df = 23; 509)

Note: *p<0.1; **p<0.05; ***p<0.01

Table 3.3: Regression Results for Two Subgroups with Different Levels of Internal Capital

	Dependent variable:			
	Insurance Price by Line			
	Low Internal Capital		*High Internal Capital*	
	OLS	*Panel FE*	*OLS*	*Panel FE*
	(1)	(2)	(3)	(4)
Internal Capital	−0.562***	−0.602***	0.028	0.536**
	(0.131)	(0.208)	(0.121)	(0.243)
Asset Size	−0.000	−0.000*	−0.000	−0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Asset Risk (OAR)	0.676	1.053	1.714***	0.713
	(0.780)	(0.805)	(0.567)	(0.619)
Constant	15.218***		3.678	
	(2.901)		(2.832)	
Observations	302	302	258	258
R^2	0.182	0.143	0.120	0.119
Adjusted R^2	0.118	0.122	0.038	0.100
Residual Std. Error	1.874 (df = 279)		1.306 (df = 235)	
F Statistic	2.827*** (df = 22; 279)	1.957*** (df = 22; 259)	1.458* (df = 22; 235)	1.329 (df = 22; 217)
Note:				*p<0.1; **p<0.05; ***p<0.01

Chapter 4

Insurance Solvency Regulation

4.1 Introduction

There are three sources of inefficiencies in the unregulated economy as analyzed in the previous section that stem from the fact that markets are incomplete. First, each insurer makes its insurance supply decisions and investment decisions incorporating its *individual* asset return distribution without fully internalizing the potential correlation of asset returns across insurers arising from the fact that a proportion τ of insurers is exposed to a common shock. Without considering aggregate risk, insurers may hold insufficient liquidity reserves and over-invest their capital in risky assets. Second, insurees' idiosyncratic losses may not be fully insured by insurers when insurers' internal capital is relatively low. Insurees bear insurers' default risk driven by the asset side of their balance sheets when there are no effective risk sharing mechanisms among insurers to share their asset risk because there are no traded Arrow-Debreu securities contingent on insurers' individual asset realizations or the realization of the aggregate shock. Third, insurees do not have direct access to the risky assets with insurance firms also serving as intermediaries that channels the insurees' capital into more productive risky assets. Insurers, however, cannot effectively share the investment risk with insurees through the insurance policies that only protect insurees' losses without combining investment returns to insurees.

The equilibrium price and insurance coverage level in the unregulated economy, therefore,

do not internalize the externalities created by aggregate risk of insurers' assets and the lack of instruments that achieve full risk-sharing. Consequently, we potentially have a misallocation of insuree capital to the purchase of insurance and misallocation of insurer capital to safe and risky assets. Regulatory intervention could improve allocative efficiency by internalizing the externalities created by aggregate risk, imposing necessary liquidity reserve requirements to influence insurers' investment decisions, and also providing *risk sharing mechanisms* through *ex post* taxation transfers among insurers.

In this chapter, we first proceed to analyze the implications of our framework for the solvency regulation of insurers by analyzing the benchmark "first best" economy in which aggregate risk is fully internalized and there is perfect risk-sharing among insurees and insurers. We derive the Pareto optimal allocation of insurer capital between the safe asset (liquidity reserves) and risky assets as well as the sharing of risk between insurers and insurees. When the aggregate risk is low, there is sufficient aggregate capital in the economy to provide full insurance to insurees so that insurers bear all the aggregat risk. Further, because the expected return from risky assets exceeds the risk-free return, it is optimal to allocate all capital to risky assets so that neither insurers nor insurees have holdings in the risk-free asset. When aggregate risk takes intermediate values, insurees cannot be provided with full insurance because of the limited liability of insurers in the bad aggregate state. Consequently, insurers and insurees share aggregate risk, but it is still optimal to exploit the higher expected surplus generated by the risky assets so that all the capital in the economy is invested in the risky assets. When aggregate risk is very high, however, risk-averse insurees would bear excessively high losses in the bad aggregate state if all capital were invested in risky assets. Consequently, both insurees and insurers hold positive liquidity reserves, and share aggregate risk.

We also demonstrate that a regulator/social planner can implement the first-best allocation policies through a combination of comprehensive insurance policies sold by insurers that combine insurance with investment, reinsurance, a minimum liquidity requirement, and ex post budget-neutral taxation that is contingent on the aggregate state. The comprehensive insurance policies provide direct access to the risky assets for insurees. Reinsurance achieves

risk-sharing among insurers, while ex post taxation transfers funds from solvent to insolvent insurers. The minimum liquidity requrement, which is only imposed when aggregate risk exceeds a threshold, forces insurers to maintain the first best level of liquidity reserves.

4.2 Benchmark First Best Scenario

We begin by studying a hypothetical benchmark scenario that full internalizes the inefficiencies in the unregulated economy due to aggregate risk and imperfect risk sharing mechanisms among insurees and insurers. In this benchmark economy, there is perfect sharing of the idiosyncratic risk of insuree losses among insurees and idiosyncratic risks of asset returns among insurers. Consequently, insurers and insurees are only exposed to the aggregate shock. Without loss of generality, we can assume that there is a single representative risk averse insuree with 1 unit of the capital good and a single representative risk neutral insurer with K units of the capital good. Both the insuree and the insurer have access to risky assets that may be subject to aggregate shocks.

We examine efficient (Pareto optimal) allocations in the benchmark economy. Pareto optimal allocations must only be contingent on the aggregate state of the economy. With probability q, the economy is in the "bad" aggregate state where a proportion τ of risky investments earn a low return R_L. In the bad aggregate state, the return per unit of capital invested is M^L, where $M^L = (1-q)(1-\tau)R_H + q(1-\tau)R_L + \tau R_L$. With probability $1-q$, the economy is in the "good" aggregate state where a proportion τ of the risky investments earn a high rate of return R_H. In the good aggregate state, the return per unit capital invested is M^H, where $M^H = (1-q)(1-\tau)R_H + q(1-\tau)R_L + \tau R_H$. The insurer provides insurance to cover the insuree's loss, but also shares the aggregate risk associated with investments in the risky assets.

Let C^H and C^L be the representative insurer's combined returns from investing capital in the risky technology and selling insurance in the good and bad aggregate states, respectively. The representative insurer invests a proportion α of its capital in the safe asset and the remaining proportion $1-\alpha$ in the risky asset. The insuree invests a proportion β of its

capital in safe assets and the rest in purchasing risky insurance. Let D^H and D^L be the net payoffs received by the representative insuree in the good and bad state, respectively, which includes individual losses and returns from risky assets and/or insurance. We focus on the Pareto optimal allocation in which the representative insurer receives its autarkic payoff. Consequently, the planning problem is

$$\max_{\beta,\alpha,D^L,D^H} \underbrace{q\ln(\beta R_f + D^L)}_{\text{bad aggregate state}} + \underbrace{(1-q)\ln(\beta R_f + D^H)}_{\text{good aggregate state}} \tag{4.1}$$

subject to

$$\alpha K R_f + [(1-q)C^H + qC^L] = K[(1-q)R_H + qR_L] \tag{4.2}$$

$$D^L + C^L = [(1-\beta) + (1-\alpha)K]M^L - pl \tag{4.3}$$

$$D^H + C^H = [(1-\beta) + (1-\alpha)K]M^H - pl \tag{4.4}$$

$$\alpha K R_f + C^L \geq 0 \tag{4.5}$$

$$\alpha K R_f + C^H \geq 0 \tag{4.6}$$

Equations (4.3) and (4.4) capture the fact that the total payoffs to the representative insuree and insurer in the two aggregate states must equal the aggregate payoff from the investments net of the loss incurred by the insuree. Because there is perfect sharing of insuree loss risks, the total loss incurred by the representative insuree is pl. Equations (4.5) and (4.6) are limited liability constraints for the representative insurer in the two aggregate states.

The following proposition shows the optimal asset allocation between risky and safe assets, and the optimal risk allocation among the representative insuree and insurer.

