

Summer 8-13-2013

# The Effects of Using Direct Instruction and the Equal Additions Algorithm to Promote Subtraction with Regrouping skills of Students with Emotional and Behavioral Disorders with Mathematics Difficulties

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## ACCEPTANCE

This dissertation, THE EFFECTS OF USING DIRECT INSTRUCTION AND THE EQUAL ADDITIONS ALGORITHM TO PROMOTE SUBTRACTION WITH REGROUPING SKILLS OF STUDENTS WITH EMOTIONAL AND BEHAVIORAL DISORDERS WITH MATHEMATICS DIFFICULTIES, by ANGELA CHRISTINE FAIN, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree, Doctor of Philosophy, in the College of Education, Georgia State University.

The Dissertation Advisory Committee and the student's Department Chairperson, as representatives of the faculty, certify that this dissertation has met all standards of excellence and scholarship as determined by the faculty. The Dean of the College of Education concurs.

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## ABSTRACT

### THE EFFECTS OF USING DIRECT INSTRUCTION AND THE EQUAL ADDITIONS ALGORITHM TO PROMOTE SUBTRACTION WITH REGROUPING SKILLS OF STUDENTS WITH EMOTIONAL AND BEHAVIORAL DISORDERS WITH MATHEMATICS DIFFICULTIES

by  
Angela Christine Fain

Students with emotional and behavioral disorders (E/BD) display severe social and academic deficits that can adversely affect their academic performance in mathematics and result in higher rates of failure throughout their schooling compared to other students with disabilities (U.S. Department of Education, 2005; Webber & Plotts, 2008). Furthermore, students with E/BD are at a greater risk of being served in more exclusionary and restrictive settings compared to their peers as a result of their poor social skills and chronic disruptive behaviors (Gagnon & Leone, 2005; Furney, Hasazi, Clark-Keefe, & Hartnett, 2003; U.S. Department of Education, 2005; Whorton, Siders, Fowler, & Naylor, 2000). This is of great concern as students with E/BD often receive lower grades, fail more classes, have higher drop-out rates, have fewer employment opportunities, and have increased involvement in the legal system (Bullock & Gable, 2006; Cullinan & Sabornie, 2004; Jolivette, Stichter, Nelson, Scott, & Liaupsin, 2000; Kauffman, 2001). The purpose of this study was to analyze the effect of the equal additions algorithm on subtraction with regrouping on the subtraction performance of fourth-grade students with E/BD and mathematics difficulties. The equal additions algorithm was taught using a direct instruction technique. This study investigated 3 participants at the fourth grade level in a residential treatment facility which serves students with E/BD. A multiprobe multiple baseline across participants design was used

for this study. Assessments used for this study included (a) *Woodcock Johnson III* (*WJIII*), (b) the *ENRIGHT*, (c) a student questionnaire, (d) baseline probes, and (e) an error analysis student profile. Data was analyzed by visual analysis. The results suggest that when the equal additions algorithm was systematically implemented students were able to successfully complete subtraction with regrouping problems and errors dramatically decreased. Limitations and future for research directions are discussed.



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A Dissertation

Presented in Partial Fulfillment of Requirements for the  
Degree of  
Doctor of Philosophy  
in  
Education of Students with Exceptionalities  
in  
the Department of Educational Psychology and Special Education  
in  
the College of Education  
Georgia State University

Atlanta, GA  
2013

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2013

## ACKNOWLEDGMENTS

There are many people who helped support me through my doctoral studies leading up to this point that I would like to thank. First, I would like to thank all of the professors at Georgia State University who helped me along the way, providing their expertise and guidance in their courses, especially Dr. Fredrick, Dr. Houchins, Dr. Alberto, Dr. Chahine, Dr. Calhoon, Dr. Jolivette, and Dr. Patton-Terry. I want to thank Dr. Viel-Ruma for being a great supporter and supervisor. I also want to thank Drs. Leontovich and Laushey for being such great listeners and for being so supportive throughout the process. I thank all the members of my committee for their time and valuable feedback. I also want to thank Dr. Chahine for being so supportive and encouraging over the years. I would like to thank my advisor, Dr. Kathryn Heller. You supported me, encouraged me, and helped me through this process and I will forever be grateful. Finally, I want to thank Dr. Gallagher for being such a great mentor and advisor. You have given me so much support and advice throughout the years and it has meant the world to me.

I want to thank all of my colleagues and friends that I have shared this experience with. We have created bonds that will last a lifetime. Thank you to my dear friend of many years, Christina, for always being there and for supporting me. I would like to thank Katy for always having such a great attitude and for being a great friend. I look forward to working with you for a long time. To my dear friend, Ellen, who has been my study partner from the first semester of our program, thank you for everything and for being such a great support throughout the years. To Joanna, words cannot express how grateful I am to you for everything that you have done to help me through this process and how lucky I am to have a friend like you.

Lastly, I would like to thank all of my friends and family. To all my friends who have supported me through this process, thank you. Your support has meant the world to me. I would like to thank my family for their support, especially my mom. Thank you for your unconditional love and support. I am forever grateful.

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## ABBREVIATIONS

CBA	Curriculum-Based Assessment
CCC	Cover, Copy, Compare
CRA	Concrete-Representational-Abstract
E/BD	Emotional and Behavioral Disorder
IDEA	Individuals with Disabilities Education Act
LD	Learning Disabilities
MD	Mathematics Disabilities



## CHAPTER 1

### MATHEMATICS INTERVENTIONS FOR STUDENTS WITH EMOTIONAL AND BEHAVIORAL DISORDERS

Students with emotional and behavioral disorders (E/BD) have deficits in behavioral performance, academic achievement, and social skills that greatly interfere with their educational performance in school (Rutherford, Quinn, & Mathur, 2004). These students display chronic disruptive behaviors that are generally identified after repeated academic failure and/or chronic disruptive behavior (Kauffman, 2001), and their deficits tend to maintain across grade levels and content areas (Nelson, Benner, Lane, & Smith, 2004). Compared to their peers in other disability categories, students with E/BD are more likely to have lower grades, fail more classes, be retained, be served in restrictive settings, and drop out of school (Bullock & Gable, 2006; Jolivette, Stichter, Nelson, Scott, & Liaupsin, 2000). Furthermore, students with E/BD often have fewer employment opportunities, increased involvement in the legal system, and increased chances for negative experience within the community (Cullinan & Sabornie, 2004; Jolivette et al., 2000; Kauffman, 2001).

One of the academic areas of particular concern is mathematics. Research shows that students with E/BD perform more than one year below their non-disabled peers in these areas (Cullinan, 2002) and achieve well below national averages in mathematics (Anderson, Kutash, & Duchnowski, 2001). Furthermore, the severe social and academic deficits of students with E/BD can adversely affect their academic performance and result in higher rates of academic failure throughout their schooling compared to other students with disabilities (U.S. Department of Education, 2005; Webber & Plotts, 2008).

The purpose of this literature review was to examine the variety of mathematics strategies that have been applied to teach subtraction to students with E/BD and mathematics difficulties. Studies that present strategies on subtraction for students with mild disabilities are examined for a number of reasons: (a) There are only a few studies available that specifically focus on students with E/BD; and (b) they provide a basis for better understanding effective strategies that may be appropriate for students with E/BD.

### **Defining E/BD**

The Individuals with Disabilities Education Act (IDEA, 2004) has defined an emotional or behavioral disability as exhibiting one or more of the following characteristics: (a) an inability to learn not explained by intellectual, sensory, or health factors; (b) an inability to build or maintain satisfactory relationships with peers and teachers; (c) inappropriate behaviors or feelings under normal circumstances; (d) a pervasive mood of unhappiness or depression; and/or (e) a tendency to develop physical symptoms or fears associated with personal or school problems. These criteria are identified as having been displayed to a marked degree and over an extended period of time.

Because of the emotional and behavioral characteristics displayed by students with E/BD, they may be educated in a variety of settings including the general education classroom, resource, self-contained classrooms, self-contained schools, and residential settings. According to IDEA (2004) and No Child Left Behind (NCLB, 2001), all students are entitled to have access to the general education curriculum within a least restrictive environment (LRE) to the maximum extent appropriate with those who are not disabled. Only when a student is considered to have a disability in which the nature or

severity of the disability is such that education in a general education class with supplementary aids and services will not satisfactorily meet the needs of the student is the student considered for placement in an alternative setting such as a special class or separate school.

## **Settings**

### **Public School Settings**

Students with E/BD face educational challenges such as preparing and organizing their materials for numerous courses, listening to lectures, taking notes, actively participating in class, mastering a wide variety of academic content, and studying for tests (Mastropieri & Scruggs, 2001). Placement in inclusion classrooms can be difficult for these students and their teachers and placement in these settings may lead to inconsistent academic success (Nelson et al., 2004). However, with adequate support from administrators and special education teachers, appropriate curriculum, positive classroom environments, and effective teaching strategies, students with disabilities can succeed in a general education environment (Mastropieri & Scruggs, 2001; Villa, Thousand, Nevin, & Liston, 2005).

Students with E/BD are increasingly being served in exclusionary settings such as self-contained classrooms and self-contained schools (Furney, Hasazi, Clark-Keefe, & Hartnett, 2003; Whorton, Siders, Fowler, & Naylor, 2000). More restrictive environments can be beneficial in that they offer smaller class sizes, support from other professionals in the classroom such as paraprofessionals, teachers with social skills training and classroom management skills (Singer, Butler, Palfrey, & Walker, 1986). Furthermore, teachers in more restrictive settings are trained to implement modifications and

accommodations and offer more diverse instructional strategies to meet the individual needs of the students (Meadows, Neel, Scott, & Parker, 1994). In more restrictive settings such as self-contained schools, students may receive more behavioral and therapeutic support not often found in general education classrooms (Lane, Wehby, Little, & Cooley, 2005).

Very little literature is available on the performance of students with E/BD in self-contained settings. In 2005, Lane and colleagues conducted a study that compared the academic, behavioral, and social deficits of students with disabilities educated in self-contained classrooms and a self-contained school. Seventy-two students with high incidence disabilities, primarily emotional disturbances, educated in either self-contained classrooms or a self-contained school were evaluated using the *Woodcock-Johnson III Test of Achievement (WJ-III)*, curriculum-based measures of oral reading fluency and reading comprehension, and two subtests from the *Wechsler Intelligence Scale for Children-Third Edition (WISC-III)*. Teachers completed the *Social Skills Rating System (SSRS)* and the *Walker-McConnell Scale of Teacher and Peer Preferred Social Behavior and School Adjustment* behavior rating scales. A series of one-way, fixed-effects MANOVAs and fixed-effects univariate ANOVAs were conducted. Results indicated that students educated in the self-contained classrooms had higher academic skills in reading comprehension, oral reading fluency, oral language, written expression, broad math, and broad reading than those educated in a self-contained school. Behaviorally, students in the self-contained school received significantly more disciplinary contacts and negatively worded items in their folders.

## **Residential Settings**

Students with E/BD are often placed in exclusionary settings that provide intensive therapeutic and behavioral support such as residential treatment centers far more than are students in any other disability category (Gagnon & Leone, 2005; U.S. Department of Education, 2002). According to the U.S. Department of Education (2002), some 77,000 students with E/BD are educated in residential treatment centers. According to the National Center on Education Statistics (NCES, 2010) 2% of the students with E/BD in the public school population were served in a separate residential facility in 2008. A *residential school* is a comprehensive, therapeutic, educational school setting that provides students with 24-hour monitoring that addresses the social, emotional, and educational needs of students (AWMC Working Party of Residential Resources, 1984; Kauffman & Smucker, 1995). To meet the individualized needs of a student with E/BD, special education services are designed to provide behavioral, academic, and social interventions designed to enhance success in school (Gunter, Jack, DePaepe, Reed, & Harrison, 1994). As a result, residential settings often have a greater emphasis on behavioral issues opposed to academic gains (Grizenko & Sayegh, 1990; Kotsopoulous, Walker, Beggs, & Jones, 1991; Wehby, Symons, & Shores, 1995).

Despite the increase in the number of students being served in residential settings, little information is available regarding the quality of education students receive. Students in residential treatment settings often receive less instruction compared to students in other educational settings which is concerning as these students often return to their public schools (Grizenko, Sayegh, & Papineau, 1994; Katsiyannis, 1993). Instruction in residential settings is often designed as individual, independent work with an emphasis

on paper and pencil tasks (Wheby et al., 1995). Poor instructional strategies like these can lead to fewer academic and behavioral achievements (Lane et al., 2005). With a history of inadequate educational services for students in residential settings, there is a concern that students with E/BD in these settings may not be receiving the educational supports and opportunities necessary for academic achievement (Gagnon & Leone, 2005). The quality of instruction students receive in a residential setting is of great concern, especially when combined with the fact that as students with E/BD struggle to achieve in core academic subjects, particularly mathematics.

### **Overall Instruction of Students with E/BD**

There are two common functions of problem behaviors associated with poor academic performance among students with E/BD. Some students may choose to engage in inappropriate behaviors due to the fact that the academic task is too difficult and disruption allows them to escape the demand of the task. Other students are able to successfully perform the tasks, but choose to engage in negative behaviors (Witt, VanDerHeyden, & Gilbertson, 2007). While a skill-deficit problem requires attention focused on instructional strategies and a performance deficit requires attention directed at antecedents and consequences of behavior, opportunities for incentives can be provided to increase work productivity. Researchers have found that frequent feedback and praise from teachers and peers have positive effects on behavior and academic performance (Graham, 1999, Sutherland & Welby, 2001a; Waxman & Walberg, 1991).

Of the research on effective teaching strategies related to students with behavioral problems, active student responding is highly correlated to academic achievement (Brophy, 1986; Gettinger & Stroiber, 1999; Greenwood, 1996). In addition, teachers who

spend more time providing guided practice on initial tasks have students who are more engaged (McKee & Witt, 1990) and when teachers implement a variety of instructional response modes or supports students are more likely to be on-task. Research suggest that few teachers of students with E/BD use effective teaching strategies in the classroom (Gunter & Denny, 1996), however, if effective instructional strategies are implemented most children will be actively engaged in the learning process, behave appropriately, and learn (Witt, VanDerHeyden, & Gilbertson, 2007).

### **Overall Mathematics Performance of Students with Disabilities**

It is estimated that 5 to 10% of students in elementary schools have a mathematics disability (MD) and that nearly 50% of all students have difficulty in mathematics (Badian, 1983; Geary, 2003; Kosci, 1974; Rivera, 1997; Siegler, 2007). Students who demonstrate difficulty in mathematics may or may not have been diagnosed with another disability (i.e. emotional and behavioral disorder, mild intellectual disability). According to the *National Assessment of Educational Progress at Grades 4 and 8* (NAEP; 2009), the fourth-grader's mathematics assessment showed students with disabilities had an average performance score of 220 in 2007, placing them in the bottom of the *Basic* level of achievement compared to their nondisabled peers averaging 242. Sadly, these students did not show a change in their scores in 2009. Furthermore, in the number properties and operations section of the mathematics assessment, 4<sup>th</sup> grade students with disabilities versus their peers without disabilities showed a significant decrease in performance of 20 points. Specifically, in a subtraction with regrouping question from the number properties and operations section, 33% of fourth-grade students who were asked to subtract a two-digit number from a three-digit number did so incorrectly. Further, fourth-grade students

who performed at *Basic* level or *Below Basic* level of the NAEP answered incorrectly 36% of the time (NAEP).

Within mathematics, computation is a foundational skill found in every math content area (e.g., measurement, geometry, algebra) (U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistic (NAEP), 2009 Mathematics Assessment) and has been an area of great concern for teachers and researchers (Cawley, Parmar, Yan, & Miller, 1996; Rivera & Smith, 1988).

Computational skills are typically defined by the accuracy and fluency with simple arithmetic problems (Siegler, 1988). Researchers have observed that children having difficulty in mathematics exhibit multiple computation errors, including repetitive use of incorrect algorithms (Coker, 1991; Resnick, 1984), ineffective strategy use, and poor recall for basic facts (Russell & Ginsberg, 1984). A recent report from the National Mathematics Advisory Panel (NMAP; 2008) indicated that computational ability is dependent upon basic fact recall which requires fluency with the standard algorithms for addition, subtraction, multiplication, and division. Furthermore, conceptual understanding of mathematical operations, fluent execution of procedures, and the ability to recall basic facts support effective and efficient problem solving.

Failure to develop sound computational skills may impact these students as they progress through school. As knowledge develops cumulatively in mathematics, the acquisition of basic skills is critical for students in the primary grades and according to Woodward (2004), as academically low-achieving students move through the early grades they face a number of difficulties as they encounter increasingly complex mathematical tasks. A descriptive study by Calhoon, Emerson, Flores, and Houchins



(2007) of high school students with mathematics disabilities (MD) reveals these students continue to show a lack of computational fluency in a majority of mathematics areas at the 4<sup>th</sup> grade level. More specifically, their study showed these high school students demonstrated profound difficulties in subtracting multiple digits with regrouping. Results suggested that the retention of fourth-grade-level computational skills may present difficulties in learning higher order math skills for students with MD.

### **Academic Characteristics of Students with E/BD**

#### **General Performance Across Content Areas**

It is widely accepted that students with E/BD perform below grade level in reading, spelling, and mathematics for a variety of reasons (Lane, 2007; Mastropieri, Jenkins, & Scruggs, 1985; Nelson et al., 2004; Osher et al., 2007; Reid, Gonzalez, Nordness, Trout, & Epstein, 2004). Students with E/BD perform more than one year below their non-disabled peers in these areas (Cullinan, 2002) and achieve well below national averages in reading and mathematics (Anderson et al., 2001). Academically, these students often have difficulty attending to tasks, completing tasks in a timely manner, staying on-task, and completing tasks independently (Cancio, West, & Young, 2004; Lane, Carter, Pierson, & Glaeser, 2006).

