# Standards-Based Instruction: A Case Study of a College Algebra Teacher 

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## ACCEPTANCE

This dissertation, STANDARDS-BASED INSTRUCTION: A CASE STUDY OF A COLLEGE ALGEBRA TEACHER, by ANTHONIA EKWUOCHA, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree Doctor of Philosophy in the College of Education, Georgia State University.

The Dissertation Advisory Committee and the student's Department Chair, as representatives of the faculty, certify that this dissertation has met all standards of excellence and scholarship as determined by the faculty. The Dean of the College of Education concurs.

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#### Abstract

\title{ STANDARDS-BASED INSTRUCTION: A CASE STUDY OF A COLLEGE ALGEBRA TEACHER } by Anthonia Ekwuocha

The lecture method has dominated undergraduate mathematics education (Bergsten, 2007). The lecture method promotes passive learning instead of active learning among students, thus contributing to attrition in undergraduate mathematics. Standards-based instruction has been found to be effective in reducing students' attrition in undergraduate mathematics (Ellington, 2005). College algebra is gatekeeper for higher undergraduate mathematics courses (Thiel, Peterman \& Brown, 2008). Research indicates that if college algebra is taught with standards-based teaching strategies, it will help reduce students' attrition and encourage more students to take higher level mathematics courses (Burmeister, Kenney, \& Nice, 1996). Standards-based instructional strategies include but are not limited to real life applications, cooperative learning, proper use of technology, implementation of writing, multiple approaches, connection with other experiences, and experiential teaching (American Mathematical Association of Two-Year Colleges (AMATYC), 2006).

Despite all effort to improve undergraduate mathematics instruction, there are still limited empirical studies on standards-based instruction in college algebra. Research in undergraduate mathematics education is a new field of study (Brown \& Murphy, 2000). Research reported that overall students' attrition in college algebra could be as high as $41 \%$ in a community college (Owens, 2003). This high attrition rate in college algebra


may impact students' continuation in higher mathematics courses and their interest in the field of mathematics. As a result more research efforts must be focused on ways to improve college algebra instruction. Thus, the purpose of this study was to explore the teaching practices of a college algebra teacher who adopts standards-based techniques in his classroom. The research questions that guided the study were: What teaching practices are used in the mathematics classroom of a college algebra teacher? How are the teaching practices of the teacher aligned with the characteristics of standards-based instruction?

The participant of the study was a college algebra teacher who was identified as a standards-based teacher. The teaching practices of the teacher were analyzed and presented using a qualitative single case study method. Data were collected from interviews with the teacher, classroom observations, and artifacts. The research project was drawn from the frameworks of culturally relevant pedagogy theory, symbolic interaction theory, experiential teaching theory, and standards-based instruction.

Analysis of the data showed that the teaching practices of the participant were mathematical communication, proper use of technology in instruction and assessment, building mathematical connections, multiple representations, motivating students to learn mathematics, and repetition of key terms. The teaching practices aligned with the characteristics of standards-based instruction. Findings from the study suggest that standards-based instruction strategies should be used in undergraduate mathematics education, especially in teaching college algebra to alleviate some of the problems. Moreover, university administrators at college level should organize workshops and professional development about standards-based instruction strategies for their teachers.

# STANDARDS-BASED INSTRUCTION: A CASE STUDY OF 

 A COLLEGE ALGEBRA TEACHERby
Anthonia Ekwuocha

## A Dissertation

Presented in Partial Fulfillment of Requirements for the
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in
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in
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## CHAPTER 1

## INTRODUCTION

## Problem

The problem that leads to this study is student attrition due to poor quality of instruction in undergraduate-level mathematics (Seymour \& Hewitt, 1997). Mathematics education promotes active learning as opposed to the typical lecture method. The lecture method has dominated undergraduate mathematics education (Bergsten, 2007). Bergsten further emphasizes that in mathematics, the lecture method is a traditional method of teaching, which is a one-way communication from the teacher to the students. The lecture method promotes passive learning instead of active learning among students; it does not stimulate higher order thinking and lacks social activities and creativity on the part of the students (Bergsten, 2007), thus contributing to attrition in undergraduate mathematics. Standards-based instruction strategies, such as real-life applications, cooperative learning, proper use of technology, implementation of writing, and experiential teaching, may be possible ways to combat student attrition in undergraduate mathematics (Ellington, 2005).

