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Output Dynamics, and
Balanced Budget Fiscal Rules**

**Pedro R. D. Bom
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GeorgiaState
University

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OF POLICY STUDIES

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Public Infrastructure Investment, Output Dynamics, and Balanced Budget Fiscal Rules*

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Abstract

We study the dynamic output and welfare effects of public infrastructure investment under a balanced budget fiscal rule, using an overlapping generations model of a small open economy. The government finances public investment by employing distortionary labor taxes. We find a negative short-run output multiplier, which (in absolute terms) exceeds the positive long-run output multiplier. In contrast to conventional results regarding public investment shocks, we obtain dampened cycles in output and the labor tax rate. The cyclical dynamics are induced by the interaction of households' finite life spans, the wealth effect on labor supply, and the balanced budget fiscal rule. Finally, we show that, for a plausible calibration of our model, households' lifetime welfare improves.

JEL codes: E62, F41, H54

Keywords: Infrastructure capital, public investment, distortionary taxation, fiscal policy, Yaari-Blanchard overlapping generations

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1 Introduction

Many governments of industrialized nations have resorted to fiscal stimulus packages to help weather the current global economic crisis. Public infrastructure investment—which is narrowly defined to include highways, airports, bridges, railways, sewerage and water systems, and dams other flood control structures (Bom and Ligthart, 2008)—were a key component of the fiscal stimulus measures. Without any offsetting tax and expenditure measures, more public investment boosts public spending, which in turn causes the short-run fiscal balance to deteriorate. In view of rising fiscal deficits in various OECD countries, some governments have been discussing whether to put legal bounds on their annual budget balance. The debate in the United States, for example, has focused on balanced budget amendments for the federal government.¹ More recently, in Europe, the political leaders of France and Germany called for all eurozone nations to enact constitutional amendments requiring balanced budgets.² Can public investment be effective in stimulating output and in improving welfare if the government has to adhere to a balanced budget rule? What do the transitional dynamics induced by a public investment impulse look like? The present paper addresses these questions.

Most contributions on the dynamic macroeconomic effects of public investment employ an infinitely-lived representative agent framework for a closed economy without a leisure-labor choice. Baxter and King (1993) and Turnovsky and Fisher (1995), however, endogenize labor supply, but do not discuss second-best welfare effects. Other contributions explicitly focus on the growth effects of public capital by assuming constant returns to scale in reproducible factors of production. Key contributions in this area are those by Barro (1990) and Glomm and Ravikumar (1994, 1997). The theoretical literature has not yet paid much attention to the output dynamics of public investment when households are finitely lived. Such a specification does not only provide a realistic description of the household sector, but is also instrumental in arriving at an endogenously determined (non-hysteretic) steady state in a small open

¹See Poterba (1995) for a background to this discussion. Note that the majority of the states are required to balance their current budgets at the end of the fiscal year, whereas some states balance their budgets on a two-year cycle (cf. Poterba and Rueben, 2001). States can borrow, however, for capital account purposes.

²Eurozone countries have adopted the euro and have signed on to the Stability and Growth Pact, which prescribes a ceiling on annual budget deficits of 3 percent of Gross Domestic Product (GDP).

economy context.³ Heijdra and Meijdam (2002) and Bom, Heijdra, and Ligthart (2010) employ models of finitely-lived households, but assume public investment to be financed by lump-sum taxes.⁴ In practice, countries do not have access to lump-sum taxes and fund their spending by distortionary taxes. In fact, labor income taxes account for about one third of total tax revenues and almost 10 percent of GDP in OECD countries (OECD, 2010). Labor income taxes affect households' consumption-leisure tradeoff and therefore have important labor market and welfare effects. Our paper investigates how labor market distortions interact with the positive spillovers of public investment. In view of this labor market and welfare focus, it is pivotal to provide a realistic description of households' preferences.

So far, the public capital literature has employed a rather restrictive specification of households' preferences and therefore has not come to grips with the labor dynamics of public investment. Heijdra and Meijdam (2002) ignore the household's labor-leisure choice by assuming exogenous labor supply. Heijdra, Van der Horst, and Meijdam (2002) endogenize labor supply, but assume Greenwood, Hercowitz, and Huffman (1988) preferences, which do not feature a wealth effect on labor supply. Bom, Heijdra, and Ligthart (2010) in turn employ a Cobb-Douglas utility function. In this paper, we employ a more general preference specification, that is, the constant elasticity of substitution (CES) utility function, which allows us to separate the *intra*temporal substitution effect on labor supply from the *inter*temporal substitution effect on labor supply. This distinction is important in view of the emphasis the Real Business Cycle (RBC) literature has put on the intertemporal labor supply effect for shock propagation (cf. Prescott, 2006). More important, recent empirical evidence (cf. Kimball and Shapiro, 2008) shows that the size of the intertemporal substitution effect in labor supply is non-negligible.

We develop a dynamic macroeconomic model of a small open economy that includes a public capital spillover on the production side. On the household side, we build a labor-leisure tradeoff into the Yaari (1965)-Blanchard (1985) framework of finitely-lived households. The

³Small open economy models of the Ramsey type yield a hysteretic steady state, reflecting the requirement that the rate of interest should equal the pure rate of time preference for a meaningful steady state to exist.

⁴A notable exception is the unpublished paper by Heijdra, Van der Horst, and Meijdam (2002), who employ a very restrictive preference specification based on Greenwood, Hercowitz, and Huffman (1988). See the discussion below.

government adheres to a balanced budget fiscal policy rule by employing distortionary labor taxes to finance public investment. To avoid trivial capital dynamics, we postulate adjustment costs of both private and public investment. In line with the literature on open economy macroeconomics, there is an internationally traded bond, which guarantees that households can use the current account of the balance of payments to smooth private consumption. Although a large number of our key results can be derived analytically, we provide a numerical simulation based on plausible parameters for a typical small open economy in the OECD area.

We find that a balanced budget increase in public investment induces dampened cycles in output and other key macroeconomic variables, whereas existing public investment studies obtain monotonic impulse responses. The dampened cycles arise from the interaction of households' finite planning horizons, the wealth effect on labor supply, and the government's balanced budget rule. The non-monotonic transition paths do not depend on the presence of the public capital externality. However, in a framework of infinitely-lived households the cycles disappear, owing to the absence of a wealth effect on labor supply during transition. We show that private investment, employment, and output fall in the short run, reflecting the reduction in labor supply caused by distortionary labor taxes. However, more public investment increases long-run output. In the benchmark case, we find an output multiplier of 2.25, which falls short of the value of 2.71 found in the lump-sum tax financing case (cf. Bom, Heijdra, and Ligthart, 2010). In the long run, employment increases as long as the elasticity of substitution between consumption and leisure is larger than unity. On the one hand, this positive employment effect reinforces the long-run output effect. On the other hand, the higher intratemporal elasticity of labor supply increases labor market distortions and exacerbates the short-run output contraction. Finally, our numerical analysis reveals that a balanced budget public investment impulse improves households' lifetime welfare in the benchmark calibration. This result suggests that public investment should be encouraged even if labor tax financing is distortionary.

The paper proceeds as follows. Section 2 sets out the dynamic macroeconomic framework for a small open economy. Section 3 analyzes the steady state and its stability and presents a simple graphical framework. Section 4 studies the long-run macroeconomic and welfare

effects of a balanced budget public investment impulse. Section 5 analyzes numerically the transitional dynamics and long-run effects of a public investment shock. Finally, Section 6 concludes the paper.

2 The Model

This section develops a micro-founded dynamic macroeconomic model for a typical industrialized small open economy. Subsequently, it discusses the behavior of individual households, aggregate households, firms, and the government.

2.1 Individual Households

The economy is inhabited by finitely-lived households, who face a constant probability of death equal to their rate of birth (denoted by β). Because the population size is constant, we can normalize it to unity. There are infinitely many disconnected generations, reflecting the absence of bequests. Expected lifetime utility at time t of a household born at time $v \leq t$ is given by an additively time-separable utility function:

$$\Lambda(v, t) \equiv \int_t^\infty \ln U(v, \tau) e^{-(\alpha+\beta)(\tau-t)} d\tau, \quad \alpha > 0, \beta \geq 0, \quad (1)$$

where α is the pure rate of time preference and $U(v, t)$ represents a CES sub-utility index:

$$U(v, t) \equiv \left[\varepsilon_C C(v, t)^{\frac{\sigma_C-1}{\sigma_C}} + (1 - \varepsilon_C) [1 - L(v, t)]^{\frac{\sigma_C-1}{\sigma_C}} \right]^{\frac{\sigma_C}{\sigma_C-1}}, \quad 0 < \varepsilon_C < 1, \sigma_C \geq 0, \quad (2)$$

where ε_C is the consumption weight in utility, $C(v, t)$ denotes private consumption, $L(v, t)$ is hours of labor supplied,⁵ and σ_C is the elasticity of substitution between private consumption and leisure. By choosing a CES specification of sub-utility, we model nonseparability between consumption and labor and embed the Cobb-Douglas specification for $\sigma_C = 1$. Equation (2) introduces a wealth effect on labor supply; that is, labor effort depends on the intertemporal consumption-savings choice.

⁵The total time available to the household has been normalized to unity so that $1 - L(v, t)$ denotes leisure.

We define ‘full’ consumption as the market value of private consumption and leisure:

$$X(v, t) \equiv P(t)U(v, t) = C(v, t) + \bar{w}(t) [1 - L(v, t)], \quad (3)$$

where $P(t)$ is the utility-based consumer price index (see [11] below), $\bar{w}(t) \equiv w(t)[1 - t_L(t)]$ is the after-tax real wage rate, $w(t)$ represents the before-tax real wage rate, $t_L(t)$ is a proportional labor income tax, and r denotes the exogenously given world rate of interest. We use private consumption as numeraire commodity, whose price has been normalized to unity. The household’s flow budget constraint is:

$$\dot{A}(v, t) = (r + \beta)A(v, t) + \bar{w}(t) - X(v, t), \quad (4)$$

where $\dot{A}(v, t) \equiv dA(v, t)/dt$, with $A(v, t)$ denoting real financial wealth. In keeping with Blanchard (1985), households contract actuarially fair ‘reverse’ life insurance. While alive, households receive an effective rate of return $r + \beta$ on their financial wealth. In the event of death, the insurance company appropriates all the wealth of the household.

The representative household of cohort v , who is endowed with perfect foresight, maximizes lifetime utility (1)–(2) subject to its budget identity (4) and a no-Ponzi game solvency condition. We solve the household’s problem by two-stage budgeting. In the first stage, the household decides on its consumption over time, yielding the *individual* full consumption Euler equation:

$$\frac{\dot{X}(v, t)}{X(v, t)} = \frac{\dot{U}(v, t)}{U(v, t)} + \frac{\dot{P}(t)}{P(t)} = r - \alpha. \quad (5)$$

We study the case of a patient nation (i.e., $r > \alpha$), which generates rising individual consumption profiles. Equation (5) says that full consumption growth rises with the real rate of interest and falls with the pure rate of time preference. By integrating (4), we obtain full consumption as a constant proportion of the household’s wealth portfolio:

$$X(v, t) = (\alpha + \beta) [A(v, t) + H(v, t)], \quad (6)$$

where $H(v, t)$ denotes lifetime human wealth of vintage v at time t :

$$H(v, t) \equiv \int_t^\infty \bar{w}(\tau) e^{-(r+\beta)(\tau-t)} d\tau, \quad (7)$$

which equals the present discounted value of the current and future after-tax returns to labor.