Behaviorally, students with E/BD exhibit inappropriate classroom behaviors during academic tasks; they may be anxious and nervous (Ashcroft, Krause, & Hopko, 2007; Liaupsin, Jolivette, & Scott, 2007; Wright, 1996), non-compliant (Osher et al., 2007), and/or verbally or physically aggressive to divert attention from their academic difficulties and/or escape task demands (Fuchs, Fuchs, & National Center on Student Progress, 2001; Garnett, 1987; Lane, 2007; Osher et al.). Task difficulty can impact how

these students respond, as often times they display disruptive or noncompliant behaviors in an effort to escape task demands (Mayer, 2001; Van Acker, 2002). These inappropriate behaviors often result in lower teacher expectations, removal from a desired task, or removal from the educational setting (Colvin, 2004; Nelson, 1997; Van Acker).

Furthermore, these students often display poor social skills that can affect their ability to complete academic tasks (Colvin, 2004; Sutherland, Lewis-Palmer, Stichter, & Morgan, 2008). Social skill deficits in students with E/BD include misreading social cues from peers and teachers, inappropriately responding to directives, and the inability to maintain appropriate peer and adult relationships (IDEA, 2004). Students with E/BD have difficulty taking turns, appropriately seeking teacher attention, maintaining appropriate peer interactions, and responding appropriately in social situations (Cook et al., 2008; Cullinan & Sabornie, 2004; Kauffman, 2001).

### **Mathematics**

It is well known that students with E/BD demonstrate deficits in mathematics achievement compared to their non-disabled peers (McLaughlin, Krezmien, & Zablocki, 2009; Wagner, Kutash, Duchnowski, Epstein, & Sumi, 2005). This is not surprising as academic underachievement is one of the identifying criteria by IDEA (2004) for E/BD. Due to the severe social and academic deficiencies that adversely affect the academic performance of these students, they demonstrate higher rates of academic failure that persist throughout their schooling compared to other students with disabilities (Webber & Plotts, 2008).

Researchers have shown that students with E/BD demonstrate significant mathematics deficiencies in elementary school, performing 1 to 2 grade levels behind

their peers (Templeton, Neel, & Blood, 2008; Trout, Nordness, Pierce, & Epstein, 2003). Greenbaum and colleagues (1986) found that 97% of students with E/BD, ages 12 to 14, were performing below grade level in mathematics. Consistent with these findings, Nelson et al. (2004) conducted a cross-sectional study of 155 K-12 students with E/BD and found they experienced academic achievement deficits in mathematics and that these deficits appear to broaden over time. Results indicated that approximately 56% of the children and 83% of the adolescents scored below the mean of their non-disabled peers on the Broad Math cluster of the WJ-III.

### **Computational Skills**

While most of the research on academic interventions in mathematics focus on basic math fact recall, basic computational skills, and problem solving (Garnett, 1987; Miller, Strawser, & Mercer, 1996; Montague, 2008; Montague & Brooks, 1993), there is little research that addresses more advanced computational skills such as subtraction with regrouping. Moreover, the current research seems to be primarily focused on behavioral issues (Hodge, Riccomini, Buford, & Herbst, 2006). Research to date has shown several strategies such as concrete-representational-abstract (CRA) teaching sequence and strategy instruction to be effective for teaching single digit subtraction (Maccini & Ruhl, 2000; Mercer & Miller, 1992). However, further research is needed to address academic interventions that are effective for teaching higher-level computation skills such as subtraction with regrouping.

Of particular concern, is the inability of students with E/BD to master basic math skills as it often results in school failure and may result in failure as an adult as these skills are fundamental to success in everyday situations (Gunter & Denny, 1998;

Meadows et al., 1994). Students who struggle with mathematics in the elementary grades often demonstrate difficulty with *arithmetic combinations* (problems involving addition and subtraction that can be solved using a number of strategies and are not always retrieved as basic fact answers) (Brownell & Carper, 1943; Gersten, Jordan, & Flojo, 2005). According to the National Council for Teachers of Mathematics (NCTM, 2009), computation skills are the basis for the five mathematical standards including number sense, geometry, algebra, measurement, and data analysis and probability. Students with mathematics difficulties who perform poorly on computation skills are more at risk of having difficulty in life skills, such as the workplace and money, and maintaining a social life (McCloskey, 2007). Furthermore, efficiency and fluency of basic mathematics facts is required for successful independent living (Patton, Cronin, Bassett, & Koppel, 1997), a foundation for applications related to time, money, and problem solving (Daly, Martens, Barnett, Witt, & Olson, 2007), and for comprehending underlying mathematical concepts (Gersten & Chard, 1999).

In a review of instructional interventions in mathematics for students with E/BD, Hodge and colleagues (2006) identified 13 studies that addressed basic computation skills in mathematics. Various interventions were used to increase student achievement in basic computation skills during independent work. Interventions included self-monitoring, self-management, peer tutoring, mnemonics, concrete-representation-abstract (CRA), cover-copy-compare (CCC), error analysis, direct instruction, and alternative algorithms.

### **Strategies for Subtraction for Students with Mild Disabilities**

Currently, there is a paucity of research on mathematics interventions aimed at improving academic performance of students with disabilities (Bryant et al., 2008;

Gersten et al., 2005). It is of particular importance that effective instructional strategies in mathematics are identified for students with E/BD (Mooney, Epstein, Reid, & Nelson, 2003; Pierce, Reid, & Epstein, 2004; Wehby, Lane, & Falk, 2003). Of the existing academic intervention research with students with E/BD, some common practices are prevalent. Effective instructional interventions include positive interactions, high rates of engagement, self-monitoring, peer-assisted learning, organizational supports, and direct instruction (Gunter, Denny, & Venn, 2000; Skinner, Bamberg, Smith, & Powell, 1993; Spencer, Scruggs, & Mastropieri, 2003). In addition, several meta-analyses have been conducted (Coddling, Burns, & Lukito, 2011; Hodge, et al., 2006; Kroesbergen & Van Luit, 2003) which compare various interventions across mathematics skills. Common findings among the research suggest that while and drill and practice (Ashcraft, 1987; Goldman, Mertz, & Pellegrinio, 1986, 1989) and modeling (Daly et al., 2007, Fuchs et al., 2008; Rivera & Bryant, 1992) tend to be most effective, a combination of interventions leads to better outcomes than a single treatment (Coddling et al.).

Similarities in academic performance between students with LD and E/BD have been identified. Both groups demonstrate below-average performance in content areas, deficits in basic academics, and low motivation (Dunlap et al. 1993; Fulk, Bringham, & Lohman, 1998; Ruhl & Berlinghoff, 1992). Within the domain of subtraction, a limited number of studies on effective instructional interventions exist; however, due to the similarities in academic performance, some researchers have suggested that instructional strategies found to be effective for students with LD may apply to students with E/BD (Bauer, Keefe, & Shea, 2001; Henley, Ramsey, & Algozzine, 1999).

Instructional strategies in subtraction consist of self-management, peer tutoring,

mnemonics, Concrete-Representational-Abstract (CRA), Cover, Copy, and Compare (CCC), and error analysis. Instructional strategies that involve thinking about the thought processes involved in solving problems are called meta-cognitive strategies. Examples of meta-cognitive strategies are self-monitoring, self-checking, and structured organizers. Problem solving strategies include George Polya's 4-step process (Van de Walle, 1998), FOPS (Jitendra & Star, 2008), Verbal Rehearsal (Montague, 2006), and Solve It! (Montague, 2007). According to Montague (1997), cognitive instruction is designed to provide scaffolding using systematic modeling, interactive dialogue, practice, and opportunities for students to share why they used a specific strategy. Examples of cognitive instruction include direct instruction, schema-based strategy instruction, subtraction strategies (i.e. alternative algorithms), and Cover, Copy, and Compare (CCC). Additional strategies include scaffolding, automaticity of basic facts through rules and relationships, visualizing strategies, and analysis of student work.

### **Self-Regulation/Self-Management**

Behavioral and educational researchers have devoted a great amount of research to identifying instructional techniques that enhance the ability of a student with a disability to learn and perform academic tasks in a consistent manner. Self-regulation is defined as the ability to regulate one's cognitive activities (Flavell, 1976) and includes strategies such as self-instruction, self-questioning, self-monitoring, self-evaluation, and self-reinforcement (Montague, 2008). These strategies are designed to help students gain access to cognitive processes that promote learning.

One of the strategies that has received considerable attention is referred to as self-monitoring, which is one of several strategies included in the self-regulation research

(Rosenbaum & Drabman, 1979). Self-monitoring approaches have been used with populations ranging from typically achieving students to students with ID (Ballard & Glynn, 1975; Whitman & Johnston, 1983). Researchers have shown that using self-monitoring can increase accuracy in many content areas because they provide students with instructional cues that allow for self-initiated responding and produce specific response strategies (Harris, 1986; Kneedler & Hallahan, 1981).

Dunlap and Dunlap (1989) evaluated the effectiveness of a self-monitoring package that was applied to two, three, and four digit subtraction with regrouping problems. Using a multiple baseline across students design, three students, ages 10 -13 with LD were provided with didactic explanations, verbal feedback, and a point incentive during the baseline conditions and individualized self-monitoring checklists during the self-monitoring package phase. The individualized self-monitoring checklists were developed for each student based on an error analysis conducted on each student's previous errors. During maintenance, the checklists were removed and the students continued to work under the previous conditions. Results indicated the use of the self-monitoring package produced immediate and substantial gains for each student. Furthermore, all of the students continued to perform better during the maintenance condition.

### **Peer Tutoring**

Peer assisted learning strategies are instructional strategies designed to improve math performance and behavior through peer tutoring, group rewards, and self-management procedures. The teacher is responsible for training the students on the process of peer tutoring and role of tutor or tutee. Students are assigned partners by the

teacher and follow highly structured tutoring procedures. Tutors present material previously covered by the teacher and provide feedback to the tutee. Students take turns as the tutor and tutee while the teacher circulates the room. Examples of peer assisted learning include Peer Assisted Learning Strategies (PALS), a highly structured format (Fuchs & Fuchs, 2001), and Reciprocal Peer Tutoring (RPT), designed to assist students working in small groups (Fantuzzo, King, & Heller, 1992).

Researchers have found peer tutoring to be effective in increasing academic performance in mathematics for children at different ability levels (Calhoon & Fuchs, 2003; Fuchs, Fuchs, Phillips, Hamlett, & Karns, 1995). Student learning has been found to be dependent on the type and quality of interactions during peer-mediated learning (Slavin, 1996; Webb, 1985). Research has shown that when students receive explicit instruction and practice using peer-mediated strategies performance is enhanced (Fuchs, Fuchs, Hamlett, Phillips, & Bentz, 1994).

Researchers have investigated cross-age tutoring (Bar-Eli & Raviv, 1982; Beirne-Smith, 1991) as well as within-class peer assisted learning (Bahr & Reith, 1991; Calhoon & Fuchs, 2003; Fuchs et al., 1995) in computation for students with LD. In a meta-analysis, Gersten and colleagues (2009) found the above mentioned studies to show consistently more modest effect sizes than other mathematics interventions analyzed. However, results also indicated stronger findings for cross-age tutoring interventions. In 1995, Harper, Mallette, Maheady, Bentley, and Moore (1995) evaluated the effects of using class-wide peer tutoring with three elementary students with mild disabilities to teach subtraction computational skills. Using an alternating treatment design, rate of correct responses and short and long term retention were assessed. Results indicated that



peer tutoring was effective in increasing students' accuracy, rate of responding, and retention with basic subtraction facts.

### **Mnemonics**

Mnemonics is one type of instructional strategy used to support the learning of specific skills. Examples of mathematics mnemonics include Slobs & Lamps and PEMDAS. Slobs & Lamps is a mathematics strategy designed to help students remember the regrouping process of borrowing and carrying. Slobs is used in subtraction where students follow a series of steps to solve a subtraction problem: 1) look at the top right number, 2) see if it is smaller or larger than the lower number, 3) cross off the number in the next column, 4) borrow one ten from that column by reducing the number by one and adding ten to the number in the right column, and 5) subtract the lower number from the top number. A mnemonic used for solving mathematical problems with multiple operations is PEMDAS, Please Excuse My Dear Aunt Sally. Students use the mnemonic as a way to solve a problem in a specific order by solving the items in parenthesis first, followed by items that contain exponents, multiplication, division, addition, and subtraction.

In a review of the literature of mnemonic interventions on academic outcomes, Wolgemuth and Cobb (2008) found that the use of mnemonic interventions was positively correlated with student achievement for students across disabilities and academic disciplines. Manalo, Bunnell, and Stillman (2000) investigated the effects of using process mnemonics for teaching computational skills to 8<sup>th</sup> grade students with mathematics disabilities (MD). In the first experiment, students were randomly assigned to either process mnemonics, demonstration-imitation, study skills, or no instruction. In

experiment 2, instructors were used to teach the skills opposed to the first author.

Students in the process mnemonics group made significant improvements in subtraction that were maintained through the follow up stage for both experiments.

### **Concrete-Representation-Abstract**

The Concrete-Representation-Abstract (CRA) teaching sequence supports the learning of a variety of mathematical skills for students with LD using a graduated instructional sequence (Ketterlin-Geller, Chard, & Fien, 2008). In the first stage, this teaching sequence promotes learning through *concrete* or hands-on instruction using manipulatives. As students progress, pictorial *representations* of the previously used manipulative objects are introduced. Students' learning is advanced through the final *abstract* stage of instruction which uses numbers and operational symbols to present the mathematical concept (Witzel, Riccomini, & Schneider, 2008).

Research has shown CRA to be effective for teaching basic mathematics facts, fractions, algebra, and place value to students with LD, E/BD, and ID (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Mercer & Miller, 1992; Peterson, Mercer, & O'Shea, 1988). In 1987, Peterson, Mercer, Tragash, and O'Shea evaluated the effectiveness of teaching initial place value skills using two different teaching methods to twenty-four students, ages 8 – 13, with learning disabilities (LD). Students in the control group received instruction on an abstract level while students in the treatment group received instruction in a concrete, semiconcrete (representational), abstract teaching sequence. Students in the intervention group received three lessons using concrete manipulative devices, three lessons using semiconcrete or pictorial representations, and three lessons that included abstract level instruction, while the control group received nine lessons all

at the abstract level of instruction. Using a 2x3 mixed design with one between (treatment) and one within (performance over time) group factor to examine skill acquisition, maintenance, retention and generalization, results indicated that students using the concrete-semiconcrete-abstract teaching sequence acquired initial place value skills better than their peers in the control group. In addition, they found the CRA teaching sequence had positive effects on the students' ability to maintain this skill over time.

In 2009, Flores studied the effects of CRA when it is used to teach subtraction with regrouping to six third-grade students who were failing mathematics. Of the six participants, four were identified as having LD. Using a multiple-probe across groups design, students received instruction 3 days a week for 30 minutes each day. The probes used to measure student progress consisted of 30 two-digit minus two-digit subtraction with regrouping problems. Results indicated that CRA instruction produced academic gains in subtraction with regrouping across all students. Five of the six students maintained performance at or above the criterion level during maintenance.

### **Cover, Copy, Compare**

Cover, Copy, and Compare (CCC) is a self-managed strategy that has been shown to be effective for mathematics (Skinner, Shapiro, Turco, Cole, & Brown, 1992; Skinner, Turco, Beatty, & Rasavage, 1989). It is a five-step procedure that provides students with increased opportunities to respond to mathematics material and self-evaluate their progress. It requires student's to 1) review a problem and its solution on the left side of the paper, 2) cover the problem and solution with an index card, 3) solve the problem on the right side of the paper, 4) uncover the problem and solution on the left side, and 5)

evaluate their response and make corrections to the response if it is incorrect by copying the correct problem and response a number of times. The use of CCC has been effective for increasing student engagement and providing immediate corrective feedback as it provides numerous opportunities that students are presented with academic stimuli, to respond to those stimuli (Siegler & Shrager, 1984; Skinner et al., 1989).

### **Error Analysis**

Error analysis has been used in several studies as an assessment strategy to identify specific errors in conjunction with other intervention strategies. Researchers have found that evaluating students' mathematical errors can help improve student outcomes (Riccomini); provide valuable information for assessment, instruction, and curriculum development (Mercer & Mercer); provide modifications in instructional methodology; and provide information to develop a specific plan for teaching and learning (Ashlock, 2006; Bley & Thorton, 1995; Fernandez & Garcia, 2008; Van Lehn, 1982). Furthermore, analysis of student performance increases understanding and the prediction of math performance (Balacheff, 1990; Romberg & Carpenter, 1986) and provides teachers with the precise area of difficulty the student is having so they can focus on teaching the specific skill (Parmar & Cawley, 1997; Riccomini).

Higher level computation skills such as subtraction with regrouping are essential for learning more complex mathematics and students are dependent upon these skills to be successful in school and the community. Identifying errors in such computation skills may thus be a valuable source of information about student's procedural and computational knowledge and provide relevant information for instructional decisions (Resnick, 1984). Studies on calculation show most errors demonstrated by students are

systematic (Cox, 1974; Clements, 1982; Graeber, 1992) the result of mistaken or missing knowledge (Van Lehn, 1982). Furthermore, errors tend to increase in frequency as problems become more complex and involve multi-digit problems in computation (Babbitt, 1990; Calhoun et al., 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2007). Errors in computation are typically classified as fact errors, operation errors, procedural errors, wrong operation, defective algorithm, incomplete algorithm, grouping error, inappropriate inversion, identity error, zero error, random response, and careless error (Ashlock, 1990; Englehardt, 1977).

Although studies have shown that students with mathematics difficulties demonstrate difficulty with single- and multi-digit mathematical problems (Geary, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan, 2003); little research using error analysis to determine the type of errors students with mathematics difficulties make exists (Geary et al., 2007; Raghubar et al., 2009). Furthermore, only a few studies have focused on errors in subtraction among students with disabilities (Skrtic, Kvam, & Beals, 1983).