Previous research has reported high levels of student attrition in undergraduate mathematics (Seymour, 2001). Astin and Astin (1993) (as cited in Seymour, 2001), reported that there was a high amount of student decline (40\%) in taking undergraduate mathematics and other mathematics-related subjects in America. In addition, Seymour and Hewitt (1997) reported that the highest decline in the number of students in undergraduate mathematics was seen among first-year students. Previous studies have reported that one of the reasons for student attrition in undergraduate mathematics is
"poor quality of undergraduate mathematics instruction" (Linn \& Kessel, 1996, p. 101). Seymour (2001) confirmed that poor teaching was the students' major concern about undergraduate mathematics education.

Researchers have identified lecture as a poor teaching method (Linn \& Kessel, 1996; Twigg, 2004). Astin and Astin (1993) further discussed the poor teaching of undergraduate mathematics. Although the Astin and Astin study is 17 years old, it is still significant because current studies frequently reference their work. Astin and Astin (1993) revealed that most university professors utilize the lecture method. They stand in front of the class without engaging their students. Additionally, several professors asked questions, which they quickly answered themselves, discouraging student participation. Students were bored, some arrived late, and some left early. Twigg (2004) also stressed that the disadvantage of the lecture method is that it treats all students the same, regardless of their different learning styles, academic preparations, interests, motivation to learn, and abilities to learn.

Responding to the poor teaching in mathematics classrooms, the National Council of Teachers of Mathematics (NCTM) (2000). Principles and standards call for standardsbased instruction in school mathematics. The Mathematical Association of America's (MAA's) Committee for Undergraduate Program in Mathematics (CUPM) published the CUPM Guide in 2004, which lists specific recommendations on the ways to improve undergraduate mathematics instruction. In addition, the American Mathematical Association of Two-Year Colleges (AMATYC) developed and published standards for instruction for lower-level college mathematics courses. Therefore, AMATYC (2006)
and CUPM (2004) are aligned with the NCTM (2000) ideas on its recommendations for standards-based instruction.

As one of many mathematics courses offered in colleges and universities, college algebra is often considered to be one that helps students develop further interest in mathematics and lays a foundation for advanced courses. In addition, college algebra, in most two-year colleges, is designed for students majoring in different subjects including mathematics, science, engineering, behavioral science, and liberal arts (Gallo \& Odu, 2009). Hence, college algebra is significant for many students. Unfortunately, college algebra instruction is of poor quality. Harvey, Waits, and Demana (1995) mentioned that "one of the most common complaints that we hear from our colleagues about college calculus students is that the [college] algebra preparation for calculus of their students is very poor" (p. 75). Due to the poor quality of instruction in undergraduate mathematics courses (including college algebra), the attrition rate in undergraduate mathematics at most colleges and universities is high (Seymour \& Hewitt, 1994). As mentioned earlier, college algebra was identified as a high-attrition course (Waller, 2006). For example, Owens (2003) illustrated the high attrition in college algebra by stating that: "Fall 2002, the overall attrition rate at [a Community College] for college algebra was $41 \%$ ranging from $13 \%$ to $81 \%$ per section ...." (p. 1). This high attrition rate in college algebra affects students' continuation in upper-level mathematics courses and their interest in the field of mathematics, which may affect college attrition overall. Hutchison (2006) reported that very few Americans are successful in mathematics and too few are earning advanced degrees in mathematics. Therefore, to address the problem of low retention of students in undergraduate mathematics, it is important to start from a basic course such as college
algebra. As seen in previous studies, researchers have considered college algebra the gatekeeper for upper-level mathematics courses (Thiel, Peterman, \& Brown, 2008).

