# Essays on Uncertainty in Public Economics and Cooperative Bargaining 

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# ESSAYS ON UNCERTAINTY IN PUBLIC ECONOMICS AND COOPERATIVE BARGAINING <br> BY OMER FARUK BARIS 

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree<br>of<br>Doctor of Philosophy<br>in the<br>Andrew Young School of Policy Studies<br>of<br>Georgia State University

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Omer Faruk Baris
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## ACCEPTANCE

This dissertation was prepared under the direction of the candidate's Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics in the Andrew Young School of Policy Studies of Georgia State University.

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Any faults that remain in this dissertation are, of course, my own.

## This Dissertation is Dedicated to My Parents Mehmet and Fatma Baris

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ABSTRACT<br>ESSAYS ON UNCERTAINTY IN PUBLIC ECONOMICS AND BARGAINING<br>BY<br>OMER FARUK BARIS

July 2012

Committee Chair: Dr. Yongsheng Xu
Major Department: Economics
This dissertation consists of two parts. The theme connecting the two parts is the role of uncertainty. The first part focuses on the role of uncertainty in cooperative bargaining and public decision making. I provide an axiomatic characterization of the normalized utilitarian solution to bargaining problems involving uncertainty. In addition to three basic axioms that are common in the bargaining literature, I propose the axiom of weak linearity to characterize the solution.

In the second part I study uncertainty in non-cooperative games by designing a principal agent model of public bailouts. The first essay in this part sets up the model and shows that the moral hazard problem, namely the Samaritan's dilemma, exists without an altruistic principal. The second essay in this part builds upon the previous essay and focuses on the informational elements in a bailout game. Mainly, I show the existence of a separating equilibrium, where public bailouts serve as a mechanism to reveal essential information to outsiders and in which the good-type agents can benefit from rejecting a bailout offer.

## Chapter I

## INTRODUCTION

This dissertation consists of two parts. In a very broad sense, the theme connecting both parts is the role of uncertainty in economic decisions.

The first essay (Chapter II) concerns the role of uncertainty in cooperative bargaining. In this essay, I provide an axiomatic characterization of the normalized utilitarian solution for convex bargaining problems. Normalized utilitarian solution maximizes the sum of proportional gains as compared to ideal payoffs. The literature often refers to the normalized utilitarian solution as the relative utilitarian solution, since gains are evaluated in proportion to the maximal (ideal) payoffs.

I use four axioms for the characterization of the normalized utilitarian solution: Pareto Optimality, Anonymity, Invariance to Equivalent Utility Representations and Weak Linearity. The first three of these axioms are widely used in the bargaining literature. The fourth axiom, Weak Linearity is introduced in this essay as a weaker variant of the linearity property, often referred to as the "no timing effect condition".

Weak Linearity requires that the timing of the agreement be ignored over a lottery of convex bargaining problems when these problems have the same ideal point. Unlike the Linearity axiom, the Weak Linearity axiom is compatible with the property of invariance to equivalent utility representations. The solution, which satisfies the no timing effect condition, then is ex ante efficient.

The second part of this dissertation studies the role of uncertainty in public decision making regarding bailouts in two essays. In particular, the moral hazard problem and the case of signaling in public bailouts are examined.

In the first essay in this part (Chapter III), I analyze public bailouts by taking a non-cooperative game-theoretic approach and constructing a principal-agent model. I translate the original the Samaritan's Dilemma to the case of a non-altruistic government bailing out private companies. The case is popularly termed as too big to fail). The main result in this chapter is that the moral hazard problem is unavoidable in public bailouts when we relax the assumptions of complete information and an altruistic principal. When the public bailout process is examined as a sequential game with uncertainties, the information gap between agents and the policymaker leads to the moral hazard problem in which an agent exerts less effort once insulated from the risk of failure through a bailout mechanism.

In the following essay (Chapter IV), I address the informational aspects of bailout policies as a continuation of Chapter III. In particular, I show that there exists a separating equilibrium in the bailout game when the game design allows good agents to reveal their true types by rejecting the bailout offer. The main result in this essay is the case that, under certain conditions, the bailout mechanism gives rise to an equilibrium in which the good-type agents separate themselves from the bad-types by rejecting the bailout offer. This type of efficiency-enhancing signaling mechanism mitigates and reverses the moral hazard problem in public bailouts. As a result, the bailout game serves as a signaling mechanism under which efficiency gains are possible.

## Chapter II

## TIMING EFFECT IN BARGAINING AND THE NORMALIZED UTILITARIAN SOLUTION

## II. 1 Introduction

The axiomatic bargaining literature concerns the implications of several axioms that describe the desirable properties of a cooperative outcome. The original framework pioneered by Nash (1950) considers bargaining situations under perfect and complete information in which individuals are expected utility maximizers as described by von Neumann-Morgenstern. In search of a cooperative solution, the main question is clearly related to the issue of fairness. This framework of cooperative bargaining can be applied to several real life situations: division of bequests, gifts, household chores, divorce settlements, wage bargaining, arbitration mechanisms, etc.

This paper addresses the role of uncertainty in bargaining problems. ${ }^{\top}$ The issue of uncertainty has been studied from several dimensions in the literature. In general, the players' risk perception and sensitivity plays a crucial role in bargaining situations, as individual preferences are assumed to be represented by von-Neumann Morgenstern utilities.

A common approach is to analyze the behavior of bargaining solutions when the risk preferences of players are changed. For example, a relatively risk averse player

[^0]is replaced with a player who is less risk averse. Among others, Kihlstrom et al. (1980) analyzes the effect of risk aversion on several solution concepts, Riddell (1981) studies bargaining problems under uncertain states of nature, and Bossert et al. (1996) examines efficiency under uncertainty and characterizes monotone path solutions using non-probabilistic decision rules.

With a special emphasis on the timing of the agreement, Myerson (1981) considers two important properties: linearity and concavity. Linearity is referred to as the "no timing effect" condition, implying that the bargainers cannot strategically delay or expedite the bargaining process. Concavity favors earlier (ex ante) agreements against delayed (ex post) agreements. Another similar axiom is additivity which concerns simultaneity of different bargaining problems, introduced by Perles and Maschler (1981) (see also Peters 1986).

In this paper I follow Myerson (1981) and provide a characterization of the normalized utilitarian solution using a weaker version of linearity property (described in the next section). The normalized utilitarian solution is obtained by maximizing the sum of bargainers' proportional payoffs with respect to the ideal point. The ideal point (also referred to as the utopia point $(\mathrm{Yu} 1973)$ ) is the vector of maximum feasible (and individually rational) payoffs for each bargainer in a bargaining problem. In this respect, the normalized utilitarian solution uses the same information as the Kalai and Smorodinsky (1975) solution.

In order to motivate the issue of uncertainty in bargaining and the normalized utilitarian solution, first consider the implications of different bargaining solutions to a hypothetical scenario described below:

Example 1. Consider the case of two individuals having lost their way in the Sahara Desert. They are at different locations looking for a water supply and there is a fixed supply of water at a different location, which is unknown to both. The amount of water is fixed at 10 gallons. Further, suppose that Traveler A has a
container with a capacity of 9 gallons, and Traveler B has a smaller container with a capacity of only 3 gallons. If Traveler A discovers the water supply first she gets 9 gallons of water from the source; then only one gallon of water will be left for Traveler B. Conversely, if Traveler B discovers the water supply first, she gets 3 gallons of water, and 7 gallons of water will be left for Traveler A. A bargaining situation will arise if they simultaneously discover the water supply. Then how are the travelers going to allocate the water?

The bargaining problem in Example 1 is illustrated in Figure 1.


Figure 1: Travelers' bargaining problem.

Let us consider some common solutions that are offered in the bargaining literature. These solutions are summarized in Table 1.

A classic egalitarian gives 3 gallons of water to each traveler. Traveler A is not allowed to get more water under this solution because it would violate the principle of equal distribution. It is clearly a fair outcome, but it is inefficient to waste (or to leave) 4 gallons of valuable water in the desert. The problem of inefficiency in egalitarianism is an issue of concern for most economists. Nevertheless, there is a room for improvement.

Relaxing the rigid classic egalitarian solution, a maximin (Rawlsian) egalitarian would instead ensure that Traveler A and Traveler B each get at least 3 gallons of water. The Rawlsian solution allow Traveler A to get more water, but that is not an improvement as long as Traveler B stays with 3 gallons of water.

On the other hand, a utilitarian proposes a series of solutions in which all of the water is used, securing efficiency. The utilitarian, however, is indifferent regarding the distribution of water. Therefore, the continuum of $(10-x, x)$ with $1 \leq x \leq 3$ are all possible utilitarian solutions. Accordingly, the utilitarian is indifferent between a situation of Traveler A getting 7 gallons of water and Traveler B getting 3 gallons and a situation of Traveler A getting 9 gallons and Traveler B getting only 1 gallon. In Rawlsian and utilitarian solutions the outcome is multi-valued.

| Solution | Outcome | Unused capacity <br> (proportional) |
| :--- | :---: | :---: |
| Egalitarian | $(3,3)$ | $(2 / 3,0)$ |
| Rawlsian | $(3+x, 3), 0 \leq x \leq 4$ | $(6-x / 9,0)$ |
| Utilitarian | $(10-x, x), 1 \leq x \leq 3$ | $(x-1 / 9,3-x / 3)$ |
| Nash | $(7,3)$ | $(2 / 9,0)$ |
| Kalai-Smordinsky | $(15 / 2,15 / 2)$ | $(1 / 6,1 / 6)$ |
| Normalized utilitarian | $(7,3)$ | $(2 / 9,0)$ |

Table 1: Solutions to travelers' bargaining problem in Example 1.

The two most common single-valued solutions in the bargaining literature are the Nash (1950) and the Kalai and Smorodinsky (1975) solutions. The Nash solution resolves the conflict between the egalitarian and the utilitarian by utilizing an alternative approach. Accordingly, the product of individual quantities is maximized, splitting the watter supply as $(7,3)$. Note that this solution is consistent with both the Rawlsian and utilitarian solutions.

Another alternative apprach is provided by Kalai and Smorodinsky (1975), which utilizes the proportionality principle. Accordingly, the proportional gains (or losses) are equalized. The Kalai-Smorodinsky solution dissents from the Nash
solution because at $(7,3)$ Traveler B's container is full but Traveler A's container is only $7 / 9$ full. Making the proportions equal, a split of $(15 / 2,5 / 2)$ is the outcome according to the Kalai-Smorodinsky solution, so that each traveler fills up 5/6 of their containers and there is no waste of water. I need to note that this solution is efficient in a weaker sense for bargaining problems with more than two individuals. As it utilizes the equality of proportions as a principle, the Kalai-Smorodinsky solution is also referred to as the relative egalitarian solution in the literature.

The utilitarian counterpart to the Kalai-Smorodinsky solution is developed as the normalized utilitarian solution. This solution maximizes the sum of proportional gains (or equivalently minimizes the sum of proportional losses). Accordingly, the normalized utilitarian solution is $(7,3)$, which in this case coincides with the Nash solution. Although $(7,3)$ is the common solution of the Rwalsian, utilitarian and Nash solutions, further complications to the problem in Example 1 point out the disagreement between these solutions.

Example 2 (Travelers' bargaining, revisited). Suppose now that Traveler B's 3 -gallon container is replaced with a 2 -gallon leaky container and a 1-gallon bottle. Unfortunately for Traveler B, when the 2-gallon leaky container is filled, water is spilled and therefore wasted. Hence, Traveler B suffers from low productivity. Note that the total capacity for Traveler B remains unchanged. Suppose for every one gallon that is put in Traveler B's 2-gallon container, exactly one gallon of water is wasted. In light of the new information, the question is whether the allocation of water should change.

The new bargaining problem in Example 2 is illustrated in Figure 2 and the solutions are summarized in Table 2.

Let us consider the changes in the solutions I discussed above. The Egalitarian solution remains unchanged, ensuring that each traveler gets exactly 3 gallons of water. To give 3 gallons of water to Traveler B, 2 gallons of water will be wasted


Figure 2: Travelers' bargaining problem revisited.
during the transfer and 2 gallons will be left in the desert as unused. The Rawlsian solution is between $(3,3)$ and $(5,3)$ causing a waste of 2 gallons during transfer and up to 2 gallons may be left unused.

The Utilitarian solution is now $(9,1)$ so that no water is wasted or left behind.
The Nash solution is $\left(\frac{11}{2}, \frac{11}{4}\right)$ maximizing the product of quantities. Again, no water is left behind according to Nashs solution, however, $7 / 4$ gallons are wasted during the transfer.

The Kalai-Smorodinsky solution is $\left(\frac{33}{5}, \frac{11}{5}\right)$, so that both travelers fill exactly $11 / 15$ of their total capacities, respectively. With the Kalai-Smorodinsky solution, total waste is $6 / 5$ gallons due to the spills during the transfer to Traveler B's leaky container and no water remains unused in the desert.

Finally, the normalized utilitarian solution suggests a split of $(5,3)$, leaving no unused water in the well and causing a waste of 2 gallons during the transfer to Traveler B's leaky container. The motivation behind the normalized utilitarian solution concerns the fairness argument as well as the timing issue and the reasoning is as follows: Consider the original situation that neither Traveler A nor Traveler B has any containers. Instead, the containers are also in the desert at a

| Solution | Outcome | Unused capacity <br> (proportional) |
| :--- | :---: | :---: |
| Egalitarian | $(3,3)$ | $(2 / 3,0)$ |
| Rawlsian | $(3+x, 3), 0 \leq x \leq 2$ | $(6-x / 9,0)$ |
| Utilitarian | $(9,1)$ | $(0,2 / 3)$ |
| Nash | $(11 / 2,11 / 4)$ | $(7 / 18,1 / 12)$ |
| Kalai-Smordinsky | $(33 / 5,11 / 5)$ | $(4 / 15,4 / 15)$ |
| Normalized utilitarian | $(5,3)$ | $(4 / 9,0)$ |

Table 2: Solutions to travelers' bargaining problem in Example 2.
location unknown to both travelers. At one location, there is a 9-gallon container, while at the other location there lies the leaky 2-gallon container and the 1-gallon bottle. If Traveler A and Traveler B are equally likely to discover each location of containers, then they should be able to reach a bargaining solution before the discovery of the containers. Furthermore, the solution should not change regardless of the timing of the settlement, whether it was reached before or after the discovery of the containers.

From the fairness perspective, the normalized utilitarian solution argues that the two individuals in fact share 10 gallons of water equally $(5,5)$ ex ante; so that all of the water is allocated. Although 2 gallons of the water will be wasted during the transfer, the unlucky traveler who discovers the leaky container is not be punished again because of her misfortune. They will each get 5 gallons from the well, but the final allocation will be either $(5,3)$ or $(3,5)$, depending on who ends up with the leaky container. Note that the sum of unused capacity is minimized by the normalized egalitarian solution at $\frac{4}{9}$.

In this paper, I address the timing effect in convex $n$-person bargaining problems (Nash 1950) without requiring interpersonal utility comparisons. I provide an axiomatic characterization of the normalized utilitarian bargaining solution using four axioms : Pareto Optimality (PO), Anonymity (AN), Invariance to Equivalent Utility Representations (INV) and Weak Linearity (WLIN). The first two of these
axioms ( PO and AN ) are standard and ubiquitous axioms in axiomatic bargaining. Axiom INV is also well known in the literature. It simply rules out interpersonal comparisons of cardinal utilities and it is used by Nash (1950) and Kalai and Smorodinsky (1975), among others, in characterizing their solutions.

The fourth axiom, Axiom WLIN, is a weaker variant of the Linearity (LIN) axiom and the central contribution of this paper to the literature. As mentioned above, linearity in the bargaining literature is introduced by Myerson (1981) as the no timing effect condition: "the timing of the bargaining can be ignored if and only if the solution is linear."

The linearity condition as defined in Myerson (1981) is not compatible with scale invariance axiom. The WLIN axiom I propose in this paper mitigates this compatibility problem. This is in parallel to the result that the normalized utilitarian solution does not require interpersonal comparisons of utilities.

The type of restriction (the ideal point restriction) imposed on linearity condition by WLIN is quite common in the bargaining literature. The following are some examples of axioms that apply the ideal point restriction to different axioms $\mathbf{2}^{2}$ Restricted monotonicity (Roth 1979, Thomson 1980, 2010) is an application of the monotonicity axiom to the pairs of bargaining problems that have the same ideal point. In characterization of the equal-loss solution, Chun (1988) assumes equality of ideal points. Based on contraction independence axiom, Xu and Yoshihara (2006) use the same ideal point restriction for the axiom of weak contraction independence, in characterizing the Kalai-Smorodinsky solution for non-convex bargaining problems.