Among the studies, inversion errors have been found to be the most common type of all systematic errors. Inversion errors occur when the minuend is subtracted from the subtrahend in subtraction problems requiring regrouping, or borrowing (Buswell & John, 1926; Cox, 1975; Smith, 1968). In 1978, Blankenship investigated the acquisition, generalization, and maintenance of skills among 9 students with LD who made systematic inversion errors in subtraction when borrowing. Results indicated that using a demonstration plus feedback technique to teach the decomposition method of subtraction reduced students' inversion errors in subtraction. Overall systematic inversion errors decreased from 86.7% to 6.7% and accuracy increased from 0% to 86.2%. In 1982,

Frank, Logan, and Martin investigated the subtraction errors of 94 elementary students with LD using subtraction tests constructed to maximize the subjects' opportunity to demonstrate proficiency in subtraction skills. Results indicated that in subtraction problems requiring regrouping, one of the most common errors was inversion. Sugai and Smith (1986) conducted an error analysis on the types of error made by three girls and four boys with LD in the 3<sup>rd</sup>, 4<sup>th</sup>, and 5<sup>th</sup> grades using the equal additions algorithm to teach subtraction. Before training, results showed that 6 of the 7 students made the same type of error (reversing the order of subtraction) when computing subtraction with regrouping. After training was initiated, reversal errors decreased significantly. Cawley and colleagues (1996) compared the computation performance of students with LD to normally achieving (NA) students. Students ranged in age from 7 to 14 and the computation measures included addition, subtraction, multiplication, and division. Overall, students with LD made more algorithmic errors than the NA students.

### **Direct Instruction/Strategy Instruction**

Direct instruction is the explicit teaching of rules and strategies combined with immediate, corrective feedback through guided practice (Gersten, Carnine, & White, 1984). The direct approach is teacher led, wherein the teacher controls the instructional goals and pace, chooses the appropriate materials, and provides immediate corrective feedback to the student. In a meta-analysis of mathematics interventions, Kroesbergen and van Luit (2003), found direct instruction approaches to be more effective for basic skills acquisition for students with disabilities. Other meta-analyses and studies have found similar effects with the use of direct instruction (Carnine, 1997; Swanson, Carson, & Lee, 1996; Swanson & Hoyskn, 1998). Among students with E/BD, there is a limited

amount of research defining effective instruction; however, direct or explicit instruction for students with E/BD has been identified as one of the most beneficial forms of instruction for students and teachers (Gunter, Coutinho, & Cade, 2002; Pierce et al., 2004). Research has repeatedly demonstrated that students make quicker gains and learn more effectively when instruction is systematic, explicit, unambiguous, well designed, and monitored (Gunter et al., 2002; Lane et al., 2006; Pierce et al., 2004; Woodward, 2004). When combined with effective strategy instruction, direct instruction has been established as an effective evidence-based practice for struggling learners in diverse educational settings (Jolivette et al., 2008).

The direct instruction model is based on six components: 1) gaining students' attention, 2) reviewing past learning, 3) presenting new information through demonstration or modeling, 4) assisting students through guided practice, 5) evaluating student performance, and 6) reviewing the lesson. Researchers agree that a direct instruction approach that is clear, presents materials in a structured and systematic manner, provides daily review of previously learned concepts, provides ample opportunities for students to respond, and provides repeated opportunities for practice is best suited for students with E/BD (Gunter, Hummel, & Venn, 1998; Martella, Nelson, & Marchand-Martella, 2003; Scott & Shearer- Lingo, 2002; Sutherland & Wehby, 2001b; Yell, 2009).

Only a few studies using direct instruction have focused on the acquisition of mathematics skills. Researchers have investigated the use of an explicit instructional approach to teach multiplication preskills (Carnine, 1980), basic facts (Carnine & Stein, 1981), and word problems (Darch, Carnine, & Gersten, 1984). Kameenui, Carnine,

Darch, and Stein (1986) used a direct instruction approach conceptualized from the Project Follow Through direct instruction model that Gersten and Carnine (1984) describe to teach subtraction. Twenty-three first graders identified as low performers were randomly assigned to either a Project Follow Through direct instruction group or a comparison group. The strategy used for the Project Follow Through direct instruction group was a semi-concrete, line drawing strategy, using clearly articulated teaching sequences that contained explicit, step-by-step teacher modeling and assessment of student mastery at each step of development. The comparison group was taught the concept of subtraction using pictures and teacher discussion. Results indicated that the students who received the explicit strategy benefited more than the students in the comparison group.

### **Alternate Algorithms**

Alternative algorithms are strategies designed to improve academic performance. Several alternative algorithms/methods have been identified to help students compute subtraction problems more efficiently and effectively: the Austrian algorithm, counting-up algorithm, low-stress algorithm, the additive method, the inverse relation method, indirect addition, and the equal additions algorithm. However, in the United States, three different algorithms were commonly used until the 1940s: the decomposition, the Austrian method, and the equal additions.

**Decomposition.** While the decomposition method was fast becoming the predominant method of subtraction in the U.S., when Brownell modified the decomposition algorithm using a crutch technique in 1939, use of other algorithms became almost extinct in mathematics textbooks. According to Wilson, who conducted a



nationwide survey, by 1934 the decomposition algorithm was used two and a half times as often as the equal additions algorithm.

The decomposition algorithm of subtraction is commonly known as the *borrowing* method, which requires a student to subtract the subtrahend from the minuend, borrowing from the tens and adding to the ones as needed to complete the problem. See Figure 1 for an example. The decomposition algorithm was not the only method of subtraction used to teach subtraction with regrouping in the United States. While the take-away, or decomposition method, has been advocated for students with disabilities since the 1920's, it is inconsistent with the definitions of subtraction among special educators. Dating back to 1849, Burnham defined subtraction as the process of finding the difference between two numbers, which is consistent with teaching the *big ideas* in special education (Carnine, Jones, & Dixon, 1994).

$$\begin{array}{r}
 5 \ 11 \\
 \cancel{61} \\
 - \ \underline{35} \\
 26
 \end{array}$$

*Figure 1.* Decomposition Method.

**Austrian method.** The Austrian method of subtraction is also known as the additions method, as it makes a more precise connection between addition and subtraction in that it gets one to think of what needs to be added to the minuend to get the difference (Ross & Pratt-Cotter, 2000). It more directly relates addition with subtraction than other algorithms. In this method, the solution is found by directly relating the answer to addition. Students start with the smaller number and decide what number, when added to the smaller number, will give you the larger number. For example, when given the problem  $13 - 7$ , the student should think, "7 and what gives you 13?". Finding the missing addend in this case helps connect the concepts of addition to subtraction.

Illustrated in Figure 2 is one of the earliest uses of the Austrian algorithm in the United States, which was found in a 1902 textbook from Wentworth & Smith. The top number was underlined in the original text in order to have the form of inverted addition. While the explanation of the process is the Austrian method, the method of underlining the top number is not typical of the algorithm. In most textbook examples, the line is drawn under the second number.

Similar to the Austrian method, indirect addition has been found to be an efficient strategy for subtraction with small differences in recent research (Threllfall, 2002; Torbeyns, DeSmedt, Ghesquière, & Verschaffel, 2009; Treffers & Buys, 2001; Wittmann & Muller, 1990). Using indirect addition, the solution to the problem is found by calculating the difference of two numbers. Students start with the smaller number and add, or count, up to the larger number. While indirect addition has been shown to be effective, it is rarely taught or used among traditionally schooled children (Heirdsfield & Cooper, 2004; Selter, 2001; Torbeyns et al.). For example, in 2010, De Smedt, Torbeyns, Stassens, Ghesquire, and Verschaffel investigated the development of indirect addition as an alternative for solving multi-digit subtraction for 35 third-graders. Students were assigned to either an explicit or implicit learning environment that aimed to encourage the development of indirect addition. Results revealed that students in both groups rarely used the indirect addition method throughout the study. However, when indirect addition was used, it was executed very efficiently.

Furthermore, Selter (2001) conducted a study and found students used indirect addition on three-digit subtractions only 1% of the time. In a study where participants were assigned to either choice or no-choice groups, Torbeyns et al. (2009) found that in

From 94,275 take 67,492

94275 Thus: 2 and 3 are 5; 9 and 8 are 17, carry 1 to

67492 4 as in addition, making it 5, 5 and 7 are 12;

26783 carry 1 to 7; 7 and 2 are 9.

Figure 2. Austrian Method.

the no-choice condition, participants' who were instructed to apply the indirect addition strategy demonstrated significantly better performance in terms of speed and accuracy.

**Equal additions algorithm.** Today, some texts introduce the equal additions algorithm, but the decomposition is still the predominant algorithm. The equal additions algorithm can be traced back to the 15<sup>th</sup> and 16<sup>th</sup> centuries (Johnson, 1938) and is commonly referred to as *the borrow and repay* method. In this method of subtraction a power of ten is borrowed to add to the necessary place in the minuend and repaid by adding to the digit in the next place of the subtrahend (see Figure 3). For example, when the given the problem  $95 - 28$ , 8 cannot be subtracted from 5, therefore 10 units must be added to 5 in the top number to form 15 and 10 units added to 20 in the bottom number which adds up to 30. The answer (67) remains the same because we have added 10 to the both the top number and bottom number. According to Ross and Pratt-Cotter (2000), this method is more representative of the term *borrow* than the decomposition algorithm, as a power of ten is *borrowed* from the minuend and then added to the subtrahend.

$$\begin{array}{r}
 95 \\
 - 28 \\
 \hline
 67
 \end{array}$$

The image shows a subtraction problem using the equal additions algorithm. The minuend is 95 and the subtrahend is 28. A red '1' is written above the 5, and a red '3' is written above the 2. A red diagonal line is drawn through the 2 and the 8. The result is 67.

*Figure 3. Equal Additions Algorithm*

An extensive search of the literature has produced a limited amount of empirical research studies on subtraction with regrouping for students with BD and LD. From the limited amount of research found, the equal additions algorithm has been shown to be as effective, if not superior, to the decomposition algorithm in several studies and was the primary method of subtraction taught in the United States until the 1940's. Results from several studies have found that students in grades 2-5 with and without disabilities made significant gains using the equal additions algorithm over the decomposition algorithm. In 1914, Ballard investigated the effects of the three algorithms methods and found that the equal additions method was superior to the decomposition method.

In 1918, McClelland studied the methods of subtraction used by 143 children, ages 12 -13 and found the equal additions algorithm to be superior in speed, accuracy, and adaptability to new conditions. Similarly, Winch (1919) found the equal additions algorithm showed a decided advantage in accuracy and speed over the decomposition algorithm with younger children and children with mathematics weaknesses.

In 1947, Brownell showed the equal additions algorithm to be a more significantly favorable method of subtraction over the decomposition algorithm for 3<sup>rd</sup> grade students being introduced to borrowing. Using four experimental groups, two for decomposition and two for equal additions algorithm, subjects were either taught the procedure in a rational way or a mechanical way for three weeks. Intelligence tests were administered and students were given a computational test of simple addition and subtraction. Students were matched to groups based on IQ and test scores. Computational tests on borrowing, a six week retention test, and interviews at the end of intervention and retention were held. Results showed both equal additions algorithm groups significantly outperformed the decomposition mechanical method. There was no significant difference between the decomposition rational group and the equal additions rational group.

Results of a research to practice study conducted by Hoppe (1975) indicated positive outcomes for 2<sup>nd</sup> grade students using the equal additions algorithm over the decomposition algorithm. Hoppe conducted the study with her 2<sup>nd</sup> grade students teaching subtraction requiring renaming in the minuend using the decomposition and equal additions algorithm. Children were randomly assigned to groups based on IQ scores from the Otis-Lennon Mental Ability Test and gender. Both groups received instruction twenty minutes per day using concrete and semi-concrete materials existing of

sets of sticks, a large counting frame, dimes and pennies, place value charts, and individual abacuses for fourteen days. Student achievement was tested using subtraction with renaming and regrouping problems every day from the third to the seventh lesson and three weeks later to determine retention. The equal additions algorithm was mastered and retained by students at a rate of 77% compared to 46% for the students using the decomposition algorithm from the fourth lesson on.

Most recently, Sugai and Smith (1986) conducted a study with three girls and four boys in the 3<sup>rd</sup>, 4<sup>th</sup>, and 5<sup>th</sup> grades using the equal additions algorithm to teach subtraction with a specific modeling technique. All of the students were identified as LD and were receiving special education instruction in a resource classroom. Students received 15-minute instruction sessions daily on four types of subtraction with regrouping problems until mastery for three consecutive days at 90% was obtained for each student. The teacher taught subtraction with regrouping using the equal additions algorithm and demonstrated the algorithm with one problem that was left on the board to serve as a model. Students then worked four problems on the board and if they got them correct they began working on a worksheet. Students who made errors were corrected with oral prompting, referral back to the model, and the use of fingers. Then they were given another problem and asked to verbalize as they completed it. Students were required to work another four problems on the board until they reached 100% accuracy. The same correction procedure was used for the worksheets consisting of 10 trained problems and 2 untrained and review problems each. The probe problems were mixed with the training problems on the worksheets to determine if scores improved because of training. After training of each type of problem, a 20-problem probe was given to each student with

problems of all types to determine what students could correctly answer. After all problem types were trained and mastered, students worked on baseline worksheets that contained all four problem types for three consecutive days. Results for all students showed an increase in the percentage correct of subtraction problems requiring regrouping using the equal additions algorithm. To achieve 90% accuracy for any problem type, the minimum number of days required to teach the equal additions algorithm was three and the maximum was 13.

### **Conclusion**

There is a limited amount of research on academic interventions for students with E/BD. In a review of the academic intervention research, Nelson et al. (2004) found only 55 studies have been conducted in the past 30 years. Of the limited research, most attention has been placed on student-directed interventions as opposed to teacher directed (Hodge et al., 2006). As students with E/BD exhibit academic deficits early on in their schooling, more research needs to be conducted to determine the effectiveness of current instructional programs and interventions (Nelson et al.). While researchers and educators are aware that many students experience difficulties in mathematics, instruction for these students has not received the attention given to reading instruction (Gersten et al., 2009).

Furthermore, most research in this area is focused on basic mathematics skills, and has failed to investigate effective interventions in problem-solving and higher order mathematics skills for students with E/BD. The limited research in strategy instruction to improve basic computation skills needs to be extended to problem-solving and higher order mathematics skills for students with E/BD. Further investigation into whether effective mathematics interventions for students with other mild disabilities are effective



for students with E/BD should be considered as students with mild disabilities share many similar academic deficits.

When considering researching effective mathematics strategies for students with E/BD, an approach may be considered that incorporates instructional techniques and strategies. Mathematics educators and school psychologists have been emphasizing the use of multiple strategies to improve mathematics education for many years (Brownell, 1947; Jolivette et al., 2008). Recently, curriculum reform documents, new curricula, textbooks, software, and other instructional materials have focused on promoting a variety of strategies for students, especially those with mathematics difficulties (Baroody, 2003; Verschaffel, Greer, & De Corte, 2007; Verschaffel, Greer, & Torbeyns, 2006).

Teaching students with E/BD the equal additions method using a direct instruction technique may be effective. The direct instruction approach is consistent with the tenants of special education, has a proven track record of success among other content areas, and has been shown to be effective in teaching a variety of basic mathematics skills (Gersten, Fuchs, Williams, & Baker, 2001; Swanson & Hoskyn, 1998). In a meta-analysis of mathematics instruction for students with LD, Gersten and colleagues (2009) found that studies that incorporated direct, or explicit instructional strategies resulted in significant effects and produced some of the largest effect sizes. Specifically, when studies focused on teaching a single mathematical proficiency or to solve a wide variety of problem types that included multi-digits, the results indicated large effects.

Error analysis has been identified as one of the main principles for remedial education for students with LD (Salvia & Hughes, 1990; Salvia & Ysseldyke, 2004). The identification and evaluation of students' mathematical errors can help improve student

outcomes (Riccomini, 2005); provide valuable information for assessment, instruction, and curriculum development (Mercer & Mercer, 1998); provide modifications in instructional methodology; and provide information to develop a specific plan for teaching and learning (Ashlock, 2006; Bley & Thorton, 1995; Fernandez & Garcia, 2008; Van Lehn, 1982). However, little research has been conducted to support the use of error analysis in the identification of specific mathematical errors of students with disabilities.

With regards to subtraction with regrouping, very little emphasis has been placed on defining effective instruction strategies to assist students who struggle with this concept. Of the three algorithms used to teach subtraction with regrouping, the decomposition method is clearly the most commonly used strategy in the U.S. However, many students continue to struggle with this concept. Alternate algorithms, such as the equal additions algorithm, may prove to be an effective alternative for students with disabilities. In fact, the equal additions algorithm actually has been shown to be as effective as, if not more so, in improving student efficiency and effectiveness.

Of all students with disabilities, students with E/BD may present the most unique and challenging characteristics when it comes to improving academic outcomes. Due to the nature of the disability, these students often present behaviors that interfere with their academic success and they are often left to complete paper and pencil tasks in independent seat work. This seems to be especially true for students in residential settings, as their emotional and behavioral deficits are often a priority. As a result, much of the existing literature is aimed at providing self-regulation strategies that address behavioral and academic concerns. While this may be due to the fact that, more often than not, these students are educated in environments other than the general education

setting, academic strategies that improve student performance need to be further investigated. Additional research for students with E/BD in residential settings using a systematic approach of direct instruction with equal additions and error analysis may be effective and warrants further analysis.