Ellington (2005) showed that standards-based instructional strategies have helped improve student attrition in college algebra. Burmeister, Kenney, and Nice (1996) also explained that teaching college algebra with standards-based teaching strategies helps to reduce student attrition and leads to more students taking upper-level mathematics courses. Despite efforts to improve undergraduate mathematics instruction, there limited study of standards-based instruction in college algebra. Research in undergraduate mathematics education is a new field of study (Brown \& Murphy, 2000). Most research in mathematics education has been with grade schools (Selden \& Selden, 1993).

There are limited qualitative studies in standards-based instruction in college algebra. To address the issue of student attrition in undergraduate mathematics, more research is needed in standards-based college algebra instruction. The current qualitative study was designed to describe the actions of a college algebra teacher, describe a classroom following a standards-based approach, and highlight experiences of the students in such a classroom. This study is significant because it will contribute to the literature on standards-based instruction. The results of this study can have an effect on decisions made about college algebra instruction.

## Purpose

Several organizations, such as NCTM, CUPM, and AMATYC, have suggested the use of standards-based instruction in mathematics classrooms. According to the AMATYC (1995, 2006), these standards-based instruction techniques are to: (a) connect mathematics with other experiences; (b) use multiple approaches; (c) allow students to
experience mathematics; (d) use multiple instructional strategies that encourage active student learning and address different learning and teaching styles; (d) integrate technology as a tool to help students discover and understand key mathematical concepts; and (e) align technology tools for assessment with instruction.

Although there is a push for the implementation of standards-based instruction, the lecture method still dominates mathematics classrooms in college-level courses (Bergsten, 2007). Therefore, the purpose of the current study is to explore the teaching practices of a college algebra teacher who utilizes standards-based techniques in his classroom; these techniques were used to analyze his teaching practices. The results of this study will contribute more effective ways of teaching mathematics at the collegiate level.

## Research Questions

The current study uses a case study to investigate the teaching practices of a college algebra teacher following a standards-based approach. The teacher was selected using purposeful sampling, and classroom observations, in-depth interviews, and field notes were used to identify his teaching practices. The primary research questions guiding this study were:

1. What teaching practices are used in the mathematics classroom of a college algebra teacher?
2. How are the teaching practices of the teacher aligned with the characteristics of standards-based instruction?

## Rationale for the Research

Previous study (Ellington, 2005) has shown that standards-based instruction provides opportunities for educators to change their teaching styles from the lecture method to more interactive modes of teaching, which reduces student attrition in college mathematics courses. Researchers have reported the negative effects of the lecture method (traditional method) in mathematics classrooms (Freire, 2000; Twigg, 2004). According to Freire (2000), education is dominated by the "banking" method (traditional method), further describing that "banking education is an act of depositing, which the students are the depositories and the teacher is the depositor. Instead of communicating, the teacher issues communiqués and makes deposits which the students patiently receive, memorize, and repeat" (p. 72). Skovsmose (2005) noted that in traditional mathematics classrooms, "the mathematics textbooks dominate the class teaching which the teachers follow page-by-page" (p. 9). Skovsmose (2005) further explained that exercises in mathematics textbooks do not require creativity from the students. Gutstein and Peterson (2005) also examined the negative impact of traditional mathematics teaching strategies, mentioning that "traditional forms of mathematics are often too abstract, promote student failure and self-doubt, and frankly are immoral in a world as unjust as ours. Traditional mathematics is bad for students and bad for society" (p. 5).