In the second stage, the household allocates $C(v, t)$ and $1 - L(v, t)$ such that (2) is maximized subject to (3). Combining the first-order conditions gives rise to: $\frac{C(v, t)}{1-L(v, t)} = \left(\frac{\varepsilon_C}{1-\varepsilon_C}\right)^{\sigma_C} \bar{w}(t)^{\sigma_C}$. By substituting this optimality condition into (3), we obtain:

$$C(v, t) = [1 - \omega_N(t)]X(v, t), \quad (8)$$

$$\bar{w}(t)[1 - L(v, t)] = \omega_N(t)X(v, t), \quad (9)$$

where $\omega_N(t)$ is the (time-varying) share of leisure in full consumption:

$$\omega_N(t) \equiv (1 - \varepsilon_C)^{\sigma_C} \left(\frac{\bar{w}(t)}{P(t)}\right)^{1-\sigma_C}, \quad 0 < \omega_N(t) < 1. \quad (10)$$

Equations (8) and (9) relate private goods consumption and leisure consumption to the level of full consumption. Households supply more hours of labor if gross wages rise, the labor tax rate falls, the share of leisure in full consumption drops, or full consumption falls. By substituting (8) and (9) into (2), we obtain the utility-based consumer price index:

$$P(t) \equiv \begin{cases} [\varepsilon_C^{\sigma_C} + (1 - \varepsilon_C)^{\sigma_C} \bar{w}(t)^{1-\sigma_C}]^{\frac{1}{1-\sigma_C}} & \text{for } \sigma_C \neq 1 \\ \left(\frac{1}{\varepsilon_C}\right)^{\varepsilon_C} \left(\frac{\bar{w}(t)}{1-\varepsilon_C}\right)^{1-\varepsilon_C} & \text{for } \sigma_C = 1 \end{cases}. \quad (11)$$

2.2 Aggregate Households

The size of cohort v at time t is a fraction $\beta e^{\beta(v-t)}$ of the total population.⁶ Therefore, the relationship between aggregate full consumption and individual full consumption of each

⁶We assume large cohorts, so that frequencies and probabilities coincide by the law of large numbers.

individual household is:

$$X(t) = \int_{-\infty}^t X(v, t) \beta e^{\beta(v-t)} dv. \quad (12)$$

By aggregating (5) over all existing generations, we arrive at the modified Keynes-Ramsey (MKR) rule:

$$\frac{\dot{X}(t)}{X(t)} = r - \alpha - \beta(\alpha + \beta) \frac{A(t)}{X(t)} = \frac{\dot{X}(v, t)}{X(v, t)} - \beta \cdot \frac{X(t) - X(t, t)}{X(t)}. \quad (13)$$

The expression after the second equality sign says that aggregate full consumption *growth* equals individual full consumption growth (the first term) minus the ‘generational turnover effect’ (the second term), that is, the wealth redistribution caused by the passing away of generations. Intuitively, old generations have accumulated wealth over the course of their life, whereas new generations are born without financial wealth (i.e., $A(t, t) = 0$). Consequently, the full consumption *level* of new generations $X(t, t)$ falls short of the average full consumption level $X(t)$.

2.3 Firms

The representative firm hires $L(t)$ hours of labor and rents $K(t)$ units of capital services to produce homogeneous output $Y(t)$ according to a Cobb-Douglas technology:

$$Y(t) = K(t)^{\varepsilon_Y} L(t)^{1-\varepsilon_Y} K_G(t)^\eta, \quad 0 < \varepsilon_Y < 1, \eta \geq 0, \quad (14)$$

where ε_Y is the output elasticity of private capital, η is the output elasticity of public capital, and $K_G(t)$ denotes the public capital stock. The public capital stock is assumed to give rise to a positive production externality, which is measured by η .⁷ Heijdra and Meijdam (2002) and most other authors also employ a Cobb-Douglas technology, which implies that public capital augments the private factors of production in a Hicks-neutral fashion.⁸ The restriction

⁷The government cannot charge a user fee on the firm’s use of public capital.

⁸Bom, Heijdra, and Ligthart (2010) employ a more general CES technology to analyze the factor-augmenting role of public capital.

$0 < \eta + \varepsilon_Y < 1$ ensures diminishing returns with respect to private and public capital taken together, thus excluding endogenous growth.

To allow non-trivial capital dynamics, we model adjustment costs in private investment. Net capital formation is linked to gross investment $I(t)$ according to the following function:

$$\dot{K}(t) = \left[\Phi \left(\frac{I(t)}{K(t)} \right) - \delta \right] K(t), \quad \Phi(0) = 0, \quad \Phi'(\cdot) > 0, \quad \Phi''(\cdot) < 0, \quad (15)$$

where δ is the rate of depreciation of private capital and $\Phi(\cdot)$ is the installation cost function of private capital. The degree of physical capital mobility of private capital is given by $0 < \rho_A \equiv -\frac{I}{K} \frac{\Phi''(\cdot)}{\Phi'(\cdot)} \ll \infty$, where a small ρ_A characterizes a high degree of physical capital mobility.

The firm maximizes the net present value of its cash flow,

$$V(t) \equiv \int_t^\infty [Y(\tau) - w(\tau)L(\tau) - I(\tau)] e^{-r(\tau-t)} d\tau, \quad (16)$$

subject to the capital accumulation constraint (15) and the stock of public capital. Note that we have normalized the prices of final output and investment goods to unity. Solving the firm's optimization problem yields the following first-order conditions:

$$w(t) = (1 - \varepsilon_Y) \frac{Y(t)}{L(t)}, \quad (17)$$

$$1 = q(t) \Phi' \left(\frac{I(t)}{K(t)} \right), \quad (18)$$

$$\frac{\dot{q}(t)}{q(t)} + \varepsilon_Y \frac{Y(t)}{K(t)} = r + \delta - \left[\Phi \left(\frac{I(t)}{K(t)} \right) - \frac{I(t)}{K(t)} \Phi' \left(\frac{I(t)}{K(t)} \right) \right], \quad (19)$$

where $q(t)$ denotes Tobin's q , which is defined as the market value of the private capital stock relative to its replacement costs. Equation (17) describes a downward sloping labor demand relationship in the (w, L) space. Equation (18) represents the investment-capital ratio as a function of Tobin's q . Finally, equation (19) captures the evolution over time of Tobin's q , which shows that the return on investment in private capital (left-hand side)—consisting of the shadow capital gain/loss and the marginal product of private capital—should equal the

user cost of private capital (right-hand side).⁹

2.4 Government

The government invests $I_G(t)$ in infrastructure capital and consumes $C_G(t)$ goods. We study the case in which the government commits to a balanced budget at each instant of time by levying a proportional labor income tax:

$$I_G(t) + C_G(t) = t_L(t)w(t)L(t). \quad (20)$$

Just like firms, the government faces convex adjustment costs in gross investment. Public capital accumulates according to:

$$\dot{K}_G(t) = \left[\Phi_G \left(\frac{I_G(t)}{K_G(t)} \right) - \delta_G \right] K_G(t), \quad \Phi_G(0) = 0, \quad \Phi'_G(\cdot) > 0, \quad \Phi''_G(\cdot) < 0, \quad (21)$$

where δ_G is the rate of depreciation of public capital and $\Phi_G(\cdot)$ is the installation cost function of public capital. The parameter $0 < \chi_G \equiv \frac{I_G \Phi'_G(\cdot)}{K_G} \ll \infty$ represents the elasticity of the public capital installation cost function.

2.5 Foreign Sector and Market Equilibrium

Foreign financial capital $F(t)$ is perfectly mobile across borders. The change in net foreign assets is determined by the balance on the current account of the balance of payments:

$$\dot{F}(t) = rF(t) + Z(t), \quad (22)$$

where $rF(t)$ denotes the return on net foreign assets and $Z(t)$ are net exports.

The goods market, which does not feature any rigidities, clears at each instant of time, yielding $Y(t) = C(t) + C_G(t) + I(t) + I_G(t) + Z(t)$. Similarly, the labor market equilibrates

⁹Without adjustment costs, we have $\Phi(\cdot) = I(t)/K(t)$ and $\Phi'(\cdot) = 1$. Equation (18) then reduces to $q = 1$. In this case, $K(t)$ adjusts instantaneously to its steady-state level. Consequently, equation (19) reduces to $\varepsilon_Y \frac{Y(t)}{K(t)} = r + \delta$, which is the familiar expression for the rental rate derived in a static framework.

instantly via a fully flexible real gross wage. Asset market equilibrium is defined as:

$$A(t) = V(t) + F(t), \tag{23}$$

where $V(t) = q(t)K(t)$ denotes the firm's stock market value. Assets in the household's portfolio are assumed to be perfect substitutes. Initially, $A(0) = V(0) > 0$ because $F(0) = 0$ and $K(0) > 0$. Physical capital is thus fully domestically owned.

3 Solving the Model

We now turn to solving the model outlined in the previous section. Section 3.1 derives the reduced-form model, Section 3.2 analyzes numerically the model's steady state and stability, and Section 3.3 develops a simple graphical framework.

3.1 Deriving the Reduced-Form Model

We log-linearize the model around an initial steady state with $F(0) = 0$ (implying that the current account is initially balanced). The log-linearized equations are reported in Table A.1. A tilde ($\tilde{\cdot}$) denotes a relative change, that is, $\tilde{X}(t) \equiv dX(t)/X$, where X is the steady-state value of $X(t)$. Variables with a tilde and a dot represent the time rate of change relative to the initial steady state, that is, $\dot{\tilde{X}}(t) \equiv d\dot{X}(t)/X = \dot{X}(t)/X$. For financial assets and human capital, we use a slightly different notation: $\tilde{A}(t) \equiv rdA(t)/Y$ and $\dot{\tilde{A}}(t) \equiv rd\dot{A}(t)/Y$. Finally, for the labor tax rate we employ: $\tilde{t}_L(t) \equiv dt_L(t)/(1 - t_L)$.

The dynamic equations of the model can be reduced to a model in two predetermined variables (i.e., the private capital stock and financial assets) and two non-predetermined variables (i.e., Tobin's q and full consumption). By collecting relative changes of variables in the vector $\tilde{\mathbf{z}}(t) \equiv [\tilde{K}(t) \tilde{q}(t) \tilde{X}(t) \tilde{A}(t)]'$ and shock terms in the vector $\mathbf{\Gamma}(t) \equiv [0 \ \gamma_q(t) \ 0 \ \gamma_A(t)]'$, we write the reduced-form dynamic system as:

$$\dot{\tilde{\mathbf{z}}}(t) = \Delta\tilde{\mathbf{z}}(t) - \mathbf{\Gamma}(t), \tag{24}$$

where Δ is a 4×4 Jacobian matrix (see Appendix A.1).

Let us first focus on a number of special cases giving rise to characteristic roots that are real. The trivial case of exogenous labor supply yields a dynamic system that can be decomposed in two independent subsystems, that is, an investment subsystem $[\tilde{q}(t), \tilde{K}(t)]$ and a savings subsystem $[\tilde{X}(t), \tilde{A}(t)]$. The model is saddle-path stable; we obtain two positive and two negative real roots.¹⁰ If households have infinite life spans (i.e., $\beta = 0$), the generational turnover effect drops from (13). For a steady state to exist, the knife-edge condition $r = \alpha$ should hold, implying that the third row of Δ consists of zeros only. In that case, there is a zero root in full consumption, one negative real root, and two positive real roots. The model features a hysteretic steady state.

For the general case of endogenous labor supply, the dynamic system is non-recursive. The dynamic properties of the system depend crucially on ω_{LL} , σ_C , β , and t_L . The solution of the characteristic polynomial corresponding to (24) may potentially yield complex-valued roots. To get insight into the properties of the roots, we pursue a numerical analysis.

3.2 Solving the Model Numerically

This section investigates the model numerically based on plausible parameter values taken from the literature and data.

3.2.1 Parameter Values

We choose parameter values in such a way as to match the characteristics of a typical small open economy in the OECD area (Table 1). The time unit represents a year. We assume a probability of death β of 1.82 percent to reflect an average expected life span of 55 working years. The world rate of interest is fixed at 4 percent. We assume that both private and public capital depreciate at the rate of 10 percent. Following Baxter and King (1993), the ratio of public consumption to GDP (ω_G^C) is set to 20 percent. In addition, the ratio of public investment to GDP (ω_G^I) takes on a value of 5 percent, which is somewhat above the average for industrialized countries, but more closely in line with data for southern European member

¹⁰Bom and Ligthart (2011) provide the derivations.

states. Our quantitative results depend crucially on the size of the output elasticity of public capital η . Based on Bom and Ligthart's (2008) meta-analysis of estimated values of η , we employ $\eta = 0.08$. We perform a sensitivity analysis on this parameter later on.

Because our model features labor market distortions of public investment, its quantitative implications depend to a great extent on the size of the Frisch elasticity of labor supply $\bar{\omega}_{LL} \equiv \omega_{LL}[1 + (\sigma_C - 1)(1 - \omega_N)]$ (which also captures the intratemporal substitution elasticity of labor supply) and on the leisure-labor ratio $\omega_{LL} \equiv (1 - L)/L$ (which governs the intertemporal elasticity of labor supply).¹¹ Kimball and Shapiro (2008) claim that '[modest long-run elasticities of labor supply are] one of the best-documented regularities in economics' (p. 1). In our model, a zero long-run (uncompensated) elasticity of labor supply implies $\bar{\omega}_{LL} = \omega_{LL}$, which in turn requires a unitary elasticity of substitution between consumption and leisure (i.e., $\sigma_C = 1$).¹² Kimball and Shapiro (2008) report estimates of the Frisch elasticity of about one and refer to a number of papers finding smaller estimates. RBC models, on the other hand, typically require larger elasticities; Prescott (2006), for instance, assumes Frisch elasticities of at least two.¹³ However, Prescott (2006) claims that this feature of RBC models is not necessarily incompatible with the evidence at the micro level, as adjustments at the extensive margin generate larger elasticities at the aggregate level than at the individual level. In the baseline case, we assume $\bar{\omega}_{LL} = \omega_{LL} = 2$ and $\sigma_C = 1$. Later on, we investigate the sensitivity of our results to different values of ω_{LL} and σ_C .