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## CHAPTER 2

THE EFFECTS OF USING DIRECT INSTRUCTION AND THE EQUAL ADDITIONS  
ALGORITHM TO PROMOTE SUBTRACTION WITH REGROUPING SKILLS OF  
STUDENTS WITH EMOTIONAL AND BEHAVIORAL DISORDERS WITH  
MATHEMATICS DIFFICULTIES

Students with E/BD display severe social and academic deficits that can adversely affect their academic performance in mathematics and result in higher rates of failure throughout their schooling compared to other students with disabilities (Webber & Plotts, 2008; U.S. Department of Education, 2005). They typically perform 1 to 2 grade levels below their peers (Templeton, Neel, & Blood, 2008; Trout, Nordness, Pierce, & Epstein, 2003) demonstrating significant deficiencies and achieving well below national averages in mathematics (Anderson, Kutash, & Dushnowski, 2001). Of particular concern, is the inability of students with E/BD to master basic math skills as this often results in school failure and may result in failure as an adult since these skills are fundamental to success in everyday situations (Gunter & Denny, 1998; Meadows, Neel, Scott, & Parker, 1994). Specifically, computation skills, especially subtraction with regrouping, are problematic for students with E/BD. Computational skills are the basis of the five mathematical standards and used throughout schooling and more advanced mathematics courses. A systematic approach using direct instruction and an equal additions algorithm strategy combined with error analysis may be an effective strategy for students with E/BD in a residential setting.



### **Instructional Barriers**

Students with E/BD often have academic and behavioral deficits. Academically, they have difficulty attending to tasks, completing tasks in a timely manner, staying on-task, and completing tasks independently (Cancio, West, & Young, 2004; Lane, Carter, Pierson, & Glaeser, 2006). Behaviorally, they exhibit classroom behaviors that may interfere with academic tasks. For example, they may be anxious or nervous (Ashcroft, Krause, & Hopko, 2007; Liaupsin, Jolivette, & Scott, 2007; Wright, 1996), non-compliant (Osher et al., 2007), and/or verbally or physically aggressive to divert attention from their academic difficulties and/or escape task demands (Fuchs & Fuchs, & National Center on Student Progress, 2001; Garnett, 1987; Lane, 2007; Osher et al.). Furthermore, these students often display inappropriate or poor social skills that can impact their ability to complete academic tasks (Colvin, 2004; Sutherland, Lewis-Palmer, Stichter, & Morgan, 2008). Social skills deficits include misreading social cues from others, inappropriately responding to directives, and the inability to maintain appropriate peer and adult relationships (IDEA, 2004).

All of these inappropriate behaviors can result in lower teacher expectations, removal from desired task, or removal from the educational setting (Colvin, 2004; Nelson, 1997; Van Acker, 2002). Removal from a task or an educational setting results in reduced learning opportunities. Students with E/BD are already at a greater risk of being served in more exclusionary and restrictive settings compared to their peers (Gagnon & Leone, 2005; Furney, Hasazi, Clark-Keefe, & Hartnett, 2003, U.S. Department of Education, 2005; Whorton, Siders, Fowler, & Naylor, 2000). While more restrictive settings such as self-contained classrooms, self-contained schools, and residential settings

may be beneficial in that they provide the necessary behavioral supports needed to meet the individual behavioral needs of students, little research is available about the educational support students receive in these settings. Of the research available, students in residential treatment settings often receive less instruction compared to students in other educational settings, which is concerning as these students are often returned back to public settings (Grizenko, Sayegh, & Papineau, 1994; Katsiyannis, 1993).

Furthermore, the instruction in residential settings is often centered around individual, independent seat work with an emphasis on paper and pencil tasks (Wheby, Symons, & Shores, 1995). Lane, Wheby, Little, and Cooley (2005) found that students received significantly more disciplinary contacts and negatively worded items in their folders in a self-contained school than students in self-contained classrooms, suggesting that students in a more restrictive environment may be subject to more disciplinary time and removal from educational opportunities. Statistics show that more than 75,000 students with E/BD are currently educated in residential settings (U.S. Department of Education, 2002) and the minimal amount of research available on the educational supports and opportunities students with E/BD may or may not be receiving is of great concern.

With an estimated 48% of all school-aged students having difficulties in mathematics (Siegler, 2007), it is not surprising that that students with E/BD also struggle in mathematics. Students with E/BD demonstrate deficits in mathematics, beginning in the elementary school, performing 1 to 2 grade levels below their peers (Templeton et al., 2008; Trout et al., 2003). These deficits persist throughout their schooling when compared to other students with disabilities (U.S. Department of Education, 2005;

Webber & Plotts, 2008). Failure to master basic mathematics skills is of great concern as it may result in reduced success in more complex mathematics and everyday situations (Gunter & Denny, 1998; Meadows et al., 1994). An understanding of basic mathematics skills is necessary for successful independent living (Patton, Cronin, Bassett, & Koppell, 1997), a foundation for applications related to time, money, and problem solving (Daly, Martens, Barnett, Witt, & Oslon, 2007), and for comprehending underlying mathematical concepts (Gersten & Chard, 1999).

### **Instructional Strategies**

There is a paucity of research on mathematics interventions aimed at improving academic performance of students with disabilities, especially students with E/BD. While most of the current research on academic interventions in mathematics focuses on basic math fact recall, basic computational skills, and problem solving (Miller, Strawser, & Mercer, 1996; Montague & Brooks, 1993; Montague, 2008), there is little research that addresses effective instructional strategies in complex computational skills such as subtraction with regrouping. Moreover, much of the current research tends to focus on behavioral issues (Hodge, Riccomini, Buford, & Herbst, 2006).

Because of the limited number of studies in subtraction, researchers have suggested that students with E/BD may benefit from instructional strategies found to be effective for students with LD (Bauer, Keefe, & Shea, 2001; Henley, Ramsey, & Algozzine, 1999). Both students with E/BD and LD demonstrate below-average performance in content areas, deficits in basic academics, and low motivation (Dunlap et al., 1993; Fulk, Bringham, & Lohman, 1998; Ruhl & Berlinghoff, 1992). Of the existing mathematics intervention research with students with E/BD, effective instructional

practices include mnemonics, self-monitoring, peer-assisted learning, and direct instruction (Hodge, Riccomini, Buford, & Herbst, 2006). Among effective instructional strategies for mathematics, direct instruction has been found to be more effective for basic skills acquisition for students with disabilities (Carnine, 1997; Kroesbergen & Van Luit, 2003; Swanson, Carson, & Lee, 1996; Swanson & Hoyskn, 1999).

### **Direct Instruction**

The direct instruction model is based on six components: 1) gaining students' attention, 2) reviewing past learning, 3) presenting new information through demonstration or modeling, 4) assisting students through guided practice, 5) evaluating student performance, and 6) reviewing the lesson. Research has repeatedly demonstrated that students make quicker gains and learn more effectively when instruction is systematic, explicit, unambiguous, well designed, and monitored (Gunter, Coutinho, & Cade, 2002; Lane et al., 2006; Pierce, Reid, & Epstein, 2004; Woodward, 2004).

Direct or explicit instruction for students with E/BD has been identified as one of the most beneficial forms of instruction for students and teachers (Gunter et al., 2002; Pierce et al., 2004). Researchers agree that a direct instruction approach that is clear, presents materials in a structured and systematic manner, provides daily review of previously learned concepts, provides ample opportunities for students to respond, and provides repeated opportunities for practice is best suited for students with E/BD (Gunter, Hummel, & Venn, 1998; Martella, Nelson, & Marchand-Martella, 2003; Scott & Shearer-Lingo, 2002; Sutherland & Wehby, 2001; Yell, 2009). Only a few studies using direct instruction have focused on the acquisition of mathematics skills. Researchers have investigated the use of an explicit instructional approach to teach multiplication preskills

(Carnine, 1980), basic facts (Carnine & Stein, 1981), word problems (Darch, Carnine, & Gersten, 1984), and subtraction (Kameenui, Carnine, Darch, & Stein, 1986).

### **Error Analysis**

One of the main principles for remedial education for students with LD is the identification of student errors through error analysis (Salvia & Hughes, 1990; Salvia & Ysseldyke, 2004). The identification of errors for remedial education for students with E/BD is just as critical. Evaluating students' mathematical errors can help improve student outcomes (Riccomini, 2005); provide valuable information for assessment, instruction, and curriculum development (Mercer & Mercer, 1998); provide modifications in instructional methodology; and provide information to develop a specific plan for teaching and learning (Ashlock, 2006; Bley & Thorton, 1995; Fernandez & Garcia, 2008; Van Lehn, 1982). With so little time available to focus on the reteaching of skills, the identification and analysis of students' mathematical errors allows teachers to focus on and correct only the cause of the specific difficulty instead of focusing on re-teaching the entire skill (Parmar & Cawley, 1997; Riccomini, 2005).

Currently, very little research exists on the types of errors students make in subtraction. While studies have shown that students with mathematics difficulties demonstrate difficulty with single- and multi-digit mathematical problems (Geary, Hamson, & Hoard, 2000; Jordan, Hanich, & Kaplan, 2003), little research using error analysis to determine the type of errors students with mathematics difficulties make exists (Geary, Hoard, Nugent, & Byrd-Cravene, 2007; Raghubar et al., 2009). Furthermore, only a few studies have focused on errors in subtraction among students with disabilities (Skrtic, Kvam, & Beals, 1983). Common among these studies however, inversion errors

have been found to be prevalent among systematic errors (Buswell & John, 1926; Cox, 1975; Sherrill, 1979; Smith, 1968; Sugai & Smith, 1986). This occurs when the minuend is subtracted from the subtrahend in subtraction problems requiring regrouping, or borrowing.

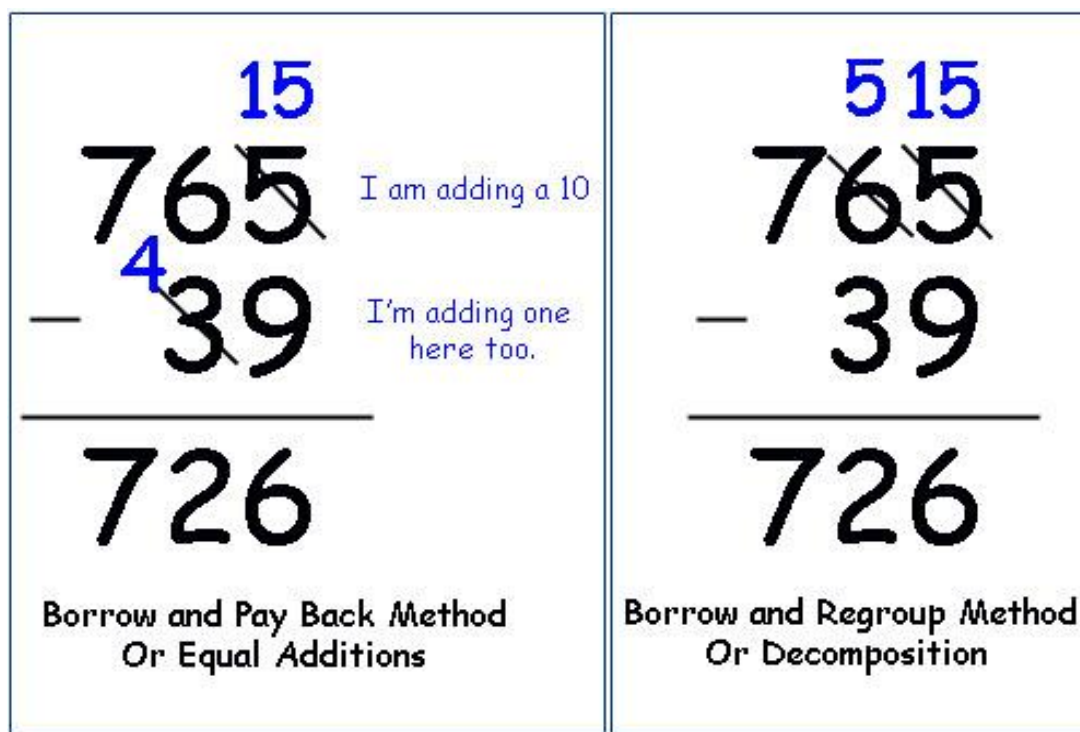
### **Algorithms and Equal Additions**

Throughout the years, several procedures have been used to teach subtraction with regrouping. The most common procedures that have been found in textbooks in the United States since the colonial times are the decomposition algorithm, the Austrian algorithm, and the equal additions algorithm. The predominant algorithm taught in the United States for the past 60 years has been the decomposition algorithm, whereas the Austrian and equal additions algorithms were rarely found in textbooks. The decomposition algorithm of subtraction is commonly known as the *borrowing* method, which requires a student to subtract the subtrahend from the minuend, borrowing from the tens and adding to the ones as needed to complete the problem. It is not clear why the decomposition algorithm became the dominant procedure, and research in this area is sparse. However, some research has shown the equal additions algorithm to be as effective, if not superior, to the decomposition algorithm.

Results from the 2011 United States Nations 4<sup>th</sup> Grade Mathematics Report Card indicate that 45% of students with disabilities are performing at the below *Basic* level. This indicates a 4% increase from 2009 and the highest it has been since 2003, when it was 49%. Fourth-grade students performing at the *Basic* level should show some evidence of understanding the mathematical concepts and procedures in the five NAEP content areas. Therefore, it can be assumed that students failing to perform at the basic

level are not acquiring the skills needed to learn more complex mathematical skills. It can also be assumed that, from this data, many students may be having difficulty learning the decomposition algorithm and could benefit from an alternative strategy such as the equal additions algorithm.

Given this valuable information, the knowledge that there are multiple ways to teach mathematical skills, and the fact that individuals learn through a variety of modalities, we should begin to investigate alternative instructional strategies to promote the successful transfer of knowledge for students with disabilities. One such strategy is the equal additions algorithm. The equal additions algorithm has been proven to be effective when teaching subtraction with regrouping in the past due to the fact that it does not involve additional concepts such as place value. This may be easier to comprehend for students with mathematics difficulties who experience frustration with using multiple concepts in mathematic computation given that the concept of regrouping is often difficult for many children with disabilities. The equal additions algorithm is commonly referred to as *the borrow and repay* method (See Figure 4). In this method of subtraction a power of ten is borrowed to add to the necessary place in the minuend and repaid by adding to the digit in the next place of the subtrahend. According to Ross and Pratt-Cotter (2000), this method is more representative of the term *borrow* than the decomposition algorithm, as a power of ten is *borrowed* from the minuend and then added to the subtrahend.



*Figure 4.* Equal Additions Algorithm vs. Decomposition Method

An extensive search of the literature has produced a limited amount of empirical research studies on subtraction with regrouping for students with LD and E/BD. From the limited amount of research found, the equal additions algorithm has been shown to be as effective as, if not superior, to the decomposition algorithm in several studies and was the primary method of subtraction taught in the United States until the 1940's (Ballard, 1914; McClelland, 1918; Winch, 1919). Results from several studies have found that students in grades 2 – 5 with and without disabilities made significant gains using the equal additions algorithm over the decomposition algorithm (Ballard; Brownell, 1947; Hoppe, 1975; McClelland; Sugai & Smith, 1986; Winch). More recently, Hoppe found 2<sup>nd</sup> grade students made greater gains using the equal additions algorithm over the decomposition



algorithm. In 1986, Sugai and Smith conducted a study with 7 3<sup>rd</sup>, 4<sup>th</sup>, and 5<sup>th</sup> grade students with LD using the equal additions algorithm with a specific modeling technique. They found all students showed an increase in the percentage correct of subtraction problems requiring regrouping using the equal additions algorithm.

Research suggests that using a combination of interventions leads to better outcomes as opposed to a single treatment for students struggling in computational fluency (Coddling, Burns, & Lutkito, 2011). It is further evidenced that students with disabilities can benefit from the use of multiple strategies (Woodward & Montague, 2002). Using direct instruction to teach the alternative algorithm of equal additions to students with E/BD in a residential setting may be an effective strategy for teaching subtraction with regrouping and needs further investigation.

### **Purpose**

The purpose of this study was to analyze the effect of direct instruction and the equal additions algorithm on the subtraction with regrouping performance of fourth-grade students with E/BD and mathematics difficulties in a residential setting. The equal additions algorithm was taught to these students using a direct instruction technique. An error analysis was conducted to investigate and identify individual student errors on all items. In addition, social validity was examined through Likert-scale questionnaires before and after the study. The students answered questions regarding the need for, ease of, and preference for the equal additions algorithm. The following research questions were investigated:

1. Do students with emotional and behavioral disorders experiencing difficulties in mathematics who receive direct instruction using the equal

- additions algorithm increase their ability to solve subtraction with regrouping problems when regrouping is necessary for the one's place?
2. Do students with emotional and behavioral disorders experiencing difficulties in mathematics who receive direct instruction using the equal additions algorithm increase their ability to solve subtraction with regrouping problems when regrouping is necessary for the ten's place?
  3. What errors are commonly identified among students with emotional and behavioral disorders experiencing difficulties in mathematics when performing subtraction with regrouping?
  4. Do students with emotional and behavioral disorders with mathematics difficulties report high, medium, or low levels of satisfaction related to the equal additions algorithm?

## **Methodology**

### **Population**

Participants in the study were students at a residential treatment facility that provides 24-hour/7-day week support services for an average of 75 boys and girls, ages 6 to 21. Students attend school all year, five days per week for five hours a day.

### **Participants**

Students were eligible for the study based on initial criteria that included (a) ability to correctly perform subtraction without regrouping in ones and tens places on the *ENRIGHT* diagnostic test, (b) inability to demonstrate subtraction with regrouping in ones and tens place on the *ENRIGHT* diagnostic test. Additional criteria included: (a) currently being taught mathematics in a fourth- or fifth-grade residential self-contained

special education classroom and (b) obtaining written permission for testing from each participant's legal guardian. Students who did not meet the inclusionary criteria and/or students with Deaf/Hard of Hearing (D/HH), Autism, Visually Impairments (VI), Moderately Intellectually Disabled (MOID), Severely Intellectually Disabled (SID), Physically Impaired (PI), and Profoundly Intellectually Disabled (PID) were not included in the study.

Six students originally met criteria for the study. However, two students left the facility before baseline treatment began and one student remained on the unit due to severe behaviors and did not attend the school facility to participate in the study. The remaining three students in the fourth-grade met criteria and participated in the study: 1 with E/BD and OHI (Danny), 1 with E/BD and SLI (Jeremy), and 1 with E/BD and SLD (Jeremiah). All students met the state criteria for emotional disturbance and are served in a 24-hour a day/ 7-day a week residential facility. Once students met all criteria and were eligible to participate in the study, students were administered the Calculation and Math Fluency subtests of the *Woodcock Johnson III (WJIII; Appendix A)* for demographical data. Table 1 presents students' evaluation data.

