Fall 10-24-2013

Dynamic Models of the Insurance Markets

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DYNAMIC MODELS OF THE INSURANCE MARKETS

BY

Ning Wang

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree

Of

Doctor of Philosophy

In the Robinson College of Business

Of

Georgia State University

GEORGIA STATE UNIVERSITY
ROBINSON COLLEGE OF BUSINESS
2013

This dissertation was prepared under the direction of the Ning Wang's Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctoral of Philosophy in Business Administration in the J. Mack Robinson College of Business of Georgia State University.

H. Fenwick Huss, Dean

DISSERTATION COMMITTEE
      Dr. Martin Grace (Chair)
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      Dr. George Zanjani
      Dr. Tyler Leverty (External)

ABSTRACT

DYNAMIC MODELS OF THE INSURANCE MARKETS

BY

Ning Wang

August 2013

Committee Chair: Dr. Martin Grace

Major Academic Unit: Department of Risk Management and Insurance

This is a multi-essay dissertation in the area of dynamic models of the insurance markets. I study issues in insurance markets by examining individual behavior and industry performance in dynamic settings. My first essay studies household life insurance demand and saving decisions by applying a heterogeneous-agent life cycle model with wage shocks and mortality shocks. This essay proposes the most important determinants of household life insurance demand, and shows the joint decision of life insurance purchase between couples. My second essay focuses on the property-liability insurance market, and aims to study the impact of one catastrophe event on an insurer's underwritings and capital raising strategy. The two-period cash flow model is built to also explore what kind of insurers can benefit from catastrophic risk underwritings. My third essay extends the second essay by incorporating a dynamic cash flow model with a series of loss shocks. I find the dynamic interaction between the insurer's balance sheet and its capital rationing resulting from loss shocks. The model generates a non-cyclical behavior of output changes in the insurance market, and this suggests the current asymmetric, unpredictable and random underwriting cycles are temporary responses to loss shocks.

Acknowledgements

It has been a real honor and a great pleasure to work with the faculty, staff, and students at Georgia State University as a doctoral student in the Department of Risk Management and Insurance. My first great thanks go to Dr. Martin Grace, my dissertation committee chair and long-time advisor. Thanks a lot to the professional mentoring, boundless kindness and considerable patience offered by Dr. Martin Grace! It is him who gives me the freedom to pursue my research interests and provides me with strong support throughout my research agenda.

I have also received generous guidance and useful advices from other committee members. I really appreciate Dr. Daniel Bauer's big help with my research project and paper writing all the time, and I want to send my best wishes to his incoming twin children. I greatly appreciate Dr. George Zanjani's valuable comments and brilliant help with my research direction. I also greatly thank Dr. Florin Bidian for his useful advices about the research methodology. As an external committee member, Dr. Tyler Leverty gives me great kindness and helpful instructions not only on my research but also on my job-hunting.

In addition, I have benefited a lot from the help and the support of many other faculty members. Special thanks go to Dr. Shinichi Nishiyama for his helpful seminar course and in-detailed instruction on my first essay. I would like to highly thank Dr. Ajay Subramanian, for his timely encouragement and always being ready to help Ph.D. students. I also greatly thank Dr. Richard Phillips, Dr. Shaun Wang, Dr. Kaiji Chen, Dr. Conrad Ciccotello, Dr. Robert Klein, Dr. Zhiyong Liu, and Mr. Chris Lemley, for their tremendous help with my research, teaching and the Ph.D. life. I also want to show my multiple appreciations to all Ph.D. students in the department for studying and discussing together, and to all the department staff for their kind help and support.

Moreover, I would like to thank all my church friends for their great kindness, timely care, and consistent prayer. My deepest gratitude is to my parents, who are always right here to support, love and help me though all my life. They show great patience and encouragement to me during my Ph.D. study period. Finally, I would like to leave my most special thanks to my husband, Dr. Feng Qin, who helps me overcome all the difficulties and gives me strong arms in any case. Thank him for all his unconditional love, accompanying, and understanding.

In all, without the support and the encouragement from so many wonderful persons, I never would have made it.

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1

Overview

This dissertation aims to study issues in financial and insurance markets by examining individual behavior and industry performance in dynamic settings. In life insurance market, I focus on the household life insurance demand and saving decisions. I also explore catastrophic risk and underwriting cycles in the property-liability insurance industry.

There is no consensus about the amount and the distribution of household life insurance holdings in empirical research due to the limited data of household life insurance purchases at the policy level (see Chambers, Schlagenhauf and Young, 2003; Grace and Lin 2007). I apply dynamic models with stochastic process to my research in exploring the relation of household life insurance purchases and household characteristics by a heterogeneous-agent life cycle model (see Chambers, Schlagenhauf and Young, 2009, 2011; Nishiyama, 2010). In my first essay, I construct a life cycle model of heterogeneous married households with wage shocks and

mortality shocks to quantitatively analyze household life insurance demand. Although the life insurance demand results in the model are higher than the observed data in Chamber, Schlagenhauf, and Young (2003), it is lower than their model results in 2009 and 2011. I also discuss the reasons in this essay. My results suggest that the most important determinants of life insurance demand are financial vulnerability, the amount of financial support needed and life insurance premium. Moreover, this paper can contribute to the simulation of the joint decision of life insurance purchases between married couples, and analyze risk sharing between couples with wage shocks and mortality shocks. I find that if one receives a good wage shock, she/he will increase her/his labor time and life insurance coverage holding, while her/his spouse will decrease his/her labor time and life insurance coverage holding.

In the property-liability insurance market, sharp price increases and large capacity swings follow catastrophic loss shocks, such as those caused by a catastrophic natural disaster or a significant macro economic event (see Winter 1988; Gron 1994; Grace and Hotchkiss, 1995; Gron and Winton 2001; Cummins and Nini, 2002; Doherty, Lamm-Tennant, and Stark, 2003; Grace and Klein, 2009). With the possibility of more frequent and severe catastrophe events, it is vital to understand how insurers and the insurance industry can respond in the post-catastrophe period.

In my second essay, I extend a two-period risky debt model (see Cummins and Danzon, 1997) into a two-period cash flow model with one catastrophic risk for an insurer by involving both a reinsurance market and a costly external capital market. I focus on analyzing an insurer's optimal strategy with capacity constraints or without

capacity constraints in an environment of catastrophic shocks. The model contributes to suggest that the insurer has an optimal capital structure in costly capital market, and the solvency ratio plays an important role in the interaction between its ability to sell new business and to raise external capital. I find that in the situation of a tight capital supply and high insurance demand, the positive relationship between catastrophic losses and insurance prices and the negative relationship between losses and insurance coverage capacity can be observed. The model also implies that one catastrophe event could act as an accelerated trigger, splitting insurers into high-quality ones and low-quality ones with respect to different underwriting efficiencies and capital raising abilities. The results indicate that the differences between good and bad insurers will be larger with more volatile catastrophes.

Underwriting cycles in early studies are always described as smooth, symmetric and predictable curves (see Venezian, 1985; Cummins and Outreville, 1987; Chen, Wong, and Lee, 1999; Meier, 2006). In recent years, ups and downs of underwriting cycles are more likely to be asymmetric, unpredictable and random (See Boyer, Jacquier and Van Norden, 2012). In my third essay, I extend the model developed in the second essay by incorporating dynamic settings to explore an alternative source of "underwriting cycles". I look at the "underwriting cycles" in output markets in the insurance industry by using a dynamic model inspired by the Real Business Cycle literature (see Winter, 1994; Kiyotaki and Moore, 1997). I claim the unpredictable "underwriting cycles" as temporary responses of the industrial coverage capacity to insured losses. I build a dynamic cash flow model of an insurer with a series of

catastrophe events in an environment of the costly external capital and insurance regulation to simulate the insurer's optimal catastrophic risk intermediation strategy. The model contributes to show that the dynamic interaction between the insurer's capital rationing and its balance sheet can generate the non-cyclical behavior of output changes if the insurer experiences a series of unexpected catastrophic shocks.

2

The Life Insurance Demand in a Heterogeneous-Agent Life Cycle Economy

Term life insurance can be purchased to mitigate financial problems resulting from premature death risk. By holding term life insurance, a household can hedge against the decline in total household income due to the death of a wage earner, and parents can provide financial security for dependents after their death.

There is no consensus on the amount of household life insurance holdings in empirical research due to the limit of data sources. In this paper, I construct a heterogeneous-agent life cycle model of married households with wage shocks and mortality shocks to quantitatively analyze the life insurance demand by heterogeneous households. I focus on exploring the relation of household life insurance purchases and household characteristics. Moreover, this paper contributes to the understanding of the joint decision of life insurance purchases between married couples, and thus to the analysis of financial risk sharing between couples in a household with wage

shocks and mortality shocks.

In my model, the peak of the household's life insurance coverage holdings in the economy is on average $370,000 occurring at age 33. My results suggest that the most important determinants of life insurance demand are financial vulnerability, the amount of financial support needed and life insurance premium. I also find that the peak of life insurance demand for single-parent households is well before couple households. In addition, increasing the number of children attributes a large increase of life insurance demand in single-parent households, but has no significant effect on couple households. Moreover, I discuss the impact of wage shocks on the joint decision of life insurance purchases between couples: one's good wage shock results in an increase of one's working hours and life insurance demand, but a decrease of spouse's working hours and life insurance demand.

2.1 Introduction

A household's life insurance demand depends on the household characteristics and the economic situation. There is no consensus about the amount and the distribution of household life insurance holdings in empirical research since the data of household life insurance purchases is limited.

Chambers, Schlagenhauf, and Young (2003) examine life insurance data from the SCF[1] for 1995, 1998 and 2001, and find that the peak of life insurance holdings is on average $250,000[2] in year 2001 dollars occurring around 50 years old. Grace and Lin

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[1] Survey of Consumer Finances
[2] $250,000 in 2001 can be adjusted to be $302,000 in 2009 by inflation rates.

(2007) examine SCF data for 1992, 1995, 1998, and 2001, and show the mean of the face value of household term life insurance in selected data is $366,263[3] in year 2001 dollars. They create a new financial vulnerability index, and find that the most significant relationship between life insurance holdings and financial vulnerability is among younger households from age 20 to 34. In addition, they find that older households from age 50 to 64 tend to use less life insurance to protect a certain level of financial vulnerability than middle-aged households from age 35 to 49. LIMRA International[4] (2004) reports that the average life insurance coverage needed for a typical household is $459,000[5] while the average life insurance owned is actually $126,000, which means the average underinsurance is more than $300,000.

Chambers, Schlagenhauf, and Young (2009, 2011) construct an overlapping-generations (OLG) model to find an economic puzzle that life insurance holdings simulated in their model are much larger than their observed data in Chambers, Schlagenhauf, and Young (2003). The peak of life insurance holdings is twice as much as their empirical study in 2003, occurring at age 30 instead[6]. In this paper, I construct a dynamic model with household earnings simulated by stochastic process to quantitatively analyze life insurance holdings of heterogeneous households, and compare my results with the empirical study. As a result of my simulation, the peak of household life insurance holdings in the model economy is around $370,000

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[3] Their study shows the median, the mean, and the maximum of term life insurance holdings in their selected data are $56,700, $366,263 and $80,000,000 in year 2001 dollars, which can be respectively adjusted to be $68,700, $443,700 and $97,000,000 in year 2009 dollars.

[4] Life Insurance Marketing and Research Association International

[5] $459,000 in 2004 can be adjusted to be $ 521,300 in year 2009 dollars. Similarly, the following $126,000 and $300,000 in 2004 can be adjusted to be $143,100 and $ 340,700 in year 2009 dollars.

[6] They choose a small value of risk aversion (1.5) in their OLG model to provide a lower bound for estimation.

occurring at age 33. The simulated result is lower than the theoretical study from Chambers, Schlagenhauf, and Young (2009). I also discuss reasons why the life insurance holdings in my model turn out to be higher than the empirical study from Chambers, Schlagenhauf, and Young (2003).

The SCF data contains information only on the total amount of life insurance held by each household, and not on the division of life insurance between couples. The dynamic model developed in this chapter can also be used to analyze the joint life insurance purchases decision of married couples.

The literature on dynamic models with stochastic process shed light on my research. Hong and Rios-Rull (2007) build an OLG model to analyze the join decision of social security, life insurance and annuities for households. In their model, they assume that agents have a bequest motive and focus on the implications of social security under a variety of baseline economies that differ in the extent to which life insurance and annuities are available. Chambers, Schlagenhauf, and Young (2009) construct an OLG model to find the economic puzzle mentioned above, but they do not aim to explore the join decision of life insurance holdings in a household. Nishiyama (2010) develop an OLG model with uninsurable wage shocks to analyze the effect of spousal and survivors benefits on the labor supply of married couples. He extends a dynamic general-equilibrium OLG model with heterogeneous households and incomplete markets, calibrated to the 2009 U.S. economy, to study to what extent the spousal and survivors benefits possibly distort the joint labor supply decision of married households.

In this paper, I build on the model of Nishiyama (2010). Compared with his paper, I focus on studying life insurance demand of married couples with heterogeneous households and partial market equilibrium. To the best of my knowledge, few papers has analyzed the effect of wage shocks and mortality shocks on the join life insurance holding decision of married households by using a heterogeneous-agent life cycle model.

Specifically, I construct a heterogeneous-agent life cycle model of married households with market wages and mortality shocks to quantitatively analyze the life insurance demand for heterogeneous households. In the model, parents are both altruistic towards each other as well as towards their children. They choose optimal consumption, working hours, and life insurance purchases to maximize their expected lifetime utility. Here I introduce the number of children by ages calibrated by USA data into household characteristics. The dynamic model in this paper can further help explore the relation of a household's life insurance demand to its specific household characteristics and the economic situation. Household characteristics in this paper can include the marital status, the number of children, mortality risk, household wealth, household income, household age, household risk attitude and so on.

My model is to explore the impact of some factors in benchmark economy in the model on life insurance demand. First, the life insurance demand by household ages suggests that the most important determinants of life insurance demand are financial vulnerability, the amount of financial support needed and life insurance premium. I find that financial vulnerability is the primary determinant of life insurance demand in

a household during its early ages when the household has low wage earning and saving wealth; while life insurance premium is the primary determinant during its late ages when the household faces highest mortality risk. Second, the results show that the peak of life insurance demand for single-parent households is well before couple households. In addition, an increasing birth rate can attribute a large increase of life insurance demand in single-parent households, but has no significant influence on couple households. Finally, I discuss the joint decision of life insurance purchases between couples in couple households: if one receives a good wage shock, she/he will increase her/his labor time and life insurance coverage holding, but her/his spouse will decrease his/her labor time and life insurance coverage holding.

The rest of this chapter is structured as follows. Section 2.2 develops the heterogeneous-agent life cycle model in detail. Section 2.3 is the model calibration to U.S. data. In Section 2.4, I show the main numerical results, and analyze how household characteristics and economic factors can affect the life insurance demand in a household. Section 2.5 provides conclusions and discussions. Algorithm methodology and optimization solutions for this life cycle model are present in Appendix 2A and Appendix 2B.

2.2 Heterogeneous-Agent Life Cycle Model

In this section, I build a heterogeneous-agent life cycle model to quantitatively derive the optimal decisions of life insurance holdings, consumption expenditures and labor time in a household. The focus is the optimal decision-making for life insurance

purchases among heterogeneous households.

2.2.1 Heterogeneous Households and Utility Functions

The households in this model economy are heterogeneous with respect to several factors. One factor is household age, denoted by $k = kmin, kmin+1...kmax$. The household enters the economy when the husband's age is over 20. For simplicity, I assume that the husband and the wife in a household are at the same age, and never get divorced.

The number of children in each household is related to household age in the calibration. Note that the child in the model refers to an individual who is still younger than 20 years old. The wage rates per efficient unit of labor for each gender, w_1 and w_2, also vary at different household ages. In this model, the wage rate refers to the wage for one unit of labor hour and one unit of working ability. So one's wage earning is the product of labor hours, wage rate per efficient unit of labor, and working ability.

Two other heterogeneous factors are the husband's and the wife's working ability, denoted by e_1 and e_2 respectively, both of which are assumed to follow a Markov process and to be independent of each other.

The parameter m has four values to specify four heterogeneous martial statuses among households: a married-couple household if $m=0$, a single-father household if $m=1$, a single-mother household if $m=2$, and a kids-only household if $m=3$. In this economy, I assume that all households are married couples at the very beginning. The

calibration of the marital status movement among heterogeneous households over the life cycle time is based on the mortality rate data.

The beginning-of-period household wealth, *a,* is another heterogeneous factor. It changes according to the household optimal saving decision in last period, and the life insurance payment if the household marital status changes in last period. Here I should note that, compared with the beginning-of-period household wealth, $\tilde{a}$ is the end-of-period wealth and denotes the optimal saving decision in the end of each period. In each period, it is determined by the net difference between household cash flow-in, including beginning-of-period household wealth, parents' wage earnings if any, social security payment if any, and cash flow-out, including household consumption, and household life insurance purchasing cost if any.

Therefore, we let *s* be the individual sate vector of a household in the model economy, $s = (a, e_1, e_2, m, k)$.

In this model economy, I assume the household's utility function in each year to be a Cobb-Douglas and CRRA function, which depends on its current marital status *m* and the number of children *n*. I construct utility functions for heterogonous households by marital status as follows.

First, utility functions of single-parent households *(m=1 or m=2)* are,

$$U(c, l_1; n, m=1) = \frac{\left[\left(\frac{c}{(1+n/2)^{\eta}} \right)^{\alpha} (l_1)^{1-\alpha} \right]^{1-\gamma}}{1-\gamma},$$

$$U(c, l_2; n, m=2) = \frac{\left[\left(\frac{c}{(1+n/2)^{\eta}} \right)^{\alpha} (l_2)^{1-\alpha} \right]^{1-\gamma}}{1-\gamma},$$

where *c* is household consumption at household age *k*; l_1 is leisure time of the

husband, which is set to be 1 in the single-mother family; l_2 is leisure time of the wife, which is set to be 1 in the single-father family; n is the number of children calibrated in this household year; η is the index of the economy scale between 0 and 1, and it implies that two-adult household spends 2^η times as much as a one-adult household for the same level of living standard since I assume that the couple can share consumption expenditures; the child-adult equivalency factor is $1/2$; the relative risk aversion is γ; α and ($1-\alpha$) are elasticity of consumption and leisure time in the utility function.

Then the following couple household's utility function *(m=0)* is the sum of two utility functions above with a slight modification that the number of equivalent adults in a couple-household becomes *(2+n/2)*,

$$U(c, l_1, l_2; n, m=0) = \frac{\left[\left(\frac{c}{(2+n/2)^\eta}\right)^\alpha (l_1)^{1-\alpha}\right]^{1-\gamma}}{1-\gamma} + \frac{\left[\left(\frac{c}{(2+n/2)^\eta}\right)^\alpha (l_2)^{1-\alpha}\right]^{1-\gamma}}{1-\gamma}.$$

Finally, the utility function of children-only households *(m=3)* is,

$$U(c; n, m=3) = n \times \frac{\left[\left(\frac{c}{n/2}\right)^\alpha\right]^{1-\gamma}}{1-\gamma},$$

where the number of equivalent adults in a kids-only household becomes *n/2*; leisure time for mother and father are both set to be *1*; and all consumption commodities are going to the children. Here I set the economy scale η to be 1 since I assume that children do not share consumption commodities without parents' custody. Then I sum up all equivalent adults' utility to get total utility for the children-only household.

2.2.2 Household Optimization Problem of Expected Lifetime Utility

In the model, household fund sources for each year are wags earnings, life insurance payments if any death and social security payments if any retirement, and funds are annually distributed into three categories: consumption expenditure, end-of-period wealth saved, and life insurance purchases. The household chooses the optimal decision path for each specific fund source and fund usage to maximize its expected utility over the lifetime.

I assume that initial households are all married couples with some children at age 20. In year k, each household receives working ability e_1 for husband and e_2 for wife, and faces mortality rates $(1-\varphi_{1,k})$ for the husband and $(1-\varphi_{2,k})$ for the wife. To maximize expected lifetime utility of a household, the adults will choose the following optimal decision rules together in each year: household consumption c, husband's leisure time l_1, wife's leisure time l_2, end-of-period saving wealth $\tilde{a}$, husband's life insurance coverage d_1 and wife's life insurance coverage d_2. In this model, d_1 and d_2 are viewed as life insurance demand of the husband's and the wife's. Here I assume that life insurance can be purchased at the actuarially fair cost.

If a household happens to become a single-father household, the amount of the wife's life insurance payment d_2 will be added into the household saving wealth. The single-father will then choose the optimal decision rules each year to maximize household utility in the following life cycle time: household consumption c, end-of-period saving wealth $\tilde{a}$, his leisure time l_2, and his life insurance coverage d_1. The single-mother households in the model follow the same logic.

If a household becomes a kids-only household, the children will only choose the amount of household consumption and end-of-period saving wealth to maximize household utility.

Let $V(s)$ be value function of the household in individual state s. The optimization problem for the household in the life cycle model is as follows,

$$V_k(s) = \max\left[\, U(c,\, l_1, l_2;\, n_k, m) + \beta \int V_{k+1}(s')\; d\Pi_k(e_1', e_2', m' | e_1, e_2, m)\,\right]$$

(1) Control variables' constraints,

$$c > 0;$$

$$0 < l_1 \le 1;\ 0 < l_2 \le 1;$$

$$d_1 \ge 0;\ d_2 \ge 0;$$

$$l_1 = 1,\ d_1 = 0,\ if\ m = 2\ or\ 3;$$

$$l_2 = 1,\ d_2 = 0,\ if\ m = 1\ or\ 3.$$

(2) The law of motion of household end-of-period wealth,

$$\tilde{a} = (1+r)\,a + w_{1,k}\,e_1\,(1-l_1) + w_{2,k}\,e_2\,(1-l_2) + \mathbb{1}_{k>65}\,\mathbb{1}_{m<3}$$

$$(1+\mathbb{1}_{m=0})\,ss - c - (1-\varphi_{1,k})\,d_1 - (1-\varphi_{2,k})\,d_2.$$

(3) The law of motion of household beginning-of-period wealth,

$$a' = \tilde{a}(s) + d_1,\ if\ m = 1\ and\ m' = 3\ or\ m = 0\ and\ m' = 2;$$

$$a' = \tilde{a}(s) + d_2,\ if\ m = 2\ and\ m' = 3\ or\ m = 0\ and\ m' = 1;$$

$$a' = \tilde{a}(s) + d_1 + d_2,\ if\ m = 0\ and\ m' = 3;$$

$$a' = \tilde{a}(s),\ \text{otherwise.}$$

(4) The law of motion of household state variables, $s' = (a',\ e_1',\ e_2',\ m',\ k+1).$

Here, r is the interest rate; $w_{1,k}$ is husband's wage rate per efficient unit of labor at

age k; $w_{2,k}$ is wife's wage rate per efficient unit of labor at age k; $\mathbb{1}_{m=0}$, $\mathbb{1}_{k>65}$ and $\mathbb{1}_{m<3}$ are all indicator functions; ss is social security payment per person above 65 years old; $\varphi_{1,k}$ is survival rate for the husband at the end of age k; $\varphi_{2,k}$ is survival rate for the wife at the end of age k; $\Pi_k(e_1', e_2', m'|e_1, e_2, m)$ is the transition probability function in the optimization, which will be calibrated in Section 2.3.

