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The Effect of Aleks on Students' Mathematics Achievement in an Online Learning Environment and the Cognitive Complexity of the Initial and Final Assessments

Eze Nwaogu

ACCEPTANCE

This Dissertation, THE EFFECT OF ALEKS ON STUDENTS' MATHEMATICS ACHIEVEMENT IN AN ONLINE LEARNING ENVIROMENT AND THE COGNITIVE COMPLEXITY OF THE INITIAL AND FINAL ASSESSMENTS, by EZE NWAOGU, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree of Doctor of Philosophy in College of Education, Georgia State University.

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ABSTRACT

THE EFFECT OF ALEKS ON STUDENTS' MATHEMATICS ACHIEVEMENT IN AN ONLINE LEARNING ENVIROMENT AND THE COGNITIVE COMPLEXITY OF THE INITIAL AND FINAL ASSESSMENTS
by
Eze N. Nwaogu

For many courses, mathematics included, there is an associated interactive e-learning system that provides assessment and tutoring. Some of these systems are classified as Intelligent Tutoring Systems. MyMathLab, Mathzone, and Assessment of LEarning in Knowledge Space (ALEKS) are just a few of the interactive e-learning systems in mathematics. In ALEKS, assessment and tutoring are based on the Knowledge Space Theory. Previous studies in a traditional learning environment have shown ALEKS users to perform equally or better in mathematics achievement than the group who did not use ALEKS.

The purpose of this research was to investigate the effect of ALEKS on students' achievement in mathematics in an online learning environment and to determine the cognitive complexity of mathematical tasks enacted by ALEKS's initial (pretest) and final (posttest) assessments. The targeted population for this study was undergraduate students in College Mathematics I, in an online course at a private university in the southwestern United States. The study used a quasi-experimental One-Group non-randomized pretest and posttest design.

Five methods of analysis and one model were used in analyzing data: *t*-test, correlation analysis, simple and multiple regression analysis, Cronbach's Alpha reliability test and Webb's depth of knowledge model. A *t*-test showed a difference between the pretest and posttest reports, meaning ALEKS had a significant effect on

students’ mathematics achievement. The correlation analysis showed a significant positive linear relationship between the concept mastery reports and the formative and summative assessments reports meaning there is a direct relationship between the ALEKS concept mastery and the assessments. The regression equation showed a better model for predicting mathematics achievement with ALEKS when the time spent learning in ALEKS and the concept mastery scores are used as part of the model.

According to Webb’s depth of knowledge model, the cognitive complexity of the pretest and posttest question items used by ALEKS were as follows: 50.5% required application of skills and concepts, 37.1% required recall of information, and 12.4% required strategic thinking: None of the questions items required extended thinking or complex reasoning, implying ALEKS is appropriate for skills and concepts building at this level of mathematics.

THE EFFECT OF ALEKS ON STUDENTS' MATHEMATICS ACHIEVEMENT IN AN ONLINE LEARNING ENVIROMENT AND THE COGNITIVE COMPLEXITY OF THE INITIAL AND FINAL ASSESSMENTS

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ABBREVIATIONS

AI	Artificial Intelligence
ALEKS	Assessment of LEarning in Knowledge Space
CAI	Computer Assisted Instruction
CBE	Computer-Based Education
DOK	Depth of Knowledge
DSL	Digital Subscriber Line
HBI	Hypermedia Based Instruction
ITS	Intelligent Tutoring System
KST	Knowledge Space Theory
MML	MyMathLab
PC	Personal Computer
SAM	Student Assessment Manager
UM	University of Memphis

CHAPTER 1

INTRODUCTION

With the introduction of computers in the 1980s and 1990s, methods of teaching and learning basic mathematics have undergone several changes. The current trend of distance learning across most subject areas involves using computers, the Internet, and online interactive learning technologies. This trend has led to an explosion of online courses and programs across colleges and universities in the United States. For many online courses, there is an associated interactive e-learning system that plays the role of a tutor and instructor. A good example is Student Assessment Manager (SAM), which provides instruction and tutoring on basic computer applications in word processing, spreadsheet, database, and graphics presentation.

The number of courseware programs in the educational software industry has extensively increased in the past 10 years. In particular, there are many hypermedia courseware resources available in the market for almost every educational subject (Elissavet & Economides, 2003). Some of these interactive e-learning systems are classified as Intelligent Tutoring Systems (ITSs). An ITS assesses and tutors students in different subject matters. In particular, undergraduate mathematics has several popular web-based interactive learning systems, such as MyMathLab (MML), Mathzone, and Assessment of LEarning in Knowledge Spaces (ALEKS).

As an undergraduate mathematics instructor, I have facilitated online math classes for several years using various ITSs, and during this period I have seen student successes and failures in mathematics achievement. Some students have achieved high grades with the use of ITS, while other students have withdrawn or done very poorly in the course.

One of the ITSs I have used is ALEKS. ALEKS claims to have based its assessment and teaching strategy on Knowledge Space Theory (KST). ALEKS also claims to reveal the knowledge state of a learner and provide instruction based on that knowledge (ALEKS, 2010). ALEKS is used at several higher institutions and public schools. Results of mathematics achievement at institutions such as Louisiana Technical University (2006), Black Hills State University (2005) and University of Memphis (2008) indicate that learning is very efficient because of the accuracy of the assessment (Falmagne, Cosyn, Doignon, & Thiery, 2004). Also, ALEKS has been shown to help less prepared students reach success in beginning algebra (Allen, 2007). Because of ALEKS's reported success and my experience as a mathematics instructor observing the learning outcomes of student enrolled in my course, I became interested in investigating the effect of ALEKS on mathematics achievement.

In this chapter, I discuss the background of the proposed study, the problem statement, rationale, the theoretical framework, and the operational definition of terms. The background of the study includes technology enhancements, computers, and distance education institutions while the problem statement and rationale look at issues facing the achievement in mathematics. Finally, the theoretical framework introduces the main theories that underlie this research study. Operational definitions of terms describe terms that apply to this research study.

Background of the Study

Technology Enhancement

In the 1980s, the major drawback for implementing interactive learning systems was not only the sluggishness of the Internet but also the slowness of Internet-accessing technologies. However, in recent years, the introduction of high-speed fiber optic network and the use of broadband technology like DSL and cable-modem in residential areas have made accessing and navigating the Internet much faster. According to U.S. Federal Communications Commission (2011), Internet connections are growing fast. For example, the number of connections over 200 kbps in at least one direction increased by 28% in 2010 to nearly 169 million. Furthermore, other recent technologies like WI-FI and dish antennas have made it easier to access and use web-based interactive learning systems on the Internet. This technological growth has resulted in development and the use of various applications for online learning.

Computers and Distance Education Institutions

Personal Computers (PCs) in use reached nearly one billion units worldwide at year-end in 2006. The United States continues to lead the world in PC use and the total number of Internet users. With only 4.6% of the world's population, the United States accounts for over 24% of all PCs in use (Juliussen, 2007). The United States retains a large PC-usage lead with over three times as many PCs as the second place nation, Japan. The proliferation of computers has led to the explosion of online courses.

With the global increase in PC use, many people are using the Internet for different purposes, and the United States is leading the pack. The number of Internet users worldwide surpassed 1.2 billion in 2006—up from only about 2 million in 1990, 45 million in 1995, and 430 million in 2000. Worldwide yearly increase in Internet users is predicted to be 140 to 145 million in the next 5 years, which means the 2-billion mark will likely occur in 2012. Many of these users focus on academics.

According to the U.S. Department of Education's National Center for Education Statistics, during the 12-month 2000–2001 academic year, 56% (2,320) of all 2-year and 4-year Title IV-eligible, degree-granting institutions offered distance education courses for any level or audience (i.e., all types of students, including elementary and secondary, college, adult education, continuing and professional education; U.S. Department of Education, 2003). The next section discusses the issues surrounding the teaching and learning of mathematics online.

Problem Statement

Based on personal and professional experience and a review of related research, I began this research with the assumption that there are problems facing the learning of basic mathematics. One such problem is that mathematics is explained by strict rules and axioms (Stemhagen, 2003). Even in the traditional mathematics classroom, students have a difficult time following these rules. Online students face even greater challenges because in most circumstances they are learning the concepts on their own through the use of computer based systems. Depending on the effectiveness of the computer-based system, many students learning mathematics online either withdraw or receive poor grades. Smith and Ferguson (2005) showed large differences between average attrition rates for mathematics versus non-mathematics online courses (0.30 versus 0.18). Clearly, there is a need for the evaluation of these computer-based learning systems to determine their effects on learning, and this study addressed that need. Flag (1990) noted that systematic evaluation of the effectiveness of computer-based education (CBE) in all its various forms (including integrated learning system, interactive multimedia, interactive learning environments, and micro worlds) often lags behind in development efforts.

Although there are several reasons for this lack of evaluation, one important reason is that consumers of technological innovations for education seem to assume that because these innovations are advertised as effective, they are effective (Revees, 1997). Because the instructional impact of an ITS is dependent on how well it was designed, formative and summative evaluation of an ITS is important (Polson & Richardson, 1988). Most of the evaluation on the ITS is based on its sufficiency rather than its educational impact or effect on teaching and learning (Corbett, Koedinger, & Anderson, 1997). According to Mark and Greer (1993), as intelligent tutoring system issues are investigated and ITS are developed, evaluation methodology becomes important, and, until recently, little attention has been paid to evaluation of intelligent tutoring systems. As a result, educational evaluation of ITS like ALEKS is important to determine its effect on teaching and learning of mathematics. So the question becomes, “What is the effect of ALEKS on students’ mathematics achievement?”

Another issue facing the learning of mathematics is the lack of higher level cognitive complexity for mathematical tasks that are used in assessment items. Lower level cognitive-demanding mathematical tasks would lead to plain memorization of mathematics concepts. Zelkowoski (2009) noted that little learning is accomplished by only incorporating low-level cognitive tasks into teaching and assessment. Stein, Grover and Henningsen (1996) emphasized the importance of incorporating cognitively demanding mathematical tasks because of their impact on student learning. The 1999 Trends in International Mathematics and Science Study, which looked at the ways that mathematics instruction differs among seven countries, found that the United States (the lowest performer in the study) rarely enacted tasks at a high level of cognitive demand

(Zurawsky, 2006). To learn mathematics and make connections between concepts and meanings, educators must present mathematical tasks or assessment items to the students using a higher level cognitive demand than at present. Therefore, determining the cognitive complexity of the pretest and posttest assessments used by ALEKS in this study is important.

Rationale for the Study

Current research on the effectiveness of ALEKS (Allen 2007; Hagerty & Smith, 2005; Hampikian et al., 2006; Hanna & Carpenter, 2006; Lavergne, 2007; Taylor, 2008) has shown an increased average success learning rate in different learning contexts and subject. However, most of these studies conducted on ALEKS have been on its use as a supplemental or remediation tool in a traditional, web-enhanced or hybrid environment. These learning environments have the potential of exposing the students to other sources of additional math tutoring beside ALEKS. Also, none of the previous research studies investigated the complexity of mathematical tasks enacted by ALEKS in the pretest and posttest assessments. Thus, this study was designed to look at the effect of ALEKS on mathematics achievement in an online learning environment and the cognitive complexity of the pretests and posttests.

Significance of this Study

This study is significant for four reasons. Online courses and their associated intelligent tutoring systems are growing across disciplines; hence, the findings of this research will inform instructional designers, faculty teaching mathematics online, students learning mathematics online and concerned administrators.

First, knowing the effect of ALEKS on students' mathematics achievement will help instructional designers incorporate ALEKS in their design of online mathematics courses.

Second, faculty armed with information on the effect of ALEKS on students' mathematics achievement and the cognitive complexity of mathematical task enacted by ALEKS in the pretest and posttest assessments will be in a better position to guide students' learning during their online learning experience.

Third, students are more likely to gain in learning and retaining mathematical knowledge when the decision to adopt an ITS for teaching and learning mathematics online is based on research.

Fourth, the findings will also provide useful information for the administrators interested in increasing retention through reducing attrition rate in mathematics courses taught online.

Purpose of the Study

A number of studies show that ALEKS users have performed equally or better in mathematics achievement than the group who did not use ALEKS (Allen, 2007; Hagerty & Smith, 2005; Hampikian et al., 2006; Hanna & Carpenter, 2006; Hu et al., 2008; Lavergne, 2007; Taylor, 200). However, none of these studies have specifically investigated the effect of ALEKS on students' achievement in mathematics in online environments or investigated the complexity of mathematical tasks enacted by ALEKS in the pretest and posttest assessments. Hence, the purpose of this research was to investigate the effect of ALEKS on students' achievement in mathematics in an online environment and to determine the cognitive complexity for mathematical tasks enacted by ALEKS' pretest and posttest assessments.

Research Questions and Hypothesis Testing

This research tested one null hypothesis and attempted to answer two questions.

Hypothesis

H_0: There is no difference between students' achievement as measured by students' scores on pretest (Baseline Assessment) and posttest (Final Assessment) from ALEKS.

Question I

What are the factors contributing to students' mathematics achievement in using the ALEKS?

1. Is there a relationship between weekly Concept Mastery and the achievement score in weekly formative assessments?
2. Is there a relationship between the Time Spent in ALEKS per week and the achievement score in weekly formative assessments?
3. Is there a relationship between the Total Time Spent in ALEKS and Final Concept Mastery?
4. Is there a relationship between the final Concept Mastery score and the Posttest?
5. Is there a relationship between the Total Time Spent in ALEKS and the Posttest scores?

Question II

What is the cognitive complexity of mathematical tasks enacted by ALEKS on the pretest and posttest assessments?

Theoretical Framework

The theoretical framework that guided this study was based on Falmagne, Cosyn, Doignon, and Thiery's (2004) Knowledge Space Theory (KST) and Norman Webb's (1997) Depth of Knowledge (DOK). This section will discuss these theories and the reason for selecting them for this study.

Knowledge Space Theory

By default, KST is the framework underlying ALEKS's design. KST explains how to reveal a learner's knowledge structures and achievement in a particular subject domain, in this case mathematics (Falmagne et al., 2004). The two major concepts of KST are the 'knowledge state,' a particular set of problems or skills that some individual is capable of solving or performing correctly, and the 'knowledge structure,' which is a collection of these knowledge states (Conlan, O'Keeffe, Hampson, & Heller, 2006). Before learning commences, ALEKS uses the principles of the KST to determine the knowledge state of the student in the subject domain and ultimately creates a knowledge structure from that knowledge state.

Depth of Knowledge (DOK)

Adopted to guide the analysis of ALEKS' pretest and posttest assessments' cognitive complexity is Webb's (1997) Depth of Knowledge. DOK is the degree of depth or cognitive complexity of knowledge required by standards and assessments; cognitive complexity refers to the cognitive demand of tasks associated with the standards (Florida

Department of Education, 2008). There are different DOK for different content areas. The content area for this research study is introductory college algebra. The DOK descriptors for mathematics are shown in Appendix A.

There are four levels of depth of knowledge for mathematics: Level one – Recall; Level two-Basic Application of Skills and Concept; Level three-Strategic Thinking; and Level four-Extended Thinking (Webb, Depth of Knowledge Levels for Four Content Areas, 2002). These levels are used to ensure that the intent of the standard and the level of student demonstration required by that standard match the assessment items. As further discussion will show, each level of the depth of knowledge is similar to Bloom's (1956) taxonomy of learning and the cognitive demand for mathematical task of Stein, Smith, Henningsen, and Silver (2000). DOK descriptor was used to determine the cognitive complexity of mathematical tasks enacted by ALEKS in pretest-posttest assessment.

Finally, the goal for using the DOK models to frame this research study was to provide explanation for cognitive skill assessment used by ALEKS and to determine the cognitive complexity of mathematical tasks enacted by ALEKS on the pretest and posttest assessments.

Operational Definitions of Terms

College Mathematics I

This course is the first half of the college algebra sequence, which is completed in College Mathematics II.

Doing Mathematics

This is work that involves mathematical tasks that require complex and non-algorithmic thinking.

Final ALEKS Concept Mastery Report

The ratio of the number of the total topics learned for the course to the total number of topics assigned for the course expressed in percentage.

Formative Assessment

Weekly quizzes administered by ALEKS at the end of each week.

Hypermedia

This is a hypertext which is not constrained to be text: it can include graphics, video and audio.

Hypertext

This is a text which contains links to other texts.

Intelligent Tutoring System (ITS)

According to Polson and Richardson (1988), an "Intelligent Tutoring System (ITS) is a computer program that: 1) is capable of competent problem solving in a domain, 2) can infer a learner's approximation of competence: and 3) is able to reduce the difference between the competence of ITS and that of the student through application of various tutoring strategies.

Online Learning Environment

This term describes education that occurs only through the Web. That is, it does not consist of any physical learning materials issued to students or actual face to face contact. Purely online learning is essentially the use of eLearning tools in a distance education mode using the Web as the *sole* medium for all student learning and contact.

Online Learning System (OLS)

An Online Learning System (OLS) is a learning management system that can be any form of educational material, which is readily available for distribution on the Web or privately over an internal network.

Posttest or Summative Assessment

This is ALEKS' Final Assessment (scheduled assessment) administered at the end of the course.

Pretest

This is ALEKS' Initial Assessment administered at the beginning of the course before learning begins.

Ready to Learn

This is the most efficient path provided by ALEKS to the student in order to master the domain of learning.

Summative Assessment

ALEKS' final assessment or posttest assessment administered at the end of the course.

Total Time in ALEKS Report

The total number of hours spent learning in ALEKS for the course.

Weekly ALEKS Concept Mastery Report

The ratio of the number of topics learned for the week to the total number of topics assigned for the week expressed in percentage.

Weekly Time in ALEKS Report

The number of hours spent learning in ALEKS per week.

Summary

The purpose of this chapter was to discuss the role of computers in distance education, the problem statement and rationale for the study, the significance of this study and the purpose of the study, theoretical framework and operational definitions of terms used. The advancement of the Internet technologies has promoted distance education and the use of Internet Intelligent Tutoring Systems such as ALEKS in teaching and learning. ALEKS has provided a medium for the teaching and learning of mathematics. With all the challenges facing the teaching and learning of mathematics, the use of ALEKS has shown to be effective to students learning mathematics traditionally and in a hybrid learning environment.

This study was focused on students who were studying mathematics in an online environment. I investigated the effect of ALEKS on students' mathematics achievement and in addition determined the cognitive complexity for mathematical tasks enacted by ALEKS in pretest and posttest assessments.

The skill assessment technology used in ALEKS is based on KST; thus, the cognitive complexity for mathematical tasks enacted by ALEKS on the pretest and posttest assessments was determined by using the DOK model. Finally, to facilitate the collection of meaningful data, the operational definition section described terms as it applies to this research study.

CHAPTER 2

REVIEW OF THE LITERATURE

The purposes of this chapter are to establish the importance of the study and to provide a benchmark for comparing the results of this study. Hence, the review of literature starts by looking at the conceptual framework of the theory and model that frames this research: Knowledge Space Theory (KST) and Norman Webb's Depth of Knowledge Model (DOK). Then from the instructional design perspective, I discuss task analysis as it pertains to this research study**.** I also identify Computer Based Learning Environments in Mathematics; describe the ITS, ALEKS; and discuss the use of ALEKS in teaching and learning mathematics and related subjects at different institutions.

KST and DOK Model

A theoretical framework serves as a basis for conducting research, while a conceptual framework shows the operationalization of such theories (Khan, 2007). This research study was framed by KST and DOK models: KST explains how to reveal a learner's knowledge structures and achievement in a particular subject domain while Depth of Knowledge provides the degree of depth or cognitive complexity of knowledge required by standards and assessments. In this section, I discuss KST, the DOK model, task analysis and their operationalization in this study.