The respective installation cost functions for private and public investment are:

$$\Phi\left(\frac{I}{K}\right) \equiv \bar{z} \ln \frac{\frac{I}{K} + \bar{z}}{\bar{z}}, \quad \Phi_G\left(\frac{I_G}{K_G}\right) \equiv \bar{z}_G \ln \frac{\frac{I_G}{K_G} + \bar{z}_G}{\bar{z}_G}, \quad (25)$$

where \bar{z} and \bar{z}_G are constants. From (25) and the definitions of ρ_A and χ_G , we derive $\rho_A = (I/K)/(I/K + \bar{z})$ and $\chi_G = (I_G/K_G)\bar{z}_G/(I_G/K_G + \bar{z}_G)$. Setting $\bar{z} = 0.532$ and using $\delta = 0.10$ yields $I/K = 0.11$ in the steady state. The latter together with \bar{z} implies steady-

¹¹The Frisch labor supply elasticity holds the marginal utility of wealth constant.

¹²Much of the microeconomic evidence points to a σ_C smaller than one (Pencavel, 1986), which is at odds with the implied elasticity of aggregate labor supply. However, RBC models are often calibrated with values of σ_C close to one. We follow the latter approach.

¹³Uhlig (2010) works with a Frisch elasticity of labor supply of unity.

state adjustment costs of about 0.2 percent of GDP. Similarly, choosing $\bar{z}_G = 0.532$ and using $\delta_G = 0.10$ gives rise to $I_G/K_G = 0.11$. In this way, we arrive at adjustment costs of similar size for public capital. These parameters imply $\rho_A = 0.171$ and $\chi_G = 0.091$.

Given the fixed rate of interest, our calibration yields rising individual consumption profiles, where α is used as a calibration parameter to arrive at $A = qK$.¹⁴ Once the parameters are set, all other information on the relevant macroeconomic ratios, initial tax rate, and technology and preference parameters can be derived. By setting the output share of private consumption to 0.55, we find a ratio of investment to output of 0.20. The implied output elasticity of private capital is 0.29, implying that the condition $\eta < 1 - \varepsilon_Y = 0.71$ is easily met. The implied ratio of output to private capital is 0.55, which is slightly lower than the value found by Cooley and Prescott (1995). For the public capital stock, we derive $Y/K_G = 2.20$, which is roughly in line with Kamps (2006), who finds a value of around 2. In keeping with the average for OECD countries, the balanced budget labor income tax rate is 0.35. The implied preference parameters are: $\omega_N = 0.63$ and $\varepsilon_C = 0.37$.

3.2.2 Roots and Stability

Panel (a) of Figure 1 analyzes model stability for various values of ω_{LL} and σ_C . The negatively sloped solid curve represents the upper bound on the parameter region that yields a stable solution. Provided ω_{LL} is not too large for a given σ_C , the model has a unique and locally saddle-point stable steady state. We find two negative roots and two positive roots that are potentially complex valued. In the stable complex case (in which case the roots feature two negative and two positive real parts), the analytical solution for the transition paths of the variables includes cosine and sine terms, which give rise to endogenously determined dampened oscillations in key variables (Bom and Ligthart, 2011). The dotted line demarcates the upper bound of the stable, non-cyclical region; it approaches the σ_C -axis only if $\sigma_C \rightarrow \infty$, whereas it intersects the vertical axis at the benchmark value of ω_{LL} . Point C ($\sigma_C = 1, \omega_{LL} = 2$) indicates the benchmark calibration, which lies within the stable, cyclical region. The solid line distinguishes the stable region with dampened cycles from the unstable region. To obtain

¹⁴For the special case of the infinite-horizon model, we set $r = \alpha$.

cyclical dynamics, a smaller value of σ_C needs to be compensated by a higher ω_{LL} .

Panel (b) of Figure 1 shows that, for the case of infinitely-lived households (i.e., $\beta = 0$) and small values of the intertemporal elasticity of labor supply, we never end up in the cyclical region, reflecting the real nature of the roots. For $\beta = 0$ and $\omega_{LL} = 2$, the infinite-horizon model is unstable. However, in the calibration point C ($\beta = 0.018, \omega_{LL} = 2$) we find stable, cyclical dynamics. The dotted line shows that for smaller values of β , a higher value of ω_{LL} is needed to take the economy into the stable region with dampened cycles. Conversely, the solid line indicates that for smaller values of β , a smaller value of ω_{LL} is needed to stay within the stable, cyclical zone.

Figure 2 studies stability for various combinations of t_L and σ_C . The dotted line represents the upper bound of the stable, non-cyclical region, whereas the lower solid line demarcates the lower bound of the unstable region. The dark grey region in between the dashed lines shows combinations of points on the negatively sloping segment of the Laffer curve [see (30) and (33) below]. Areas with light grey coloring are unstable. The white upper north-east corner in the figure—which is bounded by the solid and dashed lines—represents a stable, non-cyclical region. However, this zone does not yield economically meaningful parameter combinations. The dotted line shows that for smaller values of σ_C , a higher value of t_L is required to yield a stable, cyclical outcome. The calibration point C ($\sigma_C = 1, t_L = 0.35$) is located in the stable, cyclical region, where the economy operates on the upward sloping segment of the Laffer curve.

In sum, endogenous labor supply, finite planning horizons, and a sufficiently high initial labor tax rate are necessary to give rise to dampened cyclical dynamics. See Section 5 for an economic explanation. Because we consider a Cobb-Douglas production function, the characteristic roots do not depend on the size of the production externality. Intuitively, in the Cobb-Douglas case, a rise in the size of the spillover effect does not induce a direct substitution between private capital and labor.¹⁵ Once we introduce public debt to keep the labor tax rate constant—and thus relax the balanced budget rule—the cyclical dynamics disappear. However, this tax smoothing scenario takes us beyond the scope of the current

¹⁵If one considers a more general production function, such as a CES, public capital yields a direct substitution effect. See Bom, Heijdra, and Ligthart (2010) for an exposition of this case under lump-sum taxation.

paper.¹⁶

3.3 Graphical Framework

We develop a simple graphical apparatus that will help us in analyzing the transitional dynamics of a public investment impulse. More specifically, the framework describes the dynamic interaction between full consumption, financial assets, and the labor market.

Panel (a) of Figure 3 depicts equilibrium in the labor market (E_0) conditional on the private capital stock, the public capital stock, the labor tax rate, and full consumption. Equation (17) describes the labor demand curve (labeled L_0^d), which is a positive function of both private and public capital. The aggregate version of (9) yields the compensated or Frisch labor supply curve (labeled L_0^s), which depends negatively on full consumption and the labor tax rate. The slope of the labor supply curve assumes a positive (intra-temporal) substitution effect on labor supply (see Section 4 for a further discussion).

Panel (b) of Figure 3 displays the savings system—consisting of the variables $X(t)$ and $A(t)$ —conditional on the private capital stock. The MKR locus presents the modified Keynes-Ramsey (MKR) rule, which corresponds to (13) in steady state. The household budget identity (HBI) locus is given by the steady-state aggregate version of (4). The intersection of the MKR and HBI loci determines the (X_0, A_0) equilibrium.

4 Analytical Long-Run Effects of Public Investment

This section studies the long-run allocation and welfare effects of an unanticipated and permanent increase in public investment (i.e., $dI_G > 0$). We assume a second-best world in which the government has to resort to a distortionary labor income tax to balance its budget at each instant of time. The policy shock occurs at time $t = 0$ and the economy reaches a new steady state at $t \rightarrow \infty$. The policy change is unanticipated in the sense that it is simultaneously announced and implemented.

¹⁶The tax smoothing scenario requires public debt to be introduced into the analysis. The results and Matlab program are available upon request.

4.1 Capital and Labor Markets

The increase in public investment boosts—via the accumulation function of public capital—the long-run stock of public capital:

$$\frac{dK_G(\infty)}{dI_G} = \frac{1}{\Phi_G^{-1}(\delta_G)} > 0. \quad (26)$$

We note that (15), (21), and (18) imply that the I/K ratio, the I_G/K_G ratio, and Tobin's q are fixed in the long run. By using (19) it also follows that the marginal product of private capital is fixed. Hence, we find:

$$\frac{dK(\infty)}{dI_G} = \frac{\eta}{(1 - \varepsilon_Y)\bar{y}\omega_G^I} + \frac{K}{L} \frac{dL(\infty)}{dI_G}, \quad (27)$$

where $\bar{y} \equiv Y/K$ and we have made use of (26). Equation (27) shows a positive relationship between the long-run private capital multiplier and the long-run employment multiplier. The size of the intercept is positively affected by the size of the public capital externality and negatively by the public investment-to-GDP ratio.

By totally differentiating (19), while using (26) and (27), we find the long-run gross wage multiplier:

$$\frac{dw(\infty)}{dI_G} = \frac{Y_G}{L} \frac{1}{\Phi_G^{-1}(\delta_G)} = \frac{\eta}{L\omega_G^I} \geq 0. \quad (28)$$

The long-run wage multiplier is always positive as long as there is a public capital externality. Clearly, if $\eta = 0$, the gross wage rate is fixed by the world rate of interest.

As derived in Appendix A.2, the employment multiplier is given by:

$$\frac{dL(\infty)}{dI_G} = \frac{\bar{\sigma}_L}{\omega_G^I w} \left[\frac{\eta - \omega_G^I}{1 - t_L(1 + \bar{\sigma}_L)} \right], \quad (29)$$

where $\bar{\sigma}_L = \bar{\omega}_{LL} - \omega_{LL}$ denotes the uncompensated wage elasticity of labor supply. Using the

definition of $\bar{\omega}_{LL}$, we find:

$$\bar{\sigma}_L \equiv \omega_{LL}(\sigma_C - 1)(1 - \omega_N) \begin{cases} < 0 & \text{if } 0 < \sigma_C < 1 \\ = 0 & \text{if } \sigma_C = 1 \\ > 0 & \text{if } \sigma_C > 1 \end{cases} ,$$

for $\omega_{LL} > 0$. The employment multiplier is zero if $\bar{\sigma}_L = 0$, which is the case if the elasticity of substitution between private consumption and labor supply is unity (in which case the substitution effect of an after-tax wage change is exactly offset by the income effect) or the *intertemporal* labor supply effect is zero (i.e., $\omega_{LL} = 0$) or both. Panel (a) of Figure 1 shows that a rise in public investment shifts the labor demand curve to the right and moves the labor supply curve to the left, leaving the long-run level of employment unchanged (i.e., $L_0 = L_\infty$). The sign of the bracketed term in (29) depends on the chosen parametrization. The numerator of the expression is positive if public capital is sufficiently productive (i.e., $\eta > \omega_G^I$), where $\eta = \omega_G^I$ corresponds to the ‘golden rule’ of public investment in a first-best world (Fisher and Turnovsky, 1995, p. 771). The denominator is positive if σ_C is smaller than the following upper bound:

$$\sigma_C^U \equiv 1 + \frac{1}{\omega_{LL}(1 - \omega_N)\theta_L} = \frac{1 - t_L[1 - \omega_{LL}(1 - \omega_N)]}{t_L\omega_{LL}(1 - \omega_N)} > 1. \quad (30)$$

Assuming that $\eta > \omega_G^I$, we can consider three cases: (i) if $0 < \sigma_C < 1$, which represents the perverse case of a backward bending uncompensated labor supply curve, it follows that $dL(\infty)/dI_G < 0$; (ii) if $1 < \sigma_C < \sigma_C^U$, we find $dL(\infty)/dI_G > 0$; and (iii) if $\sigma_C > \sigma_C^U$, we arrive at $dL(\infty)/dI_G < 0$. For unproductive public capital ($\eta = 0$) in special case (ii), the employment multiplier is negative. Intuitively, private factors of production do not benefit from public capital spillovers, but are negatively affected by the distortionary labor tax. To foot the bill of the rise in public spending, the labor tax rate has to rise, which induces households to work less.

