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Mortality Risk Management

Yijia Lin

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MORTALITY RISK MANAGEMENT

BY

Yijia Lin

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree

Of

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In the Robinson College of Business

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ACCEPTANCE

This dissertation was prepared under the direction of Yijia Lin's Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Robinson College of Business of Georgia State University.

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ABSTRACT

MORTALITY RISK MANAGEMENT

BY

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2006

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This is a multi–essay dissertation in the area of mortality risk management. The first essay investigates natural hedging between life insurance and annuities and then proposes a mortality swap between a life insurer and an annuity insurer. Compared with reinsurance, capital markets have a greater capacity to absorb insurance shocks, and they may offer more flexibility to meet insurers' needs. Therefore, my second essay studies securitization of mortality risks in life annuities. Specifically I design a mortality bond to transfer longevity risks inherent in annuities or pension plans to financial markets. By explicitly taking into account the jumps in mortality stochastic processes, my third essay fills a gap in the mortality securitization modeling literature by pricing mortality securities in an incomplete market framework. Using the Survey of Consumer Finances, my fourth essay creates a new financial vulnerability index to examine a household's life cycle demand for different types of life insurance.

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I also thank my parents for always being there when I needed them most, and for supporting me through all these years. Finally, this dissertation is dedicated to my greatest blessing, my husband, the most decent and loving person I have ever known. Going through difficulties hand by hand with him gives me magic strength and courage. Thank him for all his unconditional love and understanding. A lifetime with him will always be too short.

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1

Introduction and Overview

Mortality risk management is the process of identifying mortality risk exposures faced by an insurance company, a pension plan or an individual and selecting the most appropriate techniques for treating them (Rejda, 2005). Mortality risk management is critical to the financial stability of an insurance company, a pension plan or a household. My dissertation is a four–essay dissertation. It proposes new techniques for insurers and pension plans to manage mortality risk and examines households' life insurance demand.

Froot and Stein (1998) develop a framework for analyzing the capital allocation and capital structure decisions facing financial institutions. Their model suggests that the hurdle rate of an investment opportunity consists of two parts: the standard market-risk factor and a factor for unhedgeable risk. Until now, little attention has been paid to the risk premium of unhedgeable mortality risks. In the first essay, I find evidence that annuity writing insurers who naturally hedge their annuity risk by also writing life insurance are able to charge lower premiums than do otherwise similar insurers. The evidence suggests that insurers who utilize natural hedging, a form of business diversification, have a competitive advantage.

In the second part of my first essay I note that, in reality, the mix of an insurer's life and annuity businesses is not likely to provide an optimal hedge because of technical difficulties. Moreover, the insurer may prefer business focus rather than diversification, as expanding the number of activities of the firm is not without costs (Denis et al., 1997; Comment and Jarrell, 1995; Berger and Ofek, 1995; Lamont and Polk, 2002; Servaes, 1996; Scharfstein and Stein, 2000). Thus, I design a set of securities that I call mortality swaps and demonstrate that insurers can use the swaps to achieve the benefit of natural hedging without actually "diversifying" the business of the insurers.

An important recent innovation in financial markets is the securitization of mortality risks for catastrophic events such as natural disasters or exceptional improvements in life expectancy. Mortality securitization has the potential to enhance the capacity of the insurance industry and allow it to efficiently spread risks beyond life insurance markets. Moreover, it provides an additional way to diversify an investor's portfolio as mortality risk may be uncorrelated to or have low correlation with any other financial risk that underlies stock or bond price movements. The purpose of my second essay is to explain the rationale for the existence of mortality-based securities and to develop an equilibrium model that can be used to price the proposed mortality securities. I focus on individual annuity data, although the modeling techniques could be applied to other lines of annuity or life insurance.

The securitization of mortality risks is gaining more attention from investors. The first two publicly known mortality securities are the Swiss Re mortality bond and the European Investment Bank (EIB) longevity bond. Morgan Stanley reports that investors' appetite for the Swiss Re bonds was strong while, on the other hand, the EIB bond has not sold well. The purpose of my third essay is to explain the interesting yet opposite market outcomes for these two securities.

I begin by developing a mortality stochastic model that allows for jump risk. I then consider the problem of pricing contingent claims on mortality bonds in an incomplete market framework. My results suggest that the Swiss Re bonds compensated investors with a risk premium that was approximately 90 percent higher than our model. On the other hand, the model suggests the EIB bonds charged UK pension plans a higher risk premium than did the insurance market for a similar level of longevity risk. Thus, the model explains why investors had a strong appetite for the Swiss Re bonds and not the EIB bonds. The research should also enable investors of future mortality bonds to understand better the uncertainty and pricing associated with catastrophic mortality risk securities.

Using the Survey of Consumer Finances my fourth essay examines a life cycle demand model for different types of life insurance. Specifically, I test for consumers' avoidance of income volatility as a result of the death of a wage-earning household member through the purchases of life insurance. The primary innovation in this research is that I develop a financial vulnerability index to control for the risk to a household and show, in contrast to previous research that there is a positive relationship between financial vulnerability and the amount of term life or total life insurance purchases. In addition, I find older consumers use less life insurance to protect a certain level of financial vulnerability than do younger consumers. My results are robust no matter whether or not I take into account non-monetary contribution of family members by imputing their housework value.

In summary, my dissertation examines mortality risk management for insurers, pension plans and individuals. My first essay investigates natural hedging and proposes and prices a mortality swap between a life insurer and an annuity insurer. My proposed mortality swaps may help stabilize the cash outflows of life insurers and annuity insurers. Compared with insurance markets, financial markets have much greater capacity to absorb catastrophe risks. My second essay suggests that life insurer may transfer mortality risks to capital markets via a mortality bond to increase their underwriting capacity. If so, life insurers are able to share the "bigger cake" of annuity markets if the U.S. reforms its social security system. Mortality securitization modeling is still an open question in the asset pricing literature. My third essay proposes a model to price mortality securities in an incomplete market framework with jump processes. This model nicely explains the opposite market outcomes of the first two mortality bonds. Similar to insurance companies, households also need to manage their mortality risks that arise from the death of wage-earning members. My fourth essay proposes a new financial vulnerability index to explore whether there is a life cycle relation between a household's financial vulnerability and its life insurance holdings. The Overall, my dissertation will fill several gaps in the literature on mortality risk management.

2

Natural Hedging of Life and Annuity Mortality Risks

The values of life insurance and annuity liabilities move in opposite directions in response to a change in the underlying mortality. Natural hedging utilizes this to stabilize aggregate liability cash flows. I find empirical evidence that suggests that annuity writing insurers that use natural hedging also charge lower premiums than otherwise similar insurers. This indicates that insurers that are able to utilize natural hedging have a competitive advantage. In addition, I show how a mortality swap might be used to provide the benefits of natural hedging.

2.1 Introduction

If future mortality improves relative to current expectations, life insurer liabilities decrease because death benefit payments will be later than expected. However, annuity writers have a loss relative to current expectations because they have to pay annuity benefits longer than expected. If mortality experience deteriorates, the situation is reversed: life insurers have losses and annuity writers have gains. Natural hedging utilizes this interaction of life insurance and annuities to a change in mortality to hedge against unexpected changes in future benefit payments.

The purpose of this paper is to study natural hedging of mortality risks and to propose mortality swaps as a risk management tool. Few researchers investigate the issue of natural hedging. Most of the prior research explores the impact of mortality changes on life insurance and annuities separately, or investigates a simple combination of life and pure endowment life contracts (Frees et al., 1996; Marceau and Gaillardetz, 1999; Milevsky and Promislow, 2001; Cairns et al., 2006). Studies on the impact of mortality changes on life insurance focus on "bad" shocks while those on annuities focus on "good" shocks.

Wang et al. (2003) analyze the impact of the changes of mortality factors and propose an immunization model to hedge risks based on mortality experience in Taiwan. Life insurance and annuity mortality experience can be very different, so there is "basis risk" involved in using annuities to hedge life insurance mortality risk. Their model cannot pick up this basis risk.

Marceau and Gaillardetz (1999) examine the calculation of the reserves in a stochastic mortality and interest rates environment for a general portfolio of life insurance policies. In their numerical examples, they use portfolios of term life insurance contracts and pure endowment polices, like Milevsky and Promislow (2001). They focus on convergence of simulation results. There is a hedging effect in their results, but they do not pursue the issue.

This paper proceeds as follows: In Section 2.2, I use an example to illustrate the idea of natural hedging. In Section 2.3, using market quotes of single–premium immediate annuities (SPIA) from A. M. Best, I find empirical support for natural hedging. That is, insurers that naturally hedge mortality risks have a competitive advantages over otherwise similar insurers. In Section 2.4, I propose and price a mortality swap between life insurers and annuity insurers. Section 2.5 is the conclusion and summary.

2.2 Example

This example illustrates the idea of a natural hedge. Consider a portfolio of life contingent liabilities consisting of whole life insurance policies written on lives age 35 and immediate life annuities written on lives age 65. If mortality improves, what happens to the insurer's total liability? We know that on average, the insurer will have a loss on the annuity business and a gain on the life insurance business. If mortality declines, the effects are interchanged. This example shows what can happen if mortality risk increases as a result of a common shock. Here are my assumptions:

1. Mortality for age 35 is based on the 1990-95 Society of Actuaries Male Basic Table and

the table for age 65 is based on the 1996 US Individual Annuity Mortality Male Basic Table.

2. The annuity has an annual benefit of 510 and it is issued as an immediate annuity at age 65.

3. The face amount of life insurance on age 35 is $100,000$ and the life insurance is issued at age 35. For this amount of insurance, the present value of liabilities under the life insurance and under the annuity are about equal.

4. Life insurance premiums and annuity benefits are paid annually. Death benefits are paid at the end of the year of death.

5. The initial number of lives insured, ℓ_{35}, is 10,000 which is the same as that of annuitants ℓ_{65}.

6. The mortality shock ϵ is expressed as a percentage of the force of mortality μ_{x+t}, so it ranges from -1 to 1, that is, $-1 \leq \epsilon \leq 1$ with probability 1. Without the shock, the survival probability for a life age x at year t is $p_{x+t} = \exp(-\mu_{x+t})$. With the shock, the new survival probability $p^{'}_{x+t}$ can be expressed as:

$$p^{'}_{x+t} = (e^{-\mu_{x+t}})^{1-\epsilon} = (p_{x+t})^{1-\epsilon}.$$

   If $0 < \epsilon \leq 1$, mortality experience improves. If $-1 \leq \epsilon < 0$, mortality experience deteriorates.

7. The term structure of interest rates is flat; there is a single interest rate, $i = 0.06$.

2.2.1 Life Insurance

For the life insurance, the present value of 1 paid at the end of the year of death is v^{k+1} and the expected present value is

$$A_x = \sum_{k=0}^{\infty} v^{k+1} {}_kp_x q_{x+k}$$

where x is the age when the policy issued ($x = 35$ in this example). For a benefit of f the expected present value is fA_x.

The present value of 1 per year, paid at the beginning of the year until the year of death, is

$$\ddot{a}_{\overline{K(x)+1}|} = \frac{1 - v^{K(x)+1}}{d}.$$

The expected present value

$$\ddot{a}_x = \mathrm{E}\left[\ddot{a}_{\overline{K(x)+1}|}\right] = \sum_{k=0}^{\infty} v^k {}_kp_x.$$

The net annual premium rate for 1 unit of benefit is determined so that the present value of net premiums is equal to the present value of benefits. This means

$$P_x \ddot{a}_x = A_x$$

and for a benefit of f the annual premium is

$$fP_x = fA_x/\ddot{a}_x.$$

If the insured dies at $K(x) = t$, then the insurer's net loss is the present value of the payment, less the present value premiums. For a unit benefit, the loss is

$$L = v^{K(x)+1} - P_x \ddot{a}_{\overline{K(x)+1}|} = v^{K(x)+1} - P_x \frac{1 - v^{K(x)+1}}{d}.$$

It follows from the definition of the net premium P_x that the expected loss is zero. For a benefit of f, the loss is fL. Of course, the loss can be negative in which case the result turned out in the insurer's favor. On average, the loss is zero.

2.2.2 Annuities

For an annuitant age y, the present value of 1 per year paid at the beginning of the year is

$$\ddot{a}_{\overline{K(y)+1}|} = \frac{1 - v^{K(y)+1}}{d}.$$

The expected present value

$$\ddot{a}_y = \mathrm{E}\left[\ddot{a}_{\overline{K(y)+1}|}\right] = \sum_{k=0}^{\infty} v^k {}_kp_y.$$

The policy is purchased with a single payment of $\ddot{a}_y$. In my example $y = 65$ and the mortality table is based on annuity experience. For an annual benefit of b, the net single premium is $b\ddot{a}_y$. The company's loss per unit of benefit is

$$\ddot{a}_{\overline{K(y)+1}|} - \ddot{a}_y = 1/d - \ddot{a}_y - v^{K(y)+1}/d.$$

The expected loss is zero.

2.2.3 Portfolio

The portfolio has a life insurance liability to pay a benefit of f at the end of the year of the death and a liability to pay a benefit of b at the beginning of each year as long as the annuitant is alive. The total liability is

$$fv^{K(x)+1} + b\ddot{a}_{\overline{K(y)+1}|}.$$

To offset the liability the company has

$$fP_x\ddot{a}_{\overline{K(x)+1}|} + b\ddot{a}_y.$$

The difference is the total loss:

$$L = fv^{K(x)+1} + b\ddot{a}_{\overline{K(y)+1}|} - fP_x\ddot{a}_{\overline{K(x)+1}|} - b\ddot{a}_y.$$

The expected loss is zero. However, this expectation is calculated under the assumption that the mortality follows the tables assumed in setting the premiums. If we replace the before–shock lifetimes with the after shock lifetimes, what happens to the loss?

2.2.4 Results

Figure 2.1 shows the percentage deviation of the present value of benefits from the life insurance premiums and that of annuity payments from the total annuity premium collected at time $t = 0$. I also show the percentage of deviation from the present value of total premiums collected. Each result includes a shock improvement or deterioration relative to the table mortality, modelled by multiplying the force of mortality by a factor $1 - \epsilon$ in each year. With a small mortality improvement shock $\epsilon = 0.10$, annuity insurers will lose 2.0 percent of their expected total payments. In this scenario, life insurers will gain 5.0 percent of their expected total payments. If the above life insurance and annuity are written by the same insurer, the shock has a much smaller effect on its business (a 1.5 percent gain). With a small mortality bad shock $\epsilon = -0.10$, annuity insurers will gain 1.9 percent of their expected total payments. In this scenario, life insurers will lose 4.8 percent of their expected total payments. If the above life insurance and annuity are sold by the same insurer, a bad shock has little effect on its business (a 1.4 percent loss). When there is a big good shock $\epsilon = 0.50$, the present value of total annuity payments will increase by 12.5 percent and the

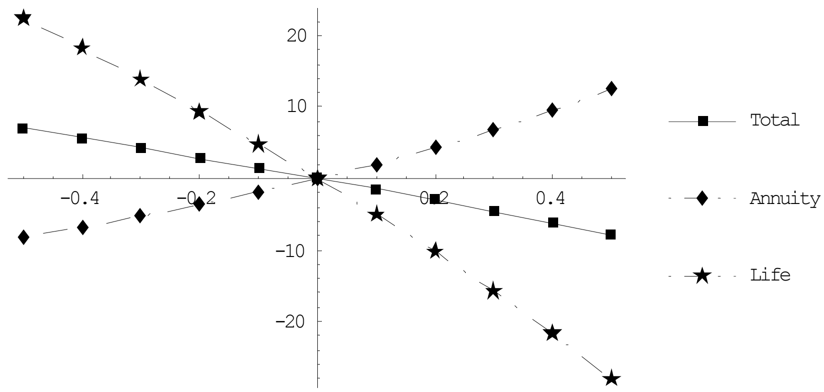

Figure 2.1: The percentage deviation of the present value of benefits from the life insurance premiums, that of annuity payments from the total annuity premium collected and that of total premiums collected at time $t = 0$. The y–axis represents the percentage loss and the x–axis represents different levels of shock improvement or deterioration factor ϵ.

life insurer will gain 28.0 percent of their total expected payments on average. The overall effects will be 7.9 percent gain on a big good shock. Writing both life and annuity business reduces the impact of a big bad shock $\epsilon = -0.50$ to a 7.0 percent loss. I have illustrated the idea of natural hedging and I conclude that writing both life and annuities does indeed reduce the insurer's aggregate mortality risk.

2.3 Empirical Support For Natural Hedging

Life insurance and annuities have become commodity-like goods, meaning that the price variable is a primary source of competition among insurance industry participants. Through various marketing campaigns, consumers are well aware whether a price offered by an insurer is attractive.

Financial theory tells us that systematic risk cannot be diversified away because systematic risk influences all businesses. This argument, however, was originally intended for security managers, not corporate managers (Chatterjee and Lubatkin, 1992). Action by corporate managers may alter the underlying systematic risk profiles of their portfolio of business (Chatterjee and Lubatkin, 1990; Helfat and Teece, 1987; Peavy, 1984; Salter and Weinhold, 1979). Thus, while mortality risk may not be hedgeable in financial markets, it may be reduced or eliminated by some insurers through reinsurance, natural hedging, and, I propose, mortality swaps in Section 2.4.

The industrial organization literature offers numerous theoretical and empirical models that link integration to a reduction of environmental uncertainty (Arrow, 1975; Carlton, 1979; Mitchell, 1978). Applied to the insurance market, these results suggest that an insurer writing both life insurance and annuities will have reduced risk relative to similar firms writing one of these products.

2.3.1 Pricing of Unhedgeable Risks

Froot and Stein (1998) investigate the pricing of non–hedgeable risks. We can adapt their model to an insurer that writes life and annuity business. At the beginning of each of two periods, defined by time 0, 1 and 2, the company sells policies with uncertain payoffs Z_0 and Z_1 at time 2. That is, at the time the decisions are made the payoffs are random variables with known distributions. Each investment payoff is decomposed as a hedgeable and non–hedgeable component:

$$Z_j = Z_j^H + Z_j^N,$$

where $j = 0$ or 1. The initial portfolio of exposure will result in a time 2 random payoff of $Z_0 = \mu_0 + \varepsilon_0$, where μ_0 is the mean and ε_0 is a mean-zero disturbance term. In our case, the initial portfolio of exposure is the original business composition of life insurance and annuities. The firm invests in a new investment at time 1, e.g. selling new annuity business. The new investment offers a random payoff of Z_1 at time 2, which can be written as $Z_1 = \mu_1 + \varepsilon_1$, where μ_1 is the mean and ε_1 is a mean-zero disturbance term. The risks can be classified into two categories: (i) perfectly tradeable exposures, which can be unloaded frictionlessly on fair-market terms, and (ii) completely non-tradeable exposures, which must be retained by the financial intermediaries no matter what. The disturbance terms, that is, the pre-existing and new risks, ε_0 and ε_1, can be decomposed as:

$$\varepsilon_0 = \varepsilon_0^T + \varepsilon_0^N,$$

$$\varepsilon_1 = \varepsilon_1^T + \varepsilon_1^N,$$

where ε_0^T is the tradeable component of ε_0, ε_0^N is the non-tradeable component, and so forth. The intermediary's realized internal wealth at time 1 is denoted by w. The investment at time 1 requires a cash input of I, which is funded by the internal sources w and external money e. Thus $I = w + e$. The investment yields a gross return of $F(I)$. The convex costs for raising external finance e are given by $C(e)$. This means that the larger the amount that must be financed externally, the more costly the funds are to raise, i.e. $C'(e) > 0$. And the change rate of cost of funds increases with size of funds raised, i.e. $C''(e) > 0$. The solution to this intermediary's time 2 problem can be denoted by the *ex post* value of the firm, $P(w)$, as follows:

$$P(w) = \max_e F(I) - I - C(e), \text{subject to } I = w + e.$$

Froot et al. (1993) and Gron and Winton (2001) show that the company's required rate of return μ for pricing its products at time 1 has the form

$$\mu = \gamma \text{cov}(Z_1^H, M) + G\text{cov}(Z_1^N, Z_0^N). \tag{2.1}$$

where $G > 0$ is the company's risk aversion factor, γ is the market price of systematic risk, and M the market return. The second factor in Equation (2.1) shows that μ increases with the correlation between pre-existing and new unhedgeable risks. If a life insurer is able to realize natural hedging, then the correlation is reduced, the company required rate of return on its products decreases, and therefore it can offer lower prices. In other words, its annuity prices will be lower than otherwise similar insurers which cannot naturally hedge their business.

2.3.2 Data, Measures and Methodologies

Data and Measures

The annuity prices are market quotes for single premium immediate annuities (SPIAs) for a 65–year–old male from 1995 to 1998 (Kiczek, 1995, 1996, 1997; A.M.Best, 1998). Each year the A.M. Best Company surveyed about 100 companies to obtain quotes for the lifetime–only monthly benefit paid to a 65–year–old male with \$100,000 to invest. The performance data of insurers is obtained from the National Association of Insurance Commissioners (NAIC). I manually match the A.M. Best data and the NAIC data. The sample includes 322 matched observations from which I extracted two sub–samples of companies that write more than a small amount of annuity business (defined below).

I transform each quote from a rate of payment per month to an equivalent price, as follows. If m is the monthly payment rate per \$100,000 and Price is what the company would charge for an annuity of 1 per year for the annuitant's lifetime, then

$$100,000 = 12m\,\text{Price} \qquad \text{and}$$
$$\text{Price} = \frac{100,000}{12m}.$$

The ratio of life insurance reserves to total annuity and life insurance reserves, denoted "Ratio", reflects the level of natural hedging provided by life insurance business to annuity business. The idea is that, of two otherwise similar annuity writers, the one with the higher Ratio value has a better hedge against longevity risk. The better hedge may allow for lower provision for risk in its premiums, and thus lower prices. In addition, this ratio determines the degree to which the insurer writes annuity business. For example, if the ratio is less than

0.90, the company annuity reserve is more than 10 percent of the total. The one sample regression includes sample observations for which Ratio < 0.90, *i.e.*, at least 10 percent of the company's reserve is for its annuity business. This sample rules out companies with very little annuity business. The sample size is $N = 299$. The other sub–sample consists of those observations for which the company has Ratio < 0.75 so that its life insurance reserves are less than 75 percent of its total reserves. The size of this sample is $N = 243$.[1]

In an efficient, competitive insurance market, the price of insurance will be inversely related to firm default risk (Phillips et al., 1998; Cummins, 1988; Merton, 1973). That is, a company with more default risk would have lower annuity prices. I use the A. M. Best rating for the year prior to the quote as a measure of default risk of annuity insurers. I use the rating to define a numerical variable, Lrate, as follows: if A.M. Best rating equals to "A++" or "A+", then Lrate =1; if A.M. Best rating equals to "A" or "A-", then Lrate =2; if A.M. Best rating equals to "B++" or "B+", then Lrate =3; if A.M. Best rating equals to "B" or "B-", then Lrate =4; and so on. This leads to the hypothesis that higher values of Lrate occur with lower annuity prices.

Other factors which may affect the annuity prices are also included in my regression model. I use the logarithm of the company's total gross annuity reserve, log(Resann), to represent the degree to which the company writes annuity business. The logarithm of the company's total assets, log(Tasset), controls for the size of the company. The sum of commissions and expenses divided by total written premium, Comexp, measures the company's expenses. Higher expenses should be related to higher annuity prices. Panel A and B in Table 2.1 report the summary statistics of my two sub-samples.

[1] If we eliminate more companies in the range $0.5 <$ Ratio < 0.75, where we might see a stronger effect of natural hedging, then the sample sizes are too small to get adequate significance.

Table 2.1: Summary Statistics from 1995 to 1998 (dollar amounts in millions)

Panel A

Variable	Description	Ratio < 0.90 ($N = 299$)			
		Mean	Minimum	Median	Maximum
m	Annuity payment rate	765.17	653.00	766.75	992.00
Price	Equivalent price	10.93	8.40	10.87	12.76
Resann	Annuity gross reserve	3,490,298	5,192	1,093,109	43,011,379
Reslife	Life insurance gross reserve	2,741,170	0	718,905	45,174,284
Ratio	$\frac{\text{Reslife}}{\text{Reslife+Resann}}$	0.44	0.00	0.47	0.89
Tasset	Total assets	9,069,208	33,351	2,996,356	127,097,380
Lrate	Lagged A. M. Best rating	1.59	1	2	4
Comexp	$\frac{\text{Commissions+expenses}}{\text{Net premiums}}$	23.21%	2.50%	21.20%	102.20%

Panel B

Variable	Description	Ratio < 0.75 ($N = 243$)			
		Mean	Min	Median	Max
m	Annuity payment rate	765.00	653.00	767.00	992.00
Price	Equivalent price	10.93	8.40	10.86	12.76
Resann	Annuity gross reserve	4,082,579	10,535	1,466,230	43,011,379
Reslife	Life insurance gross reserve	2,412,583	0	629,510	45,174,284
Ratio	$\frac{\text{Reslife}}{\text{Reslife+Resann}}$	0.35	0.00	0.37	0.75
Tasset	Total assets (in millions)	9,079,913	33,351	3,256,612	127,097,380
Lrate	Lagged A. M. Best rating	1.58	1	2	4
Comexp	$\frac{\text{Commissions+expenses}}{\text{Net premiums}}$	21.95%	2.50%	20.30%	93.50%

Methodologies

I use the pooled ordinary least square technique to investigate the relation between annuity prices and natural hedging, controlling for size, default risk, expenses and year effects. My regression model is expressed as follows:

$$\begin{aligned}\text{Price} = \alpha + \beta\,\text{Ratio} + \gamma_1 \log(\text{Resann}) + \gamma_2 \log(\text{Tasset}) + \gamma_3\,\text{Lrate} \\ + \gamma_4\,\text{Comexp} + \delta_1 D_{1996} + \delta_2 D_{1997} + \delta_3 D_{1998} + \epsilon,\end{aligned} \tag{2.2}$$

where D_{1996}, D_{1997} and D_{1998} are year dummies and ϵ is the error term.

Under my hypothesis, the coefficient β should be negative indicating that natural hedging allows for lower annuity prices. I run the regression for two sub–samples determined by the proportion of annuity business the company has written, as represented by Ratio.[2]

2.3.3 Findings and Implications

The coefficient estimates are presented in Table 2.2. White statistics (Greene, 2000) indicate that the distribution of errors is not heteroscedastic; OLS is appropriate. The signs of all the coefficients are consistent with my hypothesis. Since β is negative, annuity writers with more life insurance business tend to have lower annuity prices than similar companies with the same size and rating but less life insurance business. My story about natural hedging is consisistent with this empirical conclusion. The regression results suggest that annuity writing companies benefit from natural hedging, although these companies may not be making explicit natural hedging decisions.

When annuity business increases relative to life insurance business, the need for longevity risk hedging increases. When I focus on those observations with the proportion of annuity reserve to the sum of annuity reserve and life insurance reserve higher than 25 percent, that is Ratio < 0.75, the absolute value of its coefficient (-0.5487) controlling year effects is larger than that of the coefficient (-0.4777) obtained for the case when Ratio < 0.90. This suggests that when an annuity writer sells relatively more annuities, the increase in the life insurance hedge has a higher marginal effect in lowering the annuity price.

[2]The proportion of annuity reserves to the sum of annuity reserves and life insurance reserves measures the longevity risk exposure of an annuity insurer; this ratio is the complement of the variable Ratio. $\frac{\text{Resann}}{\text{Reslife}+\text{Resann}} = 1 - \text{Ratio}$.

Table 2.2: Pooled OLS regression—Relationship between annuity price and natural hedging

The dependent variable "Price" is what the company would charge for an annuity of 1 per year for the annuitant's lifetime. The ratio of life insurance reserves to total annuity and life insurance reserves is denoted "Ratio". I use the logarithm of the company's total gross annuity reserve, "log(Resann)", to represent the degree to which the company writes annuity business. The logarithm of the company's total assets, "log(Tasset)", controls for the size of the company. I use the A.M. Best rating to define a numerical variable, "Lrate", as follows: if A.M. Best rating equals to "A++" or "A+", then Lrate =1; if A.M. Best rating equals to "A" or "A-", then Lrate =2; if A.M. Best rating equals to "B++" or "B+", then Lrate =3; if A.M. Best rating equals to "B" or "B-", then Lrate =4; and so on. The sum of commissions and expenses divided by total written premium, "Comexp", measures the company's expenses.

Variables	Price					
	ALL		Ratio $<$ 0.90		Ratio $<$ 0.75	
Year Effects	No	Yes	No	Yes	No	Yes
Intercept	9.8563***	9.5744***	9.8234***	9.6432***	10.4060***	10.2900***
Ratio	**-0.1432**	**-0.1308**	**-0.4539****	**-0.4777*****	**-0.5782*****	**-0.5487*****
log(Resann)	-0.0400	-0.0230	-0.1746**	-0.1648**	-0.3138***	-0.3171***
log(Tasset)	0.0787	0.0581	0.2127***	0.1956***	0.3284***	0.3223***
Lrate	-0.1315**	-0.1242***	-0.1112*	-0.1073**	-0.1178*	-0.1209**
Comexp	0.7700***	0.7108***	0.8437***	0.8045***	1.0837***	0.9803***
N	322		299		243	
White test p-value	0.5513	0.2432	0.1284	0.5132	0.5472	0.2586
Adjusted R^2	0.0244	0.2598	0.0319	0.2717	0.0619	0.3122

Note: Standard errors are presented below the estimated coefficients;
*** Significant at 1% level;
** Significant at 5% level;
* Significant at 10% level.

How do we interpret the coefficient of Ratio (-0.5487) when Ratio < 0.75? Suppose an life insurer has 5 percent of its business in life insurance and 95 percent of its business in annuities. It sells \$100,000 life-only SPIAs to males aged 65 at the market average monthly payouts \$765. If it can realize full natural hedging, that is, 50 percent of business in life insurance and 50 percent business in annuities, its SPIA monthly payouts can be increased by \$18, that is, from \$765 to \$783 because it can reduce the risk premium in its price. Its SPIA prices can be more attractive than other similar competitors.

The signs of other variables are consistent with my expectations. The regression results suggest that when a life insurer writes more annuities, its annuity price goes down because the sign of the annuity business variable, log(Resann), is negative. It implies that economic scale exists in annuity sales. The size of an insurer is positively related to the price of its SPIA which may reflect the market power of bigger firms. This is consistent with prior research (Sommer, 1996; Froot and O'Connel, 1997). My default risk measure Lrate has a negative and significant coefficient, consistent with previous literature (Berger et al., 1992; Sommer, 1996). It implies that the higher default risk is associated with lower price all else equal. The sign of the coefficient of the expense variable log(Comexp) is positive and significant. Higher expenses are consistent with higher prices. The coefficients of year dummies are all significant at 1 percent level but not reported here.