Knowledge Space Theory

In 1985, Falmagne, Doignon, and associates developed a theory of knowledge representation called Knowledge Space Theory. KST is based on precedence relation. It is evident, especially in mathematics, that some levels of knowledge normally precede other levels because of prerequisite requirement, logical steps or pedagogical ease.

According to Falmagne et al. (2004), precedence relation may be used to design effective and efficient assessment mechanics. There are three assumptions in precedence relations:

1. From mastery of one problem, the mastery of other problems is assumed or "surmised."
2. Dependency relations exist between problems of a set.
3. If a learner is capable of mastering a problem *d*, then he or she will also be capable of mastering problems *b* and *c.* (see Figure 1)

KST-based System

Besides ALEKS, another KST based system is the Relational Adapting Tutoring Hypertext (RATH). RATH version 0.1 is a prototype for a Relational Adaptive Tutoring Hypertext in WWW-Environment (Hockemeyer, Held, & Albert, 1998). According to Hockemeyer et al., Relational Adaptiving Tutoring Hypertext combines mathematical models for the structure of hypertext with the theory of Knowledge Spaces from mathematical psychology; it uses prerequisite relationship and items in the domain knowledge and student's current knowledge state to present the student with links in a hypertext document for which the student fulfills the prerequisite relationship. Consequently, the student should be able to understand the information provided by the linked sites. This first prototype of RATH was applied to the field of elementary probability theory.

Application of KST in Science

Tóth (2007) used KST analysis to answer the following research question: Is there any similarity or difference between the students' groups from two different secondary schools in the cognitive organization of the basic concepts of density, mass percent,

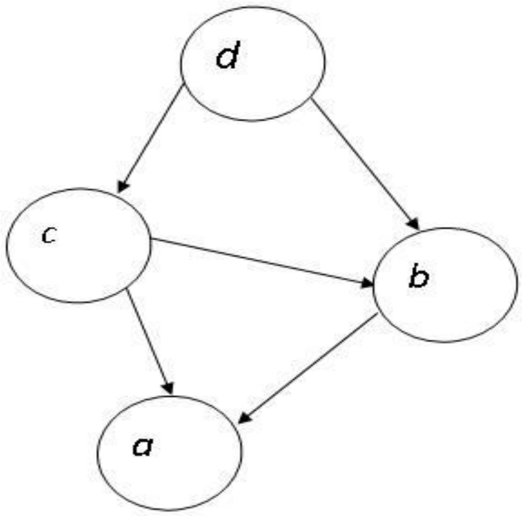

Figure 1. Precedence diagram for four types of algebraic skills. Precedence relations between problems are represented by downward arrows. Problem (*d*) is preceded by problems (*b*), (*c*), and (*a*). The mastery of problem (*d*) implies the mastery of (*b*), (*c*), and (*a*).

molar mass, molar volume and their application in calculations? The first group used KST to map students' knowledge structures in calculating density, mass-percent; molar mass and molar volume while the second group learned the concepts of density, molar mass, molar volume and mass percent by rote-learning using mnemotechnics. With the first group, there was a strong connection between the concepts of density, molar mass, molar volume and the calculation of gas volume, while with the second group there was no such connection. The research concluded that the reason for this disconnected cognitive structure is the difference in the learning method between the two groups.

Taagepera et al. (1997) used KST analysis to construct students' knowledge structures and suggested tentative critical learning pathways for each of three concepts (pressure, density and conservation of matter). For pretest, the same multiple-choice questions were administered to all (4th through 12th graders) before the topics were formally taught and a posttest was given to the same grade levels. KST analysis was used to construct knowledge structures and suggested learning path. The result found KST as a valuable quantitative assessment method for evaluating and suggesting the most feasible learning pathways taken by the students.

Arasasingham et al. (2004) used KST to assess student understanding of stoichiometry by examining the ability of beginning college chemistry students to make connections among the molecular, symbolic, and graphical representations of chemical phenomena, as well as to conceptualize, visualize, and solve numerical problems. Students took a test designed to follow conceptual development; the cognitive organization of the material or thinking patterns was analyzed by applying knowledge space theory. The results indicated that KST was a useful tool for revealing various aspects of students' cognitive structure in chemistry and could be used as an assessment tool or as a pedagogical tool to address a number of student-learning issues.

Illustration of Knowledge State

In KST, an *Item* is considered the basic unit of knowledge. In this research study, an *Item* could be a mathematics problem requiring varying skills of difficulty, a set of math problems, tasks like graphing, or an applied problem or problems. A body of knowledge consists of a set of items called a *Domain*. An example of a *Domain* would be an instructional unit, such as lesson, topic or subtopic with a learning goal. The following example of learning the quadratic formula concept illustrates this concept of KST (see Table 1). As is shown in later sections, the terms *state*, *use, apply* and *integrate* would be similar to *knowledge*, *comprehension*, *analysis* and *evaluation* levels of Bloom’s taxonomy of learning.

Table 1

Four Types of Skills in Learning the Quadratic Formula

a.	*b.*	*c.*	*d.*
State Quadratic formula	Use Quadratic formula	Apply Quadratic formula	Integrate Quadratic Formula

The student's *knowledge state* is defined as the collection of items the student is capable of performing (Giovanni, Roberto, & Riccardo, 2008). For example, the knowledge state (*a, b, c*) corresponds to a student who can perform Items *a, b* and *c* but who cannot perform Item *d*. Not all subsets of items are considered to be feasible states (Villano & Bloom, 1992). For example, if a student is capable of performing Item *d* then one may be able to infer that the student can perform (Item *b*) and thus, any state that contained Item *d* would contain Item *b* also. One also might not expect to find a student who could perform Item *d* but none of the other items; thus (*d*) would not be considered a feasible state. The collection of all feasible states is called the *knowledge structure.* A knowledge structure must contain the null state, Ø, which corresponds to the student who fails all the items, and the domain (*Q*), which corresponds to the student who has mastered all the items. An important special case of a knowledge structure called Knowledge Space occurs when the collection of knowledge states is closed under union (Albert & Hockemeyer, 1997a). The application of the knowledge space framework for skill assessment and tutoring makes it possible to obtain learning paths that describe the knowledge paths from the total novice learner through the knowledge space to the complete expert of the given domain. A student is capable of changing its knowledge state by following a learning path (Giovanni et al., 2008).

An analysis of the precedence diagram shows there are six different feasible knowledge states induced by the surmised relationship: $K = \{ \emptyset, \{a\}, \{ab\}, \{abc\}, \{abcd\}, Q\}$

Learning Paths

The knowledge structure allows several learning paths. An example of possible learning paths to this knowledge structure for the four items *a, b, c, d* are

1: $\{\emptyset\} \subset (a) \subset \{a, b\} \subset \{a, b, d\} \subset \{a, b, d, c\}$.

2: $\{\emptyset\} \subset (a) \subset \{a, b\} \subset \{a, b, c\} \subset \{a, b, c, d\}$.

3: $\{\emptyset\} \subset (a) \subset \{a, c\} \subset \{a, c, d\} \subset \{a, c, d, b\}$.

4: $\{\emptyset\} \subset (a) \subset \{a, c\} \subset \{a, c, b\} \subset \{a, c, b, d\}$.

Outer Fringe of a Knowledge State

With the exception of the topmost knowledge state in the knowledge structure, each knowledge state has at least one immediate successor. For example, the knowledge state *abc* in the knowledge structure *k*, has *abcd* as immediate successor. In this case, the item *d* is considered an outer fringe of *abc*. Teaching, learning and knowledge acquisition take place in the outer fringe.

Inner Fringe of a Knowledge State

Also with the exception of the empty state, each knowledge state has at least one predecessor state which is the state containing exactly the same problems except one. For example, the knowledge state *abc* has *ab* as predecessor. The item *c* is considered an inner fringe of the knowledge state *abc*. If a student is having a problem mastering the outer fringes, reviewing the previous states takes place in the inner fringes.

Validation of Knowledge Spaces

To build knowledge structures, subject matter experts such as teachers and text-book writers are questioned for prerequisite relationships. Using a computer-aided procedure, the experts are queried on prerequisite relationships in learning objectives from the particular concept. From the judgments of each expert, a precedence diagram and a knowledge space representing these prerequisite relationships are derived. The experts' knowledge spaces are integrated subsequently into knowledge spaces representing only those prerequisite relationships on which all, or a majority, of the experts agreed. For validating subspaces of these knowledge spaces, test data from actual students are collected and used to refine the knowledge structure obtained from the experts. The results from querying the experts and from the validation study are used to advance application of these knowledge spaces for knowledge assessment of students (Baumunk & Dowling, 1997).

Uncovering Knowledge State in a Knowledge Structure

With the knowledge space in place, the students are subjected to an assessment procedure with specific questioning and updating rules to help uncover their knowledge states. Each response (Correct, Incorrect, or Don’t Know) increases or decreases the likelihoods of certain knowledge states. The system at the same time keeps track of the uncertainty of the assessment system regarding the student’s knowledge state. When there are no more useful questions left to ask and the uncertainty of the assessment system regarding the student’s knowledge state is at its lowest, the assessment stops and the computer selects the most likely knowledge state for the student. When there is no more useful question, it means that all problems have either a very high probability or a very low probability of being answered correctly. This process ensures that few knowledge states are left to be selected. Because of the random nature of the assessment, it is very likely that the selected knowledge state may contain problems to which the student gave a false response because of careless errors. Additionally, all problems are open-ended (no multiple choice), with multiple possible solutions, and minimal correct guesses.

In summary, KST shows how to capture a learner's knowledge state for instructional intervention. KST can be used for the assessment of misconceptions and mental states for guidelines, for a description of knowledge acquisitions and for the definition and design of intelligent tutoring system (Lukas & Albert, 1999). Because ALEKS uses KST to access students' knowledge before providing instructional intervention, it is a KST-based system.

Norman Webb's DOK Model

Norman Webb (2002) of the Wisconsin Center for Educational Research, University of Wisconsin–Madison stated that the alignment of the content standards for student learning with assessments for measuring students' attainment of these expectations is an essential component of an effective standards-based education system. Project Lead the Way (PLTW) (2008) used Webb's DOK to complete a depth of knowledge analysis on a course called Introduction to Engineering Design (IED). Minnesota Department of Education (MDE; 2007), Idaho Mathematics Content Standards (2007) and Florida Department of Education (FDE; 2008) have adopted the DOK model in the different content areas. This section discusses how the DOK model was used in the IED's depth of knowledge analysis and the alignment of standards and assessments at different school districts.

PLTW (2008) used Webb's DOK model to complete a depth of knowledge analysis on an IED course. The analysis is intended to provide feedback to PLTW leaders regarding the relative level of cognitive rigor promoted as established in the course objectives. According to PLTW (2008), all course objectives were reviewed to identify those objectives that most emphasized mathematics and/or science concepts and skills. Eventually nationally recognized standard frameworks for both science and mathematics were used to guide the categorization process. In the original analysis each course objective was assigned a score using Webb's (1997, 2002) Depth of Knowledge model.

Using descriptive statistics, the DOK levels assigned to objectives on mathematics or science were analyzed. The result showed that out of 168 objectives in the IED course, 108 (64.28%) were identified for emphasizing one or more of the mathematics standards established by the National Council of Teachers of Mathematics (NCTM); 114 (67.85%) objectives were identified for emphasizing one or more of the stated science standards established by the National Research Council.

The MDE alignment study of its Minnesota Comprehensive Assessment-II (MCA-II) for grades 3-8 and 11 used procedures based on the DOK's alignment model developed by Webb (1997). The methodology for this alignment used an independent panel of experts to examine MCA-II tests in mathematics and the corresponding state content standards for mathematics. The state benchmarks and core test items from the MCA-II math tests were rated at three different cognitive levels followed by a mapping of a test item to each benchmark. These ratings were variously applied to four alignment criteria: cognitive consistency, categorical concurrence, range-of-knowledge, and balance-of-representation. According to the report, *Cognitive consistency* compared coded ratings of cognitive complexity in each content standard and test item, while *Categorical concurrence* provided a very general indication of whether both tests and standards incorporate the same content. *Range-of-Knowledge* was used to examine whether a comparable span of knowledge expected of students by a standard is the same as, or corresponds to, the span of knowledge that students need to correctly answer the assessment items. *Balance-of-Representation* was used as a proportional index that represents the distribution of content domains between content standards and assess-ments. The results showed that the 2006 MCA-II were highly aligned for categorical concurrence and range-of-knowledge but alignment for cognitive consistency and balance-of-representation had mixed results.

Idaho Mathematics Content Standards (2007) report consists of a description of the four criteria used to judge the alignment between grades 3 through 8 and 10 Idaho content standards, and the test questions found in the mathematics Idaho Standards Achievement Tests (ISAT). According to the report, the mathematics content standards were used to describe the expectations for what students are to know and do. The reviewers determined the alignment of test questions to the five content standards using DOK's model as the platform for the alignment; the final results of this study indicated that there was alignment between the Idaho mathematics Grade 3 through 8 and 10 content standards, goals, and objectives and the mathematics ISAT.

The Florida Comprehensive Assessment Test (FCAT; 2008) is based on DOK Cognitive complexity, or the cognitive demand associated with an item. According to the report, in the early years of the FCAT program, the FDE used Bloom's Taxonomy to classify test items but changed in 2004 to a new cognitive classification system based upon Webb's DOK levels. The rationale behind classifying items by their level of complexity is to focus on the expectations of the item, not on the ability of the student. The result of this classification is that items are chosen for the FCAT based on standards and grade-level appropriateness, but the complexity of the items remains independent of the particular curriculum a student has experienced.

The FCAT report identified three categories: low complexity, moderate complexity, and high complexity to form an ordered description of the demands an item may make on a student. Low complexity items may require a student to solve a one-step problem; moderate complexity items may require multiple steps, while high complexity items may require a student to analyze and synthesize information. These distinctions made in an item complexity ensure that items will assess the depth of student knowledge at each benchmark.

Webb (1997) has developed a process for aligning standards and assessments; in addition, the process and criteria have demonstrated application on analyzing the depth of knowledge and reviewing of curricular alignment as well as cognitive rigor in assessment items. Webb's body of work offers the Depth of Knowledge (DOK) model a platform employed to analyze the cognitive expectation demanded by standards, curricular activities and assessment tasks (Webb, 1997). DOK's model is based upon the assumption that curricular elements may all be categorized based upon the cognitive demands required to produce an acceptable response and that each grouping of tasks reflects a different level of cognitive expectation, or depth of knowledge, required to complete the task (PLTW, 2008).

Hence, I selected the DOK model as a basis for use in this research study for several reasons. It can answer the question “What is the cognitive complexity of mathematical tasks enacted by ALEKS on the pretest and posttest assessments?” The DOK model focuses on complexity rather than difficulty of a test item. For example, a level one task can ask the student to recall or restate a more complex concept making the latter more difficult. The rationale for classifying items by their level of complexity is to focus on the expectations of the item, not the ability of the student (Florida Department of Education, 2008).

According to Hess (2008), depth of understanding of a concept is required to be able to explain how/why a concept works (level two), apply it to a real world situation with justification and supporting evidence (level three), or to integrate one concept with other concepts or other perspectives (level four). Consequently, DOK levels are used by schools districts to develop curriculum materials and performance assessments to demonstrate learning.

In addition to DOK’s four levels, the model also defines four alignment criteria: DOK consistency, categorical concurrence, range-of knowledge correspondence, and balance of representation (Webb, 2002, p. 3). Webb defined alignment as the degree to which expectations and assessments are in agreement and serve in conjunction with one another to guide the system toward students learning of what they are expected to know and do. For the purpose of this study, Webb’s criterion of DOK consistency is used. According to Webb (2002), DOK consistency between content standards and test items indicates alignment if what is elicited from students on the test is as demanding cognitively as what students are expected to know and do as stated in the content standards. In this research study, I looked at whether what is elicited in the pretest and posttest items was as cognitively demanding as what the students were expected to know or do.

Hence, in this research study, the cognitive complexity of pretest and posttest items was determined by aligning the pretest and posttest items with Webb's (1997) DOK levels. In the next section, I discuss Webb's (1997) DOK cognitive domain, Stein, Smith, Henningsen, and Silver's (2000) cognitive demand for mathematical tasks, and Bloom's (1956) taxonomy.

Cognitive demand addresses the kind of thinking processes involved in solving a given task (Zurawsky, 2006). In mathematics, these thinking processes include memorization, the use of procedures, algorithms and formulas, and complex thinking and reasoning strategies that would be typical of "doing math," such as conjecturing, justifying and interpreting (Henningsen & Stein, 1997). Stein et al. (2000) defined cognitive demand for a mathematical task as "the kind and level of thinking required of students in order to successfully engage with and solve the task" (p. 11). The model of Stein et al. delineates four categories of cognitive demand for tasks: lower-level demands of Memorization and Procedures without Connections, and higher-level demands of Procedures with Connections and "Doing Mathematics." "Doing Mathematics" is more than mere calculations and deductions; it includes observation of patterns, testing of conjectures and estimation of results (Schoenfeld, 1992).

Bloom's (1956) taxonomy is a classification system of educational objectives based on the level of student understanding necessary for achievement or mastery. Bloom and colleagues have suggested six different cognitive stages in learning. These categories are Knowledge, Comprehension, Analysis, Application, Synthesis, and Evaluation. For the purpose of this research study, Table 2 shows that different learning objectives as shown by Bloom require different cognitive demands for mathematical tasks and different cognitive complexity levels of Norman Webb's Depth of Knowledge.

Because of the dependence relationship in mathematics, it is important that the learning objective of every concept/topic be in higher percentage. In this study, ALEKS uses the Concept Mastery Report (percentage indicating the mastery of concept/topic) to show the level of mastery of each student before advancing to the next concept/topic. The expectation is that outstanding concept mastery would reduce frequent reviewing of previous concepts and increases the student's progression along the learning path provided by ALEKS. Those students with good mastery of the previous concept/topic are likely to succeed in the next concept/topic. One of the objectives of this study is to show the relationship between Concept Mastery (ALEKS Mastery Report), Quizzes and posttest.

In summary, the operationalization of these theories that form this research shows that ALEKS is a KST-based system. DOK levels provide a platform to determine the cognitive complexity of pretest and posttest items. Additionally, similarities have been shown between Webb's (1997) Depth of Knowledge levels, Bloom's (1956), taxonomy of learning objectives, and cognitive demand for mathematical tasks of Stein et al. (2000).

Table 2

Relationship between Webb's DOK Model, Cognitive Demand for Mathematical Tasks, and Bloom's Taxonomy Level of Learning

Webb's DOK Cognitive Domain	Cognitive Demand for Math Tasks Categories	Bloom's Cognitive Domain	Objectives	Learning Outcomes Example: Quadratic formula
Recall	Memorization	Knowledge	State	Learner memorizes the formula
Basic Applica-tion of Skills and Concept	Procedures without connections to concepts or meaning	Comprehension	Use	Learner knows how to substitute values into quadratic formula and come up with answer(s)
Strategic Thinking	Procedures with connections to concepts or meaning	Analysis Application	Differentiate and Apply	Learner is able to break down the formula and apply it to another areas
Extended Thinking	Doing Mathematics	Synthesis/Evaluation	Integrate and Judge	Integrate the formula to other similar problems and judge appropriateness of its use/solution

Task Analysis

"Task analysis for instructional design is a process of analyzing and articulating the kind of learning that you expect the learners to know how to perform" (Jonassen, Tessmer, & Hannum, 1999, p. 3). As noted by Jonassen et al., the process of task analysis, which developed from the behaviorist era, has followed the paradigm shifts from cognitivism onto constructivism, but, regardless of the learning theory, a task analysis is needed for an in-depth understanding of the learning that is to take place. The purpose of task analysis in this study was to define and describe the tasks, subtasks, and sequence or path of instruction that would best facilitate learning. Based on this purpose, the appropriate format of task analysis for this study was hierarchical task analysis.

