

Spring 5-16-2014

# Meta-Didactical Slippages: A Qualitative Case Study of Didactical Situations in a Ninth Grade Mathematics Classroom

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## ACCEPTANCE

This dissertation, META-DIDACTICAL SLIPPAGES: A QUALITATIVE CASE STUDY OF DIDACTICAL SITUATIONS IN A NINTH GRADE MATHEMATICS CLASSROOM, by NATHAN JOSEPH WISDOM, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree, Doctor of Philosophy, in the College of Education, Georgia State University.

The Dissertation Advisory Committee and the student's Department Chairperson, as representatives of the faculty, certify that this dissertation has met all standards of excellence and scholarship as determined by the faculty. The Dean of the College of Education concurs.

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Nathan J. Wisdom

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### PROFESSIONAL SOCIETIES AND ORGANIZATIONS

20011-Present	American Educational Research Association
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ABSTRACT  
META-DIDACTICAL SLIPPAGES: A QUALITATIVE CASE STUDY OF  
DIDACTICAL SITUATIONS IN A NINTH GRADE  
MATHEMATICS CLASSROOM

by  
Nathan J. Wisdom

Research on the mathematical behavior of children over the past forty decades has considerably renewed and augmented the body of evaluative tests of the results of learning (Lester, 2007). Research however, has provided very little knowledge about the means of improving students' performance on these tests. Nevertheless teachers, students, and others are being pressured to improve students' performance, but in order to concentrate on basic skills, the learning itself is made more difficult and slower. The combination of requirements has led to a variety of uncontrolled phenomena such as meta-didactical slippage (Brousseau, 2008).

The purpose of this study was to: (a) understand the nature of meta-didactical slippage that occurred in a ninth grade predominantly African American mathematics classroom; and (b) describe how these meta-didactical slippages affect students conceptual understanding on a unit of study of ninth grade mathematics. The study was a descriptive, qualitative, case study that employed ethnographic techniques of data collection and analysis. The theory of didactical situations in mathematics (Brousseau, 1997) served as the theoretical lens that grounded the interpretation of the data, because it enabled the researcher to isolate moments of instruction, action, formulation, validation, and institutionalization in the mathematics teaching and learning process. The study was conducted over a period of 15 weeks in one, ninth grade class of 23 predominantly African American students at a high school in a southeastern state. Data was crystalized using multiple data collection techniques: (a) collection of document artifacts, which

included student work samples and teacher lesson plans; (b) interviews conducted with the teacher; (c) researcher introspection; and (d) direct observation. Data was analyzed using ethnographic and discourse analysis techniques, including domain analysis, coding, situated meaning, and the big “D” discourse tool. The study found four themes, which illustrated the nature meta-didactical slippages: (a) over-teaching, (b) situational bypass, (c) language and symbolic representation, and (d) the design of didactical situations.

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DIDACTICAL SITUATIONS IN A ninth GRADE  
MATHEMATICS CLASSROOM

by  
Nathan J. Wisdom

A Dissertation

Presented in Partial Fulfillment of Requirements for the  
Degree of  
Doctor of Philosophy  
in  
Teaching and Learning  
in  
the Department of Middle and Secondary Education  
in  
the College of Education  
Georgia State University

Atlanta, GA  
2014



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## ACKNOWLEDGMENTS

I would first like to thank God, whose grace and mercy brought me this far. I do not have words to express my gratitude. Many times my strength failed but He reminded me that He is strong.

Secondly I want to thank my wife Tonya, who stood by me through the most challenging and difficult times. She endured my frequent absence from home, for even when I was home I was not home. She watched as I left early in the mornings, and returned late at nights. I thank her for believing in me and for loving me. I also want to express gratitude to my four children, Jeremiah, Sarah, Christen, and Christina. They have waited long for their dad to come out and play. They have waited long for their dad to come home. Thank you Christen and Christina for decorating my books, so the words are easier for me to find.

I must say that I had the best dissertation committee in the world. I want to acknowledge and express gratitude to my committee, Dr. Chahine, Dr. Fournillier, Dr. Thomas, and Dr. Vidakovic. Each of you has contributed in your own way to my growth and development as a scholar. Thank you Dr. Chahine, for instilling in me the value of hard work, and dedication. You have challenged me to persist and persevere, to go the extra mile. I thank you for introducing me to the works of Guy Brousseau, which inspired me to think differently about the teaching and learning of mathematics. My greatest debt is to Dr. Fournillier, who taught me the value of “reading and re-reading”, and pushed me to “think small and dig deep”. She introduced me to the world of qualitative research and inspired to broaden my perspective of the world. Dr. Fournillier guided me (since the first day we met), as an instructor, a councilor, and a friend. To Dr. Thomas, I express gratitude. Since my first semester Dr. Thomas has been there for me. She insisted that I immerse in the literature, and endeavor to do my very best. Dr. Thomas, always have a kind word, she knows when I needed a scaffold to stand on, and she was always available to point me in the right direction. She afforded me the opportunity to grow as a scholar and helped me to learn the “deep magic” of academia. I express gratitude to Dr. Vidakovic who helped me to think deeply about the mathematics, and to apply the knowledge of research to the practice of teaching. Borrowing from Dr. Fournillier, I can truly say, “I am who I am because of who you are”.

I express my deepest gratitude to the large list of friends, professors, colleagues and family members, who kept me going. To Dr. Stinson who told me, “No one earns a Ph.D. alone!” To Kori and Patrice who paid the price of being a friend. To my colleagues, Cristy, Owen, and Rohan who kept asking, “how is it going Wisdom?” To the many classmates, friends and family member I say thank you.

Finally, I am in debt to the participating teacher, and the 23, ninth grade students who allowed me into their space. They have accepted me as a member of their community. Without them this project could not be completed.

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## ABBREVIATIONS

CCSSM	Common Core State Standard for Mathematics
COREM	<i>Centre d'Observation et de Recherches sur l'Enseignement des Mathématiques</i> (Center for Observation and Research on the Teaching of Mathematics)
IREM	<i>Institutets de Recherche sur l'Enseignement des Mathematiques</i> (Institute for Research on the Teaching of Mathematics)
NCTM	National Council of Teachers of Mathematics
TDSM	Theory of Didactical Situations in Mathematics

## CHAPTER 1

### INTRODUCTION

This dissertation is a report of a qualitative case study of meta-didactical slippages that occurred in one, ninth grade mathematics classroom. The study was based primarily on the direct observation of the interaction between the teacher and students as they engaged in the teaching and learning process. This first chapter of the dissertation presents the background of the study, specifies the problem of the study, presents the research questions that guided the study, and describes its significance. The chapter concludes by presenting an overview of the theoretical framework, and defining special terms used.

#### **Background of The Study**

Mathematics teaching is a complex practice, because teachers have to balance multiple goals and constraints as they decide “how to respond to students’ questions, how to represent a given mathematical idea, how long to pursue discussion of a problem, or how to make use of available technologies to develop the richness of an investigation” (Martin & Herrera, 2007, p. 18). Mathematics teachers are also responsible for developing students’ mathematical reasoning skills. Mathematical reasoning or learning occurs within a context that is determined by a set of implicit and explicit rules, circumstances, and interactions among several systems such as the teacher system, the student system, and the milieu (Brousseau, 1997). Despite these complexities, a significant responsibility is placed on the teacher to ensure that students are able to do mathematics. Thus mathematics teachers have to create meaningful didactical situations, in order to facilitate the process of doing mathematics. Nevertheless, it is in the didactical



situations that complexities and inherent difficulties of the teaching and learning process occur.

According to National Council of Teachers of Mathematics (NCTM) (Mathematics, 2000), “a significant challenge to realizing the vision portrayed in *Principles and Standards* is disengagement” (p. 371). Moreover, disengagement is often reinforced in both overt and subtle ways by the attitudes and actions of adults who have influence over students. For instance, in a study on the influence of classroom practice on the development of subject-matter understanding, Schoenfeld (1988) argued that “despite the fact that the class was well taught, and the students did well on relevant performance measures, the students learned some inappropriate and counterproductive conceptions about the nature of mathematics, as a direct result of the instruction” (p. 146). Research carried out on the teaching of rational numbers during the period 1970-1980, uncovered several phenomena connected with the teaching and learning of mathematics (Brousseau, 1997). These phenomena which occur from the interplay of relationships and constraints between the teacher, students, and mathematical content may produce certain unwanted effects (e.g. the *Topaze* effect, the *Jourdain* effect, the Metacognitive shift, the aging of teaching situations, the improper use of analogy, and the meta-didactical slippage. A definition of these terms is provided in an upcoming section) (Brousseau, 1997, 2008; Brousseau, Brousseau, & Warfield, 2009). Although these effects are inappropriate for the learning, they are often inevitable, and sometimes unknown (Brousseau, 2008; Schoenfeld, 1988).

Over the past 15 years, I observed mathematics classes taught in three different countries by both veteran and novice teachers. I also reflected on my own teaching and found several instances of these unwanted effects in my mathematics classroom.

Moreover, a number of studies reported that students in mathematics classrooms are not engaged in doing mathematics (Attard, 2012; Brown, 2009; Hiebert, 2003; Mathematics, 2000). They are instead disengaged, which often leads to disruption, and failure. Hence, I agree with the vast call for reform, that there is a need for change in how, and what mathematics is taught. Closest to my heart is the issue of achieving a balance between conceptual understanding, and procedural fluency, because according to Hiebert (2003) “Well-designed and implemented instructional programs can facilitate both conceptual understanding and procedural skills” (p. 16). In this study, I argue that in order to improve students’ performance in problem solving situations, we need a better understanding of the didactical situations in the mathematics classroom.

### **Problem Statement**

In a longitudinal, experimental study of the teaching of rational and decimal numbers, Brousseau et al. (2009) identified several phenomena that manifest during the teaching and learning sequences. One of these phenomena is the effect of *meta-didactical slippage*. According to Brousseau et al. (2009) a meta-didactical slippage is the replacement of a situation, by one of its meta-situation. In other words, the teacher teaches some alternate objective rather than the objective that was initially presented to the students. This phenomenon occurs primarily because teachers have the tendency to take all mathematics activity as an object of study and of teaching. This often leads them to intervene, and “replace an initial mathematical situation that would have permitted an authentic activity on the part of the student, by a study of the mathematical circumstances, and a lesson about that” (Brousseau et al., 2009, p. 113). The nature of these slippages is unknown, and its manifestation in the mathematics classroom is often undetected. For instance, in pointing out the difficulties associated with implementing

high-level tasks, Henningsen and Stein (1997) explained that these situations evoke in students a “desire for a reduction in task complexity that, in turn, can lead them to pressure teachers to further specify the procedures for completing the task” (p. 526). Additionally, Brousseau (2008) point out that it is possible to observe meta- didactical slippages that concern a whole society and extend uncorrected through many years. These slippages not only rob students of the opportunity to learn conceptual understanding of the content, but also limit students’ engagement in the mathematics classroom.

Student engagement has been problematized in school meetings, professional development meetings, and research reports. For example, Sowder and Schappelle (2002) provided summary reports of several researches, conducted on a variety of educational problems. These reports consisted of research related to teaching, learning, curriculum, assessment, and technology. In each category of research, there were important lessons to be learned that can inform the teaching of the classroom teacher, but they all report a lack of student engagement in the mathematics classroom. In Sowder, and Schappelle’s (2000) research synthesis, the persistent, and common theme of student engagement, essentially stated that students in mathematics classrooms are not engaged, was too strong to ignore. Student engagement however was not explicitly studied, and most of the claims about student engagement were made in the discussion, or implication sections of the reports. Additionally, the claims about student engagement in mathematics were made on the basis of classroom observations. Classroom observations, according to (Brousseau, 2008) reveled disastrous consequences, one of which is meta-didactical slippage. Moreover, the process through which stakeholders enact these consequences is recursive: failures provoke the proliferation of standardize testing; and the reinforcement

of inadequate teaching methods (Brousseau, 2008). Thus, in the absence of sufficient knowledge about the process of teaching, meta-didactical slippages occur even though they produce effects contrary to the didactical objective. Research on meta-didactical slippages is very sparse. This paucity of research should be addressed if we are to understand the impact of meta-didactical slippages in the mathematics classroom. Thus, the purpose of this case study is to: (a) understand the nature of meta-didactical slippages that occurred in a ninth grade predominantly African American mathematics classrooms; and (b) describe the affects of theses meta-didactical slippage on a unit of study of ninth grade mathematics.

### **Research Questions**

The following questions guided the study:

1. What is the nature of meta-didactical slippages that emerge in the practice of teaching mathematics?
2. In what ways do these slippages affect students' conceptual understanding of a unit of ninth grade mathematics?

### **Significance**

This study has theoretical and practical significance. First, this study used a theoretical framework (see next section) that is rarely used in the mathematics education research literature. English (2002) attributed this paucity to the fact that researchers using the theoretical framework are predominantly from France or Spain. Consequently, the bulk of the literature is not in English. This study explicates the theory of didactical situation in mathematics and provides an example of how it may be applied to increase understanding of didactical situations. Research showed that understanding didactical situations in the mathematics classroom is valuable to both mathematics teachers and

mathematics education researchers (Brousseau et al., 2009; Bussi, 2005; Schoenfeld, 1988; Sowder & Schappelle, 2002). Schoenfeld (2012) described didactical situations as situations that support student engagement with rich mathematics. Additionally, when students engage in these rich mathematical and pedagogical situations, they develop deep understanding of the mathematics (Schoenfeld, 2012). Therefore, this study adds to the body of theoretical literature on the didactics of mathematics. Furthermore, findings from this study increased our understanding of the phenomenon of meta-didactical slippages that occurred in the mathematics classroom.

Secondly, this study was conducted in a “real-life” mathematics classroom. The majority of classroom research analyzed test scores, as the major source of data. This study used focus ethnographic observations, which incorporated videotaping. This allowed a more fine-grained analysis of a very complex and nuanced setting using classroom situations as the unit of analysis. Thus, this study adds to the methodological literature on didactical situations in mathematics classroom.

The findings from this study are also significant to practitioners, because it helps to heighten mathematics teachers’ awareness of the phenomenon of meta-didactical slippages in the mathematics classroom. This increased awareness of the phenomenon can positively influence teachers’ didactic decisions as they plan and implement mathematical lessons.

Finally, this study focused on the nature of meta-didactical slippages. Therefore, special focus was on the genesis, the affordances, and the characteristics of the phenomenon. This type of analysis helped to illuminate the phenomenon so that practitioners can be better prepared to deal with the effects of meta-didactical slippages in the mathematics classroom. It has been argued that the phenomenon occurs primarily

because teachers have the tendency to take all mathematics activities as an object of teaching, which often leads them to intervene and “replace an initial mathematical situation that would have permitted an authentic activity on the part of the student, by a study of the mathematical circumstances” (Brousseau et al., 2009, p. 113). Ultimately, this study will help to improve the teaching and learning of school mathematics.

### **Theoretical Framework**

This study draws upon Guy Brousseau’s *theory of didactical situations in mathematics* (TDSM). The TDSM framework includes specific grammar with specific meanings, for terms such as *didactical situation*, *adidactical situation*, *didactical contract*, *milieu*, and *didactical transposition* (Brousseau, 1997).

Brousseau (1997) argued that TDSM assumes that the way in which an individual progresses from using natural thought to using logical thought, which is associated with mathematical reasoning, is accompanied by construction, rejection, and the use of different methods of proof. Methods of proof could be rhetorical, pragmatic, semantic, and syntactic. Furthermore, providing the child a chance to discover errors is necessary for the construction of knowledge. Drawing from Piaget’s ideas of knowledge construction, Brousseau (1997) adds that knowledge is constructed through involvement with the milieu, particularly after the start of schooling. Constructing mathematics is primarily a social activity and not an individual one. In this regard, Sriraman and English (2010) argue that TDSM espouse a social constructionist epistemology.

### **Components of TDSM**

TDSM is comprised of five major “situations”: situation of instruction, situation of action, situation of formulation, situation of validation, and situation of institutionalization. Each situation determines different types of knowledge, such as

implicit models, languages, theorems, and proofs. Before discussing each of these components, it is important to understand how these components operate together to achieve learning, in a fundamental situation.

To describe a fundamental teaching situation, Brousseau (1997) used the concept of a game which is specific to the target knowledge among different subsystems, such as the student system, the educational system and the milieu. The particular game used is so that the knowledge to be learned must appear as the solution to the problem or as the winning strategy. Moreover, the game must be designed to allow for multiple representations, and must provide a means for students to learn some form of the target knowledge. According to Brousseau (1997) didactics must allow for the construction of meanings to these multiple representations (or strategies) and for the explanation and prediction of the effects of these meanings on the type of learning that they allow the student to acquire.

The notion of a game is commonly used, for example in economics, to model situations in which intelligent individuals interact with one another in an effort to achieve their own goals (Rabin, 1993). Often times an individual's goal for playing is pleasure seeking. Thus playing the game provides pleasure for the player. According to Brousseau (1997) five definitions are required in order to model the notion of a situation with that of a game (see Table 1).

Table 1

*Definitions of the Notion of a Game*

Definition Number	Definition Statement
1	The entirely free physical or mental activity generally based on conventions or fiction, which in the mind of the one who perform this activity has no other purpose than “itself”, no other goal than the pleasure it provides.
2	The game is the organization of this activity within a system of rules defining a success and a failure, a gain and a loss
3	The instrument of the game, or whatever is used for playing the game, and occasionally one of the states of the game, determined by a particular setting of the instrument
4	The way that one plays, “the play”. Tactics or strategies for cases where procedure is involved.
5	The set of positions from which the player can choose, in a given state of the game (following definition 2)

*Note.* Adopted from “Theory of Didactical Situations in Mathematics” by G. Brousseau, 1997, pp. 48-49.

The first definition essentially presents a person that is capable of taking pleasure in doing a real world activity. The decision to participate in a mathematical activity is motivated by pleasure, which is very problematic in the context of ninth grade mathematics, because mathematics is a compulsory subject. Nevertheless, knowing mathematics can be personally satisfying and empowering because everyday life is increasingly mathematical and technological (Mathematics, 2000). Furthermore, according to Mathematics (2000) mathematics is a part of cultural heritage, and that there is a great need to understand and be able to use mathematics in everyday life and in the workplace.



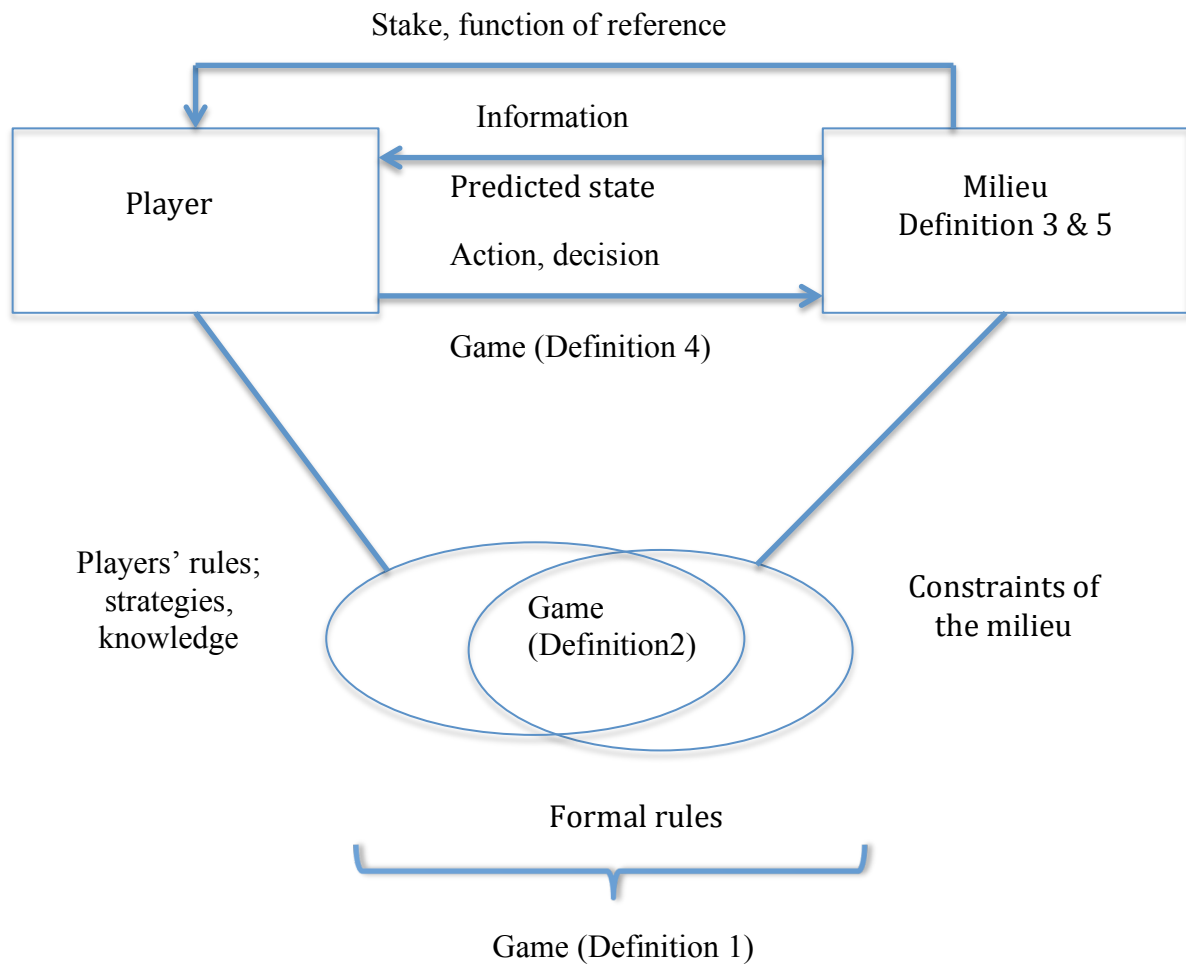


Figure 1. The relationships among the different definitions. Adopted from (Brousseau, 1997, p. 49)

It is worth noting that the different components of TDSM are best understood by examining the relationships among the different definitions (see Figure 1). The first component is a situation of *instruction* (Brousseau, 1997). Using the game metaphor, the initial entry to the game is done by the teacher giving instruction on the rules of the game. In Figure 1, this instruction phase would be at Definition 2. For example in the game of chess, the teacher instructs the class on the different pieces of the game, the rules for moving the pieces, and the object of the game. That is to say, how do you know when the game is won or is lost. This stage should not contain any new words, nor any new

knowledge (Brousseau, 1997). It is assumed that the students understand the terms used in this phase.

According to TDSM, teachers must transpose the authentic practice of mathematicians into classroom contexts (Definitions 2-5). This process is called *didactical transposition* (Brousseau, 1997). In the classroom setting, it is assumed that the students will acquire mathematical knowledge and skills, which they will in turn use to participate in doing mathematics. Acquired mathematical knowledge can appear in many different forms (D. Tirosh, 1999). Knowing mathematics is not simply learning definitions and theorems in order to recognize when to use them and apply them. An authentic reproduction of a mathematical object by the student would require the student to produce, formulate and prove new mathematical ideas. An authentic reproduction also requires the student to construct models, languages, concepts, and theories. In order to facilitate this activity, the teacher must plan and present to the students situations within which they can live, and within which the knowledge will appear as the optimal and discoverable solution to the problem posed. These simulated situations are called *didactical situations* (Brousseau, 1997).