2.2.3 Population Distribution and Aggregation

For population aggregation, I construct the population distribution function for heterogeneous households in different states. One household sate includes wealth amount, husband/wife's working ability, marital status and household age.

Let $x(s)$ be the household population probability density function at age k, and let $X(s)$ be the corresponding cumulative distribution function. The household population for each age is normalized to unity,

$$\sum_{m=0}^3 \int_{A*E^2} dX(s) = 1, \text{ where } s = (a,\, e_1,\, e_2,\, m,\, k)$$

The law of motion of the household population distribution is as follows,

$$x(s')ds' = \sum_{m=0}^3 \int_{A*E^2} \mathbf{1}\{a' = a'(\tilde{a}(s),\, m')\} * \Pi_k(e_1', e_2', m'|e_1, e_2, m)\, dX(s);$$

$$\text{where } s' = (a', e_1', e_2', m', k+1).$$

Then the aggregated values of the optimal wealth, consumption and life insurance demand by each household age for the whole population are as follows,

$$W_k = \sum_{m=0}^3 \int_{A*E^2} a(s)\, dX(s)$$

$$C_k = \sum_{m=0}^3 \int_{A*E^2} c(s)\, dX(s)$$

$$D_k = \sum_{m=0}^2 \int_{A*E^2} [d_1(s) + d_2(s)]\, dX(s)$$

2.3 Model Calibrations

In this section, I calibrate the model to match pertinent U.S. data. My calibration addresses preference parameters, household demographic distribution and household income distribution. Note that the consistency of the wealth distribution with U.S. data is essential to the model.

2.3.1 Main Preference Parameters

Table 2.1: Main Preference Parameters

Parameters	Notations	Values
Interest rate	r	0.05
Time discount factor	β	0.94;
Share of consumption in utility	α	0.36;
Index of household scale economies	η	0.678
Coefficient of relative risk aversion	γ	4.00
The number of wealth nodes	$imax$	20
The number of wage shock nodes	$jmax$	5
The number of marital status types	$mmax$	4
Initial household age	$kmin$	1 (Real age 20)
Retirement age	k_r	45 (Real age 65)
Maximum household ages	$kmax$	80 (Real age 99)
Social security payment	ss	$16,500

Table 2.1 is a list of the main economic parameters in this model. These parameter values are all consistent with either the economic literature or U.S. historical data.

In addition, I assume that all households have the same number of children for each age. I use the average number of children estimated by Nishiyama (2010), which is calibrated from the data of fertility rates at mothers' ages. Figure 2.1 shows the estimated average number of children by household ages in this paper.
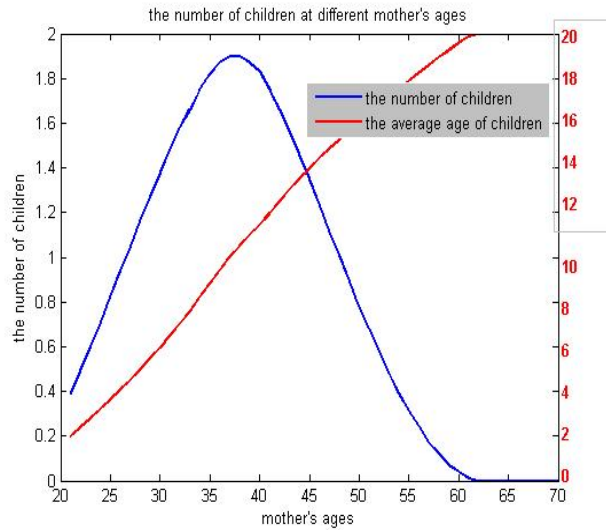
Figure 2.1: The Number of Children by Household Ages

2.3.2 Marital Status Transition and Calibration

For simplicity, I assume that both the husband's working ability and the wife's working ability in this life cycle model are independent of his/her mortality rate. Thus the state transition function for households can be obtained by the following formula,

$$\Pi_k(e_1', e_2', m'|e_1, e_2, m) = \Pi_k(m'|m)\, \Pi\,(e_1'|e_1)\ \Pi(e_2'|e_2)$$

In this section, I focus on the marital status transition process and its calibration. All households are initially married couples. With certain probabilities and evolving

paths, initial couple-households turn to be heterogeneous with different marital statuses over the life cycle time.

To simply specify probabilities of marital status movement, I assume that the husband's mortality rate and the wife's mortality rate are independent of each other. Here, survival rates by ages are cited from Table 4 of the 2010 Annual Statistical Supplement of the Social Security Administration. The household marital status transition probability matrix from state m at age k to state m' at age $(k+1)$ is $\Pi_k(m'|m)$. Then we have,

$$\Pi_k(m'|m)=\begin{pmatrix} \varphi_{1,k}\,\varphi_{2,k} & \varphi_{1,k}\,(1-\varphi_{2,k}) & (1-\varphi_{1,k})\,\varphi_{2,k} & (1-\varphi_{1,k})\,(1-\varphi_{2,k}) \\ 0 & \varphi_{1,k} & 0 & 1-\varphi_{1,k} \\ 0 & 0 & \varphi_{2,k} & 1-\varphi_{2,k} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
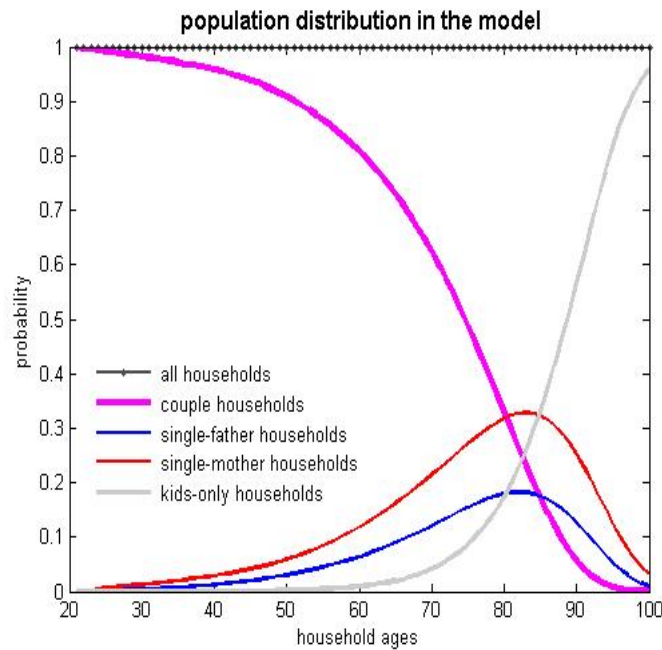
Figure 2.2: Population Distribution with Marital Status by Ages

Figure 2.2 shows the population distribution with respect to marital status

calibrated in the model.

2.3.3 Working Ability Transition and Wage Rate Calibration

As I mentioned in Section 2.2.1, one's wage earning is the product of working ability, wage rate, and labor hours. In this section, I build up the working ability transition process for both husbands and wives, and also calibrate the wage rate by ages for two genders by U.S. data.

With idiosyncratic wage shocks for each household state, wage earners have access to different levels of working ability in the model. The motions of the husband's working ability e_1 and the wife's working ability e_2 are both assumed to follow Markov chains, and these two stochastic processes are independent of each other. The stochastic processes of e_1 and e_2 are as follows,

$$ln\ e_{1,j+1} = ln\ e_{1,j} + z_{1,j+1}$$

$$ln\ z_{1,j+1} = \rho\,z_{1,j} + \varepsilon_{1,j+1} \quad \text{where } \varepsilon_{1,j} \sim N(0,\ \sigma_1^2)$$

$$ln\ e_{2,j+1} = ln\ e_{2,j} + z_{2,j+1}$$

$$ln\ z_{2,j+1} = \rho\,z_{2,j} + \varepsilon_{2,j+1} \quad \text{where } \varepsilon_{2,j} \sim N(0,\ \sigma_2^2)$$

where z_1 and z_2 are persistent wage shocks of men and women.

Nishiyama (2010) calibrates the income distribution for men and women by estimating these parameters above, which is consistent with the data from weekly earnings in CPS[7]. This numerical approximation yields the vector of persistent wage shock nodes and the Markov working-ability transition probability matrix for each

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[7] Current Population Survey

gender as follows,

$$z_1 = [0.3103, 0.5801, 1.0000, 1.7240, 3.2229] ;$$

$$z_2 = [0.3322, 0.5988, 1.0000, 1.6701, 3.0099] ;$$

$$\Pi(e_1'|e_1) = \Pi(e_2'|e_2) = \begin{pmatrix} 0.8979 & 0.1021 & 0.0000 & 0.0000 & 0.0000 \\ 0.0308 & 0.8902 & 0.0790 & 0.0000 & 0.0000 \\ 0.0000 & 0.0518 & 0.8964 & 0.0518 & 0.0000 \\ 0.0000 & 0.0000 & 0.0790 & 0.8902 & 0.0308 \\ 0.0000 & 0.0000 & 0.0000 & 0.1021 & 0.8979 \end{pmatrix}$$

Using the data of 2009 weekly earnings in CPS, first of all, I calibrate median wage rates for both husband and wife from age 21 to 65. Here one is assumed to retire at age 65. The data shows that the median annual wage earning of a full-time employee for both genders is 739×52. Correspondingly in this model, median annual income is the product of benchmark wage rate per efficient unit of labor, median labor hours, and median working ability (1.0000). I assume that median labor time is 1/3 out of 1. So the benchmark wage rate in this economy should be $\bar{w} =$ $\$ (739 \times 52)/ [(1/3) \times 1.0000] = \$1.15284 \times 10^5$.

Then I calculate the wage rate for each gender at each age by multiplying the benchmark wage rate $\bar{w}$ and the ratio of his/her median weekly wage at each age to the median full-time employee's weekly wage ($739). Figure 2.3 shows original and OLS-adjusted wage rates for both husband and wife over the life cycle time.
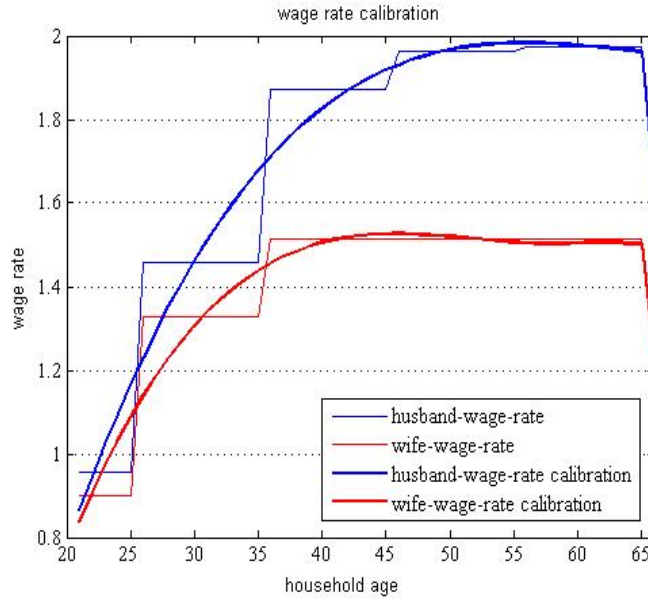
Figure 2.3: Wage Rates by Ages and Genders

Note that I cannot obtain the upper tail of the income distribution and wealth distribution in the model economy since the working ability transition probability function cannot produce a quite high income. Except the upper tail, both of the income distribution and the wealth distribution by ages calibrated in the model are consistent with U.S. data.

2.4 Household Life Insurance Demand Results and Analysis

In this section, I show the numerical results of the Heterogeneous-Agent Life Cycle model economy, and explore the determinants of life insurance holdings for heterogeneous households. I also analyze the impact of some factors in benchmark economy in the model on life insurance demand.

Figure 2.4 shows the peak of life insurance demand in our model is around $370,000 occurring at age 33, and the demand begins decreasing rapidly after age 54 by more than 10% per year.

The life insurance demand in the model is higher than observed data in Chamber, Schlagenhauf, and Young (2003). First, I assume that all households initially are married couples, which can increase overall life insurance holdings since a single household should have relatively less life insurance demand.
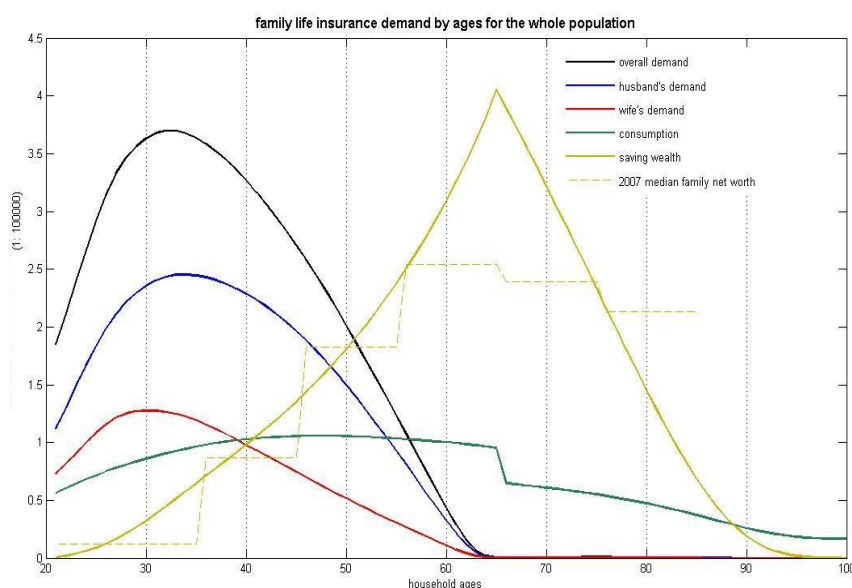
Figure 2.4: Household Life Insurance Demand for Aggregate Population

Second, the wage calibration is based on the income data of full-time employee in 2009. This attributes a higher median value of wage earning than their data, and thus a higher amount of life insurance purchases.

Third, savings in the model are smaller than their data, so households tend to purchase more life insurance to protect their financial security. I have the two following reasons. Considering that the wealth distribution in the real world is skewed, I calibrate the wealth distribution using the median value of household net worth in 2007 from SCF instead of the mean value. The fact that the median value of net worth is less than the mean value leads to a smaller amount of saving wealth calibrated in

my model than the survey data. Additionally, the upper tail of the wealth distribution cannot be calibrated due to the limit of my model.

Finally, the risk aversion coefficient is consistently equal to a high level of 4.0 over the lifetime in my model, which may push life insurance demand up. If I change the coefficient of risk aversion to the same value (1.5) as Chambers, Schlagenhauf, and Young (2009), the peak of life insurance holding decreases to $320,000 occurring household age 37.
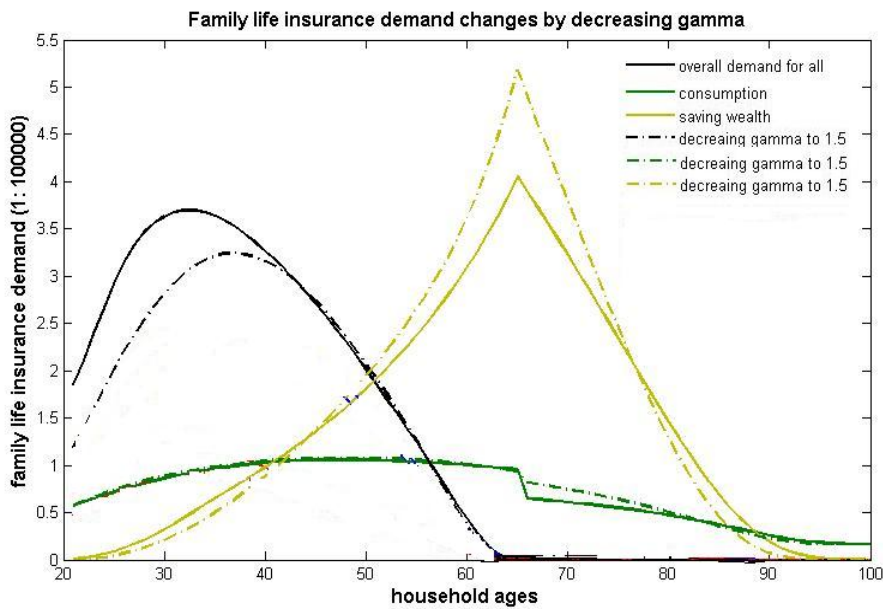
Figure 2.5: Household Life Insurance Demand with Gamma being 1.5

Figure 2.5 provides life insurance demand with a low risk aversion, 1.5. The results are smaller than the theoretical estimation in Chambers, Schlagenhauf, and Young (2009) and fit their observed data in 2003 better. However, in this way, there is no consistence of the calibrated wealth distribution in the model with the U.S. household net worth data in 2007.

2.4.1 Household Age

Figure 2.4 shows the life insurance demand by household ages. During early ages from 20 to 30, households hold a relatively small amount of wealth and have relatively low earnings, and have an increasing number of children. They tend to save money and purchase a high level of life insurance holdings to provide financial security. On one hand, households continually increase life insurance purchases to satisfy their increasing financial support needed. On the other hand, wage earners of young households have the lowest mortality risk, so households choose to purchase high but not the highest life insurance coverage despite of quite a low life insurance premium.

From age 31 to 40, households have the largest number of children and need a large amount of financial support from both saving wealth and life insurance purchases. Therefore, households keep purchasing high life insurance coverage to hedge mortality risk and wage shocks, and continue to save a lot to increase economic strength. The peak of life insurance holdings occurs at age 33. Since the number of children begins to decrease from age 37, life insurance holdings starts decreasing by less than 3% per year in the late of this period.

From age 40 to 54, the number of children in a household continues to decrease and wealth is continuously accumulated. Households decrease life insurance purchases by between 3% and 10% per year. Figure 2.5 shows there is a much larger chance for people above 55 years old to die than those below 55, and mortality rates quickly increase after age 55. However, from age 55 to 65, households have almost no

children and possess the largest amount of wealth to provide financial security. Considering life insurance premium is much more expensive for them than before, they largely decrease life insurance purchases by above 10% per year as a result.

The age distribution of life insurance holdings in the model suggests that the avoidance of financial vulnerability due to mortality shocks and wage shocks, the amount of financial support needed and the life insurance premium are the most important determinants of life insurance demand.

2.4.2 Mortality Shock

Figure 2.6 shows that life insurance demand greatly changes when I decline survival rates by 5%[8] in the model. In this case, the household is not able to earn as much money as in the benchmark economy, and household consumption and welfare are both dropping considerably.
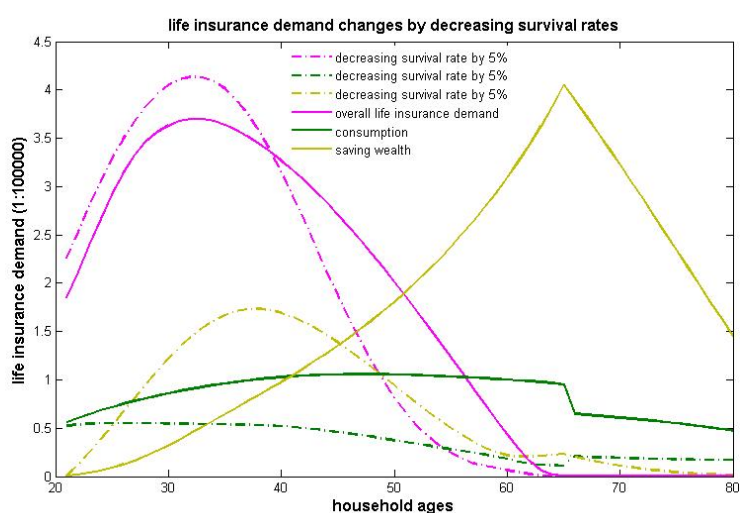
Figure 2.6: Household Life Insurance Demand with Lower Survival Rates

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[8] Although it is unrealistic that the survival rate can be reduced by 5%, I test it to examine the impact of a mortality shock on life insurance demand.

In spite of a low income, households during early ages save much more money than in the benchmark economy, and purchase more life insurance coverage to provide financial security to protect households from the increased mortality risk. In contrast, households tend to have significantly less life insurance holdings than in the benchmark economy during late ages due to much higher life insurance premium.
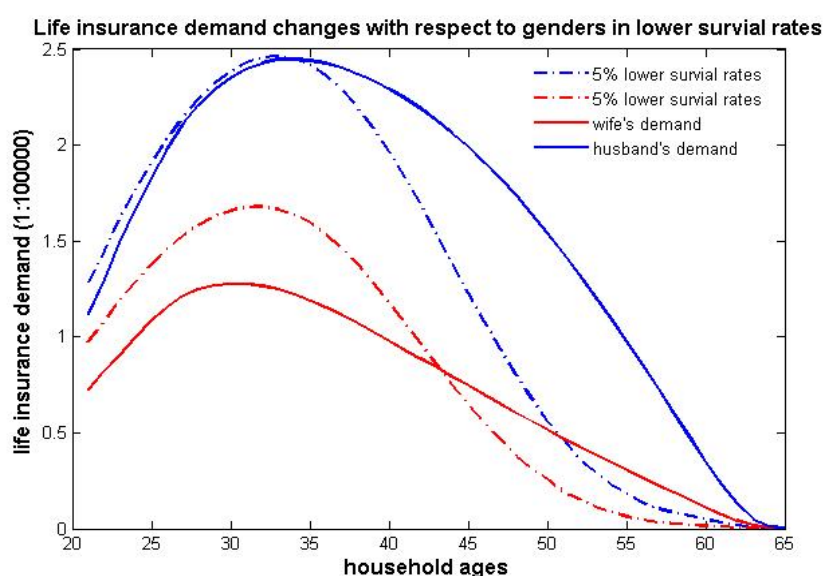
Figure 2.7: Life Insurance Demand by Genders with Lower Survival Rates

Figure 2.7 further provides the changes in the life insurance demand for each gender. I notice that the wife's life insurance holdings during early ages jump extraordinarily so as to hedge greater financial risk due to her higher death probability. I can also notice a large decrease of life insurance holdings for husbands among old households due to the much higher insurance premium.

Based on the fact that survival rate for the wife is higher than for the husband in each household year, I can strengthen our inference that the avoidance of financial vulnerability is the dominating factor of life insurance purchases during household early ages and instead life insurance premiums are the dominating concern during

household late ages.

2.4.3 Household Marital Status

Figure 2.8 shows life insurance demands with respect to the different situations of marital status. Couple households follow the same tendency of life insurance demand movement as the whole population.