Attempting to overcome the problems of traditional teaching, some researchers have focused on standards-based instruction techniques, including group work (Burmeister, Kenney, \& Nice, 1996; Rogers, Davidson, Reynolds, Czarnocha, \& Aliaga, 2001); proper use of technology (Mayes, 1995; Thiel, Peterman, \& Brown, 2008); implementation of writing (Kagesten \& Engelbrecht, 2006); real-life applications
(Austin, Berceli, \& Sarae, 1999); and experiential teaching (Fowler, 1991; McGlinn, 1999). Despite all these techniques, the literature (Engelbrecht \& Harding, 2009; Pierce, Turville, \& Giri, 2003; Rasmussen \& Kwon, 2007) shows that student attrition in undergraduate mathematics still exists; some researchers (Fargasz \& Leder, 2000; Seymour, 2001) have suggested "poor teaching" as one of the reasons for this attrition. Limited studies have been conducted to identify ways to reduce student attrition through use of standards-based instruction in college algebra classrooms. The current study is designed to add to extant literature by reporting the teaching practices of a college algebra teacher using standards-based instruction.

## Conceptual Framework

## Overview of Standards-Based Instruction

The conceptual framework that guided this study was standards-based instruction, which served as a rubric for data collection and analysis. Use of this framework is defensible because it comprises the components of the theoretical frameworks of this study. The theoretical frameworks used in the current study were culturally relevant pedagogy theory, symbolic interaction theory, and experiential instruction theory. As previously mentioned, the three major advocates of standardsbased instruction are the NCTM, AMATYC, and MAA. These advocates are all professional organizations dedicated to mathematics teaching and learning, which highlights the importance of students' active participation in the learning process. Teachers should deviate from what Freire (2000) called "banking education" and give students the opportunity to be a part of the teaching and learning process. Each of these organizations will be discussed in more depth in the literature review section.

The National Council of Teachers of Mathematics (NCTM) (2000) reports that it is important for teachers to connect new knowledge to prior knowledge of students, engage students through the questioning method, and expose students to challenging tasks that involve real-world experience. Also, to teach for understanding, teachers have to observe, listen, and interact with their students. The NCTM (2000) notes the need for teachers to create an environment where students can discuss, collaborate, justify, and experiment with different mathematical methods. Careful planning and questioning from the teachers creates a more conducive teaching and learning environment. The NCTM (2000) emphasizes that it is crucial for teachers to use proper assessment techniques, such as "open-ended questions, constructed-response tasks, selected-response items, performance tasks, observations, conversations, journals, and portfolios" (p. 23). The NCTM points out that proper use of technology should be part of the teaching and learning process.

Similarly, the Mathematical Association of America (MAA) (2004), through their Committee for Undergraduate Program in Mathematics (CUPM), provides recommendations on ways to improve undergraduate mathematics instruction. Their recommendations include developing "mathematical thinking (pattern recognition, generalization, abstraction, problem solving, careful analysis, rigorous argument) and communication skills" and using "computer technology to support problem solving and to promote undergraduate understanding" and involving students in applying and connecting mathematics knowledge to real world life and other disciplines (p. 5). According to the CUPM (1981, p. 578):

- A mathematical sciences program should use interactive classroom teaching to involve students actively in the development of new material.
- Applications should be used to illustrate and motivate material in abstract and applied courses.
- Students should have an opportunity to undertake "real-world" mathematical modeling projects.

The American Mathematical Association of Two-Year Colleges (AMATYC)
$(1995,2006)$ promotes standards-based instruction to improve teaching and learning in lower-level mathematics courses, which justifies use of AMATYC as a conceptual framework because the current study is focused on college algebra. The AMATYC's suggestions for standards-based instruction are aligned with the core values of NCTM (2000) and CUPM $(1981,2004)$. The AMATYC's standards-based instructional strategies, which map faculty guidelines, are described here:

Connect mathematics with other experiences, use multiple approaches, allow students to experience mathematics, use multiple instructional strategies that encourage active student learning and address different learning and teaching styles, integrate technology as a tool to help students discover and understand key mathematical concepts, align technology tools for assessment with instruction. (AMATYC, 1995, pp.16-17; AMATYC, 2006, p. 59)