Didactical situations in TDSM are representations of a real world situation, which have no teaching context or intension. The didactical situation is contrived by the teacher in order for the students to gain the knowledge and skills necessary to solve a similar problem in its real life context. It is the real world or adidactical situation that characterizes the knowledge at stake, thus the teacher must always help the student to “strip the situations of all the didactical artifices, as quickly as possible so as to leave her with personal and objective knowledge” (p. 31).

The next component of TDSM is a situation of *action*. In this phase the student begins to play. When the opponent plays, the student must analyze the situation and draw information from it, which then leads to decision-making and action. After each move, the situation is modified, and the student must continue to learn from the situation. The space in which this learning occurs is called the milieu. In TDSM, the *milieu* is everything that acts on the student, or everything that the student acts on within a situation of action. It is important to note that the teacher is a part of the milieu.

In the milieu, there are implicit rules that determine what each partner, the teacher and or the student will be responsible for managing. There are also implicit rules that determine what responsibility each partner has to the other person. This system of reciprocal obligation is called a *didactical contract*. This contract is not a real contract; it has never been contracted in any form between the teacher and the student. However, according to TDSM the breaking of this contract has serious consequences. The issue of the didactical contract will be addressed in an upcoming section.

The third component of TDSM is a situation of *formulation*. In order to win a game such as chess, it is not sufficient to just know the rules of the game. The student must begin to anticipate oppositions or problems. They must be conscious of the set of strategies that he or she can and would use. Consequently, the student must formulate these strategies and apply them within the game. These strategies, if validated, will become part of the student's repertoire of strategies. According to Brousseau (1997) the student is subject to two types of feedback: an immediate feedback at the time of formulation from the teacher or other students in the class (who show that they do or do not understand the suggestion) and a feedback from the milieu at the time of the next round of play to determine whether the formulated strategy was a winning strategy or not.

A fourth component is a situation of *validation*. After formulating a set of strategies, the reasons that the students give to convince another student, or the teacher “must be drawn out progressively, constructed, tested, formulated, discussed and agreed upon” (Brousseau, 1997, p. 15). Furthermore, doing mathematics involves more than simply receiving, learning, and sending correct mathematical messages. It is not just getting a right answer to a problem. Therefore the child should not only know mathematics, but must also be able to use mathematics as a reason for accepting or rejecting a proposition, a theorem, a strategy, or a model. This activity requires an attitude of proof. According to TDSM, this attitude is not innate, and therefore it must be developed and sustained by particular didactical situations. Table 2 summarized the major observed behaviors in the different situations. Table 2 also provides researchers and mathematics teachers an instrument to classify observed behaviors in the classroom, according to the type of knowing that is manifested and the specific situation in which the behavior occurs.

The final component is a situation of *institutionalization*. In TDSM institutionalization is the process by which a social knowledge become persistent, and exist as cultural facts. In institutionalization, the teacher selects an assign status to those parts of the learning that has been validated, and or valued. The teacher then ensures that the students practice these skills so that the knowledge becomes a permanent part of the students’ culture.

Table 2

*Observed Behaviors Classified by type of Situation and Knowings*

Types of knowings	Types of Situations			
	Situation of Action	Situation of formulation	Situation of validation	Situation of institutionalization
Procedure	Know-how. Implement the procedure; choose it in preference of another	Detailed descriptions Designation	Justification of the relevant procedure which is adequate correct and optimal	Canonization of the procedure and algorithm) drill and practice)
Implicit model	Make choices		Contingent proof;	
Property	make decisions motivated by the related knowing without being able to formulate it		Experimental proof;	
Relation			Proofs by exhaustion	
Representation				
Knowings	Apply a knowing; the knowing could be formulated	Statement of the property or of the relationship. More correct formulation	Proofs; Mathematical proofs; More convincing translation; Organization; Axiomatization	Canonization of a theory, of a knowing; Didactical transposition
Statement				
Theory				
Language	Use language for explaining. Behavior shows a division into objects corresponding to signs and words	Use of a language of a formal system, of a formation for communicating, speaking know-how	Justification of a word of a language, of a formal model (relevance, adequacy, optimization) definitions; Metalinguistic activities	Choice of definitions, linguistic and grammatical conventions

*Note.* Adopted from “Theory of Didactical Situations in Mathematics” by G. Brousseau, 1997, p. 216.

### **Definition of Terms**

The following definitions were adopted from (Brousseau, 1997) and are applicable to this study:

**Institutionalization.** This term refers to the teacher defined set of allowable relationships, between the students' mathematical construction, the scientific knowledge, and the didactical project. In other words the teacher gives status to the students' mathematical productions, and ensures that the students practice these skills.

**Adidactical situations.** Adidactical situations are situations that allow the student to use mathematical knowledge outside of a teaching context. These are social, or cultural problems that exist in a real live context, devoid of any teaching and learning intentions.

**Didactical situations.** Didactical situations are mathematical tasks that the teacher contrives in order to model real life situations. The target knowledge is represented as a solution to this task. The teacher usually instructs the students using a variety of techniques, and usually controls the situation.

**Knowings.** These include individual intellectual cognitive, often unconscious, constructs.

**Connaissance.** This term refers to socially shared and recognized cognitive constructs, which must be made explicit (knowledge).

**Savoir.** A collection of knowings required to interpret and recognized connaissances. These knowings could be described as 'knowing how', in the sense that when you *know how* to solve a problem, you can *know* the solution to the problem.

**Meta-didactical slippage.** Whenever the teacher takes a means of teaching as a new object to be taught; either the whole situation, part of the situation, or the resolution of the situation (Brousseau, 2008).

**Perceived failure.** When the students explicitly expressed disagreement with, or lack of understanding of the target objective of the lesson (can also be manifested by incorrect solution, or justification to problems in student work)

**Milieu.** The middle space, or entire learning environment including the didactical situations, that sustains the teaching and learning process.

## CHAPTER 2

### LITERATURE REVIEW

This review provides an overview and synthesis of the research relating to didactical situations in the mathematics classroom. The review further explicates the didactical contract in the teaching and learning endeavor, as well as delineates the historical development of TDSM. The literature review also seeks to answer these specific questions:

- 1) What methodologies are used in studies on didactical situations in mathematics?
- 2) What are the findings related to the ways teachers and students construct knowledge in the classroom context?
- 3) What gaps emerge in the existing literature as it relates to both the methodologies, and TDSM?

#### **Background**

Schools in general, mathematics classrooms in particular, are the primary institutions for individuals to become acculturated into the complex web of human competence and social network of the mathematical community. Researchers studying the mathematics classroom identified complexities and pointed out the need for deep understanding of the teaching and learning process in the mathematics classroom (Balacheff, 1990; Brousseau, 1997; Brousseau et al., 2009; Brousseau & Gibel, 2005; Brousseau & Warfield, 1999; Cobb, Wood, Yackel, & McNeal, 1992; Cobb, Yackel, & Wood, 1992; Devichi & Munier, 2013; G. Harel & Koichu, 2010; Herbst, 2003; Margolinas, Coulange, & Bessot, 2005; McNeal & Simon, 2000; Nunokawa, 2005; Schoenfeld, 2012; Steinbring, 2005). Guy Brousseau (1997) developed the theory of



didactical situations in mathematics while conducting research and classroom observations over a number of years. In reviewing the literature, I focused on the interplay between the teacher, the student, and the mathematical content, in what Brousseau (1997) calls the “didactical situation”.

The review is organized as follows: (a) The literature search strategy employed to locate relevant literature; (b) a description of didactical situations, (c) the complexities that occurs in didactical situations, (d) types of knowledge, (e) mathematical objects, (f) the dominant methodologies that was used in studying didactical situations in mathematics, (g) strengths and weaknesses of the methodologies used in the literature, and (h) gaps in the literature. Finally, there is a summary of salient issues in the review.

### **Literature Search Strategy**

Following procedures outlined by Garson (2012) , a systematic review was conducted in January 2013, with the purpose of identifying articles and books bearing directly on the nature of meta-didactical slippages in mathematics teaching and the consequences that they have on students’ conceptual understanding of mathematics. A two-stage strategy was pursued involving an expert chain-of-citations approach followed by a keyword-based computerized search of the literature. In both stages, all years were searched with no publication date limit.

In the expert chain-of-citations stage, faculty and researchers known to me were consulted for recommendations of articles or books dealing with the research topic. This resulted in a list of three books and one article. Each of these four sources was examined to identify further citations related to the topic and to identify keywords useful for subsequent computer searches. When a citation was identified, it was used in the same manner to identify further citations. To limit the chain-of-citations search, the process

was allowed to branch three levels deep. At the end of this stage, a total of 6 books and 25 articles were identified dealing directly with didactical situations in mathematics.

The computerized search stage employed keywords derived from the chain-of-citations stage and from my research questions. Keyword searches involved a word from each of three word groups: (1) Didactic of mathematics, (2) mathematical knowledge, and (3) Common misconceptions. Keyword searches were then undertaken using six databases: (1) Academic Search Complete, (2) ERIC, (3) JSTOR, (4) PsycINFO, (5) Web of Science and (6) the GSU Library.

### **Didactical Situations**

Schoenfeld (2012) described didactical situations as situations that support student engagement with rich mathematics. Furthermore, the author explained that when students engage in these rich mathematical and pedagogical situations, they develop a deep understanding of the mathematics (Schoenfeld, 2012). For Brousseau (1997), didactical situations are representations of real-world situations which have no teaching context or intention. Brousseau referred to the real-world situations as adidactical situations. The didactical situation then is contrived by the teacher in order for the students to gain the knowledge and skills necessary to solve a similar problem in its real life context. It is the real-world or adidactical situation that characterizes the knowledge at stake, thus the teacher must always help the student to “strip the situations of all the didactical artifices, as quickly as possible so as to leave her with personal and objective knowledge” (Brousseau, 1997, p. 31). Stripping a situation of all didactical artifices is better understood in light of Freudenthal’s thirteen major problems of mathematics education (Freudenthal, 1981), which was presented at the Fourth International Congress on Mathematics Education at Berkeley in 1980. Table 3 is a catalogue of the problems.

Table 3

*Summary of Freudenthal's 13 problems of mathematics education*

Problem number	Major Problem of Mathematics Education
1	Why can Jennifer not do arithmetic?
2	How do people learn?
3	How to use progressive schematization and formalization in teaching any mathematical subject whatever?
4	How to keep open the sources of insight during the training process, how to stimulate the retention of insight, in particular in the process of schematizing?
5	How to stimulate reflecting on one's own physical, mental and mathematical activities?
6	How to develop mathematical attitude?
7	How is mathematical learning structured according to levels, and can this structure be used in attempts at differentiation?
8	How to create suitable contexts in order to teach mathematizing?
9	Can you teach geometry by having the learner reflect on his spatial intuitions?
10	How can calculators and computers be used to arouse and increase mathematical understanding?
11	How to design educational development as a strategy for change?
12	Where can we find the nerve fiber to influence education?
13	Educational research itself is a major problem of mathematics education

*Note.* Adopted from "Major Problems of Mathematics Education" by Hans Freudenthal, 1981, *Educational Studies in Mathematics*, 12(2).

Freudenthal's list of thirteen problems are still unsolved, and according to Adda (1998), are still of interest to mathematics educators and researchers. Of particular interest to my study is "how to create suitable contexts in order to teach mathematizing?" (Adda, 1998, p. 50). I believe that mathematizing is a social activity. There is no simple solution to this problem, however, research on didactical situations tend to illuminate the problem in different ways. It is interesting to note that the problems are not independent. For instance, my interest in the eighth problem is interconnected with the second

problem. Further, the literature on didactical situations has exposed a number of complexities associated with mathematics teaching and learning.

### **Complexities in Didactical Situations**

It is in the didactical situations that complexities and inherent difficulties occur. As stated earlier, mathematics teaching is a complex practice because teachers have to balance multiple goals and constraints as they decide “how to respond to students question, how to represent a given mathematical idea, how long to pursue discussion of a problem, or how to make use of available technologies to develop the richness of an investigation” (Martin & Herrera, 2007, p. 18). Several studies using the theory of didactical situations in mathematics as analytical tools point out the complexities of the teachers’ work in the classroom (Brousseau & Gibel, 2005; Brousseau & Warfield, 1999; Hersant & Perrin-Glorian, 2005; Kontorovich, Koichu, Leikin, & Berman, 2012). However, Brousseau (1997) explained several observed cases of the complexity of the teachers’ work. What follows is a description of some of the most salient cases from Brousseau’s work.

**The Topaze effect.** The Topaze effect, named after a French play entitled *Topaze*, by Marcel Pagnol requires the teacher to obtain a predetermined answer from the student (Brousseau, 1997). When the student fails to produce the required answer, the teacher responds by asking probing questions. Determined to get the students to produce the right answer, the teacher chooses easier and easier questions to guide the students to the answer. As this process continues, the target knowledge sometimes disappears. Whenever and “if the target knowledge disappears completely, we have the Topaze effect”(Brousseau, 1997, p. 25).

An example of the Topaze effect can be seen in a study conducted by Robert and Rogalski (2005). In this study the authors conducted independent analyses of the same observed lesson using two different approaches; a didactical one based on the theory of didactical situations in mathematics, and a psychological one based on a socio-constructivist theoretical framework. Both analyses showed that tasks were fragmented into a number of sub-tasks, in order to lead students through a predetermined cognitive route. For instance the authors state:

We have to stress first that all the tasks  $T_i$  were almost immediately followed by interventions from the teacher proposing a series of sub-tasks. This simplified the tasks for the students, and it forced them to use the formulas given in the lessons on absolute value (initial property, or the series of equivalences), in some cases while it was still in the process of being learnt (Robert & Rogalski, 2005, p. 277).

The task of solving absolute value problems was reduced to finding a match with one of the equivalence relationships that was defined by the teacher, thus the knowledge at stake in the conceptual field of absolute value disappears at least partially if not completely.

**The Jourdain effect.** The Jourdain effect is named after the main character in the French play entitled *The Bourgeois gentleman*, by Moliere (Brousseau, 1997). In the Jourdain effect, according to Brousseau, rather than acknowledging failure of a teaching attempt, the teacher accepts an incorrect answer as legitimate and validates the process through which the student obtained the answer. For instance, the teacher tries to teach the student a particular concept, but the student does not show any evidence of understanding. The teacher avoids debating the knowledge with the student and rewards the student for giving the trivial response. The teacher also legitimizes the student's trivial answer as authentic mathematical activity.

**The metacognitive shift.** The metacognitive shift is a case where the teaching activity failed, but unlike the Jourdain effect, the teacher acknowledged the failure and respond by “shifting” the object of study from genuine mathematical knowledge (Brousseau, 1997). In some cases, the teacher may take his or her teaching method as the object of study.

**Improper use of analogy.** Improper use of analogy is the effect whereby the teacher responds to a failed attempt by pointing the student to a similar problem (Brousseau, 1997). The student is able to solve the problem not by engaging with the problem per say, but by applying a familiar algorithm. For instance, the teacher replaces the numbers in a word problem that was presented as an example. The students easily recognize the problem as similar to the example, and replaced the new numbers in the example problem. The teacher accepts this solution as a legitimate indication that the student has learned the target knowledge. Thus the student is able to produce the correct solution because the problem conforms to a given model.

**The aging of teaching situations.** The aging of teaching situations (Brousseau, 1997) is another important effect of the teaching and learning process. This occurs because an exact reproduction of a lesson does not have the same effect, even if the students are different. The teacher, having experienced the interaction of the lesson with students, modified the lesson by either removing superfluous information from the problem or supplying missing information to the problem in order to limit the level of students’ uncertainty.

In a study to investigate the kinds of phenomena that can be reproduced when the same class situation is implemented in two different classes Arsac, Balacheff, and Mante (1992) wrote that their precautions which consisted of carefully presenting the situation

and the theoretical ideas behind it were not sufficient to avoid difficulties in reproducing the lesson as intended. The difficulties were attributed to three main categories: (a) constraints, such as time or epistemological responsibility; (b) personal ideas of the teacher, such as ideas about proofs in geometry, or about management of the class; and (c) problems of control of the actual effects of micro-decisions of the teacher (Arsac et al., 1992). Due to the difficulty of the teacher to reproduce the lesson in its original form, it is not always known what really is being produced during the course of a lesson. For Brousseau, the object of *didactique*, is “knowing what is being produced in a teaching situation” (Brousseau, 1997, p. 29). What follows is a discussion of the different types of knowledge.

### **Types of Knowledge**

The literature in mathematics education describe many different types of knowledge (English, 2002). For example, constructs such as relational, instrumental, conceptual, procedural, implicit, explicit, elementary, advanced, algorithmic, formal, intuitive, visual, situated, knowing that, knowing how, knowing why, and knowing to are discussed in the literature (Brousseau, 1997; English, 2002; Henningsen & Stein, 1997; Hiebert, Stigler, & Jacobs, 2005; Kilpatrick, 1992; Knuth, Stephens, McNeil, & Alibali, 2006; Martin & Herrera, 2007; Mathematics, 2000; Sfard, 2003; Sinclair, 2010). The notion of knowledge, understanding, and knowing is very complex, but given its’ saliency in the literature, it is significant to both mathematics teachers and mathematics education researchers.

The literature however, discussed these forms of knowledge as either a dichotomy or a continuum (Even & Tirosh, 2002). Each of these themes has for me a fundamental limitation in that they portray an “either-or”, or “both” philosophy. In either case, one

form is privileged over the other. The conflict between the perspectives of the different forms of knowledge only adds to the complexity of the teaching and learning process.

A notable example of the conflict between different types of knowledge is Boaler's three-year case study of two schools using different mathematics teaching approaches (Boaler, 1998). According to Boaler (1998), the Phoenix Park method is to encourage students to take responsibility for their own actions and to be independent thinkers. This teaching method was based on the philosophy that students should encounter a need to use mathematics in situations that were realistic and meaningful to them. This philosophy is echoed within the National Council of Teachers of Mathematics (NCTM) standards and the Common Core State Standard for Mathematics (CCSSM). Whereas it is probably not practical for most teachers to teach the Phoenix Park way all of the time, there are lessons to be learned from the Phoenix Park story.

The first lesson is that the Phoenix Park students performed as well or even better on high stakes national tests as their peers. This is an extraordinary result because they did not place emphasis on test taking strategies and procedural fluency as their peers, yet they were still able to outperform their peers who spent a significant amount of time on procedural fluency and test preparation. Therefore, a lesson here is that, if mathematics teachers teach students to think and if students understand the mathematics, they are learning, and then they will do well on tests. Thus mathematics teachers do not need to spend so much time focusing on the test preparation.

Another lesson from the Phoenix Park's story is that students develop the desire and ability to think about mathematics, and represent mathematical ideas in multiple ways. According to Mathematics (2000) students should be encouraged to use multiple representations. The Phoenix Park story is an example of students using multiple



representations to solve problems. The students had a vested interest in their problems and a genuine desire for a solution. This in turn can result in student motivation and improved engagement.

One important lesson that stood out to me in the Phoenix Park story was that any teacher can teach the Phoenix Park way. According to Boaler (1998), the teachers were not regarded as exceptional. The teachers were ordinary teachers with typical problems shared by teachers in any school. The teachers who participated in the study included “newly qualified” teachers, teachers with “classroom management issues,” and teachers that had difficulty fitting in with the process based approach. This means therefore, that to teach the Phoenix Park way is attainable in any school as long as there are a few hard working and committed teachers.

Although Boaler’s work was notable for the findings, depth of analysis, and insights, I find the conflict between procedural knowledge and conceptual understanding very problematic and limited. The French tradition of mathematics education research provides a more broad meaning to the relationship between and among the types of knowledge.

In the French tradition, two words emerged as significant in dealing with the different forms of knowing, *connaissance* and *savoir*. *Connaissance* and *savoir* both translate to “knowledge” from the Collins French-English dictionary, but they describe very distinct aspects of knowledge. These words form a kind of interdependent pair, which is essential in understanding the teaching and learning process. The English language does not have such word pairs to differentiate the different forms of knowledge. Understandably, this is probably the reason that only descriptions of these constructs are provided in the literature.

## **Connaissance and Savoirs**

Connaissances are exposed when classroom situations or events provoke students to react. When students react by making declarations, reflecting, and learning, their intellectual activity is manifested (Brousseau, 1997; Brousseau et al., 2009). For Brousseau, what students do, their intensions, their decisions, their perceptions, their beliefs, their language, and their reasoning, reveals their connaissances. The reality however, is that “only one part of these connaissances is recognized as expressible, and expressed whether by the student, by other students, by the teacher, or by society” (Brousseau et al., 2009, p. 110). These connaissances are recognized with the help of *reference connaissances*, such as customs, language, established definitions and theorems, logic, beliefs, culture, etc. According to Brousseau (1997), these reference connaissances are the savoirs.

Savoirs are the essential means of expressing connaissances. In other words, savoirs are the skills, techniques, or devices used in a particular field or occupation. A student’s repertoire of savoirs may be different from that of other students. In order for the class to have effective communication, there must be a common repertory of savoirs (Brousseau et al., 2009). Furthermore, an environment of connaissance that makes it possible to use them connects savoirs. Any connaissances that are not connected by any savoirs will disappear (Brousseau et al., 2009).

Connaissance and savoir do not implicate the binary or continuum relationship that is often discussed in the literature. For instance, “conceptual understanding” and “procedural knowledge” are framed as opposites, where conceptual understanding is privileged over procedural knowledge. Similarly, elementary and advanced knowledge form another opposite in which advanced knowledge is more desirable. In the same vein,

knowing *why* is superior to knowing *how*. It is also important to note that several synonyms for *knowledge* appear in the literature. For instance, algorithmic knowledge is the same as procedural knowledge, which is sometimes referred to as *know how* (English, 2002).

The importance of *connaissance* and *savoir* lies in the state of the knowledge. In the mathematical situation, knowledge is formulated by the student or by the teacher. This knowledge could then evolve into deeper understanding. In the teaching and learning process, the student learns to pose questions, to distinguish givens, to analyze texts, and to discard useless information (Brousseau et al., 2009). These are examples of *connaissance*. It is imperative that the teacher does not teach these *connaissances* as *savoirs*. To do so could undermine the learning goal of the mathematics lesson.

Learning, according to Brousseau et al. (2009), is manifested in the appearance of new *connaissances* and new *savoirs*. Furthermore, *connaissances* may be “exact or false, approximate, or dubious, conscious or unconscious” (p. 110). Only the *connaissances* that are recognizable with the help of *savoirs* can be communicated. Learning also occurs when the status of a *connaissance* changes to a *savoir*, or when *savoirs* are employed in situations to form new *connaissances*. These learning in the mathematics classroom are represented as mathematical objects. A discussion of the most salient issues related to mathematical objects follows.

### **Mathematical Objects**

The nature of mathematical objects is essential in understanding the didactics of mathematics. A review of the literature in mathematics education research revealed that the most common aim of the field of mathematics education research is to study the factors that affect the teaching and learning of mathematics and to develop programs to

improve the teaching of mathematics (Sierpinska, & Kilpatrick, 1998). Broadly speaking, this aim has been tackled by defining and clarifying the nature of mathematical objects. Sierpinska and Kilpatrick (1998) identified some key assumptions about the nature of mathematical objects: (a) Mathematics can be seen as a human activity involving the solution of socially shared problem situations; (b) Mathematical activity creates a symbolic language in which problem situations and their solutions are expressed; and (c) Mathematical activity aims at the construction of logically organized conceptual systems.