2.4 Mortality Swaps

In section 2.3, I conclude that natural hedging may allow an annuity writer to lower its prices. However, it may be too expensive and unrealistic for an annuity writer to utilize natural hedging by changing its business composition. If we consider a corporate pension plan as an annuity writer, it may not even be legal for it to issue life insurance. Even for an insurer specialized in annuities, entering the life insurance business may not be practical. Moreover, natural hedging is not a static process. Dynamic natural hedging is required for new business. If an insurer is able to take advantage of natural hedging at a low cost by financial innovation, it can gain competitive advantage in the market by selling annuity products at lower prices. I propose mortality swaps to accomplish this goal.

2.4.1 Basic Ideas

I am suggesting that a market for mortality swaps may develop in which brokers and dealers offer swaps to annuity writers and separately to life insurers. The broker may match each annuity deal with a life deal or manage its portfolio of mortality swaps on an aggregate basis. As a start toward development of such a market, I propose a market–based approach to valuing each side of a mortality swap. The annuity (or survivor risk) side is priced in a way that is consistent with observed prices in the annuity market. Similarly the mortality (or death risk) side is consistent with the life insurance market.

Dowd et al. (2004) propose the possible uses of survivor swaps as instruments for managing, hedging and trading mortality-dependent risks. Their proposed survivor swap involves transferring a mortality risk related to a specific population with another population, that is, one specific longevity risk for another specific longevity risk. Their mortality swap can be used to diversify the longevity risks. However, the survivor swap does not hedge a good shock or a bad shock that strikes across populations (a systematic risk). My approach is different and it can hedge these shocks. Without any collateral, the swap payments are subject to counter–party risk. That is, one party, the broker or the insurer, may default. I ignore this issue and assume all parties fulfill their contractual obligations. Mortality swaps are described as follows.

Each year the annuity writer pays floating cash flows to the life insurer based on the actual number of deaths in the life insurer's specified portfolio of policies. This provides a benefit to the life insurer if mortality deteriorates, which the annuity writer may pay from its gain due to reduced annuity benefits.

At the same time the life insurer pays an annuity benefit based on the actual number of

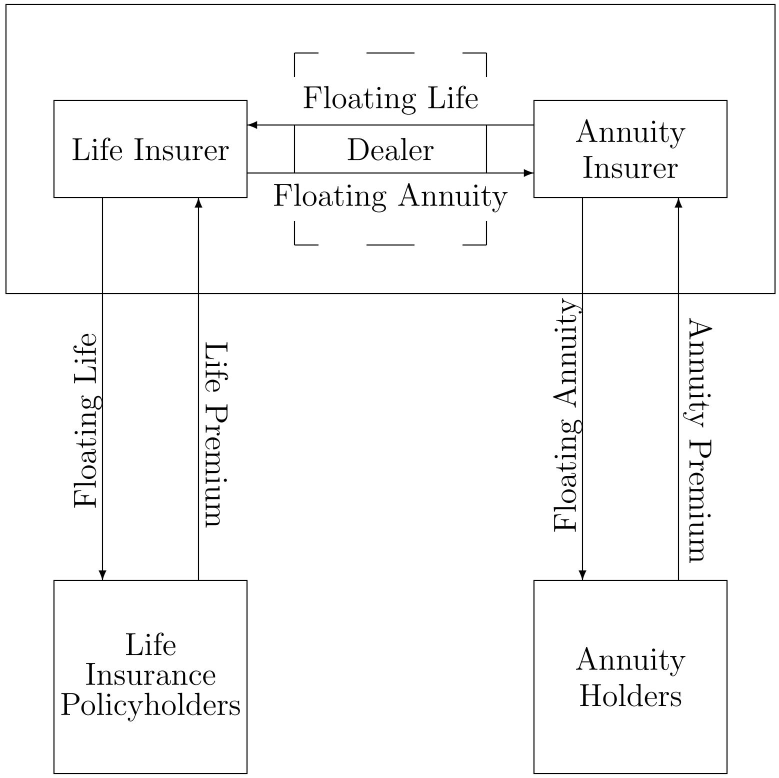

Figure 2.2: Mortality Swap Diagram.

survivors in the annuity writer's specified portfolio of annuities. If, for example, mortality improves, the life insurer pays the annuity writer but it has a gain on its own life insurance policies. There are no other swap payments. Figure 2.2 illustrates the mortality swap cash flows.

2.4.2 Mortality Swap Design

In order to keep the notation fairly simple, I assume that all of the lives in the life insurance portfolio are subject to the same mortality table, denoted by (x), for pricing purposes. At time 0 we have a portfolio of ℓ_x lives each insured for an amount f. The random number of survivors[3] to age $x+k$ is denoted ℓ_{x+k}. The number of deaths in the year $(k, k+1)$ is $d_{x+k} = \ell_{x+k} - \ell_{x+k+1}$. The distribution of d_{x+k} is binomial with parameters $m = \ell_x$

[3]This is a slight abuse of standard actuarial notation in which ℓ_{x+k} denotes the expected number of survivors.

and $q = {}_kp_x - {}_{k+1}p_x$, with q calculated using the mortality table specified at time 0.[4] The life insurer's aggregate death benefit payment in year $k+1$ amounts to fd_{x+k}. At time 0, the expected value is $\mathrm{E}(fd_{x+k}) = fm({}_kp_x - {}_{k+1}p_x)$. The insurer's risk is that actual life insurance benefit payments exceed the expected value and so it swaps, agreeing to pay annuity payments (defined below) in exchange for a payment in year $k+1$. The payment from the swap is

$$f(d_{x+k} - \mathrm{E}(d_{x+k}))_+ = \begin{cases} f(d_{x+k} - \mathrm{E}(d_{x+k})) & \text{if } d_{x+k} > \mathrm{E}(d_{x+k}) \\ 0 & \text{otherwise} \end{cases} \quad \text{for } k = 0, 1, \ldots. \tag{2.3}$$

The swap payment limits the net aggregate life insurance benefit from the life insurer to the expected value in each year. Of course, there is a price: The life insurer must provide annuity swap payments when the annuity writer needs them.

The annuity writer pays a benefit b per year to each survivor of an initial cohort of ℓ_y lives all subject to the same table at time 0 (but usually different from the lives on which the life insurance is written). The distribution of ℓ_{y+k} is binomial with parameters $m = \ell_y$ and $q = {}_kp_k$. The total benefit paid at time k is $b\ell_{y+k}$ and the annuity writer needs relief when it exceeds its expected value. This defines the life insurer's swap payment to the annuity writer:

$$b(\ell_{y+k} - \mathrm{E}(\ell_{y+k}))_+ = \begin{cases} b(\ell_{x+k} - \mathrm{E}(\ell_{x+k})) & \text{if } \ell_{y+k} > \mathrm{E}(\ell_{y+k}) \\ 0 & \text{otherwise} \end{cases} \quad \text{for } k = 0, 1, \ldots. \tag{2.4}$$

This is the net cash flow the annuity writer pays in the year $(k, k+1)$:

[4] In Section 4.5 in Chapter 4, I show how to model the shift of the selected mortality table by using a compound Poisson process.

Annuity benefits	$b\ell_{y+k}$
To the life insurer	$f(d_{x+k}-\mathrm{E}(d_{x+k}))_+$
From the life insurer	$b(\ell_{y+k}-\mathrm{E}(\ell_{y+k}))_+$
Total	$\begin{cases} b\ell_{y+k} & \text{if } d_{x+k}\le \mathrm{E}(d_{x+k}), \ell_{y+k}\le \mathrm{E}(\ell_{y+k}) \\ b\ell_{y+k}+f(d_{x+k}-\mathrm{E}(d_{x+k})) & \text{if } d_{x+k}> \mathrm{E}(d_{x+k}), \ell_{y+k}\le \mathrm{E}(\ell_{y+k}) \\ b\mathrm{E}(\ell_{y+k}) & \text{if } d_{x+k}\le \mathrm{E}(d_{x+k}), \ell_{y+k}> \mathrm{E}(\ell_{y+k}) \\ b\mathrm{E}(\ell_{y+k})+f(d_{x+k}-\mathrm{E}(d_{x+k})) & \text{if } d_{x+k}> \mathrm{E}(d_{x+k}), \ell_{y+k}> \mathrm{E}(\ell_{y+k}) \end{cases}$

Here is the life insurer's net cash flow in the year $(k, k+1)$:

Death benefits	fd_{x+k}
From annuity writer	$f(d_{x+k}-\mathrm{E}(d_{x+k}))_+$
To annuity writer	$b(\ell_{y+k}-\mathrm{E}(\ell_{y+k}))_+$
Total	$\begin{cases} fd_{x+k} & \text{if } d_{x+k}\le \mathrm{E}(d_{x+k}), \ell_{y+k}\le \mathrm{E}(\ell_{y+k}) \\ f\mathrm{E}(d_{x+k}) & \text{if } d_{x+k}> \mathrm{E}(d_{x+k}), \ell_{y+k}\le \mathrm{E}(\ell_{y+k}) \\ fd_{x+k}+b(\ell_{y+k}-\mathrm{E}(\ell_{y+k})) & \text{if } d_{x+k}\le \mathrm{E}(d_{x+k}), \ell_{y+k}> \mathrm{E}(\ell_{y+k}) \\ f\mathrm{E}(d_{x+k})+b(\ell_{y+k}-\mathrm{E}(\ell_{y+k})) & \text{if } d_{x+k}> \mathrm{E}(d_{x+k}), \ell_{y+k}> \mathrm{E}(\ell_{y+k}) \end{cases}$

In each year the swap rearranges the sum of annuity and life insurance benefit payments. The sum is always $b\ell_{y+k}+fd_{x+k}$, but the parties swap adverse outcomes. They need not swap all of their business. The swap contract could be adjusted so that the swap payment triggers are higher than the expected values, for example, or the contract could specify upper bounds on the annual swap payments.

The underlying portfolios of insured lives and annuitants should be selected so that they are subject to the same general mortality change factors. Change factors can move mortality either way. The 1918 flu epidemic is an example which, although the effects varied by age, had a negative impact across populations. Providing pure water and sanitary sewers in

European and American cities improved mortality at all ages. The invention of penicillin had a positive effect across populations.

2.4.3 One-Factor Wang Transform

Cairns et al. (2006) discuss a theoretical framework for pricing mortality derivatives and valuing liabilities which incorporate mortality guarantees. Their stochastic mortality models require certain reasonable criteria in terms of their potential future dynamics and mortality curve shapes. Mortality changes in a complex manner, influenced by socioeconomic factors, biological variables, government policies, environmental influences, health conditions and health behaviors. Not all of these factors improve with time and, moreover, opinions on future mortality trends vary widely (Buettner, 2002; Hayflick, 2002; Goss et al., 1998; Rogers, 2002). Even if we could settle on a such a dynamic framework, estimation of the parameters may be very difficult. I take a different, static approach.

Wang (1996, 2000, 2001) has developed a method of pricing risks that unifies financial and insurance pricing theories. This method can be used to price mortality bonds (Lin and Cox, 2005). Now I apply this method to mortality swaps. Consider a random payment X paid at time T. If the cumulative distribution function is $F(x)$, then a distorted or transformed distribution $F^*(x)$ is determined by parameter λ according to the equation

$$F^*(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]. \tag{2.5}$$

where $\Phi(x)$ is the standard normal cdf. The idea is to determine λ so that the time 0 price of X is its discounted expected value using the transformed distribution. Then the formula for the price is

$$v_T \mathrm{E}^*(X) = v_T \int x dF^*(x) \tag{2.6}$$

where v_T the discount factor determined by the market for risk free bonds at time 0. Thus, for an insurer's given liability X with cumulative density function $F(x)$, the Wang transform will produce a risk–adjusted density function $F^*(x)$. The mean value under $F^*(x)$, denoted by $\mathrm{E}^*[X]$, is the risk–adjusted fair-value of X at time T. Wang's paper describes the utility of this approach. It generalizes well known techniques in finance and actuarial science. My idea is to use observed annuity prices to estimate the market price of risk for annuities, then use the same distribution to price the annuity side of a mortality swap. For the life insurance side I use the same idea. Term life insurance prices to determine the market price of risk in the life insurance market, giving us the appropriate distribution for pricing the life insurance side of the swap.

The Wang transform is based on the idea that the life insurance or annuity market price takes into account the uncertainty in the mortality table, as well as the uncertainty in the lifetime of a life insured or an annuitant once the table is given. The market price of risk does not and need not reflect the risk in interest rates because I am assuming that mortality and interest rate risks are independent. Moreover, I am assuming that investors accept the same transformed distribution and independence assumption for pricing mortality swaps.

The Wang transform is an equilibrium model that can recover the CAPM model. Under the normal distribution assumption, the market price of risk of an asset in the classical CAPM, $\lambda = [\mathrm{E}(r) - r_f]/\sigma$, is the excess return per unit of volatility. For insurance risks, it is the risk load per unit of volatility. The Wang transform takes the equilibrium perspective of the CAPM model. If the return of an asset or risk r is normally distributed with mean μ and standard deviation σ, then the transformed distribution is also normal with mean $\mu^* = \mu - \lambda\sigma$ and standard deviation $\sigma^* = \sigma$.

Proof. If r is normal with mean μ and variance σ^2, then

$$F(r) = \Phi\left(\frac{r-\mu}{\sigma}\right).$$

The Wang transform is given by

$$
\begin{aligned}
F^*(r) &= \Phi[\Phi^{-1}(F(r)) + \lambda] \\
&= \Phi\left(\frac{r-\mu}{\sigma} + \lambda\right) \\
&= \Phi\left(\frac{r-(\mu-\lambda\sigma)}{\sigma}\right).
\end{aligned}
$$

That is, the transformed distribution preserves the normal distribution with mean

$$\mu^* = \mu - \lambda\sigma \tag{2.7}$$

and standard deviation $\sigma^* = \sigma$. $\square$

In equilibrium, I expect that the risk-adjusted return μ^* equals to the risk free rate r_f. Therefore I can rewrite equation (2.7) as

$$\lambda = \frac{\mu - \mu^*}{\sigma} = \frac{\mu - r_f}{\sigma}. \tag{2.8}$$

This is the market price of risk in the CAPM. Moreover, the Wang transform can reproduce the Black–Scholes model with lognormally–distributed assets (Wang, 2002).

The Wang transform has a more desirable property than the CAPM. The CAPM cannot be applied to the situation when the distributions are not normally distributed. So it limits the CAPM's application in the insurance area where insurance loss distributions are skewed. Contrast to that of the CAPM, the cumulative distribution function $F_X(x)$ in the Wang transform can be some distributions other than normal distributions.

Market price of risk

For the annuity distribution function $F_a(t) = {}_tq_{65}$, I use the 1996 IAM 2000 Basic Table for a male life age sixty–five. Then assuming an expense factor equal to 4 percent, I use the 1996 market quotes of qualified immediate annuities (Kiczek, 1996) and the US Treasury yield curve on December 30, 1996 to get the market price of risk λ_a by solving the following equation[5] numerically for λ:

$$1000(1-0.04) = 7.48\sum_{j=1}^{\infty} v_{j/12}[1-F_a^*(j/12)] \tag{2.9}$$

The variable $v_{j/12}$ is the discount factor at time 0 for a payment of 1 after $j/12$ years. And $[1-F_a^*(j/12)]$ is the survival probability usually denoted ${}_{j/12}p_{65}$, but using the transformed distribution. This has to be solved numerically for λ_a.

Thus I determine the market price of risk for annuitants is $\lambda_a = 0.2134$. I think of the 1996 IAM 2000 Basic Table as the actual or physical distribution, which requires a distortion to obtain market prices. Figure 2.3 shows the graphs of $F_a(t)$ and $F_a^*(t)$. The distorted distribution lies to the right and above the physical distribution. This indicates that the market view of annuitant mortality is that it will improve relative to the base table.

Similarly, for the life insurance distribution function $F_l(t) = {}_tq_{35}$, I use the 1990-95 SOA Life Insurance Basic Table for a male life age thirty–five. A.M.Best (1996) reports market quotes for both preferred non-smokers and standard smokers. The 1990-95 SOA Basic Table is created based on a mixture of smokers and non-smokers. I calculate a weighted average of term life insurance prices (\$507.48) using weights based on the incidence of smoking in the US population as reported by the Center for the Disease Control[6] in 1995. Assuming an expense factor equal to 10 percent, I use the 1996 average market quotes of ten–year level \$250,000 term life insurance (A.M.Best, 1996) based on 97 companies and the US Treasury yield curve on December 30, 1996 to get the market price of risk $\lambda_l = 0.1933$ by solving the

[5]The analogous equations in Lin and Cox (2005) are incorrect, although the solutions to the equations as stated here, are correct.

[6]Source: www.cdc.gov. About 57 percent former and current male smokers and 43 percent male non-smokers.

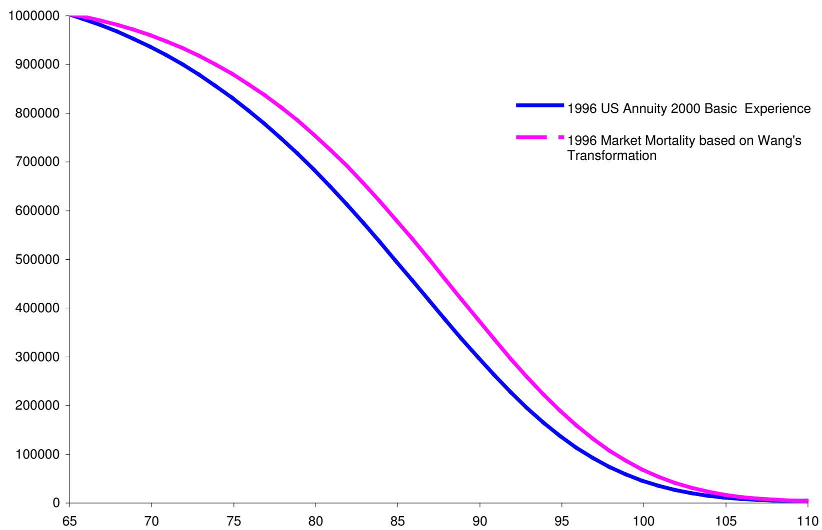

Figure 2.3: The result of applying the Wang transform to the survival distribution based on 1996 IAM experience for males age 65 and prices from Kiczek (1996).

following equation:

$$507.48 \times (1 - 0.10) = 250,000 \sum_{k=0}^{9} v_{k+1} \left[F^*(k+1) - F^*(k)\right] \tag{2.10}$$

where v_{k+1} is the discount factor for a payment after $k+1$ years. A positive life insurance market price of risk means the market anticipates improved mortality for insured lives, relative to the base table.

Mortality Swap Pricing

I can price each side of the swap now. This will allow me to determine factors b and f for which the two market prices are equal so no cash is paid by either party to initate the contract; they make only contractual swap payments.

The market price of the payment $f(d_{x+k} - \mathrm{E}(d_{x+k}))_+$ for year k is its discounted expected value $f\mathrm{E}^*[(d_{x+k} - \mathrm{E}(d_{x+k}))_+]$. The "*-distribution" of d_{x+k} is binomial with parameters $m = \ell_x$ and $q = F^*(k+1) - F^*(k)$.[7] For large m (over 20 is often suggested), the binomial distribution is approximately normal and I am thinking of portfolios of hundreds of lives. Thus, to a very good approximation,

$$\mathrm{E}^*[(d_{x+k} - \mathrm{E}(d_{x+k}))_+] \approx \sigma_k \mathrm{E}[(X - x_k)_+]$$

where X is a standard normal variable, $\mu_k = mq$ and $\sigma_k = \sqrt{mq(1-q)}$ are the mean and standard deviation of d_{x+k} and

$$x_k = \frac{\mathrm{E}(d_{x+k}) - \mu_k}{\sigma_k}.$$

For a standard normal variable X, there is a formula for this (Lin and Cox, 2005):

$$\mathrm{E}[(X - x)_+] = \phi(x) - x[1 - \Phi(x)] \tag{2.11}$$

where $\phi(x) = \Phi'(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$. Thus we can calculate the market price of the life insurance benefit swap payments as

$$f\sum_{k=0}^{\infty} v_{k+1}\mathrm{E}^*[(d_{x+k} - \mathrm{E}(d_{x+k}))_+] = f\sum_{k=0}^{\infty} v_{k+1}\sigma_k[\phi(x_k) - x_k[1 - \Phi(x_k)]]. \tag{2.12}$$

I apply this same technique to calculate the market value of the annuity benefit swap payments. I only change the definitions of μ_k and σ_k.

$$b\sum_{k=0}^{\infty} v_{k+1}\mathrm{E}^*[(\ell_{y+k} - \mathrm{E}(\ell_{y+k}))_+] = b\sum_{k=0}^{\infty} v_{k+1}\sigma_k[\phi(x_k) - x_k[1 - \Phi(x_k)]] \tag{2.13}$$

The distribution of ℓ_{y+k} is binomial with $m = \ell_y$ and $q = 1 - F^*(k)$. The mean are standard

[7] The "*-distribution" shows the market expectation of future mortality table shift.

deviation are, again, $\mu_k = mq$ and $\sigma_k = \sqrt{mq(1-q)}$. The remaining parameter is

$$x_k = \frac{\mathrm{E}(\ell_{y+k}) - \mu_k}{\sigma_k}.$$

I calculate these values for a 10–year swap, using $m = 10,000$ lives in the annuity and life insurance portfolios. The market value of the life insurance payments, for a death benefit of \$1,000,000, is \$52,758. This is about 1.6 percent of the net term life insurance premium.

For the annuity payment side of the swap, the market value per one dollar of annual benefit, again for 10,000 lives, is \$1,690. To adjust the average amount of insurance f per term policy to make each side of the swap have the same price, we have

$$1,690 = f\frac{52,758}{100,000}$$

or $f = 3,203$. Thus for each dollar of annual annuity benefit we must have \$3,203 of death benefit to make the prices equal at time 0. Then each party will have the benefit of natural hedging for 10 years. Even though the prices are equal at time 0, the mortality may move one way or the other so that the future market value favors one party. The swap has to be revalued each year to properly reflect each company's position as it may be either an asset or liability.

2.5 Conclusions and Discussion

Natural hedging utilizes the opposite reaction of life insurance and annuities to the same mortality change to stabilize aggregate cash outflows. My empirical evidences suggest natural hedging is an important factor contributing to annuity price differences after I control for other variables. These differences become more significant for those insurers selling relatively more annuity business. I expect that life insurers may reach the same conclusion.

Most insurance companies still have considerable net exposures to mortality risks even if they reduce their exposure by pooling individual mortality risk and by balancing their annuity positions against their life positions (Dowd et al., 2004). Natural hedging is feasible and mortality swaps make it available widely.

I show how to design a mortality swap between a life insurer and annuity writer to create a natural hedge. Compared with traditional reinsurance and other derivatives, such as mortality bonds, mortality swaps may be arranged at possibly lower costs and in a more flexible way to suit diverse circumstances. Thus, there are good reasons to anticipate increased activity in mortality swaps between life insurers and annuity insurers.

3

Securitization of Mortality Risks in Life Annuities

Securitization with payments linked to explicit mortality events provides a new investment opportunity to investors and financial institutions. Moreover, mortality-linked securities provide an alternative risk management tool for insurers. The purpose of this paper is to study the securitization of mortality risks, especially the longevity risk inherent in a portfolio of annuities or in a pension plan.

3.1 Introduction

Securities with mortality risk as a component have been around a long time. These securities arise as securitization of portfolios of life insurance or annuity policies. The risks underlying a life insurance or annuity portfolio include interest rate risk, policyholder lapse risk, as well as mortality or longevity risk. In these transactions, the positive future net cash flow from the policies is dedicated to pay the bondholders. Therefore, they are similar to asset securitization. Cummins (2004) surveys recent life insurance securitization transactions, including these asset-type securities.

However, securitization of *pure* mortality or longevity risk is a recent and potentially important innovation in financial markets. Pure mortality or longevity securitization is more like property-linked catastrophe bonds than the common asset-type life insurance securitization. This is because, like that of a property-linked catastrophe bond based on earthquake or hurricane losses, the payment of a mortality security is subject only to a well-defined risk. In the case of a mortality bond, the event might be a sudden spike in death rates, which may be caused by a flu epidemic.

Catastrophes impose a big potential problem for a life insurer's solvency since fatalities from natural and man–made disasters may be tremendous. For example, the earthquake and tsunami in southern Asia and eastern Africa in December 2004 killed 182,340 people and resulted in 129,897 missing (Guy Carpenter, 2005). Although most of the victims did not own life insurance, the life insurance industry may not have enough capacity to cover this type of catastrophe losses if such an event were to occur in a more economically developed region where most people buy life insurance. Cummins and Doherty (1997) noted that "a closer look at the industry reveals that the capacity to bear a large catastrophic loss is actually much more limited than the aggregate statistics would suggest." Securities linked to catastrophe death risk, such as the Swiss Re bond, are discussed in Chapter 4.

This paper focuses on the other side of mortality risk — longevity risk. Although mortality improves over time, future rates of improvement are uncertain. At the same time we are seeing, especially in the U.S., a trend to shift longevity risk to individuals. In the US, defined benefit pension plans are converting to defined contribution plans. Proposed Social Security reforms further shift mortality risk to individuals. Thus, there should be an increased demand for individual annuities. As demand for annuities increases, the annuity insurers' need for risk management of potential mortality improvements will increase.

As a new risk management tool, mortality securitization enhances the capacity of the life insurance industry by transferring its catastrophic losses to financial markets. Jaffee and Russell (1997) and Froot (2001) argue that insurance securitization offers a potentially more efficient mechanism for financing catastrophe losses than conventional insurance and reinsurance. Securitization brings more capital and provides innovative contracting features for the life insurance industry to bear potential mortality shocks, thus avoiding the market disruptions caused by disruptive reinsurance price and availability cycles. Moreover, because mortality securities may be uncorrelated with financial markets, they provide a valuable new source of diversification for market participants (Cox et al., 2000; Litzenberger et al., 1996; Canter et al., 1997). Finally, Cummins (2004) categorizes securitization as arbitrage opportunities or new classes of risk that enhance market efficiency. This paper proposes a feasible way to securitize longevity risk.

In section 3.2, I describe securitization of longevity risk with a mortality bond or a mortality swap. In section 3.3, I illustrate how insurers (or reinsurers or pension plans) can use mortality–based securities to manage longevity risk. In section 3.4, I show how good is the hedge provided by my proposed mortality bond. In section 3.5, I describe the difficulties arising in making mortality projections. I discuss annuity data, including the Individual Annuity Mortality tables and the Group Annuity Experience Mortality (GAEM) reports from Reports of the Transactions of the Society of Actuaries (TSA). Section 3.6 is for discussion and conclusions.

3.2 Mortality Securitization

I propose a new type of mortality bond which is similar to the Swiss Re deal discussed in Section 4.3.1 but focused on longevity. The structure is similar to other deals. Generally, the life-based securitizations follow the same structure as the so called catastrophe-risk bonds. There have been more than thirty catastrophe bond transactions reported in the financial press and many papers written about them. Mortality bonds are different in several important ways. For example, deviation from mortality forecasts may occur gradually over a long period, as opposed to a sudden property portfolio loss. However, in both transactions, costs are likely to be high relative to reinsurance on a transaction basis.

In both transactions, the insurer (reinsurer or annuity provider) purchases reinsurance from a special purpose company (SPC). The SPC issues bonds to investors. The bond contract and reinsurance convey the risk from the annuity provider to the investors. The SPC invests the reinsurance premium and cash from the sale of the bonds in default-free securities. I will show how this can be set up to allow the SPC to pay the benefits under the terms of the reinsurance with certainty.

3.2.1 Example

As an example of a mortality securitization, consider an insurer[1] that must pay immediate life annuities to ℓ_x annuitants[2] all aged x initially. Set the payment rate at 1,000 per year per annuitant. Let ℓ_{x+t} denote the number of survivors to year t. The insurer pays $1,000\ell_{x+t}$ to its annuitants, which is random, as viewed from time 0. I will define a bond contract to hedge the risk that this portion of the insurer's payments to its annuitants exceeds an agreed upon level.

The insurer buys insurance from its SPC for a premium P at time 0. The insurance

[1]The "insurer" could be an annuity writer, annuity reinsurer or private pension plan. The counter party could be a life insurer or investor.

[2]The security could be based on a mortality index rather than an actual portfolio. This will avoid the moral hazard problem, but it introduces basis risk.

contract has a schedule of fixed trigger levels X_t such that the SPC pays the insurer the excess of the actual payments over the trigger. In year t the insurer pays amount $1000\ell_{x+t}$ to its annuitants. If the payments exceed the trigger for that year, it collects the excess from the SPC, up to a maximum amount. Let us say that the maximum is stated as a multiple of the rate of annuity payments $1000C$. Thus in each year $t = 1, 2, \ldots, T$ the insurer collects the benefit B_t from the SPC determined by this formula:

$$B_t = \begin{cases} 1000C & \text{if } \ell_{x+t} > X_t + C \\ 1000(\ell_{x+t} - X_t) & \text{if } X_t < \ell_{x+t} \leq X_t + C \\ 0 & \text{if } \ell_{x+t} \leq X_t \end{cases} \tag{3.1}$$

The insurer specifies the annuitant pool in much the same way that mortgage loans are identified in construction of a mortgage–backed security. The insurer's cash flow to annuitants $1000\ell_{x+t}$ at time t is offset by positive cash flow B_t from the insurance:

$$\begin{aligned} \text{Insurer's Net Cash Flow } &= 1000\ell_{x+t} - B_t \qquad (3.2) \\ &= \begin{cases} 1000(\ell_{x+t} - C) & \text{if } \ell_{x+T} > C + X_t \\ 1000X_t & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ 1000\ell_{x+t} & \text{if } \ell_{x+t} \leq X_t \end{cases} \end{aligned}$$

For this structure, there is no basis risk in the reinsurance. Basis risk arises when the hedge is not exactly the same as the reinsurer's risk. The mortality bond covers the same risk, so there is no basis risk. This is in contrast to the Swiss Re deal, which is based on a population index rather than a portfolio of Swiss Re's life insurance polices (or its clients' policies). While there is no basis risk, the contract does not provide full coverage. I will study the distribution of the present value of the excess later.