Proposition 9 (Benchmark Asset allocation and Risk Sharing among Insurees and Insurers)

1. *Suppose* $K \geq \frac{ER-R_L}{R_L}$. *Regardless of the value of* τ, $\beta^* = 0, \quad \alpha^* = 0$, *that is, both the insuree and insurer invest nothing in the safe asset. The representative insuree is fully insured against losses and investment returns, that is, the returns to the insuree per*

unit of capital invested in the good and bad aggregate states are equal.

$$D^H = D^L = D^* = ER - pl \tag{4.7}$$

2. *Suppose*

   (a) (*i.*) *either* $q < 0.5$, *and* $K < \min\left(\frac{ER-R_L}{R_L}, \frac{(ER-pl)(ER-R_f)}{ER\cdot R_f}\frac{1-q}{1-2q}\right)$, *or* (*ii.*) $q > 0.5$ *and* $K < \frac{ER-R_L}{R_L}$

   (b) $(ER + R_f - R_H - R_L)R_f + pl(ER - R_f) < 0$

   - *When* $\tau \leq \tau_1$, *where* $\tau_1 = \frac{K\cdot ER}{(1+K)(ER-R_L)}$, $\beta^* = 0$, $\alpha^* = 0$, *that is, both the insuree and insurer invest nothing in the safe asset. The representative insuree is fully insured against losses and investment returns, that is, the returns to the insuree per unit of capital invested in the good and bad aggregate states are equal, same as* (4.7)

   - *When* $\tau_1 < \tau < \tau_2$, *where*

$$\begin{aligned}\tau_2 &= \frac{(1-q)(1+K)(R_H+R_L-2ER)ER + q\cdot ER\cdot K(ER-R_L)}{2(1-q)(1+K)(R_H-ER)(ER-R_L)} \\ &+ \frac{\sqrt{\begin{array}{c}\Big((1-q)(1+K)(R_H+R_L-2ER)ER + q\cdot ER\cdot K(ER-R_L)\Big)^2 \\ -4\Big((1-q)(1+K)(R_H-ER)(ER-R_L)\Big)\Big(\big(qK \\ -(1-q)(1+K)\big)\cdot ER + pl(1-q)\Big)(ER-R_f)\end{array}}}{2(1-q)(1+K)(R_H-ER)(ER-R_L)},\end{aligned}$$

   $\beta^* = 0$, *and* $\alpha^* = 0$, *that is, the insuree and insure continue to invest nothing in the safe asset. Insurees are imperfectly insured; the returns per unit of capital invested in the good and bad aggregate states are, respectively*

$$D^H = (1+K)M^H - pl - \frac{ER}{1-q}K, \qquad D^L = (1+K)M^L - pl$$

The insurer's limited liability constraint 4.5 binds, and its returns per unit of capital

of invested in the in good and bad aggregate states are, respectively

$$C^H = \frac{ER}{1-q}K \qquad\qquad C^L = 0$$

- *when $\tau > \tau_2$, there is nonzero investment in the safe asset with*

$$\beta^* + \alpha^* K = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)}$$

The insuree is imperfectly insured, and its returns per unit of capital invested in the good and bad aggregate states are, respectively

$$D^H = \big(1 + K - (\beta^* + \alpha^* K)\big)M^H - pl - \frac{ER}{1-q}K + \alpha^* K R_f$$
$$D^L = \big(1 + K - (\beta^* + \alpha^* K)\big)M^L - pl + \alpha^* K R_f$$

The insurer's limited liability constraint 4.5 binds, and its returns per unit of capital of investing in the good and bad aggregate states are, respectively

$$C^H = \frac{ER}{1-q}K - \alpha^* K \cdot R_f \qquad\qquad C^L = -\alpha^* K \cdot R_f$$

The above proposition shows the effects of aggregate risk on the optimal asset allocation and risk sharing among insurees and insurers. When the internal capital, K, is greater than the threshold level $\frac{ER-R_L}{R_L}$ [1], insurers always have adequate capital to insure its promised payments to insurees even in the "bad" state where aggregate shocks to the asset occur. Thus it is optimal to invest all social capital in risky assets to produce the highest expected returns from investments. Insurees are fully insured by insurers, and the aggregate shocks are completely borne by insurers.

However, suppose the internal capital, K, is below the threshold level, there are three subcases relying on the measurement of aggregate shocks, τ. When the aggregate risk τ is

[1] $\frac{ER-R_L}{R_L} \geq \overline{K}_1$. When insurers sell comprehensive insurance contracts, the minimum level of internal capital to keep insurer solvency in "bad" state is higher than that in the unregulated economy

relatively low, insuree's idiosyncratic losses and returns from investment in risky assets can be fully insured by insurers. Thus it is optimal to invest all social capital in risky assets to produce the highest expected returns from investments as the situation where insurers are endowed with sufficient capital. When the aggregate risk measure τ takes intermediate values, there may not be enough capital in the bad aggregate state to cover insuree losses. The representative insurer, therefore, defaults and its limited liability constraint in that state is binding. It is, however, still optimal for all capital to be invested in risky assets.[2] When the aggregate risk τ is above a high threshold, however, the marginal increase in the expected return from investments in the risky assets is insufficient to compensate for the disutility to the representative insuree arising from the imperfect insurance payoffs due to aggregate shocks. It is, therefore, optimal to hold a certain amount in safe assets, that is, to maintain a nonzero liquidity buffer. Figure 4.1 summarizes the relationship between aggregate risk measure τ and the optimal investment in safe assets. It reflects the tradeoffs between total allocative returns from investments and the risk sharing among insurees and insurers. When the aggregate risk is low, the total allocative capital reaches the maximum level, and insurees are fully insured, and insurers take all aggregate risk. When the aggregate risk is in the intermediate level, the total allocative capital also reaches the highest level, and insurees are imperfectly insured, and insurees and insurers share the aggregate asset risk. When the aggregate risk is high, the marginal decrease in the total allocative capital due to some investment in safe assets trades off the wedge between insurance claims received by insurees in good and bad aggregate states.

We next analyze how the benchmark level of investment portfolios and risk sharing can be implemented through regulatory intervention.

[2]When asset default probability is sufficiently high and insurer's internal capital is relatively low, the marginal increase in total expected allocative capital returns from risky assets may be insufficient to compensate the disutility arising from the imperfect insurance payoffs due to aggregate shocks. It, thus may be optimal to hold some safe assets as in the third case.

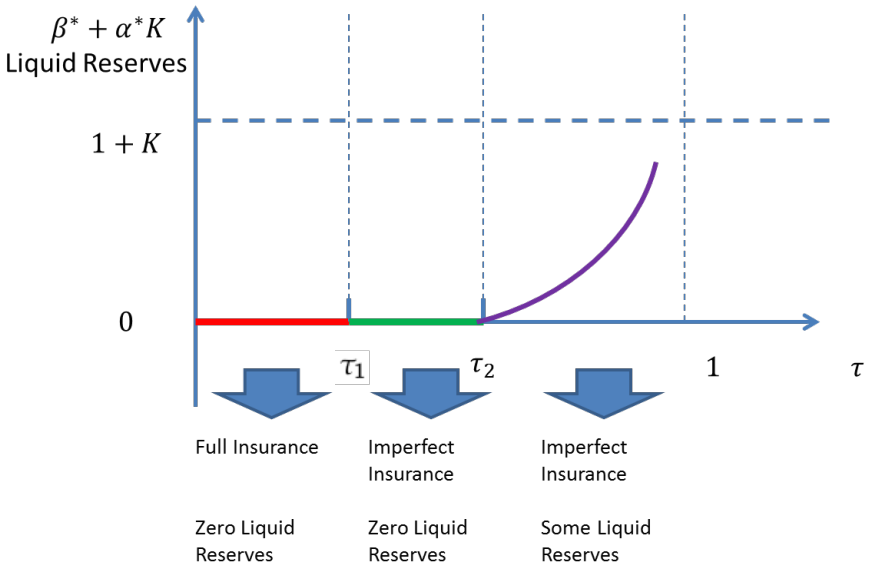

Figure 4.1: Aggregate Risk and Liquidity Reserves Buffer

4.3 Regulatory Intervention

As discussed earlier, the inefficient investment allocation and imperfect risk-sharing in the unregulated economy relative to the first-best benchmark arises from three factors: the imperfect sharing of idiosyncratic loss risk among insurees, imperfect asset risk sharing among insurees and insurers, and the incomplete internalization of the effects of aggregate risk on insurers' investment portfolios and the provision of insurance. The above three factors provide regulators the room to reduce the market inefficiency using comprehensive tools.