4.2 Labor Taxes and Output

Differentiating $T(t) \equiv t_L(t)w(t)L(t)$ with respect to I_G gives the slope of the long-run Laffer curve:

$$\frac{dT(\infty)}{dI_G} = wL \left[\frac{dt_L(\infty)}{dI_G} + t_L \left(\frac{dw(\infty)}{dI_G} \frac{1}{w} + \frac{dL(\infty)}{dI_G} \frac{1}{L} \right) \right]. \quad (31)$$

The first term between straight brackets represents the tax rate effect, whereas the second and third terms capture tax base effects (which only materialize for $t_L > 0$). If the term between brackets is positive, the long-run Laffer curve is upward sloping in the (T, t_L) -space. The labor tax rate is adjusted to keep the government budget balanced, implying that the left-hand side of (31) is set to zero. Using (29) and (28) into (31), imposing $dT(\infty)/dI_G = 0$, and rewriting gives the long-run effect of public investment on the labor tax rate:

$$\frac{dt_L(\infty)}{dI_G} = \frac{1 - t_L}{wL\omega_G^I} \left[\frac{\omega_G^I - \eta t_L(1 + \bar{\sigma}_L)}{1 - t_L(1 + \bar{\sigma}_L)} \right]. \quad (32)$$

If initial labor tax rates are zero, the term in square brackets in (32) is unambiguously positive, so that a rise in public investment increases the labor tax rate. Hence, the economy operates on the upward-sloping segment of the Laffer curve. If $t_L > 0$, the sign of the numerator is ambiguous. By setting this expression to zero, we can derive a lower bound on the elasticity of substitution between private consumption and leisure:

$$\sigma_C^L \equiv \frac{\omega_G^I - \eta t_L[1 - \omega_{LL}(1 - \omega_N)]}{\eta t_L \omega_{LL}(1 - \omega_N)} \begin{cases} < 1 & \text{if } \omega_G^I < \eta t_L \\ = 1 & \text{if } \omega_G^I = \eta t_L \\ > 1 & \text{if } \omega_G^I > \eta t_L \end{cases}. \quad (33)$$

If $t_L > 0$ and $\eta > \omega_G^I$, then $\sigma_C^L < \sigma_C^U$. In this case, the long-run labor tax multiplier (32) is positive if $\sigma_C < \sigma_C^L$ or $\sigma_C > \sigma_C^U$ and negative if $\sigma_C^L < \sigma_C < \sigma_C^U$. Note that the region $\sigma_C > \sigma_C^U$ is not very meaningful from an economic point of view. The threshold cases $\sigma_C = \sigma_C^L$ and $\sigma_C = \sigma_C^U$ give rise to horizontal and vertical long-run Laffer curves, respectively.

Having derived the long-run changes in all inputs, we can now derive the long-run output

multiplier. By totally differentiating (14), using (26)–(27), and (29), we get:

$$\frac{dY(\infty)}{dI_G} = \frac{1}{(1 - \varepsilon_Y)\omega_G^I} \left[\eta + \frac{\bar{\sigma}_L(\eta - \omega_G^I)}{1 - t_L(1 + \bar{\sigma}_L)} \right], \quad (34)$$

where the first term in brackets corresponds to the private capital effect and the second term describes the employment effect. If public capital is unproductive (i.e., $\eta = 0$), only the negative part of the employment effect remains, so that the output multiplier is also negative (provided $\bar{\sigma}_L > 0$). If household preferences are Cobb-Douglas (i.e., $\bar{\sigma}_L = 0$), the employment effect drops completely, implying that the long-run output effect is not affected by the size of the intertemporal labor supply elasticity.

4.3 Full Consumption, Net Foreign Assets, and Welfare

Plugging (28) and (29) into the differentiated household budget constraint gives the effect of public investment on full consumption:

$$\frac{dX(\infty)}{dI_G} = \frac{\omega_X}{\omega_A} \frac{rdA(\infty)}{dI_G} = \frac{\omega_X}{(1 - \varepsilon_Y)\omega_G^I} \left[\frac{\eta - \omega_G^I}{1 - t_L(1 + \bar{\sigma}_L)} \right], \quad (35)$$

where $\omega_A \equiv rA/Y$ denotes the output share of asset income and $\omega_X \equiv X/Y$ denotes the output share of full consumption. If $\eta > \omega_G^I$, the full consumption multiplier is positive provided that $\sigma_C < \sigma_C^U$ and negative if $\sigma_C > \sigma_C^U$.

Finally, to derive the effect on long-run foreign assets, we totally differentiate (23), while using (27) and (35) and noting that Tobin's q is at its initial value in the long run:

$$\frac{dF(\infty)}{dI_G} = -\frac{\omega_A}{r(1 - \varepsilon_Y)\omega_G^I} \left[\eta - \frac{(1 - \bar{\sigma}_L)(\eta - \omega_G^I)}{1 - t_L(1 + \bar{\sigma}_L)} \right], \quad (36)$$

where we have used that $A = qK$ in the initial steady state.

The long-run *instantaneous* welfare effect of public investment follows from totally differentiating $X(t) = P(t)U(t)$ with respect to I_G and using multipliers (28) and (35):

$$\frac{dU(\infty)}{dI_G} = \frac{(1 - t_L)[L\beta(\alpha + \beta) + (1 - L)r(r - \alpha)](\eta - \omega_G^I)}{PL\omega_G^I[1 - t_L(1 + \bar{\sigma}_L)][\beta(\alpha + \beta) - r(r - \alpha)]}, \quad (37)$$

which implies that public investment is welfare improving in the long run if and only if $\eta > \omega_G^I$, provided that $\sigma_C < \sigma_C^U$. Clearly, unproductive public investment (i.e., $\eta = 0$) decreases long-run welfare.

5 Quantitative Dynamic Effects of Public Investment

To quantify and visualize the dynamic macroeconomic effects of an unanticipated, permanent, and balanced budget increase in public investment, we perform a simulation analysis based on the parameter setting of Section 3.2.1. Section 5.1 illustrates the transitional dynamics and Section 5.2 presents numerical results on both the short-run and long-run effects.

5.1 Impulse Responses

In generating the impulse responses of a public investment shock, we use the analytical transition paths derived in Bom and Ligthart (2011), together with the steady-state log-linearized equations of Table A1. To accommodate differences in the adjustment speed of variables, we plot impulse response functions for 200 time periods. The public investment impulse amounts to $\tilde{I}_G = 0.1$ and occurs at time $t = 0$. Because the labor tax base changes over time, the labor tax rate is endogenously varied to keep the government budget balanced at each instant of time.

5.1.1 Allocation Effects in the Benchmark Case

Figure 4 shows the impulse responses for various values of the intertemporal labor supply elasticity; that is, ω_{LL} takes on values of 2.00 (solid line), 1.00 (dashed line), and 0 (dotted line). Let us first focus on the benchmark scenario of $\omega_{LL} = 2$. On impact, employment falls, the gross wage rate rises, and the labor tax rate increases. In terms of Panel (a) of Figure 3, the labor supply curve L_0^s shifts to the left to L_1^s , whereas the labor demand curve L_0^d remains unaffected, thereby pushing up the gross wage rate. Intuitively, the rise in the labor tax rate that is required to balance the government budget induces households to substitute toward more leisure consumption. However, the fall in wealth—which prompts households to work

harder and consume less private goods—alleviates the drop in employment. In Panel (b) of Figure 3, the economy moves along the dotted dynamic path to point E_1 . Given that private capital is a predetermined variable, the private capital-labor ratio rises. On impact, Tobin’s q jumps down—reflecting a fall in the (future) marginal product of private capital—thereby depressing private investment. Although both short-run domestic absorption and output fall, the latter dominates so that short-run net imports rise. It is important to note that the short-run drop in private consumption and private investment follows from the distortionary nature of labor taxes; indeed, the opposite result obtains if lump-sum tax financing is considered (cf. Bom, Heijdra, and Ligthart, 2010).

Shortly after the shock, the private capital stock starts falling, reflecting depressed private investment. Because labor and private capital are cooperative factors of production, the labor demand curve shifts to the left from L_1^d to L_2^d in Panel (a) of Figure 3. The private capital-labor ratio rises further, taking the economy from E_1 to E_2 via the dotted dynamic path. The rise in gross wages associated with the larger K/L -ratio increases full consumption; see the move from E_1 to E_2 along the dynamic path in Panel (b) of Figure 3. Tobin’s q eventually recovers, thereby increasing private investment and thus boosting private capital accumulation. Together with the continuous expansion of the stock of public capital, this accumulation of private capital causes employment to rise, which is represented by a shift of the labor demand curve from L_2^d to L_3^d . Consequently, output increases substantially. The labor tax base expands, allowing a reduction in the labor tax rate during periods 20 to 60. Because the after-tax return on working increases, households supply more labor; that is, the labor supply curve shifts to the right from L_2^s to L_3^s . The employment increment reaches its maximum in point E_3 , which roughly coincides with the peaks of private capital stock and output at about period 35. Panel (b) of Figure 3 reveals that financial assets and full consumption also increase as the economy moves from point E_2 to E_3 .

The economy enters into a new cycle in which the absolute increment in the capital stock and employment is smaller than in the previous cycle. Intuitively, the labor tax rate needs to rise to offset the fall in the labor tax base. However, to balance the public budget, the tax base falls by less than in the previous cycle. In Panel (a) of Figure 3, the labor supply curve

moves to the left to eventually—after going through a number of smaller oscillations—settle in the new steady state. Panel (b) of Figure 3 shows that full consumption and financial assets also spiral toward E_∞ . In the new steady state, employment is not affected by the public investment impulse, whereas output, the public capital stock, the private capital stock, private consumption, and the labor tax rate are larger than in the initial steady state.¹⁷ In addition, the country has accumulated foreign debt in the new equilibrium. Both gross and after-tax wages have risen, whereas Tobin’s q returns to its initial steady-state value.

The benchmark case of endogenous intertemporal labor supply shows dampened cyclical dynamics. Assuming the economy is within the stable region, a larger intertemporal labor supply elasticity increases the amplitude of the cycles. Ignoring the intertemporal margin of labor supply (i.e., $\omega_{LL} = 0$) yields monotonic transition paths for all the variables. Because all three cases presented in Figure 4 assume $\sigma_C = 1$, long-run employment is not affected by the public investment impulse. Nevertheless, long-run output rises, reflecting the increased stocks of private and public capital.

5.1.2 Other Specifications

Panels (a)–(c) of Figure 5 show the responses of output and private consumption to a permanent public investment impulse for the special cases of infinitely-lived households (i.e., $\beta = 0$), unproductive public spending (i.e., $\eta = 0$), and elastic substitution between private consumption and leisure (i.e., $\sigma_C > 1$). In each case, we allow the intertemporal elasticity of labor supply to assume the values $\omega_{LL} = 0$ (dotted line), $\omega_{LL} = 1$ (dashed line), and $\omega_{LL} = 1.75$ (solid line).¹⁸ Panel (a) shows that the cycles disappear in the infinite-horizon model.¹⁹ Because intergenerational spillovers are absent, all future costs and benefits of public investment accrue to the infinitely-lived representative agent, who adjusts full consumption once and for all at the time of the shock. The wealth effect triggers a negative response of

¹⁷The positive relationship between changes in private consumption and output is in line with evidence from vector autoregressive (VAR) models (cf. Perotti, 2004).

¹⁸Note that the benchmark value of the intertemporal elasticity of labor supply (i.e., $\omega_{LL} = 2$) gives rise to unstable dynamics in the infinite-horizon model (i.e., $\beta = 0$); see Panel (b) of Figure 1.

¹⁹In contrast to the findings of Schmitt-Grohe and Uribe (2003), the hysteretic and non-hysteretic model give rise to very different transitional dynamics. Schmitt-Grohe and Uribe (2003) employ a stochastic framework and speak of non-stationary and stationary models. Whether the model is stochastic or deterministic does not affect their key point.

labor supply—which in turn causes a temporary drop in employment and output—but only at impact. During transition, the wealth effect is switched off, thereby eliminating the cyclical responses of employment and output. Notice that the absence of cycles is independent of ω_{LL} .

Panel (b) depicts the case of unproductive public spending (i.e., $\eta = 0$). If labor supply is inelastic, output is insensitive to unproductive public spending. Private consumption, however, falls over time, reflecting the higher labor tax rate required to balance the government budget. If labor supply is elastic, unproductive public spending generates a small temporary fall in output, which returns to its initial steady state in the long run. In this case, private consumption drops at impact, but decreases only slightly over time to the new (lower) steady state. The dampened cycles remain as long as labor supply is sufficiently elastic, pointing to the role of distortionary labor taxes in generating non-monotonic dynamics.²⁰

Finally, Panel (c) considers a larger elasticity of substitution between leisure and private consumption (i.e., $\sigma_C = 1.25 > 1$). Values of σ_C larger than unity—which correspond to a positive uncompensated wage elasticity of labor supply—increase the amplitude of the output effect. Intuitively, the labor supply elasticity $\bar{\sigma}_L$ increases, yielding a more elastic labor supply response to a change in the tax rate. As a result, the labor tax base becomes more elastic too, which generates a larger tax base effect. To balance the government budget, the labor tax rate has to change by more than under a small uncompensated wage elasticity of labor supply. The assumption of $\omega_{LL} = 1.75$ gives rise to output dynamics qualitatively similar to the benchmark case, reflecting the fact that a larger σ_C substitutes for a lower ω_{LL} . In contrast to the benchmark case, however, the larger value of σ_C generates a positive short-run effect of public investment on private consumption.