According to Brousseau (1997) mathematical objects are classified in three main groups. These groups describe the work of mathematicians, the work of students, and the teacher's work. For mathematicians the formulation of knowledge has a complex history that includes a succession of difficulties and questions which promote a fundamental concept, the rejection of false claims, the inclusion of techniques from other areas, and so on (Brousseau, 1997). Thus, there is a network of activities that provide for the mathematician origin, meaning, motivation, and use.

The student must reproduce this mathematical activity of the mathematician. A proper reproduction of genuine mathematics by the student would require the student to produce, formulate, prove, and construct models, languages, concepts and theories (Brousseau, 1997). The student must also share these objects with others, recognize those which conform to the culture, and incorporate those from other contexts which are useful.

The teacher must transpose the scientific work of the mathematician into the classroom context so that the students can engage and reproduce the work of mathematicians. The complexities of these activities however, can only be understood by understanding the nature of the didactical contract.

## The Didactical Contract

The concept of *didactical contract* is a major component of TDSM and is central to the analysis of the workings of the didactical systems. The didactical contract is akin to the social contract, but differs in important ways. A generally accepted notion is that teaching is an intentional activity. One can assume that the teacher simply wants the student to learn. According to Uljens (1998), this learning process requires the participation of the student and the teacher. Moreover, there are socially accepted roles that the two parties have. Uljens (1998) contended that the social contract in a school necessarily requires that every student is an intentional learner, and the teacher has a right to expect an interested attitude from the student. The didactical contract is more complex, in that it is not explicit, and in most cases it is not known. For example, if the teacher finds that a student is breaking the social contract, the teacher can appeal to the parents of that student, because the parents also enter into the social contract. But in the didactical contract, the teacher cannot appeal to the student, the administrator, or the parent for breaking the contract. The teacher revolt in many different ways, and the student is confused. Similarly, the student has no one to appeal to if the teacher breaks the contract, the student revolts, and the teacher is confused.

In TDSM, part of the didactical contract is specific to the target mathematical knowledge. This implies that the didactical contract is different for each mathematical concept. Moreover, the didactical contract is different for different students. Therefore, it is difficult to fully describe the didactical contract. However, according to Brousseau (1997) it is not essential to describe the didactical contract because it is the breaking of the contract that is important. In order to examine some of the immediate consequences of breaking the didactical contract Brousseau (1997) assumed that:

- The teacher is supposed to create sufficient conditions for the appropriation of knowledge and must “recognize” this appropriation when it occurs.
- The student is supposed to be able to satisfy these conditions.
- The didactical relationship must “continue” at all costs.
- The teacher therefore assumes that earlier learning and the new conditions provide the student with the possibility of new learning.

In the didactical situation, if the teacher perceives a failure in the learning, the student is put on trial for not fulfilling the expected learning objective (Brousseau, 1997). Implicitly, the teacher is also put on trial for not fulfilling what that student expected. In TDSM, the didactical contract is manifested when a failure occurs. The student is surprised because he or she does not know how to solve the problem, and thus rebels against what the teacher could not provide (Brousseau, 1997). Similarly, the teacher is surprised because of what the student fails to do. The teacher revolts, negotiates, and searches for a new contract (Brousseau, 1997). There are a number of paths that the teacher can take to continue the didactical relationship. No path is known a priori. A summary of the phenomena related to the negotiation of the didactical contract is shown in Figure 2.

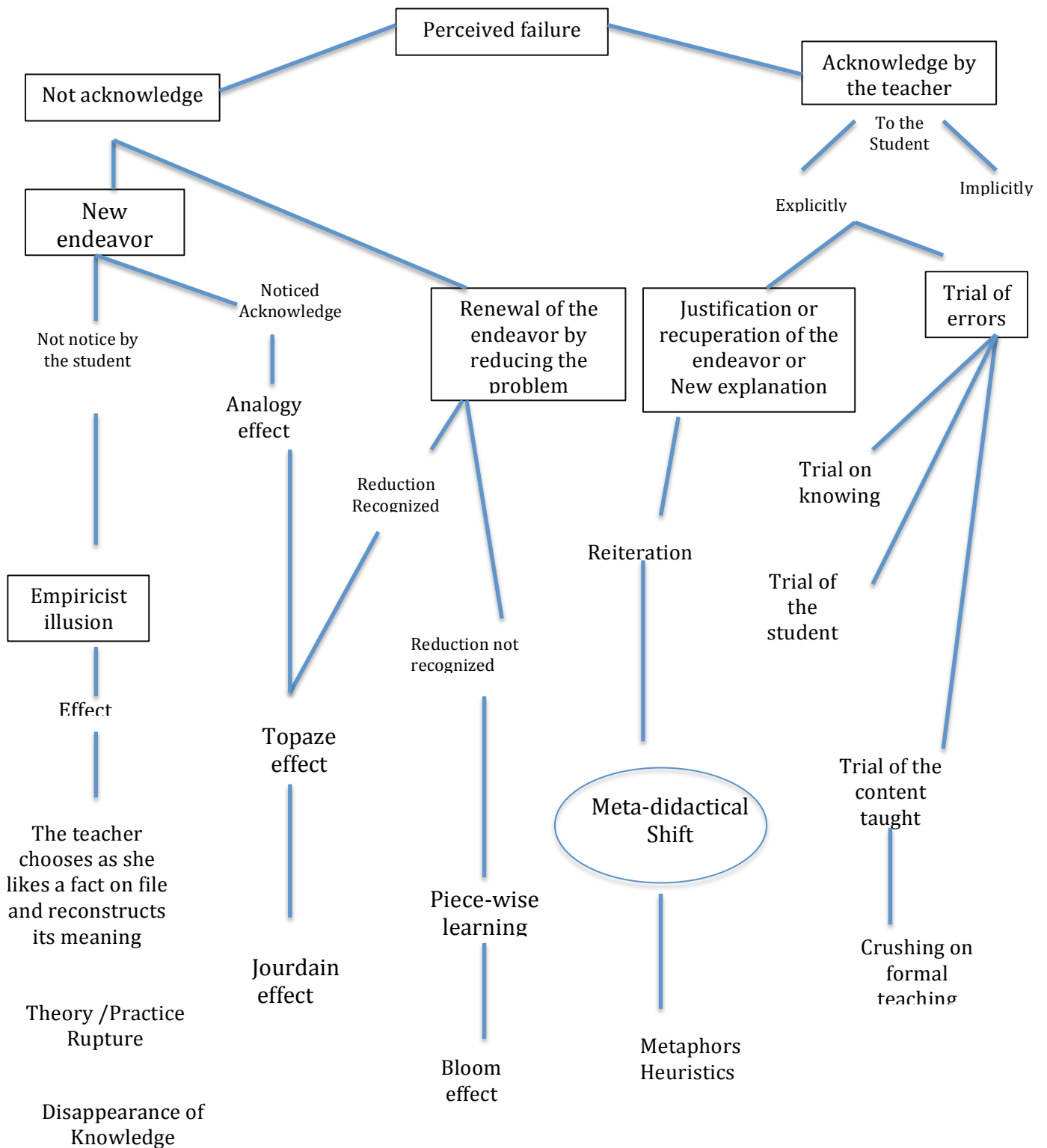


Figure 2. The teaching endeavor. Adopted from “Theory of Didactical Situations in Mathematics” by G. Brousseau, 1997, p.247.

### Historical Development of TDSM

According to Brousseau, Brousseau, and Warfield (2004) several factors of the late sixties motivated the work on which TDSM is based. For example, public opinion in the sixties was exerting pressures on the mathematics taught in school to resemble the mathematics practiced by mathematicians. Additionally, the widely held belief that understanding a mathematical concept implies that at the end of the learning process, the student has at his or her disposal a collection of widely varied, and logically interlinked pieces of knowledge. Brousseau (1997) rejected these notions on the basis that mathematical concepts are constructed in the course of a situation whereby a rich collection of reasons come to bear. Moreover, Brousseau believed that there is no mathematical activity that a teacher can present that is independent of a teaching objective (Brousseau et al., 2004).

The development of TDSM draws on a wide range of theoretical ideas. According to Brousseau (1997) TDSM was influenced by Piaget's theorization of cognitive development as a process of constructive adaptation. The theory was later refined by incorporating the theoretical ideas of the French epistemologist Gaston Bachelard (Ruthven, Laborde, Leach, & Tiberghien, 2009), who posits that knowledge advances through epistemological obstacles.

The concept of *epistemological obstacle* enabled original approaches to be developed concerning conceptual difficulties and analysis of students' errors (Brousseau, 1997). This concept has been particularly productive in the analysis of the difficulties that students experienced when moving from whole numbers to decimals (Brousseau et al., 2004; Brousseau, Brousseau, & Warfield, 2007, 2008; Brousseau et al., 2009). In TDSM obstacles are manifested by errors, but according to Brousseau (1997) these errors are not



due to chance, and they are not necessarily explainable. Accordingly, much research is needed to distinguish, recognize, list, and to examine the relationships and causes of epistemological obstacles in mathematics (Brousseau, 1997).

Another refinement to TDSM is the addition of the concept of didactical transposition. *Didactical transposition* is a concept that was originally developed by Yves Chevallard in the early 80s to explain the transformations that mathematical subjects undergo when they enter a didactical system (Brousseau, 1997). In TDSM, this concept is defined and activated by the notion of the *fundamental situation* for a piece of knowledge, which constitutes special study tool of phenomena involving transposition by defining the conditions for preserving the meaning of knowledge at the moment of transposition. According to Sriraman and English (2010), Yves Chevallard also extended TDSM from within the institutional setting to the much wider “Institutional” setting. Therefore, whereas Brousseau’s theory focuses on the classroom teaching and learning of mathematics, Chevallard’s approach focuses on the mathematics in a much broader context, which involves, scholars/mathematicians, curriculum/policy makers, teachers and students (English, 2002). Chevallard’s approach is known as the anthropological theory of didactical situations.

Using a constructivist approach to learning, Brousseau designed teaching experiments with an initial aim to develop an existence theorem, and to clarify and complete TDSM (Brousseau et al., 2004). The lessons were to be studied and criticized using robust theoretical, pragmatic and methodological instruments. These instruments came from TDSM, however they were modified during the course of the experiments (Brousseau et al., 2004).

Armed with the positive results of the first three modules of the lesson sequence Brousseau continued to work on TDSM (Brousseau et al., 2004). Brousseau assumed that students expand their knowledge through interaction with problems that offers both resistance and feedback. The resistance and feedback then affects the mathematical knowledge at stake. Additionally, Brousseau proposed that children, in suitably carefully arranged circumstances could build their own knowledge of mathematics. Among his many objective was to prove that under these conditions all children could “create, understand, learn, use, and love some mathematics that has a reputation for being difficult” (Brousseau et al., 2007, p. 281). After 20 years of teaching the same sequence of lessons between the period 1974-1997, new research questions came to the fore which resulted in increased clarifications to TDSM (Brousseau et al., 2009).

There are two institutions that were very significant to the historical development of TDSM (Brousseau, 1997). These institutions are *Institutets de Recherche sur l’Enseignement des Mathematiques* (Institute for Research on the Teaching of Mathematics - IREM), and *Centre d’Observation et de Recherches sur l’Enseignement des Mathématiques* (Center for Observation and Research on the Teaching of Mathematics - COREM). A brief description of these institutions follows.

### **IREM**

IREMs are French institutions, within universities that are created to develop research in mathematics education and to participate in in-service training of mathematics teachers (Brousseau, 1997). Each IREM has three components: (a) a colloquium open to all teachers of mathematics, (b) teaching information and documentation of professors of mathematics, and (c) research on the teaching of mathematics (Brousseau, 1997).

According to Sierpinska and Kilpatrick (1998) the meeting of theoreticians and teachers

in these institutes has been one of the levers of research in mathematics education in the history of French didactique.

## **COREM**

The COREM is an original institution created by Guy Brousseau in 1972 (Sierpinska & Kilpatrick, 1998). According to Brousseau (2008) the COREM was created to permit mathematicians to carry out different sorts of systematic and sustained observations. It was made up of three contractual entities: a research laboratory, a technical team and a whole school (14 classes) with an adapted status. The COREM was located in a school for 3-10 year-old children from ordinary backgrounds, where there is no pre-selection. The school has a special building, which is a big classroom where video and audio recorders are available. Mathematics lessons conducted in this room would be recorded and observed. The school, according to Brousseau (1997) was not to be an experimental school, but essentially a center for observation. Specifically, the school was designed with an aim to permit observers to pick out behaviors while influencing them as little as possible. More importantly, “it aimed to *make it necessary* on their part to produce didactical knowledge *subject to a pragmatic restriction* for a short term on the part of the system observed” (Brousseau, 2008, p. 1). Research on the teaching of the natural numbers was carried out at the COREM from 1970-1974 (Brousseau, 1997) and on rational numbers and decimals from 1973-1980.

Thus far, we see that the key notion of TDSM is that of situations. Moreover, the possibility of isolating, in the specially constructed situations, moments of *action*, *formulation*, *validation* and the tools involved at each of these moments constituted a major part of the work carried out for more than thirty years on various mathematical

topics (Brousseau, 1997). The bulk of this work is still in the French language.

Notwithstanding, several scholars have added, or clarified different aspects TDSM.

### **Application of TDSM in Mathematics Education Research**

A recent application and extension of TDSM was reported in a study by Hersant and Perrin-Glorian (2005). In this study, the didactical contract was divided into three levels: the *macro-*, the *meso-*, and the *micro-contract*. According to Hersant and Perrin-Glorian (2005) the macro-contract is mainly concerned with the teaching objective, the meso-contract with the realization of an activity, and the micro-contract deals with the for example, a concrete question in an exercise.

Using TDSM as theoretical frame, Hersant and Perrin-Glorian (2005) characterized an ordinary mathematics teaching practice, called *interactive synthesis discussion* (ISD), using the three levels of the didactical contract. They presented two case studies with experienced teachers, one in grade 8, and the other in grade 10. In the study they presented a modified framework of the didactical contract which was used to analyze the lessons (Hersant & Perrin-Glorian, 2005). This modified framework provided an analytic lens, which researchers could use to understand didactical situations in the teaching and learning process of already existing practices.

One of the most significant applications of TDSM to mathematics education research is that it provides an analytic tool to understand didactical phenomena. TDSM also provides specific assumptions an analytic tool to guide researchers and mathematics educators in studying the teaching and learning of mathematics in an institution. The mathematics teacher has the role of transforming a real world situation, (a didactical situation), into the classroom in order to provide students with the experience of solving problems in the real world. The transformation causes a decontextualization, which

usually strip the situation of the historical context (Brousseau, 1997). For the mathematics teachers, only problem solving can demonstrate the student has learned the desired mathematical knowledge. Consequently, Brousseau (1997) differentiates between different sub systems of didactical situations:

- a classification of the interactions of the subject with the didactical milieu;
- a classification of types of organization of this milieu;
- a classification of types of function of a piece of knowledge; and
- a classification of modes of spontaneous evolution of knowledge (p. 60).

According to Brousseau (1997) each classification must sufficiently justify itself within its own domain by (a) the considerable and obvious difference between the objects classified; (b) the simplification that it can provide in their description, their analysis, and their understanding; (c) the relevance of this classification (and its importance to other possible classifications) for each domain concerned; and (d) its completely exhaustive character. For example, a certain type of interaction is specific to one type of social and material organization, because it favors a certain form of knowing, and can cause the form of knowing to evolve.

Interactions, according to TDSM, are the relationship between a student and the milieu. Brousseau (1997) identified three main categories: exchange of judgment, *validation*; exchange of information coded into language, *formulation*; and exchange of information that is not coded, or is without a language: these could be actions or decisions that act directly on the other performer, *action*.

The form of knowledge, which controls the student's interactions is judged using two categories: (a) it must be composed of a description, or model expressed in a certain "language" or theory; and (b) it must be composed of a statement about the adequacy of the description, whether it is a contingent or a necessity and whether it is consistent with

the student's knowledge or the milieu (Brousseau, 1997). See Table 2 for a summary of this analytic tool.

The final analytic tool is the evolution of forms of knowledge. In TDSM, this is the notion of learning. Knowledge evolves according to complex processes, which cannot be explained only by the interactions with the milieu. However, evolution of the forms of knowledge can be observed by considering the type of milieu and the type of situation (action, formulation, or validation) (Brousseau, 1997). The milieu can be either an objective milieu in that it provides built in feedback, or a milieu where feedback is handled by didactical means (Brousseau & Gibel, 2005), whereby the teacher is the primary if not only, source of feedback. In the next section I discuss advantages and disadvantages of TDSM in examining the complexities of the mathematics classrooms.

### **Strengths and Weaknesses of TDSM**

**Strengths.** TDSM can assist researchers and mathematics teachers to study and construct theoretical models of situations that produce effective learning because “it is an instrument for the construction of minimal explanation of newly observed facts that would be compatible with newly established knowledge” (Brousseau & Gibel, 2005, p. 17). TDSM also provides instruments to study the complexity of situations that involves the interaction of teacher, student, and content in the mathematics classroom.

Another strength of TDSM is that it allows for the careful analysis of a teaching sequence to provide understanding of the forms and states of knowledge that manifest during the teaching and learning process. According to Brousseau (1997) TDSM provides knowledge about teaching which concerns different aspects of the teaching and learning process. For instance: (a) knowledge concerning students, their behaviors and their mathematical understanding in different teaching conditions; (b) knowledge related to

conditions to be created in teaching and learning situations; and (c) knowledge concerning conditions to be maintained in the management, or implementation of the teaching (Brousseau, 1997).

Finally, an important advantage of TDSM is that it can help the teacher to change his or her status, training and relationship with society (Brousseau, 1997), by acting directly on the knowledge that the teacher uses, and by acting on the knowledge of professional partners, parents, and the general public.

**Weaknesses.** Although TDSM provides knowledge of problem effects in the classroom, “it cannot produce a solution to such problems by mere engineering adjustments” (Brousseau, 1997, p. 260). For instance, in the negotiation of the didactical contract, TDSM does not provide an optimal path for the teacher to take if a specific failure occurred (see Figure 2).

Another weakness of TDSM is with the dissemination of didactique (Brousseau, 1997). This disadvantage could possibly be due to (at least in the English-Speaking circles) the difficulty with the French language, and with the difficulty with the concept in general. For example, Bussi (2005) pointed out that the theoretical sophistication of TDSM is huge and not easy to communicate.

### **Methodological Considerations**

Several environmental forces shaped the development of research in mathematics education (Kilpatrick, 1992). For instance, the requirement for university faculty members to conduct research, Klein’s reform movement in mathematics curriculum in the 1900s, psychological research into mathematical thinking, child studies, the testing movement; and several other forces helped to shape research in mathematics education.

According to Crotty (1998) the research method or technique used to gather and analyze data must be linked to some research question; therefore, the aims of research drive the methodology selection. Furthermore, the methodology is the plan of action lying behind the choice of particular method. For example, in a discussion of the historical roots, philosophical roots, and the emergence of a profession in the field of mathematics education, Kilpatrick (1992) pointed out that research in mathematics education has dealt primarily with problems of learning and teaching as defined by the researchers. These researches delved into the question of what mathematics is taught and learned and how the content is taught and learned. Table 4 is a summary of the main research perspectives and aims in mathematics education research.

Table 4

*Research perspectives and their aims*

Main Aims	Research Perspective
To predict, explain, or control	Empirical-analytical (Experimental, intervention, innovation)
To understand the meanings of the learning and teaching of mathematics for participants in these activities.	Ethnographic, anthropology approaches (Observational, mostly)
To improve practice and involve the participants in the improvement	Action research (Participant observation)

*Note.* Adopted from “A history of research in mathematics education “, by J Kilpatrick, 1992, pp. 3-4.



Methodologically speaking, the two major themes that emerge in the literature on didactical situations in the mathematics classrooms are didactic engineering and lesson study (Adda, 1998; Brousseau, 1997; Kilpatrick, 1992). Didactical engineering has to do with designing lessons intended to produce a desired outcome, whereas lesson study involves observing a teaching sequence to understand what happened. Both of these classes of research designs examine the classroom with an aim of ultimately improving classroom practices, but with different foci.

**Didactical engineering.** Didactical engineering is aimed at innovations by controlling the ‘how’ of teaching to produce an effect on the ‘what’ is learned. Didactical engineering studies used a more empirical design following the traditional paradigm of scientific research. Indeed, Guy Brousseau’s work, which the theory of didactical situations in mathematics is based, was of the experimental perspective (Brousseau, 1997; Brousseau et al., 2004, 2007, 2008; Brousseau et al., 2009). Classroom lessons were carefully designed, implemented, and observed under laboratory conditions. These lessons were referred to as teaching experiments (Brousseau, 1997).

Teaching experiments or lessons designed to produce a given outcome, served as treatment in these empirical designs. What is now known as situations, was initially considered as didactical variables to be manipulated and controlled as in the case of a statistical experimental design (Brousseau, 1997). Researchers in the paradigm of didactical engineering aimed at reproducing results by manipulating one or more variables but, as can be seen in the study by Arzac et al. (1992), the task of reproducing a teaching and learning outcome was opposed by both time constraints and epistemological obstacles. Moreover, the learning that occurred in the classroom was not easily observable. In this study the case study methodology was used. The researchers had

seven different observers, one for each of six groups of students and another for whole class discussion. The sessions were video and audio taped. The teacher was also interviewed after watching the video recording. Data was analyzed using the theory of didactical situations as a framework. The results showed that it was probably not possible to control teaching situations and to control their effects on students' learning (Arsac et al., 1992).

**Lesson study.** Lesson study focuses on what occurs during the teaching in order to understand the teaching and learning process. Unlike didactical engineering, the researcher's aim is not to manipulate, or control any constraints of the teaching. For instance, in a study using theory of didactical situation to characterize a mathematics teaching practice used in secondary schools, Hersant and Perrin-Glorian (2005) stated that "the aim of this research is to gain knowledge and understanding of teaching phenomena. It is not to produce immediate action or to improve teaching in a direct way. Moreover our project is not one of didactic engineering" (p. 114). Consequently, a qualitative case study was chosen. The data, which consisted of classroom episodes, was collected through passive classroom observations and analyzed with reference to the didactical contract.

It is important to note that observation of classroom activities is not without complexities because actions by the teacher, the student, or the observer can be interpreted differently (Brousseau & Gibel, 2005). It is not always possible to claim that a given observable behavior is a sign of reasoning. In order to study students reasoning in the teaching and learning process, Brousseau and Gibel (2005) went beyond the formal definition of reasoning and examined conditions in which a presumed reasoning can be considered an actual reasoning. Classroom episodes were analyzed using the theory of

didactical situations in mathematics. Brousseau and Gibel (2005) showed that although the students produced forms of reasoning when faced with a problem situation, they did not make progress in their practice of reasoning. For, “they have not reflected back on their reasoning, on its validity, relevance, or adequacy because the teacher was not able to process it” (p. 54).