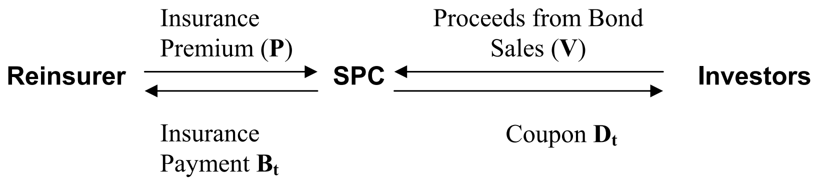

Figure 3.1: Mortality Bond Cash Flow Diagram.

3.2.2 Bonds

Here is a description of the cash flows between the SPC, the investors and the insurer as illustrated in Figure 3.1. The SPC payments to the investors are:

$$D_t = \begin{cases} 0 & \text{if } \ell_{x+t} > C + X_t \\ 1000C - B_t & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ 1000C & \text{if } \ell_{x+t} \leq X_t \end{cases} \tag{3.3}$$

$$= \begin{cases} 0 & \text{if } \ell_{x+t} > C + X_t \\ 1000(C + X_t - \ell_{x+t}) & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ 1000C & \text{if } \ell_{x+t} \leq X_t \end{cases} \tag{3.4}$$

where D_t is the total coupon paid to investors. The maximum value of ℓ_{x+t} is ℓ_x, attained if nobody dies, but from the perspective of 0, it is a random value between 0 and ℓ_x. Let the market price of the mortality bond be denoted by V. The aggregate cash flow out of the SPC is

$$B_t + D_t = 1000C \tag{3.5}$$

for each year $t = 1, ..., T$ and the principal amount $1000F$ at $t = T$. The SPC will perform on its insurance and bond contract commitments with probability 1, provided $P + V$ is at least equal to the price W of a default free fixed coupon bond with annual coupon $1000C$ and principal $1000F$ valued with the bond market discount factors:

$$P + V \geq W = 1000Fd(0,T) + \sum_{k=1}^{T} 1000Cd(0,k). \tag{3.6}$$

The discount factors $d(0,k)$ can be taken from the bond market at the time the insurance is issued. In other words, if the insurance premium and proceeds from sale of the mortality bonds are sufficient, the SPC can buy a straight bond and have exactly the required coupon cash flow it needs to meet its obligation to the insurer and the investors. Each year SPC receives $1000C$ as the straight bond coupon and then pays D_t to investors and B_t to the insurer. It is always the case that $1000C = D_t + B_t$ is exactly enough to meet its obligations.

Thus we see how to set up a longevity risk bond contract for which the longevity risk over T years is passed to the capital market almost completely. Of course, the price of the mortality bond is yet to be addressed. We need to see how likely it is that some payments will be covered. That is, how good is the insurance coverage? From the investor's perspective, how likely is it that they will miss a coupon? At time T, the SPC will have the accumulated value of $P + V - W$ and this is positive with probability 1. This future value belongs to the insurer since it is the sole owner of the SPC. For this paper I assume $P + V = W$.

3.2.3 Swaps

The same cash flows, B_t to the insurer and D_t to the bondholders, can be arranged with swap agreements and no principal payment at time T. However, without the principal as collateral, the swap payments are subject to counter–party risk. Assuming there is no counter–party risk, the equivalent swaps contracts are described as follows. Since there is no counter–party risk, the insurer's payment of P at time 0 can be replaced by level annual payments of x

where

$$P = x \sum_{k=1}^{T} d(0,k).$$

Then each year, the insurer pays x to the SPC (or a swap originator) and gets a floating benefit $B_t, t = 1, 2, ..., T$. There are no other payments. This is a fixed for floating swap from the insurer's perspective. So long as there is no counter–party risk, the insurer can get essentially the same reinsurance benefit from a swap. The swap might be provided by a broker or investment banker.

The same analysis applies to the bondholder's cash flows. In place of paying V for the mortality bond, they can pay a fixed amount y each year in order to receive variable coupons D_t, where

$$y \sum_{k=1}^{T} d(0,k) + 1000Fd(0,T) = \sum_{k=1}^{T} \mathrm{E}^*[D_t]d(0,k) + 1000Fd(0,T).$$

So

$$y \sum_{k=1}^{T} d(0,k) = \sum_{k=1}^{T} \mathrm{E}^*[D_t]d(0,k).$$

The mean value $\mathrm{E}^*[D_t]$ defines the risk–adjusted fair value of D_t at time t. Then in each year, the SPC gets $x + y$, exactly enough to finance its obligation $B_t + D_t$. The only difference is collateral. If there is no possibility of default on the fixed payments, then SPC will always have just enough cash to meet its floating payment obligations. In this case, swaps can replace the mortality bond. This may save transaction costs. The trade–off is that it introduces default risk.

3.3 Pricing Mortality Risk Bonds

I apply the one-factor Wang transform (Wang, 1996, 2000, 2001) introduced in Section 2.4.3 to price mortality risk bonds. Let $\Phi(x)$ be the standard normal cumulative distribution function with a probability density function

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

for all x. Wang defines the distortion operator as

$$g_\lambda(u) = \Phi[\Phi^{-1}(u) - \lambda] \tag{3.7}$$

for $0 < u < 1$ and a parameter λ.

3.3.1 Market Price of Risk

First I estimate the market price of risk λ. I define my transformed distribution F^* as:

$$F^*(t) = g_\lambda(F)(t) = \Phi[\Phi^{-1}({}_tq_{65}) - \lambda] \tag{3.8}$$

For the distribution function $F(t) = {}_tq_{65}$, I use the 1996 IAM 2000 Basic Table for a male life age 65 and, separately, for a female life age 65. Assuming an expense factor equal to 6 percent, I use the August 1996 market quotes of qualified immediate annuities (Kiczek, 1996) and the US Treasury yield curve on August 15, 1996 to get the market price of risk λ by solving the following equations numerically:

$$\begin{aligned} 1,000 \times (1 - 0.06) &= 7.48 \times 12a_{65}^{(12)} \qquad \text{for males,} \\ 1,000 \times (1 - 0.06) &= 6.94 \times 12a_{65}^{(12)} \qquad \text{for females.} \end{aligned} \tag{3.9}$$

The market price of risk for males and females respectively is shown in Table 3.1. The market price of risk is 0.1792 for male annuitants and 0.2312 for female annuitants.

Table 3.1: The market price of risk, determined by the 1996 IAM 2000 Basic Table, the US Treasury constant maturity interest rate term structure on August 15, 1996, and annuity market prices from Best's Review (August 1996) net of my assumed expense factor 6 percent. The payment rate is the dollars per month of life annuity per $1,000 of lump–sum life annuity premium at the issue age. The market value is the price (net of annuity expenses) for $1 per month of life annuity.

	Payment Rate	Market Value	Market price of risk
Male (65)	7.48	125.73	0.1792
Female (65)	6.94	135.25	0.2312

3.3.2 Mortality Bond Strike Levels

A designed portfolio of annuities underlies the mortality bond. The mortality bond contract may set several strike levels X_t. In my example, I set three different improvement levels for male and female age 65 immediate annuities which determine the strike levels. I use the Renshaw et al. (1996)'s method to predict the force of mortality and discuss this method in section 3.5. The improvement levels in Table 3.2 are determined by the average of 30–year force of mortality improvement forecast for age group 65–74, age group 75–84 and age group 85–94 respectively based on the 1963, 1973, 1983 and 1996 US individual annuity mortality tables.

Table 3.2: Three different improvement levels determine the strike levels.

Age Range	Change of Force of Mortality
65–74	-0.0070
75–84	-0.0093
85–94	-0.0103

Including the above improvement factors, the corresponding strike level for each age will be X_t. The number of survivors ℓ_{65+t} is the number of lives attaining age in the survivorship group set in the contract. This means that I set the strike levels X_t as follows:

$$X_t = \begin{cases} \ell_x \, {}_tp_x \, e^{0.0070t} & \text{for } t = 1, ..., 10 \\ \ell_x \, {}_tp_x \, e^{0.07} e^{0.0093(t-10)} & \text{for } t = 11, ..., 20 \\ \ell_x \, {}_tp_x \, e^{0.163} e^{0.0103(t-20)} & \text{for } t = 21, ..., 30 \end{cases}$$

where ${}_tp_x$ is the survival probability for males or females from the 1996 IAM 2000 Basic table.

In the annuity market, the price of an immediate annuity is the discounted expected cash flow to a random lifetime of annuitant. The random cash flows are $\{\frac{1}{12}1000\ell_{x+t/12}|t = 1, 2, ...\}$. The observed price allows me to calculate the market price of risk λ in the Wang transform. The market price of risk λ is obtained from the following equation:

$$12\ell_x a_{65}^{(12)} = \sum_{t=1/12}^{\infty} \mathrm{E}^*[\ell_{x+t}]d(0, t) \qquad (3.10)$$

where $\ell_x a_{65}^{(12)}$ is the total immediate annuity premium net of the insurer's expenses from a initial number of annuitants ℓ_x and $\mathrm{E}^*[\ell_{x+t}]$ is the transformed expected number of survivors to time t.

In the bond market I have cash flows $\{D_t\}$ which depend on the same distribution of survivors. I assume that investors accept the same pricing method so that the bond price is

$$V = Fd(0, T) + \sum_{t=1}^{T} \mathrm{E}^*[D_t]d(0, t) \qquad (3.11)$$

where D_t is defined in (3.3) and $d(0, t)$ is the discount factor based on the risk free interest rate term structure at the time the bond is issued. The face amount F is not at risk; it is paid at time T regardless of the number of surviving annuitants. The discount factors are from the US Treasury interest rate term structure on August 15, 1996. The survival distribution in equation (3.11) is the distribution derived from the annuity market. It is based on the 1996 US Annuity 2000 Basic Mortality Tables and equation (3.8) with $\lambda = 0.1792$ for male annuitants and $\lambda = 0.2312$ for females.

$\mathrm{E}^*[D_t]$ is calculated as follows. From (3.3) we can write the coupon payment as

$$\frac{1}{1000} D_t = \begin{cases} 0 & \text{if } \ell_{x+t} > C + X_t \\ C + X_t - \ell_{x+t} & \text{if } X_t < \ell_{x+t} \leq C + X_t \\ C & \text{if } \ell_{x+t} \leq X_t \end{cases} \tag{3.12}$$

$$= C - \max(\ell_{x+t} - X_t, 0) + \max(\ell_{x+t} - X_t - C, 0)$$

$$= C - (\ell_{x+t} - X_t)_+ + (\ell_{x+t} - X_t - C)_+ \tag{3.13}$$

Therefore

$$\frac{1}{1000} \mathrm{E}^*[D_t] = C - \mathrm{E}^*[(\ell_{x+t} - X_t)_+] + \mathrm{E}^*[(\ell_{x+t} - X_t - C)_+].$$

The distribution of ℓ_{x+t} is the distribution of the number of survivors from ℓ_x who survive to age $x+t$, which occurs with probability ${}_tp_x^*$ where ${}_tp_x^*$ is the transformed survival probability. Therefore ℓ_{x+t} has a binomial with parameters ℓ_x and ${}_tp_x^*$. We have a large ℓ_x value so ℓ_{x+t} has approximately a normal distribution with mean $\mathrm{E}^*[\ell_{x+t}] = \mu_t^* = \ell_{x t}p_x^*$ and the variance $\mathrm{Var}^*[\ell_{x+t}] = \sigma_t^{*2} = \ell_{x t}p_x^*(1 - {}_tp_x^*)$.[3] From equation (2.11), I get

$$\mathrm{E}[(X - k)_+] = \Psi(k) = \int_k^\infty [1 - \Phi(t)] dt$$
$$= \phi(k) - k[1 - \Phi(k)].$$

This is a useful form since the functions $\phi(k)$ and $\Phi(k)$ can be calculated with *Excel*. Then

[3] The calculation is done separately for males and females although the notation does not reflect the difference. We can easily adjust this for a mixture of males and females.

I can calculate components of $\mathrm{E}^*(D_t)$:

$$\begin{aligned}
\mathrm{E}^*[(\ell_{x+t} - X_t)_+] &= \mathrm{E}^*[(\ell_{x+t} - \mu_t^* - (X_t - \mu_t^*)_+] \\
&= \sigma_t^* \,\mathrm{E}^* \left[\left(\frac{\ell_{x+t} - \mu_t^*}{\sigma_t^*} - k_t \right)_+ \right] \\
&= \sigma_t^* \Psi(k_t)
\end{aligned}$$

where $k_t = (X_t - \mu_t^*)/\sigma_t^*$. Similarly

$$\mathrm{E}^*[(\ell_{x+t} - X_t - C)_+] = \sigma_t^* \Psi(k_t + C/\sigma_t^*)$$

Finally I have the formula:

$$\mathrm{E}^*[D_t] = 1000 \left\{ C - \sigma_t^* \left[\Psi(k_t) - \Psi(k_t + C/\sigma_t^*) \right] \right\}. \tag{3.14}$$

Consider an initial cohort of 10,000 annuitants all the same sex, $\ell_{65} = 10,000$. Table 3.3 shows prices for mortality bonds and reinsurance for a group of 10,000 male and female annuitants respectively with \$1,000 annual payout per person, with the strike levels defined above, the annual aggregate cash flow out of the SPC \$700,000 (=1000C) and a 7 percent coupon rate for both straight bond and mortality bond. The price of the mortality bond on male age 65 immediate annuitants is \$998.85 per \$1000 of face value. Similarly, the bond price for the female age 65 immediate annuitants is \$995.57 per \$1000. With the above setup, the reinsurance price is \$11,493 for male age 65 and \$44,337 for female age 65. It gives the insurer 30–year protection. If the number of survivors exceeds the strike level X_t in year t, the SPC will pay the insurer the excess (B_t) up to \$700,000 and the total coupon the investors will get that year is $\max[0, 700,000 - B_t]$. Compared with the total immediate annuity premium the insurer collects from the annuitants (\$99,650,768 for male age 65 and \$107,232,089 for female age 65), the reinsurance premium the insurer pays the SPC is only a negligible proportion of the total annuity premium (0.012 percent for male and 0.041 percent

Table 3.3: Mortality Bond Price and Reinsurance Premium

	Male (65)	Female (65)
Market price of risk (λ)	0.1792	0.2312
Number of annuitants	10,000	10,000
Annuity annual payout per person	1,000	1,000
Total premium from annuitants	99,650,768	107,232,089
Improvement level age 65 - 74	-0.0070	-0.0070
Improvement level age 75 - 84	-0.0093	-0.0093
Improvement level age 85 - 94	-0.0103	-0.0103
Face value of straight bond	10,000,000	10,000,000
Face value of mortality bond	10,000,000	10,000,000
Coupon rate of straight bond and mortality bond	0.07	0.07
Annual aggregate cash flow out of SPC (1000C)	700,000	700,000
Straight bond price	10,000,000	10,000,000
Mortality bond price	9,988,507	9,955,663
Reinsurance premium	11,493	44,337

for female).

3.4 How Good is the Hedge?

Given the distribution of survivors, there is very little variance in the cash flows. For example, given the survivor function ${}_tp_x$ of ℓ_{x+t}, we can describe ℓ_{x+t} as a binomial distribution. It is the number of successes in $N = \ell_x$ trials with the probability of a success on a given trial of ${}_tp_x$. The distribution of ℓ_{x+t} is approximately normal with parameters $\mathrm{E}[\ell_{x+t}] = N{}_tp_x$ and $\mathrm{Var}[\ell_{x+t}] = N{}_tp_x(1 - {}_tp_x)$. The coefficient of variation is the ratio of σ_t/μ_t. The graph of the coefficient of variation of the number of survivors for an initial group of 10,000 annuitants, based on the 1994 GAM female age 65 survival distribution is shown in Figure 3.2. Note that for a bond of duration 30-years, the coefficient of variation rises to a maximum of about 1 percent , so there is little risk, *given the table.* In calculating the bond value, we have to evaluate the expected value $\mathrm{E}(\ell_{x+t})$ carefully. It is not enough to estimate a mortality table and then estimate the expected value. That approach would ignore the uncertainty in the table.

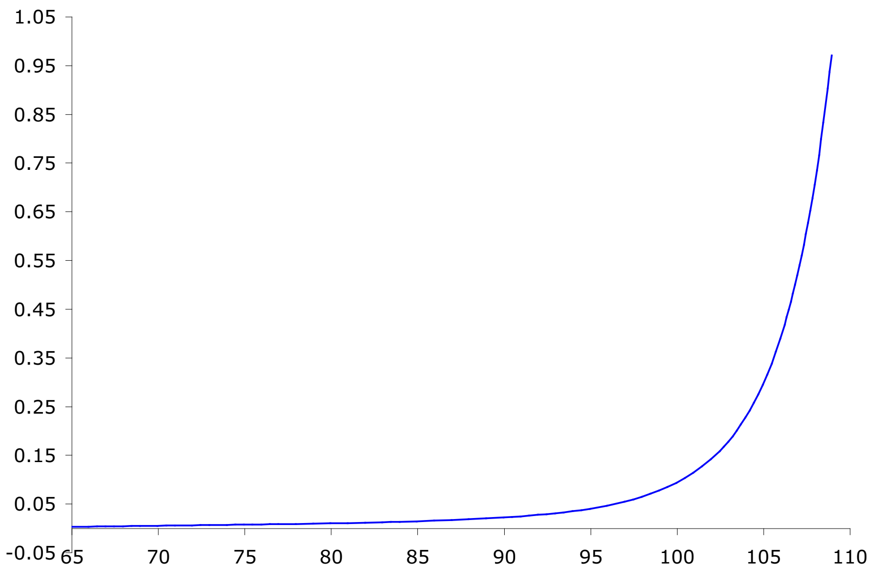

Figure 3.2: The ratio of standard deviation to expected number of survivors of an initial group of 10,000 annuitants, based on the 1994 GAM female age 65 mortality distribution.

In order to illustrate this further, suppose that the possible tables are labelled with a random variable θ. The the conditional distribution $\ell_{x+t}|\theta$ depends on θ. The unconditional moments are

$$\begin{aligned} \mathrm{E}[\ell_{x+t}] &= \mathrm{E}[\mathrm{E}[\ell_{x+t}|\theta]] = N\mathrm{E}[\mathrm{E}[{}_tp_x|\theta]] \\ \mathrm{Var}[\ell_{x+t}] &= \mathrm{E}[\mathrm{Var}[\ell_{x+t}|\theta]] + \mathrm{Var}[\mathrm{E}[\ell_{x+t}|\theta]]. \end{aligned} \tag{3.15}$$

Even if, as in Figure 3.2, there is very little variance in $\mathrm{E}[\ell_{x+t}|\theta]$ for all θ and the range of $t \leq 30$, there is still variance due to table uncertainty (the second term in equation (3.15)). I have little experience to guide me in estimating the terms $\mathrm{E}[\mathrm{E}[{}_tp_x|\theta]]$ and $\mathrm{E}[\mathrm{Var}[{}_tp_x|\theta]]$. Of course, this uncertainty occurs in all kinds of mortality calculation, not just mortality bonds.

I use the simulation to examine the impact of mortality shocks which shift the mortality tables to the insurer and the investors. With the setup shown in Table 3.3, I assume that the uncertainty υ_t at time t in the mortality table follows a normal distribution with mean 0 and variance 1. The distribution of mortality shocks ε_t at time t is a beta distribution with parameters a and b. The mortality improvement shock ε_t is expressed as a percentage of the force of mortality μ_{x+t}, so it ranges from 0 to 1, that is, $0 < \epsilon_t < 1$ with probability 1.

Before performing the simulation, I first examine the mean and standard deviation of the annual percentage mortality improvement based on the US 1963, 1973, 1983 and 1996 IAM tables for the males aged from age 65 to age 94. I conclude that its mean μ_m is equal to 0.0122 and the standard deviation σ_m is 0.0099. In the following simulation, I assume the coefficient of variation CV in different shock scenarios is constant, that is,

$$CV = \sigma_m/\mu_m = 0.0099/0.0122 = 0.8139.$$

Without the shock, the one-year survival probability for an age $x+t$ with the market expectation is $p^*_{x+t} = \exp(-\mu^*_{x+t})$. With the shock, the new survival probability p'_{x+t} can be expressed as:

$$p'_{x+t} = (e^{-\mu^*_{x+t}})^{1-\varepsilon_t} = (p^*_{x+t})^{1-\varepsilon_t}.$$

The random number of survivors ℓ'_{x+t+1} at time $t+1$ is conditional on last period's survival number ℓ'_{x+t}, the shock parameters ε_t and the mortality table random parameters υ_t:

$$\ell'_{x+t+1} = \ell'_{x+t}p'_{x+t} + \upsilon_t\sqrt{\ell'_{x+t}p'_{x+t}(1-p'_{x+t})}.$$

Table 3.4 presents the results of simulations of the number of survivors ℓ_{85} at time $t = 20$, the present value of annuity payments and the present value of cash flows to bondholders. Each simulation includes a shock improvement to market mortality, modeled by multiplying

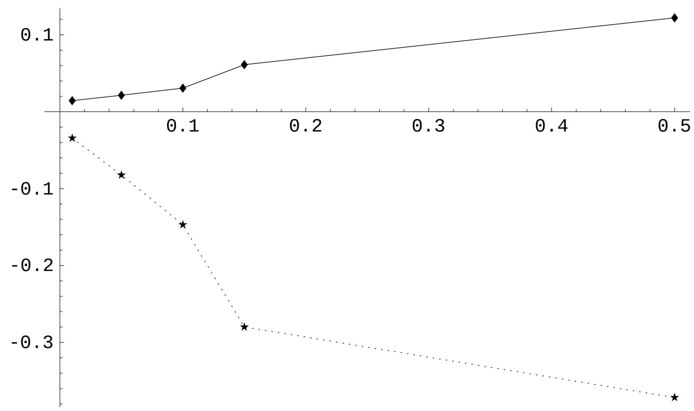

Figure 3.3: The change in expected present values of annuity payments (solid line) and bondholder payments (broken line) are shown as a function of the mortality shocks $\mathrm{E}[\varepsilon_t]$. The numerical values are shown in Table 3.4.

the force of mortality by a factor $1 - \epsilon_t$ in each year. With a small mortality improvement shock $\mathrm{E}[\varepsilon_t] = 0.01$ (Table 3.4), that is, $a = 1.49$ and $b = 147.51$, the present value of total annuity payments increase from 99,650,768 without shock to 101,081,752 on average. In this scenario, investors will lose 3.43 percent [=(9,988,507 -9,646,354)/9,988,507] of their expected total payments. When there is a big shock $\mathrm{E}[\varepsilon_t] = 0.5$, the present value of total annuity payments will increase by 12.21 percent and the investors will lose 37.61 percent of their total expected payments on average. The impact of different mortality shock is illustrated in Figure 3.3. The mortality bond coupons are reduced as the SPC pays reinsurance benefits to the insurer. This hedges the insurer's risk that the number of survivors exceeds the market's expected value.

The mortality bond price and reinsurance premium are very sensitive to an insurer's expense rate. With a given annuity market quote and a given strike level, the net annuity premium increases with a decrease in the expense factor and thus the market price of risk λ increases. This implies that the market predicts a higher future survival rate ${}_tp_x^*$ and anticipates that the number of survivors is more likely to exceed the given strike level X_t. The mortality bond price goes down because the investors are more likely to lose higher proportion of their coupons and the reinsurance premium correspondingly goes up. The results for an increase in the expense rate are just on the opposite. See Table 3.5.

As Edwalds (2003) notes, longevity risk could easily extend over 50 years or more. Most long term bonds mature within 30 years. It is conceivable that a reinsurer can issue a very long term bond (through the SPC), essentially fully collateralized with cash flows only depending on mortality risk, which would appeal to investors. This would increase the reinsurer's capacity to issue long term contracts to its client companies.

Reinsurers may find annuity securitization to be an efficient means of increasing capacity despite transaction costs, simply because reinsurers must hold more capital to write the same risk. With greater capacity, better contracting terms (longer terms, for example) and potentially lower cost (more efficient use of capital), securitization may be a feasible tool for reinsurer to hedge its mortality risks.

For investors, the risk of losing a large proportion of annual coupon is relatively low (e.g. in my setup), even if for a big mortality improvement shock. The mortality bond may be a good candidate for the investors to diversify their investment portfolio.

3.5 Difficulties in Accurate Mortality Projection

General and insured population mortality has improved remarkably over the last several decades. For example, the force of mortality for male age 65 decreases from 0.0222 based on the US 1963 IAM Table to 0.0111 based on the US 1996 IAM Table. At old age probabilities

Table 3.4: Simulation results for mortality shocks of 1%, 5%, 10%, 25% and 50% improvements in excess of market expectation (10,000 trials).

		Present Value		Percentage Change	
	l_{85}	Annuity Payments	Coupons and Principal	Annuity Payments	Coupons and Principal
Shock parameters:	$a = 1.49$, $b = 147.51$, $\mathrm{E}[\varepsilon_t] = 0.01$, $\sigma[\varepsilon_t] = 0.0081$				
Mean	5,882	101,081,752	9,646,354	1.44%	-3.43%
Maximum	6,061	102,034,832	9,733,716	2.39%	-2.55%
95th percentile	5,934	101,356,632	9,724,704	1.71%	-2.64%
5th percentile	5,854	100,930,352	9,494,340	1.28%	-4.95%
Minimum	5,850	100,910,696	9,022,497	1.26%	-9.67%
Standard deviation	26	138,312	76,634		
Shock parameters:	$a = 1.38$, $b = 26.30$, $\mathrm{E}[\varepsilon_t] = 0.05$, $\sigma[\varepsilon_t] = 0.0407$				
Mean	6,014	101,784,128	9,166,880	2.14%	-8.23%
Maximum	6,906	106,551,160	9,733,260	6.92%	-2.56%
95th percentile	6,282	103,214,240	9,691,183	3.58%	-2.98%
5th percentile	5,867	100,998,600	7,989,176	1.35%	-20.02%
Minimum	5,850	100,911,688	6,277,006	1.27%	-37.16%
Standard deviation	135	720,514	538,599		
Shock parameters:	$a = 1.26$, $b = 11.37$, $\mathrm{E}[\varepsilon_t] = 0.10$, $\sigma[\varepsilon_t] = 0.0814$				
Mean	6,187	102,709,848	8,521,295	3.07%	-14.69%
Maximum	7,933	112,101,464	9,733,680	12.49%	-2.55%
95th percentile	6,754	105,734,760	9,653,101	6.11%	-3.36%
5th percentile	5,881	101,073,520	6,599,827	1.43%	-33.93%
Minimum	5,850	100,910,768	4,901,722	1.26%	-50.93%
Standard deviation	285	1,522,121	995,280		
Shock parameters:	$a = 0.88$, $b = 2.65$, $\mathrm{E}[\varepsilon_t] = 0.25$, $\sigma[\varepsilon_t] = 0.2035$				
Mean	6,753	105,752,992	7,192,883	6.12%	-27.99%
Maximum	9,954	123,485,544	9,733,768	23.92%	-2.55%
95th percentile	8,372	114,514,088	9,624,507	14.92%	-3.64%
5th percentile	5,891	101,128,584	4,562,061	1.48%	-54.33%
Minimum	5,850	100,910,584	3,818,042	1.26%	-61.78%
Standard deviation	790	4,266,107	1,668,137		
Shock parameters:	$a = 0.25$, $b = 0.25$, $\mathrm{E}[\varepsilon_t] = 0.50$, $\sigma[\varepsilon_t] = 0.4070$				
Mean	7,847	111,815,112	6,232,152	12.21%	-37.61%
Maximum	10,005	123,776,936	9,733,797	24.21%	-2.55%
95th percentile	10,003	123,768,536	9,733,007	24.20%	-2.56%
5th percentile	5,850	100,912,208	3,808,961	1.27%	-61.87%
Minimum	5,850	100,910,520	3,808,223	1.26%	-61.87%
Standard deviation	1,696	9,326,669	2,436,069		

Table 3.5: The sensitivity of mortality bond price and reinsurance premium to the change of an insurer's expense rate

	Male		Female	
Expense Factor	Mortality Bond Price	Reinsurance Premium	Mortality Bond Price	Reinsurance Premium
4%	9,316,726	683,274	9,279,932	720,068
6%	9,988,507	11,493	9,955,663	44,337
8%	10,000,000	0	10,000,000	0

of death are decreasing, increasing the need for living benefits. The calculation of expected present values (needed in pricing and reserving) requires an appropriate mortality projection in order to avoid underestimation or overestimation of future costs which will jeopardize an insurer's profit or its market share.

Rogers (2002) shows that mortality operates within a complex framework and is influenced by socioeconomic factors, biological variables, government policies, environmental influences, health conditions and health behaviors. Not all of these factors improve with time. For example, for biological variables, recent declines in mortality rates were not distributed evenly over the disease categories of underlying and multiple causes of death. According to Stallard (2002), successes against the top three killers (heart diseases, cerebrovascular diseases and malignant neoplasms) did not translate into successes against many of the lower ranked diseases. Moreover, Olshansky (2004) points out, a projected "quantum leap" in mortality depends on new biomedical technologies, administered to enough people to have an impact on the population. This may be difficult to do, even if these were a technological breakthrough.

3.5.1 Different Opinions on Mortality Trend

Improvement

Buettner (2002) concludes that there are today two alternative views about the future improvement of mortality at older ages: compression vs. expansion (sometimes also called

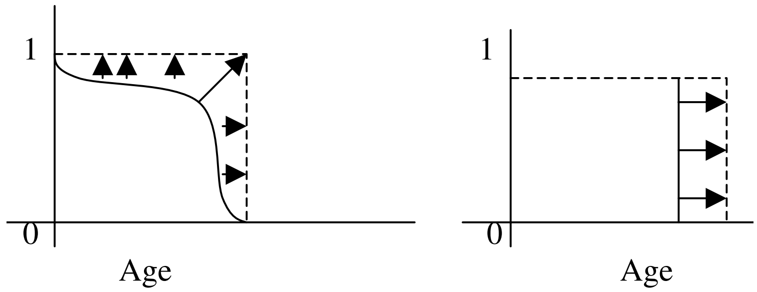

Figure 3.4: Two views of mortality improvement, rectangularization on the left and steady progress on the right.

rectangularization vs. steady progress), illustrated in Figure 3.4. Mortality compression occurs when age-specific mortality declines over a widening range of adult ages, but meets natural limits for very advanced ages. As a result, the survivor curve would approach a rectangle and mortality across countries may indeed converge to similar patterns.

In the case of steady progress, there are no natural limits to further reductions in mortality at higher ages. The age at which natural limits set in does not exist. Consequently, all age groups, especially higher age groups, would continue to experience declining mortality. The Human Genome Project is producing a rapidly expanding base of knowledge about life processes at their most fundamental level. Some experts have predicted that the genes for the aging process will be identified and drugs to retard the aging process will be developed in the not distant future. It is worth noting that genetic technology, including the mapping of the human genome, has developed much faster than forecasts. Anti–aging drugs may be available sooner than anyone forecasts.