4.3.1 Taxation and Idiosyncratic Risk

In the unregulated economy, there is no effective idiosyncratic risk sharing mechanism among insurers. It follows that insurees bear the default risk driven by the idiosyncratic component of an insurer's asset risk when the insurer's internal capital is sufficiently low. In the regulated economy, the regulators can serve as a "reinsurer" by taxing the insurers whose risky assets succeed and reinsuring the insurers whose risky assets fail. This *ex post* taxation contingent on the aggregate state is very similar to "insurance guarantee funds" run by state regulators. This mechanism can fully insure insurer's idiosyncratic asset risk, but not the aggregate risk.

Taxation and reinsurance, therefore, depend on the aggregate state of the economy. Let T_S^H and T_F^H be the taxation transfers from successful and failed insurers, respectively in the good aggregate state. Also, let T_S^H and T_F^H be the taxation transfers from successful and failed insurers, respectively in the bad aggregate state. A positive transfer means receiving a subsidy, while a negative transfer means a tax payout. Thus the tax balance condition in both good and bad aggregate state are:

$$\big((1-q)(1-\tau)+\tau\big)T_S^H + q(1-\tau)T_F^H = 0$$
$$(1-q)(1-\tau)T_S^L + \big(q(1-\tau)+\tau\big)T_F^L = 0$$

4.3.2 Comprehensive Insurance and Optimal Risk Sharing

In the unregulated economy, insurers provide insurance to cover individual insuree losses, and also serve as financial intermediaries to channel insuree capital to more productive assets. Because asset markets are incomplete, there is imperfect sharing of aggregate asset risk among insurees and insurers. In the regulated economy, we can implement the first best allocation if insurers sell *comprehensive insurance* policies that combine loss protection and investment returns. Let d_l^H/d_{nl}^H be the returns per unit of capital invested in comprehensive insurance policies in the good aggregate state where insurees incur idiosyncratic loss/no loss, and d_l^L/d_{nl}^L be the returns per unit of capital invested in comprehensive insurance policies in the bad aggregate state where insurees incur idiosyncratic loss/no loss.

4.3.3 Liquidity Requirement and Aggregate Risk

Proposition 9 and Figure 4.1 show the optimality of investing a nonzero amount of the total capital in the safe asset when the measure of aggregate shocks is above the threshold, τ_2. The regulator can enforce this asset allocation by imposing a minimum liquidity requirement when aggregate risk is high enough. It is worth emphasizing here that what matters for the allocation of capital is the *total amount*, $\beta^* + \alpha^* K$, in the safe asset. The regulator can also implement this outcome through *ex ante* taxation. Specifically, the regulator can tax insuree capital at the rate β^*, insurer internal capital at the rate α^*, and invest the proceeds in the

safe asset. The regulator can then use the proceeds from this investment to make transfers to insurers and insurees and, thereby, implement the efficient allocation.

4.3.4 Comprehensive Regulatory Intervention

The following proposition describes how the above comprehensive tools can be used to achieve the first best benchmark level of investment allocation and aggregate risk sharing among insurees and insurers.

Proposition 10 (Regulatory Intervention) *Suppose*

1. *(i.) either* $q < 0.5$*, and* $K < \min\left(\frac{ER-R_L}{R_L}, \frac{(ER-pl)(ER-R_f)}{ER \cdot R_f}\frac{1-q}{1-2q}\right)$*, or (ii.)* $q > 0.5$ *and* $K < \frac{ER-R_L}{R_L}$
2. $(ER + R_f - R_H - R_L)R_f + pl(ER - R_f) < 0$

- *When* $\tau \leq \tau_1$*, the regulator imposes no liquidity requirement. Insurees and insurers invest everything in risky assets so that*

$$\beta^* = 0, \qquad \alpha^* = 0$$

The optimal returns per unit of capital invested in the comprehensive insurance policy in the good and bad aggregate states are the same, that is

$$d_{nl}^H = d_{nl}^L = ER - pl \qquad d_l^H = d_l^L = ER + (1-p)l$$

Insurees are fully insured against idiosyncratic losses and asset risk. Insurers bear idiosyncratic and aggregate asset risk through the following taxation scheme

$$\begin{cases} T_S^L &= (1+K)(M^L - R_H) \\ T_F^L &= (1+K)(M^L - R_L) \\ T_S^H &= (1+K)(M^H - R_H) \\ T_F^H &= (1+K)(M^H - R_L) \end{cases} \tag{4.8}$$

- *When $\tau_1 < \tau \leq \tau_2$, the regulator imposes no liquidity requirement. Insurees and insurers invest everything in risky assets so that*

$$\beta^* = 0, \qquad \alpha^* = 0$$

The optimal returns per unit of capital invested in the comprehensive insurance policy in the good and bad aggregate states are unequal and are given by

$$d_{nl}^H = (1+K)M^H - \frac{ER}{1-q}K - pl \qquad d_{nl}^L = (1+K)M^L - pl \tag{4.9}$$

$$d_l^H = (1+K)M^H - \frac{ER}{1-q}K + (1-p)l \qquad d_l^L = (1+K)M^L + (1-p)l \tag{4.10}$$

Insurers bear the idiosyncratic loss risk of insurees as well as idiosyncratic asset risk through the taxation scheme as 4.8. Aggregate risk is, however, shared among insurees and insurers through the comprehensive insurance policy 4.9.

- *When $\tau > \tau_2$, the regulator imposes a liquidity requirement on insurers and insurees that is given by*

$$\alpha^* \in \left(\max\left\{ \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{K(M^H - R_f)(R_f - M^L)} - \frac{1}{K},\ 0 \right\},\ 1 \right)$$

$$\beta^* = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha^* K,$$

Alternately, the regulator can levy ex ante taxes at the rate $\alpha^*$for insurers and $\beta^*$for insurees and invest the proceeds in the safe asset.

- *The optimal return per unit of capital invested in the comprehensive insurance policy*

in the good and bad aggregate states are unequal and given by

$$\begin{aligned} d_{nl}^{H} &= \frac{(1+K)M^{H}+(\beta+\alpha K)(R_f-M^{H})-\frac{ER}{1-q}K-\beta^{*}R_f-pl}{1-\beta^{*}} \\ d_{nl}^{L} &= \frac{(1+K)M^{L}+(\beta^{*}+\alpha^{*}K)(R_f-M^{L})-\beta^{*}R_f-pl}{1-\beta^{*}} \end{aligned} \tag{4.11}$$

$$\begin{aligned} d_{l}^{H} &= \frac{(1+K)M^{H}+(\beta+\alpha K)(R_f-M^{H})-\frac{ER}{1-q}K-\beta^{*}R_f+(1-p)l}{1-\beta^{*}} \\ d_{l}^{L} &= \frac{(1+K)M^{L}+(\beta^{*}+\alpha^{*}K)(R_f-M^{L})-\beta^{*}R_f+(1-p)l}{1-\beta^{*}} \end{aligned} \tag{4.12}$$

Insuree' idiosyncratic losses and idiosyncratic asset risk are fully taken by insurers through the taxation scheme as follows:

$$\begin{cases} T_S^L &= (1+K-(\beta^{*}+\alpha^{*}K))(M^{L}-R_H) \\ T_F^L &= (1+K-(\beta^{*}+\alpha^{*}K))(M^{L}-R_L) \\ T_S^H &= \big(1+K-(\beta^{*}+\alpha^{*}K)\big)(M^{H}-R_H) \\ T_F^H &= \big(1+K-(\beta^{*}+\alpha^{*}K)\big)(M^{H}-R_L) \end{cases} \tag{4.13}$$

aggregate risk is shared by insurees and insurers through the comprehensive insurance policy as 4.12.