"A hierarchy is an organization of elements that, according to prerequisite relationships, describes the path of experiences a learner must take to achieve any single behavior that appears higher in the hierarchy" (Seels & Glasgow, 1990, p. 94)". Thus, in a hierarchical analysis, the instructional designer breaks down a task from top to bottom, thereby, showing a hierarchical relationship amongst the tasks, and then instruction is sequenced from the bottom up. A hierarchical task analysis (also known as a prerequisite task analysis) answers the following question: "What must the learner know or be able to do to achieve this task?" Some of the principles that set hierarchical task analysis apart from the other formats of analysis are as follows:

1. A hierarchical task analysis is developed from bottom up, from general to specific.
2. A hierarchical task analysis is based on learning taxonomies, starting from the most complex to the least complex. The nature of the terminal task determines at which level in the taxonomy one should start breaking down the task from more complex to less complex, going through each of the learning levels.
3. A hierarchical task analysis is represented in terms of levels of tasks. Each level should (more or less) represent one learning level (e.g. problem-solving, concept learning, etc.). The highest level is the most complex. Lower levels form prerequisite skills for higher levels. Lines connect tasks

between levels. Each task can be broken down into one or more tasks from one level to the next.

4. A hierarchical task analysis is read bottom-up. Arrows pointing upwards are used to connect the tasks towards the terminal task.

5. In a hierarchical analysis, each task is a prerequisite to the task directly above it. Tasks that can happen concurrently with other tasks are put on the same level in the hierarchy.

Learning Hierarchy Analysis

When an instructional objective indicates that the learner will use a concept, apply a rule, or solve a problem, a learning hierarchy analysis can identify the prerequisite skills to perform that objective (Jonassen et al., 1999). A learning hierarchy shows prerequisites in an ordered relationship where lower skills on the chart will be learned before the higher-ranking ones until the objective is met. Because of the prerequisite relationship, learning hierarchy analysis is also referred to as prerequisites analysis.

In 1962, Robert Gagne introduced the learning hierarchy concept. The basis for the concept of learning hierarchy is a dependence relationship among intellectual skills which stipulates that there are a set of prerequisite skills for any higher order intellectual skill and the mastery of prerequisite skills facilitates learning of higher skills. According to White and Gagne (1978), the development of relationship among intellectual skills has made the method of constructing a learning hierarchy an ideal method for analyzing instructional content, particularly when instructional designers are faced with the task of developing instructional material.

In addition, developing a learning hierarchy defines what must be taught and the sequence in which to teach it. For example, in Figure 2, task four of integrating quadratic formula to other similar problems and judging the appropriateness of its use/solution has been decomposed into the following enabling tasks: task three of application of quadratic formula to other areas, task two of substituting values into the quadratic formula and coming up with answer, and task one of memorizing of the quadratic formula. The implication is that the learner cannot perform the third task until he/she has performed the first and second tasks respectively.

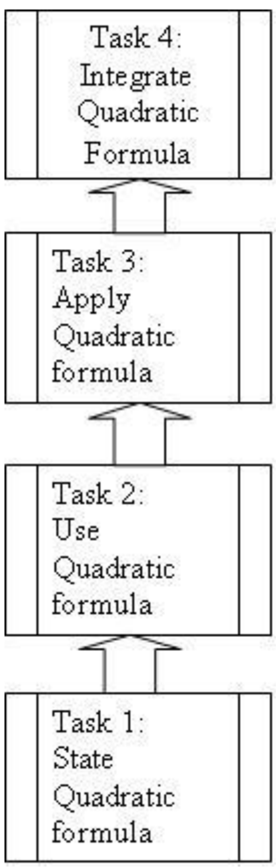

Figure 2: Hierarchical Relationship among the Tasks

Learning hierarchy analysis is appropriate for this study because it shares similar concepts with the Knowledge Space Theory. Both KST and learning hierarchy analysis are based on dependency and prerequisite relationship among intellectual skills. In each case, mastering the lower intellectual skills facilitates the learning of the higher intellectual skills. In addition, learning hierarchy analysis explains the instructional activities and strategies that determine the sequence of course content.

Computer Based Learning Environments in Mathematics

Handal and Herrington (2003) identified different categories of computer-based learning in mathematics and their associated learning outcomes. They argued that the sequence of progression from Drills and Tutorials to Games and Simulation and finally to Hypertext and Hypermedia based instruction is reflective of the progression from behaviorist to constructivist learning approaches. Behaviorism is based on observable changes in behavior; Cognitivism is based on the thought process behind the behavior, and Constructivism is based on the premise that individuals all construct their own perspectives of the world through individual experiences and schema (Ertmer & Newby, 1993).

Drills and Tutorial

Drills and tutorial are based on behaviorist learning philosophy and are used to teach declarative skills (Lawrence, 1997). The expected outcome is that the gradual increase in the difficulty of drilling activity will also increase the mathematical know-ledge of the students. Tutorials are enhanced drill and practice activity because they provide guidance, structure, sequence and immediate feedback. Drills and tutorial have the advantage of filling in for the instructor, providing individualized instruction, and supporting already learned material. While providing opportunities to enrich the understanding of mathematical concepts, drill and tutorial have the potential to use multimedia capabilities to motivate students in an online learning environment (Handel & Herrington, 2003).

According to Hasselbring's (1988) report, when prior training for developing a declarative knowledge network is implemented, using computer-based drill and practice is effective in developing mathematics automaticity or fast recall of mathematics facts in learning for handicapped children. An experimental mathematics program, which was called "Fast Facts," successfully developed the recall of basic mathematics facts in 160 mildly handicapped and nonhandicapped students aged 7-14.

Behaviorism is based on observable changes in behavior, and the mind is treated as a passive black box that receives knowledge by transmission (Mergel, 1988). Behaviorist learning models include drill and practice and programmed instruction. One of the instructional design approaches includes generative computer assisted instruction (CAI). Generative instructional strategy is similar to showing flash cards, and feedback is either right or wrong. Urban-Lurain (2004) discussed the progression from CAI to ITS. With the strides made in the Artificial Intelligence (AI) community in the 1960s and 1970s, CAI improved from generative to adaptive, but the adaptive nature used only observable behavior and not the knowledge state of the learner. During the same period, cognitive scientists started looking at how the brain emerges from the mind in the form of information processing. The merging of research from cognitive science and artificial intelligence led to the development of ITS from CAI.

Games and Simulation

The concept of information processing led into instructional design approaches that are based on cognitive learning theory and advancement in using structured games and simulations to assist in the learning of mathematics concepts. Games are goal-oriented activities that use multimedia technology to simplify reality, while simulations are used to facilitate learning through artificial situations when it is not possible to perform the real situation (Handal & Herringhton, 2003). Games are governed by rules that involve competition with win or lose situations. Certain skills and practices are assumed in order for the learner to win. Simulation is very similar to gaming in that it is goal oriented but also different in that there are no rules or competition. The idea is that the learner will gain some knowledge while playing the game or while following the simulation. Games and simulations are structured to follow the cognitive learning model, and participants gain factual information and learn procedural sequences (Walcott & Walcott, 1976).

Lucas (1974) investigated the effect of using simulation-gaming techniques on the acquisition and cognitive retention of concepts, facts and principles in a study with 295 participants. The experimental group received instruction in a simulation gaming technique, while the other group received instruction in a lecture-discussion format. Even though both groups did well in the cognitive achievement, the students in the experimental group did better in delayed posttest results, showing simulation-gaming as a teaching tool that enhances learning.

Hypertext and Hypermedia

Hypertexts provide clickable buttons to other nodes, and implementation of good principles of hypertext interface design helps avoid navigational problems and hence maximize learning (Koneman & Jonassen, 1994). Hypermedia Based Instruction (HBI) is based on the constructivist philosophy of learning (Gabbard, 2000). It is very similar to previously discussed computer based instructions. The major difference is that while the computer-based instructions present information in structured and linear sequence, hypermedia present information in a node-and-link structure by mixing hypertext and multimedia. The use of hypertext and hypermedia introduces teachers to two innovations that offer students an opportunity to create their own meaningful learning environments (Blanchard & Rottenberg, 1990).

Hypermedia Based Instruction is closely related to constructivist learning principle and has been claimed to be very effective and successful in reaching a variety of learning styles because it is more media rich than the traditional computer based instructions (Handal & Herringhton, 2003). According to Handal and Herringhton, clickable thesauruses or dictionaries embedded within a learning environment are examples of HBI applications.

Intelligent Tutoring Systems

According to Polson and Richardson (1988), an "Intelligent Tutoring System (ITS) is a computer program that: 1) is capable of competent problem solving in a domain; 2) can infer a learner's approximation of competence; and 3) is able to reduce the difference between ITS' competence and the student's through application of various tutoring strategies". An ITS is made up of four major components: Expert Model, Student

Model, Pedagogical Model and Task Environment (Corbett, Koedinger, & Anderson, 1997; see Figure 3).

The Expert Model or the domain model is used to store the knowledge of the instructional domain and interpret student's solution (Pramuditha, Antonija, & Brent, 2006). In an educational environment, this would represent the course, such as College Algebra. ITS uses the processes of knowledge engineering to build the concepts that are contained in the Expert Model. Based on the content of the expert model, the Student Model diagnoses, stores, and tracks a student's cognitive state in the subject matter.

The Student Model uses an Overlay, Differential, or Perturbation principle to perform the diagnosis while the Pedagogical model provides tutoring support through diagnostic and didactic support (Urban-Lurain, 2004). In order to determine the student's present cognitive state, the Overlay principle treats the student's present knowledge of math as a subset of the expert model. The Differential principle uses the missing concept from the student's math knowledge to determine the student's present math cognitive state. The Perturbation principle uses the student's misconceptions in the student's math knowledge to determine the student's present math cognitive state.

The Pedagogical Model or teaching model provides instructional interventions taking into account the knowledge base and the student model (Albert & Schrepp, 1999). Two kinds of instructional support are available**:** diagnostic support and didactic support. Levels of diagnostic support are Behavioral, Epistemic and Individual. Behavioral support is based on observable behaviors only; epistemic support is based on both observable behavior and the knowledge state of the student, while Individual support involves observable behaviors, knowledge state and the affective behaviors of the

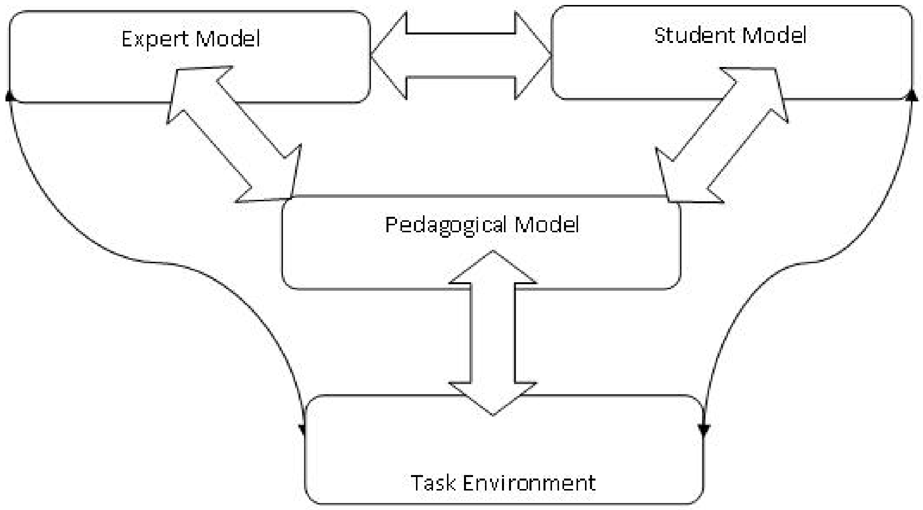

Figure 3: Components of an Intelligent Tutoring System.

student. Didactic supports are derived from curriculum and instruction. Curriculum support handles the scope and sequence of the course while instructional support covers issues such as demonstration, monitoring, and exploration.

The Task Environment or the Interactive Human Computer Interface presents information to and receives information from the student (Albert & Shrepp, 1999). The principle design for the interface is contextualization and facilitation of learning. The ITS task environment calls for facilitation of learning in a contextualized environment. In a constructivist approach, knowledge is constructed with active participation and negotiation by the learner and the mind is treated as an active entity (Lefoe, 1998). Other constructivist models include generative learning, cognitive flexibility and cognitive apprenticeship. Compared to other constructivists' models, generative learning theory gives more emphasis on the generation of new conceptual understandings by the learner (Wittrock, 1990).

ITS leverages the generative theory by the way it asks students questions in so called application of concepts problems. Application problems are real life problems. Cognitive flexibility encourages the display of concepts in more than one form. Learning activities must provide multiple representations of content (Spiro, Feltovich, Jacobson, & Coulson, 1992). Because hypermedia supports the linking of graphics, audio, text and video elements in a node-like structure, its use in ITS gives it the cognitive flexibility.

Cognitive apprenticeship is a process where the master of a skill demonstrates that skill to an apprentice (Colins, Brown, & Newman, 1989). In this learning environment, the ITS assumes the role of a coach while the students are the apprentices.

In summary, generative learning, cognitive apprenticeship and cognitive flexibility play a very important role in how the ITS facilitates learning in an online environment. Generative learning encourages active participation of the student by the ITS, while cognitive apprenticeship would encourage the ITS to demonstrate and expect the students to do the same.

In this study, participants used a particular ITS, ALEKS, in learning mathematics. ALEKS is an intelligent tutoring system that was built on the work of a team of cognitive scientists, software engineers, and mathematicians from New York University and University of California at Irvine with funding provided by the National Science Foundation (ALEKS, 2010). ALEKS uses knowledge space theory to assess a student's knowledge state and prescribe targeted instruction on topics/concepts a student is ready to learn (Falmagne et al., 2004). ALEKS assesses the student's current course knowledge by asking the student a number of content area questions (usually 20-30 questions). ALEKS avoids multiple-choice questions but chooses each question on the basis of the student's answers to all the previous questions. By the time the student has completed the assessment, ALEKS has developed a precise picture of the student's knowledge of the course material, knowing which topics the student has mastered and which topics the student is ready to learn. The student's knowledge is represented by a multicolor pie (See Appendix B).

The pie chart is also the student's point of entry into the Learning Mode. The Learning Mode is the interface that provides instruction based on what the student is ready to learn. In the Learning Mode, the student is offered a choice of topics that he or she is ready to learn. Based on the diagnosis, the student has the prerequisite knowledge to learn these topics successfully. When a student is working on a particular problem, the student can access an explanation to that problem by clicking on the "Explain" button. The explanation typically provides a step-by-step solution to the problem, with commentary. In some cases, an alternative or more detailed explanation is also available. The student receives immediate feedback, suggestions for correcting mistakes or to improve student's progress such as looking up definitions in ALEKS on-line dictionary. ALEKS may propose that the student temporarily abandon the problem and work on a related problem. If the student is successful in solving the new problem, the system will generally offer two or three more instances of the same topic to make sure the student has mastered it. To ensure knowledge retention, ALEKS periodically reassesses the student, using the results to adjust the student's knowledge of the course.

ALEKS teaches mathematics through continuous involvement of the student. For example, ALEKS does not use True/False or Multiple Choice problems. Because ALEKS measures the learning rate from the active time in the learning mode, it shuts off after a certain time of non-activity in that mode. ALEKS expects continuous engagement and generation of the student's ideas while in the learning mode. As already discussed in the previous sections, depending on the student's learning preferences, instructional delivery demonstrated through ALEKS learning mode could be in different formats: text, audio, simulation and video. After an ALEKS demonstration of problem solving steps, the student is asked to work out a similar problem.

Before progressing to another topic, the ALEKS learning mode uses drill and tutorial to ensure concept mastery. This involves repeated questions on a particular concept or topic until ALEKS is satisfied that the learner has mastered the concept. For areas that use mathematical instruments, ALEKS simulates such instruments and such simulation environments include ALEKS's pencil, paper, ruler, and eraser. ALEKS uses hypertext to link together its supplemental and additional resources and to ensure a non-linear instructional sequence; ALEKS incorporates hypermedia-based instruction in the form of ALEKS's mathematics dictionary and thesaurus.

The Use of ALEKS in the E-learning of Mathematics

Researchers at several universities in the United States have published research results on the use of ALEKS. Hu, Luellen, Okwumabua, Xu, and Mo (2008), of the University of Memphis (UM), explored the effectiveness of using ALEKS to close the racial score gaps in an undergraduate behavioral statistics course. This observational study focused on 548 UM undergraduate students who completed a statistics course under the same professor between Spring 1995 and Fall 2005 terms. In their study, 137 students took the course in an online format that used ALEKS, and the other 411 students took the course in a conventional lecture format.

Hu et al. (2008) compared the academic performance of students from online ALEKS-using sections of statistics to a retrospective comparison group comprised of students who took statistics under a conventional lecture format. When the performance of students enrolled in the online ITS sections was considered, the racial disparity observed for Black and White students enrolled in the lecture formatted sections did not hold. The study reported that ALEKS closed the racial gap by eliminating one letter grade between groups of students in this course at this university.

In another study, Hanna and Carpenter (2006) used ALEKS to provide tutoring for precalculus students that were in Calculus I and II courses at Louisiana Technical University. Students were required to use the program outside of class for at least 3 hours a week and had to make 6% progress in new material learned with ALEKS each week. Students' progress was checked weekly. In the (Calculus I course), 91% of students who used ALEKS for 23.5 hours or more during the term (107 students) received an A, B, or C. Only 9% of these ALEKS users received a D or F or withdrew from the course. Additionally 55% of students using ALEKS less than 23.5 hours (218 students) received an A, B, or C, and 45% received a D, F, or withdrew from the course. These results showed higher achievement for students who spent more time in ALEKS. In Calculus II course, students who used ALEKS (n = 30) were compared with students who did not use ALEKS at all (n = 45).Overall, 90% of students who used ALEKS received an A, B, or C, and only 10% received a D, F, or withdrew from the course. Of the students who did not use ALEKS, only 53% received an A, B, or C, and 47% received a D, F, or withdrew from the course. These results showed that those students who used ALEKS performed better than those who did not.

Hagerty and Smith (2005) used ALEKS to replace traditional homework assignments in a college algebra course at Black Hills State University. Four sections of the course used ALEKS (the experimental group) and four sections of the course were taught in a traditional manner (the control group). Students (*n* = 251) were randomly assigned to one group or the other. Three of the four ALEKS sections dramatically outperformed the control groups in gains between the pretest and the posttest. The exception was one ALEKS section that was the only "night" section in the experimental group. Students did not have sufficient computer access and were allowed to switch to a traditional format early in the course.

Hampikian, Moll, Gardner, and Schrader (2006), at Boise State University, used ALEKS as a key component in two new introductory engineering courses offered concurrently with the students' first mathematics course. One group of students (*n* = 17) took Precalculus concurrently with Engineering 110 (using ALEKS), and another group (*n* = 28) took Calculus I paired with Engineering 120 (using ALEKS). Grade performance of the students taking the ALEKS-oriented courses was compared with that of students who took only Precalculus or Calculus I alone. Of the students using ALEKS in Engineering 110, ~41% received an A or B in Precalculus, and ~59% received an A, B, or C. Among students in Precalculus alone (no ALEKS), ~27% received an A or B, and ~52% received an A, B, or C. Of the students using ALEKS in Engineering 120, ~79% received an A, B, or C in Calculus I. Approximately 49% of the students in Calculus I alone (no ALEKS) received an A, B, or C.