Danny was a 9-year, 6-month old male White student in the 4<sup>th</sup> grade. He was diagnosed with E/BD and OHI, being served under attention deficit disorder (ADD), and had been a resident of the school/facility for one year. His treatment plan behaviors included disruptive behaviors, childhood traumas, and mood instability.

Jeremy was a 9-year, 9-month old male White student in the 4<sup>th</sup> grade. He had been a resident of the school/facility for 7 months. He was diagnosed with E/BD and SLI and his treatment plan behaviors included disruptive behaviors, mood instability, and

childhood traumas.

Jeremiah was a 10-year, 6-month old African American student in the 4<sup>th</sup> grade. He was diagnosed with E/BD and SLD and had been a resident of the school/facility for 16 months. His treatment plan behaviors included disruptive behaviors and childhood traumas.

Table 1

*Participants' Evaluation Data*

Name	Age (yrs)	KBIT2	WJIII GE	WJIII GE	WJIII AE	WJIII AE
		Composite IQ	Math Fluency	Calculation	Math Fluency	Calculation
Danny	9.6	96	2.1	3.8	7-6	9-4
Jeremy	9.9	94	3.0	4.9	8-5	10-5
Jeremiah	10.6	90	2.0	3.2	7-5	8-8

**Setting and Personnel**

The residential facility provides 24-hour/7-day week support services for an average of 75 boys and girls, ages 6 to 21. Students attend school all year, five days per week for five hours a day and are assigned based on grade level. Students were pulled from the classroom for approximately 20 minutes of individualized, one on one instruction by the researcher.

**Materials**

Data collection materials for this study included (a) the *ENRIGHT* (Appendix B), (b) questionnaire (Appendix C), (c) direct instruction scripts (Appendix D), (d) a fidelity checklist (Appendix E), (e) baseline probes (Appendix F), (f) error analysis student profiles (Appendix G), and (g) behavior management with positive reinforcement sheets (Appendix H).

Three types of subtraction problems, adapted from Sugai and Smith (1986) were used: 1) regrouping necessary for one's place, two digits minus one digit and three digits minus one digit, 2) regrouping necessary for one's place, two digits minus two digits and three digits minus two digits, and 3) regrouping necessary for ten's place, three digits

minus two digits (See Table 2).

A pool of subtraction problems were gathered for the study and used for: 1) baseline and probes, 2) guided instruction, and 3) daily worksheets. Thus there were three different sets of subtraction problems to ensure that overlap did not occur during the study. These problems were randomly selected from [www.interventioncentral.com](http://www.interventioncentral.com) and

Table 2

*Types of Subtraction Problems*

Type	Amount of Regrouping	Number of Digits	Example
1	Regrouping necessary for ones place	2-1, 3-1	32-7, 456-9
2	Regrouping necessary for ones place	2-2, 3-2	45-27, 632-17
3	Regrouping necessary for tens place	3-2	746-83

each pool consisted of approximately 20–50 problems. The baseline probes consisted of a variety of each of the three problem types, whereas the guided instruction problems and daily worksheets consisted of problems specific to the problem type being explicitly taught during that session.

### Procedure

**Preintervention measures.** To determine eligibility for the study, students were assessed on initial abilities in subtraction with whole numbers using the *ENRIGHT Diagnostic Inventory of Basic Arithmetic Skills (1983)*. The *Inventory* is a simple measure that thoroughly assesses, diagnosis, and analyzes 144 basic computation skills. The Skill Placement Test in subtraction of whole numbers was used for this study. The Skill Placement Tests are designed to identify, within a specific skill sequence, the step that needs to be further tested. They are designed from simplest to more complex, can be administered to individuals or to groups, are provided with two formats (form A and B), and can be answered orally or in written format.

The Skill Placement Test consisted of 10 problems ranging from 1-digit minus 1-

digit to 3-digits minus 3-digits. Students were expected to correctly answer B1, B2, B5, and B6 incorrectly answer B7, B8, and B10. Items B3, B4, and B9 were not considered to be crucially important to determining eligibility for the study. See Table 3 for a complete description of the problems on the Skill Placement Test.

Table 3

*Skill Placement Test Subtraction of Whole Numbers*

Type	Subtracts whole numbers with . . .
B1	1 digit from 1 digit
B2	1 digit from itself
B3	zero from a 1-digit number
B4	1 digit from 2 digits, less than 20
B5	1 digit from 2 digits with no regrouping
B6	2 digits from 2 digits with no regrouping
B7	1 digit from 2 digits, regrouping tens
B8	2 digits from 2 digits, regrouping tens
B9	3 digits from 3 digits with no regrouping
B10	3 digits from 3 digits, regrouping tens



A researcher developed pre/post questionnaire was administered to students to assess students' mathematical abilities and preferences and was comprised of questions which target mathematical skills. Responses were given on a 5-point Likert scale (See Appendix C). The questionnaire served as part of a social validity measure. (See Social Validity for more information).

**Intervention procedure.** The intervention consisted of teaching of the equal additions algorithm using a direct instruction approach.

Direct instruction is designed to provide instruction that helps students acquire, retain, and generalize new learning as efficiently and effectively as possible through basic instructional design elements: (a) sequence of skills and concepts, (b) explicit instructional strategies, (c) preskills, (d) example selection, and (e) practice and review (Stein, Kindler, Silbert, & Carnine, 2006). The scripts for direct instruction of the equal additions algorithm were designed by the researcher and adapted from *Designing Effective Mathematics Instruction: A Direct Instruct Approach* (Stein et al.).

Each daily lesson included a scripted lesson to help ensure the delivery of systematic and explicit instruction in the use of the equal additions algorithm. Instruction lasted for approximately 20 minutes. Students were led through each of the parts of the script (A – D). Lesson components included four parts designed to guide the student from when to identify, when to use the equal additions algorithm, and to independently use equal additions as follows: (a) teacher led instruction with the student identifying when to use equal additions, (b) teacher led instruction on the steps of equal additions with the teacher modeling, (c) a worksheet with guided practice, and (d) an independent worksheet (see Appendix D for examples of each part of the script).

In part A, one subtraction problem was provided as an example for determining when the equal additions algorithm is used. This example problem changed depending on the type of subtraction problem being taught (e.g. number of digits being taught such as 2-digits minus 1-digit). As seen in appendix D, the teacher began each lesson by reviewing the rule about equal additions. Then the student stated the rule back to the teacher. The student was shown two other problems and asked the same series of questions, which required them to determine if regrouping, and the use of equal additions, was necessary for discrimination practice.

In part B, the teacher explained the steps required to effectively use the equal additions algorithm. First, the teacher wrote the first problem on the board and had the student read it aloud. Then the teacher asked the student what the ones column tells them to do (is the number on the bottom bigger than the number on the top?) Then the teacher explained to the student the steps to add to the top and add to the bottom using the model problem on the board. As the teacher did this, she paused and checked for understanding and had the student periodically repeat the steps back to her. Once the teacher had completed the steps with the model problem, she repeated the steps with two additional problems. An example of a lesson teaching the equal additions algorithm using direct instruction is presented in Figure 5.

TEACHER	STUDENT
1. (Write these problems on the board.) $\begin{array}{r} 23 \\ -5 \\ \hline \end{array}$ $\begin{array}{r} 16 \\ -7 \\ \hline \end{array}$ $\begin{array}{r} 34 \\ -8 \\ \hline \end{array}$	
(Point to the first problem.) Read this problem. The ones column tells us to start with 3 and take away 5. What does the ones column tell us to do?	23 – 5 Start with 3 and take away 5
Do we have to add to the top and bottom? ( <b>Pause and signal</b> )	Yes
Right. 5 is bigger than 3 so we are going to have to add to the top and add to the bottom. ( <i>To correct: What are we starting with in the one's column? Is the number on the bottom bigger than the number on the top? So do we add to the top and add to the bottom if the number on the bottom is bigger than the number on the top?</i> )	
2. Here's how we add to the top and add to the bottom: First, we add to the top by adding ten ones to the 3 ones. What do we do first? ( <b>Repeat steps 1 and 2 with the second and third problems</b> )	Add ten ones to the 3 ones
Right, we add ten ones to the 3 ones. We do this by crossing out the 3 and writing 13 above it. How do we do this? ( <b>Cross out the 3 and write 13</b> )	We cross out the 3 and write 13 above it
What are 13 ones minus 5 ones? ( <b>Write 8</b> )	8
3. Next, we add to the bottom. We add ten to the left of the next digit in the bottom number. What do we do?	We add a ten to the left of the next digit in the bottom number.
What digit is in the bottom number?	5
Right ( <b>point to the 5</b> ) As you can see, there isn't a number next to the 5 for us to add ten to. But that's ok because we can add ten to zero. Right?	Yes
What is ten plus zero?	10
Right. And if we add ten to here, that would mean that a 1 would go here. ( <b>Insert the 1</b> )	
What are 2 tens minus 1 ten? Right	1
4. Now, let's go to the next problem: 16-7. What do the numbers in the ones column tell us? Is the number on the bottom bigger than the number on the top? Do we need to add to the top and bottom?	Yes  Yes
5. Tell me how to add to the top and bottom. What do we do first?	Cross out the 6 and add ten ones to get 16: 16-7 = 9
What do we do next?	Then add ten to the bottom, placing a 1 to the left of the 7. 1-1 = 0.
<b>(Repeat with last problem)</b>	

Figure 5. Sample Direct Instruction Script.

Part C consists of a worksheet of three problems with guided practice. While students completed the problems on the worksheet, the first example problem used during part A & B remained on the board as a model for the student. As students completed the problems, teachers guided the students through the steps of the equal additions algorithm using a series of questions. The students had to successfully complete all 3 problems before moving on to part D. If a student made an error on any of the 3 problems, the following correction procedures were used: a) defective algorithm errors were addressed using the teacher led guided practice (b) part of the script wherein the incorrect problem was reviewed with the student, and (c) defective algorithm errors were referred back to the model on the board (i.e., to cue marks added).

During the teacher led instruction parts of the script (A & B), guided practice was used to explain the steps of the equal additions algorithm to the students and to support the student to prevent errors. If an error occurred, the teacher went back to the first step and prompted the student by asking a series of questions to guide the student through the steps of using the equal additions algorithm correctly. For example, if a student incorrectly responded when prompted by the teacher to add to the top and to the bottom, the teacher asked the student the following questions: *What are we starting with in the one's column? Is the number on the bottom bigger than the number on the top? So do we add to the top and add to the bottom if the number on the bottom is bigger than the number on the top?*

Upon completion of section C, students were given a worksheet which is labeled part D. The worksheet consisted of six problems consistent with the problem type taught for that session. Students were told to do their best and tell the instructor when they were

finished. The instructor identified which problems were solved incorrectly and immediately went through teacher led guided practice (B) with the student over those problems.

**Curriculum-based assessments (CBA).** Part D of each lesson were independent lessons consisting of 6 practice problems which were CBAs generated from [www.interventioncentral.com](http://www.interventioncentral.com) by the researcher. Any worksheet generating a problem containing a zero was eliminated and replaced with another problem because zeros were not being assessed during this intervention. Any problem that showed up in the practice problems or baseline data set were replaced as well. Each CBA, or student assessment, consisted of 6 problems specific for the problem type being taught (See Appendix C). This CBA was designed to assess the participants' ability to successfully solve subtraction problems with regrouping, a single skill set, and it was not timed. This CBA, therefore, was considered an untimed-focused curriculum-based assessment (Hudson & Miller, 2006).

**Baseline and probes.** Baseline probes consisted of 12 subtraction problems across the 3 problem types. Four problems of each of the three types of problems were provided because generalization to the untrained problem types is expected with this method of subtraction. The problems were selected from the pool of randomly selected problems collected from [www.interventioncentral.com](http://www.interventioncentral.com). Based on the types of problems used (refer to Table 2), the baseline probe consisted of 2 2-digits minus 1-digit items, 2 3-digits minus 1-digit items, 2 2-digits minus 2-digits items, 2 3-digits minus 2-digits items, and 4 3-digits minus 1-digits items. Data from the students were recorded during baseline and during probe sessions, which occurred after reaching criteria on each type of

problem. The order of the twelve problems was randomly rearranged during each baseline and probe session.

Probe data were collected to ascertain mastery of problem taught within each skill of problem type. In addition, it was expected that some students may be able to generalize the skills learned using the equal additions algorithm to other problem types. This was examined using the baseline probe given during preintervention and after successful mastery of each problem type.

**Error analysis.** An error analysis was conducted during the preintervention phase on the *ENRIGHT* measure, during the intervention phase on the baseline probes and daily worksheets, and during the maintenance phase on the error patterns. Data were collected on all errors for each student and recorded on an error analysis worksheet. Every incorrect problem was recorded on the error analysis worksheet, exactly as the student did according to the error type, for each session. Table 4 provides a detailed description and example of each error type.

### **Reinforcers**

Throughout intervention, students were provided with behavior specific positive reinforcement for their participation after the completion of each session, based on hard work, good attitudes, and success in learning the equal additions algorithm. In addition, positive reinforcement was used to increase on-task behaviors and completion of sessions (Maag, 2004). Students were reinforced for completion of each session. As there are few universal reinforcers, students were interviewed to determine what is reinforcing for them (Walker & Shea, 1995) and the administration and researcher decided on appropriate

Table 4

*Error Analysis*

Error Type	Example	Description
Basic fact	$\begin{array}{r} 256 \\ - 74 \\ \hline 186 \end{array}$	An error is made recalling a basic number fact
Operation	$\begin{array}{r} 73 \\ - 8 \\ \hline 81 \end{array}$	The student attempts to solve the problem by using another operation sign, in this case addition
Inversion (reversal)	$\begin{array}{r} 24 \\ - 9 \\ \hline 25 \end{array}$	Occurs when the minuend (top number) is subtracted from the subtrahend (bottom number) in subtraction problems requiring regrouping
Fails to add to subtrahend	$\begin{array}{r} 18 \\ 58 \\ - 9 \\ \hline 59 \end{array}$	Student adds to the minuend (top numbers) but fails to add to the subtrahend (bottom number)
Fails to add to minuend, but adds to the subtrahend	$\begin{array}{r} 58 \\ - 19 \\ \hline \end{array}$	Student fails to add to the minuend (top number) but adds to the subtrahend
Adds in the wrong place in the subtrahend	$\begin{array}{r} 18 \\ 158 \\ - 129 \\ \hline 39 \end{array}$	Student adds to the minuend (top number) but adds to the wrong place in the subtrahend (bottom number)
Decomposition	$\begin{array}{r} 4 \ 18 \\ 158 \\ - 29 \\ \hline 129 \end{array}$	Reduces a digit in the minuend to borrow from
Random Response	$\begin{array}{r} 25 \\ - 9 \\ \hline 41 \end{array}$	The student demonstrates little understanding of how to solve the problem and writes numbers randomly
Other Defective Algorithm	$\begin{array}{r} 4 \ 18 \\ 158 \\ - 29 \\ \hline 119 \end{array}$	The student attempts to use the correct operation but uses the wrong procedure for solving the problem

reinforcers for the students. Examples of reinforcers included printable color sheets and printable origami crafts. Daily reinforcers were tracked using behavior management positive reinforcement reward sheets (Appendix H).

### **Research Design**

A multiprobe multiple baseline across participants design was used for this study. Use of a multiple baseline design does not require the withdrawal of the intervention, which is useful in a case where a newly learned behavior (e.g., requesting materials) cannot be unlearned. The independent variable is the direct instruction of the equal additions algorithm and the dependent variable is the percentage of correct problems of subtraction with regrouping.

**Baseline.** Baseline was taken until a stable baseline was reached for 5 sessions or more (Alberto & Troutman, 2009). Baseline data consisted of the baseline/probe data set of 12 subtraction problems across the 3 problem types. The first student moved to intervention after stable baseline was reached. The other students continued having probe baseline data collected. The second student did not leave baseline until the first student had a positive trend in their first phase of intervention or until the first student reached criteria. A positive trend is defined as three ascending consecutive data points. Similarly, the third student did not start intervention until the second student had a positive trend in their first phase in intervention or until they reached criteria. The order of the students was based upon random assignment.

**Probe Data Pattern.** After all students met entry level criterion, a baseline condition lasting for five days began. Students were probed intermittently for the remainder of the study. One student was given a baseline probe every other session and



the other student completed a baseline probe every three days. In the event that a probe had not been given to a student within two sessions before starting intervention, a final baseline data point was taken before intervention started.

**Intervention type 1.** Intervention 1 consisted of problem type 1 which were 2 digits minus 1 digit and 3 digits minus 1 digit subtraction problems with regrouping necessary for the one's place. Students were given 6 type 1 problems per session as a worksheet. Students reached criteria when they answered 5 out of 6 (83%) or better correct for 2 out of 3 consecutive sessions. Students who did not reach criteria after 20 sessions could be excused from the study.

**Probe 1.** After reaching criteria on type 1 problems, students were given a probe. This contained the same problems given during baseline. The probe 1 lasted for one session. No instruction or correction was given at this time.

**Intervention type 2.** After completing probe 1, students received instruction in type 2 problems which consisted of 2 digits minus 2 digits and 3 digits minus 2 digits subtraction problems with regrouping necessary for the one's place. Students were given 6 type 2 problems per session as a worksheet. Students reached criteria when they answered 5 out of 6 (83%) or better correct for 2 out of 3 consecutive sessions. Students who did not reach criteria after 20 sessions could be excused from the study.

**Probe 2.** After reaching criteria on type 2 problems, students were given a probe. This contained the same problems given during baseline. The probe 2 lasted for one session. No instruction or correction was given at this time.

**Intervention type 3.** After completing probe 2, students received instruction in type 3 problems which consisted of 3 digits minus 2 digits problems with regrouping

necessary for the ten's place. Students were given 6 type 3 problems per session as a worksheet. Students reached criteria when they answered 5 out of 6 (83%) or better correct for 2 out of 3 consecutive sessions. Students who did not reach criteria after 20 sessions could be excused from the study.

