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### STELLAR VARIABILITY: A BROAD AND NARROW PERSPECTIVE

by

### JAMES ROBERT PARKS IV

Under the Direction of Russel J. White

### ABSTRACT

A broad near-infrared photometric survey is conducted of 1678 stars in the direction of the  $\rho$  Ophiuchi ( $\rho$  Oph) star forming region using data from the 2MASS Calibration Database. The survey involves up to 1584 photometric measurements in the J, H and  $K_s$  bands with an ~1 day cadence spanning 2.5 years. Identified are 101 variable stars with  $\Delta K_s$  band amplitudes from 0.044 to 2.31 mag and  $\Delta(J-K_s)$  color amplitudes ranging from 0.053 to 1.47 mag. Of the 72  $\rho$  Oph star cluster members, 79% are variable; in addition, 22 variable stars are identified as candidate members. The variability is categorized as periodic, long timescale, or irregular based on the  $K_s$  time series morphology. The dominant variability mechanisms are assigned based on the correlation between the stellar color and single band variability. Periodic signals are found in 32 variable stars with periods between 0.49 to 92 days. The most common variability mechanism among these stars is rotational modulation of cool starspots. Periodic eclipse-like variability is identified in 6 stars with periods ranging from 3 to 8 days; in these cases the variability mechanism may be warped circumstellar material driven by a hot proto-Jupiter. Aperiodic, long time scale variability is identified in 31 stars with time series ranging from 64 to 790 days. The variability mechanism is split evenly between either variable extinction or mass accretion. The remaining 40 stars exhibit sporadic, aperiodic variability with no discernible time scale or variability mechanism.

Interferometric images of the active giant  $\lambda$  Andromedae ( $\lambda$  And) were obtained for 27 epochs spanning November. 2007 to September, 2011. The *H* band angular diameter and limb darkening coefficient of  $\lambda$  And are 2.777  $\pm$  0.027 mas and 0.241  $\pm$  0.014, respectively. Starspot properties are extracted via a parametric model and an image reconstruction program. High fidelity images are obtained from the 2009, 2010, and 2011 data sets. Stellar rotation, consistent with the photometrically determined period, is traced via starspot motion in 2010 and 2011. The orientation of  $\lambda$  And is fully characterized with a sky position angle and inclination angle of 23° and 78°, respectively.

### STELLAR VARIABILITY: A BROAD AND NARROW PERSPECTIVE

by

### JAMES ROBERT PARKS IV

A Dissertation Submitted in Partial Fulfillment of Requirements for the Degree of

Doctor of Philosophy

in the College of Arts and Sciences

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### STELLAR VARIABILITY: A BROAD AND NARROW PERSPECTIVE

by

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## DEDICATION

This dissertation is dedicated to my brother Victor J. Ricchezza who has been an inspiration to me on being forthright and courageous in the face of new challenges.

First and foremost I would like to thank Dr. Russel White for his insight, encouragement, and above all, patience on this project. His tutelage has been invaluable in not only crafting this document, but also in crafting myself into a better scientist and writer. I owe my future in astronomy to you, Fearless Leader.

I would like to thank Dr. Peter Plavchan who was instrumental in the completion of the  $\rho$  Ophiuchus variability study. He made my fellowship at the Infrared Processing and Analysis Center both instructive and memorable.

I would like to extend my thanks to Dr. John Monnier for his role in the  $\lambda$  Andromeda interferometric imaging project. He proved to be my interferometry grandmaster from whom I have learned nearly all I know about extracting as much information as one possibly can from smashing two or more beams of light together.

My thanks to Dr. Fabien Baron, the Yoda of interferometric imaging. Dr. Baron was irreplaceable in his instruction of interferometric image reconstruction and statistical modeling.

I would also like to thank the rest of my committee, Dr. Doug Gies, Dr. Bill Hastings, and Dr. Hal McAlister for all of their sagacious questions and comments concerning this dissertation.

I would like to thank Dr. Brian Kloppenborg for his expertise in interferometric observing, interferometric modeling and the Linux operating system in general. He is a valued colleague and friend.

My thanks to Dr. Mary Barsony needs to be extended for her many insightful questions and suggestions concerning the  $\rho$  Ophiuchus variability study.

In addition, I would like to thank Nicole "Charlie" Cabrera, Justin Cantrell, and Sam Quinn for their helpful suggestions to improving this document. I would thank Jeremy Jones for his stellar (pun intended) evolution modeling for  $\lambda$  Andromeda and for being an excellent sounding board for ideas concerning interferometry. In particular I would like to thank Cassy Davison for not only providing the same, but also for keeping me sane and motivated. Cassy, you have been a phenomenal office mate.

My thanks goes out to the entire CHARA team without whom starspots on  $\lambda$  Andromeda might never have been imaged. Specific thanks goes out to Dr. Chris Farrington and P. J. Goldfinger for being the best two telescope operators in the business ensuring observing went smoothly and enjoyably with righteous tunes all night long. Very special thanks to Dr. Gail Scheafer whose expertise with both CHARA and the MIRC beam combiner was invaluable. Gail, thank you very much for holding my hand through those first couple observing runs.

I would also like to thank Dr. Ming Zhao and Dr. Xiao Che for their expertise with the MIRC data reduction pipeline and improvements to MIRC's software and hardware.

I would like to thank Austin "Trae" Cooper III, Arielle Kass, Laura Kertz, and Ryan Nock for proofreading this tome and providing valuable suggestions as to all things grammatical.

My thanks to Emily Manne-Nicholas for her support during a particularly bleak time during this adventure.

Last, but certainly not least, my thanks to Lynn Mullins. While she arrived late to this party, she gave me the strength to see this project through to the very end. I love you, Lynn.

And now for the official sounding bit:

This dissertation makes use of data products from the Two Micron All Sky Survey, which is a joint project of the University of Massachusetts and the Infrared Processing and Analysis Center/California Institute of Technology, funded by the National Aeronautics and Space Administration and the National Science Foundation. This research has made use of the NASA Exoplanet Archive, which is operated by the California Institute of Technology, under contract with the National Aeronautics and Space Administration under the Exoplanet Exploration Program. I would like to acknowledge the Infrared Processing and Analysis Center's Visiting Graduate Student program for providing the opportunity and funding for this publication.

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# LIST OF ABBREVIATIONS

2MASS Two Micron All-Sky Survey

# 2MASS Cal-PSWDB

	Two Micron All-Sky Survey Calibration Point Source Working Database
AU	Astronomical Unit
$\mathbf{A}_V$	Extinction in V band
BLS	boxcar Lomb-Scargle
CCD	Charged Coupled Device
CHARA	Center for High Angular Resolution Astronomy
CoRoT	Convection, Rotation, and planetary Transits
CPU	central processing unit
FWHM	full-width at half-maximum
Kepler	NASA Kepler satellite
IDL	Interactive Data Language
IR	infrared
LBI	Long Baseline optical/near-IR Interferometry
LOS	line-of-sight
$\mathbf{LS}$	Lomb-Scargle
LTV	long timescale variable
mas	milliarcsecond
MIRC	Michigan Infra-Red Combiner
NASA	National Aeronautics and Space Administration
рс	parsec

PPAPlavchan-Parks AlgorithmRAMrandom access memory $\rho$  Oph $\rho$  Ophiuchi $\lambda$  And $\lambda$  AndromedaeSEDspectral energy densityYSOyoung stellar object

### INTRODUCTION

– 1 –

Stellar variability is a ubiquitous phenomenon during every phase of a star's lifetime. From the cradle to the grave, a star will change its appearance to the outside observer. One method by which this variability is detected is through observed changes in a star's brightness over time. These changes are not tracing variations in the internal nuclear production rate, but instead are a consequence of temporal variations in the surface or circumstellar phenomena. Observing how and why a star's brightness varies over time is the first step to understanding such topics as the chemical composition and structure within stellar envelopes, meridional flows with stellar envelopes, origins of magnetic dynamos, astroseismology, starspot characteristics, circumstellar disk structure and evolution, and planetary formation, just to name a few. Besides the astrophysical implications of stellar variability, this variability can also inhibit precise measurements of a star's fundamental parameters (e.g. radius, effective temperature) and interfere with exoplanet surveys.

This dissertation strives to characterize stellar variability from a broad and narrow perspective using stars both young and old. The broad perspective is provided by a long term photometric study of young stars in the  $\rho$  Oph star forming region. This study will provide insight on variability time scales ranging from days to years as well as insight into potential variability mechanisms. The narrow perspective is provided by interferometically imaging the surface of  $\lambda$  Andromedae, a known photometrically variable evolved star. These images, obtained with the Georgia State University Center for High Angular Resolution Astronomy Array, will allow for precise characterization of starspots without certain initial assumptions and limitations inherent to previous methods of starspot investigation.

#### 1.1 Young Stellar Objects

The physical mechanisms behind photometric variability in young stellar objects (YSOs) include, but are not limited to: rotational modulation of magnetically and accretion induced starspots, evolution of the circumstellar environment and/or interstellar extinction, variable mass accretion, transit events and stellar pulsation. Large sample variability studies indicate these mechanisms often operate concurrently resulting in very complex photometric time series in young stars (Herbst et al. 1994). Interpretations are also inhibited because the broad band photometry acquired by seeing limited ground based telescopes is unable to resolve spatially the inner regions around young stars, let alone the stellar surface, in order to identify unambiguously the cause of the variability; Sun-like stars in nearby star forming regions have sizes that are submilliarcsecond in angular diameter.

Both recent surveys and modeling efforts have discovered that these variability mechanisms operate on very specific periodic or aperiodic time scales. Magnetically induced cool starspots are expected to produce periodic variability with periods less than two weeks that are stable on month long time scales (Rebull 2001; Berdyugina 2005). Accretion induced hot starspots, on the other hand, will have periods in the same range, but with less consistent stability (Gullbring et al. 1996; Basri et al. 1997; Smith et al. 1999). Photometric variability due to extinction from circumstellar material can be either periodic or aperiodic with periods ranging from days to years depending on the occulter's distance from the star (assuming Keplerian rotation). AA Tau and UX Ori systems are examples of short period (P  $\sim$  few days) that are periodic and quasi-periodic, respectively. The time scales of variability due to changes in mass accretion through a circumstellar disk are related to the physics (e.g. disk viscosity, time variable magnetic field) causing the change (Terquem & Papaloizou 2000; Carpenter et al. 2001; Bouvier et al. 2007). These time scales can range from ~1 day (Eiroa et al. 2002) to many years (Armitage 1995; Kenyon et al. 1996). Two types of high amplitude  $(\Delta V \sim 4\text{-}5 \text{ mag})$  photometrically variable YSOs are FU Ori and EXor outbursters. The time scale of the outburst could last for months in the case of the EXors (Lorenzetti et al. 2012) or even decades (Hartmann & Kenyon 1996). These various variability mechanisms give rise to a distinct morphology of light curves that are just now being quantified (Wolk et al. 2013; Cody et al. 2014). High cadence, long temporal baseline photometric surveys can temporally resolve variable YSOs, help identify dominant variability mechanisms, and help establish a definitive light curve morphology scheme.

Multiwavelength observations are also useful in helping to distinguish between various variability mechanisms. Attempts have been made to model the affect of these mechanisms on correlations between single band photometric variability to color variability (Carpenter et al. 2001; Scholz et al. 2009; Rice et al. 2012; Faesi et al. 2012). For instance, variability due to extinction will cause a star to redden as it dims. However, for cool starspots observed in the near-IR, the contrast between the starspot and surrounding photosphere is small, thus producing a colorless variability (Vrba et al. 1985). In addition, multiwavelength observations have been used to identify if a YSO is variable in the first place via the Stetson Index (Carpenter et al. 2001, 2002; Plavchan et al. 2008b; Cody et al. 2014). The Stetson Index measures the correlation between the time series measured in two separate passbands.

Intensive photometric monitoring in the near-IR has the advantage of probing both the stellar surface and the inner circumstellar regions from  $\sim 0.01$  to 1 AU for low mass stars (Dullemond & Monnier 2010). This region is of particular interest as it contains both the

corotation and dust sublimation radius. It is at these radii that the accretion funnel onto the star begins and where Type II planetary migration halts giving rise to "hot" Jupiters.

The  $\rho$  Ophiuchi ( $\rho$  Oph) cluster makes an excellent laboratory to test high cadence, long temporal baseline, multiwavelength near-IR observations to distinguish between variability mechanisms in young stars.  $\rho$  Oph is a dense star forming region containing a few hundred known YSOs with ages ranging from 0.3 to 3 Myr. The region is rich with variable stars; previous surveys having identified more than 100 photometrically variable stars (Greene & Young 1992; Barsony et al. 1997, 2005; Bontemps et al. 2001; Wilking et al. 2005; Alves de Oliveira & Casali 2008). In addition, the region is heavily embedded in dust with the amount of visual extinction ranging from  $A_V = 5$  to 25 mag in the cloud core (Cambrésy 1999).

Plavchan et al. (2008b hereafter, P08) carried out a pilot study of 57 stars in the  $\rho$ Oph field using photometry collected by the Two Micron All-Sky Survey Calibration Point Source Working Database (2MASS Cal-PSWDB). That study identified periodic variability in two YSOs from a sample of candidate M stars. This dissertation expands on the initial pilot study performed in P08 and includes the full  $\rho$  Oph field data set from the 2MASS Cal-PSWDB to understand better the variability of young stars in this cloud.

#### 1.2 Magnetically Active Stars

Aristotelian philosophy stated that the Sun was a perfect, unchanging glowing orb in the heavens. This belief proved so prevalent that it was not until 1611, when Galileo used his telescope to observe sunspots, that this belief was proven incorrect. Spots on other stars was first hypothesized by Kron (1947). In the decades since, starspots have been studied, in detail, on scores of other stars ranging in age, spectral type, and luminosity class (Strassmeier 2009b and references therein). More recently, space missions, such as NASA's Kepler Spacecraft have increased this number to potentially tens of thousands (Basri et al. 2011). One motivation for studying starspots is a better understanding of stellar interiors, particularly the origins of magnetic dynamos. Another motivation is that starspots complicate measurements of fundamental stellar properties (i.e., effective temperature, luminosity, radial velocity, etc.). Besides the astrophysical implications, if a spotted star happens to harbor orbiting planets, the increased uncertainties in the stellar properties will translate directly to increased uncertainties in the exoplanet properties (i.e., mass, radius). With the advent of millimagnitude photometry, meter per second radial velocity surveys and direct milliarcsecond resolution interferometric imaging, this "second-order" effect can no longer be ignored.

In order to use starspots as probes of stellar and dynamo astrophysics or to correct for their effects on exoplanet properties, the starspots themselves must be properly characterized. Unfortunately, for almost all stars other than the Sun, starspot properties (i.e., size, temperature, location, number) have been determined from two broad category indirect methods: light curve inversion and Doppler imaging.

### 1.2.1 Light Curve Inversion

Light curve inversion (LCI) is a broad term covering a number of different techniques to produce maps of starspots on a stellar surface via the inversion of broad band photometry. One method is the use of a simple two temperature model with one temperature corresponding to the stellar photosphere and the other to the starspot. Due to the poor spatial resolution provided by photometry, the observed flux is assumed to be a combination of both the photosphere and starspot modulated by the amount of surface coverage provided by the starspot. Fig. 1.1 shows examples of light curves and the resulting inversion maps for the RS CVn type star  $\sigma$  Gem showing starspots as a function of longitude. One limitation of this method is the lack of latitudinal information contained in the map. The rotational inclination must be assumed if the starspot latitude is to be estimated. This method also provides no information as to the shape of starspots. Additionally, there is a degeneracy between the starspot coverage and the starspot temperature. The use of spectroscopic line ratios between lines sensitive and insensitive to temperature has been used to break this degeneracy and provide starspot temperatures with a precision of less than 10 K (Gray 1996). As a starspot rotates into view, the surface averaged effective temperature will decrease. This will cause the depth of the temperature sensitive line to decrease thus changing the line depth ratio with regards to the temperature insensitive line. This technique coupled with simultaneous photometric time series has been used to estimate starspot sizes and temperatures for a number of RS CVn binaries (Frasca et al. 2005, 2008). This technique still suffers from assumptions of the rotational inclination axis and starspot shape. In addition this technique is limited to stars with a nonasimuthally symmetric starspot distribution capable of producing detectable photometric variability.

A different multiwavelength LCI technique that does not depend on *a priori* knowledge of starspot number or shape and does not suffer from a degeneracy in starspot temperature and size is called *matrix light curve inversion*. First developed by Wild (1989) and then refined by Harmon & Crews (2000), it inverts the light curve onto a stellar surface subdivided by "spherical rectangles" bounded by circles of latitude and meridians of longitude. Each



Figure 1.1: An example of light curve inversion used to analyze properties of starspots on the RS CVN binary  $\sigma$  Gem. The first and third columns are maps of the starspot coverage of the stellar surface. Darker regions indicate higher coverage. The second and fourth columns contain observed (crosses) and calculated (solid line) V-band light curves corresponding to the surface map. (Berdyugina 2005)

rectangle is modeled to be uniformly illuminated across its face. This allows for any number of starspots on different latitudes to be modeled provided the total specific intensity of the stellar surface reproduces the observed light curve. The input parameters for the model are the rotational inclination, a limb darkened model, the spot to photosphere intensity ratio per filter, the ratio of photosphere intensity for filter n to the reference filter, and an estimate of the noise variance in the observed light curves. The method assumes dark starspots by biasing against bright patches. While this method does provide more information than other LCI methods without the need of spectral information, it does suffer from certain limitations. Rotation inclinations  $\geq 60^{\circ}$ do not yield very reliable results for starspot latitudes. Furthermore, the rotation inclinations need to be accurate to within 10° to avoid "not terri-



Figure 1.2: An example of the stroboscopic effect produced as a transiting planet occults starspots on Kepler-17b. Left: A sequence of combined and binned transit light curves with the best fit model over-plotted in red. Occulted starspots are revealed in the combined curves since the stellar rotation period is eight times the planets orbital period. The same starspots are crossed every eight transits at a similar orbital phase. Right: The residuals of the best fit model subtracted from each individual combined light curve modulo 8. The vertical dashed lines correspond to the beginning and the end of the transits. Five occulted starspots are indicated on the residuals (A, B, C, D, and E) as they appear transit after transit at phase positions expected from the stellar rotation period (Désert et al. 2011).

bly detrimental" results. The results are also biased on knowing the starspot temperatures to within an accuracy of 250 K.

A third LCI method to characterize starspots have been employed recently to both CoRoTand *Kepler* data (e.g. Wolter et al. (2009); Désert et al. (2011)). The high precision photometry allows some study of starspots for stars without a transiting exoplanet (Basri et al. 2011). The observational signature for starspots in this instance is periodic sinusoidal-like variability. This variability leads to the identification of stellar rotation periods and photospherically active regions (Harrison et al. 2012). However, despite the millimagnitude precision, insight into starspot properties is still limited by the one-dimensional nature of broad band photometry. In the case where a transiting exoplanet does exist and occults a starspot, the time series brightens during the time of transit (see Fig. 1.2) producing a *stroboscopic* effect. This effect refers to the deformations in the transit light curve depths that can be inverted to make maps of the stellar surface along the transit path. Long term monitoring of these occultations can provide information on starspot lifetimes and positions along with estimates on potential differential rotation (Désert et al. 2011).

### 1.2.2 Doppler Imaging

Doppler imaging (DI) is a spectroscopic technique that requires frequent observations of a star over one or ideally many rotation periods. Fig. 1.3 shows a schematic representation of the principle behind Doppler imaging. Cool starspots are detected as asymmetric distortions in certain spectral line features. The line profile is shallower where the starspot is located relative to rotation axis. This asymmetry moves from the blue to the red side of the line profile as the star's rotation carries the starspot from the preceding to the receding stellar limb. The starspot latitude is directly proportional to the velocity amplitude, or the length the distortion propagates through the line profile. The major limitations to Doppler imaging include precise information of stellar parameters, accurate stellar atmosphere models, and accurate atomic and molecular line lists. Inaccurate line profiles, rotational velocities, and stellar effective temperatures can lead to artifacts in the surface maps such as polar starspots and/or latitudinal starspot belts (Unruh & Collier Cameron 1997; Berdyugina & Tuominen 1998). While these concerns have been largely addressed (Unruh 1996; Rice 2002), a more direct method for imaging starspots would bolster confidence in the present results. A related technique known as Zeeman Doppler Imaging, introduced by Semel (1989) and



Figure 1.3: A schematic demonstrating the principle behind Doppler imaging. The dashed line indicates the rotationally broadened spectral line profile of an unspotted rapidly rotating star. The solid line indicates the effect on this spectral line as a cool starspot moves across the stellar surface. (Berdyugina 2005)

further developed by Donati et al. (1989), Semel et al. (1993), Brown et al. (1991), and Donati & Brown (1997), produces maps of the stellar magnetic field distribution as opposed to starspots. This is done by inverting the Stokes V parameter is an analogous way to traditional DI. Inverting the Stokes I parameter provides a surface map of the temperature distribution.

These techniques have provided a picture of starspots which in many cases is contrary to the behavior of spots on the Sun. In terms of lifetimes, the large starspots responsible for sinusoidal-like photometric variability can persist from months to years. For the Sun, typical sunspots live on average for only days to weeks. The covering factor, or percentage of the visible surface covered by spots, is far larger for active stars (10% to 50%) than for the Sun where the covering factor never exceeds 0.2% (Cox 2000). In addition, at times where the covering factor is largest, the overall luminosity of active stars decreases substantially  $(\Delta V \leq 0.6)$  whereas the overall luminosity of the Sun actually increases. The activity cycle in the Sun corresponds to the time from one period of sunspot minimum to the next. The cycle length ranges between 9 to 13.5 yrs with an average period of 11.1 yr. Activity cycles in active stars have been detected through photometric and Ca II emission variability. These cycles are periodic on time scales from 3 to 21 years although some active stars have been known to exhibit double periodic cycles or not cycle at all (Baliunas et al. 1995; Frick et al. 2004; Lockwood et al. 2007). Perhaps the most dramatic differences between starspot and sunspot behavior are the locations where the spots emerge on the photosphere. At the beginning of a solar cycle, sunspots appear an approximate latitude of 30° symmetric about the equator. As the cycle progresses, sunspots migrate toward the equator stopping at an approximate latitude of 8° (Babcock 1961 and references therein). Starspots have been observed to reside anywhere from low to high latitudes or at the poles (Strassmeier 2009a and references therein). Unfortunately imaging efforts do not yet have the temporal baseline to investigate starspot position as a function of activity cycle.

Models have been created to reconcile the differences in spot behavior between the Sun and active stars, particularly in formation location. A nonlinear flux tube instability has been used to explain high latitude starspots (Schuessler & Solanki 1992; Schuessler et al. 1996). For rapid rotators, the dynamo generating the magnetic field responsible for the starspots operates at the base of the convection zone providing the time necessary for the Coriolis force to carry the flux tube toward the pole as the tube is carried to the photosphere via magnetic buoyancy. Increasing the rotation rate has the effect of shifting the emergent starspots to higher latitudes with an absence of equatorial starspots (Granzer et al. 2000). However these models are not able to explain adequately polar sunspots in main-sequence

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stars. Berdyugina (2005) notes the flux tube concept in these models for heavily spotted stars implicitly assumes that the large identified starspots are not monolithic, but represent starspot groups. These groups are composed of Sun-like spots created by smaller flux tubes.

To help verify the present understanding of starspots and to explore magnetic activity from active stars to the Sun, a direct method of determining starspot properties is required. The best strategy would be to actually image the stellar surface. Fortunately this is possible via long baseline optical/infrared interferometry (LBI). By combining the light, akin to Young's double slit experiment, from multiple, widely spaced telescopes, angular resolutions down to less than 1 mas can be achieved. Images of stellar surfaces, rapidly rotating stars, binary stars, and star+disk systems are growing more commonplace over the past decade (Tuthill et al. 2001; Monnier et al. 2007; Kloppenborg et al. 2010; Che et al. 2011; Baron et al. 2012). Bright, convection-induced starspots have been imaged using LBI on the surfaces of Betelgeuse and T Per (Haubois et al. 2009; Baron et al. 2014). At present, the angular resolution of the longest baseline interferometer is ~0.4 mas in the H band. The median angular diameter for surveys of A, F, and G main sequence stars is 0.991 mas or ~ 2.5 resolution elements (Baines et al. 2008; Boyajian et al. 2012). Therefore this technique is currently only viable for giant stars and close early type dwarfs.

In Chapter 2, the photometric survey of  $\rho$  Ophiuchi cluster will be discussed. This will include descriptions of the sample selection, criteria for variability, and time series analysis methods for both periodic and long time scale variability. Chapter 3 will discuss both the morphology and potential mechanisms for the variability identified in  $\rho$  Oph. A primer on long baseline interferometry is located in Chapter 4 including how the presence of starspots will affect interferometric observables. Chapter 5 is a discussion of both the interferometric
and photometric observations obtained for  $\lambda$  Andromedae. This chapter also describes the two methods used to create images of the stellar surface and how starspot characteristics (e.g. size, location, temperature) are measured. Chapter 6 discusses the results from the observations spanning 2007 to 2011 including the potential tracing of stellar rotation via starspot motion. A complete summary of this dissertation is located in Chapter 7. -2 -

# THE PHOTOMETRIC SURVEY OF $\rho$ OPHIUCHI CLUSTER

This dissertation begins with a broad perspective on stellar variability. This perspective is gained through a high cadence, long baseline, multiwavelength photometric survey of young stars in the  $\rho$  Ophiuchi cluster. This survey has the potential to investigate numerous forms of stellar variability, many of which operate concurrently in a single star system (Herbst et al. 1994). This chapter discusses the observations and the stars to be surveyed for variability. It goes on to identify 3 methods for identifying stellar variability and describes the final variability catalog. The effect of the observing strategy on identifying fully and measuring the full amplitude of variability is explored. The chapter concludes with discussions on 2 methods used to identify variability timescales within the final variable catalog.

## 2.1 $\rho$ Ophiuchi Molecular Cloud

The environment of  $\rho$  Ophiuchi ( $\rho$  Oph) makes an excellent laboratory to test the ability of high cadence, long temporal near-IR observations to distinguish between variability mechanisms in young stars. The  $\rho$  Oph cluster is a dense star forming region containing a few hundred known young stellar objects (YSOs) with ages ranging from 0.3 to 3 Myr. The region is rich with variable stars; previous surveys having identified more than 100 photometrically variable stars (Greene & Young 1992; Barsony et al. 1997, 2005; Bontemps et al. 2001; Wilking et al. 2005; Alves de Oliveira & Casali 2008). Photometric surveys are limited to the near- to far-IR due to large amounts of visual extinction ranging from  $A_V = 5$  to 25 mag in the cloud core (Cambrésy 1999). This complex interstellar environment could itself be responsible for detected photometric variability. Plavchan et al. (2008b hereafter P08) carried out a pilot study of 57 stars in the direction of the  $\rho$  Oph field using photometry collected by the Two Micron All Sky Survey Calibration Point Source Working Database (2MASS Cal-PSWDB). That study identified periodic variability in two YSOs among a sample of candidate M stars. The study presented here expands on the initial pilot study performed in P08 and will include the full  $\rho$  Oph field data set from the 2MASS Cal-PSWDB to better understand the variability of young stars in this cloud.

## 2.2 Observations

The Two Micron All Sky Survey (Skrutskie et al. 2006 2MASS) imaged nearly the entire sky via simultaneous drift scanning in three near-infrared bands  $(JHK_s)$  between 1997 and 2001. Observations were taken at the northern Mt. Hopkins Observatory and the southern CTIO facility. Photometric calibration for 2MASS required hourly observations of 35 calibration fields split evenly between the northern and southern hemispheres. Each calibration field is 1° in length and 8.5′ wide. One calibration field lies in the direction of the Ophiuchus constellation. This field is centered at  $\alpha = 16^{h}27^{m}15.6^{s}$  and  $\delta = -24^{\circ}41'23''$  (J2000) and covers part of the  $\rho$  Oph L1688 cloud core (Bok 1956). These data have an observing cadence of ~1 epoch per day over an ~2.5 year temporal baseline. A complete observation is comprised of six consecutive 1.3 second scans in declination with a nearly constant right ascension. Each scan is offset by 5″ in right ascension to minimize errors from pixel effects. The six scans, or "scan group", are finally coadded to minimize short time scale and systematic variations. A complete scan group is obtained in approximately 8 minutes (Cutri et al. 2006 §III.2b). The maximum number of scans for a single star is 1584 divided by 6 or 264 scan groups. Photometry is extracted from the calibration field via the 2MASS Point Source Catalog (2MASS PSC) automated processing system. Details of the system's implementation are described in Cutri et al. (2006); here a brief summary is given. Photometry for sources fainter than J = 9, H = 8.5 and  $K_s = 8$  mag, are extracted by profile fitting. Profile fitting compares the source flux to a pregenerated point spread function (PSF) via  $\chi^2$  minimization. The PSFs are selected from a lookup table with respect to a dimensionless seeing index that is updated regularly during each scan. The seeing index characterizes the atmospheric seeing during specific observations. The library of PSFs is generated by empirically fitting the 50 brightest stars in a single 2MASS calibration scan with a specific average seeing index. This scan is not necessarily of the  $\rho$  Oph field, but a calibration field containing a different slice of the sky. An error at the few percent level may be present in the resulting photometry due to mismatched PSFs arising from rapid seeing variations.

For the few sources brighter than the above cut off magnitudes, photometry is performed using a 4" fixed aperture corrected using a curve of growth. Atmospheric seeing conditions can place as much as 15% of the flux from a point source outside this fixed aperture. A curve of growth correction is a constant factor added to the measured photometry to simulate measurements taken using an "infinite" aperture. The benefit of this method is avoiding decreased signal-to-noise and potential source confusion arising from large aperture photometry. However, curve of growth corrections assume the sources are unresolved single stars that can be approximated by a PSF. Therefore photometry for extended sources (i.e., stars embedded in bright nebular emission) or multiple systems are not properly characterized with this method. All the data scans are compiled in the 2MASS Cal-PSWDB.

## 2.3 Source Identification

The source selection in the  $\rho$  Oph field is similar to that described in P08, which is summarized here. A *parent sample* catalog of 7815 sources is constructed from a coadded deep image of the field (Cutri et al. 2006). For each target in the parent sample, the 2MASS Cal-PSWDB is searched for detections within a 2" matching radius. This radius is several  $\sigma$  larger than the 2MASS Point Source Catalog (PSC) astrometric precision and astrometric bias between the PSWDB and PSC (Zacharias et al. 2005; Skrutskie et al. 2006). This ensures confidence that all Cal-PSWDB detections for the parent 7815 sources are found within the PSC astrometric precision.

Of the 7815 stars identified in the parent sample, 1678 stars have a sufficient number of detections for variability and periodic analysis. This sample of 1678 stars is henceforth referred to as the *target sample*. A "sufficient number" is defined as stars detected in  $\geq 10\%$  of the observations in either J, H or  $K_s$  and  $\geq 50$  detections in the J band. The first constraint ensures a sufficient number of data points for a robust periodogram computation. The 10% limit is an *ad hoc* limit chosen to reduce the noise present in the variability statistics. The second constraint removes sources near the FOV edges that are not present in most scans.

Finally, despite the success of the 2MASS prescription to produce high quality photometric measurements, occasionally photometry affected by latent image artifacts, spurious detections and poor quality detections still persists in the database. The reader is referred to P08 for a full treatment on how sources with poor photometry are characterized and excluded. Cutri et al. (2006) describe the different varieties of latent image artifacts arising from a number of phenomenon associated with the optical system. These artifacts are identified and removed via visual inspection. Multiple simultaneous detections found within the 2" search radius of a target, which are typically spurious byproducts of the source extraction pipeline, are eliminated. Simultaneous detections are when two (or more) detected sources are identified with a single source in the 2MASS Cal-PSWDB. Secondary detections are typically ~0.5 - 1.5 magnitudes fainter than the primary detection. In addition, they are typically detected in only one passband and only in one of the six scans. Unaccounted for spurious detections can give the appearance of variability and introduce systematic noise into any underlying periodic signals. Photometric measurements with poor spatial fits to the model PSF are also excluded from our analysis. A poor spatial fit occurs when the  $\chi^2$  value between the observed stellar profile and a model PSF is > 10. This is flagged as "E" quality photometry within the Cal-PSWDB. Image saturation, cosmic rays, hot pixels, extended emission or partially resolved doubles could account for this poor quality fit to the photometry (Cutri et al. 2006). Photometry with poor spatial fits are systematically brighter by a few tenths of a magnitude, and this can falsely trigger the identification of variability.

## 2.3.1 Detection and Completeness Limits

For nonvariable stars, the photometric measurement uncertainty is characterized by the standard deviation of all photometric measurements in a particular band. P08 showed that this photometric standard deviation as a function of apparent magnitude, for 2MASS photometry, follows the form of two distinct power laws. One power law describes brighter sources, where Poisson statistics dominate the uncertainty, while the second describes the dimmer sources, where the uncertainty is dominated by instrumental noise. The point of intersection between these two power laws, or "break point", designates the survey completeness limit where source detection drops below 100%. This power law model is used to predict the photometric scatter for a star, and any star that has a dispersion significantly (>  $5\sigma$ ) above this is identified as a candidate variable. The model, as a function of apparent magnitude m, is given by the following expression:

$$10^{[\sigma_{m,model} \pm \nu_{m,model}(m)]} = b_{m,l} \pm \sigma_{b_{m,l}} + (a_{m,l} \pm \sigma_{a_{m,l}}) 10^{0.4m}$$
(2.1)

where  $a_{m,l}$ ,  $b_{m,l}$ ,  $\sigma_{a,m,l}$ , and  $\sigma_{b,m,l}$  represent the slope, intercept and respective errors for each fit in each band over magnitude region l. This model is first applied to our sample of 1678 stars using coefficients derived by P08 from the entire 2MASS Calibration Field data set. These coefficients, however, yield a relatively poor fit to the  $\rho$  Oph calibration field. The lower noise in the  $\rho$  Oph data is attributed to better average seeing conditions during these observations. As a result, the model is refit on the  $\rho$  Oph data set alone to derive a new set of coefficients. The new coefficients with errors are listed in Table 2.1. Fig. 2.1 shows the best fit model along with the observed photometric scatter in each band.

Table 2.1: Model Fit Parameters for Observed Photo-

metric Scatter

Band	Range	$a_{m,l} \pm \sigma_{a_{m,l}}$	$b_{m,l} \pm \sigma_{b_{m,l}}$
J	<16.63	$(3.046 \pm 0.043) \times 10^{-8}$	$1.01326 {\pm} 0.00087$
J	>16.63	$(1.484 \pm 0.083) \times 10^{-8}$	$1.08343 {\pm} 0.00046$
H	$<\!\!15.75$	$(6.467 \pm 0.055) \times 10^{-8}$	$1.01444 {\pm} 0.00041$
H	>15.75	$(4.03 \pm 0.16) \times 10^{-8}$	$1.0628 {\pm} 0.0059$
$K_s$	$<\!15.10$	$(1.2247 \pm 0.0094) \times 10^{-7}$	$1.0134{\pm}0.0036$
$K_s$	>15.10	$(4.98 \pm 0.25) \times 10^{-8}$	$1.0934 \pm 0.0042$



Figure 2.1: Photometric standard deviation versus apparent magnitude derived from up to 1584 observations of each sample star; there are 1678 sample stars in total. The solid red line corresponds to the photometric model fit to this sample. The dashed green line marks the break magnitude, where the detection rate drops below 100%, in each band. The break magnitudes are J = 16.63, H = 15.75, and  $K_s = 15.10$  mag.

The model yields completeness limits for this survey of 16.63, 15.75 and 15.10 mag in J, H and  $K_s$ , respectively. These are significantly fainter limits than the 2MASS PSC as a whole, which are 15.8, 15.1 and 14.3 mag in J, H and  $K_s$ , respectively. The approximate detection limits for this study, found by averaging the apparent magnitudes for the 10 faintest objects meeting our detection criteria, are 17.7, 16.7 and 16.0 mag in J, H and  $K_s$  respectively.

#### 2.4 Selection Criteria for Variability

Numerous surveys have used time series analysis on multiwavelength photometry to characterize young star variability (Mathieu et al. 1997; Carpenter et al. 2001, 2002; Grankin et al. 2007, 2008; Morales-Calderón et al. 2011; Findeisen et al. 2013; Wolk et al. 2013 and references therein). The methods for identifing stellar variability are nearly as numerous as the variability studies themselves. These include, but are not limited to, the Stetson index, excess photometric dispersion,  $\chi^2$  statistic, crosscorrelation and Fourier analysis (Stetson 1996; Carpenter et al. 2001; Barsony et al. 2005; Alves de Oliveira & Casali 2008).

Variable stars are identified in this work through 3 complementary methods that are sensitive to different types of variability. A full description of these techniques are presented in P08. The same terminology used in P08 is adopted in this work. Here a summary is presented along with specifics regarding this sample. The first and second methods, "flickering" and "excursive", identify variability in each band individually. The third method uses the Stetson index to identify correlated variability between bands.

# 2.4.1 Flickering Variability

Flickering variability describes when the star's photometric scatter significantly differs from the predicted scatter. Flickering variability is sensitive to continuous variability, as consistent, substantial variations are needed to significantly increase the observed photometric dispersion above the expected nonvariable value. To identify flickering variables, an observed dispersion is calculated for all scan measurements of a star prior to conbining as a scan group. This is then compared to the star's expected dispersion with associated uncertainty,  $\sigma$ , calculated using the noise model described in § 2.3.1 (Eqn. 2.1 and Fig. 2.1). If the observed dispersion exceeds the expected dispersion by more than  $5\sigma$ , the star is a candidate variable. This search is done separately for each of the 3 bands  $(J,H,K_s)$ ; a star can thus be flagged as a flickering variable in 1, 2 or all 3 bands. Following this criterion, 17 stars flag in only a single band, 23 flag in two bands and 54 flag in all three bands. If variability is intrinsic to the star, the expectation is the flickering will occur in more than one band. Low signal-tonoise photometry (the number of observations,  $N_{obs}$ , is typically less than 500) might likely account for the 9 candidate variables that flicker in the K band only. It might also account for the 11 candidate variables that flicker in both the H and  $K_s$  bands. However there is no obvious explanation for the 17 candidate variable stars that flicker only in the J or H, J and H, or J and  $K_s$  bands. The average dispersions for these variable stars in J, H and  $K_s$  are  $0.12 \pm 0.46$ ,  $0.12 \pm 0.43$  and  $0.11 \pm 0.35$  mag, respectively. The listed errors are the standard deviation of the average dispersion. These values represent the dispersion intrinsic to the source, or specifically the dispersion after the predicted nonvariable measurement dispersion is subtracted in quadrature from the observed dispersion.

# 2.4.2 Excursive Variability

Excursive variability describes when the average magnitude of a individual scan group is significantly deviant from the mean of all the star's scan groups. Excursive variability is sensitive to short time scale variations such as a single eclipse event or flare. Excursive candidate variables are identified if the average magnitude for a single scan group exceeds the global mean by more than  $5\sigma$ , where here  $\sigma$  is the coadded uncertainty in the scan group photometry. As with flickering variability, this search is done separately for each of the 3 bands. From the final variable catalog, 21 stars flag in only a single band, 19 flag in two bands and 41 flag in all three bands. Low signal-to-noise photometry (N<sub>obs</sub> typically less than 500) might account for the 10 candidate variables that are excursive in the  $K_s$  band only. It might also account for the 12 candidate variables that are excursive in both the *H* and  $K_s$  bands. However there is no obvious explanation for the 19 candidate variable stars that are excursive only in the *J* or *H*, *J* and *H*, or *J* and  $K_s$  bands. The average number of deviant scan groups per star in our variable catalog is 24, 42 and 57 in J, H and  $K_s$  bands, respectively.

## 2.4.3 Welch-Stetson Index

The Welch-Stetson index (Welch & Stetson 1993) describes the correlation in a star's photometric variation between different bands. The Welch-Stetson index is sensitive to variability whose amplitude is not significantly different between photometric bands. For example, the Welch-Stetson index is not sensitive to a strong increase in the  $Br\gamma$  emission line strength that might only affect 1 band. This index has been previously used on other molecular cloud 2MASS variability surveys in Orion A and Chameleon I (Carpenter et al. 2001, 2002). The Welch-Stetson index is computed for all 1678 stars; a star is considered a candidate variable if this index is > 0.2. P08 determined this criterion based on 18 of 23 periodic variables, in that work, having indices above this value. The same index is adopted here since the observing methodology is identical in both works. This index is smaller than those adopted for the Orion A (0.55) and Chameleon I (1.00) surveys. The Orion A survey contained 29 epochs over a 36 day temporal baseline and the Chameleon I survey contained 15 epochs over 5 months. The smaller number of observed epochs in each case causes these surveys to be less sensitive to variability and thus in need of a higher index. A Welch-Stetson index of zero indicates random noise or no correlation between the photometry in different bands. A positive index indicates correlation between the photometry in two bands. The higher the index, the greater the correlation between the photometry. Using the Welch-Stetson index 57 stars flag as variable.

### 2.4.4 Excluding Seeing Induced Variables

A common way in which a nonvariable star is misidentified as variable is from photometric variations caused by changing atmospheric seeing. Both photometric techniques used here (PSF fitting, fixed sums) are susceptible to this, especially in regions that are crowded or where there is bright nebular emission. Seeing estimates, corresponding to the average FWHM for each calibration scan, are provided for the Cal-PSWDB photometry. The typical seeing values range between 2.5" to 2.7" over the entire observing season (Cutri et al. 2006).

The possibility of changes in brightness being correlated with changes in the seeing are first investigated. This is done by computing the Pearson r-correlation statistic for each star, *n*. The statistic is given by the following:

$$r_{n} = \frac{\sum_{t=1}^{N_{m,n}} (m_{n,t} - \overline{m}_{n}) (S_{m,t} - \overline{S_{m}})}{\sqrt{\sum_{t=1}^{N_{n,t}} (m_{n,t} - \overline{m}_{n})^{2}} \sqrt{\sum_{t=1}^{N_{n,t}} (S_{m,t} - \overline{S_{m}})^{2}}}$$
(2.2)

where *m* is the band,  $S_{m,t}$  is the *m*-band seeing FWHM in arcseconds at epoch *t* and  $\overline{S}$  is the average seeing in *m*-band. The separate quantities are summed over all  $N_{m,n}$  *m*-band observations for star, *n*. This statistic spans the range from -1 to 1 with negative values indicating inversely correlated variations and positive values corresponding to directly correlated variations. An inverse correlation means as the seeing worsens the star gets brighter. A direct correlation refers to the opposite effect. Since in Eqn. 2.2, the photometry comparison (numerator) is computed in magnitudes and the photometric standard deviation (denominator) is computed first in flux units then converted to magnitudes, this can result in *r* values slightly outside the -1 to 1 range. A slight trend exists in the sample of 1678 stars toward an inverse seeing correlation in each band. The average *r* statistics in *J*,

H and  $K_s$  are -0.12, -0.11 and -0.05, respectively. Inverse correlation is likely caused by crowded fields where as the seeing worsens, flux from surrounding stars may encroach into the measured star's aperture or spatial profile. While these correlations are not very significant in most cases, it is noted the seeing in one band is slightly correlated with the seeing in another band. This is consistent with multiband photometry taken simultaneously. To look for correlations between bands, the Pearson index for J and H are plotted in Fig. 2.2. To characterize and flag seeing induced variability, a single seeing test is constructed to provide an estimate of seeing effects on measured photometry. Each correlation statistic  $(r_J, r_H, r_K)$  is considered a component of a single "seeing vector". This vector is rotated and transformed from cartesian to cylindrical coordinates so the z-axis corresponds to  $r_J = r_H = r_K$ . This representation causes the seeing correlation to be axisymmetric about the z-axis, thus reducing the characterization of multi-band seeing correlation by one dimension. A "seeing ellipse" is described by

$$\frac{z_n^2}{\sigma_z^2} + \frac{\rho_n^2}{\sigma_\rho^2} = 1 \tag{2.3}$$

where  $z_n$  is the component of the seeing vector for star n, with standard deviation  $\sigma_z$ , along the z-axis.  $\rho_n$  is the component for star n along the  $\rho$ -axis, with the standard deviation  $\sigma_{\rho}$ . Both  $\sigma_z$  and  $\sigma_{\rho}$  are determined from the distribution of the ensemble 1678 stars. A candidate variable is flagged as seeing correlated when the seeing vector length is larger than the seeing ellipse for the ensemble. This is the case when the left hand side of Eqn. 2.3 is greater than unity. Of the 1678 stars, 19 stars fail this test indicating the variability of these



Figure 2.2: The seeing correlation between the J and H bands for the 1678 sample stars. The dashed green line is a 1:1 correlation between the seeing correlation in the J band,  $r_J$ , and H band,  $r_H$ . The line also corresponds to the projected z-axis as described in the text just prior to Eqn 3

stars is likely solely caused by fluctuations in atmospheric seeing. These 19 stars are not included into the final variable catalog.

# 2.4.5 Final Variable Catalog

From the target sample of 1678 stars, 101 stars (6%) are identified as variable. These variable stars are referred to as the *variable catalog*. The variable catalog is listed in Table 2.2. The full set of light curves, color curves and color-color plots for all variables stars is only available online. For inclusion into the variable catalog, a star must not exhibit seeing correlated photometry (see § 2.4.4) and must meet 2 of the 7 variability criteria (see § 2.4.1 - 2.4.3). In addition, the 2 criteria must be met in different bands or in a single band along with the Welch-Stetson criterion. This last condition is imposed in order to prevent identifying

variability due to poor quality or spurious photometry that is missed by the previous filters. The amplitudes of variability for stars within the variable catalog span a wide range. The range in  $\Delta K_s$  spans 0.04 to 2.31 mag and  $\Delta(H-K_s)$  varies from 0.01 to 1.62 mag. The variable catalog contains 47 stars with  $\Delta K_s > 0.25$  mag and 66 stars with  $\Delta(H-K_s) > 0.1$ mag.