Life Table Entropy

Life table entropy refers to a phenomenon that further improvement of already high life expectancies may become increasingly more difficult. The gains in survival a century ago were greater than they have been more recently. For instance, Rogers (2002) shows that the survival gains achieved between 1900 and 1920 are large compared to the modest gains realized between 1980 and 1999. Hayflick (2002) suggests that,

> ... Those who predict enormous gains in life expectation in the future based only on mathematically sound predictions of life table data but ignore the biological facts that underlie longevity determination and aging do so at their own peril and the peril of those who make health policy for the future of this country.

Deterioration

Although general population mortality has improved over time, the improvement may be overstated. Substantial mortality improvements often come after periods of mortality deterioration. For example, between 1970 and 1975, males aged 30-35 saw annual mortality improvement of over 2 percent, but this may be an adjustment to the 1.5 percent annual mortality decline that occurred during the previous five-year period. Moreover there is still a chance for a resurgence of infectious diseases, such as the 1918 worldwide flu.

Figures 3.5, 3.6, 3.7, 3.8, 3.9 and 3.10[4] show the impact of 1918 world-wide flu on the population mortality in the United States, France and Switzerland. The horizontal axis in Figures 3.5, 3.6, 3.7, 3.8, 3.9 and 3.10 stands for the age and the vertical axis is the force of mortality. Each graph except the last one in each figure shows the mortality experience in one particular year and is ordered according to time. The last shows all of the preceding graphes together. There is an obvious spike around year 1918 for all these three countries. The 1918 flu pushed up the mortality rates dramatically. However, the force of mortality went back to normal afterward. Similar epidemics in the future may strike life insurance industry hard since the financial capacity of the life insurance industry to pay catastrophic death losses is limited.

Moreover, deaths due to influenza could increase with the introduction of new influenza strains or with shortages of the influenza vaccine. Rogers (2002) argues that although HIV is now controlled, it is not eradicated and could expand, or variants of HIV could develop that

[4]Source: Human Mortality Database. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at *www.mortality.org* or *www.humanmortality.de* (Data downloaded November 1 – 10, 2004).

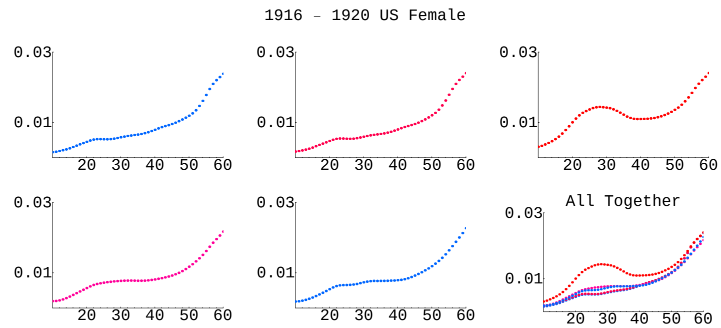

Figure 3.5: 1916–1920 US Female Population q_x.

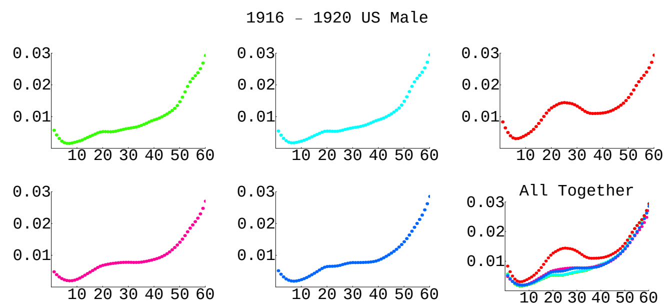

Figure 3.6: 1916–1920 US Male Population q_x.

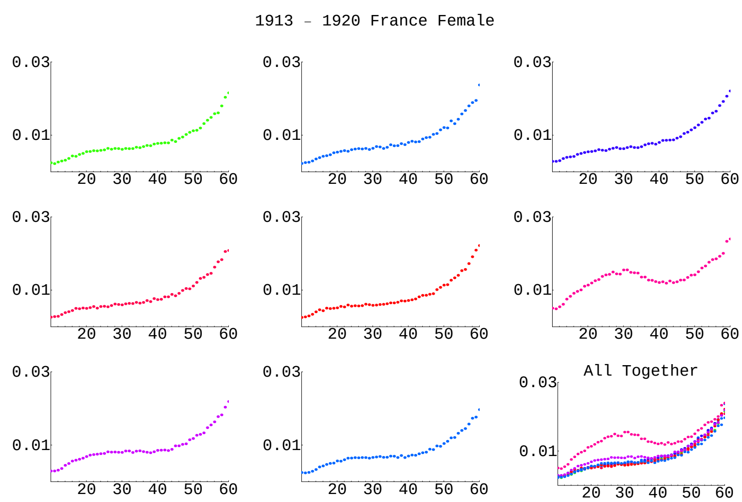

Figure 3.7: 1913–1920 France Female Population q_x.

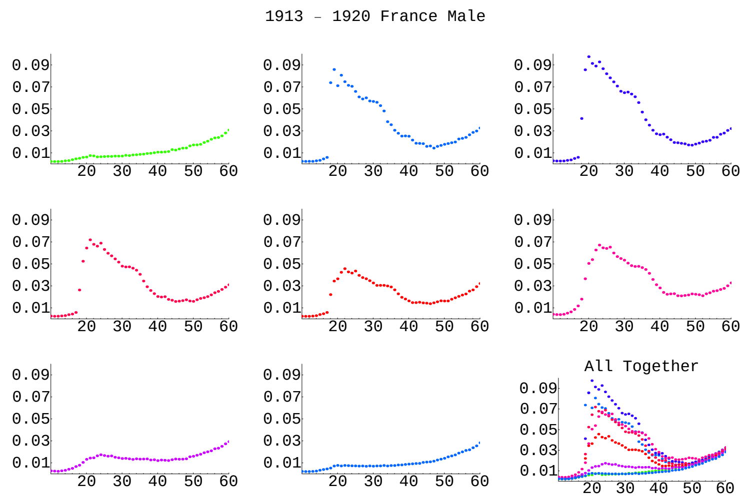

Figure 3.8: 1913–1920 France Male Population q_x.

Figure 3.9: 1916–1920 Switzerland Female Population q_x.

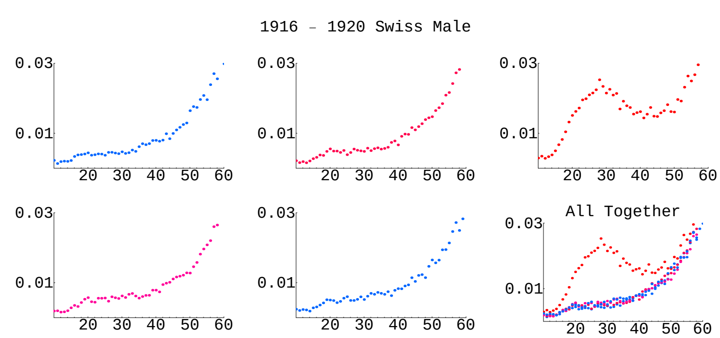

Figure 3.10: 1916–1920 Switzerland Male Population q_x.

Table 3.6: December 2004 Earthquake and Tsunami Death Toll and Percentage Excess Death Rates by Country

Country	Confirmed deaths[a]	Missing[a]	% Excess Death Rate[b]
Indonesia	127,420	116,368	16.58%
Sri Lanka	38,195	4,924	33.81%
India	10,779	5,614	0.18%
Thailand	5,395	2,991	1.90%
Somalia	298	-	0.21%
Myanmar	90	-	-
Maldives	82	-	3.25%
Malaysia	68	-	0.06%
Tanzania	10	-	-
Bangladesh	2	-	-
Kenya	1	-	-
Total	182,340	129,897	-

[a]Source: *Associated Press*

[b]Based on the authors' calculation. The percentage excess death rate is equal to the excess death rate from tsunami and missing divided by the normal population death rate.

The normal population death rate is obtained from *education.yahoo.com*.

could increase mortality. Drug resistant infectious diseases like tuberculosis could increase. Goss et al. (1998) find that age–adjusted annual death rates for ages 85 and over in the United States actually deteriorated by 0.72 percent per year for males and by 0.52 percent for females during the observation period 1990-94.

In addition, catastrophe death losses from natural disasters should not be ignored. A more recent example of unanticipated catastrophe death losses is the December 26, 2004 earthquake and tsunami. It caused massive devastation across southern Asia and eastern Africa (Guy Carpenter, 2005). The earthquake damage in Indonesia was obscured by the subsequent tsunami that hit 15–20 minutes later. The death and missing count has now exceeded 300,000 (Table 3.6). The last column in Table 3.6 shows that the 2004 Indonesian population death index increased by 16.58 percent relative to the 2003 level. The excess population mortality death rate is even higher for Sri Lanka, about 34 percent. It raises a realistic question for life insurers, "Are you well prepared to handle such disasters in the future"?

In sum, there is no agreement among experts on the future of mortality. Steady improvement is the trend, but changes in either direction are feasible.

3.5.2 Technical Difficulties in Mortality Projections

Quality of Data

Good quality, complete data are prerequisites for reliable mortality projections. However, in reality, it is not easy to obtain data for research. For example, although detailed data on old-age mortality are collected in most countries of the developed world, they are not so commonly available for developing countries. Buettner (2002) claims that even in developed countries, the quality of age reporting deteriorates among the very old.

The Society of Actuaries' series of studies of life annuity experience is of limited value for several reasons. First, it is not timely. Second, it is appropriate only for the products the policy holders owned (whole life, term life, or annuities, for example). So it cannot be used directly to assess mortality for new products or similar products issued on a new basis (e.g., underwriting annuities for select mortality).

Thulin et al. (2002) note that complexity of annuity products nowadays often makes mortality projection difficult. Sometimes, an insurer has to introduce new entries with different mortality assumptions into the insured pool. For instance, trends in the marketplace are blurring traditional distinctions in the following two key areas:

(1) Work site products sold on an individual basis increasingly show features traditionally associated with group products.

(2) Group products sold on the basis of individual election in the workplace (voluntary products) with minimal participation requirements compete directly with individual products.

They severely limit insurers' ability to underwrite to discern mortality differentials. New sources of underwriting information are becoming a way of life for insurers, as pressure on costs and hastened issue pressure create an underwriting environment with less documentation and information. One solution is making more data available to researchers and making it available sooner.

The Society of Actuaries publishes tables and mortality reports from time to time. The individual annuity mortality (IAM) tables are intended for estimation of insurance company liabilities and these tables are based on actual insurance industry experience. I use the projected IAM tables to determine the strike levels for my annuity mortality bond in section 3.3.

The Society also published periodic group annuity mortality reports of actual experience. While the reports do not contain complete mortality tables, they are not adjusted and not as conservative as the IAM tables. Moreover, the experience reports were made more frequently than the IAM tables were constructed. In this section, I use the group annuity mortality tables for the illustration and prediction of future mortality trends although the same skill can be applied on the individual annuity mortality tables.

The *GAM Experience Reports* (Committee on Annuities, 1952, 1962, 1975, 1983, 1984, 1987, 1990, 1994, 1996) describe the mortality improvement from 1951–1992. The *Reports* give the number of deaths observed among a cohort of annuitants in 5–year age groups observed for one year. The observations of deaths and exposures are summarized in the appendix to this paper. The *Reports* provide data, but do not construct mortality tables. I show graphs of this experience in Figures 3.11. For male and female data, the survival curves generally rise with the observation period. The change between 1981 and 1991 for females is an exception since there is some deterioration at the later ages. That is, the lowest line at each age are for the 1951 observations, the next to lowest are for 1961, and so on. The trend in improvement is increasing on average, with the largest increase occurring between 1971 and 1981 for males and females.

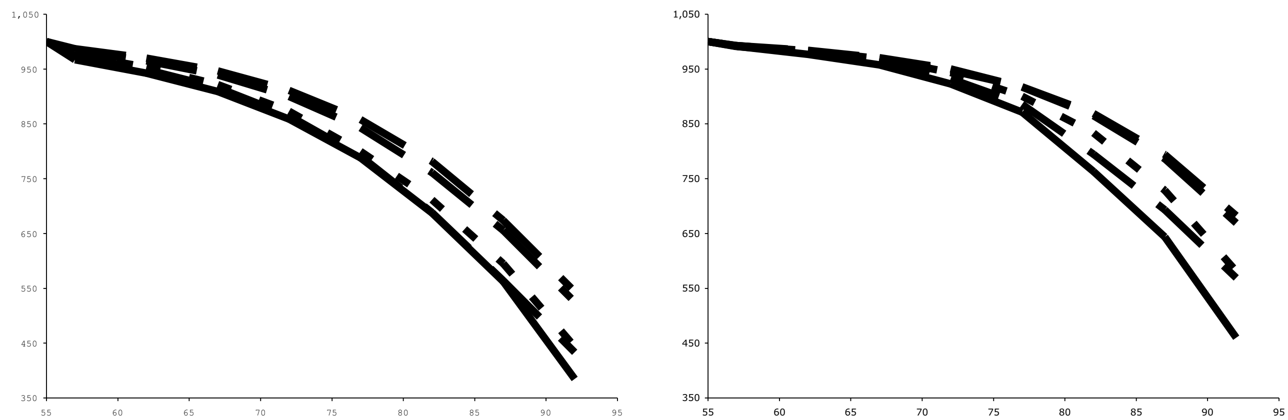

Figure 3.11: Number of survivors (vertical axis) of an initial cohort of 1,000 male (left) and female lives at age 55, based on the Society of Actuaries *TSA Reports* for 1951, 1961, 1971, 1981, and 1991 on group annuity experience, without adjustments. The horizontal axis stands for age.

Projection Models

Recent changes in mortality challenge mortality projection models. The competitive nature of the insurance market means that an insurer cannot raise its price at will. A sound projection model is crucial. However, the revealed weakness and problems of poor fitting may arise because most projection models do not capture the dynamics of mortality that is changing in a dramatic and fundamental way.

Renshaw et al. (1996) suggest a generalized linear model that showed mortality declining over time with the rates of decline not being necessarily uniform across the age range. It incorporates both the age variation in morality and the underlying trends in the mortality rates. The advantage of this model is that the predictions of future forces of mortality come directly from the model formula. I adopt this model for investigating the performance of mortality derivatives based on a portfolio of life annuities.

During a certain period, the force of mortality, $\mu(x,t)$, at age x, in calendar year t, is modeled using the following formula:

$$\begin{aligned}\mu(x,t) &= \exp\left[\beta_0 + \sum_{j=1}^{s}\beta_j L_j(x') + \sum_{i=1}^{r}\alpha_i t'^i + \sum_{i=1}^{r}\sum_{j=1}^{s}\gamma_{i,j}L_j(x')t'^i\right] \\ &= \exp\left\{\sum_{j=0}^{s}\beta_j L_j(x')\right\}\exp\left\{\sum_{i=1}^{r}\left(\alpha_i + \sum_{j=1}^{s}\gamma_{ij}L_j(x')\right)t'^i\right\} \end{aligned} \tag{3.16}$$

where

$$t' = \frac{t-1971.5}{20.5} \quad \text{and} \quad x' = \frac{x-74.5}{17.5}.$$

Sithole et al. (2000) use the same model. They note that first factor in (3.16) is the equivalent of a Gompertz-Makeham graduation term. The second multiplicative term is an adjustment term to predict an age-specific trend. The γ_{ij} terms may be pre-set to 0. The age and time variables are rescaled to x' and t' so that both are mapped onto the interval $[-1,+1]$ after transforming ages and calendar years. $L_j(x)$ is the Legendre polynomial defined below:

$$\begin{aligned} L_0(x) &= 1 \\ L_1(x) &= x \\ L_2(x) &= (3x^2-1)/2 \\ L_3(x) &= (5x^3-3x)/2 \\ &\vdots \\ (n+1)L_{n+1}(x) &= (2n+1)xL_n(x) - nL_{n-1}(x) \end{aligned}$$

where n is a positive integer and $-1 \leq x \leq 1$.

The data are the actual group annuity mortality experience for calendar years $t = 1951, 1961, 1971, 1981, \ldots, 1992$. Since the GAM Experience Reports are five-year age group results, I assume that the ratio of the total number of deaths in each group over the total number of exposures in that group (the average death rate in that group) represents the death rate of the middle-point age of that group. I use the middle-point age as my observation in the regression. The experience was analyzed for the middle-point age ranges $x = 57$ to 92 years for both male and female, giving a total of 120 data cells for males and 120 for females.

In fitting the equation (3.16), I find that when the parameter $\gamma_{1,2}$ is excluded from the formula (for male and female), all of the remaining six parameters in the model are statistically significant. Although the six–parameter model which excludes the quadratic coefficient in age effects from the trend adjustment term was next fitted to the data, the revised models seem to be appropriate for making predictions of future forces of mortality.

$$\mu(x,t) = \exp\left[\beta_0 + \beta_1 L_1(x') + \beta_2 L_2(x') + \beta_3 L_3(x') + \alpha_1 t' + \gamma_{11} L_1(x') t'\right] \tag{3.17}$$

Details of the revised fit are given in Table 3.7.

Table 3.7: Group annuities, 6-parameter log-link model. All of the coefficients are significant at the 1% level.

	Male		Female	
Coefficient	Estimate	Std. Error[a]	Estimate	Std. Error[a]
β_0	-2.7744	0.0087	-3.3375	0.0111
β_1	1.3991	0.0139	1.7028	0.0179
β_2	0.1053	0.0114	0.1543	0.0146
β_3	-0.1073	0.0127	-0.0872	0.0163
α_1	-0.2719	0.0116	-0.2660	0.0149
$\gamma_{1,1}$	0.0839	0.0178	-0.1294	0.0228
Adjusted R^2	0.9944		0.9930	

Note: [a]Standard Error.

Figure 3.12 shows the male group annuity predicted forces of mortality based on the 6–parameter model given by equation (3.17). All of the predicted forces of mortality progress smoothly with respect to both age and time. There are errors in the estimate which should tell us how confident we can be in projecting mortality into the future, assuming the dynamics of mortality improvement continues as it has in the observation period. This is potentially dangerous. As I have pointed out earlier, there is a good bit of controversy with regard to the dynamics of mortality improvement.

I note also that these results are based on group annuity experience. Individual annuity experience may be very different. For example, adverse selection should be a much more important issue. As the market for individual immediate annuities develops, insurers will have to adjust their estimates to reflect the change in the market mortality. They may have to apply underwriting techniques and control for moral hazard and adverse selection when they issue annuities, just as they now do for life insurance. Since individual annuity mortality tables are more likely to capture the information asymmetry, in section 3.3 I use the projection based on individual annuity mortality tables to determine different mortality improvement levels for different age groups specified in the mortality bond contract.

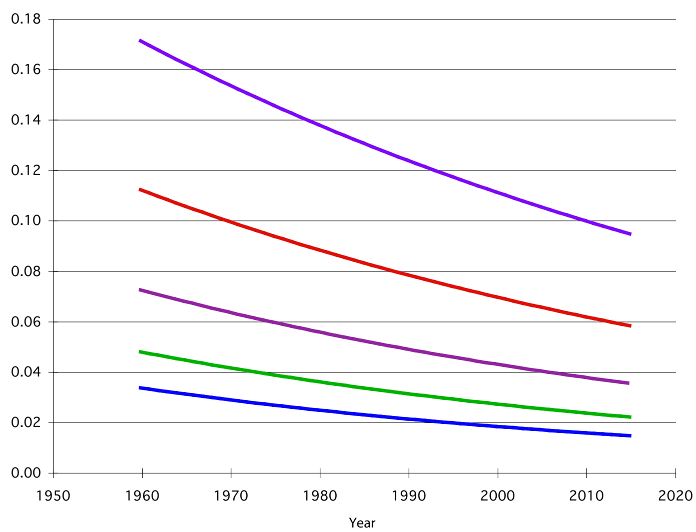

Figure 3.12: Male group annuity predicted forces of mortality based on 6-parameter log-link model and *TSA Reports* 1951 - 1992. The top curve is the force of mortality for age 85, the one just below it is for age 80, then 75, 70 and the bottom curve is for age 65. The greatest improvement (steepest slope) is at age 85.

3.6 Conclusions and Discussion

Financial innovation has led to the creation of new classes of securities that provide opportunities for insurers to manage their underwriting and to price risks more efficiently. Cummins and Lewis (2002) establish that risk expansion helps to explain the development of catastrophic risk bonds and options in the 1990s. A similar expansion is needed to manage longevity risk. There is a growing demand for a long term hedge against improving annuity mortality. I have shown how innovation in swaps and bond contracts can provide new securities which can provide the hedge insurers need.

There is a trend of privatizing social security systems with insurers taking more longevity risk. Moreover, the trend to defined contribution corporate pension plans is increasing the potential demand for immediate annuities. This is an opportunity and also a challenge to insurers. Insurers will need increased capacity to take on longevity risks and securities markets can provide it. This will allow annuity insurers to share this "big cake." Securitization of mortality risks has long duration, high capacity and possibly low cost. Demand for new securities arises when new risks appear and when existing risks become more significant in magnitude. And we now have the technology to securitize the mortality risks based on modern financial models. Securitization in the annuity and life insurance markets has been relatively rare, but I have argued that this may change. I explore the securitization of mortality risks showing how it can help solve the difficulties in managing annuity mortality risks.

4

Mortality Securitization Modeling

As a step toward understanding mortality securities, I develop an asset pricing model for mortality-based securities in an incomplete market framework with jump processes. My model nicely explains opposite market outcomes of two existing pure mortality securities.

4.1 Introduction

The first pure death-risk linked deal was the three-year Swiss Re bond issued in December 2003 (Swiss Re, 2003; MorganStanley, 2003; The Actuary, 2004). Almost one year later, the European Investment Bank (EIB) issued the first pure longevity-risk linked deal— a 25-year 540 million-pound (775 million-euro) bond as part of a product designed by the

BNP Paribas aimed at protecting UK pension schemes against longevity risk.[1] In 2005, the Swiss Re Company issued two more mortality bonds. The Swiss Re deal is a hedge against catastrophic loss of insured lives that might result from natural or man-made disasters in the US or Europe. The EIB deal is a security to transfer the other tail of mortality risk, longevity risk, to the capital market.

Interestingly, the market outcomes of the first two mortality bonds are opposite. According to MorganStanley, "the appetite for this security [the Swiss Re bonds] from investors was strong." This is the same reaction investors have had to the so–called catastrophe bonds based on portfolios of property insurance. The strong appetite for mortality securities may indicate a growing potential market for mortality securities (Lin and Cox, 2005). On the other hand, the EIB bond has not sold very well at all.

It is important to understand why investors viewed the Swiss Re bond price favorably, yet did not buy the EIB bond. To evaluate these two bonds, we need a model that can capture and price mortality risks. I notice there are only a few preliminary papers in this area. Developing asset pricing theory in this area is important as it will help market participants better understand these new financial instruments. Most of the existing mortality securitization pricing papers have two major shortcomings: first, they ignore mortality jumps (Lee and Carter, 1992; Lee, 2000; Renshaw et al., 1996; Sithole et al., 2000; Milevsky and Promislow, 2001; Olivieri and Pitacco, 2002; Dahl, 2003; Cairns et al., 2006). Mortality jumps should not be ignored in mortality securitization modeling since the rationale behind selling or buying mortality securities is to hedge or take catastrophe mortality risks (i.e., too many people die); second, they use complete market pricing methodology. I doubt that pure mortality risk bonds can be replicated with traded securities. Therefore, I propose to price mortality bonds in an incomplete market framework with the jump processes and the Wang transform.

My idea is to derive the two-factor Wang transform parameters from insurance markets

[1] From *www.IPE.com* on November 8, 2004.

then use the same parameters for mortality-linked security pricing. I also use a jump model for mortality dynamics. My model enables investors to better understand why the Swiss Re deal is an attractive investment despite the uncertainty associated with catastrophic mortality risks while the EIB bond has not sold very well.

The paper proceeds as follows: In section 4.2 I highlight some shocks in the mortality evolution. Section 4.3 describes the current mortality securitization market and the designs of the first two mortality bonds — the Swiss Re and EIB bonds. The two-factor Wang transform as an incomplete market pricing method is introduced in Section 4.4. In Section 4.5, I propose a mortality stochastic model with jumps and price the Swiss Re and EIB bonds. I show that the jump process plays an important role in mortality securitization modeling. Moreover, my model nicely explains the opposite market outcomes of these two bonds. Section 4.6 is a final discussion and conclusion.

4.2 Mortality Shocks

Mortality operates within a complex framework and is influenced by socioeconomic factors, biological variables, government policies, environmental influences, health conditions and health behaviors (Rogers, 2002). As no one can accurately predict mortality dynamics, unexpected mortality shocks can devastate the insurance pooling mechanism. In the case of life insurance, the "bad" risk is that the lives insured die sooner than forecast when the policies were issued (I call it "bad shock"). Examples of bad shocks, such as the 1918 worldwide flu and the 2004 earthquake and tsunami, are shown in Section 3.5.1. For an annuity portfolio, the "bad" risk is that the annuitants will live longer than expected when the annuities were issued (I call it "good shock").

Both kinds of mortality changes reflect the dynamics of the underlying table. Unanticipated changes might be shocks like a pandemic flu. It could also be a more gradual, but still unanticipated dynamic. For example, mortality gradually improved in the twentieth century as cities began to provide sanitary sewers and clean water. Technological changes, such as development of vaccines and antibiotics also improved longevity. Insurers and pension plans are keenly interested in understanding the future course of mortality, as well as the protection provided by hedging, asset allocation strategies, reinsurance, and securitization. Among these risk management tools, mortality securitization is a fairly new tool which should gain more attention.

4.3 Mortality Securitization Markets

Lane and Beckwith (2005) describe recent activity in the insurance securitization market. Over the past ten years, insurance and capital market are converging. Capital market investors search for uncorrelated risk for diversification and the risk–adjusted excess return "α". Insurance-linked securities have low or no correlation with financial markets, providing diversification. Moreover, the existing insurance-linked securities provide high risk-adjusted excess return. They attract more and more investors. For example, hedge funds increased their investment in insurance–linked securities. At the same time, insurers are looking for new sources of risk financing in the capital markets.

Table 4.1 shows all of the property and life insurance securitization issuances in dollar terms and by number of transactions through March 2005. In general, insurance securitization increased in both dollar amounts and the number of issues especially in the last two years. The life securitization includes three "pure" mortality bonds in the US and one in the Europe since December 2003. The first two publicly known pure mortality securities are the Swiss Re bond issued in December 2003 (Swiss Re, 2003; MorganStanley, 2003; The Actuary, 2004) and the European Investment Bank (EIB) longevity risk bond issued in November 2004.

Table 4.1: Property and Life Insurance Securitization Issuance

Period	Issuance in Millions	Number of Deals
Pre 3/1998	\$886.1	7
4/1998 – 3/1999	\$1,366.9	7
4/1999 – 3/2000	\$1,219.4	11
4/2000 – 3/2001	\$1,126.0	10
4/2001 – 3/2002	\$826.9	6
4/2002 – 3/2003	\$832.20	25
4/2003 – 3/2004	\$1,894.60	27
4/2004 – 3/2005	\$1,803.30	21

Source: Lane Financial L.L.C., April, 2005.

4.3.1 Design of the Swiss Re Bond

The financial capacity of the life insurance industry to pay catastrophic death losses from hurricanes, epidemics, earthquakes, and other natural or man-made disasters is limited. To expand its capacity to pay catastrophic mortality losses, the Swiss Reinsurance Company, the world's second–largest reinsurance company, obtained \$400 million in coverage from institutional investors after its first pure mortality security. Swiss Re issued the bond in late December 2003. It matures on January 1, 2007. So it is a three-year deal. The principal is exposed to mortality risk. The mortality risk is defined in terms of an index q based on the weighted average annual population death rates in the US, UK, France, Italy and Switzerland. If the index q exceeds 130 percent of the actual 2002 level, q_0, then the investors will have a reduced principal payment. The following equation describes the principal repayment mathematically.

$$\text{Maturity value} = \begin{cases} 400,000,000 & \text{if } q \leq 1.3q_0 \\ 400,000,000\dfrac{1.5q_0 - q}{0.2q_0} & \text{if } 1.3q_0 < q \leq 1.5q_0. \\ 0 & \text{if } q > 1.5q_0 \end{cases}$$

where q = weighted average population mortality in the US, UK, France, Italy, and Switzerland. q_0 = Year 2002 level, q_i = Year $(2002 + i)$ level, and $q = \max(q_1, q_2, q_3)$.

If the maturity value of the Swiss Re deal were based on the Sri Lanka's population index, the excess death rate 33.81 percent caused by the earthquake and tsunami in 2004 (Table 3.6) would exceed the trigger level 30 percent. Then the maturity value would be lower than the face value.

4.3.2 Design of the EIB Longevity Bond

About one year after the Swiss Re bond issue, in November 2004, the EIB issued a longevity bond to provide a solution for financial institutions looking for instruments to hedge their long-term systematic longevity risks. This bond is the result of the co-operation between the BNP Paribas as structurer/manager, the European Investment Bank (EIB) as issuer and the PartnerRe as the provider of analysis, expertise and risk taking capacity. The total value of the issue was £540 million (775 million-euro). It was primarily intended for purchase by UK pension plans (Cairns et al., 2005). The term of the EIB bond is 25 years. Potential buyers of the EIB bond are pension plans, as the bond transfers longevity risks to investors.

Here is how the EIB bond works: The bond's cash flows will be based on the actual longevity experience of the English and Welsh male population aged 65 years old, as published annually by the Office for National Statistics. The future cash flows of the bond will be equal to the amount of a fixed annuity, £50 million, multiplied by the percentage of the reference population still alive at each anniversary. The cash flows, therefore, decline over time. Figure 4.1 shows projected coupons (payable annually) based on the projected survival rates produced by the UK Government Actuary's Department (GAD).