The lesson study that was commonly used in the literature that employed the theory of didactical situations in mathematics could be classified as either classroom observation or a participant observation. This is because lesson study was an approach that originated in Japan that is primarily used for professional development (Hart, Alston, & Murata, 2011). This version is very similar to didactical engineering in that the lessons studied were designed specifically for the purpose of observation and investigation. Moreover, according to Hart et al. (2011) the lesson study goes through a study cycle in order to revise and reteach the research lesson to a new group of students. Therefore the lesson study paradigm, although has roots in the Japanese tradition, is more akin to the ethnographic method of participant observation.

### **Strengths and Weaknesses of Methodologies**

**Strengths.** Two of the main aims of research in mathematics education were to understand and to improve the teaching and learning of mathematics (see Table 1). The methodologies used in the literature on didactical situations in mathematics were well suited for these aims. For instance, in a study aimed at exploring the possibilities of making transition and connection between arithmetical and algebraic practices, Sadovsky and Sessa (2005) implemented a didactical engineering project which allowed them to present ideas for teaching, and conditions that enable effective teaching and learning. Using the theory of didactical situations Sadovsky and Sessa (2005) showed that it is

possible to obtain an adidactic milieu which generated questions. Moreover, “the move from arithmetic to algebra is nurtured by questions framed in the social space of the classroom as a consequence of the work proposed by the teacher” (Sadovsky & Sessa, 2005, p. 107).

The methodologies used in the literature were also well suited for providing insights, descriptions and understandings on teaching phenomena and complexities in the mathematics classroom, because classroom observation is the dominant method used to collect data. Classroom observations were used both in the didactic engineering paradigm and the lesson study paradigm. Classroom observations provide the researcher with possibilities of probing into the contexts, the meanings, and the processes of production of mathematical knowledge as it occur in the mathematics classroom.

**Weaknesses.** Classroom observations are local, and thus inevitably biased by local constraints. Therefore the notion of replicability and generalizability from the experimental sciences is not appropriate. Consequently, researchers have to provide lengthy discussions of the theoretical and methodological frames that undergird their studies. The theory of didactical situations in mathematics, which is the dominant theory that is used in the literature on didactical situations, is very sophisticated and contained a number of difficult concepts.

Another weakness with the methodologies as cited in the literature, particularly with those used in the paradigm of didactical engineering, is that the classical experimental method, which involved carrying out different statistical tests, does not fit the purpose of research in didactical situations. For instance, Brousseau (1997) pointed out that the use of statistical tests is not ethically admissible because no professional can

agree a priori to teach in order to see what would happen. Moreover educational systems usually react to its own results by modifying its teaching conditions (Brousseau, 1997).

### **Gaps in The Literature**

The literature on didactical situations in mathematics identified several phenomena of the teaching and learning situation, especially with regards to the didactical contract. However, the literature does very little in describing the nature of these phenomena, and the consequence that they have on future learning of the target knowledge. For instance, in a study focusing on the development of mathematical understandings that took place in a 10<sup>th</sup> grade geometry class, Schoenfeld (1988) pointed out that despite the fact that the class was well taught, and the students did well on relevant performance measures, the students learned some inappropriate and counterproductive conceptions about the nature of mathematics. Furthermore, these inappropriate conceptions were as a direct result of the instruction.

Other phenomena such as the complexities in didactical situations identified by Brousseau (1997) provide significant insights into the complexities of the teaching and learning of mathematics. But the literature on didactical situations in mathematics fails to examine these phenomena, and how they serve in the construction and sustenance of mathematical knowing. Consequently, research investigating the nature of didactical phenomena is well needed.

### **Summary and Conclusion**

The purpose of this review was to provide an overview and synthesis of the research relating to didactical situations in the mathematics classroom. Didactical situations in mathematics are situations designed by the teacher in order to enculturate

students in an important part of human culture, that of mathematics. Several phenomena of the teaching and learning process serve as obstacles to the learning of mathematics.

The two dominant methodologies that emerged in the literature in studying didactical situations in mathematics were didactical engineering and lesson study. The main data collection technique used in these methodologies was classroom observation. Data were analyzed using the theory of didactical situations in mathematics as the theoretical lens. Strength and weaknesses of the methodologies used in studying didactical situations were discussed and important gaps in the literature were identified. Finally, the literature review called for more research on didactical situations in the mathematics classroom, and demonstrated the need to research investigating the nature of the phenomenon of meta-didactical slippages in the mathematics classroom.

## CHAPTER 3

### METHODOLOGY

This chapter explains the methods used to carry out the study, providing special emphasis to the analysis of data. This chapter of the dissertation presents a review of the purpose statement and research questions, the theoretical framework that guided the methodology of the study, and the epistemology that informed the theoretical perspective. The remainder of the chapter presents (1) the design that governed my choice and use of participant observation and open-ended interviews as methods of data collection, (2) the data collection, (3) the data analysis techniques, (4) trustworthiness, (5) limitations of the study, and (6) a brief summary of the chapter.

#### **Purpose and Research Questions**

The purpose of this study was to (a) understand the nature of meta-didactical slippages and how meta-didactical slippages occur in a ninth grade predominantly African American mathematics classroom; and (b) describe the consequence of meta-didactical slippage on a unit of study of ninth grade mathematics.

The following questions guided the study:

1. What is the nature of meta-didactical slippages that emerge in the practice of teaching mathematics?
2. In what ways do these slippages affect students' conceptual understanding of a unit of ninth grade mathematics?

## **Theoretical Perspective**

The philosophical stance of this study was rooted in the perspective of cultural anthropology. This perspective views culture as a set of cognitive structures that children learn as they grow up in a particular community, and that they use to make decisions about their own behaviors and that of the people around them. This stance spawned the theory of didactical situations in mathematics (Brousseau, 1997), which is the theoretical frame used in this study. Meanings are constructed in a social situation, and the meanings change from culture to culture and from individual to individual. The theory of didactical situations in mathematics helped me to isolate particular meanings that teachers and students construct in didactical situations in the mathematics classroom (see chapter 1 for a more detailed discussion).

## **Epistemology**

This study was grounded in a social constructionist epistemology. Mathematics teachers and students construct meanings from both the situation and from the act of teaching and learning mathematics. Moreover, according to Crotty (1998), because of the essential relationship that human experience bears to its object, no object can be adequately described in isolation from its conscious being experiencing it, nor can any experience be adequately described in isolation from its object.

Constructionist epistemology consists of at least two schools of thought. These schools of thought are sometimes called empirically oriented constructivism and radically oriented constructivism. The former holds that knowledge is anchored in the external environment and exists independently of the learner. The latter maintains that knowledge resides in the constructions of the subject. In this study, I follow the latter.



## **Study Design**

This study was a descriptive, qualitative, case study conducted in one ninth grade mathematics classroom over a 15-week period. The case study was selected because it is well suited to take into account the complexity of didactical interactions between teacher, student, and the content of mathematics. These interactions are intangible, yet the impact of these interactions on the student, the teacher, the institution, and hence the society at large, are very tangible. The case study is grounded in the lived reality and can help us to understand complex inter-relationships (Hays, 2004). Furthermore, the case study according to Hays (2004) seeks to answer focused questions by producing in-depth descriptions and interpretations over a short period of time. Thus, in order to probe beneath the surface of the didactical situations, and to get a better understanding of meta-didactical slippages, the qualitative case study methodology is well suited.

Qualitative researchers use multiple methods. The use of multiple methods reflects the researchers' aim to secure an in-depth understanding of the phenomenon in question. This process of using multiple methods is referred to as triangulation in the literature (Berg, 2009; Cohen, Manion, & Morrison, 2013; DeMarrais & Lapan, 2004; Denzin & Lincoln, 2005; Hays, 2004; Yin, 2002). According to Denzin and Lincoln (2005), this combination of multiple methodological practices, empirical materials, perspectives, and observers is understood as a strategy that adds depth, rigor, breadth, complexity, and richness to an inquiry. Since didactical situations in the mathematics classroom are examples of a complex situation that involves multiple representations, the qualitative case study is an ideal research methodology.

The case study is one of the many strategies of inquiry that the qualitative bricoleur can use to conduct research. According to Yin (2009) a rationale for selecting

the case study is when the researcher is studying contemporary events but “the relevant behaviors cannot be manipulated” (p. 11). Additionally, the research questions, that is to say the substance (what the research is about) and form (“who”, “what”, “where”, or “how”, questions) of the research questions, provide a good rationale for choosing the case study. The case study recognizes and accepts that there can be many factors operating in a single case. Accordingly, many types of data can be incorporated into a case study such as interviews, participant observations, documents, and quantitative data to provide rich and vivid descriptions of events relevant to the case.

There are several typologies of case studies in the literature (Berg, 2009; DeMarrais & Lapan, 2004; Hays, 2004; Stake, 1995; Thomas, 2011; Yin, 2009). However, Yin (2009) identified three main types in terms of the intended aims: (a) exploratory, (b) descriptive, and (c) explanatory. Thus case studies can serve to explain, describe, illustrate, and enlighten. By studying didactical situations in the mathematics classroom, my aim was to describe the real-life, complex dynamic unfolding interaction of the phenomena in its natural occurring environment. Consequently, the qualitative case study is well suited to study didactical situations in mathematics classrooms.

The descriptive case study design required that the researcher presents a priori, a descriptive theory, which served as a framework for the study (Yin, 2009). In this study the theory of didactical situations in mathematics served as the descriptive theoretical framework that guided the study (Brousseau, 1997). Once the study is grounded in a theoretical framework, Yin (2009) identified five components of the research design: (1) the studies questions; (2) the studies propositions, if any; (3) the study’s unit(s) of analysis; (4) the logic linking the data to the propositions; and (5) the criteria for

interpreting the findings. In this study no propositions was formulated, because the primary goal was to describe the phenomenon as it occurred in its natural environment.

## **Methods**

The intent of this investigation was to describe the nature of meta-didactical slippages that occurred in in a ninth grade mathematics classroom, and to investigate how those slippages affected students' constructions, and productions on a unit of study. The analysis of the didactic classroom interactions determined the occurrence of slippages. The study thus maintained clear focus on the interactions between the students, teacher and the mathematics. To set a context for the study, I begin with a description of the school, classroom and mathematics content.

**School context.** The investigation of this study took take place in one 9th grade mathematics classroom, located in a high school in a southeastern state. In the fall of 2013 enrollment in this school was approximately 1400 students. The ethnic makeup of the school population was 70% African American, 20% White, 5% Hispanic and 5% Multiracial. The school is on an A/B block schedule, each class meets for 90 minutes every other day. Approximately 450 of the student population were ninth graders, and the school enrolled 20-32 students in each of ninth grade mathematics class.

Choosing the ninth grade was important because studies (e.g. Styron and Peasant (2010)) pointed out that ninth grade students struggled with the transition from middle school to high school because of higher expectations from teachers, additional homework, and the freedom of selecting the most appropriate classes and activities to prepare them for life after high school. In the study school the ninth grade mathematics was considered to be an area of weakness, based on performance on the State's mandatory End of the Course Tests. Additionally, I chose the ninth grade because I

inferred that an understanding of the phenomenon of meta-didactical slippage would inform the design and implementation of mathematics teaching and learning in subsequent years.

**Classroom context.** The study class consisted of 23 students, 14 females and 9 males. Of the 23 students enrolled in the study class, 21 were African American, 1 White, and 1 Hispanic. The teacher was an African American male, who taught high school mathematics for over 12 years. The teacher was considered to have high mathematical content knowledge and pedagogical skills. The classroom was equipped with a large dry erase board, digital projector and projector screen. The classroom was also equipped with a number of small individual dry erase boards which students often used at their desks. The large dry erase board and projector were the dominant modes of presenting information to the class. Students frequently used the large dry erase board in the front of the room to report on their solution, and or give plenary discussion after small group. Students predominantly sat in pairs in rows facing the dry erase board. Student was allowed to move at will to other parts of the room either to sharpen pencil, or to consult with other students not in their pair, or their group. Although, the school was on A/B block schedule, the ninth grade mathematics classes met every day. This was a district's initiative in order to improve the mathematics performance on the state's mandatory end of the course tests.

**The mathematics content.** As mentioned above, the school district added a "double dose" of mathematics for the ninth grade classes. All ninth grade mathematics class met for 90 minutes every day. The school was in their second year of implementing the Common Core State Standards for Mathematics. The target unit in this study dealt with relationships between quantities. This was the first of three units of study for the

semester, but the school district decided that students would only focus on the relationship between quantities unit of study for the semester. Additionally, the students would review pre-requisite skills for the ninth grade curriculum. Appendix E is a description of the curriculum standards for the relationship between quantities unit, which represented the target mathematical knowledge for ninth grade students.

**Selection of participants.** Criterion sampling was used to select a school in the southern region of the United States. The study school met the following criteria: (a) the principal allowed access to the school (b) the teacher and students consented to participate in the study, (c) the teacher had a minimum of three years of teaching experience, and (d) the study class was a ninth grade mathematics class that was implementing the Common Core State Standards for Mathematics. In this study, the teacher participant selected the study class from among six sections of ninth grade classes that were taught by the teacher.

**Data collection and instrumentation.** To answer the research questions proposed, I used four data collection techniques: (a) collection of document artifacts, which included student work samples and teacher lesson plans; (b) direct observation (c) open ended interviews, conducted with the teacher; and (d) researcher introspection. Data collection instruments include the interview protocol, the observation log, and the documents artifacts.

**Document artifacts.** I collected artifacts such as teacher lesson plans and lesson notes (content notes, tasks, problem sets, etc.) and students' work sample from the site. The documents produced by the teacher provided *a priori* information of the intended knowledge that was at stake for a particular lesson. I used this a priori information to determine whether or not a meta-didactical slippage occurred. I used the students' work

sample to identify evidence of the affects of any slippages, and to provide insights about students' conceptions about the mathematics content.

***Classroom observations.*** Participant observation was the main data collection technique. The study class met 5 days per week for 90 minutes. This allowed me more sustained observations in the classroom. The classroom visits were videotaped in order to facilitate analysis of the data. During observation of the lessons, I recorded data using an observation log derived from Table 2 (see Appendix A). I also used a composition notebook to record observations that I could not immediately place on the observation log. I observed generally, but more focused observations were triggered by a perceived failure in a didactic sequence. During observations, I looked for two specific observable instances to indicate a perceived failure in didactic sequence. One instance was by students' didactic questions, which indicated a failure, and the other was whenever the teacher re-explained a didactic sequence, or provided more explanation, for content that was not asked for by any student. I also used gestures and other visible body language that may indicate that the teacher perceived a failure. Once I perceived a failure, I described that episode according to the type of situation and the type of knowing that was manifested. I observed a total of 30 classroom sessions.

***Interviews.*** Three formal open-ended interviews were conducted with the classroom teacher over the study period. Interviews with the teacher were videotaped and transcribed to facilitate analysis. The first interview was conducted at the beginning of the study. The second interview was conducted after analysis of initial data, and the final interview was conducted towards the end of the study. I held ongoing informal conversations with the teacher for the duration of the study. During the informal conversations, I asked the teacher to watch video clips from the class to focus our

conversations on specific interactions or specific mathematical concept. I also inquired about student representations on student work, or asked about the objective of the lesson and the target knowledge for a particular lesson. Some times the teacher insisted on discussing particular students, particular moments, or feelings that were associated with an episode.

The goal of this study was to generate depth of understanding. Thus, the interviews were open-ended and in-depth, following what Rubin and Rubin (2005) called responsive interviewing. These unstructured interviews with the teacher helped me to achieve depth of understanding by going over context, dealing with complexities of overlapping themes, and paying attention to meanings and situations (Rubin & Rubin, 2005). I used an interview protocol to guide with the interviews (see Appendix B), but the specific observations, and classroom situations were the main guide for the interviews.

To structure the interview with the teacher, I asked three types of questions: descriptive questions, structured questions, and contrast questions (Spradley, 1979). Descriptive questions were designed to explore the broad topics in the research, and enable me to collect ongoing samples of the informants' language (Spradley, 1979). For instance, a descriptive question was "Can you describe in detail all of the manipulative you use in your mathematics classroom?" The questions in the diagonal of the description matrix in Appendix C are also examples of descriptive questions. Structured questions enabled me to discover basic units in the participant's cultural knowledge. An example of a structured question was "How does your teaching incorporate the use of student mini dry erase boards?" Finally, contrast questions probed into the dimension of meanings that the participants used to distinguish objects and events (Spradley, 1979). For example,

different actions or feelings could be associated with a single domain. An example of a contrast question was “what are the ways that students use calculators in the mathematics classroom?”

*Researcher introspect.* In addition to my field notes where I wrote what I saw, I also kept a journal of my reflections. I wrote about my challenges, my fears, my mistakes, and things that I could do better. I also reflected on how I was feeling in the moment. At first my journal reflections was predominantly disappointments that felt. For example, my first few days I was overwhelmed. I felt like I was wasting time. I did not know what to observe. After reading what I was thinking, I became more focused in my observations. Thus, I used the data from my journal to improve the process of observing and being in the classroom. Over time I began to write analytic notes, and ideas that I got in the moment.

All records taken from the site were kept private to the extent allowed by law. Pseudonyms were used on study records, and only the researchers had access to the information provided. The video files and typed notes were stored on a password protected external hard drive, kept in a locked drawer in my office. The school name, the names of participants, or other facts that might point to the participant’s identity were not reported. The findings were summarized and reported in-group form. Table 5 is a summary of all the data I collected.



Table 5

*Summary of the Data Collected from the Research Site.*

Data/Evidence Collected	Purpose
1) Videotapes in classroom	Record classroom in action a) Student and teacher interaction b) Verification of classroom observation c) Look for instances of perceived failure d) Look for evidence of affects of slippages on students' learning
2) Classroom Observations	a) Record my observation of classroom b) Record my reflections c) Look for evidence of the affects of slippages on students' learning d) Verification of video data and document data
3) Interview with Teacher	a) Member Checking b) Conformation of themes from video, and observations c) Probing issues d) Understand teacher's perspective
3) Documents (Student work sample) (Teacher Lesson Plan/Notes)	a) Identify instances of possible failure b) Identify possible (mis)conceptions c) Identify aims/target knowledge d) Provide insight on students' construction e) Verification of classroom observation

### **Procedure**

**Initiating entry.** In order to gain access to the study site, I contacted the principals in the school district to obtain permission to work in the school, and to obtain an estimate of the number of ninth grade mathematics teachers that met my study criteria. One of the principals in the school district gave approval to conduct research in the

school. After obtaining approval from the school principal, I contacted the school district, and completed the required documents in order to obtain permission to conduct research in the school district. Once the district IRB was granted, I completed the required IRB application for Georgia State University's IRB department, and was granted IRB approval (see Appendix D).

This study was conducted over 15 weeks, for 5 days per week. On my first day, I obtained a signed consent form from the teacher participant and had a short conference with the teacher before the class began. While I waited outside the classroom, the teacher informed the students in the study class that they would have an opportunity to engage in research, and that the researcher was coming to talk to the class. I was invited inside and the teacher introduced me as a student researcher and mathematics teacher. I explained the research process and provided each student with an approved consent form for his or her parents. I read and explained the consent process to the students and assured them that they did not have to participate, that non-participation would not affect their grades, and they would not be treated differently in any way if they chose not to participate in the study. I also explained the child assent procedure, and explained that they could not be forced to participate even if their parents signed the consent form. I provided the child assent forms and advised them to take it home and return it with the parent consent form. I designated a tray for the consent forms, and asked the student to return the forms in sealed envelopes to the designated area. I did not collect any consent forms on the first day and no videotaping was done on the first day.

**Conducting the study.** On the second day, I collected 15, signed parental informed consent and 15 child assent forms. Students placed the forms in a designated spot in the classroom. I took out the forms and left the envelopes in the tray. I did not

videotape on the second day, but I stayed in the classroom and wrote field notes. For the remainder of the first week I observed the class and took field notes but I did not videotape. By the end of the first week, I obtained informed consent and child assent forms from all 23 students in the class.

The second week I began to videotape. I conducted classroom visits every day that the class met for the duration of the study period. Each class lasted for 90 minutes. The class worked on a unit of study entitled “Relationships between Quantities”. Appendix F is description of the unit. During each classroom visit, I observed the interactions between the student, the teacher, and the content of mathematics. I also observed students working during independent work and collected student work samples (e.g., student response to problem sets).

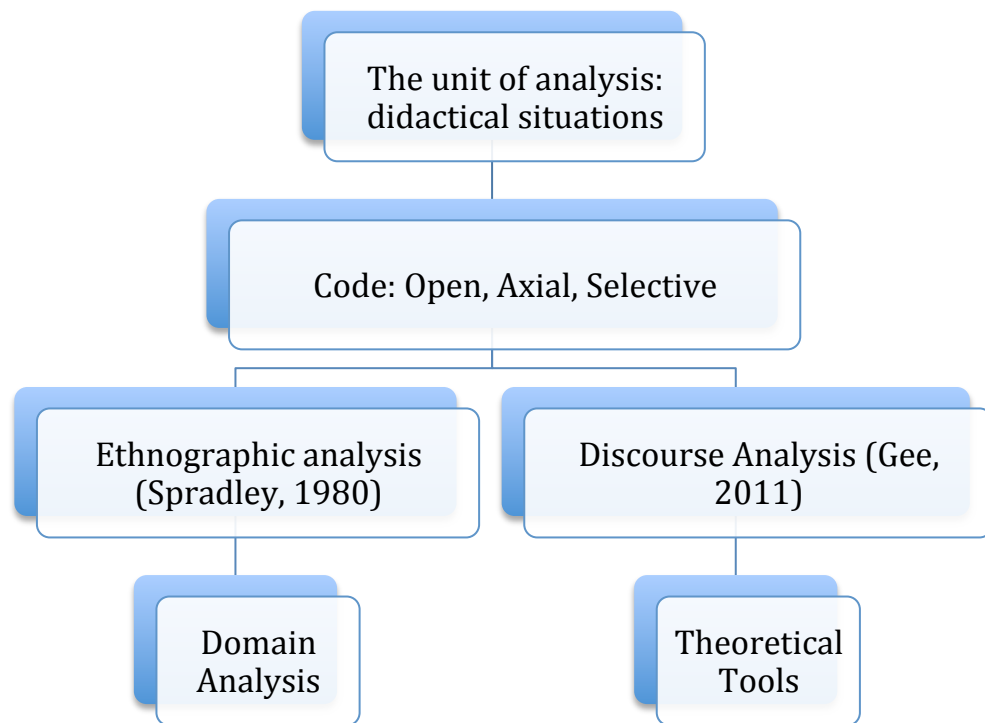
At the end of each day, I watched the video recordings in order to get a sense of what went on in the class. I also created clips of episodes from the class that I did not understand, in order to view with the classroom teacher at a later time. I then transferred the video recordings from the camera to my computer for data reduction and editing.

### **Data Analysis**

The analysis of the data began with transcribing the field notes and video taped interviews with the teacher. As I transcribed the field notes and interviews, I noted key ideas, phrases, and mathematical concepts that emerged. The unit of analysis for this study was the classroom didactical situations (Bussi, 2005). More specifically, I parsed the didactical situations (or episodes) for instances of a perceived failure in the teaching endeavor (Brousseau, 1997). After watching the entire videotape for the day, I partitioned the videotapes into 5-second nodes, using Apple’s iMovie video editing software. The video editing software allowed me to select nodes for which a perceived failure occurred.