The above proposition implies that the comprehensive tools can be used by regulators to reduce the inefficiencies of unregulated economy. *Ex post* taxation contingent on the aggregate state, plays the role of "insurance guarantee funds", which induces insurers to fully absorb insurees' idiosyncratic loss risk when their internal capital is relatively low. *Comprehensive insurance policies* combining insurance with investment, together with *ex post* taxation, enhance aggregate risk sharing between insurees and insurers. The *liquidity requirement* adjusts inefficiencies arising from insurer's misallocation of their assets and the optimal aggregate risk sharing among insurees and insurers. Thus, when aggregate risk is high enough, the optimal investment allocation reflects the tradeoff between the growth of total assets and insurees' aversion to aggregate risk.

4.4 Conclusions

We derive insights into the solvency regulation of insurers by deriving the Pareto optimal allocation of insurer capital to liquidity reserves and risky assets as well as risk sharing among insurees and insurers. We show that, when aggregate risk is below a threshold, it is Pareto optimal for insurers and insurees to hold zero liquidity reserves, insurees are fully insured, and insurers bear all aggregate risk. When aggregate risk takes intermediate values, both insurees and insurers still hold no liquidity reserves, but insurees partially share aggregate risk with insurers. When the aggregate risk is high, however, both insurees and insurers hold nonzero liquidity reserves, and insurees partially share aggregate risk with insurers. We demonstrate that the efficient allocation can be implemented through regulatory intervention that comprises of comprehensive insurance policies that combine insurance and investment, reinsurance, a minimum liquidity requirement when aggregate risk is high, and ex post budget-neutral taxation and subsidies contingent on the realized aggregate state.

In future research, it would be interesting to develop a dynamic structural model of insurance markets. The analysis of such a model that is suitably calibrated to data could generate quantitative insights into the optimal regulation of insurance markets.

4.5 Appendix: Proofs

Proof of Proposition 9

Proof. We show the Pareto optimal allocation planning problem is maximizing (4.1) subject to (4.2),(4.3),(4.4),(4.5) and (4.6).

We substitute D^L and D^H with C^H and C^H using the relationships implied by(4.3) and (4.4). (4.6) can be omitted if $C^H \geq C^L$. Let λ and μ are the Lagrangian multiplier associate with (4.2) and (4.5), respectively. Thus

$$\begin{aligned}\mathcal{L} &= q\ln(\beta R_f + W^L - C^L) + (1-q)\ln(\beta R_f + W^H - C^H) \\ &\quad + \lambda\{\alpha K R_f + [(1-q)C^H + qC^L] - K[(1-q)R_H + qR_L]\} + \mu\{\alpha K R_f + C^L\}\end{aligned}$$

The first order condition with respect to C^H and C^L are, respectively:

$$\begin{aligned}\partial C^L &: -\frac{q}{\beta R_f + W^L - C^L} + \lambda q + \mu = 0 \\ \partial C^H &: -\frac{1-q}{\beta R_f + W^H - C^H} + \lambda(1-q) = 0\end{aligned} \tag{4.14}$$

We first suppose $\mu = 0$. Equations (4.14) imply $W^H - C^H = W^L - C^L$, and the relationship between C^H and C^L is

$$C^H = C^L + [(1-\beta) + (1-\alpha)K]\tau(R_H - R_L)$$

Plugging above relation into equation (4.2), we have

$$\begin{aligned}C^{L*} &= K\big(ER - \alpha R_f\big) - (1-q)[(1-\beta) + (1-\alpha)K]\tau(R_H - R_L) \\ C^{H*} &= K\big(ER - \alpha R_f\big) + q[(1-\beta) + (1-\alpha)K]\tau(R_H - R_L) \\ D^* &= D^{H*} = D^{L*} = (1-\beta)ER - \alpha K\big(ER - R_f\big) - pl\end{aligned}$$

where $ER = (1-q)R_H + qR_L$ as defined in Section 2.3.1. The insuree is fully insured, and its utility is:

$$\begin{aligned}EU_{\text{insuree}} &= \ln\big(\beta R_f + D^*\big) \\ &= \ln\big(-\beta(ER - R_f) - \alpha K(ER - R_f) + ER - pl\big)\end{aligned}$$

We now derive the optimal level of investment in safe assets.

$$\max_{\alpha,\beta} \ln\big(-(\beta + \alpha K)(ER - R_f) + ER - pl\big) \tag{4.15}$$

subject to

$$\begin{aligned}\alpha K R_f + C^{L*} &\geq 0 \\ 0 \leq \beta + \alpha K &\leq 1 + K\end{aligned}$$

Since the objective function, (4.15), is a decreasing function of $(\beta + \alpha K)$, thus $\beta^* + \alpha^* K = 0$.

The above constraint (4.16) can be simplified as follows:

$$\beta + \alpha K \leq \frac{KER}{(ER - R_L)\tau} - (1 + K)$$

Suppose $K \geq \frac{ER-R_L}{R_L}$, $\frac{KER}{(ER-R_L)\tau} - (1+K) \leq 0$ for any value of τ. In other words, (4.16) can be omitted, and the optimal level of investment in safe assets is zero. The Part 1 of Proposition 9, thus holds.

Suppose $K < \frac{ER-R_L}{R_L}$, $\frac{KER}{(ER-R_L)\tau} - (1+K) \leq 0$ still holds for any τ such that $\tau \leq \tau_1$ where $\tau_1 = \frac{K \cdot ER}{(1+K)(ER-R_L)}$. Similarly, (4.16) can also be omitted, and the optimal level of investment in safe assets is zero. The first case of Part 2 of Proposition 9 holds.

However, if $\tau > \tau_1$, then the optimal level of investment in safe assets is determined by (4.16) when it binds. which contradicts with $\mu = 0$. Consequently, there does not exist the case where insurees are fully when $\tau > \tau_1$.

Now we suppose $\mu > 0$, and limited liability constraint of insurers in "bad" aggregate state, (4.5), binds; that is $\alpha K R_f + C^L = 0$. Thus $C^L = -\alpha K R_f$ and $C_H = \frac{ER-(1-q)\alpha R_f}{1-q} K$. It is easy to show

$$\begin{aligned} D^L &= W^L - C^L = [1 + K - (\beta + \alpha K)]M^L - pl + \alpha K R_f \\ D^H &= W^H - C^H = [1 + K - (\beta + \alpha K)]M^H - pl - \frac{ER}{1-q} K + \alpha K R_f \end{aligned}$$

Thus insurees' total capital in "good" and "bad" aggregate states, receptively, are

$$\begin{aligned} N^L &= \beta R_f + D^L = (1 + K)M^L + (\beta + \alpha K)(R_f - M^L) - pl \\ N^H &= \beta R_f + D^H = (1 + K)M^H + (\beta + \alpha K)(R_f - M^H) - pl - \frac{ER}{1-q} K \end{aligned}$$

We now solve for the optimal level of investment in safe assets

$$\begin{aligned} \max_{\alpha,\beta} q \ln\big((1 + K)M^L + (\beta + \alpha K)(R_f - M^L) - pl\big) + (1 - q)\ln((1 + K)M^H \\ +(\beta + \alpha K)(R_f - M^H) - pl - \tfrac{ER}{1-q} K) \end{aligned}$$

subject to

$$0 \leq \beta + \alpha K \quad \leq \quad 1 + K$$

The Lagrangian function is

$$\begin{aligned} \mathcal{L} = q \ln\big((1 + K)M^L + (\beta + \alpha K)(R_f - M^L) - pl\big) + (1 - q)\ln((1 + K)M^H \\ + (\beta + \alpha K)(R_f - M^H) - pl - \frac{ER}{1-q} K) - \lambda_1(1 + K - (\beta + \alpha K)) - \lambda_2(\beta + \alpha K) \end{aligned}$$

The first order condition with respect to $(\beta + \alpha K)$ that is

$$\frac{q(R_f - M^L)}{N^L} - \frac{(1 - q)(M^H - R_f)}{N^H} + \lambda_1 - \lambda_2 = 0$$

Suppose $\lambda_1 = \lambda_2 = 0$, then $\frac{q(R_f - M^L)}{N^L} = \frac{(1-q)(M^H - R_f)}{N^H}$ That is

$$\begin{aligned} & q(R_f - M^L)\big((1+K)M^H + (\beta + \alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K\big) \\ = \;& (1-q)(M^H - R_f)\big((1+K)M^L + (\beta + \alpha K)(R_f - M^L) - pl\big) \end{aligned}$$

Rearrange the above equations, we have

$$\beta + \alpha K = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)}$$

Now we have to check $0 < \beta + \alpha K < 1 + K$.