4.4 Incomplete Market Pricing Method—Two-Factor Wang Transform

Pricing derivative securities in complete markets involves replicating portfolios. For example, if we have a traded bond and stock index, then options on the stock index can be replicated

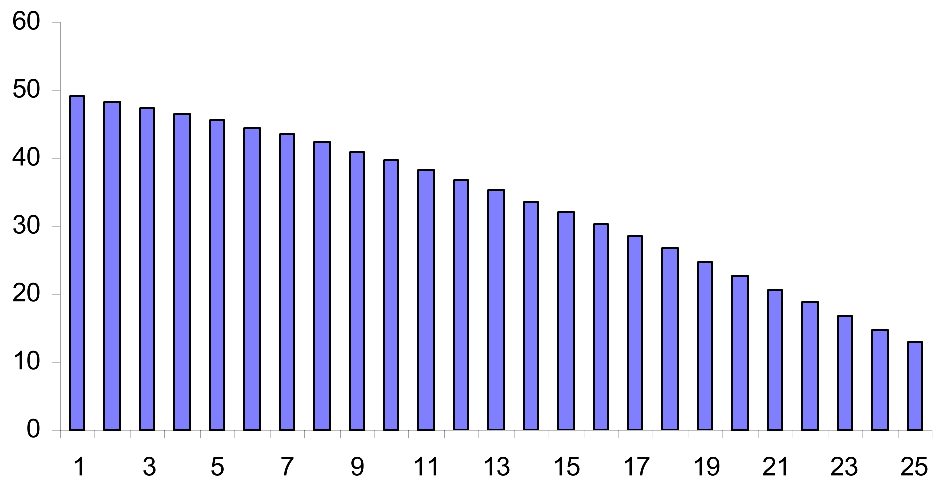

Figure 4.1: Projected coupons of the EIB longevity bond. The vertical axis shows projected cash flows (millions in pounds) and time is on the horizontal axis.

by holding bonds and the index, which are priced. The analogy for the Swiss Re bond does not work. The bond is a mortality derivative, but we have no efficiently traded mortality index with which to create a replicating hedge. Situations like this are called incomplete markets. Pricing in this situation must rely on some other assumption — there is no traded underlying security.

In my application, I use the observed prices of life insurance and annuities in the retail market to derive those parameters by using the two-factor Wang transform. Then I use the same model to price mortality securities.

4.4.1 Two-Factor Wang Transform

The one-factor Wang transform in equation (2.5) assumes that the true distribution is known. However, in reality, we can at best estimate the parameters of a probability density function by a sample out of the population. The two-factor Wang transform allows for parameter uncertainty:

$$F_X^*(x) = Q[\Phi^{-1}(F_X(x)) + \lambda] \tag{4.1}$$

where Q is the t-distribution. As in the one-factor model in equation (2.5), the discounted expected value under the transformed distribution $F_X^*(x)$ in equation (4.1) is the price of X.

Suppose an insurer transfers its longevity risk to the financial market by issuing a longevity bond. We can derive the market price of risk λ based on the annuity retail market, then use the same λ to price the longevity bond. If there are no transaction costs between the insurance market and the financial market and annuities were actually traded, my method guarantees no arbitrage opportunity between these two markets. There is another way to view this. Insurers price these life and annuity obligations using a distribution (usually privately held) of future life time. I get to observe the insurers' prices and use an industry market distribution (known to all) $F(x)$. From the prices, I am able to derive the market price of risk λ so that the observed retail life insurance or annuity prices are discounted expected values using $F^*(x)$. Then I transfer the same λ to the bond market and use the $F^*(x)$ to price mortality-linked bonds. As a result, the insurer uses the same mortality assumptions to price the mortality bond as it uses in pricing retail life insurance or annuities.

4.4.2 Why Transform?

According to the classical CAPM with the complete market assumption, the risk premium of an asset should be zero if its payoffs are uncorrelated with those of the market portfolio. Insurance market is an incomplete market which violates the complete market assumption in the CAPM. Therefore, the CAPM cannot explain positive and very high risk premium of insurance-linked securities whose risk has no or low correlation with that of financial markets. By maximizing the *ex post* value of the firm, Froot and O'Connel (1997), Froot and Stein (1998) and Froot (2003) suggest that high risk premium of insurance-linked securities or reinsurance reflects risk aversion of insurers or investors when they face unhedgeable insurance risks. Risk aversion of the insurers may arise from the fact that the true economic capital requirements of insurance/reinsurance business are not straightforward and potential financial distress costs are very high (Minton et al., 2004). On the other hand, risk aversion of investors may arise from their loss aversion and/or default risk, potential moral hazard behavior and basis risk of the insurance-linked security issuing firm (Doherty, 1997). Consistent with the theories of Froot and O'Connel (1997), Froot (2003) and Doherty (1997), the transformed distribution in the Wang transform reflects risk aversion of insurers and investors to unhedgeable risks. In Section 4.5, I show that the transformed mortality distribution has a longer tail (i.e. higher probability of having catastrophes) than the physical distribution. Evidently insurers and investors are risk averse to catastrophic mortality events.

4.4.3 Estimation

I show how to estimate the market price of risk of longevity based on the two-factor Wang transform. Suppose an annuity insurer sells the single premium immediate annuities (SPIA) on lives age 65 in 1996.

I define my transformed distribution F^* as:

$$F^*({}_tq_{65}) = {}_tq^*_{65} = Q[\Phi^{-1}({}_tq_{65}) - \lambda] \tag{4.2}$$

where ${}_tq_{65}$ is the probability that a person aged 65 dies before age $65 + t$. To calculate the values of ${}_tq_{65}$ for male and female aged 65, I use the same data, expenses assumption (i.e. 6 percent) and equation (3.9) as those in Section 3.3.1.

The market prices of risk for males and females by applying the two-factor Wang transform equation (4.2) respectively are shown in Table 4.2. The market price of risk is 0.2318 for male annuitants and 0.3286 for female annuitants which is higher than those (i.e. 0.1792 for male and 0.2312 for female) in Table 3.1 when I use the one-factor Wang transform. It implies higher required risk premiums due to parameter uncertainty.

Table 4.2: The market price of risk by applying the two-factor Wang transform, determined by the 1996 IAM 2000 Basic Table, the US Treasury interest rate term structure on August 15, 1996, and the annuity market prices from Kiczek (1996). The single premium is the lump-sum life annuity premium at the issue age 65. The payment rate is the dollars per month of life annuity per $1,000 single premium.

	Single Premium	Monthly Payment Rate	Market price of risk λ
Male (65)	$1,000	$7.48	0.2318
Female (65)	$1,000	$6.94	0.3286

4.5 Mortality Securitization Modeling

Mortality securitization modeling depends on two indispensable parts: (1) mortality forecasting theory and (2) incomplete market pricing theory. First, the principal or coupons of a mortality security are determined by future mortality levels. For example, the principal of the Swiss Re bond will be reduced if the future population mortality index increases by more than 30 percent relative to the 2002 level. Therefore I need a model to describe future mortality stochastic processes. Second, the insurance market is an incomplete market. If I transfer insurance risks to the financial market by selling insurance-linked securities, I should use an incomplete market pricing method to price these securities.

4.5.1 Existing Mortality Securitization Modeling

Existing mortality forecasting literature

Mortality securitization modeling is based on the analysis of future mortality dynamic processes. The mortality dynamics include normal deviations from the trend and unanticipated mortality shocks. Since the rationale behind selling or buying mortality securities is to hedge or take catastrophe mortality risks, a good mortality stochastic model should take into account mortality jumps – which might be caused by epidemics, wars, or natural catastrophes such as tsunamis.

Most mortality forecasting papers do not explicitly model mortality jumps when they describe future mortality stochastic processes. Dahl (2003) and Milevsky and Promislow

(2001) model the force of mortality as an Itô–type stochastic process. Therefore, they are the models with continuous time and continuous sample paths. Cairns et al. (2006) apply the pricing framework of positive interest rate model (Flesaker and Hughston, 1996; Rogers, 1997; Cairns, 2004) in their mortality model to get positive survival probabilities. However, they do not show how to estimate their model. This is also a continuous time model with continuous sample paths. Econometric methods, like the Renshaw's method (Renshaw et al., 1996; Sithole et al., 2000) and the Lee-Carter model (Lee and Carter, 1992; Lee, 2000), also do not explicitly take into account mortality jumps when they model future mortality dynamics. To improve the existing mortality securitization modeling, I propose and estimate a mortality securitization model with jumps.

Existing mortality security pricing literature

There are only a few asset pricing papers in mortality securitization. Cairns et al. (2006) price mortality securities with the risk-neutral measure. I propose to use an incomplete market pricing technique (the Wang transform) to price mortality securities.

4.5.2 Model

Most of the existing mortality securities (e.g. the Swiss Re bond) link their payoffs to the population mortality index since the population index is more transparent. Therefore, the following discussion shows how to describe the dynamics of the US population mortality index based on my model although my method can be also applied in other situations.

Model

Biffis (2005) addresses the risk analysis and market valuation of life insurance contracts in a jump-diffusion setup. My approach combines a Brownian motion and a compound Poisson process. The probability of a jump (e.g. a big change in mortality) occurring during a time interval of $(t, t+h)$ (where h is as small as you like) can be written as:

$$\begin{aligned} \Pr[\text{No event occurs in } (t, t+h)] &= 1 - \Lambda h + o(h) \\ \Pr[\text{One event occurs in } (t, t+h)] &= \Lambda h + o(h) \\ \Pr[\text{More than one event occurs in } (t, t+h)] &= o(h), \end{aligned} \tag{4.3}$$

where Λ is the mean number of arrivals per unit time, where $o(h)/h$ tends to 0 as h tends to 0. N_t represents the total number of jumps a time interval of $(0, t)$. The stochastic process N_t and the standard Brownian motion W_t (described below) are independent.

I describes the US population mortality index q_t dynamics at time t as the combination of a Brownian motion and a compound Poisson process as follows:

$$\frac{\mathrm{d}q_t}{q_t} = \begin{cases} (\alpha - \Lambda k)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t, \text{ if the Poisson event does not occur at time } t; \\ (\alpha - \Lambda k)\,\mathrm{d}t + \sigma\,\mathrm{d}W_t + (Y-1), \text{ if the Poisson event occurs at time } t. \end{cases} \tag{4.4}$$

where α is the instantaneous expected force of the US population mortality index; σ is the instantaneous volatility of the mortality index, conditional on no jumps. W_t is a standard Brownian motion with mean 0 and variance t.

The quantity $(Y-1)$ is an impulse function producing a finite jump in q_t to q_tY. I can get $k \equiv \mathrm{E}(Y-1)$ where $\mathrm{E}(Y-1)$ is the expected percentage change in the mortality index if a Poisson event occurs.

The "σdW_t" part describes the instantaneous part of the unanticipated "normal" mortality index change, and the "$Y-1$" part describes the part due to the "abnormal" mortality shocks. If $\Lambda = 0$, then $Y-1=0$. And it is the same as the standard stochastic model without jumps.

The mortality index, q_t, will be continuous most of the time with finite jumps of differing signs and amplitudes occurring at discrete points of time. If α, Λ, k, and σ are constants, I can solve the differential equation (4.4) as

$$\frac{q_t}{q_0} = \exp\left[(\alpha - \frac{1}{2}\sigma^2 - \Lambda k)t + \sigma W_t\right] Y(N_t), \tag{4.5}$$

where N_t is the total number of mortality jumps with parameter Λt during a time interval of length t. And it follows the independent Poisson process described in equation (4.3). The cumulative jump size $Y(N_t) = 1$ if $N_t = 0$ and $Y(N_t) = \prod_{j=1}^{N_t} Y_j$ for $N_t \geq 1$ where the size of the j^{th} jump, Y_j, is independently and identically distributed.

From equation (4.5), I can derive the index value q_{t+h}, given q_t resulting in

$$q_{t+h}|\mathcal{F}_t = q_t \exp\left[(\alpha - \frac{1}{2}\sigma^2 - \Lambda k)h + \sigma \Delta W_t\right] \prod_{j>N_t}^{N_{t+h}} Y_j \tag{4.6}$$

where $\mathcal{F}_t$ is the information set up to time t.

I assume Y_j is log-normally distributed with parameters m and s, that is,

$$Y_j = e^{m+su}, \text{ where } u \sim N(0,1). \tag{4.7}$$

If Y_j are log-normally distributed, then the distribution of $\dfrac{q_{t+h}}{q_t}$ will be log-normal too. After taking logarithm on both sides of equation (4.6), I obtain

$$\begin{aligned} Z(h) &= \log q_{t+h} - \log q_t \\ &= (\alpha - \frac{1}{2}\sigma^2 - \Lambda k)h + \sigma \Delta W_t + \sum_{j>N_t}^{N_{t+h}} \log(Y_j). \end{aligned} \tag{4.8}$$

If the variable $\Delta N_h = N_{t+h} - N_t$ is the number of events during the period h, the variable $Z(h)|(\Delta N_h = n)$ will be normally distributed with mean $M_n = (\alpha - \frac{1}{2}\sigma^2 - \Lambda k)h + nm$ and variance $S_n^2 = \sigma^2 h + ns^2$. From $\mathrm{E}[Y_j] = \exp(m + s^2/2)$, I get $k \equiv \exp(m + s^2/2) - 1$ since the expected value of the mortality index percentage change $k \equiv \mathrm{E}[Y_j - 1]$ if the Poisson event occurs.

The density function of $Z(h)$, $f_{Z(h)}(z)$, can be written in terms of the conditional density of $Z(h)|(\Delta N_h = n)$, denoted $f_{Z(h)}\left(z|\Delta N_h = n\right)$, which has a normal distribution:

$$\begin{aligned} f_{Z(h)}(z) &= \sum_{n=0}^{\infty} f_{Z(h)}\left(z|\Delta N_h = n\right) \Pr\left(\Delta N_h = n\right) \\ &= \sum_{n=0}^{\infty} f_{Z(h)}\left(z|\Delta N_h = n\right) \frac{e^{-\Lambda h}\left(\Lambda h\right)^n}{n!} \\ &= \sum_{n=0}^{\infty} \frac{1}{S_n\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z-M_n}{S_n}\right)^2} \frac{e^{-\Lambda h}\left(\Lambda h\right)^n}{n!}. \end{aligned} \tag{4.9}$$

If I have a time series of K observations of q_t where $t = 0, 1, 2, ..., K-1$, there will be $K-1$ observations of z's with time interval equal to $h = 1$. In each time interval of length $h = 1$, I assume that the probability of an event from time t to $t+h$ is Λ and the probability of more than one event during such a time interval is negligible. I can estimate the parameters $\Lambda, \alpha, \sigma, m$ and s by maximizing the following loglikelihood function (4.10) based on observations $z_1, z_2, ..., z_{K-1}$:

$$\begin{aligned} &\sum_{i=1}^{K-1} \log f_{Z(1)}(z_i) = \sum_{i=1}^{K-1} \log\left(\sum_{n=0}^{\infty} f_{Z(1)}\left(z_i|\Delta N_h = n\right) \Pr\left(\Delta N_h = n\right)\right) \\ &= \sum_{i=1}^{K-1} \log\left(\sum_{n=0}^{\infty} \frac{1}{S_n\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_i-M_n}{S_n}\right)^2} \frac{e^{-\Lambda h}\left(\Lambda h\right)^n}{n!}\right) \\ &\approx \sum_{i=1}^{K-1} \log\left(\sum_{n=0}^{10} \frac{1}{S_n\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_i-M_n}{S_n}\right)^2} \frac{e^{-\Lambda h}\left(\Lambda h\right)^n}{n!}\right), \end{aligned} \tag{4.10}$$

where $M_n = (\alpha - \frac{1}{2}\sigma^2 - \Lambda k)h + nm$ and variance $S_n^2 = \sigma^2 h + ns^2$. For example, when $n =$

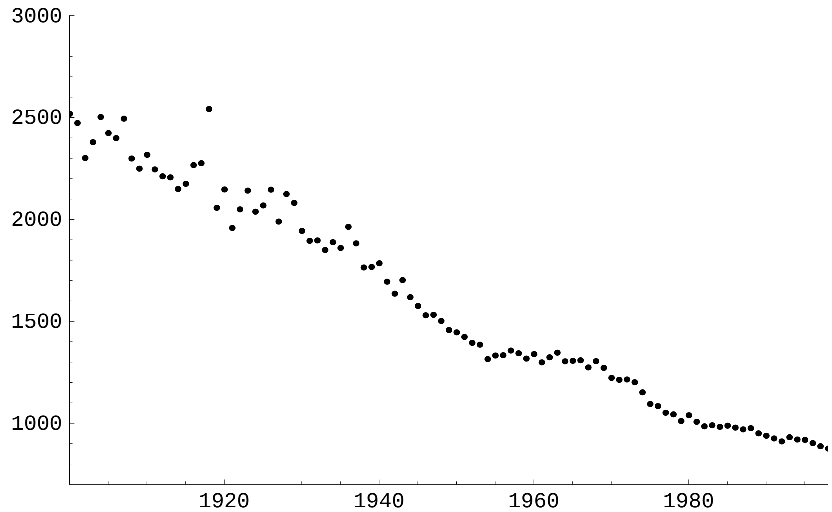

Figure 4.2: 1900 – 1998 US Total Population Death Rate per 100,000 (= $100,000q_t$ where $t = 1900, 1901, ..., 1998$).

0 or 1, I can get

$$M_0 = \alpha - \frac{1}{2}\sigma^2 - \Lambda\left[\exp(m + s^2/2) - 1\right],$$
$$M_1 = \alpha - \frac{1}{2}\sigma^2 - \Lambda\left[\exp(m + s^2/2) - 1\right] + m,$$
$$S_0^2 = \sigma^2,$$
$$S_1^2 = \sigma^2 + s^2.$$

Data

My data are obtained from the Vital Statistics of the United States (VSUS).[2] The VSUS reports the United States age-adjusted death rates per 100,000 standard million population (2000 standard) for selected causes of death. Age-adjusted death rates are used to compare relative mortality risks across groups and over time; they are the indexes rather than the direct measures. I plot my data from 1900 to 1998 in Figure 4.2.

Figure 4.2 shows that the mortality stochastic process does not follow a mean-reverting

[2]Source: *http://www.cdc.gov.*

Table 4.3: Maximum Likelihood Parameter Estimates Based on the US Population Mortality Index 1900 –1998. The likelihood ratio test rejects the model without jumps at the significance level of 0.1 percent and the Schwartz-Bayes criterion also rejects the model without jumps.

Parameter	Estimate	Parameter	Estimate
α	-0.0100	m	-0.0233
σ	0.0302	s	0.1028
Λ	0.0536	k	-0.0178

process.[3] Moreover, there are several jumps in the US population mortality evolution which should be captured by a good mortality stochastic model. Mortality shocks may cause financial distress or bankruptcy of insurers or pension plans and they are also the risks underlying the mortality securities.

Estimation results

Based on the US population mortality index q_t from 1900 to 1998 shown in Figure 4.2, Table 4.3 reports my maximum likelihood estimation results. The instantaneous expected force of mortality index α is equal to -0.0100. The negative sign of α suggests the US population mortality improves over time. The instantaneous volatility of the mortality index, conditional on no jumps, σ is equal to 0.0302. The estimate of the Poisson parameter Λ implies that the mortality jump is approximately a one–in–twenty–years ($1/\Lambda \approx 20$) event. My likelihood ratio test rejects the model without jumps at the significance level of 0.1 percent.

4.5.3 Market Price of Risk of the Swiss Re Bond

Based on equations (4.2) and (4.6) and the US population mortality index from 1900 to 1998, my estimated market price of risk λ of the Swiss Re deal is 0.8657. Figure 4.3 shows that the transformed probability density function (PDF) $f^*(q)$ with $\lambda = 0.8657$ by using the two-factor Wang transform lies on the right of the PDF of the simulated US population

[3]However, it could be a process that is mean reverting around a trend.

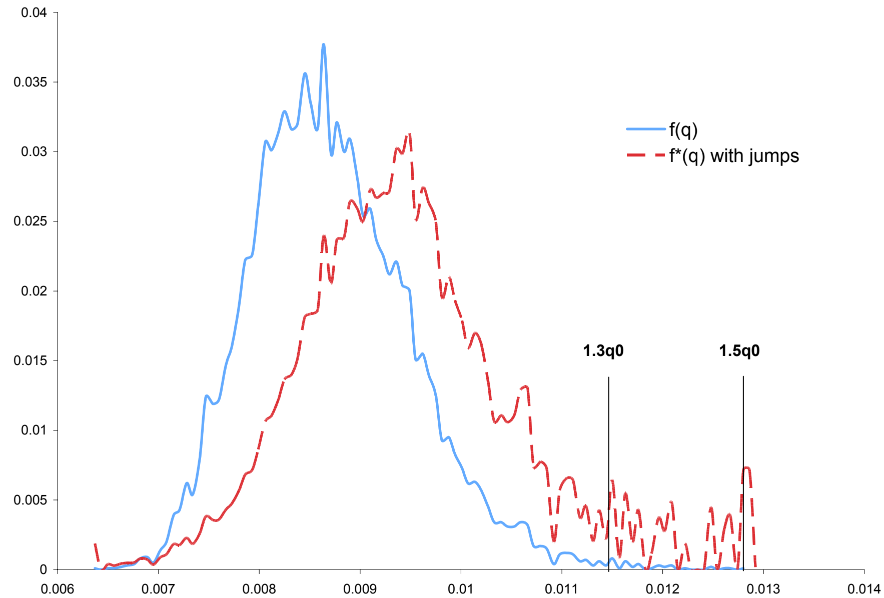

Figure 4.3: Two-factor Wang transformed probability distribution of q with $\lambda = 0.8657$ and 6 degrees of freedom (shown as broken line) and physical probability distribution of q (shown as solid line). The horizontal axis is the one-year US population death rate and the vertical axis stands for the probability.

mortality index $f(q)$. After transforming the data, I put more weight on the right tail. It implies that the market expects a higher probability of having a big loss than the actual probability suggests.

Is the jump process important?

Most of the existing mortality stochastic models do not consider the jump process. In Section 4.5.2, I use the compound Poisson process to model the dynamics of the US population mortality index. To prove the jump process is important in the mortality securitization modeling, I compare the market price of risk *without* mortality jumps with that *with* jumps.[4]

The mortality stochastic model *without* jumps is shown as follows:

$$\frac{\mathrm{d}q_t}{q_t} = \alpha_n \, \mathrm{d}t + \sigma_n \, \mathrm{d}W_t, \tag{4.11}$$

[4] I thank Patrick Brockett for his suggestion to add this part to the paper.

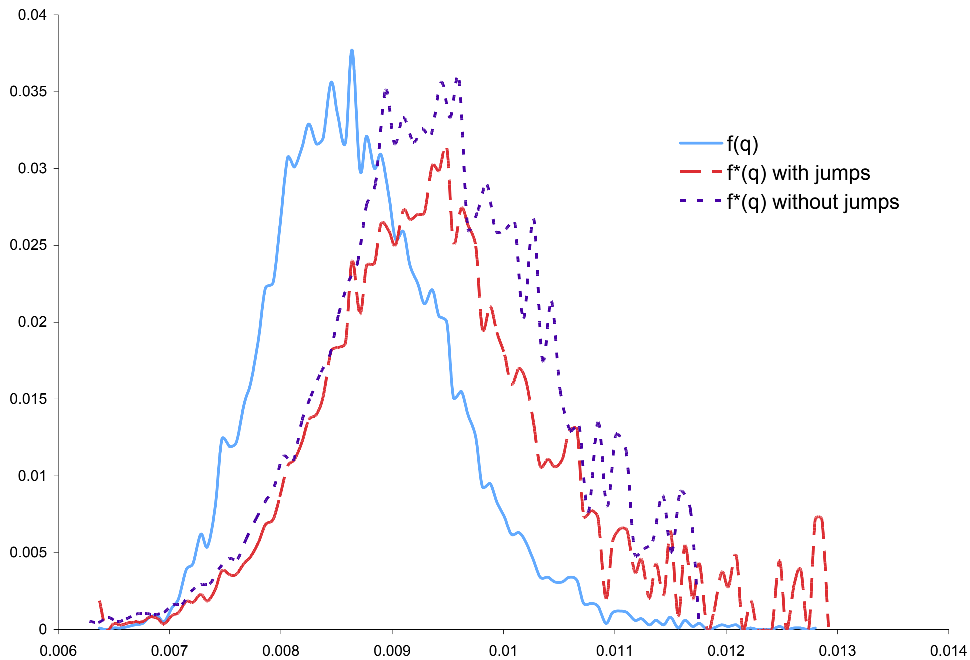

Figure 4.4: Two-factor Wang Transformed Probability Distribution of q with $\lambda = 0.8657$ and 6 degrees of freedom (shown as "f$^*(q)$ with jumps ") in the jump model and that with $\lambda = 1.2523$ and 6 degrees of freedom (shown as "f$^*(q)$ without jumps ") in the model without jumps. The x-axis is the one-year US population death rate and the y axis stands for the probability.

where α_n and σ_n are the expected force and volatility of the US population mortality index in the model without jumps. Based on the same data shown in Section 4.5.2, my maximum likelihood estimate of α_n is -0.0100 and 0.0388 for σ_n. Without jumps, my estimated market price of risk for the US–based Swiss Re bond equals to 1.2523 which is 44 percent higher than the market price of risk 0.8657 when I model the US population mortality index with jumps. Moreover, I plot the transformed distributions with and without jumps in Figure 4.4. Figure 4.4 shows that the transformed distribution with jumps has a fatter tail than that without jumps. It implies that the model without jumps underestimates the probability of having a catastrophe death event. Failing to model jumps leads to a big deviation from the right market price of risk and the correct transformed distribution. I conclude that the jump process plays an important role in the mortality securitization modeling.

Is the Swiss Re bond a good deal for investors?

Wang (2000) reports that the average market price of risk of property catastrophe bonds is about 0.45.[5] Bantwal and Kunreuther (1999) found the spread of property catastrophe bonds are too high to be explained by standard financial theory. Here I find the market price of risk of the Swiss Re bond, 0.8657, is even higher than that of the property catastrophe bonds, 0.45. Although the high risk premium of the Swiss Re deal may suggest high transaction costs of the first mortality security, it may also be interpreted as the Swiss Re overcompensates the investors for their taking its mortality risks. The risk premium of the Swiss Re bond is much higher than my model suggests. So it explains why the "appetite" for the Swiss Re bond was strong.

Why did the Swiss Re company pay such a high risk premium to the investors? The Swiss Re Company's life reinsurance business accounted for 43 percent of its group revenues in 2002, up from 38 percent in 2001 (MorganStanley, 2003). Although capital is crucial for a firm to absorb mortality shocks, the true economic capital requirements of life reinsurance business is not straightforward. Moreover, the costs of potential financial distress are high. Minton et al. (2004) conclude that securitization of financial institutions is a contracting innovation aimed at lowering financial distress costs. Therefore, MorganStanley (2003) concludes that Swiss Re must be taking a view that the cost of capital that is relieved via this transaction exceeds the effective net cost of servicing the bond. Moreover, insurance companies pay high risk premiums to develop a mortality securitization market. If catastrophes deplete traditional reinsurance risk-taking capacity, the insurers can turn to the mortality security market for protection. In all, the Swiss Re mortality bond is a good deal to the investors.

[5] Applying the two-factor Wang transform with $\lambda = 0.45$, my calculated par spread for the Swiss Re bond is 0.76 percent which is lower than that of the Swiss Re bond 1.35 percent. The difference may arise from the fact that I use the US population index as the benchmark while the Swiss Re deal is based on the weighted average of five developed countries. If I use the weighted index, I expect that my calculated par spread will be even lower because of diversification effect of mortality risks among these five countries.

4.5.4 Market Price of Risk of the EIB Bond

Applying the two-factor Wang transform based on the realized mortality rates of English and Welsh males aged 65 and over in 2003 and the gilt STRIPS on November 18, 2004, I get the market price of risk for the EIB bond $\lambda = 0.2408$. It is not surprising that the market price of risk of the EIB bond (0.2408) is lower than that of the Swiss Re bond (0.8657) since longevity risks have much less immediate and dramatic impact on annuity business or pension plans (25 years or longer) than catastrophic death risks caused by disasters like flu (less than a year) on life insurance business. The interesting question here is whether the market price of risk of the EIB bond, $\lambda = 0.2408$, is too high for the potential bond buyers—the UK pension plans.

Let's compare the market price of risk of the EIB bond $\lambda = 0.2408$ with that of private annuities. My calculated market price of risk of the average SPIA for the US male annuitants aged (65) in Section 4.4.3 is 0.2318. The market price of risk reflects the costs of adverse selection. Adverse selection is the tendency of persons with a higher-than-average chance of loss to seek insurance at standard rates, results in higher-than-expected loss levels (**?**). For example, healthy people purchase more annuities while those in poorer health buy more life insurance. Prior literature concludes that pension plans have much less adverse selection problem than commercial annuity insurers because both healthy and less healthy employees participate in pension plans. Therefore the longevity risk for a pension plan should be lower than that for a commercial annuity insurer. Moreover, since mortality experiences of the English and Welsh population improve much less than that of the US population in the past 50 years (Cox et al., 2006), theoretically, the market price of risk for the EIB bond should be lower than that for the US annuity business, 0.2318. However, it is not true for the EIB bond. Therefore, it explains why the UK pension plans, the potential *buyers* of the EIB bond, are not willing to buy the EIB bond since they can get cheaper protection from the annuity or reinsurance markets.

4.6 Conclusions and Discussion

A market for mortality-based securities will develop if the prices and contracting features make the securities attractive to potential buyers and sellers. The Swiss Re bond sold well but the EIB bond did not. I explain these opposite market outcomes by looking at their risk premiums. To calculate the risk premiums I need models, analogous to the term structure on interest rate models. The mortality bond market will be richer in that, in addition to default free zero coupon bonds, it will have bonds which will be redeemed at face value only if a specified number of lives survives or dies to the maturity date. I find only a few preliminary papers on this topic. Development of the theory in this direction is important as an extension of traditional bond market models and it would be very useful in explaining mortality market risk to potential market participants.

My model shows that the Swiss Re mortality bond offers a higher risk premium to investors than the property-linked catastrophe bonds. However, the EIB charges a very high risk premium to take longevity risks in the UK pension plans. Since the price of the EIB bond is not attractive, no UK pension plan buys this bond until now!

Someone may argue that the index–linked mortality securities are subject to unacceptable levels of basis risk. The basis risk is low for the Swiss Re bond because the Swiss Re Company is a global insurance leader occupying 25 percent of all global reinsurance business. It is appropriate for it to link its mortality bond payments to the population indices. Moreover, using the population indices is transparent to investors. Therefore it has no moral hazard problem. Lastly, insurers may not be willing to disclose their underwriting experiences to the public. If they used an index linked to their business, they would be forced to do so. On the other hand, the EIB bond does not provide a good hedge for a pension plan: there exist significant basis risk between the reference population mortality and that of an individual pension plan. Basis risk problem further reduces the attractiveness of the EIB bond.

Moreover, the EIB bond requires a large up-front cash outflow to buyers. The hedgers (e.g. the UK pension plans) have to put up the principal to the EIB for insurance. But it would make more sense for them just pay premium. The longevity bond I propose in Chapter 3 is more feasible since it is structured like an option. Therefore, its premium will be much lower than the principal of the EIB bond. The payoffs of the option would help the hedgers to pay their annuity payments if the number of survivors exceeds the strike level.