$\begin{array}{c} \mathrm{RA}^{\mathrm{a}}\\ \mathrm{(degrees)} \end{array}$	${ m Dec}^{ m a}$ (degrees)	Variability Flags <sup>b</sup>	$\begin{array}{c} \Delta \mathbf{K}_s \\ (\mathrm{mag}) \end{array}$	$\begin{array}{c} \Delta(J\text{-}H)\\ (\text{mag}) \end{array}$	$\begin{array}{c} \Delta(H\text{-}K_s) \\ (\text{mag}) \end{array}$	Type	YSO Class <sup>c</sup>	'On Cloud' <sup>d</sup>
246.732269	-25.137508	1101010	0.114	0.150	0.123	irregular		no
246.736557	-24.230934	1111111	0.476	0.235	0.173	periodic		ves
246.737396	-24.922001	1000010	0.167	0.153	0.180	irregular		no
246.738800	-24.594057	0010010	0.166			irregular	II	yes
246.739273	-24.883030	0001010	0.160	0.105	0.139	irregular		no
246.739410	-24.880718	1111110	0.854			irregular		no
246.741165	-24.964417	0001100	0.108	0.093	0.124	irregular		no
246.743301	-24.358301	1111111	0.292	0.140	0.257	periodic	II	yes
246.743774	-24.760160	1111111	0.500	0.415	0.270	LTV	II	yes
246.744202	-24.767410	0010100	0.131	0.198	0.168	irregular		yes
246.744324	-24.309591	1111111	0.294	0.241	0.180	irregular	II	yes
246.746017	-24.599096	1111111	0.218	0.185	0.380	LTV	II	yes
246.746155	-24.787884	1111110	1.109			irregular		yes
246.746490	-24.582909	0000010	0.631			irregular	Ι	yes
246.748657	-24.261997	1111100	0.078	0.124	0.073	irregular		yes
246.752335	-24.273695	1100000	0.061	0.082	0.066	irregular		yes
246.752975	-24.774199	1111101	0.061	0.091	0.084	LTV		yes
246.756760	-24.360228	1111111	0.078	0.074	0.180	LTV	III	yes
246.759720	-24.624172	1111100	0.082	0.083	0.176	irregular		yes
246.761093	-24.776232	1101001	0.049	0.061	0.122	LTV		yes
246.761490	-25.045506	1100000	0.053	0.062	0.083	irregular		no
246.761902	-24.315140	0001110	0.060	0.086	0.064	irregular		yes
246.764984	-24.334810	1110111	0.894		0.707	LTV	II	yes
246.767090	-24.474903	1110111	0.294	0.275	0.523	LTV	II	yes
246.768814	-24.716524	1111111	0.084	0.051	0.065	periodic	III	yes

Table 2.2: Catalog of Variable Stars in  $\rho$  Oph

Table 2.2 – Continued

RA <sup>a</sup>	$\mathrm{Dec}^{\mathrm{a}}$	Variability Flags <sup>b</sup>	$\Delta K_s$	$\Delta(J-H)$	$\Delta(H-K_s)$	Type	YSO Class <sup>c</sup>	'On Cloud' <sup>d</sup>
(degrees)	(degrees)		(mag)	(mag)	(mag)			
246.768997	-24.703840	1111111	0.086	0.047	0.050	periodic	III	yes
246.769058	-24.454285	1111111	0.411	0.232	0.329	periodic	II	yes
246.770874	-25.146795	1100000	0.052	0.084	0.077	irregular		no
246.771545	-24.335421	1110000	0.049	0.079	0.078	irregular		yes
246.773605	-24.670246	0010010	0.337			periodic	II	yes
246.774628	-24.993830	1100000	0.044	0.078	0.055	irregular		no
246.774902	-24.476698	0000110	0.182		0.157	irregular	II	yes
246.777481	-24.696856	1111111	0.224	0.128	0.099	periodic	II	yes
246.778244	-24.637451	0110111	1.125		1.065	LTV	Ι	yes
246.784149	-24.707903	1001001	0.069	0.061	0.195	irregular		yes
246.787857	-24.200172	1111111	0.508	0.242	0.204	periodic	II	yes
246.787964	-24.568890	1111111	0.330	0.137	0.102	periodic	II	yes
246.788971	-24.672836	0110110	0.195		0.119	LTV	II	yes
246.789230	-24.621819	1111111	1.636		0.628	periodic	Ι	yes
246.791824	-24.486958	1111111	0.334		0.366	irregular	II	yes
246.792862	-24.320118	1110010	0.198	1.272	0.098	LTV	II	yes
246.795700	-24.758245	1000001	0.056	0.073	0.074	irregular		yes
246.796570	-24.679569	1110111	0.984		1.123	LTV	II	yes
246.798706	-24.394924	1111001	0.058	0.067	0.122	LTV	III	yes
246.798828	-24.642199	0110111	1.012		0.784	LTV	II	yes
246.798920	-24.786337	1010001	0.069	0.079	0.074	irregular		yes
246.800537	-24.580280	1111111	0.729	0.346	0.402	periodic	II	yes
246.803055	-25.067175	1111111	0.339	0.157	0.112	periodic		no
246.807236	-24.304626	1111111	0.155	0.085	0.090	irregular	II	yes
246.807388	-25.095842	1110000	0.059	0.065	0.091	irregular		no
246.807602	-24.725399	1111111	0.560	0.249	0.133	LTV	II	yes
246.807755	-24.262215	0110000	0.205	—	0.291	irregular		yes

Table 2.2 – Continued

RA <sup>a</sup>	$\mathrm{Dec}^{\mathrm{a}}$	Variability Flags <sup>b</sup>	$\Delta K_s$	$\Delta(J-H)$	$\Delta(H-K_s)$	Type	YSO Class <sup>c</sup>	'On Cloud' <sup>d</sup>
(degrees)	(degrees)		(mag)	(mag)	(mag)			
246.808533	-24.252649	1110000	0.109	0.200	0.149	irregular		yes
246.813034	-24.860764	1111111	0.330	0.213	0.111	periodic		no
246.813858	-24.264278	1010000	0.069	0.099	0.088	irregular		yes
246.814423	-24.444342	0110111	0.562		0.850	LTV	II	yes
246.814636	-24.514885	0010010	0.445			LTV	II	yes
246.815674	-24.645327	1111111	0.299	0.262	0.297	LTV	II	yes
246.816071	-24.645321	1111111	0.213	0.258	0.305	periodic/LTV	II	yes
246.816208	-24.420513	0110010	0.356			periodic	II	yes
246.816925	-24.250999	0110000	0.352		0.534	irregular		yes
246.816971	-24.271143	1010000	0.391		0.517	irregular		yes
246.821976	-24.374475	0110010	0.163		1.137	LTV	0	yes
246.822739	-24.218828	0100000	0.155	0.217	0.315	irregular		yes
246.823074	-24.482300	0110111	0.728		0.813	LTV	Ι	yes
246.826492	-24.914923	1111111	0.807	0.354	0.327	periodic		no
246.826584	-24.407238	0110111	0.135		0.371	periodic/LTV	III	yes
246.826599	-24.654037	1110111	0.528		0.009	periodic	Ι	yes
246.827026	-24.484921	1111111	0.640	0.161	0.186	periodic	II	yes
246.831299	-24.694487	0001111	0.092	0.057	0.071	periodic	III	yes
246.839462	-24.695250	1111111	0.215	0.276	0.158	irregular	II	yes
246.840378	-24.363819	0100000	0.050	0.184	0.069	irregular	III	yes
246.840836	-24.498091	0110111	1.199		1.256	LTV	Ι	yes
246.840942	-24.726538	0010000	0.062		0.056	periodic	III	yes
246.843735	-25.126837	1111111	0.700		0.751	irregular		no
246.845520	-24.299223	1111111	0.130	0.044	0.071	periodic	III	yes
246.845718	-24.801941	1111111	0.134	0.097	0.062	LTV		yes
246.846909	-24.809896	1001001	0.074	0.099	0.094	irregular		yes
246.848297	-24.207954	1111001	0.052	0.066	0.080	LTV		yes

Table 2.2 – Continued

$RA^{a}$ (degrees)	$\frac{\rm Dec^{a}}{\rm (degrees)}$	Variability Flags <sup>b</sup>	$\begin{array}{c} \Delta \mathbf{K}_s \\ (\mathrm{mag}) \end{array}$	$\begin{array}{c} \Delta(J\text{-}H)\\ (\mathrm{mag}) \end{array}$	$\begin{array}{c} \Delta(H\text{-}K_s) \\ (\text{mag}) \end{array}$	Type	YSO Class <sup>c</sup>	'On Cloud'd
246.852676	-24.684278	0010010	0.749			LTV	Ι	yes
246.852737	-24.493141	0010010	0.094		0.327	periodic		yes
246.854782	-24.775953	0100101	0.065	0.089	0.173	LTV		yes
246.855560	-25.105873	1111111	0.402	0.272	0.295	periodic		no
246.859329	-24.323023	0111111	0.326	0.432	0.175	irregular	II	yes
246.859558	-24.712914	0010010	0.098		0.200	periodic	Ι	yes
246.860397	-24.656380	1111111	0.318	0.185	0.207	periodic	II	yes
246.860779	-24.431711	1111111	0.211	0.099	0.096	periodic	II	yes
246.862335	-24.680904	0110110	0.393		0.927	LTV	Ι	yes
246.862778	-24.538191	0010010	0.090	—	0.478	periodic		yes
246.864105	-24.521235	1111111	0.305	0.121	0.082	periodic	II	yes
246.866684	-24.659260	0110111	0.784	—	0.549	periodic	Ι	yes
246.872681	-24.654474	1111111	2.312		1.318	LTV	Ι	yes
246.875793	-24.462006	1111111	0.155	0.110	0.233	LTV	II	yes
246.877228	-24.542961	1100000	0.070	0.075	0.082	irregular		yes
246.878510	-24.790745	1110101	0.067	0.057	0.057	periodic	III	yes
246.878571	-24.415533	1111111	0.282	0.106	0.160	LTV	II	yes
246.878784	-24.459188	0010000	0.926			LTV	Ι	yes
246.879166	-25.065256	1111110	1.057			irregular		no
246.879456	-24.567505	1111001	0.058	0.054	0.071	periodic	III	yes
246.880157	-25.071445	1111110	0.566	0.386	0.473	irregular		no
246.883682	-25.148535	1110000	0.613	0.318	0.679	irregular		no

Table 2.2 – Continued

$\begin{array}{cccc} RA^{a} & Dec^{a} & Variability Flags^{b} & \Delta K_{s} & \Delta (J-H) & \Delta (H-K_{s}) & Type & YSO \ Class^{c} & On \ Omega{} \\ (degrees) & (degrees) & (mag) & (mag) \end{array}$
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<sup>&</sup>lt;sup>a</sup>Epoch 2000

<sup>&</sup>lt;sup>b</sup>First three flags correspond to flickering variability. The second three flags correspond to excursive variability. The seventh flag corresponds to the Stetson Index. Flag is set to 1 when true; 0 otherwise.

<sup>&</sup>lt;sup>c</sup>(Bontemps et al. 2001; Gutermuth et al. 2009) <sup>d</sup> "On Cloud" indicates the star is north of  $\delta = -24^{\circ}51'$ . The amount of visual extinction exceeds  $A_V = 5$  north of this demarcation.

Fig. 2.3 contains the coadded calibration field in the direction of  $\rho$  Oph; the target sample of 1678 stars and the variable catalog of 101 stars are plotted to show their spatial distribution. It is clear that target stars are not evenly distributed in the field. A demarcation line at  $\delta = -24^{\circ}51'$  is set as an ad hoc determination of cloud membership. North of this limit is considered "on cloud" while anything south is classified as "off cloud". This demarcation corresponds roughly to where  $A_V = 5$  mag (Cambrésy 1999). Comparing the variability north and south of this demarcation, the "on-cloud" variable fraction increases to 15% while the variable fraction for the "field" drops to a mere 1%. This is consistent with the expectation young stars are are more often found spatially close to molecular clouds and are more variable than field stars.



Figure 2.3:  $\rho$  Ophiuchi field. A: The field is split into "North" and "South" panels. The 1678 source sample is overlaid in yellow. B: The same field overlaid with all variable sources. Green - periodic variables (§5.1). Red - time scale variables (§5.2). Yellow - irregular variables (§5.3). C: Same field overlaid with all classified YSO sources (§3.3). Yellow - Class I. Green - Class II. Red - Class III. The green line in all "South" panels represents a demarcation at  $\delta$ : -24° 51′ where  $A_V = 5$  mag (Cambrésy 1999). North of this demarcation contains higher visual extinction.

## 2.5 Known Young Stars in the $\rho$ Oph Field

Clues to the formation and evolution of young stars may be revealed by relating the variability to the stars' evolutionary states. As originally proposed by Lada (1987), young stars are classified into four evolutionary stages or *classes* (Class 0, Class I, Class II and Class III). Class assignment is typically based on photometry through the infrared slope index in the wavelength range from 2 to 25  $\mu$ m. Class 0 stars represent cloud cores undergoing the initial stages of protostellar collapse. Class I stars are heavily embedded protostars with infalling material from a circumstellar envelope forming an accretion disk. Class II stars are fully assembled stars with accretion primarily from the circumstellar disk channeled onto the star along magnetic field lines; classical T Tauri (CTTS) stars are another name for Class II stars. The last stage, Class III, represents stars yet to reach the main-sequence with depleted or no accretion disks due to mass accretion onto the star, photoevaporation, or planet formation. They may nevertheless retain debris disks or disks with depleted inner holes. These stars are also known as weak lined T Tauri stars (WTTS).

To identify if any of the 1678 sample stars have a previously assigned evolutionary class, the sample is cross-referenced with the  $\rho$  Oph L1688 cloud core mid-IR surveys by Bontemps et al. (2001 hereafter, B01) and Gutermuth et al. (2009 hereafter, G09). The measurements obtained by B01 were taken with the *ISO* ISOCAM LW2 and LW3 broad band cameras centered on 6.7  $\mu$ m and 14.7  $\mu$ m, respectively. The G09 survey obtained measurements in the *Spitzer* IRAC 3.6, 4.5, 5.8 and 8.0  $\mu$ m bands. For stars with no detections in either 5.8 or 8.0  $\mu$ m, the photometric measurements were complemented by *J*, *H* and *K<sub>s</sub>* 2MASS data. In addition, 24  $\mu$ m *Spitzer* MIPS data is also used to verify YSO classifications in cases with high SNR ( $\sigma < 0.2$  mag) and star luminosity ([24] < 7 mag).

The B01 survey provides YSO classifications for 54 of the 1678 target sample stars, while G09 provides classifications for 58 stars. However, overlapping targets between these surveys results in 40 stars classified by both B01 and G09, yielding YSO classifications for only 72 target sample stars. For 5 stars classified by both B01 and G09, the two surveys disagree on the classification. G09 classifies these 5 stars as belonging in an earlier evolutionary stage by one class than B01 (i.e., WL 22 is classified as Class I by G09 and a Class II by B01). In these cases, the classification by G09 is adopted because of the broader wavelength coverage utilized. Assuming the B01 survey identified all the young stellar objects in the  $\rho$  Oph region (425 YSOs), this survey contains ~17% of these YSOs. Of the 72 stars with YSO classifications, 79% are identified as variable stars. As a function of YSO class, 92%of both Class I (12 of 13) and Class III (11 of 12) are variable stars. The variable fraction decreases to 72% (34 of 47) for Class II stars. The majority (14 stars) of the nonvariable YSOs are Class II while ISO-Oph 99 is Class I. All of these stars are located "on cloud". As a YSO evolves in time the median brightness and color variability amplitudes decrease. The median peak-to-trough  $\Delta K_s$  amplitude for Class I, II, and III stars are 0.77, 0.31, and 0.08 mag, respectively. The median peak-to-trough  $\Delta(H-K_s)$  color amplitudes are 0.81, 0.21, and 0.07 mag for each class, respectively.

## 2.6 Advantages of High Cadence Variability Studies

In this section, the advantages of high cadence, long temporal baseline observations in variability studies are investigated. The results of this work are compared to the Alves de Oliveira & Casali (2008 hereafter AC08) survey of the  $\rho$  Oph central cloud core. The AC08 survey searched for variability in thousands of target stars within a ~0.8 deg<sup>2</sup> field of view. These stars were observed in the H and  $K_s$  bands during 14 epochs spanning May, June and July 2005 and 2006. The magnitudes of target stars fell within 11 to 19 mag in H and 10 to 18 mag in  $K_s$ .

This survey and AC08 have 464 stars in common. The prescription for identifying variables in AC08 is based on  $\chi^2$  fitting and crosscorrelations between the H and K<sub>s</sub> photometry. Comparing the number of variables detected from the 464 stars, AC08 identifies 32 (7%) variables while this work identifies 82 (18%). The larger fraction of detected variables by this survey could be attributed to the higher sampling over a longer temporal baseline or from different sensitivities in the adopted variability criteria. To determine which explanation is more probable, histograms of the  $\Delta K_s$  peak-to-trough amplitudes for the variables identified by both this work and AC08 within the joint 464 star sample are computed. Fig. 2.4 contains these histograms as well as the histograms for the  $\Delta(H-K_s)$  peak-to-trough color amplitudes. It is clear from Fig. 2.4 the fraction of variables with  $\Delta K_s < 0.5$  mag detected by each survey is nearly identical. The same is true for variables with  $\Delta(H-K_s) < 0.55$  mag. Therefore, the higher fraction of variables detected, as compared to AC08, is most likely a consequence of the higher observing cadence. It is worth noting that 7 stars within the joint sample are identified as variable by AC08, but are not in this work. This work identified 5 of these stars as having photometry correlated with seeing. Therefore these stars may have been intrinsically variable within the observing window, however this variability could not be confidently confirmed.

While the detection fraction of low amplitude variables is nearly identical between surveys, the detection fraction of high amplitude variability stars is not. AC08 does not detect variables with  $\Delta K_s > 0.7$  mag or  $\Delta (H-K_s) > 0.55$  mag. This work finds 5.25% of detected



Figure 2.4: Top: Histograms of  $\Delta K_s$  for AC08 (red forward hatching) and this work (green backward hatching). The two distributions are statistically indistinguishable. Bottom: Histograms of  $\Delta(H-K_s)$  for AC08 and this work. Each survey is represented the same as the top plot. This work detects larger amplitudes in both  $\Delta K_s$  and  $\Delta(H-K_s)$  than AC08.

variables have  $\Delta K_s$  amplitudes greater than these upper limits. In addition, 6% of detected variables have  $\Delta(H-K_s)$  color amplitudes greater than these upper limits. Within the 464 star joint sample, 25 stars are identified as variable in both this work and AC08. Strong correlations exist between the difference in amplitudes measured between surveys and the amplitudes measured in this work (see Fig. 2.5). Sparsely sampled photometry will underestimate the amplitude of variability in both magnitude and color.

High cadence, long temporal baseline observations are vital for fully characterizing the variability of young stars. It increases the detection fraction of the survey allowing for more accurate statistics, such as the incidence of variable stars and distribution of variability amplitudes. In addition, this strategy is needed to sample the full amplitude of variability.



Figure 2.5: The differences between the light and color amplitudes measured by AC08 and this work. *Top:* The comparison between measured  $K_s$  variability. *Bottom:* The comparison between measured  $(H-K_s)$  color variability. In both cases, there is good agreement between the surveys for low amplitude variability. However as the amplitude increases, AC08 underestimates the variability. The dashed line in both plots indicates a difference of zero.

### **2.7** New Candidate $\rho$ Oph Members

As photometric variability is an ubiquitous characteristic of young stars, it is a useful tool for assessing youth and potential membership in the  $\rho$  Oph star forming region. However, variability alone is not sufficient evidence for identifying potential members and additional constraints are needed, such as spatial location and location on a color-magnitude diagram. Candidate  $\rho$  Oph membership is first determined by crossreferencing the final variable catalog with previous surveys to identify previously known  $\rho$  Oph members (Strom et al. 1995; Barsony et al. 1997, 2005; Grosso et al. 2000; Ozawa et al. 2005; Wilking et al. 2005; Pillitteri et al. 2010). These are the same surveys used by AC08 to assign membership to their variable stars. This identifies 62 of the 101 variable stars as confirmed members of  $\rho$  Oph, which are plotted on a  $K_s$  versus (*H*- $K_s$ ) color-magnitude diagram in Fig. 2.6. For comparison, the 53 variable stars determined as  $\rho$  Oph members by AC08 are also plotted. Both surveys identify 11 as  $\rho$  Oph members. A dashed line connects the data for these stars as observed by AC08 and this work. The solid black line indicates a 3 Myr isochrone constructed using NextGen models for masses between 0.02 to 1.4  $M_{\odot}$  at a distance of 129 pc (Baraffe et al. 1998). The distance is the weighted average between previous measurements (Loinard et al. 2008; Mamajek 2008). A star is classified as a new candidate member if it is located "on cloud" (see § 2.4.5) and is brighter and redder than the 3 Myr isochrone (see Fig. 2.6). Table 2.3 contains the 22 stars identified as candidate  $\rho$  Oph members from the previously unassociated 39 stars. Candidate member 2MASS J16270597-2428363 is classified as a Class II YSO thereby increasing the likelihood of membership. Follow up spectroscopic observations in the mid-IR for the remaining candidates to determine whether these stars are YSOs will provide additional evidence for membership.

RA (dograda)	Dec	Catalog ID <sup>a</sup>	J <sup>b</sup>	H <sup>b</sup>	$K_s^{\rm b}$	(J-H)	$(H-K_s)$
(degrees)	(degrees)		(mag)	(mag)	(mag)	(mag)	(mag)
246.744202	-24.76741	65861-2446029	$15.392{\pm}0.001$	$14.000 \pm 0.001$	$13.364 {\pm} 0.001$	1.392	0.636
246.746155	-24.787884	70054-2446444	$16.935 {\pm} 0.032$	$16.255 {\pm} 0.011$	$15.567 {\pm} 0.012$	0.680	0.688
246.748657	-24.261997	65967 - 2415433	$14.229 {\pm} 0.001$	$12.404{\pm}0.001$	$11.642 {\pm} 0.001$	1.825	0.762
246.752335	-24.273695	70055 - 2416255	$13.432{\pm}0.001$	$11.840 {\pm} 0.001$	$10.997 {\pm} 0.001$	1.592	0.842
246.752975	-24.774199	70072-2446272	$13.670 {\pm} 0.001$	$12.002 \pm 0.001$	$11.233 {\pm} 0.001$	1.668	0.769
246.761093	-24.776232	70266-2446345	$13.348 {\pm} 0.001$	$11.596{\pm}0.001$	$10.665 {\pm} 0.001$	1.752	0.930
246.761902	-24.31514	70285-2418546	$13.090 {\pm} 0.001$	$11.049 {\pm} 0.001$	$10.096 {\pm} 0.001$	2.041	0.953
246.771545	-24.335421	70516 - 2420077	$12.700{\pm}0.001$	$10.440 {\pm} 0.001$	$9.341 {\pm} 0.001$	2.260	1.099
246.774902	-24.476698	70597 - 2428363	$16.905 {\pm} 0.005$	$14.467 {\pm} 0.001$	$13.029 {\pm} 0.001$	2.438	1.439
246.784149	-24.707903	60819-2442286	$15.365 {\pm} 0.001$	$12.252 {\pm} 0.001$	$10.723 {\pm} 0.001$	3.113	1.529
246.795700	-24.758245	71096 - 2445298	$13.011 {\pm} 0.001$	$11.056 {\pm} 0.001$	$10.156 {\pm} 0.001$	1.955	0.900
246.807755	-24.262215	71384 - 2415441	$16.760 {\pm} 0.005$	$15.065 {\pm} 0.002$	$14.251{\pm}0.002$	1.696	0.814
246.808533	-24.252649	71404 - 2415096	$15.356 {\pm} 0.002$	$13.899 {\pm} 0.001$	$13.274{\pm}0.001$	1.457	0.625
246.813858	-24.264278	71531 - 2415515	$13.990{\pm}0.001$	$12.532{\pm}0.001$	$11.865 {\pm} 0.001$	1.458	0.667
246.816925	-24.250999	71605 - 2415039	$16.829 {\pm} 0.005$	$15.461 {\pm} 0.003$	$14.768 {\pm} 0.003$	1.368	0.693
246.816971	-24.271143	71604 - 2416163	$17.056 {\pm} 0.007$	$15.708 {\pm} 0.004$	$15.007 \pm 0.004$	1.348	0.701
246.821976	-24.374475	71726-2422283	$17.341 {\pm} 0.124$	$15.567 {\pm} 0.003$	$13.418 {\pm} 0.001$	1.774	2.149
246.822739	-24.218828	71744 - 2413079	$15.993 {\pm} 0.002$	$14.378 {\pm} 0.002$	$13.707 {\pm} 0.001$	1.614	0.671
246.845718	-24.801941	72297 - 2448071	$10.922{\pm}0.001$	$9.832 {\pm} 0.001$	$9.336 {\pm} 0.001$	1.089	0.496
246.846909	-24.809896	72325-2448357	$14.124{\pm}0.001$	$12.613 {\pm} 0.001$	$11.982{\pm}0.001$	1.511	0.631
246.848297	-24.207954	72357-2412288	$12.805 {\pm} 0.001$	$10.796 {\pm} 0.001$	$9.851 {\pm} 0.001$	2.009	0.945
246.854782	-24.775953	72514 - 2446335	$15.527 {\pm} 0.001$	$12.957 {\pm} 0.001$	$11.683 {\pm} 0.001$	2.569	1.275

Table 2.3: Candidate $\rho$  Ophiuchus Members

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<sup>&</sup>lt;sup>a</sup>The catalog ID has been truncated by 2MASS J162 for 2MASS catalog stars. <sup>b</sup>Unweighted mean apparent magnitude of Cal-PSWDB photometry



Figure 2.6: Candidate  $\rho$  Oph membership. The filled blue circles indicate variables previously identified as  $\rho$  Oph members. The green filled circles mark the 22 new candidate  $\rho$  Oph members. The blue asterisks indicate the AC08 measured  $K_s$  and  $(H-K_s)$  for variables previously identified as  $\rho$  Oph members. The dashed lines connect the same variable as it is detected in this work. The solid black line is a 3 Myr isochrone constructed using NextGen models for masses between 0.02 and 1.4 M<sub> $\odot$ </sub> for a distance of 129 pc (Baraffe et al. 1998). The red arrow corresponds to a reddening vector for  $A_V = 10$  mag.

#### 2.8 Time Series Analysis

Characterizing the amplitude, time scale, and form (e.g. periodic vs. aperiodic) of variability provides valuable insights into the underlying physical mechanism(s) causing the brightness variations. Period searching algorithms have been very helpful in this regard (e.g. Lomb (1976); Scargle (1982)). In this section, two separate methods for measuring the time scales of variability are discussed.

## 2.8.1 Periodicity Analysis via the Plavchan-Parks Algorithm

A novel period searching algorithm, henceforth called the Plavchan-Parks algorithm (PPA), is implemented to detect periodicity in identified variable stars. The algorithm described

below is a more mature version than the one used in Plavchan et al. (2008b). The version of the algorithm is used in the NASA Exoplanet Archive periodogram tool (von Braun et al. 2009; Ramirez et al. 2009). Tens of thousands of test periods are investigated by the PPA algorithm with a uniform frequency sampling between 0.1 and 1000 days. For each trial period,  $P_j$ , the PPA starts by generating a phase folded light curve from the time series photometry. A phase is defined as the time  $(t_i)$  modulo the test period  $(P_i)$ . This light curve is smoothed via boxcar smoothing with a phase width, p = 0.06. This smoothed light curve is designated as the prior, or reference curve. When the measured photometry for a periodic source is folded to the test period, the photometry is assumed to be approximately continuous and smoothly varying over the phased cycle. The difference between the measured photometry and the prior is computed for every photometric measurement,  $m_i$ . This difference is compared to the difference between the measured photometry and a "nonvariable" straight line, defined by the photometric mean (see Fig. 2.7). A poor fit results when these two differences are equal or nearly equal to each other. A good fit results when the difference between the data and the smoothed prior is smallest. This normalization removes the dependence on the absolute value and dispersion in  $m_i$ . A quality of fit,  $\chi^2_{n_0}$ , is computed by Eqn. 2.4 only over the 40 data points with the poorest fits  $(n_0 = 40)$  (i.e., the epochs with the largest difference between the data,  $m_i$ , and the prior (average) in the denominator (numerator)):

$$\chi_{n_0}^2 = \frac{\sum_{i=1}^{n_0} (m_i - \overline{m})^2}{\sum_{i=1}^{n_0} (m_i - m_{prior_i})^2}$$
(2.4)



Figure 2.7: A demonstration of Plavchan-Parks Algorithm on ISO-Oph 96. Top: Light curve phased to a period of 16.6672 days This period is considered insignificant. BOTTOM: phased to a period of 3.5285 days. This is the most significant period from the periodogram. The dotted line indicates the mean magnitude for this star. The red lines in the middle and bottom panels is the prior generated for each period. Computing the  $\chi^2_{40}$  for the 3.5285 and 16.6672 day periods indicates the power value for the former is ~9x larger, implying a much larger statistical significance.

where the prior term,  $m_{prior_t}$ , is the mean of  $m_i$  if  $m_i$  is within the boxcar smoothing window. The summations in the numerator and denominator in Eqn. 2.4 are over independent sets of poorly fit measurements, since the poorest fit measurements by the prior might not be the same as the measurements that deviate the most from the mean. The best fit periods have the largest  $\chi^2_{n_0}$  value. In other words,  $\chi^2_{n_0}$  represents the power of the periodic signal. The power indicates, for the PPA, the relative improvements of the prior compared to a straight line for a given test period  $P_i$ .

To evaluate the statistical significance of the power value for a peak period in the periodogram, or in other words to compute a false alarm probability (FAP), there are several possible quantitative methodologies to arrive at an appropriate probability distribution. The approaches include one, an analytic derivation from first principles; two, a Monte Carlo simulation of periodograms generated by randomly swapping measurement values at each epoch; three, the distributions of power values at other periods in the same (adequately sampled) periodogram; and four, the distribution of maximum power values for all sources in an ensemble (mostly nonvariable) survey. The first approach is rarely used in the literature, with the noted exception of the Lomb-Scargle periodogram (Scargle 1982). In the case of the Lomb-Scargle periodogram and typical radial velocity surveys, however, systematic errors in the velocity measurements can invalidate the assumptions in the first approach. The second Monte Carlo approach is often used as a more reliable method for Lomb-Scargle periodograms (Marcy & Butler 1998), and is equally applicable to the PPA periodogram. In this section, the third method to evaluate a period's statistical significance is discussed. This third method is readily applicable to most time series and is the method used in this work for computing the FAP for found periods. In the Appendix, the fourth method is discussed. The fourth method is survey dependent, but provides the insight that the PPA periodogram is "well behaved" with respect to changes in data values, number of observations, and algorithm parameters p and  $n_0$ .

The distribution of power values in an adequately sampled PPA periodogram for a nonvariable source is best described by a lognormal distribution. In this instance, adequately sampled means covering a broad dynamic range of periods and sampling the periodogram at a large number of periods representative of the expected frequency resolution dictated by the cadence. Fig. 2.8 contains the periodogram for the nonperiodic star 2MASS J16265576-2508150. The power values vary about a mean value, or a "significance floor". The distribution is slightly asymmetric with a slight bias towards power values greater than the mean, consistent with a normal distribution in log space. Fig. 2.9 shows this distribution is very similar to the periodogram power value distribution for the boxcar least squares (BLS) periodogram applied to the same source (Kovács et al. 2002), albeit with a different mean and standard deviation. The BLS periodogram traditionally makes the assumption of a normal distribution for evaluating the statistical significance of a peak period in the distribution of power values from an adequately sampled periodogram. However, again, a lognormal distribution is a more appropriate prescription for the BLS distribution (von Braun et al. 2009; Ramirez et al. 2009). While the assumption of a normal distribution of power values is probably adequate for both algorithms, a normal distribution will ascribe a greater statistical significance (i.e., a smaller FAP) to a peak period than a lognormal distribution. Therefore, the more conservative lognormal distribution is adopted in evaluating the statistical significance of peak periods in both the BLS and PPA periodograms.

To determine if a period is statistically significant for a given source is this survey, the log of power values from the PPA periodogram are computed, as well as the mean and standard deviation of the log distribution. Power values that are  $5\sigma$  outliers in the periodogram are identified as statistically significant periods with low FAP. Each of these significant periods are investigated via visual inspection of the photometry folded to the period in question. Finally the statistical significance of the derived period is confirmed by either the Lomb-Scargle or BLS algorithms, depending on the folded light curve shape. The Lomb-Scargle algorithm is optimized to identify sinusoidal-like periodic variations, while the BLS algorithm is better equipped in identifying eclipse-like periodic variations. Thus, the PPA periodogram excels at identifying periodic signatures from both sinusoidal-like and eclipse-like time series periodic variations (Plavchan et al. 2008b). The period error is derived from the 1 $\sigma$  width of



Figure 2.8: *Left:* Periodogram for the irregular variable 2MASS J16265576-2508150 using the PPA. *Right:* The histogram of periodogram power values used to determine the significance of calculated periods. The solid line indicates a log-normal distribution fit to the histogram values and the dashed line indicates a normal distribution fit. The log-normal fit is used as it results in a more conservative higher false alarm probability.



Figure 2.9: Same as Figure 2.8 however using the BLS algorithm
a Gaussian fit to the period's peak in the periodogram. For certain stars, the most significant peak in the periodogram is closely surrounded by less significant peaks that are false positive periods. To avoid contamination from these other peaks, only the section of the periodogram within  $\pm 3\%$  of the most significant peak is used to determine the period error. An upper bound to confident periods is placed at 200 days. Stars are rejected as truly periodic with larger periods since the star will complete at most 3 cycles within the observing baseline. These "periods" are reported as timescales and described in § 3.3.

It should be noted that 6 periodic variables (YLW 1C, ISO-Oph 102, 2MASS J16271513-2451388, YLW 10C, YLW 13A, 2MASS J16272658-2425543) have identified periods within 1% of an integer value. Given the 1 day cadence, an integer period could be attributed to aliasing in the time series. These 6 stars were visually scrutinized and determined the shape of the periodicity is due to an astrophysical phenonoma as opposed to the cadence.

From the 101 variables, 32 stars (32%) are identified to exhibit periodic variability with periods ranging from 0.49 to 92 days. Table 3.1 contains the list of periodic variables.

### 2.8.2 Detecting Secondary or Masked Periodic Variability

The PPA found two statistically distinct (> $20\sigma$ ) periods for the young stellar object YLW 1C. The time series folded to the shorter period (5.7792 days) exhibits a sinusoidal-like shape. The time series folded to the longer period (5.9514 day) exhibits an "eclipse-like" shape where the star periodically dims from a near constant continuum flux. This prompted a search for secondary periods in the other 5 stars that exhibit eclipse-like periodic variability. We found 3 stars (YLW 1C, 2MASS J16272658-2425543, YLW 10C) to vary periodcally at two distinctly different periods; sinusoidal-like variability at one period and eclipse-like variability at the other. Initially the secondary period is not statistically significant; it is only discovered when the time series of the eclipse event is removed. The PPA is run only on the time series preceding each eclipse ingress and after each eclipse egress. A small number  $(\sim 10)$  of sharp drops outside the eclipse events in the time series for 2MASS J16272658-2425543 and YLW 10C are also omitted from the PPA analysis. Errors in the secondary periods are determined in the same manner as the primary periods.

Since multiple variability mechanisms may be common in variable stars (Herbst et al. 1994; Morales-Calderón et al. 2011), we attempted to search for periodic variability in stars where the variability was complex. For 6 variable stars, the stellar brightess fluxuates about a mean level for one or two consecutive years. During the remaining time, a large amplitude variation is observed lasting longer than 50 days. The PPA is run on the nearly constant time series omitting the large amplitude variation event. In 2 stars (WL 20W and ISO-Oph 126), the PPA found a significant period in the "whitened" time series. The time series folded to the appropriate period results in sinusoidal-like variability with an amplitude  $\sim 50\%$  smaller than the large amplitude variation. This larger amplitude variation effectively masked the smaller amplitude periodic signal. For each star, the periodic variability could not be recovered during the large amplitude variation. Fig. 2.10 contains the  $K_s$  light curves for WL 20W and ISO-Oph 126, as well as the  $K_s$  light curves folded to the identified periods. For WL 20W, 93 out of 262 scan groups were removed before the PPA analysis. For ISO-Oph 126, 149 out of 262 scan groups were removed. Fig. 2.11 shows the periodograms for both stars using the full time series and the pre-whitened time series.



Figure 2.10: *TOP*: The  $K_s$  light curves for WL 20W and ISO-Oph 126. Both light curves display a large amplitude long time scale variation. *BOTTOM*: The folded  $K_s$  light curves for WL 20W (P = 2.1026 ± 0.0060 days) and ISO-Oph 126 (P = 9.114 ± 0.90 days). The periods are only detected once the photometry affected by the large amplitude variation is removed. Those data are not included in the folded light curves.



Figure 2.11: Periodograms for WL 20W and ISO-Oph 126 including and excluding the long time scale event photometry. *TOP*: The periodograms from running the PPA on the full data set including the large amplitude long time scale variation. *BOTTOM*: The periodograms from the PPA only on the photometry not affected by the large amplitude variation. The 2.1026 day period is only seen and significant in the lower periodogram for WL 20. The same is true for the 9.114 day period of ISO-Oph 126. Additionally, this period peak power value is nearly 3x more significant than any power value detected using the complete set of photometry.

### 2.8.3 Measuring Long Time Scale Variability

The long temporal baseline of the photometric time series allows for the analysis of variability on month and year time scales which are time scales not well explored for young stars. Long time scale (>50 days) variability differs from periodic or irregular variability in that the mean flux value may not remain nearly constant from season to season. In addition, the photometry in one season may systematically brighten or dim while remaining constant in the other two seasons. Examples of these two phenomenon in the time series for WL 20W and ISO-Oph 126 are shown in Fig. 2.10. Fig. 2.11 shows how the periodograms for each star are effected once the long time scale photometry is ignored by the PPA. The intention in this section is to measure the time scale of the single largest amplitude aperiodic or irregular variation.

Two criteria are used to identify stars exhibiting long time scale variability. The first criterion is the difference between the photometric mean magnitude from one season to either of the remaining two seasons must be greater than  $3\sigma$ , where  $\sigma$  is the average photometric error of the data over the entire temporal baseline (see Fig. 3.11, WL 6). The second criterion is that the slope in the photometry in at least one season must be greater than  $\pm 5^{\circ}$ . The quality of the line fit determining the slope is assessed by visual inspection. The motivation for the second criterion is illustrated by WL 14 (see Fig. 3.9). An obvious decreasing trend in the photometry is seen in the third season, however the sharp flux drop in the second season causes the mean flux between the two seasons to not satisfy the first criterion. Of the 101 variables, 31 stars (31%) satisfy at least one of these criteria and are designated long time scale variables (LTVs).

A differencing technique is employed to measure the time scale over which a LTV changes from one extreme in flux to the other. Fig. 2.12 provides a visual demonstration of this method. In the top panel of Fig. 2.12, a gradual dimming over the entire data set is observed. This global trend is seen in the time series of 68% of the LTVs. Two different types of variability are believed responsible for the global trend and the long time scale variation. Removal of the global trends provides an unbiased analysis of the shorter time scale variation in the time series superimposed on these trends. The global trend is a sustained, but small amplitude effect superimposed over the time series including the larger amplitude, long time scale variation. LTVs with these global trends are split evenly with 50% dimming over time and 50% brightening. The amplitude of the global trends range from 7.5 to 330 millimag/year, with a median value of 26 millimag/year. The median value corresponds to a change in the stellar flux of ~60 millimag over the temporal baseline.

An accurate time scale measurement for the largest amplitude variation can be complicated by the presence of small time scale variability. The middle panel of Fig. 2.12 shows how the light curve is smoothed with a 50 day moving median filter. The length of 50 days is chosen by visual inspection of the smoothed light curves; this timescale suppresses the smaller amplitude, shorter time scale variability while preserving the shape of the long time scale variation. The time scale for the long time scale variation is set to be the time difference between when the LTV is at one extreme in flux (i.e., brightest state) to the opposite extreme (i.e., dimmest state). This time scale is determined by subtracting the smoothed magnitude found at time i with the smoothed magnitude found at time j using the following:

$$M_{i,j} = \sum_{i=1}^{N_{obs}} \sum_{j=1}^{N_{obs}} (m_j^* - m_i^*)$$
(2.5)

where  $M_{i,j}$  is the  $\Delta$ mag between time j and time i,  $m_j^*$  is the magnitude at time j,  $m_i^*$  is the magnitude at time i and  $N_{obs}$  is the total number of observations. The time between the largest  $\Delta$ mag is recorded as the time scale. In many cases, the full time scale of the variation cannot be measured due to the data sampling. The bottom panel of Fig. 2.12 shows the quantity  $M_{i,j}$  as a function of times between measurements i and j. The "landscape" shows multiple peaks each corresponding to various time scales of variability. The highest peak is considered as the time scale associated with the greatest change in magnitude (i.e., largest  $M_{i,j}$ ). Either extreme flux state may fall within a gap in the photometry or outside the date range of observations. Therefore these time scales should be treated as lower bounds. The variability time scales range from 64 to 790 days. Not all LTVs display only one discrete long time scale variation. ISO-Oph 119 clearly shows two distinct long time scale variations. For ISO-Oph 119 and similar cases, only the time scale for the largest amplitude variation is measured. Figs. 3.9 to 3.12 contain the  $K_s$  light curves for these LTVs.

Despite observations spanning  $\sim 2.5$  years, in most cases it is not possible to conclude whether or not long time scale variability is periodic. However, 6 LTVs have photometry suggestive of periodic behavior based on visual inspection of the stars' folded light curves corresponding to periods ranging from 207 to 589 days. The light curves are folded to the most significant period found by the PPA. These candidate periodic stars are identified in the first column of Table 3.3. These sources are not included with the periodic variables as the found periods are greater than the 200 day confidence limit (see § 2.8.1).



Figure 2.12: A demonstration of the method used to estimate the variability time scale of LTVs. Top: The  $K_s$  light curves for 2MASS J16271726-2422283. The gray dashed line is a linear least-squares fit to the data. Middle: The same light curve after the data are smoothed and the linear fit is removed. The smoothing is done using a moving median filter with a 50 day width. Bottom: This shows the  $\Delta$ mag as a function of the time between individual photometric measurements,  $m_j^*$  and  $m_i^*$ . The recorded 132 day time scale corresponds to highest peak, or largest  $\Delta$  mag occurring in the middle plot. This time scale describes the star flux decrease from ~400 to ~525 2MASS modified Julian Date.

– 3 –

# VARIABILITY OF YOUNG STELLAR OBJECTS

In the previous chapter, the methods for measuring the amplitude and timescales of stellar variability, in particular for young stars, was discussed. The goal is to use this information, in turn, to place constraints on the physical mechanisms responsible for the identified variability. This chapter will begin by detailing the observational characteristics associated with 5 variability mechanisms. A discussion of the variability characterization subdivided into 3 subcategories based on variability timescales follows. The chapter concludes by discussing where 2 distinct variability mechanisms can be estimated for a single star.

### 3.1 Variability Mechanisms

Empirical methods based on correlations between observed magnitudes and color have been employed to characterize stellar variability of young stars (Carpenter et al. 2001, 2002; Alves de Oliveira & Casali 2008). These methods consider variability due to rotational modulation of hot or cool starspots, variable extinction, variable mass accretion and structure changes in the circumstellar environment. Cool starspots are believed to be caused by localized magnetic inhibition of convection energy transport. Hot starspots, on the other hand, result from either surface flaring or heating by mass accretion onto the surface along magnetic field lines. Extinction may occur from asymmetries in an accretion disk or even from isolated dense regions of the parent molecular cloud passing through the line of sight. Variable mass accretion rates can cause the star brightness to vary through the clearing of the inner circumstellar disk. In addition, variability may be caused by energy released as material in an accretion disk moves toward a star by viscous processes. Finally, these mechanisms are not mutually exclusive and are often seen to exist simultaneously (Herbst et al. 1994).

Each of the above variability mechanisms can be distinguished based on the temporal nature of the variability and correlations between color variability to stellar brightness.<sup>1</sup> The following set of qualitative observables are developed to classify the observed variability and to connect these variations to physical mechanisms.

- Long lived cool starspots result in periodic variability with periods consistent with the rotational periods of young stars ( $\leq 14$  days) (Rebull 2001). This variability is often sinusoidal in shape. At the temperature range of most YSOs, the near-IR wavelength regime samples the Rayleigh-Jeans tail of the stellar energy distribution where the contrast between the starspot and surrounding photosphere is small (e.g. (Vrba et al. 1985)). Therefore, the (*J-H*) and (*H-K<sub>s</sub>*) colors should remain constant (within photometric errors) as the brightness varies.
- Variability by hot starspots can either result in periodic or irregular variability. Long lived hot starspots caused by accretion onto the stellar surface may result in periodic variability. However it should be noted that accretion induced hot spots may display aperiodic behavior due to a stochastic accretion rate. Variability caused by flares will be aperiodic and will have time scales on the order of hours to days. As with cool starspots, the period of variability will be consistent with the rotational periods of young stars. In both cases the affected photosphere should be hotter than the

<sup>&</sup>lt;sup>1</sup>The correlations between stellar color and brightness are based on models in Carpenter et al. (2001 and references herein).

surrounding surface resulting in the star becoming bluer as the star brightens (Rodonò & Cutispoto 1988; Panagi & Andrews 1995; Yu & Gan 2006).

- Variable extinction can result in either periodic or long time scale variability. Variability caused by asymmetries in the inner circumstellar disk, if present, may be periodic with periods from days to weeks. Unlike variability caused by starspots, periodic variable extinction need not appear sinusoidal but present more likely as eclipse-like features. These eclipse-like features are sharp drops or "dips" in the stellar flux with a regularity dependent on the observing cadence. Variability caused by asymmetries in the outer circumstellar disk (> 1 AU) will not be periodic within the temporal baseline of this study due to long period of revolution around the host star. This variability and variable extinction from inter cloud material can occur on long time scales, however as the time scale depends on the system geometry, there is no expectation as to its duration. Variable extinction causes the star to redden as the star dims.
- Variability caused by a variable accretion rate within the circumstellar disk is not expected to be periodic. The time scale of variability does place constraints on the physics causing this rate change (e.g. disk viscosity, time variable magnetic field) (Armitage 1995; Mahdavi & Kenyon 1998; Lai 1999; Terquem & Papaloizou 2000; Carpenter et al. 2001). During times of lower accretion rates, the inner disk cools and the inner hole becomes larger. This, in turn, decreases the contribution of dust reradiation, particularly in the  $K_s$  band, to the overall energy budget of the star and circumstellar disk system. Therefore while the total system flux drops, a larger percentage of emitted radiation is from the star causing the system to become bluer as the system dims. However, if

the inner circumstellar disk edge is dominated by the dust sublimation temperature, a observationally similar effect will result. In this case, an increased accretion rate raises the star's effective temperature in turn increasing both the distance to the circumstellar disk inner rim and disk vertical height. The result would be that the system would become brighter as it reddens. Both physical scenarios produce a qualitatively identical result to the observed correlation between brightness and color.

In an attempt to identify the dominant variability mechanism, stars in the variable catalog are placed into subclasses based on the observed shape and time scale of variability. These subclasses are: periodic, long time scale and irregular. These classifications along with the above criterion identified the likely dominant variability mechanism for 53 of the 101 stars in the variable catalog. The type of variability associated with each star is listed in Table 2.2 and each sub class is described in the following subsections. The periodic sub class accounts for 32% of the variable catalog with the majority (88%) lying "on cloud". Long time scale variables make up 31% of the variable catalog. All LTVs reside "on cloud". The irregular subclass contains the most members comprising 40% of the variable catalog. Only 68% of irregular variables lie "on cloud". These subclasses are rough descriptions and are by no means mutually exclusive. For instance, WL 20W and ISO-Oph 126 are placed into both the periodic and long time scale subclasses.