5.1.3 Welfare Effects

Panel (a) of Figure 6 depicts the dynamic (instantaneous) welfare effects of a change in public investment for $\omega_{LL} = 2$ (solid line), $\omega_{LL} = 1$ (dashed line), and $\omega_{LL} = 0$ (dotted line). If the intertemporal labor supply elasticity is zero (i.e., $\omega_{LL} = 0$), the welfare profile is monotonically

²⁰Heijdra and Ligthart (2010) show that external economies of scale cause Schmitt-Grohe and Uribe's (2003) key result to break down. The cycles in their framework disappear once the effect of external economies of scale is switched off.

rising starting from zero at impact to a positive long-run value. For $\omega_{LL} > 0$, however, the dynamic welfare effects are non-monotonic, being positive in the short run and long run but negative in the medium run. This negative medium-run welfare loss increases with ω_{LL} , being especially pronounced for the case giving rise to dampened cyclical dynamics (i.e., $\omega_{LL} = 2$).

The question arises as to how the discounted sum of welfare gains/losses is affected by parameter changes. Panel (b) of Figure 6 shows, for various values of ω_{LL} , the second-best optimal public investment-to-GDP ratio, that is, the value of ω_G^I that maximizes the present discounted value of instantaneous welfare given the government's budget constraint. For $\omega_{LL} = 0$ —in which case labor taxes do not distort the labor market—the optimal level of public investment is around 7.5 percent of GDP. For $\omega_{LL} > 0$, the optimal share of public investment decreases to about 6 percent, slightly increasing with ω_{LL} up to about $\omega_{LL} = 2.2$, above which the dynamic system is no longer stable. The benchmark calibration point C lies within the area of welfare gains, suggesting that, from a welfare perspective, public investment should be increased to around 6 percent of GDP.

The welfare effects of public investment critically depend on the size of the public capital externality. Panel (c) displays the welfare effects for $\eta = 0.10$ (dotted-dashed line), $\eta = 0.08$ (solid line), $\eta = 0.05$ (dashed line), and $\eta = 0$ (dotted line). Again, the welfare profiles display dampened cycles. If public capital is unproductive, the welfare effects are always negative. Productive public investment generates welfare gains in the both the short and long run and welfare losses in the medium run. Note that larger values of η increase not only the positive short- and long-run welfare effects but also—due to the wealth effect on labor supply—the medium-run welfare losses. Nevertheless, as shown in Panel (d), the second-best optimal ω_G^I rises linearly with the size of the public capital spillover. Note that the slope of the line is below unity, showing that a given η sustains a smaller second-best optimal GDP share of public investment, reflecting the deadweight loss of labor tax financing.

5.2 Quantitative Short-Run and Long-Run Effects

Table 2 presents the short-run and long-run effects of the balanced budget public investment shock. In the benchmark case, the long-run output multiplier amounts to 2.25, which falls

naturally short of the value of 2.71 obtained by Bom, Heijdra, and Ligthart (2010), who assume a non-distortionary financing scenario. The positive output multiplier reflects the larger stocks of public and private capital in the long run. In fact, the long-run private capital multiplier amounts to 4.08, owing to ‘crowding-in’ of public investment by private investment. In contrast, because the benchmark case sets $\sigma_C = 1$ (so that $\bar{\sigma}_L = 0$), the long-run employment multiplier is zero [see (29)]. The long-run output expansion comes at the cost of a much larger short-run contraction, however. Indeed, the distortionary nature of labor taxes together with the wealth effect on labor supply generate a short-run employment multiplier of -1.72 and a short-run output multiplier of -3.68. Moreover, compared with Bom, Heijdra, and Ligthart (2010), the labor market distortions give rise to stronger short-run crowding-out of private investment by public investment.

Varying the intertemporal elasticity of labor supply affects only the wage multiplier in the long run. In fact, because Cobb-Douglas preferences are assumed (i.e., $\sigma_C = 1$), the long-run employment multiplier is zero irrespective of ω_{LL} . Consequently, the long-run multipliers of private capital, output, private consumption, private investment, foreign assets, and labor tax rate are also independent of ω_{LL} . In the short run, however, the employment multiplier is less negative for lower values of ω_{LL} . As a result, the short-run output contraction is much less severe for $\omega_{LL} = 1$ than for $\omega_{LL} = 2$ and absent altogether for $\omega_{LL} = 0$. Likewise, in absolute terms, the short-run negative multiplier of private investment falls substantially for $\omega_{LL} = 1$, and even turns positive for $\omega_{LL} = 0$.

An elasticity of substitution between consumption and leisure (σ_C) slightly larger (smaller) than one yields a positive (negative) long-run employment multiplier. As a consequence, all long-run multipliers excepting that of the real wage are (in absolute value) somewhat larger (smaller) than in the benchmark case. In the short run, the economically meaningful case of a small positive uncompensated wage elasticity of labor supply (i.e., $\sigma_C = 1.25$) yields a positive effect of public investment on private consumption, although at the cost of further depressing employment, private investment, and output.

Under Cobb-Douglas preferences, unproductive public spending (i.e., $\eta = 0$) does not affect the steady-state level of output, since neither employment nor capital react to public

investment in the long run. As the size of the public capital externality increases, the long-run output multiplier expands, although again at the cost of an even larger short-run output contraction. As in the case of $\sigma_C > 1$, a sufficiently large value of η generates a positive short-run response of private consumption, but exacerbates the negative short-run effects on employment and private investment. The long-run multiplier of the stock of foreign debt is negative and decreases with η .

The bottom section of Table 2 displays the short-run and long-run effects on instantaneous aggregate utility—which are denoted by $\tilde{U}(0)$ and $\tilde{U}(\infty)$, respectively—and follow from using $\tilde{U}(t) = \tilde{X}(t) - \tilde{P}(t)$. In addition, the table presents the change in lifetime utility of an infinitely-lived representative agent, $d\Lambda_R(0)$, which is derived from (1) with $\beta = 0$ imposed. To assess the welfare costs of labor taxation, we report the welfare effects of public investment under lump-sum (or head) tax financing and labor tax financing (labeled ‘H’ and ‘L,’ respectively). The results show that, despite the short-run contraction in output and private consumption, instantaneous welfare rises both in the short and in the long run in the cases where public capital is productive. In the short run, instantaneous utility rises by more under labor tax than lump-sum tax financing, owing to the boost in leisure consumption. In the long run, however, the labor market distortions cause the instantaneous welfare effect to be larger in the lump-sum tax financing scenario. Similarly, lifetime utility only rises if public capital is sufficiently productive. In the benchmark case, the lifetime utility losses from labor market distortions amount to 58 percent of the lifetime welfare gains.

6 Conclusions

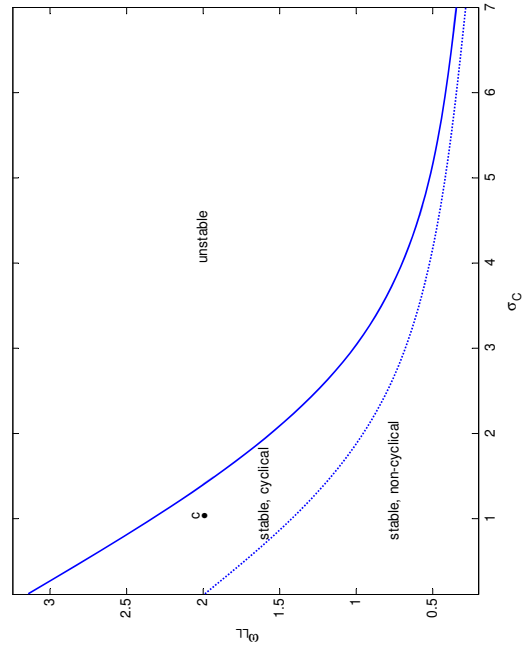
The paper studies the dynamic macroeconomic and welfare effects of public investment in a micro-founded model of a small open economy in the OECD area. The government keeps the budget balanced by employing distortionary labor taxes to finance public investment. The household sector of the model extends a Yaari-Blanchard model of overlapping generations by introducing an intertemporal labor supply effect, public capital, and distortionary taxation. On the one hand, public capital generates positive spillovers to private production. On the

other hand, the labor tax distorts the labor market.

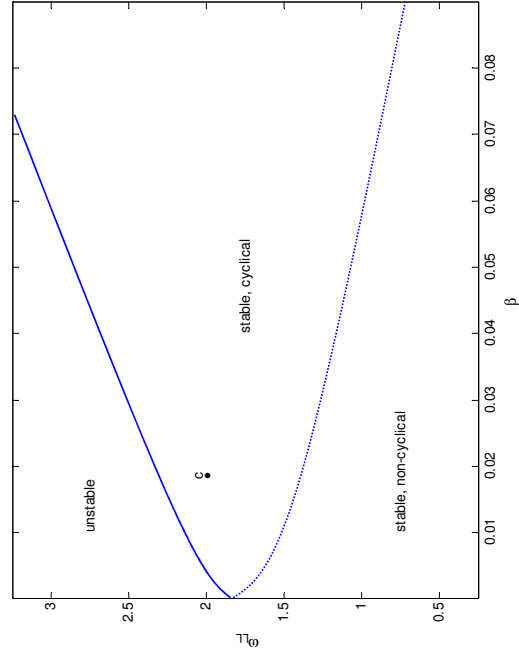
For a plausible calibration of the model, we find dampened cyclical dynamics in key macroeconomic variables. The cycles are induced by the combination of finite planning horizons of households, the wealth effect on labor supply, and the balanced budget fiscal rule. A balanced budget permanent impulse to public investment increases long-run output and private consumption, which is in line with empirical evidence. The benchmark case of Cobb-Douglas preferences yields a long-run output multiplier of 2.25. In the short run, however, the strong decrease in employment causes an even larger output contraction, which is accompanied by a decrease in private investment and private consumption. An elasticity of substitution between consumption and leisure larger than unity or a sufficiently large output elasticity of public capital or both increase the long-run output multiplier above the benchmark value and exacerbate the negative short-run effect on employment, private investment, and output. Short-run private consumption rises, however.

Finally, our numerical analysis reveals that a balanced budget public investment impulse improves households' lifetime welfare in the benchmark analysis. Bom and Ligthart's (2008) short-run estimate of the output elasticity of public capital of 0.08 implies an optimal public investment ratio of about 6 percent of GDP, which is well above the OECD average. Therefore, our results suggest that, from a welfare point of view, public investment should be encouraged even in the second-best scenario in which governments have to resort to distortionary labor tax financing.