As previously described, the three major advocates of standards-based instruction are the NCTM, AMATYC, and MAA. The theoretical frameworks of this study are components of the study's conceptual framework. This relationship between the theoretical framework and conceptual framework justifies the use of standards-based instruction as the lens for data collection and analysis. In particular, the AMATYC's standards-based instruction strategies served as rubric for data collection and data analysis (see Literature Review). The use of the AMATYC's standards-based instruction
techniques was justified because the study focuses on lower-level college mathematics course, which is also the AMATYC's focus.

## Theoretical Framework

Kilbourn (2006) stated that a "theoretical framework represents a point of view that legitimizes the manner in which the interpretations are justified or warranted" (p. 533). According to Fowler (2006), the purposes of theoretical frameworks are to help researchers focus their study, develop research questions, plan data collection, and structure data analysis. Henstrand (2006) added that theoretical frameworks assist researchers in managing their subjectivities. In this section, I introduce the theories that informed my research on standards-based instruction. These theoretical frameworks were (a) culturally relevant pedagogy theory; (b) symbolic interaction theory; and (c) experiential teaching theory. These theories guided my data collection and analysis and helped me focus on the aspects of my study that were relevant to the theory.

## Culturally Relevant Pedagogy

Culturally relevant pedagogy theory was one of the frameworks selected for my study. This theory was chosen because standards-based instruction emphasizes that it is important to relate instruction to students' culture. Culturally relevant pedagogy is a grounded theory developed by Ladson-Billings (1995a) on the basis of research conducted concerning cultural discontinuities and successful teachers of African American students. Ladson-Billings (1990, 1992, 1995a, 1995b, 2000) narrated reasons for culturally relevant pedagogy among African American students, reporting that students who are not part of the white middle class experience difficulties in school due to the fact that educators have not attempted to insert their culture into the education
system. Specifically, Ladson-Billings (2000) criticized the use of "one best system" (p. 207), which, based on the 19th-century Americanization model, focused primarily on learning styles of immigrants and cultural groups from Europe while neglecting immigrants from Africa.

Ladson-Billings (1995a) developed culturally relevant pedagogy by studying eight exemplary teachers of African American students. Participants of that study (Ladson-Billings, 1995a) were selected as "excellent teachers" through student and parent nominations and principal recommendations. The parents' guidelines for nominating the teachers included "respect by the teacher, student enthusiasm toward school and academic tasks, and students' attitudes toward themselves and others" (Ladson-Billing, 1995b, p. 471). The principals' criteria for nomination were student performance, classroom management strategies, and classroom observations of teaching techniques. Teachers' names appearing on both parents' and principals' lists were included in the study. Altogether, 5 African American and 3 White females who had 12 to 40 years of teaching experience participated in the study. During the 3 -year ethnographic study, data were collected through interviews and unannounced classroom observations of the teachers. During these observations, classes were videotaped and field notes were taken. Although each of the teachers used different strategies to improve student success in their classrooms, Ladson-Billings (1995a) proposed this grounded theory, culturally relevant pedagogy. She defined this as a "kind of teaching that is designed not merely to fit the school culture to the students' culture but also to use student culture as the basis to helping students understand themselves and others, structure social interactions, and conceptualize knowledge" (Ladson-Billings, 1992, p.
314). The three themes of culturally relevant pedagogy are academic success, cultural competence, and critical consciousness, meaning that these are three things that teachers should attend to while they implement culturally relevant pedagogy.