The rest of this paper is organized as follows: Section 2 introduces the notation and definitions. Section 3 discusses the axioms. Section 4 provides the characterization of the normalized utilitarian solution and checks the independence

[^1]of the axioms used. Section 5 provides a discussion and concludes. All proofs are provided in the Appendix.

## II. 2 Definitions

Let $N=\{1,2, \ldots, n\}$ be the set of players with $n \geq 2$. Let $\mathbb{R}$ be the set of all real numbers, $\mathbb{R}_{+}$be the set of all non-negative real numbers and $\mathbb{R}_{++}$be the set of all positive numbers. Let $\mathbb{R}^{n}$ (resp. $\mathbb{R}_{+}^{n}$ and $\mathbb{R}_{++}^{n}$ ) be the $n$-fold Cartesian product of $\mathbb{R}$ (resp. $\mathbb{R}_{+}$and $\mathbb{R}_{++}$). For any $x, y \in \mathbb{R}_{+}^{n}$, I write $x>y$ to mean $x_{i} \geq y_{i}$ for all $i \in \mathbb{N}$ and $x \neq y$, and $x \gg y$ to mean $x_{i}>y_{i}$ for all $i \in \mathbb{N}$.

For any subset $S \subseteq \mathbb{R}_{+}^{n}, S$ is said to be non-trivial if there exists $s \in S$ such that $s \gg 0$ and comprehensive if for all $s, t \in \mathbb{R}_{+}^{n}, s>t$ and $s \in S$ then $t \in S$. For any two sets $S, T \subseteq \mathbb{R}_{+}^{n}$ and for any number $\lambda$, I define $\lambda S+(1-\lambda) T$ to be the set $\lambda S+(1-\lambda) T=\{\lambda s+(1-\lambda) t \mid s \in S$ and $t \in T\}$. A positive affine transformation $\sigma$ on $\mathbb{R}^{n}$ is a mapping $\mathbb{R}^{n} \rightarrow \mathbb{R}^{n}$ such that, for some $\alpha \in \mathbb{R}_{++}^{n}$ and $\beta \in \mathbb{R}^{n}$, $\sigma(x)=\alpha x+\beta$.

The comprehensive hull of a set $S \subset \mathbb{R}_{+}^{n}$ is the smallest comprehensive set containing $S$ :

$$
\operatorname{compS} S=\left\{z \in \mathbb{R}^{n} \mid z \leq x \text { for some } x \in S\right\}
$$

The convex-comprehensive hull of a set $S \subset \mathbb{R}_{+}^{n}$ is the smallest convex comprehensive set containing $S$ :

$$
\operatorname{convS}=\{\lambda x+(1-\lambda) y \mid x, y \in S \text { and } \lambda \in[0,1]\}
$$

For any closed set $S \subseteq \mathbb{R}_{+}^{n}$, a point $s \in S$ is a boundary point of $S$ if every ball with $s$ as the center intersects both $S$ and its complement. I denote the set of boundary points of $S$ as $\mathcal{B}(S)$. I define $a \in S$ as an extreme point of a convex set $S$, if for any $b, c \in S, a=\lambda b+(1-\lambda) c$ if and only if $a=b=c$.

A bargaining problem is defined as a pair $(S, d)$ where $S \subseteq \mathbb{R}_{+}^{n}$ is the feasible set, $d \in S$ is the disagreement point. The disagreement point represents the payoffs to be received by each player if no agreement is reached. It is assumed that there exists $s \in S$ such that $s \gg d$. Throughout the discussion I will assume that the disagreement point is fixed.

Let $\Sigma$ be the set of all non-trivial, convex, compact and comprehensive subsets of $\mathbb{R}_{+}^{n}$. A bargaining solution $F$ over $\Sigma$ assigns a non-empty subset $F(S, d) \subset S$ for every bargaining problem $(S, d), S \in \Sigma$. Note that, this is different from Nash's original definition of a solution, since the bargaining solution may be multi-valued.

Let $\pi$ be a permutation of $N$ and $\Pi$ denote the set of all permutations of $N$. For all $x=\left(x_{i}\right)_{i \in N} \in \mathbb{R}_{+}^{n}$, let $\pi(x)=\left(x_{\pi(i)}\right)_{i \in N}$. For all $S \in \Sigma$ and any permutation $\pi \in \Pi$, let $\pi(S)=\{\pi(s) \mid s \in S\}$. For any $S \in \Sigma, S$ is said to be symmetric if $S=\pi(S)$ for all $\pi \in \Pi$.

For all $S \in \Sigma$ and all $i \in N$, let $m_{i}(S, d)=\max \left\{s_{i} \mid\left(s_{1}, \ldots, s_{i}, \ldots, s_{n}\right) \in S\right\}$. Therefore, $m(S, d)=\left(m_{i}(S, d)\right)_{i \in N}$ is the ideal point of the bargaining problem $S$. Note that $m(S, d) \gg d$ for any $S \in \Sigma$.

Definition 1 (Normalized Utilitarian Solution). A solution $F$ over $\Sigma$ is the normalized utilitarian solution $F^{N U}$ if for all $S \in \Sigma$

$$
F^{N U}(S, d)=\left\{s \in S \left\lvert\, \sum_{i \in N} \frac{m_{i}-s_{i}}{m_{i}-d_{i}} \leq \sum_{i \in N} \frac{m_{i}-x_{i}}{m_{i}-d_{i}}\right. \text { for all } x \in S\right\}
$$

Note that the above definition uses the minimization of the sum of proportional losses, which is equivalent to the following standard 'maximization' definition of the normalized utilitarian solution:

$$
F^{N U}(S, d):=\left\{s \in S \left\lvert\, \sum_{i \in N} \frac{s_{i}-d_{i}}{m_{i}-d_{i}} \geq \sum_{i \in N} \frac{x_{i}-d_{i}}{m_{i}-d_{i}}\right. \text { for all } x \in S\right\}
$$

The following definition generalizes the normalized utilitarian solution:

Definition 2 (Generalized Normalized Utilitarian Solution). Let $\mu=\left(\mu_{1}, \ldots, \mu_{n}\right)$ $\in \mathbb{R}_{+}^{n}$ be a vector. The generalized normalized utilitarian solution $F_{\mu}^{G N U}$ over $\Sigma$ is defined by

$$
F_{\mu}^{G N U}(S, d):=\arg \max _{s \in S} \sum_{i \in N} \mu_{i} \frac{s_{i}-d_{i}}{m_{i}-d_{i}}
$$

where $m_{i}=\max _{s \in S} s_{i}$ for all $i \in N$.

## II. 3 Axioms

Pareto Optimality (PO) For all $S \in \Sigma$ and for all $s \in F(S, d)$, if $x>s$ then $x \notin S$.

Anonymity (AN) For all $S \in \Sigma$ and for all $\pi \in \Pi, F(\pi(S), \pi(d)=\pi(F(S, d))$.

Invariance to Equivalent Utility Representations (INV) For all $S \in \Sigma$ and for all $\alpha \in \mathbb{R}_{++}^{n}$ and $\beta \in \mathbb{R}^{n}, F(\alpha S+\beta, \alpha d+\beta)=\alpha F(S, d)+\beta$.

The first two axioms above are standard in the literature. PO ensures that the gains from cooperation are fully exhausted. AN is a stronger version of Nash's symmetry axiom. It states that the solution should not depend on any particular attributes (e.g. the names) of the bargainers.

The INV axiom requires the bargaining solution to be invariant to equivalent utility representations and only the utility gains of the bargainers over the disagreement point is meaningful in determination of the bargaining solution. INV rules out interpersonal comparisons of utility (1) levels (as required by egalitarian and maximin solutions), and (2) scales (as required by classic utilitarian solution). In the literature this axiom is sometimes decomposed into translation invariance ( $\beta$ ) and scale invariance $(\alpha)$.

The next axiom is due to Myerson (1981).

Linearity (LIN) For any $S, T \in \Sigma$

$$
F(\lambda S+(1-\lambda) T, d)=\lambda F(S, d)+(1-\lambda) F(T, d)
$$

for every $\lambda \in[0,1]$.

LIN states that the timing of the agreement is irrelevant over a bargaining problem which is obtained through a lottery over two bargaining problems. When the players face one of the two bargaining problems $S$ and $T$ (with the same disagreement point) with probability $\lambda$ and $(1-\lambda)$ respectively, the timing of the agreement should not matter. Myerson (1981) combined LIN axiom with PO to characterize the (generalized) utilitarian solution. It is also known that LIN is not compatible with INV Myerson 1978, Pechersky 2006. ${ }^{3}$

By weakening the LIN axiom in the following way, it is possible to avoid this impossibility.

Weak Linearity (WLIN) For any $S, T \in \Sigma$, if $m(S)=m(T)$, then

$$
F(\lambda S+(1-\lambda) T, d)=\lambda F(S, d)+(1-\lambda) F(T, d)
$$

for every $\lambda \in[0,1]$.

WLIN restricts LIN for the pairs of bargaining problems which have the same ideal point. That is, the timing effect is irrelevant only if a bargainer expects the same maximum feasible gain from each of the uncertain bargaining problems $(S, d)$ and ( $T, d$ ).

The bargaining problem tomorrow will be $S$ with probability $\lambda$ and $T$ with probability $(1-\lambda)$. Thus, the solution to the bargaining problem will be $F(S)$ with

[^2]probability $\lambda$ and $F(T)$ with probability $(1-\lambda)$. The bargainers can reach an agreement today by considering a new bargaining problem $\lambda S+(1-\lambda) T$. WLIN ensures that bargainers will not gain or lose by either delaying or expediting the agreement.

## II. 4 Characterization and Independence of Axioms

The following theorems characterize the generalized normalized utilitarian solution and the normalized utilitarian solutions respectively. Proofs are provided in the Appendix.

Theorem 1 (Generalized normalized utilitarian solution). A solution $F$ over $\Sigma$ is the generalized normalized utilitarian solution $F^{G N U}$ if and only if it satisfies $P O$, INV and WLIN.

Theorem 2 (Normalized utilitarian solution). The only GNU bargaining solution which satisfies $A N$ is the $N U$ solution $F^{N U}$.

To check the independence of the axioms, first drop AN. The first theorem (Theorem 1) characterizes the generalized form of the normalized utilitarian solution using PO, INV, and WLIN, and demonstrates the independence of AN from PO, INV and WLIN. Theorem 2 demonstrates that the normalized utilitarian solution is indeed a special case of the generalized form when $\mu_{i}=\mu_{j}$ for all $i, j \in N$.

To check the independence of PO , consider the disagreement point $d$ as a solution. It is easy to verify that $d$ satisfies AN, INV, and WLIN but not PO.

When INV is dropped, the classic utilitarian solution satisfies PO, AN, and WLIN, but clearly it is not equivalent to the normalized utilitarian solution $F^{N U}$.

Finally, when WLIN is dropped, consider the Nash solution. The Nash solution satisfies AN, PO, and INV, but clearly it is not equivalent to the normalized utilitarian solution $F^{N U}$.

## II. 5 Discussion

The timing of the agreement becomes important if the bargaining problem is obtained from a lottery of two (or more) bargaining problems. My characterization in this chapter is built upon Myerson (1981)'s result that the timing effect can be ignored only if the bargaining solution is linear. The linearity axiom is thus called the no timing effect condition. Together with Pareto Optimality, linearity implies that the bargaining solution is the generalized utilitarian solution (Myerson 1981). The incompatibility of the linearity axiom with the property of invariance to equivalent utility representations (INV) is to be expected since the utilitarian solution implicitly assumes interpersonal comparisons of utility (see also Myerson 1978, Pechersky 2006).

To overcome the incompatibility between linearity and the property of invariance to equivalent utility representations, I use a weaker version of the linearity axiom. As discussed in the text weak linearity axiom weakens the linearity axiom in such a way that interpersonal comparisons of utilities are not required but the timing effect is still critical. This axiom restricts attention to the specific bargaining problems which have the same ideal point. The linearity axiom imposes a linearity condition over a lottery of bargaining problems, which can be chosen arbitrarily. In contrast, the weak linearity axiom requires the "no timing effect" condition to hold only if these problems agree on the same ideal point. When two or more bargaining problems agree on the same ideal point, a bargainer will expect the same maximum payoff from each of these problems.

The contrast between the linearity and weak linearity axioms is akin to the ideal point restriction applied by Kalai and Smorodinsky (1975) to the monotonicity axiom. In this respect, for bargaining problems with uncertainty, the contrast between the normalized utilitarian and Kalai-Smorodinsky solutions can be seen as
an extension of the well-known trade-off between efficiency and equality. The former is ex ante efficient while the latter is only ex post efficient (in addition to being egalitarian). The individual monotonicity axiom (Kalai and Smorodinsky 1975) imposes ideal point restriction on strong monotonicity, and the normalization of individual payoffs with respect to the ideal point takes the egalitarian solution to the Kalai-Smorodinsky solution. Similarly, by the ideal point restriction on the linearity axiom, the normalization of individual payoffs with respect to the ideal point takes the utilitarian solution to the normalized utilitarian solution.

In connection with the impartial observer theorem of Harsanyi (1953), the ideal point restriction imposed by the weak linearity axiom on bargaining problems also relates to the impartiality restrictions discussed by Karni (1998). Harsanyi's utilitarianism defines impartiality by assigning equal weights to individual utilities. However, as Karni (1998) argues, the equality of weights is not meaningful when individual utilities can be manipulated by positive affine transformations. By assigning probabilities to different bargaining problems, linearity implicitly imposes comparability between such bargaining problems. This is problematic since these comparisons are not meaningful when each bargaining problem can be manipulated by rescaling. However, I will argue that these comparisons are meaningful only between bargaining problems that have the same ideal point, as required by the weak linearity axiom. Each individual's payoff is compared intra-personally between different bargaining problems $\hat{母}^{母}$ Intra-personal comparisons of individual utilities are meaningful in expected utility theory while interpersonal comparisons are not allowed.

Suppose two bargainers have "something" over which they bargain but they do not get anything if they fail to reach an agreement. 5 The two obvious allocations are (i) all to first player (ii) all to the second player; hence the ideal point is a function

[^3]of the "something". Now, it is possible to identify games as different allocation rules or mechanisms: all will have the same ideal point since all have the option of giving everything to one player, but will have different bargaining sets. There is a fixed "something" they bargain and naturally, linearity applies only to such sets.

As a result of the weak linearity axiom, the normalized utilitarian solution is ex ante efficient for convex bargaining problems involving uncertainty. The Nash and Kalai-Smorodinsky solutions fail to satisfy this property even for two person bargaining problems.

To illustrate ex ante efficiency, consider the two-person bargaining problem $S$ in Figure 3, where $a, b$, and $c$ denote the outcomes reached by Kalai-Smorodinsky solution, the Nash solution, and the normalized utilitarian solution, respectively.


Figure 3: Three bargaining solutions to $S$ : $a$-Kalai Smorodinsky solution, $b$-the Nash solution, $c$-the normalized utilitarian solution.

Recall that all of the three solutions satisfy the three main axioms of Pareto


Figure 4: Ex ante efficiency.
optimality, symmetry, and invariance to equivalent utility representations. Suppose also that the bargaining problem (the shape of $S$ ) is known today, but the bargainers are uncertain about their roles (on which axis they will be represented) tomorrow. With probability $\lambda$, Player A will be located on the horizontal axis, Player B will be on the vertical axis, and with probability $(1-\lambda)$ vice versa. The players can wait until tomorrow so that the uncertainty will be resolved, or they may use the available information in order to reach an agreement today.

Let $S^{\prime}$ be the symmetric counterpart of $S$ along the $45^{\circ}$ line. Then, the expected bargaining problem will be $\lambda S+(1-\lambda) S^{\prime}$. This is shown in Figure 4 with $\lambda=\frac{1}{2}$.

Let the three solutions corresponding to $S^{\prime}$ are defined similarly as $a^{\prime}, b^{\prime}$, and $c^{\prime}$ respectively and let the players agree to bargain over the expected bargaining problem $\lambda S+(1-\lambda) S^{\prime}$. In that case the expected outcomes under Kalai-Smorodinsky and Nash solutions, are $b^{*}=\lambda b+(1-\lambda) b^{\prime}$ and $a^{*}=\lambda a+(1-\lambda) a^{\prime}$, respectively. The expected outcome under the normalized utilitarian solution is $c^{*}=\lambda c+(1-\lambda) c^{\prime}$. Note that $c^{*}>b^{*}>a^{*}=a$ indicate that $c^{*}$ is ex ante efficient (PO over the expected bargaining problem $\left.\lambda S+(1-\lambda) S^{\prime}\right)$ but $a^{*}$ and $b^{*}$ are not ex ante efficient. In parallel to Pivato (2009), the long-term expected utility is maximized only through the normalized utilitarian solution over a random sequence of future bargaining problems.