In an action research project, LaVergne (2007) discussed the impact of ALEKS on standardized math scores of students in algebra. In the study, LaVergne placed all students in Algebra 1A on ALEKS two class periods per week in addition to their standard curriculum. The average improvement score on a standardized test was higher than the district and national averages for the students using ALEKS for a certain amount of time within the week. Students who used ALEKS for 31-60 minutes per week (n = 83) and students who used ALEKS for 61-90 minutes a week (n = 15) both improved by an average of 2.7 points.

Taylor (2008) explored the differences in mathematical achievement of under-prepared college freshmen in an intermediate algebra course using different teaching approaches based on students' demographics, algebra tests, mathematics anxiety, and mathematics attitude. In this study, 54 freshmen who enrolled in a course using ALEKS and 39 freshmen students who enrolled in a traditional lecture course without ALEKS were investigated for the effects of this web-based technology centric course. The result showed that ALEKS intermediate algebra students performed as well as the traditional group taught by lecture. The anxiety of the ALEKS group decreased by more than that of the traditional group, and the ALEKS group attitude towards math improved at a greater rate.

Allen (2007), from the Community College of Rhode Island, studied grade distribution of students in Elementary Algebra who used ALEKS (McGraw-Hill) and those who used MyMathLab (Addison-Wesley). A total of 210 student records were analyzed for this study. Of these, 107 used ALEKS and 103 used MyMathLab.

Allen (2007) showed that 34% of students who used ALEKS and 24% of students who used MyMathLab received the grades of A, B or C, while 38% of students who used ALEKS and 45% of students who used MyMathLab received a grade of D or F. Although a numbers of factors influencing success make it impossible to claim that ALEKS outperformed MyMathLab, based on this result, it seems reasonable to conclude that that ALEKS was able to assist less prepared students to reach success in elementary algebra because of ALEKS's emphasis on repetition of all algebraic skills and continuous assessment.

There has been at least one case in an introductory graduate level statistics course where the use of ALEKS did not make a difference between a hybrid course and a traditional face to face course. Xu, Meyer, and Morgan (2008), at the University of Memphis, used a mixed-methods approach to evaluate a hybrid teaching format that incorporated ALEKS to address students' learning needs in a graduate-level introductory statistics course. Student performance in the hybrid course with ALEKS was found to be no different from the same course taught in a traditional face-to-face format. Survey and focus group interviews revealed that students' experience with ALEKS and learning of statistics varied systematically across performance levels. Both quantitative and qualitative data suggested that (a) class format may not be as important as students' mathematical ability and skills for their success in introductory statistical courses, and (b) a teaching approach that addresses the underlying determinants of learning behaviors would be more effective than simply delivering the material in a different format.

Research results have shown that majority of the institutions that used ALEKS for remediation or as a supplement in different mathematics subjects and in different learning environments had an increased learning rate. The same research results have shown that the group that used ALEKS performed as well as or better than the group who did not use

ALEKS; however, none of the research findings focused on a basic mathematics course that was taught completely online.

Summary

In summary, this review of literature discussed the operationalization of the KST and DOK model in this research study. From the instructional design perspective, this review of literature discussed task analysis and its similarity with the KST. It also described ALEKS as an example of an intelligent tutoring system and its use in the teaching and learning of Mathematics at different institutions. Finally, different computer based learning environments in mathematics and their associated learning theories were identified and discussed.

CHAPTER 3

METHODOLOGY

The purpose of this research was to investigate the effect of ALEKS on students' achievement in mathematics in an online environment and to determine the cognitive complexity for mathematical tasks enacted by ALEKS' pretest and posttest assessments. This chapter will present the research methodology, population, procedure, and instruments that were used in collecting data. The methods of data analysis, assumptions and limitations of the study are also addressed.

This is a quasi-experimental study that used the one-group, non-randomized, pretest-posttest design. Quasi-experiments are studies that aim to evaluate interventions but do not use randomization (Harris et al., 2006). This research design was selected for this study because of the difficulty in randomizing participants. In the present university and the online learning environment, ALEKS is the primary source for assessment, teaching, and learning of mathematics, so ethical considerations typically will not allow random withholding of ALEKS.

This category of design is most frequently used when it is not feasible for the researcher to use random assignment, and it is commonly employed in the evaluation of educational programs when random assignment is not possible or practical (Gribbons & Herman, 1997). Because of the already established course schedule by the college and students' self-selection into particular sections of the course, it was impossible to apply random sample without upsetting the student's course schedule. So, in order not to disrupt students' course schedules, this research used the intact classes established by the university for the College Mathematics I course.

Research Design

As noted by Mertens (2005), one-group, pretest-posttest design method might be necessary in a situation where the school does not allow for differential provision of services. Also, according to Conttrell and Mckenzie (2011), schools or groups may not find it ethical or permissible to allow research where some students are treated differently. In addition, this method was selected because students who are learning College Mathematics I in this online environment rely on the ALEKS system for instruction, so designing a control group by removing the ALEKS system would introduce a differential provision of services.

Borg and Gall (1989) state that the one-group, pretest-posttest design is justified in circumstances in which one is attempting to change attitude, behavior or knowledge that is unlikely to change without introduction of experimental treatment. In this study, the research design was justified by the use of ALEKS as an instructional intervention which was used to try to enhance students' mathematical knowledge.

The schematic for one-group, pretest-posttest design is as follows:

- A group of participants is measured twice 0_1X0_2
    - There is no control group
- The treatment effect is computed as $0_2 - 0_1$

Where:

- 0_1 and 0_2 are observations of the dependent variables (Pretest and Posttest respectively)
- X is the experimental treatment, which is learning with ALEKS

According to Creswell (2003), quantitative studies use research questions and hypotheses to shape and focus their study. This research tested one null hypothesis and answered two questions:

H_0: There is no difference between students' achievement as measured by students' scores on pretest (Baseline Assessment) and posttest (Final Assessment) from ALEKS.

Question I. What are the factors contributing to students' mathematics achievement in using the ALEKS?

Question II. What is the cognitive complexity of mathematical tasks enacted by ALEKS on the pretest and posttest assessments?

Method

Population

The research took place within the online campus of a large private, multi-campus, 4-year urban university which offers mathematics in the College of Arts and Sciences. The main campus of the university institution is situated in the southwestern region of the United States. The school offers courses in both online and ground format. There are about 210,000 online students at all program levels (associate's, baccalaureate, master's, doctoral). However, only about 70,000 are online bachelor degree students who might be taking College Mathematics I.

The gender make-up of the university in 2010 was 63% female and 37% male. The race distribution was as follows: 54.7% White, 22.9% African American, 12.6% Hispanic, 1.2% Native American/Native Alaskan, 4.8% Asian Pacific Islander, and 3.8% others/unknown. Most of the students are working professionals taking courses or completing degree requirement. The average age of students is 35 years and the age range is from 25 years to 50 years.

On average, students who take the mathematics course are nontraditional students, White women of approximately 35 years of age. The students who take College Mathematics I are generally undergraduates majoring in the arts, business, health Science or humanities. College Mathematics I is for all students who have not taken any College Mathematics course. These students are required to take the course at any point in their degree program.

Sample

A purposeful sampling was used in this study. A total of 80 students who enrolled in the College Mathematics I course during five different 5-week sessions were recruited for the study. These five sessions were taught consecutively or concurrently over a 7-month period and the average size for each class was 18. However, because of different class size and attrition, the final number of participants varied. According to Bartlett, Kotrlik, and Higgins (2001), in order to use multiple regression analysis the ratio of observations to the independent variable should not fall below 5 because if this minimum is not followed the research study could run the risk of "overfitting." That is, providing a result that is specific to the sample (Halinski & Feldt, 1970). But Miller and Kunce (1973) and Halinski and Feldt (1970) reported a more conventional and optimal approach of at least10 observations per independent variable.

Procedures

The instructional role of the researcher in this study was to facilitate the online class by posting and grading discussion questions, grading individual and team assignments, providing feedback, guidance and direction to the students throughout the course. All reports and assessments required for this study were graded and reported by ALEKS. In this study, I served as both the researcher and the instructor. In my role as a researcher, I directed the ALEKS system to provide me with the initial and final assessment reports, weekly quizzes reports and skill mastery reports. I had no influence over the reports provided by ALEKS because as noted in previous chapters, ALEKS reports are automatically generated based on student's skill assessment, tutoring, and learning progression.

All activities pertaining to assessment, tutoring and learning of College Mathematics I were conducted online inside the ALEKS system. Numerical codes were used to identify students' scores from ALEKS.

Determination of Student's Knowledge State by ALEKS

Knowledge state plays an important role in learning with ALEKS. In this research study, the initial contact of the learner with ALEKS begins with the determination of the learner's knowledge state of College Mathematics I.

When a student first logs on to ALEKS, a brief tutorial shows him or her how to use the ALEKS answer input tools. The student then begins the ALEKS Initial Assessment. In a short period of time (about 45 minutes for most courses), ALEKS assesses the student's current course knowledge by asking a number of questions (usually 20-30). The student's knowledge is represented by a multicolor pie chart which is also the student's entry into the Learning Mode. The pie chart displays current progress state (see Figure 4).

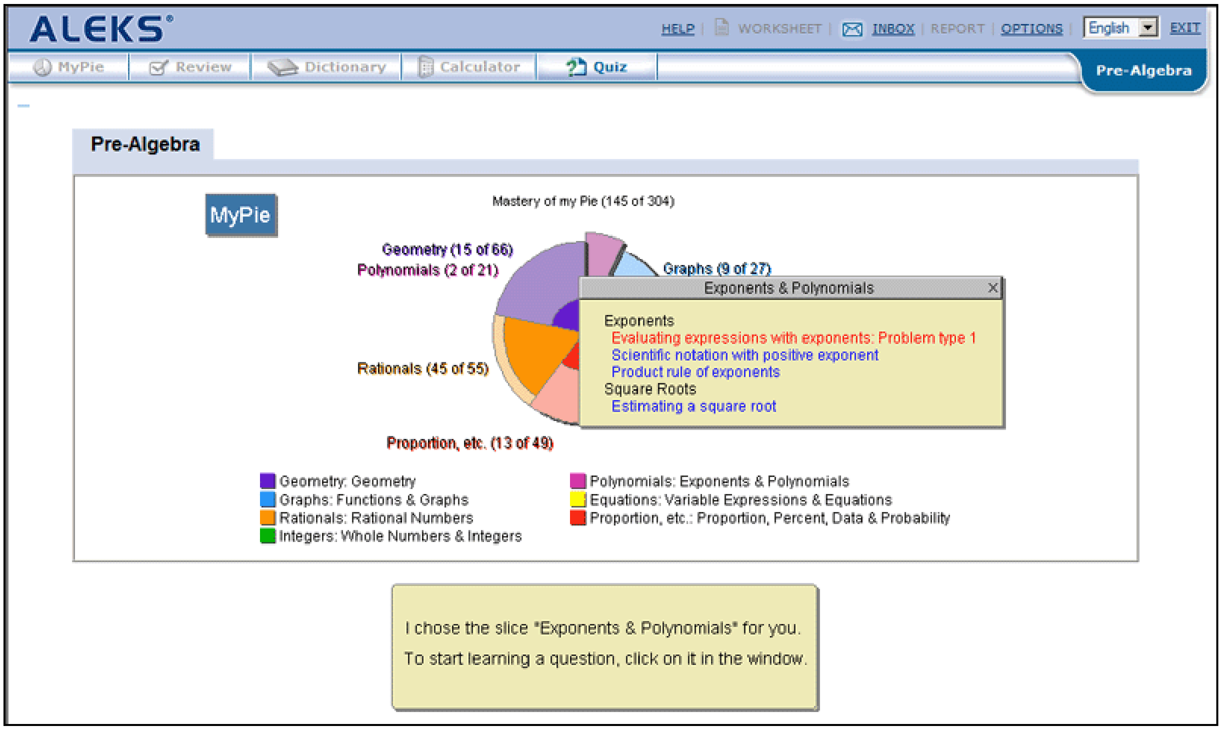

Figure 4. Multicolor Pie Chart Representing Student's Knowledge. Source: http://www.aleks.com/highered/math/tour_math_pie

The colored pie slices represent the concepts covered in the syllabus of College Mathematics I. Beside each pie section is the topic of the concept cluster, the number of concepts in that pie section, and the number of concepts the student has mastered. Darker colors in each pie section indicate concepts that have been mastered.

To avoid the problem of the participants becoming test-wise, the pretest and posttest were algorithmic questions. A pretest (initial assessment) was administered by ALEKS to all students prior to tracking their learning progress. The pretest was helpful in assessing students' prior knowledge of College Mathematics I.

Data were collected from individual participants using ALEKS's reports instrument for the 5-week duration of the course. Each week the students completed the pie slice assigned, quizzes, individual homework and learning team homework. They also completed main forum discussion from week 1 to week 4 and a final assessment in week 5. Initial assessment was administered and data collected in week 1. At the end of each learning module or week, the quiz report and weekly time spent in ALEKS report were made available by ALEKS to the researcher. During the fifth week, the final assessment and final examination are administered by ALEKS (see Table 3).

ALEKS provided data on each participant in 10 different areas: Initial Assessment Report and Final Assessment Report, Weekly Concept Mastery Report, Weekly Quiz Report, Weekly Time in ALEKS Report, Final Concept Mastery Report, Total Time in ALEKS Report, and list of pretest and posttest questions enacted by ALEKS. Through these areas, data were analyzed to determine the effect of ALEKS on students' achievement in mathematics in the online environment and to determine the cognitive complexity of mathematical tasks enacted by ALEKS' pretest-posttest assessments (see Table 4).

Research Ethics

Institutional Review Board Approval

In compliance with research ethics, prior to commencing data collection and analysis I obtained two Institutional Review Board (IRB) approvals: one from the site where the research was conducted and the other from Georgia State University.

Validity and Reliability

Validity does not only ensure reliability, but it also remains the most important characteristic a test or a measuring instrument can possess (Gay & Airasian, 2003). The main variable that was measured in this study was mathematics achievement.

Table 3

Data Collection Procedure Chart

Activity	Reports	Week 1 Real Numbers	Week 2 Solving Linear Equations/ Inequalities	Week 3 Graphing Linear Equation /Inequalities	Week 4 Systems of Equation	Week 5 Review
Pretest	ALEKS Initial Assessment Report	Initial Assessment				
Concept Skill Mastery	Weekly ALEKS Concept Mastery Report	Complete Real Numbers pie slice	Complete Solving Linear Equations/ Inequalities pie slice	Complete Graphing Linear Equations/ Inequalities pie slice	Complete Solving Systems of Equation pie slice	Review
Formative Assessment/- Quizzes	Weekly ALEKS Quiz Report	Week 1 Quiz	Week 2 Quiz	Week 3 Quiz	Week 4 Quiz	
Time Spent in ALEKS	Weekly Time in ALEKS Report	Week 1 Time	Week 2 Time	Week 3 Time	Week 4 Time	
Summative Assessment/ Posttest	ALEKS Final Assessment Report					Final Assessment

Table 4

Relationship between Data Collection Process and Tools

Data	Collection Process	Tools
Weekly Concept Mastery score	Weekly ALEKS Concept Mastery Report: Each week ALEKS calculates in percent the ratio of number of topics mastered to the number of topics assigned for week. (Number of topics mastered per week).	ALEKS
Weekly Time in ALEKS	Weekly Time in ALEKS Report: Each week ALEKS calculates the number of hours spent in learning mode. (Number of hours per week).	ALEKS
Weekly Quiz grade	Weekly ALEKS Quiz Report: Formative assessments in form of Weekly Quizzes are given, graded and scored in percents by ALEKS.	ALEKS
Final Concept Mastery score	Final ALEKS Concept Mastery Report: ALEKS calculates in percentage the ratio of the total number of topics mastered to the total number of topics assigned for the course. (Total Number of topics mastered for the course).	ALEKS
Total Time in ALEKS	Total Time in ALEKS Report: ALEKS calculates the total number of hours spent in learning mode. (Number of hours in learning mode for course).	ALEKS
Initial Assessment grade	ALEKS Initial Assessment Report: Pretest in form of initial assessment is given, graded and scored in percents by ALEKS.	ALEKS
Final Assessment grade	ALEKS Final Assessment Report: Posttest in form of Final assessment is given, graded and scored in percents by ALEKS	ALEKS
Sample of Pretest and Posttest Questions	Pretest and Posttest Questions used ALEKS used Initial and Final Assessment	ALEKS
ALEKS related Discussions	Discussions in OLS Main Forum: Main forum discussions related to ALEKS will be analyzed for students' view of learning with ALEKS.	OLS

To determine mathematics achievement in College Mathematics I, this research used pretest, posttest, quizzes, and concept mastery reports provided by ALEKS as valid measures for these criteria.

History and Maturation

Gay and Airasian (2003) state that the longer a study lasts, the more likely history and maturation will be a threat. Each class session lasted for 5 weeks and sections were taught concurrently and or subsequently for 7 months. The class duration helped minimize the threat of history and maturation.

Testing and Instrumentation

Pretest sensitization tends to be a problem when the duration between pretest and posttest are close (Bonate, 2000). In this research study, pretest and posttest were 4 weeks apart. In addition, ALEKS uses algorithmic questions. In algorithmic questions, the difficulties of the mathematics questions are preserved while the numerical constants that appear in the questions are changed. As stated earlier, ALEKS does not use multiple choice or true false questions. As a result, memorizing the questions from the pretest would not help the participant on the posttest. Because the ALEKS system was the primary instrument for collecting data in this research, lack of consistency or unreliable data collection from the measuring instrument that could occur with humans was not an issue.

Instruments

In this research the main instruments used from the ALEKS system were the ALEKS Initial Assessment Report (pretest) in week 1, and ALEKS Final Assessment

Report (posttest) in week 5 (See Table 5). Between Initial and Final assessment the following instruments were used to collect data from ALEKS:

Table 5

Relationship between Instruments and Subquestions

Instruments	*Subquestions*
Weekly ALEKS Concept Mastery Report and Weekly ALEKS Quiz Report	Is there a relationship between weekly Concept Mastery and weekly formative assessments?
Weekly Time in ALEKS Report and Weekly ALEKS Quiz Report	Is there a relationship between the Time Spent in ALEKS per week and achievement score in weekly formative assessments?
Total Time in ALEKS Report and Final ALEKS Concept Mastery Report	Is there a relationship between the Total Time Spent in ALEKS and the Final Concept Mastery score?
Final ALEKS Concept Mastery Report and ALEKS Final Assessment Report	Is there a relationship between the Final Concept Mastery score and the Posttest score?
Total Time in ALEKS Report and ALEKS Final Assessment Report	Is there a relationship between the Total Time Spent in ALEKS and the posttest score?
ALEKS Initial Assessment Report and ALEKS Final Assessment Report	Are there any differences in students’ achievement scores between the pretest and posttest assessments?
Sample of Pretest-Posttest Questions (SPPQ)	What is the cognitive complexity of mathematical tasks enacted by ALEKS on the pretest and posttest assessments?

Weekly Time in ALEKS Report, Weekly ALEKS Concept Mastery Report, Weekly ALEKS Quiz Report, ALEKS Final Concept Mastery Report, Total Time in ALEKS Report, and Pretest and Posttest questions items.