We first whether $\beta + \alpha K > 0$ holds, that is

$$(1-q)(1+K)(R_H - ER)(ER - R_L)\tau^2 - \Big((1-q)(1+K)(R_H + R_L - 2ER)ER$$
$$+q \cdot ER \cdot K(ER - R_L)\Big)\tau + \Big(\big(qK - (1-q)(1+K)\big) \cdot ER + pl(1-q)\Big)(ER - R_f) > 0$$

We need $\tau \le \tau_2'$ or $\tau \ge \tau_2$ to make above inequality hold, where

$$\begin{aligned} \tau_2' \;=\; & \frac{(1-q)(1+K)(R_H + R_L - 2ER)ER + q \cdot ER \cdot K(ER - R_L)}{2(1-q)(1+K)(R_H - ER)(ER - R_L)} \\ & - \frac{\sqrt{\begin{array}{c}\Big((1-q)(1+K)(R_H + R_L - 2ER)ER + q \cdot ER \cdot K(ER - R_L)\Big)^2 \\ -4\Big((1-q)(1+K)(R_H - ER)(ER - R_L)\Big)\Big(\big(qK \\ -(1-q)(1+K)\big) \cdot ER + pl(1-q)\Big)(ER - R_f)\end{array}}}{2(1-q)(1+K)(R_H - ER)(ER - R_L)} \end{aligned}$$

$$\begin{aligned} \tau_2 \;=\; & \frac{(1-q)(1+K)(R_H + R_L - 2ER)ER + q \cdot ER \cdot K(ER - R_L)}{2(1-q)(1+K)(R_H - ER)(ER - R_L)} \\ & + \frac{\sqrt{\begin{array}{c}\Big((1-q)(1+K)(R_H + R_L - 2ER)ER + q \cdot ER \cdot K(ER - R_L)\Big)^2 \\ -4\Big((1-q)(1+K)(R_H - ER)(ER - R_L)\Big)\Big(\big(qK \\ -(1-q)(1+K)\big) \cdot ER + pl(1-q)\Big)(ER - R_f)\end{array}}}{2(1-q)(1+K)(R_H - ER)(ER - R_L)} \end{aligned}$$

We now compare τ_1 and τ_2'. When $\tau = \tau_1$, we check the value of $\beta + \alpha K|_{\tau=\tau_1}$, that is,

$$\beta + \alpha K|_{\tau=\tau_1} = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)}$$

that is whether

$$(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q} < 0$$

The above inequality is equivalent to

$$\big(1 + K\frac{1-2q}{1-q}\big) \cdot ER \cdot R_f < ER^2 - pl(ER - R_f)$$

Therefore, when $q < 0.5$, and $K < \min\Big(\frac{ER-R_L}{R_L}, \frac{(ER-pl)(ER-R_f)}{ER \cdot R_f}\frac{1-q}{1-2q}\Big)$, or when $q > 0.5$, and $K < \frac{ER-R_L}{R_L}$, we have $\beta + \alpha K|_{\tau=\left(\frac{K}{1+K}\right)\left(\frac{ER}{ER-R_L}\right)} < 0$. In other words, $\tau_2' < \tau_1$.

We next check when $\tau = 1$, whether $\beta + \alpha K < 1 + K$.

When $\tau = 1$, we have $M^L = R_L$, and $M^H = R_H$.

$$\begin{aligned}
\beta + \alpha K - (1+K) &= \frac{(1+K)(ER \cdot R_f - R_H R_L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - R_L)}{1-q}}{(R_H - R_f)(R_f - R_L)} - (1+K) \\
&= \frac{(1+K)(ER \cdot R_f - R_H R_L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - R_L)}{1-q} + (1+K)(R_f - R_H)(R_f - R_L)}{(R_H - R_f)(R_f - R_L)}
\end{aligned}$$

To show $\beta + \alpha K - (1+K) > 0$ which is equivalent to show

$$(1+K)(ER \cdot R_f - R_H R_L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - R_L)}{1-q} + (1+K)(R_f - R_H)(R_f - R_L) < 0$$

that is,

$$pl(ER - R_f) + \big((ER + R_f - R_H - R_L)R_f\big) < K\big(\frac{q}{1-q}ER(R_f - R_L) - (ER + R_f - R_H - R_L)R_f\big)$$

Suppose

$$(ER + R_f - R_H - R_L)R_f + pl(ER - R_f) < 0$$

$\beta + \alpha K - (1+K)|_{\tau=1} > 0$ holds. In other words, $\tau_2 < 1$. Therefore, when $\tau > \tau_2$, the optimal level of investment in safe assets is

$$\beta^* + \alpha^* K = \frac{(1+K)(ER \cdot R_f - M_H M_L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M_L)}{1-q}}{(M_H - R_f)(R_f - M_L)}$$

Insuree and insurer both hold positive amount of safe assets, insuree and insurer share the aggregate shocks. However, when $\tau_1 \le \tau \le \tau_2$, it is optimal that insurees and insurers still invest nothing in safe assets, that is, $\beta^* + \alpha^* K = 0$, but they share the aggregate asset

shocks. ■

Proof of Proposition 10

Proof. We first consider the case when $\tau < \tau_1$, the representative insuree is fully insured, we have the following system of equations for each state:

$$\begin{aligned}
\beta R_f + d_{nl}^H(1-\beta) &= ER - (\beta + \alpha K)(ER - R_f) - pl \\
\beta R_f + d_{nl}^L(1-\beta) &= ER - (\beta + \alpha K)(ER - R_f) - pl \\
\beta R_f + d_{l}^H(1-\beta) - l &= ER - (\beta + \alpha K)(ER - R_f) - pl \\
\beta R_f + d_{l}^L(1-\beta) - l &= ER - (\beta + \alpha K)(ER - R_f) - pl
\end{aligned}$$

Thus

$$d_{nl}^H = d_{nl}^L = \frac{(1-\beta)ER - (\beta + \alpha K)(ER - R_f) - pl}{1-\beta} d_l^H = d_l^L = \frac{(1-\beta)ER - (\beta + \alpha K)(ER - R_f) + (1-p}{1-\beta}$$

So insuree's utility is

$$\max_{\beta} \ln\big(ER - (\beta + \alpha K)(ER - R_f) - pl\big)$$

subject to

$$0 \le \beta \le 1$$

Thus

$$\beta^* = 0$$

Regulator's problem is

$$\max_{\alpha} \ln\big(ER - \alpha K(ER - R_f) - pl\big)$$

subject to

$$0 \le \alpha \le 1$$

Thus

$$\alpha^* = 0$$

Therefore, the optimal insurance contract is

$$d_{nl}^{L*} = d_{nl}^{H*} = ER - pl \qquad d_{nl}^{L*} = d_{nl}^{H*} = ER + (1-p)l$$

Now we solve for the optimal tax/subsidy depends on the realized aggregate states. In bad aggregate state, the successful insurer's payoff is $\big(1+K\big)R_H + T_S^L - D^L$, while failed insurer's payoff is $\big(1+K\big)R_L + T_F^L - D^L$

each insurer does not bear idiosyncratic risk

$$\begin{aligned}
& \big(1+K\big)R_H + T_S^L - D^L = \big(1+K\big)R_L + T_F^L - D^L \\
= \;& C^{L*} = K \cdot ER - (1-q)(M^H - M^L)(1+K) \\
\Rightarrow \;& \begin{cases} T_S^L & = C^{L*} + d^L - (1+K)\cdot R_H = (1+K)(M^L - R_H) < 0 \\ T_F^L & = C^{L*} + d^L - (1+K)\cdot R_L = (1+K)(M^L - R_L) > 0 \end{cases}
\end{aligned}$$

Thus the tax/subsidy for insurers whose assets success or fail, respectively, are:

$$\begin{cases} T_S^L &= (1+K)(M^L - R_H) \\ T_F^L &= (1+K)(M^L - R_L) \end{cases}$$

The tax budget is balance neutral because $\big(q(1-\tau)+\tau\big)\cdot T_F^L + (1-q)(1-\tau)\cdot T_S^L = (1+K)(M^L - M^L) = 0$.