To my knowledge, $\mathrm{CaO}(1982)$ is the first in the literature to introduce the normalized utilitarian solution (defined as the modified Thomson solution). As mentioned earlier, this solution is also referred to as the relative utilitarian solutionin the social choice literature. However, as Segal (2000) points out, this solution is not utilitarian in nature. The sort of interpersonal comparisons of cardinal utility, which are fundamental to classical utilitarianism, are ruled out in this solution. However, I follow the existing literature in this paper and use the term normalized utilitarian solution (Miyagawa 2002, Thomson 2010) in reference to this solution (Dhillon 1998, Dhillon and Mertens 1999, Segal 2000, Sobel 2001, Pivato 2009).

In general, earlier characterizations of the normalized utilitarian solution (CaO 1982, Dhillon 1998, Dhillon and Mertens 1999) are based on a version of the monotonicity axiom. Segal (2000) introduces the dictatorship indifference axiom, and a weaker form of the dictatorship indifference axiom is used by Pivato (2009) to characterize the normalized utilitarian solution (together with the axioms of strong linearity and Pareto optimality).

The title (Let's agree that all dictatorships are equally bad) of Segal (2000) is somewhat misleading. The dictatorship indifference axiom is interpreted in a way
that pure dictatorship allocations (i.e. giving everything to a single player) are "equally undesirable" (Segal 2000, pp. 581). As a matter of fact, the normalized utilitarian solution does not rule out dictatorship allocations in every case. In a case that the Pareto frontier of the bargaining problem is parallel to the hyperplane determined by the dictatorial points, the normalized utilitarian solution is obviously multi-valued. Furthermore, if this hyperplane is identical to the entire Pareto frontier, then pure dictatorship solutions are not ruled out.

Consider, for example, a two-person convex bargaining problem given by $S=\left\{x \mid a x_{1}+b x_{2} \leq 1\right\}$. The normalized utilitarian solution for $S$ is the entire Pareto frontier given by the equation $a x_{1}+b x_{2}=1$. The pure dictatorial solutions $\left(\frac{1}{a}, 0\right)$ and $\left(0, \frac{1}{b}\right)$ belong to the set of normalized utilitarian solutions. None of the axioms rule out these solutions.

Pivato (2009) uses an approach that is closer to mine in terms of interpretation, although the axioms and the formal characterization are different.

As a final remark, the risk properties of von Neumann-Morgenstern utility scales play a crucial role in the normalized utilitarian solution. Once there is some uncertainty regarding the bargaining problem, the significance of the weak linearity condition becomes more evident. Using this phenomenon, every bargaining problem can be thought as a combination of two or more bargaining problems with different probabilities. In this context, the normalized utilitarian solution provides ex ante Pareto optimal solutions.

Accordingly, the contrast between the normalized utilitarian and the Kalai-Smorodinsky solutions is a natural extension of the well-known efficiency and equality trade-off utilitarianism and egalitarianism. The Kalai-Smorodinsky solution, as referred to as the relative egalitarian solution, is weakly Pareto optimal for bargaining problems with more than two players. After normalization of individual utilities to $[0,1]$ scale, the Kalai-Smorodinsky solution has an egalitarian
objective: the proportionate losses are equal among individuals. The normalized utilitarian solution uses the same normalization and it has a utilitarian objective: the sum of proportionate losses among individuals is minimized.

## Chapter III

## GOVERNMENT'S DILEMMA IN PUBLIC BAILOUTS

## III. 1 Introduction

The idea of public bailouts is more than a century old. During a time of bank runs and financial panic, Sir Walter Bagehot (1873) - when discussing last-resort lending - advised on rescuing the good banks and letting bad banks go insolvent. Today, the concept of providing public assistance to troubled agents in the private sector in times of financial distress has been extended to non-financial firms facing the risk of bankruptcy during economic downturns and countries whose reserves prove insufficient ${ }^{6}$ Recent events across the globe (e.g. the 2008 recession in the U.S. and the Euro Zone's troubles with Greece) have once again demonstrated that bailouts are still crucial assignments in public policy-making.

From an economic perspective, one can provide different justifications, both in favor of public bailouts and against them. Arguably the most severe criticism is prominently described by the Samaritan's Dilemma (cf. Buchanan 1975), where the strategic interaction between an agent and an altruistic principal causes moral hazard: Anticipating financial assistance by the principal when in need, agents engage in excessively risky behavior. In the words of Bagehot (1873): "Any aid to a

[^4]present bad bank is the surest mode of preventing the establishment of a future good bank." ${ }^{7}$

To discuss further, Buchanan (1975) demonstrates how modern-day welfare policies lead to a dilemma when they become the rule of conduct. Calling it the Samaritan's Dilemma, he provides insights into the inefficiency of the outcome that arises through a donor's decision to supply assistance to a recipient in need. The dilemma occurs whenever a donor and a recipient interact strategically for at least two periods $\sqrt{8}^{8}$ Acting in the first period, the recipient can potentially manipulate the donor's second-period decision. This strategic interaction leads to moral hazard, as the recipient finds it beneficial to exert less effort (i.e. behave in a more risky way), in expectation of more assistance. On the other hand, the donor (policymaker), who cannot control the recipient's actions, is caught in a trap, the Samaritan's Dilemma, due to her altruism and her rational decision-making.

To address this moral hazard problem in public bailouts, I consider a sequential game that captures the informational gap between agents and a public policymaker. My model builds upon the Samaritan's Dilemma problem, but I relax the underlying assumptions of altruism and complete information. Therefore, it is an extension to, not of, the Samaritan's Dilemma.

The most critical assumption in the literature on the Samaritan's Dilemma is the altruism of the principal (public policymaker) who engages in a strategic interaction with an agent. In this paper, I demonstrate that the moral hazard problem does not vanish, even when the principal is not altruistic, but is instead concerned with the negative externalities resulting from an agent's failure. Moreover, I show that this result holds whether or not the agent's effort is observable by the principal. In that sense, my findings apply to all cases of principal-agent problems of financial

[^5]assistance, where the financing is done by a third party (e.g. taxpayers or other member nations). When there are multiple agents with interdependent prospects, the problem aggravates: the absence of bailouts leads to inefficient behavior by each agent as a Nash equilibrium and its presence leads to the well-known moral hazard problem. Hence the government's dilemma.

The efficiency-enhancing solutions proposed in the literature for the Samaritan's Dilemma include pre-commitment (Buchanan 1975, Chami 1996, Dijkstra 2002), in-kind or tied transfers (Bruce and Waldman 1990, 1991, Coate 1995). The pre-commitment solutions require either strong assumptions on the credibility of these commitments or a strict enforcement mechanism, which is hard to employ in practice. In parallel to that, in-kind (as opposed to direct financial) transfers are not practically feasible for public bailouts. Even for the so-called "Rotten Kid Theorem" (Becker 1974, 1981, Bergstrom 1989), the savings by the selfish kid are too low relative to the efficient level, and it is hard to attain the Pareto frontier (Bruce and Waldman 1990).

At this point, it is necessary to explain why the assumption of a non-altruist principal for the case of public bailouts is critical. The public bailouts in practice are not due to the government's altruism towards the private companies that are rescued. Instead, these banks and firms are rescued by the government from bankruptcy in order to avoid collateral damage. The argument in favor of a bailout is that large institutions cannot be allowed to go bankrupt, as their failure might put the entire (financial) system in jeopardy ("too big to fail", see e.g. Stern and Feldman 2004, Gup 2004). This is particularly true in times of a financial crisis due to an expected contagion effect when the failure of one institution increases the risk for all others. The main problem is not the altruism of the principal, but is instead the negative externalities (e.g. systemic risk), and the expected social cost (e.g. unemployment) after a firm's bankruptcy. Combining all these third-party costs
together, a more appropriate term is the dead-weight costs of failure. Based on a cost-benefit analysis, a public bailout should be offered when the dead-weight costs of failure in the economy exceed the resulting inefficiency and moral hazard costs.

I also need to point out that bailouts are often political considerations rather than economic decisions. In general, the justifications in favor of bailouts and the benefits thereof are highly speculative. For that reason, I limit the discussion around the conventional justifications for government intervention that are broadly agreed on in public economics (cf. Tresch 2002): A public bailout is justified when
(i) there is a market failure,
(ii) the mechanism is efficiency enhancing,
(iii) the mechanism minimizes the moral hazard problem,
(iv) negative externalities linked to the mechanism are minimized.

In this chapter, I identify the market failure (i) and address the moral hazard problem (iii) stemming from the presence of asymmetric information and risk of failure that has negative externalities (iv). A special case of efficiency enhancing solution (ii) is laid out in the next chapter.

An overwhelming majority of the literature on public bailouts focuses on the financial sector. This is not surprising given the importance and frequency of bank bailouts and the relative proportion of financial sector firms among bailout recipients. Most of these studies include some variant of a deposit insurance mechanism (see e.g. Bryant 1980, Diamond and Dybvig 1983). There is also an abundance of research on the effects of banking bailouts (see e.g. Boyd et al. 2004, Bryant 1980, Butkiewicz and Lewis 1991, Diamond and Dybvig 1983, Diamond and | Rajan 2002, Gorton and Huang 2004, Kho et al. 2000, Yaron 2005) |
| :---: | :---: | :---: |

Moreover, there is extensive list of papers searching for an optimal bailout policy and on the effect of different remedies (cf. Ringbom et al. 2004, Rochet and Vives

2002, among others). In relevance to my study, I would like to point to Ghatak et al. (2001), who argue using an overlapping generations model that efforts against risk are negatively affected by a public assistance program. Shim et al. (2008) extends the pre-commitment option to a framework of an IMF-type institution with voluntary coinsurance arrangements between agents. Using a model with stochastic outside shocks, the author demonstrates that ex ante loan contracts are superior to ex post loans. However, the credibility of these contracts remains mostly unanswered. As another approach, Dijkstra (2002) shows that changing the sequence of events and moves also alters the inefficient outcome in the backward induction game.

My approach differs fundamentally from these models examining bank bailouts, as I do not require borrowing and lending, and there is no lender of the last resort. As a result, the bailout mechanism is not restricted to a deposit insurance institution or a central bank. This flexibility allows me to extend the analysis to include non-financial sector bailouts, including country bailouts, which are equally common.

The remainder of the chapter is organized as follows: Section III.2 introduces my basic model that captures the bailout dilemma of a non-altruistic principal under plausible assumptions. In Section III.3, the model is extended to the case of multiple agents and the presence of systemic risk. Finally, Section III. 4 provides a brief discussion of my findings and concludes. All proofs are provided in the Appendix.

## III. 2 The Moral Hazard Problem in Bailouts

As mentioned in the previous section, the Samaritan's Dilemma is a textbook case of moral hazard arising from the altruism of a principal. However, altruism is not a requirement for the presence of moral hazard. It can arise even when the principal is not concerned with the direct costs of bankruptcy. Instead, there are indirect
(dead-weight) costs associated with an agent's failure that harm third-party individuals (citizens) and therefore enter the principal's welfare function directly.

In my basic model, an agent (firm) and a principal (government) are involved in a sequential game. The agent faces a risk of failure (e.g. bankruptcy) and the principal can provide financial assistance to reduce that risk (bailout). Their utilities are denoted by $U$ for the agent and $W$ for the principal, respectively. The game follows the sequence of events described below.

1. The agent moves first and chooses a level of effort $e>0$, which reduces the risk of bankruptcy in case of an external shock. Effort is costly and generates disutility $v(e)$. The specific assumptions on $v$ are described in Assumption 2 . The effort may be thought of as the level of precautionary measures and risk management activities carried out by a firm, or savings by an individual, or reserves by a country in the respective cases.
2. Nature rolls its dice, and with probability $\pi$, an exogenous shock (financial crisis) occurs; otherwise the game ends and the agent collects a positive payoff $y>0$.
3. In case of an exogenous shock, the principal transfers $b \geq 0$ to the agent (bailout).
4. The agent fails with probability $p(e, b)$ and survives with probability $1-p(e, b)$. The agent receives zero payoff in case of a failure and a strictly positive payoff of $y>0$ otherwise. The agent's failure causes a social cost of $c$, a negative externality, which is borne by the principal.

I make the following assumptions regarding model parameters.

Assumption 1. The agents' utility function is additive and separable:
$U(e)=[1-\pi p(e, b)] \cdot u(y)-v(e)$ with $u(0)=0, u^{\prime}>0 u^{\prime \prime} \leq 0$.

Assumption 2. The cost of effort, $v(e)$, is non-negative, increasing and convex in $e: v(0)=0, v^{\prime}>0, v^{\prime \prime}>0$.

Assumption 3. The probability of failure, $p$, is decreasing and convex in $e$ and also in $b$.
(a) $\frac{\partial p}{\partial e}<0$ and $\frac{\partial^{2} p}{\partial e^{2}}>0$
(b) $\frac{\partial p}{\partial b}<0$ and $\frac{\partial^{2} p}{\partial b^{2}}>0$
(c) $\frac{\partial^{2} p}{\partial b \partial e}>0$

Assumption 3 states that both effort and bailout reduce the probability of failure but with diminishing marginal rates. It further implies that a bailout has less of an impact on the probability of default, the higher the level of effort by the agent.

In this model, the principal offers to help the agent because she is concerned with the well-being of society as a whole (or more precisely, with the well-being of the third-party individuals) and thus wants to avoid the social costs caused by the agent's failure. In this context, the well-being of society is denoted by $W$.

The expected utility of the agent is given by

$$
\begin{equation*}
U(e)=u(y)-v(e)-\pi p(e, b) u(y) \tag{1}
\end{equation*}
$$

and the agent chooses $e$ to maximize (1). Correspondingly, the principal's expected welfare is given by

$$
\begin{equation*}
W(b)=W-\pi b-\pi p(e, b) c \tag{2}
\end{equation*}
$$

and the principal chooses $b$ to maximize (2).

## III.2.1 Complete Information

I first consider the case where the principal can observe $e$ : The agent chooses effort $e$ in order to maximize (1), and after observing the agent's choice of effort, $e^{*}$, the principal chooses bailout transfer $b$ to maximize (2).

When the agent's choice of effort is monitored by the principal, Proposition 1 shows that the agent, in expectation of higher transfers, exerts less effort than without transfer, provided that the social costs of failure are large enough to justify a bailout. This result identifies the moral hazard problem in parallel to the Samaritan's Dilemma. The key difference here is that the result does not require the principal to be altruistic, and thus it captures the practical bailout motives.

The following lemma states that the more effort the agent exerts, the less assitance he can expect from the principal.

Lemma 1. When the principal observes the agent's effort, higher effort chosen by the agent leads to lower bailout transfers.

Proposition 1. If the principal can observe the agent's effort, and if $b^{*}>0$, the agent exerts effort below the efficient level.

As a parallel result to the Samaritan's Dilemma, Proposition 1 identifies the moral hazard implications of public bailouts when the principal is not altruistic but the failure of the agent has negative externalities.

## III.2.2 Incomplete Information

I shall now demonstrate that the result in the previous section extends to the case of incomplete information. I do this by extending the Samaritan's Dilemma in a second direction: by relaxing the assumption of complete information. Specifically, the principal selects bailout transfer $b$ in order to maximize Equation (2) without
observing the agent's choice of $e^{*}$. The following theorem describes the moral hazard problem of bailouts under incomplete information:

Theorem 3. If the social costs of failure, $c$, are sufficiently large to justify a bailout (that is, $b^{*}>0$ ), the agent's optimal effort $e^{*}$ is below the efficient level.

Unlike the complete information case, the moral hazard problem is now conditional when the agent's effort choice is not observed by the principal. For small values of social cost $c$, which is public information, the social cost $c$ is negligible by the principal, the marginal cost of effort is significantly higher than the marginal benefit for the agent, the expected bailout money is considerably small $b^{*}$, and there is no moral hazard. On the other hand, the agent's first condition requires that the optimal level of effort $e^{* *}$ under the principal's pre-commitment policy is chosen when the marginal cost of the effort $v^{\prime}(e * *)$ is equal to the marginal benefits $-\frac{\partial p}{\partial e} \pi u(y)$.

The strategic considerations by the principal must be so strong (credible) that the moral hazard problem can be avoided. In theory, the pre-commitment option works so that $b=\bar{b}$ and $\frac{d e}{d b}=0$. In practice, however, especially when $c$ is considerably high, the agent can always call the principal's bluff. For instance, by knowing that higher employment will harm the government's popularity, the management of the bailout firm may dismiss any kind of pre-commitments announced by the government and therefore choose risky investments and expect to be rescued in case of a bankruptcy.