Assumptions of the Study

In this study, I made the following assumptions:

1. The technical performance of the OLS and ALEKS systems during the session did not affect the students’ performance. OLS and ALEKS are computer systems, and there are always possibilities of malfunction. The breakdown of any of these systems could disrupt the research in the following way: inability to take quizzes, examinations, pretest, and posttest.
2. The anxiety that comes with learning in an online environment is not a major factor in learning mathematics with ALEKS. As stated earlier in the problem statement, the issue of learning how to navigate two systems (OLS and ALEKS) could raise some anxiety and hence affect students’ performance in the learning of mathematics.
3. The anxiety that comes with learning mathematics was not a major factor. Bowers (2001) reports that mathematics anxiety temporarily disrupts mental processes needed for doing arithmetic and drags down mathematics competence.
4. The Initial Assessment, Final Assessment, ALEKS Concept Mastery, Quizzes, and Time Spent in ALEKS measure the effect ALEKS on students’ math achievement. This research assumed there are no other variables that measure the effect of ALEKS on students’ math achievement besides those mentioned.

5. Participants in this study are those who began and completed the course. That is, only students who took the final exam and final assessment in the course were included in the statistical analysis of the research. In this study, completing the course meant taking the pretest, all the quizzes, and the posttest.

6. Participants used the ALEKS system and the prescribed learning path as their primary source of learning College Mathematics I. The major theoretical framework of this study was that ALEKS prescribes a learning path based on the knowledge space theory, so not using ALEKS or following the prescribed learning path will flaw the research study.

Limitations of the Study

This research was constrained by the following:

1. As the instructor and researcher, I was aware that my personal bias could affect the design, sampling, measurement and interpretation of data collected in this study. I have taught mathematics in face-to-face environments for many years. I have also facilitated online mathematics classes for several years using various ITS. During this period I have seen students' successes and failures in both face-to-face and online environments. One of the ITS I have used and still continue to use is ALEKS, which as noted earlier is a KST based system. Because KST is an assessment theory that reveals the knowledge state of a learner and provides a focused instruction based on the knowledge state of the learner, I expected that students who use ALEKS would learn and retain mathematical

knowledge. I also expected those students who have followed the prescribed learning path provided by ALEKS (i.e., completing each week's pie slice) would perform better in the formative and summative assessments provided by ALEKS than those who did not. However, because ALEKS produced all the necessary data required for this research study, I am confident that this reduced potential researcher's bias in this study.

2. The study was constrained by the number of students who participated in the final assessment. Participants who completed the final assessment were used in the statistical analysis of the study. In online environments students are likely to drop out because of reasons other than mathematics difficulty, and hence not having enough students complete the final assessment will affect the total number of participants. As stated earlier in the introduction, attrition in the online mathematics learning environment is a known problem and was an issue in this research study. College Mathematics I is not an exception to attrition: from my experience in the College Mathematics I courses I have taught, an average of 33% of the students drop out before the end of the course.

3. As an instructor I had no control over what additional resources, beyond ALEKS, participants used to gain mathematics knowledge. For example, concept discussions in OLS, books, or personal tutoring could have been used to gaining the mathematical knowledge of College Mathematics I.

4. The findings of this research were limited to ALEKS rather than other ITSs because ALEKS is a KST based system. There are other ITSs that are not based on KST, so it is inappropriate to generalize this study to such systems.

5. Purposeful sampling was used. Therefore, the generalizations of the results from this study are limited to a group similar to the subjects used in this research. Other generalizations may or may not apply.

Data Analysis

Five methods of analysis and one model were used in analyzing the data: *t*-test, correctional analysis, simple regression analysis, multiple regression analysis, Cronbach's Alpha reliability test and Webb's depth of knowledge model. A paired-samples *t*-test compares the means of two scores from related sample while simple and multiple linear regression analysis allows the prediction of one variable from several other variables (Cronk, 2004). In this research study, I used a paired-samples *t*-test to test the hypothesis, correctional analysis to determine the relationship between the independent and dependent variables, and simple and multiple linear regression analysis to predict students' achievement in College Mathematics I based on concept mastery and/or time spent learning in ALEKS. Cronbach's alpha measures the internal consistency of a data set (Cronk, 2004). In this research, Cronbach's Alpha was used to determine the degree to which the pretest and posttest items measure achievement.

To answer the second question, "What is the cognitive complexity of mathematical tasks enacted by ALEKS on the pretest and posttest assessments?" I used

Webb’s (1997) four levels of DOK (see Appendix A) to determine the cognitive complexity of the pretest and posttest assessment.

Summary

This chapter discussed research methodology, population, procedure and instruments that were used in data collection. The assumptions, limitations and constraints of the study were also discussed. The methods of data analysis showed it was a quantitative study. All data except for the cognitive complexity of mathematical tasks enacted by ALEKS on the pretest and posttest assessments were collected through ALEKS reports. In the following chapter, data were collected, results were analyzed and presented.

CHAPTER 4

RESULTS

The purpose of this research was to investigate the effect of ALEKS on students' achievement in mathematics in an online learning environment and to determine the cognitive complexity for mathematical tasks enacted by ALEKS' pretest and posttest assessments. Data were collected from College Mathematics I courses participants' gender, race, age distribution and the number of students within each class session. For the purpose of this research a class session or session is interpreted as an academic term of 5-weeks' duration. In this chapter, I present data to show the assumptions that were confirmed for the different statistical analysis I used in this research. Then I present the reliability of the pretest and posttest question items. This study was guided by two research questions, and the results of these research questions are organized, presented and discussed. Finally, students' responses to an open-ended question posted in the OLS were analyzed and presented.

Participants

This research was conducted during fall and spring semesters of the academic year 2010-2011. Data were collected from College Mathematics I, and there were a total of five class sessions involved in this research. There were 16 students per session for a total of 80 students. Eighty students took the pretest (16 from each session), but 59 took the posttest or the final exam. Out of the 59, 11 enrolled in session I, 12 in session II, 11 in session III, 13 in session IV, and 12 in session V of the College Mathematics I. The following number of students did not complete the course: Five out of 16 (31.25%) in session I, 4 out of 16 or (25.00%) in session II, 5 out of 16 (31.25%) in session III, 3 out of 16 (18.75%) in session IV, and 4 out of 16 (25.00%) in session V. The majority of the participants were female (72.27%); 23.72% were male (see Table 6). More than 94% were over 22 years old. More than 42% of the participants identified themselves as White, 20.0% identified themselves as Black, 3.4% identified themselves as Hispanic, 8.5% identified themselves as Asian, and 25.0% did not report their race (See Table 7).

Testing Assumptions about Data Used in the Analyses

Before conducting the analysis, data were first screened for any missing data and outliers, resulting in 56 students who completed the pretest and posttest assessments. In this section, I discuss the assumptions regarding the data used in this research study. According to Osborne and Waters (2002), when these assumptions are not met, it could result in Type I or Type II errors or in an underestimation of the significance effect or effect size(s).The ANOVA, *t*-test, correlation and regression analysis were used to analyze the data in this research. The *t*-test provided the differences between the pretest and the posttest; the correlation analysis provided the relationship between the independent and the dependent variables, while the regression analysis which included ANOVA results provided the model for predicting mathematics achievement with ALEKS. Because of these statistical analysis used in this research, the assumptions of normality, homogeneity of variances, multiple collinearity, linearity and homoscedasticity are discussed.

Assumptions of Normality

For all posttest and pretest data, the Kolmogorov-Smirnov test of normality was used to determine whether the distribution of values was normal ($p > 0.05$) or not normal ($p < 0.05$) and to indicate whether parametric or nonparametric statistical analysis should

Table 6

Gender Distribution of Participants

Class Session	Male	Female	Total
Session I	2	9	11
Session II	3	9	12
Session III	4	7	11
Session IV	2	11	13
Session V	3	9	12
Total	14	45	59

Table 7

Race Distribution of Participants

Class Session	White	Black	Hispanic	Asian	Not Reported	Total
Session I	3	2	1	1	4	11
Session II	7	4	0	0	1	12
Session III	6	3	0	1	1	11
Session IV	6	1	0	2	4	13
Session V	3	2	1	1	5	12
Total	25	12	2	5	15	59

be used to analyze the test results. The Kolmogorov-Smirnov test of normality showed 0.716 and 0.497 for pretest and posttest respectively, indicating that $p > 0.05$ (see Table 8), hence satisfying the assumptions for normality.

Regression assumes that variables have normal distributions because a non-normally distributed variable distorts the relationship and significance tests. If skewness

Table 8

One-Sample Kolmogorov-Smirnov Test

		Pretest	Posttest
N		56	56
Normal Parameters[a,b]	Mean	41.09	75.64
	Std. Deviation	23.428	19.928
Most Extreme Differences	Absolute	.093	.111
	Positive	.093	.111
	Negative	-.061	-.101
Kolmogorov-Smirnov Z		.697	.829
Asymp. Sig. (2-tailed)		.716	.497

a. Test distribution is Normal.

b. Calculated from data.

= 0, the data are perfectly symmetrical, but a skewness of exactly zero is unlikely for real-world data, so Bulmer (1979) suggests the following rule of thumb:

- If skewness is less than −1 or greater than +1, the distribution is skewed.
- If skewness is between −1 and −½ or between +½ and +1, the distribution is moderately skewed.
- If skewness is between −½ and +½, the distribution is approximately symmetric.

Following the above rule, the variables posttest, concept mastery, and total time spent learning in ALEKS are skewed with the statistics of approximately -1.3, -2.2, and 1.1, respectively, while the pretest is approximately symmetric skewed with a statistic of 0.3 (see Table 9). In addition, posttest (-1.3), concept mastery (-2.2) are negatively skewed while total time spent learning in ALEKS (1.1) and pretest (0.3) are positively skewed. According to Brown (1997), a skewed distribution may actually be a desirable outcome on a criterion-referenced test; violations of assumption of normality are only problematic

Table 9

Descriptive Statistics on Skewness

	N	Minimum	Maximum	M	SD	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	Std. Error	Statistic	Std. Error
Pretest	56	1	94	41.09	23.428	.366	.319	-.607	.628
Posttest	56	10	100	75.64	19.928	-1.264	.319	2.390	.628
Concept_Mastery	56	10	100	86.79	20.815	-2.193	.319	4.978	.628
Total_Time	56	7	98	33.73	22.055	1.143	.319	.845	.628
Valid N (listwise)	56								

if the test is norm-referenced and being used for norm-referenced. The evaluation used in this research attempted to uncover the strengths and weakness of a student in terms of what he or she knows or does not know, understands or does not understand, or can do or cannot do, as measured against a benchmark or standard, and hence it was a criterion referenced test.

Assumptions of Homogeneity of Variances

Levene's test is an inferential statistic used to assess the equality of variances in different samples. It tests the null hypothesis that the population variances are equal (called homogeneity of variance). ANOVA and *t*-test assume that variances of the populations from which different samples are drawn are equal. If Levene's test statistic is significant (i.e., $p \leq .05$), then the two variances are significantly different. If it is not significant (i.e., $p > .05$), the two variances are not significantly different; that is, the two variances are approximately equal. If the Levene's test did not produce significant results, this research would have met the homogeneity of variances assumption. In this study, Levene’s Test was used to determine the assumption that the variance on the dependent variable was met, and the test showed a significance of .763 for the dependent variables posttest (see Table 10). Thus, it can be assumed that the variance was approximately equal hence meeting the homogeneity of variances assumption.

Assumptions of Multicollinearity

The Variance Inflation Factor (VIF) which is used to identify multicollinearity, is a measure of how highly correlated each independent variable is with the other predictors in the model. If the value of VIF is larger than 10 for a predictor, this implies large inflation standard errors of regression coefficient and large value of inflation standard

Table 10

Test of Homogeneity of Variances

	Levene Statistic	df1	df2	Sig.
Posttest	.554[a]	6	30	.763

a. Groups with only one case are ignored in computing the test of homogeneity of variance for Posttest.

errors lead to small t-statistics for partial regression coefficients and wider confidence intervals. The VIF of the independent variables concept mastery and total time spent in ALEKS were 1.036 each (see Table 11). Hence, the assumption of multicollinearity in this research study was satisfied.

Assumptions of Linearity

Multiple linear regressions can only accurately determine the relationship between dependent and independent variables if the relationship is linear in nature. According to Osborne and Watters (2002), non-linearity results in misestimating the true relationships. In a multiple regression, under-estimation of the true relationship increases the chance of Type I errors. Pedhazur (1997) suggested the examination of the residual plots of the standardized residuals as a function of standardized predicted values as a method of detecting non-linearity. In this research the plots of the standardized residuals versus the standardized predicted suggested a linear relationship between the independent variables (concept mastery and total_time) and the dependent variable (posttest) (see Figure 5).

Table 11

Variance Inflation Factor Coefficients

		Unstandardized Coefficients		Standardized Coefficients			Collinearity Statistics	
Model		B	Std. Error	Beta	t	Sig.	Tolerance	VIF
1	(Constant)	4.835	6.082		.795	.430		
	Concept_Mastery	.816	.068	.852	11.968	.000	1.000	1.000
2	(Constant)	8.888	5.359		1.659	.103		
	Concept_Mastery	.865	.060	.903	14.361	.000	.965	1.036
	Total_Time	-.246	.057	-.272	-4.328	.000	.965	1.036

a. Dependent Variable: Posttest

Assumptions of Homoscedasticity

Homoscedasticity means that the variance of errors is the same across all levels of the independent variable, but when the variance of errors differs at different values of the independent variable heteroscedasticity is indicated (Osborne & Waters, 2002). According to Tabachnick and Fidell (1996), minor heteroscedasticity has no effect on the significance tests but when heteroscedasticity is evident, it can increase the possibility of Type I error. Homoscedasticity assumption can be checked by a visual examination of a plot of the standardized residuals by the regression standardized predicted values. Heteroscedasticity is indicated when the residuals are not evenly scattered around zero or the horizontal line. In this research study, the residuals are somewhat randomly scattered around 0 (the horizontal line) showing relatively even distribution (see Figure 6).

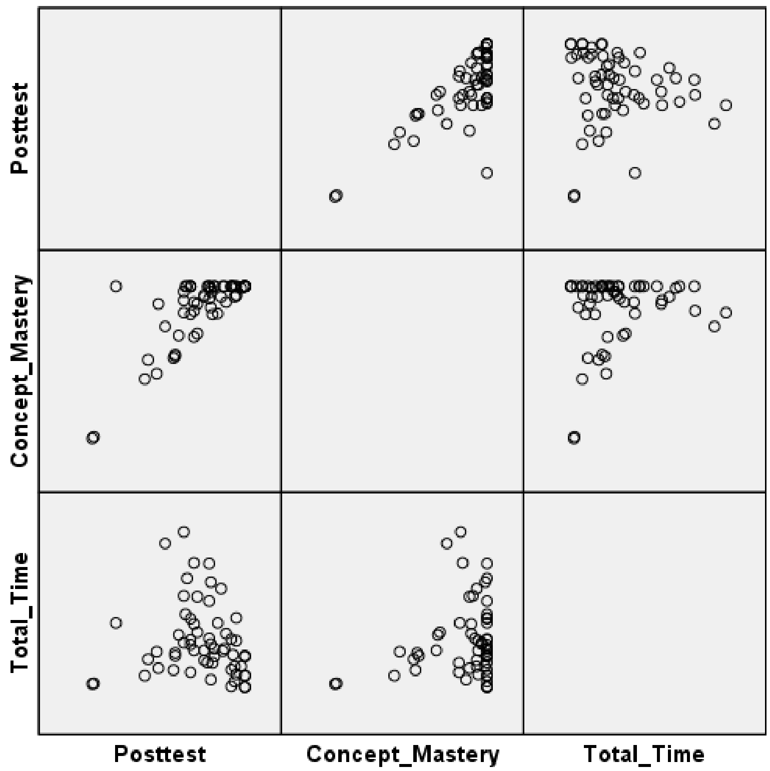

Figure 5. Linearity.

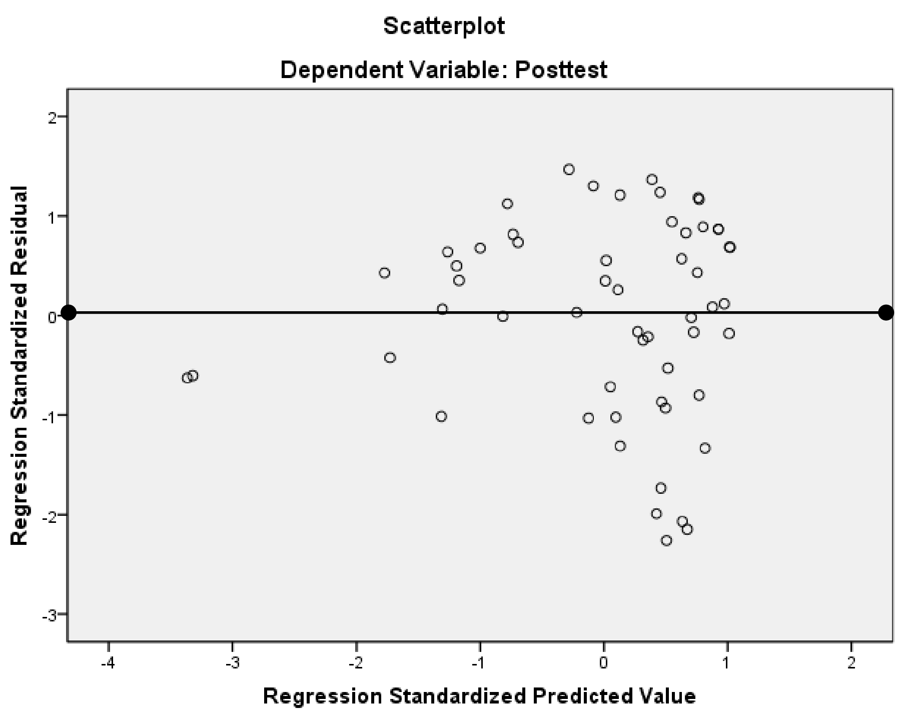

Figure 6. Homoscedasticity. Chart of Regression Standardized Residual versus Standardized Predicted value.

Reliability of ALEKS's Pretest and Posttest

In this study, the construct of mathematics achievement was operationally defined as scores in the weekly quizzes and posttest. The weekly quizzes were generated by ALEKS. The test items from each quiz matched the course weekly objectives, while the posttest items matched the course objectives for the College Mathematics I. This provided face validity.

The reliability of the pretest and posttest was analyzed using the Cronbach's alpha. Cronbach's alpha calculates the mean of all possible split-half correlations and is preferred by many researchers when the internal consistency of test items is to be determined (Ary, Jacobs, Razavieh, & Sorensen, 2006). Reliability of the pretest-posttest is the extent to which these exams yielded consistent results. Ideally, the reliability coefficient should be close to one. Cronbach's alpha for the pretest-posttest was 0.984 based on scores from a sample of 97 questions used on the pretest-posttest assessment, while the "Cronbach's alpha if item is deleted" was at least 0.984 for pretest and posttest. The Cronbach's alpha provided content validity.

Testing of Hypothesis

H_0: There is no difference in the measures between the pretest (Baseline Assessment) and the posttest (Final Assessment) from using the ALEKS mathematics tutoring system.

For this hypothesis, the independent variables were the concept mastery and time spent learning in ALEKS. The dependent variables were mathematics achievements defined as scores on the quizzes and posttest. After verifying that the assumptions were met, a paired sample *t*-test was run to compare the mean pretest score to the mean posttest score. SPSS was used for the analysis and the alpha level was set at 0.05. The mean on the pretest was 41.09 (SD = 23.43), and the mean on the posttest was 75.64 (SD = 19.93; see Table 12). A significant increase from the pretest to posttest was found ($t(55) = -12.256$, $p < .001$). Therefore, the null hypothesis was rejected, meaning there was statistically significant difference in mathematics achievement between the pretest and posttest assessment administered in the College Mathematics I.