These criteria do not always allow for the dominant variability mechanism to be identified. The main reasons preventing an estimate of the mechanism are: the time scale/period or color correlation is contrary to the above diagnostics, no dominant amplitude variability is clearly evident, or the photometry in J and H is below the completeness limits in each band resulting in no useful color information. Mechanisms appended with a question mark in Table 2.2 either possess a marginal color correlation via visual inspection, or the diagnostics did not definitively differentiate between proposed mechanisms.

## 3.2 Periodic Variables

The PPA identifies 32 of 101 stars (32%) within the variable catalog as periodic with periods ranging from 0.49 to 92.28 days. Table 3.1 contains the list of periodic variables. The light curves for certain subsets of periodic variables are very similar in form when phased to the identified period. This allows for periodic variables to be separated into two subcategories: sinusoidal-like and eclipse-like. Assignment to a particular subcategory is based upon visual inspection of the folded light curve in the band with the highest signal-to-noise.

Table 3.1: Periodic Variables

Catalog ID <sup>a</sup>	Period <sup>b</sup> (days)	$\frac{\Delta K_s}{(\mathrm{mag})}$	$\begin{array}{c} \Delta(J\text{-}H)\\ (\mathrm{mag}) \end{array}$	$\begin{array}{c} \Delta(H\text{-}K_s)\\ (\text{mag}) \end{array}$	YSO Class	Sub-Category	Var. Mech. <sup>c</sup>
ISO-Oph 83	$25.554{\pm}0.071$	0.476	0.235	0.173		Sinusoidal	Extinction
YLW 1C	$5.7753 {\pm} 0.0085$	0.292	0.140	0.257	II	Sinusoidal	Hot $Starspot(s)$
	$5.9514 {\pm} 0.0014$	0.29				Eclipse	Extinction
ISO-Oph 96	$3.5285 {\pm} 0.0032$	0.084	0.051	0.065	III	Sinusoidal	Cool Starspot(s)
ISO-Oph 97	$14.520 {\pm} 0.088$	0.086	0.047	0.050	III	Sinusoidal	Cool $Starspot(s)$
ISO-Oph 98	$5.9301{\pm}0.0092$	0.411	0.232	0.329	II	Sinusoidal	Cool Starspot(s)
ISO-Oph 100	$3.682 {\pm} 0.002$	0.337			II	Sinusoidal	Unknown
ISO-Oph 102	$3.02173 {\pm} 0.00044$	0.224	0.128	0.099	II	Eclipse	Cool $Starspot(s)$ ?
ISO-Oph 106	$3.4370 {\pm} 0.0012$	0.508	0.242	0.204	II	Eclipse	Extinction
WL 10	$2.4149 {\pm} 0.0027$	0.330	0.137	0.102	II	Sinusoidal	Cool $Starspot(s)$ ?
WL 15	$19.412 {\pm} 0.085$	1.636		0.628	Ι	Sinusoidal	Unknown
WL 11	$3.0437 {\pm} 0.0038$	0.729	0.346	0.402	II	Sinusoidal	Hot $Starspot(s)$ ?
65744 - 2504017	$0.83141 {\pm} 0.00030$	0.339	0.157	0.112		Sinusoidal	Hot $Starspot(s)$ ?
71513 - 2451388	$8.004 {\pm} 0.046$	0.330	0.213	0.111		Eclipse	Extinction
WL $20W$	$2.1026 \pm 0.0060$	0.213	0.258	0.305	II	Sinusoidal	Cool Starspot(s)
YLW 10C	$2.9468 {\pm} 0.0029$	0.356			II	Eclipse	Extinction?
	$3.0779 {\pm} 0.0025$	0.28				Sinusoidal	Cool $Starspot(s)$ ?
71836 - 2454537	$2.7917 {\pm} 0.0017$	0.807	0.354	0.327		Sinusoidal	Hot $Starspot(s)$ ?
ISO-Oph 126	$9.114 \pm 0.090$	0.135	—	0.371	III	Sinusoidal	Cool Starspot(s)
ISO-Oph 127	$6.365 {\pm} 0.014$	0.528		0.009	Ι	Sinusoidal	$Cool \ Starspot(s)$
WL $4$	$65.61 {\pm} 0.40$	0.640	0.161	0.186	II	Inverse Eclipse	Circumbinary Disk
YLW 13A	$7.0270 {\pm} 0.0056$	0.092	0.057	0.071	III	Sinusoidal	$Cool \ Starspot(s)$
ISO-Oph 133	$6.354{\pm}0.011$	0.062		0.056	III	Sinusoidal	$Cool \ Starspot(s)$
ISO-Oph 135	$5.536 {\pm} 0.019$	0.130	0.044	0.071	III	Sinusoidal	Cool Starspot(s)
72463-2429353	$6.581 {\pm} 0.012$	0.094		0.327	0	Sinusoidal	$Cool \ Starspot(s)$

Continued on Next Page...

Table 3.1 – Continued

Catalog ID <sup>a</sup>	$\begin{array}{c} \text{Period}^{\text{b}} \\ \text{(days)} \end{array}$	$\Delta K_s$ (mag)	$\begin{array}{c} \Delta(J\text{-}H)\\ (\mathrm{mag}) \end{array}$	$\begin{array}{c} \Delta(H\text{-}K_s) \\ (\text{mag}) \end{array}$	YSO Class	Sub-Category	Var. Mech. <sup>c</sup>
72533-2506211	$0.485143 \pm 0.000050$	0.402	0.272	0.295		Sinusoidal	Unknown
ISO-Oph 139	$3.7202 {\pm} 0.0041$	0.098		0.200	Ι	Sinusoidal	Cool $Starspot(s)$
YLW 16C	$1.14182 {\pm} 0.00043$	0.318	0.185	0.207	II	Sinusoidal	Cool Starspot(s)
72658 - 2425543	$2.9602 {\pm} 0.0013$	0.211	0.099	0.096	II	Eclipse	Extinction
	$1.52921{\pm}0.00065$	0.17				Sinusoidal	Cool $Starspot(s)$
72706-2432175	$18.779 {\pm} 0.099$	0.090		0.478		Sinusoidal	Cool $Starspot(s)$
WL 13	$23.476 {\pm} 0.077$	0.305	0.121	0.082	II	Sinusoidal	Cool $Starspot(s)$
YLW 16A	$92.28 \pm 0.84$	0.784		0.549	Ι	Inverse Eclipse	Circumbinary Disk
ISO-Oph 149	$1.24505 {\pm} 0.00039$	0.067	0.057	0.057	III	Sinusoidal	Cool $Starspot(s)$
ISO-Oph 148	$3.5548 {\pm} 0.0039$	0.058	0.054	0.071	III	Sinusoidal	$Cool \ Starspot(s)$

<sup>&</sup>lt;sup>a</sup>The catalog ID has been truncated by 2MASS J162 for 2MASS catalog stars <sup>b</sup>The FAP for all periods are <1% <sup>c</sup>A question mark denotes a variability mechanism that is uncertain due to insufficient color information

#### 3.2.1 Sinusoidal-like Periodic Variables

Figs. 3.1 to 3.4 contain the  $K_s$  light curves for sinusoidal-like periodic variables. This subcategory of sinusoidal-like periodic variables includes the most periodic variables (25 stars) with periods ranging from 0.49 to 25.55 days. The peak-to-trough  $\Delta K_s$  amplitudes range from 0.06 to 1.64 mag, with a median value of 0.29 mag. The peak-to-trough  $\Delta (H-K_s)$  color amplitudes range from 0.01 to 0.63 mag, with a median value of 0.19 mag. Typically, the light curve folded to the most significant period shows only one sinusoidal cycle. However, four stars (ISO-Oph 100, WL 10, WL 13, YLW 13A) show what could be interpreted as a second cycle at half the frequency (i.e., double the period).

Probable variability mechanisms are identified for these stars by applying the criteria discussed in § 3. Correlations are qualitatively examined between the ensemble  $K_s$  photometry and stellar colors when folded to the star's period. Fig. 3.6 illustrates examples of



Figure 3.1: The folded  $K_s$  light curves for 6 sinusoidal-like periodic variables. The red line indicates the star's mean magnitude.



Figure 3.2: The folded  $K_s$  light curves for 6 sinusoidal-like periodic variables. The catalog name for stars labeled with a 2MASS designation have been truncated by 2MASS J162. The red line indicates the star's mean magnitude.



Figure 3.3: The folded  $K_s$  light curves for 6 sinusoidal-like periodic variables. The catalog name for stars labeled with a 2MASS designation have been truncated by 2MASS J162. The red line indicates the star's mean magnitude.



Figure 3.4: The folded  $K_s$  light curves for 6 sinusoidal-like periodic variables. The red line indicates the star's mean magnitude.

these interpretations for both cool and hot starspots. For 18 of the sinusoidal-like periodic variables (72%), the variability in the (J-H) and  $(H-K_s)$  colors does not correlate with the  $K_s$  variability. This favors rotational modulation by cool starspots as the dominant variability mechanism. For 3 sinusoidal-like periodic variables (12%), the (J-H) and  $(H-K_s)$ colors become bluer as the star brightens. This is consistent with the behavior expected from rotational modulation by an accretion induced hot starspot. The (J-H) and  $(H-K_s)$ colors become redder as the star dims for 1 (4%) sinusoidal-like periodic variable. This favors variable extinction as the dominant variability mechanism. Finally for the remaining 3 sinusoidal-like periodic variables (12%), no dominant variability mechanism could be assigned using the adopted criteria.

All sinusoidal-like periodic variables except 3 (2MASS J1625744-2504017, 2MASS J16271836-2454537 and 2MASS J16272533-2506211) are located "on cloud". This subcategory contains



Figure 3.5: Left: The folded  $K_s$  and color curves for the sinusoidal-like periodic variable ISO-Oph 135. The lack of near-IR color changes with brightness changes favors cool starspots as the variability mechanism. Right: The folded  $K_s$  and color curves for the sinusoidal-like periodic variable WL 11. The stellar color becomes bluer as the  $K_s$  photometry becomes brighter. This favors rotational modulation of accretion induced hot starspots as the dominant mechanism. The red line in each plot indicates the mean value.

19 stars with a YSO classification: 3 Class I (25%), 8 Class II (24%) and 8 Class III (73%).<sup>2</sup> The variability mechanism for 2 of the Class I stars is cool starspots, while the mechanism could not be identified for the third. Of the Class II stars, 5 vary due to cool starspots, 1 from an accretion-induced hot starspot and 1 is unknown. The variability of all Class III stars is caused by cool starspots.

Two key points can be made by analyzing the 18 sinusoidal-like periodic variables where cool starspots is the believed variability mechanism. First, cool starspots on young stars persist on preferential longitudes on year timescales. This is evidenced by a lack of phase drift in the folded light curves. This phenomenon of preferential or *active* longitudes has

<sup>&</sup>lt;sup>2</sup>The percentages indicate the percentage of variable stars in each class that are sinusoidal-like periodic variables.

been associated with a number of chromospherically active stars (e.g. RS CVns, FK Com) (Strassmeier et al. 1988; Zeilik et al. 1988; Henry et al. 1995; Jetsu 1996). Second the variability amplitude due to cool starspots changes on much shorter timescales as evidenced by the significant scatter within the folded light curves. This amplitude variability is likely caused by an evolving starspot covering factor and/or starspot temperature. The covering factor is defined as the area of the observed stellar disk covered by the starspot(s).

ISO-Oph 96, ISO-Oph 133, ISO-Oph 149 and 2MASS J16272533-2506211 differ from the remaining sinusoidal-like periodic variables as the light curves for these 4 stars are asymmetric (i.e., they have a sharp increase in flux then decrease more slowly). The (J-H) and  $(H-K_s)$  color variability are not correlated to the  $K_s$  variability for the first three asymmetric sinusoidal-like periodic variables. This favors a dominant variability mechanism of rotational modulation by cool starspots. Asymmetric light curves have been observed for both WTTS and chromospherically active dwarf stars (Cutispoto et al. 2001, 2003; Grankin et al. 2008; Frasca et al. 2009). In both cases, the variability is believed to be caused by magnetically generated cool starspots. "Reverse" asymmetric light curves with a slow rise in source flux followed by a steep drop are also observed. Frasca et al. (2009) is able to closely model the *RIJH* asymmetric light curves of the WTTS V1529 Ori by rotating a stellar surface with two cool starspots of unequal areas separated by ~130° in longitude. The size of the leading cool starspot determines if a "forward" or "reverse" asymmetric light curve is seen.

2MASS J16272533-2506211, hereafter designated 'J211', is peculiar due to its unique and difficult to interpret brightness and color variations. Fig. 3.6 contains  $K_s$ , (J-H) and  $(H-K_s)$  photometry for J211 folded to P = 0.485143 \pm 0.000050 days. This is the shortest period star among the periodic variables. The peak-to-trough  $K_s$  amplitude is 0.40 mag and



Figure 3.6: Left: The J, H and  $K_s$  light curves of 2MASS J16272533-2506211 folded to a period of 0.485143  $\pm$  0.000050 days. Right: The  $K_s$ , (J-H) and  $(H-K_s)$  light and color curves folded to the above period. The significant difference in form of the J folded light curve to the other two suggests the cause for the variability in the (J-H) color arises mainly from the J band. The red line in each plot indicates the mean value in each case.

the  $\Delta(H-K_s)$  color amplitude is 0.30 mag. The (J-H) color for J211 clearly becomes bluer for a phase duration of ~0.3 (~3.5 hrs) centered approximately on the times of maximum brightness. Somewhat surprisingly, however, no similar variation is seen in the  $(H-K_s)$  color during the same period. The data are deemed reliable as the J and H photometry are significantly brighter than the survey completeness limits. The variability mechanism is not identified for J211 as this  $K_s$ -color behavior is inconsistent with any criteria discussed in § 3. The shape of the light curve coupled with the short period suggests J211 might be a RR Lyrae variable. However the peculiar color behavior is not expected in these stars.

#### 3.2.2 Eclipse- and Inverse Eclipse-like Periodic Variables

Eclipse-like periodic variables possess photometry containing sharp periodic drops in source flux. The duration of these drops, or possibly "eclipses", is in all cases less than a phase of 0.3 when the photometry is folded to the most significant period. Fig. 3.7 contains the folded light curves for the 6 eclipse-like periodic variables. These eclipse-like periodic variables are assigned to this subclass by visual inspection; it is not possible to confidently state that these sharp changes in photometry are *bona fide* occultations of star light (i.e., true eclipses).

The periods for the eclipse-like periodic variables range from 2.95 to 8.00 days. The duration of these eclipses range from 5.8 to 12.7 hours. The  $\Delta K_s$  amplitudes range from 0.21 to 0.51, with a median value of 0.31 mag. The  $\Delta(H-K_s)$  color amplitude range from 0.10 to 0.25 mag, with a median value of 0.11 mag. These amplitudes in both magnitude and color represent the total change in stellar flux and they do not necessarily represent eclipse depths since there is considerable scatter in the out-of eclipse photometry. The eclipse depths and how they are determined are described below. The variability mechanism for the eclipse is determined in the same manner as with the sinusoidal like periodic variables. Correlations between the  $K_s$  photometry and stellar colors (J-H) and  $(H-K_s)$  for the eclipse event are assessed visually and compared with the criteria discussed in  $\S$  3. For 4 (66%) eclipse-like periodic variables, the (J-H) and  $(H-K_s)$  colors become redder as the star dims. The color correlation coupled with the short, periodic behavior favor extinction, possibly by the inner region of a circumstellar disk, as the dominant variability mechanism causing the eclipse. The variability in the colors during the eclipse for ISO-Oph 102 is not correlated with the  $K_s$ photometry, consistent with variability caused by rotational modulation of cool starspots.



Figure 3.7: The folded J and  $K_s$  light curves for 6 eclipse-like periodic variables. The catalog name for stars labeled with a 2MASS designation have been truncated by 2MASS J162. The red line indicates the star's mean magnitude

Unfortunately, both the J and H photometry for YLW 10C are dimmer than the survey completeness limits and the lack of color information prevents confident identification of a variability mechanism. All 6 eclipse-like periodic variables are classified as YSO Class II  $(15\%)^3$ . All stars in this subcategory are located "on cloud" except 2MASS J16271513-2451388.

Morales-Calderón et al. (2011, YSOVAR) qualitatively identified a number of similar eclipse-like variables in their mid-IR variability survey of YSOs within the Orion Nebula Cluster. The survey found 38 stars exhibiting brief, sharp drops in stellar flux. These stars are identified as AA Tau or "dipper" variables. The variability mechanism is believed to be due to high latitude warps in the inner accretion disk periodically occulting the star (Bertout

<sup>&</sup>lt;sup>3</sup>The percentage indicates the percentage of variable Class II stars that are eclipse-like periodic variables.

2000; Bouvier et al. 2003). The 4 eclipse-like variables that show evidence of extinction could be considered AA Tau variables, and possibly all 6 eclipse-like systems.

Under the assumption all 6 eclipse-like periodic variables are AA Tau variables, constraints on the spatial location within the circumstellar disk and size of the hypothetical occulter are investigated. Marsh et al. (2010) performed a deep mid-IR imaging survey of the  $\rho$  Oph 2MASS Calibration field used in this work. They computed the T<sub>eff</sub>, A<sub>V</sub> and mass for 5 of the 6 eclipse-like periodic variables. This was done by fitting model spectra to observed SEDs. The SEDs were computed from photometry in the *J*, *H*, *K*, [3.5] and [4.5] bands. The model spectra were obtained using the COND, DUSTY and NextGen models. T<sub>eff</sub> and A<sub>V</sub> are found by minimizing the following equation:

$$\phi(T_{eff}, \alpha, A_V) = \sum_{\lambda=1}^{5} \frac{1}{\sigma_{\lambda}^2} [f_{\lambda}^{obs} - \alpha 10^{-0.4r_{\lambda}A_V} f_{\lambda}^{mod}(T_{eff})]^2 - A_V \beta$$
(3.1)

where  $\alpha$  is a flux scaling factor,  $f_{\lambda}^{obs}$  and  $f_{\lambda}^{obs}(T_{eff})$  are the respective observed and model fluxes at wavelength  $\lambda$ ,  $\sigma_{\lambda}$  is the flux uncertainty,  $r_{\lambda}$  is the absorption at wavelength  $\lambda$  relative to  $A_V$  and  $\beta$  is a constant penalty parameter. The right side of Eqn. 3.1 is not included in the summation over  $\lambda$ . In the Rayleigh-Jeans regime a degeneracy exists between  $T_{eff}$  and  $A_V$ such that a high temperature star seen through high extinction will have a SED similar to that of a low temperature star seen through low extinction. The parameter  $\beta$  is used to break this degeneracy by penalizing solutions with low values of  $A_V$ . This parameter was optimized by minimizing Eqn. 3.1 for a sample of 124 low mass stars with known spectroscopically determined  $T_{eff}$ . This procedure was performed using a range in  $\beta$  from 0 to 1.5. A final  $\beta = 0.7$  was selected as this produced the smallest residuals between the estimated and the spectroscopic values of  $T_{eff}$ . Values of  $A_V$  have errors between 1 to 2.7 while  $T_{eff}$  is accurate to within 860 K. The COND and DUSTY models then yield a model unique mass for each star based upon the assumed age of 1 Myr. They used a mass-temperature relationship to derive the mass for hotter stars fit by the NextGen models. This relationship is derived from observations of pre-main-sequence stars by Greene & Meyer (1995) in the  $\rho$  Oph cloud core. They conclude an accuracy in the mass estimate to within a factor of ~2-3.

For the case of ISO-Oph 106, a 1 Myr isochrone from Siess et al. (2000) is used to determine the  $T_{eff}$  and radius for ISO-Oph 106 by assuming a mass of 0.5  $M_{\odot}$ . The stellar radius for the remaining eclipse-like periodic variables is computed by  $R_{\star} = \left(\frac{L_{\star}}{T_{eff}^4}\right)^{\frac{1}{2}}$ , where each quantity is in solar units. The stellar luminosity,  $L_{\star}$ , is computed using the prescription outlined in Natta et al. (2006) except in the case of YLW 10C. The prescription relates the stellar luminosity as a function of J magnitude and extinction  $A_J$ . The extinction is computed using the (J-H) and (H-K) colors corrected into the CIT system using the  $\rho$ Oph extinction law by Kenyon et al. (1998) and the CTTS locus defined by Meyer et al. (1997). The J band photometry for YLW 10C is below the survey completeness limit. The luminosity for this star is found using the 1 Myr isochrone mentioned above and the mass determined by Marsh et al. (2010). The estimated mass,  $T_{eff}$  and  $R_{\odot}$  are in Table 3.2.

Assuming the occulter has negligible mass and orbits under Keplerian rotation, the occulter's distance from the host star can be computed for each eclipse-like periodic variable. The diameter of the occulter is computed from the duration of the eclipse event and its location in the circumstellar disk. Strictly speaking, the computed diameters are along the orbital path. No assumption concerning the occulter geometry (i.e., spherical, ellipsoidal) is made. This diameter is considered a strict lower bound as the disk geometry, occulter impact parameter and occulter opacity are unknown. The eclipse depth, however, is determined by first removing the eclipse event photometry from the time series data (see § 2.8.1). The eclipse depth is then determined to be the difference between the maximum magnitude in the eclipse feature relative to a median non-eclipse mean magnitude. This calculation results in occulter distances, a, that range between 1.83 to 7.21 R<sub>\*</sub>, with a median value of 3.20 R<sub>\*</sub>. The occulter size, D, ranges from 0.58 to 3.63 R<sub>\*</sub>, with a median value of 2.19 R<sub>\*</sub>. The range in  $\Delta K_s$  eclipse depth is 0.12 to 0.51 mag, with a median value of 0.27 mag. Table 3.2 summarizes the results of this investigation.

It should be noted that the lack of identified eclipsing binaries is not unexpected. The pilot study done by P08 contained 7554 stars and only found 3 eclipsing systems. Given a 0.04% detection fraction, this survey would need to contain 2518 stars which is larger than 1678 stars studied.

Table 3.2: Summary of Eclipse-Like Periodic Variable

Characteristics

Star	$\overset{\rm L}{({\rm L}_{M_{\odot}})}$	$\stackrel{\rm M}{}_{({\rm M}_{M_{\odot}})}$	$\begin{array}{c} T_{eff} \\ (K) \end{array}$	$\begin{array}{c} \mathbf{R}_{\star} \\ (\mathbf{R}_{M_{\odot}}) \end{array}$	$^{\mathrm{a}}_{\mathrm{(R_{\star})}}$	$\begin{array}{c} \mathrm{D} \\ \mathrm{(R_{\star})} \end{array}$	$\Delta K_s$ (mag)
YLW 1c	1.81	1.16	4738	2.01	7.21	2.04	0.29
2MASS J16271513-2451388	0.52	0.11	3033	2.63	3.09	0.58	0.26
2MASS J10272038-2423343 ISO Oph 102	0.002 0.22	0.043 0.050	2183	1.07	2.84 1.82	2.00 1 37	0.17 0.12
ISO-Oph 102 ISO-Oph 106	0.22 $0.88$	0.039 0.50	3769	2.06	3.69	3.48	$0.12 \\ 0.51$
YLW 10c	0.746	0.35	3901	1.90	3.21	3.63	0.28

Inverse eclipse-like periodic variables are similar to eclipse-like periodic variables, but the "eclipse" is an increase in source flux rather than a decrease. Fig. 3.8 contains the folded  $K_s$  and color curves for WL 4 and YLW 16A, the 2 inverse eclipse-like periodic variables in the variable catalog. WL 4 is a Class II YSO whose period of variability is  $65.61 \pm 0.40$  days. The peak-to-trough  $\Delta K_s$  amplitude is 0.67 mag and the peak-to-trough  $\Delta (H-K_s)$  color amplitude is 0.19 mag. The (J-H) color for WL 4 becomes redder as the star brightens during the inverse eclipse event. However, the (J-H) color change starts just prior and ends just after the inverse eclipse event. The difference in each case is ~0.1 in phase or ~6.6 days. The  $(H-K_s)$  color is not correlated with the  $K_s$  variability.

The period of variability for YLW 16A is longer than WL 4 at 92.28  $\pm$  0.84 days. The amplitudes of variability for YLW 16A, a Class I YSO, are also larger with peak-to-trough  $\Delta K_s$  and  $\Delta (H-K_s)$  amplitudes of 0.95 and 0.34 mag, respectively. As the *J* band photometry is dimmer than the survey completeness limits, no reliable (*J-H*) color information is available. The (*H-K<sub>s</sub>*) color variability is sinusoidal-like, but is not aligned with the  $K_s$ variability. Both stars reside "on cloud".

The variability mechanism for both WL 4 and YLW 16A is believed to be related, and similar to the interpretations proposed in separate letters (Plavchan et al. 2008b, 2013). Here the proposed variability mechanism is summarized. Both systems contain a visual binary companion detected through high resolution direct imaging (Ratzka et al. 2005; Plavchan et al. 2013). The two visible components for WL 4 are separated by 0.176" and separated by 0.3" in the case of YLW 16A. This corresponds to a projected linear separations of 23 AU and 39 AU, respectively, given a mean distance of 129 pc (see § 2.7). In each system, the large amplitude variability is believed to be intrinsic to one of the visible pair. This



Figure 3.8: The folded  $K_s$  and color curves for the inverse eclipse-like periodic variables WL 4 (P = 65.6 days) and YLW 16A (P = 92.3 days). The red line in each plot indicates the mean value in each case.

component is, in turn, hypothesized to be a close binary surrounded by a circumbinary disk; this system is thus a triple system. The influence of the wide companion has caused the plane of the circumbinary disk to be inclined to the orbital plane of the inner binary. The variability results when each component of the inner binary is periodically obscured by the circumbinary disk as the binary orbits around the barycenter. Kusakabe et al. (2005) proposed a similar model to explain the variability for KH-15D.

### 3.3 Long Time Scale Variables

The largest amplitude variability in long time scale variables is not observed to be periodic, but show consistent trends, unlike irregular variables. The  $K_s$  light curves are shown in Fig. 3.9 to 3.12; all 31 LTVs (31% of the variable catalog) are listed in Table 3.3.



Figure 3.9: The J or  $K_s$  light curves for 6 long time scale variables. The TS corresponds to the variability time scale as described in § 2.8.3. The highest signal-to-noise light curve of these 2 is illustrated.



Figure 3.10: The J or  $K_s$  light curves for 6 long time scale variables. The TS corresponds to the variability time scale as described in § 2.8.3. The highest signal-to-noise light curve of these 2 is illustrated.



Figure 3.11: The  $K_s$  light curves for 6 long time scale variables. The TS corresponds to the variability time scale as described in § 2.8.3.



Figure 3.12: The J or  $K_s$  light curves for 7 long time scale variables. The TS corresponds to the variability time scale as described in § 2.8.3. The highest signal-to-noise light curve of these 2 is illustrated.

Catalog ID <sup>a</sup>	Time-scale	$\Delta K_s$	$\Delta(J-H)$	$\Delta(H-K_s)$	YSO Class	Var. Mech. <sup>c</sup>
	(days)	(mag)	(mag)	(mag)		
ISO-Oph 88	310	0.500	0.415	0.270	II	Extinction
WL14	384	0.218	0.185	0.380	II	Extinction
70072-2446272	543	0.061	0.091	0.084		Extinction
ISO-Oph 91	143	0.078	0.074	0.180	III	Extinction
70266-2446345	313	0.049	0.061	0.122		Extinction
ISO-Oph 94	64	0.894		0.707	II	Extinction
WL $1^{\rm b}$	226	0.294	0.275	0.523	II	Accretion?
WL $17^{\rm b}$	578	1.125		1.065	Ι	Unknown
ISO-Oph 107	355	0.195		0.119	II	Accretion
YLW 8A	355	0.198	1.272	0.098	II	Accretion
ISO-Oph $112^{\rm b}$	207	0.984		1.123	II	Extinction
ISO-Oph 113	120	0.058	0.067	0.122	III	Unknown
WL $19^{\rm b}$	589	1.012	—	0.784	II	Accretion?
ISO-Oph 117	530	0.560	0.249	0.133	II	Accretion
$YLW \ 10B^{b}$	578	0.562		0.850	II	Unknown
ISO-Oph 119	76	0.445	—		II	Unknown
WL $20E$	81	0.299	0.262	0.297	II	Accretion
WL $20W$	122	0.213	0.258	0.305	II	Extinction
71726-2422283	132	0.163	—	1.137		Unknown
YLW 12A	92	0.728		0.813	Ι	Accretion
ISO-Oph 126	349	0.135	—	0.371	III	Extinction
WL 6	172	1.199		1.256	Ι	Accretion?
72297 - 2448071	327	0.134	0.097	0.062		Accretion?
72357-2412288	354	0.052	0.066	0.080		Extinction
ISO-Oph 137	89	0.749	—		Ι	Unknown
72514 - 2446335	750	0.065	0.089	0.173		Unknown
YLW $15A$	478	0.393		0.927	Ι	Extinction
YLW $16B$	140	2.312		1.318	Ι	Extinction
$YLW 17B^{b}$	516	0.155	0.110	0.233	II	Accretion
ISO-Oph $151$	398	0.282	0.106	0.160	II	Accretion
ISO-Oph 150	239	0.926			Ι	Unknown

Table 3.3: Time-Scale Variables

<sup>&</sup>lt;sup>a</sup>The catalog ID has been truncated by 2MASS J162 for 2MASS catalog stars.

<sup>&</sup>lt;sup>b</sup>Candidate sinusoidal-like periodic LTV.

<sup>&</sup>lt;sup>c</sup>A question mark denotes a variability mechanism that is uncertain due to insufficient color information.

In § 2.8.3, a method for quantifying the time scale of the extended brightness variations is detailed. These time scales range from 64 to 790 days, the latter being near the full duration of the observing campaign. The peak-to-trough  $\Delta K_s$  amplitudes for all LTVs range from 0.05 to 2.31 mag, with a median value of 0.29 mag. The peak-to-trough  $\Delta(H-K_s)$  color amplitudes range from 0.06 to 1.32 mag, with a median value of 0.23 mag. The two most probable mechanisms for these aperiodic variations with time scales much longer than typical stellar rotation periods are variable extinction and variable mass accretion rates. Fig. 3.13 shows two examples of the change in  $K_s$  brightness and stellar color caused by these two mechanisms. Extinction causes the star to become redder as the star dims. Changes in the mass accretion rate cause stars to become bluer as the star dims. For 12 LTVs (39%), the (J-H) and  $(H-K_s)$  colors become redder as the star dims favoring variable extinction as the dominant variability mechanism. The (J-H) and  $(H-K_s)$  colors become bluer as the star dims in 11 LTVs (34%) favoring variable mass accretion rates as the dominant variability The remaining 7 LTVs (23%), either do not have useful color information mechanism. because the J or H (or both) photometry is below the survey completeness limits, or the brightness-color correlation does not agree with any of the 4 listed variability criteria. No dominant variability mechanism is assigned to these stars.

One intriguing scenario to explain why LTVs do not seem to favor one variability mechanism over another is the viewing angle. For LTVs where variable accretion is the favored mechanism, the system could be more face-on providing a clearer view of the inner disk hole. Variable extinction due to circumstellar disk asymmetries is more easily seen at higher disk inclinations where the "puffed up" outer disk attenuates the light from the inner disk. As these two mechanisms have opposing brightness-color correlations, systems with no measured



Figure 3.13: Left: The  $K_s$  light, (J-H) color curve, and  $(H-K_s)$  color curve for the long time scale variable WL 14. This is an example of variability caused by extinction. As the  $K_s$ magnitude drops the colors become redder. Right: The same light and color curves for the long time scale variable ISO-Oph 117. This is an example of variability caused by variable mass accretion. As the  $K_s$  magnitude drops the colors become bluer.

correlations may represent intermediate viewing angles. In this case, the measured effects from variable accretion will "cancel" out or confuse the measured effects from variable extinction.

All LTVs are located "on cloud" and 25 stars of the 31 LTVs are classified as a YSO: 7 Class I (58%), 15 Class II (44%) and 3 Class III (27%).<sup>4</sup> The favored variability mechanism in the Class I LTVs is variable extinction for 2 stars, variable mass accretion for 2 stars and unidentified for 3 stars. The variability in the Class II LTVs is consistent with variable extinction in 5 stars, variable mass accretion in 8 stars and is not identified for 2 stars. The

<sup>&</sup>lt;sup>4</sup>The percentages indicate the percentage of variable stars in each class that are sinusoidal-like periodic variables.

 $K_s$ -color correlation in 2 Class III LTVs favors variable extinction as the dominant variability mechanism while the mechanism for variability in the third Class III LTV is not identified.

Based upon visual inspection of folded light curves, 6 LTVs are considered candidate periodic variables. These candidate periodic LTVs are denoted in Table 3.3 and Fig. 3.14 contains their folded  $K_s$  light curves. The variability time scales, ranging from 207 to 589 days, for the candidate periodic LTVs are measured using the Lomb-Scargle algorithm. The PPA does not find the time scales found by Lomb-Scargle to be significant. The stars, on average, have higher flux and color amplitude variability than the LTVs taken as a whole. The peak-to-trough  $\Delta K_s$  amplitude for these candidate periodic variables range from 0.16 to 1.13 mag, with a median value of 0.77 mag. The peak-to-trough color amplitude range from 0.23 to 1.12 mag, with a median value of 0.82 mag. For 3 candidate periodic LTVs, the (J-H)and  $(H-K_s)$  colors become bluer as the star dims favoring a variable mass accretion rate as the dominant variability mechanism. The  $(H-K_s)$  color of ISO-Oph 112 reddens as the star dims. This is consistent with variable extinction as the dominant variability mechanism. A combination of J band photometry below the survey completeness limits and the  $K_s$ -(H- $K_s$  color correlation not matching any of the 4 criteria precludes the identification of the dominant variability mechanism for 2 candidate periodic LTVs. All the candidate periodic LTVs are classified as a YSO: 1 Class I star and 5 Class II stars.



Figure 3.14: The  $K_s$  folded light curves for 6 candidate periodic long time scale variables. The red line indicates the star's mean magnitude

## 3.4 Irregular Variables

The variable catalog contains 40 stars (40%) that are clearly variable, but the largest amplitude variability is not periodic or coherent on long time scales. The  $K_s$  light curves are located in Figs. 3.15 to 3.19. Table 3.4 contains the list of irregular variables.


Figure 3.15: The J or  $K_s$  light curves for 8 irregular variables.



Figure 3.16: The J or  $K_s$  light curves for 8 irregular variables.



Figure 3.17: The J or  $K_s$  light curves for 8 irregular variables.



Figure 3.18: The  $K_s$  light curves for 8 irregular variables.



Figure 3.19: The J or  $K_s$  light curves for the 8 irregular variables.

Catalog ID <sup>a</sup>	$\Delta K_s$ (mag)	$\frac{\Delta(J-H)}{(\mathrm{mag})}$	$\frac{\Delta(H-K_s)}{(\mathrm{mag})}$	YSO Class	Var. Mech. <sup>b</sup>
65576 2508150	0.114	0.150	0 1 2 3		Unknown
65600 2455102	$0.114 \\ 0.167$	0.150 0.153	0.120	_	Unknown
WL 21	0.107	0.100	0.100	II	Unknown
65744-2452589	0.100	0 105	0 139		Unknown
65789-2452371	0.100 0.854		0.105		Unknown
65789-2457518	0.004	0 093	0.124		Unknown
65861-2446029	0.100	0.000	0.121		Accretion?
ISO-Oph 87	0.101 0.294	0.130 0.241	0.180	II	2 "flare" events
70054-2446444	1 109				2 Hare events Unknown
WL 22	0.631			T	Unknown
65967-2415433	0.078	0.124	0.073		Unknown
70055-2416255	0.061	0.082	0.066		Unknown
WL 16	0.082	0.083	0.176		Unknown
70276-2502437	0.053	0.062	0.083		Unknown
70285-2418546	0.060	0.086	0.064		Unknown
70501-2508484	0.052	0.084	0.077		Unknown
70516-2420077	0.049	0.079	0.078		Unknown
70591-2459376	0.044	0.078	0.055		Unknown
70597-2428363	0.182		0.157	II	Unknown
60819-2442286	0.069	0.061	0.195		Unknown
71003-2429133	0.334		0.366	II	Unknown
71096-2445298	0.056	0.073	0.074		Unknown
71173-2447109	0.069	0.079	0.074		Unknown
ISO-Oph 116	0.155	0.085	0.090	II	Accretion
71377 - 2505450	0.059	0.065	0.091		Unknown
71384 - 2415441	0.205		0.291		Unknown
71404 - 2415096	0.109	0.200	0.149		Unknown
71531 - 2415515	0.069	0.099	0.088		Unknown
71605 - 2415039	0.352		0.534		Unknown
71604 - 2416163	0.391		0.517		Unknown
71744-2413079	0.155	0.217	0.315		Unknown
YLW $13B$	0.215	0.276	0.158	II	Accretion
ISO-Oph 131	0.050	0.184	0.069	III	Unknown
72330-2507282	0.700		0.751		Unknown
72325-2448357	0.074	0.099	0.094		Unknown
ISO-Oph 138	0.326	0.432	0.175	II	Unknown
73052-2432347	0.070	0.075	0.082		Unknown
73107-2504004	1.057				Unknown

Table 3.4: Irregular Variables

Continued on Next Page...

Catalog ID<sup>a</sup>  $\Delta(J-H)$  $\Delta(H-K_s)$ YSO Class Var. Mech.<sup>b</sup>  $\Delta K_s$ (mag)(mag)(mag)Unknown 73122-2504172 0.5660.3860.473Unknown 73208 - 25085450.6130.3180.679

Table 3.4 – Continued

<sup>&</sup>lt;sup>a</sup>The catalog ID has been truncated by 2MASS J162 for 2MASS catalog stars.

<sup>&</sup>lt;sup>b</sup>A question mark denotes a variability mechanism that is uncertain due to insufficient color information.

The  $\Delta K_s$  amplitude ranges from 0.04 to 1.11 mag, with a median value of 0.14 mag. The  $\Delta(H-K_s)$  color amplitude ranges from 0.05 to 0.75 mag with a median value of 0.14 mag. Using the variability criteria, discussed in  $\S$  3, the primary variability mechanism is only identified for 4 irregular variables (2MASSJ16265861-2446029, ISO-Oph 116, YLW 13B, ISO-Oph 87). The first three 3 exhibit a long time scale variation in at least one observing season where the star becomes bluer as it dims. This is indicative of variable mass accretion as the variability mechanism. These stars are not considered LTVs as this variability is not the largest amplitude variability in the time series. The variable accretion for YLW 13B is identified to occur for ~ 115 days in the second year with a  $\Delta K_s \sim 0.15$  mag and  $\Delta (H K_s$  0.11 mag (see Fig. 3.18). ISO-Oph 116 varies via variable accretion at least twice (see Fig. 3.16). The first time occurs for  $\sim 170$  days in the first year with  $\Delta K_s = 0.14$  mag and  $\Delta(H-K_s) = 0.07$  mag. The second time occurs for  $\sim 70$  days in the third year with  $\Delta K_s$ ~0.11 mag and  $\Delta(H-K_s) = 0.07$  mag. The average error in both  $K_s$  and  $(H-K_s)$  is 0.01 mag for both YLW 13B and ISO-Oph 116. In the case of 2MASSJ16265861-2446029, only the (J-H) color becomes bluer making the mechanism identification tentative. The YSO classification for this star is unknown. The J band photometry is too dim in 11 irregular variables to identify the variability mechanism. Of these 11 variables, the H band is also too dim in 5 stars.

The variability in ISO-Oph 87 is peculiar due to two flare-like events that occur in the  $K_s$  photometry on approximately 1400 and 1700 2MASS MJD.. Fig. 3.20 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry for this Class II YSO. If the events are truly related to an increase in stellar activity, the star is expected to become bluer in both (J-H) and  $(H-K_s)$ . However, no change is seen in the (J-H) color and the star reddens in  $(H-K_s)$ . The first



Figure 3.20: The K<sub>s</sub>, (J-H) and  $(H-K_s)$  photometry for ISO-Oph 87. The photometry contains at least two "flare" events, where the star brightens sharply in  $K_s$ , occurring at 1400 and 1700 2MASS MJD. Two other possible "flare" events occur between 1600 and 1700 2MASS MJD. The events become redder as the star brightens rather than becoming bluer as expected for stellar flares.

event lasts for ~10 days with  $\Delta K_s = 0.20$  mag and  $\Delta (H-K_s) = 0.13$  mag. The second event occurs for ~6 days and  $\Delta K_s = 0.18$  mag and  $\Delta (H-K_s) = 0.16$  mag. Two additional, lower amplitude spikes in the  $K_s$  photometry between 1600 and 1700 2MASS MJD might also be similar flare-like events.

Irregular variables have the smallest percentage (68%) of stars located "on cloud". Only 9 stars in this subcategory are classified as a YSO: 1 Class I (8%), 7 Class II (21%) and 1 Class III (9%).<sup>5</sup> Most (16 stars) of the 22 candidate  $\rho$  Oph members are irregular variables. If these candidate members are YSOs, then the fraction of YSO irregular variables is comparable to the fraction of periodic and LTV YSOs.

<sup>&</sup>lt;sup>5</sup>The percentages indicate the percentage of variable stars in each class that are irregular variables.

#### 3.5 Examples of Multiple Variability Mechanisms

As discussed in the Introduction, variability studies indicate young stars sometimes exhibit complex photometric behavior believed to result from multiple variability mechanisms acting concurrently. For most stars in this survey, only the highest amplitude variability can be confidently characterized. However, a lower amplitude, second type of variability is definitely seen in 7 variable stars. Four of these seven stars (YLW 1C, 2MASS J16272658-2425543, YLW 10C, WL 4) show evidence for two separate, yet statistically distinct periodic variations. The remaining three stars (WL 20W, ISO-Oph 126, WL 15) are periodically variable underneath a higher amplitude, long time scale variation. The methods used to identify both the primary and secondary variabilities in these stars are discussed in § 2.8.1. The following subsections contain detailed discussions for each of these stars except WL 4 which is described in Plavchan et al. (2008b).

• YLW 1C (ISO-Oph 86): This CTTS exhibits both sinusoidal-like and eclipse-like periodic variability at 2 distinct periods; the periods are distinct from each other to a  $20\sigma$  confidence level. Fig. 3.21 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry folded to the sinusoidal-like period, P = 5.7792  $\pm$  0.0085 days. The peak-to-trough  $\Delta K_s$  amplitude for this variability is 0.14 mag. The variability in both the (J-H) color and  $(H-K_s)$  color is not correlated with the  $K_s$  variability. This favors variability caused by rotational modulation of a cool starspot(s). Fig. 3.22 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry folded to the eclipse-like period, P = 5.9514  $\pm$  0.0014 days. The  $\Delta K_s$  eclipse depth is 0.29 mag. The  $(H-K_s)$  color reddens during the eclipse event consistent with variability caused by extinction. This behavior is not seen in the (J-H)



Figure 3.21: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for YLW 1C phased to the 5.7752  $\pm$  0.0085 day sinusoidal-like period. The red line indicates the mean value in each panel. The lack of color correlation with  $\Delta K_s$  points to rotational modulation of cool starspots as the variability mechanism.

color. Since the J band photometry is near the survey completeness limits, the absence of a clear reddening trend may be due to low signal-to-noise in the color curve.

The existence of two periods for YLW 1C is consistent with the interpretation that these events are true occultation events, as proposed for AA Tau (see § 3.2.2). The short period, arising from a stellar surface feature(s), traces the stellar rotation rate. The longer period suggests the occultation of the star by an obscuration located just beyond the circumstellar disk corotation radius. Following the analysis described in § 3.2.2, the occulter of YLW 1C is located 7.2 R<sub>\*</sub> from the host star and the duration of the occultation is ~6.4 hours corresponding to a minimum occulter diameter of ~2.0 R<sub>\*</sub>. The reader is reminded this diameter represents the extent within the orbital path



Figure 3.22: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for YLW 1C phased to the 5.9514  $\pm$  0.0014 day eclipse-like period. The red line indicates the mean value in each panel. The  $(H-K_s)$  become redder as  $K_s$  dims indicating variable extinction as the likely variability mechanism.

and makes no claim on any preferential occulter shape. The eclipse depth is  $\Delta K_s = 0.29$  mag.

The large occulter diameter argues against the direct detection of a hot protoplanet. However recent imaging results suggest that gas giant planets maybe considerably extended in the mass accretion phase (Quanz et al. 2013; Kraus & Ireland 2012). If true in this case, this would demonstrate the existence of a hot protoplanet with a period of 6 days very near the peak in the period distribution for exoplanets (Wright et al. 2012). Alternatively, the event could be caused by an occultation of a warped portion of a circumstellar disk. This scenario has been proposed to explain the near- to mid-IR variability in LRLL 31 (Flaherty & Muzerolle 2010; Flaherty et al. 2012). While most YSO disk models invoke axisymmetry, objects such as YLW 1C are prompting the creation of more complex models.

• 2MASS J16272658-2425543: This CTTS, designated 'J543' hereafter, is another star exhibiting both sinusoidal-like and eclipse-like variability with two distinctly different periods. Fig. 3.23 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry folded to the sinusoidal-like period, P = 1.52921 ± 0.00065 days. The peak-to-trough  $\Delta K_s$  amplitude for this variability is 0.20 mag. The variability in the (J-H) and  $(H-K_s)$  colors are not correlated with the K<sub>s</sub> photometry, which favors variability caused by rotational modulation of a cool starspot. Fig. 3.24 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry folded to the eclipse-like period, P = 2.9602 ± 0.0013 days. The  $\Delta K_s$  eclipse depth is 0.17 mag. Both the (J-H) and  $(H-K_s)$  colors redden during in the eclipse event consistent with variable extinction.

The physical interpretation for the observed variability is identical to that of YLW 1C. The 1.6 day period corresponds to the stellar rotation rate and the 3.0 day period arises from a periodic occultation by an asymmetry in the inner circumstellar disk. The occulter size and distance are 2.8 R<sub>\*</sub> and  $\sim$ 3.0 R<sub>\*</sub>. Differing from YLW 1C, the occulter for J543 is located approximately a stellar radius beyond the corotation radius.

YLW 10C (ISO-Oph 122): YLW 10C is the third CTTS where two distinct periods are identified. Fig. 3.25 contains the K<sub>s</sub>, (J-H) and (H-K<sub>s</sub>) photometry folded to the sinusoidal-like period, P = 3.0779 ± 0.0025 days. The peak-to-trough ΔK<sub>s</sub> amplitude for this variability is 0.25 mag. Fig. 3.26 contains the K<sub>s</sub>, (J-H) and (H-K<sub>s</sub>) photometry folded to the eclipse-like period, P = 2.9468 ± 0.0029 days. The ΔK<sub>s</sub> eclipse



Figure 3.23: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for J543 phased to the 1.52921  $\pm$  0.00065 day sinusoidal-like period. The red line indicates the mean value in each panel. The lack of color correlation with  $\Delta K_s$  points to rotational modulation of cool starspots as the variability mechanism.