The discussion above demonstrates that the moral hazard problem in the classical Samaritan's Dilemma is also present under more realistic assumptions in the context of bailouts: the presence of a bailout mechanism reduces the agent's incentives to take precautions independent from whether the principal is altruistic or concerned with the social costs of the agent's failure, and whether effort is public
information or not. The next section extends this analysis to multiple agents and interdependent failure risk.

## III. 3 Multiple Agents and Systemic Risk

In this section, the bailout model of Section III.2 is extended to include multiple agents. While my illustration is with two agents interacting with each other and a principal, it can be extended to an arbitrary number of agents.

Agents and principal are involved in a bailout game with the sequence of events as in the single-agent model (cf. Section III.2). I adjust the notation to the new circumstances by adding subscript $i \in\{1,2\}$ to the agents' utilities ( $u_{i}, v_{i}$ ) and payouts $\left(y_{i}\right)$, the probabilities of failure $\left(p_{i}\right)$, bailout transfers $\left(b_{i}\right)$ and social costs $\left(c_{i}\right)$. Again, $U_{i}$ and $p_{i}$ are subject to Assumptions 1, 2 and 3. An additional assumption can be introduced as the principal satisfies the budget constraint

$$
\begin{equation*}
b_{1}+b_{2} \leq B \tag{3}
\end{equation*}
$$

for some $B>0$ which is exogenously chosen as the upper limit of available funds for bailouts.

To introduce systemic risk and to capture the agents' interdependence, let the probability of an exogenous shock be $\pi=\pi(e)$ (where $e=\left(e_{1}, e_{2}\right)$ ) depend on both agents' choices of effort. The mapping $\pi: \mathbb{R}_{+}^{2} \rightarrow[0,1]$ is subject to the following assumptions:

Assumption 4. (a) $\pi\left(e_{1}, e_{2}\right)=1$ when $\left(e_{1}, e_{2}\right)=0$.
(b) $\frac{\partial \pi}{\partial e_{i}}<0$ for $e \gg 0$.
(c) $\frac{\partial^{2} \pi}{\partial e_{i}^{2}}>0$ and $\frac{\partial^{2} \pi}{\partial e_{i} \partial e_{j}}>0$ for $e \gg 0$.

Assumption 4 requires that the shock is inevitable if one of the agents (or both) chooses zero effort. Furthermore, the probability of a shock is decreasing in the agents' effort choices provided that each agent exerts a positive level of effort.

Following the two agents' simultaneous choice of effort levels, if there is am exogenous shock, the principal transfers the optimal bailout amount to each agent, before each agent fails with probability $p_{i}\left(e_{i}, b_{i}\right)$. In particular, I assume for simplicity that the agents' failure probabilities are independent.

## III.3.1 Optimal Effort without Bailouts

First, as a baseline case, I consider the scenario where the principal precommits to a bailout policy with $b_{1}=b_{2}=0$. The agents' expected utilities are then given by

$$
\begin{equation*}
U_{i}\left(e_{i}\right)=u_{i}\left(y_{i}\right)-\pi\left(e_{i}, e_{-i}\right) p_{i}\left(e_{i}, 0\right) u\left(y_{i}\right)-v_{i}\left(e_{i}\right) . \tag{4}
\end{equation*}
$$

Without the principal's involvement in the game, the two agents optimize their own efforts by taking each other's best response effort as given. Accordingly, Agent 1 solves

$$
\max _{e_{1}} u_{1}\left(y_{1}\right)-v_{1}\left(e_{1}\right)-\pi\left(e_{1}, e_{2}^{*}\right) p_{1}\left(e_{1}, 0\right) u_{1}\left(y_{1}\right) .
$$

The first order condition for an interior solution is

$$
\begin{equation*}
\frac{\partial \pi\left(e_{1}, e_{2}\right)}{\partial e_{1}} p_{1}\left(e_{1}, 0\right)+\frac{\partial p_{1}\left(e_{1}, 0\right)}{\partial e_{1}} \pi\left(e_{1}, e_{2}\right)=-\frac{v_{1}^{\prime}\left(e_{1}\right)}{u_{1}\left(y_{1}\right)} . \tag{5}
\end{equation*}
$$

Similarly, the first order condition for Agent 2's maximization problem is

$$
\begin{equation*}
\frac{\partial \pi\left(e_{1}, e_{2}\right)}{\partial e_{2}} p_{2}\left(e_{2}, 0\right)+\frac{\partial p_{2}\left(e_{2}, 0\right)}{\partial e_{2}} \pi\left(e_{1}, e_{2}\right)=-\frac{v_{2}^{\prime}\left(e_{2}\right)}{u_{2}\left(y_{2}\right)} \tag{6}
\end{equation*}
$$

Equations 5 and 6 give us the best agents' response functions $e_{1}\left(e_{2}\right)$ and $e_{2}\left(e_{1}\right)$. Under Assumption 4, one agent's best response effort is decreasing in the other agent's choice of effort. The following proposition establishes this result:

Proposition 2. Under Assumption 4, $\frac{d e_{i}\left(e_{j}\right)}{d e_{j}}<0$.
Let $\left(e_{1}^{*}, e_{2}^{*}\right)$ be the corresponding equilibrium outcome, that is $e_{1}^{*}=e_{1}\left(e_{2}^{*}\right)$ and $e_{2}^{*}=e_{2}\left(e_{1}^{*}\right)$. It follows immediately from Proposition 2 that $e_{1}^{*}<e_{1}(0)$. Similarly, a cooperative outcome would enforce both agents to pre-commitments, leading to $\frac{d e_{i}\left(e_{j}\right)}{d e_{j}}=0$, and thus a Pareto improvement compared to $\left(e_{1}^{*}, e_{2}^{*}\right)$. This result can be considered as the typical case for under-provision of a public good if one considers the reduction in the likelihood of a crisis as a public good consumed by both agents. The main obstacle, however, is the absence of an enforcement mechanism against cheating by the agents.

From an economic point of view, this may help in rationalizing bailouts in the same way the provision of public goods by a government is justified; however, bailouts give rise to the moral hazard problem as described in Proposition 1 and Theorem 3.

## III.3.2 Bailouts under Complete Information

To consider public bailouts with multiple agents, first suppose that the principal can observe the agents' levels of effort. By backward induction, the principal's optimal transfers $b_{i}\left(e_{i}\right)$ are determined as functions of effort, and subsequently the agents' maximization problem is solved for the optimal level of effort, assuming a transfer function $b_{i}\left(e_{i}\right)$.

The principal's maximization problem is

$$
\max _{b_{1}} W\left(b_{1}\right)=\max _{b_{1}} W_{0}-\pi\left(e_{1}, e_{2}\right)\left[b_{1}+b_{2}+p_{1}\left(e_{1}, b_{1}\right) c_{1}+p_{2}\left(e_{2}, b_{2}\right) c_{2}\right]
$$

which yields first order conditions

$$
\begin{equation*}
\frac{\partial p_{i}\left(e_{i}, b_{i}\right)}{\partial b_{i}}=-\frac{1}{c_{i}}, \tag{7}
\end{equation*}
$$

for $i \in\{1,2\}$.
Conversely, Agent $i \in\{1,2\}$ chooses his level of effort by maximizing

$$
\max _{e_{i}} U_{i}\left(e_{i}\right)=\max _{e_{i}} u_{i}\left(y_{i}\right)-v_{i}\left(e_{i}\right)-\pi\left(e_{i}, e_{-i}^{*}\right) p_{i}\left(e_{i}, b_{i}\left(e_{i}\right)\right) u_{i}\left(y_{i}\right) .
$$

As in Lemma 1, with one step of backward induction, $b_{1}$ and $b_{2}$ can be interpreted as functions of $e_{1}$ and $e_{2}$ respectively.

Rewriting Agent $i$ 's expected utility function by updating Equation (4)

$$
\begin{equation*}
E U_{i}\left(e_{i}\right)=u_{i}\left(y_{i}\right)-\pi\left(e_{i}, e_{-i}\right) p_{i}\left(e_{i}, b_{i}\left(e_{i}\right)\right) u\left(y_{i}\right)-v_{i}\left(e_{i}\right) \tag{8}
\end{equation*}
$$

The two agents choose their effort levels simultaneously in order to optimize their expected utilities by taking each other's best response effort as given. The first order condition for Agent 1 is

$$
\begin{equation*}
\frac{\partial \pi\left(e_{1}, e_{2}\right)}{\partial e_{1}} p_{1}\left(e_{1}, b_{1}\left(e_{1}\right)\right)+\frac{\partial p_{1}\left(e_{1}, b_{1}\left(e_{1}\right)\right)}{\partial e_{1}} \pi\left(e_{1}, e_{2}\right)=-\frac{v_{1}^{\prime}\left(e_{1}\right)}{u_{1}\left(y_{1}\right)} \tag{9}
\end{equation*}
$$

Agent 2 has a similar first order condition and these conditions give us the best response functions $e_{1}\left(e_{2}\right)$ and $e_{2}\left(e_{1}\right)$.

Using this information, and by differentiating principal's first order condition in (7) totally with respect to $e_{1}$ and $e_{2}$ I find:

$$
\begin{align*}
& \frac{d b_{1}}{d e_{1}}=\frac{\left(\frac{\partial \pi}{\partial e_{1}}+\frac{\partial \pi}{\partial e_{2}} \frac{d e_{2}}{d e_{1}}\right)-\pi c_{1}\left(\frac{\partial^{2} p_{1}}{\partial b_{1} \partial e_{1}}\right)}{\frac{\partial^{2} p_{1}}{\partial b_{1}^{2}} \pi c_{1}}  \tag{10}\\
& \frac{d b_{2}}{d e_{2}}=\frac{\left(\frac{\partial \pi}{\partial e_{2}}+\frac{\partial \pi}{\partial e_{1}} \frac{d e_{1}}{d e_{2}}\right)-\pi c_{2}\left(\frac{\partial^{2} p_{2}}{\partial b_{2} \partial e_{2}}\right)}{\frac{\partial^{2} p_{2}}{\partial b_{2}^{2}} \pi c_{2}} \tag{11}
\end{align*}
$$

At the same time, using the best response functions $e_{1}\left(e_{2}\right)$ and $e_{2}\left(e_{1}\right)$ I differentiate one agent's first order condition with respect to other agent's optimal choice of effort to find

$$
\frac{d e_{1}}{d e_{2}}=-\frac{p \frac{\partial^{2} \pi}{\partial e_{1} \partial e_{2}}+\frac{\partial p_{1}}{\partial e_{1}} \frac{\partial \pi}{\partial e_{2}}}{p \frac{\partial^{2} \pi}{\partial e_{1}^{2}}+\frac{\partial \pi}{\partial e_{1}}\left(\frac{\partial p_{1}}{\partial e_{1}}+\frac{\partial p_{1}}{\partial b_{1}} \frac{d b_{1}}{d e_{1}}\right)+\pi\left(\frac{\partial^{2} p_{1}}{\partial e_{1} \partial b_{1}} \frac{d b_{1}}{d e_{1}}+\frac{\partial^{2} p_{1}}{\partial e_{1}^{2}}\right)+\frac{\partial p_{1}}{\partial e_{1}} \frac{\partial \pi}{\partial e_{1}}+\frac{v^{\prime \prime}}{u_{1}\left(y_{1}\right)}}
$$

Combining the last equation with 10 and solving them recursively, I find that $\frac{d b_{i}}{d e_{i}}<0$ and $\frac{d e_{i}}{d e_{j}}<0$ under Assumption 4 .

Let $e^{* *}=\left(e_{1}^{* *}, e_{2}^{* *}\right)$ be the vector of optimal efforts chosen by both agents. Comparing it to the outcome $e^{*}=\left(e_{1}^{*}, e_{2}^{*}\right)$ in the previous section, I see that $e^{* *} \ll e^{*}$. As a result, the presence of a public bailout mechanism further deteriorates the equilibrium.

Theorem 4. When the agents' effort levels are observable by the principal and $b=\left(b_{1}, b_{2}\right)>0$, both agents show less effort than the efficient level.

Proof of Theorem 4. Follows from the previous discussion.

## III.3.3 Bailouts under Incomplete Information

The result in the previous section can be extended to a case with incomplete information, provided that there exists an imperfect market indicator observed by the principal. Typically, this type of an aggregate indicator is obtained after
analyzing aggregate market data, which does not allow the principal to identify each agent's choice of effort level separately. For example, when each agent borrows some amount from the money market, and that borrowing amount is linked to the agent's effort level, it is possible to estimate the total borrowing amount by looking at market data (e.g. money supply, interest rates) although it is not possible to identify each and every agent's borrowing. Similarly, when every agent in the market involves in riskier investments (that is low effort), the market data will imperfectly reveal the aggregate trend of risk perception, although it is not possible to identify individual risk levels.

I introduce such an indicator $\lambda=\lambda(e)$ to the model, which is observed by the principal where $e=\left(e_{1}, e_{2}\right)$ is the vector of the agents' choices of effort and $\lambda: \mathbb{R}_{+}^{2} \rightarrow[0,1]$. I assume that the mapping $\lambda$ satisfies the following:

Assumption 5. $\lambda$ is monotonically increasing in $e$.

Lemma 2. The probability of a shock is decreasing in agents' effort choices, hence decreasing in $\lambda: \frac{d \pi}{d \lambda}<0$

Proof of Lemma 2 . Follows immediately from Assumption 5.

By Lemma 2, the result established in Theorem 4 can be extended to the incomplete information case where the agents' efforts are not directly observable by the principal but an imperfect and aggregate market indicator exists.

## III. 4 Further Remarks

In my model, first note that the principal has a participation constraint in order to offer a bailout policy. That is, the principal needs to expect a higher level of welfare compared to the case of not bailing out the agent. Let the optimal levels of effort be $e^{*}$ (with a bailout) and $e^{* *}$ without a bailout. The participation constraint for the
principal is

$$
E W\left(b^{*}\right)=W-\pi b^{*}-\pi p\left(e^{*}, b^{*}\right) c>W-\pi p\left(e^{* *}, 0\right) c=E W(0)
$$

simplified to

$$
\begin{equation*}
b^{*}<\left(p\left(e^{* *}, 0\right)-p\left(e^{*}, b^{*}\right)\right) c \tag{12}
\end{equation*}
$$

Equation 12 can be interpreted in the following way: If any of the terms on the right-hand-side of (12) converges to zero, then a bailout is not a socially preferable action. When the principals transfer of bailout funds to the agent does not yield any significant effect on the probability of success, i.e. if $p\left(e^{*}, b^{*}\right)=p\left(e^{* *}, 0\right)$, then a bailout decision is not socially preferable and the agent should not be bailed out. On the other hand, if the social cost of a bankruptcy is negligible for the principal, that is $c$ close to zero, then no funds should be transferred to the agent either.

One interpretation of the result being dependent on the value of $c$ is popularly known as the case of "too big to fail". In practice, $c$ is linked to the size of a firm and there are many ways to measure that size, e.g. the number of employees hired by the firm. It is possible to argue that the failure of the firm should be prevented to avoid a substantial increase in unemployment. Consider for example the auto bailouts in Michigan in 2008. The market share of Michigan-based automobile manufacturers have been declining for many decades, but they remained as the main employer of auto-workers in that state. Then, one can argue that, during the automotive industry crisis of 2008-10, the main factor that motivated the public bailout of these manufacturers was to avoid bankruptcy and ensuing layoffs.

I would like to point out that this part of discussion on hypothetical values of $c$ is often political rather than relying on economic reasoning, unless there are accurate estimations of $c$. In practice, these estimates are not available and the exact values of $c$ are more often than not speculative. The assessment relies heavily
on hypothetical and untestable "what if" scenarios: Once a bailout proposal is put on the agenda, the costs are always claimed to be "too big", and a government intervention through a bailout seems and absolute necessity. This situation can be confirmed merely by following the narrative of the bailouts, as one will often hear the words "unprecedented challenges" faced by the private firms and the government's objective to "save the economy from another catastrophe like the Great Depression". From this perspective, the discussion is fruitful only after careful estimates on the social cost $c$. Consider the cases of Lehman Brothers (LB) and Bear Sterns (BS) are considered, where the former was let fail and the latter was rescued although there were marginal difference in their respective externality $\operatorname{costs} c_{L B}$ and $c_{B S}$. Once $c$ is assumed to be sufficiently high so that a bailout is justified, then the moral hazard result is unavoidable. In the opposite case where $c$ is sufficiently low and negligible, the moral hazard is not a problem anyway because there will be no bailouts.