Studies have reported ways for teachers to achieve academic success with their students. For instance, the teachers in Ladson-Billings's study (1995a) were able to help their students perform at higher levels. Ladson-Billings (1995b) stressed that "students demonstrated an ability to read, write, speak, compute, pose and solve problems at sophisticated levels-that is, pose their own questions about the nature of teacher- or text-posed problems and engage in peer review of problem solutions" (p. 475). In addition, culturally relevant teachers set high expectations, believe that their students can achieve, and connect new knowledge to students' previous knowledge (Ladson-Billings, 1992; Lipman, 1995; Standford, 1997). Hemming (1994) also revealed that academic success was also achieved through conversational methods and classroom discussions. The teachers in this study "listened, offered advice, and used highly supportive, nonconfrontational means to foster students' academic and personal growth" (Hemming, 1994, p. 8).

Cultural competence indicates that students should be able to maintain their cultural identity as well as academic excellence. Relating student culture to teaching makes learning easier for students and helps motivate and engage them. Culturally relevant teachers should include their students' culture(s) in teaching. Jordan (1985) made it clear that:

Educational practices must match with the [students'] culture in ways that ensure the generation of academically important behaviors. It does not mean that all school practices need be completely congruent with natal cultural practices, in the sense of exactly or even closely matching or agreeing with them. The point of
cultural compatibility is that the natal culture is used as a guide in the selection of educational program elements so that academically desired behaviors are produced and undesired behavior are avoided. (p. 110)

It is argued by advocates of culturally relevant pedagogy that teachers should allow their students to use their native languages in the classroom but at the same time encourage them to learn the standard language (Ladson-Billings \& Henry, 1990; Howard, 2001; Ladson-Billings, 1992; Ladson-Billings \& Hemming, 1994; Lipman, 1995). For instance, Lipman (1995) mentioned that one of his participating teachers allowed the students to use "Black English," but he still maintained the importance of standard English. The teacher corrected his students' oral and written grammar and assigned weekly words. Studies focusing on culturally relevant teachers also showed that they integrated students' cultures into classroom teachings through the use of art, music, and literature (e.g., Ladson-Billings, 1994, 1995). Ladson-Billings and Henry (1990) reported that one of the teachers in their study used "rap" to help students master spelling rules and multiplication tables. Hilliard, a participant in the Ladson-Billings (1995a) study, even allowed students to bring in sample lyrics of rap of songs and connected these lyrics to school learning.

Critical consciousness is another component of culturally relevant pedagogy. According to Ladson-Billings (1995a), culturally relevant pedagogy should be more than academic success and cultural competence; it also involves students' ability to develop sociopolitical consciousness, which helps them to critique social inequalities. That is, culturally relevant pedagogy should also challenge students to critique information in the textbook, inequality in school system, and the society. Gutstein and Peterson (2005) discussed how he raised the level of critical consciousness in his mathematics teaching, which helped his students read and write the world with mathematics. Gutstein and

Peterson (2005) asked students to analyze World Map Mercator Projection, finding that his "students determined for themselves that the Mercator Map did not show equal areas equally" (p. 114). In another study (Ladson-Billings and Henry, 1990), the level of critical consciousness was raised when one of the teachers reported inappropriate curriculum for her students. The teacher fought extensively for culturally related curriculum. The teacher wrote to the school board about the inappropriate curriculum; she also requested a textbook change, which she believed would improve student performance (Ladson-Billings \& Henry, 1990).

Ladson-Billings (1995a) further explained that the teachers in her study (leading to the grounded theory of culturally relevant pedagogy) shared three beliefs and characteristics in terms of their conceptions of self and others, social interactions, and conceptions of knowledge. As for conceptions of self and others, these exemplary teachers all believed that "all the students were capable of academic success," they all "saw their pedagogy as art-unpredictable," they were "always in the process of becoming," and they all "believed in a Freirean notion of 'teaching as mining' or pulling knowledge out" (p. 478). In terms of social interactions with their students, these teachers all successfully "maintain fluid student-teacher relationships, demonstrate a connectedness with all of the students, develop a community of learners, and encourage students to learn collaboratively and be responsible for another" (p. 480). In terms of beliefs about knowledge, these teachers shared the view that "knowledge is not static; it is shared, recycled, and constructed; knowledge must be viewed critically" (p. 481). All participating teachers were passionate about knowledge and learning. They also believed that it was teachers' responsibility to "scaffold, or build bridges, to facilitate learning"
and that an "assessment must be multifaceted, incorporating multiple forms of excellence" (p. 481). Ladson-Billing (1995a) also highlighted that culturally relevant pedagogy is just "good teaching" that is not only centered on African American students but also on other students who are not reached by the school system.