Figure 3.24: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for J543 phased to the 2.9602  $\pm$  0.0013 day eclipse-like period. The red line indicates the mean value in each panel. Both colors become redder as  $K_s$  dims indicating variable extinction as the likely variability mechanism.



Figure 3.25: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for YLW 10C phased to the 3.0779  $\pm$  0.0025 day sinusoidal-like period. The red line indicates the mean value in each panel. The lack of reliable J and H photometry prohibits a confident estimate of the variability mechanism.

depth is 0.28 mag. Unfortunately, both the J and H photometry are below the survey completeness limits preventing the identification of either variability mechanism.

Given the sinusoidal-like and eclipse-like variability is very similar to both YLW 1C and J543, the same physical interpretation is proposed for this star. However, unlike YLW 1C, the sinusoidal-like variability, presumed to trace the stellar rotation rate, has a longer period by 3.1 hours than the periodic occultations. This places the hypothetical occulter *within* the corotation radius. The size and distance to the occulter are  $3.31 \text{ R}_{\star}$  and  $3.75 \text{ R}_{\star}$ . The occulter is located within the dust sublimation radius as computed using the formalism of Jura & Turner (1998). This formalism is only an approximation as it does not take into account dust evaporation and condensation rates, grain size, or grain composition.



Figure 3.26: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for YLW 10C phased to the 2.9468  $\pm$  0.0029 day eclipse-like period. The red line indicates the mean value in each panel.

• WL 20W (YLW 11B, ISO-Oph 126): This CTTS is both periodically variable and variable over a long time scale. As such, WL 20W is designated both a periodic variable and a LTV. Fig. 3.27 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry for this star. The long time scale variability begins on ~1600 2MASS MJD and has a timescale of 122 days. The  $\Delta K_s$  depth is 0.26 mag. Both the (J-H) and  $(H-K_s)$  colors become redder during the long time scale variation, consistent with variable extinction. The periodic signal is not significant unless the time series affected by the long time scale variability is omitted from analysis by the PPA (see Fig. 2.11). Fig. 3.28 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry folded to the sinusoidal-like period, P = 2.1026  $\pm$  0.0060 days. The peak-to-trough  $\Delta K_s$  amplitude for the sinusoidal-like variability is 0.19 mag. Neither the (J-H) nor the  $(H-K_s)$  color is correlated to the  $K_s$  variability. This favors rotational modulation by cool starspots as the variability mechanism.



Figure 3.27: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for WL 20W. The long time scale variation is easily seen beginning at ~1600 2MASS MJD. Both the (J-H) and  $(H-K_s)$  become redder as  $K_s$  dims indicating variable extinction as the likely variability mechanism.



Figure 3.28: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for WL 20W folded to the period 2.1026 days. The red line indicates the mean value in each panel. No correlation between  $\Delta K_s$  and the change in colors points to rotational modulation of cool starspots as the likely variability mechanism. Only the photometry before the long time scale variation is plotted.

The variability of AA Tau has been cited as an explanation for the periodic eclipsing variables in this survey. However recently it has been discovered that AA Tau is exhibiting a long time scale dimming on the order of 2-3 mag in the V band (Bouvier et al. 2013). This long time scale variation is superimposed on top of the periodic variability. Additionally the system appears to become bluer in this dim state; a phenomenon seen in UX Ori-type variables (Grinin et al. 1991; Herbst et al. 1994). The physical interpretation for UX Ori-type variability is that the star dims due to an asymmetric optically thick occulter beyond the inner circumstellar disk. The bluer color represents a larger contribution of scattered starlight off the occulting material. This scenario is an alternative explanation than variable mass accretion for the long time scale variation observed in WL 20W. The eclipse depth corresponds to a 2.9 mag

dimming when converted to the V band by using the extinction coefficients given in Cohen et al. (1981); this is consistent with the AA Tau long time scale variation. The (J-H) and  $(H-K_s)$  colors do become bluer during the long time scale variation also consistent with the observations of AA Tau. Our overall interpretation for the variability in WL 20W is a central star rotating with a 2 day period that is occulted by a pocket of optically thick material located beyond the inner circumstellar disk.

It is worth noting this star belongs to a triple system that is spatially resolved in the mid-IR. Ressler & Barsony (2001) show the most variable member is, in fact, WL 20S. They classify this source as Class I through SED fitting of mid-IR photometry. They show that WL 20E and WL 20W are nearly constant on decadal timescales whereas the flux of WL 20W increased sixfold in 15 years. As the largest separation between

these components is 3.66'', the large aperture size in our work (4'') includes all three stars and thus cannot rule out the possibility that the measured variability arises from this southern component.

• ISO-Oph 126: This WTTS is similar to WL 20W in that it exhibits both periodic variability and a long time scale variation. ISO-Oph 126 is also designated both a periodic variable and a LTV. Fig. 3.29 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry for this star. The J band photometry is below the survey completeness limits. Therefore the (J-H) color is deemed unreliable for analysis. The long time scale variation dominates the photometry prior to 1400 2MASS MJD with a time scale of 349 days. The  $\Delta K_s$ depth of this variation from the continuum brightness is 0.10 mag. The  $(H-K_s)$  color becomes redder during the long time scale variation as the star dims. This is consistent with extinction as the variability mechanism. The PPA identifies a significant periodic signal when only the portion of the time series after 1400 2MASS MJD is analyzed (see Fig. 2.11). Fig. 3.30 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry folded to the sinusoidal-like period,  $P = 9.114 \pm 0.090$  days. The peak-to-trough  $\Delta K_s$ amplitude for the sinusoidal-like variability is 0.06 mag. The  $(H-K_s)$  color becomes bluer as  $K_s$  brightens favoring a variability mechanism of rotational modulation by accretion-induced hot starspots.

While the origin of the periodic variability can be attributed to stellar surface features, the favored interpretation of extinction in this case potentially challenges the class identification as a disk-less WTTS. While Barsony et al. (2005) could not provide a YSO classification for ISO-Oph 126, the authors could place an upper limit to the



Figure 3.29: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for ISO-Oph 126. The long time scale variation is easily seen prior to ~1400 2MASS MJD. Both the (J-H) and  $(H-K_s)$  become redder as  $K_s$  dims indicating variable extinction as the likely variability mechanism.



Figure 3.30: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for ISO-Oph 126 folded to the period 9.114 days. The red line indicates the mean value in each panel. The  $(H-K_s)$  color becomes bluer as  $K_s$  brightens favoring a variability mechanism of rotational modulation by accretion-induced hot starspots.

spectral index at  $\leq$ -0.88. This allows for the possibility this star is Class II and surrounded by an optically thick accretion disk.

One intriguing option is the occultation by a disk surrounding an orbital companion. This scenario is invoked to explain long time scale variability in evolved star systems  $\epsilon$  Aur (Guinan & Dewarf 2002; Kloppenborg et al. 2010; Stencel et al. 2011), EE Cep (Mikolajewski & Graczyk 1999; Graczyk et al. 2003; Mikolajewski et al. 2005; Gałan et al. 2010) and most recently in the young star system 1SWASP J140747.93-394542.6 ('J1407') (Mamajek et al. 2012). Photometric variations within the long time scale variations are believed to arise from structure within the occulting disk. This structure may represent new planets (EE Cep; Gałan et al. (2010)), or it may represent planetary moons (J1407; Mamajek et al. (2012)). It is noted that there is significant scatter in the  $K_s$  time series during the first half of the long time scale variation in comparison to the second half of this variation (see Fig. 3.31). High resolution imaging or radial velocity monitoring may help to confirm the existence of a companion to ISO-Oph 126.

WL 15 (YLW 7A, ISO-Oph 108): This star is one of the brightest at K<sub>s</sub> (K<sub>s</sub> = 7.05 mag) and the reddest ((H - K<sub>s</sub>) = 4.01 mag) in the variable catalog. WL 15 is a Class I YSO. Similar to WL 20W and ISO-Oph 126, this star exhibits a large amplitude, long time scale variation overtop of a smaller amplitude periodic variability. Unlike WL 20W and ISO-Oph 126, the photometry during the long time scale variation is too sparse to confidently identify a time scale. Therefore WL 15 is only designated a periodic variable. Fig. 3.32 contains the K<sub>s</sub>, (J-H) and (H-K<sub>s</sub>) photometry for WL 15. The J band photometry is below the survey completeness limit and is deemed



Figure 3.31: The  $K_s$  and  $(H-K_s)$  photometry for the long time scale variation in ISO-Oph 126. The  $K_s$  shows considerable structure during ingress, but is nearly stable during the "eclipse" egress. The red line indicates the mean magnitude of the continuum photometry.

unreliable for analysis. The long time scale variation is observed between 1396 and 1443 2MASS MJD with a  $\Delta K_s$  amplitude of ~1 mag. The mean  $(H-K_s)$  color does not change as the star dims during this event. Even including the long time scale variation, the PPA found a significant periodic signal. Fig. 3.33 contains the  $K_s$ , (J-H) and  $(H-K_s)$  photometry for WL 15 folded to the sinusoidal-like period, P = 19.412  $\pm$  0.085 days. This variability has a peak-to-trough  $\Delta K_s$  amplitude of 0.90 mag. The  $(H-K_s)$  color is not correlated to the  $K_s$  photometry.

While the colorless periodic variability favors rotational modulation by cool starspots, the amplitude of variability does not. The highest amplitude variability confirmed to as caused by cool starspots, to date, is  $\Delta V = 0.63$  mag (Strassmeier et al. 1997). This amplitude is nearly 0.3 mag lower than that observed for WL 15. In addition,



Figure 3.32: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for WL 15. The ~19 day period is clearly evident and appears to continue even through a ~1 mag drop in  $K_s$  band flux. The photometry during the larger amplitude flux decrease is too sparse to determine confidently a time scale. A lack of a trend in the  $(H-K_s)$  color during this event highly suggests against extinction except by an opaque occulter.



Figure 3.33: The  $K_s$ , (J-H) and  $(H-K_s)$  photometry for WL 15 folded to the period 19.412 days. The red line indicates the mean value in each panel.

the contrast between the starspot and surrounding photosphere increases toward bluer wavelengths. Therefore the amplitude will be even larger in the optical. The origin behind the long time scale variation is equally peculiar. As the  $(H-K_s)$  color does not become redder as the star dims, this seems to eliminate extinction as variability mechanism. However, variability by rotational modulation of surface features seems implausible given the timescale and amplitude of the variation. – 4 –

# THEORY BEHIND LONG BASELINE INTERFEROMETRY

The remainder of this dissertation focuses on stellar variability with a narrow perspective. In particular, the focus will be to fully characterize a specific variability mechanism through direct imaging. The method by which to measure the properties of starspots on the magnetically active giant  $\lambda$  Andromedae is long baseline near-IR interferometry (LBI). The chapter begins by discussing the theory behind LBI with specific attention paid to the 2 interferometric observables visibility and closure phase. The chapter then goes on to explore the effects of starspots on these 2 observables. A discussion of th tools necessary for obtaining interferometric measurements concludes the chapter. These tools are the Center for High Angular Resolution Astronomy interferometric telescope array (the CHARA Array) and the Michigan Infra-Red beam Combiner (MIRC). For a more detailed perspective on the basics of interferometry, the reader is directed to the excellent reviews by Quirrenbach (2001) and Monnier (2003). More comprehensive descriptions of the CHARA Array and the MIRC beam combiner can be found in ten Brummelaar et al. (2005) and Monnier et al. (2006).

# 4.1 Young's Double Slit Experiment

Young's double slit experiment provides a framework to explain the basic concepts of long baseline interferometry. In the early  $19^{th}$  century, the physicist Thomas Young passed monochromatic nearly plane-parallel light through a mask containing two parallel slits and discovered, on the other side of the mask, that the light combined to create a fringe pattern; this pattern could be visualized by projecting it onto a screen. A schematic of this



Figure 4.1: Youngs double slit interference experiment (monochromatic light) presented to illustrate the basic principles behind stellar interferometry. *Left:* The case for a single point-source. *Right*: The case for a double source with the angular distance being half the fringe spacing.

experiment is shown in Fig. 4.1. This experiment convincingly demonstrates the wave and interference properties of light, and is still taught in introductory physics labs today.

This idealized model is realized in a practical interferometer by collecting light with two telescopes separated by some distance and bringing the light together in a beam combination facility for interference. The interference is due to the light waves propagating from each telescope (slit) to the beam combination facility with different relative path lengths causing the light to both constructively and deconstructively interfere at different points along a detector (screen). One can write down the condition for constructive interference; the fringe spatial frequency (fringes per unit angle, expressed in Eqn. 4.1) of the intensity distribution on the detector is equal to to the projected telescope separation, or baseline B, in units of the observed wavelength  $\lambda$ .

fringe spatial frequency 
$$\equiv u = \frac{B}{\lambda} rad^{-1}$$
 (4.1)

The ability to discern the two components of a binary star system is often used to gauge the spatial resolution of an instrument, be it a conventional circular aperture telescope or a separated element interferometer. Classical diffraction theory has established the "Rayleigh Criterion" for defining the (diffraction limited) resolution of a filled circular aperture of diameter D:

resolution of telescope 
$$\equiv \Delta \Theta_{telescope} = 1.22 \frac{\lambda}{D} rad$$
 (4.2)

This criterion corresponds to the angular separation on the sky when one stellar component is centered on the first null in the diffraction pattern of the other; the binary is then said to be just resolved. A similar criterion can be defined for an interferometer: an equal brightness binary is resolved by an interferometer if the fringe contrast goes to zero at the longest baseline. This can be visualized by returning to the framework of Young's double slit experiment. Imagine another point source of light (of equal brightness, but incoherent with the first) located at an angle  $\lambda/(2B)$  from the first source (see right panel of Fig. 4.1). The two fringe patterns are 180° out of phase, hence canceling each other out and presenting a uniform illumination on the detector. Hence,

resolution of interferometer 
$$\equiv \Delta \Theta_{interferometer} = \frac{\lambda}{2B}$$
 (4.3)

While these two criteria are somewhat arbitrary, they are useful for estimating the angular resolution of an optical system and are in widespread use by the astronomical community. Since the baseline of an interferometer can be made much larger than a single segmented mirror, the advantage of using an interferometer is clear. The 10 m Keck telescope can reach an angular resolution of 13.8 mas in the visual regime (0.55  $\mu$ m). However the Keck interferometer, which utilizes both Keck 10 m telescopes with a baseline of 85 m, could achieve a resolution of 3.3 mas at the same wavelength.

## 4.2 The Van Cittert-Zernike Theorem

Given that an interferometer functions in the same manner as the Young's double slit experiment, how can a fringe pattern be used to determine information on the source's morphology? The translation is done using the van Cittert-Zernike theorem. This theorem states that the contrast and location (phase) of the fringes, i.e. the complex visibility, corresponds to the Fourier transform of the source intensity distribution on the sky at the spatial frequencies corresponding to the baseline projected on the sky. The complex visibility, V(u,v) is expressed mathematically by the following:

$$V(u,v) = \frac{\iint d\alpha d\beta I(\alpha,\beta) e^{-2\pi i (\alpha u + \beta v)}}{\iint d\alpha d\beta I(\alpha,\beta)}$$
(4.4)

where  $\alpha$  and  $\beta$  are the spherical spatial coordinates on the sky, I is the source intensity function, and u and v are the spatial frequencies in the x (East-West) and y (North-South) directions, respectively (see Eqn. 4.1).

#### 4.2.1 Visibility Amplitude

From a practical standpoint, the two measured quantities from a interference pattern (fringes) are the fringe amplitude or "Michelson visibility", which is a function of the amount of constructive and destructive interference, and the fringe location or phase. The visibility is defined simply as:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{4.5}$$

To simplify the nomenclature in this paper, I will refer to the fringe amplitude simply as "visibility" despite the term being more applicable to complex visibility. The visibility is, in reality, the amplitude modulus of the complex visibility.

For very simple source morphologies, such as a point source, uniform disk, or binary system, the relationship between the observed visibility and the brightness distribution,  $I(\alpha,\beta)$ , is an analytically function. For example, the following equation describes the visibility as a function of spatial frequency, u, for a limb darkened disk.

$$V(u) = \Lambda(n+1) \frac{|2J_n(\pi a u)|}{(\pi a u/2)^n}$$
(4.6)

where  $n = (\alpha+2)/2$ , *a* is the stellar angular diameter,  $\alpha$  is the power limb darkening coefficient,  $J_n$  is a  $n^{th}$  order Bessel function and  $\Lambda$  is a Gamma function. Fig. 4.2 shows how the visibility as a function of spatial frequency differs between a uniform stellar disk and a binary system. More complicated morphologies, such as a star with starspots, lead to even more



Figure 4.2: Example of the difference between the visibility curves for a single star and a binary system. The dotted line indicates the curve for a single star with  $\theta$ =1.0 mas, while the solid line represents the curve for a binary system with the following parameters:  $\theta_{primary}$ =1.0 mas,  $\theta_{secondary}$ =0.5 mas,  $\alpha$ =10 mas, and  $\Delta K$ =2.0 mag. (Baines et al. 2008)

complicated visibility functions that cannot be determined analytically, but can be used to reconstruct source morphology through parametric modeling or image reconstruction.

Fig. 4.3 illustrates the meaning of certain terms when discussing visibility curves. The point where the visibility stops decreasing in value and begins to increase is called a *null*. For a uniform disk, the baseline position of the  $1^{st}$  null is a measurement of the disk size. As the angular size of an object decreases, the location of the  $1^{st}$  null is located at larger baseline positions and *vise versa*. Therefore to resolve fully smaller and smaller stars, the



Figure 4.3: Explanation of the terms *lobe* and *null* in regards to a visibility curve. A *null* corresponds to the point where the visibility is zero. The position of the  $1^{st}$  *null* is related to the angular diameter of a uniform disk. A *lobe* refers to the visibility curve between *nulls*; higher order *lobes* contain information on smaller scales. The exception is the  $1^{st}$  lobe which is defined as the curve between a baseline of zero to the  $1^{st}$  *null*.

baseline needed needs to be longer and longer. This also applies to attempts to resolve close binary stars or surface features on a resolved source. A visibility *lobe* is the region of the visibility curve between nulls with the  $1^{st}$  lobe located between the zero baseline position to the  $1^{st}$  null.

### 4.2.2 Closure Phases

If the source morphology is point symmetric, then visibility measurements alone are adequate to describe the brightness distribution. If the morphology is not point symmetric, the phase of the interference pattern is required to describe asymmetries within the brightness distribution. A few examples where phase information is required are stars with surface fea-



Figure 4.4: Atmospheric time delays or phase errors at telescopes cause fringe shifts, as can be seen through analogy with Young's double slit experiment. (Monnier 2003)

tures (e.g. starspots), warped circumstellar disks where one edge is brighter than the other, and binary systems with unequal brightness ratios or with tidally distorted companions.

Unfortunately, phase information from a single telescope pair is lost due to atmospheric disturbances. The atmosphere can be envisioned as composed of numerous small pockets of material. The optical characteristics are different from one pocket to the next. The pockets are small enough in the optical/near-IR regime that each telescope will be receiving light through different atmospheric pockets than the others. Fig. 4.4 is a schematic of this effect using the double slit experiment as a framework.

Unlike with two telescopes, a limited amount of phase information can be obtained when combining light from 3 or more telescopes. Although each pair of telescopes has a band phase information, the sum of the phase information from 3 or more telescopes provides a combined phase measurement, known as *closure phase*. The is expressed mathematically for 3 telescopes as follows:

$$\Phi(1-2) = \Phi_0(1-2) + [\phi(2) - \phi(1)]$$
(4.7a)

$$\Phi(2-3) = \Phi_0(2-3) + [\phi(3) - \phi(2)]$$
(4.7b)

$$\Phi(3-1) = \Phi_0(3-1) + [\phi(1) - \phi(3)]$$
(4.7c)

The first quantity represents the measured phase, the second represents the intrinsic phase, and the third represents the phase shift due to the atmosphere. By adding the three equations together, the atmospheric terms cancel each other. The number of independent closure phases is given by:

$$\frac{N-1}{2} = \frac{N-1)(N-2)}{2} \tag{4.8}$$

where N is the number of telescope beams combined. Obviously the more telescopes used the more intrinsic phase information is recovered. Table 4.1 quantifies how much phase information is recovered as a function of the number of telescopes.

Number of telescopes	Number of independent closure phases	Percentage (%) of phase information
3	1	33
7	15	71
21	190	90
27	325	93
50	1176	96

Table 4.1: Phase information contained in the closure phases alone.

A source with point symmetry (e.g. an unspotted star) has a closure phase equal to zero. Any nonzero closure phase indicates some asymmetry (e.g. stellar companion, starspot) in the source brightness distribution. An exception is if the source is large enough with respect to the baseline for the visibility to reach a null. In this case, a zero closure phase will rapidly transition to  $\pm 180^{\circ}$  as one moves through the null in spatial frequency. This does *not* indicate an asymmetry. For the remainder of this paper, I treat  $\pm 180^{\circ}$  as "zero" closure phase for simplicity. The reader is directed to the excellent review by Monnier (2003 and references therein) for an additional qualitative and quantitative discussion of closure phase.

## 4.2.3 Aperture Synthesis

The main advantage of an interferometer over a single aperture telescope, as discussed in § 4.1, is the increased spatial resolution. An interferometer obtains approximately the same resolution as a single telescope with an aperture equal to its longest baseline. However, the trade off for this increased resolution is loss of spatial frequency information. The visibility in the [u,v] plane is the Fourier transform of a spatial brightness distribution with coordinates  $\alpha$  and  $\beta$ . If you imagine a single mirror telescope being composed of an infinite number of point mirrors, then each pair of point mirrors acts in the same way as an interferometer and the baseline between these two points corresponds to a single point in the [u,v] plane. Therefore, the [u.v] plane for a single mirror telescope is completely filled in a single observation providing complete spatial frequency information with respect to the aperture size. This is, however, not the case for an interferometer where only discrete points in the [u,v] plane are sampled for a single observation. The process of filling the [u,v] plane in order to recover this spatial information is known as *aperture synthesis*.



Figure 4.5: Upper Left: a snapshot with 2 telescopes. Upper Right: 3 observations with 2 telescopes taken 1.5 hours apart. Lower Left: multiple observations with 2 telescopes taken 30 minutes apart. Lower Right: observations with 2 telescopes taken 30 minutes apart over the entire night. (Millour et al. 2008)

There are three primary methods for aperture synthesis when observing celestial objects; these may be used singly or in combination. The first is to observe the target over the course of a night. As the Earth rotates, the baseline between different telescopes will change with respect to the perspective of the brightness distribution. This change in the apparent baseline will sample different parts of the [u,v] plane. Fig. 4.5 demonstrates this effect.



Figure 4.6: Upper Left: a snapshot with 2 telescopes. Upper Right: a snapshot with 2 telescopes through JHK filters. Lower Left: 6 observations with 2 telescopes through JHK filters. Lower Right: 6 observations with 3 telescopes through JHK filters.

In the case of two interferometers at two different latitudes, the closer the interferometer is to the equator, the more linear the [u,v] sampling, while an interferometer near one geographic pole will sample the [u,v] plane in a more circular fashion. The second method is to observe the target using multiple telescopes separated by nonredundant distances. The larger the baseline between with telescope pair, the larger the spatial frequency (smaller spatial scale) sampled on the [u,v] plane and *vise versa*. The third method is to observe the target at different wavelengths. Since the spatial frequency is set by the ratio of baseline to wavelength, observations acquired at different wavelengths will sample different positions in the [u,v] plane. Fig. 4.6 demonstrates how the [u,v] plane is filled using additional telescopes and also by observing at different wavebands.

#### 4.3 The Effect of Starspots on Interferometric Observables

Given the challenge of interpreting visibility and closure phase directly in terms of fundamental stellar and starspot properties, illustrative examples on how these observables change based on the presence of starspots and the properties of those starspots are offered. The two interferometric observables considered here are visibility and closure phase. Fig. 4.7 illustrates the visibility over different spatial frequencies for an unspotted star, a star with a cool starspot near the western limb, and the same star with the starspot on the eastern



Figure 4.7: The left most panel shows model images of an unspotted star (top), a star with a starspot on the western hemisphere (middle), and a star with a starspot on the eastern hemisphere (low). The right most panel shows the distribution of visibilities over the entire [u,v] plane for each model image.


Figure 4.8: The left most panels show orthogonal visibility cuts from the corresponding model images from Fig 4.7. The starspot is seen as a change in the visibilities between the two cuts. The right most panels show the closure phases extracted from the corresponding model images. Note the nonzero closure phases for the unspotted star. The closure phases for the two spotted stars are near mirror images of each other thus breaking the 180° ambiguity found in the visibility measurements.

limb. Fig. 4.8 contains the visibility curves for both a north-south oriented and an east-west oriented baseline. Also in Fig. 4.8 are the closure phases for each stellar surface for a densely sampled [u,v] plane. For the unspotted star, the visibility distribution resembles a classic circular aperture diffraction pattern. The visibility cuts along the orthogonal directions show the pattern is axisymmetric. Since an unspotted star is point symmetric, the closure phases are zero as expected. When a starspot is present, the visibility pattern changes as shown in the visibility cuts. For the baseline oriented along a starspot, the 1<sup>st</sup> null is located at a larger baseline position indicating the starspot is giving the illusion that the star is smaller along this axis. In addition, the null does not reach zero visibility and the amplitude of the visibility lobes is diminished. In contrast, the 1<sup>st</sup> null in the visibility curve along the

orthogonal baseline is at a slightly smaller baseline position indicating the star has an apparent larger angular diameter on this axis. Also, the amplitude of  $1^{st}$  visibility lobe is higher than in the absence of the starspot. What is important to notice is that at longer baselines the visibility curves show closer agreement. The reason is that the longer baselines sample smaller spatial scales that are less sensitive to larger scale brightness variations such as due to a starspot. The spatial scales affected by the starspot size, however once the starspot size is below the array's angular resolution, the starspot contribution to the visibility ends. Another important point is that the visibility response is identical regardless on which limb the starspot is located. This is because visibility measurements alone possess an  $180^{\circ}$  degeneracy. This is not the case with the closure phase measurements as the starspot location produces drastically different results (see Fig. 4.8).

As it is clear both the stellar and starspot properties alter the observed visibilities and closure phases, the specific effects as a function of each property is now discussed. The parameters considered are the star's angular diameter,  $\theta$ , the limb darkening coefficient,  $\alpha$ , the starspot covering factor,  $\phi$ , the starspot latitude, b, the starspot longitude, l, and the starspot intensity ratio, f. The limb darkening coefficient is characterized as  $I(\mu) = I(1)\mu^{\alpha}$ , where  $\mu$  is equal to  $\cos\theta$  (Michelson & Pease 1921; Hestroffer 1997). The quantity  $\mu$  refers to the projected viewing of the emerging intensity with  $\mu = 0$  corresponding to intensity from the disk center and  $\mu = 1$  corresponding to intensity from the limb. Lacour et al. (2008) found little statistical difference between a single parameter power law prescription versus other multiparameter limb darkening prescriptions. The starspot covering factor is defined as the ratio of the starspot size to the visible stellar disk. The starspot intensity ratio is the ratio of the starspot intensity to the intensity of the photosphere at the same position. As



Figure 4.9: Response in the visibilities based on changes to the stellar diameter (top) and the limb-darkening coefficient (bottom). The visibility response is qualitatively similar between these two parameters on the first lobe. On subsequent lobes it is clear, changes in the stellar diameter cause the lobes to change width and changes in the coefficient cause the lobe amplitude to change.

the focus of this work is cool starspots, only starspots that are fainter than the surrounding photosphere are considered. For each test, each of the other parameters are kept fixed and the effects of 3 different values for the parameter of interest is investigated. In addition, the spatial frequencies sampled in each test run from 0 to 250 mega $\lambda$ , which corresponds to baselines between 0 to 324 meters in the H band. The results are grouped based on stellar characteristics, starspot characteristics and starspot location.

Angular Diameter and Limb darkening Coefficient: Fig. 4.9 shows the results of changing the angular diameter and limb darkening coefficient on the visibilities. In these cases the closure phase is zero since the angular diameter and limb darkening are assumed to be circularly symmetric. The test angular diameters,  $\theta = 2.50$ , 2.75, and 3.00 mas, are selected as they are close to the predicted diameter of  $\lambda$  Andromedae (2.77 mas), the target star for the interferometeric imaging project. A range of predicted values of  $\alpha$  for giant stars have yet to be computed. This range is found for this study by computing the  $\alpha$  that matches the fit from a four parameter prescription of a star with  $\log(g) = 3.0$  and  $T_{eff} = 3500$  K and a star with  $\log(g) = 5.0$  and  $T_{eff} = 50,000$  K (Claret & Bloemen 2011). Both stars have solar metallicity and are observed in the *H* band. These stars are expected to be at opposite extremes in regards to the degree of limb darkening. The best fit values for  $\alpha$  are then 0.0420 and 0.2414, thus I select values of 0.0, 0.1, and 0.2 as the  $\alpha$  parameters to be tested.

The 1<sup>st</sup> visibility null is located at larger baseline positions as  $\theta$  becomes smaller. This is akin to stating that longer baselines are needed to resolve fully smaller stars. The null moves in a similar fashion as  $\alpha$  becomes greater. A greater degree of limb darkening will make the star appear smaller to the observer and thus needs a longer baseline to resolve fully. Aside from the magnitude, differences in the effect of these two parameters become clear by looking at the visibility lobes. As  $\alpha$  increases, the amplitudes of the  $2^{nd}$  and subsequent lobes decrease. The amplitudes remain constant for increasing  $\theta$ . Conversely as  $\theta$  increases, the width of each lobe broadens while the width is roughly constant for changes in  $\alpha$ . Quantitatively a mistake in characterizing  $\alpha$  does not highly affect the measurement of  $\theta$  in the H band. To demonstrate this consider four stars with  $\theta = 0.5, 1.0, 2.0$  and 4.0 mas and an  $\alpha = 0.1$ . A best fit diameter is determined for each synthetic star by assuming 2 different values of  $\alpha$ , 0.0 and 0.2. The result of undervaluing  $\alpha$  translates into  $\theta$  being smaller than its "true" value by  $\sim 1\%$ . The opposite is true with an overvalued  $\alpha$  resulting in an  $\sim 1\%$  overestimate in  $\theta$ . This is independent of the stellar angular diameters considered here.

Covering Factor and Intensity Ratio: The effects for various covering factors and intensity ratios are explored by analyzing the visibilities and closure phases from single starspot models with a range of each parameter. For these tests, the starspot is held at a fixed position of  $b = 0^{\circ}$  and  $l = 45^{\circ}$ . As the covering factor is varied the intensity ratio is fixed at 0.6. As the intensity ratio is varied the covering factor is fixed at 0.3 or 30%. Fig. 4.10 contains the effect these tests have on visibility while Fig. 4.11 contains the effects on closure phase. The covering factor,  $\phi$ , can range from 0.0 to 1.0 (0% to 100%) but in practice for active stars, this covering factor can range from 10% to 50% (Berdyugina 2005). Therefore, the  $\phi$  test values are 0.1, 0.3, and 0.5 for a single starspot. The intensity ratio is the ratio between the intensity at starspot center and the unspotted intensity at the same point. Defined this way, this ratio is independent of limb darkening. The test values for intensity ratio, 0.3, 0.6, and 0.9, are taken from the measured starspot temperatures on active stars (Berdyugina 2005). Increasing either parameter does not change the  $1^{st}$  null position (and hence the apparent stellar angular diameter), however the  $2^{nd}$  null position occurs at a slightly higher spatial frequency. As a note, both parameters do change the  $1^{st}$  null position in regards to an unspotted star. For  $\phi$ , the increase causes the 1<sup>st</sup> null visibility to become more nonzero, while the same is true of decreasing f. Increasing  $\phi$  increases the amplitude of the  $2^{nd}$  lobe and decreases the amplitude of the  $3^{rd}$ . Changing f does not affect the  $2^{nd}$  lobe amplitude, but the  $3^{rd}$  lobe becomes slightly amplified with decreasing f. The closure phase becomes more nonzero as the starspot becomes larger or the intensity ratio decreases. This similarity in behavior indicates a possible degeneracy between covering factor and intensity ratio. The presence of this degeneracy is discussed within the presentation of the final interferometric imaging results in Ch. 6.



Figure 4.10: Response in visibility based on changes in the starspot covering factor (top left), intensity ratio (top right), latitude (bottom left), and longitude (bottom right). See text for a full description of how the visibility responds to changes in these parameters.



Figure 4.11: Response in closure phase based on changes in the starspot covering factor *(top left)*, intensity ratio *(top right)*, latitude *(bottom left)*, and longitude *(bottom right)*. The colors correspond to the same parameter values shown in Fig. 4.10. See text for a full description of how the closure phase responds to changes in these parameters.

Latitude and Longitude: The effects for various starspot latitudes and longitudes are explored by analyzing the visibilities and closure phases from single starspot models with a range of each parameter. For these tests, the starspot covering factor and intensity ratio are fixed at  $\phi = 0.3$  and f = 0.6. As the latitude is varied the longitude is fixed at 45°. As the longitude is varied the latitude is fixed at  $45^{\circ}$ . Fig. 4.10 contains the effect these tests have on visibility while Fig. 4.11 contains the effects on closure phase. As the rotation axis orientation is not known a priori, the latitude, b, is set to run from  $-90^{\circ}$  in the south and  $+90^{\circ}$  in the north with respect to the sky. Likewise the longitude, l, runs from  $-90^{\circ}$  in the east to  $+90^{\circ}$  in the west. This is a relative coordinate system applicable to starspots only seen on the same night. In both cases, the range in values chosen are 0, 45, and  $90^{\circ}$ . Negative values of this range are identical to the following results. The visibility curves are identical between the b and l tests. When the starspot is at the disk center, the  $1^{st}$ null occurs at a smaller spatial frequency with respect to an unspotted star. Additionally, the lobe amplitudes are enhanced. The closure phases are also zero as expected from point symmetry.<sup>1</sup> When the starspot is located closer to the limb, the visibility curve closely resembles that of an unspotted star. Likewise the closure phases are close to zero due to the starspot's shrunken presence by geometric effects. The visibility curve, also, resembles an unspotted star for a starspot halfway between these two extremes, with the exception that the  $1^{st}$  null does not reach zero. It, in fact, resembles the intermediate solutions for  $\phi$  and f. This intermediate solution produces the most nonzero closure phase. Unlike the visibilities, the change in closure for differing b and l are not identical, but appear to mirror



Figure 4.12: Results of a Monte Carlo simulation on the effects of starspots on stellar diameter measurements. The test used 5000 simulated stellar surfaces each containing a single cool starspot with randomly generated characteristics (i.e., covering factor, location, temperature). This shows that starspots do not affect stellar diameter measurements to a limiting accuracy of 2.5%.

each other. This demonstrates the importance on closure phase information in order to determine starspot locations.

Starspot Properties and Angular Diameter: Additionally, how starspots affect the measurement of interferometically measured angular diameters is investigated. A Monte Carlo simulation is performed by generating 5000 artificial stellar surfaces with  $\theta$  ranging from 1.0 to 5.0 mas and  $\alpha = 0.24$  containing a single cool starspot. The parameters of this starspot are randomized given the parameter ranges in line with literature values (Berdyugina 2005). The covering factor is allowed to range from 0.1 to 0.5, b and l range from -90° to +90°, and f ranges from 0.2 to 0.8. These 5000 surfaces are forced to fall evenly into four  $\Delta$  magnitude bins of 0.05 to 0.10, 0.10 to 0.15, 0.15 to 0.20, and 0.20 to 0.25 magnitudes. The  $\Delta$  mag-

<sup>&</sup>lt;sup>1</sup>The few nonzero phases are believed to be caused by pixelization within the model of the stellar surface.

nitude is computed by taking the ratio of the surface flux with a starspot with regards to the surface flux without a starspot; this serves as a proxy for the amount of "spottedness" produced through the combined effect of these parameters. This is to help quantify the affect starspots have on  $\theta$  measurements as a function of starspot  $\phi$ , b, l, and intensity ratio. Fig. 4.12 shows that starspots limit the accuracy of measuring angular diameters to 2.5%. This limit is *not* a function of starspot  $\Delta$  magnitude (over the range 0.05 to 0.25 mag) and by extension starspot parameters.

### 4.4 The Center for High Angular Resolution Astronomy Array

The Center for High Angular Resolution Astronomy operates an optical/near-IR interferometer located on the top of Mt. Wilson in California. The CHARA Array obtains funding from the National Science Foundation, Georgia State University, the W. M. Keck Foundation, and the David and Lucile Packard Foundation. The interferometer is composed of six



Figure 4.13: A schematic of the CHARA array and its surroundings. North points to the lower right.

1-m aperture telescopes in a "Y"-shaped nonredundant array. The baseline lengths range from 34 to 331 meters making this the largest optical/near-infrared interferometer in the world (ten Brummelaar et al. 2005). The longest baselines provide for angular resolutions of ~0.7 mas in the K band and ~0.4 mas in the H band. Telescopes are given an alpha numeric code where the letter indicates the cardinal direction (East, West, South) and the number indicates the baseline position (1 is exterior to 2). For example, the longest baseline is provided by observing with the E1-S1 telescope pair. Fig. 4.13 shows an overhead schematic of the CHARA Array.

The CHARA Array functions by combining light from 2 or more telescopes in real time using one of five different beam combiners. Each of the beam combiners is designed for specific science goals in mind and operates either in the optical or near infra-red. These science goals include, but are not limited to, the determination of fundamental stellar properties, characterizing circumstellar disks, high precision monitoring of Cepheid variable radii, and interferometric imaging of binaries and stellar surfaces.

#### 4.5 The Michigan Infra-Red Beam Combiner

The beam combiner relevant to this dissertation is the Michigan Infra-Red Combiner (MIRC). The primary goal of MIRC is interferometric imaging. MIRC has been used successfully to image the surfaces of rapidly rotating stars (Zhao et al. 2009; Che et al. 2011), interacting binaries (Zhao et al. 2008; Baron et al. 2012), circumstellar disks (Schaefer et al. 2010), stellar winds (Richardson et al. 2013), and a stellar eclipse by a companion star with a pronounced circumstellar disk (Kloppenborg et al. 2010).



Figure 4.14: A schematic of the MIRC beam combination. Light from each beam is passed through fibers placed on v-groove with non-redundant spacing and are combined using a spherical mirror. The fringes are passed through a cylindrical lens for proper orientation before passing into the spectrograph. (Monnier et al. 2004)

Fig. 4.14 shows a schematic of the MIRC optical design. MIRC is an image plane combiner, combining up to all 6 CHARA telescopes simultaneously to provide a maximum of 15 visibilities and 20 closure phases (see Table 4.1). MIRC operates in the H and K bands with the ability to disperse spectrally the combined light using three different modes (R = 42, 150, and 400). MIRC utilizes single-mode fibers to filter spatially out atmospheric turbulence. The fibers are arranged on a v-groove array in a nonredundant pattern to provide each fringe with a unique spatial frequency signature. The beams exiting the fibers are collimated by a microlens array and then focused by a spherical mirror to interfere with each other. The interference fringes are compressed and focused by a cylindrical lens in order to pass properly through the slit of the spectrograph. After the dispersed fringes leave the spectrograph, they are detected by a PICNIC camera (Monnier et al. 2004, 2006).

# LONG BASELINE INTERFEROMETRY AS A TOOL TO STUDY STARSPOTS

In the previous chapter, the principles behind long baseline near-IR interferometry were discussed. This chapter turns the attention to the object of study  $\lambda$  Andromedae beginning with a discussion of the star's physical characteristics. A summary of the interferometric observations, including the methods employed to estimate error and remove miscalibrated data, is presented. The chapter continues by describing the photometric observations taken as a check to the final results. The chapter concludes describing the 2 independent methods used to convert visibility and closure phase into images of the stellar surface.

# 5.1 The Chromospherically Active Giant $\lambda$ Andromedae

 $\lambda$  Andromedae ( $\lambda$  And; HD 222107) is an ideal candidate for interferometric imaging.  $\lambda$  And is a bright (V = 3.872 mag), G8 giant (IV-III) classified as a RS CVn type variable star in the Third Catalog of Chromospherically Active Binaries (Eker et al. 2008). Calder (1935) first discovered the photometric variability of  $\lambda$  And with a historical peak amplitude of  $\Delta V$ ~0.3 magnitudes. Henry et al. (1995) conducted a 15 year photometric monitoring campaign finding periodic variability of 53.95 days over an 11.1 year stellar activity cycle.  $\lambda$  And was found by Walker (1944) to be a SB1 with an orbital period of 20.5212 days; Donati et al. (1995) identifies the companion to be a low main-sequence dwarf or high mass brown dwarf with  $M = 0.08 \pm 0.02$  M<sub> $\overline{\mathcal{C}}$ </sub>. The high flux contrast between the two components of  $\lambda$  And will preclude the companion affecting the photometric or interferometric observations. Using the angular size-color relations of van Belle (1999) and the trigonometric distance 26.41  $\pm$  0.15 pc measured by van Leeuwen (2007), the angular diameter of  $\lambda$  And is predicted to be 2.75 mas based on the (V-K) color of 2.406 mag. This angular diameter is  $\sim 5x$  the resolution provided by the CHARA interferometer in the *H* band. In short,  $\lambda$  And is an effectively single, interferometrically large, bright star with significant variability strongly believed to arise from cool starspots.

Doppler imaging (§ 1.2) has not been done on  $\lambda$  And since its relatively slow projected rotational velocity ( $v\sin i = 6.5 \text{ m/s}$ ; Donati et al. (1995)) makes it difficult to detect the deformations of absorption lines arising from starspots with current facilities (Strassmeier 2009a). On the other hand, light curve inversion (§ 1.2) has been modestly successful in studying starspots on  $\lambda$  And. Frasca et al. (2008) created a surface map based on optical photometric and spectroscopic monitoring. Their best fit inversion suggested the presence of two cool starspots separated by 81° in longitude each covering ~8% of the visible surface with temperatures ~880 K cooler than the photosphere. Based on the relative temporal variation of the optical light curve and H $\alpha$  emission, they conclude that each starspot is embedded within an active region that is of comparable size, but leading the starspot.

#### 5.2 Interferometric Observations

In the hopes of confirming the presence of cool starspots on the surface of  $\lambda$  And, the star was observed on 27 nights between 2007 and 2011 using the CHARA Array (§ 4.4). The data were collected using the MIRC beam combiner (see § 4.5 for details) in the *H* band using the low spectral dispersion mode (R = 40). At the longest baselines, the angular resolution is ~0.4 mas. Prior to 2011, MIRC was able to combine 4 telescope beams simultaneously. Upgrades to MIRC in 2011 now allow allow for the combination of light from all 6 CHARA Array telescopes simultaneously. Table 5.1 lists the date of the observations, the baselines utilized, the number of [u,v] points, and the calibrators used on each night. The parenthetical number besides a calibrator is the number of observations during the night of that particular calibrator.

The star 2 Aur (HD 30384) was observed on Nov  $7^{th}$ , 2009 using the same techniques employed for  $\lambda$  And for that year. The second epoch on Nov  $8^{th}$  was lost due to weather preventing a combination of two consecutive nights. This single K3 giant does not have a record of photometric variability and thus was observed as a "control" star to test the fidelity of the imaging techniques.

Date	Baselines	# of (u,v)	Calibrators
2007			
Nov $17^{th}$	S2-E2-W1-W2	96	$\sigma$ Cyg (3), $v$ And (2), $\zeta$ Per (2)
2008			
Aug $17^{th}$	S1-E1-W1-W2	96	37  And  (2), 45  Per  (3)
Aug $18^{th}$	S1-E1-W1-W2	144	$\gamma$ Lyr, 7 And (2), 37 And, $\zeta$ Per (2)
Aug $19^{th}$	S1-E1-W1-W2	48	7 And, $\zeta$ Per (2)
Aug $20^{th}$	S1-E1-W1-W2	96	7  And  (2), 37  And  (2), 45  Per  (3)
Aug $21^{st}$	S1-E1-W1-W2	96	7 And (2), 37 And (2), $\zeta$ Per, 45 Per
Sep $20^{th}$	S1-E1-W1-W2	72	7 And (2), $\zeta$ Cas, $\delta$ Per (2)
Sep $27^{th}$	S1-E1-W1-W2	72	$\sigma$ Cyg, 37 And (2), $\zeta$ Per (2), tet Gem (3)
2009			
Aug $24^{th}$	S1-E1-W1-W2	272	7 And (3), 37 And (2)
8	S2-E2-W1-W2		
Aug $25^{th}$	S1-E1-W1-W2	432	7 And (4), 37 And (2), HR 75
	S2-E2-W1-W2		
2010			
Aug $2^{nd}$	S1-E1-W1-W2	168	7 And (2), 37 And
	S2-E2-W1-W2		
Aug $3^{rd}$	S1-E1-W1-W2	456	$\sigma$ Cyg, 7 And (3), 37 And (2)
	S2-E2-W1-W2		
Aug $10^{th}$	S1-E1-W1-W2	432	$\sigma$ Cyg, 7 And (3), 37 And (4)
	S2-E2-W1-W2		
Aug $11^{th}$	S1-E1-W1-W2	288	$\sigma$ Cyg, 7 And (4), 37 And (2)
	S2-E2-W1-W2		
Aug $18^{th}$	S1-E1-W1-W2	432	$\sigma$ Cyg, 7 And (3), 37 And (5)
	S2-E2-W1-W2		
Aug $19^{th}$	S1-E1-W1-W2	432	$\sigma$ Cyg (2), 7 And (5), 37 And (7)
	S2-E2-W1-W2		
Aug $24^{th}$	S1-E1-W1-W2	528	$\sigma$ Cyg (2), 7 And (6), 37 And (6)
	S2-E2-W1-W2		
Aug $25^{th}$	S1-E1-W1-W2	384	$\sigma$ Cyg (2), 7 And (5), 37 And (2)
	S2-E2-W1-W2		
Sep $2^{nd}$	S1-E1-W1-W2	528	7 And (7), 37 And (6)
	S2-E2-W1-W2		
Sep $3^{rd}$	S1-E1-W1-W2	600	7 And (9), 37 And (3)
	S2-E2-W1-W2		

Table 5.1: CHARA Observing Log

Continued on Next Page...

Calibrators Date Baselines # of (u,v)Sep  $10^{th}$ S1-E1-W1-W2 3367 And (6), 37 And (2) S2-E2-W1-W22011 Sep  $2^{nd}$  $\sigma$  Cyg, 7 And (2), 22 And (3), HR 653 S1-S2-E1-E2-W1-W2 360 Sep  $6^{th}$  $\sigma$  Cyg, 7 And (2), 22 And (3), HR 653 S1-S2-E1-E2-W1-W2 392Sep  $10^{th}$ 7 And (2), 22 And S1-S2-E1-E2-W1-W2 360 Sep  $14^{th}$ 7 And (4), 22 And S1-S2-E1-E2-W1-W2864 Sep  $19^{th}$ S1-S2-E1-E2-W1-W27 And (3), 22 And, HR 653 (2) 808 Sep  $24^{th}$ S1-S2-E1-E2-W1-W27 And, 22 And, HR 653 (2), eta Aur 200

Table 5.1 – Continued



Figure 5.1: The [u,v] coverage obtained for the 2007 and 2008 observing runs. 2007: A - Nov  $17^{th}$ ; 2008: B - Aug  $17^{th}$ ; C - Aug  $18^{th}$ ; D - Aug  $19^{th}$ ; E - Aug  $20^{th}$ ; F - Aug  $21^{st}$ ; G - Sep  $20^{th}$ ; H - Sep  $27^{th}$ 

Interferometric data are collected when the light path difference between each telescope pair is zero. When this occurs, the light from each telescope combines as an interference fringe or "fringe". The MIRC combiner is then set to track these fringes while the data and calibration frames are taken. Collection of a single block of data, typically, does not exceed 30 minutes.