## Chapter IV

## EFFICIENCY ENHANCING SIGNALING IN BAILOUTS

## IV. 1 Introduction

Since the 2008 recession, the idea of public bailouts has once again been on the minds of economists, policymakers, and concerned citizens across Europe and the U.S. The subject remains widely disputed: Advocates tend to argue that bailouts could prevent the dead-weight costs of market failure, while opponents point to the moral hazard problem that arises when agents expect to be bailed out in case of failure. Rather than taking sides in this debate, I aim to add to the discussion by exploring the signaling effect of a bailout rejection in this chapter.

I design a sequential bailout game, in which a principal offers the agents financial assistance to reduce their risk of failure. I show that under certain conditions, the bailout offer induces a separating equilibrium in which the offer is accepted by the bad-type agents and rejected by the good-types. The rejection sends a signal of financial strength and self-confidence to the markets, and thus bailout mechanisms provide an opportunity for a good-type agent to reveal his type to the markets. Moreover, this signaling property of bailouts can actually mitigate the moral hazard problem: By endogenizing the agent's type (through costly effort), I demonstrate that the presence of a bailout mechanism may induce some agents to increase their level of effort (by anticipating and subsequently rejecting the bailout offer).

The insights from the model are not limited to financial sector bailouts. The following cases can be considered as typical examples: individuals refusing assistance out of pride $\sqrt[9]{ }$ countries rejecting financial support from the IMF, members of a currency union (e.g. Euro Zone) rejecting assistance in order to avoid lower demand at the next bond auction and a potential drop in foreign investments, firms (in financial as well as non-financial sectors) rejecting a government bailout that would potentially damage their reputation and raise questions about their future profitability. On the final case, consider the following extract from a newspaper article reporting on the events during the recent U.S. bank bailout:
"The chief executives of the nine largest banks in the United States trooped into a gilded conference room at the Treasury Department at 3 p.m. Monday. To their astonishment, they were each handed a one-page document that said they agreed to sell shares to the government, then Treasury Secretary Henry M. Paulson Jr. said they must sign it before they left. The chairman of Wells Fargo, Richard M. Kovacevich, protested strongly that, unlike his New York rivals, his bank was not in trouble because of investments in exotic mortgages, and did not need a bailout, according to people briefed on the meeting." (Drama Behind a Banking Deal, New York Times, Page A1, October 15, 2008.)

At first glance, a bailout offer may be seen from the agent's perspective as "free money", and its rejection counterintuitive. Signaling theory (introduced e.g. Akerlof 1970, Spence 1973, 1974, 1981), however, helps us explain why good-type agents may engage in seemingly wasteful actions in order to reveal their unobservable quality. Also the Counter-Signaling theory (cf. Feltovich et al. 2002) extends this standard implication to the case of a self-confident agent sending a signal by not sending any

[^6]signal. The analysis in this chapter links to both cases. In the context of public bailouts, the financial assistance provided by a central bank may look like a "free lunch" for a mediocre bank. Is it worth putting its public image and perception at risk by accepting the offer and being pooled together with the most risky banks?

In addition to applying a variant of Counter-Signaling theory to public bailouts, I enhance the model with moral hazard, which has interesting implications.

Intuitively, the moral hazard problem in bailouts stems from agents benefiting from assistance primarily when they are in bad shape facing a risk of failure, as I have shown in the previous chapter. With the signaling opportunity in presence, the bailout mechanism provides a benefit for good-type agents as well, and thus gives agents additional incentives to become a "good-type". Hence, moral hazard and signaling work in opposite directions.

The equilibrium outcome heavily depends on the specifications of the payoffs (market valuation) and the cost structure in the marketplace. In a reputation-driven market, for instance, signaling is quite valuable, and agents are more likely to exert high effort when a bailout mechanism is in place, so that they can be later distinguished.

As discussed in the previous chapter, the textbook case for the moral hazard problem is identified with the Samaritan's Dilemma (cf. Buchanan 1975), which provides a practical framework for bailout situations. As a reminder, efficiency-enhancing solutions to the Samaritan's Dilemma in the literature are pre-commitment (see e.g. Buchanan 1975, Chami 1996, Dijkstra 2002) and in-kind or tied transfers (cf. Bruce and Waldman 1990, 1991, Coate 1995). The credibility of these commitment contracts and their enforceability are questionable, and in-kind transfers are not practically feasible for the case of public bailouts.

Lagerlof (2004) demonstrates that the moral hazard problem is mitigated when the assumption of complete information is relaxed in a way that gives the agent's
level of savings (effort) a signaling value. My intuition is similar in nature, but I employ a different model specification and signaling mechanism. For instance, as in the previous chapter, the principal in my model is not altruistic per se, but concerned with the externality costs of the agent's failure. More important, the agent's choice of effort remains private information, and only his response to the bailout offer can serve as a signal.

The common ground, however, is the presence of adverse selection that is necessary for the signaling to work. When for some agents - but not all - the appreciation in market value exceeds the bailout funds, a separating equilibrium arises where the bailout proposal is accepted only by the less financially sound agents. For them, it is not possible to mimick the good-type agents since rejecting the bailout is too costly. In my simplified model, the agent's type is thus truthfully revealed, and his market value will adjust as a result of investor demand.

The remainder of the paper is organized as follows: Section IV. 2 develops the basic model of adverse selection in bailouts. In Section IV. 3 I extend this model to incorporate the moral hazard problem. Section IV. 4 provides the general comparative static results and discusses policy implications, and Section IV. 5 concludes. All proofs are provided in the Appendix.

## IV. 2 Adverse Selection and Signaling in Bailouts

To address the problem of adverse selection in public bailouts, I design a principal-agent model with hidden information, where the agent faces a risk of failure (bankruptcy) and the principal can provide financial assistance (bailout). The agent subsequently chooses whether to accept or reject the bailout offer.

The agent can be one of two types, $\theta \in \Theta=\{H, L\}$, that differ by their respective failure rates and payoffs. I denote the probability that the agent is of high type by $\operatorname{Pr}(\theta=H) \equiv \mu$. Specifically, the agent fails with probability $p_{\theta}(b)$ if he
accepts the bailout offer $b$, and $p_{\theta}(0)$ if he declines. Thereby, $b \geq 0$ denotes the bailout amount the principal offers to the agent.

I make the following straightforward assumption.

## Assumption 6.

(a) $0<p_{H}(b)<p_{L}(b) \leq 1$, for any $b \geq 0$.
(b) For any given $b \geq 0, p_{L}^{\prime}(b)<p_{H}^{\prime}(b)<0$.
(c) $p_{\theta}^{\prime \prime}()>$.0 .

Assumption 6 (a) states that given the same financial assistance, the low-type agent is more likely to fail. Part (b) implies that a larger bailout reduces the risk of failure further, and that a bailout is more effective for the low-type agent. Finally, (c) expresses the diminishing marginal value of a bailout: an additional dollar of help is less effective when the agent has already received a substantial amount.

Assumption 6 immediately implies the following result:

## Lemma 3.

$$
\frac{1-p_{H}(b)}{1-p_{H}(0)}<\frac{1-p_{L}(b)}{1-p_{L}(0)}
$$

The immediate and direct costs of bankruptcy are borne by the agent: His failure yields a payoff of 0 , but he receives a positive payout $y_{\theta \phi}>0$ otherwise. Thereby $\theta$ is the agent's true type, and $\phi \in\{H, L, M\}$ denotes how the agent is perceived by the market, with $\phi=M$ standing for the market being unable to distinguish between the two types. The agent's well-being is given by the expected utility over his payoff. I require his utility function $U(y)$ to be well-behaved, that is:

## Assumption 7.

$$
U(0)=0, U^{\prime}(.)>0, \text { and } U^{\prime \prime}(.) \leq 0
$$

To simplify notation, I define $u_{\theta \phi} \equiv U\left(y_{\theta \phi}\right)$ for the purpose of my analysis. Naturally, on the basis of survivability, the high-type agent should be better off than the low-type agent, under the same market perception. I also require that a more favorable perception by the market is reflected in the agent's payoffs. That is:

## Assumption 8.

(a) $u_{H \phi}>u_{L \phi}$ for any $\phi$, and
(b) $u_{\theta H}>u_{\theta M}>u_{\theta L} \quad$ for any $\theta$.

The latter restriction reflects that e.g. a firm with good reputation has easier and cheaper access to borrowing or having more customers and is overall more profitable.

While not being altruistic per se, the principal has to carry the indirect (that is, social) costs of the agent's failure, which I denote by $c>0$. The principal chooses $b$ in order to maximize her well-being, which - in the case where only the low-type agent accepts the offer - is given by

$$
W(b)=W_{0}-(1-\mu) b-(1-\mu) p_{L}(b) c .
$$

In sum, I propose a bailout game with the following sequence of events:

1. The agent observes its type drawn from $\Theta$. He is of type $H$ with probability $\mu$ and type $L$ with probability $1-\mu$.
2. Without observing the agent's type, the principal offers bailout transfer $b$. The bailout transfer lowers the risk of failure as discussed above.
3. The agent decides whether to Accept or Reject the bailout offer.
4. Nature rolls its dice and the payoffs are realized. The agent fails with probability $p_{\theta}(b)$ and survives with probability $1-p_{\theta}(b)$. If the agent fails, he
receives a payoff of 0 and causes a social cost equal to $c$. In case of survival, there is no social cost, and the agent receives payoff $y_{\theta \phi}>0$, with corresponding utility $u_{\theta \phi}$.

Since there are two types of agents, each with a choice between two alternatives (namely whether to Accept or Reject the bailout proposal), there are four potential equilibrium outcomes: Two pooling equilibria, where either both types Accept or Reject; and two separating outcomes where one type Accepts the bailout and the other Rejects it. I define $\Psi \in\{A, R\}^{2}$ as the two-dimensional response set of a high-type and a low-type agent to the bailout proposal. For instance, I denote the case where the high-type Accepts and the low type agent Rejects the bailout with $\psi=(A, R)$, and so forth. Therefore, a pooling equilibrium requires a response of $(A, A)$ or $(R, R)$, and the potential separating equilibria entail $(A, R)$ or $(R, A)$. I now consider each case individually.

## IV.2.1 Pooling Equilibria

$\psi=(A, A)$

In a pooling equilibrium where both types Accept, an agent will have a payoff of $y_{\theta M}$. Conversely, if the agent were to Reject the bailout, the market values him as a high-type, so that his payoff is $y_{\theta H} \sqrt{10}$

For $\psi=(A, A)$ to be an equilibrium outcome, both agents would need to Accept the bailout offer $b_{A A}^{*}$; and $b_{A A}^{*}$ must maximize the principal's welfare function, given that both types Accept the bailout. When the agent is of high type, $\theta=H$, he Accepts the bailout proposal $b_{A A}^{*}$ if and only if

$$
p_{H}\left(b_{A A}^{*}\right) 0+\left(1-p_{H}\left(b_{A A}^{*}\right)\right) u_{H M}>p_{H}(0) 0+\left(1-p_{H}(0)\right) u_{H H},
$$

[^7]which simplifies to
\[

$$
\begin{equation*}
\frac{1-p_{H}\left(b_{A A}^{*}\right)}{1-p_{H}(0)}>\frac{u_{H H}}{u_{H M}} . \tag{13}
\end{equation*}
$$

\]

Similarly, a low-type agent Accepts $b_{A A}^{*}$ if and only if

$$
\begin{equation*}
\frac{1-p_{L}\left(b_{A A}^{*}\right)}{1-p_{L}(0)}>\frac{u_{L H}}{u_{L M}} \tag{14}
\end{equation*}
$$

On the other hand, the principal chooses the bailout proposal $b=b_{A A}^{*}$ so as to maximize

$$
W_{A A}(b)=\mu\left[W_{0}-b-p_{H}(b) c\right]+(1-\mu)\left[W_{0}-b-p_{L}(b) c\right] .
$$

This yields the first-order condition

$$
\begin{equation*}
\mu p_{H}^{\prime}(b)+\left.(1-\mu) p_{L}^{\prime}(b)\right|_{b=b_{A A}^{*}}=-\frac{1}{c} \tag{15}
\end{equation*}
$$

and the following result is obtained.

Proposition 3. There exists a pooling equilibrium where both types Accept the bailout offer $b_{A A}^{*}$ given as the solution to Equation (15), if the following conditions hold:

- $b_{A A}^{*}>0$.
- $\frac{1-p_{H}\left(b_{A A}^{*}\right)}{1-p_{H}(0)}>\frac{u_{H H}}{u_{H M}}$.
- $\frac{1-p_{H}\left(b_{A A}^{*}\right)}{1-p_{H}(0)}>\frac{u_{L H}}{u_{L M}}$.

Proof. Follows from the previous discussion.
$\psi=(R, R)$
When both types Reject the bailout, the principal's expected welfare is

$$
\begin{equation*}
W_{R R}(b)=\mu\left[W_{0}-p_{H}(0) c\right]+(1-\mu)\left[W_{0}-p_{L}(0) c\right] \tag{16}
\end{equation*}
$$

which is independent of $b$, as expected. Nonetheless, the bailout offer needs to induce both types to Reject it. I therefore set $b_{R R}^{*}=0$, since this is the weakest restriction to induce compliance by the agent. Hence no bailout is offered, and the resulting equilibrium is a trivial one.

## IV.2.2 Separating Equilibria

$\psi=(A, R)$
First consider the separating case $(A, R)$, where the high-type Accepts the bailout and the low-type Rejects. Clearly, this is not a feasible outcome: the low-type will be better off by mimicking the high-type by Accepting the bailout, which would not only reduce his risk, but also increase his market value.
$\psi=(R, A)$
Lastly, I address the most interesting case where only the low-type agent Accepts the bailout offer. In the corresponding (separating) equilibrium, the agent will be valued by the market as a high-type (and get utility $u_{\theta H}$ ) if he Rejects the bailout, and as a low-type (with utility $u_{\theta L}$ ) if he Accepts the proposal. Hence, for a high-type agent to Reject the bailout offer $b_{R A}^{*}$, it is needed

$$
p_{H}(0) 0+\left(1-p_{H}(0)\right) u_{H H}>p_{H}\left(b_{R A}^{*}\right) 0+\left(1-p_{H}\left(b_{R A}^{*}\right)\right) y_{H L}
$$

which simplifies to

$$
\begin{equation*}
\frac{1-p_{H}\left(b_{R A}^{*}\right)}{1-p_{H}(0)}<\frac{u_{H H}}{u_{H L}} \tag{17}
\end{equation*}
$$

Similarly, a low-type agent Accepts $b_{R A}^{*}$ if and only if

$$
\begin{equation*}
\frac{1-p_{L}\left(b_{R A}^{*}\right)}{1-p_{L}(0)}<\frac{u_{L H}}{u_{L L}} \tag{18}
\end{equation*}
$$

This implies the following lemma.

Lemma 4. It is optimal for a low-type agent to Accept $b_{R A}^{*}$ and for a high-type agent to Reject the bailout if and only if Inequalities (17) and (18) are satisfied.

Proof. Follows from the previous discussion.

I now turn to the principal's optimization problem:

Lemma 5. If $\psi=(R, A)$, the principal's welfare is maximized at $b_{R A}^{*}$ given by

$$
p_{L}^{\prime}\left(b_{R A}^{*}\right)=-\frac{1}{c}
$$

These findings are combined to obtain the conditions for a separating equilibrium. For simplicity, I summarize the underlying assumptions, before stating the main theorem of this section.

## Assumption 9.

(a) $b_{R A}^{*}=p_{L}^{\prime-1}(-1 / c)>0$.
(b) $\frac{1-p_{H}\left(b_{R A}^{*}\right)}{1-p_{H}(0)}<\frac{u_{H H}}{u_{H L}}$.
(c) $\frac{u_{L H}}{u_{L L}}<\frac{1-p_{L}\left(b_{R A}^{*}\right)}{1-p_{L}(0)}$.

Theorem 5. (Efficiency Enhancing Signaling)
Under Assumption 9, there exists a separating equilibrium where the low-type agent Accepts the bailout proposal $b_{R A}^{*}=p_{L}^{\prime-1}(-1 / c)$, and the high-type agent Rejects the offer.

In the present case, the high-type agent takes advantage of the signaling mechanism. By rejecting the bailout offer, he reveals his type, sending a message that he does not need any assistance. In contrast, rejecting the bailout offer is too costly for the low-type agent, and he takes advantage of the bailout assistance in order to reduce the risk of failure.