My results show a slightly different situation for single-parent households. The single-parent household tends to save much more than couple households at the very early ages to avoid potentially high financial vulnerability due to the last adult's death. However, the peak of life insurance purchases for single-parent households (around age 28) is well before couple households (at age 33) since the amount of financial support needed for single-parent households afterwards is smaller than that of couple households as the number of children decreases with household ages.
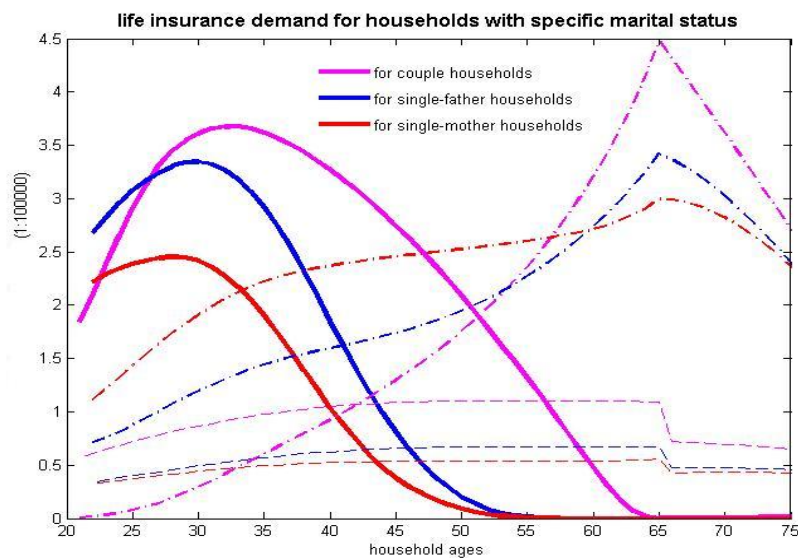
Figure 2.8: Household Life Insurance Demand by Martial Status

2.4.4 The Number of Children

Figure 2.9 shows the impact of the number of children in a household on life insurance demand. Compared to a slight increase in couple household's life insurance purchases with the increasing number of children, there is a big jump of life insurance holdings for the single-parent households.

The results show that raising the number of children in each household by 10% leads to 12.8% higher life insurance demand of single-father households and even 15.6% higher life insurance demand of single-mother households. However, the life insurance demand of couple households increases only by 1.6% in this case. The results suggest that increasing the number of children attributes a high increase of life insurance demand in single-parent households, but has no significant influence on couple households.
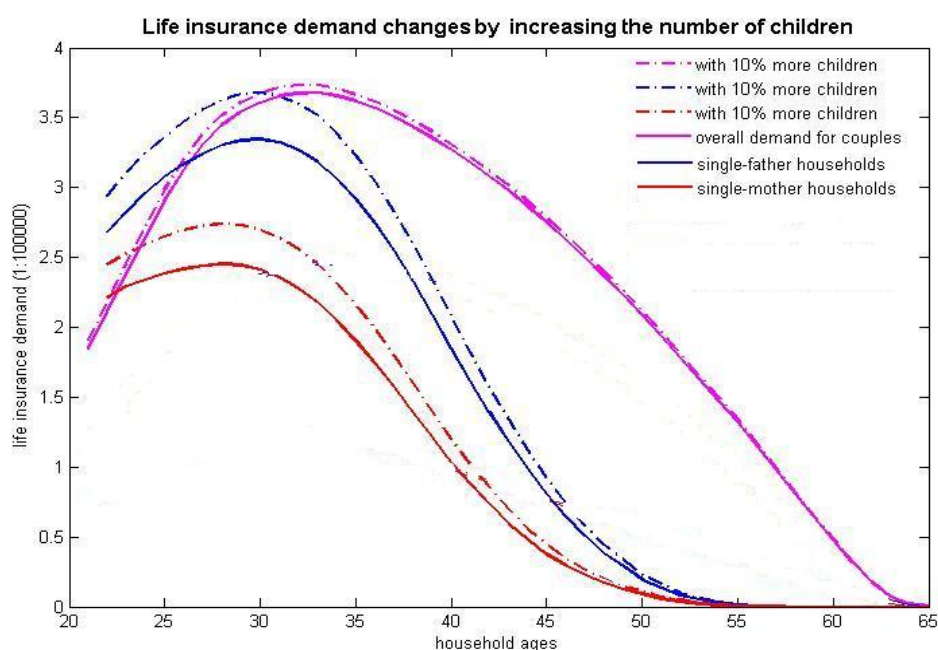
Figure 2.9: Household Life Insurance Demand with More Children

2.4.5 Household Income and Wealth

Figure 2.10 shows the relationship between the husband's life insurance demand and household wealth and his working ability at age 39. When the husband's working ability becomes higher, his life insurance demand is increased which can provides stronger financial security in order to guarantee that remaining household members are able to keep the same level of living standard before and after his death.

Figure 2.11 shows the husband's life insurance demand is reduced when the wife's working ability is higher. It is because of his prediction of less financial risk after his death.
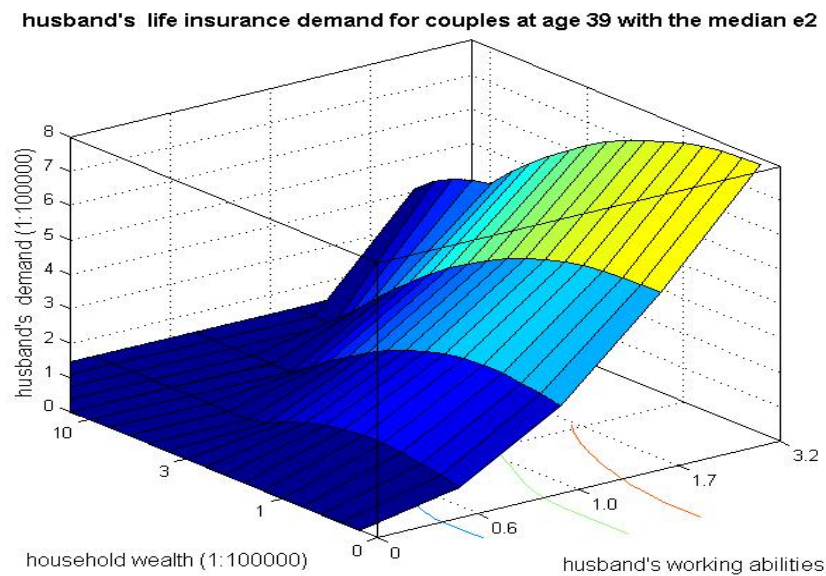
Figure 2.10: Husband's Life Insurance Demand in Couple Households by Household Wealth and His Working Ability at Age 39

For the aggregate population, Figure 2.12 illustrates the simulation results of the joint decision of life insurance purchases between couples in couple-households. It implies that the more important wage earner holds higher life insurance coverage. It shows that with the wife's working ability increasing by 5%, the husband's life

insurance demand reduces on average by 4.7% and the wife's life insurance demand grows up by 9.5%. Therefore, if one wage-earner receives a good wage shock, she/he will increase her/his working hours and life insurance coverage, and her/his spouse will decreases his/her working hours and life insurance coverage to maximize household utility.
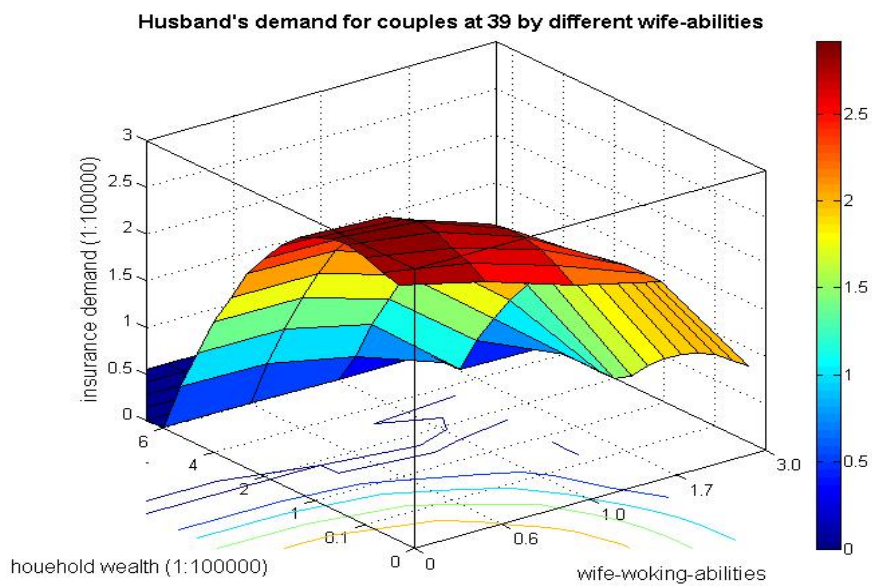
Figure 2.11: Husband's Life Insurance Demand in Couple Households by Household Wealth and His Wife's Working Ability at Age 39
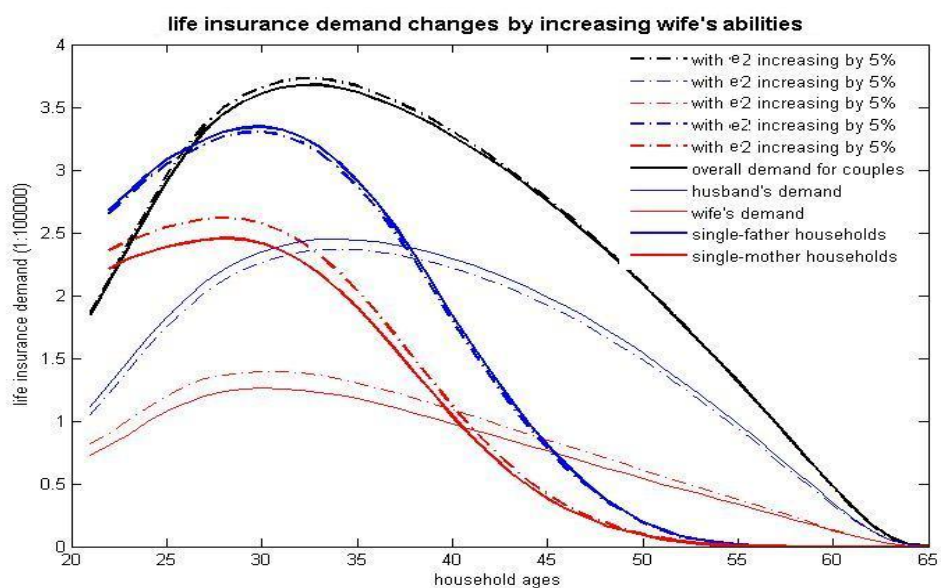
Figure 2.12: Life Insurance Demand with a Higher Wife's Working Ability

2.4.6 Household Welfare

Figure 2.13 shows couple household's welfare at age 39. It implies that household welfare is increasing with working ability increasing and wealth rising.
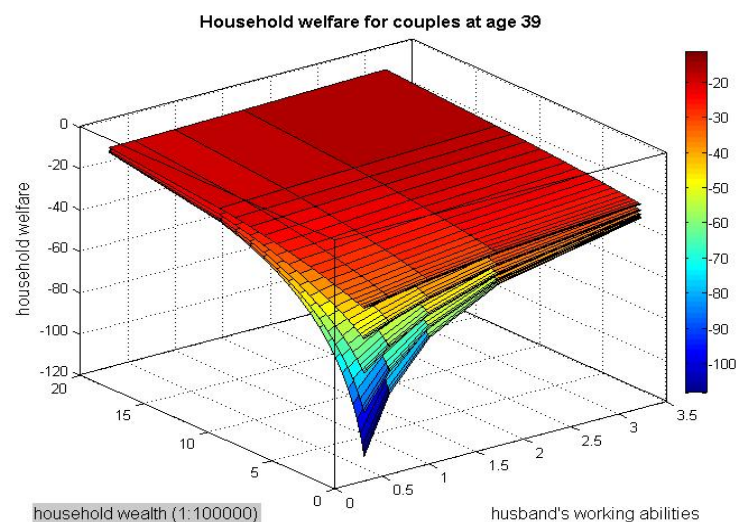
Figure 2.13: Couple-Household Welfare by Wealth and Working Ability at 39

2.5 Conclusions and Discussions

In this paper, I construct a heterogeneous-agent life cycle model with market wage shocks and mortality shocks to explore the relation of a household's life insurance demand to its specific household characteristics and the economic situation. The dynamic model and its calibration are discussed and the optimal decisions of saving wealth and life insurance purchases among heterogeneous households are found by numerical approximation.

I discuss the impact of mortality shocks, marital status, the number of children, and wage shocks on life insurance holdings among heterogeneous households. The results suggest that the most important determinants of life insurance demand are

financial vulnerability, the amount of financial support needed and the level of life insurance premium. More importantly, this paper provides one way to simulate the join decision of life insurance purchases between married couples and show risk sharing within a household.

The results in this chapter show us the same puzzle as Chambers, Schlagenhauf, and Young (2009) do, that the peak of that in the model economy is occurring much earlier than their observed data from SCF in 2003, and that the simulated amount is larger than their empirical study in 2003, although I obtain smaller results than them. This comes up with several interesting questions to explore: whether the U.S. households underinsured or not, whether people are rational or not towards household financial products, and what kind of factors can bridge the gap between empirical research and theoretical model in studying the life insurance demand.

In future research, it is worth matching this heterogeneous household life cycle model economy more exactly with the recent SCF data economy, which can make the simulation result more convincing and the comparison result more useful. I will focus on examining the demographic distribution, income distribution, wealth holdings and life insurance holdings in data economy, and also improving the calibration to match the observed data. In addition, I will try to introduce funeral cost, annuities, divorce and remarriage into the model economy in order to find a way to perform the matching experiment. I also aim to conduct sensitivity analysis to explore the impact of some other factors on life insurance holdings, such as risk attitude, social security

payment, interest rate, annuities holdings, divorce rate and remarriage rate. It is also interesting to examine the change of household welfare by dismissing life insurance holdings in the model.

3

A Two-Period Cash Flow Model with Catastrophic Risk

In the property-liability insurance market, sharp price increases and large capacity swings follow catastrophic loss shocks. Taking Hurricane Katrina (2005)[9] for instance, some insurance companies stopped insuring homeowners in the disaster area because of the high costs from Hurricanes Katrina, or raised homeowners' insurance premiums to cover their risk.

At the firm level, after a catastrophe event, insurers turn out to have different post-catastrophe performances. For example, eleven property/casualty insurers[10] became insolvent resulting from Hurricane Andrew (1992). Some of the state's largest homeowners insurers had to obtain resources from their parent companies and others had to use their surplus to pay Hurricane Andrew claims. Allstate, as an example, paid out $1.9 billion, $500 million more than it had made in profits from its Florida

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[9] Hurricane Katrina, the fourth catastrophe, is not only the most expensive natural disaster on record but also an event that intensified discussion nationwide about the way natural and man-made disasters are managed.
[10] Ten in Florida and one in Louisiana.

operations from all types of insurance and investment income on those funds over the 53 years it had been in business.[11]

With the possibility of more frequent and severe catastrophe events, it is important to understand how insurers and the insurance market respond. In this paper, I study an insurer's optimal strategy in a two-period cash flow model with capacity constraints and without capacity constraints, given the possibility of catastrophic shocks. I further analyze how catastrophic shocks can affect the industrial organization of the property-liability insurance market by examining the insurers' post-catastrophe performance.

The model contributes by suggesting that the insurer has an optimal capital structure in a world where capital is costly. Further, the firm's solvency ratio plays an important role in the interaction between its ability to sell new business and to raise external capital. I find that the insurer's supply capacity is decreased and the external capital shrinks due to capacity constraints after catastrophic shocks.

I also find that one catastrophic event could act as an accelerated trigger, splitting insurers into high-quality ones and low-quality ones with respect to different underwriting efficiencies and capital raising abilities. I claim that a well-capitalized insurer could have advantages in both the ability to sell new business and the ability to raise external capital. Such an insurer may even gain additional profit when it can take advantage of the insurance price increase and the insured's loyalty after catastrophic shocks.

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[11] "Catastrophes: Insurance Issues", Insurance Matters, May 2013.

3.1 Introduction

Both the Capacity Constraint Theory (Winter, 1988; Gron, 1994) and the related Risk Over Hang Theory (Gron and Winton 2001) suggest that sharp price increases and large capacity swings will follow a capital shock, such as those caused by a large natural disaster or a significant macro economic event. This is, in part, due to relatively high capital adjustment costs (see Winter 1988, 1991; Gron, 1994). In the property-liability insurance market, the mismatch between an unexpected catastrophe loss and loss reserves could cause a capital shortfall and a premium increase for the entire insurance industry (see Gron 1994; Gron and Winton 2001; Cummins and Nini, 2002; Doherty, Lamm-Tennant, and Stark, 2003).

To examine the effect of catastrophe events on the insurance industry, it is essential to understand how insurers and the insurance market respond to catastrophic shocks in different environments.

One can imagine that, once a catastrophe occurs, the demand expansion and the supply reduction turn out to cause premiums to grow sharply and then gradually moderate as the insurance industry becomes sufficiently recapitalized. During this process, insurers with a comparative advantage in intermediating catastrophic risks may take advantage of the market price increase and relatively low cost of external capital, while other insurers may encounter insolvency or significant financial stress resulting from capital insufficiency. Further, one catastrophic event could act as trigger, splitting insurers into high-quality ones and low-quality ones with respect to different underwriting efficiencies and capital raising capabilities. Meanwhile, new

investors, who would supply capital to incumbent insurers and new insurers, may enter the insurance market after the event. With incumbent insurers categorized by their ability to withstand serial catastrophes and new comers continually entering into the market, changes in the insurance industry are sequentially occurring.

In this paper, I construct a two-period cash flow model with catastrophic risk for an insurer in order to find whether and how catastrophic shocks can influence insurance prices and the industrial organization in the property-liability insurance market. The focus of the model is to understand how a catastrophe event can affect the insurer's underwriting decision and capital structure with capacity constraints.

I find the profit-maximizing insurer has an optimal capital structure in an environment with costly capital. The model suggests that in the situation of a tight capital supply and high insurance demand, a positive relationship between catastrophic losses and insurance prices and the negative relationship between losses and insurance coverage capacity can be observed.

The two-period model contributes by showing that the insurer's solvency ratio plays an important role in the interaction between the insurer's balance sheet and external capital rationing. I also find that the insurer with a good solvency position prior to the shock could obtain advantageous position in both the ability to sell new business and the ability to raise external capital. This indicates that the difference between good and bad insurers will be larger with more volatile catastrophes. In the future, I also quantitatively derive the condition in which the insurer can benefit from underwriting catastrophic risk.

This chapter is structured as follows. Section 3.2 is literature review. Section 3.3 develops a two-period cash flow model with catastrophic risk for an insurer. In section 3.4, I solve the model and also analyze the insurer's optimal catastrophic risk intermediation strategy in two different cases: without capacity constraints and with capacity constraints. In section 3.5, I show some implications developed from this model for a potential empirical study. Section 3.6 provides conclusions and discussions.

3.2 Literature Review

By the Capacity Constraint Theory and the related Risk Over Hang Theory, the short-run insurance industry's supply curve is upward sloping when a capacity constraint becomes binding, and that it is costly for insurers to raise new capital immediately following a negative capital shock because of agency and bankruptcy costs. Negative shocks to claims or industry capital can substantially reduce industry capacity, shifting the supply curve to the left to push up the price (see Winter, 1988, 1991, 1994; Gron, 1994).

In the literature, many papers provide support for some of the findings of the Capacity Constraint Theory in the property-liability insurance industry. Several studies have found that unanticipated decreases in the insurance industry capacity can cause higher profitability and prices (see Winter 1988, 1994; Gron, 1994; Doherty and Garven, 1995). Grace and Hotchkiss (1995) show the great effects of shocks to the general economic variables on the insurance underwriting performances measured by

profitability. Doherty, Lamm-Tennant, and Starks (2003) check the temporal and cross sectional variation in insurance company stock prices after 9/11, and find insurers suffering the lowest losses with less leverage were able to exploit the post-loss hard market. This implies that insurers could make profit if they can develop a successful catastrophic risk intermediation strategy. Grace and Klein (2009) indicate that insurers have substantially raised insurance rates and reduced their exposures after the intense hurricane seasons of 2004 and 2005, and that there has been substantial market restructuring in Florida but significantly less so in other states. This is really due to the fact that Florida's market is subject to many more shocks than other markets. They also show the evidence that catastrophes can influence the insurance industrial organization.

Researchers have also built models to study the relationship between shocks and capitalization. Froot, Scharfstein, and Stein (1993) develop a portfolio model of corporate risk management to show that capital-market imperfections can make risk-neutral insurers appear to be risk averse and to be more risk averse if there is a negative shock to internal capital. This portfolio model is extended to research shocks in the insurance industry. Gron and Winton (2001), for example, conclude that nonlife-insurers will reduce their willingness to engage in correlated business activities when past risks cannot be easily diversified or hedged. This kind of model suggests that negative shocks to capital can decrease the industry capacity.

Cagle and Harrington (1995) and Cummins and Danzon (1997) both develop cash flow models to predict an ambiguous relationship between the insurance price and a

loss shock based on different assumptions about the effects of shocks on demand elasticity (also see Grace, Klein and Kleindorfer, 2004). Specifically, the model of Cagle and Harrington (1995) is a one-period cash flow model for the insurance market equilibrium with the costly capital market assumption. Cummins and Danzon (1997) build a two-period risky debt model for an insurer with new equity endogenously issued in the second period. In their model, the costly capital market and the capacity constraint are not emphasized.

In this paper, I extend these two models into a two-period cash flow model with catastrophic risk for an insurer by involving both a reinsurance market and an external capital market. In this model, I aim to study the impact of catastrophic shocks on the insurer's next-period optimal strategic choices of the underwriting capacity quantity and the capital structure under different environments of financial markets.

3.3 Two-Period Cash Flow Model with Catastrophe Risk

0	Unit loss l^i with the probability of P^i	1		2
e_0 $\pi Q(\pi, b)$	$-l^i Q$	$\pi^i Q^i(\pi^i, b^i)$		$-Q^i(\pi^i, b^i)$
e	$-R(e, b)$	e^i		$-R^i(e^i, b^i)$
$-C(b)\beta Q(\pi, b)$	$\beta l^i Q$	$-C^i(b^i)\beta^i Q^i(\pi^i, b^i)$		$\beta^i Q^i(\pi^i, b^i)$
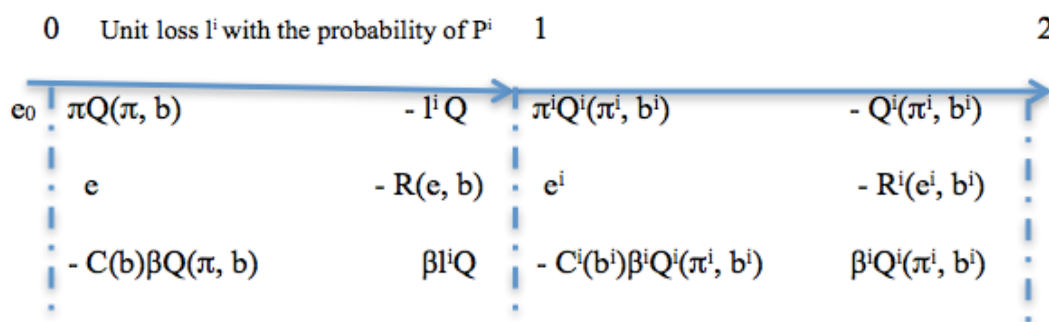
Figure 3.1: Time Line of the Two-Period Cash Flow Model for an Insurer

In this section, I develop a two-period cash flow model with one catastrophic event for an insurer to explore the insurer's optimal catastrophe risk intermediation strategy.

In this two-period model, the insurer originally has retained earnings e_0 as initial

endowment, and one catastrophe event occurs during the first period. Figure 3.1 shows the time line in this model.

At the beginning of each of these two periods, the insurer collects annual premium πQ from the insured, where π is the insurance premium per unit of coverage and Q is the total insurance coverage. I assume that the premium π is exogenously determined in the first period, and the insurer chooses its optimal post-catastrophe premium in the second period.