I collected these nodes and used them to create a separate video file for more focused analysis. The phenomenon of interest was meta-didactical slippage. The aim was to improve our understanding of meta-didactical slippages to inform what we know about didactical situations in mathematics, and more generally, the teaching and learning process.

In order to achieve a more fine grained analysis two analytic techniques were used: ethnographic analysis using Spradley's (1998) model and discourse analysis using Gee's (2011) model. Episodes from the classroom were coded using the theory of didactical situations in mathematics to guide the construction of codes. A summary of my analytic procedure is shown in Figure 3. In order to maintain focus throughout the analysis, I asked the following questions of the data: (a) what is the genesis of these slippages? (b) how may this slippage be identified? (c) what are their attributes? (d) what are their affordances? (e) can they be predicted? and (f) how can they be controlled if possible? I used a combination of hand coding and computer qualitative software coding. I used the ATLAS.ti qualitative software to manage the data files and to retrieve codes quickly.



*Figure 3.* Summary of analytic procedure followed in this study.

### **Coding**

In both analytic techniques, I used three coding techniques: a) open coding, which involves initial identification of topics; b) axial coding, where categories are specified in terms of the actions/interactions that give rise to it; and c) selective coding, which entails identification of the core category on which the analysis is focused (Ezzy, 2002). The first phase of coding was open coding as specific domains and/or themes emerged during the didactical interactions. This first phase of coding was followed by axial coding and then by selective coding.

**Open coding.** Broadly speaking, open coding is the initial identification of topics, which consists of: (a) exploring the data; (b) identifying the units of analysis; (c) coding for meanings, feelings, and actions; (d) experimenting with the codes; (e) compare and contrast events, actions, and feelings; (f) integrating codes into more inclusive codes; and

(f) identifying the properties of codes (Ezzy, 2002). In this phase of my coding I used concepts derived from TDSM, and I identified perceived failures, according to the affordance and genesis.

**Axial coding.** Axial coding according to Ezzy (2002) is to integrate codes around the axis of central categories. Axial coding involves: (a) exploring the codes identified in the open coding phase, (b) examining the relationships between the codes, (c) specifying the conditions associated with a code, and compare codes with preexisting theory. In this phase I combined classroom codes, which were essentially synonyms. I also checked with the teacher on my selected episodes and the particular codes that emerged. This resulted in further reduction of the data, because the teacher requested that some episodes not be included in my analysis.

**Selective coding.** Selective coding involves the identification of the core categories around which the analysis is focused. This final stage of coding consisted of: (a) identifying the core code or central story in the analysis; (b) examining the relationship between the core codes and other codes; and (c) comparing the coding scheme with preexisting theory. In this process I identified four themes that emerged as slippages. Each of the four themes emerged as meta-didactical slippages. At this stage I went back to watching the tapes, reading the transcripts, looking at student work, and looking at teacher lesson notes.

### **Ethnographic Analysis**

Ethnographic analysis is one way of analyzing, and making meaning of social settings. According to Spradley (1980), the goal in ethnography is to “discover the cultural patterns people are using to organize their behavior, to make and use objects, to arrange space, and to make sense out of their experience” (p.130). The mathematics

classroom is considered to be a form of culture. Spradley (1980) identified nine major dimensions of every social situation: (1) *space*, the physical place or places; (2) *actor*, the people involved; (3) *activity*, a set of related acts people do; (4) *object*, the physical things that are present; (5) *act*, single action that people do; (6) *event*, a set of related activities that people carry out; (7) *time*, sequencing that takes place over time; (8) *goal*, the things people are trying to accomplish; and (9) *feeling*, the emotions felt and expressed.

These dimensions served as a guide as the classroom interaction was observed and analyzed. As part of the observation and analysis, a “descriptive question matrix” (Spradley, 1980, p. 80) was used (see Appendix A) to capture and probe the interrelation among the nine dimensions.

According to Spradley (1979), domains are the first and most important units of analysis. Domain analysis was conducted to discover units of meaning that unfold in the didactical situation of the mathematics classroom. Domain analysis “involves a search for the larger units of cultural knowledge called domain” (Spradley, 1979, p. 94). An ethnographic domain analysis was appropriate for this study because I was interested in looking for the units of meaning that can be attributed to the cultural category of meta-didactical slippage. In working through the data, I focused on the following questions: What act, or action indicates a perceived failure? What are specific units of this domain? And what feelings or actions are associated with each unit? This process helped me to describe the nature of meta-didactical slippage.

I first made a preliminary domain search to analyze the interview data by using the verbatim interview transcript to search for names for things (mathematical concepts, skills, etc.) within the transcripts. I then look for possible names for categories of cultural

knowledge, followed by common terms that belong to the categories already identified (Spradley, 1979). From this preliminary search, I selected the semantic relationships that seemed important to conduct a domain analysis, following Spradley's (1979) six-step method. The six steps are: (a) selecting a single semantic relationship, (b) preparing a domain analysis worksheet, (c) selecting a sample of informant statements, (d) searching for possible cover terms and included terms that appropriately fit the semantic relationship, (e) formulating structural questions for each domain, and (f) making a list of all hypothesized domains (Spradley, 1979).

In order to maintain focus, and to increase validity, I initially followed a sequence of peer debriefings along with checking interpretations with the participating teacher, transcribing, reading, and coding early data, and writing journals and memos (Ezzy, 2002). The data was sorted, resorted, organized, reorganized, labeled, and relabeled in order to answer the research questions in a meaningful way by providing thick descriptions of the occurrence and nature of meta-didactical slippage.

The document artifacts were analyzed by focusing on the processes of production, consumption, and exchange (Prior, 2003). Specifically, I asked questions such as, "What was the student trying to convey?" "What conception is demonstrated by this production?" and "For what reason was this document produced?" I also parsed student work samples for mathematical errors. When an error was located, I tried to reconstruct the sequence of constructions that possibly occurred in order to produce the written product. Doing this allowed me to identify, conceptions, that possibly produced the written product.



## Discourse Analysis

Discourse, for Gee (2011), is any meaningful use of language, including gestures, where the primary function is to support the performance of social activities, social identities, and human affiliation within cultures, social groups, and institutions. Thus, discourse refers to the ways of representing, believing, valuing, and participating with all of the sign systems that people have at their disposal.

Gee (2011) argued that language is very important in social situations because: (a) language is connected to engagement of social activities (e.g., classroom lessons), (b) language is connected to formation of social identities (e.g., students as learners), (c) language is connected to interactions of social groups (e.g., classroom communities), and (d) language is connected to the founding of social institutions (e.g., schools).

The analytic technique is comprised of two levels. In the first level the researcher reads the data to identify what is represented, what is not represented, and what broad themes and patterns emerged. In the second phase, the researcher employs tools of discourse analysis to analyze how the structure and form of language expresses meaning (Gee, 2011). These tools are specific questions that the researchers ask of the data. Gee (2011) presented 27 tools and suggested that the researcher apply all 27 tools to the data. However, a study can focus on different tools depending on the research questions posed.

This study used four of the 27 tools. These tools are, what Gee classified as theoretical tools: situated meaning (how is the language used in context?), social languages (what are the varieties of the language between participants?), figured worlds (are there taken-for-granted theories embedded in the language?), and discourses (what identities are enacted in time and space in this exchange?) (Gee, 2011).

**Situated meaning tool.** The situated meaning tool, which draws on cognitive psychology (Gee, 2011), states that “for any communication, ask of words and phrases what situated meanings they have. That is what specific meaning do listeners have to attribute to these words and phrases, given the context and how the context is constructed?” (p. 153). This tool helped me to make sense of the situated meanings that the teacher and students constructed in the teaching and learning process.

**The social language tool.** The social language tool draws from sociolinguistic theories, which helps us to understand language work to allow humans to carry out and enact different types of social work and socially situated identities (Gee, 2011). The social language tool states that “for any communication, ask how it uses words and grammatical structure (types of phrase, clauses, and sentences) to signal and enact given social language” (Gee, 2011, p. 161).

**The figured world’s tool.** The figured worlds tool, according to Gee (2011) draws on theories from psychological anthropology about how groups of people use narratives and images to make sense of the world. This tool states that:

for any communication, ask what typical stories or figured worlds the words and phrases of the communication are assuming and inviting listeners to assume. What participants, activities, ways of interacting, forms of language, people, objects environments, and institutions, as well as values are in the figured worlds? (p. 171).

The figured worlds tool then can play an important role in the mathematics classroom, because much of the communication takes place in the mathematics classroom uses symbols. Some symbols have meanings to a broad cultural group; however in the mathematics classroom, teachers and students construct local symbols, which are known to the group but not to outsiders.

**The “big D” discourse tool.** The final discourse tool I chose is the “big D” discourse tool, which draws from a variety of areas, such as cultural anthropology, psychology, philosophy, etc. to show that meanings goes beyond human minds and language to include objects, tools, technologies, and network of people collaborating with each other (Gee, 2011). The tool asks from the communication:

How the person is using the language, as well as ways of acting, interacting, believing, valuing, dressing, and using various objects, tools and technologies in certain types of environments to enact a specific socially recognizable identity and engage in one or more socially recognizable activities (Gee, 2011, p. 181).

Thus, it is important to identify what sorts of identity the speaker is enacting in the classroom. Moreover, what actions, interactions, values, and beliefs are associated with the communication? This allowed me to identify the in-depth details of the phenomenon in order to provide rich description of the nature of meta-didactical slippages in the mathematics classroom.

### **Researcher Role**

The primary goal of the qualitative researcher is to better understand human behavior and experience, as well as to grasp the process by which people construct meaning and to observe what those meanings are (Bogdan & Biklen, 2007). Qualitative research according to Denzin and Lincoln (2005) is a situated activity that locates the observer in the participants’ world. Moreover, qualitative researchers study things in their natural setting, attempting to make sense of or interpret the phenomena in terms of the meanings people bring to them (Denzin & Lincoln, 2005).

I visited the study classroom every day for the duration of the study. During the visits I observed the classroom interactions from the position of a passive observer. I did not interfere or assist with the planning or instruction of the lessons. I did not engage in

conversations with the students, and I did not attempt to help the students on problems. I also did not interfere with any classroom management issues. Passive observation was necessary, in order to not disrupt the didactical contract of the classroom. Any active participation would not only disrupt the existing contract, but would also establish a new didactical contract.

During the study period, the students became increasingly conscious of the camera, and some students wanted to operate the camera and move around the classroom with the camera. Other students wanted to perform in front of the camera, which would block the view of the classroom. At the teacher's request, I allowed the students to take turns recording. This seemed to accelerate the students' acceptance of the camera equipment, and researcher in the classroom. Students eventually lost interest in the camera.

The qualitative researcher is seen as *bricoleur* (Denzin & Lincoln, 2005). A bricoleur is essentially a person who works with his or her hands to get the job done. For Denzin and Lincoln (2005), there are many types of bricoleurs, such as interpretive, narrative, theoretical, and methodological. The interpretive bricoleur produces a "pieced-together set of representations that is fitted to the specifics of a complex situation" (Denzin & Lincoln, 2005, p. 4). I used a bricolage of analysis and interpretation to help me to probe deep into the phenomenon of meta-didactical slippage in the classroom. I frequently made comparisons of the different forms of data, such as the observation and the interview data, along with the different analytic techniques in order to attend to issues of validity.

## **Trustworthiness**

Validity and reliability has very different meanings in qualitative and quantitative research paradigm. For instance in quantitative research, principles such as generalizability, neutrality, controllability, and replicability are essential (Bogdan & Biklen, 2007; Cohen et al., 2013; Denzin & Lincoln, 2005). In qualitative research, however, validity and reliability are replaced with principles such as credibility, consistency, applicability, trustworthiness, and dependability (Denzin & Lincoln, 2005). The central imaginary of validity according to Richardson (2000) is that of the crystal, which combines symmetry, and substance with a variety of shapes, multidimensionalities, and angles of approach. Therefore this study strived for crystallization, because crystallization “provides us with a deepened, complex, thoroughly partial understanding of the topic” (Richardson, 2000, p. 934). In this qualitative case study, several techniques were used to enhance the trustworthiness of the data and interpretation. These techniques included prolonged engagement, persistent observation, member checking, and crystallization (Berg, 2009; Bogdan & Biklen, 2007; DeWalt & DeWalt, 2002; Rubin & Rubin, 2005; Spradley, 1980; Yin, 2009). The trustworthiness of the research is increased when the researcher follow clearly defined principles. For instance, Yin (2009) established three principle of data collection which helps to deal with problems of validity and reliability of the case study. The principles are: (a) using multiple source of evidence, (b) creating a case study database, and (c) maintaining a chain of evidence.

Using multiple sources of evidence and multiple analytic techniques helped to increase the trustworthiness of the study, because it allowed me to address a broader range of historical, attitudinal, and behavioral issues (Cohen et al., 2013). According to

Yin (2009) the findings or the conclusions in a case study are more likely to be more convincing and accurate if it is based on several different sources of information. As stated earlier, this study used four sources of data and two analytic techniques.

The second principle was accomplished by creating a data inventory. Each datum, such as interview transcripts, observational transcripts, and document collected in the field, was carefully annotated and recorded in the data inventory. For data security, the inventory does not contain the actual data. The actual data was stored in a formal, presentable database (Yin, 2009).

In order to accomplish the third principle, I carefully document pertinent information, such as time, place, and conditions under which the data was collected, for each piece of datum. I took precaution to ensure that the data presented in the final report was indeed the data that was collected (Yin, 2009). Additionally, I ensured that each conclusion was tied to the research question, and that evidence from the data was clearly identified. I used file names to identify the data.

### **Study Limitations**

The following limitations applied to this study:

- Meta-didactical slippage is a complex construct. The complexity of situations examined in this case study was difficult to represent simply. The task of identifying and describing them was very challenging. For example, by providing thick descriptions of one aspect of a situation simultaneously encapsulate other areas. It was not known a priori when meta-didactical slippages would occur, or if one would occur. Furthermore according to the theoretical framework, the teacher could follow different paths if the teacher

perceived a failure. Thus, there was no guarantee that the teacher's negotiation of the didactical contract would result in a meta-didactical slippage.

- The volume of data produced in this case study made it difficult for data analysis. This possibly resulted in over simplification of the situation under investigation, and the inevitable reduction of data.
- The presence of the researcher and camera equipment in the classroom possibly changed the behavior of the teacher and students in ways that may have hindered the authenticity of the data collected.
- This study only investigated one unit of instruction, over a 15-week period. Studies over a longer period of time would reveal more about the nature of meta-didactical slippages.

## CHAPTER 4

### RESULTS

This study examined the nature of meta-didactical slippages that occurred in a ninth grade mathematics classroom, and catalogued any affects of these slippages on students' learning on one unit of ninth grade mathematics. The study was a qualitative case study conducted in one, ninth grade mathematics classroom with special focus on the “relationships between quantities” unit of study. The chapter is organized in terms of the two specific research questions posed. The chapter first reports on the nature of meta-didactical slippages that occurred in the classroom, and then it presents the ways that those slippages affected students' conceptual understanding on the “relationships between quantities” unit.

#### **Research Questions**

The research questions that guided this study were:

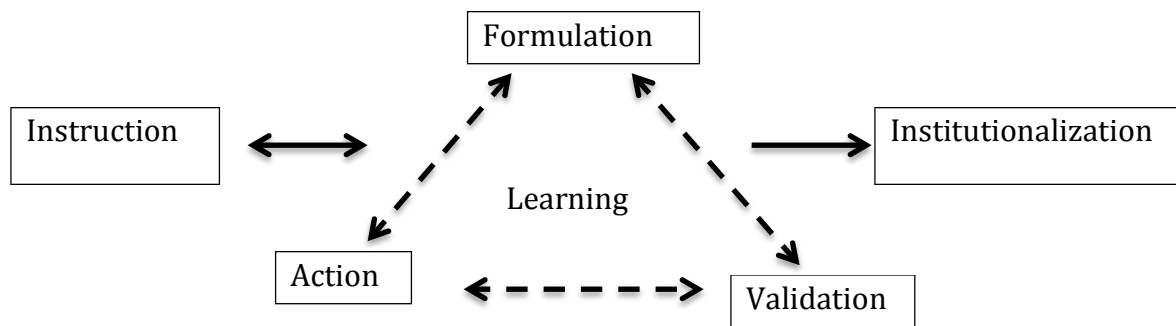
1. What is the nature of meta-didactical slippages that occurred in a ninth grade mathematics classroom?
2. How did the teacher perceive these meta-didactical slippages affect teaching and learning of a unit of analysis of 9th grade mathematics?

Previous conception of the effect of meta-didactical slippage was that it is a modality, which the teacher used to recuperate a perceived failed teaching. Thus, in the mathematics classroom meta-didactical slippage was based on the assumption that the teacher perceived a failure in a didactic sequence, and made an attempt to correct the error. However, the findings of this study showed that meta-didactical slippages in the mathematics classroom are non-trivial phenomena, which have profound impact on



student learning and retention of mathematics. Moreover, meta- didactical slippages often go unnoticed and thus uncorrected.

In this study I used TDSM as theoretical lens for my analysis and interpretation of the data. Figure 4 is a summary of the conceptual framework, which guided my interpretation and representation of the data. In the conceptual framework, learning predominantly takes place in situations of formulation, action, and validation.



*Figure 4.* Summary of the conceptual framework, which guided the interpretation of the data.

In the next section I report the findings. To report the findings, I selected classroom episodes for which it was relatively simple to describe the mathematical ideas at stake and the milieu that sustained the didactical situation. Moreover, I selected episodes that the teacher approved.

### Findings

In this study four themes emerged as illustrative of the nature of meta-didactical slippages: (1) over-teaching, (2) situational bypass, (3) language and symbolic representation, and (4) the design of didactical situation. Each theme emerged as an instance of meta-didactical slippage. For the purpose of this report I will number the slippages. I will also provide two episodes from the classroom to instantiate each slippage. The episodes are also numbered. The numbering is purely stylistic. Finally,

there is a linkage between slippages. Thus, the episodes selected to illustrate a particular slippage, can simultaneously illustrating other slippages. Moreover, the slippages do not always operate independently in the mathematics classroom.

### **Slippage 1: Over-Teaching**

Over-teaching emerged in my analysis as a slippage. The word “teaching” is a very complex and nuanced word in the mathematics classroom. There are different meanings when the teacher and students use the word “teaching”. Furthermore, the word teaching has different meanings when the same student used it at different moments in the mathematics classroom. The meaning of teaching in this context refers to the teacher “giving instruction”, often at the board. Therefore, the teacher “giving instruction to all the students at the board” manifests the slippage of over-teaching. This slippage however, cannot be identified by classroom observation alone. For, it only became known through the teacher’s reflection on the classroom episode.

**Episode 1.** This episode is an example of the slippage of over-teaching. What follows is a description of the context, and the narrative and transcript from the videotapes.

**Context.** This episode occurred on my 10<sup>th</sup> observation of the classroom. In the episode the teacher presented a linear equation with variables on both sides, to be solved using the method of graphing. The problem was first presented to the class as an opening activity, at the beginning of class to be solved algebraically. The students worked on the problem individually for five minutes. After the five minutes the teacher asked for a volunteer to come to the board. One student volunteered to present her solution on the board. As she presented her solution on the board, the students sitting at their desks, asked her to explain her solution. The student at the board was able to explain all the

steps that she took to complete the problem. After a healthy applause from the teacher and the other students, the student at the board went back to her seat. The teacher now asked the students to solve the problem graphically. The teacher's lesson notes showed that the problem was designed to address two objectives: (a) to choosing an appropriate scale to graph linear equations, and (b) to separate an equation with variables on both sides of the equation into two functions of the form  $f(x) = g(x)$ . To achieve the objectives the students were required to graph both functions on the same pair of axes in order to locate the solution. The teacher wrote the problem situation on the board as: Solve  $8x - 4 = -10x + 50$  by graphing.

*Narrative and transcript.* After presenting the problem on the board the teacher elicited feedback from the class as he went through a moment of instruction. The teacher reminded students that “*here they have two expressions of the form  $y = mx + b$* ”, and that they were to plot them on the same pair of axes. The teacher also reminded students that the solution is where the two graphs meets, and that they already knew what the solution was:

- Teacher: at this point you guys should know what to do because we spoken on that. If am graphing it what's my first step anybody remembers?
- Student 1: Variables on one side
- Student 2: Isolate the y
- Teacher: Isolate the y? Where is the y?
- Student 2: No where
- Teacher: [smiling at the response then wrote on the board  $8x - 4 = y$ ], You remember we say you gonna set up two equations...
- Student 3: Where did you get the y from?
- Teacher: We forgot that is the generic.... If we have the... [interrupted]
- Teacher: one second... If I have this, right? equals y [wrote  $8x - 4 = y$  on the board] and I also have negative  $10x + 50 = y$  [wrote equation on the board] the fact is they are both equals y so we say we could set them equal to each other and solve algebraically... but if I want to solve it graphically we said you have to go back into your two separate equations...
- Student 4: You lost me! Where do you get the y from?

- Teacher: I am coming here again ok fine... I see your hand yes... Yes [pointing to a student]
- Student 5: [not a didactic question]
- Teacher: Can we finish this first? Ok... We have two expressions that are equal to each other okay. This is an expression [circled  $8x-4$ ] and this is an expression [*Circled the  $-10x+50$* ]. I am saying both expressions are equal to  $y$ . Cause again that's how I get my output, so I can set them equal to  $y$  in which case that gives me two equations and now I am going to graph each equations so that's what we are going to do...[*overlap by student question where did you get the  $y$ ?*] okay... So Let's go to the graph sheet now and for your ... [*Teachers draws  $x$  and  $y$  axes showing positive quadrant*]
- Student 6: So I can graph negative 4...[*This was more of a question, but the response of the teacher indicated that it was not a question*]
- Teacher exactly... exactly now we could graph each equation separately... So you with me, right? My  $x$  value I am going to have increments of 1, 2, 3, 4, 5... and for this right here I am going to suggest we go up by 5 [*marking the  $y$  axis with 5, 10, 15... as he speaks*]. So could you just set your graph up for me please... we gonna go by fives! (Transcript of Video SL\_12-5-13)

The problem was not an unfamiliar situation, because the class had already discussed the process involved in solving such an equation in a previous lesson. The silence met by the teacher's comment that at this point they should know what to do, was broken by the teacher's almost desperate plea for anyone to tell what is the first step in graphing a linear equation. At this point, the students as if felt compelled to answer the teacher's question started to respond. Student 1 suggested getting the variables on one side but the teacher did not acknowledge the response. While viewing the episode the teacher explained, "*I ignored that response because the students already solved the problem algebraically. Getting the variable on one side would be an algebraic technique, but the focus was to solve graphically*" (INT\_11-18-2013). The teacher responded when student 2 said, "isolate the  $y$ ", with the question "*where is the  $y$ ?*" The question was not posed only to student 2, but to the whole class. This was an indication that the teacher perceived that the students did not sufficiently understand what to do or did not

remember the previous lessons where they discussed similar situations. The teacher wrote the two equations on the board and told students to remember that this was discussed before. The two equations that the teacher wrote on the board prompted other student questions, for instance student 3 asked, “*Where did you get the y?*”

Experience with this curriculum demonstrated that separating a linear equation into two linear functions of the form  $f(x) = g(x)$  is a conceptual challenge for ninth grade students. Evidenced by the students’ questions, it appeared that although the teacher presented and explained the process of obtaining two linear equations, the students still had difficulty with the concept. For, as soon as the explanation was given, another student asked the question again using the same words as before, “*where do you get the y from?*” The teacher recuperated the situation by repeating the instruction. The repetition of the instruction however, seemed to do very little to help the situation, and the episode continued with the teacher giving instruction at the board.