Similarly, if in the good aggregate state, successful insurer's payoff is $\big(1+K\big)R_H + T_S^H - d^H$, while failed insurer's payoff is $\big(1+K\big)R_L + T_F^H - d^H$. each insurer does not bear idiosyncratic shocks, and the following equation holds.

$$\begin{aligned} & \big((1+K-t\cdot K\big)R_H + T_S^H - d^H = \big(1+K-t\cdot K\big)R_L + T_F^H - d^H \\ = \;& C^{H*} = K\cdot ER + q(M^H - M^L)(1+K) \\ \Rightarrow \;& \begin{cases} T_S^H &= C^{H*} + d^H - (1+K)\cdot R_H = (1+K)(M^H - R_H) < 0 \\ T_F^H &= C_H^* + d^H - (1+K)\cdot R_L = (1+K)(M^H - R_L) > 0 \end{cases} \end{aligned}$$

The taxes/subsidies for insurers whose assets succeed or fail are

$$\begin{cases} T_S^H &= (1+K)(M^H - R_H) \\ T_F^H &= (1+K)(M^H - R_L) \end{cases}$$

In good aggregate state, the taxation is also budget neutral since $\big((1-q)(1-\tau)+\tau\big)\cdot T_S^H + q(1-\tau)\cdot T_F^H = (1+K)(M^H - M^H) = 0$ Therefore, the taxation schem is

$$\begin{cases} T_S^L &= (1+K)(M^L - R_H) \\ T_F^L &= (1+K)(M^L - R_L) \\ T_S^H &= (1+K)(M^H - R_H) \\ T_F^H &= (1+K)(M^H - R_L) \end{cases}$$

We now consider the second case where $\tau_1 \le \tau \le \tau_2$, insurees cannot be perfectly insured, the insuree's payoffs in the two aggregate states are:

$$\begin{aligned} \beta R_f + d^L(1-\beta) - pl &= (1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl \\ \beta R_f + d^H(1-\beta) - pl &= (1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K \end{aligned}$$

The insuree's problem is:

$$\begin{aligned} \max_{\beta} (1-q)\ln\big((1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K\big) \\ + q\ln\big((1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl\big) \end{aligned}$$

subject to

$$0 \le \beta \le 1$$

Thus

$$\begin{aligned}\mathcal{L} &= (1-q)\ln\big((1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K\big)\\ &\quad +q\ln\big((1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl\big)+\lambda_1\beta+\lambda_2(1-\beta)\end{aligned}$$

It follow that the first order condition is:

$$\begin{aligned}&\frac{(1-q)(R_f-M^H)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}\\ &+\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}+\lambda_1-\lambda_2=0\end{aligned}$$

that is Suppose $\lambda_2=\lambda_1=0$, that is $0<\beta<1$, However, we can solve the solution to function

$$\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}=\frac{(1-q)(M^H-R_f)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}$$

such that $\beta*<0$, which violates $0<\beta<1$.

Since $\frac{q(R_f-M^L)}{(1+K)M^L+(\beta+\alpha K)(R_f-M^L)-pl}-\frac{(1-q)(M^H-R_f)}{(1+K)M^H+(\beta+\alpha K)(R_f-M^H)-pl-\frac{ER}{1-q}K}$ is a decreasing function of β, thus we need $\lambda_2=0$, and $\lambda_1>0$, that is

$$\beta^*=0$$

Similarly, the optimal investment of insurer in safe asset is as follows:

$$\max_{\alpha}(1-q)\ln\big((1+K)M_H+\alpha K(R_f-M_H)-pl-\frac{ER}{1-q}K\big)+q\ln\big((1+K)M_L+\alpha K(R_f-M_L)-pl\big)$$

subject to

$$0\le\alpha\le 1$$

In the similar way, we can solve the optimal α^*, that is $\alpha^*=0$.

Therefore the optimal insurance contracts are:

$$d^L_{nl}=(1+K)M^L-pl\qquad d^H_{nl}=(1+K)M^H-\frac{ER}{1-q}K-pl$$

$$d^L_{l}=(1+K)M^L+(1-p)l\qquad d^H_{l}=(1+K)M^H-\frac{ER}{1-q}K+(1-p)l$$

$d^L=(1+K)M^L$

Now we derive the **optimal taxation scheme**: In the bad aggregate state, the payoff of the insurer whose assets succeed is $\big(1+K\big)R_H+T^L_S-d^L=(1+K)(R_H-M_L)+T^L_S$, while the payoff of the insurer whose assets fail is $\big(1+K\big)R_L+T^L_F-d^L=(1+K)(R_L-M_L)+T^L_F$. Since regulator can reinsure the idiosyncratic shocks to insuerers through tax, each insurer

does not bear idiosyncratic risk. In other words,

$$(1+K)R_H + T_S^L - d^L = (1+K)R_L + T_F^L - d^L = C^{L*} = 0$$

Thus the taxation/subsidy among insurees are:

$$\begin{cases} T_S^L &= d^L - (1+K)R_H = (1+K)(M^L - R_H) < 0 \\ T_F^L &= (1+K)M^L - (1+K)R_L = (1+K)(M^L - R_L) > 0 \end{cases}$$

In bad aggregate state, the tax transfers satisfy the following budge neutral constraint:

$$\big(q(1-\tau)+\tau\big)\cdot T_S^L + (1-q)(1-\tau)\cdot T_F^L = (1+K)(M^L - M^L) = 0$$

Similarly, if in the good aggregate state, the payoff of insurers whose assets succeed is $\big((1+K\big)R_H + T_S^H - d_H$, while the payoff of the insurers whose assets fail is $\big(1+K\big)R_L + T_F^H - d_H$.Each insurer do not bear idiosyncratic risk, then

$$\big((1+K\big)R_H + T_S^H - d^H = \big(1+K\big)R_L + T_F^H - d^H = C^{H*} = \frac{ER\cdot K}{1-q}$$

$$\begin{cases} T_S^H &= d^H + \frac{ER\cdot K}{1-q} - (1+K)R_H = (1+K)(M^H - R_H) < 0 \\ T_F^H &= d^H + \frac{ER\cdot K}{1-q} - (1+K)R_L = (1+K)(M^H - R_L) > 0 \end{cases}$$

In good aggregate state, the taxation is budget budget neutral since

$$\big((1-q)(1-\tau)+\tau\big)\cdot T_S^H + q(1-\tau)\cdot T_F^H = (1+K)(M^H - M^H) = 0$$

Therefore, the optimal tax scheme is:

$$\begin{cases} T_S^L &= (1+K)(M^L - R_H) \\ T_F^L &= (1+K)(M^L - R_L) \\ T_S^H &= (1+K)(M^H - R_H) \\ T_F^H &= (1+K)(M^H - R_L) \end{cases}$$

In this case,it is still optimal for insurees invest all their capital in buying risky insurance contracts, insurers invest all their capital in risky assets, and regulators's taxes transfers are given as above.