Research Question I

"What are the factors contributing to students' mathematics achievement in using the ALEKS?" After I determined from the hypothesis that there is a significant difference in mathematics achievement between the baseline (pretest) and final assessment (posttest) with ALEKS instructional intervention between the pretest and posttest, I proceeded to answer the overarching question. The sub-questions 1 through 5 and research question II were used to answer this question.

Subquestion 1: Is there a relationship between the weekly Concept Mastery and the achievement scores in weekly formative assessments (quizzes)?

The Pearson Correlation coefficient was used to determine if there were linear relationships between the weekly concept mastery, weekly time spent in ALEKS and the weekly quizzes (see Tables 12, 13, 14, & 15). From the tables, the results showed a linear, significant and positive relationship between the weekly concept mastery and the weekly quiz grades for all 4 weeks indicating that participants with higher concept mastery scored more on the Quiz formative assessments.

Table 12

Correlation results between Weekly Concept Mastery, Weekly Time Spent in ALEKS and Weekly Quiz grades from Week 1

		WK1_Concept Mastery	WK1_Time_ Spent	WK1_Quiz_ Grade
WK1_Concept Mastery	Pearson Correlation	1	.195	.510**
	Sig (2-tailed)		.150	.000
WK1_Time_ Spent	Pearson Correlation	.195	1	.117
	Sig (2-tailed)	.150		.389
WK1_Quiz_ Grade	Pearson Correlation		.117	1
	Sig (2-tailed)	.000	.389	

**.Correlation is significant at the 0.01 level (2-tailed). *N* = 56.

Table 13

Correlation results between Weekly Concept Mastery, Weekly Time Spent in ALEKS and Weekly Quiz grades from Week 2

		WK2_Concept Mastery	WK2_Time_ Spent	WK2_Quiz_ Grade
WK2_Concept Mastery	Pearson Correlation	1	.196	.454**
	Sig (2-tailed)		.140	.000
WK2_Time_ Spent	Pearson Correlation	.196	1	.059
	Sig (2-tailed)	.140		.661
WK2_Quiz_ Grade	Pearson Correlation		.059	1
	Sig (2-tailed)	.000	.661	

**.Correlation is significant at the 0.01 level (2-tailed). *N* = 56.

Table 14

Correlation results between Weekly Concept Mastery, Weekly Time Spent in ALEKS and Weekly Quiz grades from Week 3

		WK3_Concept Mastery	WK3_Time_ Spent	WK3_Quiz_ Grade
WK3_Concept Mastery	Pearson Correlation	1	.156	.773**
	Sig (2-tailed)		.251	.000
WK3_Time_ Spent	Pearson Correlation	.156	1	.037
	Sig (2-tailed)	.251		.786
WK3_Quiz_ Grade	Pearson Correlation		.037	1
	Sig (2-tailed)	.000	.786	

**.Correlation is significant at the 0.01 level (2-tailed). *N* = 56.

Table 15

Correlation results between Weekly Concept Mastery, Weekly Time Spent in ALEKS and Weekly Quiz grades from Week 4

		WK4_Concept Mastery	WK4_Time_ Spent	WK4_Quiz_ Grade
WK4_Concept Mastery	Pearson Correlation	1	.169	.632**
	Sig (2-tailed)		.213	.000
WK4_Time_ Spent	Pearson Correlation	.169	1	−.077
	Sig (2-tailed)	.213		.573
WK4_Quiz_ Grade	Pearson Correlation		−.077	1
	Sig (2-tailed)	.000	.573	

**.Correlation is significant at the 0.01 level (2-tailed). *N* = 56.

Each week showed a linear and significant relationship between the concept mastery and quiz scores; however, week 3 showed the strongest relationship with ($r(54) = .773$, $p < .001$), while week 2 showed the weakest relationship ($r(54) = .454$, $p < .001$). In order to understand what happened in weeks 2 and 3, I looked at the skill sets required for the topics covered in weeks 1, 2, and 3. As already discussed in the methodology, the following topics were covered from week 1 to week 4 and in the following order: 1) Real Numbers; 2) Solving Linear Equations and Inequalities; 3) Graphing Linear Equations and Inequalities; and 4) Systems of Linear Equations.

Real numbers, the topic covered in week 1, required reading and comprehension skills; solving linear equation and inequalities, the topic covered in week 2, required solving of equations using the properties of real numbers. Graphing linear equations and inequalities, covered in week 3, required the skills of creating graphs from data generated from solving equation and inequalities. This would mean that the sudden skills change from reading and comprehension in week 1 to solving equations in week 2 could account for the weakest relationship experienced in week 2. The skill of graphing linear equations and linear inequalities used in week 3 was an extension of week 2 skills but more visual, practical and hands-on. For example, solving equation for its coordinates, which is part of week 2 skills, is required for graphing linear equation and inequalities in week 3. This would mean that the extension of week 2's concept and skills could account for the strongest relationship experienced in week 3. In addition to these extreme relationships experienced in weeks 2 and 3, all participants with higher concept mastery scored more on the Quiz formative assessments.

Subquestion 2: Is there a relationship between the time spent learning in ALEKS per week and the achievement score in weekly formative assessments?

The Pearson Correlation coefficient was used to determine the relationship between the times spent learning in ALEKS and the quiz scores from week 1 through week 4 (see Tables 12-15). In order to answer research subquestion 2, I collected all the participants' weekly quiz scores and the time spent learning in ALEKS and analyzed them in SPSS. None of the weeks showed a significant relationship between the time spent in ALEKS and quiz scores. Hence, the time spent learning in ALEKS was not related to the quiz scores.

However, the results showed a steady decline in the correlation coefficient between the time spent in ALEKS and the quiz scores from week 1 to week 4: ($r(54) = .117$; $r(54) = .059$; $r(54) = .037$; $r(54) = -.077$, respectively) at the .05 significance level. Because of the dependency relationship between mathematical concepts, the topics covered in college mathematics I are organized in their degree of difficulty from the least difficult in week 1 to the most difficult in week 4. Consequently, it was evident from these results that when students failed to master the basis in the first week, the time spent in ALEKS continued to show weaker relationship with the quiz formative assessments up to the point of a negative correlation in week four. This outcome argues for the importance of laying a solid foundation at the beginning of the course in order to facilitate a better understanding of subsequent concepts.

Subquestion 3: Is there a relationship between the Total Time Spent in ALEKS and Final Concept Mastery?

A Pearson correlation coefficient was calculated for the relationship between participants' total time in ALEKS and final concept mastery. See Table 15 for the results.

In order to answer this question all the participants' total time spent learning in ALEKS and their total concept mastery for the course were collected from ALEKS and analyzed in SPSS. The result showed a very weak correlation (.188) between the total time spent learning in ALEKS and the final concept mastery. Because of the weak correlation between time spent learning in ALEKS and the concept mastery in the summative assessment (posttest), it would mean that participants' total time spent in ALEKS is not related to the concept mastery. This would also mean that concept mastery and time spent learning in ALEKS would not be confounding factors when studying mathematics achievement with ALEKS.

Subquestion 4: Is there a relationship between the final Concept Mastery score and the Posttest?

In order to answer this question a posttest was administered by ALEKS in week 5 and all the participants' final concept mastery scores and their posttest scores were collected from ALEKS and analyzed in SPSS. A Pearson correlation coefficient was calculated for the relationship between participants' final concept mastery and posttest scores (see Table 16). A strong positive correlation was found ($r(54) = .852$, $p < .001$), indicating a significant linear relationship between the two variables.

The result showed a very strong correlation (.852) between the final concept mastery scores and their posttest scores. The strong and positive correlation result

Table 16

Correlation Result between Posttest, Final Concept Mastery, and Total Time

		Posttest	Concept Mastery	Total Time
Posttest	Pearson Correlation	1	.852**	-.103
	Sig. (2-tailed)		.000	.451
	N	56	56	56
Concept Mastery	Pearson Correlation	.852**	1	.188
	Sig. (2-tailed)	.000		.166
	N	56	56	56
Total Time	Pearson Correlation	-.103	.188	1
	Sig. (2-tailed)	.451	.166	
	N	56	56	56

**Correlation is significant at the 0.01 level (2-tailed).

between final concept mastery and posttest scores in the summative assessments (posttest) indicates that participants' total time spent learning in ALEKS is related to the final concept mastery meaning that participants who scored high on concept mastery also scored high on the posttest.

Subquestion 5: Is there a relationship between the Total Time Spent in ALEKS and the Posttest scores?

A Pearson correlation coefficient was calculated for the relationship between participants' Total Time spent in ALEKS and posttest scores (see Table 16). A weak correlation that was not significant was found ($r(54) = -.103$, $p > .05$).

To answer this question, I had ALEKS administer a posttest in week 5. I then collected data on each participant's total time spent learning in ALEKS and his or her posttest score. The analysis results showed a weak and negative correlation (−.103) between the Total Time Spent learning in ALEKS and the Posttest scores. Because of the weak correlation between these two variables, it would mean that participants' time spent in ALEKS is not related to the performance on the posttest. In fact, the negative sign indicates that without mastering the concepts students may start scoring less in the assessments regardless of the time spent learning in ALEKS.

To further explore research Question I, "What are the factors contributing to students' mathematics achievement in using the ALEKS?" I attempted to fit the research data into two simple linear regression models and a multiple regression model.

A simple linear regression was run to predict participants' achievement in mathematics based on their concept mastery. A significant regression equation was found ($F(1,54) = 143.223$, $p < .001$) with an R^2 of .726 (see Table 17). Participants' predicted achievement was equal to $4.835 + .816 \times$ (Concept Mastery), where achievement and Concept Mastery were measured in percentage points.

Another simple linear regression was calculated to predict participants' mathematics achievement based on Time Spent in ALEKS. A nonsignificant regression equation was found ($F(1,54) = .577$ $p > .05$) with an R^2 of .011 (see Table 17). Participants' predicted achievement was equal to $78.775 - (.093 \times$ (Time Spent in ALEKS)), where achievement was measured in percentage points and Time spent in ALEKS was measured in hours.

Finally, a multiple linear regression was calculated to predict participants' achievement based on the concept mastery and the total time spent in ALEKS. A significant regression equation was found ($F(2,53) = 104.496$, $p < .001$; see Table 17), with an R^2 of .798. Participants' predicted achievement was equal to 8.888 – (.246 × (Time Spent in ALEKS) + (.865 × (Concept Mastery)), where achievement and Concept Mastery were measured in percentage points and Time spent in ALEKS was measured in hours.

Table 17

Results of Simple and Multiple Regression Model

Model Summary Posttest and Concept Mastery

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.852[a]	.726	.721	10.524

a. Predictors: (Constant), Concept_Mastery

b. Dependent Variable: Posttest

Model Summary Posttest and Total_Time_Spent in ALEKS

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.103[a]	.011	-.008	20.006

a. Predictors: (Constant), Total_Time

b. Dependent Variable: Posttest

Model SummaryPosttest, Total_Time andConcept_Mastery

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.893[a]	.798	.790	9.131

a. Predictors: (Constant), Total_Time, Concept_Mastery

b. Dependent Variable: Posttest

The simple linear regression analyses of the participants' mathematics achievement based on concept mastery or time spent in ALEKS showed R^2 of .726 and .011, respectively; thus, 72.6% of the variation in achievement can be explained by differences in concept mastery (higher concept mastery score leads to higher achievement score), while only 1.1% of the variation in achievement can be explained by differences in time spent in ALEKS. The simple linear regression analyses of the participants' mathematics achievement based on concept mastery showed that participants average percentage achievement increased by .816 for each percentage increase in concept mastery while the simple linear regression analyses of the participants' mathematics achievement based on time spent in ALEKS decreased by .093 for each hourly increase.

However, the multiple regression analysis showed an R^2 of .798; thus, 79.8% of the variation in achievement can be explained by differences in concept mastery and time spent in ALEKS. From the regression results, it mean that the multiple linear regression equation (Mathematics Achievement = 8.888 – (.246 × (Time Spent in ALEKS)) + (.865 × (Concept Mastery)) showed a better model for predicting math achievement in College Mathematics I (R^2 =.798) than either of the simple linear regression equations. In this case, the regression weights of both concept mastery and time spent in ALEKS are significant when used together as predictors of students' mathematics achievement in College Mathematics I than when concept mastery or time spent in ALEKS were separate predictors.

In the multiple regression equation, the regression weight for concept mastery was positive (.865) and the regression weight for time spent in ALEKS was negative (−.246). In this case, I argue that the variance in concept mastery that does not account for variance in mathematics achievement was suppressed by the time spent in ALEKS. As already shown for Subquestion 2 and for Subquestion 5, time spent in ALEKS was not correlated to mathematics achievement. It is reasonable then to infer that the suppression of time spent in ALEKS from the multiple regression equation resulted in a better prediction of the achievement. It also means that spending time without mastering the concepts does not lead to mathematics achievement.

Research Question II

What is the cognitive complexity of mathematical tasks enacted by ALEKS on the pretest and posttest assessments? A sample of 97 questions items and solutions from ALEKS's pretest and posttest were printed and given to three professors of mathematics to rate according to the questions' cognitive complexity using Webb's (1997) Depth of Knowledge levels (see Chapter 2). The mathematics professors were also given DOK levels descriptors (see Appendix A). All three professors independently analyzed each question and solution and rated it according to DOK level one, two, three or four. According to Shrout and Fleiss (1979), it is important to assess the reliability of judgments made by raters in order to know the extent the measurement are measuring anything and Intraclass correlation coefficients has been shown to provide such measures of reliability. Hence, a reliability test was run on the ratings of the three professors to determine interrater reliability. Intraclass Correlation (ICC) was used to measure the interrater reliability. The average measure of the ICC was 0.987; indicating inter-rater consistency on all the four DOK levels (see Table 18). Next, with direction of the researcher, the three professors held a meeting and discussed question items for which they did not agree on the same DOK level. The professors used the method of consensus

Table 18

Intraclass Correlation Coefficients

	Intraclass Correlation[a]	95% Confidence Interval		F Test with True Value 0	
		Lower Bound	Upper Bound	Value	df1
Single Measures	.962[b]	.947	.973	78.161	96
Average Measures	.987	.982	.991	78.161	96

Intraclass Correlation Coefficient

	F Test with True Value 0	
	df2	Sig
Single Measures	192	.000
Average Measures	192	.000

Two-way random effects model where both people effects and measures effects are random.
a. Type A intraclass correlation coefficients using an absolute agreement definition.
b. The estimator is the same, whether the interaction effect is present or not.

to reach agreement on these questions they did not agree on individually. See Appendix F for samples of question items that the professors did not agree on. All data were transcribed into an Excel spreadsheet (see Appendix D).

The distribution of the 97 pretest and posttest items consisted of the following: 26 questions about Fractions, Signed Numbers, Percents and Geometry; 18 questions about Real Numbers; 26 questions about Solving Linear Equations and Inequalities; 22 questions about Graphing Linear Equations and Inequalities; and 5 questions about

Systems of Linear Equations. The analysis of the pretest and posttest test items according to the DOK's levels showed 36 questions (37.1%) in DOK level 1, 49 questions (50.5%) in DOK level 2, 12 questions (12.4%) in DOK level 3, and none (0.0%) in DOK level 4 (see Table 19). A graphical display of the topics and their DOK's levels are shown in Appendix E.

Additional Findings

In this study, I also analyzed additional qualitative data in order to answer the guiding research questions. In order to gain more insight on what participants report to be the contributing factors to their performance in College Mathematics I, the following question was placed in the OLS discussion forms. "Discuss at least one main factor (Textbook, ALEKS, Team, Instructor, Tutor etc.) that contributed to your performance in this math class or what could you have done differently to perform better?" This question was asked at the end of each 5-week session after the posttest. In a regular class session, discussion was not compulsory in the final week of the course so this affected the number of responses. Fifty out of the 59 students (84.7%) responded. These responses were collated and analyzed and the following themes emerged from the data. While the information was used to gain a more holistic view of the research findings, it was not used to interpret the primary data. Rather, it provided further understanding of student experiences using ALEKS and an avenue for future research.

A majority of the students perceived ALEKS as a significant and a useful helper, especially with its continuous practice over their study plan or learning path. For example, a student said, "For me Aleks was huge, I feel like I spent the most amount of time with it and by being able to practice over and over it really helped me." One student,

Table 19

Pretest & Posttest Topics and Depth of Knowledge (DOK) Distribution

Topics	DOK Level 1	DOK Level 2	DOK Level 3	DOK Level 4	Total
Fractions, Signed Numbers, Percents & Geometry	12	10	4	0	26
Real Numbers	11	7	0	0	18
Solving Linear Equations & Inequalities	6	19	1	0	26
Graphing Linear Equations & Inequalities	6	13	3	0	22
Systems of Linear Equations	0	1	4	0	5
Total	35	49	12	0	97

who identified herself as visual learner, said, "The biggest contribution was the use of ALEKS. I am a visual learner and having the ability to not just focus on a textbook is nice. It took me a bit to get the hang of some subjects and ALEKS helped to explain it better than the textbook." Alongside ALEKS, some students found their coworkers, colleagues and family members as very crucial to their performance in this class. For example, one student said, "I also used the help of a couple of engineering students from work. They explained a couple problems to me."

Of those that responded, there were a couple of students who did not find ALEKS helpful in their performance. For these students, ALEKS was not only complicated and not-user-friendly but it also added to their learning curve. One of these students said "I started getting stressed out over the time factors. There is so much to learn in such a short amount of time" and the other student said, "I had very little success in this class, the book example help a lot more, and at times putting them together with the lecture help me a lot especially on my homework assignments. I really didn't do well on ALEKS, I find it to be challenging. My success would be the book example because it breaks it down for you, for each problem."

Finally, many students identified time management and procrastination as what they could have done differently to improve their performance. For example, one student said, "I could have had a higher grade right now if I would have allowed myself more time and if I would have used that time to retake the exams until I got A's on all of them. When taking any class, one must not procrastinate. It is better to do a little everyday to have a better understanding and stay ahead of the game."

The open-ended question used to gain more insights into the students' feeling about ALEKS has shown that majority of the students liked ALEKS, supporting the result of the research question one and the decision to reject the null hypothesis.

Summary

The analysis in this study was carried out based on the stated assumptions; hence, this chapter discussed the assumptions made in this research and presented the results and the findings of the study. This chapter also answered and discussed each of the research subquestions. The result showed a strong and significant relationship between the concept mastery and achievement scores: quizzes and posttest. The higher the concept mastery scores, the higher the achievement scores. The results did not show any significant relationship between the variables (a) time spent learning in ALEKS and (b) achievement scores. In other words, times participants' spent learning in ALEKS was not identifiably related to their performance on any of the achievement scores. Further, the results did not show any relationship between the two independent variables: concept mastery and time spent learning in ALEKS.

Simple and multiple regressions equation were fitted to predict participants' mathematics achievement based on their concept mastery and/or time spent in ALEKS. As shown in the models, the multiple regressions which used concept mastery and the time spent in ALEKS as independent variables to predict mathematics achievement provided a better model than the simple regression models which used either the concept mastery or time spent in ALEKS to predict mathematics achievement.

The findings of this research placed the cognitive complexity of half the pretest and posttest assessment items as questions that require basic reasoning. This was followed by questions that require recall and reproduction, and the least significant were questions that require strategic thinking. None of the question items required extended thinking or complex reasoning.

Finally, the additional findings showed that most students liked ALEKS and identified ALEKS as the major contributor to their success in College Mathematics I. However, there were some students who used other sources beside ALEKS or who did not like ALEKS at all. But in all cases, ALEKS provided what needs to be learned, the learning path, and validated that the material has been learned.