**Probe 3.** After reaching criteria on type 3 problems, students were given a probe. This consisted of the same problems given during baseline. The probe 3 lasted for one session. No instruction or correction was being given at this time.

**Reliability and Procedural Fidelity.** Inter-rater reliability was conducted for 20% of the tests by a second observer for a 90% or greater agreement. A graduate student was trained through demonstration and practice in the scoring procedures. They had to demonstrate 100% achievement on scoring procedures. Inter-rater reliability was determined by dividing the total number of agreements between the graduate student and the researcher by the total number of observations, and then multiplied by 100.

To ensure accurate implementation of direct instruction, fidelity was assessed for 25% of the sessions. A copy of the direct instruction script with a checklist (see Appendix E) was completed to determine if all of the intervention steps were completed accurately. Fidelity was calculated by dividing the total number of observed steps by the total number of steps. All validity and fidelity was conducted by three researchers who have completed CITI training and were trained in using the treatment checklist. The percentage for each student was: Danny, 99% (98% to 100%); Jeremy, 100% (100%); and Jeremiah 99% (98% to 100%). Interobserver agreement and fidelity was conducted for 20% of the fidelity checks by a second observer on all scored probes. Percentage of agreement was calculated by dividing the total number of agreements by the total number

of agreements plus disagreements and multiplying by 100%. The percentage for each student was 100%.

**Social Validity.** Social validity was examined through Likert-scale questionnaires before and after the study. The students answered questions regarding the need for, ease of, and preference for the interventions used. The questionnaires were given at pretest and upon the completion the maintenance assessment and probes. The questionnaires were administered following the completion of the study to determine whether any change had occurred during the course of study in the students' self-assessment of their mathematical abilities and preferences.

### **Data Analysis**

Visual analysis of graphed results was used to evaluate the effectiveness of the intervention (see Figure 6). Specific characteristics of the data points were carefully analyzed and examined including level, trend, variability, overlap, immediacy of effect, and consistency of data patterns across similar phases (Kratochwill, et al., 2010).

## **Results**

All three students reached criterion, suggesting that a functional relationship exists between the direct instruction of the equal additions algorithm and acquisition of the subtraction with regrouping. On all three problem types, Danny reached criterion within 6 sessions, Jeremy reached criterion in 13 sessions, and Jeremiah reached criterion in 14 sessions.

### **Danny**

After a baseline condition of 5 days in which all three problem types were tested, Danny remained at 0% across all baseline conditions (see Figure 6). After 2 sessions of

training on problem type 1 (two digit minus one digit and three digit minus one digit problems with regrouping necessary for the ones place), Danny reached criterion, scoring 6 out of 6 (100%) correct in the first and second sessions. The following probe condition

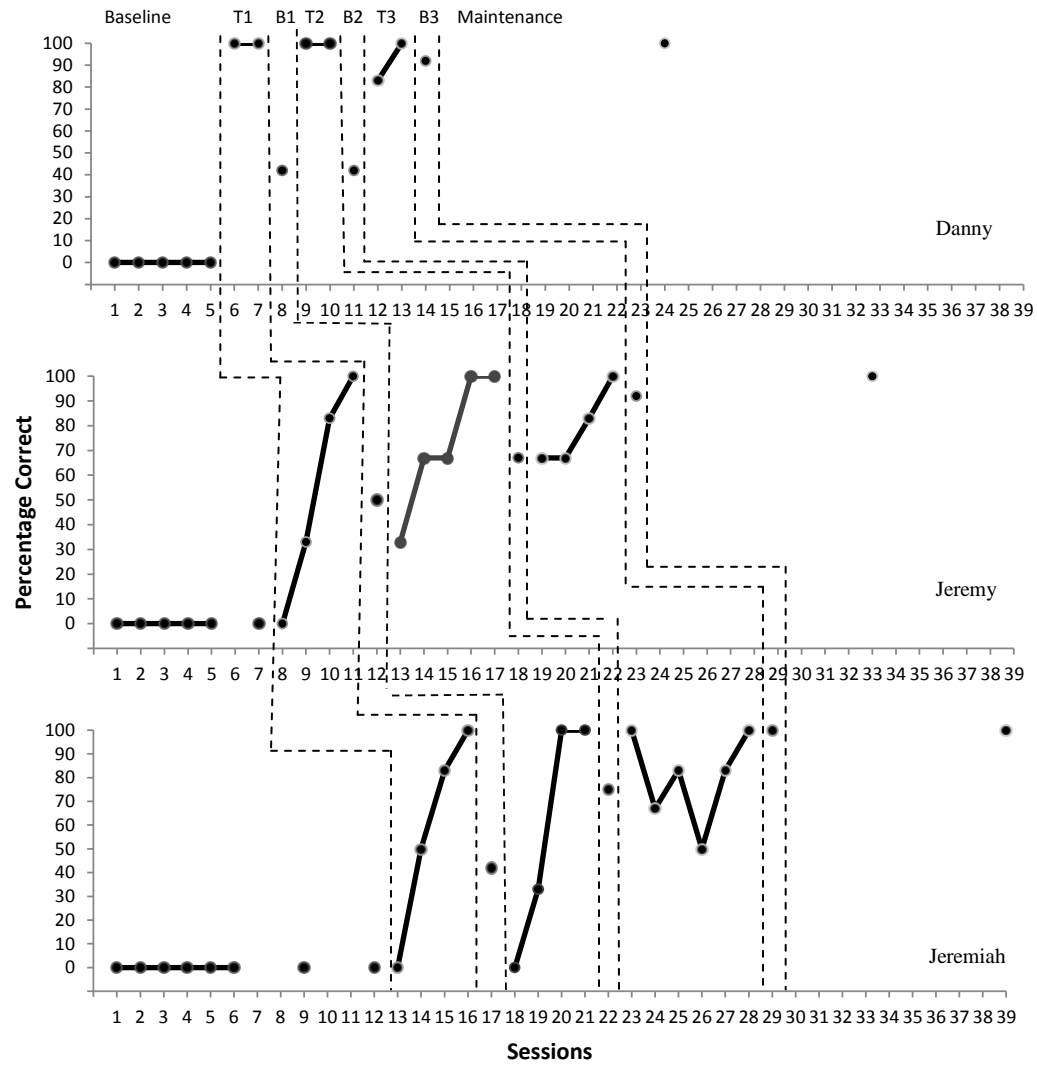


Figure 6. Percentage of subtraction with regrouping problems correct.

indicated that Danny successfully computed all type 1 problems, scoring 5 out of 12 (42%).

Training was initiated on type 2 problems (two digit minus two digit and three digit minus two digit with regrouping necessary for the one's place) and Danny reached criterion in 2 sessions, scoring 6 out of 6 (100%) correct in both sessions. On the next probe, Danny scored 5 out of 12 (42%). Training was initiated for type 3 problems (three digit minus two digit with regrouping necessary for the ten's place) and Danny reached criterion in 2 sessions, scoring 5 out of 6 (83%) and 6 out of 6 (100%).

Before training, Danny attempted 60 problems and made 95% inversion errors and 5% random response errors (see Table 5). Inversion errors occur when the order of subtraction is reversed and the student subtracts the minuend from the subtrahend because it is the smaller of the two numbers. On the first baseline probe, Danny attempted one type 2 problem but did not complete the problem, indicating he was unable to compute type 2 or type 3 problems without training.

On the second probe, Danny attempted all type 1, 2, and 3 problems on the probe. He was still able to compute type 1 problems correctly but he did not compute any type 2 or type 3 problems correctly. For the probe error analysis indicated 29% were inversion errors, 57% of the errors were the result of failing to add to the subtrahend after adding the minuend, and 14% of the errors occurred when he borrowed when unnecessary.

Table 5

*Percentage of errors for Danny*

	Baseline	Type 1/ Probe 1	Type 2/ Probe 2	Type 3/ Probe 3	Maintenance
Inversion	95%		29%		
Fails to add to subtrahend			57%	100%	
Borrows when unnecessary			14%		
Random response	5%				

No errors were demonstrated during type 3 phase. On the third probe he was able to compute all problem types successfully, computing 11 out of 12 problems successfully. All errors (100%) in the type 3 phase and probe were the result of failing to add to the subtrahend after successfully adding to the minuend. No errors were demonstrated on maintenance.

**Jeremy**

After a baseline condition lasting 7 sessions and during which data was collected across all problem types for 6 probe sessions, Jeremy remained at 0% across all baseline conditions. Jeremy received training on all problem types for a total of 13 sessions (see Figure 6). Jeremy received training on problem type 1 (two digit minus one digit and three digit minus one digit problems with regrouping necessary for the one's place), and reached criterion after 4 sessions scoring 0 out of 6 (0%), 2 out of 6 (33%), 5/6 (83%), and 6/6 (100%). The probe condition indicated a score of 6/12 (50%) with maintenance of all type 1 problems and his ability to compute some type 2 problems without training.

Jeremy reached criterion for type 2 problems (two digit minus two digit and three digit minus two digit with regrouping necessary for the one's place) after 5 sessions scoring 2/6 (33%), 4/6 (67%), 4/6 (67%), 6/6 (100%), and 6/6 (100%). The following probe indicated a score of 8/12 (67%) with maintenance on type 1 and type 2 problems. Criterion was reached on type 3 problems (three digit minus two digit with regrouping necessary for the ten's place) after four sessions with scores of 4/6 (67%), 4/6 (67%), 5/6 (83%), and 6/6 (100%).

Before training, Jeremy attempted 84 problems and 100% of the errors were the result of an inversion error (see Table 6). During training of type 1 problems, his errors

Table 6

*Percentage of errors for Jeremy*

	Baseline	Type 1/ Probe 1	Type 2/ Probe 2	Type 3/ Probe 3	Maintenance
Basic Fact		20%	25%	50%	
Operation			8%	33%	
Inversion	100%		25%		
Fails to add subtrahend			42%	17%	
Adds in wrong place in subtrahend		30%			
Adds wrong amount to subtrahend		50%			100%

consisted of basic facts (20%), adding in the wrong place in the subtrahend (30%), and adding the wrong amount to the subtrahend (50%). No errors were recorded on probe 1 because all of the problems he attempted he got correct.

Errors during training of type 2 problems and on probe 2 included basic facts (25%), operations (8%), inversions (25%), and failing to add to the subtrahend (42%). Errors on type 3 problems and on probe 3 included basic facts (50%), operations (33%), and failing to add to the subtrahend (17%). On maintenance, Jeremy demonstrated one error, adding the wrong amount to the subtrahend (100%).

### **Jeremiah**

During a baseline condition of 10 sessions, 8 baseline probes were given where he remained at 0% across all baseline conditions (see Figure 6). After receiving training on type 1 problems (two digit minus one digit and three digit minus one digit problems with regrouping necessary for the one's place), Jeremiah reached criterion after 4 sessions, scoring 0 out of 6 (0%), 3 out of 6 (50%), 5 out of 6 (83%), and 6 out of 6 (100%). The subsequent probe condition indicated maintenance of type 1 problems, scoring 5 out of 12 (42%). He did not attempt any type 2 or 3 problems.

Jeremiah reached criterion for type 2 problems (two digit minus two digit and three digit minus two digit with regrouping necessary for the one's place) after 4 sessions, scoring 0 out of 6 (0%), 2 out of 6 (33%), 6 out of 6 (100%), and 6 out of 6 (100%). The following probe condition showed mastery of problem types 1 and 2 with a score of 9 out of 12 (75%) problems correct.

After training on type 3 problems (three digit minus two digit with regrouping necessary for the ten's place), Jeremiah reached criterion after 6 sessions, scoring 6 out of



6 (100%), 4 out of 6 (67%), 5 out of 6 (83%), 3 out of 6 (50%), 5 out of 6 (83%), and 6 out of 6 (100%). The following probe condition showed mastery of all problem types, scoring 12 out of 12 (100%).

Before training, Jeremiah attempted 72 problems and demonstrated errors in basic facts (59%) and inversion errors (34%; see Table 7). After the first session, Jeremiah continued to make only inversion errors (60%), but then the majority of his errors were basic facts (30%) and adding in the wrong place in the subtrahend (10%).

Errors on type 2 problems and probe 2 were the result of inversion errors (9%), failing to add to the subtrahend (73%), adding in the wrong place in the subtrahend (9%), and failing to complete the problem (9%). On type 3 problems the errors were basic facts

Table 7

*Percentage of errors for Jeremiah*

	Baseline	Type 1/ Probe 1	Type 2/ Probe 2	Type 3/ Probe 3	Maintenance
Basic Fact	59%	30%		57%	
Inversion	34%	60%	9%	13%	
Fails to add subtrahend			73%		
Adds in wrong place in subtrahend		10%	9%		
Borrows when unnecessary				15%	
Adds wrong amount when borrowing				15%	
Random response	7%				
Did not attempt	54%		9%		

(57%), inversion (13%), borrowing when unnecessary (15%), and adding the wrong amount when borrowing (15%). No errors were demonstrated on probe 3 or maintenance.

**Questionnaire.** All of the students indicated that they thought learning the equal additions method of subtraction made learning to do subtraction with regrouping easier and had overall positive responses to the questionnaires (see Table 8). In addition, two students ranked their overall abilities to perform subtraction with regrouping problems as better than when they began the study. One student ranked his overall feelings for math as much higher on the positive side after the study, originally marking that he strongly disagreed with the statement that ‘he likes doing math’ and agreeing with this statement at the end of the study. He also had a more positive outlook by the end of the study

Table 8

*Social Validity Questionnaire*

Item	Danny		Jeremy		Jeremiah		Mean	
	Pre-	Post-	Pre-	Post-	Pre-	Post-	Pre-	Post-
1. I like doing math.	5	5	5	5	1	4	3.6	4.6
2. I feel I am good at math.	5	5	5	5	3	5	4.3	5.0
3. I think learning math is hard for me.	1	1	1	1	3	4	1.6	2.0
4. I like doing subtraction.	1	1	5	5	5	5	3.6	3.6
5. I think I a good at doing subtraction problems that involve regrouping in the ones place.	3		5		1		3.0	
6. I think I am good at subtraction with regrouping.		5		5		5		5.0
7. The new method that I have been using made learning to do this type of subtraction easier.		5		5		5		5.0

5=Strongly agree, 1=Strongly disagree

regarding his perception of his overall abilities in math.

### **Discussion**

The purpose of the study was to determine the effects of using direct instruction and the equal additions algorithm on the subtraction with regrouping performance of fourth-grade students with E/BD and mathematics difficulties in a residential setting. The results of the study found that all three students reached criteria. Three demonstrations of effect across three tiers provide evidence of a functional relationship between the instruction of the equal additions algorithm and the acquisition of the targeted skill of subtraction with regrouping. Data showed that when the equal additions algorithm was systematically implemented, students successfully completed subtraction with regrouping problems on an average of 97% and inversion errors decreased by an average of 63%. These results confirm Sugai's previous study that demonstrates using the equal additions algorithm with a demonstration-plus-permanent model technique is effective in increasing student's abilities in subtraction with regrouping (Sugai & Smith, 1986).

### **Acquisition**

According to visual analysis, Danny responded more quickly to the equal additions algorithm method and met criterion more quickly on all problem types compared to Jeremy and Jeremiah. Danny did not have higher general achievement math scores on the WJIII compared to his peers but he was extremely excited and motivated to learn and easily picked up the method. Both Jeremy and Jeremiah met criterion on all three problem types and made steady progress on all three probes. Jeremiah demonstrated more difficulty reaching criterion with type 3 problems than the other two students, taking six sessions, which is likely due to his lack of basic facts and that he is functioning

at a lower age and grade level in mathematics. All students successfully demonstrated the ability to solve subtraction with regrouping problems when necessary in the ones and ten's place using the equal additions algorithm. These results address the first two research questions demonstrating that students with E/BD could successfully solve subtraction problems when regrouping is necessary in the ones and tens place.

### **Error Analysis**

The error analyses provided information regarding the nature of the individual student's errors on subtraction with regrouping problems. Data were collected during baseline, treatment, and maintenance to determine the types of errors the students made and if student errors decreased as a result of the intervention.

During baseline, the most common errors demonstrated by students were inversion errors and/or basic facts. These findings are consistent with previous studies conducted by Frank, Logan, and Martin (1982) and Sugai and Smith (1986) who found one of the most common errors on subtraction with regrouping problems was inversion. Inversion errors occur when the order of subtraction is reversed and the student subtracts the minuend from the subtrahend because it is the smaller of the two numbers. Inversion errors are the most common type of systematic errors that are demonstrated by students with and without disabilities (Blankenship, 1978; Cox, 1975; National Research Council, 2002; Resnick, 1982).

Basic facts have been found by researchers as the main cause of errors in subtraction when examining error patterns (Cawley, Parmar, Yan, & Miller, 1996). When students are not proficient in basic mathematics skills they demonstrate numerous mathematics errors (Cawley, Parmar, Foley, & Salmon, 2001; Marchand-Martella,

Slocum, & Martella, 2004; Resnick & Omanson, 1987). Janke and Pilkey (1985) found that basic fact errors made up more than half of the computation errors for children in 2<sup>nd</sup> through 6<sup>th</sup> grade.

Throughout the study, Jeremiah continued to demonstrate basic fact errors, unlike the other two students, which is most likely due to his overall mathematics functioning. At the time of the study, he was functioning at the second grade level in math fluency and the third grade level in math calculation. He employed finger counting ineffectively, which has been found to be one of the developmentally immature counting strategies that children with disabilities often rely on when they are struggling with basic fast knowledge (Geary, 2004; Woodward & Montague, 2002). He often relied on this strategy and used it incorrectly, resulting in incorrect answers. In phase 3, Jeremiah demonstrated more difficulty than his peers, which may be the result of an increase in digits. Researchers have found that students make more errors as problems become more complex (Babbitt, 1990; Geary, Hoard, Nugent, & Byrd-Craven, 2007).