The decision to remain in a situation of instruction therefore prevented the situation from progressing to a situation of action, formulation, and validation. This decision however, was not intentional. It was the teacher’s intension to “stop teaching”, and to transfer responsibility of the problem to the students. In an interview, the teacher expressed his frustration with this episode:

*I definitely over teach this one. I really don’t know why I pushed pass the goal of the problem. I intentionally selected this problem so that the students would have to choose a scale for the y-axis that is different from 1, 2, 3, which we always use. Because, I know the standard clearly ask them to be able to choose an appropriate scale, and that is what they are going to be tested on. I did not expect the students to have so many questions on separating the two equations, because we spoke on that before. So I just rushed through it... like ok fine (INT\_11-18-2013).*

The teacher's decision to remain in a situation of instruction was therefore influenced by the questions that the students were asking. The teacher did not expect the students to have difficulty separating the equation into the two functions of the form  $f(x) = g(x)$ . Students' questions thus, accounted for the teacher's decision to re enter a situation of instruction. For example after student 3, asked, "*where did you get the y from?*", the teacher initiated a didactic sequence to explain. Immediately after the explanation another student, who seemed to be engaged in the lesson, asked, "*Where did you get the y from?*" The teacher at this point chose not to re-explain. The teacher advanced the solution of the problem to the graphing of the two equations. Still in a situation of instruction, the teacher suggested what scale to use for the x-, and y-axes. This according to the teacher was a contradiction to the intended purpose of the problem, and was not his intension. Therefore, the kinds of questions that students asked in a didactical situation are affordances to the slippage of over teaching. Additionally, intentionality is a major affordance to this slippage. The next episode is another example of the slippage of over teaching with a different affordance.

**Episode 2.** This is second episode that illustrated the slippage of over-teaching. The episode occurred towards the end of the teachers planned curriculum for the semester. The lesson was the 21<sup>st</sup> lesson that I observed.

**Context.** In this second episode the teacher presented a situation that can be modeled by an exponential function of the form  $f(x) = a(b)^x$ . The teacher wanted the students to solve the problem for a given value of  $f(x)$ , by using a method of "guess and check". The teacher presented the problem on the overhead projector: *In the absence of predators, the natural growth rate of rabbits is 4% per year. A population begins with 100 rabbits. The function  $f(x) = 100(1.04)^x$  gives the population of rabbits after  $x$*

years. (a) How long will it take for the population to double? (b) How long will it take for the population to reach 1000? In this situation the students did not have the knowledge of solving exponential equation using properties of logarithms and exponents. Therefore, the only way that they would be able to solve it is by “guess and check”.

**Narrative and transcript.** After the teacher presented the problem on the overhead projector, he instructed the students to use the method of guess and check. They were also required to document their process. The teacher paused for two minutes to allow students to work the problem. It appeared that the teacher perceived that the students did not know what to do because he began to explain the first part of the question on the board. After completing part (a) on the board the teacher pause again, but this time only for one minute. The class was silent and all the students seemed to be engaged with the problem. The silence was broken as the teacher began an instructional sequence for part (b). I thought the teacher did not wait long enough because I was also working the problem in my notes and I did not complete the problem. I selected the transcript for part (a) because it is shorter and because the two instructional sequences progressed in a similar mode:

Teacher [wrote the problem situation on the board: growth rate of rabbits:  $f(x) = 100(1.04)^x$  ], how long will it take to double? How long will it take the population to reach 1000? The way to do this is again by guess and check... yes, guess and check!

Student A: So what if you get wrong... guess and check?

Teacher: So what do you want to plug in for x?

Student B: 5

Teacher: let's plug in 5 and see what you get. What do you get?

Student C: 121

Teacher: 121? 121 point something? [*Overlapping talk*]. So in that case 122. Stop there for a moment for me. We got 121, what do you think we should try next?

Student D: 6

Teacher: Let's not try 6, because going up by 1 it's going to work but lets take a leap of faith and try something big

Student D: 8...9...

Teacher: 8? ok. Now if it is too big we can come back down [*teacher erase the 5, and the 122, then wrote 8 for exponent*] now what do you get?

Student C: 136

Teacher: you got 136. So we had 122, now we have 136 so we should kinda go a bit higher. Maybe we should go above 10...

Student E: 12

Teacher: Let's try 12. That's all I would do in that case

Student D: higher!

Teacher: you want to go higher still? Perfect, so how high?

Student E: 15

Teacher: 15. Again it's going to take us a little time but in this case... So what do you get?

Student F: higher, 180

Teacher: That give us 180, so we are close right?

Student D: we should try 17

Student E: no we should go higher! 18

Teacher: 18? Ok. So we get 202. So about 18 years. Obviously it will be a little less than 18 years but this will do. (Transcript of Video PSL\_12-5-13)

In this episode five students participated in answering questions, or posing a response. The other students sat silent, and wrote in their notebooks. The situation was designed to allow students to formulate hypotheses, act on their formulations, and validate their formulations with the feedback from their calculations by comparing their results with the 200 rabbits. This situation had an objective milieu in that students would get feedback from the situation as to the validity of their solutions.

The teacher asked the students to select a starting number. The students selected 5. This indicated that the students were familiar with the process of guess and check, because 5 seemed to be sufficiently arbitrary. The teacher asked the class to act on the 5, and "*see what you get*". After the students reported 121, the teacher elicited another formulation. The students selected 6, but the teacher rejected the 6 with no explicit explanation. Choosing the 6, would be ideal for the students to gain experience with the process of guess and check, because students would begin to see that the growth is slow by the next round of play. According to the teacher's plan, the teacher wanted students to



begin to select “*better numbers to try based on the answers they were getting*”. The teacher “over-teaches” in this situation by asking the students not to select 6. By examining, student work, and for the remainder of the semester, and my data collection period, I did not find any evidence that indicated that the students understood why the teacher did not want them to select 6, and 7, and so on. The teacher simply asked the students to take a “leap of faith” and go higher. It appeared that students were not ready to enter a situation of formulation with part (b) of the problem, because they did not begin to work on the assigned problem for more than 3 minutes. The teacher walked around the room and possibly observed that students appeared to be waiting on him to work the problem on the board. I was not sure at this point if the students did not know what to do or if they collectively know that the teacher would re enter a situation of instruction, but after about five minutes, the teacher went to the board and begin the problem in a situation of instruction. Thus the teacher restricted the progression of situation by entering an instruction sequence. Moreover the situation stayed in instruction, which lasted for 6 minutes. In this sequence the teacher suggested the numbers to use. This instruction therefore prevented the situation from progressing to a situation of action, formulation, and validation.

Conversation with teacher after watching the episode and discussing what happened, the teacher had this to say:

*I tell you... I felt a shift... I know what I wanted to happen, but I just did not know how to make it happen. I did not want them to quit on me either. So the big question is what is the optimal time to stop teaching? Should the teacher allow the situation to drop? And then try to recuperate it another time? What must the teacher do if after you pass the problem to the students they don't progress with the situation? (INT\_12-19-2013)*

When the teacher said I “felt a shift”, I coded this as being influenced by the constraint of time, because it was only three weeks before the semester ends. Thus the lesson was very close to the end of the semester, and close to a county mandated benchmark test. Of the three weeks left in the semester, the last two weeks was scheduled for review and testing. However, when the teacher asked, “*what is the optimal time to stop teaching?*” I coded this as an example of a more complex experience in the mathematics classroom. The concerns that the teacher expressed is kin to what Brousseau (1997) called a paradox of devolution. In this paradox the teacher wants the student to find the answer by his or her self, but at the same time the teacher wants the student to find the correct answer. Thus when the teacher asked, “*should you allow the situation to drop?*” the teacher is questioning the social responsibility of wanting the students to get the correct answer. Consequently, the slippage of over-teaching which restricts the progression of a situation from entering into situations of formulation, action, and validation is also afforded through the legitimate social responsibility of wanting the students to produce the correct knowledge.

### **Slippage 2: “Then We Can Practice Some”: Situational Bypass**

Practice emerged in the data as an important theme. Practice refers to the process by which the teacher assigns a problem-set for the students to complete in order to concretize, and formalize a skill that was previously learned. In this portion of my analysis I selected two episodes from the class. In these episodes the teacher forced the situation to progress directly from a situation of instruction to a situation where the students practice the skill. According to TDSM, practice is a situation of institutionalization whereby there is a canonization of a procedure or algorithm (see Table 2). Without going through situations of action, formulation, or validation, a

situation of institutionalization emerged as a meta-didactical slippage. I called this slippage a situational bypass.

**Episode 1.** This first episode is an introductory lesson. Examination of the teacher's lesson notes showed that the teacher planned to introduce exponential function, as well as to have students interpret each part of the exponential expression.

Unbeknownst to the teacher, the situation progressed from instruction directly to practicing. Moreover, the situation shifted from interpreting the parts of the expressions to evaluating exponential functions.

**Context.** This episode was the first lesson in a sequence of lessons dealing with exponential functions. This occurred on my 18<sup>th</sup> classroom observation. As this was an introductory lesson, the teacher presented a short PowerPoint presentation with the definition of exponential function, and some basic examples of exponential phenomena. The teacher gave examples such as the amount of money in a retirement account. Unlike solving linear equations and simplifying linear expressions, the exponential function was never introduced in lower grades. This was their first encounter with exponential functions. The students, sitting in pairs, copied the notes from the board and were expected to ask questions of the teacher as he goes through the presentation.

**Narrative and transcript.** After presenting the "notes" as the teacher called it, the following occurred:

Teacher:	All right let's move on
Student1:	No we're not ready
Teacher:	(120.0) Alright let us go on...Again all I am going to do is just evaluate...so we just going to evaluate
Student1:	We don't have to write the bottom one? [ <i>The bottom contained: An exponential function is of the form <math>f(x) = ab^x</math>, where <math>a \neq 0</math>, <math>b \neq 1</math>, and <math>b &gt; 0</math></i> ]
Teacher:	You should... again that is the generic formula...
Student2:	Question! What does x represent?

- Teacher: The exponent...Base on this right here... I'll show you in a minute...
- Student 2: I don't need it right now I just need to know what it is...
- Teacher: Once we get to an example it will become clear. Today all I am going to do is to show you how to plug-in into the formula. We are going to evaluate... I think we ready to move on...
- Students: Nooooo!... hold on!
- Student1: Now you see? I am not the only one
- Teacher: Alright... Let's go back one... Now If you copy the table, from the previous page, you will notice that this formula represents the table on the previous page... So why don't we go back a second, let me show you. [*A student sitting at the computer clicked the slides back to the table*]...  $f(x)$  is simply another way of saying  $y$ , right output  $f(x)$ . What's the initial value from the table?
- Student1: 2
- Teacher: 2? Alright 2 that's my a... What's my common ratio? [*the teacher referred to mathematical content from a previous lesson*]
- Student1: 3
- Teacher: So we multiplying by 3 each time, and of course raised to the  $x$  power [*teacher writes the function on the board*] ok, that's it so that's how we came up with the formula... initial value times the common ratio raised to a power... now go back... you have one minute you all. So J... [*student 2*], Your question was "What does the  $x$  represents? So if I am evaluating, we going to look at an example now, show you what the  $x$  is.. Could we move on?
- Students: No!
- Teacher: (10.0) Sounds like you are ready. [*The teacher presented problem situation: The function  $f(x) = 500(1.035)^x$  models the amount of money in a certificate of deposit after  $x$  years. How much money will there be in 6 years?*]...So again that is all I am going to do today, just evaluating... J you remember order of operation? What comes first? What operation do you see?
- Student2: Parenthesis
- Teacher: You see parenthesis? What is inside the parenthesis?
- Student2: 1.035
- Teacher: So there is really nothing to do in there... So two operations
- Student 2: The second one is exponent
- Teacher: The second one is exponent what is the first one?
- Student 2: Parenthesis
- Teacher: Parenthesis is not! Is nothing in the parenthesis! What does it imply?
- Student 3: Multiplication
- Teacher: Multiplication! That is what it implies right? Which one would you do first?
- Student 4: Exponent
- Teacher: Exponents... So you have to do exponents before you multiply [*Writes  $f(6) = 500(1.035)^6$  on the board*]. Now do you see the 6, we are going to replace  $x$  with 6 which is exactly what you said, and that's all we gonna do

- today which is evaluate. So 500 times 1.035 raised to the 6 power is 1.229 so basically exponents is first and now multiply so  $f(6) = 614.63$
- Students: [talking among themselves] that is not a lot after 6 years?
- Teacher: You all want to go into that? [Teacher heard the student comment and responded] That is for tomorrow. I just want to show you how to put it in, I am not going to give you too much right now... but it is based on the common ratio, so the bigger the common ratio the larger the output... At least it is a positive growth anyway so we will take that... Now let me just say one thing... what does the 6 represents?
- Student 5: Exponent
- Teacher: Your exponent? Your x-value, which is your input and what does this represent? Your y-value, which is your output. So unlike what we were doing before. I can tell what the input is and what the output is. Why would that be important?
- Student 6: Cause it helps you figure out what your coordinate is.
- Teacher: It helps you figure out your coordinates. Now let's talk about how to put this on the calculator. Then we can practice some. (Transcript of Video PSL\_12-5-13)

This episode occurred immediately after the PowerPoint presentation. The teacher wanted to provide an example of how to evaluate an exponential function, but the students, were not ready to move on. From the presentation the teacher introduced the topic and provided a table that shows the relationship between the input and output of an exponential function. The teacher also provided examples of the application of exponential functions. At the end of the presentation the teacher changed the situation to evaluating exponential functions. When the teacher said, "*let's move on*", Student 1 answered for the class, "*we are not ready*". Analyzing the classroom discourse showed that the response of student 1 was not simply an indication that they were still copying the notes, but rather an indication that the content was unfamiliar, and that they did not understand. This was a subtle indication of a failure in the teaching sequence.

Immediately after the teacher's response to student1, student 2 asked, "*what does x represent?*" Examination of the teacher's notes indicated that this question was consistent with the content standard that the teacher presented in passed lessons, with linear

relationships and again with exponential relationships at the beginning of the current lesson. The standard required students to interpret parts of an expression, such as terms, factors, and coefficients. They were also required to interpret expressions that represent a quantity in terms of its context. Therefore, the students were indicating that the content was complex, and the symbolic representation was not clear.

The students were aware of the teacher's intention to continue with the lesson, because student 2, immediately postponed the question by saying, "*I don't need it right now, I just need to know what it is*". It appeared to me that the teacher wanted to advance the sequence, and the students did not want to interrupt. My observation at that moment was that the students and the teacher negotiated equilibrium of a didactical contract. The question that student 2 asked was consistent with both the culture of the class, and the expectation that the teacher established. However, at this time the students opted to postpone the issue. Thus, the teacher pushed the lesson to a situation where the students must "practice some" problems similar to the example in order to gain the necessary skill. This was an instance whereby the situation jumped from instruction to a situation of institutionalization without allowing the situation to go through the situations of action, formulation, or validation.

In an interview later that day the teacher spoke about the need for pushing the students:

*The County has a benchmark coming up in two weeks, and all they want is for the students to plug-in, into the formula, so all I wanted to show them is just to evaluate. I figured that the only way to get them to do this is for them to practice. We will have to come back to the concept another time. But right now I got to move. (INT\_12-9-2013)*

The teacher attributed this bypass of situation to actors outside the classroom. The county's benchmark was coming up in two weeks. The benchmarks were important

because the student scores on the benchmarks were used to evaluate the teacher. Moreover, the benchmark does not require students to have a deep understanding of the mathematics. The benchmark only required students to “*plug-in*”. This was my 18<sup>th</sup> classroom observation and I formulated from my observations that this teacher was deeply concerned about the conceptual understanding of mathematics. Furthermore, the teacher’s comment that “*we will have to come back to the concept another time*” indicated that the teacher believed that mere practicing evaluating exponential functions was not sufficient for the students to construct the knowledge. I considered the constraints of standardize testing to be affordances to the slippage of situational bypass. Moreover, the situated, cultural use of the test scores was also affordances to this slippage.

**Episode 2.** In this next episode, the teacher started the class in a situation of institutionalization. This was a common practice in this classroom and classrooms in this school, because it was a required part of the teacher lesson plans that teachers submit to the principal. Therefore, the first 5 minutes of every class was devoted to a “Warm-up” activity. In this mathematics classroom, the warm-up activity was invariably a situation in which students are given a set of problems to practice for a standardized test. The episode shows another instance of situational bypass.

**Context.** This episode occurred towards the end of data collection period, and the day following episode 1 above. The teacher began to prepare students for a benchmark test, which would be administered in two weeks. The activity was a review and skills practice activity. The activity was designed to address difficulties that students were having the previous day, particularly with the order of operations. The teacher presented eight problems on the overhead for the students to quickly work through. The problem-

set is shown in Figure 5. After about five minutes, the teacher provided the answers to all the problems on the board. The teacher directed the students to check their work with the solutions presented and to correct their errors if any. The teacher also encouraged students to ask for an explanation of any discrepancies.

Warm-Up	
Find the value of each expression	
1. $2^5$	5. $-3^4$
2. $2^{-5}$	6. $(-3)^4$
3. $(0.2)^3$	7. $7(-4)^2$
4. $15\left(\frac{1}{3}\right)^3$	8. $12(-0.4)^3$

Figure 5. Practice problem set used for warm-up and review activity.



*Narrative and transcript.* The following transcript shows the didactic sequence that occurred beginning with a student question:

- Student 1: I have one more question...
- Teacher: one more question? Let's go
- Student 1: You know how you have the negative inside the parenthesis. In number 8... you have the answers up there so I don't know...
- Teacher: It does not matter, we could still talk about the process of how we got it... so number 8 what about it?
- Student 1: You know how like for number 5, and number 6 you have the negative.... You know like in number 6 you don't have the negative sign in the parenthesis.... I don't even know what I want to say any more.... [*Student typed numbers on calculator*] so this? I don't understand how this I get it out positive but it is coming up negative...
- Teacher: What answer?
- Student 1: For number 8. So when it is inside the parenthesis it is negative and when it is outside
- Teacher: No no [*looking at student work*] Be careful...careful...look at the exponent, the exponent is odd... so when you multiply  $-0.4$  times  $-0.4$  it becomes positive, and when you multiply by  $-0.4$  again it becomes negative again, because you doing it three times. So not only must it have to be parenthesis, the exponent has to be even for you to get an even output you see? But in this case it is odd
- Student 2: So you can't just make that decision?
- Student 1: But ok for number 6, and number 8, not 8 number 5 [ $-3^4 = -81$ ], and 6 You see how you say the negative sign is not in the parenthesis so it is going to be negative and for number 6 [ $(-3)^4 = 81$ ] it is inside the parenthesis so it is going to be positive?
- Teacher: Yes
- Student 3: Will that always happen in those situations?
- Teacher: It will, but there is what I am saying now ok... imagine I take this even number out and I put an odd number [changed  $(-3)^4$  to  $(-3)^5$ ] what's going to happen now? It going to be negative... so what I am trying to show you is that for it to become positive the exponent has to be even. If the exponent is odd it is going to be negative even if you put it in parenthesis... you gonna get a negative. Because the exponent ... [*demonstrate on board*] but the only time I really care to put parenthesis is if the exponent is even. But we'll practice some more (Transcript of Video PSL\_12-5-13).

Although this situation started in institutionalization, it was transposed to a situation of instruction. Based on students' questions, it appeared that students were having disagreement with whether or not the result was negative or positive. I observed

that students had the correct numerical value on their papers, but the incorrect signs. I was however, not sure if the incorrect signs was due to a mathematical conception or from the syntax of the calculators the students were using. The calculator syntax has different meanings to the expressions  $-m^n$  and  $(-m)^n$ . However, analysis of the classroom discourse showed that the students often treat both expressions as equivalent. Moreover, the students often disregard the difference of say  $-a$ , and  $+a$ , for they often argued that, "*it is the same thing*".

One student started to formulate an idea that if a negative is outside the parenthesis then the result is negative, and if the negative is inside the parenthesis, then the result is positive. That student acted on this formulation for question 8, but when the teacher disclosed the correct result for question 8, it was different than what the student got. The student made the formulation public, and requested an explanation. The teacher however, explained the situation by didactical means. After the teacher's explanation, student 2 who sat behind student 1 commented, "*So you can't just make that decision?*". This was an indication that student 2 also made the same formulation as student 1. Following the same argument, student 3 who was sitting in the back of the classroom asked, "*will that always happen in those situations?*". The students did not collaborate on their work because it was an independent activity, thus I presumed that the class had similar conceptions. Examining students' work showed that many other students formed the same conception, because they had identical errors in their calculations.

At the end of the episode the teacher said, "*We'll practice some more*". This was the same statement the teacher made the previous day with the same content, and almost identical situation. I could tell that the teacher wanted to begin the lesson for that day, but the issue was not resolved, because the students continued to ask questions about their

work. Therefore, the teacher went to a large bookshelf at the side of the classroom to get a stack of worksheets. He gave the worksheets to a student to distribute to the class. I could tell that the worksheets were not the intended problem set for that day, because the teacher had to search through a large stack of papers to find them. The work sheets contained 30 questions similar to Figure 6. For the remainder of the period the students worked in pairs to complete the worksheet. The teacher walked around the room, and provided individual instruction to students.

In TDSM, learning occurs when new *connaissances* and *savoirs* appear. In a situation of practice especially when the student associate a “grade” to their production, it is difficult for learning to take place, for there is seldom any need for the appearances of new *connaissances* and *savoirs*. It is only through the process of formulation, action, and validation, that new *connaissances* and *savoirs* are formed. Therefore I consider bypassing these moments an instance of meta-didactical slippage.

For the next two weeks, of the semester and my data collection period the teacher spent a significant amount of class time on evaluating exponential expressions, in an effort to correct the conception that students had regarding parenthesis, and negative signs. By the end of data collection students still had the initial conception formulated by student 1. In a conversation with the teacher at the end of data collection, I asked about the conception that I thought the students made. The teacher explained that he heard the formulation but tried to quickly dispel the notion. He said that he could not understand why the students still kept that misconception after so many opportunities to practice. The teacher recalled, that at the time he thought the formulation was corrected by the practice problems and that he did not realize that “*such a simple thing could have such a big effect*”. I told the teacher that I thought the problem could be that the students uncritically

accept what the calculator output. While watching the episode the teacher recounted that such moments are frequent in the classroom, and that in those moments he is totally unsure of the source of the error.

### **Slippage 3: Language and Symbolic Representation**

Language emerged as a major theme in the data. In the mathematics classroom the means of communication is often symbolic. Students drew symbols in the air with their fingers to communicate their ideas. Students also wrote on the board, or often students call the teacher to come to their desks to look at what they were “*trying to say*”. Other times it was the teacher that drew pictures, and symbols in order to communicate. In this portion of my analysis, I considered episodes in which the language or the symbolic representation emerged as an important theme to understand the nature of meta-didactical slippages.