In summary, I contribute to the mortality securitization literature by proposing a mortality stochastic model with jumps and pricing the mortality securities in an incomplete market framework. My model nicely explains the opposite market outcomes of the Swiss Re deal and the EIB bond. Finally I comment on the basis risk problem of these two bonds. I also point out the design problem of the EIB bond. Again, it shows the attractiveness of the Swiss Re deal but not for the EIB bond.

5

Household Life Cycle Protection: Life Insurance Holdings, Financial Vulnerability and Portfolio Implications

Using the Survey of Consumer Finances I examine the life cycle demand for different types of life insurance. Specifically I test for the consumer's avoidance of income volatility resulting from the death of a household's wage-earner through the purchase of life insurance. I first develop a financial vulnerability index to control for the risk to the household. I then examine the life cycle demand for life insurance using several definitions of life insurance. I find, in contrast to previous research, that there is a relationship between financial vulnerability and the amount of term life or total life insurance purchased. In addition, I find older consumers use less life insurance to protect a certain level of financial vulnerability than younger consumers. Secondly, my study provides evidence that life insurance demand is jointly determined as part of a household's portfolio. Finally, I consider the impact of family members' non-monetary contribution on the household's life cycle protection decision. My results provide some evidence households take into account the value of non-monetary contribution in their insurance purchase.

5.1 Introduction

A household's demand for life insurance depends on its economic and demographic structure. Using the Survey of Consumer Finances, my study examines the life cycle demand for different types of life insurance. First, I propose a financial vulnerability index to capture a household's financial insecurity. I define financial vulnerability as the household's living standard volatility as a result of the death of wage-earning household member. It is determined by the labor and non-labor income and death probability of each adult household member, the consumption to income ratio, and the effect of age on future consumption needs. I examine the demand for life insurance using several types of life insurance. Secondly, I examine household financial portfolios to see the relationship between insurance and other assets. Finally, I consider the impact of family members' non-monetary contribution (e.g. housework) on the household's life cycle protection decision.

Merton (1975) indicated that the usual sources of consumer uncertainty include uncertainty about future capital income, future labor income (human capital), age at death, investment opportunities, and relative prices of consumer goods. Holden et al. (1986) and Hurd and Wise (1989) document sharp declines in living standards and increases in poverty rates among women whose husbands passed away. Analyzing data gathered during the 1960s from households in middle-age through early retirement, Auerbach and Kotlikoff (1987, 1991a,b) found that roughly one-third of wives and secondary earners would have seen their living standards decline by 25 percent or more had their spouses actually died. While I know that life insurance can be demanded for a number of reasons, I look in particular at the life cycle income protection rationale for demanding life insurance.

Based on the Survey of Consumer Finances (SCF), my paper studies the relationship between a household's financial vulnerability and its total life insurance held on the lives of both spouses. I focus explicitly on those households with a married couple, with both spouses between 20 and 64 years of age, and at least one of the spouses having regular earnings as an employee. The key determinant of the demand for life insurance is the effect of the insured's death on the future consumption of the other household members. My financial vulnerability index measures the financial vulnerability by the volatility of a household's living standard as a whole. It fits into the SCF data nicely because the SCF reports the total amount of life insurance held by each household, and not on the individual demand for life insurance by each spouse. My financial vulnerability index does a good job in explaining the financial vulnerability of a household because it is transparent, easy to implement and based on weaker assumptions. In contrast to previous research, e.g. Bernheim et al. (2001), I find a relationship between financial vulnerability and purchases of term life insurance and a relationship between vulnerability and total (sum of term life and whole life net amount at risk) purchases while this positive relationship is not consistently significant for net amount at risk of whole life insurance. Moreover, my life cycle empirical results show that the sensitivity of total life insurance to financial vulnerability decreases for older households. It suggests younger households are likely to use more life insurance to manage their financial vulnerability but the household substitutes away from higher priced life insurance towards other assets as it ages.

My empirical analysis of consumer portfolios suggests that individual retirement accounts are complements to total life insurance for the young- and middle-aged and real estate are complements to total life insurance only for the youngest. However, bonds are a substitute for term life insurance for older households.

Finally, I examine whether the household's life cycle protection decision takes into account non-monetary contribution of family members. I use the Heckman (1979)'s two-step method to impute the value of housework. My results provide some evidence households take into account the value of non-monetary contribution in their insurance purchase. The households are slightly less sensitive to their financial vulnerability as measured by lower insurance demand in this case because the "imputed" housework value increases their wealth and makes them appear less risk averse.

The paper is organized as follows. Section 5.2 provides my method for measuring financial vulnerability, and section 5.3 describes the data, hypotheses and methodology. Section 5.4 shows the results of the life cycle relationship between households' life insurance holdings and financial vulnerability. I then examine the household's financial portfolio to see the life cycle relationship between life insurance and other assets. I impute the value of the non-monetary contribution and retest the household's life cycle protection decision in Section 5.5. The final section summarizes the study.

5.2 A Different Strategy for Measuring Financial Vulnerability

Bernheim et al. (2001) adopt a yardstick to measure a household's financial vulnerability: the percentage decline in an individual's sustainable living standard that would result from a spouse's death. To calculate this decline, they make use of a relatively sophisticated, but proprietary life cycle consumption model embodied in the financial planning software, Economic Security Planner (or ESPlanner).[1] They do not find a relationship between a

[1] Economic Security Planner, Inc. provides free copies of the software for academic research: www.ESPlanner.com.

household's life insurance holding and its financial vulnerability.

My financial vulnerability index is also a life cycle measure as it reflects different living standards among households, different living standards for households before and after a spouse dies, absolute consumption needs of a surviving spouse (and other members of the family) and age effects. However, my financial vulnerability index is different from that of Bernheim et al. (2001). It has several advantages: my index is more transparent and easier to implement. I can directly use it to study households' life-cycle protection by using different types of life insurance. Moreover, my index is based on weaker assumptions.

5.2.1 Financial Vulnerability Index

One of the primary assumptions regarding a couple's standard of living involves determining the relative cost savings from living together versus separately. There are fixed costs of operating a household which can be "shared" between spouses. I use the value 0.678 which was suggested by Bernheim et al. (2001) to indicate the household scale economies.[2] It implies that a two-adult household must spend 1.5999 ($=2^{0.678}$) times as much as a one-adult household to achieve the same living standard. In other words, the two-adult household spends 0.4001 (= 2 - 1.5999) less than that if they live separately. Bernheim et al. (2001) further considered the effects of the number of the children and use OECD child-adult equivalency factor 0.5. I also use this equivalency factor.

Moreover, households with different income levels consume different proportion of their income. The US Department of Labor provides the annual income and expenditures survey report, the *Consumer Expenditures*, based on approximately 7,500 sample households (5,000 prior to 1999) every year.[3] I observe, in general, low income people spend all of their income while high income households are able to save. That is, the ratio of consumption to labor and

[2] The OECD uses a value of 0.7 for the exponent (see Ringen (1991)).

[3] From *http://www.bls.gov/cex/home.htm*. Annual income and expenditures integrated from the *Interview and Diary* surveys in varying detail, classified by income, age, consumer unit size, and other demographic characteristics of consumer units, since 1984. Annual income and expenditures from the *Interview and Diary* surveys by selected consumer unit characteristics, since 1980.

non-labor income before taxes and deductions varies for each household. The relationship between a household's overall consumption ($\hat{C}_i$) and labor and non-labor income before taxes and deductions (Tincome_i) for household i is shown as

$$\hat{C}_i = \alpha_i * \text{Tincome}_i, \tag{5.1}$$

where α_i is so-called consumption to income ratio. Based on Table 2 in the *Consumer Expenditures* each year,[4] I calculate the annual consumption to income ratios in equation (5.1) for households in different levels of income before taxes. That is, household i's consumption to income ratio α_i may be different from household j's ratio α_j. For example, in 1998, if the households earn income before taxes in the range of \$40,000 - \$49,999, their annual consumption to income before taxes ratio is 0.90. It suggests that these households save 10 percent of their income before tax. This ratio decreases to 0.84 for the households with the income before taxes in the range \$50,000 - \$69,999. Thus higher income households save more. However, the households with the income less than \$30,000 consume all of their income and save nothing. In my model, it implies that their consumption to income ratio is 1.

Moreover, the consumption to total income ratio is also not constant before and after a spouse dies. That is, a wife's consumption to total income ratio if her husband dies $\beta_{\text{wife},i}$ and that for a husband if his wife dies $\beta_{\text{hus},i}$ vary with reduced total income if one of spouses dies. Suppose a household's total income before taxes in the 1998 Survey of Consumer Finances is \$55,000. It includes the husband's labor income of \$40,000, the wife's labor income of \$10,000 and the household's overall non-labor income of \$5,000. If both of the

[4] Before 2003, the *Consumer Expenditures* reported the detailed annual consumption and income information for the households with complete reporting income up to \$70,000. From 2003, the US Department of Labor began to report detailed annual consumption and income for higher income households. I notice that the consumption to income before taxes ratios of households in the same income levels do not vary a lot among years. In order to get a more complete picture of households' consumption and income, I assume that the consumption to income ratios for households with income before taxes higher than \$70,000 per year in 1992, 1995, 1998 and 2001 follows that in 2003 after I adjust all consumption and income in 2001 dollars.

spouses survive, $\alpha_i = 0.84$. If the husband dies, the household income falls to \$15,000 (= \$55,000 - \$40,000). So $\beta_{\text{wife},i} = 1$. If the wife dies, the household income falls to \$45,000 (= \$55,000 - \$10,000) and $\beta_{\text{hus},i} = 0.90$. If household j has a different total income before taxes, its α_j, $\beta_{\text{wife},j}$ and $\beta_{\text{hus},j}$ may be different from α_i, $\beta_{\text{wife},i}$ and $\beta_{\text{hus},i}$ of household i.

When both of spouses are alive, the living standard of the household i is

$$C_i = \alpha_i \frac{\text{Tincome}_i}{(2 + \frac{N}{2})^{0.678}}. \tag{5.2}$$

The variable Tincome_i is the labor and non-labor income before taxes and deductions of household i in \$100,000s and C_i is the living standard of household i when both of spouses are alive. N is the number of the dependent children, and $2^{0.678}$ measures the household scale economies.

When the husband dies, the living standard of the wife $C_{\text{wife},i}$ becomes

$$C_{\text{wife},i} = \beta_{\text{wife},i} \frac{\text{Tincome}_i - Y_{\text{hus},i}}{(1 + \frac{N}{2})^{0.678}}. \tag{5.3}$$

The variable $Y_{\text{hus},i}$ is the husband's total employment income of the household i in \$100,000s and $\beta_{\text{wife},i}$ is the wife's (and other family members') consumption to total income ratio if the husband dies. The reason why I only deduct the husband's labor income $Y_{\text{hus},i}$ from the household's total income, Tincome_i, is that although the wife (and other members of the family) loses the husband's labor income, $Y_{\text{hus},i}$, she inherits her husband's non-labor income, e.g. from financial assets.

The impact on the household i if the husband dies ($\text{IMPACT}_{\text{wife},i}$) can be expressed as the percentage decline in its living standard:

$$\text{IMPACT}_{\text{wife},i} = \frac{C_{\text{wife},i}}{C_i} - 1 = \frac{\beta_{\text{wife},i}(\text{Tincome}_i - Y_{\text{hus},i})(2+\frac{N}{2})^{0.678}}{\alpha_i \text{Tincome}_i (1+\frac{N}{2})^{0.678}} - 1. \tag{5.4}$$

Correspondingly, when the wife dies, the living standard of the husband $C_{\text{hus},i}$ is

$$C_{\text{hus},i} = \beta_{\text{hus},i} \frac{\text{Tincome}_i - Y_{\text{wife},i}}{(1+\frac{N}{2})^{0.678}}, \tag{5.5}$$

where the variable $Y_{\text{wife},i}$ is the wife's employment income of the household i in \$100,000s and $\beta_{\text{hus},i}$ is the husband's (and other family members') consumption to total income ratio if the wife dies.

The impact on the household i if the wife dies ($\text{IMPACT}_{\text{hus},i}$) is given by

$$\text{IMPACT}_{\text{hus},i} = \frac{C_{\text{hus},i}}{C_i} - 1 = \frac{\beta_{\text{hus},i}(\text{Tincome}_i - Y_{\text{wife},i})(2+\frac{N}{2})^{0.678}}{\alpha_i \text{Tincome}_i (1+\frac{N}{2})^{0.678}} - 1. \tag{5.6}$$

The variables $\text{IMPACT}_{\text{wife},i}$ and $\text{IMPACT}_{\text{hus},i}$ defined in equations (5.4) and (5.6) respectively only reflect the *relative* household living standard decline if one of the spouses dies. However, the surviving spouse (and other members of the family) cares about *absolute* consumption. The lost absolute labor income of the dead spouse causes an *absolute* living standard decline for the surviving spouse (and other members of the family). All else equal, the wife with the higher labor income from her husband will incur a higher absolute consumption decline than the other wife with lower husband's labor income. To see how this matters, assume that two wives in households i and j respectively will incur the same *relative* living standard decline if their husbands die, that is,

$$\text{IMPACT}_{\text{wife},i} = \text{IMPACT}_{\text{wife},j} \Rightarrow \frac{C_{\text{wife},i}}{C_i} - 1 = \frac{C_{\text{wife},j}}{C_j} - 1. \tag{5.7}$$

However, each wife's *absolute* living standard decline may be different and determined by the lost labor income from her husband. To account for this possibility, I multiply the relative living standard decline measure of the wife $\text{IMPACT}_{\text{wife},i}$ by the husband's labor income $Y_{\text{hus},i}$ to capture the *absolute* living standard decline of the wife. The *absolute* living standard decline of the husband is similarly defined.

I also consider an age effect. The effect of age on future consumption needs is not the same for the old family and the young household. An older family maybe much less vulnerable than a younger family to the death of its more important wage-earner because of the smaller expected number of surviving years left for the survivor. The idea is shown in Figure 5.1. The upper graph in Figure 5.1 shows the effect of husband's age x on the household's future consumption needs. Assume that the husband, age x, plans to retire at age 65. If the husband dies at age x, the household will incur annual absolute living standard decline ($\text{IMPACT}_{\text{wife},i} \cdot Y_{\text{hus},i}$) for $(65 - x)$ years. Assuming a flat interest rate 5 percent,[5] an annuity factor, $a_{\overline{65-x}|}$, captures the age effect of the husband's death at age x. The higher $a_{\overline{65-x}|}$ implies a larger effect of his age on future consumption needs of other family members. Similarly, $a_{\overline{65-y}|}$ represents the wife's age effect and is illustrated in the lower graph in Figure 5.1.

Taking into account all of the above factors, my index of financial vulnerability (IMPACT_i) of the household i is then defined as

$$\text{IMPACT}_i = \sqrt{q_{x,i}^{\text{hus}}(\text{IMPACT}_{\text{wife},i} \cdot Y_{\text{hus},i} \cdot a_{\overline{65-x}|})^2 + q_{y,i}^{\text{wife}}(\text{IMPACT}_{\text{hus},i} \cdot Y_{\text{wife},i} \cdot a_{\overline{65-y}|})^2}. \tag{5.8}$$

The index I defined is similar to the definition of standard deviation. The variable $q_{x,i}^{\text{hus}}$ is the one-year death probability of the husband aged x of the household i in the survey year and $q_{y,i}^{\text{wife}}$ the one-year death probability of the wife aged y of the household i in the

[5] My results are robust to other interest rate assumptions.

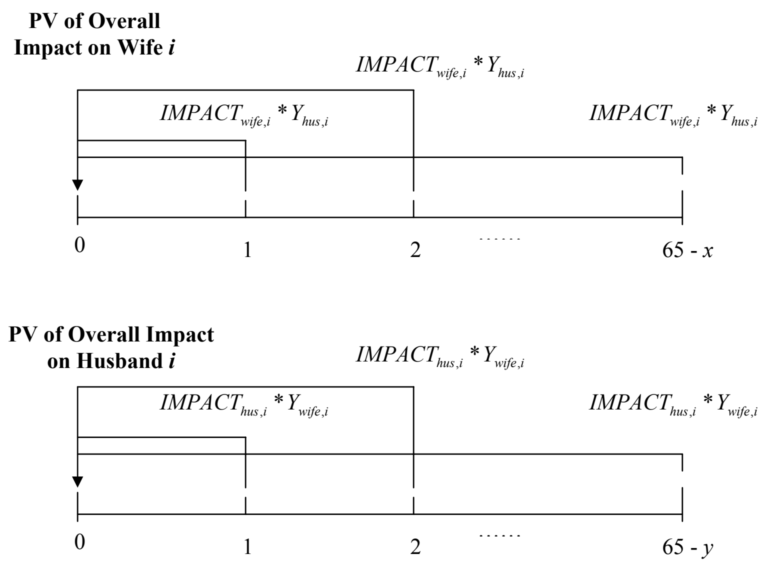

Figure 5.1: The age effect of spouses on future consumption needs of other family members

survey year. I use the 1990-1995 US SOA Life Insurance Basic Mortality Table to capture the mortality experience of the observed household. The reason why I use a one-year death probability is that the current life insurance holding reflects the household's expectation of its potential risks if one of spouses dies in the foreseeable future, e.g. one year. The variables $Y_{\text{hus},i}$ and $Y_{\text{wife},i}$ are the husband's and wife's total employment income of the household i in \$100,000s respectively. The variable $\text{IMPACT}_{\text{wife},i}$ ($\text{IMPACT}_{\text{hus},i}$) accounts for the relative living standard decline of household i if the husband (the wife) dies. The product in the first pair of brackets in equation (5.8) reflects the overall impact of husband's death at age x on the household. It equals to annual absolute living standard decline (or deviation from current absolute living standard) $\text{IMPACT}_{\text{wife},i} \cdot Y_{\text{hus},i}$ times the age effect, $a_{\overline{65-x}|}$. The overall effect of wife's death at age y is similarly defined and is the product in the second pair of brackets in equation (5.8). Moreover, my index solves one of the main problems of using the Survey of Consumer Finances as it reports the results of the survey based on a household instead of an individual. My index measures the financial vulnerability by the volatility of a couple's living standard as a whole. Thus, IMPACT_i captures the volatility of a household's financial situation if one spouse dies.

5.2.2 Advantages of Proposed Financial Vulnerability Index

Based on the 1995 Survey of Consumer Finances (SCF), Bernheim et al. (2001) also study households' financial vulnerabilities by comparing an individual's highest sustainable living standard when both of spouses survive, C^*, with C_s^n which represents the living standard the husband (when $s =$ husband) or the wife (when $s =$ wife) would enjoy without life insurance. They define their financial vulnerability index as follows:

$$\text{IMPACT}_{\text{Bernheim}} = \frac{C_s^n}{C^*} - 1. \tag{5.9}$$

Bernheim et al. (2001) run the regression of the households' actual life insurance holdings on their benchmark life insurance (or benchmark life insurance to household income) rather than on their financial vulnerability measure in Equation (5.9). Their benchmark life insurance quantity is obtained from Economic Security Planner (or ESPlanner). Different financial planning software often gives us very different life insurance purchase suggestions. An important issue is that if the ESPlanner is inaccurate, it will come up with an incorrect benchmark life insurance and thus will lead to incorrect conclusions. While I do not suggest ESP is inaccurate, I cannot tell how the model underlying their conclusions is set up. My financial vulnerability index is different from theirs and can act as a compliment to their method. Compared with Bernheim et al. (2001)'s financial vulnerability measure, my index has the following advantages:

- Transparency: Bernheim et al. (2001) use a life-cycle financial planning software, Economic Security Planner (or ESPlanner) to determine the benchmark life insurance demand. Their method is not transparent: it is hard to tell how the ESPlanner works and what its underlying assumptions are. The key determinant of the demand for life insurance is the effect of the insured's death on the future consumption of the other household members because they will lose the insured's labor income forever. My financial vulnerability index transparently measures this effect by including the lost absolute consumption, the consumption to labor earnings ratios α_i, and the life-cycle effects.

- Implementation: The cost associated with compiling data and modeling households' financial vulnerabilities is a substantial obstacle in investigating the relationship between household's life insurance holdings and financial insecurity. A complete description of Bernheim et al. (2001)'s model would be "prohibitively lengthy" (p.7 in Bernheim et al. (2001)). I present a method of estimating financial vulnerabilities that is easily implemented using data from the SCF and *Consumer Expenditures* and is easily understood.

- Broader product comparison possible: There are differences between term life insurance and whole life insurance which are overlooked in Bernheim et al. (2001). First, whole life insurance has a cash value while term life insurance has no cash value. Second, the duration of whole life insurance is generally much longer than term insurance. Third, term life insurance is naturally suited for ensuring that mortgages and other loans are paid on the debtor/insured's death and as a vehicle for ensuring that education or other needs are available if death were to cut short the period needed for the provider/insured to earn the needed funds. Further, whole life insurance can serve as a quasi-forced savings plan (Black and Skipper, 2000). The differences between the two types of insurance may lead to differences in the household's insurance purchasing behavior. Bernheim et al. (2001) fail to make distinctions between term life insurance and whole life insurance demand in their analysis. However, our financial vulnerability index is able to do so.

- Weaker assumptions: Bernheim et al. (2001) state that, " [T]his is a measure of the percent by which the survivor's living standard would, with no insurance protection, fall short of or exceed the couple's *highest* sustainable living standard (p. 6)." If I understand their "highest sustainable living standard" correctly, my concern is whether it is appropriate to use the highest sustainable living standard to obtain the benchmark. In reality, people normally lead a life style below their highest living standard. If consumers are prudent, they will set aside some money for a "rainy day" (Kimball, 1990). My measure is based on a more typical consumption level.

5.3 Data, Hypotheses and Methodology

I now turn to an empirical examination of a household's life cycle protection. First, I investigate the effects of financial vulnerability on the households' demand for net amount at risk, term life insurance and total life insurance. Net amount at risk is my measure of households' long-term pure life insurance demand. Term life insurance measures the quantity of short-term life insurance demanded. And total life insurance is my measure of households' total life insurance demand. Second, households hold various asset items including life insurance in a portfolio. Life insurance demand is determined, in part, by households' financial asset allocation decisions. I empirically test how each asset item may influence life insurance holdings. Finally, a family member who has zero earnings may make non-monetary contributions to the family. For instance, one spouse who stays at home cleans, cooks and cares for children contributes to household income. The family would suffer a serious financial setback if this spouse dies. Therefore I retest the household life cycle protection by imputing a value for uncompensated housework in Section 5.5.

5.3.1 Data Description

The sample for my study consists of the 1992, 1995, 1998 and 2001 years of the Survey of Consumer Finances. In each of these four years, the survey covered over 4,000 households. The data includes demographic, income, wealth, debt and credit, pensions, attitudes about financial matters, the nature of transactions with various types of financial institutions, housing, real estate, business, vehicles, health and life insurance, current and past employment, current social security benefits, inheritances, charitable contributions, education, and retirement plans. The architects of the SCF data files imputed missing information, supplying five "implicates" for each household.[6] Bernheim et al. (2001) use the first implicate. I also use the first implicate in this study to compare my results and theirs.[7]

I focus on households with married couples. Because I are looking at those who potentially have a need for life insurance I restrict the ages of both spouses to a range from 20 to 64. Following Bernheim et al. (2001), I exclude those observations where neither spouse had regular labor earnings. Accurate measurement of life insurance holdings is particularly critical for my analysis. Fortunately, the SCF data match up reasonably well with other sources of information concerning this life insurance.[8] Moreover, including consistent labor income measures is also important because they are two major variables determining my financial vulnerability index. I delete those observations with labor income paid by lump sum, one payment only/in total, by the piece/job and if the pay varies. The SCF reports the husband's labor income and the wife's labor income respectively. It also reports the household's total labor income. I double check to make sure that the sum of the husband's labor income and that of the wife is the same (or almost the same) as the household's total labor income. In all, I identify around one percent of my sample as outliers.

An important characteristic of the SCF is that it contains information only on the total amount of term life insurance and total amount of whole life insurance held by each household, and not on the division of this insurance between spouses. Bernheim et al. (2001) estimate a regression model explaining the fraction of a couple's total life insurance held

[6]Kennickell (1994) provides a description of the imputation procedure.

[7]The problem of using only one of the five imputed data sets is that the true variances of the estimated coefficients are inflated (Bernheim et al., 2003). This implies that the standard errors from my paper are too large but I note significance in many important variables.

[8]Bernheim et al. (2001) made some comparisons between statistics on life insurance coverage (including all individual and group policies) drawn from the SCF and from a survey fielded by the life Insurance Marketing Research Organization (LIMRA). Furthermore, they computed the aggregate amount of in-force life insurance implied by the SCF survey responses, and compared this with total in-force life insurance reported by the industry (obtained from the ACLI (1999)). They concluded that there is no indication that the SCF understates life insurance coverage.

on the life of husband as a function of the age of each spouse, the husband's earnings, the husband's share of the couple's total non-asset income, family size, and the husband's share of the couple's total benchmark life insurance. Due to the nature of the data, this type of estimation may be biased because it does not look at "household" purchases of insurance (Lewis, 1989). It could lead to the conclusion that there is no correlation between life insurance demand and a household's financial vulnerability. Thus, I explore the relation between different types of life insurance demand and financial vulnerability directly based on the structure and characteristics of the household.

5.3.2 Dependent Variables and Hypotheses

The face value of life insurance is the amount an insurer will pay to the beneficiary when the insured dies. The face value reflects the amount a household perceives is appropriate to manage its financial vulnerability. However, there is a problem with face value of whole life insurance because the actual amount of pure whole life insurance protection at any point is the difference between the policy cash value (saving component) at that point and the face amount. This difference is called the net amount at risk (Black and Skipper, 2000). Therefore, the net amount at risk is a more appropriate proxy of the quantity of whole life insurance demanded than the whole life insurance face value. From the perspective of consumers, I consider the "net amount at risk" of whole life insurance as the proxy of whole life insurance quantity demanded and the face value of term life insurance as the proxy of term life insurance quantity demanded. Because of the skewness of the face value of term life insurance or the net amount at risk, I use a logarithmic transformation. Since Bernheim et al. (2001) explore the relationship based on the total insurance demand, I also study this relationship by using the sum of term life insurance face value and the net amount at risk of whole life insurance.

According to Ando and Modigliani (1963)'s life cycle theory, an individual's income will be low in the beginning and end stages of life and high during the middle income-earning years of life. Term insurance can be useful for persons with low incomes and high insurance needs because of its relatively low price (Black and Skipper, 2000). Since younger families have lower income and less wealth accumulation, they may desire lower-cost insurance protection. On the other hand, older households maybe less vulnerable than younger families to the death of the more important wage-earner because of the smaller expected number of surviving years left for the survivor. Moreover, older households may be less risk averse because they have already accumulated a certain amount of wealth. In addition, Chen et al. (2001) find that baby boomers tend to purchase less life insurance than those in previous generations. Baby boomers are in the middle-age and older-age groups in my study. I predict that there will be a more significant relationship between younger household's life insurance holdings and its financial vulnerability.

5.3.3 Other Explanatory Variables and Hypotheses

In addition to independent variable IMPACT_i, other differences, such as demographic characteristics, financial situation and obligations, among couples are expected to affect life insurance demand. Identifying those factors will give us a clearer picture on the relationship between life insurance demand and financial vulnerability and between life insurance holdings and other assets.

Assets

Intuitively, the wealth a person holds will influence life insurance purchases. The relation between the demand for life insurance and wealth is ambiguous as it depends upon a consumer's risk tolerance. It is possible that an individual increases his life insurance demand with increasing wealth. It is also possible that a person will mainly put the incremental wealth into savings because he thinks he can handle risks with his improved economic strength. If so, life insurance can be an inferior good. Fortune (1973) found that per capita wealth was related negatively to "net" life insurance in force. This was attributed to the fact that increases in wealth lead to decreases in aversion to risk.

Life insurance demand is determined by a household's asset allocation decisions. Fortune (1973) concludes life insurance is a substitute for financial assets such as equities or other lower risk assets. Headen and Lee (1974) propose a four-component interrelated household asset model including primary securities (corporate stocks and bonds), money (currency and demand deposits), time deposits (all savings shares and various time deposits) and life insurance sales (ordinary). However, their results on the relationship between life insurance demand and other financial assets are inconclusive. They find limited evidence that life insurance decreases with investment in stocks, bonds and increases with saving account holdings.

Individual retirement accounts are also saving accounts with tax advantages. Higher rates of net saving is assumed to be positively correlated with life insurance demand since life insurance is a primary financial asset alternative for low-asset holders (Headen and Lee, 1974). Household financial assets also include annuities. Brown (1999) finds evidence that many households simultaneously hold term life insurance and private annuities. Moreover, Bernheim et al. (2001) state that housing may also affect life insurance needs. Although they treat housing as fixed consumption, I look at real estate holdings as a part of household assets. Households often purchase term life insurance to ensure that mortgages are paid on the debtor/insured's death.

In order to identify the effect of different types of assets on the different types of life demand, I split the assets into several categories. I include cash and cash equivalents, mutual funds, stocks, bonds, annuities, individual retirement accounts, real estate and other assets. All of the above assets are all measured based on the unit of the household using the log value.

Debts

Good risk management principles suggest the family unit should be protected against catastrophic losses. Life insurance can be a way to ensure that mortgages and other obligations are paid on the insured's death. Again, it is ambiguous whether there is a positive relationship between life insurance holdings and debts of a household.

Education

Education tends to be a good predictor of earning ability over the long term. It is also associated with wealth, financial vulnerability and life insurance demand. Burnett and Palmer (1984) show that higher education is associated with higher life insurance demand even allowing for the higher incomes. However, Goldsmith (1983) concludes that households with a more educated wife, *ceteris paribus*, have a lower likelihood of purchasing term insurance on the husband. Thus the overall effect of education on a household's insurance holdings is uncertain.

Inheritance, Obligations, Bequests and Emergencies

In the SCF, there is a question concerning an expected inheritance. Thus, I am able to control for a potential substitute for life insurance. Also the survey asked whether there are any foreseeable major financial obligations expected to be met in the future such as educational expenses, health care costs and so forth. I control for fixed obligations that life insurance may finance if one of spouses dies. Finally, I consider a household's desire to leave a bequest and also include it as one of independent variables.