CHAPTER 5

CONCLUSION

This study was designed to determine the effect of ALEKS on mathematics achievement in College Mathematics I. The findings reported in the previous chapter indicate that ALEKS showed a significant effect on students' achievement in College Mathematics I. These findings offer insight on how to interpret and use some of the ALEKS reports. In this chapter, I provide a summary of the study and discuss what the results mean in the context of using ALEKS as a tutoring system for mathematics. Finally, I provide a conclusion and recommendations for further research in the area of teaching and learning with intelligent tutoring systems.

Summary of the Study

The purpose of this research was to investigate the effect of ALEKS on students' achievement in mathematics in an online learning environment and to determine the cognitive complexity for mathematical tasks enacted by ALEKS' pretest and posttest assessments. The participants of this study were enrolled in one of five different sessions of the College Mathematics I in a 4-year private university located in the southwestern region of United States. I taught all the five sessions. The theoretical frameworks underlying the present study were Knowledge Space Theory and Webb's (1997) Depth of Knowledge Model (1997). KST explains how to reveal a learner's knowledge structures and achievement in a particular subject domain, while Depth of Knowledge is a scale of cognitive demand. ALEKS's design is based on Knowledge Space Theory. Webb's DOK Model was used to determine the cognitive complexity of ALEKS's pretest-posttest question items.

Two instructional media were used in this study: (a) the Online Learning System, a learning management system that was used for discussion, assignment submission and for providing information and document to the students; and (b) ALEKS, a mathematics tutoring system used to tutor and assess students achievement in College Mathematics I. ALEKS provided all the instruments for the data collection that was used to answer questions I and II, while OLS provided students response to the additional findings. The study was guided by two research questions:

Research Question I: What are the factors contributing to students' mathematics achievement in using the ALEKS? To answer this question, I developed five research subquestions and one additional question:

1. Is there a relationship between weekly Concept Mastery and the achievement score in weekly formative assessments?
2. Is there a relationship between the Time Spent in ALEKS per week and the achievement score in weekly formative assessments?
3. Is there a relationship between the Total Time Spent in ALEKS and Final Concept Mastery?
4. Is there a relationship between the final Concept Mastery score and the Posttest?
5. Is there a relationship between the Total Time Spent in ALEKS and the Posttest scores?

Research Question II: What is the cognitive complexity of mathematical tasks enacted by ALEKS on the pretest and posttest assessments?

Quantitative research methodologies, *t*-test, regression and correlation, were used in this study to analyze the data related to the research question 1, while Webb's (1997) depth of knowledge levels was used to analyze and answer the research question 2. For additional insight into the results, students' written responses to the open-ended questions were collated and analyzed for emerging themes.

Discussion

The significant differences between the pretest and posttest in this research results showed that ALEKS had a significant effect on students' mathematics achievement. For example, the negatively skewed posttest and positively skewed pretest as shown in the assumption of normality indicate that the teaching, materials, and student learning are all functioning very well. The difference between the positively skewed distribution at the beginning of a course and the negatively skewed distribution at the end of a course would be an indication of how much the students had learned while the course was going on (Brown, 1997). This would be true because the students had previously scored poorly in a positively skewed distribution (with students generally scoring very low) at the beginning of the course on a similar test.

As already stated in the theoretical framework, Knowledge Space Theory is one of the theories that frame this research. According to Falmagne et al. (2004), KST explains how to reveal a learner's knowledge structure and achievement in a subject domain. ALEKS used KST in College Mathematics I to assess and provide learning paths. Through individual's knowledge structure, ALEKS made sure that students were presented with information they were ready to learn. Hence, this result would mean that ALEKS was successful in assessing and providing learning paths for the students. It also means that defining learning paths for individuals based on their knowledge structure leads to mathematics achievement.

Jonassen et al. (1999) described task analysis for instructional design as a process of analyzing and articulating the kind of information that one expects the learners to know and perform. Learning hierarchy analysis uses dependency and prerequisite relationship among intellectual skills to determine what is to be learned and the sequence it is to be learned (Seels & Glasgow, 1990). Hence, it would appear from the result of the posttest that using task analysis to recommend content level and learning sequence facilitates mathematics achievement.

The main ALEKS report that predicted student achievement was the concept mastery reports. There was a significant and positive relationship between weekly ALEKS concept mastery reports and the weekly ALEKS quiz reports. Also, there was a strong, significant, and positive relationship between final ALEKS concept mastery report and the posttest assessments, referred in this report as ALEKS final assessment report. These results showed that the weekly ALEKS concept mastery report and the final ALEKS concept mastery report are the major predictors of students' achievement when learning with ALEKS. This would mean that students gained mathematical knowledge in College Mathematics I between the pretest and posttest assessments through the use of ALEKS. In addition, the ALEKS concept mastery was the only ALEKS report in this study that predicted mathematics achievement in College Mathematics I, implying that instructors could use the concept mastery report to guide students' learning. Furthermore faculty could use the weekly concept mastery reports to identify students who are at the risk of failing and provide appropriate help and advice. For example, a less than satisfactory score in the concept mastery on the first week is an indication of serious issues on mathematics achievement in the subsequent weeks.

However, the quizzes and the posttest assessments results did not show that the time spent learning in ALEKS each week and the total time spent learning in ALEKS for the course have any effect on students' achievement. Instead, the results showed a non-significant and weak relationship between the weekly time in ALEKS reports and weekly ALEKS quiz reports. Also, the result showed a nonsignificant and weak relationship between the total time in ALEKS report and ALEKS final assessment report. Because it was already shown that mastering the concepts is associated with higher achievement, thse results suggest that spending time in ALEKS without mastering the concepts does not translate to mathematics achievement. This would imply that ALEKS would have to integrate in its design a way of making sure that the time spent learning has a direct relationship with concepts mastered. In addition, a constant decline in the correlation coefficient between time spent studying in ALEKS and the formative assessments (weekly quizzes) implies that week 1 or the first formative assessment is very important in catching students who are at risk of failing the course.

Even though the result of this research study supports previous studies (Allen, 2007; Hagerty & Smith, 2005; Hampikian et al., 2006; Hu et al., 2008; Lavergne, 2007; Taylor, 2008) of higher mathematics achievement when using ALEKS, this study differs with some other previous studies (Hanna & Carpenter, 2006; Stillson & Alsup, 2003) that showed higher test scores are associated with time spent using ALEKS. Stillson and Alsup, in their study of the effectiveness of teaching Basic Algebra using the interactive learning system ALEKS to supplement traditional instruction, found that higher test scores were associated with more time spent learning in ALEKS. In another study, Hanna and Carpenter used ALEKS to provide tutoring for precalculus students that were in Calculus I and II courses; their results showed higher achievement for students who spent more time in ALEKS. But in my current study, the results did not show any relationship between the time spent in ALEKS and mathematics achievement. The major differences between my current study and these previous studies (Hanna & Carpenter; Stillson & Alsup) are that the previous studies took place in a traditional learning environment where ALEKS was used as a supplement to traditional instruction, while the current study was conducted in an online learning environment and ALEKS was used as the primary source of instruction. It is possible that in the previous studies that the assistance of the instructor in the classroom made the students stay on task and also provided just-in-time assistance, thus helping the students master more concepts in a shorter period of time. But in the online learning environment, students are left to monitor their time in ALEKS and would only receive delayed assistance mostly electronically when mathematics problems arise. Therefore, comparing previous studies and this study, it would appear that the time spent in learning ALEKS correlates to higher mathematics achievement when the use of ALEKS is monitored, for example, in a traditional learning environment.

The regression equation showed a better model for predicting mathematics achievement in ALEKS when the time spent learning in ALEKS was suppressed from the equation, implying that mastering the concept is more important for mathematics achievement in ALEKS than the time spent learning in ALEKS. The findings of this study have shown that ALEKS concept mastery report is important when predicting students' mathematics achievement in formative and summative assessments. Because some studies (Hanna & Carpenter, 2006; Stillson & Alsup, 2003) have shown direct relationship between time spent in ALEKS and mathematics achievement in a traditional learning environment, this would imply that ALEKS designers would find a way of assisting an online learner master more concepts in a shorter period of time.

The cognitive complexity or the depth of knowledge of most of the pretest-posttest questions item was at level two. According to Webb's (1997) depth of knowledge model, question items in level two require the application of skills and concepts or engagement of some mental processing beyond a habitual response. A level two assessment item requires students to make some decisions as to how to approach the problem or activity. According to the result, there were no question items in level four or question items that required extended thinking over an extended period of time, complex reasoning, planning, and developing. So, the cognitive complexity of most test items required application of Skills and Concepts, followed by question items that require a recall of information and few of the questions that required Strategic Thinking, but none required Extended Thinking. As shown in the literature review, Recall corresponds to Memorization (in the cognitive demand for math task domain) or Knowledge (in Bloom's, 1956, cognitive domain); Application of Basic Skills and Concepts corresponds to Procedures without Connections to Concepts and Meaning (in the cognitive demand for math task domain) or Comprehension (in Bloom's cognitive domain); Strategic Thinking corresponds to Procedures with Connections to Concepts and Meaning (in the cognitive demand for math task domain) or Analysis and Application (in Bloom's cognitive domain); while Extended Thinking corresponds Doing Math (in the cognitive demand for math task domain) or Synthesis and Evaluation (in Bloom's cognitive domain). So, following these correspondences, most of the assessment items fall within Procedures without Connections or Comprehension followed by Memorization or Knowledge, few of the questions required Procedures with Connection with Meaning or Analysis and Application. None of the items required Doing Math or Synthesis and Evaluation. The results might suggest that most of the questions items presented by ALEKS pretest and posttest assessment in College Mathematics I is appropriate for skills and concept building.

Finally, the qualitative findings indicated that most students liked ALEKS as the major source of instruction. However, there were some students who used other resources beside ALEKS or who did not like it, citing the reason that it was challenging. This implies that students have different preferences and learning styles (Pashler, McDaniels, Rohrer, & Bjork, 2009). Rakap (2010), in a study of the learning styles and computer skills of adult students' learning online, showed that learning styles/preferences had significant effect on students' knowledge acquisition. This means that ALEKS should not be the only source of instruction in the online learning environment: Provision should be made for real time instruction (synchronously or face to face).

Implications for Practice

Baroody and Coslick (1998) identified skill and concept approach as important to mathematics instruction. Skill approach focuses on the memorization of basic skills, while concept approach emphasizes meaningful memorization of skills (Baroody & Dowker, 2003). Because beginning-level mathematics classes like Introduction to Algebra normally emphasize skill and concept building and this research showed that

ALEKS's pretest and posttest cognitive complexity focused more on skill and concept building, instructional designers who are interested in skill and concept building may consider incorporating ALEKS in their design.

As concept mastery has proved to be effective in predicting students' achievement, faculty teaching mathematics with ALEKS could use the concept mastery reports to guide student learning by requiring students to score at a certain level on the concept mastery before attempting any of the formative or summative assessments. In addition, a mathematics instructor could use concept mastery to provide feedback and study plan for a student. Also, knowing the cognitive complexity of the pretest and posttest assessments will help the instructor determine explicitly what the student must do in order to demonstrate learning.

The result of this research will assist the administrators who are interested in reducing mathematics attrition in online learning environment by making decision on which mathematics intelligent tutoring system to adopt. Students are likely to gain in learning when the decision to adopt a mathematics tutoring system is based on research.

Recommendations for Further Study

The following recommendations for further investigation are based on the findings of this study. To validate the findings of this research further, the study should be replicated in a controlled environment and with a larger sample and in other introductory algebra mathematics courses. Additional research should be conducted comparing ALEKS with other intelligent tutoring systems. In part such studies would add to what is known about the effect of ALEKS on mathematics achievement while providing an insight to the knowledge base about the role of intelligent tutoring system in teaching and learning.

In order to find out the role of instructors in mastering mathematics concepts, this study should be replicated in a controlled environment with and without instructor led sections. The findings of such study will provide insight on the role of instructor in concept mastery.

The present study did not take into consideration the role participants' technology skills play in online learning environment. For example, two systems that require technological skills were used in this research study: OLS and ALEKS. As a result, a study that examines the effect of technology skills on mathematics achievement in online learning environments will provide a stronger model for predicting mathematics achievement.

Because this research study supports the evidence of mathematics achievement as shown in the review of literature, there should be a study to compare the achievement level of a class taught with ALEKS and another class taught without ALEKS in an online learning environment. Such a study would not only show whether there is achievement but the level of achievement.

Because mathematics anxiety has shown to affect mathematics achievement in a traditional learning environment, a study on the effect of mathematics anxiety on mathematics achievement in an online learning environment using ALEKS is also recommended.

Conclusion

The purpose of this research was to investigate the effect of ALEKS on students' achievement in mathematics in an online learning environment and to determine the cognitive complexity for mathematical tasks enacted by ALEKS' pretest and posttest assessments. The participants in this study were enrolled in a College Mathematics course in a 4-year private university located in the southwestern region of United States. The finding of this study shows that ALEKS had a significant effect on students' mathematics achievement in the College Mathematics course, and the main ALEKS report that predicted students' achievement in mathematics was the concept mastery report. Time spent learning in ALEKS was not a predictor of mathematics achievement.

The cognitive complexity or the depth of knowledge of most of the pretest and posttest questions used in this study required the application of skills and concepts or engagement of some mental processing beyond a routine response. None of the pretest-posttest questions required extended thinking over a period of time, complex reasoning, planning and developing.

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APPENDIXES

APPENDIX A

Depth of Knowledge Descriptors

Math/Sci Examples	Webb's Depth of Knowledge Levels			
Bloom's Taxonomy	**Level 1** Recall & Reproduction	**Level 2** Skills & Concepts	**Level 3** Strategic Thinking/ Reasoning	**Level 4** Extended Thinking
Knowledge Define, duplicate, label, list, memorize, name, order, recognize, relate, recall, reproduce, state	○ Recall, recognize, or locate basic facts, ideas, principles ○ Recall or identify conversions between and among representations or numbers, or within and between customary and metric measures			
Comprehension Classify, describe, discuss, explain, express, identify, indicate, locate, recognize, report, restate, review, select, translate	○ Make conversions between and among representations or numbers, or within and between customary and metric measures ○ Evaluate an expression ○ Locate points on a grid or number line ○ Solve a one-step problem	○ Specify and explain relationships (cause-effect; why or how; non-examples/examples) ○ Make and record observations ○ Take notes to organize information/ideas ○ Summarize results or concepts ○ Make basic inferences or logical predictions from data/observations	○ Use concepts to solve non-routine problems ○ Explain, generalize, or connect ideas using supporting evidence ○ Make or justify conjectures ○ Explain thinking when more than one response is possible ○ Explain phenomena in terms of concepts	○ Relate mathematical or scientific concepts to other content areas or concepts ○ Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Application Apply, choose, demonstrate, dramatize, employ, illustrate, interpret, practice, schedule, sketch, solve, use, write	○ Follow simple procedures (recipe-type directions) ○ Calculate, measure, apply a rule ○ Apply an algorithm or formula (area, perimeter, etc.) ○ Represent in words or diagrams a scientific concept or relationship	○ Select a procedure according to criteria and perform it ○ Solve routine problem applying multiple concepts or decision points ○ Retrieve information from a table, graph, or figure and use it solve a problem requiring multiple steps	○ Design investigation for a specific purpose or research question ○ Conduct a designed investigation ○ Use concepts to solve non-routine problems ○ Use reasoning, planning, and evidence	○ Select or devise approach among many alternatives to solve a problem ○ Conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analysis Analyze, appraise, calculate, categorize, compare, criticize, discriminate, distinguish, examine, experiment	○ Retrieve information from a table or graph	○ Categorize, classify materials based on characteristics ○ Compare/ contrast figures or data ○ Select appro graph and display data ○ Interpret data from a simple graph ○ Extend a pattern	○ Compare information within or across data sets or texts ○ Analyze and draw conclusions from data ○ Generalize a pattern ○ Interpret data from complex graph	○ Analyze multiple sources of evidence ○ analyze complex/abstract themes ○ Gather, analyze, and evaluate information
Synthesis Rearrange, assemble, collect, compose, create, design, develop, formulate, manage, organize, plan, propose, set up, write	○ Brainstorm ideas, concepts, or perspectives related to a topic	○ Use models to represent mathematical concepts	○ Synthesize information within one source or text ○ Formulate an original problem, given a situation ○ Develop a scientific/mathematical model for a complex situation	○ Synthesize information across multiple sources or texts ○ Design a mathematical model to inform and solve a practical or abstract situation
Evaluation Appraise, argue, assess, attach, choose compare, defend estimate, judge, predict, rate, core, select, support, value, evaluate			○ Cite evidence and develop a logical argument for concepts ○ Describe, compare, and contrast solution methods ○ Verify reasonableness of results	○ Gather, analyze, & evaluate information to draw conclusions ○ Apply understanding in a novel way, provide argument or justification for the application

6 Cognitive complexity: Applying Webb DOK Levels to Bloom's Taxonomy Karin K. Hess, National Center for Assessment, Dover, NH 2005 updated 2006

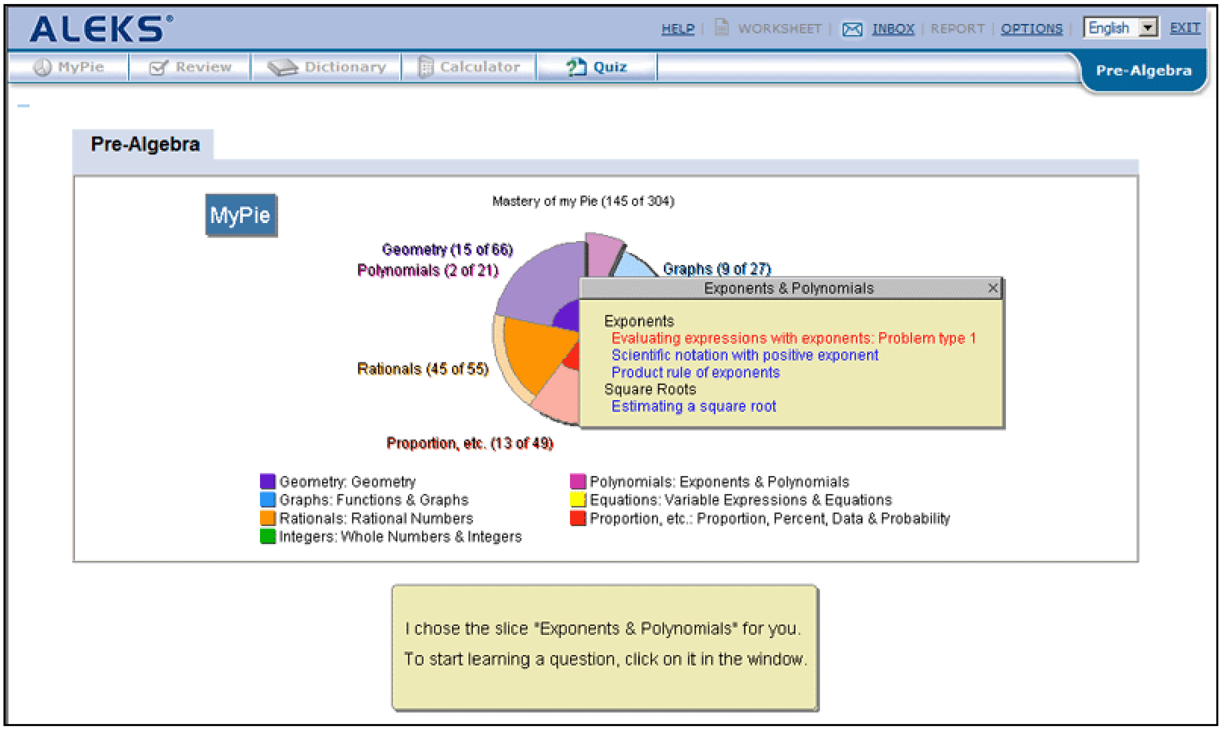
Table 1: Applying Webb's Depth of Knowledge Levels for Mathematics