Figure 1: Stability Regions for Various Values of ω_{LL} , σ_C , and β



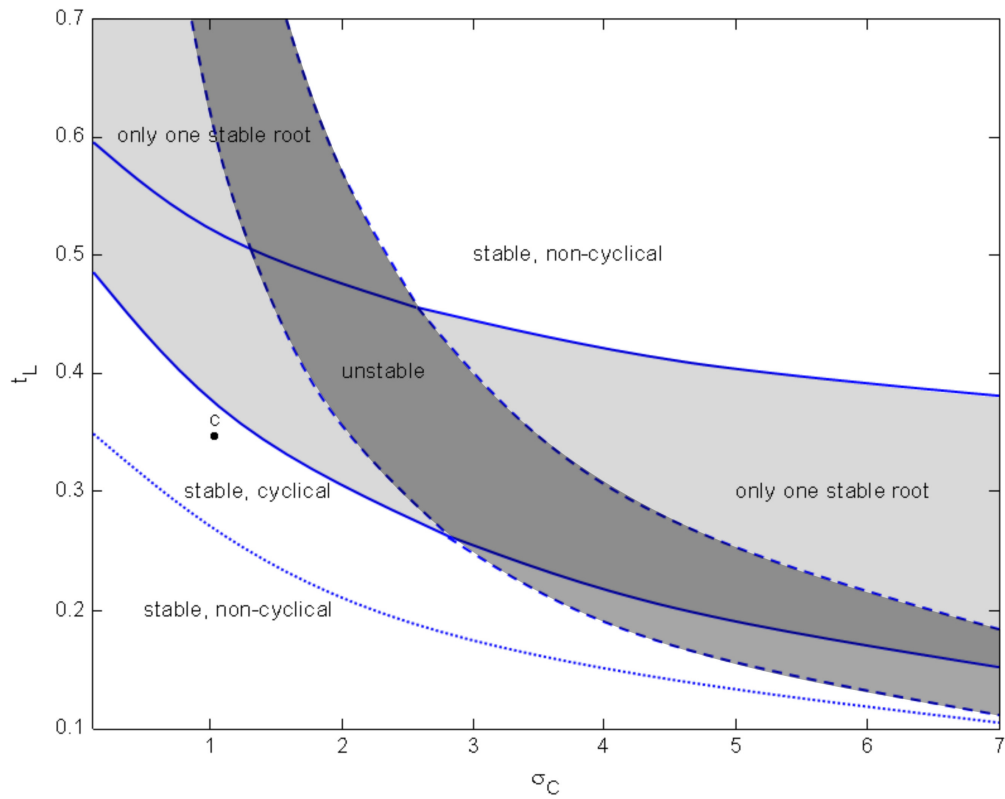
Panel (a): ω_{LL} and σ_C



Panel (b): ω_{LL} and β

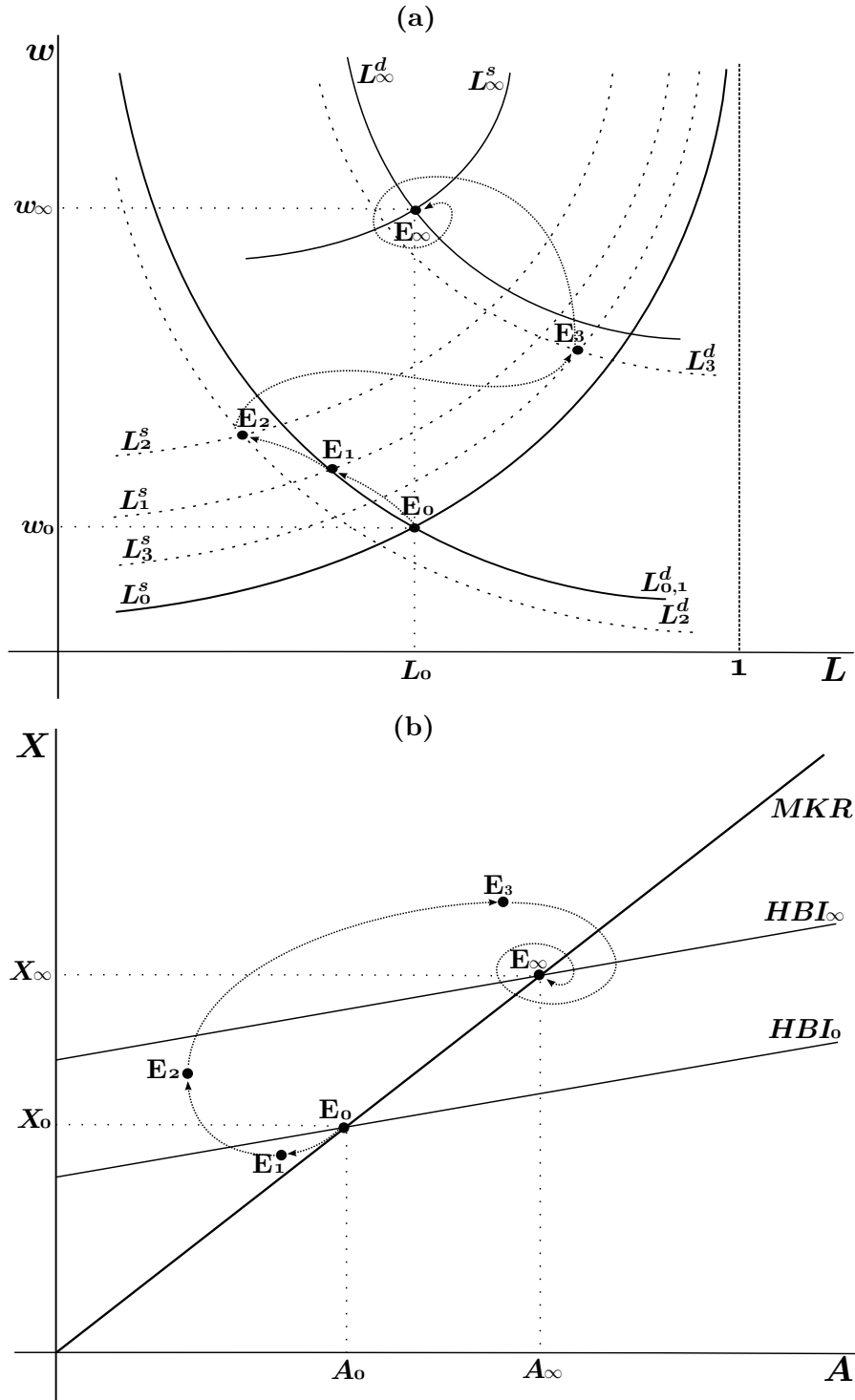
Notes: The dotted line represents the upper bound of the stable, non-cyclical region and the solid line demarcates the lower bound of the unstable region. The area in between the solid line and the dotted line represents parameter combinations for which the model yields stable, cyclical dynamics. Point C denotes the benchmark calibration.

Figure 2: Stability Regions for Various Values of t_L and σ_C



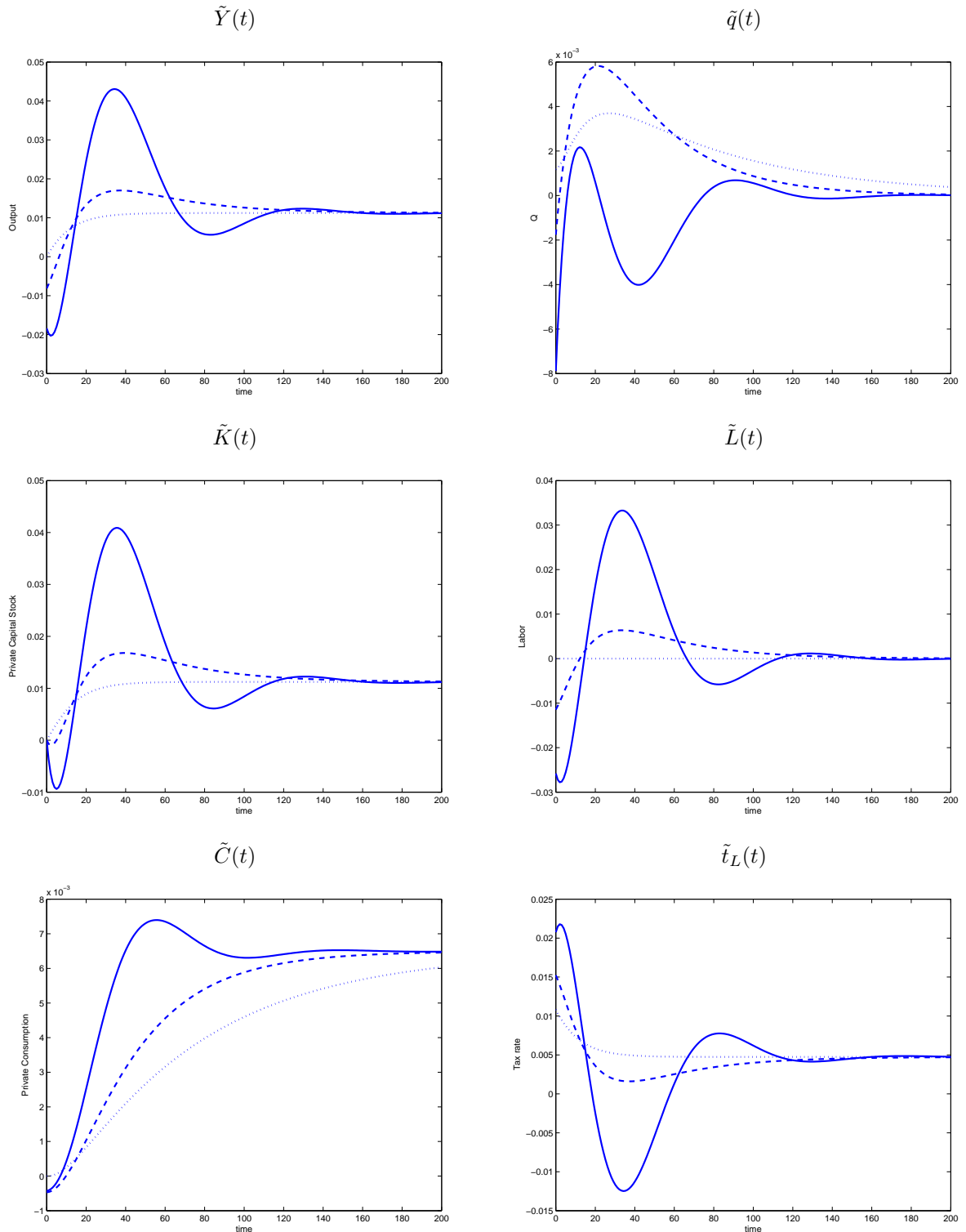
Notes: The dotted line represents the upper bound of the stable, non-cyclical region and the lower solid line demarcates the lower bound of the unstable region. The light grey area represents an unstable zone and the dark grey area in between the dashed lines shows combinations of points on the negatively sloped segment of the Laffer curve [see (30) and (33)]. The upper north-east corner—which is bounded by the solid and dashed lines—represents a stable, non-cyclical region. Point C denotes the benchmark calibration.

Figure 3: Long-Run Effects of a Public Investment Impulse:
The Labor Market and Savings System



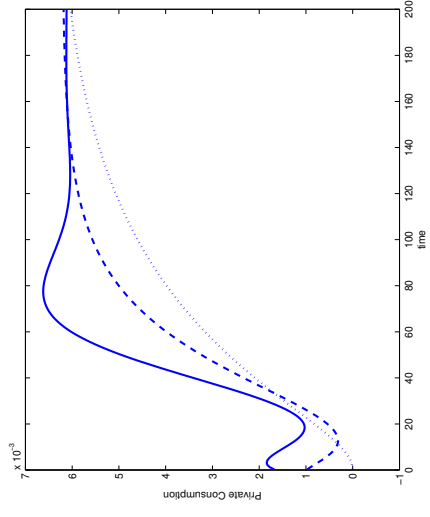
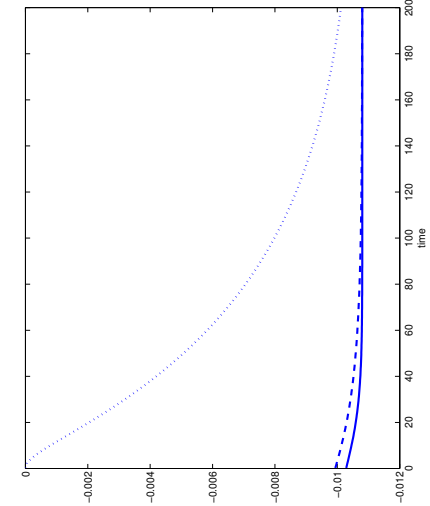
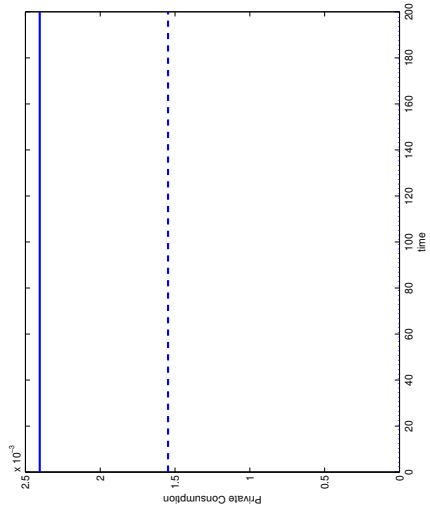
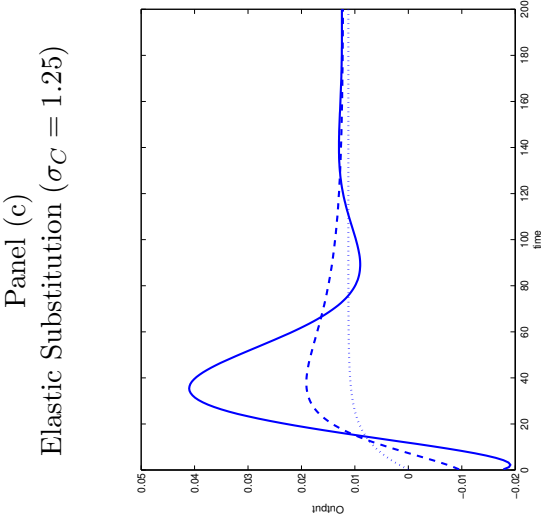
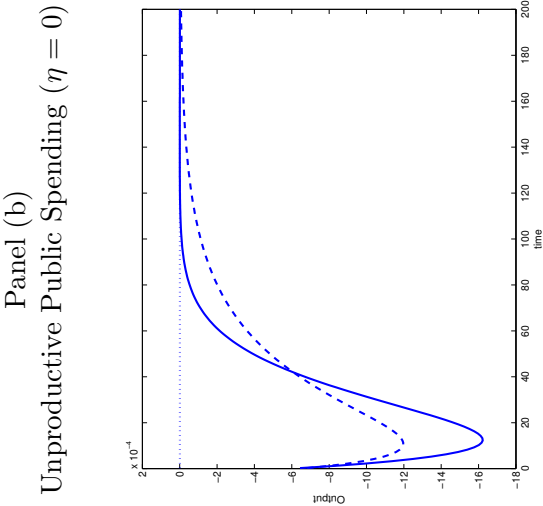
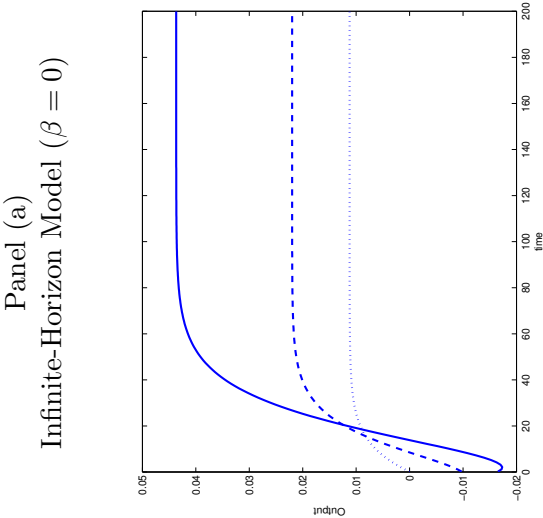
Notes: The top panel depicts labor market equilibrium. Aggregating equation (9) yields the labor supply curve L^s , whereas equation (17) gives the labor demand curve L^d . The bottom panel displays the savings system. The MKR locus denotes the modified Keynes-Ramsey rule (13) and the household budget identity (HBI) is given by the aggregate version of (4).

