TITLE: Fan-Linear Maps and Fan Algebras

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Abstract: Fan algebras arise from fan-linear maps, a special class of functions defined on partitions of the nonnegative integer lattice in the plane. These algebras are natural objects to study in commutative algebra as they include many classical examples commutative rings. Additionally, the ubiquity of this structure has only been recently identified, therefore little is known regarding the properties of these algebras.

We begin our study by classifying all fan-linear maps via the conditions imposed on them by their domains. This classification includes a general result regarding all semigroup homomorphisms from finitely generated subsemigroups of the nonnegative integer lattice into the integers. We then go on to show that the set of all fan-linear maps on any fixed partition is necessarily a finitely-generated affine semigroup. Finally, this leads to the conclusion that the set of fan algebras corresponding to a fixed partition and a fixed set of ideals forms a finitely generated semigroup. This is accomplished through the identification of generating maps in the semigroup of all fan-linear maps with generating algebras and the description of a natural additive operation.