In the following section, I enhance the model by endogenizing the agent's type through costly effort. I demonstrate in this context that a bailout mechanism may also cause moral hazard, in parallel to the findings of the previous chapter.

However, I observe that the presence of a bailout offer can also have a positive effect on the agents' level of effort, namely by allowing high-type agents to signal their financial superiority to the market.

## IV. 3 Bailouts with Moral Hazard and Adverse Selection

I want to explore the effect of the signaling mechanism described in Section IV.2 on the moral hazard problem that is so common in bailouts. For the signaling mechanism to work, I require a separating equilibrium, where only the low-type agent Accepts the principal's bailout proposal. Focusing on this scenario, I expand the principal-agent model from Section IV. 2 by allowing the agent to select his type through costly effort.

I describe conditions that shape the agent's optimal effort choice and compare those to the optimal effort level without the bailout mechanism. Then it is possible to identify the conditions under which the existence of a bailout plan gives rise to
moral hazard. Perhaps more interestingly, it is also possible to describe scenarios where the signaling opportunity mitigates or even reverses the moral hazard problem.

## IV.3.1 A Separating Equilibrium with Bailouts

I assume a continuum of agents that first choose whether to exert high effort $e_{H}>0$ and become a high-type agent or instead choose zero effort to become a low-type agent. Agents differ (solely) in how costly it is for them to exert high effort, and their disutility is given by $v_{H}^{\tau} \equiv v_{\tau}\left(e_{H}\right)$, where $\tau$ denotes the agent's type in regard to productivity. For simplicity I also assume that $v_{\tau}(0)=0$ for all $\tau$. Note that the agent's type, $\theta \in \Theta=\{H, L\}$, depends solely on his choice of effort. Moreover, as in the previous section, the agent's type is the only determinant for his payout, and his probability of failure.

The remaining steps of the model coincide with the sequential game from Section IV.2. That is:

1. Each agent decides whether to become a high-type or a low-type by choosing the respective level of effort. His choice, as well as his productivity-type $\tau$ remain private information.
2. The principal offers bailout transfer $b>0$. The bailout reduces the probability of failure, which is consistent with Assumption 6.
3. The agent Accepts or Rejects the bailout offer. If he Accepts, the market will view him as a low-type so that he receives a utility of $u_{\theta L}$ ). Conversely, he will be considered a high-type agent with utility $u_{\theta H}$ if he Rejects the bailout offer. In particular, I require Assumptions 7 and 8 to hold.
4. Nature rolls its dice and the payoffs are realized. As before, an agent's failure results in personal utility 0 and social costs $c$. If he survives, he gains utility $u_{\theta \phi}$, and there are no social costs.

Lemma 4 describes the condition under which an agent Accepts the bailout if he initially choses low effort and Rejects the bailout after choosing high effort. In this separating equilibrium, the principal's optimal bailout offer $b^{*}$ is again given by Lemma 5

Combining these results essentially characterizes a separating equilibrium, akin to Section IV.2, for the second part of the game. The novel feature of this section is the choice of effort that precedes the separation and signaling process and that specifies an agent as a high-type or as a low-type. The analysis yields the following result:

Theorem 6. Under Assumption 9, there exists a separating equilibrium under which the principal offers bailout $b^{*}$ given by the identity $p_{L}^{\prime}\left(b^{*}\right)=-\frac{1}{c}$, and where it is optimal for the agent to ...
(a) ... choose low effort and Accept the bailout if

$$
\begin{equation*}
v_{H}^{\tau}>\left[1-p_{H}(0)\right] \cdot u_{H H}-\left[1-p_{L}\left(b^{*}\right)\right] \cdot u_{L L} . \tag{19}
\end{equation*}
$$

(b) ... choose high effort and Reject the bailout otherwise.

I now consider the optimal choice of effort when no bailout mechanism is present.

## IV.3.2 Optimal Effort without Bailouts

In the absence of a bailout mechanism, an agent has no opportunity to signal his type and he will be treated as an "average"-type by the market. Similar to Section IV.2, his utility in case of survival is given by $u_{H M}$ if he exerts high effort, and $u_{L M}$
otherwise. In particular, the conditions specified in Assumption 8 hold. Hence, when exerting low effort, the agent's expected utility is given by

$$
E U_{L} \equiv p_{L}(0) \cdot 0+\left[1-p_{L}(0)\right] \cdot u_{L M}
$$

and in the case of high effort he receives

$$
E U_{H} \equiv p_{H}(0) \cdot 0+\left[1-p_{H}(0)\right] \cdot u_{H M}-v_{H}^{\tau}
$$

This immediately leads to the following proposition.

Proposition 4. It is optimal for the agent to exert low effort if and only if

$$
\begin{equation*}
v_{H}^{\tau}>\left[1-p_{H}(0)\right] \cdot u_{H M}-\left[1-p_{L}(0)\right] \cdot u_{L M} . \tag{20}
\end{equation*}
$$

Proof. Follows from the previous discussion.

## IV.3.3 Effect of Bailout Mechanism on Optimal Effort

To analyze the impact of a bailout mechanism on an agent's optimal level of effort, a comparison is needed between Equations (19) and (20). If the agent is inefficient and effort is extremely costly, $v_{H}^{\tau}$ is large and both inequalities are satisfied. In that case, zero effort is optimal both with and without a bailout offer. Conversely, for efficient agents both inequalities are violated, and the agent exerts high effort in both cases.

More interestingly, it is also possible for one inequality to be satisfied while the other is violated. The following two theorems characterize the respective outcomes:

Theorem 7 (Samaritan's Dilemma, revisited). Under Assumption 9, and if

$$
\begin{equation*}
\left[1-p_{H}(0)\right] \cdot u_{H M}-\left[1-p_{L}(0)\right] \cdot u_{L M}>v_{H}^{\tau}>\left[1-p_{H}(0)\right] \cdot u_{H H}-\left[1-p_{L}\left(b^{*}\right)\right] \cdot u_{L L} \tag{21}
\end{equation*}
$$

the introduction of a bailout mechanism reduces the agent's optimal level of effort. Theorem 8 (Efficiency-Enhancing Signaling, revisited). Under Assumption 9, and if

$$
\begin{equation*}
\left[1-p_{H}(0)\right] \cdot u_{H M}-\left[1-p_{L}(0)\right] \cdot u_{L M}<v_{H}^{\tau}<\left[1-p_{H}(0)\right] \cdot u_{H H}-\left[1-p_{L}\left(b^{*}\right)\right] \cdot u_{L L} \tag{22}
\end{equation*}
$$

the presence of a bailout mechanism increases the agent's optimal level of effort from 0 to $e_{H}$.

Theorems 7 and 8 demonstrate that the bailout mechanism can both cause and reverse moral hazard, depending on market specifications. Provided that Assumption 9 holds, and if

$$
\begin{equation*}
\left[1-p_{H}(0)\right] \cdot u_{H M}-\left[1-p_{L}(0)\right] \cdot u_{L M}>\left[1-p_{H}(0)\right] \cdot u_{H H}-\left[1-p_{L}\left(b^{*}\right)\right] \cdot u_{L L}, \tag{23}
\end{equation*}
$$

the moral hazard problem dominates: medium-efficient agents, for which Inequality (21) holds, settle for becoming a low-type. However, they would have exerted high effort if it were not for the bailout mechanism. In addition, the more efficient agents, for which $v_{H}^{\tau}<\left[1-p_{H}(0)\right] \cdot u_{H H}-\left[1-p_{L}\left(b^{*}\right)\right] \cdot u_{L L}$, will choose high effort regardless of the bailout; conversely, the less efficient agents, characterized by $v_{H}^{\tau}>\left[1-p_{H}(0)\right] \cdot u_{H M}-\left[1-p_{L}(0)\right] \cdot u_{L M}$, will exert no effort in both cases.

When Inequality 23 is reversed, the signaling opportunity reverses the moral hazard problem for some agents, namely those that satisfy Inequality (22). Similary to the previous case, the inefficient agents $\left(v_{H}^{\tau}>\left[1-p_{H}(0)\right] \cdot u_{H H}-\left[1-p_{L}\left(b^{*}\right)\right] \cdot u_{L L}\right)$ will shirk either way, and the most efficient agents $\left(v_{H}^{\tau}<\left[1-p_{H}(0)\right] \cdot u_{H M}-\left[1-p_{L}(0)\right] \cdot u_{L M}\right)$ exert high effort, regardless of the bailout situation.

It remains to be shown that it is possible for both scenarios to arise. For that matter, note that Inequality (23) can be rewritten as

$$
\begin{equation*}
\left[1-p_{H}(0)\right] \cdot \frac{u_{H H}-u_{H M}}{u_{L L}}+\left[1-p_{L}(0)\right] \cdot \frac{u_{L M}-u_{L L}}{u_{L L}} \lesseqgtr p_{L}(0)-p_{L}\left(b^{*}\right) \tag{24}
\end{equation*}
$$

Both fractions are positive by Assumption 8. The right-hand side is also positive by Assumption 6. The sign of this inequality depends on the differences in payoffs relative to the impact of a bailout: if for instance the payoff levels are rather similar (that is, if the fractions in Equation (24) are small), or if an agent who chose a low level of effort and who would consequently benefit significantly from the bailout (that is, $p_{L}(0)-p_{L}\left(b^{*}\right)$ is large), then the conditions for Theorem 7 may be satisfied. In that case, the anticipation of a bailout leads to moral hazard as the agent reduces its level of effort.

Conversely, if the return to signaling $\left(u_{H H}-u_{H M}\right)$ is considerable, or if the optimal bailout amount reduces the failure rate of the agent only slightly (that is if $p_{L}(0)-p_{L}\left(b^{*}\right)$ is small), the right-hand side of Equation (21) exceeds the left-hand side. If the disutility of effort $\left(v_{H}^{\tau}\right)$ matches as well, Theorem 8 applies: the agent exerts a high level of effort when anticipating a bailout offer because it intends to use the offer (by rejecting it) in order to signal its type to the market. In essence, the bailout mechanism gives the agent an opportunity to signal the market that he exerted a high level of effort. Without the bailout mechanism, he could not send this signal and therefore would initially exert less effort.

## IV. 4 Comparative Statics and Welfare Analysis

It is possible now to evaluate the welfare implications of these results. In this section, I study how the choice of equilibrium varies as the specific parameters of
the model change. To do so, I will separately look at the impact of productivity, payoffs, market characteristics and the social cost.

## IV.4.1 Equilibrium Selection

The principal's concern for the social cost $c$ will be reflected in her choice of the bailout transfer $b^{*}$. For small values of $c$, the optimal bailout decision is trivially $b^{*}=0$, and the results shown in Section IV.3.2 apply. Thus, from now on, I assume that $c$ is not negligible to the extent that $p_{\theta}^{\prime-1}(-1 / c)>0$.

From an agent's perspective, the selection of the equilibrium outcome depends on two crucial parameters: his productivity and his expected payout structure. For instance, his productivity, that is the costliness of effort $\left(v_{\theta}^{\tau}\right)$, determines both the choice of effort $e$ and the final decision to Accept or Reject the bailout. Moreover, the payoff and utility structure of the agent play a crucial role in effort choice and equilibrium selection.

Starting with the latter, note that it is necessary to distinguish the difference between the two types of agents (i.e. $u_{H \phi}-u_{L \phi}$ ) from the impact of market perception (i.e. $u_{\theta H}-u_{\theta L}$ ). More generally, the equilibrium outcome can be affected by the agent's risk aversion: Since risk-neutral agents are more prone to failure, an aversion to risk tends to lower the cost of a bailout in either equilibrium by pushing the agent towards a higher level of effort.

Another critical factor is the market perception of the agent. As demonstrated at the end of Section IV.3, if the payoff structure is heavily influenced by outside opinion, the return to signaling is high, and a separating equilibrium becomes more likely. Such a situation is more likely to occur in the financial services industry than for manufacturers' whose asset values generally exhibit less volatility and can be determined more accurately.

Signaling can also be quite valuable for firms that are publicly traded, as they tend to put more emphasis on their outside image, in addition to their internal operations and risk management. It further matters how perfectly the respective market sector relays information: If every piece of public information about the agent is captured by the stock price, then the possibility of a separating equilibrium through signaling will be higher as well.

From the conditions of Theorem 8, it is possible to deduce that a separating equilibrium is more likely if the dispersion in $v_{\theta}^{\tau}$ is high in an economy with a continuum of agents. This is particularly true when some agents are significantly more productive than others and thus have cheaper access to precautionary measures. Since Inequality 19 is immediately violated, it is optimal for a productive agent to exert high effort. Trivially, this outcome is efficient: neither the principal nor the agent can be made better off. Conversely, in sectors of the economy where productivity is widely homogeneous, the margin for a separating equilibrium is very narrow. An empirical investigation is necessary if one wishes to identify and contrast the cases of the bailouts in different sectors, but this is beyond the scope of the current study.

## IV.4.2 An Illustration of Bankruptcy Risk

For illustrative purposes, consider the following functional form for the agent's bankruptcy risk:

$$
\begin{equation*}
p\left(b ; \alpha_{\theta}, \beta_{\theta}\right)=\frac{\alpha_{\theta}}{(1+b)^{\beta_{\theta}}} . \tag{25}
\end{equation*}
$$

which asymptotes to a probability of failure zero with $p(0)=\alpha$. Here, $\alpha \leq 1$ can be used to constrain agents to a maximum level of risk while $\beta_{\theta}>0$ indicates the relative efficacy of the bailout money on the probability of failure. In this formulation, one can pick $\alpha_{H}$ for agents with high effort and $\alpha_{L}$ for low (or zero) effort (with $\alpha_{H}<\alpha_{L}$ ). Although, a similar argument can be made for the values $\beta$
can take, Assumption 6 implies $\beta_{H}=\beta_{L} \equiv \beta \cdot{ }^{11}$ A graphic illustration of $p(\cdot)$ is provided in Figure 5.


Figure 5: The risk of failure with $1>\alpha_{L}>\alpha_{H}>0$ and $\beta_{H}=\beta_{L}$.

Lemma 5 specifies an internal solution to the principal's optimization problem under a separating equilibrium. When social costs are identical across types (as assumed), it follows that $b_{L}^{*}>b_{H}^{*}$. That is, if the agent's type was known by the principal, a low-type agent should expect a larger bailout offer than a high-type. See Figure 6 for an illustration.

With the functional form assumption of Equation 25, it follows that

$$
b^{*}=\left(\alpha_{L} \beta c\right)^{\frac{1}{1+\beta}}-1
$$

[^8]

Figure 6: Optimal bailout transfers when social cost $c$ is identical across types.
as the optimal bailout offer to the low-type agent. For an agent to actually choose low (or zero) effort and subsequently Accept the bailout offer, it is required that

$$
\begin{equation*}
v_{H}^{\tau}>u_{H H}\left(1-\alpha_{H}\right)-u_{L L}\left(1-\frac{1}{\beta c}\right) . \tag{26}
\end{equation*}
$$

Since the right-hand side of (26) is decreasing in $c$, this condition is more likely to be satisfied when the social cost of the agent's failure is substantially large. In other words: ceteris paribus, an agent whose failure is significantly harmful to society tends to exert less effort than an agent whose failure is relatively lest critical.

The intuition is that the agent can expect a larger bailout offer (note that $b^{*}$ is also increasing in $c$ ), and therefore transferring some of the risk onto the public by reducing risk-management activities is preferable. If at the same time

$$
\begin{equation*}
v_{H}^{\tau}<\left(1-\alpha_{H}\right) u_{H M}-\left(1-\alpha_{L}\right) u_{L M}, \tag{27}
\end{equation*}
$$

then the agent would have exerted high effort were it not for the (rational) anticipation of a bailout. This is the moral hazard problem that is a common concern surrounding public bailouts.

Conversely, if the agent's failure is not that costly to society, so that Inequality (26) is violated, the agent - anticipating only a small bailout - optimally exerts high effort. Note that by the separating equilibrium conditions, he still benefits from the bailout offer, namely by signaling his type through a rejection. More important, if Inequality 27 is violated as well, the agent would have chosen low effort, had the bailout mechanism not been in place. This situation captures the efficiency-enhancing signaling property of public bailouts described in Section IV.3.

Overall, it is observed that the signaling effect is less probable when the agent's failure causes tremendous social cost, as the agent is more prone to moral hazard. I now contrast the two outcomes described in Theorems 7 and 8 in regard to the principal's welfare.

## IV.4.3 Principal's Welfare

If effort is extremely costly, so that Inequality (26) is satisfied and Inequality (27) is violated, the agent will shirk regardless of a bailout proposal. Hence the bailout makes the principal worse off, since the (low-type) agent will accept it. If, on the other hand, effort is very cheap and Inequality (26) is violated and Inequality (27) is satisfied, the agent will exert high effort in either case and reject the bailout, if offered. Hence, the principal does not gain or lose by proposing a bailout.