The standard MIRC pipeline was used for data reduction (Monnier et al. 2007). The frames containing the fringe pattern in each block of data were coadded. These coadded frames are corrected for instrumental effects through a background frame subtraction and foreground frame normalization. The background frame is used to eliminate both the bias level and any dark current. The foreground frame is used to flatten the response across the CCD allowing for unbiased measurements of the fringe amplitudes. The fringe amplitudes and phases measured from a Fourier transform of these corrected, coadded frames are used to form the raw squared visibilities, triple amplitudes and closure phases. Photometric calibration due to differences in the flux amplitude per telescope beam was performed via real time flux estimates derived from choppers or through the use of a beam splitter following spatial filtering (Che et al. 2010). The former is used prior to 2010 and the latter after 2010. The data were then transformed from relative measurements to absolute measurements through observations of a calibration star or "calibrator". A calibrator is a star of known size that is typically on the order or smaller than the Array's resolution limit and is within a few degrees of the target on the sky. Calibrator observations were taken roughly half a dozen times during the night intermixed between observations of science targets.

The 2007 data were collected with the S2-E2-W1-W2 telescope configuration, while the 2008 data were obtained with the S1-E1-W1-W2 telescopes. These observations involved 1 to 3 "snapshot" measurements. A snapshot measurement is a single block of data collected interspersed with observations of other science targets and calibrators. Fig. 5.1 shows the [u,v] coverage obtained for both the 2007 and 2008 data sets. Each block of data in 2007 and 2008 is composed of 6 visibilities, 4 closure phases, and 4 triple amplitudes in 8 narrow spectral channels.

The 2009 and 2010 observing runs employed a different observing strategy designed to maximize coverage in the [u,v] plane.  $\lambda$  And was observed using the S1-E1-W1-W2 set of telescopes from the beginning of the night until delay was no longer available; typically this occurred around midnight. The same targets were then observed using the S2-E2-W1-W2 telescope array for the remainder of the night. The change in telescope configurations typically took 30 minutes. In all cases the  $\lambda$  And measurements were bracketed by measurements of a calibrator star. The data from both telescope configurations were then combined into



Figure 5.2: The [u,v] coverage obtained for the 2009 observing run as well as the first two epochs of the 2010 observing run. 2009: A - Aug 24<sup>th</sup>; B - Aug 25<sup>th</sup>; 2010: C - Aug 2<sup>nd</sup>; D - Aug 3<sup>rd</sup>; E - Aug 10<sup>th</sup>; F - Aug 11<sup>th</sup>

a single oifits file; oifits is the standard file system for storing interferometric observations (Pauls et al. 2005). This strategy relies on the assumption that the surface features of  $\lambda$  And do not change on a few hour time scale, which is supported by its low rotational period (~ 54 days). Each block of data using this strategy yields 11 visibilities, 8 closure phases, and 8 triple amplitudes per spectral channel. The [u,v] coverage improved by a factor of 2 to 6 from that obtained in 2007 and 2008.

Both the 2009 and 2010 data sets consist of observations on sequential nights. Due to the slow rotation period (~54 days) for  $\lambda$  And, the position of starspots only drift by 6.6° between nights. Therefore model and reconstructed images from each sequential night are expected be nearly identical. The small rotational drift is not expected to cause the imaging methods to fit poorly the data. This strategy provides both an increased [u,v] coverage and a sanity check for the imaging methods. The data collected on subsequent nights are combined



Figure 5.3: The [u,v] coverage obtained for the second half of the 2010 observing run. A - Aug  $18^{th}$ ; B - Aug  $19^{th}$ ; C - Aug  $24^{th}$ ; D - Aug  $25^{th}$ ; E - Sep  $2^{nd}$ ; F - Sep  $3^{rd}$ 

into a single oifits file. Fig. 5.2 shows the [u,v] coverage obtained during the 2009 observing run. The only exception to this strategy is on Sept 10<sup>th</sup> and 11<sup>th</sup> 2010, as poor weather prevented observations on the 11<sup>th</sup>. Each pair of sequential observations spans ~13% of the measured rotation period. This cadence was chosen in hopes of tracing stellar rotation via starspot motion over a substantial fraction of the star's rotation period. The 11 epochs (5 pairs plus 1 night) span 39 days or ~72% of a full rotation. Figs. 5.2 and 5.3 show the obtained [u,v] coverage for each night, as well as the combined [u,v] coverage for each pair of nights, except Sept 10<sup>th</sup>. One key difference between the 2009 and 2010 observations is the addition of photometric channels to MIRC after to the 2010 observations. The photometric channels allow for better calibration of visibilities and are now a standard component of the data collection process (Che et al. 2010).



Figure 5.4: The [u,v] coverage obtained for the 2011 observing run. A - Sep  $2^{nd}$ ; B - Sep  $6^{th}$ ; C - Sep  $10^{th}$ ; D - Sep  $14^{th}$ ; E - Sep  $19^{th}$ ; F - Sep  $24^{th}$ 

The 2011 observing run was the first to benefit from the MIRC upgrade that allowed light from all 6 telescopes to be combined simultaneously.  $\lambda$  And was observed each night for as long as the delay lines would permit (typically ~ 6 hrs). On each night  $\lambda$  And measurements were bracketed by measurements of a calibrator star. Each block of data yields 11 visibilities, 20 closure phases, and 20 triple amplitudes. The observing cadence was shortened to ~4 days, ~7% of a full rotation, to increase the temporal sampling; the 6 epochs span ~41% of a full rotation period. Fig. 5.4 contains the [u,v] coverage for the 2011 data set. Since observations on sequential nights were not combined, the [u,v] coverage is approximately half of that obtained for the 2010 data set, despite using two additional telescopes.

The star 37 And was one of two or three calibrators for both the 2009 and 2010 data sets. Unfortunately it was discovered that 37 And may be a high flux contrast binary based on an apparent sinusoidal-like variation in the closure phase with a few degree amplitude (Che 2012, private communication). It is unclear how or to what extent this will affect modeling and image reconstruction without knowing the relative brightness and orientation of this putative companion. Possible biases are discussed in § 5.5.1.

#### 5.2.1 Assigning Appropriate Errors to the Interferometric Measurements

Two types of error are applied to the calibrated squared visibilities and triple amplitudes to appropriately account for systematics. Additive errors account for two different behaviors in the data. In certain data sets, the calibrated squared visibility and/or triple amplitude falls below zero. As this is a nonphysical solution, the constant is made large enough to enlarge the error to include zero. The squared visibilities and triple amplitudes are expected to increase or decrease monotonically as a function of wavelength. Therefore the errors are enlarged by the constant to account for any abnormal structures (e.g. step functions) found in the data across the 8 spectral channels. Typical additive errors for the squared visibility and triple amplitude are  $2 \times 10^{-4}$  and  $1 \times 10^{-5}$ , respectively. Multiplicative errors are applied to account for any systematics in the calibration process. The multiplicative errors improved after 2010 due to better photometric calibration provided by the photometric channels. The typical multiplicative errors in squared visibility and triple amplitude are 15% (10%) and 20% (15%), respectively prior to 2010 (after 2010).

A typical error of  $1^{\circ}$  is added to the closure phase errors, as suggested by Zhao et al. (2011). However, poor data quality has warranted increasing the closure phase additive error to as much as 5°. In addition to this additive error, two additional closure phase errors are incorporated in order to avoid poor model fits due to calibration systematics. These new errors are important in the low signal to noise (S/N) regime near to visibility null crossings. As correlated camera readout noise dominates the closure phase measurements at low S/N, minimum closure phase errors are applied when the S/N in the triple amplitude signal is  $\lesssim$ 1. Finite time averaging and spectral bandpass effects are accounted for by an error term proportional to  $\Delta CP_{\lambda}$  across each spectral channel.  $CP_{\lambda}$  corresponds to the closure phase as a function of wavelength. These two errors are applied to the closure phase noise via the following equation  $\sigma_{CP} > MAX((30^{\circ}/(S/N_{T3amp}^2), 0.2\Delta CP_{\lambda}))$ , where  $S/N_{T3amp}$  is the signal to noise in the triple amplitude measurement.

#### 5.2.2 Identification and Removal of Poor Data

Fig. 4.3 shows that the visibility of a resolved disk is a smooth Bessel function. § 4.3 demonstrates that the amplitude of the visibility lobes are affected by the various starspot parameters. Depending on the position of the starspot on the stellar surface, certain baselines will resolve these starspot parameters better than others. This can result in a visibility amplitude spread on the second and third lobes. However, the visibilities should still resemble a Bessel function.

In a few data sets in 2010 and 2011, the visibility in certain blocks of data on certain baseline pairs are not consistent with a simple Bessel function or how this function is affected by starspot parameters. The reason for these discrepant data blocks is most likely due to poor calibration data on that baseline either prior to or after the data block was acquired. These discrepancies are found almost exclusively on the short baseline pairs (e.g. S1-S2, W1-W2, E1-E2). In all discrepant cases, the visibilities were lower than the visibilities in the other blocks on the same baseline. Data were judged to be discrepant via visual inspection and removed prior to either modeling or image reconstruction. At most, the rejected data only amounted to 1% of the total data for any epoch.

#### 5.3 Photometric Observations

The photometric observations of  $\lambda$  And were obtained using the 0.4 m automated telescope at Fairborn Observatory operated by Tennessee State University.  $\lambda$  And was observed 583 times over 3 years ranging from Sept 2007 to Jan 2011. The time series was observed in two bands, corrected for atmospheric extinction and transformed into the Johnson *B* and *V* filter system. The typical photometric errors are 6.3 and 6.0 millimag for the *B* and *V* filter, respectively. These errors are estimated from the standard deviations of the check star  $\kappa$ And (B = 4.076, V = 4.137) photometric time series. Fig. 5.5 shows the complete *V* band time series photometry with the times of interferometric observations indicated by dashed, vertical lines. The time series is measured differentially with respect to the companion star  $\psi$  And (B = 6.067, V = 4.982).



Figure 5.5: time series photometry for  $\lambda$  And ranging from Sep 2007 to Jan 2011. The red dashed line indicates the 2007 interferometric observation. The green dashed lines indicate the 2008 interferometric observations. The blue dashed line indicates the 2009 interferometric observations. The yellow dashed lines indicate the 2010 interferometric observations. The brightening trend of the time series is due to the 11.1 yr stellar cycle (Henry et al. 1995)

### 5.3.1 The Optical Light Curve of $\lambda$ And Over 4 Years

Fig. 5.5 displays the  $\Delta V$  band (V-C) time series of  $\lambda$  And over a 3.4 yr span from Sept  $30^{th}$ , 2007 to Jan  $20^{th}$ , 2011. The time series has a  $\Delta V = 0.2$  mag between the brightest and faintest points over this span. The upward trend in the time series and changes in variability amplitude are consistent with  $\lambda$  And's 11.1 yr stellar cycle. Four seasons of time series photometry can be seen with an interferometric observing run occurring in each. The time series in each of these seasons is analyzed separately to gain a better understanding of the photometric variability near the times of interferometric observations.

Fig. 5.6 displays light curves for each season folded to the most significant period using only that particular season's time series. Each period is found using the Plavchan-Parks



Figure 5.6: Top Left: Season 1 time series folded to a period of  $26.978 \pm 0.032$  days. Top Right: Season 2 time series folded to a period of  $54.25 \pm 0.91$  days. Bottom Left: Season 3 time series folded to a period of  $55.0 \pm 1.1$  days. Bottom Right: Season 4 time series folded to a period of  $54.8 \pm 1.9$  days. A global increase in brightness of 0.33 millimag/day was removed prior to plotting the data. The lines indicate the same as they did in Fig. 5.5.

algorithm (Parks et al. 2014). Uncertainties in the period are set by the widths of Gaussian fits to the most significant period for each season. The four determined periods are 26.978  $\pm 0.032$ ,  $54.25 \pm 0.91$ ,  $55.0 \pm 1.1$ , and  $54.8 \pm 1.9$  days. A likely explanation why the season 1 period is approximately half the believed rotation period of ~54 days is that starspots exist on longitudes separated by ~180°. In the remaining seasons, the starspots are predominantly limited to a single hemisphere. Doubling the measured period yields a season 1 rotational period of 53.956  $\pm 0.045$  days, where there is a second less significant peak in the periodogram. The new error is the old error added to itself in quadrature. The average rotation period of  $\lambda$  And is  $54.5 \pm 1.2$  days, where the reported error is the propagated seasonal mean error.

As will be discussed in  $\S$  6.2 and  $\S$  6.3, the interferometric data in 2007 and 2008 were not able to produce self-consistent models or image reconstructions. Since the time series does not extend past Jan. 2011, the analysis is focused on seasons 3 (2009) and 4 (2010). Season 3 spans 130.7 days or 2.4 rotation periods and has a  $\Delta V = 0.154$  mag. From one rotation to the next, the starspot properties do not appear to change significantly as illustrated by the low scatter compared to observation errors in the folded light curve. Season 4 spans 121.8 days or 2.2 rotation periods and has a  $\Delta V = 0.099$  mag. The folded light curve varies from one rotation to the next, indicating a more rapid evolution of starspot properties than in season 3. Also the light curve seems bimodal suggesting starspots on two active longitudes. Active longitudes have been associated with magnetically active stars (Berdyugina 2005). These longitudes are places of preferential starspot formation. Active longitudes are believed to be permanent, but can migrate with respect to the star's rotational frame of reference (Jetsu et al. 1993; Lanza et al. 1998; Berdyugina & Tuominen 1998). The active longitude is migrating from a phase of  $\sim 0.3$  to a phase of  $\sim 0.6$  in season 4. The phase is computed by subtracting the time of each observation by the time of the first observation and then dividing this by the identified period for that season.

Another distinguishing feature of season 4 is a slight upward trend in the time series. This trend is 0.33 millimag/day based on a linear fit to the data. The period is identified without removing this trend, but the trend is removed to measure the amplitude of variability. The trend is removed by subtracting a linear fit from the time series.

#### 5.4 A Parametric Model of a Spotted Star

Two different techniques are employed to characterize starspots from the observed interferometric data: a parametric model and image reconstruction. The parametric model of a stellar surface is computed using an IDL code written by the author. This code is capable modeling any number of cool or hot starspots on a user defined, limb darkened surface. In addition, the code accounts for the effects of foreshortening on starspots located away from the substellar point. The free parameters are the same as discussed in § 4.3: stellar angular diameter ( $\theta$ ), limb darkening coefficient ( $\alpha$ ), starspot covering factor ( $\phi$ ), starspot latitude (b) and longitude (l), and starspot intensity ratio. The code extracts model interferometric data by computing the Fourier transform from an artificially generated stellar surface. The sampling for the Fourier transform is taken from the [u,v] coverage of the observed data being modeled. The goodness of fit parameter is the equally weighted average reduced  $\chi^2$ between observed and modeled visibilities, closure phases and triple amplitudes.

Changes in the angular diameter and, to a lesser extent, the limb darkening coefficient have a large effect on modeled visibilities at spatial scales smaller than the first visibility lobe. As the starspot information is contained at these small spatial scales, accurately determining these stellar properties prior to searching for the starspot properties is needed. This is done by first combining all the interferometric data into a single oifits file. The data on the first visibility lobe is modeled using a multiparameter minimization routine (AMOEBA; Press et al. (1992)) producing a measured  $\theta$  and  $\alpha$ . The uncertainty in these values is found by holding one parameter fixed and stepping through the other parameter until the reduced  $\chi^2$  increases by unity. In the primer (Chapter 4), it is shown that starspots can limit the accuracy of stellar diameter measurements by 2.5%. Therefore, an uncertainty of 2.5% is added in quadrature to the uncertainly found via this method. Closure phases are not considered at this stage;  $\lambda$  And is likely not rotationally ( $v \sin i = 6.5$  km/s) or Roche lobe distorted.

Once  $\theta$  and  $\alpha$  are known, model solutions utilizing the complete data for each night are computed. Model solutions with one, two, and three starspots are run with the preferred model yielding the lowest  $\chi^2$  statistic. A fourth starspot model is only investigated if the presence of the additional starspot is consistent with the prior and subsequent epochs. Only cool starspots are modeled as these are the type to persist on time scales of a stellar rotation. Early attempts with the AMOEBA algorithm on starspot models demonstrated that the solutions are biased by initial parameters and search scales. This is indicative of numerous local reduced  $\chi^2$  minima along with a deeper global minimum. The search scale employed, or the amount AMOEBA can change a parameter during a search, is roughly 10% of the physical range for each parameter. For example, the range in allowable intensity ratios is from 0.5 to 1.0 so the search scale is set to 0.05. AMOEBA is only very proficient at finding an accurate solution once the search occurs in the global minimum. Therefore, a genetic algorithm is employed prior to running AMOEBA to start the AMOEBA search in the global minimum.

A genetic algorithm (GA) is an iterative process through which a best solution is found by "evolving" an initial set of randomly chosen model solutions (members). The fitness, or chance it will be used in the subsequent iteration, of each member is determined based on the member's reduced  $\chi^2$ . The "survival" process is determined via a roulette wheel scheme. The wheel is spun a number of times equal to the population size. The probability the wheel will choose a member to survive is proportional to the member's fitness. Therefore the next population will, in theory, be composed of model solutions with lower  $\chi^2$  on average. This new population is "evolved" via two different random methods: crossover and mutation. Crossover takes sections of a parameter value and swaps it with another parameter value. For example, solution A has a latitude of 45.12° and solution B has a longitude of 12.57°. Crossover can swap the digits after the decimal to yield a new latitude of 45.57° and longitude of 12.12°. Mutation causes a section of the parameter value to change randomly. Using the previous example, the latitude 42.12° could mutate to become 49.12°. Both crossover and mutation are applied with a frequency of 90% and 1%, respectively. The fitness of the new population is determined and the entire process is iterated until the average fitness drops below a convergence criterion.

The population size is a balance between parameter space coverage and computing time. The larger the population, the more the parameter space is sampled, but the computing time required for a final solution is longer. Early experiments suggested 1000 members per parameter searched is sufficient to explore adequately the  $\chi^2$  space. While an exponential increase in population as a function of parameter number is likely a more appropriate strategy, this approach would be prohibitively expensive in terms of the required computing time. The fitness, F, of each member is evaluated using a Boltzmann weighting,  $F = \exp(-E/\delta E)$ , where E is the reduced  $\chi^2$  and  $\delta E$  is the total range in reduced  $\chi^2$ .

The reason AMOEBA is still needed after the GA is the GA's inability to converge to the exact minima in  $\chi^2$  space (Charbonneau 1995). During the GA and AMOEBA implementations, both the stellar angular diameter and limb darkening coefficient are kept fixed. Errors to model starspot parameters are found by randomly varying each best fit final parameter and then running the AMOEBA search algorithm.  $\phi$  and f are varied by  $\pm 0.1$ and the b and l are varied by 1.8°. This procedure is run ten times. The parameter errors correspond to the standard deviation of the ten trial values. If a trial solution has a  $\chi^2$  better than the initial final solution, this trial solution becomes the final solution.

As the model represents the monochromatic flux in the Rayleigh-Jeans tail, the intensity ratio becomes the temperature ratio between the starspot and the photosphere. The error in the temperature ratio,  $T_R$  is simply the error determined for the intensity ratio.

### 5.5 SQUEEZE: Image Reconstruction

Parametric modeling is a very effective tool in determining starspot properties; however it is limited by the assumptions used to create the model (e.g. circular starspots). Image reconstruction on the other hand has more freedom to portray more realistic starspot shapes and sizes. The main hurdle faced by image reconstruction is an incompleteness problem. A typical image will be composed of thousands of pixels while the typical interferometric data set will contain only hundreds of data points. In other terms, while LBI attempts to fill in a complete aperture through aperture synthesis, there are still significant gaps as defined by the [u,v] coverage. Reconstruction programs overcome this hurdle by reconciling a  $\chi^2$  statistic with a regularization statistic, or regularizer, modulated by a user defined weighting parameter. Image reconstructions abide by only two assumptions: 1) the intensity in a particular pixel must be positive and 2) the flux of the reconstructed image is normalized to unity.

The image reconstruction code SQUEEZE written by Fabien Baron is used on the  $\lambda$  And data sets (Baron et al. 2010). SQUEEZE begins by setting an initial state that is a 2-D array of pixels with each pixel filled by a user defined number of intensity elements. For all data sets, 4,000 elements per pixel is used. The reader is directed to § 5.5.1 for a discussion on image artifacts. The initial 2-D array state is defined to be a uniform disk of angular diameter 2.777 mas, which in turn sets the image pixel scale to 0.1108 mas/pixel. After the initial state is set, intensity is randomly moved from pixel to pixel iteratively based on a probability given by the  $\chi^2$  statistic and the regularizer. The  $\chi^2$  statistic represents the quality of fit between the image and the measured data. The regularizer contains all the a*priori* knowledge concerning the source brightness distribution. The regularizer is necessary to prevent "overfitting" the image to regions well described by the data. In addition, it minimizes the amount of small scale, unresolvable structures in the image. No prior distribution is used for any night to constrain the image shape. Use of a prior heavily penalizes the movement of intensity outside the distribution defined by that prior. The best fit parametric model is not used as either an initial state or a prior in order to ensure the two methods are independent. The *total variation* regularizer is designed to minimize brightness gradients across the surface. Thus the regularizer favors a conservative stance with a few large starspots as opposed to many small starspots.

A final image reconstruction is the average of ten iterations through SQUEEZE. This is an attempt to minimize the effect of artifacts caused by the reconstruction process. Starspot parameters are extracted by fitting a circular aperture over identified starspots. The aperture size provides the covering factor and the location of the aperture center provides the starspot latitude and longitude. As the starspot edge is difficult to quantify and the starspot may



Figure 5.7: Shown is a closeup of the SQUEEZE reconstruction for the Sep  $2^{nd}$ , 2011 data near an apparent starspot. The black circle on the right shows the aperture used to extract starspot properties from reconstructed image. The black circle on the left shows the aperture over the "quiet" photosphere. The "quiet" photosphere is defined as a part of the stellar surface devoid of flux gradients. The size of the aperture is identical to the minimum achievable angular resolution. For more detailed information see § 5.5.

be irregular in shape,  $\phi$  is considered a lower bound. The intensity ratio is calculated by dividing the intensity at the aperture center with a intensity measurement of the "quiet" photosphere. The quiet photosphere is identified as a part of the stellar surface devoid of intensity gradients. This area is selected based on the absence of gradients rather than the projected angle  $\mu$ . As the reconstructed images do not reproduce well the stellar limb darkening, the intensity measured is not considered a strong function of viewing angle. The circular aperture is fit to the reconstructed starspots by eye. Errors for the reconstructed starspot parameters are determined by extracting parameters from the ten reconstructions that are averaged to compose the final image. The parameter errors correspond to the standard deviation of these 10 sets of extracted parameters. In addition to the creation of a final averaged reconstructed image, an image representing the standard deviation of the ten iterations is created. The detection strength of the starspot is computed using this mean standard deviation. A circular aperture is placed on the quiet photosphere with a size equal to the minimum angular resolution. The detection strength is the mean intensity within this aperture subtracted from the mean intensity within the starspot aperture and then divided by the standard deviation.

### 5.5.1 Identification of Artifacts within Image Reconstructions

One very important and fundamental question to ask concerning reconstructed images is, "What can be believed?". Essentially, are all features present in reconstructed images (larger than the resolution limit) real surface features or are they artifacts introduced by miscalibrated observables, sparse [u,v] coverage, the reconstruction process, etc.? And to what extent are some features real and others mere artifacts? Described below is one methodology to help answer this question.

The problem that arises is what does the star actually look like at the time of the observations (the "true" image), hence the cause for the observations in the first place. Therefore the features to be believed *a priori* cannot be known. However, if the features of the true image could be known, then by comparing the reconstructed image to this image, those features not present in the true image can be eliminated as artifacts. One way to approximate the true image is by extracting simulated interferometric observables from the best fit parametric model for a particular night using the same [u,v] coverage and applying the same observational errors. A simulated reconstruction is then created using the identical method to create the final image reconstruction. Artifacts due to miscalibrated observables will be

features seen in the final reconstruction, but are absent in both the simulated reconstruction and the model image. Artifacts due to the [u,v] coverage and the reconstruction process will be features seen in both reconstructions, but not in the true image.

A discussion of artifact identification for a specific night will be included in the relevant section in Chapter 6.

- 6 -

# INTERFEROMETRIC IMAGING OF $\lambda$ ANDROMEDAE

The final chapter on the narrow variability perspective describes the results from the monitoring of  $\lambda$  And between 2007 and 2011. The observing strategy evolved over time to improve the inconsistent results measured 2007 and 2008 as well as due to improvements made to MIRC. The final strategy allows for a compelling picture of not only starspots, but also of tracing stellar rotation via starspot motion across the visible surface. It is worth noting that the final images typically contain anywhere from one to four large starspots. For simplicity, each starspot will be referred to in the singular as no claim can be confidently made whether the detected starspots are monolithic in nature or a localized grouping of many smaller starspots. While the angular resolution of the CHARA Array is unprecedented, it is still too large to resolve this question. The results of this monitoring are then compared to previous starspot studies of  $\lambda$  And made by indirect LCI techniques. Finally, the results for 2 Aur, a believed unspotted giant star, are explored serving as a check to the methodology described in the previous chapter.

#### 6.1 $\lambda$ Andromedae Stellar Properties

The angular diameter and limb darkening coefficient are determined via the modeling described in § 5.4. The initial value of  $\theta$  for the AMOEBA code is set to 2.75 mas as determined from the  $\lambda$  And  $(V-K_s)$  color and V magnitude (van Belle 1999). An initial  $\alpha$  is found by matching a power law fit to a four parameter fit from Claret & Bloemen (2011) given the coefficients for a  $T_{eff} = 4750$  K and  $\log(g) = 3.0$  star. This yielded a result of  $\alpha = 0.22$ consistent with results from other power law fits to interferometric data of late type giants
(Wittkowski et al. 2002, 2006). The search scales were roughly 10% of the initial values. The final results are  $\theta = 2.777 \pm 0.027$  and  $\alpha = 0.241 \pm 0.014$ . The errors are determined by altering the parameter in question while keeping the other fixed until the reduced  $\chi^2$ increases by one.

The angular diameter was interferometrically measured by Nordgren et al. (1999) using the Navy Prototype Optical Interferometer. The measurements were made in the optical (649 to 850 nm) using 3 nonredundant baselines with a maximum baseline of 37.5 m. The limbdarkening was determined through an involved process using quadratic R and I coefficients taken from Claret et al. (1995). Therefore no comparison to the limb darkening coefficient measured here can be made. Nordgren et al. (1999) measured a limb darkened angular diameter of 2.66  $\pm$  0.08 mas. This value is 1.6 $\sigma$  smaller than angular diameter measured in this work. Two possibilities for the discrepancy are that the optical limb darkening is too severe causing a prematurely truncated angular diameter or the affect starspots have on accurate interferometric angular diameter measurements is larger in the optical.

The linear stellar radius of  $\lambda$  And is 7.886  $\pm$  0.077  $R_{\odot}$ . This was computed by projecting the measured angular diameter to a Hipparcos trigonometric distance of 37.87  $\pm$  0.21 mas (van Leeuwen 2007). This radius is slightly larger than the radius, R = 7.51  $R_{\odot}$ , derived by Frasca et al. (2008). This discrepancy is even larger considering F08 used the old Hipparcos calculations which undervalued the distance by ~0.6 pc.

Fig. 6.1 shows a model spectral energy distribution fit to the observed *UBVRIJHK* photometry from Ducati (2002); the model atmospheres are generated using the MARCS code (Gustafsson et al. 2008). Using the Hipparcos distance to  $\lambda$  And, these energy distribution fits then provide an estimate of its luminosity, which is determined to be 47.86  $\pm$  1.35  $L_{\odot}$ .



Figure 6.1: The spectral energy distribution fit for  $\lambda$  Andromedae. The red circles indicate the observed photometry and the blue boxes are the modeled SED points. The gray line is the best fit MARCS stellar atmosphere model.



Figure 6.2:  $\lambda$  Andromedae plotted on a H-R diagram. The plot contains mass tracks ranging from 0.9 to 1.4 M<sub> $\odot$ </sub>. The point represents a metal rich star ([Fe/H] = 0.176) with a mass of ~1.1 M<sub> $\odot$ </sub> and an age of ~9.0 Gyr.

The measured luminosity and radius provide an independent estimate of the photospheric temperature following Stephan's Law, which is 4626±35 K. Using these photospheric values, the mass and age of  $\lambda$  And are estimated from comparisons with MESA evolutionary models (Paxton et al. 2011, 2013). These models yielded an age of ~9.0 Gyr and a mass of ~1.1  $M_{\odot}$  assuming a metallicity of [Fe/H] = 0.176 dex as seen in Fig. 6.2. Unfortunately the appropriate metallicity of the models to compare to is somewhat uncertain. The photospheric [Fe/H] has been measured to be -0.50 dex, while the lighter metals range from -0.2 to -0.3 dex (Donati et al. 1995). Maldonado et al. (2013) computed similar abundances and using the PARAM code (da Silva et al. 2006) computed an age and mass for  $\lambda$  And of 8.71 ± 1.87 Gyr and 1.01±0.06  $M_{\odot}$ , respectively.

Parameter		Value
$\begin{array}{c} \text{Distance} \\ \theta \end{array}$	(pc) $(mas)$	$26.41 \pm 0.15$ (fixed) $2.777 \pm 0.027$ (fixed)
$T_{eff} \\ L_{\star} \\ \log(g) \\ A_V$	$({\rm K}) \\ ({\rm L}_{M_{\odot}} \\ ({\rm cms}^{-2}) \\ ({\rm mag})$	$\begin{array}{r} 4618^{+27}_{-31} \\ 47.86{\pm}1.35 \\ 4.0^{0.8}_{-0.6} \\ 0.007^{0.009}_{-0.007} \end{array}$

Table 6.1: SED Model Parameters

## 6.2 $\lambda$ Andromedae Starspot Properties: 2007 Data Set

The first CHARA Array observations of  $\lambda$  And were taken on Nov 17<sup>th</sup>, 2007 using the S2-E2-W1-W2 telescopes. This snapshot observation resulted in 96 [u,v] points with a configuration shown in Fig. 5.1.

The measured nonzero closure phases, as shown in Fig. 6.3, indicate the presence of surface asymmetries. An unspotted model image yields an extremely poor fit (reduced  $\chi^2 = 40$ ) to the interferometric data.

The best fit parametric model (reduced  $\chi^2 = 4.61$ ) contains three cool starspots. Fig. 6.3 contains the best fit model image along with the model fits to the visibilities, triple amplitudes and closure phases. The starspot properties are listed in Table 6.2. A modeled starspot, with  $\phi = 19.2 \pm 3.0\%$  and  $T_R = 0.906 \pm 0.069$ , is located on the southeastern limb. Another



Figure 6.3: The best fit results for the Nov  $17^{th}$  2007 data set. Top Left: The model image. Top Right: The observed minus modeled visibilities as a function of baseline. Bottom Left: The closure phase as a function of spatial frequency. The orange asterisks indicate observed data and the red diamonds are the modeled fit. Bottom Right: The triple amplitudes as a function of spatial frequency. The symbols mean the same as in the closure phase plot.



Figure 6.4: Results for the Nov  $17^{th}$ , 2007 data set, including the model image (*left*), reconstructed image (*middle*), and simulated image (*right*) images. The white dot in the lower right hand corner represents the resolution limit for the CHARA array.

modeled starspot is nearly centrally located with a  $\phi = 6.8 \pm 4.0\%$  and  $T_R = 0.719 \pm 0.055$ . The third modeled starspot is located on the northwestern limb and has a  $\phi = 14.1 \pm 5.5\%$  and a  $T_R = 0.778 \pm 0.062$ . Fig. 6.4 compares the model image to the reconstructed image. The white dot in the lower right of these images corresponds to 0.4 mas or one resolution element. Only the central starspot is visible in the SQUEEZE reconstruction. The two modeled limb starspots are hinted at in the reconstruction, but cannot be conclusively confirmed. The parameters for the central starspot extracted from the reconstructed image are also listed in Table 6.2. The covering factor and location for the reconstructed starspot are consistent with the modeled central starspot. A discrepancy between the two is the reconstructed starspot is 180 K warmer than the modeled starspot.

		Nov $17^{th}$			
Parameter		Model	SQUEEZE		
$\phi_1$	(%)	$19.2 \pm 3.0$			
$b_1$	$(^{\circ})$	$-33.4 \pm 1.1$			
$l_1$	$(^{\circ})$	$-63.40 {\pm} 0.98$			
$T_{R1}$		$0.906 {\pm} 0.069$			
$\sigma$					
$\phi_2$	(%)	$6.8 {\pm} 4.9$	6.0		
$b_2$	$(^{\circ})$	$-5.7 \pm 1.4$	-9.2		
$l_2$	$(^{\circ})$	$-0.7 \pm 1.3$	-7.0		
$T_{R2}$		$0.719 {\pm} 0.055$	0.892		
σ			5.0		
$\phi_3$	(%)	$14.1 \pm 5.5$			
$b_3$	$(^{\circ})$	$43.8 \pm 1.4$			
$l_3$	$(^{\circ})$	$73.0 {\pm} 1.5$			
$T_{R3}$		$0.778 {\pm} 0.062$			
σ					
Reduced $\chi^2$		4.61	0.88		

Table 6.2: 2007 Starspot Properties

As described in § 5.5.1, simulated interferometric data is created from the best fit parametric model to produce a simulated reconstructed image. This image is used in an attempt to identify artifacts that could arise from the reconstruction process or from limited [u,v]sampling. While no obvious artifacts are present in the reconstructed image, the central starspot is not circular and appears to be connected to another starspot on the eastern limb. It is expected that if this feature was solely an artifact due to the reconstruction process, then it would appear in the reconstructed image of the parametric model (the simulated image). The simulated image could not reproduce the connection between the central starspot and the limb. Therefore it remains possible this feature is a true representation of the stellar surface.

The phased photometric time series indicates the interferometric observation was taken near maximum brightness (see Fig. 5.6). The presence of starspots on the visible surface during maximum photometric brightness is not an inconsistency.  $\lambda$  And is approximately 0.1 mag in V dimmer during this time as it is during times of maximum brightness in other seasons. The apparent V magnitude at the time of maximum brightness varies from 3.783 to 3.693 mag across the four seasons of photometric data. In § 5.3.1, the possibility the short rotational period (26.978 days) is due to starspots on opposite hemispheres is discussed. Hypothetically, the model image is consistent with the photometry if a rotation east to west or west to east is assumed. In this case, two starspots are on opposite hemispheres (east vs. west) accounting for the observed shorter periodic variability and the smaller central starspot will make the star appear dimmer at maximum brightness. However, this is all speculative due to presence of only one epoch of interferometric data.

# 6.3 $\lambda$ Andromedae Starspot Properties: 2008 Data Set

In 2008, two observing runs of  $\lambda$  And were performed with one in August and the other in September. Both runs employed snapshot observations using the S1-E1-W1-W2 telescopes. The August run was composed of observations taken on five consecutive nights between the 17<sup>th</sup> and the 21<sup>st</sup>. The [u,v] coverage achieved ranged from 48 to 144 data points with the densest coverage obtained on Aug 18<sup>th</sup>. Fig. 5.1 contains the plots of these [u,v] configurations. The September run was composed of two observations taken a week apart on the 20<sup>th</sup> and the 27<sup>th</sup>. The [u,v] coverage achieved was 72 points for each night. Fig. 5.1 contains the [u,v] configurations for these nights.



Figure 6.5: The best fit results for the Aug  $17^{th}$  data set. Top Left: The model image. Top Right: The observed minus modeled visibilities as a function of baseline. Bottom Left: The closure phase as a function of spatial frequency. The orange asterisks indicate observed data and the red diamonds are the modeled fit. Bottom Right: The triple amplitudes as a function of spatial frequency. The symbols mean the same as in the closure phase plot.



Figure 6.6: The observed closure phases for the Aug 2008 data sets. Red Cross: Aug  $17^{th}$ . Orange Asterisks: Aug  $18^{th}$ . Yellow Squares: Aug  $19^{th}$ . Green Diamonds: Aug  $20^{th}$ . Blue Triangles: Aug  $21^{st}$ . The distinct non-zero closure phase signature points to surface asymmetries. The similarity in the closure phase between nights indicates a consistent asymmetric surface pattern from night to night.

#### 6.3.1 The August Observations

Fig. 6.5 shows a distinct nonzero closure phase signature pointing to the existence of surface asymmetries. This signature is present in all five nights and shows consistent behavior as seen in Fig. 6.6. The observed closure phases lend support to the hypothesis of an asymmetric starspot configuration that evolves slowly compared to the rotation period. An unspotted model image yields an extremely poor fit to the interferometric data for each epoch with the reduced  $\chi^2$  ranging between 5.6 to 18.

Fig. 6.5 contains the best fit model image for Aug  $17^{th}$  along with the model fits to the visibilities, triple amplitudes and closure phases. The starspot properties are listed in Table 6.3. Fig. 6.7 contains the model, reconstructed, and simulated images for Aug  $17^{th}$ ,



Figure 6.7: Results from the Aug 17<sup>th</sup>, Aug 18<sup>th</sup>, and Aug 19<sup>th</sup>, 2008 data sets, including the model images (top row), reconstructed images (middle row), and simulated images (bottom row). The hexagonal appearance of  $\lambda$  And in the reconstructed image is due to the sparse [u,v] sampling. The white dot in the lower right hand corner represents the resolution limit for the CHARA Array.

 $18^{th}$ , and  $19^{th}$ . Fig. 6.8 contains the model, reconstructed, and simulated images for Aug  $20^{th}$ and  $21^{st}$ . For each night, except Aug  $21^{st}$ , the best fit model images contain two starspots. Three starspots are visible in the Aug  $21^{st}$  model image. The reduced  $\chi^2$  for these models are all below 2.85 with the lowest reduced  $\chi^2$  (1.14) occurring on Aug  $19^{th}$ . Given the measured rotation period, starspots would move only 6.6° across the surface from one night to the next. Therefore, the starspot configuration is expected to change only slightly from night to night. The best fit model images, however, do not present a consistent starspot configuration. Starspot evolution on time scales less than a day is not typical for magnetically active stars suggesting that the best fit model images do not accurately represent the true surface of the star. Additionally, the reconstructed images do not contain any conclusive evidence for



Figure 6.8: Results from the Aug 20<sup>th</sup> and Aug 21<sup>st</sup>, 2008 data sets, including the model images (top row), reconstructed images (middle row), and simulated images (bottom row). The hexagonal appearance of  $\lambda$  And in the reconstructed image is due to the sparse [u,v] sampling. The white dot in the lower right hand corner represents the resolution limit for the CHARA Array.

starspots. The noncircularity in the stellar disk is most likely due to the sparse [u,v] sampling as demonstrated by the simulated images.

The likely factors contributing to the nondetection of a consistent starspot configuration include, but are not limited to, poor [u,v] sampling, miscalibration and the influence of the binary 37 And used as a calibrator. 37 And was not used as a calibrator for the 2007 observation where the [u,v] sampling was sparse but consistency between the model and reconstructed images exist.

Parameter		Aug $17^{th}$	Aug $18^{th}$	Aug $19^{th}$	Aug $20^{th}$	Aug $21^{st}$	Sep $20^{th}$	Sep $27^{th}$
$\phi_1$	(%)	$42.4 \pm 6.8$	$12.1 \pm 5.2$	$35.2 \pm 4.1$	$2.6{\pm}1.9$	$44.5 \pm 4.6$	$25.5 {\pm} 6.0$	$38.6 \pm 3.0$
$b_1$	$(^{\circ})$	$-51.1 \pm 1.2$	$-15.67 \pm 0.88$	$-5.0 \pm 2.2$	$-11.51 \pm 0.67$	$18 \pm 10$	$-62.1 \pm 6.0$	$18 \pm 14$
$l_1$	$(^{\circ})$	$64.7 \pm 1.1$	$55.6 \pm 2.8$	$16.3 \pm 3.8$	$-42.3 \pm 1.1$	$-84.9 \pm 2.5$	$-86.7 \pm 3.9$	$-21 \pm 37$
$T_{R1}$		$0.732{\pm}0.069$	$0.881 {\pm} 0.056$	$0.890 {\pm} 0.045$	$0.500 {\pm} 0.032$	$0.850 {\pm} 0.062$	$0.742{\pm}0.045$	$0.771 {\pm} 0.074$
$\phi_2$	(%)	$15.2 \pm 3.8$	$8.1 \pm 8.1$	$45 \pm 17$	$43 \pm 21$	$26.0 \pm 5.8$	$8.4 {\pm} 4.1$	
$b_2$	$(^{\circ})$	$-61.48 \pm 0.66$	$-20.18 \pm 1.00$	$-81.3 \pm 9.8$	$-61.4 \pm 8.7$	$-24.95 \pm 7.7$	$35.9 {\pm} 1.8$	
$l_2$	$(^{\circ})$	$11.98 {\pm} 0.83$	$-41.2 \pm 1.6$	$-3.9 \pm 2.4$	$55.09 {\pm} 4.36$	$73.2 \pm 8.5$	$51.8 \pm 4.4$	
$T_{R2}$		$0.505 {\pm} 0.033$	$0.852 {\pm} 0.048$	$0.550 {\pm} 0.034$	$0.819 {\pm} 0.057$	$0.947 {\pm} 0.146$	$0.673 {\pm} 0.050$	
$\phi_3$	(%)					$16.0 \pm 3.2$		
$b_3$	$(^{\circ})$					$-7.8 \pm 1.1$		
$l_3$	$(^{\circ})$					$-40.1 \pm 2.1$		
$T_{R3}$						$0.798 {\pm} 0.056$		
Reduced $\chi^2$		2.66	2.85	1.14	1.19	1.75	0.73	8.33

Table 6.3: 2008 Starspot Properties

The phased photometric time series indicates these interferometric observations where taken  $\sim 11$  days after maximum brightness. While starspots may exist even during maximum brightness, the effect of said starspots on interferometric observables would be minimal in comparison to when starspots cause a more substantial drop in the stellar brightness. This might also explain the lack of a consistent starspot detection in this data set.

Thus, despite strong evidence for starspots on the surface of  $\lambda$  And during these epochs, from both measured nonzero closure phases and the variable light curve, the [u,v] coverage using 4 telescopes on a single night is insufficient to determine confidently starspot properties.



Figure 6.9: The best fit results for the Sep  $20^{th}$ , 2008 data set. Top Left: The model image. Top Right: The observed minus modeled visibilities as a function of baseline. Bottom Left: The closure phase as a function of spatial frequency. The orange asterisks indicate observed data and the red diamonds are the modeled fit. Bottom Right: The triple amplitudes as a function of spatial frequency. The symbols mean the same as in the closure phase plot.



Figure 6.10: Results from the Sep  $20^{th}$  and Sep  $27^{th}$ , 2008 data set, including the model images (top row), reconstructed images (middle row), and simulated images (bottom row). The hexagonal appearance of  $\lambda$  And in the reconstructed image is due to the sparse [u,v] sampling. The white dot in the lower right hand corner represents the resolution limit for the CHARA Array.

# 6.3.2 The September Observations

Fig. 6.9 shows a distinct nonzero closure phase across nearly all sampled spatial scales. An unspotted model image does not fit the interferometric data well with a reduced  $\chi^2 = 22$  for Sep 20<sup>th</sup> and 14 for Sep 27<sup>th</sup>.

Fig. 6.9 contains the best fit model image (reduced  $\chi^2 = 0.74$ ) for Sep 20<sup>th</sup> along with the model fits to the visibilities, triple amplitudes and closure phases. The starspot properties are listed in Table 6.3. Fig. 6.10 contains the model, reconstructed and simulated images for Sep 20<sup>th</sup> and 27<sup>th</sup>. The best fit model image for Sep 20<sup>th</sup> contains two starspots. A starspot, with  $\phi = 25.5 \pm 6.0\%$  and  $T_R = 0.850 \pm 0.062$ , is located near the northwestern limb. The second starspot, with  $\phi = 8.4 \pm 4.1\%$  and  $T_R = 0.947 \pm 0.146$ , is barely visible on the

southeastern limb. While the reconstructed image shows indications of the same starspots seen in the model image, these starspots cannot be confidently confirmed. In addition the reconstructed image is not circular potentially due to the sparse [u,v] coverage. This effect is also seen in all of the August reconstructed images. If  $\lambda$  And rotates roughly from east to west, the northwestern starspot rotating out of view between Sep  $20^{th}$  and Sep  $27^{th}$  is consistent with the photometric time series. However, the southeastern starspot rotating into view is not consistent with this picture. The best fit model image (reduce  $\chi^2 = 5.20$ ) for Sep 27<sup>th</sup> contains one starspot with  $\phi = 38.6 \pm 3.0\%$  and  $T_R = 0.771 \pm 0.074$  seen in the eastern hemisphere. The reconstructed image is not consistent with the model image. As with the reconstructed image of Sep  $20^{th}$ , the image is not circular and potentially covered in artifacts, in particular the bright starspots in the northwest and southeast. The starspot in the model image would point to a south to north rotation, however the rotation rate would have to be approximately twice the one measured in order for the starspot to move from the southeast limb (Sep  $20^{th}$ ) to where it is now (Sep  $27^{th}$ ). In addition, this motion is inconsistent with the photometric time series.

The phased photometric time series indicates the observation on Sep 20<sup>th</sup> was taken ~8 days after minimum brightness and the observation on Sep 27<sup>th</sup> was taken ~5.5 days before maximum brightness.  $\lambda$  And had rotated 220° and 266° since the first observation on Aug 17<sup>th</sup>. By measuring the drop in flux of the model image with respect to an unspotted star, a rudimentary light curve can be produced from the parametric models. Unfortunately, with only two data points and an inconsistent picture of the starspot coverage in these epochs, nothing would be gained by comparing the interferometric light curve to the observed photometric light curve. As with the August data set, despite evidence for starspots from both closure phase information and the light curve, the extracted starspot properties cannot be confidently identified as genuine. The reasons is poor agreement between the model and reconstructed images along with the obvious artifacts in the reconstructed images. These inconsistencies are again due to the limited [u,v] coverage provided by snapshot observations with only 4 telescopes.