Term Life Insurance

Term life insurance furnishes protection for a limited number of years at the end of which the policy expires, meaning that it terminates with no maturity value. The face amount of the policy is payable only if the insured's death occurs during the stipulated term, and nothing is paid in case of survival. Term insurance can be the basis for one's permanent insurance program through a so-called buy-term-and-invest-the-difference (BTID) arrangement. The difference between the higher-premium cash-value policy and the lower-premium term policy is to be invested separately, such as in a mutual fund, savings account, an annuity, or other investment media. The hope is that the term plus the separate investment will outperform the cash-value life insurance policy (Black and Skipper, 2000). Thus, I predict that the term life insurance is a substitute for the whole life insurance. So I use the log value of the term life face value in the net amount at risk function and the log value of the net amount at risk value in term life insurance demand function.

Age

The relationship between age and life insurance is ambiguous. Burnett and Palmer (1984) do not find a significant relationship between age and life insurance holdings. For older people, they may have a greater desire to leave a bequest. However, they may have a binding budget constraint when approaching retirement. Following Bernheim et al. (2001), I control for the effects of age by two ways. In the first way, I include linear and quadratic terms in the couple's average age. On the other hand, in the second way, I divide my sample into three age groups. Three age dummies are assigned to these three groups.

Income

I include household labor income in my model. Income, like wealth, may have ambiguous effects on the life insurance demand. If the consumer has decreasing absolute risk aversion, he will purchase less insurance at higher levels of income due to decreasing marginal utility of income. However, I know that as income increases new types of risks arise. For example, consumers may buy bigger houses and may incur more expensive obligations. Thus, one could hypothesize a positive relationship between income and insurance demand. Burnett and Palmer (1984) find a significant and positive relationship between income and life insurance holdings.

5.3.4 Summary Statistics

My final sample consists of 6,755 married couples for the 1992, 1995, 1998 and 2001 years of the Survey of Consumer Finances. Variables in dollars are all in year 2001 dollars. Table 5.1 shows the descriptive statistics for my sample. "Net amount at risk" is the difference between face value of whole life insurance and whole life cash value. "Salary and wage" refers to labor income. "Cash and cash equivalent" includes checking accounts, saving accounts, money market deposit accounts, money market mutual funds, call accounts at brokerages and certificates of deposit. "Mutual fund" includes stock mutual funds, tax-free bond mutual funds, government bond mutual funds, other bond mutual funds, combination and other mutual funds and total directly-held mutual funds, excluding market-money mutual funds. "Stock" refers to the publicly traded stock. "Bond" includes tax-exempt bonds (state and local bonds), mortgage-backed bonds, US government and government agency bonds and bills, corporate and foreign bonds and savings bonds. "A household's individual retirement account" includes individual retirement account, thrift accounts and future pensions. "Individual annuity not including job pension" refers to other managed assets such as trusts, annuities and managed investment accounts in which a household has equity interest. "Real estate" is the sum of the value of primary residence, other residential real estate and net equity in nonresidential real estate. If a household only owns a part of the property, the value reported should be only the household's share. "Other assets" are a household's total assets excluding whole life cash value, cash, mutual fund, stock, bond, individual annuity not including job pension, individual retirement account and real estate. The education level of respondents and spouses reflects the number of years of schooling.

5.3.5 Estimation Methodology

The regression on the SCF data using ordinary least squares (OLS) are potentially problematical because there is about 35 percent zero term life face value, 58 percent zero whole life face value and 16 percent zero total life insurance face value. Tobit models under this situation will give us consistent estimates. So I employ the ordinary tobit estimation of the life insurance demand model. In addition to the dependent variable to measure whole life insurance demanded (log of net amount at risk of whole life), I estimate another two quantities: log of term life face value and log of sum of term life face value and whole life net amount at risk. Since there are many zero values in my dependent variables, I add a relatively small value (0.00001) to those with zero. I then test for the sensitivity with respect to adding this small value and find that the results are robust to size of the data transformation.

Table 5.1: Summary Statistics for the Survey of Consumer Finances 1992, 1995, 1998 and 2001 Waves

Variable Description	**Mean**	**Standard Deviation**	**Minimum**	**Median**	**Maximum**
Dependent Variables					
Net amount at risk (NAR)	$248,892	1,930,412	0	0	86,786,560
Face value of term life insurance	$366,263	1,995,366	0	56,700	80,000,000
Term life face value + whole life NAR	$615,155	2,871,676	0	126,000	93,746,560
Independent Variables					
Financial vulnerability index	0.2350	0.3065	0.0000	0.1229	1.9930
Salary and wage of the respondent before taxes	$79,490	134,360	0	41,737	2,261,997
Salary and wage of the spouse before taxes	$25,572	46,311	0	14,351	1,122,372
A household's total salary and wage before taxes	$105,062	144,084	5,039	64,258	2,261,997
A household's total income before taxes	$313,055	1,258,928	5,450	74,430	28,188,000
Average age of couple	42	10	20	43	64
Sizable inheritance expected	$103,740	1,514,129	0	0	112,140,000
Cash + cash equivalent	$117,729	1,027,846	0	6,496	60,152,200
Mutual fund	$95,447	830,297	0	0	44,385,080
Stock	$520,522	6,849,555	0	0	288,828,200
Bond	$163,355	2,306,561	0	0	135,465,200
Individual annuity not including job pension	$187,984	3,735,184	0	0	163,500,000
A household's individual retirement account	$129,938	508,276	0	13,000	20,390,480
Real estate	$589,868	2,798,397	0	140,000	126,633,780
Other assets	$2,215,470	15,987,826	0	24,360	500,876,000
Total debt of the household	$131,536	584,411	0	51,500	36,523,000
Education level of the respondent	14.0691	2.6268	1	14	17
Education level of the spouse	13.8343	2.6221	0	14	17
Number of Dependent Children	1.2762	1.2383	0	1	8
Desire to leave a bequest*	39.14%	48.81%	0	0	1
Foreseeable major financial obligations*	59.19%	49.15%	0	1	1
Baby boom indicator of the respondent*	56.23%	49.61%	0	1	1
Baby boom indicator of the spouse*	58.87%	49.20%	0	1	1

Number of observations: 6,755. *: 1 = Yes, 0 = No.

Moreover, the pooled results will be overstated if the inclusion of the same household for multiple years results in observations that are not independent (although the SCF data is not a panel data). Therefore, I run the regressions and report the estimates by year.[9]

5.4 Life Cycle Protection Analysis

5.4.1 Financial Vulnerability

I first analyze households' life-cycle life insurance holding and financial vulnerability without including asset items. I run the tobit regressions by year. The dependent variables are three definitions of life insurance demanded (net amount at risk of whole life, term life face value and sum of whole life net amount at risk and term life face value respectively). There are two separate set of results. For the first set of regression, I include linear and quadratic terms in the couple's average age.

$$Log(\mathbf{LifeIns}) = \alpha_3 + \gamma \text{IMPACT} + \beta_1 \text{Age} + \beta_2 \text{Age}^2 + \varepsilon_1, \tag{5.10}$$

where **LifeIns** stands for three different dependent variables representing quantity of insurance demanded. IMPACT is my financial vulnerability index. I expect the coefficient γ is positive which means that a household increases its life insurance holdings with increasing financial vulnerability. The variable Age stands for the couple's average age.[10]

In the second set of regression, I adopt a more flexible functional specification proposed by Bernheim et al. (2001). That is, I divide my sample into three age groups and assign a group dummy for each group. Moreover, I interact these group dummies with my financial

[9] My results are even stronger when I pool four waves of the survey.

[10] Originally, I also included an interaction term between age and my financial vulnerability index to investigate the hypothesis that the correlation between insurance holdings and vulnerabilities changes with age. However, including this interaction term caused a serious multi-collinearity problem since the Pearson's correlation between the interaction term and financial vulnerability index was as high as 0.98.

vulnerability index.

$$\begin{aligned} Log(\textbf{LifeIns}) =& \alpha_1 \mathrm{A}_{20-34} + \alpha_2 \mathrm{A}_{35-49} + \alpha_3 \mathrm{A}_{50-64} \\ &+ \gamma_1 \mathrm{IMPACT} \cdot \mathrm{A}_{20-34} + \gamma_2 \mathrm{IMPACT} \cdot \mathrm{A}_{35-49} + \gamma_3 \mathrm{IMPACT} \cdot \mathrm{A}_{50-64} + \varepsilon_2, \end{aligned} \tag{5.11}$$

where A_{20-34}, A_{35-49} and A_{50-64} are age dummies for age group 20 - 34, 35 - 49 and 50 - 64 respectively.

In the first separate set of age definition, the regression results in Table 5.2 show that there is a significant and positive relationship between a household's financial vulnerability and net amount at risk as the marginal effects of the financial vulnerability index are all significant in most cases except year 1998. Regression results based on a more flexible functional specification suggest that the positive and significant relationship in 1992, 1995 and 2001 is driven by the middle and older age groups because the interaction terms of age group dummy and financial vulnerability index are positive and significant for those two groups but not for the younger families. It implies that younger households do not purchase whole life insurance to manage its potential financial insecurity caused by the death of the income earner(s).

An important conclusion to be drawn from Table 5.3 is that there is a positive and significant relationship between a household's term life insurance and its financial vulnerability in all four time periods in the first specification. Moreover, the second specification suggests that younger households tend to purchase term life insurance, an opposite picture to what I conclude for the whole life insurance from Table 5.2. It is not a surprising result. Whole life insurance is more expensive than term life insurance. Younger households have not accumulated enough wealth (or consume too much annual income) to purchase whole life insurance and therefore prefer term life insurance.

Table 5.4 shows the results when the dependent variable is the sum of net amount at risk and term life insurance. Both the first specification and the second specification in all four years tell us the same story: the higher volatility of potential living standard implies more total life insurance purchases. In general, households in different ages use a combination of term life and net amount at risk to reduce their potential financial vulnerability. Moreover, according to the results of the second specification, younger households are more sensitive to financial vulnerability than older households because the coefficient of interaction term of financial vulnerability index and age group 20 - 34 is higher than the interaction terms for the other two age groups (e.g. 6.4232 for younger group vs. 4.6381 and 4.9168 for middle and older groups in 2001) except year 1992. I conclude that there is a life cycle relationship between a household's life insurance holdings and its financial vulnerability. My results are opposite to Bernheim et al. (2001)'s conclusion because they do not find this relationship specifically as they only look at total life purchases in 1995.

Table 5.2: Life Cycle Relationship: Net Amount at Risk and Financial Vulnerability Index

	1992	1995	1998	2001	1992	1995	1998	2001
Independent Variables	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]
Intercept	-30.33***	-21.51***	-32.85***	-32.91***				
	(5.49)	(4.67)	(5.10)	(4.68)				
Financial vulnerability index	3.90***	3.83***	0.43	2.11***				
	(1.06)	(0.96)	(0.96)	(0.78)				
Average age of couple	0.97***	0.47**	1.02***	0.96***				
	(0.27)	(0.23)	(0.25)	(0.22)				
(Average age of couple)^2	-0.01***	-0.01	-0.01***	-0.01***				
	(0.00)	(0.01)	(0.00)	(0.00)				
Age 20 - 34 (group 1)					-9.42***	-10.57***	-11.55***	-11.47***
					(0.79)	(0.74)	(0.81)	(0.92)
Age 35 - 49 (group 2)					-5.76***	-7.24***	-6.55***	-7.83***
					(0.60)	(0.51)	(0.51)	(0.44)
Age 50 - 64 (group 3)					-3.71***	-5.77***	-5.18***	-6.47***
					(0.80)	(0.69)	(0.65)	(0.57)
Financial vulnerability index					4.53	5.37	-1.21	-10.61*
interacted with age group 1					(3.08)	(3.76)	(4.00)	(5.74)
Financial vulnerability index					4.26***	3.15**	1.07	2.98***
interacted with age group 2					(1.58)	(1.37)	(1.34)	(1.04)
Financial vulnerability index					4.42**	5.37***	0.91	2.65**
interacted with age group 3					(1.78)	(1.61)	(1.50)	(1.32)
Number of observations	1496	1745	1723	1791	1496	1745	1723	1791
Log-likelihood	-3694	-3786	-3652	-3222	-3708	-3803	-3655	-3231

Standard errors are presented below the estimated coefficients; [a] Marginal effects (M. E.) of tobit model are computed at the mean of Xs; *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level.

Table 5.3: Life Cycle Relationship: Term Life Insurance and Financial Vulnerability Index

	1992	1995	1998	2001	1992	1995	1998	2001
Independent Variables	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]
Intercept	-14.95***	-18.44***	-18.56***	-18.12***				
	(5.25)	(4.74)	(5.29)	(5.06)				
Financial vulnerability index	2.53**	4.34***	3.6713***	3.63***				
	(1.10)	(1.06)	(1.07)	(0.97)				
Average age of couple	0.78***	0.88***	0.85***	0.85***				
	(0.26)	(0.23)	(0.26)	(0.25)				
(Average age of couple)^2	-0.01***	-0.01***	-0.01***	-0.01***				
	(0.00)	(0.00)	(0.00)	(0.00)				
Age 20 - 34 (group 1)					0.85	-1.75**	-2.51***	-2.23***
					(0.81)	(0.79)	(0.84)	(0.84)
Age 35 - 49 (group 2)					1.90***	0.83	-1.01	0.57
					(0.65)	(0.60)	(0.62)	(0.59)
Age 50 - 64 (group 3)					0.36	0.81	-1.86**	-1.25*
					(0.84)	(0.78)	(0.78)	(0.75)
Financial vulnerability index					-2.48	9.01**	8.41**	8.31**
interacted with age group 1					(3.16)	(3.90)	(3.73)	(3.54)
Financial vulnerability index					2.98*	5.84***	5.10***	2.66**
interacted with age group 2					(1.58)	(1.45)	(1.47)	(1.29)
Financial vulnerability index					3.73**	2.37	1.55	4.56***
interacted with age group 3					(1.85)	(1.81)	(1.75)	(1.69)
Number of observations	1496	1745	1723	1791	1496	1745	1723	1791
Log-likelihood	-4753	-5469	-5196	-5516	-4755	-5471	-5196	-5518

Standard errors are presented below the estimated coefficients; [a] Marginal effects (M. E.) of tobit model are computed at the mean of Xs; *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level.

Table 5.4: Life Cycle Relationship: Sum of Net Amount at Risk and Term Life Insurance and Financial Vulnerability Index

	1992	1995	1998	2001	1992	1995	1998	2001
Independent Variables	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]
Intercept	-9.39***	-17.37***	-17.78***	-21.10***				
	(3.63)	(3.59)	(4.31)	(4.26)				
Financial vulnerability index	3.80***	4.13***	3.34***	4.58***				
	(0.76)	(0.81)	(0.88)	(0.82)				
Average age of couple	0.74***	1.02***	0.99***	1.10***				
	(0.18)	(0.18)	(0.21)	(0.21)				
(Average age of couple)^2	-0.01***	-0.01***	-0.01***	-0.01***				
	(0.00)	(0.00)	(0.00)	(0.00)				
Age 20 - 34 (group 1)					6.04***	3.20***	2.18***	1.16
					(0.57)	(0.62)	(0.71)	(0.72)
Age 35 - 49 (group 2)					8.14***	7.22***	5.70***	5.56***
					(0.46)	(0.46)	(0.52)	(0.50)
Age 50 - 64 (group 3)					8.42***	7.75***	6.57***	5.48***
					(0.60)	(0.60)	(0.65)	(0.64)
Financial vulnerability index					3.64*	8.67***	5.86*	6.42**
interacted with age group 1					(2.17)	(3.05)	(3.15)	(3.04)
Financial vulnerability index					3.76***	4.30***	4.75***	4.64***
interacted with age group 2					(1.14)	(1.14)	(1.23)	(1.10)
Financial vulnerability index					4.65***	4.29***	2.18	4.92***
interacted with age group 3					(1.32)	(1.39)	(1.42)	(1.43)
Number of observations	1496	1745	1723	1791	1496	1745	1723	1791
Log-likelihood	-4977	-5794	-5755	-5940	-5015	-5471	-5196	-5953

Standard errors are presented below the estimated coefficients; [a] Marginal effects (M. E.) of tobit model are computed at the mean of Xs; *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level.

5.4.2 Portfolio Implications

Headen and Lee (1974) explore the linkage between life insurance demand and household financial assets. They construct a four–component interrelated household asset estimated using primary securities, money, time deposits and ordinary life insurance sales. They use another four variables reflecting household expectations and current economic conditions in financial markets: net savings, the consumer sentiment index (reflecting household expectations of future prices, income, and general economic conditions), interest rates (on high grade bonds) and index of security prices. Given the low t-ratios for lagged alternative assets, they conclude that the evidence concerning the relation of life insurance demand and other alternative financial assets is not certain. The variables Headen and Lee (1974) define are macroeconomic oriented. For example, they use ordinary life insurance sales as life insurance demand and they investigate stocks and bonds quarterly flowing to household sector. However, they do not explore the life cycle effects and do not study the term insurance and whole life insurance separately.

I, on the other hand, investigate a household's life insurance purchasing behavior from a microeconomic perspective and treat each household as a unit. Although I have found a life cycle relationship between life insurance and financial vulnerability, it may be interesting to further determine the life cycle relationship between a household life insurance holding and other assets. My regression is shown as follows:

$$
\begin{aligned}
Log(\mathbf{LifeIns}) = {} & \gamma_4 \mathrm{IMPACT} + \alpha_4 \mathrm{A}_{20-34} + \alpha_5 \mathrm{A}_{35-49} + \alpha_6 \mathrm{A}_{50-64} \\
& + \sum_i^n \delta_{1i} \mathrm{Asset}_i \cdot \mathrm{A}_{20-34} + \sum_i^n \delta_{2i} \mathrm{Asset}_i \cdot \mathrm{A}_{35-49} + \sum_i^n \delta_{3i} \mathrm{Asset}_i \cdot \mathrm{A}_{50-64} \\
& + \beta X' + \varepsilon_3,
\end{aligned}
\tag{5.12}
$$

where the variable Asset_i stands for asset item i including cash, stocks, bonds, mutual fund, annuities, individual retirement accounts and real estate. The vector X includes other control variables.

I focus on the relationship of a household's life insurance holdings with its different kinds of financial assets: cash, stocks, bonds, mutual fund, annuities, individual retirement accounts and real estate. I run the tobit regressions by year and get similar results as those for year 2001 shown in Table 5.5. My results in Table 5.5 suggest that most of the life-cycle relationships between life insurance holdings and different asset items are insignificant because the interaction terms of different assets and age group dummies are not significant. I find limited positive (negative) relationship between individual retirement accounts, annuities and real estate (bonds) respectively with life insurance holdings which is consistent with the existing literature (Fortune, 1973; Headen and Lee, 1974; Brown, 1999; Bernheim et al., 2001).

For some financial assets (i.e. cash, stock, individual retirement accounts and real estate) and the age groups, the interaction terms of financial vulnerability index and age group dummies are highly correlated with the interaction terms of financial assets and the same age group dummies. To solve the multi-collinearity problem, I include the financial vulnerability index rather than financial vulnerability index and age group dummy interaction terms in equation (5.12). It dramatically reduces the multi-collinearity problem. The coefficients of financial vulnerability index in Table 5.5 are positive and significant at around 6 percent for term life insurance and total life insurance respectively but not for net amount at risk. It is not surprising that the coefficients of financial vulnerability index is less significant and have smaller magnitude than those shown in Table 5.2, 5.3 and 5.4. The household will be less vulnerable to the death of breadwinner if it has more financial assets. Therefore, it will buy less life insurance.

The education levels of the respondent in the three tobit models in Table 5.5 are all positive and statistically significant, consistent with Burnett and Palmer (1984). It suggests that a more educated household has a greater likelihood of understanding the need for insurance. The results also provide some evidence that life insurance demand is related to the bequest motive for the term life insurance regression and the sum of term life and net amount at risk regression. If a couple desires to leave an estate, the evidence suggests a positive relationship between a bequest and the demand for total life insurance although its coefficient is not statistically significant.

I also note the negative relationship between term and whole life insurance in the demand equation. Term life insurance is negatively related to cash value and net amount at risk respectively of whole life insurance which means that whole life insurance is a substitute for term life insurance. Another finding is that labor income of both spouses in most cases is positively related to the net amount at risk, the term life and the total insurance demand in my tobit models. Foreseeable major financial obligations expected to be met in the near future such as educational expenses, health care costs and so forth are also positively and significantly related to the net amount at risk, the term life and the total life insurance demand. In this sense, people tend to use life insurance to manage their current or short-term obligation.[11]

[11]I also explore the impact of the baby boom cohort on the life insurance demand. The Baby Boom generation refers to the cohort born between 1946 and 1964. Contrary to the finding of Chen et al. (2001), I do not find a significant difference of the baby boom cohort's life insurance purchasing behavior from earlier or later counterparts. Since controlling for whether a householder belongs to the baby boom cohort does not improve my regression results, I do not include it in my regression model.

Table 5.5: Life Cycle Relationship: Life Insurance and Household's Portfolio in 2001

Independent Variables	Net Amount at Risk		Term Life		NAR + Term Life	
	OLS	TOBIT	OLS	TOBIT	OLS	TOBIT
	Estimate	M. E.[a]	Estimate	M. E.[a]	Estimate	M. E.[a]
Financial vulnerability index	-0.1171	-0.1869	1.8974**	1.9318*	1.5160**	1.5686*
	(0.838)	(0.213)	(0.871)	(1.021)	(0.713)	(0.838)
Age 20 - 34 (group 1)	-2.4542***	-4.1447***	-16.5513***	-24.6087***	-10.2308***	-15.9931***
	(0.914)	(0.951)	(2.213)	(2.755)	(1.808)	(2.247)
Age 35 - 49 (group 2)	-2.3969***	-3.0802***	-13.5899***	-20.2894***	-7.7396***	-12.1368***
	(0.879)	(0.721)	(2.138)	(2.625)	(1.747)	(2.137)
Age 50 - 64 (group 3)	-3.5604***	-3.9223***	-9.8748***	-15.4132***	-5.1059**	-8.4841***
	(1.055)	(0.831)	(2.587)	(3.152)	(2.111)	(2.541)
Log(cash+cash equivalent) interacted	-0.0518	-0.0012	0.3025	0.4593*	0.3036*	0.4541**
with age group 1	(0.090)	(0.112)	(0.222)	(0.275)	(0.181)	(0.225)
Log(cash+cash equivalent) interacted	0.0232	-0.0146	-0.0271	-0.0299	0.2140	0.2718
with age group 2	(0.075)	(0.053)	(0.183)	(0.225)	(0.150)	(0.182)
Log(cash+cash equivalent) interacted	0.0932	0.0412	-0.1606	-0.2139	0.1469	0.1696
with age group 3	(0.098)	(0.071)	(0.241)	(0.294)	(0.197)	(0.237)
Log(stock) interacted with age group 1	0.1103*	0.1232**	0.1282	0.1286	0.1790	0.2088
	(0.065)	(0.052)	(0.159)	(0.186)	(0.130)	(0.153)
Log(stock) interacted with age group 2	-0.0380	-0.0208	-0.0645	-0.0846	-0.1049	-0.1298*
	(0.033)	(0.021)	(0.082)	(0.096)	(0.067)	(0.079)
Log(stock) interacted with age group 3	0.0652*	0.0281	0.1148	0.1288	0.0513	0.0501
	(0.039)	(0.024)	(0.096)	(0.114)	(0.079)	(0.093)
Log(bond) interacted with age group 1	-0.0291	-0.0161	0.0081	0.0054	0.1209	0.1462
	(0.088)	(0.063)	(0.217)	(0.256)	(0.178)	(0.209)
Log(bond) interacted with age group 2	0.0411	0.0393	-0.0361	-0.0392	-0.0133	-0.0142
	(0.039)	(0.024)	(0.097)	(0.114)	(0.079)	(0.093)
Log(bond) interacted with age group 3	-0.0136	-0.0126	-0.3304***	-0.3883***	-0.1378	-0.1529

Table 5.5: Life Cycle Relationship: Life Insurance and Household's Portfolio in 2001 **(Continued)**

Independent Variables	Net Amount at Risk		Term Life		NAR + Term Life	
	OLS	TOBIT	OLS	TOBIT	OLS	TOBIT
	Estimate	M. E.[a]	Estimate	M. E.[a]	Estimate	M. E.[a]
	(0.044)	(0.026)	(0.107)	(0.128)	(0.088)	(0.103)
Log(mutual fund) interacted with age group 1	-0.0623 (0.077)	-0.0218 (0.064)	0.1359 (0.189)	0.1165 (0.220)	0.1395 (0.154)	0.1432 (0.182)
Log(mutual fund) interacted with age group 2	-0.0096 (0.033)	-0.0121 (0.020)	0.0881 (0.081)	0.0819 (0.096)	0.0105 (0.067)	0.0020 (0.078)
Log(mutual fund) interacted with age group 3	-0.0229 (0.037)	-0.0193 (0.021)	0.0338 (0.090)	0.0328 (0.107)	0.0018 (0.073)	-0.0018 (0.086)
Log(annuity) interacted with age group 1	0.1131 (0.110)	0.0485 (0.074)	-0.1840 (0.271)	-0.2370 (0.324)	-0.0553 (0.222)	-0.0873 (0.263)
Log(annuity) interacted with age group 2	0.0949** (0.045)	0.0229 (0.024)	-0.0580 (0.112)	-0.0662 (0.132)	-0.0930 (0.091)	-0.1204 (0.107)
Log(annuity) interacted with age group 3	0.0433 (0.040)	0.0152 (0.023)	-0.0166 (0.099)	-0.0202 (0.117)	0.0821 (0.081)	0.0878 (0.095)
Log(a household's individual retirement account) interacted with age group 1	0.0116 (0.054)	0.0649 (0.052)	0.5083*** (0.131)	0.6265*** (0.157)	0.5499*** (0.107)	0.6975*** (0.130)
Log(a household's individual retirement account) interacted with age group 2	-0.0272 (0.038)	-0.0290 (0.026)	0.4236*** (0.092)	0.5023*** (0.110)	0.2769*** (0.075)	0.3236*** (0.090)
Log(a household's individual retirement account) interacted with age group 3	-0.0607 (0.048)	-0.0389 (0.030)	0.2997** (0.117)	0.3527** (0.140)	0.1524 (0.095)	0.1770 (0.113)
Log(real estate) interacted with age group 1	-0.0277 (0.043)	-0.0596 (0.040)	0.1665 (0.104)	0.2027 (0.126)	0.0911 (0.086)	0.1095 (0.104)
Log(real estate) interacted with age group 2	0.0071 (0.041)	0.0026 (0.029)	0.2118** (0.101)	0.2596** (0.122)	0.2030** (0.082)	0.2474** (0.099)
Log(real estate) interacted with age group 3	0.0125 (0.070)	0.0067 (0.056)	0.0571 (0.171)	0.0613 (0.207)	0.0222 (0.140)	0.0134 (0.168)

Table 5.5: Life Cycle Relationship: Life Insurance and Household's Portfolio in 2001 **(Continued)**

Independent Variables	Net Amount at Risk		Term Life		NAR + Term Life	
	OLS	TOBIT	OLS	TOBIT	OLS	TOBIT
	Estimate	M. E.[a]	Estimate	M. E.[a]	Estimate	M. E.[a]
Log(sizable inheritance expected)	0.0076	0.0124	-0.0008	0.0030	-0.0082	-0.0056
	(0.022)	(0.014)	(0.055)	(0.065)	(0.045)	(0.053)
Log(total debt of the household)	0.0116	0.0141	0.2128***	0.2565***	0.1797***	0.2251***
	(0.029)	(0.019)	(0.071)	(0.086)	(0.058)	(0.069)
Log(other assets)	0.0424	0.0101	-0.1096	-0.1340	0.0738	0.0935
	(0.043)	(0.032)	(0.106)	(0.128)	(0.087)	(0.105)
Log(salary and wage of the respondent)	0.0547*	0.0396*	0.1442*	0.1692*	0.1415**	0.1634**
	(0.032)	(0.022)	(0.079)	(0.095)	(0.065)	(0.077)
Log(salary and wage of the spouse)	-0.0066	0.0119	0.1105*	0.1315*	0.0775	0.0972*
	(0.024)	(0.016)	(0.060)	(0.071)	(0.049)	(0.058)
Education level of the respondent	0.0918*	0.0635*	0.3190	0.3627**	0.3152***	0.3687***
	(0.051)	(0.037)	(0.124)	(0.150)	(0.101)	(0.122)
Education level of the spouse	-0.0375	-0.0214	0.1669	0.2199	0.0198	0.0276
	(0.049)	(0.035)	(0.121)	(0.147)	(0.099)	(0.119)
Desire to leave a bequest	0.0557	-0.1172	0.7805	0.8786	0.3750	0.3552
	(0.229)	(0.158)	(0.563)	(0.667)	(0.461)	(0.547)
Foreseeable major financial obligations	0.5854***	0.4081***	1.2215**	1.4633**	1.3311***	1.5945***
	(0.217)	(0.153)	(0.533)	(0.637)	(0.435)	(0.519)
Log(cash value)	0.9512***	0.4028***	-0.2262***	-0.2714***	0.1986***	0.2284***
	(0.011)	(0.039)	(0.061)	(0.075)	(0.021)	(0.025)
Log(face value of term life insurance)	-0.0066	0.0007				
	(0.010)	(0.006)				
Log(Net Amount at Risk)			-0.0397	-0.0410		
			(0.059)	(0.072)		
Adjusted R-square	0.8425		0.1450		0.2234	

Table 5.5: Life Cycle Relationship: Life Insurance and Household's Portfolio in 2001 **(Continued)**

Independent Variables	Net Amount at Risk		Term Life		NAR + Term Life	
	OLS	TOBIT	OLS	TOBIT	OLS	TOBIT
	Estimate	M. E.[a]	Estimate	M. E.[a]	Estimate	M. E.[a]
Log-likelihood		-2064.82		-5387.18		-5769.86

Number of observations: 1,791; Standard errors are presented below the estimated coefficients;

[a] Marginal effects (M. E.) of tobit model are computed at the mean of Xs;

*** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level.

5.5 Robustness Check: Valuing Non-monetary Contribution

One may argue that the non-monetary contribution of a spouse who stays at home should be considered as income as a family will also suffer a financial loss if the spouse were to die. To impute the value of household services I divide the sample into two parts. It is interesting to examine whether the positive and significant relationship between financial vulnerability index and insurance holdings I find in Section 5.4 is driven by families' different choices between earning two incomes and earning one income.