The insurer also raises external capital. Here I treat external capital as one-period debt, which is issued by the insurer at an amount of e at the beginning of each period and is repaid with a total amount of external capital cost, R, at the end of the period. In the real world, the insurer can raise the capital both from debt holders with interest cost and equity holders with agency cost and adjustment cost. Here I use debt holders instead of equity holders because it is easy to calculate the cost of the capital in each period.

Meanwhile, the insurer purchases reinsurance coverage of βQ, where β is the ratio of reinsurance coverage to the total coverage, and C denotes the reinsurance premium per unit of reinsurance coverage. Here β is between 0 and 1, and $\beta = 0$ means no reinsurance while $\beta = 1$ means full reinsurance.

At the end of each period, the insurer indemnifies the insured for covered losses lQ and receives the reimbursement of βlQ from the reinsurer. Here l can be viewed as unit loss that is the loss incurred per dollar insured. We can denote $L = lQ$ as the loss incurred. At the same time, the insurer repays the due debt R back to external capital

holders.

In this model, the insured event occurring during the first period can cause different levels of losses: $l^i Q$ with probability of P^i, and $l^i < l^j$, if $i < j$, where $i, j = 1, 2, \ldots I$. Here l^i is the loss incurred per dollar insured in case i. I assume that the expected value of loss incurred is equal to the total insurance coverage, $\sum_{i=1}^{I} P^i l^i Q = Q$. Correspondingly, each economic variable in the second period would have different states with superscript "i". Here we can also denote $L^i = l^i Q$ as the total loss incurred in case i. If we set $\bar{L}$ to be the threshold for the amount of catastrophic loss, a catastrophe event in this paper could refer to the event, whose loss, L^i, is more than $\bar{L}$.

In this model, b is defined as the ratio of assets to liabilities, and I also refer to it as the solvency ratio. If b is equal to or more than 1, the insurer is solvent. Here, b impacts the insurance coverage Q, the external capital cost R, and the reinsurance premium C. Let b^i denote the same ratio for each state i in the second period. Similarly, Q^i, R^i, C^i, e^i, and β^i all denote the same economic variables as previous ones for each state in the second period. Thus b^i should have impact on Q^i, R^i, C^i, e^i, and β^i in the second period.

I also make assumptions with regards to the following functions. I assume that the insured will purchase more insurance with a lower premium π and a higher solvency ratio b. Therefore, the demand function for insurance coverage $Q(\pi, b)$ is a concave function with $Q_\pi < 0$, $Q_{\pi\pi} < 0$, $Q_b > 0$, $Q_{bb} < 0$, where subscripts are used to denote partial derivatives. The cost function of reinsurance per unit of coverage $C(b)$ is a convex function with $C_b < 0$, $C_{bb} > 0$. My basic assumption here is that the

reinsurance premium will increase as the insurer has a lower solvency ratio, and it increases at an increasing rate (see Froot, 2001). The cost function of external capital $R(e,b)$ is a convex function with $R_e > 0$, $R_{ee} > 0$, $R_b < 0$, $R_{bb} > 0$. Considering deadweight costs should be an increasing function of the amount of external capital, I assume that R will go up, at an increasing rate, as the insurer issues a larger amount of debt e. I also assume that issuing debt will be more costly when the insurer becomes more likely to be insolvent, and it changes at an increasing rate.

Then the insurer's expected cash flow in the first period should be expressed by $\{e_0 + [\pi - C(b)\beta - (1-\beta)\sum_{i=1}^{I} P^i l^i r_f^{-1}]Q(\pi,b) + e - R(b, e)r_f^{-1}\}$, where r_f is the risk-free rate. In the second period, the cash flow of state i in this model economy should be $\{[\pi^i - C^i(b^i)\beta^i - (1-\beta^i) r_f^{-1}] Q^i(\pi^i, b^i) + e^i - R^i(b^i, e^i)r_f^{-1}\}$. To maximize the profit within two periods, the insurer would choose the optimal amount of external capital $\{e, e^i\}$ and reinsurance ratio $\{\beta, \beta^i\}$ for each state i in both of these two periods, and set up the optimal premium $\{\pi^i\}$ for each state i in the second period. The optimization problem of the profit for the insurer in this model is as follows,

Max Profit $= e_0 + [\pi - C(b)\beta - (1-\beta)\sum_{i=1}^{I} P^i l^i r_f^{-1}]Q(\pi,b) + e - R(b, e)r_f^{-1}$

$\{e, e^i, \beta, \beta^i, \pi^i\}$ $\quad + r_f^{-1}\sum_{i=1}^{I} P^i\{[\pi^i - C^i(b^i)\beta^i - (1-\beta^i) r_f^{-1}]Q^i(\pi^i, b^i) + e^i - R^i(b^i, e^i)r_f^{-1}\}$

s.t.

$$b = \frac{e_0 + (\pi - C\beta)Q + e - Rr_f^{-1}}{(1-\beta)Qr_f^{-1}}$$

$$b^i = \frac{[r_f(\pi - C\beta)Q - (1-\beta)L^i] + [r_f(e_0 + e) - R] + [(\pi^i - C^i\beta^i)Q^i + (e^i - R^i r_f^{-1})]}{(1-\beta^i)Q^i r_f^{-1}}$$

3.4 The Insurer's Optimal Strategy Analysis

In this section, I discuss the optimization solutions for the model in two different cases: we want to look at the insurer's choices of catastrophic risk intermediation strategy in the costly external capital market, but let us look at the risk free capital market at first.

3.4.1 Case One: Risk Free Capital Market

In the first case I examine, the cost of capital is assumed to be equal to the risk free rate. Thus conditions (3.1) and (3.2) below will hold in risk free capital market for the marginal cost of reinsurance and external capital.

$$C(b) = C^i (b^i) = r_f^{-1} \tag{3.1}$$

$$R(b,e) = R^i (b^i, e^i) = r_f \tag{3.2}$$

These two conditions imply that the insurer can choose any reinsurance ratio β^i between 0 and 1 and raise any feasible external capital e^i from external capital owners without any extra charge. In other words, there is no need for the insurer to reserve funds to prepare for future loss payments. Based on the First Order Conditions (FOCs) and the comparative statics analysis of the optimization problem under these two conditions, the following results can be obtained:

$$Q_b = C_b = R_b = Q_{b^i}^i = C_{b^i}^i = R_{b^i}^i = 0 \tag{3.3}$$

$$E_{Q^i \pi^i} = -\frac{\pi^i}{\pi^i - r_f^{-1}} \tag{3.4}$$

Equation (3.3) describes the fact that the solvency ratio of the insurer, b, has no impact on the insurance demand Q, the reinsurance cost C, or the external capital cost R, because the insurer can always raise revenues as high as it needs with no extra risk

charge.

Equation (3.4) is the price elasticity of insurance demand in each state during the second period. It implies that the second-period premium of the insurer will be determined by its specific price elasticity in each state, and has nothing to do with the previous loss payment.

All in all, in a risk free economy, the insurer's solvency position does not matter and a catastrophic shock has no effect on the insurer's underwriting and capital structure. In such a situation, neither the Capacity Constraint Theory nor the Risk Over Hang Theory has any effect at all.

3.4.2 Case Two: Costly Capital Market

The second case I examine assumes the capital is costly. When the marginal cost of capital is greater than the risk-free rate, the conditions (3.1) and (3.2) should be changed into inequities (3.5) and (3.6) such that,

$C(b), C^i (b^i) > r_f^{-1}$ (3.5)

$R(b,e), R^i (b^i, e^i) > r_f$ (3.6)

In this case, the insurer tends to choose an optimal intermediation strategy to reserve funds to make preparations for expected future loss payments. From the FOCs and the Comparative Statics Analysis of the optimization problem, I can obtain the following results with regards to the optimal catastrophic risk intermediation strategy. Note that $T^i = \pi^i - C^i \beta^i - (1 - \beta^i) r_f^{-1}$ and $i < j$ for all the equations below.

$MP_b^i = \frac{\partial \, Profit}{\partial \, b^i} = T^i Q_{b^i}^i - \beta^i Q^i C_{b^i}^i - r_f^{-1} R_{b^i}^i;$ (3.7)

Equation (3.7) is the insurer's marginal profit with respect to its solvency ratio in state i, denoted by MP_b^i. Note that this marginal profit will be increased if the insurer has a better solvency position in this model, and this is because, in this model, consumers are willing to purchase insurance from more secure insurers.

$$MP_b^i = -\frac{Q^i + T^i Q_{\pi^i}^i}{b_{\pi^i}^i} = \frac{(C^i - r_f^{-1})Q^i}{b_{\beta^i}^i} = \frac{r_f^{-1} R_{e^i}^i - 1}{b_{e^i}^i}; \qquad (3.8.1)$$

Equation (3.8.1) shows the equilibrium of these three markets in such an economy: the primary insurance market, the reinsurance market and the external capital market. It implies that the insurer's solvency position plays an important role in the interaction between the ability to sell new business and the ability to raise capital. I also claim that the insurer with a good solvency position could have relatively high marginal profit, MP_b^i, and thus obtain advantages in both the ability to sell new business and the ability to raise external capital.

$$MP_\pi^i = MP_b^i \, b_{\pi^i}^i = -Q^i - T^i Q_{\pi^i}^i; \qquad (3.8.2)$$

$$MP_\beta^i = MP_b^i \, b_{\beta^i}^i = (C^i - r_f^{-1})Q^i; \qquad (3.8.3)$$

$$MP_e^i = MP_b^i \, b_{e^i}^i = r_f^{-1} R_{e^i}^i - 1; \qquad (3.8.4)$$

Equation (3.8.1) to (3.8.4) show that the insurer has an optimal capital structure in costly capital economy. Specifically, Equation (3.8.2) shows that the marginal profit with respect to the insurance premium, MP_π^i, is equal to the marginal cost of setting up the premium π^i in the second period, which is $-Q^i - T^i Q_{\pi^i}^i$. Equation (3.8.3) states that the optimal β^i is the reinsurance ratio when the marginal cost of purchasing such reinsurance, $(C^i - r_f^{-1})Q^i$, is equal to the marginal profit of reinsurance MP_β^i. In addition, Equation (3.8.4) implies that the optimal e^i is the amount of external

capital when the marginal cost of raising such capital, $r_f^{-1}R_{e^i}^i - 1$, is equal to the marginal profit of external capital MP_e^i.

$$\frac{d\pi^i}{dL^i} = \frac{\beta^i C_{b^i}^i Q_{\pi^i}^i - (Q_{b^i}^i + T^i Q_{\pi^i b^i}^i + b_{\pi^i}^i MP_{b^i}^i)}{|SOC| * |b_{L^i}^{i-1}|} ; \tag{3.9}$$

Equation (3.9) describes the effect of losses in the last period on the next-period insurance price, the sign of which is determined by cross partial derivative $Q_{\pi^i b^i}^i$ and the first derivative of the solvency ratio with respect to premium $b_{\pi^i}^i$. Firstly, let us assume $C_{b^i}^i = 0$ in order to check the sign in a simple way. If the insurance demand becomes more price elastic in response to a lower solvency ratio, with $Q_{\pi^i b^i}^i$ and $b_{\pi^i}^i$ being both positive, the effect of losses on premium will be negative. This situation can be plausible when people turn to buy other available insurance products at the same cost from insurers with higher solvency prospects, or when people make use of other effective mechanisms to mitigate risks rather than purchase insurance.

If $Q_{\pi^i b^i}^i$ is negative, which means the insurance price elasticity of demand will be lower in response to a lower solvency ratio, the relationship between previous losses and future premiums can be positive. This situation can be valid when there is a supply shock in the insurance industry, and people cannot find any other effective risk management solutions. The insurer can increase its own premium and the insured will purchase higher priced insurance products from insurers with relatively higher solvency prospects. This positive effect can be stronger when $b_{\pi^i}^i$ is also negative, where $b_{\pi^i}^i$ denotes the relationship between premium and solvency ratio. Based on the definition of b^i in the optimization problem above, this negative relationship should be induced by a large shortfall of insurance coverage Q. Therefore, in the

extreme case with tight capital supply and high insurance demand in the insurance industry, the positive relationship of shock losses and premium can be observed.

Let us now check this effect in an economy with the reinsurance market. The positive relation between the loss payment and the premium would be greater when $C_{b^i}^i$ is large and negative (costly) in the reinsurance market. This is consistent with the statement that price spikes after a shock would be larger when the reinsurance rate is more sensitive to the insurer's solvency ratio during the period of a tight reinsurance market.

Finally, I find that the positive effect of losses on the next-period premium can shrink when $Q_{b^i}^i$, which is recalled as the first derivative of coverage demand Q with respect to solvency ratio b, is larger. This means the effect will be smaller if the insured is more sensitive to the insurer's solvency ratio. It tells us that the price spike is limited for the insurer with a relatively low solvency ratio after the shock since many customers tend to leave such an insurer. Therefore, it is more likely for these insurers to encounter insolvency after a catastrophic event. This also means the insurer's solvency prospects matters as well-capitalized insurers have an advantage over less well-capitalized insurers.

$$\frac{de^i}{dL^i} = \frac{r_f^{-1} R_{e^i b^i}^i - b_{e^i}^i MP_{b^i}^i}{|SOC| * |b_{L^i}^{i-1}|} \;;$$
(3.10)

Equation (3.10) illustrates the effect of losses on the external capital, and the sign of the effect is determined by cross partial derivative $R_{e^i b^i}^i$ and first derivative of solvency ratio with respect to external capital $b_{e^i}^i$. If $R_{e^i b^i}^i$ is negative, which means the external capital cost is more sensitive to the capital amount in response to a lower

solvency ratio, the relationship between losses and external capital can be negative. In this case, the external capital market is too tight, so the insurer tends to decrease its external capital, or makes its solvency ratio as high as possible to attract external capital. If $R^i_{e^i b^i}$ is positive, with the external capital cost being less sensitive in response to a lower solvency ratio, the external capital market is not tight yet and the insurer may directly access more external capital to cover higher losses.

$$\frac{d\beta^i}{dL^i} = \frac{(C^i - r_f^{-1})Q^i_{b^i} + Q^i C^i_{b^i} - b^i_{\beta^i} MP^i_{b^i}}{|SOC| * |b^{i-1}_{L^i}|} \tag{3.11}$$

Equation (3.11) provides the effect of losses on the next-period reinsurance ratio. It shows that the effect will be small if the marginal cost of reinsurance $C^i_{b^i}$ is largely negative (costly). This implies that the insurer would avoid reinsurance solutions to transfer risks when the reinsurance market is tight.

$$E_{Q^i \pi^i} = -\frac{(Q^i + MP^i_b b^i_{\pi^i})\pi^i}{T^i Q^i}; \tag{3.12}$$

Equation (3.12) is the price elasticity of coverage demand in the costly external capital market. It shows that the insurance premium in each state in the costly capital market is determined not only by its specific price elasticity but also by its overall marginal profit and its solvency position in each state. It means that changes in premium in the costly capital market can be induced by changes in the insurer's solvency position. If we let $MP^i_b = 0$, this equation will be the same as the equation (3.4) derived in the risk-free capital market, in which the insurer's solvency ratio does not matter at all.

$$\Delta^{ij} = (T^j Q^j - T^i Q^i) - [r_f^{-1}(R^j - R^i) - (e^j - e^i)] - (1 - \beta)(L^j - L^i). \tag{3.13}$$

The difference between the insurer's overall profits in two states i and j is shown

by Equation (3.13). From the previous analysis, I can conclude that there is a chance for the insurer with a high solvency ratio to have a larger expected profit when it pays larger losses in the first period, denoted by $\Delta^{ij} > 0$.

The first term in Equation (3.13), $(T^j Q^j - T^i Q^i)$, can be interpreted as underwriting premium spiking after a loss; and the term $[r^{-1}(R^j - R^i) - (e^j - e^i)]$ is the extra external capital cost due to a loss. The term $(1 - \beta)(L^j - L^i)$ is the loss payment difference between two states. This equation shows that the possibility of a positive profit difference between these two states can be increased when the highly solvent insurer can take advantage of price spikes and the insured's loyalty in post-catastrophe insurance sales, thus reducing the effect of penalties of a costly reinsurance rate and high external capital costs after shocks. This is also the condition in which the insurer can benefit from catastrophic risk coverage across these two periods in this model economy.

3.5 Implications for the Empirical Tests

Implications for the empirical study are provided in this section to examine the results developed from the two-period cash flow model in costly capital market. The tests will be finished in future research.

In the two-period model, I focus on the effect of catastrophic shocks on the changes of premium and coverage capacity for a representative insurer. By examining the implication of heterogeneous firms' post-catastrophe performance in the model, I also explore what kind of insurers with catastrophic risk exposures can obtain

advantageous position after a catastrophe event.

3.5.1 Hypothesis I and Its Empirical Testing Strategy

Recall that Equation 3.9 describes the effect of losses on the next-period insurance price. The sign of this effect is determined by $Q^i_{\pi^i b^i}$ and $b^i_{\pi^i}$. If $Q^i_{\pi^i b^i}$ is negative, this means the insured would like to purchase higher priced insurance products from insurers with relatively higher solvency prospects when there is a capital shock due to a catastrophic event. This situation can go further when $b^i_{\pi^i}$ is also negative, which implies a large shortfall of insurance coverage, Q. Further, an insurer with a low leverage ratio and a high solvency ratio can claim a relatively higher insurance price in the post-catastrophe market. Equation 3.9 also shows that the post-catastrophe insurance price will be larger when the reinsurance rate is more sensitive to the insurer's solvency ratio, with $C^i_{b^i}$ being large and negative. So the post-catastrophe insurance price spike can be strengthened in hard reinsurance market. Therefore, I can develop Hypothesis I as follows.

Hypothesis I: With an internal capital shortage after a catastrophic shock, the relationship between the insurer's losses and next-period insurance price in catastrophe prone lines will be positive.

Note that the relationship can be influenced by the insurer's underwriting portfolio, capital capacity, firm characteristics and the reinsurance market situation.

To test this hypothesis, I would use the information from all property-casualty insurers with hurricane risk exposures from 1990-2012, and define the hurricane risk

prone line of business as the sum of direct premium written in homeowners, farm owners, auto physical damage, commercial multi-peril (non-liability), and inland marine. I assume that the price elasticity of the insured's demand can be consistent within one kind of catastrophe events, the hurricanes. I use an OLS regression to test Hypothesis I. The empirical model for insurer i can be built as follows,

$$Price_{i,t} = \beta_0 + \beta_1 (Loss\ incurred/Total\ asset)_{i,\ t-1} + \beta_2 H\_index_{i,\ t-1}$$

$$+ \beta_3\ Capacity_{i,\ t-1} + \beta_4 Dummy\_hard_t + \beta_5 New\_equity_{i,\ t}$$

$$+ \beta_6 Leverage\_ratio_{i,\ t-1} + \beta_7 Solvency\_ratio_{i,\ t-1} + \beta_8 ROE_{i,\ t-1}$$

$$+ \beta_9 Log\ (asset)_{i,\ t} + \beta_{10} Dummy\_single_{i,\ t} + \beta_{11} Dummy\_public_{i,\ t}$$

$$+ \beta_{12} Dummy\_rating_{i,\ t} + \beta_{13} Reinsurance_t + \varepsilon_{i,t.}$$

In this regression, the dependent variable, *Price*, is the insurance price of hurricane risk prone line. *Price* is the ratio of (net premium written − underwriting expenses − dividends to policyholders) to the present value of accident year losses incurred (See Cummins and Danzon 1997). The explanatory variable is the ratio of losses incurred by hurricanes to total asset in the last period, denoted by (*Loss incurred/Total asset)*.

Many control variables will be chosen to describe the insurer's underwriting diversity, capital capacity, financial quality, firm characteristics and the reinsurance market situation. The *H_index* is the Herfindahl index that indicated an insurer's underwriting diversification in hurricane prone lines. For capital capacity, I develop three alternative proxy variables: *Capacity* is the ratio of equity capital amount K_{t-1} to the average of K_{t-1}, K_{t-2}, and K_{t-3} (See Winter 1994, Cummins and Danzon 1997);

Dummy_hard is one if the primary insurance market is hard, zero otherwise; *New_equity* is the amount of newly raised equity, including new equity issues and transfers from noninsurance parent corporations. Three proxy variables are incorporated to represent an insurer's financial quality: *Leverage_ratio* is the ratio of liability to the policyholder's surplus; *Solvency_ratio* is the total surplus to liabilities; *ROE* is the net income before dividend and tax divided by total equity capital. I also include the firm size as measured by *Log (asset)*, a dummy for a single unaffiliated firm, and a dummy for a public firm to distinguish firm characteristics. *Dummy_rating* is one if the insurer's A.M. Best rating is no lower than "A - -", zero otherwise. Finally I add *Reinsurance* that is measured by the catastrophe reinsurance price index to represent to some extend the reinsurance market is hard.

In addition, I develop the following two empirical tests for Hypothesis I by incorporating the previous OLS model into different subgroups of the original sample.

First, I find in the two-period model that the positive effect of losses on the next-period premium can shrink if the insured is more sensitive to the insurer's solvency ratio. Since the capital elasticity of demand for the commercial line is higher than the personal line, I can extend Hypothesis I that the positive effect between losses and next-period insurance price, indicated by the coefficient β_1, can be larger for a commercial line insure than a personal line insurer. We can divide the sample into two subgroups, commercial line insurers and personal line insurers. Here I define an insurer as a commercial line insurer if the direct premiums written in catastrophe-related commercial lines of business are more than 50% of the insurer's

total premiums written, while a personal line insurer is the insurer that concentrate more on personal lines of business. To test this extension hypothesis, the same regression as the one applied to Hypothesis I above can be conducted for both subgroups, and then the estimated coefficients can be compared with each other.

Second, considering the price elasticity of demand for hurricane risk is higher in Florida than in New York (see Grace and Klein, 2004), the positive effect above can be predicted to be larger for insurers that underwrite hurricane risk mainly in Florida than those that underwrite hurricane risk mainly in New York. The sample can also be divided into two subgroups, insurers with more hurricane risk prone exposures in Florida and insurers with more exposures in New York. Another regression can be conducted to these two subgroups with the same methodology and proxy variables as those used to test Hypothesis I. Then the estimated coefficients, β_1, can be compared.

3.5.2 Hypothesis II and Its Empirical Testing Strategy

The previous two-period cash flow model implies that high-quality insurers may benefit from catastrophe events by taking advantage of price spikes and the insured' loyalty in post-catastrophe underwritings, and by enjoying a relatively low reinsurance rate and a relatively low capital cost (see Equation 3.13). Therefore, one catastrophe event could act as an accelerated trigger, splitting insurers into high-quality ones and low-quality ones with respect to different levels of underwriting efficiencies and capital raising abilities. Therefore, I develop the following Hypothesis II.

Hypothesis II: A higher-quality insurer defined by firm performance during a catastrophic event possesses better catastrophic risk underwriting technology and higher capital rising ability.

Note that better catastrophe risk underwriting technology will be indicated by wider diversification, larger amount of assets, lower combined ratios, and a longer catastrophic risk underwriting history. Higher capital rising ability will be indicated by a higher solvency ratio, a lower-risk investment strategy, a lower leverage ratio, a higher rating rank.