To analyze the principal's well-being in the more interesting cases where Inequalities (26) and (27) are either both satisfied or both violated, I denote the principal's (expected) welfare by $W_{N B}$ under a no-bailout policy and by $W_{M H}$ and $W_{S}$ in cases where the bailout causes moral hazard and allows for signaling, respectively. As discussed previously, the latter two outcomes are mutually
exclusive, and it depends to a certain extent on the social cost of and which of the two outcomes are possible.

Considering first the case of moral hazard, that is when both inequalities are satisfied, note that the agent would exert high effort without the bailout, but low effort otherwise. The principal's welfare without bailout is thus

$$
W_{N B}=W_{0}-p_{H}(0) c=W_{0}-\alpha_{H} c
$$

while the presence of a bailout yields

$$
W_{M H}=W_{0}-p_{L}\left(b^{*}\right) c-b^{*}=W_{0}-\left(\frac{\alpha_{L} c}{\beta^{\beta}}\right)^{\frac{1}{1+\beta}}-\left(\alpha_{L} c \beta\right)^{\frac{1}{1+\beta}}+1
$$

Substituting $b^{*}=\left(\alpha_{L} \beta c\right)^{\frac{1}{1+\beta}}-1$ and simplifying

$$
W_{M H}=W_{0}-1-c^{\frac{1}{1+\beta}} \alpha_{L}^{\frac{1}{1+\beta}}\left(\left(\frac{1}{\beta}\right)^{\frac{1}{1+\beta}}+\beta^{\frac{1}{1+\beta}}\right) .
$$

The sign of $W_{M B}-W_{N B}$ is ambiguous in general. Note that $W_{N B}$ and $W_{M H}$ are indirectly dependent on the choice of agent's effort $e$ since Inequalities (26) and 27) need to be satisfied and $e$ will determine $\alpha_{L}$ and $\alpha_{H}$. If $e$ is fixed so that both inequalities are satisfied, then $W_{M H}$ and the difference $W_{M B}-W_{N B}$ will depend on the magnitude of the social cost parameter $c$. First, note that when $c=0$, $W_{N B}>W_{M H}$ trivially. Second, note that both $W_{N B}$ and $W_{M H}$ are decreasing in $c$.

Furthermore, while $W_{N B}$ is linear with a slope of $\alpha_{H}$, which is a constant, $W_{M H}$ is convex in $c$ when $\beta>0$. Therefore, one can deduce that the gap between $W_{N B}$ and $W_{M H}$ is widening for small values of $c$, but for larger values of social cost $c$ this is no longer the case. The parameters $\alpha$ and $\beta$ are critical, since when $W_{M H}>W_{N B}$ is the case, the principal may be better off by offering a bailout, even with the moral
hazard problem. For illustrative purposes, Figures 7 and 8 demonstrate various cases depending on the exogenous parameters.


Figure 7: Principal's welfare under high risk of failure ; $1>\alpha_{L}>\alpha_{H}>0.5 . W_{N B}$ denotes the expected welfare when the principal commits to a no-bailout policy, $W_{M H}$ denotes the expected welfare when a bailout policy leads to moral hazard.

Conversely, when Inequalities (26) and (27) are both violated, the presence of a bailout mechanism introduces a signaling opportunity. The principal's respective welfares under no bailout and under a bailout offer with signaling are

$$
W_{N B}=W_{0}-\alpha_{L} c,
$$

and

$$
W_{S}=W_{0}-\alpha_{H} c .
$$

Clearly, the principal is better off when offering the bailout (since it induces high effort and will be rejected). The agent is also better off with the bailout (after all,


Figure 8: Principal's welfare under low risk of failure ; $\alpha_{L}>0.5>\alpha_{H}>0 . W_{N B}$ denotes the expected welfare when the principal commits to a no-bailout policy, $W_{M H}$ denotes the expected welfare when a bailout policy leads to moral hazard.
he could simply ignore it). In fact, when Inequalities (26) and (27) are both violated, the bailout offer $b^{*}$ is efficient.

## IV. 5 Conclusion

Market failures and the presence of social costs serve as the main justifications for government protection for private firms in case of bankruptcy risks. However, public bailouts remain controversial since they have unintended consequences. The asymmetries of information and action (or incentive) lie at the heart of the bailout question leading to problems of adverse selection and moral hazard.

From a game-theoretic setup, the principal-agent models help us to analyze bailout situations in order to guide both policy and practice, although tracing the full consequences of bailout policies in practice is a difficult task. The model I
discussed in the chapters III and IV follow the literature on principal-agent models by encompassing asymmetric information, hidden action and endogenous risk of failure.

I argue that the bailout offers made by the public policymakers to private agents create an immediate and costly signaling opportunity for the participants. By accepting the government protection plan, an agent explicitly or implicitly reveals critical information about his financial situation, which is private information. The same logic is used for a financially sound and a self-confident agent who rejects the bailout offer in expectation for an appreciation in market valuation.

## Appendix A

## APPENDIX TO CHAPTER II

## A. 1 Proofs of Theorems in Chapter II

Proof of Theorem 1. It can be checked that $F^{G N U}$ satisfies PO, INV, and WLIN. I will show that PO, INV and WLIN imply $F=F^{G N U}$.

First, by translation invariance, $d$ can always be normalized to the origin $(d=0)$ so that a bargaining problem can be denoted simply as $S$ instead of $(S, d)$. Note that scale invariance is equivalent to the axiom INV, when $d$ is normalized to the origin.

Next, let $\Sigma^{\prime}$ denote the set of $n$-person bargaining problems with $d=(0, \ldots, 0)$ and $m=(1, \ldots, 1)$. By INV, any bargaining problem $S \in \Sigma$ can be represented by a bargaining problem $S^{\prime} \in \Sigma^{\prime}$. Specifically, for any $S \in \Sigma$ I define

$$
S^{\prime}=\left\{\left.\left(\frac{s_{i}}{m_{i}}\right)_{i \in N} \right\rvert\, s \in S\right\}
$$

Note that WLIN is equivalent to LIN when restricted to $\Sigma^{\prime}$.
Then by PO, WLIN, and Theorem 1 of Myerson (1981), there exists a vector $\mu \in \mathbb{R}_{+}^{n}$ such that $\mu F\left(S^{\prime}\right) \geq \mu s^{\prime}$ for all $s^{\prime} \in S^{\prime}$. By INV and letting $\mu^{\prime}=\left(\frac{\mu_{i}}{m_{i}}\right)$, it follows that $\mu^{\prime} F(S)=\mu\left(\frac{F_{i}(S)}{m_{i}}\right) \geq \mu^{\prime} s=\mu\left(\frac{s_{i}}{m_{i}}\right)$ for all $s \in S$. That is, $F$ over $\Sigma$ must be generalized normalized utilitarian.

$$
F(S)=\arg \max _{s \in S} \sum_{i \in N} \mu_{i} \frac{s_{i}}{m_{i}}
$$

Another proof of Theorem 1. Rescale any bargaining problem using INV. For any $S \in \Sigma$, define $T=\Phi(S), T^{k}=\Phi^{k}(T)$ as defined previously and for some $\mu>0$, $\sum_{i} \mu_{i}=1$ define $\Delta_{\mu}=\mu_{1} T^{1}+\ldots+\mu_{n} T^{n}$.

By PO, the elements of $F\left(\Delta_{\mu}\right)$ must lie on the Pareto frontier of $\Delta_{\mu}$ and $F\left(\Delta_{\mu}\right) \subseteq \mathcal{B}\left(\Delta_{\mu}\right)$. Then, by the Supporting Hyperplane Theorem (see Rockafellar 1970, Section 11, Corollary 11.6.2), for any $\delta^{*} \in F\left(\Delta_{\mu}\right)$ there exists a linear function $f$ that achieves its maximum over $\Delta_{\mu}$ at $\delta^{*}$. So for some $\gamma \gg 0, f(\delta)=\gamma \cdot \delta$ and $\delta^{*} \in \arg \max _{\Delta} f(\delta)$. The elements of $F\left(\Delta_{\mu}\right)$ maximize the sum $\sum_{i} \mu_{i} t_{i}$ for all $t \in T$ since $\delta_{i}=\sum_{i} \mu_{i} t_{i}$. By WLIN, it must also be true that the elements of $F(S)$ maximizes the sum $\sum_{i} \mu_{i} \frac{s_{i}}{m_{i}}$.

Proof of Theorem 2. As in Theorem 1, let $S^{\prime}=\left\{\left.\left(\frac{s_{i}}{m_{i}}\right)_{i \in N} \right\rvert\, s \in S\right\}$. For any $s^{*} \in F(S), F$ is $G N U$, there exists $\mu \in \mathbb{R}_{+}^{n}, \mu \neq 0$ such that $\mu \cdot\left(\frac{s_{i}^{*}}{m_{i}}\right)_{i \in N}=\max _{s^{\prime} \in S^{\prime}}\left(\mu \cdot s^{\prime}\right)$, for all $S \in \Sigma$. Then, by AN, $\mu$ can be rewritten as $\mu=\bar{\mu} e$ for some number $\bar{\mu}>0$ with $e$ denoting the vector of ones $e=(1, \ldots, 1)$. Otherwise, $\mu \cdot \pi\left(\frac{s_{i}^{*}}{m_{i}}\right)_{i \in N}$ is not maximal for some $\pi \in \Pi$.

Then

$$
F(S)=\arg \max _{s \in S} \bar{\mu} \sum_{i \in N} \frac{s_{i}}{m_{i}}=\arg \max _{s \in S} \sum_{i \in N} \frac{s_{i}}{m_{i}}=F^{N U}(S)
$$

Another proof of Theorem 2. It is straightforward to check that $\Delta_{\mu}$ is symmetric when $\mu_{i}=\mu_{j}$ for all $i, j \in N$. By PO, the elements of $F\left(\Delta_{\mu}\right)$ must lie on the Pareto frontier of $\Delta_{\mu}$ and $F\left(\Delta_{\mu}\right) \subseteq \mathcal{B}\left(\Delta_{\mu}\right)$. Since $\Delta_{\mu}$ is symmetric, we have $\delta^{*} \in F\left(\Delta_{\mu}\right)$ when $\delta_{i}^{*}=\delta_{j}^{*}$ for all $i, j \in N$ by AN.

If $\delta^{*}$ is an extreme point of $\Delta_{\mu}$, then $F\left(\Delta_{\mu}\right)$ is singleton and $\delta^{*}$ is the unique solution where all coordinates are equal.

If $\delta^{*}$ is not an extreme point then $\delta^{*}=\lambda \delta^{\prime}+(1-\lambda) \delta^{\prime \prime}$ for some $\delta^{\prime}, \delta^{\prime \prime} \in F\left(\Delta_{\mu}\right)$. Note also that by AN, we have $\pi(\delta) \in F\left(\Delta_{\mu}\right)$ for all $\pi \in \Pi$ for any $\delta \in F\left(\Delta_{\mu}\right)$. Then $F\left(\Delta_{\mu}\right)$ is the collection of points on the Pareto frontier that lie on the hyperplane passing through $\delta^{*}$. In this case the solution is multi-valued.

In each case $F\left(\Delta_{\mu}\right)$ maximizes the sum $\frac{1}{n} \sum_{i} \delta_{i}$, implying $\mu=\left(\frac{1}{n}, \ldots, \frac{1}{n}\right)$.

## Appendix B

## APPENDIX TO CHAPTER III

## B. 1 Samaritan's Dilemma

The Biblical parable of the Good Samaritan appears in Luke 10:25-37. Traveling from Jerusalem to Jericho, the Samaritan encounters a Jewish traveler, who was beaten, robbed, and "left half dead" along the road. While a priest and then a Levite pass by and refuse to lend a hand, the Samaritan helps the Jewish traveler and his act is highly applauded as a selfless act.

The Samaritan's Dilemma, introduced by Buchanan (1975), reflects the characteristics of the modern welfare state and emphasizes the role of the time-inconsistency problem. I will briefly describe the bailout game in Buchanan's framework by considering a simple bailout game played by the government (the principal) and a firm (the agent).

The firm, as a nature of its business activities, faces the risk of bankruptcy. Against this risk, the firm engages in activities related to risk management in various levels. The degree of risk management is reduced to two levels for simplicity. The government wants to help the firm against the risk of bankruptcy in order to avoid the dead-weight costs, has an option to help the firm. The government's preferences are ranked from highest to lowest as following:
(3) The firm chooses high effort and the government helps.
(2) The firm chooses low effort and the government helps.
(1) The firm chooses high effort and the government does not help.
(0) The firm chooses low effort and the government does not help.

In response, the agent's preferences are ranked from highest to lowest as:
(3) The firm chooses low effort and the government helps.
(2) The firm chooses high effort and the government helps.
(1) The firm chooses high effort and the government does not help.
(0) The firm chooses low effort and the government does not help.

|  | Low Effort | High Effort |
| :--- | :---: | :---: |
| Help | 2,3 | 3,2 |
| Do Not Help | 0,0 | 1,1 |

Table 3: The bailout game and the Samaritan's Dilemma

Given these preferences, the game is set out in normal form in Table 3, in which the government is the row player and the firm is the column player. Checking the payoff matrix, the firm does not have a dominant strategy. If the government does not help, the firm's best response is to choose high effort. If the government decides to help, the firm is better of by choosing low effort. The government, however, has a dominant strategy to help to the firm. Whether the firm chooses low or high effort, the best response of the government is to help. Given that the firm is informed about the government's preferences, $(2,3)$ is a Nash equilibrium in which neither party can improve independently deciding to do something else.

Consider the following modification to the game which involves strategic considerations on the part of the government. Suppose that the government convincingly announces that the Low Effort-Help option is the least preferred outcome. A credible pre-commitment to an alternative ranking of preferences by principal is as follows:
(3) The firm chooses low effort and the government does not help.
(2) The firm chooses high effort and the government helps.
(1) The firm chooses high effort and the government does not help.
(0) The firm chooses low effort and the government helps.

|  | Low Effort | High Effort |
| :--- | :---: | :---: |
| Help | 0,3 | 2,2 |
| Do Not Help | 3,0 | 1,1 |

Table 4: The modified bailout game

In the modified version of the game, as depicted in Table 4, neither the government nor the firm has a dominant strategy. On the one hand, if the firm chooses high effort, the government's best response is to help, and on the other hand if the firm chooses low effort, the government's best response is not to help. The agent's best responses are the same as in the original version. But the best possible outcome $(0,3)$ for the firm is ranked at the bottom of the government's preferences. The second best option for each player coincides at $(2,2)$ where the firm puts high effort and the government helps. Note that this option is ranked as the best outcome in government's original preferences.

This solution heavily relies on the credibility of the government's pre-commitment to the modified ranking of preferences. If the firm believes strongly that the government's true preferences are as in the original version, the government's pre-commitment is no longer credible. The agent may simple call the government's bluff and chooses low effort.

## B. 2 Proofs of Theorems in Chapter III

Proof of Lemma 1. From (2) the principal's first order condition is

$$
\begin{equation*}
W_{b}=-\pi-\pi \frac{\partial p}{\partial b} c=0 \tag{28}
\end{equation*}
$$

and the second order condition is

$$
\begin{equation*}
W_{b b}=-\pi \frac{\partial^{2} p}{\partial b^{2}} c<0 \tag{29}
\end{equation*}
$$

Note that, by Assumption 3, the second order condition is satisfied. Since each player moves only once, with one step of backward induction, we can treat $b$ as a function of $e$. Since Equation (28) is an identity, differentiating $W_{b}(b(e), e)=0$ totally with respect to $e$ we obtain

$$
-\pi c\left(\frac{\partial^{2} p}{\partial b^{2}} \frac{d b}{d e}+\frac{\partial^{2} p}{\partial b \partial e}\right)=0
$$

that is

$$
\frac{d b}{d e}=-\frac{\frac{\partial^{2} p}{\partial b \partial e}}{\frac{\partial^{2} p}{\partial b^{2}}} .
$$

By Assumption 3 the last expression is negative so we have $b^{\prime}(e)<0$.

Proof of Proposition 1. By Equation (1), the agent's first order condition is

$$
\begin{equation*}
-v^{\prime}(e)-\pi\left[\frac{\partial p(e, b)}{\partial e}+\frac{\partial p(e, b)}{\partial b} \frac{d b}{d e}\right] u(y)=0 \tag{30}
\end{equation*}
$$

Let $b^{*}$ and $e^{*}$ denote the (joint) solutions to Equations (28) and (30).