In treatment, a shift in errors occurred that may be reflective of learning the new algorithm. For example, failing to add to the subtrahend, adds the wrong amount to the subtrahend, and adds in the wrong place in the subtrahend were all common errors that occurred while learning the new algorithm. This may be because the students had difficulty remembering the steps to the new algorithm, and again, further instruction was warranted. Furthermore, inversion errors continued to occur for all students throughout treatment, although they were eliminated for Danny and Jeremy after type 2 problems were taught and they reduced dramatically for Jeremy by the end of treatment. Danny demonstrated an increase in basic fact errors in phase 3, which was likely due to his

growing impatience throughout the study.

### **Generalization**

The purpose of the probes was to examine if they were able to generalize from the types of problems in treatment phase to problems presented in baseline. In addition, we were examining if students were able to generalize between what they were taught and other groups of problems that they were not taught. For example, during phase one they were taught type 1 (two digit and three digit minus 1 digit regrouping in the ones place) subtraction with regrouping problems and we wanted to see if they were able to generalize to type 2 (2 digit and three digit minus two digit regrouping in the ones place) or type 3 (three digit minus two digit regrouping in the tens place) subtraction with regrouping problems.

After students reached criterion on type 1 problems, all three students were able to demonstrate mastery on type 1 problems on the first probe. Jeremy was the only student who attempted one additional type 2 problem and answered it correctly. He answered the problem correctly using the traditional form of decomposition to solve it. This is interesting because he did not attempt this with any other problems on the probe or throughout the remainder of the study and he had not previously attempted to use the decomposition method when solving problems before treatment.

After students mastered all type 2 problems in phase 2, Jeremiah and Jeremy attempted all problems and demonstrated mastery of type 2 problems, maintenance of type 1 problems, but did not demonstrate generalization of type 3 problems. Danny attempted all problems and demonstrated maintenance of type 1 problems, and failed to demonstrate mastery of type 2 problems or generalize to type 3 problems. This is

probably due to the fact that there was not a minimum number of sessions in treatment and he was not given enough time to practice and master the skills he was learning.

After reaching criterion in all types of problems during phase 3, all three students demonstrated mastery of all problems. All three students attempted all problems and Jeremy and Jeremiah demonstrated mastery of type 3 problems and maintenance of type 1 and type 2 problems. Danny demonstrated mastery of type 3 problems and maintenance of type 1 problems. He made an error on a type 2 problem, failing to add to the subtrahend.

While students were able to demonstrate generalization across problems within the same problem type, generalization across problem types did not occur. Generalization from type 1 to type 2 problems may not have occurred due to the nature of the algorithm. Due to the nature of the problem sets, type 1 problems were designed as two and three digit problems with only a subtrahend in the ones place. This method allowed for scaffolding of instruction and control of the problem sets to ensure that students were learning that you always add a one to the 'empty' space next the number on the bottom. Due to this fact, it may not be reasonable to expect students to be able to generalize from type 1 problems to type 2 problems on their own without instruction because this requires them to add a one to a number in the subtrahend.

Generalization from type 2 (regrouping in the ones place) to type 3 (regrouping in the tens place) problems did not occur either. Students either did not attempt type 3 problems on the probes or made inversion errors when attempting type 3 problems. Due to the complexity of this algorithm and the short amount of time that the students were able to successfully solve subtraction with regrouping problems, I am not sure that

generalization across problem types may not have been a reasonable expectation. Perhaps with more time to master the skills for each problem type, the students would have been able to show generalization.

On maintenance, Danny and Jeremiah demonstrated maintenance on all problem types. Jeremiah demonstrated maintenance on all problem types but incorrectly answered a type 2 problem, adding the wrong amount to the subtrahend. All students demonstrated mastery of type 1, 2, and 3 problems. On average the students successfully completed subtraction with regrouping problems 97% and inversion errors decreased by 63%. These results support Sugai and Smith's (1986) previous study that the equal additions algorithm is effective in increasing students' abilities in subtraction with regrouping.

### **Social Validity**

A final component of this study was the collection of social validity data to assess the student's feelings and preferences in mathematics and this investigation. Pre and post social validity data indicated very little change for Danny and Jeremy (see Table 8). Results indicate that both Danny and Jeremy may have difficulties with self-perceptions and their abilities to self report accurately, based on the positive results they made from this study. Jeremiah's results appear more accurate based on some of the responses. It is encouraging that the students report positive results regarding their experiences with this intervention and the use of this algorithm.

### **Limitations and Future Directions**

Several limitations related to implementing a multiple baseline study in a residential school were experienced. First, the number of students who met criteria and were available for the duration of the study was a challenge. This was due, partly, to the



frequency with which the participants were discharged from the facility during the summer months, when the study took place. After initial testing to determine eligibility, 6 students met the requirements to participate in the study but two were released from the residential school before treatment began. One other student was detained on a behavioral unit for the duration of the study and could not participate in the study. Future researchers should investigate other populations, including students with LD and MID to determine its effectiveness. Further repetition of the study is necessary to more closely examine the types of errors students make while performing subtraction with regrouping problems. This would provide more data for teachers when considering using the equal additions algorithm in the classroom and help to add to the research literature, which is greatly lacking in effective mathematics strategies for students with E/BD.

Second, while the amount of time needed to facilitate the sessions with the student only took about 15 to 20 minutes, the amount of time needed to create the direct instruction scripted lessons was very time intensive. This was due to the fact that the equal additions algorithm is not a well known algorithm nor is it taught frequently in common United States textbooks. Therefore, preparation of materials was quite intensive as all of the materials had to be created by the researcher. Future researchers should assess using an explicit method to teach the algorithm without using scripted lessons as a more practical approach for the classroom teacher as the equal additions algorithm may be a more efficient and effective procedure for students to learn.

Third, while this study completed an error analysis to determine the types of errors that students made while subtracting with regrouping, an analysis of the errors was not conducted to drive instruction and provide remediation. Adding a remediation

component may help to provide more realistic results and better mimic what truly occurs in a classroom. For example, if error analysis indicates that a student is using an ineffective/incorrect counting procedure, teachers should be able to incorporate time to remediate and correct the problem to ensure that the student is able to compute basic facts before proceeding with the intervention. Future researchers should consider looking at how to provide quick remediation strategies for students who are struggling with basic facts and are using immature counting strategies ineffectively such as finger counting.

Fourth, due to the fact that generalization did not occur across problem types during the study and this is an important skill for students, it is important to consider the length of the study and minimum number of sessions for mastery. Future researchers should consider implementing a minimum number of sessions for each problem type to ensure mastery of each type of problem and promote generalization of skills. Researchers may also want to reconsider the problem type redistribution and type of problems taught during each phase to promote generalization across problem types.

### **Conclusion**

Students with E/BD may present the most challenging characteristics when it comes to improving academic outcomes. Due to the nature of the disability, students with E/BD often present behaviors that interfere with their academic success. This seems to be especially true for students in residential settings, as their emotional and behavioral deficits are often a priority and therefore academics are not the focus. Furthermore, the type of interventions aimed at helping students with E/BD in these settings are often independently driven on self-management strategies and paper and pencil seat work. As a

result, much of the existing literature for students with E/BD is aimed at providing behavioral strategies and self-regulation strategies that address academic concerns.

This study focused on the effects of using explicit instruction to teach the equal additions algorithm to students with E/BD in a residential treatment school, who had additional disabilities in ADD, E/BD, and SLD with disruptive behaviors, childhood traumas, and mood instability that were so significant that it resulted in them being served in a residential treatment facility/school. Results from this study suggest that using direct instruction to teach the equal additions algorithm was effective for students in this study and that it can be an effective intervention for students in a residential facility/school. These data contribute to the limited research on effective instructional strategies for students with E/BD who are struggling in mathematics and provide educators with an alternative method for teaching subtraction with regrouping.

Findings from this study help to demonstrate the effectiveness of direct instruction and the use of scripted lessons to teach more complex mathematics skills to students with E/BD and mathematics difficulties. In addition, these students were able to successfully demonstrate how to use the equal additions algorithm across three problem types in a relatively short amount of time with a high rate of accuracy. These results suggest that this intervention would be an effective alternative for teaching subtraction with regrouping in the future.

Another important aspect of this research was the error analysis. Error analysis is an effective strategy to help identify specific errors to help remediate instruction for students with disabilities. This study contributes to the limited research on the error patterns of students with E/BD in subtraction with regrouping and supports the current

literature that most students' errors are the result of basic facts and/or inversion errors. Results from this study suggest that basic fact knowledge is a skill that students with E/BD struggle with and that immature and ineffective counting strategies can continue to hinder their success as they get older, making remediation a key factor of any successful intervention. In addition, this study demonstrated that the intervention was successful in greatly reducing, if not completely, the number of inversion errors demonstrated by students. Finally, results from this study showed that students greatly reduced, or eliminated, the number of errors they made when they used the equal additions algorithm versus the decomposition method.

Teachers of students with E/BD need strategies and interventions that have been proven to be effective for these students in improving outcomes in teaching subtraction with regrouping. Results from this study demonstrate that students can successfully acquire the skills to implement the equal additions algorithm through explicit, direct instruction to correctly answer subtraction with regrouping problems accurately.

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APPENDIXES

## APPENDIX A

## Test 5 Calculation

A. <input type="text"/>	B. <input type="text"/>	1. $2 + 2 = \square$	2. $1 + 1 = \square$	3. $2 + 1 = \square$
4. $6 + 1 = \square$	5. $2 + 4 = \square$	6. $\begin{array}{r} 3 \\ - 2 \\ \hline \end{array}$	7. $\begin{array}{r} 5 \\ - 2 \\ \hline \end{array}$	8. $3 - 1 = \square$
9. $5 - 1 = \square$	10. $\begin{array}{r} 9 \\ + 7 \\ \hline \end{array}$	11. $\begin{array}{r} 17 \\ - 9 \\ \hline \end{array}$	12. $\begin{array}{r} 89 \\ - 18 \\ \hline \end{array}$	13. $\begin{array}{r} 5 \\ \times 3 \\ \hline \end{array}$
14. $\begin{array}{r} 476 \\ 61 \\ + 2,611 \\ \hline \end{array}$	15. $2\overline{)8}$	16. $\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	17. $\begin{array}{r} 13 \\ \times 7 \\ \hline \end{array}$	
18. $\begin{array}{r} 48 \\ - 19 \\ \hline \end{array}$	19. $\begin{array}{r} 14 \\ \times 6 \\ \hline \end{array}$	20. $\begin{array}{r} \frac{2}{3} \\ - \frac{1}{3} \\ \hline \end{array}$	21. $42\overline{)126}$	
22. $48\overline{)288}$	23. $\begin{array}{r} \frac{7}{8} \\ - \frac{2}{8} \\ \hline \end{array}$	24. $25\overline{)3250}$	25. $\begin{array}{r} 2\frac{3}{4} \\ + 4\frac{1}{8} \\ \hline \end{array}$	

Go to the next page →

26. $\begin{array}{r} 1.05 \\ \times .2 \\ \hline \end{array}$	27. $\begin{array}{r} 2\frac{6}{7} \\ + 5\frac{1}{2} \\ \hline \end{array}$	28. $\begin{array}{r} -18 \\ + 12 \\ \hline \end{array}$	29. $\begin{array}{r} -6 \\ \times 7 \\ \hline \end{array}$
30. $\frac{4}{7} \div \frac{1}{2} =$	31. $\sqrt[3]{125} =$	32. $\begin{array}{l} 2x + 4y = 16 \\ 3x - y = 3 \\ x = \\ y = \end{array}$	33. Solve for x: $x^2 + 2x - 3 = 0$
34. $12\% \text{ of } 6.0 =$	35. $\left(\frac{4b}{3y}\right) \left(\frac{-4y}{12b^2}\right) =$	36. $8\frac{1}{2} \div 4\frac{1}{8} =$	37. $\log_b 81 = 4$ $b =$
38. Simplify: $\sqrt{\frac{4a^2}{16}} =$	39. $\sqrt{0.0025} =$	40. $\begin{array}{l} f(x) = 6x^3 \\ f'(x) = \end{array}$	41. $\begin{array}{l} \cos \theta = \frac{\sqrt{3}}{2} \\ \theta = \end{array}$
42. $2y = 6x + 8$ Slope = y intercept =	43. Evaluate: $\begin{vmatrix} 8 & 2 \\ -4 & 1 \end{vmatrix} =$	44. $\int_1^3 3x^2 dx =$	45. $\begin{array}{l} \tan \theta = 1 \\ \sin \theta = \end{array}$



## Test 6 Math Fluency

$\frac{1}{-1}$	$\frac{0}{+3}$	$\frac{2}{+2}$	$\frac{4}{-2}$	$\frac{2}{+1}$	$\frac{3}{-3}$	$\frac{0}{+0}$	$\frac{3}{-0}$	$\frac{2}{-1}$	$\frac{2}{+4}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{5}{+0}$	$\frac{3}{-1}$	$\frac{1}{+6}$	$\frac{4}{+4}$	$\frac{5}{-0}$	$\frac{1}{+1}$	$\frac{6}{-1}$	$\frac{3}{+5}$	$\frac{4}{-1}$	$\frac{5}{-2}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{3}{-2}$	$\frac{5}{+1}$	$\frac{6}{-3}$	$\frac{2}{-2}$	$\frac{7}{+1}$	$\frac{4}{-4}$	$\frac{1}{+8}$	$\frac{4}{-3}$	$\frac{7}{+2}$	$\frac{4}{+1}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{2}{+5}$	$\frac{8}{-1}$	$\frac{5}{-4}$	$\frac{3}{+3}$	$\frac{10}{-2}$	$\frac{3}{+6}$	$\frac{7}{-2}$	$\frac{2}{+8}$	$\frac{3}{+1}$	$\frac{9}{-4}$
----------------	----------------	----------------	----------------	-----------------	----------------	----------------	----------------	----------------	----------------

$\frac{6}{-2}$	$\frac{4}{+6}$	$\frac{9}{+3}$	$\frac{8}{-6}$	$\frac{7}{+5}$	$\frac{10}{-10}$	$\frac{2}{+6}$	$\frac{5}{-3}$	$\frac{6}{-6}$	$\frac{3}{+4}$
----------------	----------------	----------------	----------------	----------------	------------------	----------------	----------------	----------------	----------------

$\frac{5}{+5}$	$\frac{8}{-3}$	$\frac{5}{-1}$	$\frac{8}{+0}$	$\frac{7}{-4}$	$\frac{1}{+9}$	$\frac{10}{-6}$	$\frac{8}{+4}$	$\frac{6}{+8}$	$\frac{9}{-9}$
----------------	----------------	----------------	----------------	----------------	----------------	-----------------	----------------	----------------	----------------

$\frac{1}{\times 1}$	$\frac{4}{+5}$	$\frac{7}{+7}$	$\frac{2}{\times 3}$	$\frac{10}{-5}$	$\frac{1}{\times 2}$	$\frac{3}{+2}$	$\frac{8}{-8}$	$\frac{9}{+5}$	$\frac{5}{\times 1}$
----------------------	----------------	----------------	----------------------	-----------------	----------------------	----------------	----------------	----------------	----------------------

$\frac{3}{+7}$	$\frac{1}{\times 4}$	$\frac{9}{-2}$	$\frac{9}{+8}$	$\frac{3}{\times 3}$	$\frac{6}{-5}$	$\frac{2}{\times 2}$	$\frac{9}{-3}$	$\frac{3}{\times 1}$	$\frac{6}{+6}$
----------------	----------------------	----------------	----------------	----------------------	----------------	----------------------	----------------	----------------------	----------------



## APPENDIX B

## Skill Placement Test B. Subtraction of Whole Numbers Form A

**SKILL:** Subtracts whole numbers.

**GRADE LEVELS TAUGHT:** 1.0 to 3.3

**ARITHMETIC RECORD BOOK:** Page 3

**MATERIALS:** Copy of S-10 and a pencil.

**ASSESSMENT METHODS:** Individual or group written response.

**DISCONTINUE:** After each student has computed as many of the test items as he or she can.

**NEXT:**

1. Give **Basic Facts Test: Subtraction**, pages 40-41. (See **NOTE 2**.)
2. The letter and number above each test item in the **Answers** represent the matching skill test. Give the skill test that corresponds to the letter and number of the first test item computed incorrectly.

**NOTES:**

1. You may wish to use Form A of the **Skill Placement Test** as a pretest and Form B as a post test.
2. Before skill testing, the examiner should determine if the student has difficulty with basic subtraction facts. Such a weakness can interfere with the gathering of accurate information about student ability to compute.

**DIRECTIONS:** Give each student a copy of S-10 and a pencil. Tell the students to start with the first test item and to compute the test items in order. Tell the students to stop when they have computed as many of the test items as they can.