To test Hypothesis II, the sample of insures can be all property-casualty insurers with hurricane-prone line underwritings. We can focus on the data in five main hurricane seasons in U.S.: 1992 (Hurricane Andrew), 2004 (Hurricane Iva), 2005 (Hurricane Katrina), 2008 (Hurricane Ike), and 2012 (Hurricane Sandy). The following OLS regression will be applied to test Hypothesis II,

$$Quality_i = \gamma_0 + \gamma_1 Combined\_ratio_i + \gamma_2 H\_index_i + \gamma_3 Risk\_exposure_i$$
$$+ \gamma_4 Underwriting\_age_i + \gamma_5 Reinsu\_ratio_i$$
$$+ \gamma_6 Solvency\_ratio_i + \gamma_7 Leverage\_ratio_i + \gamma_8 Liquidity\_ratio_i$$
$$+ \gamma_9 Capacity_i + \gamma_{10} Invest\_risk_i + \gamma_{11} Growth\_opportunity_i$$
$$+ \gamma_{12} Log(asset)_i + \gamma_{13} Dummy\_single_i + \gamma_{14} Dummy\_public_i$$
$$+ \gamma_{15} Dummy\_rating_i + \varepsilon_i.$$

Here, the quality of insures with hurricane risk underwritings is assessed based on the change of Return on Asset (ROA). In the regression, the dependent variable, *Quality,* is measured by the difference between an insurer's post-catastrophe ROA

value and its prior-catastrophe ROA value. Therefore, insurers with high quality are defined as the ones with relatively great value of *Quality*, while low-quality insurers are the ones with low value of *Quality*.

Multiple explanatory variables will be chosen to represent an insurer's underwriting technology, financial quality, and firm characteristic. The *Combined_ratio* is the ratio of claims incurred to net premium earned; *Risk_exposure* is equal to the ratio of the premium written for hurricane-risk-exposure line to the total net premium written; *Underwriting_age* is defined as how many years the insurer has possessed hurricane risk exposures; *Reinsu_ratio* is obtained by ceded premium/gross premium; *Liquidity_ratio* is dividing the sum of cash investment and short-term investment by invested assets; *Invest_risk* is ratio of the sum of stock, real estate and junk bond to total invested assets; *Growth_opportunity* is the change of net premium written in one year. All other proxy variables have been defined in the same way as those developed to test Hypothesis I.

3.5.3 Hypothesis III and Its Empirical Testing Strategy

The two-period model also suggests the larger the losses incurred by a catastrophe event, the stronger the separation between the high-quality insurers and low-qualities ones can be observed. So Hypothesis III is as follows.

Hypothesis III: The standard deviation of insurers' quality during catastrophic events is positively related to losses of these catastrophic events.

The insurers chosen to test Hypothesis III should satisfy the following two

conditions. First, each of them should go through hurricanes in all of five hurricane seasons as listed in Hypothesis II. Second, in each seasonal year, their quality distribution is consistent with the quality distribution of all the sample insurers in Hypothesis II. Hypothesis III can then be tested through the following OLS regression with the aggregated data of these selected insurers in different hurricane seasons.

$$s.d.\_quality_t = \delta_0 + \delta_1 (Loss\ incurred/Total\ asset)_t + \delta_2 Capacity_t + \delta_3 Reinsurance_t + \varepsilon_t$$

The dependent variable, *s.d._quality*, is the standard deviation of the insurers' ROA change in the same hurricane season. The independent variable is the ratio of total loss incurred by hurricanes to the total asset for all chosen insurers in that season. The control variables are *Capacity* (see Hypothesis I) and *Reinsurance* (also see Hypothesis I). Considering the sample size is small (5 hurricane seasons are included), I may just show some descriptive statistics instead.

3.6 Conclusions and Discussions

In the property-liability insurance market, the demand expansion and the supply reduction due to a catastrophe can cause premiums to grow sharply and then gradually moderate until the insurance industry becomes sufficiently recapitalized. During this process, good insurers with a comparative advantage of intermediating catastrophic risks may make use of the price change and relatively low external capital cost, while others may encounter insolvency problems resulting from capital insufficiency. One catastrophic event could act as trigger, splitting insurers into high-quality ones and low-quality ones with respect to different underwriting efficiencies and capital raising

capabilities. Changes in the insurance industry are sequentially occurring with a series of catastrophic shocks.

The model developed in this paper contributes to find the interaction between the insurer's capital rationing and balance sheet, in which the solvency ratio plays an import role. I discuss to some extent what kind of insurer can benefit from the catastrophic risk underwriting.

In addition, I also have outlined the empirical testing strategy for three hypotheses implied by the two-period model. This study also contributes to the empirical test of Capacity Constraint Theory to find more about the impact of catastrophic shocks on the insurance industrial organization and the relation between the capital market and the insurance industry.

In this chapter, I have analyzed the static effect of one catastrophic shock on an insurer' optimal underwriting strategy and capital raising strategy in a two-period model. Beyond this two-period model, I would like to analyze the dynamic effect of a series of catastrophic shocks on the insurer's optimal output strategies through the infinite time line in the next chapter. This dynamic economy can provide us the possibility to examine the existence and the reason of the so-called "underwriting cycle" in the property-liability insurance market.

4

A Dynamic Model of Financial Markets: Catastrophes, Cycles, and Capacity Constraints

One can observe that sharp price changes and large capacity swings follow a series of catastrophic loss shocks in the property-liability insurance industry. When observed prices and converge quantities diverge from equilibrium prices and quantities, the hard or soft market is defined. In early studies, one period of "underwriting cycles" consists of one hard market and one soft market. In recent years, ups and downs of "underwriting cycles" are no longer observed as smooth and predictable curves, and they are more likely to be asymmetric and random. Under the background of more frequent and severe catastrophe events in the property-liability insurance market nowadays, it is vital to research how an insurer responses to a series of catastrophic shocks, and to explore the sources of the "underwriting cycles".

In this chapter, I look at the "underwriting cycles" in output markets in the

insurance industry by using a dynamic model inspired by the Real Business Cycle literature. I build a dynamic cash flow model of an insurer with a series of catastrophe events in an environment with costly external capital and insurance regulation to simulate the insurer's optimal catastrophic risk intermediation strategy.

The model contributes to show that the dynamic interaction between the insurer's capital rationing and balance sheet can generate the non-cyclical behavior of output changes if the insurer experiences a series of unexpected catastrophic shocks. My results cast doubt on the existence of the "underwriting cycle" in the property-liability insurance market that is defined to be cyclical and predictable, and help to explain the unpredictable "underwriting cycles" as temporary responses of the industrial coverage capacity to insured losses.

4.1 Introduction

A rich Real Business Cycle literature has developed dynamic models to understand the credit cycle, financial bubbles, and macroeconomic output behaviors. The Real Business Cycle theory views recessions and periods of economic growth as the efficient response to exogenous changes in the real economic environment. In the real business cycle model, the source of a firm's output dynamics can be the amplifying effect of shocks on the balance sheet (see Bernanke and Gertler, 1989) or the leverage (see Kiyotaki and Moore, 1997; Szemely, 2010).

In the property-liability insurance market, negative shocks to claims or industry capital caused by a large natural disaster or a significant macro economic event can

substantially reduce the insurance industry capacity and push up the price (see Winter, 1988, 1991, 1994; Gron, 1994; Cummins and Nini, 2002; Doherty, Lamm-Tennant, and Stark, 2003; Grace and Hotchkiss, 1995). Grace and Hotchkiss (1995) show the great effects of shocks to the general economic variables on the insurance underwriting performances measured by profitability. Grace and Klein (2009) indicate that insurers have substantially raised insurance rates and reduced their exposures after the intense hurricane seasons of 2004 and 2005, and they also show the evidence that catastrophes can influence the insurance industrial organization.

Considering catastrophic shocks can have impact on the insurer's balance sheet and capital raising, it is exposed to catastrophic shocks that can affect the insurer's outputs in each period. Thus the property-liability insurance market is a perfect environment to study the impact of a series of catastrophic shocks. In other words, the Real Business Cycle theory and the related dynamic model can be one way to study how a series of catastrophic shocks affect the property-liability insurance market and also to explore the sources of the insurer's output dynamics.

In the property-liability insurance industry, variations of supply capacity in the insurance market have a significant, negative effect on movements in pricing and profitability, generating the market conditions associated with the so-called "underwriting cycles". A "soft" period is a period in which premiums are low, capital base is high and competition is high. After large claims, less stable companies quit from the market and even some stable and large companies are left with less capital. Then the market hardens with rapidly rising premiums and stringent underwriting

standards. Underwriters are less likely to take on risk in such a "hard" period.

In early studies, the period of "underwriting cycles" can be predictable by empirical testing. Although the existence of underwriting cycles in property and liability insurance market is well established in the insurance economic literature, there is little evidence that insurers are able to forecast these cycles to make a profit.

In recent years, ups and downs of "underwriting cycles" seem to be less predictable. For example, significant destruction in the property-liability insurance market can be found from Hurricane Andrew (1992), the 9/11 Attacks (2001) and Hurricanes Katrina and Rita (2005). These catastrophic shocks tend to largely decrease the insurers' coverage capacity in the property-liability insurance industry, and the impacts seem to be unpredictable and the effects can remain for several years.

My study contributes to the insurance economics literature in the field of the dynamic interaction between the capital market and the insurance industry, by using the Real Business Cycle methodology to develop a dynamic model of financial markets with catastrophic shocks. In this paper, there is non cyclical "underwriting cycles" in the property-liability insurance industry any more, and instead, the non-cyclical behavior of output changes resulting from the insurer's responses to catastrophic shocks in the model economy can be observed.

The analysis focuses on how a series of catastrophic shocks affect the insurer's underwriting strategy and capital structure in a dynamic model economy. I find that the effect of a one time catastrophic shock could spread and amplify over time by a dynamic interaction between the insurer's balance sheet and capital rationing. The

simulation results show that this dynamic interaction can generate a non-cyclical behavior of output changes when the insurer experiences a series of unexpected catastrophic shocks, and thus I cast doubt on the existence of the so-called "underwriting cycle" which is cyclical and predictable. I suggest that such behavior cannot be forecasted, and unexpected shocks can change the direction of the behavior. I also claim that the ex-ante magnitude and the period of the changes can be jointly determined in the insurance market and the capital market.

The simulation results also imply that the changes of output markets can be larger when the shock is more volatile, the external capital market is tighter, and the solvency regulation is more relaxed.

This chapter is structured as follows. Section 4.2 is literature review. In Section 4.3, I develop a dynamic cash flow model with a series of catastrophic shocks for an insurer, and I focus on the dynamic interaction analysis. A linear quadratic approximation for the dynamic cash model is provided in Section 4.4. Section 4.5 simulates the insurer's optimal catastrophic risk intermediation strategy in benchmark economy, and shows a non-cyclic behavior of output fluctuations. In addition, section 4.6 compares results of the experimental economy with the benchmark economy, and analyzes the factors that affect the magnitude of the output fluctuations. I show the empirical study of impulse response analysis in Section 4.7. Section 4.8 provides conclusions and discussions.

4.2 Literature Review

In the literature, there is no consensus on the existence and the origin of the property-liability underwriting cycles. A number of rationales behind underwriting cycles exist in early studies. The "lack of pricing restraint" theory implies that the cycle is caused by the lack of pricing discipline (Stewart, 1987). Companies may price below cost to keep market share, for example. However, many would agree that an insurer with good performance could demand a relatively high premium, and does not have to lower its rate to sell more insurance, thus invoking a price war.

Cycles might arise from ratemaking methods and forecasting errors in accounting and regulation (Venezian, 1985). This scenario predicts that underwriting cycles in property-liability insurance should follow a cosine wave-like pattern. Venezian (1985) examines the cycle by a second-order auto-regression effect in underwriting profits. During the time of his study, the period of the cycle can even be predictable for single lines of coverage in some literature. However, some other tests for the causes of a cycle, such as those by Harrington (1984), fail to find the cycle's sensitivity to several related possible reasons, such as future loss expectations, adjustment lags, risk attitudes, and forecast errors.

Although autoregressive estimations (see Venezian, 1985) are then generally used to predict the cycle's period in both the U.S. and the international insurance markets (see Cummins and Outreville, 1987; Chen, Wong, and Lee, 1999; Harrington and Niehaus, 2001; also see Meier, 2006), Boyer, Jacquier and Van Norden (2012) claim that naive inference on the existence and the period of a cycle based on the point

estimates of autoregressive models is biased. They cannot find any evidence of cyclicality any longer when correcting for such a bias. Actually, with more frequent and sever catastrophe events occurring nowadays, ups and downs of the underwriting cycles are more likely to be asymmetric and random. This phenomenon also challenges the analysis of the existence, the reason and the prediction of the underwriting cycles.

To the best of my knowledge, a few papers have explored the property-liability insurance underwriting cycles by using a multi-period model. Lin (2005) applies a multi-period model of insurance market equilibrium to obtain a dynamic solution for equilibrium price and quantity. This model aims to explain the phenomena of market prices failing to achieve Pareto optimality for a single period, and provide the insights into the volatility of insurance prices related to the underwriting cycle.

Here, I should address one paper studying dynamics of insurance markets. Winter (1994) develops a dynamic cash flow model to analyze the price dynamics in competitive insurance markets. In my paper, the dynamic model is also built based on the cash flow analysis for an insurer. I also have one basic assumption the same as his: the external equity is more costly for an insurer than internal equity.

My paper is different from Winter (1994) in the following aspects. First, his paper focuses on the price dynamics due to losses, while I focus on the persistent fluctuations in the supply market of issued coverage. We have different assumptions on the pricing in the competitive insurance market. He does not assume the conventional economic theory that premiums equal the present value of expected

policy claims in competitive insurance markets, and argues that average claims cannot be predicted with certainty due to aggregate uncertainty or common factors. He concludes that the accumulation of losses can attribute to jumps in premiums. In this paper, I follow the conventional economic theory with premiums being exogenously determined by expected losses, and focus on the changes of insurance coverage supply. Therefore, both of us have the specific linear cash flow equations for an insurer, but I do not involve the analysis of non-linear demand function of the insured. Actually, it is hard to define the specific form of non-linear demand equations, and thus it is impossible to calibrate the model for simulations.

Second, we have a different source of a dynamic mechanism. In Winter's paper, the dynamic non-linearity of premiums is due to dependence among the sizes of losses, conditional upon the events of losses. While, in my paper, the output dynamics does not result from the loss itself. Instead, I find that the effect of a one time catastrophic shock could spread and amplify over time by a dynamic interaction between the insurer's balance sheet and its capital rationing. This dynamic interaction can produce a non-cyclical fluctuation of output changes in insurance market.

Third, Winter's paper implies that insurers with limited liability must maintain enough net worth to make credible to their promises to pay claims. So equity in his paper becomes a measure of capacity in the market. However, I use the Kenny Ratio, which is defined as the ratio of total premiums to total surplus, to relate the debt capital market to the insurer's underwritings strategy in an environment of costly capital and insurance regulation. Note that Kenny Ratio is generally used by insurance

regulators to indicate an insurer's solvency ratio.

In my paper, I contribute by finding that the dynamic interaction between the insurer's balance sheet and its capital rationing due to catastrophic shocks can general an output dynamics in the model. No simulation is conducted in Winter's paper since the insurance demand function is not linear by his assumption and is hard to define. However, my paper develops the calibration by a linear quadratic approximation and provides simulated results of a non-cyclical behavior of output changes in the property-liability insurance market. The simulation is to show that the dynamic interaction can generate a non-cyclical fluctuation of coverage supply when the insurer experiences a series of catastrophic shocks. The results imply that such an asymmetric, nontraditional "underwriting cycles" can be resulting from the insurers' responses to a series of loss shocks.

I refer to the Real Business Cycle literature to construct the dynamic model in this paper. Kydland and Prescott (1982) envision that technological shocks shift the constant output growth trend up or down. In the model of Kiyotaki and Moore (1997), collateral constraints amplify the effects of shocks to the real economy. They show that small and temporary shocks to technology or income distribution can generate large, persistent fluctuations in output and asset prices.

In this paper, I focus on the insurer's output responses to catastrophic shocks by analyzing the profit-maximizing insurer's optimal catastrophic risk intermediation strategies with capacity constraints in a dynamic economy. I find that a one-time catastrophic shock plays an important role in the interaction of the insurer's capital

rationing and balance sheet due to the incorporating of Kenny Ratio into capacity constraints. Actually this implication is consistent with the claim in Chapter 3 that the solvency ratio impacts the interaction between the ability to sell new business and to ability to raise external capital in each period. But, further, the dynamic model in this chapter shows that the interaction effect will amplify and spread out over time. According to such a dynamic interaction, we can observe movements of output, which is the result of responding to a series of catastrophic shocks: the insurer's supply capacity is decreased and the external capital largely shrinks due to capacity constraints after catastrophic shocks.

4.3 Dynamic Cash Flow Model with a Series of Catastrophic Shocks

In this section, I construct a dynamic cash flow model in which catastrophic shocks affect both the underwriting profit for the insurer and the capital cost in the capital market. This model extends the previous two-period model in Chapter 3 into an infinite time line model, and emphasizes the dynamic effect of the insurer's solvency position, changed with catastrophic events, on the insurer's underwritings and capital structures in an environment of the costly external capital market and the insurance regulation.

In this dynamic model, I assume that the insurance price is unchanged all through the time line, and the price-taking insurer has perfect information to make a forecast of the expected future losses. In the model economy, catastrophic shocks have a dual

impact on the insurer's cash flows: not only are they factors of the insurer's operational income in balance sheet, but they also affect the insurer's capital raising capability. The dynamic interaction between the insurer's balance sheet and capital rising rationing turns out to be an amplifying transmission mechanism, by which the effects of a one time catastrophic shock persistently spreads to the following cash flow distributions.

4.3.1 Time Line

To explain the model construction in detail, I take the cash flows during the period t as an example. Figure 4.1 below summarizes all the positive and negative cash flows for a representative insurer from period t to period $t+1$.

	t	(loss ratio α_t)		t+1	(loss ratio α_{t+1})

$\pi_{t-1}Q_t$ $\quad$ $-\alpha_t Q_t$ $\quad$ $\pi_t Q_{t+1}$ $\quad$ $-\alpha_{t+1} Q_{t+1}$

e_t $\quad$ $-R_t(\alpha_t, e_t)$ $\quad$ e_{t+1} $\quad$ $-R_{t+1}(\alpha_{t+1}, e_{t+1})$

D_{t-1} $\quad$ D_t

K_t with a return rate of r_t $\qquad$ K_{t+1} with a return rate of r_{t+1}
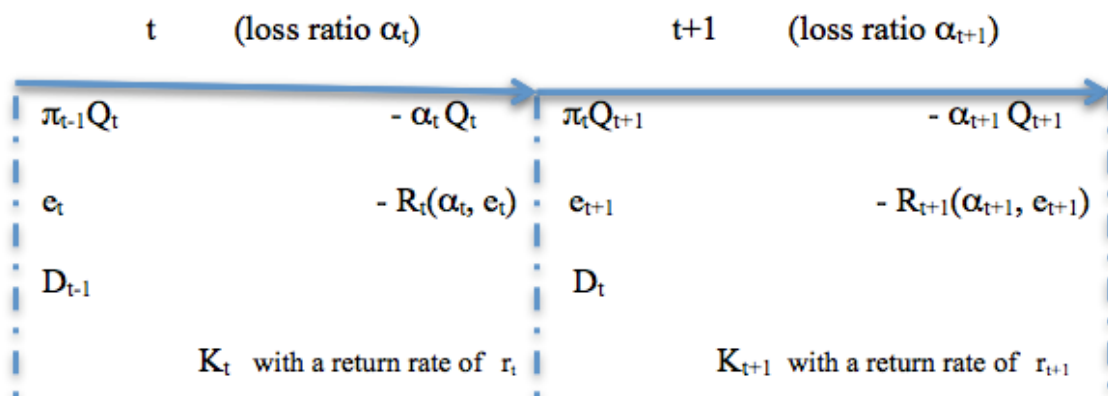
Figure 4.1: Time Line of the Dynamic Cash Flow Model for an Insurer

At the beginning of the period t, the insurer has retained earnings K_t accumulated from all previous operations, and the retained earnings continue to be accumulated at a return rate of r_t until the end of the period t.

At the end of the period t, the insurer collects the total premium of $\pi_t Q_{t+1}$ for one-period coverage policies in period $t+1$. π_t is the insurance price that is the gross

premium per unit of coverage. Meanwhile, the insurer needs to pay for the total losses of $\alpha_t Q_t$ claimed during the period t, where α_t denotes the loss ratio, which is the ratio of coverage that incurs losses.

In this model, I assume that the insurer is efficient in estimating expected losses it underwrites, with the price $\pi_t = (1+\varnothing) \, E_t \, (\alpha_{t+1})$, where $\varnothing$ is the loading rate of the insurance industry to allow for a profit. A series of α_t here follow the stochastic process, whose calibration will be discussed in section 4.4. In other words, the premium π_t in this model is exogenously determined by the insurance industry. However, the insurer cannot predict the frequency and the severity of loss shocks, which is expressed as a one time positive or negative change of α_t. Here a high positive change of α_t that is above a threshold of catastrophic loss ratio can be viewed as a catastrophic shock.

At the end of period t, the insurer also needs to repay the one-period debt e_t with a total amount of R_t. The debt is raised from the external capital market by the end of period $t-1$. Meanwhile, the insurer would raise new debt e_{t+1} with a promised repayment of R_{t+1} in the next period. In the real world, the insurer can also raise the capital from equity holders with agency cost and adjustment cost due to asymmetric information. Here I use debt holders instead of equity holders because it is easy to calculate the cost of the capital in each period.

I assume that R is a convex function with $R_e > 0$, $R_{ee} > 0$, $R_\alpha > 0, R_{\alpha\alpha} > 0$. This assumption follows a basic principal in the capital market that investors would increase (decrease) the capital cost when observing the fact that the insurer incurs

larger (smaller) losses and thus has a relatively worse (better) financial position.

After paying out the net dividend D_t to the insurer's owners at the end of the period t, the insurer would gather K_{t+1} to be the internal capital surplus at the beginning of the period $t+1$, which will be accumulated at a return rate of r_{t+1} during the period $t+1$.

In the literature, the firm's owners can be assumed to be risk averse towards investment or dividend payments (see Froot, Scharfstein, and Stein, 1993; Froot, 2001; Szemely, 2010). Here I assign a concave utility function of the dividend payments, $U(D_t)$, for the insurer's owners.

4.3.2 Optimization Problem

Then the cash flow at the end of the period t can be then derived as follows,

$$D_t = \pi_t Q_{t+1} + r_t K_t + e_{t+1} - \alpha_t Q_t - R_t(e_t, \alpha_t) - K_{t+1}^{12} \tag{4.1}$$

From Equation (4.1), one can find in this model that the insurer collects revenue from total premium written $\pi_t Q_{t+1}$, newly raised external capital e_{t+1} and the beginning-of-period internal capital K_t; and the insurer distributes the revenue into three categories: claimed loss payment $\alpha_t Q_t$, promised gross return $R_t(\alpha_t, e_t)$, and end-of-period internal capital surplus K_{t+1}.

Note that the dividend payment in each period, D_t, can be interpreted as the net cash flow after all the operations of each period (See Footnote 13).