## 6.4 $\lambda$ Andromedae Starspot Properties: 2009 Data Set

The  $\lambda$  And data set in 2009 consists of two observations on Aug 24<sup>th</sup> and Aug 25<sup>th</sup> that are combined to increase the final [u,v] coverage. As noted in § 6.3, starspots should migrate across the surface by only ~6° over 1 night, so the combination of these two nights is not believed to adversely affect the quality of the extracted properties. Each night is the combination of observations using both the S1-E1-W1-W2 and S2-E2-W1-W2 telescope arrays. This strategy resulted in 704 [u,v] points (see Fig. 5.1). Fig. 5.2 show the distribution of [u,v] coverage obtained for the pair of observations.

Fig. 6.11 clearly shows nonzero closure phases at both the lower and higher sampled spatial scales. An unspotted model image does not fit well with the measured interferometric data resulting in a reduced  $\chi^2 = 5.9$ .

The best fit parametric model (reduced  $\chi^2 = 1.44$ ) contains three cool starspots. Fig. 6.11 contains the best fit model image along with the model fits to the visibilities, triple amplitudes and closure phases. The starspot properties are listed in Table 6.4. The modeled starspot on the eastern limb was not conclusively detected in the reconstructed image. The properties of the two reconstructed starspots are nearly identical to the corresponding modeled starspots,



Figure 6.11: The best fit results for the Aug  $24^{th}$  + Aug  $25^{th}$ , 2009 data sets. Top Left: The model image. Top Right: The observed minus modeled visibilities as a function of baseline. Bottom Left: The closure phase as a function of spatial frequency. The orange asterisks indicate observed data and the red diamonds are the modeled fit. Bottom Right: The triple amplitudes as a function of spatial frequency. The symbols mean the same as in the closure phase plot.



Figure 6.12: Results from the Aug  $24^{th}$  + Aug  $25^{th}$ , 2009 data set, including the model images (left), reconstructed images (middle), and simulated images (right). The white dot in the lower right hand corner represents the resolution limit for the CHARA Array.

to within errors. The western modeled and reconstructed starspots are included in the final results despite the measured covering factor of both being close to or below the CHARA Array's angular resolution (0.4 mas or  $\phi = 2.1\%$ ). The potential starspots are accepted as the model reduced  $\chi^2$  is worse without its inclusion and the reconstructed starspot is detected with a 3.64 $\sigma$  confidence. Fig. 6.12 contains the final model, reconstructed and simulated images for the 2009 data set.

		Aug $24^{th} + 25^{th}$				
Parameter		Model	SQUEEZE			
$\phi_1$	(%)	$16.0 \pm 1.9$				
$b_1$	$(^{\circ})$	$-8.6 \pm 2.0$				
$l_1$	$(^{\circ})$	$-76.0 \pm 1.9$				
$T_{R1}$		$0.752{\pm}0.030$				
$\sigma$						
$\phi_2$	(%)	$4.1 \pm 6.8$	3.6			
$b_2$	$(^{\circ})$	$-2.3 \pm 1.9$	-3.4			
$l_2$	$(^{\circ})$	$-13.4 \pm 5.3$	-26.8			
$T_{R2}$		$0.852{\pm}0.076$	0.850			
$\sigma$			4.8			
$\phi_3$	(%)	$2.2{\pm}2.2$	2.0			
$b_3$	$(^{\circ})$	$-0.8 \pm 1.4$	-2.3			
$l_3$	$(^{\circ})$	$22.9 \pm 2.0$	21.1			
$T_{R3}$		$0.918 {\pm} 0.036$	0.916			
σ			3.6			
Reduced $\chi^2$		1.44	0.93			

Table 6.4: 2009 Starspot Properties

Two bright starspots are visible on the western limb; each starspot lies nearly equidistant above and below a east-west "equator". These starspots are not visible in the simulated image. This allows for the possibility that these starspots are genuine surface features.

Based on the phased time series, the interferometric observations were taken near maximum brightness. These results provide additional support to the claim that the surface of  $\lambda$  And contains cool starspots at maximum brightness.

The observing strategy of combining 2 consecutive nights of data provides a near 500% gain in the best [u,v] coverage obtained in 2008. This has produced a much improved consistency between the model image and the reconstructed image along with a better quality of fit in both cases. However, the lack of multiple epochs does not provide consistency checks or a measure of the rotation period.

# 6.5 $\lambda$ Andromedae Starspot Properties: 2010 Data Set

Between Aug  $2^{nd}$  and Sep  $11^{th}$  2010, 6 epochs of data were obtained for  $\lambda$  And in an identical fashion as in 2009 with the exception of Sep  $10^{th}$ ; weather prevented the second observation on Sep  $11^{th}$  and therefore Sep  $10^{th}$  is analyzed as a single epoch. Table 5.1 contains the number of [u,v] points per observation. The number of [u,v] points obtained for each of the combined 5 epochs ranged from 624 to 1128 with the densest converge obtained by the combination of Sep  $2^{nd}$  and  $3^{rd}$ . Fig. 5.2 and Fig. 5.3 show the distribution of [u,v] coverage obtained for each pair of observations. The 6 epochs are spaced with a cadence between 6 to 9 days corresponding to 10.9% to 16.4% of the measured rotation period; significant apparent starspot motion is expected from 1 epoch to the next. The complete observing run spans 71% of one complete  $\lambda$  And rotation period.



Figure 6.13: The best fit results for the Aug  $24^{th}$  + Aug  $25^{th}$ , 2010 data sets. *Top Left*: The model image. *Top Right*: The observed minus modeled visibilities as a function of baseline. *Bottom Left*: The closure phase as a function of spatial frequency. The orange asterisks indicate observed data and the red diamonds are the modeled fit. *Bottom Right*: The triple amplitudes as a function of spatial frequency. The symbols mean the same as in the closure phase plot.

Fig. 6.13 shows a distinct nonzero closure phase signature across all sampled spatial scales pointing to the existence of surface asymmetries. This signature is present in all six epochs as shown in Fig. 6.14. Unlike in August 2008, the measured closure phase distribution differs from one epoch to another. This lends support to the hypothesis of an asymmetric starspot configuration that evolves over time as the star's rotation brings starspots into and out of view. An unspotted model image yields a poor fit to the interferometric data for each epoch in 2010 with the reduced  $\chi^2$  ranging between 3.6 and 20.

Fig. 6.13 contains the best fit model image for this epoch along with the model fits to the visibilities, triple amplitudes and closure phases. The starspot properties are listed in Table 6.5. Fig. 6.15 and 6.16 contain the model, reconstructed and simulated images for each



Figure 6.14: The observed closure phases for the 2010 data sets. Red Cross: Aug  $2^{nd}+3^{rd}$ . Orange Asterisks: Aug  $10^{th}+11^{th}$ . Yellow Squares: Aug  $18^{th}+19^{th}$ . Green Diamonds: Aug  $24^{th}+25^{th}$ . Blue Triangles: Sep  $2^{nd}+3^{rd}$ . Purple Points: Sep  $10^{th}$ . The distinct non-zero closure phase signature points to surface asymmetries. The errors bars have been excluded for clarity. The differences in the closure phase between nights indicates an evolving asymmetric surface pattern from night to night.

epoch. The best fit parametric models for each epoch contain between 2 to 4 cool starspots. The model reduced  $\chi^2$  range between 0.69 to 1.66 for these epochs with the best fit occurring for Aug 18<sup>th</sup> and 19<sup>th</sup>. As an ensemble, the covering factor,  $\phi$ , ranges from 4.0 to 21.8% with a median value of 7.6%. The errors in these values are unfortunately large ranging 18.5 to 100% with a median error of 52%. The reason for such high error in the covering factor is unknown. The temperature ratio,  $T_R$ , ranges from 0.756 to 0.925 with a median value of 0.853. Using the  $T_{eff}$  found by the SED fit (4618 K), the median temperature difference between starspot and photosphere is 679 K. The errors in both latitude and longitude are nearly identical and range from 0.75 to 7.8° with a median error of 1.9°.



Figure 6.15: Results from the Aug  $2^{th}+3^{rd}$ , Aug  $10^{th}+11^{th}$ , and Aug  $18^{th}+19^{th}$ , 2010 data sets, including the model images (top row), reconstructed images (middle row), and simulated images (bottom row). The white dot in the lower right hand corner represents the resolution limit for the CHARA Array.



Figure 6.16: Results from the Aug  $24^{th}+25^{rd}$ , Sep  $2^{nd}+3^{rd}$ , and Sep  $10^{th}$ , 2010 data sets, including the model images (top row), reconstructed images (middle row), and simulated images (bottom row). The white dot in the lower right hand corner represents the resolution limit for the CHARA Array.



Figure 6.17: The error in starspot parameters versus the starspot position for the 2010 data set. The error is normalized to the highest error for each parameter. The position is taken relative to the stellar disk center where the 0 corresponds to disk center and 1 corresponds to the limb. No correlation exists between the starspot's parameter error and the starspot's position.

One might believe a starspot near the stellar limb might have higher errors in the measured parameter due to its smaller profile in comparison to a starspot near the substellar point. This is tested by first normalizing the parameter errors using the highest error value. A distance vector is computed from the measured l and b for each starspot and then the length is normalized by the radius of the star (e.g. a starspot at the center has length 0, a starspot at the limb has length 1). A Pearson correlation test between the normalized parameter errors and the normalized starspot distance indicates no correlation exists. This is illustrated in Fig. 6.17.

Table 6.5: 2010 Starspot Properties

	Aug $2^{nd} + 3^{rd}$		Aug $10^{th} + 11^{th}$		Aug $18^{th} + 19^{th}$		Aug $24^{th}+25^{th}$		Sep $2^{nd} + 3^{rd}$		Sep $10^{th}$		
Param.		Model	Recon.	Model	Recon.	Model	Recon.	Model	Recon.	Model	Recon.	Model	Recon.
$\phi_1$	(%)	$5.3 \pm 5.3$	9.0	$7.6{\pm}4.0$	4.0	$8.4{\pm}5.0$		$7.9{\pm}3.7$	4.8	$13.8 {\pm} 4.6$	5.8	$44{\pm}12$	_
$b_1$	(°)	$-10.1 \pm 1.3$	23.6	$0.5 \pm 1.3$	-11.5	$7.1 \pm 1.2$	_	$11.5 \pm 1.1$	28.7	$4.9 \pm 1.0$	4.6	$17.55 {\pm} 0.98$	
$l_1$	(°)	$-55.12 \pm 0.84$	2.50	$-59.4 \pm 1.0$	9.4	$-64.9 \pm 5.4$		$-27.17 \pm 0.79$	7.86	$-46.8 \pm 1.8$	-34.2	$-40.9 \pm 5.6$	_
$T_{R1}$		$0.925 {\pm} 0.047$	0.855	$0.759 {\pm} 0.059$	0.857	$0.777 {\pm} 0.054$		$0.772 {\pm} 0.036$	0.724	$0.870 {\pm} 0.059$	0.817	$0.925 {\pm} 0.017$	_
σ			4.12		6.28	_			12.17		8.35		_
,	(04)				4.0				4.0				
$\phi_2$	(%)	$20.5 \pm 3.8$	_	$5.8 \pm 3.1$	4.0	$7.4 \pm 5.1$	5.76	$7.1 \pm 4.6$	4.8	$8.0 \pm 8.0$	5.8	$13.5 \pm 6.4$	6.8
$b_2$	(°)	$23.5 \pm 1.3$	_	-1.1±1.1	-2.3	$16.4 \pm 1.1$	16.3	$34.9 \pm 1.4$	3.4	$31.4 \pm 1.4$	27.4	$29.84 \pm 0.96$	21.10
$l_2$	(°)	$3.12 \pm 0.75$	_	$-19.0\pm1.6$	-27.4	$-16.4 \pm 1.7$	-22.0	$19.00 \pm 0.96$	-28.74	$24.0\pm1.4$	11.7	$9.7 \pm 4.9$	0.0
$T_{R2}$		$0.894 \pm 0.025$	_	$0.859 \pm 0.054$	0.857	$0.759 \pm 0.063$	0.720	$0.756 \pm 0.057$	0.728	$0.790 \pm 0.059$	0.761	$0.898 \pm 0.048$	0.849
σ			_		7.11	_	16.20	_	11.55	—	9.81	_	5.46
4	(07)	11 5 5 5				<b>F</b> 9   9 4	2.0	C 2   4 4					
$\phi_3$	(%)	$11.0\pm0.0$	_	$5.0\pm 2.0$	_	$0.3 \pm 3.4$	3.0 4.C	$0.3 \pm 4.4$			_		
03	(°)	$52.5\pm2.1$	_	$-0.8\pm1.0$	_	$11.0\pm1.0$	4.0	$28.3 \pm 1.3$	_		_		
13 T	(-)	$70.4\pm0.9$		$10.0\pm1.2$		$30.4\pm1.7$	20.0	$10.0 \pm 1.0$					
1 R3		$0.794 \pm 0.142$		$0.859 \pm 0.000$		$0.855 \pm 0.051$	0.850	$0.835 \pm 0.105$					
0							0.9				_		
φı	(%)			$21.8 \pm 5.8$		$4.0 \pm 4.1$	2.6	_				_	
$b_{4}$	(°)			$54.95 \pm 0.99$		$11.8 \pm 2.8$	-30.0						
14	$(\circ)$	_		$77.5 \pm 1.5$		$68.1 \pm 6.7$	2.7			_			
$T_{PA}$		_		$0.908 \pm 0.041$		$0.853 \pm 0.060$	0.914			_			
$\sigma$						_	6.16			_			—
Reduced $\chi^2$		1.50	1.00	1.37	1.00	0.69	1.03	1.61	0.98	1.06	0.97	0.95	0.99

The reduced  $\chi^2$  for each reconstructed image is at or below 1.01. Good qualitative agreement exists between the model and reconstructed images as seen in Fig. 6.15 and Fig. 6.16. The comparison between the starspot parameters extracted from the model and reconstructed images is discussed below for each epoch. Table 6.5 contains the measured starspot parameters from both the model and reconstructed images for all 6 epochs.

- Aug  $2^{nd}+3^{rd}$  (Epoch 1): The model image contains three starspots; the first starspot is on the southeastern limb, the second starspot is slightly north of center and the last starspot is on the northwestern limb. The southeastern and northwestern starspots are hinted at in the reconstructed image, but cannot be confidently identified. The central reconstructed starspot is detected with to a  $4.1\sigma$  confidence limit. The reader is directed to § 5.5 for the discussion on how starspot parameters and detection strengths are estimated. The position agreement between the reconstructed and model starspot is well within the measured errors. The reconstructed starspot  $T_R$  is lower than the model  $T_R$  by  $1.4\sigma$ . The reconstructed starspot  $\phi$  (9%) is  $3\sigma$  smaller than the modeled  $\phi$ . However, as cautioned in § 5.5, due to the noncircularity of reconstructed starspots and the lack of a quantified starspot edge, the reconstructed  $\phi$  should only be considered a lower bound.
- Aug 10<sup>th</sup>+11<sup>th</sup> (Epoch 2): The model image contains four starspots; the first starspot is located on the eastern limb, the second starspot is to the west of the first, the third starspot is just west and slightly south of the second starspot, and the last starspot is located on the northwestern limb. The modeled northwestern starspot is not seen in the reconstructed image. The remaining three starspots are potentially

seen in the reconstructed image as a "starspot belt" extending across the disk from the eastern limb. This belt-like appearance is recreated by the simulated image providing support for agreement between the model and reconstructed images. Two starspots can be estimated to exist in this belt structure. The detection strengths for the eastern and western reconstructed starspots is 6.3 and 7.1 $\sigma$ , respectively. For the eastern reconstructed starspot, the agreement with the model is to within 1 $\sigma$  for  $\phi$ , l, and  $T_R$ . The reconstructed b (-11.5°) is 4.7 $\sigma$  further south than the model. For the western reconstructed starspot, the agreement with the model is within 1 $\sigma$  for  $\phi$ , b, and  $T_R$ . The reconstructed starspot, the agreement with the model is within 1 $\sigma$  for  $\phi$ , b, and  $T_R$ .

- Aug  $18^{th}+19^{th}$  (Epoch 3): The model image contains four starspots that form a near straight line stretching from the eastern limb to the western limb. The western most modeled starspot is not seen in the reconstructed image. The existence of the eastern most modeled starspot in the reconstructed image cannot be confidently confirmed. The detection strengths for the eastern and western reconstructed starspots is 16.2 and  $8.9\sigma$ , respectively. For the eastern reconstructed starspot, the agreement with the model is within  $1\sigma$  for  $\phi$ , b, and  $T_R$ . The reconstructed l (-22°) is  $4.7\sigma$  further east than the model. For the western reconstructed starspot, the agreement is within  $1\sigma$ for  $\phi$  and  $T_R$ . The reconstructed b (4.6°) is  $4.9\sigma$  further south than the model. The reconstructed l (29°) is only  $1.6\sigma$  further east than the model.
- Aug 24<sup>th</sup>+25<sup>th</sup> (Epoch 4): The model image contains three starspots; the first starspot is located just east of center, the second starspot is located in the northwest quadrant of the disk, and the last starspot is on the northwestern limb. The western most

modeled starspot is not seen in the reconstructed image. The detection strengths for the eastern and western reconstructed starspots is  $12.1\sigma$  and  $11.6\sigma$ , respectively. For the eastern reconstructed starspot, the agreement with the model is within  $1\sigma$  for  $\phi$ and  $T_R$ . The reconstructed b (29°) is  $4.4\sigma$  further south than the model while the reconstructed l (7.9°) is  $11.5\sigma$  further east. For the western reconstructed starspot, the agreement with the model is within  $1\sigma$  for  $\phi$  and  $T_R$ . The reconstructed b (3.4°) is 7.4 $\sigma$  further south than the model while the reconstructed l (-28°) is  $1.9\sigma$  further east.

- Sep  $2^{nd} + 3^{rd}$  (Epoch 5): The model image contains two starspots; the first starspot is on the east limb and the second starspot is in the northwest quadrant of the disk. The detection strengths for the eastern and western reconstructed starspots is 9.8 and  $8.4\sigma$ , respectively. For the eastern reconstructed starspot, the agreement with the model is within  $1\sigma$  for  $\phi$  and  $T_R$ . The reconstructed b (27°) is  $2.9\sigma$  further south than the model while the reconstructed l (12°) is  $8.8\sigma$  further east. For the western reconstructed starspot, the agreement with the model is within  $1\sigma$  for  $\phi$  b, and  $T_R$ . The reconstructed l (-34°) is  $7\sigma$  further east than the model.
- Sep 10<sup>th</sup> (Epoch 6): The model image contains one starspot located north of center. The second model "starspot" is not considered here for reasons discussed later. The detection strength of the reconstructed starspot is 5.5σ. The agreement with the model is within 1σ for T<sub>R</sub>. The reconstructed φ is 1.1σ smaller than the model. The reconstructed b (21°) is 9.1σ further south than the model while the reconstructed l (0°) is 2.1σ further east than the model.

The reconstruction process does not identify starspots on the stellar limb nearly as well as the parametric model. For epochs with visual agreement between model and reconstructed starspots,  $\phi$  and T<sub>R</sub> estimates are within 1 error bar in nearly every case. The agreement suffers for position estimates with starspots in reconstructions consistently further east and/or further south than their counterparts in the model images.

The simulated images and the observing cadence can be used to help identify any reconstructed surface features that may be artifacts. In Epoch 2 through 6, a number of warm starspots are observed evenly spaced around the limb of the reconstructed star image. These are rejected as artifacts due to their symmetry and constant position contrary to what is expected on a rotating surface. The origin of these artifacts may be due to the [u,v] sampling since the pattern of the warm starspots is similar to the pattern of tightly clustered points in the [u,v] plane (see Fig. 5.3). The southern cool starspot in Epoch 2 is rejected as an artifact on the basis that covering factor is below the resolution limit. A warm starspot is observed in the disk center of the reconstructed images of Epochs 3, 4, and 5. These are rejected as artifacts as they are reproduced in the simulated images and do not move despite the star's rotation. The brighter southern pole in Epoch 6 is similarly rejected as an artifact as it too is reproduced in the simulated image. The warm starspot near the southwestern limb in Epoch 1, however, cannot be rejected as false as it is not present in the simulated image.

The model image for Epoch 6 contains two cool starspots. While, both starspots are listed in Table 6.5 the starspot located near the northeastern limb (b: 17.5°, l: -40.9°) is excluded when discussing ensemble starspot properties. The rationale is as follows: the starspot is nearly twice the size ( $\phi = 44\%$ ) as the next largest identified starspot, it is the warmest starspot ( $T_R = 0.981$ ) and this starspot is not confidently seen in the reconstructed image. This "starspot" may be a widely spread, rarified patch of starspots with the covering factor of each individual starspot falling below the resolution limit. The model attempts to reconcile the interferometric signature these starspots produce through the addition of this larger, warmer starspot.

Photometric V band time series is available beginning a few days after the last interferometric observation. This short cadence time series spans approximately two rotations of  $\lambda$  And. Fig. 6.18 shows this time series (black diamonds) plotted over modified Julian day. A modeled light curve can be constructed from the best fit parametric models for each epoch. A change in flux between an unspotted star and the modeled surface can be measured, converted into a  $\Delta$  magnitude and then scaled from comparison with the observed time series. The scaling is done through an additive constant that shifts the modeled time series to the approximate values of the photometric time series. A multiplicative constant is used to expand the amplitude of variability to be comparable to the photometric time series. This constant is required since the images represent flux in the H band as opposed to the photometric V band. In this case the magnitude values where multiplied by a factor of 8. The modeled time series is included in Fig. 6.18 represented by colored asterisks (Epoch 1 red, Epoch 2 - orange, Epoch 3 - yellow, Epoch 4 - green, Epoch 5 - blue, Epoch 6 - purple). The solid black line represents a spline fit to the photometric time series. The dashed line represents this fit shifted backward in time by 54.8 days, the rotation period identified using this time series. The modeled time series follows the behavior of the photometric time series quite well. This is further evidenced in Fig. 6.19 where the modeled time series and the photometric time series are folded by using the 54.8 day period. The sole outlier is Epoch



Figure 6.18: The Gray diamonds correspond to the V band time series taken between Sep. 20, 2010 and Jan. 20, 2011. The solid black line corresponds to a spline fit to this time series. The colored asterisks represent the photometry taken from the best fit parametric models for the 6 epochs. The dashed line corresponds to the spline fit shifted back in time by one rotation period (54.8 days).

5, which is brighter than expected. It is difficult to explain the discrepancy based on [u,v] coverage since this epoch had the densest coverage. Data quality does not seem to be a viable explanation as the errors are not significantly larger than other epochs and the model reduced  $\chi^2$  is one of the lowest all six epochs.



Figure 6.19: The gray diamonds correspond to the V band time series phased to a period of  $54.8 \pm 1.9$  days. The colored asterisks represent the scaled photometry taken from the best fit parametric models for the 6 epochs.

## 6.5.1 Tracing Rotation in the 2010 Data Set

The multiepoch starspot imaging presented above has the potential to trace the rotation of a star via starspot motion. If this motion can be observed, the stellar rotation axis can be fully described in both inclination and position angle. In addition, the multiple epochs allow us to further test the identified starspot properties by comparing the flux variability these properties would produce with contemporaneous photometric light curves. Neither Doppler imaging or light curve inversion (see § 1.2) has the capacity to determine these quantities and, in fact, the inclination angle must be assumed in both cases.

The observing baseline for the 2010 data set spans  $\sim 72\%$  of the photometrically determined rotation period. The average cadence will carry starspots  $\sim 15\%$  across the stellar surface between epochs assuming a negligible amount of differential rotation. Since the measured rotation periods for each photometric season agree within the errors, no differential rotation is expected and the following analysis assumes solid body rotation.

Starspot	Epoch Range	$\Delta \phi$ (%)	$\Delta T_R$	$\begin{array}{c} \mathbf{P}_{rot}\\ (days) \end{array}$
А	$1 \rightarrow ?$			
В	$1 \rightarrow 2$	1	0.004	46.6
С	$\begin{array}{c} 1 \rightarrow 2 \\ 2 \rightarrow 3 \end{array}$	-0.3 -1	-0.018 -0.002	$44.3 \\ 47.9$
D	$2 \rightarrow 3$ $3 \rightarrow 4$	-0.5 1	-0.002	$56.9 \\ 70.1$
Е	$2 \rightarrow 3$ $3 \rightarrow 4$	-0.2 -0.3	0.000 -0.001	$63.7 \\ 78.7$
F	$3 \rightarrow 4$ $4 \rightarrow 5$	$-0.5 \\ 0.1$	-0.002 0.006	$77.0 \\ 63.4$
G	$5 \rightarrow 6$	0	0.007	49.1

Table 6.6: Evidence for Stellar Rotation in the 2010 Data

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Stellar rotation is determined by tracing the position of starspots in one epoch to subsequent epochs by eye. For clarity, individual starspots seen in these epochs are labeled Athrough G in Fig. 6.20. Four starspots (C, D, E, F) are seen in three epochs and therefore provide the most useful constraints on the rotation and inclination angle. Starspot A is only definitely seen in Epoch 1. The timing is consistent with this starspot rotating around behind the star and appearing again as the large eastern starspot in Sep  $10^{th}$ . However, the properties of starspot A are not consistent with those of the eastern starspot which disputes this claim. The progression of each starspot is described in Table 6.6, along with the changes in  $\phi$  and  $T_R$  for a starspot and the computed rotation period based on the measured angle between starspot positions from one epoch to the next. Starspots are not expected to evolve either in size or temperature on time scales of one stellar rotation. Therefore, if the proposed scheme of identifying starspots is accurate, the change in  $\phi$  and  $T_R$  should be small. The largest change in  $\phi$  and  $T_R$  for any starspot over the observed rotation is 1.3% and 0.02, respectively. As these are below or comparable to the median  $\phi$  error of 4.6% and median  $T_R$  of 0.016, this supports the claim the starspots are not significantly evolving. The picture becomes muddled when the starspots are used to compute a rotation period based on their angular movement. The average rotation period based on starspot motion is  $60 \pm 13$  days. The error is the standard deviation of the individual rotation periods. Only the large error bar makes this consistent with the photometric rotation period of  $54.8 \pm 1.9$  days.

Fig. 6.21 shows each of the starspots plotted by Declination vs. Right Ascension overlaid by ellipse fits. Estimations of the inclination and position angle of the rotation angle can be made by measuring these elliptical paths. A starspot being carried across the stellar surface via rotation will appear to travel along an elliptical path when viewed in two dimensions.


Figure 6.20: The best fit models for each epoch in 2010. In each model, the starspot(s) are labeled (A through G) to indicate the same starspot as seen in each epoch.

The position angle,  $\Psi$ , is simply the tilt of this ellipse counterclockwise from north (up). The inclination angle, *i*, is the inverse sine of the ellipse eccentricity. If the star is viewed face-on  $(i = 0^{\circ})$ , then the starspot will appear to traverse a circular path (e = 0). Conversely if the star is viewed edge-on  $(i = 90^{\circ})$ , the starspot will appear to traverse a line (e = 1). Prior to fitting the ellipse, the *b* and *l* for each starspot are projected onto the sky becoming  $\Delta Dec$  and  $\Delta RA$ , respectively. An ellipse is fit via visual inspection only for starspots *C* through *F* as there are at least three measurements per starspot. The average  $\Psi$  and *i* are 18.5  $\pm$  8.1° and 75  $\pm$  5.0°, respectively. The errors are the standard deviations of measured values. This inclination angle is higher than 60° assumed by Frasca et al. (2008), but is consistent if the uncertainties in the previous inclination estimate  $60^{+30_{\circ}}_{-15}$  by Donati et al. (1995) are accurate. The rotation axis is measured to be coming out of the plane of the sky in the northern hemisphere.



Figure 6.21: Ellipse fits to starspot positions in the 2010 data sets. The dash dot dot line corresponds to spot F. The long dash line corresponds to spot E. The solid line corresponds to spot E. The dotted line corresponds to spot D. The dash dot line corresponds to spot C. The red circle indicates the edge of  $\lambda$  And. The average computed position angle,  $\Phi$ , and inclination angle, i, from these fits are  $18.5 \pm 8.1^{\circ}$  and  $75.0 \pm 5.0^{\circ}$ , respectively.

The 2009 observing strategy and multiple observed epochs have provided a convincing picture of starspots on the surface of  $\lambda$  And in support of the closure phase information and the variable light curve. The agreement between the modeled and reconstructed starspot properties is within one error bar, in most cases. In addition, the starspots produce a flux variability which is consistent with that observed photometrically just subsequent to the interferometric observations. There is evidence to suggest that starspots imaged in one epoch are again imaged in subsequent epochs. This provides an opportunity to trace the rotation of  $\lambda$  And and compute direct estimates for the star's rotation axis inclination and position angle in the sky.



Figure 6.22: The best fit results for the Sep  $10^{th}$ , 2011 data set. Top Left: The model image. Top Right: The observed minus modeled visibilities as a function of baseline. Bottom Left: The closure phase as a function of spatial frequency. The orange asterisks indicate observed data and the red diamonds are the modeled fit. Bottom Right: The triple amplitudes as a function of spatial frequency. The symbols mean the same as in the closure phase plot.

## 6.6 $\lambda$ Andromedae Starspot Properties: 2011 Data Set

Between Sep  $2^{nd}$  and Sep  $24^{th}$ , 2011, 6 epochs of data were obtained for  $\lambda$  And; this star was observed for as long as delay lines were available with all six telescopes simultaneously (approximately 8 hours). Since all 6 telescopes are used simultaneously, only 1 night of data was acquired per epoch. Table 5.1 contains the number of [u,v] obtains points per observation. The number of [u,v] points achieved ranged from 200 to 864 with the densest coverage obtained on Sep 14<sup>th</sup> (see Fig. 5.4). This observing run consisted of 6 epochs with a cadence of 4 or 5 days which corresponds to 7.3% and 9.2% of the rotation period. The complete observation run spans 40.4% of one rotation period.

Fig. 6.22 shows a distinct nonzero closure phase signature across all sampled spatial scales pointing to the existence of surface asymmetries. This signature is present in all



Figure 6.23: The observed closure phases for the 2011 data sets. Red Cross: Sep  $2^{nd}$ . Orange Asterisks: Sep  $6^{th}$ . Yellow Squares: Sep  $10^{th}$ . Green Diamonds: Sep  $14^{th}$ . Blue Triangles: Sep  $19^{th}$ . Purple Point: Sep  $24^{th}$ . The error bars have been removed for clarity. The distinct non-zero closure phase signature points to surface asymmetries. The differences the closure phase between nights indicates an evolving asymmetric surface pattern from night to night.

six epochs as shown in Fig. 6.23. As with the 2010 data sets, the observed closure phases point to an asymmetric starspot configuration that evolves over time as the star's rotation brings starspots into and out of view. An unspotted model image yields a poor fit to the interferometric data for each epoch in 2011 with the reduced  $\chi^2$  ranging between 4.1 to 11.

The best fit parametric models for each epoch contain between 1 to 2 cool starspots. The model reduced  $\chi^2$  range between 1.35 to 5.16 for these epochs with the best fit occurring for Aug 2<sup>nd</sup>. Fig. 6.22 contains the best fit model image for this epoch along with the model fits to the visibilities, triple amplitudes and closure phases. The starspot properties are listed in Table 6.7. Fig. 6.24 and 6.25 contain the model, reconstructed and simulated images for each epoch. As an ensemble, the covering factor,  $\phi$ , ranges from 10% to 17% with a median value of 12%. The errors in these values range from 2.5% to 6.4% with a median error of



Figure 6.24: Results from the Sep  $2^{nd}$ , Sep  $6^{th}$ , and Sep  $10^{th}$ , 2010 data sets, including the model images (top row), reconstructed images (middle row), and simulated images (bottom row). The white dot in the lower right hand corner represents the resolution limit for the CHARA array.



Figure 6.25: Results from the Sep  $14^{th}$ , Sep  $19^{th}$ , and Sep  $124^{th}$ , 2010 data sets, including the model images (top row), reconstructed images (middle row), and simulated images (bottom row). The white dot in the lower right hand corner represents the resolution limit for the CHARA array.

5.3%. The temperature ratio,  $T_R$ , ranges from 0.799 to 0.866 with a median value of 0.843. Again, assuming an  $T_{eff} = 4618$  K from the SED fit, the median temperature difference between starspot and photosphere is 725 K. The temperature ratio errors range from 0.024 to 0.057 with a median error of 0.049. The errors is both latitude and longitude are nearly identical and range from 0.63 to 5.7° with a median error of 1.4°.

As with the 2010 data set, the hypothesis that parameter error scales with starspot distance from the stellar limb is tested. A Pearson correlation test between the parameter errors and the starspot distance indicates moderate positive correlations with  $\phi$  ( $r_{\phi} = 0.75$ ) and l ( $\eta = 0.76$ ). The test also shows a slight positive correlation in b ( $r_{b} = 0.56$ ). No correlation exists for  $T_{R}$ . Fig. 6.26 shows these correlations by plotting the normalized parameter error versus the normalized starspot position.



Figure 6.26: The error in starspot parameters versus the starspot position for the 2011 data set. The error is normalized to the highest error for each parameter. The position is taken relative to the stellar disk center where the 0 corresponds to disk center and 1 corresponds to the limb. A moderate correlation exists between position and both the  $\phi$  and l parameter errors. A slight correlation exists between position and the b parameter error. No correlation is measured for the  $T_R$  parameter error.

<u></u>			,		1				. 1		. 1		. 1
	Sep $2^{nd}$		Sep $6^{th}$		Sep $10^{th}$		Sep $14^{th}$		Sep $19^{th}$		Sep $24^{th}$		
Param.		Model	Recon.	Model	Recon.	Model	Recon.	Model	Recon.	Model	Recon.	Model	Recon.
$\phi_1$	(%)	$16.9 {\pm} 5.7$		$12.5 \pm 4.8$	4.0	$11.8 \pm 3.1$	7.8	$10.2 \pm 2.5$	4.8	$10.4 \pm 6.4$	4.0	$14.5 \pm 3.8$	
$b_1$	(°)	$-1.3 \pm 2.1$	_	$4.15 {\pm} 0.93$	11.54	$11.9 \pm 1.3$	16.3	$24.9 {\pm} 0.6$	24.8	$1.30 {\pm} 0.92$	-22.33	$14.6 {\pm} 1.6$	_
$l_1$	(°)	$-60.3 \pm 5.4$	—	$-35.6 \pm 1.6$	-29.3	$-7.4 \pm 1.0$	-14.5	$11.8 \pm 1.3$	6.3	$-60.7 \pm 3.5$	14.4	$-34.1 \pm 1.2$	
$T_{R1}$		$0.823 {\pm} 0.049$	—	$0.824{\pm}0.053$	0.839	$0.780{\pm}0.045$	0.771	$0.850 {\pm} 0.024$	0.700	$0.826 {\pm} 0.037$	0.849	$0.826 {\pm} 0.054$	
$\sigma$		—	_		9.09		5.59		9.67		3.23	—	_
$\phi_2$	(%)	$10.0 {\pm} 4.5$	4.8	$10.0 \pm 5.4$	_	$14.7 \pm 5.7$	_	—	2.6	$14.8 \pm 5.3$	4.8	$15.6 {\pm} 6.0$	_
$b_2$	(°)	$3.65 {\pm} 0.95$	9.21	$20.1 \pm 1.7$		$37.7 \pm 1.4$	—		3.4	$40.5 \pm 1.4$	24.8	$62.1 \pm 2.6$	
$l_2$	(°)	$8.58 {\pm} 0.76$	12.88	$38.1 \pm 1.2$		$68.1 \pm 2.2$			-31.4	$49.8 {\pm} 2.9$	5.1	$85.6 \pm 5.7$	_
$T_{R2}$		$0.865 {\pm} 0.047$	0.859	$0.864{\pm}0.046$		$0.866 {\pm} 0.053$			0.916	$0.843 {\pm} 0.049$	0.850	$0.850 {\pm} 0.057$	_
$\sigma$		—	5.31	—		—		—	3.38		3.18		_
Dodwood 2		1 40	1.01	2.26	1.09	1 50	0.08	1 50	0.00	E 99	0.05	1.08	0.07
Reduced $\chi^2$		1.40	1.01	2.20	1.02	1.50	0.98	1.59	0.99	0.22	0.95	1.98	0.97

Table 6.7: 2011 Starspot Properties

The reduced  $\chi^2$  for each of the reconstructed images is at or below 1.01. Good qualitative agreement exists between the model and reconstructed images as seen in Fig. 6.24 and Fig. 6.25. The comparison between the starspot parameters extracted from the model and reconstructed images is discussed below for each epoch.

- Sep  $2^{nd}$  (Epoch 1): The model image contains two starspots; the first is located on the eastern limb and the second is located at the substellar point. The eastern modeled starspot is hinted at in the reconstructed image, but cannot be confidently identified. The position of this reconstructed starspot is in agreement with the model image. The western reconstructed starspot is detected with to a  $5.3\sigma$  confidence limit. The agreement with the model is within  $1\sigma$  for  $T_R$ . The reconstructed  $\phi$  (4.8%) is  $1.2\sigma$ smaller than the model. The reconstructed b (9.2°) is  $5.8\sigma$  further north than the model while the reconstructed l (13°) is  $5.7\sigma$  further west.
- Sep 6<sup>th</sup> (Epoch 2): The model image contains two starspots; the first starspots is located east of center and the second is located west of center. The western modeled starspot is not seen in the reconstructed image. The detection strength of the eastern reconstructed starspot is 9.1σ. The agreement with the model is within 1σ for T<sub>R</sub>. The reconstructed φ (4%) is 1.9σ smaller than the model. The reconstructed b (12°) is 7.9σ further north than the model while the reconstructed l (-29°) is 3.9σ further west.
- Sep 10<sup>th</sup> (Epoch 3): The model image contains two starspots; the first starspot is centrally located and the second starspot is located on the northwestern limb. The western most modeled starspot is not seen in the reconstructed image. The southwestern cool

reconstructed starspot is not analyzed since the detection strength is below  $3\sigma$ . The detection strength of the eastern starspot is 5.6 $\sigma$ . The agreement with the model is within  $1\sigma$  for  $T_R$ . The reconstructed  $\phi$  (7.8%) is  $1.1\sigma$  smaller than the model. The reconstructed b (16°) is  $3.4\sigma$  further north than the model while the reconstructed l (-15°) is  $5.5\sigma$  further east.

- Sep 14<sup>th</sup> (Epoch 4): The model image contains one starspot located slightly north and west of center. The detection strengths for the western reconstructed starspot is 9.7σ. The agreement with the model is within 1σ for b. The reconstructed φ (4.8%) is 2.1σ smaller than the model while the reconstructed T<sub>R</sub> (0.915) is 6.9σ cooler. The reconstructed l (6.3°) is 4.2σ further east than the model value.
- Sep 19<sup>th</sup> (Epoch 5): The model image contains two starspots; the first starspot is located on the eastern limb and the second starspot is located on the northwestern limb. The detection strengths for the northern and southern reconstructed starspots are both 3.2σ. While the parameters of two starspots can be estimated from the reconstructed image, these starspots are not in any agreement with the two model starspots.
- Sep 24<sup>th</sup> (Epoch 6): The model image contains two starspots; the first starspot is located east of center and the second starspot is located on the northwestern limb. There are indications that the two modeled starspots are seen in similar locations in the reconstructed image, however this cannot be confidently confirmed.

The adopted observing strategy provided reasonable agreement however some exceptions do exist. It is unclear why the western starspot seen in model images is not recovered in the reconstructed images for Sep  $6^{th}$  and  $10^{th}$ . Nor is it clear why there is poor agreement between the Sep  $19^{th}$  model and reconstructed images. In general, the agreement is not as good as it is in the 2010 data set. The reconstructed covering factor is always smaller than the modeled covering factor in each epoch, however, this is not surprising as this parameter should be considered a lower bound. In an opposite trend than 2010, if reconstructed *b* does not agree with the model, the reconstructed starspot is more north. No trend exists for when the agreement in *l* is poor.

The reconstructed images in Epochs 1, 3, and 5 contain a ring of warm starspots around the stellar limb. These starspots are rejected as artifacts caused by [u,v] sampling due to their symmetry and constant location between epochs, which is contrary to the expectation of starspots on a rotating surface. In Epochs 2 and 4, the warm starspots in the northeast and southwest are rejected as artifacts. The warm northeast starspot in Epoch 3, the warm central starspot in Epoch 4, and the cool southern starspot in Epoch 5 are all rejected as artifacts. The rejection in each of these cases is motivated by the presence of similar features in the respective simulated images. The noncircular stellar disk in Epoch 6 is most certainly an artifact due to the limited [u,v] sampling (200 points). In addition, the shape of this disk resembles the configuration of [u,v] points (see Fig. 5.4).

No photometric observations are available near the time of the interferometric observations. However, a modeled light curve is again computed via the method described in § 6.5. Fig. 6.27 is a plot of this light curve versus modified Julian date. A sinusoid, represented by the black solid line, with a period of 54.8 days is phased to best fit the modeled points via visual inspection. The peak-to-trough amplitude of the curve is set to 0.1 mag. For Epochs 1, 2, 3, and 6, the flux drops as expected for starspots transiting across a rotating surface. However, Epochs 4 and 5 are significantly deviant from this scenario by having a much higher flux than expected. This deviation does not appear to be caused by poor [u,v] sampling or data quality. Epochs 4 and 5 have the densest [u,v] coverage in the data set as well as the best data quality. A sinusoid with a period of 27.4 days is also plotted. This represents the scenario described for the 2007 data set where  $\lambda$  And possesses two cool starspots separated by ~ 180° in longitude. The amplitude of this sinusoid has been doubled to 0.2 mag and phased by eye to best fit the data. In this case, again four of the epochs fit the curve well with Epochs 1 and 3 as the exceptions. Error bars have been plotted based solely on the range in magnitude given the starspot flux ratio errors in each epoch. Given this and the fact that the  $\lambda$  And light curve is more irregular than a smooth sinusoid, it is more probable that Epoch 1 fits with this second scenario. The take away point is that the modeled light curve is not completely inconsistent with a sinusoidal-like variability with the same rotational period as  $\lambda$  And.



Figure 6.27: The black asterisks represent the modeled photometry for each the 6 epochs. The open diamonds represent the magnitude change in the model images corresponding to the error in the starspot flux. The solid line indicates a sine curve with a period of 54.8 days and  $\Delta \text{ mag} = 0.1 \text{ mag}$ . The dotted line indicates a sine curve with a period of 27.4 days and  $\Delta \text{ mag} = 0.2 \text{ mag}$ . The phasing is fit by eye to best fit the point.

### 6.6.1 Tracing Rotation in the 2011 Data Set

The observing baseline for the 2011 data set spans  $\sim 41\%$  of the photometrically determined rotation period. The cadence will carry starspots  $\sim 7\%$  across the stellar surface between epochs assuming a negligible amount of differential rotation. Fig. 6.28 shows a compelling pattern of stellar rotation by three starspots labeled A through C. Starspot B is seen in all 6 epochs and provides the best estimates of both the stellar rotation and rotation axis. The observing strategy behind the 2011 data set was designed to provide an increased number of measurements for any individual transiting starspot(s). This is done to shrink the uncertainties in the estimates of the rotation axis computed from the 2010 data set. The uncertainties arose from a sparse number of measures per transiting starspot.



Figure 6.28: The best fit models for each night in 2011. In each model, the starspot(s) are labeled (A, B, and C) to indicate the same starspot as seen in each epoch.

Starspot	Epoch Range	$\Delta \phi$ (%)	$\Delta T_R$	$\begin{array}{c} \mathbf{P}_{rot}\\ (days) \end{array}$
А	$1 \rightarrow 2$	0	0.000	43.4
	$2 \rightarrow 3$	5	0.001	45.9
В	$1 \rightarrow 2$	-4	0.000	56.7
	$2 \rightarrow 3$	-1	-0.008	49.8
	$3 \rightarrow 4$	-2	0.018	64.6
	$4 \rightarrow 5$	5	-0.005	51.1
	$5 \rightarrow 6$	1	0.002	59.2
С	$5 \rightarrow 6$	5	0.000	61.2

Table 6.8: Evidence for Stellar Rotation in the 2011 Data

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Figure 6.29: Ellipse fits to starspot positions in the 2011 data sets. The dashed line corresponds to the fit to spot B. The dash dot dot line corresponds to spot A. The red circle indicates the edge of  $\lambda$  And. The average computed position angle,  $\Phi$ , and inclination angle, i, from these fits are  $25.1 \pm 5.1^{\circ}$  and  $68.2 \pm 2.0^{\circ}$ , respectively.

Table 6.8 contains the changes in  $\phi$  and  $T_R$  for starspots A, B, and C and the computed rotation period based on the measured angle between starspot positions from one epoch to the next. The largest change in  $\phi$  and  $T_R$  for any starspot over the observed rotation is 7% and 0.018, respectively. As these are comparable to the median  $\phi$  error of 5.3% and median  $T_R$  of 0.014, the starspots do not appear to be significantly evolving. The average rotation period based on starspot motion is 54.0 ± 7.6 days. The error is the standard deviation of the individual rotation periods. This is nearly identical to the photometrically determined rotation period of 54.5 ± 2.4 days. In addition, the error in the starspot derived period is nearly half of that found by the 2010 data set.

Fig. 6.29 shows each of the starspots plotted by latitude vs. longitude overlaid by ellipse fits. For starspots A and B, an ellipse is fit via visual inspection. The C starspot is excluded

from this analysis due to having only two data points. The average  $\Psi$  and *i* are  $25 \pm 5.1^{\circ}$ and  $68.2 \pm 2.0^{\circ}$ , respectively. The rotation axis is tilted out of the plane of the sky in the northern hemisphere. These values and orientation are consistent with the estimates found by the 2010 data set to within the error bars.

In 2011, the CHARA Array gained the ability to observe using all 6 telescopes simultaneously, instead of combinations of separate 4 telescope configurations. This substantially increased the number of visibilities and closure phases obtained during each acquired block of data. However, the [u,v] coverage decreased to almost half of what was obtained in both 2009 and 2010. This explains the lesser amount of consistency between the model and reconstructed images across all epochs. However, the shorter cadence of the observations did allow for a much improved tracing of  $\lambda$  And's rotation. The analysis of the apparent starspot motion provided estimates of the rotation axis inclination and position angle that are nearly identical with the estimates from the 2010 data.

## 6.7 Comparing Results with the Literature

Having demonstrated that starspot properties can be measured for  $\lambda$  And using interferometric observations, these results are compared to the results of previous investigations. Donati et al. (1995 hereafter D95) created a surface map of  $\lambda$  And via a matrix LCI (see § 1.2) technique using Johnson BV light curves spanning one rotation period. D95 models the observed light curve using 2 starspots with a  $T_R = 0.83 \pm 0.06$ . One starspot is located at  $b = 50^{\circ}$  with a  $\phi = 8\%$ . The other starspot is located at  $b = 20^{\circ}$  with a  $\phi = 4\%$ . The starspots are separated by 140° in longitude. Both the latitudes and covering factors for these starspots are consistent with those identified in this work. However, the temperature ratio in D95 is significantly less than that measured for both the 2010 (median  $T_R = 0.853$ ) and 2011 (median  $T_R = 0.843$ ) data sets.