Other researchers also helped give voice to those educators implementing culturally relevant pedagogy. Bowers (2000) explored culturally relevant teaching, in which the participant teacher believed that all students could learn and that it is the teacher's duty to teach his or her students effectively. Thus, the teacher set high standards for the students and related teaching to students' life experiences. For instance, it was observed that students were having difficulty with multiplication and division, so the participant teacher created mathematical games and examples based on the students' cultural knowledge, which helped them better understand the concepts. Each student was required to write down his or her academic and behavioral performance goals. Then, students had to identify and assess their strengths, successes, and failures and come up with performance-improvement plans. The teacher helped the students master concepts and develop self-confidence, academic excellence, and self-esteem; her effective teaching greatly enhanced student performance. With the exception of one, all of the students in the Bowers (2000) study passed the state's standardized test.

In an interview with three African American male teachers, Lynn (2006) discovered how they implemented culturally relevant teaching in their instruction. The teachers connected curriculum to their African American students' home culture. These teachers were part of the same community and "honored" their students. These teachers were also familiar with African American history and culture, which helped them direct
the students to explore complex ideas about freedom, love, and justice. The teachers supported cultural competence by allowing their students to use the local language. These teachers believed that good teaching should be considerate of students' real lives. Student culture was used as a "bridge" in the classroom, and participant teachers communicated with students in the local language, but they emphasized the use of standard English when appropriate (Lynn, 2006).

Gay (2000) listed the following characteristics of culturally responsive teaching:

1. It acknowledges the legitimacy of cultural heritages of different ethnic groups, both as legacies that effect students' dispositions, attitudes, and approaches to learning and as worthy content to be taught in the formal curriculum.
2. It builds bridges of meaningfulness between home and school experiences as well as between academic abstractions and lived socio-cultural realities.
3. It uses a wide variety of strategies that are connected to different learning styles.
4. It teaches students to know and praise their own and each other's cultural heritages.
5. It incorporates multicultural information, resources, and materials in all the subjects and skills routinely taught in schools. (p. 29)

The characteristics mentioned in Gay (2000) promote active learning and student success in mathematics, which is the main focus of standards-based instruction. In collecting and analyzing data and answering the research questions that guided the current study, I used the characteristics and conceptions discussed in this section as evidence of culturally relevant pedagogy, which is one of the objectives of standards-based instruction.

The National Council of Teachers of Mathematics (NCTM) (2000) argues that teachers should consider students' informal knowledge while teaching. For mathematics teachers to reach all students, it is vital to consider students' culture in the teaching process (Ladson-Billings, 1994; Tate, 1995). On the other hand, "robbing students of their culture, language, history, and values, schools often reduce these students to the status of subhumans who need to be rescued from their 'savage' selves" (Bartolome, 1996, p. 233). To understand the relationship between NCTM standards and culturally relevant pedagogy, Gutstein, Lipman, Hernandez, and de los Reyes (1997) conducted a study on culturally relevant mathematics teaching using a model of culturally relevant mathematics instruction. The three essential components of the model were (a) connections between becoming critical mathematical thinkers and viewing knowledge critically in a broad sense, (b) connections between building on students' informal mathematical knowledge and building on students' cultural and experimental knowledge, and (c) orientations to students' culture and experiences.