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[12] Note that in Winter (1994), e_t is assumed to be the equity issued in the beginning of the period t and it should be subtracted from the current dividend payout , denoted by d_t in his paper, when calculating the net cash flow for an insurer in this period. So the maximization problem with Winter's equity assumption is Max $[\sum_{t=0}^{\infty} \beta^t U(d_t - R_t(e_t))]$, and the constraint can be $K_{t+1} = \pi_t Q_{t+1} + r_t K_t - \alpha_t Q_t - d_t + e_{t+1}$. So one can find that the model construction with Winter's equity assumptions is equivalent with the model with debt assumption in this paper if we set $D_t = d_t - R_t(e_t)$. Therefore, there is no issue of Ponzi schemes when treating equity as debt in this paper.

Next, let us look at the capacity constraints in this model. Here, I assume that the insurer is subject to a simple solvency regulation, expressed by the Kenny Ratio η_t that is the ratio of premiums written to policyholders' surplus. Such a ratio provides a measure of an insurer's financial stability and solvency position. The regulation turns out to be stricter when a lower Kenny Ratio is required.

The following capital rising constraint can be derived,

$$\pi_t Q_{t+1} / (K_t + e_{t+1}) \leq \eta_t \qquad\qquad (4.2)$$

Note that the constraint above is very important to help explain the dynamic interaction between the capital rationing and the balance sheet for an insurer, and is also a bridge to connect the insurance market and the capital market with capacity constraints.

For the optimization problem in this dynamic model, the insurer's owners will choose the optimal strategies of dividend payment D_t, underwriting insurance coverage Q_{t+1}, newly raised external capital e_{t+1}, and saving internal capital K_{t+1} in each period to maximize its expected utility of net dividend payments in the infinite timeline. So the optimization problem in this model can be built by (4.3) subject to (4.1) and (4.2), where β is the discount rate.

$$Max\ E[\textstyle\sum_{t=0}^{\infty} \beta^t U(D_t)] \qquad\qquad (4.3)$$

where $\{ Q_{t+1},\ e_{t+1},\ K_{t+1},\ D_t \} \in Arg\ \{ Max\ E[\textstyle\sum_{t=0}^{\infty} \beta^t U(D_t)]\}$

4.3.3 Dynamic Interaction Analysis

To know the dynamic interaction mechanism in this model, let us check the steady

state solutions *{Q*, e*, K*, D*}* at first. There is no shock in the steady state, so the steady state loss ratio equals to *E(α)*. The insurer has an incentive to make the capital rising constraint (4.2) be binding into (4.4). Equation (4.5) is then derived from cash flow equation (4.1) with binding constraint (4.4).

$$\eta(K^*+e^*) = \pi Q^* \tag{4.4}$$

$$[R(e^*, E(\alpha)) - (2-r)\, e^*] = [(\pi - E(\alpha)) + (r-1)\, \pi/\eta]\, Q^* - D^* \tag{4.5}$$

Equation (4.5) shows that the insurer's capital rising capabilities and underwriting profits are mutually dependent in the steady state. The insurer tends to raise external capital e* to expand its underwriting coverage Q* until the capital raising cost *[R(e*, E(α)) – (2- r) e*]* is covered by the difference between the underwriting expansion profits $[(\pi - E(\alpha)) + (r-1)\, \pi/\eta]\, Q^*$ and the steady state dividend payment *D**.

If there is a positive shock to the steady sate loss ratio *E(α)*, the available funds from underwriting profits to raise external capital is decreasing, and the decreasing external capital will then be reduced further with a higher external capital cost *R* along with the shock. In turn, the largely decreased external capital will reduce the future underwriting expansion and thus the future underwriting profits. This implies that small catastrophic shocks can generate large, persistent fluctuations in both underwriting profits and capital raising capabilities. This is to show the effect of a one time catastrophic shock on the cash flows in the current and the following periods.

Note that, from Equation (4.5), the insurer will raise more external capital with a higher rate of return on invested assets r or a lower Kenny Ratio η, illustrating that the insurer tends to raise more capital if it has higher investment returns or if it needs

to satisfy a stricter solvency regulation.

From (4.1) and (4.2), we can also get the motion of external capital raised as follows,

$$(r_t - 1)\, e_{t+1} = \left[\pi \left(1 + \frac{r_t}{\eta_t}\right) Q_{t+1} - \frac{\pi_{t+1}}{\eta_{t+1}} Q_{t+2} - \alpha_t Q_t - D_t\right] - R_t(\alpha_t) + e_{t+2} \qquad (4.6)$$

Similarly as the finding from Equation (4.5), a positive change of α_t can negatively impact underwriting profits, which will reduce the funds available to raise new capital e_{t+1}. Moreover, with the total capital cost R_t increased by α_t, e_{t+1} shrinks more deeply, which in turn limits the future underwriting quantity Q_{t+1}. Therefore, the insurer's current catastrophic shock will affect its future cash flows according to mutual dependence between the insurer's capital raising capabilities and underwriting profits.

From Equation (4.6), a lower η_t (stricter solvency regulation) leads to more external capital raised during the period t since the insurer needs to keep a good solvency position to expand its underwriting capacity; while a lower η_{t+1} (a future strict solvency regime) leads to less external capital raised during the period t in order to avoid a higher repayment in the next period.

From (4.5) and (4.6), such a interaction between the insurer's balance sheet and capital rising rationing turns out to be an amplifying transmission mechanism, by which the effects of a one time catastrophic shock persistently spread to the future cash flows in the dynamic economy. It implies that amplifying fluctuations of output due to a series of catastrophic shocks can be observed in the insurance market. Further, the amplifying effect will be larger (smaller) if the shock is more (less) volatile, and if

the capital regulation constraint is more relaxed (stricter), and if the capital market is more (less) sensitive to shocks, and if the insurer relies on external capital more (less) heavily.

4.4 Linear Quadratic Approximation for the Dynamic Cash Flow Model

In this section, I calibrate the dynamic model by a linear quadratic approximation in order to simulate the amplifying effect of a one time catastrophic shock due to the dynamic interaction discussed in section 4.3.

4.4.1 Dividend Utility Function and Capital Cost Function

Consistent with the Real Business Cycle literature (e.g., Szemely, 2010; Gertler and Kiyotaki, 2010), I assume that the utility function of net dividend payments for the insurer's owners is a CRRA function as follows,

$$U(D) = \frac{D^{1-\gamma} - 1}{1-\gamma}$$

As mentioned before, I assume that the cost function of the external capital (i.e. debt repayment function) in capital market is a convex function as follow,

$$R(e, \alpha) = r_e \alpha^{\theta_\alpha} e^{\theta_e}$$

The parameter r_e is the cost of external capital, and parameters θ_α and θ_e are the elasticity of the catastrophic shock and the external capital amount in the cost function. Note that the calibration of these parameters should satisfy the consumption that the external capital is more expensive for an insurer to raise than the internal

capital. I also develop the experimental economy by changing the values of parameters θ_α and θ_e.

4.4.2 Stochastic Process of Loss Ratios

The motions of the loss ratio α follow a stochastic process as follows,

$$\ln(\alpha') = \rho \ln(\alpha) + \varepsilon' \quad \text{where} \quad \varepsilon' \sim N(0, \sigma^2)$$

The parameter ρ is the autocorrelation of the loss ratios in the time line. The parameter σ measures the uncertainty of loss ratio, thus ε' can be interpreted as catastrophic shocks.

In this paper, I cite the parameters of stochastic technology shocks in the Real Business Cycle literature to be parameters ρ and σ (see Szemely, 2010). Different from the technology shock, I choose a relatively low volatility of loss ratios, σ. This is because, by assumption, the insurer is efficient in estimating the regular loss it underwrites.

4.4.3 Main Parameters

Table 4.1 is a list of main economic parameters for the benchmark economy in the dynamic model.

Table 4.1: Main Parameters in the Benchmark Model Economy

Parameters	Notations	Values
Time discount factor	β	0.95
Kenny ratio	η	2.00
Coefficient of relative risk aversion	γ	1.50
Gross return rate of investment	r	1.06
Gross return rate of external capital	r_e	$1.2\,r$
Catastrophic shock Elasticity	θ_α	1.05
External Capital Elasticity	θ_e	1.05
Autocorrelation of $\ln(\alpha)$	ρ	0.80
Standard deviation of ε'	σ	0.003
Loading rate	ϕ	0.10

4.5 Catastrophic Risk Intermediation Strategies in Benchmark Economy

I solve the insurer's certainty-equivalent steady-state equilibrium by a linear quadratic approximation and then simulate its optimal decision path. Based on the results, we find a non-cyclical behavior of output changes in both coverage capacity and external capital with a series of catastrophic shocks. The results show that such the behavior cannot be forecasted, and the insurance market and the capital market can jointly determine its ex-ante magnitude.

In the model economy, I also find that the insurer always keeps the volatilities of

dividend *D* and retained earnings *K* low enough in its optimal strategy. This can be explained by the dividend signaling theory that dividends can be used as a signal of firm quality (see Miller and Rock, 1985). Dividend decreases convey bad news of firm quality to both consumers and investors, and the impact can be amplified in the model economy. To reduce the bad signal effect of dividend decreases, the insurer would like to smooth the dividend payment. The fluctuations of the model economy are intensively expressed by changes of the coverage quantity *Q* and the capital *e*.

4.5.1 Optimal Coverage Capacity Strategy and Catastrophic Shocks

Figure 4.2 below shows us that the insurance coverage in the benchmark economy is negatively correlated with catastrophic risk.
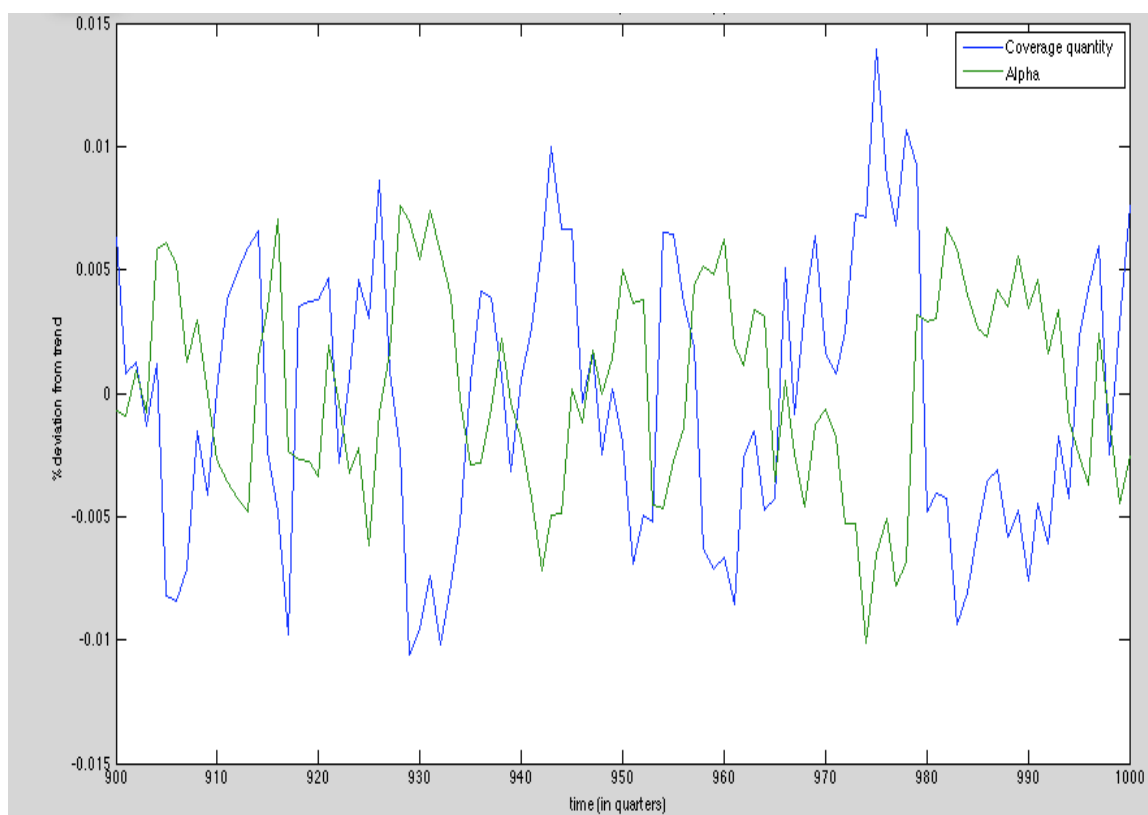
Figure 4.2: Optimal Underwriting Coverage With Catastrophic Shocks

In this figure, each peak of coverage quantity is always behind the peak of loss ratio α, which means the insurer reacts after each catastrophic risk. It supports the statement that the effect of a one time catastrophic shock can spread to the following cash flows, and the insurer tends to decrease its underwriting coverage to avoid potential large losses when they observe an occurrence of catastrophic shocks. This is consistent with the analysis that the insurer with capacity constraints would decrease its underwriting coverage when there is a large loss ratio.

Figure 4.2 also shows that fluctuations of underwriting capacity can be caused by catastrophic shocks, and more volatile than catastrophic shocks. This is resulting from the amplifying transmission mechanism by the insurer's dynamic interaction of balance sheet and capital rationing.

4.5.2 Optimal External Capital Strategy and Catastrophic shocks

In the benchmark economy with costly external capital market, Figure 4.3 shows that the amount of external capital raised is negatively correlated with catastrophic risk. This is consistent with the analysis that the external capital will shrink due to higher external capital cost and less funds for debt repayment along with an occurrence of catastrophe events.

In this figure, each peak of external capital is always behind the peak of alpha, which means the insurer's strategy of raising external capital is a reaction to catastrophic shocks. Consistent with the previous analysis, the effect of a one time catastrophic shock can spread out, and the insurer will raise less external capital due

to larger loss payments and higher capital cost after shocks.

This figure also shows that fluctuations of external capital can be caused by catastrophic shocks, and the fluctuations are more volatile than catastrophic shocks due to the dynamic interaction of underwriting profits and capital rationing.
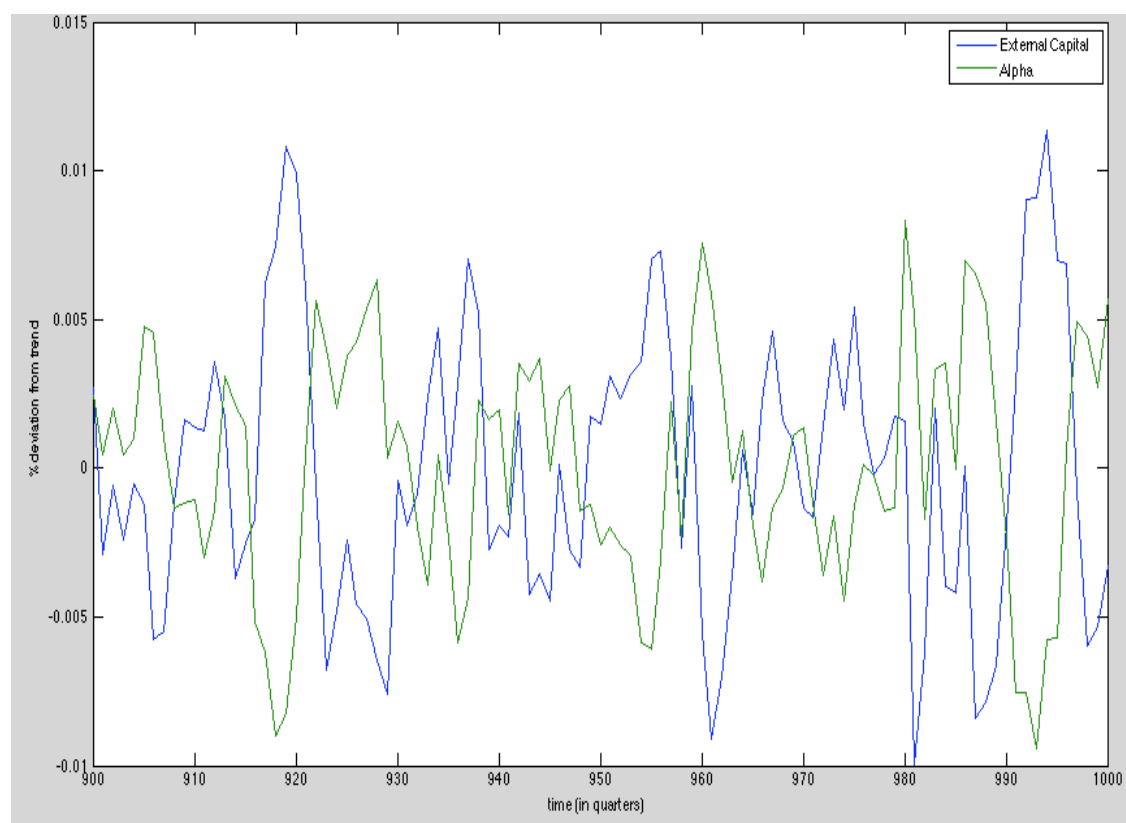
Figure 4.3: Optimal External Capital with Catastrophic Shocks

From the simulated results in benchmark economy, future distributions of the choices of underwriting capacity and external capital can be affected by a one time catastrophic shock in current period. Moreover, due to the dynamic interaction between the capital market and the insurance industry, the amplifying fluctuation of output markets responding to catastrophic shocks can be observed in the insurance industry.

Figure 4.2 and 4.3 support the capacity constraint theory that the supply capacity

is reduced in the insurance industry due to capital shortage after catastrophic risks. Further, according to the dynamic interaction discussed before, these two figures illustrate the non-cyclical behavior of output changes in both coverage capacity and external capital raised if the insurer experiences a series of unexpected catastrophic shocks. They also imply that such output changes cannot be predictable, and the unexpected catastrophic shocks can affect the direction of output changes. At this point, I doubt about the existence of cyclical and predictable "underwriting cycles".

4.6 Catastrophes and Output Fluctuations in Experimental Economy

In this section, I analyze the relation of catastrophic shocks and output fluctuations in experimental economy to determine the factors that affect the amplitude of fluctuations in the model economy.

4.6.1 Relaxed Capacity Constraints

If I reduce the cost of external capital r_e in benchmark economy by 10%, and also reduce the elasticity θ_e and θ_α by 4% respectively, the external capital turns out to be positively correlated with catastrophic risk. It is shown in Figure 4.4.

In this experimental economy, the model economy has a low capital cost and the external capital market is not sensitive to the loss ratio. In this case, the capital market is too soft in which capacity constraints cannot work, so the insurer is more likely to resort to the external capital to expand its underwritings and reserve for future losses.

In this figure, one can find that the fluctuations of external capital, in an environment of relaxed capacity constraints, become quite small due to the absence of the amplifying interaction mechanism.
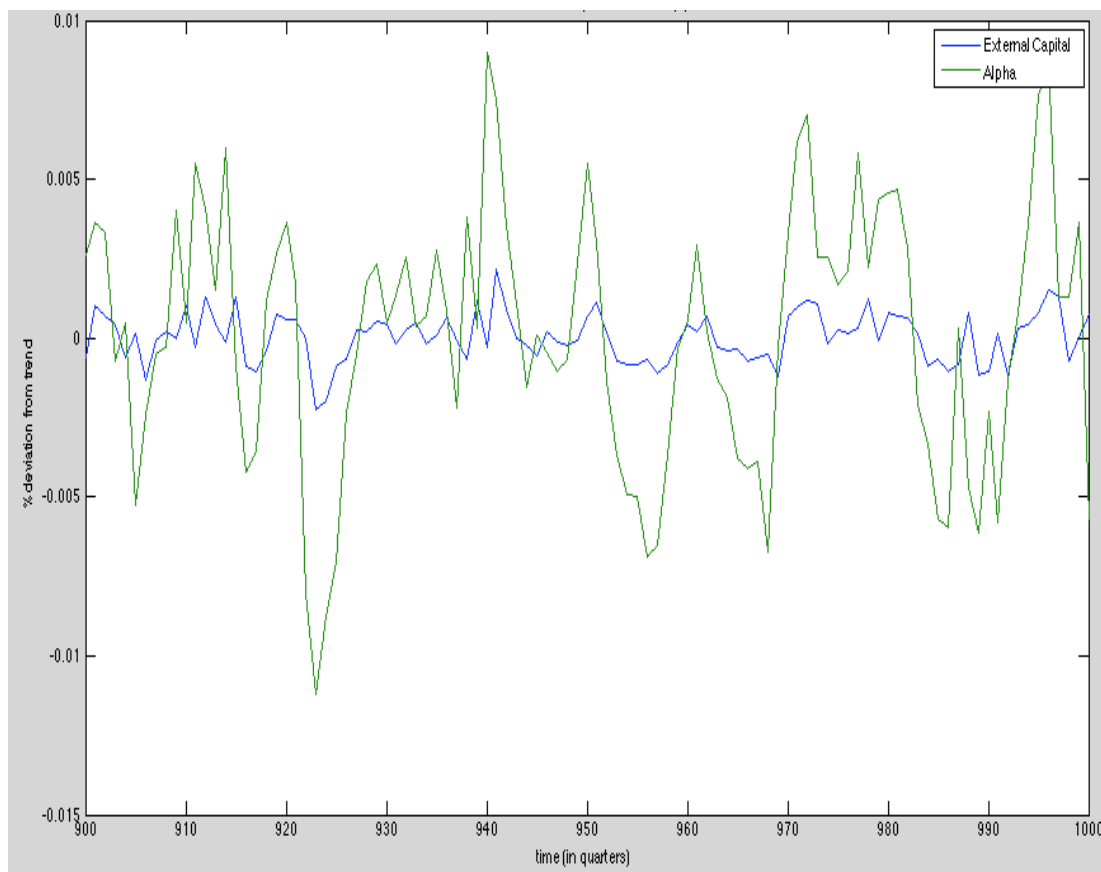
Figure 4.4: Optimal External Capital When Relaxing Capacity Constraints

4.6.2 High Volatility of Catastrophic Shocks

In the following three sub-sections, I report the amplitude of fluctuations in the insurer's underwriting quantity, external capital and internal capital surplus due to catastrophic risks in experimental economy, in order to check the factors in the model economy that can influence the magnitude of the non-cyclical output changes.

The left hand side of Table 4.2 reports the percentage standard deviations of fluctuations in the benchmark economy, while the right one reports the corresponding

amplitudes in the experimental economy with higher loss ratio volatility, σ, from

0.003 to 0.005.

Table 4.2: Standard Deviations of Fluctuations in Benchmark (left) and in
Experimental Economy with σ =0.005 (right)

	st.dev.(%)			st.dev.(%)
Coverage Q	3.112		Coverage Q	3.352
Dividend D	0.007		Dividend D	0.007
Internal Capital K	0.007		Internal Capital K	0.007
External Capital e	2.997		External Capital e	3.085
Ex Marginal Cost R_e	0.252		Ex Marginal Cost R_e	0.390
Catstrophic Risk alpha	0.380		Catstrophic Risk alpha	0.583

From Table 4.2, one can find that fluctuations of coverage quantity Q and external

capital e resulting from more volatile catastrophic shocks both become larger. This is

consistent with the previous analysis that fluctuations of output caused by catastrophic

shocks will be larger if the shock is more volatile.

4.6.3 Tight External Capital Market

Table 4.3 compares fluctuations of the benchmark economy (the left panel) with the

corresponding amplitudes in an economy with the catastrophic shock elasticity, θ_α,

increasing from 1.05 to 1.1(the right panel).