A more recent study of the starspot properties of  $\lambda$  And was performed by Frasca et al. (2008 hereafter F08). They use a 2-component LCI method using Johnson V band photometry coupled with spectral line depth ratios to create a map of starspots on  $\lambda$  And. The results of F08 are very consistent with D95 with the modeled surface containing 2 cool starspots each with a  $T_R = 0.815^{+0.064}_{-0.036}$ . The covering factors for the two starspots are 8.7% and 3.6% located at latitudes 57° and 9°, respectively. The starspots are separated by 81° in longitude. All starspot properties identified by F08 are consistent with the starspot properties measured in this work. One difference between F08 and this work, as well as D95, is the modeling of 2 plage regions by F08. These bright regions are similar in size to the modeled cool starspots. The plages are also in similar locations only offset to the starspots by ~20° in longitude and ~7° in latitude.

## 6.8 The Unspotted Giant 2 Aur

Although the overall properties of the starspots of  $\lambda$  And are supported by the general agreement between multiple epoch imaging, multiple seasons of data, its photometric light curve, and previous work, occasional inconsistencies persist. This section contains the discussion of applying the methodology described in Ch. 5 to a star not expected to have large, cool starspots. This is intended as a reality check to ensure the method doesn't fabricate starspots where none are expected.

2 Aur (HD 30834) is a bright (V = 4.787, H = 1.502) K3 giant (Jaschek et al. 1964) located at a distance of  $184 \pm 10$  pc (van Leeuwen 2007). This star is not a known X-ray



Figure 6.30: Upper Left: Model solution for the Nov  $7^{th}$  data set of 2 Aur. Upper Right: A plot of the difference in visibilities between the observed and modeled data. Lower Left: A plot of the observed and model closure phases. Lower Right: A plot of the observed and model triple amplitudes.

source suggesting it is not magnetically active. This star is, also, not a known photometric variable star.

The data were obtained on November  $7^{th}$ , 2009 and the observing strategy is identical to that employed in August of that year; the data set is comprised of observations spanning the first half of the night employing the S1-E1-W1-W2 telescope configuration combined with observations over the second half of the night employing the S2-E2-W1-W2 configuration. The [u,v] coverage (384 points) is approximately half of that obtained in August since only data for one night was collected due to weather.

Fig. 6.30 shows the fit to the visibilities, closure phases, and triple amplitude for the best fit parametric model. As seen in the Figure's lower left panel, only a slight nonzero closure phase located primarily at the smallest spatial scales.



Figure 6.31: Results from the 2 Aur data sets, including the model images (top row), reconstructed images (middle row), and simulated images (bottom row). The white dot in the lower right hand corner represents the resolution limit for the CHARA array.

A limb darkening angular diameter of  $\theta_{ld} = 2.67 \pm 0.14$  mas and a limb darkening coefficient  $\alpha = 0.20 \pm 0.20$  are modeled from the first lobe visibility data. Given an angular size of 2.67 mas projected to a distance of 184 pc, a linear radius of  $26.4^{+2.8}_{-2.0} R_{\odot}$  is computed. Koleva & Vazdekis (2012) computes a  $T_{eff} = 4256$  and  $\log(g) = 1.67$  for 2 Aur based on spectra taken by the New Generation Stellar Library.

The best fit parametric model (reduced  $\chi^2 = 0.70$ ) is for a surface with zero starspots. However, the reconstructed image, shown in Fig. 6.31, contains a starspot structure extending from the north-east limb to approximately mid-disk. The significance of this starspot is approximately 6  $\sigma$ . A simulated reconstructed image of a featureless surface reveals a large roughly circular starspot centered at mid-disk without the structure extending to the stellar limb (see Fig. 6.31). However, this starspot is concluded to be an artifact despite the difference in shape between the reconstructed image and simulated image. This is mo-

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tivated mainly from the lack of significant non-zero closure phase to explain its existence. Additionally, none of the best fit models with one, two, or three starspots resembles this reconstructed image.

# SUMMARY OF RESULTS

Presented are two in depth studies of stellar variability. The first involves a long term multiwavelength monitoring campaign of 7815 stars in the direction of the  $\rho$  Ophiuchi molecular cloud. The intention is to characterize the photometric variability in an attempt to describe the variability morphology and how this might lead to physical interpretations of the variability mechanisms. The second involves interferometric imaging of cool starspots on the surface of the chromospherically active giant,  $\lambda$  Andromedae, using long baseline near-infrared interferometry. The intention is to characterize starspot properties via a direct imaging method both as a check to results from previous starspot studies and to provide a more precise measure on how starspots affect a variety of astrophysical phenomena. Here are the summaries and seminal findings of each study.

## 7.1 Summary of the $\rho$ Ophiuchi Cluster Variability Survey

High precision, high cadence J, H,  $K_s$  photometry is obtained for 7815 stars in the direction of the  $\rho$  Oph molecular cloud with a temporal baseline of ~ 2.5 yrs. Spurious detections, partially resolved doubles, galactic contamination, and unrelated field stars are eliminated from the photometry. The target sample meeting the specifications for time series variability analysis includes 1678 stars. A seven point variability test is used to identify 101 variable stars, which is 6% of the parent sample. These tests are sensitive to variability on a variety of different time scales and forms (e.g. sinusoidal, 'eclipse-like', etc.).

Of the 101 stars in the variable catalog, 84% are located "on cloud" while only 16% lie within the "field". Location "on cloud", variability, and  $(H-K_s)$  colors redder than a 3 Myr

isochrone are used to assess membership in the  $\rho$  Oph star cluster. This method identified 22 stars new candidate  $\rho$  Oph members.

The effects of observing strategy on variability detection and measured amplitudes is investigated by comparing this work to the  $\rho$  Oph variability study performed by Alves de Oliveira & Casali (2008). These two studies have 464 stars in common; AC08 identified 7% as variable stars and this work identifies 18%. The increase in detection fraction is not caused by different sensitivities in the separate variability criteria used in each survey. This work also found both a higher  $K_s$  variability amplitude and  $(H-K_s)$  color amplitude in 25 stars identified as variable in both surveys than measured by AC08. High cadence observational monitoring is therefore a more accurate method to characterize stellar variability since it not only will discover more variables within a given set of stars, but also it is also more likely to detect intrinsically higher amplitude variability.

The  $K_s$  variability and stellar color behaviors are used to estimate the physical mechanism responsible for the variability. Rotational modulation by long lived cool starspots is expected to produce colorless, periodic variability. Rotational modulation of long lived hot starspots (e.g. accretion) is, also, expected to be periodic, while short lived starspots (e.g. flares) are not. The star becomes bluer as it brightens in both cases. Extinction induced variability can either be periodic or exhibit long time scale variation based on the geometry of the occulter relative to the star. Changes in the mass accretion rate onto the star are not expected to be periodic, but occur on time scales ranging from days to years. As this rate increases, the star becomes bluer as the star dims.

Identifying periodic variability within the variable catalog is done via a newly improved period searching algorithm, the Plavchan algorithm. The algorithm tests tens of thousands of periods with uniform frequency sampling between 0.1 to 1000 days. This is done by comparing the observed light curve to a dynamically generated prior. The statistical significance of individual periods is computed via two methods: the distribution of power values at other periods in the same periodogram and the distribution of maximum power values for all sources in an ensemble survey. The Plavchan algorithm finds periodic variability in 32% of the variable catalog with periods ranging from 0.49 to 92 days.

From cross-referencing the target sample with two previous surveys, 72 stars have been assigned a YSO classification (13 Class I, 47 Class II, 12 Class III). The variability fraction of these YSOs is 79%. The variability fraction differs according to YSO class with 92% of Class I and Class III stars identified as variable; this fraction drops to 72% for Class II stars. The amplitude of both brightness and color variability are decreasing functions of YSO class. The median peak-to-trough  $\Delta K_s$  amplitude for Class I, II and III stars are 0.77, 0.31 and 0.08 mag, respectively. In addition, the median peak-to-trough  $\Delta(H-K_s)$  color amplitudes are 0.81, 0.21 and 0.07 mag for each class respectively.

The periodic variables are split into two subcategories: sinusoidal-like and eclipse/inverse eclipse-like. Sinusoidal-like periodic variability describes a sinusoidal-like change in the observed flux when the time series is folded to the most significant period. Rotational modulation by cool starspots is believed to be the common variability mechanism in this subcategory. Sinusoidal-like periodic variables are found in each YSO class (3 Class I, 8 Class II, 8 Class III). Eclipse-like periodic variability results in discrete drops, or "dips", in the observed flux when the time series is folded to the most significant period. Periods range from 2 to 8 days with the duration of these dips lasting less than 30% of one periodic epoch. This subcategory contains 6 stars with a median peak-to-trough  $\Delta K_s$  and  $\Delta(H-K_s)$  color amplitudes of 0.31 and 0.11 mag, respectively. Variable extinction is the likely mechanism for the eclipse-like variations. All stars in this subcategory are Class II YSOs. The inverse eclipse-like variables, WL 4 and YLW 16A have periods of 65.61 and 92.3 days, respectively. The variability mechanism proposed in both cases is the periodic obscuration of one component in a close binary by a warped circumbinary disk (Plavchan et al. 2008a, 2013).

In half of the eclipse-like variables (YLW 1C, 2MASS J16272658-2425543, YLW 10C) an additional statistically significant period is identified. This sinusoidal-like periodic variability coupled with the presence of "dips" suggests a rapidly rotating spotted star occulted by a clump of optically thick material in the inner accretion disk. These stars strengthen the interpolation posed to explain the variability of other YSO AA Tau-like variables (Morales-Calderón et al. 2011). The periods corresponding to periodic occultations in YLW 1C and YLW 10C are located near their respective corotation radii. The mechanism driving these occultations could arise from a warped inner circumstellar disk caused by an inclined magnetic dipole, or could be the prenatal cloud of a forming hot Jupiter.

Long time scale variables is a variability subclass, containing 31 stars, where the measured flux increases or decreases consistently over months or years. The variability time scale is measured using a differencing technique and approximates the time between maximum and minimum brightness. The measured time scales range from 64 to 790 days. The peak-totrough  $\Delta K_s$  amplitudes range from 0.05 to 2.31 mag and the peak-to-trough  $\Delta (H-K_s)$  color amplitudes range from 0.06 to 1.32 mag. Variable extinction and variable accretion rates are both equally likely to cause long time scale variability. This subclass contains 25 known YSOs with 7 Class I, 15 Class II and 3 Class III stars. The time series of irregular variables are aperiodic and do not vary over any discernible time scale. This subclass contains more members (40) than either the periodic or long time scale subclasses. The peak-to-trough  $\Delta K_s$  amplitudes range from 0.04 to 1.11 mag and peak-to-trough  $\Delta(H-K_s)$  color amplitudes range from 0.05 to 0.75 mag. No single dominant variability mechanism explains irregular variability. Only 9 known YSOs (1 Class I, 7 Class II, 1 Class III) are irregular variables.

The CTTS WL 20W and the WTTS ISO-Oph 126 are similar in that both have a long time scale variation superimposed on a periodic signal. In both cases, the physical mechanisms for the variability are consistent with an occultation of a rapidly rotating spotted star by optically thick material outside the inner accretion disk. For WL 20W, the sinusoidallike variability has a period of 2.1026 days and a peak-to-trough  $\Delta K_s$  amplitude of 0.19 mag. The long time scale variability has a duration of 122 days with a  $\Delta K_s$  eclipse depth of 0.26 mag. For ISO-Oph 126, the sinusoidal-like variability has a period of 9.114 days and a peakto-trough  $\Delta K_s$  amplitude of 0.06 mag. The long time scale variability has a duration of 349 days with a  $\Delta K_s$  eclipse depth of 0.10 mag.

The very high amplitude periodic variability measured in the Class I star WL 15 is not consistent with any proposed mechanism. The observed 47 day colorless decrease in  $K_s$  band brightness of ~1 mag is also not easily explained.

#### 7.2 Summary of the $\lambda$ Andromedae Starspots Survey

 $\lambda$  Andromedae, a bright (V = 3.872 mag) G8 giant, has a long recorded history of consistent, sinusoidal-like photometric variability. This variability is believed to result from the rotational modulation of cool starspots. Using light curve inversion techniques, the presence of starspots has been revealed indirectly (Donati et al. 1995; Frasca et al. 2008). Long baseline optical/near infra-red interferometry has directly imaged a number of astrophysical systems (e.g. close binaries, circumstellar disks, rapidly rotating stellar surfaces) with unprecedented angular resolution.

In an attempt to confirm and directly measure the starspots on  $\lambda$  And, this star was observed using the MIRC beam combiner on the CHARA Array for 27 epochs spanning from 2007 Nov 17<sup>th</sup> to 2011 Sep 24<sup>th</sup>. The observing strategy evolved over time due to upgrades in the MIRC beam combiner. Contemporaneous photometric observations are also available from Sep 30<sup>th</sup>, 2007 to Jan 20<sup>th</sup>, 2011. The photometry provides an independent relative estimate of starspot coverage that can be compared to the imaging results.

Images are produced through two independent methods, a parametric model and an image reconstruction code. The parametric model utilizes a mixed minimization approach of an AMOEBA code coupled with a genetic algorithm to determine values of the stellar diameter and the limb darkening coefficient. In addition it determines the values of covering factor (the percentage of the visible disk covered by a starspot), latitude, longitude, and intensity ratio relative to the photosphere for any number of modeled starspots. The intensity ratio is later converted into a temperature ratio. The only assumptions are that only cool, circular starspots are present. The second imaging method is through the imaging reconstruction program SQUEEZE (Baron et al. 2010). SQUEEZE begins by assuming a circular, uniform intensity distribution on a zero intensity background. The program then uses a Markov chain Monte Carlo approach to reassign randomly intensity within the image tempered by a regularizer that forces the total variation in intensity between pixels to be minimized. The program iterates until the redistribution of intensity provides the best quality of fit compared to the measured data.

The CHARA Array observations in 2007 and 2008 of  $\lambda$  And employed 1 to 3 snapshot observations using 4 telescopes that yielded only minimal coverage in the [u,v] plane. Despite the strong evidence for starspots during these epochs from both measured nonzero closure phase and variable photometry, this minimal [u,v] coverage is insufficient to determine confidently the starspot properties.

In 2009 and 2010, a new observing strategy was employed to maximize the [u,v] coverage without compromising the observed data due to the rotation of  $\lambda$  And. Over the first half of the night, observations were obtained using the 4 S1-E1-W1-W2 telescopes. These data were combined with observations over the second half of the night obtained using the 4 S2-E2-W1-W2 telescopes. Finally this strategy was repeated on a consecutive night and the data from both nights were combined into a single epoch. The 6° rotation of  $\lambda$  Andromedae from night to night is not expected to affect adversely the quality of the final images. This strategy results in at least 10x the [u,v] coverage that was obtained in 2007 and 2008.

The model and reconstructed images resulting from the 2009 data set are consistent with each other within the error bars. The model image found three starspots forming a near straight line from the eastern limb into the western hemisphere. The reconstructed image contains the two starspots located near the disk center without a clear indication of the third modeled starspot. The single epoch, however, prevents a check of these results through comparison with other epochs acquired a short time before or after. Assuming the starspots are genuine, this would support the idea active stars have starspots on the visible surface even during photometric maximum. The 2010 data set improves on the results of the previous year through a multiepoch approach. Between one to four starspots are imaged on each of the 6 epochs of data obtained. As an ensemble the median value in the starspot covering factor is 7.6%. The median value of the temperature ratio between the starspot and the photosphere is 0.961. The starspot properties extracted from the reconstructed images are consistent with the modeled results to within the error bars.

A photometric V band time series is available beginning a few days after the last interferometric observation. This short cadence time series spans approximately two rotations of  $\lambda$ And. The flux variability in the modeled time series follows the behavior of the photometric time series quite well when the proper scaling factors are applied. This is illustrated both in plotting the modeled time series with photometric time series as a simple light curve or by plotting both on a phased light curve by folding the time series by 54.8 days. A sole outlier does exist in Epoch 5, which is brighter than expected. It is difficult to explain the discrepancy based on [u,v] coverage or data quality.

The observing cadence between the 6 epochs in 2010 is between 6 to 9 days corresponding to 10.9% to 16.4% of the rotation period. The observations span 71% of one rotation cycle. Four starspots are believed to be seen in 3 epochs and this provides a resource to both compute the rotation period via apparent starspot motion and characterize the rotation axis. The rotation period based on starspot motion is  $60 \pm 13$  days, which is consistent with the photometric rotation period of  $54.8 \pm 1.9$  days. The rotation axis is tilted out of the plane of the sky with an inclination of  $78 \pm 1.5^{\circ}$  and a position angle of  $20 \pm 6.8^{\circ}$ .

The MIRC beam combiner was upgraded in 2011 to allow the use of all 6 CHARA telescopes simultaneously.  $\lambda$  And was observed for a single night on 6 different nights.

The increased number of telescopes substantially increased the number of visibilities and closure phases obtained for each block of data, however, the [u,v] coverage proved to be approximately half that acquired in 2010 since data were only collected on a single night per epoch. As a result the parametric model and reconstructed images were not as consistent as in the 2010 data. Only one to two starspots are identified in the model images for each epoch. As an ensemble, the median value of the starspot covering factor is 12% and the median temperature ratio is 0.958.

No photometric observations are available near the time of the interferometric observations. However, a sinusoid with a period of 54.8 days is phased to the best fit modeled points via visual inspection. The peak-to-trough amplitude of the curve is set to 0.1 mag. In four epochs, the flux drops as expected for starspots transiting across a rotating surface. However, Epochs 4 and 5 are significantly deviant from this scenario by having a much higher flux than expected. This deviation does not appear to be caused by poor [u,v] sampling or data quality.

The 2011 observing cadence between the 6 epochs is between 4 or 5 days corresponding to 7.3% and 9.2% of the rotation period spanning ~40% of one rotation. While only three separate starspots can be identified migrating across the surface, one starspot is imaged in each of the 6 epochs. From this a starspot derived rotation period is  $54.0 \pm 7.6$  days which is nearly identical to the photometric rotation period of  $54.8 \pm 1.9$  days. The rotation is again found to tilt out of the plane of the sky with an inclination of  $77.98 \pm 0.18^{\circ}$  and a position angle of  $23 \pm 6.4^{\circ}$ . These values are in agreement with the orientation computed from the 2010 data set.

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Appendix

## Empirical Determination of $\chi^2_{n_0}$ Dependence on Parameters

In this appendix, an analysis of the dependence of the PA periodogram  $\chi^2_{n_0}$  power values on the number of observations and periodogram parameters  $n_0$  and p is presented. The ensemble survey of mostly non-variable stars is used to carry out this analysis and to present an alternative approach to evaluate the statistical significance of periodogram power values.

First, a random subset of 180 stars is chosen from the survey collection of 1678 stars. These stars are evenly distributed in  $N_{obs}$  and  $K_s$  magnitude. For a given set of parameters p and  $n_0$ , the maximum  $\chi^2_{n_0}$  periodogram power value is computed for the 180 stars. Since other algorithms exist that specialize in finding periodic sources with low numbers of detection (Dworetsky 1983  $N_{obs} \sim 20$ ), test cases are limited to 0.04 < p and  $\leq 0.5$  and  $12 < n_0 \leq 250$ . Fig. A.1 shows the dependence of  $\chi^2_{n_0}$  on  $N_{obs}$  for the 180 stars with a particular set of p and  $n_0$ . This dependence is somewhat expected – a smaller number of observations can result in an increase in the likelihood for false-positive periodogram peaks.

The distribution of  $\chi^2_{n_0}$  values as a function of N<sub>obs</sub> is well-described by the functional form:

$$F(N_{obs}) = \left(\frac{a}{N_{obs} - b}\right)^{1.5} + c \tag{A.1}$$

where a, b and c represent real numbers that differ for a given p and  $n_0$ . As the power law index decreases below 1.5 for p < 0.04 and/or  $n_0 < 12$ , these ranges are excluded from the analysis. The fitting parameters in Eqn. A.1 are found via trial and error to minimize



Figure A.1: Graph of the  $\chi^2_{n_0}$  value as a function of detection size,  $N_{obs}$ , for the test case with the parameters:  $n_0 = 25$  and p = 0.06. The red line represents the functional fit of Eqn. A.1 with values of (a,b,c) = (62.5495,18.4963,1.2523) where the residuals are minimized.

the residuals when compared to a variety of functional forms tested, rather than from an analytic derivation based upon first principles.

The next step is to determine how the constants a, b and c vary as functions of the parameters p and  $n_0$ . Fixing  $n_0$ , the maximum  $\chi^2_{n_0}$  as a function of  $N_{obs}$  is empirically fit to 10 chosen p values resulting in 10 different values of a, b and c. The same process is repeated except p is fixed and  $n_0$  is varied. Fig. A.2 displays the dependence of a on the parameters p and  $n_0$ . Six fits to the dependence of a, b and c on parameters p and  $n_0$  are determined empirically through trial and error to be:

$$f_a(n_0) = -0.3491(n_0 - 17.0796)e^{-0.0451n_0} + 63.4573$$
 (A.2a)

$$f_b(n_0) = 1.3023(1 - \frac{23.3762}{n_0})e^{-0.0283n_0} + 18.4347$$
(A.2b)



Figure A.2: Top: Graph of the constant a as a function of parameter p (see Eqn. A.3a). Bottom: Graph of the constant a as a function of parameter  $n_0$  (see Eqn. A.2a).

$$f_c(n_0) = 0.2796e^{-0.0381n_0} + 1.1467 \tag{A.2c}$$

$$f_a(p) = 82.6288e^{-12.1989p} + 25.4356 \tag{A.3a}$$

$$f_b(p) = 3.0791 p^{-0.6377} \tag{A.3b}$$

$$f_c p = (p - 0.0305)^{-0.0395} + 0.0905$$
 (A.3c)

The particular functional forms of Eqn. A.2a through Eqn. A.3c are again not analytically motivated, but instead minimize the residuals from a variety of functional forms tested.

These six functions of one parameter are combined into three functions of both parameters p and  $n_0$ . This is accomplished by replacing the constant term in the  $n_0$  function by the entire corresponding p function. The constant term from  $f_c(p)$  function is also dropped.

Thus, the following functions are found to adequately describe the dependence of a, b and c on parameters p and  $n_0$  for this survey:

$$f_a(n_0, p) = -0.3491(n_0 - 17.0796)e^{-0.0451n_0} + 82.6288e^{-12.1989p} + 25.4356$$
(A.4a)

$$f_b(n_0, p) = 1.3023(1 - \frac{23.3762}{n_0})e^{-0.0283n_0} + 3.0791p^{-0.6377}$$
(A.4b)

$$f_c(n_0, p) = 0.2796e^{-0.0381n_0} + (p - 0.0305)^{-0.0395}$$
(A.4c)

and therefore Eqn. A.1 can be rewritten as:

$$F(N_{obs}, n_0, p) = \left(\frac{f_a(n_0, p)}{N_{obs} - f_b(n_0, p)}\right)^{1.5} + f_c(n_0, p)$$
(A.5)

The values p = 0.06 and  $n_0 = 40$  – used throughout this paper to identify periodic variables – are an optimal choice of parameters for this survey. They yield the smallest residuals when the 180 test cases are fit in Eqn. A.5. Thus, for this survey the maximum peak power in the periodogram for a non-variable star is approximately given by the expression:

$$F(N_{obs}, 40, 0.06) = \left(\frac{63.8629}{N_{obs} - 18.6927}\right)^{1.5} + 1.2100 \tag{A.6}$$

The validity of the numerical fits (Eqns. A.4a through A.4c) is verified using an additional 18 test cases with randomly selected values of p and  $n_0$ . The  $\chi^2_{n_0}$  versus N<sub>obs</sub> distributions are again fit using Eqn. A.1, yielding 18 "observed" a, b and c values for each pair of p and  $n_0$ . Predicted values for a, b and c are found by using Eqns. A.4a through A.4c and compared to the "observed" values. The mean percent errors between the observed and predicted values for the 18 test cases are  $-1.1\pm3.4\%$  for  $f_a$ ,  $1.1\pm3.5\%$  for  $f_b$  and  $-0.1\pm1.8\%$  for  $f_c$ . Table A.1 contains the percent errors for each of the individual 18 test cases. Eqns. A.4a through A.4c therefore adequately predict the values a, b and c in Eqn. A.1 for any set of parameters p and  $n_0$  within the parameter space explored in this survey. Thus, this demonstrates the PA algorithm is reasonably "well-behaved." Eqn. A.5 could be applied to different surveys and cadences. However, the particular numerical values in Eqns. A.4a through A.4c and Eqn. A.6 for a, b and c likely depend on the specific cadence of a survey.

n <sub>0</sub>	р	Obs. Param. A	Pred. Param. A	% Error	Obs. Param. B	Pred. Param. B	% Error	Obs. Param. C	Pred. Param. C	% Error
12	0.25	31.338	30.382	-3.054	6.796	6.574	-3.259	1.205	1.239	2.815
13	0.04	80.893	76.952	-4.872	22.235	23.263	4.622	1.393	1.372	-1.500
15	0.06	64.109	65.547	2.243	17.776	18.042	1.500	1.308	1.307	-0.042
18	0.05	69.756	70.192	0.625	19.958	20.568	3.054	1.310	1.309	-0.054
18	0.06	63.138	65.036	3.006	18.173	18.284	0.610	1.287	1.290	0.208
25	0.06	62.550	64.284	2.772	18.496	18.560	0.343	1.253	1.257	0.337
25	0.40	25.656	25.169	-1.898	5.833	5.565	-4.602	1.135	1.148	1.136
25	0.10	49.530	48.938	-1.194	13.192	13.411	1.662	1.200	1.219	1.602
25	0.04	79.775	75.265	-5.653	22.927	24.024	4.784	1.290	1.310	1.534
40	0.06	62.172	63.863	2.719	18.604	18.693	0.479	1.208	1.210	0.194
60	0.24	29.562	28.859	-2.378	8.219	7.796	-5.145	1.128	1.092	-3.168
75	0.055	68.359	66.993	-1.998	18.500	19.682	6.390	1.157	1.174	1.423
125	0.30	28.994	27.429	-5.397	6.329	6.666	5.334	1.080	1.055	-2.269
140	0.45	24.910	25.700	3.171	5.127	5.144	0.335	1.067	1.036	-2.884
175	0.14	43.597	40.392	-7.351	10.569	10.796	2.143	1.090	1.092	0.145
180	0.05	71.813	70.318	-2.082	20.056	20.808	3.751	1.147	1.168	1.867
200	0.06	63.461	65.170	2.694	18.439	18.522	0.451	1.145	1.149	0.385
240	0.40	26.132	26.062	-0.268	5.800	5.525	-4.742	1.070	1.040	-2.794

Table A.1: Monte Carlo Simulation: Testing Significance Function



Figure A.3: The maximum PA periodogram power value for a star in this survey is found to be well-described by Eqn. A.5 (Fig. A.1). The scatter about the value predicted by Eqn. A.5,  $\sigma$ , is also found to be dependent on N<sub>obs</sub> as shown in this figure.

To evaluate the statistical significance of a peak period power value in a periodogram, it does not suffice to identify the expected value for the ensemble survey. The scatter about the expected value is also necessary. This scatter, or standard deviation ( $\sigma$ ), of the peak period power values about the expected value depends on N<sub>obs</sub> in a predictable fashion for this survey (Fig. A.3). To characterize this scatter, the scatter for each survey star is grouped into bins as a function of N<sub>obs</sub>, with a bin size of 25. An average  $\sigma$  is computed for each bin and an empirical fit to this distribution is made, given by Eqn. A.7:

$$\sigma(N_{obs}) = \frac{2.3790}{N_{obs} - 21.6449} + 0.0105 \tag{A.7}$$

Putting it all together, now an affirmative periodicity condition can be defined. The statistical significance of a measured period for a star in this survey is simply a function of the number of observations. Based on visual inspection of star light curves, this condition is defined by the following:

$$\frac{\chi^2_{n_0,i}}{F(N_{obs_i}, n_0, p)} - 1 > 6\sigma(N_{obs_i}) \tag{A.8}$$

Periods where this condition is met can be considered statistically significant for the star investigated. Periods found using this criteria are generally also found to be statistically significant using the methods outlined in § 2.8.1.

## Source Code for the Plavchan-Parks algorithm

This appendix contains the C source code for the Plavchan-Parks algorithm. The code included here is the backbone for the period searching tools found on the *NASA Exoplanet Archive* website: http://exoplanetarchive.ipac.caltech.edu/index.html. As such, this code is also capable of identifying periods using both the Lomb-Scargle algorithm (Scargle 1982) and the BLS algorithm (Kovács et al. 2002).

```
/* periodogram.c (-> periodogram executable)
* Description:
* Compute a periodogram using one of three algorithms: Lomb-Scargle,
* BLS, or Plavchan-Parks.
*
* Syntax:
* Usage: periodogram
* [-a <PeriodogramType (algorithm): one of ls, bls, pp>]
* [-b <NumberOfBins (-a bls only)>]
* [-d <FrequencyStepSize (-i fixedf only)>]
* [-f <FrequencyRangeMin> | -P <PeriodRangeMax>]
* [-F <FrequencyRangeMax> | -p <PeriodRangeMin>]
* [-i <PeriodStepMethod: std, exp, fixedf, pp>]
* [-K <StatNumberOfSamples>]
* [-M <StatMean>]
* [-n <NumberOfOutliers (-a pp only)>]
* [-N <NumberOfPeaksToReturn>]
* [-o <OversampleFactor (not with -i pp)>]
* [-q <FractionOfPeriodInTransitMin (-a bls only)>]
* [-Q <FractionOfPeriodInTransitMax (-a bls only)>]
* [-R <OutputDirectory>]
* [-s <PhaseSmoothingBoxSize>]
* [-S <PeakSignificanceThreshold (on power for output)>]
* [-T <Title (name of star)>]
* [-u <PeriodStepFactor (-i pp only)>]
* [-V <StatStandardDeviation>]
```

```
* [-x <TimeColumn>]
* [-y <DataColumn>]
* [-Y <DataErrorColumn>]
* <InputFile>
* [<OutputFile>]
* Switches:
* --help
*
     Returns this message.
* -a <PeriodogramType (algorithm)>
     Specifies which algorithm to run (one of ls, bls, pp).
*
* -b <NumberOfBins>
     Specifies the number of bins to use in the bls algorithm
*
* -d <FrequencyStepSize (-i fixedf only)>
     Specifies the size of the fixed frequency step
* -f <FrequencyRangeMin> | -P <PeriodRangeMax>
     Maximum period to consider (may be optionally specified as minimum freq)
* -F <FrequencyRangeMax> | -p <PeriodRangeMin>
     Minimum period to consider (may be specified as maximum freq)
* -i <PeriodStepMethod>
     Specifies which type of period stepping to use
*
     (one of std, exp, fixedf, pp)
*
* -K <StatNumberOfSamples>
*
     The number of samples to use for computation of p-values for output peaks.
     If not entered, the number of periods for which power is computed will
*
     be used.
* -M <StatMean>
     Mean to use for computation of p-values for output peaks. If not entered,
     the observed mean will be used.
*
* -n <NumberOfOutliers>
     Number of outliers to use in power calculation in the Plavchan-Parks algo
* -N <NumberOfPeaksToReturn>
     Limit on the number of top peaks to output in table.
*
* -o <OversampleFactor>J
     Increase number of periods sampled by this factor (not for use
*
     with -i pp or -d)
*
* -q <FractionOfPeriodInTransitMin>
     Minimum fraction of period in transit to consider with BLS algo
* -Q <FractionOfPeriodInTransitMax>
     Maximum fraction of period in transit to consider with BLS algo
*
* -R <OutputDirectory>
     The directory in which to put output files (periodogram,
*
     table of top periods). The default is '.'
*
```

```
* -s <PhaseSmoothingBoxSize>
     Size of box over which to average magnitudes for smoothed curve
*
* -S <PeakSignificanceThreshold>
    Maximum p-value to accept for output peaks in the power spectrum.
*
* -T <Title>
     The name of the star.
                            This will be used primarily for graphics.
* -u <PeriodStepFactor>
*
     Period increment factor for -i pp
* -V <StatStandardDeviation>
     Standard devaition to use for computation of p-values for output peaks.
*
     If not entered, the observed standard deviation will be used.
* -x <TimeColumn>
    Name of column in input file from which to read time info
* -y <DataColumn>
     Name of column in input file from which to read measurement values
* -Y <DataErrorColumn>
     Name of column in input file from which to read measurement errors
*
* Arguments:
* <input table file>
     Ascii table file containing time series data
*
* <output table file>
     [Optional] output table file containing PERIOD and POWER columns.
*
                                                                         If no
*
     file is specified, one will be constructed in the output directory with
*
     the name '<path-free name of input file>.out'
* Results:
* If successful, periodogram creates an output table file containing
* period and power, prints "[struct stat="OK", msg="<msg>"]" to stdout,
* and exits with 0. The output message contains the command line arguments
* needed to replicate the exact results, including derived quantities if any.
* Examples:
* The following example runs periodogram on a table file with the period
* range from .5 days to 1000 days and saves the output to out.tbl:
* $ periodogram test/test.tbl -p .5 -P 1000 out.tbl
* Return Codes:
* [struct stat = "OK"]
* [struct stat="ERROR", msg="Unable to allocate array"]
```

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```
* [struct stat="ERROR", msg="<general error message>"]"]
* Dependencies:
    Relies on utilities in tstools/src/util, some of which use libmtbl.a.
*
    Compile with ./funcArgs.c and ./pgramArgs.c
*
* Supported Platforms:
* Development Platform:
    linux
*
* Development History:
* Known Problems:
*/
/* " */
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <math.h>
#include <unistd.h>
#include <ctype.h>
#include <svc.h>
#include <utilMacros.h>
#include "util.h"
#include "statUtil.h"
#include "dataTbl.h"
#include "funcArgs.h"
#include "pgramArgs.h"
#include "pgramUtil.h"
/* Function prototypes: */
int computePeriodogram(struct DATA_TBL *data, struct PGRAM_ARGS *args,
                  struct PGRAM_FUNC_ARGS *fargs, char *errstr);
int main(int argc, char **argv) {
   FILE *errout = stdout;
```

```
char *errstr = calloc(MAXSTR, sizeof(char));
if (!errstr) {
    fprintf(errout,
            "[struct stat=\"ERROR\", "
            "msg=\"Cannot allocate space for error string\"]\n");
    fflush(errout);
    free(errstr);
    exit(1);
}
/* Initialize the argument structure */
struct PGRAM_ARGS args;
if (pgArgsInit(&args, errstr) == RET_ERR) {
    fprintf(errout,
            "[struct stat=\"ERROR\", msg=\"%s\"]\n", errstr);
    fflush(errout);
    free(errstr);
    exit(1);
}
/* Parse the arguments: */
if (pgArgsParse(argc, argv, &args, 1, errstr) != RET_OK) {
    pgArgsFree(&args, errstr);
    fprintf(errout, "[struct stat=\"ERROR\", msg=\"%s\"]\n", errstr);
    fflush(errout);
    free(errstr);
    exit(1);
}
/* Open the output file: */
FILE *out;
if (args.outToStdOut) {
    out = stdout;
}
else {
    char *fname = NULL;
    if (pgArgsGetOutputFile(&args, &fname, errstr) != RET_OK) {
        pgArgsFree(&args, errstr);
        fprintf(errout,"[struct stat=\"ERROR\", msg=\"%s\"]\n", errstr);
        fflush(errout);
        free(errstr);
        exit(1);
    }
    if (!(out = fopen(fname, "w+"))) {
        fprintf(errout, "[struct stat=\"ERROR\", "
```

```
"msg=\"OutputFile: can't open file %s\"]\n", fname);
        pgArgsFree(&args, errstr);
        fflush(errout);
        free(errstr);
        exit(1);
   }
}
/* Read the input table */
struct DATA_TBL data;
if (dtPopulate(&data, args.intbl, &args.xcol, &args.ycol, &args.yerrCol,
               &args.constraintCol, &args.constraintMin,
               &args.constraintMax,
               errstr) != RET_OK) {
   dtFree(&data, errstr);
   pgArgsFree(&args, errstr);
   if (out != stdout) fclose(out);;
   fprintf(errout,"[struct stat=\"ERROR\", msg=\"%s\"]\n", errstr);
   fflush(errout);
   free(errstr);
   exit(1);
}
/* Populate the PGRAM_FUNC_ARGS structure. This structure holds
* data input to the periodogram calculation and the results of
* the calculation as well */
struct PGRAM_FUNC_ARGS fargs;
memset(&fargs, 0, sizeof(fargs));
/* populate the fargs structure from args, data: */
if (populateFuncArgs(&args, &data, &fargs, errstr) != RET_OK) {
   dtFree(&data, errstr);
   freeFuncArgs(&fargs, errstr);
   pgArgsFree(&args, errstr);
    if (out != stdout) fclose(out);;
   fprintf(errout, "[struct stat=\"ERROR\", msg=\"%s\"]\n", errstr);
   fflush(errout);
   free(errstr);
   exit(1);
}
/* compute the periodogram */
if (computePeriodogram(&data, &args, &fargs, errstr) != RET_OK) {
   dtFree(&data, errstr);
   freeFuncArgs(&fargs, errstr);
```

```
pgArgsFree(&args, errstr);
    if (out != stdout) fclose(out);;
   fprintf(errout, "[struct stat=\"ERROR\", msg=\"%s\"]\n", errstr);
   fflush(errout);
   free(errstr);
   exit(1);
}
/* These are the results: */
if (!args.title) {
    if (dtGetDescription(&data, &args.title, errstr) != RET_OK) {
        dtFree(&data, errstr);
        freeFuncArgs(&fargs, errstr);
        pgArgsFree(&args, errstr);
        if (out != stdout) fclose(out);;
        fprintf(errout, "[struct stat=\"ERROR\", msg=\"%s\"]\n", errstr);
        fflush(errout);
        free(errstr);
        exit(1);
   }
}
char *argList, argHeader[MAXSTR];
if (pgArgsPrint(&args, argHeader, 1, &argList, errstr) != RET_OK) {
   dtFree(&data, errstr);
   freeFuncArgs(&fargs, errstr);
   pgArgsFree(&args, errstr);
   if (out != stdout) fclose(out);;
   fprintf(errout, "[struct stat=\"ERROR\", msg=\"%s\"]\n", errstr);
   fflush(errout);
   free(errstr);
   exit(1);
}
int nsamp = fargs.nsamp;
double *period = fargs.period;
double *power = fargs.power;
/* Print the output into a table structure (currently underpopulated) */
if (dtPrintResults(&data, out, nsamp,
                   "PERIOD", period,
                   "POWER", power,
                   argList, argHeader, errstr) != RET_OK) {
   dtFree(&data, errstr);
   freeFuncArgs(&fargs, errstr);
```

```
pgArgsFree(&args, errstr);
        if (out != stdout) fclose(out);;
        fprintf(errout, "[struct stat=\"ERROR\", msg=\"%s\"]\n", errstr);
        fflush(errout):
        free(errstr);
        exit(1);
    }
    /* show the args that were used for this run as output message: */
    fprintf(errout, "[struct stat=\"OK\", msg=\"%s\"]\n", argList);
    free(argList);
    /* cleanup */
    if (out != stdout) fclose(out);;
    freeFuncArgs(&fargs, errstr);
    dtFree(&data, errstr);
    pgArgsFree(&args, errstr);
    fflush(errout);
    free(errstr);
    exit(0);
}
#if DO_PROFILE
/* Functions for estimating the amount of time spent on each loop
 * in the calculations for the bls algorithm: there are loops
 * executed nsamp * ndata times, nsamp * nbins times, nsamp * nbins * qmax,
 * and nsamp * qmax * nbins * nbins times */
void blsL1(int b, double *bm, double *bw) {
    bm[b] = 0;
    bw[b] = 0;
}
void blsL2(int j, double *time, double p, double *wt,
           double *mag, int nb,
           double *bm, double *bw) {
    double phase = fmod(time[j], p)/p;
    int b = floor(nb * phase);
    bw[b] += wt[j];
    bm[b] += wt[j] * mag[j];
}
```

```
void blsL3(int b, int nb, double *bw, double *bm) {
    bm[b] = bm[b-nb];
    bw[b] = bw[b-nb];
}
void blsL4a(int k, double *bw, double *bm, int *bc, double *sw, double *sm) {
    (*bc)++;
    (*sw) += bw[k];
    (*sm) += bm[k];
}
#endif
/* computeLombScargle()
* Function to compute the Lomb-Scargle Periodogram for an input light curve
* ref: [Scargle, J.D., "Studies in Astronomical Time Series Analysis II.
        Statistical Aspects of Spectral Analysis of Unevenly Spaced Data."
        Astrophysical Journal 263:835-853 (1982)];
        http://adsabs.harvard.edu/full/1982ApJ...263..835S *\/
 * Periods are sampled according to the time period covered, or based on
* the input values of minperiod and maxperiod.
* The coefficients of the transform are selected so the statistical
 * distribution of powers for the unevenly spaced power spectrum is the
* same as that of the evenly spaced one.
 * At each period, a time offset is calculated to diagonalize the
* least-squares fit to sinusoids in the transform.
 * Power at period p is the magnitude of the transform at p.
* Arguments:
    DATA_TBL *data = populated DATA_TBL structure
 *
     PGRAM_ARGS *args = populated PGRAM_ARGS structure
 *
     PGRAM_FUNC_ARGS *args = PGRAM_FUNC_ARGS structure with
 *
                             ndata, time, mag, nsamp, and period set.
 *
 *
                             power will be populated by this function.
*
     char *errstr = holds error string, if any
*/
int computeLombScargle(struct DATA_TBL *data, struct PGRAM_ARGS *args,
                       struct PGRAM_FUNC_ARGS *fargs,
                       char *errstr) {
```

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```
/* check for input errors: */
   if (!errstr) return(RET_ERR);
   if (!data || !args || !fargs) NULL_ERROR(errstr);
   int ndata, nsamp;
   double *mag, *time, *period, *power;
   if ((funcArgsGetTime(fargs, &ndata, &time, errstr) != RET_OK) ||
       (funcArgsGetMag(fargs, &ndata, &mag, errstr) != RET_OK) ||
       (funcArgsGetPeriods(fargs, &nsamp, &period, errstr) != RET_OK) ||
       (funcArgsGetPower(fargs, &nsamp, &power, errstr) != RET_OK)) {
       sprintf(errstr, "%s (from %s)", errstr, __FUNCTION__);
       return(RET_ERR);
   }
   int i, j;
   /* Compute stats on magnitude: */
   double sdMag;
   if (dtGetDev(data, DATA_Y, &sdMag, errstr) != RET_OK) {
       return(RET_ERR);
   }
   if (sdMag == 0) {
       DUMP_RETURN(errstr, "InputFile: Zero deviation in data values");
   }
   /* Compute periodogram */
   double p; /* period */
   double w; /* angular freq at period p */
   double tnum, tdenom, t;
   double lnum, ldenom, rnum, rdenom;
   double s, c;
   double pi;
#if DEBUG
   /* reduced-precision pi for comparison to Peter's code */
   pi = 3.141592650000;
#else
   pi = M_PI;
#endif
#if 0
```

/\* to debug extremely slight differences between linux/solaris results:

```
* yes, the differences are in the 16th decimal place of the sines/cosines
     */
    printf("Pi: %.16f, sdmag: %.16f\n", pi, sdMag);
    for (j = 0; j < ndata; j++) {</pre>
        printf("time %d %.16f\n", j, time[j]);
        printf("sin: %.16f\n", sin(2.0*w*time[j]));
        printf("cos: %.16f\n", cos(2.0*w*time[j]));
    }
#endif
    for (i = 0; i < nsamp; i++) {</pre>
        p = period[i];
        /* angular frequency is 2*pi/p */
        w = 2.0*pi/p;
        /* identify time adjustment tau for this frequency: */
        tnum = 0;
        tdenom = 0;
        for (j = 0; j < ndata; j++) {</pre>
            tnum += sin(2.0*w*time[j]);
            tdenom += cos(2.0*w*time[j]);
        }
        t = (1/(2*w)) * atan2(tnum, tdenom);
        /* compute the coeffs at this frequency (using tau-adjusted day) */
        lnum = 0;
        1denom = 0;
        rnum = 0;
        rdenom = 0;
        for (j = 0; j < ndata; j++) {
            s = sin(w*(time[j]-t));
            c = cos(w*(time[j]-t));
            rnum += mag[j]*s;
            lnum += mag[j]*c;
            rdenom += s*s;
            ldenom += c*c;
        }
        /* compute the power at this frequency: */
        power[i] = (1/(2*(sdMag * sdMag))) *
            ((lnum * lnum)/ldenom + (rnum * rnum)/rdenom);
```