Valuing in monetary terms the time spent on productive household activities by using shadow and/or market prices and then adding this value to money income measure allow me to examine whether the households take into account housework value when they purchase life insurance. I impute housework value by using selectivity correction method of Heckman (1979). Heckman (1979)'s two-step estimation procedure including both a selection equation and regression equation:

$$\begin{aligned}
&\text{Selection equation:} && (5.13)\\
&z_s^* = \gamma_s' w_s + u_s,\ z_s = 1 \text{ if } z_s^* > 0 &&\\
&\qquad z_s = 0 \text{ otherwise} &&\\
&\qquad \text{Prob}(z_s = 1) = \Phi(\gamma_s' w_s) \text{ and } \text{Prob}(z_s = 0) = 1 - \Phi(\gamma_s' w_s) &&\\
&\qquad \text{where } s = \text{wife or hus.} &&\\
&\text{Regression equation:} && (5.14)\\
&\qquad \text{Log}(Y_s) = \beta_s' X_s + \varepsilon_s \text{ observed only if } z_s = 1 &&\\
&\qquad (u_s, \varepsilon_s) \sim \text{ bivariate normal } [0, 0, 1, \sigma_{s,\varepsilon}, \rho_s]. &&
\end{aligned}$$

The selection variable z_s^* is not observed. I only observe only whether a spouse is working or not. If the spouse is working, labor income Y_s is observed and I assign a working indicator $z_s = 1$. I first estimate the probit selection equation to obtain estimates of γ_s. Then I calculate $\hat{\lambda}_s = \phi(\hat{\gamma}_s' w_s)/\Phi(\hat{\gamma}_s' w_s)$ for each observation in the selected sample. Next I estimate β_s and $\beta_{s,\lambda} = \rho_s \sigma_{s,\varepsilon}$ by least squares regression of Log of Y_s on X_s and $\hat{\lambda}_s$.

Log of labor income is the dependent variable in the regression equation which is determined by the number of children, education and age. The results of the Heckman two-step estimation are shown in Table 5.6. As the number of children increases, the probability of the respondent[12] working outside increases while that of the spouse decreases. Higher education is associated with the higher probability of working outside and the higher salary for both respondents and spouses.

[12] In most cases the respondent is the husband.

Table 5.6: Estimation Results of the Heckman Model

Regression Equation	Log(Labor Income)	
Independent Variables	Respondent	Spouse
Intercept	8.1039***	9.2257***
	(0.274)	(0.396)
Number of Dependent Children	0.0527	-0.0034
	(0.034)	(0.049)
Education level of the respondent	0.1245***	
	(0.016)	
Age of the respondent	0.0282***	
	(0.004)	
Education level of the spouse		0.0527**
		(0.023)
Age of the spouse		0.0118**
		(0.006)
Adjusted R-square	0.1279	0.1183

Selection Equation	Working Indicator	
Independent Variables	Respondent	Spouse
Intercept	6.2064	5.6874
	(43.358)	(31.847)
Number of Dependent Children	0.0356*	-0.1243***
	(0.020)	(0.014)
Education level of the respondent	0.0427***	
	(0.009)	
Age of the respondent	-0.0224***	
	(0.002)	
Spouse working indicator	-4.8848	
	(43.358)	
Education level of the spouse		0.0629***
		(0.006)
Age of the spouse		-0.0207***
		(0.002)
Respondent working indicator		-5.1709
		(31.846)
λ	-7.5471***	-6.8699***
	(0.274)	(0.282)
Log-likelihood	-1990.90	-3777.74

Number of observations: 6,755;
Standard errors are presented below the estimated coefficients;
*** Significant at 1% level; ** 5% level; * 10% level.

If the observed labor income is less than the imputed value $\hat{Y}_s$, I set the labor income equal to the imputed value $\hat{Y}_s$ and then re-estimate the tobit regressions on life insurance demand shown in equations (5.10), (5.11) and (5.12). Table 5.7 shows the results of life cycle relationship between the sum of net amount at risk and term life insurance and financial vulnerability index with imputed housework value. The regression results with the net amount at risk or the term life insurance as the dependent variable adjusting for imputed housework value have the same sign and significance as those in Table 5.2 and Table 5.3 but not reported here. Based on equation (5.12), Table 5.8 shows the life cycle relationship between life insurance and household's portfolio with imputed housework value in 2001.

Table 5.7: Life Cycle Relationship: Sum of Net Amount at Risk and Term Life Insurance and Financial Vulnerability Index with Imputed Housework Value

	1992	1995	1998	2001	1992	1995	1998	2001
Independent Variables	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]	M. E.[a]
Intercept	-9.62***	-17.83***	-18.56***	-21.30***				
	(3.63)	(3.61)	(4.32)	(4.27)				
Financial vulnerability index	3.19***	2.89***	2.16**	3.74***				
	(0.77)	(0.81)	(0.89)	(0.84)				
Average age of couple	0.76***	1.05***	1.04***	1.11***				
	(0.18)	(0.18)	(0.21)	(0.21)				
(Average age of couple)^2	-0.01***	-0.01***	-0.01***	-0.01***				
	(0.00)	(0.00)	(0.00)	(0.00)				
Age 20 - 34 (group 1)					6.18***	3.41***	2.90***	1.56**
					(0.57)	(0.62)	(0.73)	(0.74)
Age 35 - 49 (group 2)					8.26***	7.44***	6.02***	5.45***
					(0.45)	(0.47)	(0.53)	(0.51)
Age 50 - 64 (group 3)					8.47***	8.06***	6.91***	5.59***
					(0.60)	(0.60)	(0.66)	(0.65)
Financial vulnerability index					2.97	6.94**	0.78	3.57
interacted with age group 1					(2.14)	(3.00)	(3.15)	(3.21)
Financial vulnerability index					3.13***	3.00***	3.53***	4.24***
interacted with age group 2					(1.09)	(1.11)	(1.21)	(1.10)
Financial vulnerability index					4.24***	3.23**	1.45	4.04***
interacted with age group 3					(1.24)	(1.31)	(1.41)	(1.42)
Number of observations	1496	1745	1723	1791	1496	1745	1723	1791
Log-likelihood	-4990	-5827	-5766	-5959	-4998	-5841	-5773	-5969

Standard errors are presented below the estimated coefficients; [a] Marginal effects (M. E.) of tobit model are computed at the mean of Xs; *** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level.

Table 5.8: Life Cycle Relationship: Life Insurance and Household's Portfolio with Imputed Housework Value in 2001

Independent Variables	Net Amount at Risk		Term Life		NAR + Term Life	
	OLS	TOBIT	OLS	TOBIT	OLS	TOBIT
	Estimate	M. E.[a]	Estimate	M. E.[a]	Estimate	M. E.[a]
Financial vulnerability index	-0.0846	-0.1700	1.8661**	1.8791*	1.4853**	1.5159*
	(0.873)	(0.220)	(0.908)	(1.064)	(0.743)	(0.873)
Age 20 - 34 (group 1)	-2.4519***	-4.1389***	-16.6269***	-24.6833***	-10.2903***	-16.0543***
	(0.914)	(0.951)	(2.213)	(2.756)	(1.808)	(2.248)
Age 35 - 49 (group 2)	-2.3877***	-3.0667***	-13.7938***	-20.4977***	-7.9014***	-12.3035***
	(0.880)	(0.721)	(2.140)	(2.626)	(1.748)	(2.138)
Age 50 - 64 (group 3)	-3.5456***	-3.9012***	-10.1591***	-15.7022***	-5.3316**	-8.7145***
	(1.055)	(0.830)	(2.587)	(3.152)	(2.111)	(2.541)
Log(cash+cash equivalent) interacted	-0.0521	-0.0013	0.3034	0.4607*	0.3044*	0.4552**
with age group 1	(0.090)	(0.112)	(0.222)	(0.275)	(0.181)	(0.225)
Log(cash+cash equivalent) interacted	0.0223	-0.0151	-0.0172	-0.0195	0.2219	0.2801
with age group 2	(0.075)	(0.053)	(0.183)	(0.225)	(0.150)	(0.182)
Log(cash+cash equivalent) interacted	0.0926	0.0405	-0.1554	-0.2076	0.1511	0.1741
with age group 3	(0.098)	(0.071)	(0.241)	(0.294)	(0.197)	(0.237)
Log(stock) interacted with age group 1	0.1099*	0.1229**	0.1301	0.1306	0.1806	0.2105
	(0.065)	(0.052)	(0.159)	(0.186)	(0.130)	(0.153)
Log(stock) interacted with age group 2	-0.0382	-0.0210	-0.0645	-0.0846	-0.1048	-0.1296*
	(0.033)	(0.021)	(0.082)	(0.096)	(0.067)	(0.079)
Log(stock) interacted with age group 3	0.0650*	0.0279	0.1162	0.1302	0.0524	0.0513
	(0.039)	(0.024)	(0.096)	(0.114)	(0.079)	(0.093)
Log(bond) interacted with age group 1	-0.0290	-0.0161	0.0076	0.0048	0.1205	0.1457
	(0.088)	(0.063)	(0.217)	(0.256)	(0.178)	(0.209)
Log(bond) interacted with age group 2	0.0413	0.0395	-0.0375	-0.0408	-0.0145	-0.0154
	(0.039)	(0.024)	(0.097)	(0.114)	(0.079)	(0.093)
Log(bond) interacted with age group 3	-0.0134	-0.0124	-0.3308***	-0.3885***	-0.1382	-0.1534

Table 5.8: Life Cycle Relationship: Life Insurance and Household's Portfolio with Imputed Housework Value in 2001 **(Continued)**

Independent Variables	Net Amount at Risk		Term Life		NAR + Term Life	
	OLS	TOBIT	OLS	TOBIT	OLS	TOBIT
	Estimate	M. E.[a]	Estimate	M. E.[a]	Estimate	M. E.[a]
	(0.044)	(0.026)	(0.107)	(0.128)	(0.088)	(0.103)
Log(mutual fund) interacted with age group 1	-0.0626	-0.0220	0.1369	0.1178	0.1405	0.1444
	(0.077)	(0.064)	(0.189)	(0.220)	(0.155)	(0.182)
Log(mutual fund) interacted with age group 2	-0.0098	-0.0123	0.0892	0.0832	0.0115	0.0030
	(0.033)	(0.020)	(0.081)	(0.096)	(0.067)	(0.078)
Log(mutual fund) interacted with age group 3	-0.0229	-0.0192	0.0335	0.0324	0.0016	-0.0020
	(0.037)	(0.021)	(0.090)	(0.107)	(0.073)	(0.086)
Log(annuity) interacted with age group 1	0.1132	0.0489	-0.1837	-0.2366	-0.0551	-0.0871
	(0.110)	(0.074)	(0.271)	(0.324)	(0.222)	(0.263)
Log(annuity) interacted with age group 2	0.0947**	0.0228	-0.0572	-0.0654	-0.0924	-0.1197
	(0.045)	(0.024)	(0.112)	(0.132)	(0.091)	(0.107)
Log(annuity) interacted with age group 3	0.0433	0.0153	-0.0155	-0.0191	0.0829	0.0886
	(0.040)	(0.023)	(0.099)	(0.117)	(0.081)	(0.095)
Log(a household's individual retirement account)	0.0115	0.0646	0.5100***	0.6281***	0.5513***	0.6989***
interacted with age group 1	(0.054)	(0.052)	(0.131)	(0.157)	(0.107)	(0.130)
Log(a household's individual retirement account)	-0.0274	-0.0294	0.4255***	0.5046***	0.2784***	0.3254***
interacted with age group 2	(0.038)	(0.026)	(0.092)	(0.110)	(0.075)	(0.090)
Log(a household's individual retirement account)	-0.0609	-0.0394	0.3008***	0.3538**	0.1534	0.1780
interacted with age group 3	(0.048)	(0.030)	(0.117)	(0.140)	(0.095)	(0.113)
Log(real estate) interacted with age group 1	-0.0277	-0.0595	0.1657	0.2018	0.0905	0.1089
	(0.043)	(0.040)	(0.104)	(0.126)	(0.086)	(0.104)
Log(real estate) interacted with age group 2	0.0069	0.0023	0.2134**	0.2611**	0.2043**	0.2489**
	(0.041)	(0.029)	(0.101)	(0.122)	(0.082)	(0.099)
Log(real estate) interacted with age group 3	0.0118	0.0063	0.0673	0.0715	0.0304	0.0217
	(0.070)	(0.056)	(0.171)	(0.207)	(0.140)	(0.168)

Table 5.8: Life Cycle Relationship: Life Insurance and Household's Portfolio with Imputed Housework Value in 2001 **(Continued)**

Independent Variables	Net Amount at Risk		Term Life		NAR + Term Life	
	OLS	TOBIT	OLS	TOBIT	OLS	TOBIT
	Estimate	M. E.[a]	Estimate	M. E.[a]	Estimate	M. E.[a]
Log(sizable inheritance expected)	0.0076	0.0122	-0.0008	0.0029	-0.0081	-0.0055
	(0.022)	(0.014)	(0.055)	(0.065)	(0.045)	(0.053)
Log(total debt of the household)	0.0116	0.0141	0.2132***	0.2572***	0.1801***	0.2255***
	(0.029)	(0.019)	(0.071)	(0.086)	(0.058)	(0.069)
Log(other assets)	0.0422	0.0096	-0.1065	-0.1308	0.0762	0.0963
	(0.043)	(0.032)	(0.106)	(0.128)	(0.087)	(0.105)
Log(salary and wage of the respondent)	0.0550*	0.0398*	0.1418*	0.1662*	0.1396**	0.1611**
	(0.032)	(0.022)	(0.079)	(0.095)	(0.065)	(0.077)
Log(salary and wage of the spouse)	-0.0065	0.0113	0.1221**	0.1425*	0.0867*	0.1060*
	(0.025)	(0.017)	(0.062)	(0.074)	(0.051)	(0.060)
Education level of the respondent	0.0918*	0.0642*	0.3131	0.3572**	0.3105***	0.3642***
	(0.051)	(0.037)	(0.124)	(0.151)	(0.102)	(0.122)
Education level of the spouse	-0.0378	-0.0219	0.1700	0.2232	0.0222	0.0303
	(0.049)	(0.035)	(0.121)	(0.147)	(0.099)	(0.119)
Desire to leave a bequest	0.0542	-0.1187	0.7914	0.8904	0.3839	0.3645
	(0.229)	(0.158)	(0.563)	(0.667)	(0.460)	(0.547)
Foreseeable major financial obligations	0.5854***	0.4084***	1.2134**	1.4558**	1.3245***	1.5886***
	(0.217)	(0.153)	(0.533)	(0.637)	(0.436)	(0.519)
Log(cash value)	0.9512***	0.4029***	-0.2256***	-0.2709***	0.1989***	0.2287***
	(0.011)	(0.039)	(0.061)	(0.075)	(0.021)	(0.025)
Log(face value of term life insurance)	-0.0066	0.0007				
	(0.010)	(0.006)				
Log(Net Amount at Risk)			-0.0400	-0.0413		
			(0.059)	(0.072)		
Adjusted R-square	0.8425		0.1448		0.2231	

Table 5.8: Life Cycle Relationship: Life Insurance and Household's Portfolio with Imputed Housework Value in 2001 **(Continued)**

Independent Variables	Net Amount at Risk		Term Life		NAR + Term Life	
	OLS	TOBIT	OLS	TOBIT	OLS	TOBIT
	Estimate	M. E.[a]	Estimate	M. E.[a]	Estimate	M. E.[a]
Log-likelihood		-2064.90		-5387.41		-5770.10

Number of observations: 1,791; Standard errors are presented below the estimated coefficients;
[a] Marginal effects (M. E.) of tobit model are computed at the mean of Xs;
*** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level.

In general, my results are robust with and without imputation of housework value since I obtain similar estimates and significance. In addition, I notice an interesting pattern. Compared with Table 5.4 and Table 5.5, my regression results in Table 5.7 and 5.8 show that life insurance demand is consistently slightly less sensitive to financial vulnerability in both magnitude and statistical significance when I impute the housework value in monetary terms.[13] For example, the coefficient of financial vulnerability index in 2001 in Table 5.4 is 4.5764 while it decreases by 18 percent to 3.7363 in Table 5.7 with imputed housework value. Similarly, in the portfolio implication regression, the coefficient of financial vulnerability index in 2001 in Table 5.5 is 1.5686 while it decreases to 1.5159 in Table 5.8 with imputed housework value. Moreover, the coefficient of financial vulnerability index in Table 5.8 is slightly less significant at 8 percent compared to 6 percent in Table 5.5. It is not a surprising result.

There are two possible explanations: first, the households do not buy life insurance on the housekeepers. The traditional economic justification for life insurance is to protect a person's income on which others are dependent (?). If the households follow this suggestion, no life insurance will be purchased on the housekeepers' lives because other family members do not depend on their earning capacity to sustain current living standards.

Second, if the one-income households (or so-called traditional families) recognize the economic value of housework and impute the monetary terms to it, they will treat themselves as two-income families because the housekeepers also create "income" at home. In theory, two-income households are less vulnerable to the death of an earner than one-income households in which only one parent is in the labor force. So the life insurance demand of two-income families is less sensitive to the households' financial vulnerabilities. Although the above two explanations are alternative, they are not mutually exclusive explanations for my findings. It is possible that the households realize the housework value but do not purchase life insurance on the housekeepers. The first explanation seems to support the reduced significance in the

[13] I reach the same conclusion with the net amount at risk or the term life insurance as the dependent variable in different years.

coefficients of financial vulnerability index and the second explanation provides a support for their reduced magnitude.

5.6 Conclusions and Discussion

I find a relationship between a household's financial vulnerability and the demand for life insurance. Unlike Bernheim et al. (2001), I decompose the demand for life insurance into the demand for term life and whole life insurance and take into account the vulnerability to loss of labor income to both spouses. Further, I employ an index of financial vulnerability that has several important features. First, it is transparent in the sense that I do not rely upon a proprietary model to construct it. Second, it is easy to implement since it dramatically reduces the cost associated with compiling data and modeling households' financial vulnerabilities. Third, my index can study households' life-cycle protection by using different types of life insurance respectively. Fourth, it is close to the reality in the sense that the consumption to income ratio for each household in the index is obtained from the annual survey, *Consumer Expenditures*.

Bernheim et al. (2001) found that the correlation between life insurance demand and financial vulnerability is essentially zero throughout the entire life cycle (they did not distinguish between whole life insurance and term life insurance and they based the ESPlanner to decide the benchmark life insurance). While my result also does not consistently apply to whole life insurance, I do see a strong relationship between term and total insurance and financial vulnerability in my life cycle analysis. My finding of a positive relationship between a household's financial vulnerability and the term life insurance and total life insurance holdings respectively seems reasonable in the light of the theory. My empirical analysis shows that the more volatile the living standard a household will be, the more term or total life insurance it will purchase.

Another conclusion from my life cycle regression results is that older households tend to use less life insurance to protect a certain level of financial vulnerability than younger households. This may arise from the elder's avoidance of the higher price of life insurance or decreasing absolute risk aversion since a household generally accumulate more wealth as it gets older. I also present evidence of how individual households change their portfolio over the life cycle and how this relates to the demand for life insurance. I find limited positive (negative) relationship between individual retirement accounts, annuities and real estate (bonds) respectively with life insurance holdings for some age groups. Finally, I take into account the non-monetary contribution of family members. My results are robust no matter whether I impute the value of the housework or not.

A

Summary of Data

I collected data from the Society of Actuaries' Transactions Reports for each of the years for which there were data. I used reports for calendar years published for the years 1951, 1961, 1971, and each year from 1981 to 1992. The last report is based on 1992 experience. I understand that the Society of Actuaries is reviving its experience studies.

1951						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	335.70	11.00	1174.25	10.00	1509.95	21.00
60-64	12102.34	308.00	3847.76	57.00	15950.10	365.00
65-69	39871.68	1413.00	4602.89	91.00	44474.57	1504.00
70-74	17218.98	958.00	1737.57	63.00	18956.55	1021.00
75-79	5873.40	484.00	666.00	37.00	6539.40	521.00
80-84	1774.33	226.00	209.00	26.00	1983.33	252.00
85-89	374.08	68.00	51.25	8.00	425.33	76.00
90-94	47.42	15.00	7.00	2.00	54.42	17.00

1961						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	1,371.88	36.00	2,454.63	18.00	3,826.51	54.00
60-64	23,718.46	605.00	9,902.34	116.00	33,620.80	721.00
65-69	96,620.43	3,371.00	19,390.30	333.00	116,010.73	3,704.00
70-74	60,560.45	3,371.00	10,594.01	349.00	71,154.46	3,720.00
75-79	26,772.96	2,275.00	3,901.58	195.00	30,674.54	2,470.00
80-84	7,701.84	1,002.00	1,057.17	109.00	8,759.01	1,111.00
85-89	1,717.08	310.00	275.00	35.00	1,992.08	345.00
90-94	254.42	59.00	39.00	7.00	293.42	66.00

1971						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	3,611.23	85.00	3,574.90	26.00	7,186.13	111.00
60-64	33,806.66	791.00	18,521.74	177.00	52,328.40	968.00
65-69	120,227.85	4,022.00	41,802.04	595.00	162,029.89	4,617.00
70-74	93,795.47	4,955.00	28,542.94	746.00	122,338.41	5,701.00
75-79	63,066.93	5,269.00	16,284.46	747.00	79,351.39	6,016.00
80-84	28,166.41	3,113.00	6,815.79	510.00	34,982.20	3,623.00
85-89	8,022.23	1,315.00	1,699.37	213.00	9,721.60	1,528.00
90-94	1,328.05	338.00	251.95	51.00	1,580.00	389.00

1981						
	Male		Female		Total	
Attanined Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	26,599.21	440.00	11,124.59	99.00	37,723.80	539.00
60-64	82,756.29	1,568.00	32,978.18	347.00	115,734.47	1,915.00
65-69	185,232.93	4,924.00	73,727.06	1,003.00	258,959.99	5,927.00
70-74	157,276.45	6,571.00	68,210.94	1,397.00	225,487.39	7,968.00
75-79	97,763.34	6,189.00	42,614.73	1,347.00	140,378.07	7,536.00
80-84	48,755.90	4,727.00	20,588.86	1,093.00	69,344.76	5,820.00
85-89	19,601.58	2,719.00	7,936.75	681.00	27,538.33	3,400.00
90-94	4,980.49	990.00	2,087.62	294.00	7,068.11	1,284.00

Group annuity experience 1951, 1961, 1971 and 1981

1982						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	28,631.53	453.00	11,754.62	92.00	40,386.15	545.00
60-64	89,455.43	1,753.00	35,433.49	336.00	124,888.92	2,089.00
65-69	192,308.39	5,097.00	75,640.56	985.00	267,948.95	6,082.00
70-74	162,420.78	6,740.00	72,661.69	1,354.00	235,082.47	8,094.00
75-79	103,419.33	6,465.00	48,058.37	1,540.00	151,477.70	8,005.00
80-84	52,549.11	4,861.00	23,671.10	1,231.00	76,220.21	6,092.00
85-89	21,392.48	2,989.00	9,443.51	832.00	30,835.99	3,821.00
90-94	5,716.77	1,082.00	2,526.42	322.00	8,243.19	1,404.00
1983						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	33,163.65	510.00	13,783.18	117.00	46,946.83	627.00
60-64	98,632.53	1,868.00	41,665.68	435.00	140,298.21	2,303.00
65-69	195,074.64	5,153.00	79,663.64	1,103.00	274,738.28	6,256.00
70-74	170,348.65	6,995.00	72,621.93	1,511.00	242,970.58	8,506.00
75-79	107,213.60	6,964.00	48,482.16	1,613.00	155,695.76	8,577.00
80-84	57,936.04	5,399.00	24,237.52	1,388.00	82,173.56	6,787.00
85-89	22,035.27	3,111.00	9,528.77	895.00	31,564.04	4,006.00
90-94	6,136.86	1,218.00	2,725.40	373.00	8,862.26	1,591.00
1984						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	40,574.69	580.00	16,305.25	132.00	56,879.94	712.00
60-64	119,381.14	2,212.00	48,941.94	448.00	168,323.08	2,660.00
65-69	221,883.84	5,695.00	91,062.97	1,241.00	312,946.81	6,936.00
70-74	200,590.93	8,196.00	86,304.56	1,870.00	286,895.49	10,066.00
75-79	129,357.81	8,141.00	60,361.35	2,106.00	189,719.16	10,247.00
80-84	67,297.97	6,288.00	31,781.28	1,771.00	99,079.25	8,059.00
85-89	26,575.80	3,766.00	12,400.26	1,211.00	38,976.06	4,977.00
90-94	7,743.72	1,574.00	3,681.76	573.00	11,425.48	2,147.00
1985						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	43,299.71	656.00	17,016.15	146.00	60,315.86	802.00
60-64	123,040.09	2,386.00	50,603.92	565.00	173,644.01	2,951.00
65-69	223,999.93	6,226.00	93,571.37	1,368.00	317,571.30	7,594.00
70-74	207,718.42	9,000.00	90,306.94	2,050.00	298,025.36	11,050.00
75-79	137,102.94	9,186.00	65,194.85	2,426.00	202,297.79	11,612.00
80-84	71,953.72	7,141.00	35,412.31	2,137.00	107,366.03	9,278.00
85-89	28,655.87	4,287.00	14,095.45	1,437.00	42,751.32	5,724.00
90-94	8,411.94	1,812.00	4,179.97	671.00	12,591.91	2,483.00

Group annuity experience 1982 – 1985

1986						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	44,010.72	627.00	16,677.86	112.00	60,688.58	739.00
60-64	122,620.42	2,163.00	50,381.10	476.00	173,001.52	2,639.00
65-69	227,995.35	5,699.41	95,512.26	1,261.00	323,507.61	6,960.41
70-74	216,055.50	8,098.29	93,727.78	1,966.00	309,783.28	10,064.29
75-79	146,182.97	8,610.00	68,834.32	2,324.00	215,017.29	10,934.00
80-84	78,070.67	7,153.00	38,836.55	2,108.00	116,907.22	9,261.00
85-89	31,484.42	4,005.00	15,650.49	1,406.00	47,134.91	5,411.00
90-94	9,097.10	1,678.00	4,672.65	690.00	13,769.75	2,368.00

1987						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	47,303.94	598.00	17,781.62	134.00	65,085.56	732.00
60-64	129,028.29	2,138.00	53,226.99	533.00	182,255.28	2,671.00
65-69	238,848.85	5,773.00	101,240.19	1,356.00	340,089.04	7,129.00
70-74	223,665.17	8,714.00	98,442.35	2,054.00	322,107.52	10,768.00
75-79	157,461.29	9,443.00	74,752.64	2,525.00	232,213.93	11,968.00
80-84	83,820.45	7,671.00	43,600.05	2,452.00	127,420.50	10,123.00
85-89	34,094.97	4,590.00	18,036.28	1,677.00	52,131.25	6,267.00
90-94	9,836.78	1,921.00	5,395.54	825.00	15,232.32	2,746.00

1988						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	49,424.32	683.00	18,162.87	141.00	67,587.19	824.00
60-64	132,778.58	2,252.00	53,788.54	513.00	186,567.12	2,765.00
65-69	235,874.82	5,587.00	102,022.53	1,295.00	337,897.35	6,882.00
70-74	221,164.05	8,388.00	99,853.21	2,116.00	321,017.26	10,504.00
75-79	162,202.31	9,530.00	78,542.78	2,630.00	240,745.09	12,160.00
80-84	88,225.65	8,012.00	47,418.51	2,583.00	135,644.16	10,595.00
85-89	35,929.54	4,707.00	20,142.57	1,879.00	56,072.11	6,586.00
90-94	10,484.98	2,002.00	5,926.74	845.00	16,411.72	2,847.00

1989						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	45,167.60	580.00	19,788.90	138.00	64,956.50	718.00
60-64	120,348.84	2,008.00	53,312.98	488.00	173,661.82	2,496.00
65-69	201,223.57	4,827.00	94,345.49	1,235.00	295,569.06	6,062.00
70-74	180,723.00	6,748.00	88,016.87	1,829.00	268,739.87	8,577.00
75-79	134,297.88	7,852.00	70,107.48	2,357.00	204,405.36	10,209.00
80-84	72,524.22	6,606.00	41,921.07	2,353.00	114,445.29	8,959.00
85-89	29,672.14	3,992.00	18,031.93	1,628.00	47,704.07	5,620.00
90-94	8,245.34	1,704.00	5,114.09	820.00	13,359.43	2,524.00

Group annuity experience 1986 – 1989

1990						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	53,375.95	686.00	24,851.00	174.00	78,226.95	860.00
60-64	146,190.29	2,333.00	67,235.53	596.00	213,425.82	2,929.00
65-69	258,735.98	5,949.00	122,669.86	1,562.00	381,405.84	7,511.00
70-74	238,694.07	8,911.00	116,031.28	2,327.00	354,725.35	11,238.00
75-79	189,088.76	11,105.00	95,064.28	3,186.00	284,153.04	14,291.00
80-84	109,583.14	9,912.00	62,967.19	3,520.00	172,550.33	13,432.00
85-89	48,022.47	6,572.00	30,700.37	2,778.00	78,722.84	9,350.00
90-94	14,672.14	2,842.00	10,005.89	1,445.00	24,678.03	4,287.00

1991						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	50,731.54	661.00	22,245.01	158.00	72,976.55	819.00
60-64	137,582.08	2,383.00	60,722.23	543.00	198,304.31	2,926.00
65-69	240,820.91	5,774.00	114,994.74	1,557.00	355,815.65	7,331.00
70-74	230,909.08	8,685.00	115,825.34	2,433.00	346,734.42	11,118.00
75-79	188,317.23	10,961.00	96,727.27	3,360.00	285,044.50	14,321.00
80-84	112,587.59	10,048.00	66,245.62	3,791.00	178,833.21	13,839.00
85-89	48,883.89	6,713.00	33,022.70	2,996.00	81,906.59	9,709.00
90-94	15,033.98	2,901.00	10,909.55	1,624.00	25,943.53	4,525.00

1992						
	Male		Female		Total	
Attained Age	Exposure	Deaths	Exposure	Deaths	Exposure	Deaths
55-59	47,790.52	689.00	20,925.44	156.00	68,715.96	845.00
60-64	122,033.83	2,143.00	55,616.52	466.00	177,650.35	2,609.00
65-69	216,153.60	5,124.00	107,068.38	1,429.00	323,221.98	6,553.00
70-74	212,415.17	7,526.00	111,099.67	2,260.00	323,514.84	9,786.00
75-79	173,061.53	9,440.00	91,863.84	3,044.00	264,925.37	12,484.00
80-84	106,152.91	9,177.00	63,719.81	3,349.00	169,872.72	12,526.00
85-89	47,214.93	6,190.00	33,278.32	2,984.00	80,493.25	9,174.00
90-94	15,059.41	2,859.00	11,268.86	1,634.00	26,328.27	4,493.00

Group annuity experience 1990 – 1992

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