Table 4.3: Standard Deviations of Fluctuations in Benchmark (left) and in
Experimental Economy with θ_α = 1.1 (right)

	st.dev.(%)			st.dev.(%)
Coverage Q	3.112		Coverage Q	3.211
Dividend D	0.007		Dividend D	0.007
Internal Capital K	0.007		Internal Capital K	0.007
External Capital e	2.997		External Capital e	3.021
Ex Marginal Cost R_e	0.252		Ex Marginal Cost R_e	0.297
Catstrophic Risk alpha	0.380		Catstrophic Risk alpha	0.372

It shows that fluctuations of outputs and external capital in the experimental

economy are both larger than those in benchmark economy. This illustrates that the

effect of catastrophic shocks is stronger in an economy with the capital market being

more sensitive towards catastrophic events.

4.6.4 Relaxed Solvency Regulation

Table 4.4 provides the corresponding amplitudes in an economy with a higher solvency ratio, η, from 2.0 to 2.5 in the right panel, and it shows fluctuations of coverage quantity and external capital raised are also both larger than those in benchmark economy shown in the left panel.

Table 4.4: Standard Deviations of Fluctuations in Benchmark (left) and Experimental Economy with η = 2.5 (right)

	st.dev.(%)			st.dev.(%)
Coverage Q	3.112		Coverage Q	3.128
Dividend D	0.007		Dividend D	0.007
Internal Capital K	0.007		Internal Capital K	0.007
External Capital e	2.997		External Capital e	3.012
Ex Marginal Cost R_e	0.252		Ex Marginal Cost R_e	0.282
Catstrophic Risk alpha	0.380		Catstrophic Risk alpha	0.371

It supports the implication that the effect of catastrophic shocks in the model economy is greater if solvency regulation is more relaxed. The solvency ratio acts as a bridge between the capital market and the insurance market, offering an environment in which each catastrophic shock can have an amplifying impact on these two markets.

From these three tables above, we can conclude that the ex-ante magnitude and the period of output changes in the model can be jointly determined in the insurance market and the capital market. Specifically, these results show that the changes in output markets can be larger if the shock is more volatile, if the external capital market is tighter, or if solvency regulation is more relaxed.

4.7 Impulse Response Function Analysis

In this section, I aim to apply Impulse Response Function (IRF) to analyze the impact of shocks in losses and capital capacity on the Property-Casualty (P&C) insurance industry. An impulse response refers to the reaction of any dynamic system in response to the external change. Especially in economics, IRF can describe how the economy reacts over time to exogenous impulses that are usually called "shocks".

The focus of this IRF analysis is to examine the P&C insurers' responses of insurance supply to impulses in loss payment, internal capital surplus, and reinsurance cost. I also check the causality relationship between insurance supply and the internal and external capital for insurers. The relationship between the insurance market and the reinsurance market is explored as well.

In this analysis, five aggregated factors for the P&C Insurance Industry are chosen, including Loss Incurred (Loss), Direct Premium Written (DPW), Net Premium Written (NPW), Policyholders' Surplus (Surplus) and Paragon Catastrophe Reinsurance Price Index (Rein). Correspondingly in the two-period model in Chapter 3 and the dynamic model in this chapter, αQ, πQ and K can refer to the proxy of Loss, DPW and Surplus respectively. Note that DPW is the total insurance supply, which can be split into the insurance price part π and the coverage quantity supply part Q. I also define Loss Ratio (Lossratio) as the value of Incurred Losses/Net Premium Written by years, and it corresponds to, $\alpha,$ in previous models. So we can test the hypothesis developed in previous models and further analyze the changes of insurance supply strategy under different economic situations.

4.7.1 Data Description and Transformation

The yearly index data of Paragon Catastrophe Reinsurance Price (Rein) are obtained from Paragon Risk Management Services. The index[13] (see Gron and Winton, 2001) shows changes in the price of catastrophe reinsurance relative to a base of one[14]. So they can describe the reinsurance cost for catastrophes by years. The other data employed in this analysis are yearly observation data in the industry level, the sources of which are either from Insurance Information Institution (Year 1990 - Year 1995) or the SNL database (Year 1996 - Year 2012). The time line for each variable is from Year1990 to Year 2012.
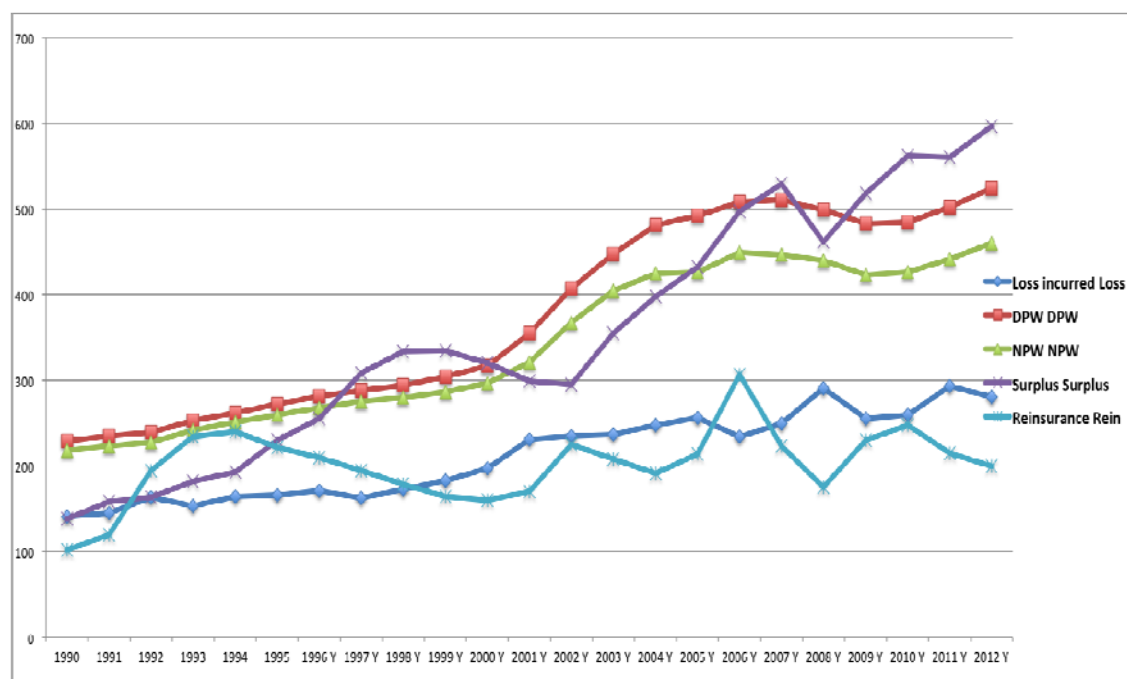

Figure 4.5: Five Aggregated Factors for the P&C Insurance Industry

I plot these five variables in Figure 4.5. Note that in order to show them in the same level of magnitude, I enlarge the Rein by 100 times, and meanwhile, shrink the

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[13] The index was also used in Congressional Budget Office.
[14] The catastrophe reinsurance price in1985 is set as the base of one.

other variables by 1000,000 times.
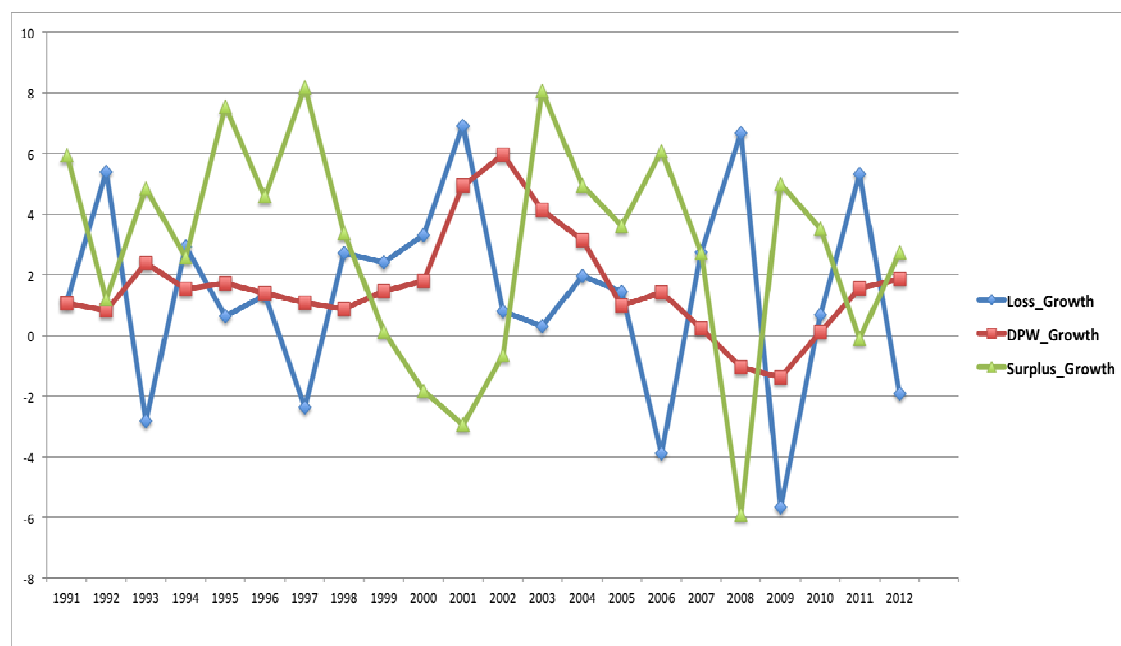
Figure 4.6: Growth Rate Data for Loss, DPW and Surplus

In order to remove the growth trend of Loss, DPW and Surplus in some following empirical regressions, I make the log transformation of these variables to get their growth rate data. For example, DPW_Growth, the growth rate of Direct Premium Written, is set to be equal to $[100 * \log (DPW_t /DPW_{t-1})]$. The growth rate data are shown in Figure 4.6.

4.7.2 VAR Modeling and Main Results

Impulse Response Function (IRF) can be often modeled in the context of Vector Auto Regression (VAR). VAR is an econometric model used to explore the correlations and interdependencies among multiple time series variables, based on its own lags and the lags of all the other variables in the model. In this subsection, I discuss two VAR model cases. In Case I, the multiple time series that I examine are Loss, DPW, Surplus

and Rein; while in Case II, I check Lossratio, DPW_Growth, Surplus_Growth, Rein. But the methodology of building VAR model for these two cases is the same.

First, I apply the commonly used lag-order selection criteria to choose the lag, based on goodness of fit measures such as AICC, SBC, FPEC and HQC. Then I use OLS to estimate the VAR. Next, I examine how well each univariate equation fits the time series data. Finally, the Granger-Causality Wald Test is conducted to explore the causal relationship between the multiple time series. This test is characterized by examining for nonzero correlations between the error processes of the cause and the effect variables to determine whether one time series is useful in forecasting another. If there is a reaction of one variable to an impulse in another variable, we may call the latter causal for the former.

Table 4.5 provides regression results for Model Case I and II. Column 2 provides the lag order chosen for each VAR model case. The third column lists all time series variables involved in the test for each case, which shows the main difference between model cases. Column 4 is the result of univariate residuals test. For example, in Case I, four OLS regressions are conducted in turn with each time series variable acting as the dependent variable while all time series with lags being independent variables. Then we can have four p-values for these four regressions. In the first row, for instance, the p-value is for the regression when the dependent variable is $Loss_t$ and the independed variabls are $Loss_{t-1}$, $Loss_{t-2}$, DPW_{t-1}, DPW_{t-2}, $Surplus_{t-1}$, $Surplus_{t-2}$, $Rein_{t-1}$ and $Rein_{t-2}$. The fifth column provides the result of Granger-Causality Wald Test. The null hypothesis of the first row in Case I, for example, is that Loss is influenced by

itself rather than the other three time series variables, such as DPW, Surplus and Rein.

Table 4.5: VAR Results of Mode Case I and II

Model Case #	Lag Order Chosen	Model Variables	Univariate Residuals Test (p-value)	Granger-Causality Wald Test (p-value)
I	VAR (2)	Loss	<0.0001***[15]	0.5072
		DPW	<0.0001***	0.0001***
		Surplus	<0.0001***	0.2725
		Rein	0.0074***	<0.0001***
II	VAR(3)	Lossratio	0.1986	0.0095***
		DPW_Growth	0.0067***	0.0010***
		Surplus_Growth	0.2178	0.0051***
		Rein	0.0859*	0.0013***

For Model Case I, one can find that each univariate model is significant. This implies that each univiate regression fits the time series data and the correlations among the multiple time series are significant within the lag order. Further, both DPW and Rein have a Granger-causal relationship with the other three variables. Loss, Surplus and Rein can be viewed to Granger-cause DPW. This means that the loss incurred, the internal capital surplus and the reinsurance cost can provide statistically significant information about future values of directed premium written. This is consistent with the previous models that the insurance underwriting supply could be affected by the loss payment, and influenced by both of the external and the internal capital situations. Additionally, the direct premium written, loss incurred, and internal capital surplus in primary insurance market can also Granger-cause the pricing of reinsurance market from the test. Thus it means that the primary insurance market should be considered when forecasting the future reinsurance cost. This is consistent

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[15] *** denotes the 1% level statistical significance; ** denotes the 5% level statistical significance; * denotes the 10% level statistical significance.

with the assumption in the two-period model of Chapter 3 that the reinsurance price will be increased/decreased in the second period if the insurer incurs large/small losses and thus has bad/good solvency status in the first period.

In Model Case II, all the involved time series variables are Lossratio, DPW_Growth, Surplus_Growth and Rein. Two univariate equations with DPW and Rein being dependent variable fit the time series data well. Although the results of the other two univariate equations are not significant, this model fits the selected data well since the value of Akaike information criterion is quite small, for instance AICC = 0.0063.

In this case, the Granger-causality Wald test results imply that these four variables can be influenced by one another, and each variable can be a reasonable factor used to predict another. This is called feedback system. It shows that loss ratio can have impact not only on the change of underwriting premium (as measured by DPW) but also on changes in internal capital surplus (as measured by Surplus) and reinsurance price (as measured by Rein). This verifies an important assumption in previous models that the loss ratio can affect both underwriting profit and capital raising. Moreover, DPW can be Granger-caused by Surplus and Rein; Rein and Surplus can also be Granger-caused by DPW. This implies that there is an interaction between underwriting new business (indicated by DPW) and capital raising situation (indicated by Surplus and Rein). It is consistent with the dynamic model as the model implies an interactive effect between the underwriting balance sheet and capital rationing for an insurer.

In next subsection, I will show the impulse response analysis for each case so as to know more about how the insurers respond to the change of each aggregated variable.

4.7.3 Impulse Response Analysis

The impulse response function is to analyze the dynamic effects of the model economy when one factor receives an impulse. Based on the estimated matrix of VAR model coefficients, we can generate IRFs to identify the consequences of a unit increase in one variable's innovation at time t for the value of another variable at time $t+lag$ holding all other innovations at all dates constant.

Figure 4.7 and Figure 4.8 are the responses to the impulse in Loss and Surplus for Model Case I. Recall that Case I involves the time series of DPW, Surplus, Loss and Rein.
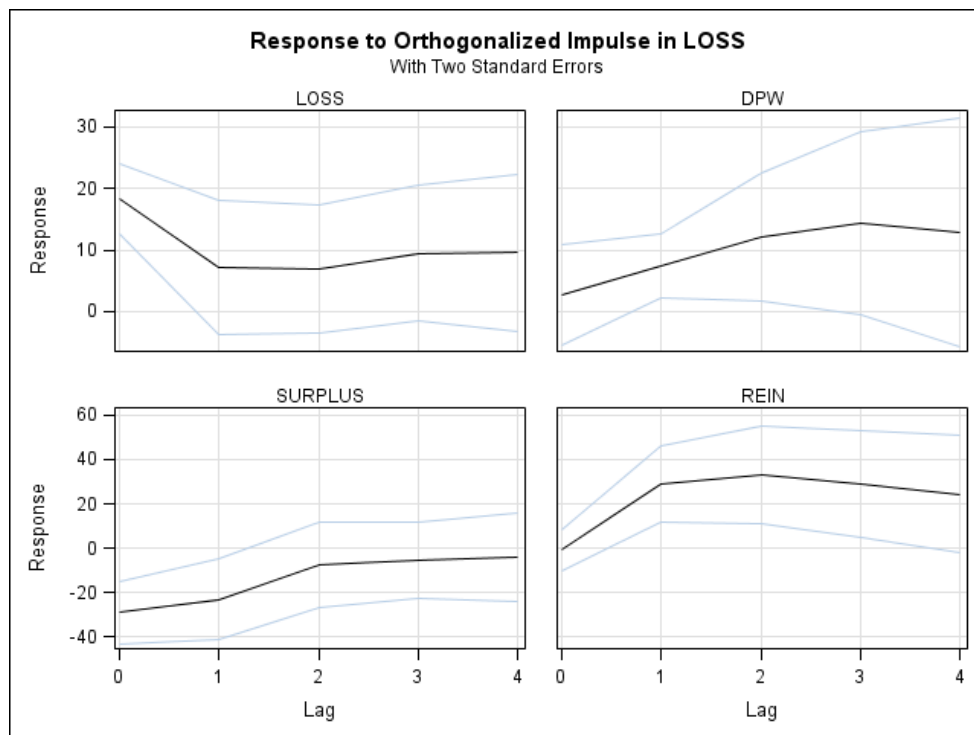
Figure 4.7: Response to Impulse in Loss Incurred for Model Case I

Figure 4.7 provides four graphs. In the top left hand corner we see the future response of loss incurred to a one standard deviation shock of losses incurred. The top right hand corner shows the response of DPW to a one standard deviation shock of losses incurred. In the bottom corners we see the response from a one standard deviation shock to losses in terms of Surplus and REIN respectively.

We can find that the response of DPW to a shock of loss incurred is positive, and the highest response is occurring in the third year. Similarly, the reinsurance price index has positive responses, but these responses are more intensive and faster than those of DPW. In the year $t+1$, the reinsurance price response is able to rise dramatically. This illustrates that price turns to be more sensitive to loss shocks. We may imply that the main reason for the increase in DPW after the loss impulse can be the increase of insurance price (premium per unit dollar of coverage). This verifies the hypothesis in Chapter 3 about the positive relationship between loss ratio and next-period insurance rate in an environment of loss shocks.

In addition, the shock to surplus has an initial negative effect that wears off after about 4 periods as the response moves to zero. It implies that insurers who can make use of insurance price increase after shocks can avoid large loss of capital surplus, which is also consistent with the hypothesis developed in Chapter 3.

Figure 4.8 shows the insurers' response to impulse in internal capital surplus from Case I. In the top left hand corner, it shows the future response of loss incurred to a one standard deviation shock of internal capital surplus. In the top right hand corner we see the response of DPW to a one standard deviation shock of internal capital

surplus.   In the bottom corners we see the response from a one standard deviation

shock to internal capital surplus in terms of Surplus and REIN respectively.

From the top right hand corner in Figure 4.8, the positive response of insurance

supply (as measured by DPW) to a shock in internal capital surplus can be observed.

It implies that the ability to underwrite new business can be enlarged with a better

solvency status, which is consistent with the previous models.
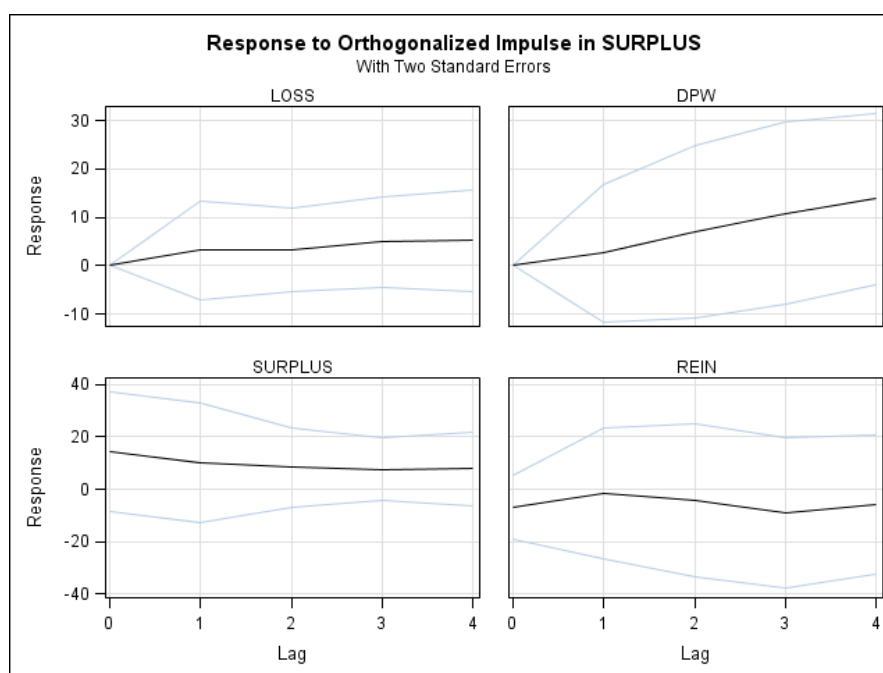
Figure 4.8: Response to Impulse in Policyholders' Surplus for Model Case I

Meanwhile, the bottom of the right hand side in Figure 4.8 shows the slight

negative response of reinsurance price index to a shock in internal capital surplus.

This slight decrease of reinsurance price index in this case can be resulting from less

demand for reinsurance capital with an impulse in internal capital surplus. This is

consistent with the assumption in previous models that insurers prefer raising capital

from the internal source to the external one. But overall, the effect of internal capital

surplus shock on reinsurance price is small.

For the model case II, Figure 4.9 below shows the responses to a shock in terms of a one standard deviation increase in the Loss Ratio.[16] Recall, CASE 2 is identified with the time series variables examined being Lossratio, DPW_Growth, Surplus_Growth and Rein. Starting with the top right hand corner, one can find that the response of the growth rate of DPW in the initial period is very high, and then decreased. The initial large response of DPW growth can be a result of high insurance price, and then its following decreasing can result from the shrinkage of insurance coverage supply when the growth-up of price turns slow. This is to support the finding in the dynamic model that the coverage quantity supply (as measured by Q) that insurers are willing to offer will be reduced with a shock of loss ratio. In the next subsection, I will further test it.
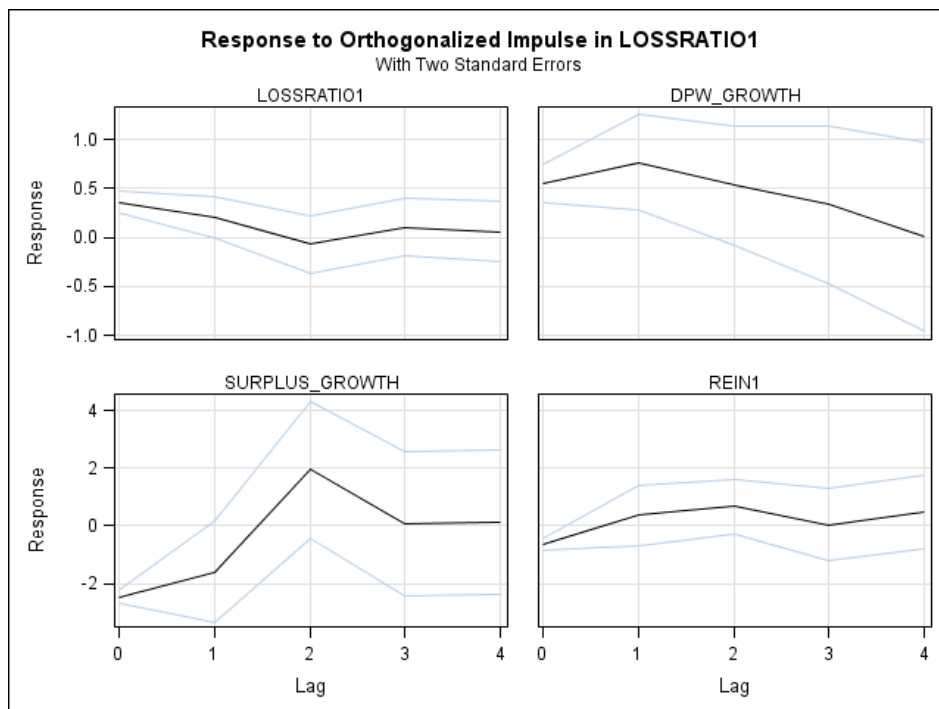
Figure 4.9: Response to Impulse to Loss Ratio for Model Case II

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[16] To keep the magnitude of loss ratio being in the same level as that of Growth Rate data, I multiple all the loss ratio data by 10 in this regression. The same methodology is applied for the Rein data. The aim is to show better and clearer figures of impulse response function.