We now proceed to the third case when $\tau > \tau_2$, insurees cannot be perfectly insured, thus the insurees' payoff in two states are: $\beta R_f + d^L(1-\beta) - pl = (1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl$ in the bad aggregate state and $\beta R_f + d^H(1-\beta) - pl = (1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K$ in good aggregate state, respectively. Thus insuree's problem is

$$\begin{aligned} \max_{\beta} (1-q)\ln\big((1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K\big) \\ + q\ln\big((1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl\big) \end{aligned}$$

subject to

$$0 \leq \beta \leq 1$$

Thus

$$\begin{aligned}\mathcal{L} &= (1-q)\ln\left((1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K\right) \\ &\quad + q\ln\left((1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl\right) + \lambda_1\beta + \lambda_2(1-\beta)\end{aligned}$$

The first order condition is

$$\begin{aligned}&\frac{(1-q)(R_f - M^H)}{(1+K)M^H + (\beta+\alpha K)(R_f - M^H) - pl - \frac{ER}{1-q}K} \\ &+ \frac{q(R_f - M^L)}{(1+K)M^L + (\beta+\alpha K)(R_f - M^L) - pl} + \lambda_1 - \lambda_2 = 0\end{aligned}$$

Suppose $\lambda_2 = \lambda_1 = 0$, that is $0 < \beta < 1$ Thus the opitmal $\beta^*$is

$$\beta^* = \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha K$$

Thus insuree' utility is given by

$$\begin{aligned}&(1-q)\ln\left((1+K)M^H - \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(R_f - M^L)}\right. \\ &\left. - pl - \frac{ER}{1-q}K\right) + q\ln\left((1+K)M^L\right. \\ &\left. + \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)} - pl\right)\end{aligned}$$

for any α, $\beta = \beta^*(\alpha)$ such that insuree's welfare will not change. Thus we need

$$\beta^* = \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha K < 1$$

If $\frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} \leq 1$, α^*can be any number between 0 and 1.

If $1 < \frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} < 1 + K$, then α^*has to be greater than $\frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{K(M^H - R_f)(R_f - M^L)} - \frac{1}{K}$.

Regulator impose the minimum requirement of liquidity buffer α^* such that any

$$\alpha^* \in \left(\max\left\{\frac{(1+K)(ER\cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER\cdot K\cdot(R_f - M^L)}{1-q}}{K(M^H - R_f)(R_f - M^L)} - \frac{1}{K},\ 0\right\},\ 1\right)$$

, in safe assets, and insurees invest $\beta^*$where

$$\beta^* = \frac{(1+K)(ER \cdot R_f - M^H M^L) + pl(ER - R_f) - \frac{qER \cdot K \cdot (R_f - M^L)}{1-q}}{(M^H - R_f)(R_f - M^L)} - \alpha^* K$$

and the opitmal insurance contracts are

$$\begin{aligned}
d_{nl}^L &= \frac{(1+K)M^L + (\beta^* + \alpha^* K)(R_f - M^L) - \beta^* R_f - pl}{1-\beta^*} \\
d_{nl}^H &= \frac{(1+K)M^H + (\beta + \alpha K)(R_f - M^H) - \frac{ER}{1-q}K - \beta^* R_f + (1-p)l}{1-\beta^*} \\
d_l^L &= \frac{(1+K)M^L + (\beta^* + \alpha^* K)(R_f - M^L) - \beta^* R_f - pl}{1-\beta^*} \\
d_l^H &= \frac{(1+K)M^H + (\beta + \alpha K)(R_f - M^H) - \frac{ER}{1-q}K - \beta^* R_f + (1-p)l}{1-\beta^*}
\end{aligned}$$

Now we derive the optimal taxation scheme. In the bad aggregate state, the payoff of insurers whose assets success is $\big(1+K-(\beta^*+\alpha^*K)\big)R_H+T_S^L-d^L(1-\beta^*)$, while the payoff of insurers whose assets fail is $\big(1+K-(\beta^*+\alpha^*K)\big)R_L+T_F^L-d^L(1-\beta^*)$ Since each insurer does not bear idiosyncratic risk:

$$\begin{aligned}
& \big(1+K-(\beta^*+\alpha^*K)\big)R_H + T_S^L - d_L(1-\beta^*) \\
= \quad & \big(1+K-(\beta^*+\alpha^*K)\big)R_L + T_F^L - d_L(1-\beta^*) \\
& = C^{L*} = -\alpha^* K R_f
\end{aligned}$$

Thus the tax schemes are:

$$\begin{cases}
T_S^L &= d_L(1-\beta^*) - \alpha^* K R_f - \big(1+K-(\beta^*+\alpha^*K)\big)R_H \\
&= (1+K-(\beta^*+\alpha^*K))(M^L - R_H) < 0 \\
T_F^L &= d_L(1-\beta^*) - \alpha^* K R_f - \big(1+K-(\beta^*+\alpha^*K)\big)R_L \\
&= (1+K-(\beta^*+\alpha^*K))(M^L - R_L) > 0
\end{cases}$$

Similarly, in good aggregate state, the payoff of insurers whose assets succeed is $\big(1+K-(\beta^*+\alpha^*K)\big)R_H+T_S^H-d^H(1-\beta^*)$, while the payoff of insurers whose assets fail is $\big(1+K-(\beta^*+\alpha^*K)\big)R_L+T_F^H-d^H(1-\beta^*)$ Since each insurer does not bear idiosyncratic risk:

$$\begin{aligned}
& \big(1+K-(\beta^*+\alpha^*K)\big)R_H + T_S^H - d_H(1-\beta^*) \\
= & \big(1+K-(\beta^*+\alpha^*K)\big)R_L + T_F^H - d_H(1-\beta^*) \\
& = C^{H^*} = \tfrac{ER \cdot K}{1-q} - \alpha K \cdot R_f
\end{aligned}$$

Therefore, the optimal tax schemes are:

$$\begin{cases} T_S^H &= d^H(1-\beta^*) + \frac{ER\cdot K}{1-q} - \alpha K \cdot R_f - \big(1+K-(\beta^*+\alpha^* K)\big)R_H \\ &= \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_H) < 0 \\ T_F^H &= d^H(1-\beta^*) + \frac{ER\cdot K}{1-q} - \alpha K \cdot R_f - \big(1+K-(\beta^*+\alpha^* K)\big)R_L \\ &= \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_L) > 0 \end{cases}$$

The tax budget neutral constraints are also satisfied in good and bad aggregate states, respectively:

$$q(1-\tau)\cdot T_F^H + \big((1-q)(1-\tau)+\tau\big)\cdot T_S^H = (1+K-(\beta^*+\alpha^* K))(M^H - M^H) = 0$$
$$(1-q)(1-\tau)\cdot T_S^L + \big(q(1-\tau)+\tau\big)\cdot T_F^L = (1+K-(\beta^*+\alpha^* K))(M^L - M^L) = 0$$

Thus we have:

$$\begin{cases} T_S^L &= (1+K-(\beta^*+\alpha^* K))(M^L - R_H) \\ T_F^L &= (1+K-(\beta^*+\alpha^* K))(M^L - R_L) \\ T_S^H &= \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_H) \\ T_F^H &= \big(1+K-(\beta^*+\alpha^* K)\big)(M^H - R_L) \end{cases}$$

■

References

1. Ahrens, F., Fuess, R., and Kestel, S. (2009). An Econometric Pricing Model for CAT Bonds and the Impact of the 2005 Hurricane Season. *Working Paper*.

2. Banks, Jeffrey and Joel Sobel, 1987, Equilibrium Selection in Signaling Games, *Econometrica*, 55, 3, 647661.

3. Bantwal, Vivek and Howard Kunreuther, 2000, A Cat Bond Premium Puzzle?, *Journal of Psychology and Financial Market*, 1, 1, 76–91

4. Baranoff, Etti, Thomas Sager, 2002, The Relations among Asset Risk, Product Risk, and Capital in the Life Insurance Industry, *Journal of Banking & Finance*, No. 26, 1181-1197.

5. Barrieu, Pauline and Henri Louberge, 2009, Hybrid Cat Bonds, *Journal of Risk and Insurance*, 76, 3, 547–578

6. Becker, Bo and Victoria Ivashina, 2013, Reaching for Yield in the Bond Market, working paper.

7. Braun, Alexander, 2014, Pricing in the Primary Market for CAT Bonds: New Empirical Evidence, *Working Paper*

8. Albert Cox, 1967, Regulation of Interest on Deposits: An Historical Review, *Journal of Finance* Vol.22, No.2, 274-296.