(Adapted from Karin Hess, Center for Assessment/NCIEA
by the Kentucky Department of Education, 2005)

Webb's DOK Levels			
Recall and Reproduction (DOK 1)	**Skills and Concepts/ Basic Reasoning (DOK 2)**	**Strategic Thinking/ Complex Reasoning (DOK 3)**	**Extended Thinking/ Reasoning (DOK 4)**
• Recall of a fact, information or procedure • Recall or recognize fact • Recall or recognize definition • Recall or recognize term • Recall and use a simple procedure • Perform a simple algorithm. • Follow a set procedure • Apply a formula • A one-step, well-defined, and straight algorithm procedure. • Perform a clearly defined series of steps • Identify • Recognize • Use appropriate tools • Measure	• Students make some decisions as to how to approach the problem • Skill/Concept • Basic Application of a skill or concept • Classify • Organize • Estimate • Make observations • Collect and display data • Compare data • Imply more than one step • Visualization Skills • Probability Skills • Explain purpose and use of experimental procedures. • Carry out experimental procedures	• Requires reasoning, planning using evidence and a higher level of thinking • Strategic Thinking • Freedom to make choices • Explain your thinking • Make conjectures • Cognitive demands are complex and abstract • Conjecture, plan, abstract, explain • Justify • Draw conclusions from observations • Cite evidence and develop logical arguments for concepts • Explain phenomena in terms of concepts	• Performance tasks • Authentic writing • Project-based assessment • Complex, reasoning, planning, developing and thinking • Cognitive demands of the tasks are high • Work is very complex • Students make connections within the content area or among content areas • Select one approach among alternatives • Design and conduct experiments • Relate findings to concepts and phenomena

APPENDIX B

Multicolor Pie Chart Representing Student's Knowledge



Source: http://www.aleks.com/highered/math/tour_math_pie

APPENDIX C

Data

Pretest, Posttest, Concept Mastery, Total Time Spent in ALEKS, Age and Gender from Researched Institution

Code	Pretest	Posttest	Concept Mastery	Total_Time	Age	Gender
1	11	61	71	37	37	2
2	60	84	84	44	26	1
3	43	65	100	50	41	2
4	25	79	93	35	25	2
5	57	95	95	34	25	1
6	65	100	100	25	41	2
7	94	92	100	7	61	2
8	57	48	49	27	48	2
9	48	80	97	32	34	2
10	87	100	100	7	21	1
11	54	88	100	29	27	1
12	7	59	59	27	30	2
13	4	41	45	13	25	2
14	21	64	85	98	35	2
15	11	87	100	28	28	2
16	18	59	60	25	31	2
17	36	79	100	57	48	2
18	39	79	100	79	44	2
19	73	94	94	19	24	2
20	24	10	10	8	48	2
21	53	82	100	25	33	2
22	36	78	100	21	32	1
23	25	92	100	35	37	2
24	22	80	99	69	50	1
25	53	68	84	15	21	2
26	34	68	100	47	44	2
27	41	89	91	38	37	2
28	1	76	94	22	36	2
29	70	100	100	13	32	2
30	39	68	100	32	44	1
31	32	72	72	39	43	2
32	41	81	84	21	34	2
33	28	68	100	34	32	2
34	24	86	94	65	47	2
35	39	98	99	19	37	2

Code	Pretest	Posttest	Concept Mastery	Total_Time	Age	Gender
36	0	49	89.7	17.7	35	1
37	49	64	92	60	30	1
38	16	70	86	80	37	2
39	22	70	91	44	26	2
40	79	94	100	10	20	2
41	72	92	100	17	35	2
42	32	72	90	60	69	1
43	55	64	97	33	35	2
44	1	66	100	71	40	2
45	0	24	100	44.4	47	1
46	31	95	100	47	32	2
47	41	43	57	23	46	2
48	67	100	100	13	29	1
49	21	53	76	91	55	2
50	22	58	58	17	30	2
51	88	80	88	11	42	2
52	18	11	11	9	29	2
53	37	93	100	26	36	2
54	65	95	95	14	31	1
55	70	100	100	25	25	1
56	78	100	100	7	43	2
57	22	76	95	28	29	1
58	0	82	100	29.2	35	2
59	43	70	70	28	29	2

Data

Weekly Concept Mastery, Time Spent in ALEKS, and Quiz Grades from Researched Institution

Code	Concept Mastery	Time Spent in ALEKS	WK1 Quiz Grade
1	100	3.4	88
2	100	8.9	88
3	100	12.3	76
4	100	10.2	84
5	100	5.3	88
6	100	6.3	88
7	100	10.2	100
8	100	2.1	96
9	94.4	2.8	64
10	100	9.5	88

11	100	3.5	100
12	100	6.6	100
13	5.6	1.3	48
14	100	20.8	88
15	100	6.1	100
16	100	7.2	88
17	100	8.3	92
18	100	24.4	96
19	100	4	96
20	100	9.3	76
21	27.8	0.4	100
22	100	3.7	80
23	94.4	2.2	88
24	100	0.8	76
25	100	8.3	84
26	94.4	15.1	96
27	100	4.3	92
28	100	8.9	88
29	100	10.7	100
30	0	8	68
31	100	1.2	100
32	100	8.4	84
33	100	0.9	80
34	100	4.9	76
35	100	10.9	84
36	100	7.6	88
37	100	1.6	84
38	100	1.4	80
39	100	19.8	84
40	100	8.2	80
41	100	3.8	96
42	100	5	88
43	100	3.8	80
44	94.4	7.6	64
45	100	3.7	60
46	100	15.6	64
47	100	10	96
48	100	1.4	88
49	16.7	8.5	56
50	33.3	1.6	48
51	100	5.4	92

52	100	2.7	96
53	100	11.1	100
54	100	2.7	92
55	100	3.5	80
56	100	6.3	80
Code	Concept Mastery	Time Spent in ALEKS	WK2 Quiz Grade
1	100	1.8	72
2	100	8.9	92
3	12.5	6.2	44
4	100	9.6	76
5	100	13.1	80
6	100	12.1	88
7	100	4	92
8	100	4.4	100
9	100	1.3	88
10	79.2	3.2	52
11	75	1.5	24
12	100	6.2	88
13	100	0.9	88
14	100	6.8	100
15	87.5	7.9	76
16	100	24.7	84
17	95.8	5.6	88
18	100	4.8	80
19	100	17.5	96
20	100	16	96
21	100	3.8	100
22	66.7	4	76
23	100	6.2	80
24	95.8	6.9	80
25	95.8	4	92
26	100	6.9	88
27	100	15.1	100
28	100	2.9	80
29	100	9.7	80
30	100	9.7	88
31	100	7.4	68
32	100	1.4	100
33	100	7.5	88
34	87.5	6.2	96
35	87.5	3.5	64

36	100	7.3	84
37	100	14	84
38	100	3.3	92
39	100	5.5	80
40	100	16.5	88
41	100	10.1	80
42	100	0.7	96
43	100	3.7	84
44	100	2.1	100
45	100	18.2	84
46	100	15.6	80
47	100	10.8	96
48	100	2.1	96
49	83.3	16.1	72
50	62.5	3.8	76
51	100	3.6	84
52	100	2.4	96
53	100	5.1	100
54	100	0.4	92
55	87.5	2.8	84
56	70.8	5.4	36
Code	Concept Mastery	Time Spent in ALEKS	WK3 Quiz Grade
1	20.8	1.4	36
2	100	12.5	68
3	8.3	13	24
4	100	11.6	96
5	100	13.7	76
6	100	9.3	84
7	100	10.5	92
8	100	5.5	100
9	100	1.2	92
10	50	5.7	24
11	8.3	0.8	12
12	91.7	7.8	88
13	100	0.4	88
14	83.3	10.5	76
15	20.8	4.4	16
16	45.8	9.6	12
17	100	5.5	88
18	100	7.8	88
19	100	15.6	88

20	100	16.3	88
21	100	5.3	92
22	100	8.6	68
23	66.7	4.5	28
24	100	5.5	64
25	100	9	76
26	100	19.3	88
27	37.5	1.1	24
28	100	12.9	68
29	100	7.4	76
30	79.2	3.5	48
31	100	3.7	100
32	100	6.4	56
33	33.3	5.4	92
34	79.2	4.6	68
35	100	10.7	52
36	79.2	17.9	56
37	100	20.8	80
38	62.5	11.9	56
39	100	2.1	82
40	100	11.8	80
41	100	6.5	88
42	58.3	15.9	44
43	87.5	12.2	28
44	100	24.8	76
45	100	16.8	96
46	100	5	92
47	37.5	21.3	12
48	100	2.9	80
49	12.5	2.5	8
50	70.8	2.3	16
51	100	3.9	72
52	100	2	100
53	100	1.3	92
54	79.2	5.1	92
55	79.2	9.8	28
56	0	1.5	28
Code	Concept Mastery	Time Spent in ALEKS	WK4 Quiz Grade
1	20	5	90
2	100	6.1	90
3	100	4.8	70

4	100	8.9	100
5	100	4.2	100
6	100	7.1	100
7	100	1	100
8	100	1.4	100
9	20	2.2	30
10	20	0.5	30
11	91.7	3.7	80
12	100	0.5	100
13	100	2.9	100
14	95.8	3.4	30
15	20.8	2	30
16	95.8	25.3	60
17	100	4.7	80
18	100	2.7	100
19	100	8.7	90
20	100	8.7	100
21	100	1.7	100
22	100	2.4	100
23	66.7	1.6	100
24	100	5.1	90
25	100	5.6	100
26	100	7.1	90
27	37.5	2.4	50
28	100	9.7	80
29	100	1.3	100
30	79.2	2.4	70
31	100	3.2	100
32	100	5.8	70
33	60	8.3	80
34	60	2.9	80
35	100	2.2	90
36	100	12.6	80
37	80	9.8	80
38	100	12.9	70
39	100	10.7	90
40	80	6.4	70
41	100	1.4	100
42	100	5.9	90
43	100	1.9	100
44	80	11.2	80

45	100	3.8	90
46	100	7.3	80
47	100	3	90
48	100	2.3	80
49	20	3.5	60
50	100	1.9	80
51	80	7.1	60
52	100	2.3	90
53	100	2.7	100
54	100	0.9	80
55	60	3.1	90
56	60	3.9	40

APPENDIX D

Raters Result on DOK levels

Test Items	DOK Level 1	DOK Level 2	DOK Level 3	DOK Level 4
Equivalent fractions	x			
Simplifying a fraction	x			
Ordering fractions		x		
Addition or subtraction of fractions with different denominators		x		
Fractional part of a circle		x		
Product of a fraction and a whole number	x			
Fraction multiplication	x			
Fraction division		x		
Integer addition: Problem type 1	x			
Integer addition: Problem type 2	x			
Integer subtraction		x		
Integer multiplication and division	x			
Signed fraction addition		x		
Signed fraction multiplication	x			
Signed decimal addition	x			
Converting between percentages and decimals	x			
Converting a percentage to a fraction	x			
Converting a fraction to a percentage		x		

Test Items	DOK Level 1	DOK Level 2	DOK Level 3	DOK Level 4
Percentage of a whole number		x		
Writing a ratio as a percentage		x		
Word problem on percentage: Problem type 1			x	
Word problem on percentage: Problem type 2			x	
Word problem on percentage: Problem type 3			x	
Computations from circle graphs			x	
Supplementary and complementary angles	x			
Sum of the angle measures of a triangle		x		
Integers and rational numbers		x		
Rational and irrational numbers	x			
Evaluating expressions with exponents: Problem type 1		x		
Substitution and evaluation	x			
Order of operations: Problem type 1		x		
Order of operations: Problem type 2		x		
Exponents and order of operations	x			
Evaluation of a linear expression in two variables	x			
Evaluation of a polynomial in one variable	x			
Writing an inequality	x			
Writing a compound inequality		x		
Writing a mathematical expression		x		

Test Items	DOK Level 1	DOK Level 2	DOK Level 3	DOK Level 4
Translating sentences into equations	x			
Introduction to algebraic symbol manipulation	x			
Distributive property: Basic	x			
Distributive property: Advanced		x		
Combining like terms: Basic	x			
Properties of addition	x			
Properties of real numbers		x		
Additive property of equality with whole numbers	x			
Additive property of equality with integers	x			
Additive property of equality with a negative coefficient		x		
Multiplicative property of equality with whole numbers	x			
Multiplicative property of equality with signed fractions		x		
Using two steps to solve an equation with whole numbers		x		
Solving a two-step equation with integers		x		
Solving a two-step equation with signed fractions		x		
Solving an equation to find the value of an expression		x		
Several occurrences of the variable		x		
Solving a linear equation with several occurrences of the variable: Problem type 1		x		
Solving a linear equation with several occurrences of the variable: Problem type 2		x		
Solving a linear equation with several occurrences of the variable: Problem type 3	x			

Test Items	DOK Level 1	DOK Level 2	DOK Level 3	DOK Level 4
Solving a linear inequality: Problem type 1	x			
Solving a linear inequality: Problem type 2		x		
Solving a linear inequality: Problem type 3		x		
Solving a linear inequality: Problem type 4		x		
Solving a word problem using a linear equation: Problem type 1		x		
Solving a word problem using a linear equation: Problem type 2		x		
Solving a word problem using a linear equation: Problem type 3		x		
Algebraic symbol manipulation	x			
Solving a triangle: Problem type 1		x		
Area and perimeter of a rectangle		x		
Finding the side length of a rectangle given its perimeter or area		x		
Word problem with linear inequalities		x		
Reading a point in the coordinate plane	x			
Plotting a point in the coordinate plane	x			
Solutions to a linear equation in two variables: Problem type 1		x		
Solutions to a linear equation in two variables: Problem type 2		x		
Graphing linear equations		x		
Graphing a line given the x- and y-intercepts		x		
Graphing a line given its equation in slope-intercept form		x		
Determining the slope of a line given its graph		x		

Test Items	DOK Level 1	DOK Level 2	DOK Level 3	DOK Level 4
Graphing a vertical or horizontal line	x			
Interpreting the graphs of two functions	x			
Graphing a compound linear inequality on the number line	x			
Y-intercept of a line	x			
Finding x- and y-intercepts of a line given the equation in standard form		x		
Finding the slope of a line given its equation		x		
Writing an equation of a line given the y-intercept and a point		x		
Writing the equation of a line given the slope and a point on the line		x		
Writing the equation of the line through two given points		x		
Writing the equations of vertical and horizontal lines through a given point		x		
Writing equations and drawing graphs to fit a narrative			x	
Application problem with a linear function: Problem type 1			x	
Application problem with a linear function: Problem type 2			x	
Writing the equation of a parallel line		x		
Classifying systems of linear equations from graphs			x	
Solving a system of linear equations		x		
Solving a word problem using a system of linear equations: Problem type 1			x	
Solving a word problem using a system of linear equations: Problem type 2			x	
Solving a word problem using a system of linear equations: Problem type 3			x	

APPENDIX E

Graphical Display of the Depth of Knowledge (DOK) Results

APPENDIX F

Sample Questions that Raters did not Agree on

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[close window]

ZM Jm

1 2 2

Properties of real numbers

Consider the following properties of real numbers:

[1] Commutative Property of Addition	[6] Commutative Property of Multiplication
[2] Associative Property of Addition	[7] Associative Property of Multiplication
[3] Additive Identity Property	[8] Multiplicative Identity Property
[4] Additive Inverse Property	[9] Multiplicative Inverse Property
[5] Distributive Property	[10] Multiplication Property of Zero

For each equation below, indicate the property that justifies the equation by filling in the box with the appropriate number.

$c+0=c$	[ ]
$3+6=6+3$	[ ]
$m\left(6\cdot\frac{4}{3}\right)=(m\cdot 6)\cdot\frac{4}{3}$	[ ]
$0=0\cdot a$	[ ]

For any real numbers x, y, and z, the properties of addition and multiplication are:

	Properties of addition:	**Properties of multiplication:**
Commutative:	$x+y=y+x$	$x\,y=y\,x$
Associative:	$(x+y)+z=x+(y+z)$	$(x\,y)\,z=x\,(y\,z)$
Identity:	$x+0=x$; $0+x=x$	$x\cdot 1=x$; $1\cdot x=x$
Inverse:	There is a unique number $-x$ such that $x+(-x)=0$; $-x+x=0$	If $x\neq 0$, there is a unique number $\frac{1}{x}$ such that $x\cdot\frac{1}{x}=1$; $\frac{1}{x}\cdot x=1$

Distributive Property:

$x(y+z)=xy+xz$ and $(x+y)z=xz+yz$

Multiplication Property of Zero:

$0\cdot x=x\cdot 0=0$

- $c+0=c$ by the Additive Identity Property.
- $3+6=6+3$ by the Commutative Property of Addition.
- $m\left(6\cdot\frac{4}{3}\right)=(m\cdot 6)\cdot\frac{4}{3}$ by the Associative Property of Multiplication.
- $0=0\cdot a$ by the Multiplication Property of Zero.

The answer is:

$c+0=c$	[3]
$3+6=6+3$	[1]
$m\left(6\cdot\frac{4}{3}\right)=(m\cdot 6)\cdot\frac{4}{3}$	[7]
$0=0\cdot a$	[10]

APPENDIX F

Sample Questions that Raters did not Agree on

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[close window]

ZM 2 1

Multiplicative property of equality with signed fractions 2

Solve for ν .

$$-\frac{7\nu}{4} = 42$$

Simplify your answer as much as possible.

We first write the equation in the following form.

$$-\frac{7}{4}\nu = 42$$

Why can we do this?

In this equation, ν is multiplied by $-\frac{7}{4}$.

We can undo this by multiplying both sides by the reciprocal $-\frac{4}{7}$. Then, we simplify.

$$\left(-\frac{4}{7}\right)\cdot\left(-\frac{7}{4}\right)\nu = \left(-\frac{4}{7}\right)\cdot 42$$

$$1\nu = -\frac{4\cdot \cancel{42}^{6}}{\cancel{7}_{1}}$$

$$\nu = -24$$

The solution is $\nu = -24$.

Checking this answer

APPENDIX F

Sample Questions that Raters did not Agree on

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[close window]

2 3 3

Solving a word problem using a linear equation: Problem type 3

Juan rented a truck for one day. There was a base fee of $18.95 , and there was an additional charge of 94 cents for each mile driven. Juan had to pay $297.19 when he returned the truck. For how many miles did he drive the truck?

We'll first make a list of what we're given.
We'll use x for the number of miles Juan drove.

Number of miles Juan drove:	x
Base fee:	$18.95
Charge per mile:	94 cents, or $0.94
Total amount Juan had to pay:	$297.19

Now we'll write an equation.
Each mile cost $0.94 and he drove for x miles.
So, the amount he paid for driving was $0.94 times x .
The total amount he paid was the base fee of $18.95 plus the amount he paid for driving.

$$18.95+0.94x=297.19$$

We solve the equation for x .

$$\begin{aligned} 18.95+0.94x &= 297.19 \\ 0.94x &= 297.19-18.95 \\ x &= \frac{297.19-18.95}{0.94} \\ x &= 296 \end{aligned}$$

Juan drove for 296 miles.

The answer is 296 miles .