}

```
return(RET_OK);
}
/* computeBLS()
* Function to compute the BLS "periodogram"
* BLS = Box-fitting Least Squares
* ref: Kovacs, G., Zucker, S. and Mazeh, T. "A box-fitting algorithm
       in the search for periodic transits." A&A 391:369-377 (2002).
 * http://adsabs.harvard.edu/abs/2002A%26A...391..369K
* The BLS algorithm starts from the premise that for a specific fraction
* of the period of an orbiting planet, the planet will transit in front
 * of its star. This time during which the star's light is
* obstructed ranges from qmin to qmax, expressed as a fraction of the
 * total period.
* For each candidate period p, the number of bins (nbins) is considered
 * to span one period: each bin corresponds to a time span of p/nbins.
 * The observed data is "folded" to match the period: observations
* at time t = p + dt are placed into the bin corresponding to dt.
* A model in which the mean signal level in the occluded phase is L and
* the level in the un-occluded phase is H is considered for each
* candidate length of the L phase (qmin * nbins to qmax * nbins).
                                                                  The
 * least squares fit is given by maximizing s**2/(r*(1-r)) where
* s is the weighted sum of magnitudes in the low period and r
 * the sum of the weights in the low period.
* Arguments:
 *
    DATA_TBL *data = populated DATA_TBL structure
 *
    PGRAM_ARGS *args = populated PGRAM_ARGS structure
    PGRAM_FUNC_ARGS *args = PGRAM_FUNC_ARGS structure with
 *
 *
                            ndata, time, mag, nsamp, and period set.
                            power will be populated by this function.
    char *errstr = holds error string, if any
*
*/
int computeBLS(struct DATA_TBL *data, struct PGRAM_ARGS *args,
              struct PGRAM_FUNC_ARGS *fargs,
              char *errstr) {
   /* check for input errors: */
```

```
if (!errstr) return(RET_ERR);
   if (!data || !args || !fargs) NULL_ERROR(errstr);
   int ndata, nsamp;
   double *mag, *time, *period, *power;
   if ((funcArgsGetTime(fargs, &ndata, &time, errstr) != RET_OK) ||
       (funcArgsGetMag(fargs, &ndata, &mag, errstr) != RET_OK) ||
       (funcArgsGetPeriods(fargs, &nsamp, &period, errstr) != RET_OK) ||
       (funcArgsGetPower(fargs, &nsamp, &power, errstr) != RET_OK)) {
       sprintf(errstr, "%s (from %s)", errstr, __FUNCTION__);
       return(RET_ERR);
   }
   double *blsR, *blsS;
   int *lowBin0, *lowBin1;
   UTIL_CALLOC(blsR, nsamp, errstr);
   UTIL_CALLOC(blsS, nsamp, errstr);
   UTIL_CALLOC(lowBin0, nsamp, errstr);
   UTIL_CALLOC(lowBin1, nsamp, errstr);
   fargs->blsR = blsR;
   fargs->blsS = blsS;
   fargs->lowBin0 = lowBin0;
   fargs->lowBin1 = lowBin1;
   int i, j, k, b;
   double *wt;
#if 0 /* in case we want weight as a function of uncertainty: */
   double *err;
   if (dtGetFilteredArray(data, DATA_Y_UNCERTAINTY, &ndata, &err, errstr)
       == RET_ERR) {
       return(RET_ERR);
   }
#endif
   UTIL_CALLOC(wt, ndata, errstr);
   double totalWt = 0;
   for (j = 0; j < ndata; j++) {</pre>
       wt[j] = 1;
       //wt[j] = err[j];
       totalWt += wt[j];
   }
```

```
/* Suggested by Peter in email 10/6/09 after conversation with
* Gaspar Bakos */
GET_BLS_NBINS(ndata, args->nbins);
int nbins = args->nbins;
if (args->debugfp) {
   fprintf(args->debugfp, "IN COMPUTE BLS: nbins = %d\n", args->nbins);
}
double qmin = args->qmin;
double qmax = args->qmax;
if (qmin <= 0 || qmax <= 0 || qmax < qmin) {
   DUMP_RETURN(errstr, "PeriodRangeMin: invalid values for qmin/qmax");
}
int minBins = qmin * nbins;
if (minBins < 1) minBins = 1;
/* I don't love that this is fixed at "5" -- if we convert to
* weighting with errors, it'll have to change */
double minWt = totalWt * qmin;
if (minWt < 5) minWt = 5; /* min weight over "low" set of bins */
/* maximum number of bins over which a "low" phase can extend:
* (this is also the amount by which we want to pad the bin array) */
int binExt = qmax * nbins + 1;
int binMax = nbins + binExt;
double *binMag, *binWt;
UTIL_MALLOC(binMag, binMax, errstr);
UTIL_MALLOC(binWt, binMax, errstr);
/* Compute periodogram */
double p; /* period */
double maxPwr; /* max power found at this period */
double pwr; /* temporary power: to max over */
int binCt, lowStart, lowEnd;
double sumWt, sumMag;
double lowWt, lowMag, phase;
```

```
long count0 = 0, count1=0, count2=0;
#endif
    for (i = 0; i < nsamp; i++) {</pre>
#if DO_PROFILE
        count0++;
#endif
        p = period[i];
        for (b = 0; b < nbins; b++) {</pre>
#if DO_PROFILE
            blsL1(b, binMag, binWt);
#else
            binMag[b] = 0;
            binWt[b] = 0;
#endif
        }
        /* "nbins" represents one period p, so enter the weights and
         * weighted magnitudes for each data point into the bin
         * corresponding to time[j] */
        for (j = 0; j < ndata; j++) {</pre>
#if DO_PROFILE
            blsL2(j, time, p, wt, mag, nbins, binWt, binMag);
#else
            /* fraction of the period elapsed at time time[j] */
            phase = fmod(time[j], p)/p;
            /* bin corresponding to that phase */
            b = floor(nbins * phase);
            binWt[b] += wt[j];
            binMag[b] += wt[j] * mag[j];
#endif
        }
        /* continue the bin arrays to binMax -- we will refer to *
         * this extension of the period below */
        for (b = nbins; b < binMax; b++) {</pre>
#if DO_PROFILE
            blsL3(b, nbins, binWt, binMag);
#else
```

binWt[b] = binWt[b-nbins]; binMag[b] = binMag[b-nbins];

```
#endif
        }
        /* Search for the "low" phase [presumed transit time] that maximizes
         * pwr = (sumMag*sumMag)/(sumWt*(totalWt-sumWt)). The low phase
         * will cover some number of bins from "minBins" to "binExt"
         * (=qmax*nbins), so evaluate pwr for each candidate starting
         * bin b in 0 to nbins and each number of additional bins
         * from 0 to binExt */
        maxPwr = 0;
        for (b = 0; b < nbins; b++) {</pre>
            /* for each starting point in the base period, consider
             * whether this might be the beginning of the "low" phase: */
            binCt = 0;
            sumWt = 0;
            sumMag = 0;
#if DO_PROFILE
            count1++;
#endif
            for (k = b; k <= b + binExt; k++) {</pre>
#if DO_PROFILE
                blsL4a(k, binWt, binMag, &binCt, &sumWt, &sumMag);
                count2++;
#else
                binCt++;
                sumWt += binWt[k]; /* "r" in paper */
                sumMag += binMag[k]; /* "s" in paper */
#endif
                if ((binCt >= minBins)
                    && (sumWt >= minWt)
                    && (sumWt < totalWt)) {
                    pwr = (sumMag*sumMag)/(sumWt*(totalWt-sumWt));
                    if (pwr >= maxPwr) {
                        maxPwr = pwr;
                        lowStart = b; /* bin # at start of "low" phase */
                        lowEnd = k; /* bin # at end of "low" phase */
                        lowWt = sumWt; /* weight of this phase (r) */
                        lowMag = sumMag; /* mag in this phase (s) */
                    }
                }
            }
        }
        maxPwr = sqrt(maxPwr);
```

```
/* Save the results (if we were able to compute them) */
        if (maxPwr > 0) {
            power[i] = maxPwr;
            blsR[i] = lowWt/totalWt;
            blsS[i] = lowMag;
            lowBin0[i] = lowStart;
            lowBin1[i] = lowEnd;
        }
    }
#if DO_PROFILE
    printf("Count0: %ld, count1: %ld, count2: %ld\n", count0, count1, count2);
#endif
    free(wt);
    free(binWt);
    free(binMag);
   return(RET_OK);
}
/* computePlavchan()
* Function to compute periodogram based on Plavchan 2008 algo
* ref: Peter Plavchan, M. Jura, J. Davy Kirkpatrick, Roc M. Cutri,
        and S. C. Gallagher, "NEAR-INFRARED VARIABILITY IN THE 2MASS
        CALIBRATION FIELDS: A SEARCH FOR PLANETARY TRANSIT CANDIDATES."
        ApJS 175:191Y228 (2008)
* For each of a set of candidate periods, this algorithm folds a light
* curve to that period and then computes a "smoothed" curve by averaging
* the curve over a box spanning a certain phase range to either side (defined
* by the parameter "smooth"). The ratio of the sum of squared deviations
* from the mean (over the "nout" worst-fitting points) is divided by
* the sum of squared deviations from the smoothed values (again, over nout).
 * The smaller the deviation from the smoothed curve, the larger this ratio
* will be, indicating that the smooth curve is a substantially better fit
 * than the straight line "mag = mean mag". This ratio is interpreted as the
* "power" at that period.
* Arguments:
    DATA_TBL *data = populated DATA_TBL structure
 *
    PGRAM_ARGS *args = populated PGRAM_ARGS structure
```

```
*
    PGRAM_FUNC_ARGS *args = PGRAM_FUNC_ARGS structure with
                          ndata, time, mag, nsamp, and period set.
*
                          power will be populated by this function.
*
*
    char *errstr = holds error string, if any
*/
int computePlavchanParks(struct DATA_TBL *data,
                  struct PGRAM_ARGS *args,
                  struct PGRAM_FUNC_ARGS *fargs,
                  char *errstr) {
   /* check for input errors: */
   if (!errstr) return(RET_ERR);
   if (!data || !args || !fargs) NULL_ERROR(errstr);
   int ndata, nsamp;
   double *mag, *time, *period, *power;
   if ((funcArgsGetTime(fargs, &ndata, &time, errstr) != RET_OK) ||
       (funcArgsGetMag(fargs, &ndata, &mag, errstr) != RET_OK) ||
       (funcArgsGetPeriods(fargs, &nsamp, &period, errstr) != RET_OK) ||
       (funcArgsGetPower(fargs, &nsamp, &power, errstr) != RET_OK)) {
       sprintf(errstr, "%s (from %s)", errstr, __FUNCTION__);
       return(RET_ERR);
   }
   int i, j, count;
   int noutliers = args->nout;
   double errval = 0; /* negative sum of squares indicates error (but use
                      * 0 so weird things don't happen downstream!) */
   /* array to hold the deviation from the smoothed curve for each
    * data point: */
   double *tmpChi;
   if (funcArgsGetChi(fargs, &ndata, &tmpChi, errstr) != RET_OK) {
       return(RET_ERR);
   }
   /* make sure we don't have more outliers than we have data points: */
   if (noutliers > ndata) noutliers = ndata;
   /* determine reference deviations (recycle "tmpChi" array): */
   double meanMag;
   if (dtGetMean(data, DATA_Y, &meanMag, errstr) != RET_OK) {
       return(RET_ERR);
```

```
}
    for (j = 0; j < ndata; j++) {</pre>
        tmpChi[j] = (mag[j] - meanMag)*(mag[j] - meanMag);
    }
    qsort(tmpChi, ndata, sizeof(*tmpChi), compare_doubles);
    /* sum the values most _poorly_ fit by the model mag = meanMag */
    double maxChi, maxStd = 0; /* maxChi is the analogous var for each pd */
    for (j = ndata-1; j >= ndata - noutliers; j--) {
        maxStd += tmpChi[j];
    }
    maxStd /= noutliers;
    /* Compute periodogram */
    double *chisq = fargs->power; /* local name for "power" */
    /* periods have been determined in wrapper function, just loop: */
    for (i = 0; i < nsamp; i++) {</pre>
        fargs->p = period[i];
        if (phaseLightCurve(fargs, errstr) != RET_OK) return(RET_ERR);
#if DEBUG
        /* track the index associated with chi values for debugging: */
        int *idxArray;
        double **sortable = fargs->sortable;
        UTIL_CALLOC(idxArray, ndata, errstr);
        for (j = 0; j < ndata; j++) {</pre>
            sortable[j][0] = tmpChi[j];
            sortable[j][1] = j;
        }
        qsort(sortable, ndata, sizeof(*sortable), compare_pairs);
        for (j = 0; j < ndata; j++) {</pre>
            tmpChi[j] = sortable[j][0];
            idxArray[j] = sortable[j][1];
        }
#else
        /* Sort the chisq values and take the noutliers worst
         * (largest) of them */
        qsort(tmpChi, ndata, sizeof(*tmpChi), compare_doubles);
#endif
```

count = 0;

```
maxChi = 0;
       for (j = ndata-1; j >= 0; j--) {
          if (tmpChi[j] != errval) {
              maxChi += tmpChi[j];
              /*
              printf("chi[%d] = %f (sum = %f) %d\n", count, tmpChi[j],
                    maxChi, idxArray[j]);
              */
              count++;
              if (count >= noutliers) break;
          }
       }
       /* at this point count is the number of valid chisq values we found,
       * <= noutliers */</pre>
       maxChi /= count;
       /* save the values we'll ultimately return: */
       if (maxChi > 0) chisq[i] = maxStd/maxChi;
       else chisq[i] = errval;
       /*
        printf("at period %d, maxChi = %g, chisq = %f\n",
               i, maxChi, chisq[i]);
       */
   }
   return(RET_OK);
}
/* computePeriodogram()
* Wrapper routine to compute the periodogram based
* on the input data. Calls populateFuncArgs to do a lot of the messy
* stuff getting data into the format each algo wants it in.
*/
#define CHANGE_LOOP 0
int computePeriodogram(struct DATA_TBL *data, struct PGRAM_ARGS *args,
                    struct PGRAM_FUNC_ARGS *fargs,
                    char *errstr) {
   if (!errstr) return(RET_ERR);
   if (!data || !args || !fargs) NULL_ERROR(errstr);
```

```
#if DEBUG
    printf("%s\n", pgArgsPrint(args, 0));
#endif
    int nproc = args->numProc;
    int idx;
    char *argstr=NULL;
    char cmd[MAXSTR];
    if (pgArgsPrint(args, NULL, 1, &argstr, errstr) != RET_OK) {
        if (argstr) free(argstr);
        return(RET_ERR);
    }
    struct SV_QUEUE myq;
    memset((&myq), 0, sizeof(struct SV_QUEUE));
#if CHANGE_LOOP
    char fname[MAXSTR];
    int nToReturn;
    int statN;
    double statMean, statSd;
    int i, j;
    int ndata;
    double *mag, *smmag;
    FILE *fp;
    nToReturn = args->nphased;
    args->nphased = 1;
    for (i = 0; i < nToReturn; i++) {</pre>
#endif
        if (args->server && (args->numProc <= 1)) {
            /* send off entire command for remote processing IF
             * we have not specified a number of processors.
             * We may have, however, specified a configuration file
             * for splitting the job once it's run on the args->server.
             *
             * If we have specified a number of processors and a server
             * on which to run, spawn that number of jobs
             * from _this_ machine ("else" below) */
            if (args->port <= 0) {
                sprintf(errstr, "Invalid port number %d on server %s",
                        args->port, args->server);
```

```
if (argstr) free(argstr);
        return(RET_ERR);
    }
    sprintf(cmd, "periodogram ");
    if (args->serverconfig) {
        if (args->port) {
            sprintf(cmd, "%s -g %s -t %d", cmd, args->serverconfig,
                    args->port);
        }
        else {
            sprintf(cmd, "%s -g %s", cmd, args->serverconfig);
        }
    }
    sprintf(cmd, "%s %s", cmd, argstr);
    if (argstr) {
        free(argstr);
        argstr = NULL;
    }
    idx = svc_remote_init(args->server, args->port);
    if (idx < 0) {
        sprintf(errstr,
                "RemoteServer: cannot make connection to %s "
                "on port %d", args->server, args->port);
        return(RET_ERR);
    }
    svc_send(idx, cmd);
    /* read results into memory with a pseudo-call to mergeJobs: */
    int indexList[1];
    indexList[0] = idx;
    if (nproc == 0) nproc = 1;
    if (mergeJobs(args, fargs, nproc, indexList, &myq, errstr)
        != RET_OK) {
        return(RET_ERR);
    }
else {
    /* we are doing any splitting of processing on this machine */
    if ((!args->serverconfig && (args->numProc <= 1))</pre>
        || (fargs->timeEst < MIN_TO_MULTI_PROC)) {</pre>
        fprintf(stderr,
            "Estimated time for processing %d periods: %.4f seconds "
```

}

```
"(%.4f minutes)\n",
        fargs->nsamp, fargs->timeEst, fargs->timeEst/60.0);
    /* compute the periodogram (results will go into
     * fargs->power) : */
    int errcode;
    if (!strcmp(args->algo, "ls")) {
        errcode = computeLombScargle(data, args, fargs, errstr);
    }
    else if (!strcmp(args->algo, "bls")) {
        errcode = computeBLS(data, args, fargs, errstr);
    }
    else if (!strcmp(args->algo, "plav")) {
        errcode = computePlavchan(data, args, fargs, errstr);
    }
    if (errcode != RET_OK) {
        return(RET_ERR);
    }
}
else {
    if (args->serverconfig) {
        if (args->debugfp) {
            fprintf(args->debugfp,
                    "serverconfig: %s\n", args->serverconfig);
        }
        if (svQueueInit(&myq, args->serverconfig, errstr)
            != RET_OK) {
            return(RET_ERR);
        }
        nproc = myq.nproc;
    }
    int *indexList;
    UTIL_CALLOC(indexList, nproc, errstr);
    if ((splitJob(args, fargs, nproc, indexList, &myq,
                  errstr)
         != RET_OK) ||
        (mergeJobs(args, fargs, nproc, indexList,
                   &myq, errstr)
         != RET_OK)) {
        if (indexList) free(indexList);
        return(RET_ERR);
    }
```
```
if (args->serverconfig) {
                    svQueueFree(&myq, errstr);
                }
                if (indexList) free(indexList);
            }
        }
        /* Look for significant peaks and output phased light curves: */
        if (findPeaks(args, fargs, errstr) != RET_OK) return(RET_ERR);
#if CHANGE_LOOP
        if (i == 0) {
            statN = args->powN;
            statMean = args->powMean;
            statSd = args->powSd;
            /* wait -- will these automatically be
             * re-used in the next loop? */
        }
        /* subtract the phased curve and store in fargs->mag */
        if (phaseLightCurve(fargs, errstr) == RET_ERR) return(RET_ERR);
        sprintf(fname, "%s.phased.%d", args->inBase, i);
        if (outputPhasedCurve(fargs, NULL, fargs->p, fname, NULL, errstr)
            != RET_OK) {
            return(RET_ERR);
        }
        if ((funcArgsGetMag(fargs, &ndata, &mag, errstr) == RET_ERR) ||
            (funcArgsGetSmoothedMag(fargs, &ndata, &smmag, errstr)
             == RET_ERR)) {
            return(RET_ERR);
        }
        for (j = 0; j < ndata; j++) {</pre>
            mag[j] -= smmag[j];
        }
        /* write temp data file: */
        sprintf(fname, "%s.tbl.%d", args->inBase, i);
        fp = fopen(fname, "w");
        if (dtPrintResults(NULL, fp, fargs->nsamp, args->xcol, fargs->time,
                           args->ycol, mag, NULL, NULL, errstr) != RET_OK) {
            return(RET_ERR);
```

```
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```

```
}
fclose(fp);
}
args->nphased = nToReturn;
args->powN = statN;
args->powMean = statMean;
args->powSd = statSd;
#endif
if (argstr) free(argstr);
return(RET_OK);
}
```

## Starspot Model IDL Code

This appendix contains the *IDL* source code used to generate the parameteric starspot model. The programs *spot\_star.pro* and *findlobe.pro* rely heavily on programs that are a part of the MIRC reduction pipeline. For access to this reduction pipeline, the reader is directed to contact John Monnier, Associate Professor of Astronomy, University of Michigan at *monnier@umich.edu*.

The *spot\_star.pro* program is the main function used to compare observed interferometric observables with "observables" extracted from a user defined sythetic stellar surface.

```
;Name:
;spot_star
; Version 1
: PURPOSE:
    Creates a 2-D model of a spotted stellar surface.
;Interferometric observables are extracted from this model and
; compared to real interferometric observables at the same
;[u,v] points.
;CALLING SEQUENCE:
;result=spot_star(param,plot=plot,filename=filename,v_rdchi2=v_rdchi2,$
                     p_rdchi2=p_rdchi2,rdchi2=rdchi2,image=image,$
                     delmag=delmag)
;
: INPUTS:
              - 1-D array containing both stellar and starspot
;
   params
;
                properties.
                params[0] = stellar size (mas)
;
                params[1] = power limb darkening coefficient
;
                params[2] = array of covering factor (one per spot)
;
                params[3] = array of starspot latitudes (one per spot)
;
                params[4] = array of starspot longitudes (one per
;
;spot)
```

```
params[5] = array of starspot flux ratios (one per spot)
;
   filename - name of oifits file to compare with model
;OPTIONAL INPUTS:
   im_name - name of image file
;
; RETURNS :
   rdchi
             - reduced chi<sup>2</sup> between input model and observations
   v_rdchi2 - reduced visibility chi<sup>2</sup>
;
   t_rdchi2 - reduced triple amplitude chi^2
   p_rdchi2 - reduced closure phase chi<sup>2</sup>
;OPTIONAL KEYWORDS:
;OPTIONAL OUTPUTS:
   v_rdchi2 - variable containing reduced visibility chi<sup>2</sup>
;
   t_rdchi2 - variable containing reduced triple amplitude chi^2
;
   p_rdchi2 - variable containing reduced closure phase chi<sup>2</sup>
   image
            - creates fits file of model surface
   delmag
             - variable containing the difference in magnitude
:
                between model and unspotted star.
; COMMENTS :
; EXAMPLES :
  result=spot_star([2.77,0.24,0.15,35.0,-45.0,0.75],$
                    filename='myoifits.oifits',/image,im_name='myimage.fits')
;
; PROCEDURES CALLED:
  stellarsurf.pro
 extract_vis2data.pro
;
 extract_t3data.pro
  findlobe.pro
 extract_data.pro
  image_cont_uv.pro
;
:REVISION HISTORY:
  This version allows for the fitting of particular lobes
  6/4/09 Added foreshortening to the spots
  6/5/09 Changed limb darkening to power law instead of claret
  6/5/09 Phased wrapped the difference in obs to mod cp so correctly
 estimate chi-squared in closure phase
; 6/10/09 Changed plots to accommadate any number of data points
  6/11/09 Changed input from cartesian x,y to long and lat.
 7/6/09 Foreshorting has been correctly applied
```

```
7/15/09 Model surface computed by stellarsurf.pro
;
; 9/10/09 Added chi2 for t3
; 9/23/09 LD coefficient kept constant
; Made sure that spot size would never go negative
; 09/25/09 Added write keyword to create fits images
  Made sure that lrat is always positive
  10/03/09 Reads in oifits from file
  11/12/09 Accepts different number of input parameters automatically
; 05/12/10 Automatically returns the appropriate chi2 depending input
; parameters
  2010Aug13 Returns weighted chi2 based on number of points in vis, cp,
; and t3
; 2011Jan21 Added windows displaying modeled surface and comparison of
  observed data with model.
 2011Jan31 Changed the weighting scheme for reduced chi<sup>2</sup>
; 2011Feb09 Code will now only search on the first lobe when searching
; over size and alpha
  2012Apr20 Code can now model any number of spots
; 2012Apr27 Changed code so that it will not compute cp or t3 when
; searching for size and alpha
  2012Aug11 Flagged data is now omitted (vis2err eq -1)
function spot_star,param,plot=plot,filename=filename,$
                      v_rdchi2=v_rdchi2,p_rdchi2=p_rdchi2,rdchi2=rdchi2,$
                      image=image,im_name=im_name,delmag=delmag
!x.style=1
!y.style=1
parnum=n_elements(param)
size=param[0]
alpha=param[1]
n_spot=(parnum-2)/4
if (n_spot eq 0) then begin
   spot_size=0.01
   lat=90.0
   long=90.0
   lrat=1.0
endif else begin
   spot_size=dblarr(n_spot)
   lat=spot_size
   long=spot_size
   lrat=spot_size
```

```
for i=0,n_spot-1 do begin
   spot_size[i]=param[4*i+2]
   lat[i]=param[4*i+3]
   long[i]=param[4*i+4]
   lrat[i]=param[4*i+5]
endfor
ellip=0.0
posang=0.0
; for use on multiple oifits files
if (keyword_set(filename) eq 0) then begin
    get_lun,u
    openr,u,'inputoifits.txt'
    filename=''
    readf,u,filename
    free_lun,u
endif
print, filename
if (parnum le 2) then lobe=1
if (parnum gt 2) then lobe=3
;fend=80
fend=(1.22/(size*1e-3))*(206265/1e6)+5
;print,fend
if (fend lt 50.32) then fend=50.328659
;if (fend gt 300.00) then fend=300.00
send=150
tend=330
print, param
normflux=stellarsurf(size,alpha,ellip,posang,spot_size,lat,long,lrat,n_spot,image=image,
diam=n_elements(normflux(0,*))
scale=2*size/diam
                                 ;star needs to be half of total flux array
; convolving model with 0.4 mas psf
;psf=psf_gaussian(npixel=100,fwhm=7.22022,/double,/normalize)
;normflux=convolve(normflux,psf)
extract_vis2data,file=filename,vis2data
extract_t3data,file=filename,t3data
```

endelse

```
obsvis2=vis2data.vis2data
obsbas=vis2data.sfu/1e6
obsvis2err=vis2data.vis2err
obscp=t3data.t3phi
obscperr=t3data.t3phierr
obseff_wave=t3data.eff_wave
obst3=t3data.t3amp
obst3err=t3data.t3amperr
cp_ndata=n_elements(obscp)
t3_ndata=n_elements(obst3)
ophi=fltarr(cp_ndata) & mcp=ophi
ucoord=vis2data.u
vcoord=vis2data.v
orderfit=findlobe(filename,lobe=lobe,fend=fend,send=send,tend=tend)
obsbas=orderfit[0,*]
obsvis2=orderfit[1,*]
obsvis2err=orderfit[2,*]
ucoord=orderfit[3,*]
vcoord=orderfit[4,*]
vis_ndata=n_elements(obsvis2)
;ucoord=findgen(200)*2-200
;vcoord=findgen(200)*2-200
;for i=0,199 do begin
     for j=0,199 do extract_data,normflux,modvis,modphase,scale=scale,u=ucoord[i],v=vcoo
;endfor
;test for theta and alpha only
if (n_spot eq 0) then begin
   extract_data,normflux,modvis2,modphase,scale=scale,u=ucoord,v=vcoord,/nohan
   modvis2=modvis2^2
   a=where(obsvis2err gt 0,complement=b)
   obsvis2=obsvis2[a]
   modvis2=modvis2[a]
   obsvis2err=obsvis2err[a]
   dum=((obsvis2-modvis2)/obsvis2err)^2.
   v_chi2=total(dum)
   v_df=vis_ndata-parnum-2
   !p.multi=[0,2,1,0,0]
```

```
image_cont_uv,normflux,/aspect,/noco,xval=-1.*(findgen(100)-50)*size/50.,$
                 vval=1.*(findgen(100)-50)*size/50.,xtit='East (mas)',$
                 ytit='North (mas)'
   visdiff=sqrt(obsvis2)-sqrt(modvis2)
   ploterror,obsbas,visdiff,sqrt(obsvis2)*0.5*obsvis2err/obsvis2,psym=1,$
             xtitle='Baseline (megalambda)',$
             ytitle='Obs Vis - Mod Vis',$
             yrange=[min(visdiff)-0.05,max(visdiff)+0.05],xrange=[0,330],/nohat
   oplot, [0,330], [0,0], linestyle=1
   v_rdchi2=v_chi2/v_df
endif else begin
                                ;test for everything
   extract_data,normflux,modvis2,modphase,scale=scale,u=ucoord,v=vcoord,/nohan
   extract_data,normflux,modvis2_1,modphi1,scale=scale,u=t3data.u1,v=t3data.v1,/nohan
   extract_data,normflux,modvis2_2,modphi2,scale=scale,u=t3data.u2,v=t3data.v2,/nohan
   extract_data,normflux,modvis2_3,modphi3,scale=scale,u=t3data.u3,v=t3data.v3,/nohan
  modvis2=modvis2^2
;SFU vector for triple amplitude closure phase
   r_sfu=sqrt(t3data.u1*t3data.u1+t3data.v1*t3data.v1+$
              t3data.u2*t3data.u2+t3data.v2*t3data.v2+$
              t3data.u3*t3data.u3+t3data.v3*t3data.v3)
  modcp=modphi1+modphi2+modphi3
   for i=0,cp_ndata-1 do if modcp[i] gt 180 then modcp[i]=modcp[i]-360
   for i=0,cp_ndata-1 do if modcp[i] lt -180 then modcp[i]=modcp[i]+360
  modt3=modvis2_1*modvis2_2*modvis2_3
   dum=(angle_diff(obscp,modcp)/obscperr)^2.
  p_chi2=total(dum)
  dum=((obst3-modt3)/obst3err)^2.
   t_chi2=total(dum)
   c_df=cp_ndata-parnum-2
   t_df=t3_ndata-parnum-2
   p_rdchi2=p_chi2/c_df
   t_rdchi2=t_chi2/t_df
```

```
;writecol,'ucoord.txt',ucoord,fmt='(a)'
;writecol,'u1.txt',t3data.u1,fmt='(a)'
;make cp all positive
;obsphi=abs(obsphi)
;modcp=abs(modcp)
;modcp=modcp mod 360
;for i=0,63 do print,modphi1[i],modphi2[i],modphi3[i],modcp[i]
;plot section
;if (keyword_set(plot) eq 1) then create_plots,size,obsvis2,modvis2,obscp,modcp,$
; obst3,modt3,obsbas,obseff_wave,obsvis2err,obscperr,obst3err,/plot
  a=where(obsvis2err gt 0,complement=b)
  obsvis2=obsvis2[a]
  modvis2=modvis2[a]
  obsvis2err=obsvis2err[a]
  dum=((obsvis2-modvis2)/obsvis2err)^2.
  v_chi2=total(dum)
  v_df=vis_ndata-parnum-2
  v_rdchi2=v_chi2/v_df
  rdchi2=((v_rdchi2)^2+(p_rdchi2)^2+(t_rdchi2)^2)/(v_rdchi2+p_rdchi2+t_rdchi2)
;rdchi2=((vis_ndata*v_rdchi2)+(cp_ndata*p_rdchi2)+(t3_ndata*t_rdchi2))/$
        (vis_ndata+cp_ndata+t3_ndata)
;Plot windows
   !p.multi=[0,2,2,0,0]
  a=sqrt(n_elements(normflux))
  image_cont_uv,normflux,/aspect,/noco,xval=-1.*(findgen(a)-a/2.)*size/(a/2.),$
                 yval=1.*(findgen(a)-a/2.)*size/(a/2.),xtit='East (mas)',$
                 ytit='North (mas)'
  visdiff=sqrt(obsvis2)-sqrt(modvis2)
  ploterror,obsbas,visdiff,sqrt(obsvis2)*0.5*obsvis2err/obsvis2,psym=1,$
             xtitle='Baseline (megalambda)',$
             ytitle='Obs Vis - Mod Vis',$
             yrange=[min(visdiff)-0.05,max(visdiff)+0.05],xrange=[0,330],/nohat
  oplot, [0,330], [0,0], linestyle=1
```

```
plot, [1.3e8,3.5e8], [-200,200], /nodata,$
```

```
xtitle='Spatial Frequency (waves)',$
        ytitle='Closure Phase (deg)'
   oploterror,r_sfu,obscp,obscperr,/nohat,psym=2,color=200
   oplot,r_sfu,modcp,psym=4,color=100
   y_max=max([max(obst3),max(modt3)])
   plot, [1.3e8,3.5e8], [0,y_max], /nodata, $
        xtitle='Spatial Frequency (waves)',$
        ytitle='Triple Amplitude'
   oploterror,r_sfu,obst3,obst3err,/nohat,psym=2,color=200
   oplot,r_sfu,modt3,psym=4,color=100
endelse
if (keyword_set(plot) ne 0) then begin
   !p.font=0
   !x.style=1
   !y.style=1
   !x.thick=4
   !y.thick=4
   !p.thick=4
   !p.charsize=1.25
   !x.margin=[10,3]
   !y.margin=[5,1]
   !p.multi=0
   set_plot,'ps'
   device,filename='fit_plot.eps',/encapsulated,/color,/landscape,$
          bits_per_pixel=8
!p.multi=[0,2,2,0,0]
   image_cont_uv,normflux,/aspect,/noco,xval=-1.*(findgen(100)-50)*size/50.,$
                 yval=1.*(findgen(100)-50)*size/50.,xtit='East (mas)',$
                 ytit='North (mas)'
   visdiff=sqrt(obsvis2)-sqrt(modvis2)
   ploterror,obsbas,visdiff,sqrt(obsvis2)*0.5*obsvis2err/obsvis2,psym=1,$
             xtitle='Baseline (megalambda)',$
             ytitle='Obs Vis - Mod Vis',$
             yrange=[min(visdiff)-0.05,max(visdiff)+0.05],xrange=[0,330],/nohat
   oplot, [0,330], [0,0], linestyle=1
   plot, [1.3e8, 3.5e8], [-200, 200], /nodata, $
        xtitle='Spatial Frequency (waves)',$
```

```
ytitle='Closure Phase (deg)'
   oploterror,r_sfu,obscp,obscperr,/nohat,psym=2,color=200
   oplot,r_sfu,modcp,psym=4,color=100
   y_max=max([max(obst3),max(modt3)])
   plot,[1.3e8,3.5e8],[0,y_max],/nodata,$
        xtitle='Spatial Frequency (waves)',$
        ytitle='Triple Amplitude'
   oploterror,r_sfu,obst3,obst3err,/nohat,psym=2,color=200
   oplot,r_sfu,modt3,psym=4,color=100
   device,/close
   !p.font=-1
   !x.style=0
   !y.style=0
   !x.thick=1
   !y.thick=1
   !p.thick=1
   !p.charsize=1
   !p.multi=1
   set_plot,'x'
endif
!x.style=0
!y.style=0
if (n_spot eq 0) then begin
   print,v_rdchi2
   return,v_rdchi2
endif else begin
   print,rdchi2,v_rdchi2,t_rdchi2,p_rdchi2
   return, rdchi2
endelse
;jump:
end
```

The *stellarsurf.pro* program is the function that creates the sythetic stellar surface based on user defined inputs for stellar angular size, the power limb darkening coefficient, the starspot covering factor, the starspot latitude, the starspot longitude, and the starspot flux ratio. This program is able to create a surface with any number of starspots.

```
;Name:
;stellarsurf
; Version 2
; PURPOSE:
    Creates a 2-D sythetic stellar surface.
;CALLING SEQUENCE
;star=stellarsurf(size,alpha,ellip,posang,spot,lat,long,lrat,n_spot,$
                    cenx=cenx,ceny=ceny,delmag=delmag,scale=scale,$
                    image=image,covfac=covfac,im_name=im_name,small=small)
; INPUTS:
   size
             - stellar angular diameter (mas)
   alpha
             - power limb darkening coefficient
;
   ellip
              - ellipicity of the star
;
   posang
              - posang of minor axis
;
              - array containing the covering factors of each starspot
;
   spot
   lat
              - array containing the latitude for each starspot
;
              - array containing the longitude for each starspot
;
   long
              - array containing the flux ratio for each starspot
   lrat
;
              - number of starspots
   n_spot
;
              - scale of image (mas/pixel)
   scale
;
:OPTIONAL INPUTS:
   im_name
              - name of the fits file containing output image
; RETURNS
              - 2-D array containing the sythetic stellar surface
   star
;OPTIONAL KEYWORDS
   image
              - creates fits file containing sythetic stellar surface
;
              - output array is 50x50 pixels as opposed to 100x100
;
   small
                pixels
;
```

```
large
              - output array is 1000x1000 pixels as opposed to
;
                100x100 pixels
;
:OPTIONAL OUTPUTS
   cenx
              - photometric center of surface in the x direction
   ceny
              - photometric center of surface in the y direction
             - scalar containing the difference in magnitude
   delmag
;
                between spotted and unspotted star
 covfac
              - scalar containing the total covering factor of
                sythetic surface regardless of spot number
; COMMENTS :
; EXAMPLE CALL:
  star=stellarsurf(2.77,0.24,0.0,0.0,0.15,35.0,-45.0,0.75,1,scale=0.1,$
                    /image,im_name='myimage.fits',/small)
; PROCEDURES CALLED:
   writefits.pro
;REVISION HISTORY:
;09/09/09 Changed size of array to reduce computing time JRPIV
          Made the code flexible to change array size based solely on
          radius and s_radius
;11/12/09 Code now computes the covering factor
;03/01/10 Relative area of spot is now input parameter
;2011Jan26 Altered spot code so spots do interfere with each other.
;Commented out spot function since it interferes with two spots on top
; of each other.
;2012Feb28 Limb darkened spots
;2012Jun25 added small keyword
;2013Sep25 added large keyword
function stellarsurf,size,alpha,ellip,posang,spot,lat,long,lrat,n_spot,$
                       cenx=cenx,ceny=ceny,delmag=delmag,scale=scale,$
                       image=image,covfac=covfac,im_name=im_name,$
                       small=small,large=large
latitude=lat*!pi/180.0d
longitude=long*!pi/180.0d
if (keyword_set(small) eq 1) then radius=50 else radius=100. ; size of array
if (keyword_set(large) eq 1) then radius=1000 else radius=100.
s_radius=radius/2.
                   ;size of star in pixels
star_r=s_radius/2.
x_pos=(radius/2.0)+(star_r*cos(latitude)*sin(longitude))
y_pos=(radius/2.0)+(star_r*sin(latitude))
```

```
;print,x_pos,y_pos
x=findgen(radius)-radius/2.+0.01 & xprime=x
y=x & yprime=xprime/(1-ellip)
xp=findgen(radius) & yp=xp
flux=dblarr(radius,radius)
normflux=flux
count_arr=flux
beta=4.0
             ;spot structure coefficent
spot_size=sqrt(spot)*s_radius
for i=0,n_spot-1 do if (spot_size[i] lt 1.0) then spot_size[i]=1.0
sr=spot_size/2.
sp_angle=atan((y_pos-(radius/2.0))/(x_pos-(radius/2.0))) ;angle around star (in plane c
phispot=asin(2*sqrt((x_pos-(radius/2.0))^2+(y_pos-(radius/2.0))^2)/s_radius)
tcf=cos(phispot)
scale=size/s_radius
s count=0.0
for i=0, radius-1 do begin
    for j=0,radius-1 do begin
       r=sqrt(xprime[i]^2+yprime[j]^2) ;conversion to polar r
       phi=asin(2*r/s_radius)
                                         ;angle from center
; condition that flux outside star is zero and inside is limbdarkened
       if (r ge star_r) then begin
          flux[i,j]=0.0
       endif else begin
          flux[i,j]=(cos(phi))^alpha
          s_count=s_count+1
       endelse
    endfor
endfor
unsp=double(total(flux))
aff=0.0
for h=0,n_spot-1 do begin
   s_rad=fltarr(2,spot_size[h])
   for i=0, spot_size[h]-1 do begin
      for j=0,spot_size[h]-1 do begin
         xx=x_pos[h]+i-sr[h]
         yy=y_pos[h]+j-sr[h]
;angles within spot
         if (abs(y_pos[h]-(radius/2.)) lt star_r and abs(x_pos[h]-(radius/2.)) $
             lt sqrt(star_r^2-(y_pos[h]-(radius/2.))^2)) then begin
            p0=double(asin((y_pos[h]-(radius/2.))/star_r))
            t0=double(asin((x_pos[h]-(radius/2.))/sqrt(star_r^2-(y_pos[h]-(radius/2.))^2
            if (abs(yy-(radius/2.)) lt star_r and abs(xx-(radius/2.))$
```

```
lt (sqrt(star_r^2-(yy-(radius/2.))^2))) then begin
               if ((yy-(radius/2.))^2 lt star_r^2) then begin
                  p1=double(asin((yy-(radius/2.))/star_r))
                  t1=double(asin((xx-(radius/2.))/sqrt(star_r^2-(yy-(radius/2.))^2)))
                  if (sin(p0)*sin(p1)+cos(p0)*cos(p1)*cos(t1-t0) gt 1.0) $
                  then rad=star_r+1 else $
;position from center of spot
                     rad=star_r*acos(sin(p0)*sin(p1)+cos(p0)*cos(p1)*cos(t1-t0))
                  if (rad ge sr[h]) then begin ; condition for filling spot flux
                     flux[xx,yy]=flux[xx,yy]
                  endif else begin
                     if (count_arr[xx,yy] eq 0.0) then begin $
                                 ;flux[xx,yy]=(lrat[h]+(rad/sr[h])^beta*(1-lrat[h]))*flux
                        flux[xx,yy]=lrat[h]*flux[xx,yy]
                        count_arr[xx,yy]=1.0
                     endif else flux[xx,yy]=flux[xx,yy]
                     aff=aff+1
                  endelse
               endif
            endif
         endif
      endfor
   endfor
endfor
covfac=aff/s_count
spotted=double(total(flux))
delmag=-2.5*alog10(spotted/unsp)
flux=rot(flux,posang)
;this normalizes flux.
fluxtot=total(flux)
normflux=flux/fluxtot
;centroiding via center of mass
topx=0.0 & topy=0.0
dumx=0.0 & dumy=0.0
weightx=dblarr(radius)
for j=0,radius-1 do begin
   for i=0, radius-1 do begin
      dumx=normflux[i,j]*i
      topx=topx+dumx
```

```
endfor
   bottomx=total(normflux[*,j])
   if (bottomx eq 0.0) then bottomx=1e10
   weightx[j]=topx/bottomx
                                 ;print,topx,bottomx,weightx[j]
   topx=0.0
endfor
weighty=dblarr(radius)
for i=0,radius-1 do begin
   for j=0,radius-1 do begin
      dumy=normflux[i,j]*j
      topy=topy+dumy
   endfor
   bottomy=total(normflux[i,*])
   if (bottomy eq 0.0) then bottomy=1e10
   weighty[i]=topy/bottomy
                                 ;print,topy,bottomy,weighty[i]
   topy=0.0
endfor
;window,0
;plot,findgen(radius),weightx,linestyle=1
;oplot,findgen(radius),weighty,linestyle=2
cenx=total(weightx)/(radius/2.)
ceny=total(weighty)/(radius/2.)
; convert cenx and ceny into offsets from center in microns
cenx=(cenx-(radius/2.))*scale*1e3
ceny=(ceny-(radius/2.))*scale*1e3
if (keyword_set(image) eq 1) then begin
   window,1,title='Model Stellar Surface'
   image_cont_uv,normflux,/aspect,/noco
   writefits, im_name, normflux
endif
return, normflux
end
```

The *findlobe.pro* program is the function that allows the user to only look at the visibility data for a particular lobe. This requires reasonably accurate values for the first, second, and

third null positions in spatial frequency units. This program allowed for the stellar angular size and limb darkening coefficient to be solved independent of starspot parameter estimation by only modelling the visibility data on the first lobe.

```
;NAME
;findlobe
; Version 1
; PURPOSE:
     This program reorders visibility data so that only certain lobes
;can be selected for modeling
;CALLING SEQUENCE:
;result=findlobe(filename,lobe=lobe,fend=fend,send=send,tend=tend,plot=plot)
; INPUTS:
    filename
               - filename of the oifits file to be modeled
    lobe
               - visibility lobe to be modeled
;
               - the end of the first visibility lobe in spatial
    fend
                  frequency units (/10<sup>6</sup>)
               - the end of the second visibility lobe in spatial
   send
;
                  frequency units (/10<sup>6</sup>)
               - the end of the third visibility lobe in spatial
   tend
;
                  frequency units (/10<sup>6</sup>)
;
;OPTIONAL INPUTS
;
; RETURNS :
   result[0] - ordered observed baseline of selected visibility lobe
;
                  in spatial frequency units (/10<sup>6</sup>)
;
   result[1]
               - ordered observed visibilities of selected visibility
;
                  lobe
   result[2] - ordered observed visibility errors of selected
;
                  visibility lobe
;
   result[3]
               - ordered u coordinates of selected visibility lobe in
                  spatial frequency units
   result[4]
               - ordered v coordinates of selected visibility lobe in
;
                  spatial frequency units
;
;OPTIONAL KEYWORDS
```

```
plot
               - creates visibility versus observed baseline of
;
                 selected visibility lobe
;OPTIONAL OUTPUTS
; COMMENTS :
; EXAMPLE CALL:
   result=findlobe('myoifits.oifits',lobe=2,fend=50,send=120,tend=200)
; PROCEDURES CALLED:
; read_oidata.pro
  extract_vis2data.pro
  extract_t3data.pro
;REVISION HISTORY:
function findlobe, filename, lobe=lobe, fend=fend, send=send, tend=tend, plot=plot
read_oidata,filename,oiarray,oitarget,oiwavelength,oivis,oivis2,oit3
extract_vis2data,vis2data,oivis2=oivis2,oiwavelength=oiwavelength,oitarget=oitarget
extract_t3data,t3data,oit3=oit3,oiwavelength=oiwavelength,oitarget=oitarget
obsvis2=vis2data.vis2data
obsbas=vis2data.sfu/1e6
obsvis2err=vis2data.vis2err
obsphi=t3data.t3phi
obsphierr=t3data.t3phierr
eff_wave=t3data.eff_wave
obst3=t3data.t3amp
obst3err=t3data.t3amperr
ucoord=vis2data.u
vcoord=vis2data.v
points=n_elements(obsvis2)
spec=n_elements(obsphi)
;sort data
a=sort(obsbas)
obsbas=obsbas[a]
obsvis2=obsvis2[a]
obsvis2err=obsvis2err[a]
ucoord=ucoord[a]
vcoord=vcoord[a]
```

```
;for i=0,points-1 do begin
     result=min(obsbas,sub)
     obsbas[sub]=1e4
;
     order[0,i]=result
;
     order[1,i]=obsvis[sub]
     order[2,i]=obserr[sub]
     order[3,i]=ucoord[sub]
     order[4,i]=vcoord[sub]
;
;endfor
count=0
;fill arrays for only first lobe
if (lobe eq 1) then begin
   a=where(obsbas le fend)
   dum0=obsbas[a]
   dum1=obsvis2[a]
   dum2=obsvis2err[a]
   dum3=ucoord[a]
   dum4=vcoord[a]
endif
; if (lobe eq 1) then begin
     for j=0,points-1 do if (order[0,j] le fend) then count=count+1
;
     orderfit=dblarr(5,count)
     for j=0, points-1 do begin
;
         if (order[0,j] le fend) then begin
;
             orderfit[0,j]=order[0,j]
;
             orderfit[1,j]=order[1,j]
;
             orderfit[2,j]=order[2,j]
;
             orderfit[3,j]=order[3,j]
             orderfit[4,j]=order[4,j]
         endif
     endfor
;endif
;fill arrays for first & second lobes
if (lobe eq 2) then begin
   a=where(obsbas le send)
   dum0=obsbas[a]
   dum1=obsvis2[a]
   dum2=obsvis2err[a]
   dum3=ucoord[a]
   dum4=vcoord[a]
```

```
;fill arrays for all lobes
if (lobe eq 3) then begin
   a=where(obsbas le tend)
   dum0=obsbas[a]
   dum1=obsvis2[a]
   dum2=obsvis2err[a]
   dum3=ucoord[a]
   dum4=vcoord[a]
; if (lobe eq 2) then begin
     for j=0,points-1 do if (order[0,j] le send) then count=count+1
     orderfit=dblarr(5,count)
     for j=0,points-1 do begin
         if (order[0,j] le send) then begin
             orderfit[0,j]=order[0,j]
             orderfit[1,j]=order[1,j]
             orderfit[2,j]=order[2,j]
             orderfit[3,j]=order[3,j]
             orderfit[4,j]=order[4,j]
         endif
     endfor
; if (lobe eq 3) then begin
     for j=0,points-1 do if (order[0,j] le tend) then count=count+1
     orderfit=dblarr(5,count)
     for j=0,points-1 do begin
         if (order[0,j] le tend) then begin
             orderfit[0,j]=order[0,j]
             orderfit[1,j]=order[1,j]
             orderfit[2,j]=order[2,j]
             orderfit[3,j]=order[3,j]
             orderfit[4,j]=order[4,j]
```

endif

endif

;

;

;

;

;

;

;

;

;

:

;endif

endif

orderfit=dblarr(5,n\_elements(a))

endfor

orderfit[0,\*]=dum0 orderfit[1,\*]=dum1

;endif

```
orderfit[2,*]=dum2
orderfit[3,*]=dum3
orderfit[4,*]=dum4
if (keyword_set(plot) eq 1) then $
plot,orderfit[0,*],sqrt(orderfit[1,*]),xtitle='SFU (MegaLambda)',ytitle='Visibilities',p
return,orderfit
```

end