**Episode 1.** In this first episode the teacher used particular calculator syntax as part of the instruction. This calculator syntax gradually became a legitimate way of representing and knowing. Moreover, the calculator function became an obstacle to student’s conceptual understanding. The episode occurred in the middle of my observations (lesson 15). It was the second lesson in a sequence designed to teach the concept of geometric sequences.

**Context.** In the previous lesson the teacher introduced geometric sequences. Students were given the common ratio, and the first term then were required to write down the sequence up to a given  $n$  terms. After the “Warm-up”, was completed and the teacher transitioned to this second lesson. The teacher wrote the objective on the board “*how to find the common ratio of a geometric sequence*”. He presented the problem in Figure 6 on the overhead.

The table shows the height of a bungee jumper's bounces.

Bounce	1	2	3
Height (Ft)	200	80	32

Figure 6. Geometric sequence problem situation (FieldNote\_10-31-20013)

**Narrative and transcript.** The teacher started in a situation of instruction. The term common ratio was defined in a lesson prior to this episode:

- Teacher: How to find the common ratio given a geometric sequence... Obviously I am going to start with my numbers. Now... 200, 80, 32
- Student 1: Start at your numbers [*student repeated what the teacher said*]
- Teacher: Because we multiply by a number to get the next term... I 'll do the opposite so I will take the second and the first term and I will divide them. So I will do  $80/200$  then I will take the third and second number and I will divide that. The reason why I am doing that is because I am looking to see if I have a common ratio... I can't do it once, for it to be common it has to be repeated a few times ok, and it has to be every time so I 'm gonna check to make sure it is common... Now  $80/200$  did I show you how to simplify that?
- Student 2: Yes! You said press simp
- Student 3: Press what?
- Student 2: S-I-M-P [*spelling out the word simp*]
- Teacher: Put  $80/200$  in the calculator [*teacher ignored the student-student interaction*]
- Student 2: do you want me to simp it? Simp it to the smallest form?
- Student 4: it will be  $2/5$
- Student 3: Where is this simp? What you guys talking about? (Transcript of Video PSL\_12-5-13)

Simplifying fractions according to the teacher was a problem for the students. Consequently, the teacher taught the students to use the calculators to simplify fractions. Observing the classroom I noticed, that students had four different types of calculators. The teacher provided two types of scientific calculators. One has a function called "simp", which simplifies a fraction to simplest form if the fraction is entered with the

backslash (/) symbol. The other calculator does not have a “simp” function, but was able to compute fractions using a function key with the symbol “ab/c”. The two types of calculators were TI-30 scientific calculators. Other students had personal calculators. Some of which had the capability to perform operations with fractions and others did not.

It seemed that student 2 was very competent with the function simp on the calculator. I presumed that the teacher only taught the class how to use the calculator that had the function simp, because student 2 answered the teacher’s question with “*yes, you said press simp*”. In this context simp is a noun, which means a key on the calculator. However, simp is used as a verb, which means to perform an action. For instance, student 2 said “*do you want me to simp it?*” Simp not only became a verb, but a mathematical discourse that had a specific meaning associated with it. Simp, which was now a legitimate language in this classroom, is simultaneously an obstacle to other students, for example, student 3 who did not have that function on her calculator. Student 2 was not aware that not on all calculators have the function “simp”, thus spelling out S-I-M-P would not help student 3 and others to find the key on their calculators.

Another use of a cultural language is for example the teacher said, “*Because we multiply by a number to get the next term... I ‘ll do the opposite so I will take the second and the first term and I will divide them*”. The first part of the statement was a referent to the previous lesson on geometric sequence. The teacher used the same phrase in the previous lesson where students were required to list the geometric sequence. The second part of the teacher’s statement was problematic, and later resulted in a misconception. The referent to the previous lesson was needed to understand the second statement. Note that the teacher said, “*We would do the opposite*”. Do the opposite of what? The teacher took for granted that students would correctly interpret the meaning of the term

“opposite”. Note also that when the teacher said, “*I will take the second and the first term and I will divide them*”, the teacher was enacting a situated meaning. That is to say, because he said the second term first, followed by the first term, then this was an indication that you should divide the second term by the first term. Nevertheless, the referent was not explicit. Therefore some of the students divided the first term by the second. For example while working on the same problem later on in the episode lesson this occurred:

Student A: I got 2.5.

Teacher: You are doing division! I don't want to do division! In order for it to work for you, you have to divide. But I am trying to keep it in a way where we are always multiplying because when it comes time to write the formula we want it in terms of multiplication...and not division. So again not that you are wrong, dividing by two and a half is perfectly fine, but instead of dividing I want to set up multiplication (Transcript of Video PSL\_12-5-13)

Student A did not interpret the teacher referent as intended. What the student did was divided 200 by 80. Student A was proposing 2.5 as the common ratio, based on the teacher's instruction to take the second term, and the first term and divide them. This was a misconception brought about by the language. The teacher also misinterpreted the student A's response, and presumed that student A was using division instead of multiplication. The common ratio was  $\frac{2}{5}$ . Therefore following the referent of the previous day, the teacher expected that students would construct  $200 \times \frac{2}{5} = 80$ . The teacher thought that student A constructed  $200 \div 2.5 = 80$ . Therefore the teacher put the student on trial for breaking the didactical contract. For, the teacher shouted, “*You are doing division! I don't want to do division!*” This however was an error brought on by the language use.

Language use was an affordance to a slippage. Examination of students' work showed that when students solved for the common ratio, the students intermittently presented the reciprocal as the common ratio. The previous lesson on geometric sequence was not a sufficient referent for students to correctly interpret the language used in discussing the common ratio. Additionally, when students presented a solution for the common ratio, the students did not validate their solutions. A situation of validation would determine if the common ratios could produce the sequence from which it came. This was also a consequence of the slippage of situational bypass discussed earlier. It was part of the classroom culture to practice a skill without actually validating the piece of knowledge. This culture was part of the didactical contract, because students were reluctant to validate their solutions even if the teacher explicitly required validation.

Language use also appeared in very complex ways. The teacher's language is often different than the students' language. Both languages are often different than the language of the mathematics. Frequently, the teacher tried to translate the mathematical language into student language in order to ease communication in the classroom. However, this transposition sometimes resulted in (mis)conception of the mathematical meaning, which could become part of the students' knowledge base.

**Episode 2.** The next episode is an instance where the language use in one situation transcended the situation to form part of the students' *connaissance*.

**Context.** After solving linear equations, the teacher instructed the students on solving literal equations. This episode occurred early in my data collection period. At this point I observed five lessons. This was my 6<sup>th</sup> classroom observation. According to the teacher, the sequencing of the lessons was critical. Solving linear equations was done before solving formulae, because the students would be able to apply their knowledge of



solving equations to solving formulae. The objective was for students to rearrange formulae to highlight a quantity of interest, using the same reasoning as in solving equations. The teacher wrote three problems on the board and selected three students to present solutions on the board. When the students finished their presentation the teacher made a short comment about their solution.

*Narrative and transcript.* A student completed her question at the board:

Teacher: Good... now what I was suggesting is that you add  $W$  on both sides of  $Ax - W = 3$ , so  $Ax = 3 + W$ , and now you subtract 3 from both sides... so  $Ax - 3 = W$ , and again it really don't matter if you put  $W$  first or last [*teacher writes  $W = Ax - 3$  or  $Ax - 3 = W$  on board*] we are interested in the process, but good job... (FieldNotes\_10-31-3013)

The teacher was probably ensuring that if different students had variations of solutions, then those students would know that their answer was also valid. The teacher made 15 similar comments over two 90-minute class periods, while reviewing the concept of transposing formulae. Each time a student presented a solution the teacher repeated a comment such as:

Teacher: And again it really don't matter if  $t$  is last or first. It's all the same! or you put the constant first [*Teacher wrote  $2n + 3 = t$  or  $t = 2n + 3$  on board*]. (FieldNotes\_10-31-2013)

On my 8<sup>th</sup> classroom observation, the teacher moved on to solving inequalities after solving equations and transposing formulae for the past four classes. In the first lesson, the teacher introduced inequality with a problem situation. After the discussing the situation the teacher extracted the symbolic representation of the problem as:

$$4000 + 10x \leq 25x$$

Teacher: So how do we solve this?  
Students: Uhhmmmmmm...

Teacher: Pretend for a second that this is an equal sign. [*Changed the inequality sign to an equal sign*](FieldNotes\_10-31-2013)

The teacher replaced the inequality symbol with an equal sign, then continued the problem in a situation of instruction. The students were able to follow the solution of the equation when the teacher replaced the inequality symbol with equal sign. The teacher wrote another symbolic problem on the board, replaced the inequality symbol with equal sign, then solve. This process was repeated four times in the lesson. The teacher provided a problem set with practice problems for the students to practice. All the problems on the problem set were written in symbolic form. The teacher walked around the room assisting students as they worked on the problem set. The teacher reminded students individually to change the inequality symbol and solve.

The next lesson the teacher presented a warm-up problem. The teacher wrote, “Solve  $\frac{-3}{5}x + 2 \geq 8$ ” on the board:

Teacher: What is different? [*Pointing to the inequality on the board  $\frac{-3}{5}x + 2 \geq 8$* ]

Student A: There is an inequality sign

Teacher: Inequality right? But again I want you to see that the same thing you were doing for an equation you do the same thing here. Is the same process, only difference is the symbol is different. But you guys should not look at it as being something that drastically different... (Transcript of Video PSL\_12-5-13)

The teacher’s advice that solving inequality was the same as solving equations resulted in a slippage of which the genesis is language use. Note in the previous nodes, the teacher replaced the inequality symbol with the equal sign. Students’ work from the classroom showed that 10 of the 23 students in the class solved the inequality  $12 < 2x - 6$  as shown in Figure 7. Only two students had the correct solution and the other students did not attempt the problem. If solving inequalities were the same as solving equations, then all 10 students who solved the problem as shown in Figure 7 would have gotten the

correct solution. The reversal of the expressions in the second line is a legitimate technique when solving equations, because equality is a symmetric relation. However, it is not so for inequalities.

It is never certain what meanings students construct from the language use in the classroom. The overuse of analogies and metaphors may result in unintended constructions that may later become obstacles for learning. When solving formulae, it appeared that the teacher preferred to have variables on the left hand side of the equation. This may account for the students reversing the expression to get the variable on the left hand side. Since equality is a symmetric relation, reversing expression will not affect the solution, but inequality is not symmetric. Thus reversing expressions would require that the inequality symbol be reversed. Observing the students as they worked showed that they replaced the inequality symbol with equal sign as they solved the problem. When they complete their solution they go back and replace all the equal signs with the original inequality symbol. It is important to note that this would be classified, as a student error on students' work but the detection of, and correction of this error would be extremely difficult.

$$12 < 2x - 6$$

$$2x - 6 < 12$$

$$\begin{array}{r} +6 \qquad +6 \\ \hline 2x < 18 \\ \hline \cancel{2} \qquad \qquad \cancel{2} \\ x < 9 \end{array}$$

Figure 7. Students' work on solving inequality (FieldNotes\_11-18-2013).

#### Slippage 4: The Design of Situations

The design of situations emerged as a major theme in understanding the nature of meta-didactical slippage. For the proper functioning of the mathematics class, the teacher has only the situations or tasks that he or she designs. The students must achieve the lesson objective by providing a response to the situation that the teacher presented. In the mathematics classroom designing a situation is not always a simple matter. Moreover, the way in which the situation was design could result in a meta-didactical slippage. In this portion of my analysis I considered the documents as the primary source of data. I used

the teacher's lesson plans/lesson notes, students' work, problem sets, and my own observations and field notes.

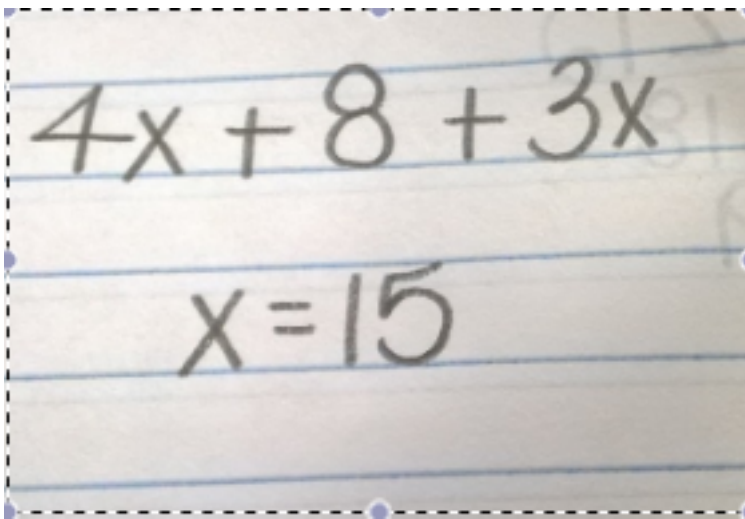
**Episode 1.** This first episode was an introductory lesson taken from the beginning of my observation period. In the episode the teacher abandoned a planned situation, for another. The teacher did not return to the abandoned situation for the remainder of the semester.

**Context.** The students completed a warm up problem in which they had to graph a linear equation of the form  $y = mx + b$ . The teacher collected the graph from each student, and then transitioned to the lesson for the day. The teacher's objective was for students to create equations and inequalities in one variable and use them to solve problems. The teacher presented a problem set for students to create equations and solve (see Figure 9).

**Narrative and transcript.** As the lesson progressed, mainly in a situation of instruction, the teacher assists the students to write an equation for the problem "*The sum of 38 and twice a number is 124. Find the number.*" After the class formulated the equation, the teacher told students to solve it on their own. The teacher walked up and down the rows looking at the students work, but not interacting with students, except to redirect off task behaviors. It appeared that the teacher perceived that students were having difficulties solving the equation  $38 + 2n = 124$ , because he stopped the class from working and went back to the board. The teacher elicited volunteers to present their solutions. One student proposed  $40n = 124$  as part of the solution to the equation. I perceived that the students were also having difficulties simplifying algebraic expressions in one

variable. The teacher asked the class to stop working on the problem set, and posed the problem “simplify  $4x + 8 + 3x$ ” for the students to solve. Three of the students in the class working independently produced Figure 8 as a solution.

Five of the students got the correct answer, and the other 15 students in the class did not have a solution on their papers. Most of the 15 students had eraser marks on their papers, which indicated that they tried to solve the problem, but I could not tell what their procedures were. The teacher abandoned the planned task of having students create equations from given situations, and transformed the lesson to simplifying algebraic expressions.



The image shows a close-up of a student's handwritten work on lined paper. The expression  $4x + 8 + 3x$  is written on the top line, and the solution  $x = 15$  is written on the line below it. The work is enclosed in a dashed black border.

*Figure 8.* Student work on simplifying algebraic expressions (FieldNotes\_10-23-2013).

For the remainder of the lesson, which lasted for 60 minutes, the teacher wrote algebraic expressions on the board for students to solve. Students worked in pairs at their desks, and then students presented their solutions at the board. It appeared that the students were able to simplify algebraic expression by the end of the class. The students did not hesitate to go to the board to display their solutions, and they raised their hands with enthusiasm as the teacher called on different students to go to the board.

The initial problem set remained on the board for the remainder of the class. Moreover, the teacher did not recuperate the situation for the remainder of the semester. This episode showed an instance where a situation was designed with specific target knowledge, but was abandoned in the milieu. The decision to abandon the situation was based on feedback from the student system. The meta-situation was more appropriate for that particular lesson, but the knowledge from the initial situation was a prerequisite knowing for other lessons later in the course.

1. The sum of 38 and twice a number is 124. Find the number.
2. The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.
3. A rectangle is 12m longer than it is wide. Its perimeter is 68m. Find its length and width.
4. The length of a rectangle is 4 cm more than the width and the perimeter is at least 48 cm. What are the smallest possible dimensions for the rectangle?
5. Find three consecutive integers whose sum is 171.
6. Find four consecutive even integers whose sum is 244.
7. Alex has twice as much money as Jennifer. Jennifer has \$6 less than Shannon. Together they have \$54. How much money does each have?
8. There are three exams in a marking period. A student received grades of 75 and 81 on the first two exams. What grade must the student earn on the last exam to get an average of no less than 80 for the marking period?

*Figure 9.* Problem-set to create and solve equations (FieldNotes\_10-23-2013).

In my field notes I coded this event as an example of meta-didactical situation, but later during my analysis recoded this situation as an instance of a much larger occurrence in the mathematics classroom. This was an indication that the initial situation was not appropriately designed. In other similar cases whereby the teacher perceived the initial situation failed the phenomenon resulted in the slippage of over-teaching.

**Episode 2.** This episode is another example in which the design of the situation resulted in a slippage.

**Context.** This was a review lesson towards the end of data collection period. The teacher completed the curriculum for the semester. According to the teacher, this activity was a review activity designed to connect several skills from the course. I perceived however, that the teacher wanted the students to evaluate exponential expressions. The



objectives for the lesson were: (a) write and graph an equation to represent an exponential relationship; (b) model a data set using an equation; and (c) Choose the best form of an equation to model exponential functions. The teacher presented the following problem on the board: *Evaluate  $3(2)^3$ .*

***Narrative and transcript.*** The students responded with the answer 216. The teacher told the class that the answer of 216 was wrong. The teacher began instruction by eliciting response from the students, “*what is the order of operation?*” The students replied “PEMDAS!” Then in a chorus the students said, “*Please Excuse My Dear Aunt Sally*”. The teacher, looking puzzled asked how did they get 216. One student shouted, almost in disbelief, “*parenthesis first!*” The teacher then proceeded to explain and correct the mistake. This particular error occurred several times in this classroom throughout the semester.

In my analysis, I found the answer of 216 not to be a mistake, but a product of students’ conceptions regarding the use of parenthesis. The students followed the rules of order of operation, which they constructed from previous years, and multiplied the 3 by the 2 to give 6. “*Parenthesis first!*” Then the students apply the exponent, 6 raised to the 3<sup>rd</sup> power is 216. Furthermore, expressions such  $3(2)$  has been constructed and passed on in the mathematics classroom culture as multiplication. Howbeit, expression such as  $2(3)$  is not a correct representation for multiplication. Parenthesis have a great many specialized meanings in mathematics, but in arithmetic expressions parentheses are used to denote modifications to the normal order of operations. This is a slippage of representation of which the genesis is very complex. Although the students corrected the error when the teacher pointed it out, they did not change their conception. Whenever a

similar problem situations reoccurred students produced a product based on their conception. For instance,

- Student A: [solving  $f(-2) = 2(3)^{-2}$ ] Look I don't know how to do that without a calculator?
- Student B: So you got to do your exponents first... hold on (typed numbers into the calculator) Wait! How do you... If the exponent is negative how do you do that again? So this is just  $2(-9)$  which is  $-18$ . You get? All you have to do is your exponent first.
- Student A: I told you that. (Transcript of Video SL\_12-5-13)

Although the negative exponent could be credited for this error, student work showed that students multiply the number outside the parenthesis first before applying the exponent. In order to address these kinds of student error, it requires situations to be designed to allow students to refute their conception. Moreover, the task should be designed to allow the students to go through the entire process of formulation, action and validation before institutionalization. The teacher revealed that to design the situation is problematic, because "*How can the teacher know what situation to design in order to get the desired target knowledge?*" Furthermore the teacher pointed out that, "*The problem is how to design situations.*" (INT\_12-9-2013)

### **Characteristics of Meta-Didactical Slippages**

This study found that the major affordances of meta-didactical slippage were (a) intentionality, (b) time constraints, (c) students' questions, and (d) situated cultural use of student test scores. These affordances operate either simultaneously, or in tandem to undermine the mathematics that is actually taught in the mathematics classroom. These factors emerged as either the genesis of meta-didactical slippage, or the force that sustained the slippages.

**Intentionality.** Intentionality emerged as a characteristic of meta-didactical slippage. Intentionality refers to the teacher formulating a conscious plan to perform or

not to perform a specified future act. The teacher's lesson plan is an explicit indication of the teacher's intentions to teach or not to teach some mathematical content. However, what intentions the teacher formulated in the mathematics classroom are not observable. Classroom observation together with interview with the teacher is needed to unearth intentionality. This study found that the teacher's intention is partly responsible for the action that the teacher performs in the classroom. Moreover, the teacher's conscious decision to perform or not to perform a specified act is connected to feedback from the milieu. For instance, if the teacher wants to complete a lesson before the class ends, the teacher may speed up the instruction, or continue to give instruction at the board, thus resulting in a slippage of over-teaching, or the slippage of situational bypass. Intentionality then is a paradox of the teaching, because the teacher has a social right to have an intention.

**Time constraint.** Time constraint is another characteristic of meta-didactical slippage. Time plays an important role in the mathematics classroom because every didactic interaction must occur within school hours, taking into account, class periods, weekly schedules, testing schedules, and holidays. This constraint is inevitable, and the teacher has no control of the time. Time constraints are interconnected with intentionality. Depending on the amount of time the teacher has to complete a specific task, the teacher formulates different intentions. For example, the teacher calculated that the county's benchmark was fast approaching and thus, decided to bypass situations of action formulation, and validation. This diversion of situation was done primarily to decrease the time required for the students to gain the target knowledge. This too is another paradox of the teaching, because in an effort to reduce the time taken to learn a piece of knowledge, the teacher effectively increased the time. This was because the

students did not construct a conceptual knowing of the target knowledge, and thus required continuous practice and revision in order to be able to do the mathematics. This study showed that although the teacher spent many days on for example, evaluating exponential expression, the students still had difficulties because they had mis(conceptions) about the order of operation.

**Students' questions.** The kinds of questions, and the number of questions that the students ask emerged as a characteristic of meta-didactical slippage. The questions that students ask provide the teacher with verbal feedback from the milieu. Frequently this feedback was an indication of a failure of the teaching sequence. The teacher then has to act on this feedback in order to recuperate the situation and to keep lesson moving. There are different ways (see Figure 3) that the teacher may choose to proceed. For instance, he may proceed in a situation of instruction, either by repeating the original information, or change the task to a meta-task. In my observation, the teacher predominantly recuperated by repeating the instruction. The decision to remain in instruction is not always intentional. This unintentional instruction results in a slippage of over teaching. In this slippage, the teacher provided the answer to the problem. The answer, according to TDSM, contained the target knowledge. Therefore, the students cannot produce the knowledge, because it has already been produced. This is another paradox of the teaching endeavor, because the teacher has a professional, and social right to provide instruction. Therefore, the slippage of over teaching, which is unearthed by the students' questions, is interconnected with intentionality. For it is identified only through observation and a posteriori discussions with the teacher. Thus the only way for the observer to determine that over-teaching occurred is by knowing the intentions of the teacher.

**Situated cultural use of test scores.** Student test scores on standardized tests are a part of the culture of the mathematics classroom. The teacher emphasized the need for students to demonstrate high scores on the benchmark tests. Moreover, the study school had a school-wide policy of doing test preparation every day in all classes. This test preparation was called “warm-up” and was done by every teacher in the school. In the slippage of situational bypass the teacher revealed his intention to have students practice skills that is required on the standardized test. The teacher also revealed that the test scores are important, because the school district uses the scores to evaluate teachers and to design programs, which were not always in the best interest of the teaching and learning process. Therefore, the cultural use of test scores was interconnected with intentionality. The teacher also disclosed that the time remaining before the test is administered influenced his decision to remain in a situation of instruction, or to prescribe practice problems for the students to gain the skills required to achieve high scores on the test. Thus, time constraints are interconnected with both intentionality and the cultural use of test scores.