Alternatively, if the principal chose not to bail out the agent regardless of effort, that is $b^{* *}(e) \equiv 0$, Equation 30 becomes

$$
\begin{equation*}
-v^{\prime}\left(e^{*}\right)-\pi\left[\frac{\partial p\left(e^{*}, 0\right)}{\partial e}\right] u(y)>0 \tag{31}
\end{equation*}
$$

by Assumption $\sqrt[3]{(c)}$ and since $\frac{d b^{* *}}{d e}=0$.
Denoting by $e^{* *}$ the level of effort that satisfies the no-bailout first order condition 31, we find $e^{*}<e^{* *}$ by Assumptions 2 and 3(c). Hence, $e^{*}$ is inefficient. Note that by pre-committing to $\bar{b}=b^{*}$, the principal is better off as well.

Proof of Theorem 3. Since effort is unobservable, the agent takes $b^{*}$ as given and chooses $e=e^{*}$ to maximize

$$
U(e)=u(y)-v(e)-\pi p\left(e, b^{*}\right) u(y)
$$

This yields the first order condition

$$
\begin{equation*}
-v^{\prime}\left(e^{*}\right)-\pi\left[\frac{\partial p\left(e^{*}, b^{*}\right)}{\partial e}\right] u(y)=0 \tag{32}
\end{equation*}
$$

If $b^{*}>0$, and similarly to the proof of Proposition 1, it follows from Assumption 3(c) that the right-hand-side of Equation 32 would be positive if no bailouts were offered. Thus, again, we find that $e^{*}<e^{* *}$, where $e^{* *}$ denotes the optimal level of effort without a bailout. That is, $e^{* *}$ is chosen to satisfy

$$
-v^{\prime}\left(e^{* *}\right)-\pi\left[\frac{\partial p\left(e^{* *}, 0\right)}{\partial e}\right] u(y)=0
$$

Lastly, we want to point out that $b^{*}$ is given as the solution to

$$
\frac{\partial p\left(e^{*}, b^{*}\right)}{\partial b}=\frac{-1}{c}
$$

By Assumption 3(b), the optimal bailout transfer $b^{*}$ is thus increasing in the social cost $c$. Hence, $b^{*}>0$ for sufficiently large $c$.

Proof of Proposition 2. Using the Implicit Function Theorem, we differentiate one agent's first order condition with respect to other agent's optimal choice of effort.

$$
\begin{array}{r}
\left(\frac{\partial^{2} \pi}{\partial e_{1} \partial e_{2}}+\frac{\partial^{2} \pi}{\partial^{2} e_{1}} \frac{d e_{1}}{d e_{2}}\right) p_{1}+\frac{\partial \pi}{\partial e_{1}} \frac{\partial p_{1}}{\partial e_{1}}+\frac{\partial^{2} p_{1}}{\partial e_{1}^{2}} \frac{d e_{1}}{d e_{2}} \pi+\frac{\partial p_{1}}{\partial e_{1}}\left(\frac{\partial \pi}{\partial e_{1}} \frac{d e_{1}}{d e_{2}}+\frac{\partial \pi}{\partial e_{2}}\right) \\
=-\frac{v_{1}^{\prime \prime}\left(e_{1}\right)}{u\left(y_{1}\right)} \frac{d e_{1}}{d e_{2}} .
\end{array}
$$

Solving for $\frac{d e_{1}}{d e_{2}}$ we obtain

$$
\frac{d e_{1}}{d e_{2}}=-\frac{\frac{\partial^{2} \pi}{\partial e_{1} \partial e_{2}} p_{1}+\frac{\partial p_{1}}{\partial e_{1}} \frac{\partial \pi}{\partial e_{2}}}{\frac{v_{1}^{\prime \prime}\left(e_{1}\right)}{u\left(y_{1}\right)}+\frac{\partial^{2} \pi}{\partial e_{1}^{2}} p_{1}+\frac{\partial \pi}{\partial e_{1}} \frac{\partial p_{1}}{\partial e_{1}}+\frac{\partial^{2} p_{1}}{\partial e_{1}{ }^{2}} \pi+\frac{\partial p_{1}}{\partial e_{1}} \frac{\partial \pi}{\partial e_{1}}}
$$

By Assumption 4, both the numerator and denominator are positive, which concludes the proof.

## Appendix C

## APPENDIX TO CHAPTER IV

## C. 1 Proofs of Theorems in Chapter IV

Proof of Lemma 3. Let $b \geq 0$ be given and fixed. Since $p_{H}^{\prime}(b)>p_{L}^{\prime}(b)$, we must have

$$
p_{L}(0)-p_{L}(b)>p_{H}(0)-p_{H}(b)
$$

Moreover, since $p_{L}(0)>p_{H}(0)$ and $p_{L}(b)>p_{H}(b)$, we get

$$
\frac{1-p_{H}(b)}{1-p_{H}(0)}=1+\frac{p_{H}(0)-p_{H}(b)}{1-p_{H}(0)}<1+\frac{p_{L}(0)-p_{L}(b)}{1-p_{L}(0)}=\frac{1-p_{L}(b)}{1-p_{L}(0)}
$$

Proof of Lemma 5. When $\psi=(R, A)$, the principal's expected welfare function is

$$
W_{R A}(b)=W_{0}-\mu p_{H}(0) c-(1-\mu)\left[b-p_{L}(b) c\right] .
$$

Taking the first-order condition with respect to $b$ yields the desired result.

Proof of Theorem 5. Follows from Lemmas 4 and 5.

Proof of Theorem 6. It is left to verify the condition for choosing low versus high effort.

Under Assumption 9, the agent's expected utility from choosing low effort is

$$
E U_{L} \equiv p_{L}\left(b^{*}\right) \cdot 0+\left[1-p_{L}\left(b^{*}\right)\right] \cdot u_{L L}
$$

and the agent's payout from exerting high effort is

$$
E U_{H} \equiv p_{H}(0) \cdot 0+\left[1-p_{H}(0)\right] \cdot u_{H H}-v_{H}^{\tau} .
$$

Since it is optimal to choose $e=0$ if and only if $E U_{L}>E U_{H}$, we obtain Inequality (19).

Proof of Theorem 7. Follows immediately from Theorem 6 and Proposition 4.
Proof of Theorem 8. Combine Theorem 6 and Proposition 4.

## References

Akerlof, G. A. (1970, August). The market for 'lemons': Quality uncertainty and the market mechanism. The Quarterly Journal of Economics 84 (3), 488-500.

Bagehot, W. (1873). Lombard Street: A Description of the Money Market. Scribner, Armstrong \& Co.

Becker, G. S. (1974). A theory of social interactions. Journal of Political Economy 82(6), pp. 1063-1093.

Becker, G. S. (1981, February). Altruism in the family and selfishness in the market place. Economica 48(189), 1-15.

Bergstrom, T. C. (1989, October). A fresh look at the rotten kid theorem-and other household mysteries. Journal of Political Economy 97(5), 1138-59.

Bossert, W., E. Nosal, and V. Sadanand (1996, June). Bargaining under uncertainty and the monotone path solutions. Games and Economic Behavior 14 (2), 173-189.

Boyd, J. H., C. Chang, and B. D. Smith (2004). Deposit insurance and bank regulation in a monetary economy: A general equilibrium exposition. Economic Theory 24 (4), 741-767.

Bruce, N. and M. Waldman (1990). The Rotten-Kid Theorem meets the Samaritan's Dilemma. The Quarterly Journal of Economics 105(1), 155-165.

Bruce, N. and M. Waldman (1991). Transfers in kind: Why they can be efficient and nonpaternalistic. The American Economic Review 81(5), 1345-1351.

Bryant, J. (1980). A model of reserves, bank runs, and deposit insurance. Journal of Banking $\mathfrak{6}$ Finance 4(4), 335-344.

Buchanan, J. M. (1975). The Samaritan's Dilemma. In E. S. Phelps (Ed.), Altruism, Morality, and Economic Theory, pp. 71-85. New York: Russell Sage Foundation.

Butkiewicz, J. L. and K. A. Lewis (1991). Bank bailouts and the conduct of monetary policy. Southern Economic Journal 58(2), 501-509.

Cao, X. (1982). Preference functions and bargaining solutions. In Decision and Control, 1982 21st IEEE Conference on, Volume 21, pp. 164-171.

Chami, R. (1996, August). King Lear's dilemma: Precommitment versus the last word. Economics Letters 52(2), 171-176.

Chun, Y. (1988). The equal-loss principle for bargaining problems. Economics Letters 26(2), 103-106.

Coate, S. (1995). Altruism, the Samaritan's Dilemma, and government transfer policy. The American Economic Review 85(1), 46-57.

Dhillon, A. (1998). Extended pareto rules and relative utilitarianism. Social Choice and Welfare 15(4), 521-542.

Dhillon, A. and J.-F. Mertens (1999). Relative utilitarianism. Econometrica 67(3), 471-498.

Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. The Journal of Political Economy 91 (3), 401-419.

Diamond, D. W. and R. G. Rajan (2002). Bank bailouts and aggregate liquidity. The American Economic Review 92(2), 38-41.

Dijkstra, B. (2002, August). Samaritan vs Rotten Kid: Another look. Royal Economic Society Annual Conference 2002 62, Royal Economic Society.

Feltovich, N., R. Harbaugh, and T. To (2002, Winter). Too cool for school? Signalling and countersignalling. RAND Journal of Economics 33(4), 630-649.

Ghatak, M., M. Morelli, and T. Sjostrom (2001, October). Occupational choice and dynamic incentives. Review of Economic Studies 68(4), 781-810.

Gorton, G. and L. Huang (2004). Liquidity, efficiency, and bank bailouts. The American Economic Review 94(3), 455-483.

Gup, B. E. (Ed.) (2004). Too Big to Fail: Policies and Practices in Government Bailouts. Westport, CT: Praeger.

Harsanyi, J. C. (1953). Cardinal utility in welfare economics and in the theory of risk-taking. Journal of Political Economy 61, 434.

Kalai, E. and M. Smorodinsky (1975, May). Other solutions to Nash's bargaining problem. Econometrica 43 (3), 513-18.

Karni, E. (1998, November). Impartiality: Definition and representation. Econometrica 66(6), 1405-1416.

Kho, B.-C., D. Lee, and R. M. Stulz (2000). U.S. banks, crises, and bailouts: From Mexico to LTCM. The American Economic Review 90(2), 28-31.

Kihlstrom, R., A. Roth, D. Schmeidler, U. of Pennsylvania. Center for Analytic Research in Economics, and the Social Sciences (1980). Risk Aversion and Solutions to Nash's Bargaining Problem. CARESS working paper. University of Pennsylvania, Center for Analytic Research in Economics and the Social Sciences.

Lagerlof, J. (2004). Efficiency-enhancing signalling in the Samaritan's Dilemma. The Economic Journal 114(492), 55-68.

Miyagawa, E. (2002). Subgame-perfect implementation of bargaining solutions. Games and Economic Behavior 41 (2), 292-308.

Myerson, R. B. (1978, March). Linearity, concavity, and scale invariance in social choice functions. Discussion Papers 321, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

Myerson, R. B. (1981, June). Utilitarianism, egalitarianism, and the timing effect in social choice problems. Econometrica 49(4), 883-97.

Nash, John F., J. (1950). The bargaining problem. Econometrica 18(2), pp. 155-162.

Pechersky, S. (2006). On the superlinear bargaining solution. In T. S. H. Driessen, G. Laan, V. A. Vasilev, E. B. Yanovskaya, W. Leinfellner, G. Eberlein, and S. H. Tijs (Eds.), Russian Contributions to Game Theory and Equilibrium Theory, Volume 39 of Theory and Decision Library, pp. 165-174. Springer Berlin Heidelberg. 10.1007/3-540-32061-X9.

Perles, M. A. (1982). Non-existence of super-additive solutions for 3-person games. International Journal of Game Theory 11, 151-161. 10.1007/BF01755725.

Perles, M. A. and M. Maschler (1981). The super-additive solution for the Nash bargaining game. International Journal of Game Theory 10, 163-193. 10.1007/BF01755963.

Peters, H. J. M. (1986, January). Simultaneity of issues and additivity in bargaining. Econometrica 54(1), 153-69.

Pivato, M. (2009). Twofold optimality of the relative utilitarian bargaining solution. Social Choice and Welfare 32(1), 79-92.

Riddell, W. C. (1981, September). Bargaining under uncertainty. American Economic Review 71 (4), 579-90.

Ringbom, S., O. Shy, and R. Stenbacka (2004, June). Optimal liquidity management and bail-out policy in the banking industry. Journal of Banking \& Finance 28(6), 1319-1335.

Rochet, J. C. and X. Vives (2002, February). Coordination failures and the lender of last resort: Was Bagehot right after all? CEPR Discussion Papers 3233, C.E.P.R. Discussion Papers.

Rockafellar, R. (1970). Convex Analysis. Princeton Mathematical Series. Princeton University Press.

Roth, A. E. (1979). An impossibility result concerning n-person bargaining games. International Journal of Game Theory 8(3), 129-132.

Segal, U. (2000). Let's agree that all dictatorships are equally bad. The Journal of Political Economy 108(3), 569-589.

Shim, I., S. Sharma, and R. Chami (2008). A model of the IMF as a coinsurance arrangement. Economics - The Open-Access, Open-Assessment E-Journal 2(14), 1-41.

Sobel, J. (2001). Manipulation of preferences and relative utilitarianism. Games and Economic Behavior 37(1), 196-215.

Spence, A. M. (1973, August). Job market signaling. The Quarterly Journal of Economics 87(3), 355-74.

Spence, M. (1974, March). Competitive and optimal responses to signals: An analysis of efficiency and distribution. Journal of Economic Theory 7(3), 296-332.

Spence, M. (1981). Signaling, screening, and information. In Studies in Labor Markets, NBER Chapters, pp. 319-358. National Bureau of Economic Research, Inc.

Stern, G. and R. Feldman (2004). Too Big to Fail: The Hazards of Bank Bailouts. G - Reference, Information and Interdisciplinary Subjects Series. Brookings Institution Press.

Thomson, W. (1980). Two characterizations of the Raiffa solution. Economics Letters 6(3), 225-231.

Thomson, W. (2010). Bargaining and the theory of cooperative games: John Nash and beyond. In W. Thomson (Ed.), Bargaining and the Theory of Cooperative Games: John Nash and Beyond. Camberly, Northampton, MA.: Edward Elgar Publishing Ltd.

Tresch, R. (2002). Public Finance: A Normative Theory. Academic Press.
Xu, Y. and N. Yoshihara (2006). Alternative characterizations of three bargaining solutions for nonconvex problems. Games and Economic Behavior 57(1), 86-92.

Yaron, L. (2005). Financial networks: Contagion, commitment, and private sector bailouts. The Journal of Finance 60(6), 2925-2953.

Yu, P. L. (1973, April). A class of solutions for group decision problems. Management Science 19(8), 936-946.

## VITA

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[^0]:    ${ }^{1}$ Here, uncertainty and risk are used as synonymously and interchangeably.

[^1]:    ${ }^{2}$ The formal definitions of these axioms are skipped here and the reader is invited to check the articles I have cited.

[^2]:    $3^{\text {Perles }} 1982$ establishes a similar impossibility result with a variant of LIN (super-additivity) for bargaining problems with $n \geq 3$.

[^3]:    ${ }^{4}$ Note that the payoffs are not compared interpersonally, so the INV axiom is not violated.
    ${ }^{5}$ I would like to thank Uzi Segal for providing this example.

[^4]:    ${ }^{6}$ The Federal Deposit Insurance Act of 1950 defines bailouts as "providing assistance, the power to support an institution through loans or direct federal acquisition of assets, until it could recover from its distress".

[^5]:    ${ }^{7}$ Naturally, the moral hazard problem is not limited to bailouts, but applies to other types of assistance, such as charity, donations, debt relief and debt forgiveness, as they also reward and encourage risky behavior.
    ${ }^{8}$ I provide a brief overview of the original model in the Appendix.

[^6]:    ${ }^{9}$ For instance, you may not want to become a welfare recipient because you worry that others might think you cannot make it on your own.

[^7]:    ${ }^{10}$ This assumption is in line with the findings (see below) that the high-type agent might be able to signal his type by rejecting the bailout.

[^8]:    ${ }^{11}$ According to Assumption 6, $\beta_{H}<\beta_{L}$ implies $p_{L}(b)<p_{H}(b)$ for some $b>0$, and $\beta_{H}>\beta_{L}$ implies $p_{L}^{\prime}(b)>p_{H}^{\prime}(b)$ for some $b>0$.