9. Julie Cagle, 1995, Insurance Supply with Capacity Constraints and Endogenous Insolvency Risk, *Journal of Risk and Uncertainty*, No.11, 219-232.

10. Chemmanur, Thomas and Paolo Fulghieri, 1994, Reputation, Renegotiation, and the Choice Between Bank Loans and Publicly Traded Debt, *Review of Financial Studies*, 7, 3, 475–506.

11. Cummins, David and Patricia Danzon, 1994, Price, Financial Quality, and Capital Flows in Insurance Markets, *Journal of Financial Intermediation*, No. 6, 3-38.

12. Cummins, David and David Sommer, 1996, Capital and Risk in Property-Liability Insurance Markets, *Journal of Banking & Finance*, No. 20, 1069-1092.

13. Cummins, David, Yijia Lin, Richard D. Phillips, Capital Allocation and the Pricing of Financially Intermediated Risks: An Empirical Investigation, 2006, working paper.

14. Cummins, David, David Lalonde, and Richard D. Phillips, 2004, The Basis Risk of Index-Linked Catastrophic Loss Securities, 2004, *Journal of Financial Economics*, 71, 77–111.

15. Cummins, David and Philippe Trainar, 2009, Securitization, Insurance, and Reinsurance, *Journal of Risk and Insurance*, 76, 3, 463–492

16. Cummins, David and Mary Weiss, 2009, Convergence of Insurance and Financial Markets: Hybrid and Securitized Risk-transfer Solutions, *Journal of Risk and Insurance*, 76, 3,493–545

17. Cummins, David, 2008, CAT Bonds and Other Risk-linked Securities: State of the Market and Recent Developments, *Risk Management and Insurance Review*, 11, 1, 23–47

18. Cummins, David, 2012, CAT Bonds and Other Risk-linked Securities: Product Design and Evolution of the Market, working paper 2012.

19. DeMarzo, Peter and Darrell Duffie, 1999, A Liquidity-Based Model of Security Design, *Econometrica*, 67, 65–99.

20. Dionne, Georges and Louis Eeckhoudt, 1984, Insurance and Saving: Some Further Results, *Insurance: Mathematics and Economics*, No. 3, 101-110.

21. Doherty, Neil and James Garven, 1986, Pricing Regulation in Property-Liability Insurance: A Contingent-Claims Approach, *Journal of Finance*, Vol. 66, No. 5, 1031-1049.

22. Drucker, Steven and Manju Puri, 2009, On Loan Sales, Loan Contracting, and Lending Relationships, *Review of Financial Studies*, 22, 2835–2872.

23. Duffee, Gregory and Chunsheng Zhou, 2001, Credit Derivatives in Banking: Useful Tools for Managing Risk, *Journal of Monetary Economics*, 48, 25–54.

24. Durbin, David (2001), Managing Natural Catastrophe Risks: The Structure and Dynamics of Reinsurance, *The Geneva Papers on Risk and Insurance,* 26: 297-309

25. Finken, Silke and Christian Laux, 2009, Catastrophe Bonds and Reinsurance: the Competitive Effect of Information-insensitive Triggers, *Journal of Risk and Insurance*, 76, 3, 579–605

26. Froot, Kenneth, 2001, The Market for Catastrophe Risk: A Clinical Examination, *Journal of Financial Economics*, 60, 529–571

27. Froot, Kenneth, 2008, The Intermediation of Financial Risks: Evolution in the Catastrophe Reinsurance Market, *Risk Management and Insurance Review*, 11, 2, 281–294

28. Gatzert, Nadine and Hato Schmeiser, 2008, Combining Fair Pricing and Capital Requirements for Non-life Insurance Companies, *Journal of Banking & Finance*, No. 32, 2589-2596.

29. Gibson, Rajna, Michel A. Habib, Alexandre Ziegler, 2014, Reinsurance or Securitization: the Case of Natural Catastrophe Risk, *Journal of Mathematical Economics*, forthcoming.

30. Gorton, Gary and George Pennacchi, 1995, Banking and Loan sales: Marketing Nonmarketable Assets, *Journal of Monetary Economics*, 35, 389–411.

31. Gron, Anne 1994, Capacity constraints and cycles in property-casualty insurance markets, *Rand Journal of Economics*, No.25, 110-127

32. Hagendorff, Bjoern , Jens Hagendorff, Kevin Keasey and Angelica Gonzalez, 2014, The risk implications of insurance securitization: The case of catastrophe bonds, *Journal of Corporate Finance* 25, 387–402.

33. Harrington, Scott and Gregory Niehaus, 1999, Basic Risk with PCS Catastrophe Insurance Derivative Contracts, *Journal of Risk and Insurance*, 66, 1, 49–82.

34. Harrington, Scott and Gregory Niehaus, 2003, Capital, Corporate Income Taxes, and Catastrophe Insurance, *Journal of Financial Intermediation*, 12, 365–389.

35. Hoerger, Thomas, Frank Sloan and Mahmud Hassan, 1990, Loss Volatility, Bankruptcy, and the Demand for Reinsurance, *Journal of Risk and Uncertainty*, 3, 3, 221–245

36. Hoy, Michael and Arthur Robson, 1981, Insurance as A Giffen Good, *Economics Letters*, No. 8, 47-51

37. Jaffee, Dwight and Thomas Russell, 1997, Catastrophe Insurance, Capital Markets, and Uninsurable risks, *Journal of Risk and Insurance*, 64, 2, 205–230

38. Jean-Baptiste, E. and Anthony Santomero, 2000, The Design of Private Reinsurance Contracts, *Journal of Financial Intermediation*, 9, 274–297

39. Lakdawalla, Darius and George Zanjani, 2012, Catastrophe Bonds, Reinsurance, and the Optimal Collateralization of Risk Transfer, *Journal of Risk and Insurance*, 79, 2, 449–476

40. Leland, Hayne and David Pyle, 1977, Informational Asymmetries, Financial Structure, and Financial Intermediation, *Journal of Finance*, 32, 2, 371–387

41. Nachman, David and Thomas Noe, 1994, Optimal Design of Securities under Asymmetric Information. *Review of Financial Studies*, 7, 1–44.

42. Niehaus, Gregory, 2002, The Allocation of Catastrophe Risk. *Journal of Banking and Finance,* 26, 584–595.

43. Parlour, Christine and Guillaume Plantin, 2008, Loan Sales and Relationship Banking. *Journal of Finance*, 63, 3, 1291–1314.

44. Parlour, Christine and Andrew Winton, 2013, Laying Off Credit Risk: Loan Sales versus Credit Default Swaps. *Journal of Financial Economics*107, 25–45.

45. Rajan, Raghuram 1992, Insiders and Outsiders: The Choice between Informed and Arm's-length Debt, *Journal of Finance*, 47, 4, 1367–1400

46. Sommer, David, 1996, The Impact of Firm Risk on Property-Liability Insurance Prices, *Journal of Risk and Insurance*, Vol. 63, No.3, 501-514.

47. Sherris, Michael, 2003, Equilibrium Insurance Pricing, Market Value of Liabilities and Optimal Capitalization, working paper.

48. Shim, Jeungbo, 2010, Capital-based Regulation, Portfolio Risk and Capital Determination: Empirical Evidence from the US Property-Liability Insurers, *Journal of Banking & Finance*, No.34, 2450-2461.

49. Thompson, James R. 2014, Credit Risk Transfer: To Sell or to Insure, *Working Paper*, University of Waterloo.

50. Taylor, Greg, 1995, An Equilibrium Model of Insurance Pricing and Capitalization, *The Journal of Risk and Insurance*, Vo. 62, No. 3, 409-446.

51. Winter, Ralph, 1990, The Dynamics of Competitive Insurance Markets, *Journal of Financial Intermediation*, No. 3, 379-415.

52. Winton, Andrew and Vijay Yerramilli, 2008, Entrepreneurial Finance: Banks versus Venture Capital, *Journal of Financial Economics*88, 51–79.

53. Zanjani, George, 2002, Pricing and capital allocation in catastrophe insurance, *Journal of Financial Economics*, No.65,238-305.