**Knowledge of situations.** Knowledge of situations emerged as a major characteristic of meta-didactical slippage. Research in mathematics often focuses on the teachers' content knowledge and pedagogical practices (Andrews & Sayers, 2012; Clarke & et al., 1993; Dowling, 2001; Flores, 2002; G. L. Harel, 2005; Harwell, Post, Maeda, Davis, Cutler, Andersen, & Kahan, 2007; Hiebert, 2003; Labato & Thanheiser, 2002; Litwiller & Bright, 2002; Dina Tirosh, 2000), but the research failed to identify knowledge of situations as fundamental to the mathematics classroom. This study found that in addition to the teachers' content knowledge, and pedagogical skills, the teacher must have a sound knowledge of situations. It is the situation that forms the core of the

milieu. The teacher gets feedback from the milieu only through interaction with the situation that the teacher contrived. Currently, the only means that the teacher has to determine whether or not the student learns a piece of knowledge is by evaluating the student work on problems. The student work is a document that the student produced. The teacher has access to this document after its production and thus cannot determine how it was produced. For example in figure 9 the teacher was able to determine that the student did not reverse the inequality symbol, but without observation the teacher could not determine that the students replaced the initial inequality symbol with the equal sign and then solved an equation. The student then erased the equal sign on the final production and rewrite the inequality symbol. Knowledge of the design of situations emerged as a meta-didactical slippage. The teacher does not always know what situations to design. Moreover the teacher does not always know what situations to design en vivo, so that students can have the opportunity to change or modify the conceptions that they hold.

The second research question was concerned with the students' conceptual understanding on the relationships and quantities unit. More specific the research question asked was: In what ways do these slippages affect students' conceptual understanding of a unit of ninth grade mathematics? To answer this question I used the unit assessment, the teacher reflections, and students' work to provide evidence.

At the end of the unit the county administered a benchmark test to assess the students understanding of the unit. The teacher also administered several formative assessments to inform his teaching of the unit. On the county's benchmark test, the average score was 40%. The minimum score was 16% and the maximum score was 61%. For test security, I was not allowed to review the test items. I was also not allowed to

observe the class during testing. The results of the benchmark test was reported to the teacher with no statistical analysis, except for description which include the average, minimum score, and maximum score. There was also a list that included the students' names and actual scores on the test.

The teacher revealed that he was disappointed with the scores, because "*I feel that students could achieve more, based on the material we covered in class, but the students are doing the work, but the understanding is not there!*" (INT\_12-9-2013). The teacher indicated that the students are doing the work, which is an indication that students are busy working on practice problems, but without a conceptual understanding. Here the teacher disclosed that the emphasis to have students practice the skills in order to do well on tests was not advantageous. This was a consequence of the slippage of situational bypass.

The teacher attributed the apparent lack of understanding of the relationships between quantities unit mainly to the slippage of over-teaching and the design of situations. For in over-teaching it is the teacher who restricts the situation from progressing to a space where learning happens. The teacher however pointed out that it was not easy to know when to stop teaching, "*So the big question is what is the optimal time to stop teaching?*" (INT\_12-19-2013). For the teacher, there is an optimal time to stop teaching a mathematics lesson. This optimal time, will allow the students to develop a conceptual understanding of the topic. If the teacher continued to provide instruction after this point, the students may not learn the target knowledge.

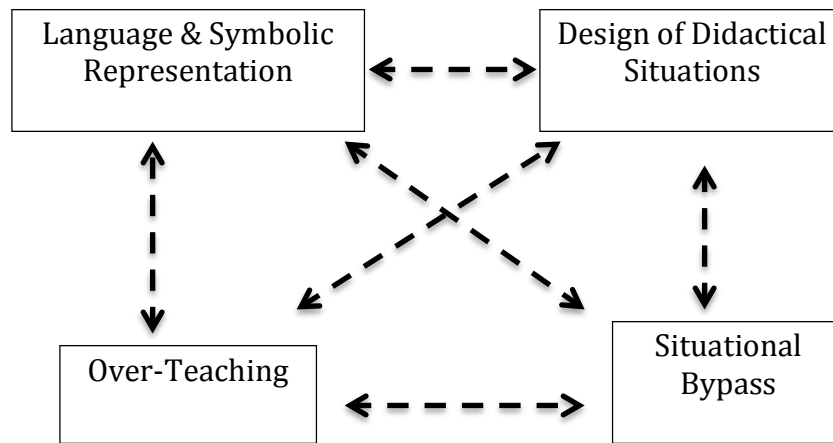
## Summary

In this chapter, I reported and described four major slippages that emerged as important to understand the nature of meta-didactical slippages in the ninth grade mathematics classroom. I selected episodes, which does not require extensive mathematical background. The teacher approved the use of these classroom activities. The major slippages were: (a) over-teaching; (b) premature practice, a situational bypass; (c) language and symbolic representation; and (c) the design of situations. I described the context in which these slippages emerged as well as the affordances. Figure 10 shows the basic relationship of these slippages to the theoretical framework, which I used to conceive them.

The slippages operate either alone or together to oppose the learning. Over teaching restricts the progression of the didactical situation whereas, situational bypass detours the situation to an algorithmic reduction of the target knowledge. The didactic situation that is contrived by the teacher can either increase the learning, or undermine the intended learning. Similarly the language use and symbolic representation can either increase the conceptual understanding of the mathematics, or undermines the learning of mathematics.

Finally, I described some of the characteristics or factors that appeared to be the genesis of meta-didactical slippages in the mathematics classroom. In chapter 5, I discuss these themes and their implications for the teaching and learning of mathematics. I also provide recommendations for the mathematics teachers, school administrators, policy makers, and future researchers.





*Figure 10.* The relationship between meta-didactical slippages and the theoretical framework.

## CHAPTER 5

### DISCUSSION AND RECOMMENDATIONS

This chapter presents a summary of the study and the conclusions drawn from the data presented in chapter 4. As an aid to the reader this chapter restates the research problem and reviews the major methods used in the study. It also provides a discussion of the implications for action and recommendations for future researchers.

#### **Summary of the Study**

##### **Overview Of The Problem**

The complexity of the teaching endeavor is a major problem for teachers as they strive to meet the demands of all stakeholders of the teaching and learning process. There are many factors that contributed to the complexity of the teaching and learning process. These factors include the nature of mathematics, the classroom culture, current curriculum reform movements, and the need to improve students' mathematics performance on state and local standardize tests. Mathematics instruction does not guarantee desired learning, because according to Schoenfeld (1988), even when the lesson is well taught undesired learning can take place as a direct consequence of the instruction. Moreover, Brousseau (1997) identified several undesired effects that may occur in the teaching and learning process. Some of these undesired effects are difficult to identify and describe. Brousseau et al. (2009) identified meta-didactical slippage as one of the undesired effects that may occur in the mathematics classroom. Research however, failed to describe how meta-didactical slippage is manifested in the mathematics classroom. Consequently, the purpose of this study was to describe the nature of meta-didactical slippage that occurred in a ninth grade mathematics classroom.

## **Research Questions**

The research questions that guided this study are:

1. What is the nature of meta-didactical slippages that occurred in a ninth grade mathematics classroom?
2. How did the teacher perceive these meta-didactical slippages affect teaching and learning of a unit of analysis of 9th grade mathematics?

## **Review of the Methodology**

The study was a descriptive qualitative case study (Yin, 2009), conducted in one ninth grade mathematics classroom in a southeastern high school. The qualitative case study is grounded in the lived reality which helped me to understand complex inter-relationships in the mathematics classroom (Hays, 2004). Furthermore, the case study according to Hays (2004) seeks to answer focused questions by producing in-depth descriptions and interpretations over a short period of time. I used the theory of didactical situations in mathematics (Brousseau, 1997) as the theoretical lens to interpret the findings.

One teacher and 23 students participated in the study. I observed and videotaped a total of 30 classroom sessions from October 14, 2013 to December 13, 2013. The classroom meets 5 day per week for 90 minutes. During the period from October to December, I interviewed the teacher three times. Moreover, I had informal conversations with the teacher on a regular basis. During the informal conversations, I used video clips from the classroom to elicit focused discussions with the teacher.

I began data analysis by writing field notes and memos. I watched the entire videotapes to get a sense of the general culture of the classroom and to get a general

sense of what happened in the classroom. I used Apple's iMovie video editing software to partition the video files into 5-second nodes. I parsed each node for instances of a perceived failure of a teaching sequence. Whenever I locate an instance, I selected the entire episode that contained the failure and created a new file. This process created two files, "possible slippages", and 'slippages'. I transcribed both files, and use them as the major video transcripts. I went back and forth from transcripts to the videos, always checking with the teacher at each stage of my analysis.

I used two analytic techniques to analyze the data, ethnographic technique (Spradley, 1980) and discourse analysis (Gee, 2011). I presented and discussed the results of my analysis in Chapter 4. In the next section I discuss the major findings.

### **Summary of Findings**

In Chapter 4, I presented four themes that emerged in the data, which illustrated the nature of meta-didactical slippage that occurred in one, ninth grade classroom. These themes were (a) over-teaching, (b) situational bypass, (c) language and symbolic representation, and (d) design of situations. The descriptions provided are far from exhaustive. However, they provide a means to begin to understand the phenomenon in the mathematics classroom. In order to understand the nature of the phenomenon, I focused on the genesis, qualities, and affordances. This study found that meta-didactical slippage is contextually nuanced and complex. Moreover, this study found that instances of meta-didactical slippages are not mutually exclusive. For, in one classroom interaction several kinds of slippages may occur which is afforded by two or more classroom factors.

### **Findings Related to The Literature**

Research on didactical situations in mathematics is growing (Schoenfeld, 2012). For instance, Bussi (2005) point out the potential impact for research in mathematics

education when classroom situations is used as the unit of analysis. The findings from this study revealed that analyzing classroom situations is necessary to uncover the nature of phenomena that may affect the teaching and learning process. For example, I discovered the slippage of over teaching only through analyzing classroom situations together with teacher reflection. Furthermore, only through discussions with the teacher, that I concluded that intentionality was a factor of that slippage. Therefore this study agrees with Bussi (2005) in calling for classroom situations to become the unit of analysis for research in didactical situations in the mathematics classroom. Moreover, this study suggests that classroom observation together with interviews can be used to describe teaching phenomena. Because I believe that classroom observation of didactical situations is fundamental to understand teaching and learning. Classroom observations can be more productive in obtaining teacher actions, intentions, and decisions in the mathematics classroom.

Findings of this study suggest that the teacher's intentions play an important role in the classroom. Intentionality was related to the literature in many ways. For instance, when Schoenfeld (1988) talked about the disasters of a well taught course, he was illustrating the nuanced connection of intentionality. In his study the teacher implemented a well taught lesson, but although the teacher had good intentions, the students learned inappropriate mathematical conception as a result of the teaching (Schoenfeld, 1988). Similarly, intentionality was also related to a study conducted by Henningsen and Stein (1997) which examined and illustrated how classroom-based factors can shape students' engagement with mathematical tasks. That study found that students' engagement in high-level cognitive processes continued or declined during classroom work on tasks. Consequently, this study suggests that open discussion of teachers' intentions in the

mathematics classroom, can provide deeper understanding of didactical phenomena, such as meta-didactical slippage.

Findings of this study suggest that slippages tend to occur whenever there are constraints operating in the classroom. These constraints, either imposed on the teacher by external actors, or imposed by actors in the milieu, served as fuel for the manifestation of meta-didactical slippages. For instance, according to BarbÉ, Bosch, Espinoza, and GascÓN (2005) observation of an empirical didactic process showed how the internal dynamics of the didactic process was affected by certain mathematical and didactic constraints that significantly determined the teacher's practice and ultimately the mathematics actually taught. This was an example of a meta-didactical slippage, because the mathematics that was taught was not the initial intention of the teacher. This finding was similar to my findings, particularly the slippage of the design of situations.

This study found that language use and symbolic representation emerged as a kind of meta-didactical slippage. This slippage is afforded through the language use, and the taken for granted meanings that was situated in the classroom. This finding was similar to the findings reported by G. Harel, Fuller, and Rabin (2008), which investigated the phenomenon of non-attendance to meaning by students in school mathematics. G. Harel et al. (2008) pointed out four particular teaching actions that de-emphasize meaning in the mathematics classroom. They categorized those teaching actions as pertaining to (a) purpose of new concepts, (b) distinctions in mathematics, (c) mathematical terminology, and (d) mathematical symbols. Moreover, when these actions were present the classroom students developed the belief that mathematics involves executing standard procedures, and that treatment of symbols was largely non-referential. These teaching actions were

instances of affordances to meta-didactical slippages, because the target knowledge was replaced with a meta-situation.

## **Conclusions**

### **Implications for Actions**

Whereas a single case study cannot provide a sound basis for the practice of teaching and learning in the mathematics classroom, this study would suggest that teachers should be more purposive in how and when they intervene in problem situation in the mathematics classroom. This is so that they do not replace an initial mathematical situation that would have permitted an authentic activity on the part of the student, by a study of the mathematical circumstances, or by reducing the cognitive demand of the task.

A second implication of this study is that the results of research on didactical situation be disseminated to mathematics teachers. I recommend that the results of this study (and other studies with similar findings) be included in professional development for mathematics teachers so that they can become aware of the phenomenon of meta-didactical slippages. The findings showed that the mathematics classroom is a very complex and highly nuanced community. Thus the increased awareness of the phenomenon should influence teachers' didactic decisions as they plan and implement mathematical lessons. In this way, the teacher is more sensitive to resist desire to take all mathematics activities as an object of teaching.

Another implication of this study is for school districts to provide professional development for mathematics teachers to learn how to design didactical situations. Moreover, I recommend continuous discourse among mathematics teachers on the design and implementation of didactical situations. The findings of this study showed that

teachers' knowledge of situations was essential to the functioning of the mathematics classroom. My recommendation is for professional development providers to provide opportunities for mathematics teachers to learn content knowledge, pedagogical skills, and how to design situations. I further recommend that content, knowledge, pedagogical skills, and design of situation to be considered and treated as one entity instead of three separate things. Finally, I recommend that teachers become more involved in classroom observation and reflection. The findings of this study showed that some classroom practices could not be identified without both observation and reflection on those observations.

### **Recommendations for Further Research**

This study was a qualitative case study conducted in one mathematics classroom over one semester. The findings of the study are thus limited in the scope of application. Consequently, research conducted over a longer time period and in different mathematics classrooms could yield greater understanding on the nature of meta-didactical slippage. I recommend a longitudinal study where the researcher observed the same students for several years of their school mathematics classrooms.

This study found that the genesis of some slippages was possibly due to past conceptions that students' hold. Therefore I recommend that this study be repeated in elementary schools where students learn fundamental concepts in mathematics for the first time.

Finally, for this study I used only one video camera. Focusing on any particular issue was at the same time overlooking others. I recommend that future studies used more than one camera so that they can capture many different perspectives of the classroom. I also recommend that further researchers increase the use of video data to study and



improve the teaching and learning of mathematics in school context. Furthermore, I recommend research in finding effective models, to aid in the use of capturing, transferring, analyzing, and disseminating video data.

### **Concluding Remarks**

This project was very difficult for the participating teacher and me. As we watched episodes from the classroom we learned more about the teaching of mathematics. We both learned that there is more to a mathematics classroom than what meets the eye. The complexity of classroom situations is more than researchers, teachers, and observers can describe. It was also difficult to not focus on mathematical mistakes, and non-didactic interactions, which happened in the mathematics classroom. I had to remain focus on the purpose of this study in order to separate the “noise” from the data, lest this study become a deficit study.

During my observation and writing my field notes, I often miss events that occurred, only to observe it on the video. Moreover, after watching the videos several times, I was able to observe events, which I missed on previous viewing of the video. In one particular session as the teacher and I watched the video, the teacher said, “I don’t recall that, but you got me on video.” There was another time where I had to stop the video, because I perceived that watching the video was invoking feelings of failure from the teacher. This prompted the very important question: how did participation in the study affect the teachers' teaching and understanding of what he does and how he does it? Additionally, how did this study affect my teaching and understanding of what I do and how do it?

As I reflect on what I observed and experienced through the research process, I thought of my own teaching of mathematics. I became more aware of the kinds of meta-

didactical slippages that occurred in my mathematics classroom. I found similarities with the number of analogies and metaphors that I use in my classroom. I became more aware of my language use and my symbolic representation of objects. As a result of this study I began to consider how the language that I used, the way in which I represent objects, and the kinds of feedback that I provide in the classroom, may affect students' conceptual understanding of mathematics. As a consequence of this study, the teacher and I have started a collaborative effort to focus on minimizing the slippage of over-teaching.

It is my hope that research such as this study, will help to combat the algorithmic reduction of mathematics that is occurring especially in school mathematics (Freudenthal, 1981; Henningsen & Stein, 1997; Herbel-Eisenmann & Cirillo, 2009; Schoenfeld, 1988). I hope that teachers will ultimately participate in the discourses of didactical situations as well as in the difficulties and challenges of classroom research.

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## APPENDIXES

APPENDIX A  
OBSERVATION LOG (ADOPTED FROM TABLE 2)

<b>Indication of Failure</b>	<b>Mathematics Task Description</b>	<b>Type of situation</b>	<b>Type of knowings manifested</b>	<b>Resolution/Decision</b>	<b>Observer Comment</b>

## APPENDIX B

## INTERVIEW PROTOCOL

Date.....  
 Time.....  
 Place.....

Interviewer: N. Wisdom

I am interested in finding out more about mathematics teacher's didactic decisions in the teaching process. We hope this will help us to better understand more about the classroom situations, in order to improve mathematics teaching and learning. Later, I would like to ask you about what initiate particular decision in the didactical situations, but first I would like start by asking you about your experience as a mathematics teacher, and your desired target knowledge for this unit of study. You have received a consent form to sign, which indicates your consent to the interview. The interview is being tape-recorded.

### Questions and Probes

1. Describe your experience as a mathematics teacher?
2. How do you define success in a mathematics teaching sequence?
3. Tell me about a time or lesson when you felt you were doing a good job in your content area, but you perceived the students were not learning your intended objective?
4. Tell me some more about...
  - a. What specific act caused you to perceive a failure?
  - b. What did you do?
5. How do you plan or contrive a didactical situation?
6. What are types of things that you look for in the didactic situation that informs you of whether or not the students are learning the mathematics that you intend for them to learn?
7. Describe in detail a successful secondary mathematics teaching and learning situation that you have had, and explain how you know it was successful?
8. Describe in detail an unsuccessful secondary mathematics teaching and learning situation that you have had, and explain how you know it was unsuccessful?
9. May I get back to you if I have questions when I go over the interview?

APPENDIX C  
OBSERVATION MATRIX

	<b>SPACE</b>	<b>OBJECT</b>	<b>ACT</b>	<b>ACTIVITY</b>	<b>EVENT</b>	<b>TIME</b>	<b>ACTOR</b>	<b>GOAL</b>	<b>FEELING</b>
<b>SPACE</b>	Can you describe in detail all the <i>places</i> ?	What are all the ways space is organized by objects?	What are all the ways space is organized by acts?	What are all the ways space is organized by acts?	What are all the ways space is organized by activities?	What spatial changes occur over time?	What are all the ways space is used by actors?	What are all the ways space is related to goals?	What places are associated with feelings?
<b>OBJECT</b>	Where are objects located?	Can you describe in detail all the <i>objects</i> ?	What are all the ways objects are used in acts?	What are all the ways objects are used in activities?	What are all the ways objects are used in events?	How are objects used at different times?	What are all the ways objects are used by actors?	How are objects used in seeking goals?	What are all the ways objects evoke feelings?
<b>ACT</b>	Where do acts occur?	How do acts incorporate the use of objects?	Can you describe in detail all the <i>acts</i> ?	How are acts a part of activities?	How are acts a part of events?	How do acts vary over time?	What are the ways acts are performed by actors?	What are all the ways acts are related to goals?	What are all the ways acts are linked to feelings?
<b>ACTIVITY</b>	What are all the places activities occur?	What are all the ways activities incorporate objects?	What are all the ways activities incorporate acts?	Can you describe in detail all the <i>activities</i> ?	What are all the ways activities are part of events?	How do activities vary at different times?	What are all the ways activities involve actors?	What are all the ways activities involve goals?	How do activities involve feelings?
<b>EVENT</b>	What are all the places events occur?	What are all the ways events incorporate objects?	What are all the ways events incorporate acts?	What are all the ways events incorporate activities?	Can you describe in detail all the <i>events</i> ?	How do events occur over time? Is there any sequencing?	How do events involve the various actors?	How are events related to goals?	How do events involve feelings?
<b>TIME</b>	Where do time periods occur?	What are all the ways time affects objects?	How do acts fall into time periods?	How do activities fall into time periods?	How do events fall into time periods?	Can you describe in detail all the time periods?	When are all the times actors are "on stage"?	How are goals related to time periods?	When are feelings evoked?
<b>ACTOR</b>	Where do actors place themselves?	What are all the ways actors use objects?	What are all the ways actors use acts?	How are actors involved in activities?	How are actors involved in events?	How do actors change over time or at different times?	Can you describe in detail all the <i>actors</i> ?	Which actors are linked to which goals?	What are the feelings experienced by actors?
<b>GOAL</b>	Where are goals sought and achieved?	What are all the ways goals involve use of objects?	What are all the ways goals involve acts?	What activities are goal-seeking, or linked to goals?	What are all the ways events are linked to goals?	Which goals are scheduled for which times?	How do the various goals affect the various actors?	Can you describe in details all the goals?	What are all the ways goals evoke feelings?
<b>FEELING</b>	Where do the various feeling states occur?	What feelings lead to the use of what objects?	What are all the ways feelings affect acts?	What are all the ways feelings affect activities?	What are all the ways feelings affect events?	How are feelings related to various time periods?	What are all the ways feelings involve actors?	What are the ways feelings influence goals?	Can you describe in detail all the feelings?





4. For any research that is conducted beyond the approval period, a Renewal Application must be submitted at least 30 days prior to the expiration date. The Renewal Application must be approved by the IRB before the expiration date else automatic termination of this study will occur. If the study expires, all research activities associated with the study must cease and a new application must be approved before any work can continue.

5. When the study is completed, a Study Closure Report must be submitted to the IRB.

All of the above referenced forms are available online at <https://irbwise.gsu.edu>. Please do not hesitate to contact Susan Vogtner in the Office of Research Integrity (404-413-3500) if you have any questions or concerns.

Sincerely,

Andrew I. Cohen, IRB Vice-Chair

**Federal Wide Assurance Number: 00000129**

APPENDIX E  
MATHEMATICS CURRICULUM FOR TARGET UNIT

**Reason quantitatively and use units to solve problems.**

**MCC9-12.N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

**MCC9-12.N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.

**MCC9-12.N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**Interpret the structure of expressions**

*Limit to linear expressions and to exponential expressions with integer exponents.*

**MCC9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context.

**MCC9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**MCC9-12.A.SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**Create equations that describe numbers or relationships**

*Limit A.CED.1 and A.CED.2 to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. Limit A.CED.3 to linear equations and inequalities. Limit A.CED.4 to formulas with a linear focus.*

**MCC9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

**MCC9-12.A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**MCC9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**MCC9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.