When checking the response of Surplus growth rate to a shock in the Loss Ratio, one can find in the left bottom panel that the responses in the first two periods are negative, but these negative responses then tend towards zero. The growth rate of Surplus peaks at the end of the second year, which is behind the peak of the growth rate of DPW. This can imply that the gradually deceased negative response of internal capital surplus to a loss shock can result from the growth of the total insurance supply (as measured by DPW_Growth).
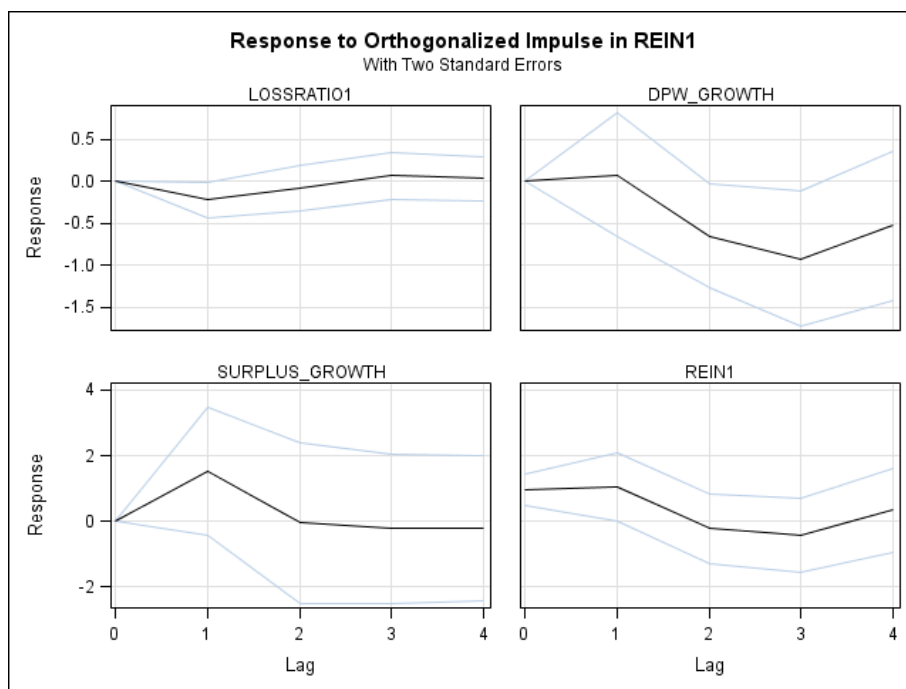
Figure 4.10: Response to Impulse in Reinsurance Price Index for Model Case II

Figure 4.10 above sheds light on how an insurer responses to a one standard deviation shock in reinsurance price index (REIN1).   The right top corner shows that the impact of a reinsurance price shock on the changes of underwriting supply (as measured by DPW) is negative. In the first period, because of possible responses of

the primary insurance price to an impulse in the reinsurance price, we can observe DPW is not changed. Then the peak of the following negative response of DPW is occurring in the end of third year. Based on the cash flow models discussed in both this chapter and Chapter 3, insurers tend to reduce the coverage quantity supply with a tight external capital market due to an interaction between the ability to underwriting new business and the ability to raise new capital.

4.7.4 Extension Model Cases

In this subsection, I try to find a way to split the total insurance supply (DPW) into two parts, the price part and the coverage quantity part. In the empirical research, it is currently impossible to know the primary insurance price since there is no easy way to access the data at the policy level.

Actually, in actuarial practice, the insurance price always can be influenced by catastrophes and the hard/soft market situation. Here I assume that the catastrophe reinsurance price index contains significant information about the primary insurance price, and I apply the reinsurance price index (Rein) to denote the insurance price index in the P&C insurance industry. In this way, I can obtain the coverage quantity in the industry level, settled by Q = Direct Premium Written /Paragon Catastrophe Reinsurance Price Index.

I have two VAR model cases in this subsection, Case III and Case IV. The methodology of developing and testing VAR model for these two cases is in the same way as Case I and II. However, in Case III, all the involved time series variables are

Loss, Q and Surplus, while Loss_Growth, Q_Growth and Surplus_Growth are examined instead in Case IV. The following Table 4.6 shows the VAR results of Model Case III and IV. Figure 4.11 and Figure 4.12 below provide the impulse response in these two cases.

From Table 4.6, the lag orders chosen for both model cases to get the smallest information criteria are larger than the previous ones. That is because there is no price adjustment effect in the model, and the effect of variables in the economy can last longer than before. In Case III, the casual relationship can be found between variables, and each univariate model is significant. In Case IV, the results show that the growth rate of insurance coverage quantity (as measured by Q_Growth) is significantly influenced by the loss changes (as measured by Loss_Growth) and the growth rate of policyholders' surplus (as measured by Surplus_Growth).

Table 4.6: VAR Results of Model Case III and IV

Model Case #	Lag Order Chosen	Model Variables	Univariate Residuals Test (p-value)	Granger-Causality Wald Test (p-value)
III	VAR(5)	Loss	0.0160**	0.0018***
		Q	0.0600*	0.0021***
		Surplus	0.0052***	<0.0001***
IV	VAR(4)	Loss_Growth	0.2006	0.0328**
		Q_Growth	0.0190**	<0.0001***
		Surplus_Growth	0.1016	<0.00001***

Figure 4.11 illustrates that the response of coverage quantity supply (as measured by Q) to a shock in Loss Incurred during the first two periods is mostly negative, and then it becomes positive in Period 3. The shape of such the response is consistent with the statement in the Capacity Constraint hypothesis that the insurance coverage quantity supply shrink sharply with loss shocks. The positive response during the

third period may be resulting in changes of the insurance demand part or the insurance

price after loss shocks.
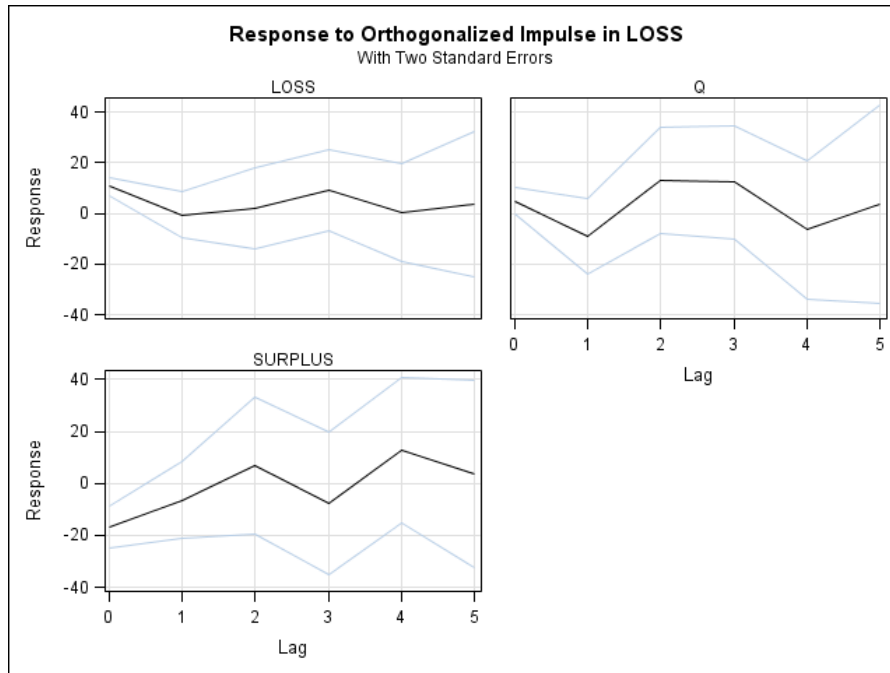
Figure 4.11: Response to Impulse in Loss Incurred for Model Case III
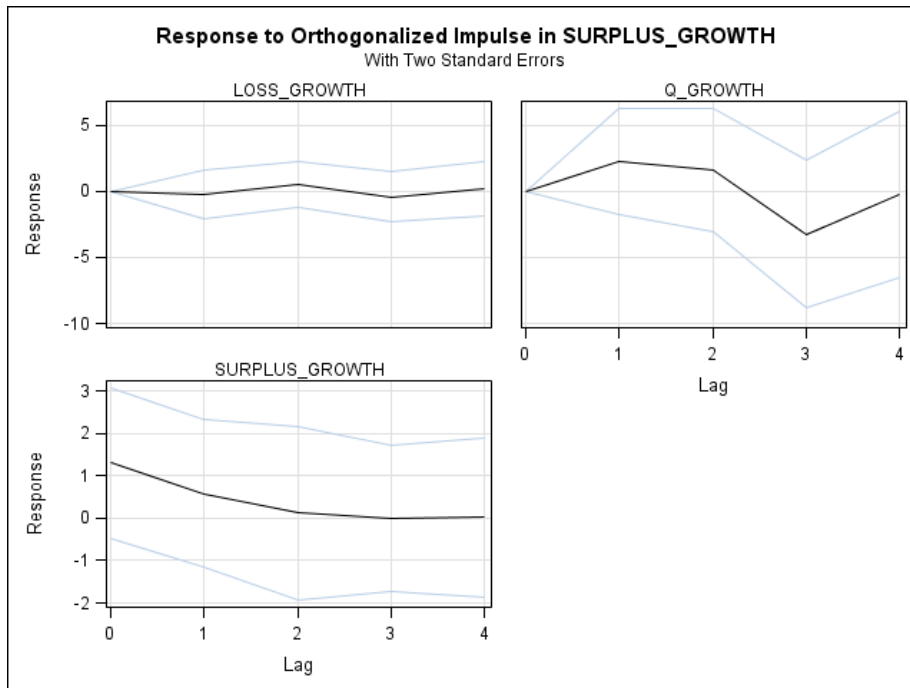
Figure 4.12: Response to Impulse in Surplus Growth Rate for Model Case IV

Figure 4.12 implies the relationship between the change of internal capital surplus and the change of insurance coverage quantity supplied. It shows that the insurer will expand the underwriting operation if there is an increase of policyholders' surplus, and the expansion would stop when the impulse disappear. It verifies the interaction between the insurer's capital rationing and underwriting balance sheet, as discussed in the previous models.

In future research, it is better to access the coverage quantity data or to develop a more reliable proxy for the insurance supply part.

4.7.5　Summary of Empirical Results

In this section, Impulse Response Function (IRF) is used to analyze the P&C insurers' responses to shocks of the loss payment and the internal and external capital. With loss shocks, we can observe a sharp decreasing in insurance coverage quantity supply (as measured by Q), and also the increase of total insurance supply (as measured by DPW) due to price spike, the increasing rate of which (as measured by DPW_Growth) becomes lower and lower as time goes by. They support the Capacity Constraint Theory (see Gron 1994; Gron and Winton 2001).

The results also show the significant Granger-causality relationship between the insurance supply (as measured by DPW in Case I or Q_Growth in Case IV) and the internal capital status and the external reinsurance price (as measured by Surplus, Rein in Case I or Surplus_Growth in Case IV). It supports the statement in pervious theoretical models that there is an interaction for an insurer between its ability to

underwriting new business and its ability to raise capital.

Finally, the results imply that the reinsurance price (as measured by Rein) can be affected by changes of internal capital surplus and losses incurred in primary insurance market. This is consistent with the assumptions in previous models.

4.8 Conclusions and Discussions

In this chapter, a dynamic cash flow model with capacity constraints is built to describe the insurer's catastrophic intermediating process towards a series of loss shocks. I focus specifically on the insurer's decision-making choices of underwriting quantity and capital structure in a dynamic economy with stochastic loss shocks, and find the dynamic interaction between the insurer's capital rationing and balance sheet, in which capacity constraints play an import role.

According to the simulation results, this paper contributes to find a non-cyclical behavior of output fluctuations in the model economy, and thus I view the unpredictable underwriting cycles as temporary responses of output markets to loss shocks. I also explore the determinants of the magnitude of output fluctuations by comparing the experimental economy with the benchmark economy.

In future work, I can develop a Heterogeneous-Agent model with recursive computational simulations to analyze different optimal decision paths of underwriting and capital structures for heterogeneous insurers. This framework allows me to quantitatively study why different insurers perform differently after catastrophes, and to explore the impact of catastrophic shocks on the industrial organization of the

insurance markets in a dynamic setting. In addition, empirical testes in the firm level can be conducted to explore how insurers respond to large losses and what kind of insurers can perform well with catastrophic risk underwriting.

Appendix 2A: Algorithm Methodology for the Life Cycle Model

The state space of heterogeneous households in this life cycle model is *imax*jmax*jmax*mmax*kmax* (20*5*5*4*80). I solve household's optimization problem backward from age *kmax* with the assumption that the value function in the period after the last period, $V_{kmax+1}(s')$, is equal to 0.

Based on fist-order conditions and the envelope condition, I construct Kuhn-Tucker conditions to figure out household's optimal decision rules in state *s*, such as $c(s)$, $l_1(s)$, $l_2(s)$, $d_1(s)$ and $d_2(s)$. Meanwhile, I update the new household value $V_{kmax}(s)$ and marginal value $V_{kmax,a}(s)$ for the *s*-sate household at corresponding time *kmax*. Then I use updated results for different household sates in period *kmax* to solve utility optimization problem at age *(kmax-1)*. The optimal decision rules for each specific household state at each year can be dynamically solved in the same way.

Appendix 2B: Optimization Solutions for the Life Cycle Model

If m = 0,

$$V(a; m=0) = U(c, l_1, l_2; n_k, m=0) + \beta*[\varphi_{1,k}*\varphi_{2,k}*V(a'; m'=0, m=0) +$$

$$\varphi_{1,k}*(1-\varphi_{2,k})*V(a'+d_2; m'=1, m=0) + \varphi_{2,k}*(1-\varphi_{1,k})*V(a'+d_1;$$

$$m'=2, m=0) + (1-\varphi_{1,k})*(1-\varphi_{2,k})*V(a'+d_1+d_2; m'=3, m=0)]$$

s.t. $\quad a' = (1+r) a + w_{1,k} e_1(1-l_1) + w_{2,k} e_2(1-l_2) + \mathbb{1}_{k>65}\, \mathbb{1}_{m<3}(1+\mathbb{1}_{m=0})ss$

$$- c - (1-\varphi_{1,k}) d_1 - (1-\varphi_{2,k}) d_2;$$

$$U_c(c, l_1, l_2; n_k, m=0) = \lambda \qquad\qquad (c)$$

$$U_{l_1}(c, l_1, l_2; n_k, m=0) = w_{1,k} e_1*\lambda \qquad\qquad (l_1)$$

$$U_{l_2}(c, l_1, l_2; n_k, m=0) = w_{2,k} e_2*\lambda \qquad\qquad (l_2)$$

$$\beta*[\varphi_{1,k}*V_a(a'; m'=0, m=0) + (1-\varphi_{1,k})*V_a(a'+d_1; m'=2, m=0)] = \lambda \qquad (d_1)$$

$$\beta*[\varphi_{2,k}*V_a(a'; m'=0, m=0) + (1-\varphi_{2,k})*V_a(a'+d_2; m'=1, m=0)] = \lambda \qquad (d_2)$$

$$\beta*[\varphi_{2,k}*V_a(a'+d_1; m'=2, m=0) + (1-\varphi_{2,k})*V_a(a'+d_1+d_2; m'=3, m=0)] = \lambda \quad (a')$$

where λ is the Lagrangian Parameter.

If m=1,

$$V(a; m=1) = U(c, l_1; n_k, m=1) + \beta*[\varphi_{1,k}*V(a'; m'=1, m=1) +$$

$$(1-\varphi_{1,k})*V(a'+d_1; m'=3, m=1]$$

s.t. $\quad a' = (1+r) a + w_{1,k} e_1 h_1 + \mathbb{1}_{k>65}\, ss - c - (1-\varphi_{1,k}) d_1;$

$$U_c(c, l_1; n_k, m=1) = \lambda \qquad\qquad (c)$$

$$U_{l_1}(c, l_1; n_k, m=1) = w_{1,k} e_1*\lambda \qquad\qquad (l_1)$$

$$\beta*V_a(a'+d_1; m'=3, m=1) = \lambda \qquad\qquad (d_1)$$

$$\beta * V_a(a'; m'=1, m=1) = \lambda \qquad\qquad (a')$$

If $m=2$,

$$V(a; m=2) = U(c, l_2; n_k, m=2) + \beta * [\varphi_{2,k} * V(a'; m'=2, m=2) +$$

$$(1-\varphi_{2,k}) * V(a'+d_2; m'=3, m=2]$$

s.t. $\quad a' = (1+r) a + w e_2 h_2 + \mathbb{1}_{k>65} \, ss - c - (1-\varphi_{2,k}) d_2$

$$U_c(c, l_2; n_k, m=2) = \lambda \qquad\qquad (c)$$

$$U_{l_2}(c, l_2; n_k, m=2) = w_{2,k} e_2 * \lambda \qquad\qquad (l_2)$$

$$\beta * V_a(a'+d_2; m'=3, m=2) = \lambda \qquad\qquad (d_2)$$

$$\beta * V_a(a'; m'=2, m=2) = \lambda \qquad\qquad (a')$$

If $m=3$,

$$V(a; m=3) = U(c; n_k, m=3) + \beta * V(a'; m'=3, m=3)$$

s.t. $\quad a' = (1+r) a - c$

$$U_c(c; n_k, m=3) = \lambda \qquad\qquad (c)$$

$$\beta * V_a(a'; m'=3, m=3) = \lambda \qquad\qquad (a')$$

Appendix 3: Optimization Solutions for the Two-Period Cash Flow Model

FOCs with β, β^i, e, e^i, π^i are as follows,

$$(TQ_b - \beta QC_b - r_f^{-1}R_b)b_\beta + (r_f^{-1} - C)Q + r_f^{-1}\sum_{i=1}^{I} P^i(T^iQ_{b^i}^i - \beta^iQ^iC_{b^i}^i - r_f^{-1}R_{b^i}^i) b_\beta^i = 0 \text{ (i)}$$

$$(T^iQ_{b^i}^i - \beta^iQ^iC_{b^i}^i - r_f^{-1}R_{b^i}^i) b_{\beta^i}^i - (C^i - r_f^{-1})Q^i = 0 \qquad\qquad \text{(ii)}$$

$$(TQ_b - \beta QC_b - r_f^{-1}R_b)b_e + (1 - r_f^{-1}R_e) + r_f^{-1}\sum_{i=1}^{I} P^i(T^iQ_{b^i}^i - \beta^iQ^iC_{b^i}^i - r_f^{-1}R_{b^i}^i) b_e^i = 0 \text{ (iii)}$$

$$(T^iQ_{b^i}^i - \beta^iQ^iC_{b^i}^i - r_f^{-1}R_{b^i}^i) b_{e^i}^i + (1 - r_f^{-1}R_{e^i}^i) = 0 \qquad\qquad \text{(iv)}$$

$$(T^iQ_{b^i}^i - \beta^iQ^iC_{b^i}^i - r_f^{-1}R_{b^i}^i)b_{\pi^i}^i + Q^i + T^iQ_{\pi^i}^i = 0 \qquad\qquad \text{(v)}$$

where $T = \pi - C\beta - (1-\beta)r_f^{-1}$ and $T^i = \pi^i - C^i\beta^i - (1-\beta^i) r_f^{-1}$

Case One: Risk Free Capital Market with $C(b) = C^i (b^i) = r_f^{-1}$, $R(b,e) = R^i (b^i, e^i) = r_f$

(i), (ii), (iii), and (iv) can imply that: $Q_b = Q_{b^i}^i = 0$,

and $TQ_b - \beta QC_b - r_f^{-1}R_b = T^iQ_{b^i}^i - \beta^iQ^iC_{b^i}^i - r_f^{-1}R_{b^i}^i = 0$;

Then $Q^i + T^iQ_{\pi^i}^i = 0$ according to (v), equivalently, it is $E_{Q^i\pi^i} = -\dfrac{\pi^i}{\pi^i - r_f^{-1}}$.

Case Two: Costly Capital Market with $C(b) = C^i (b^i) = r_f^{-1}$, $R(b,e) = R^i (b^i, e^i) = r_f$

(ii), (iv), and (v) can derive that:

$$T^iQ_{b^i}^i - \beta^iQ^iC_{b^i}^i - r_f^{-1}R_{b^i}^i = = -\frac{Q^i + T^iQ_{\pi^i}^i}{b_{\pi^i}^i} = \frac{(C^i - r_f^{-1})Q^i}{b_{\beta^i}^i} = \frac{r_f^{-1}R_{e^i}^i - 1}{b_{e^i}^i};$$

Then (v) can show that: $E_{Q^i\pi^i} = -\dfrac{(Q^i + MP_b^i b_{\pi^i}^i)\pi^i}{T^iQ^i}$;

Next, according to Comparative Statics Analysis, one can get

$$\frac{d\pi^i}{dL^i} = \frac{\frac{d(v)}{dL^i}}{|SOC|}, \quad \frac{de^i}{dL^i} = \frac{\frac{d(iv)}{dL^i}}{|SOC|}, \quad \text{and} \quad \frac{d\beta^i}{dL^i} = \frac{\frac{d(ii)}{dL^i}}{|SOC|}.$$

Appendix 4: Algorithm Methodology for the Dynamic Cash Flow Model

I solve and simulate this dynamic model with linear quadratic approximation around the steady state. First, I start with a stochastic finite horizon optimization problem, and derive the Riccati equation for this dynamic model with stochastic growth. Next, I solve the steady-state conditions for Q^*, K^*, e^*, λ^* by FOCs to obtain a certainty-equivalent steady-state equilibrium. Then I calculate the Jacobian matrix and Hessian matrix by using the log difference. Based on all the pervious steps of calculation, I approximate the return function and the state transition function around the steady state for this model, and obtain its value function and policy function that are prepared for the following simulations. I set the initial state is the steady state and simulate this stochastic growth model. Here I get the deviations from trend by using the Hodrick-Prescott filter. Finally, the standard deviations and the cross correlations with output of variables can be shown by figures or tables.

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