Figure 4: Dynamic Effects of a Permanent Public Investment Impulse:
Various Values of ω_{LL}



Notes: The vertical axis reports the relative change in the respective macroeconomic variable. The solid line denotes the benchmark scenario of $\omega_{LL} = 2$, the dashed line represents $\omega_{LL} = 1$, and the dotted line represents $\omega_{LL} = 0$. The other parameters are set at their benchmark values (Table 1). The size of the public investment impulse amounts to $\tilde{I}_G = 0.1$.

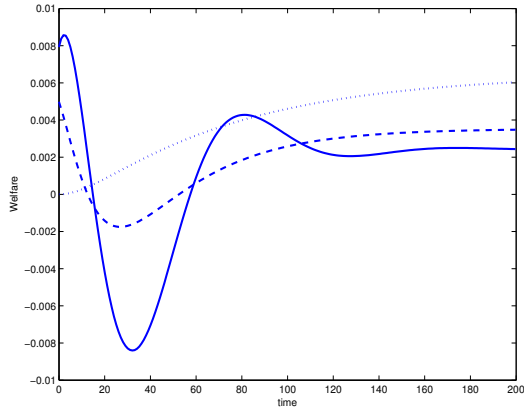
Figure 5: Other Specifications: Infinite Horizons, Unproductive Public Spending, and Elastic Substitution Between Consumption and Leisure



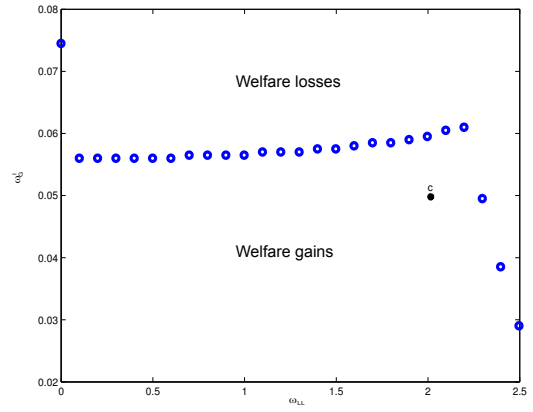
Notes: The top panels show the relative change in output and the bottom panels depict the relative change in private consumption. The solid line denotes the scenario of $\omega_{LL} = 1.75$, the dashed line represents $\omega_{LL} = 1$, and the dotted line represents $\omega_{LL} = 0$. The other parameters are set at their benchmark values. The size of the public investment impulse amounts to $\bar{I}_G = 0.1$.

Figure 6: Dynamic Welfare Effects of Public Investment and Optimal Public Investment-to-GDP Ratio

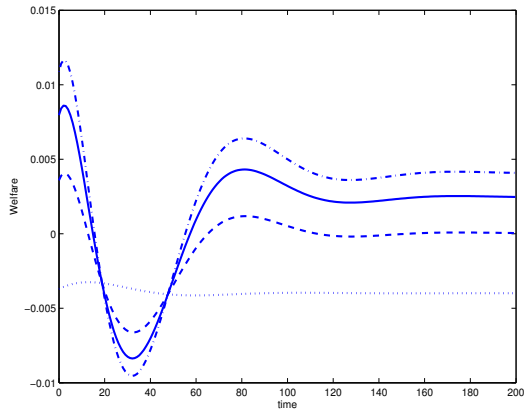
Panel (a): Welfare Effects: Various ω_{LL}



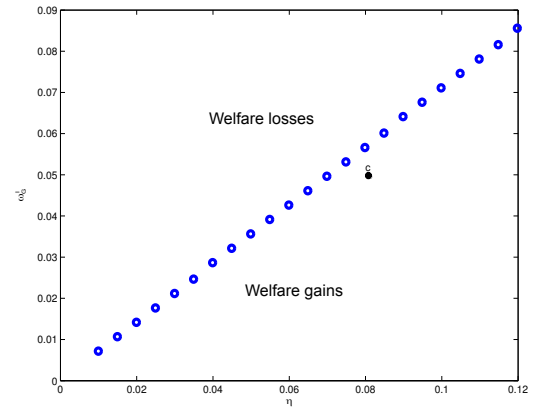
Panel (b): Optimal ω_G^I : Various ω_{LL}



Panel (c): Welfare Effects: Various η



Panel (d): Optimal ω_G^I : Various η



Notes: Panels (a) and (c) depict the dynamic welfare effects for various parameter values. Panels (b) and (d) show the optimal public investment-to-GDP ratio for various values of ω_{LL} and η , respectively. In Panel (a), the solid line denotes the scenario of $\omega_{LL} = 2$ (benchmark), the dashed line represents $\omega_{LL} = 1$, and the dotted line represents $\omega_{LL} = 0$. In Panel (c), the dotted-dashed line shows $\eta = 0.10$, the solid line depicts $\eta = 0.08$ (benchmark), the dashed line represents $\eta = 0.05$, and the dotted line represents $\eta = 0$. The other parameters are set at their benchmark values (Table 1). The area below the dots in Panels (b) and (d) denotes the parameter combinations for which a lifetime welfare gain is obtained. Point C denotes the calibration point. The size of the public investment impulse amounts to $\tilde{I}_G = 0.1$.

Table 1: Chosen and Implied Parameter Values in the Benchmark Model

| Description | Parameter/Share | Value |
|--|---------------------|-------|
| <i>Panel (a): Chosen Values</i> | | |
| Birth rate | β | 0.018 |
| Rate of interest | r | 0.040 |
| Depreciation rate of private capital | δ | 0.100 |
| Depreciation rate of public capital | δ_G | 0.100 |
| Output elasticity of public capital | η | 0.080 |
| Parameter of the installation function for private capital | \bar{z} | 0.532 |
| Parameter of the installation function for public capital | \bar{z}_G | 0.532 |
| Public consumption-to-GDP ratio | ω_G^C | 0.200 |
| Public investment-to-GDP ratio | ω_G^I | 0.050 |
| Private consumption-to-GDP ratio | ω_C | 0.550 |
| Leisure-labor ratio | ω_{LL} | 2.000 |
| Elasticity of substitution between consumption and leisure | σ_C | 1.000 |
| <i>Panel (b): Selected Implied Values</i> | | |
| Private investment-private capital ratio | I/K | 0.110 |
| Public investment-public capital ratio | I_G/K_G | 0.110 |
| Output-private capital ratio | Y/K | 0.550 |
| Output-public capital ratio | Y/K_G | 2.200 |
| Tobin's q | q | 1.210 |
| Balanced budget labor income tax rate | t_L | 0.350 |
| Output elasticity of private capital | ε_Y | 0.288 |
| Elasticity of the private accumulation function | ρ_A | 0.171 |
| Elasticity of the public accumulation function | χ_G | 0.091 |
| Preference weight of private consumption in utility function | ε_C | 0.370 |
| Leisure-full consumption ratio | ω_N | 0.630 |
| Frisch elasticity of labor supply | $\bar{\omega}_{LL}$ | 2.000 |
| Pure rate of time preference | α | 0.039 |
| Stable root 1 | h_1^* | 0.158 |
| Stable root 2 | h_2^* | 0.017 |
| Unstable root 1 | r_1^* | 0.198 |
| Unstable root 2 | r_2^* | 0.058 |

Notes: Panel (a) shows the parameters and shares of the benchmark analysis. Panel (b) presents implied values of selected economic variables and shares.

Table 2: Macroeconomic Multipliers and Welfare Effects of a Permanent Increase in Public Investment

| | Benchmark | ω_{LL} | | σ_C | | η | | |
|-----------------------------|-----------|---------------|---------|------------|---------|---------|---------|---------|
| | | 0 | 1 | 0.75 | 1.25 | 0 | 0.05 | 0.10 |
| Multipliers: | | | | | | | | |
| $\frac{dY(0)}{dI_G}$ | -3.6756 | 0.0000 | -1.6334 | -3.2082 | -4.1838 | -0.0866 | -2.3297 | -4.5728 |
| $\frac{dY(\infty)}{dI_G}$ | 2.2464 | 2.2464 | 2.2464 | 2.0265 | 2.5157 | 0.0000 | 1.4040 | 2.8080 |
| $\frac{dC(0)}{dI_G}$ | -0.0473 | 0.0000 | -0.0515 | -0.2835 | 0.2062 | -1.1629 | -0.4657 | 0.2316 |
| $\frac{dC(\infty)}{dI_G}$ | 0.7139 | 0.7139 | 0.7139 | 0.5468 | 0.9185 | -1.1898 | 0.0000 | 1.1898 |
| $\frac{dI(0)}{dI_G}$ | -1.8411 | 0.2734 | -0.4110 | -1.3845 | -2.6118 | -0.1206 | -1.1959 | -2.2712 |
| $\frac{dI(\infty)}{dI_G}$ | 0.4493 | 0.4493 | 0.4493 | 0.4053 | 0.5031 | 0.0000 | 0.2808 | 0.5616 |
| $\frac{dF(0)}{dI_G}$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $\frac{dF(\infty)}{dI_G}$ | -2.0806 | -2.0806 | -2.0806 | -1.8591 | -2.3518 | -4.7460 | -3.0801 | -1.4143 |
| $\frac{dL(0)}{dI_G}$ | -1.7202 | 0.0000 | -1.1466 | -1.5015 | -1.9580 | -0.0405 | -1.0903 | -2.1401 |
| $\frac{dL(\infty)}{dI_G}$ | 0.0000 | 0.0000 | 0.0000 | -0.0733 | 0.0897 | 0.0000 | 0.0000 | 0.0000 |
| $\frac{dK(0)}{dI_G}$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| $\frac{dK(\infty)}{dI_G}$ | 4.0839 | 4.0839 | 4.0839 | 3.6841 | 4.5733 | 0.0000 | 2.5524 | 5.1048 |
| $\frac{dw(0)}{dI_G}$ | 3.1730 | 0.0000 | 0.9400 | 2.7695 | 3.6117 | 0.0748 | 2.0112 | 3.9475 |
| $\frac{dw(\infty)}{dI_G}$ | 4.8000 | 1.6000 | 3.2000 | 4.8000 | 4.8000 | 0.0000 | 3.0000 | 6.0000 |
| $\frac{dt_L(0)}{dI_G}$ | 2.6941 | 1.4040 | 1.9773 | 2.5301 | 2.8725 | 1.4344 | 2.2217 | 3.0091 |
| $\frac{dt_L(\infty)}{dI_G}$ | 0.6155 | 0.6155 | 0.6155 | 0.6927 | 0.5210 | 1.4040 | 0.9112 | 0.4184 |
| Welfare Effects: | | | | | | | | |
| $\tilde{U}^L(0)$ | 0.0079 | 0.0000 | 0.0050 | 0.0076 | 0.0083 | -0.0038 | 0.0035 | 0.0108 |
| $\tilde{U}^H(0)$ | 0.0035 | 0.0000 | 0.0031 | 0.0034 | 0.0037 | -0.0020 | 0.0014 | 0.0049 |
| $\tilde{U}^L(\infty)$ | 0.0024 | 0.0065 | 0.0035 | 0.0022 | 0.0027 | -0.0040 | 0.0000 | 0.0040 |
| $\tilde{U}^H(\infty)$ | 0.0030 | 0.0065 | 0.0042 | 0.0028 | 0.0033 | -0.0027 | 0.0009 | 0.0044 |
| $d\Lambda_R^L(0)$ | 0.0177 | 0.0351 | 0.0185 | 0.0143 | 0.0236 | -0.0959 | -0.0249 | 0.0461 |
| $d\Lambda_R^H(0)$ | 0.0280 | 0.0351 | 0.0355 | 0.0232 | 0.0329 | -0.0674 | -0.0077 | 0.0519 |

Notes: Unless indicated otherwise, all parameters are set at their benchmark values (Table 1), where $\omega_{LL} = 2$, $\sigma_C = 1$, and $\eta = 0.08$. $\tilde{U}^i(t)$ denotes the relative change in instantaneous utility at time t for $i = \{H, L\}$, where H and L stand for lump-sum tax financing and labor tax financing, respectively. $\Lambda_R^i(t)$ represents the present discounted value of utility of an infinitely-lived representative agent: $\Lambda_R^i(t) \equiv \int_t^\infty \ln U(\tau) e^{-\alpha(\tau-t)} d\tau$.