**Answers for Form A**

$\begin{array}{r} \text{B-1}^{1.0} \\ 8 \\ -5 \\ \hline 3 \end{array}$	$\begin{array}{r} \text{B-2}^{1.0} \\ 6 \\ -6 \\ \hline 0 \end{array}$	$\begin{array}{r} \text{B-3}^{1.0} \\ 5 \\ -0 \\ \hline 5 \end{array}$	$\begin{array}{r} \text{B-4}^{1.2} \\ 15 \\ -9 \\ \hline 6 \end{array}$
$\begin{array}{r} \text{B-6}^{1.6} \\ 86 \\ -34 \\ \hline 52 \end{array}$	$\begin{array}{r} \text{B-7}^{2.0} \\ 76 \\ -8 \\ \hline 68 \end{array}$	$\begin{array}{r} \text{B-8}^{1.5} \\ 52 \\ -35 \\ \hline 17 \end{array}$	$\begin{array}{r} \text{B-9}^{1.5} \\ 675 \\ -443 \\ \hline 232 \end{array}$
$\begin{array}{r} \text{B-11}^{1.0} \\ 539 \\ -172 \\ \hline 367 \end{array}$	$\begin{array}{r} \text{B-12}^{3.1} \\ 846 \\ -379 \\ \hline 467 \end{array}$	$\begin{array}{r} \text{B-13}^{1.2} \\ 590 \\ -246 \\ \hline 344 \end{array}$	$\begin{array}{r} \text{B-10}^{1.0} \\ 753 \\ -236 \\ \hline 517 \end{array}$
			$\begin{array}{r} \text{B-14}^{1.7} \\ 806 \\ -472 \\ \hline 334 \end{array}$
			$\begin{array}{r} \text{B-15}^{1.1} \\ 600 \\ -357 \\ \hline 243 \end{array}$

**OBJECTIVE:** By \_\_\_\_\_, when given fifteen test items for subtracting whole numbers, \_\_\_\_\_ will compute with 100% accuracy. (list as appropriate)

- B-1** 1-Digit Number from a 1-Digit Number  
**B-2** 1-Digit Number from itself  
**B-3** Zero from a 1-Digit Number  
**B-4** 1-Digit Number from a 2-Digit Number Less Than 20  
**B-5** 1-Digit Number from a 2-Digit Number, with No Regrouping  
**B-6** 2-Digit Number from a 2-Digit Number, with No Regrouping  
**B-7** 1-Digit Number from a 2-Digit Number, Regrouping Tens  
**B-8** 2-Digit Number from a 2-Digit Number, Regrouping Tens  
**B-9** 3-Digit Number from a 3-Digit Number, with No Regrouping  
**B-10** 3-Digit Number from a 3-Digit Number, Regrouping Tens  
**B-11** 3-Digit Number from a 3-Digit Number, Regrouping Hundreds  
**B-12** 3-Digit Number from a 3-Digit Number, Regrouping Tens and Hundreds  
**B-13** 3-Digit Number from a 3-Digit Number with Zero in the Ones Place  
**B-14** 3-Digit Number from a 3-Digit Number with Zero in the Tens Place  
**B-15** 3-Digit Number from a 3-Digit Number with Zeros in the Ones and Tens Places

**SKILL PLACEMENT TEST**  
**B. Subtraction of Whole Numbers—Form A**

NAME: \_\_\_\_\_

1. 
$$\begin{array}{r} 8 \\ -5 \\ \hline \end{array}$$

2. 
$$\begin{array}{r} 6 \\ -6 \\ \hline \end{array}$$

3. 
$$\begin{array}{r} 5 \\ -0 \\ \hline \end{array}$$

4. 
$$\begin{array}{r} 15 \\ -9 \\ \hline \end{array}$$

5. 
$$\begin{array}{r} 77 \\ -5 \\ \hline \end{array}$$

6. 
$$\begin{array}{r} 86 \\ -34 \\ \hline \end{array}$$

7. 
$$\begin{array}{r} 76 \\ -8 \\ \hline \end{array}$$

8. 
$$\begin{array}{r} 52 \\ -35 \\ \hline \end{array}$$

9. 
$$\begin{array}{r} 675 \\ -443 \\ \hline \end{array}$$

10. 
$$\begin{array}{r} 753 \\ -236 \\ \hline \end{array}$$

11. 
$$\begin{array}{r} 539 \\ -172 \\ \hline \end{array}$$

12. 
$$\begin{array}{r} 846 \\ -379 \\ \hline \end{array}$$

13. 
$$\begin{array}{r} 590 \\ -246 \\ \hline \end{array}$$

14. 
$$\begin{array}{r} 806 \\ -472 \\ \hline \end{array}$$

15. 
$$\begin{array}{r} 600 \\ -357 \\ \hline \end{array}$$



## APPENDIX C

## Pre - Participant Social Validity Questionnaire

Student Name and # \_\_\_\_\_

Please listen to each sentence carefully. Circle the number that best fits your opinion.

1 Strongly Disagree    2 = Agree    3 = Unsure    4 = Disagree    5 = Strongly Agree



- |    |   |   |   |   |   |   |
|----|---|---|---|---|---|---|
| 1. | I like doing math.                                | 1 | 2 | 3 | 4 | 5 |
| 2. | I feel I am good at math.                         | 1 | 2 | 3 | 4 | 5 |
| 3. | I think learning math is hard for me.             | 1 | 2 | 3 | 4 | 5 |
| 4. | I like doing subtraction.                         | 1 | 2 | 3 | 4 | 5 |
| 5. | I think I am good at subtraction with regrouping. | 1 | 2 | 3 | 4 | 5 |

## APPENDIX D

SUBTRACTION WITH REGROUPING USING EQUAL ADDITIONS ALGORITHM DAY 1	
TEACHER	STUDENT
<p>PART A: WHEN TO ADD TO THE TOP AND BOTTOM</p> <p>1. (Write the following problem on the board)</p> $\begin{array}{r} 23 \\ - 5 \\ \hline \end{array}$ $\begin{array}{r} 45 \\ - 3 \\ \hline \end{array}$ $\begin{array}{r} 31 \\ - 8 \\ \hline \end{array}$ <p>Here's a rule about Equal Additions with subtraction problems:</p> <p>When the number on the bottom is larger than the number on the top, we must add to the top and add to the bottom.</p> <p>My turn. When must we add to the top and add to the bottom? When the number on the bottom is larger than the number on the top.</p> <p>Your turn. When do we add to the top and add to the bottom?</p> <p><i>(Repeat statement with student until they can say it by themselves)</i></p> <p><i>(Point to the 3).</i> What number are we starting with in the ones column?</p> <p>We're starting with 3 and taking away 5. Is the number on the bottom</p>	<p>When the number on the bottom is larger than the number than the number on the top.</p> <p>3</p>

<p>bigger than the number on the top? <i>(Pause and signal)</i></p> <p>Do we need to add to the top and add to the bottom?</p> <p>Right, we have to add to the top and add to the bottom because the number on the bottom is bigger than the number on the top.</p> <p><i>(repeat with other problems)</i></p>	<p>Yes</p> <p>Yes</p>
<p>PART B: STEPS IN ADDING TO THE TOP AND BOTTOM <span style="float: right;">DAY 1</span></p>	
<p>TEACHER</p>	<p>STUDENT</p>
<p>1. (Write these problems on the board)</p> $\begin{array}{r} 23 \\ - 5 \\ \hline \end{array} \quad \begin{array}{r} 16 \\ - 7 \\ \hline \end{array} \quad \begin{array}{r} 34 \\ - 8 \\ \hline \end{array}$ <p>(Point to the first problem) Read this problem. The ones column tells us to start with 3 and take away 5. What does the ones column tell us to do?</p> <p>Do we have to add to the top and bottom? <i>(Pause and signal)</i></p> <p>Right. 5 is bigger than 3 so we are going to have to add to the top and add to the bottom.</p> <p><i>To correct: What are we starting</i></p>	<p>23 – 5</p> <p>Start with 3 and take away 5</p> <p>Yes</p>

<p><i>with in the one's column? Is the number on the bottom bigger than the number on the top? So do we add to the top and add to the bottom if the number on the bottom is bigger than the number on the top?</i></p> <p>2. Here's how we add to the top and add to the bottom: First, we add to the top by adding ten ones to the 3 ones. What do we do first? <b><i>(Repeat steps 1 and 2 with the second and third problems)</i></b></p> <p>Right, we add ten ones to the 3 ones. We do this by crossing out the 3 and writing 13 above it. How do we do this? <b><i>(Cross out the 3 and write 13)</i></b></p> <p>What are 13 ones minus 5 ones? <b><i>(Write 8)</i></b></p> <p>3. Next, we add to the bottom. We add ten to the left of the next digit in the bottom number. What do we do?</p> <p>What digit is in the bottom number? Right <b><i>(point to the 5)</i></b></p> <p>As you can see, there isn't a number next to the 5 for us to add ten to. But that's ok because we can add ten to zero. Right?</p> <p>What is ten plus zero? Right. And if we add ten to here that would mean that a 1 would go here. <b><i>(Insert the 1)</i></b></p> <p>What are 2 tens minus 1 ten?</p>	<p>Add ten ones to the 3 ones</p> <p>We cross out the 3 and write 13 above it</p> <p>8</p> <p>We add a ten to the left of the next digit in the bottom number.</p> <p>5</p> <p>Yes</p> <p>10</p> <p>1</p>
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<p>Right.</p> <p>4. Now, let's go to the next problem: 16-7 What do the numbers in the one's column tell us? Is the number on the bottom bigger than the number on the top? Do we need to add to the top and bottom?</p> <p>5. Tell me how to add to the top and bottom. What do we do first? What do we do next?</p> <p><i>(Repeat with last problem)</i></p>	<p>Yes</p> <p>Yes</p> <p>Cross out the 6 and add ten ones to get 16.</p> $16 - 7 = 9$ <p>Then add ten to the bottom, placing a 1 to the left of the 7. <math>1 - 1 = 0</math>.</p>
<p>PART C: STRUCTURED WORKSHEET <span style="float: right;">DAY 1</span></p>	
<p>TEACHER</p>	<p>STUDENT</p>
<p>1. (Give students attached worksheets with the following problems)</p> $\begin{array}{r} 61 \\ - 2 \\ \hline \end{array} \quad \begin{array}{r} 42 \\ - 5 \\ \hline \end{array} \quad \begin{array}{r} 93 \\ - 4 \\ \hline \end{array}$ <p>Read the first problem on your worksheet</p> <p>2. What does your one's column tell you to do? Is the number on the bottom bigger than the number on the top? Do you need to add to the top and bottom?</p>	<p>61 - 2</p> <p>Start with 1 and take away 2.</p> <p>Yes.</p> <p>Yes.</p>

<p>3. What do you do first? If you add to the 1, how many will there be? So cross out the 1 and write 11 above it. (<i>check student work</i>)</p> <p>4. What is <math>11 - 2</math>?</p> <p>5. What do you do now? Do that. Add ten to the next digit in the bottom number. How many tens do you have in the tens column? (<i>check student work</i>)</p> <p>6. What is <math>6 - 1</math>? Write 5 next to the 9 under the line in the tens column.</p> <p>7. How many is 61 minus 2?</p> <p><i>(repeat steps 1 -7 for remaining problems)</i></p>	<p>Add ten to the 1 and cross out the 1.</p> <p>11.</p> <p>9.</p> <p>Add ten the left of the next digit in the bottom number.</p> <p>1</p> <p>5</p> <p>59</p>
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APPENDIX E  
Fidelity Checklist

Fidelity Checklist for SUBTRACTION WITH REGROUPING USING EQUAL ADDITIONS ALGORITHM	
TEACHER	STUDENT
PART A: WHEN TO ADD TO THE TOP AND BOTTOM	
<p>1. (Write the following problem on the board)</p> $\begin{array}{r} 23 \quad 45 \quad 31 \\ -5 \quad -3 \quad -8 \end{array}$ <p>Here's a rule about Equal Additions with subtraction problems:</p> <p>When the number on the bottom is larger than the number on the top, we must add to the top and add to the bottom.</p> <p>My turn. When must we add to the top and add to the bottom? When the number on the bottom is larger than the number on the top.</p>	
<p>2. Your turn. When do we add to the top and add to the bottom?</p> <p><i>(Repeat statement with student until they can say it by themselves)</i></p> <p><i>(Point to the 3).</i> What number are we starting with in the ones column?</p> <p>We're starting with 3 and taking away 5. Is the number on the bottom bigger than the number on the top? <i>(Pause and signal)</i></p> <p>Do we need to add to the top and add to the bottom?</p> <p>Right, we have to add to the top and add to the bottom because the number on the bottom is bigger than the number on the top.</p>	<p>When the number on the bottom is larger than the number than the number on the top.</p> <p>3</p> <p>Yes</p> <p>Yes</p>
<p>3. <i>(repeat with other problems)</i></p>	

PART B: STEPS IN ADDING TO THE TOP AND BOTTOM		DAY 1
TEACHER	STUDENT	
<p>1. (Write these problems on the board)</p> $\begin{array}{r} 23 \\ -5 \\ \hline \end{array} \quad \begin{array}{r} 16 \\ -7 \\ \hline \end{array} \quad \begin{array}{r} 34 \\ -8 \\ \hline \end{array}$ <p>(Point to the first problem) Read this problem. The ones column tells us to start with 3 and take away 5. What does the ones column tell us to do?</p> <p>Do we have to add to the top and bottom? <b>(Pause and signal)</b></p> <p>Right. 5 is bigger than 3 so we are going to have to add to the top and add to the bottom.</p> <p><i>To correct: What are we starting with in the one's column? Is the number on the bottom bigger than the number on the top? So do we add to the top and add to the bottom if the number on the bottom is bigger than the number on the top?</i></p>	23 - 5	Start with 3 and take away 5
<p>2. Here's how we add to the top and add to the bottom: First, we add to the top by adding ten ones to the 3 ones. What do we do first?</p> <p><b>(Repeat steps 1 and 2 with the second and third problems)</b></p> <p>Right, we add ten ones to the 3 ones. We do this by crossing out the 3 and writing 13 above it. How do we do this? <b>(Cross out the 3 and write 13)</b></p> <p>What are 13 ones minus 5 ones? <b>(Write 8)</b></p>	Yes	
<p>3. Next, we add to the bottom. We add ten to the left of the next digit in the bottom number. What do we do?</p> <p>What digit is in the bottom number? Right <b>(point to the 5)</b></p>	<p>Add ten ones to the 3 ones</p> <p>We cross out the 3 and write 13 above it</p> <p>8</p> <p>We add a ten to the left of the next digit in the bottom number.</p> <p>5</p>	



<p>because we can add ten to zero. Right?</p> <p>What is ten plus zero? Right. And if we add ten to here that would mean that a 1 would go here. <b>(Insert the 1)</b></p> <p>What are 2 tens minus 1 ten? Right.</p> <p>4. Now, let's go to the next problem: 16-7 What do the numbers in the one's column tell us? Is the number on the bottom bigger than the number on the top? Do we need to add to the top and bottom?</p> <p>5. Tell me how to add to the top and bottom. What do we do first? What do we do next?</p> <p><b>(Repeat with last problem)</b></p>	<p>10</p> <p>1</p> <p>Yes</p> <p>Yes</p> <p>Cross out the 6 and add ten ones to get 16. <math>16 - 7 = 9</math> Then add ten to the bottom, placing a 1 to the left of the 7. <math>1 - 1 = 0</math>.</p>
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PART C: STRUCTURED WORKSHEET		DAY 1						
TEACHER	STUDENT							
<p>1. (Give students attached worksheets with the following problems)</p> <p style="margin-left: 40px;"> <table style="display: inline-table; border: none;"> <tr> <td style="padding: 0 10px;">61</td> <td style="padding: 0 10px;">42</td> <td style="padding: 0 10px;">93</td> </tr> <tr> <td style="padding: 0 10px;"><u>-2</u></td> <td style="padding: 0 10px;"><u>-5</u></td> <td style="padding: 0 10px;"><u>-4</u></td> </tr> </table> </p>	61	42	93	<u>-2</u>	<u>-5</u>	<u>-4</u>		
61	42	93						
<u>-2</u>	<u>-5</u>	<u>-4</u>						

	Read the first problem on your worksheet	61 - 2
_____	2. What does your one's column tell you to do? Is the number on the bottom bigger than the number on the top? Do you need to add to the top and bottom?	Start with 1 and take away 2. Yes.  Yes.
_____	3. What do you do first? If you add to the 1, how many will there be? So cross out the 1 and write 11 above it. <i>(check student work)</i>	Add ten to the 1 and cross out the 1.  11.
_____	4. What is 11 - 2?	9.
_____	5. What do you do now? Do that. Add ten to the next digit in the bottom number. How many tens do you have in the tens column? <i>(check student work)</i>	Add ten the left of the next digit in the bottom number.  1
_____	6. What is 6 - 1? Write 5 next to the 9 under the line in the tens column.	5
_____	7. How many is 61 minus 2?  <i>(repeat steps 1 -7 for remaining problems)</i>	59

Total items performed correctly = \_\_\_\_\_

Total items performed incorrectly = \_\_\_\_\_

Total % items performed correctly = \_\_\_\_\_

## APPENDIX F

## Error Analysis

Equal Additions Error Analysis Form					
Student Name/Number: _____					
Types of Errors	Date:	Date:	Date:	Date:	Date:
Basic Fact					
Operation (i.e., adding)					
Inversion (reversal)					
Fails to add to subtrahend					
Fails to add to minuend					
Adds in the wrong place in the subtrahend					
Reduces a digit in the minuend					
Random Response					
Decomposition					
Other					

## APPENDIX G

Curriculum-Based Assessment Mathematics  
Single-Skill Computation Probe: Student Copy

Student: \_\_\_\_\_ Date: \_\_\_\_\_

$\begin{array}{r} 554 \\ - 5 \\ \hline \end{array}$		$\begin{array}{r} 24 \\ - 6 \\ \hline \end{array}$		$\begin{array}{r} 657 \\ - 28 \\ \hline \end{array}$		$\begin{array}{r} 328 \\ - 54 \\ \hline \end{array}$	
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$\begin{array}{r} 343 \\ - 17 \\ \hline \end{array}$		$\begin{array}{r} 654 \\ - 8 \\ \hline \end{array}$		$\begin{array}{r} 265 \\ - 8 \\ \hline \end{array}$		$\begin{array}{r} 43 \\ - 26 \\ \hline \end{array}$	
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$\begin{array}{r} 83 \\ - 66 \\ \hline \end{array}$		$\begin{array}{r} 714 \\ - 24 \\ \hline \end{array}$		$\begin{array}{r} 62 \\ - 7 \\ \hline \end{array}$		$\begin{array}{r} 637 \\ - 42 \\ \hline \end{array}$	
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## Appendix H

## Weekly Reinforcement Sheets

☺ = reward

X = no reward

	Monday _____	Tuesday _____	Wednesday___	Thursday____	Friday _____
Student 1					
Student 2					
Student 3					
Student 4					
Student 5					
Student 6					

	Monday _____	Tuesday _____	Wednesday___	Thursday____	Friday _____
Student 1					
Student 2					
Student 3					
Student 4					
Student 5					
Student 6					