Appendix

This Appendix derives a number of key expressions used in the main text. Bom and Ligthart (2011) provide further details on the derivations.

A.1 The Reduced-Form Model

The model can be condensed to

$$\dot{\tilde{\mathbf{z}}}(t) = \Delta \tilde{\mathbf{z}}(t) - \mathbf{\Gamma}(t), \quad (\text{A.1})$$

where Δ is the 4×4 Jacobian matrix:

$$\Delta \equiv \begin{bmatrix} 0 & \frac{r\omega_I}{\rho_A\omega_A} & 0 & 0 \\ \frac{r\varepsilon_Y}{\omega_A}(1 - \xi_{yk}) & r & -\frac{r\varepsilon_Y}{\omega_A}\xi_{yx} & 0 \\ 0 & 0 & r - \alpha & -\frac{r - \alpha}{\omega_A} \\ r\omega_{\bar{w}}\xi_{\bar{w}k} & 0 & r(\omega_{\bar{w}}\xi_{\bar{w}x} - \omega_X) & r \end{bmatrix},$$

where $\omega_{\bar{w}} \equiv \bar{w}/Y$ is the output share of after-tax wages and the policy shock terms are denoted by:

$$\begin{aligned} \gamma_q(t) &\equiv \frac{r\varepsilon_Y}{\omega_A} [\xi_{yg}(1 - e^{-\chi_G t}) + \xi_{yd}] \tilde{I}_G, \\ \gamma_A(t) &\equiv -r [\omega_{\bar{w}}\xi_{\bar{w}g}(1 - e^{-\chi_G t}) + \omega_{\bar{w}}\xi_{\bar{w}d}] \tilde{I}_G. \end{aligned}$$

The ξ_{yj} coefficients are as follows:

$$\begin{aligned} \xi_{yk} &\equiv \frac{\varepsilon_Y(1 + \bar{\omega}_{LL})}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, & \xi_{yx} &\equiv -\frac{(1 - \varepsilon_Y)\omega_{LL}}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, \\ \xi_{yg} &\equiv \frac{\eta(1 + \bar{\omega}_{LL})}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, & \xi_{yd} &\equiv -\frac{\bar{\omega}_{LL}\omega_G^I(1 + \bar{\theta}_L)}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, \end{aligned}$$

where $\bar{\theta}_L \equiv t_L/(1 - t_L)$. For employment, the coefficients are given by:

$$\begin{aligned}\xi_{lk} &\equiv \frac{\varepsilon_Y \bar{\omega}_{LL}(1 + \bar{\theta}_L)}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, & \xi_{lx} &\equiv -\frac{\omega_{LL}}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, \\ \xi_{lg} &\equiv \frac{\bar{\omega}_{LL}(1 + \bar{\theta}_L)\eta}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, & \xi_{ld} &\equiv -\frac{1}{1 - \varepsilon_Y} \frac{\bar{\omega}_{LL}\omega_G^I(1 + \bar{\theta}_L)}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}.\end{aligned}$$

Finally, for the wage rate the coefficients are:

$$\begin{aligned}\xi_{wk} &\equiv \frac{\varepsilon_Y(1 - \bar{\omega}_{LL}\bar{\theta}_L)}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, & \xi_{wx} &\equiv \frac{\omega_{LL}\varepsilon_Y}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, \\ \xi_{wg} &\equiv \frac{(1 - \bar{\omega}_{LL}\bar{\theta}_L)\eta}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}, & \xi_{wd} &\equiv \frac{1}{1 - \varepsilon_Y} \frac{\bar{\omega}_{LL}\varepsilon_Y\omega_G^I(1 + \bar{\theta}_L)}{1 + \bar{\omega}_{LL}[\varepsilon_Y(1 + \bar{\theta}_L) - \bar{\theta}_L]}.\end{aligned}$$

The coefficients for after-tax wages are given by:

$$\begin{aligned}\xi_{\bar{w}k} &\equiv \xi_{wk} + \bar{\theta}_L(\xi_{lk} + \xi_{wk}), & \xi_{\bar{w}x} &\equiv \xi_{wx} + \bar{\theta}_L(\xi_{lx} + \xi_{wx}), \\ \xi_{\bar{w}g} &\equiv \xi_{wg} + \bar{\theta}_L(\xi_{lg} + \xi_{wg}), & \xi_{\bar{w}d} &\equiv \xi_{wd} + \bar{\theta}_L(\xi_{ld} + \xi_{wd}) - \frac{\omega_G^I(1 + \bar{\theta}_L)}{1 - \varepsilon_Y}.\end{aligned}$$

A.2 Deriving the Long-Run Employment Multiplier

Equation (9) can be fully differentiated with respect to I_G to arrive at a general expression for the labor supply effect:

$$\frac{dL(\infty)}{dI_G} = \frac{1 - L}{w} \frac{dw(\infty)}{dI_G} - \frac{1 - L}{1 - t_L} \frac{dt_L(\infty)}{dI_G} - \frac{1 - L}{\omega_N} \frac{d\omega_N(\infty)}{dI_G} - \frac{1 - L}{X} \frac{dX(\infty)}{dI_G}. \quad (\text{A.2})$$

The first three terms capture the *intra*temporal substitution effect on labor supply and the last term captures the negative wealth effect or *inter*temporal substitution effect on labor supply.

The first two terms are derived in the main text [see (28) and (32)]. To derive $d\omega_N(\infty)/dI_G$, we fully differentiate (10):

$$\begin{aligned}\frac{d\omega_N(\infty)}{dI_G} &= \omega_N(1 - \sigma_C) \left[\frac{1}{w} \frac{dw(\infty)}{dI_G} - \frac{1}{1 - t_L} \frac{dt_L(\infty)}{dI_G} - \frac{1}{P} \frac{dP(\infty)}{dI_G} \right] \\ &= \omega_N(1 - \sigma_C)(1 - \omega_N) \left[\frac{1}{w} \frac{dw(\infty)}{dI_G} - \frac{1}{1 - t_L} \frac{dt_L(\infty)}{dI_G} \right],\end{aligned} \quad (\text{A.3})$$

where we have used the long-run change in the price index (using (11)):

$$\frac{dP(\infty)}{dI_G} = \omega_N P \left[\frac{1}{w} \frac{dw(\infty)}{dI_G} - \frac{1}{1-t_L} \frac{dt_L(\infty)}{dI_G} \right]. \quad (\text{A.4})$$

To obtain an expression for $dX(\infty)/dI_G$, we note that the steady-state version of equation (13) implies a fixed ratio of total assets to full consumption: $A/X = (r - \alpha)/\beta(\alpha + \beta)$, which can be differentiated to give:

$$\frac{dA(\infty)}{dI_G} = \frac{A}{X} \frac{dX(\infty)}{dI_G} = \frac{r - \alpha}{\beta(\alpha + \beta)} \frac{dX(\infty)}{dI_G}. \quad (\text{A.5})$$

After differentiating the steady-state (aggregate) version of the household budget identity (4), we obtain:

$$\frac{dX(\infty)}{dI_G} = \frac{X}{w} \frac{dw(\infty)}{dI_G} - \frac{X}{1-t_L} \frac{dt_L(\infty)}{dI_G}. \quad (\text{A.6})$$

Using $dt_L(\infty)/dI_G$ [from (31)], (A.3), and (A.6), we find the labor multiplier (29).

Table A1: Summary of the Log-Linearized Model

(a) *Dynamic Equations:*

$$\dot{\tilde{K}}(t) = \frac{r\omega_I}{\omega_A} \left[\tilde{I}(t) - \tilde{K}(t) \right] \quad (\text{TA.1})$$

$$\dot{\tilde{q}}(t) = r\tilde{q}(t) - \frac{r\varepsilon_Y}{\omega_A} \left[\tilde{Y}(t) - \tilde{K}(t) \right] \quad (\text{TA.2})$$

$$\dot{\tilde{X}}(t) = (r - \alpha) \left[\tilde{X}(t) - \frac{\tilde{A}(t)}{\omega_A} \right] \quad (\text{TA.3})$$

$$\dot{\tilde{A}}(t) = r \left[\tilde{A}(t) + \omega_{\tilde{w}}\tilde{w}(t) - \omega_X\tilde{X}(t) \right] \quad (\text{TA.4})$$

$$\dot{\tilde{K}}_G(t) = \chi_G \left[\tilde{I}_G - \tilde{K}_G(t) \right] \quad (\text{TA.5})$$

(b) *Static Equations:*

$$\tilde{q}(t) = \rho_A \left[\tilde{I}(t) - \tilde{K}(t) \right] \quad (\text{TA.6})$$

$$\tilde{w}(t) = \tilde{Y}(t) - \tilde{L}(t) \quad (\text{TA.7})$$

$$\tilde{Y}(t) = \varepsilon_Y\tilde{K}(t) + (1 - \varepsilon_Y)\tilde{L}(t) + \eta\tilde{K}_G(t), \quad (\text{TA.8})$$

$$\tilde{L}(t) = \omega_{LL} \left[\tilde{w}(t) - \tilde{\omega}_N(t) - \tilde{X}(t) \right] \quad (\text{TA.9})$$

$$\tilde{C}(t) = -\frac{\omega_N}{1 - \omega_N}\tilde{\omega}_N(t) + \tilde{X}(t) \quad (\text{TA.10})$$

$$\tilde{F}(t) = \tilde{A}(t) - \omega_A \left[\tilde{q}(t) + \tilde{K}(t) \right] \quad (\text{TA.11})$$

$$\tilde{t}_L(t) = \frac{1}{(1 - \varepsilon_Y)(1 - t_L)} \left[\omega_G^I\tilde{I}_G + \omega_G^C\tilde{C}_G \right] - \frac{t_L}{1 - t_L} \left[\tilde{w}(t) + \tilde{L}(t) \right] \quad (\text{TA.12})$$

(c) *Definitions:*

$$\tilde{P}(t) = \omega_N\tilde{w}(t) \quad (\text{TA.13})$$

$$\tilde{\omega}_N(t) = (1 - \sigma_C) \left[\tilde{w}(t) - \tilde{P}(t) \right] \quad (\text{TA.14})$$

$$\tilde{\tilde{w}}(t) = \tilde{w}(t) - \tilde{t}_L(t) \quad (\text{TA.15})$$

Notes: The following definitions are used: $\omega_A \equiv r(qK/Y)$, $\omega_I \equiv I/Y$, $\omega_G^C \equiv C/Y$, $\omega_G^I \equiv I_G/Y$, $\omega_w \equiv \bar{w}/Y$, $\omega_{LL} \equiv (1 - L)/L$, $\omega_X \equiv X/Y$, $\rho_A \equiv -(I/K)(\Phi''/\Phi') > 0$, and $\chi_G \equiv I_G\Phi'_G(\cdot)/K_G > 0$. A tilde ($\tilde{\cdot}$) denotes a relative change, for example, $\tilde{C}(t) \equiv dC(t)/C$. However, for financial assets we scale by steady-state output and multiply by r (e.g., $\tilde{A}(t) \equiv rdA(t)/Y$) and for labor taxes we use $\tilde{t}_L(t) \equiv dt_L(t)/(1 - t_L)$.

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