The first component of the model of culturally relevant mathematics instruction is to build connections between becoming critical mathematical thinkers and viewing knowledge critically in a broad sense. According to Gutsein et al. (1997), critical mathematical thinking refers to several abilities listed in NCTM standards documents (NCTM, 1989, 1991). These abilities imply that students are able to understand and apply reasoning processes, create and judge mathematics arguments, and validate their own thinking and answers (Gutstein et al., 1997). Critical thinking also implies that students are able to make and explore conjecture, question their peers and teachers, and use mathematical evidence to validate knowledge (Gutstein et al., 1997). Nevertheless,

Skovsmose (2005) noted that traditional mathematics classrooms do not support critical thinking and "the mathematics textbooks dominate the class teaching which the teachers follow page-by-page" (p. 9). Skovsmose (2005) further explained that exercises in mathematics textbooks limit student creativity. Gutstein and Peterson (2005) also examined the negative impact of traditional mathematics. exclaiming that "traditional forms of mathematics [teaching] are often too abstract, promote student failure and selfdoubt, and frankly are immoral in a world as unjust as ours. Traditional mathematics is bad for students and bad for society" (p. 5). However, Gutstetin et al.'s (1997) model helps students think critically and actively participate in the learning process, and supporting the student-teacher relationship.

The second component of the model of culturally relevant mathematics instruction refers to connections between building on students' informal mathematical knowledge as well as their cultural and experimental knowledge. Relating teaching to students' informal knowledge includes connecting instruction to students' cultural knowledge. Gutstein et al. (1997) emphasized the importance of linking new concepts to students' prior knowledge. Thus, it is important to use students' informal mathematical knowledge as a starting point when teaching mathematics. Skovsmose (2005), stressing the importance of relating teaching to students' previous knowledge, mentioned that "teaching should be adjusted to the students' knowledge and pre-understanding. We must consider the students from where they are" (p. 183). Skovsmose (2005) also argued that "if we want to understand the actions of learners we have to pay attention to their background..." (p. 182). In addition, NCTM (2000) standards support the idea that teachers should consider students' informal knowledge. Other studies have also attached
importance to making a connection between prior knowledge of mathematics and new knowledge (Davis, 1984; Fennema \& Franke, 1992; Hiebert \& Carpenter, 1992; Schoenfeld, 1987). In particular, Hiebert and Carpenter (1992) stated that understanding mathematics concepts involves both internal and external representations of these concepts. That is, mathematics can only be fully understood when learner's inside knowledge is related to their outside knowledge. Therefore, mathematics knowledge can only be fully understood when "it is linked to existing networks with stronger or more numerous connections" (p. 67). Fennema and Franke (1992) also noted that "mathematics must be translated for [students] so that they can see the relationship between their knowledge and the new knowledge that they are to learn" (p. 153).

The third component of the model of culturally relevant mathematics instruction is orientation to student culture and experiences, with special emphasis on empowerment. This empowerment mandates "establishing solidarity with students and their families," "seeing culture dialectically," and "going beyond traditional boundaries and providing academic challenges" (Gutstein et al., 1997). Gutstein and Peterson (2005) pointed out that teachers should view "students' home cultures and languages as strengths upon which to build, rather than as deficits for which to compensate" (p. 3).

## Symbolic Interaction Theory

Symbolic interaction theory was also chosen as a theoretical lens for the current study because according to Voigt (1996), this theory allows analysis of social interaction in classroom instruction. Voigt (1996) further explained that symbolic interaction theory emphasizes individual meaning making through social interaction with others, meaning that individuals develop personal meanings when they participate in discussions in the
mathematics classroom. Jeon (2004) stressed that for a researcher to understand a social process, the experiences of the participants must also be understood within the research context. The researcher has to understand the participants' actions and interactions, as each participant develops through social interaction. The impact of social interaction on self is relevant in studying the ways students interact with teachers. The current study of standards-based instruction stresses the use of instructional practices such as social interaction among students and teachers. Thus, the symbolic interaction perspective provided relevant guidance in investigating the participant teacher's actions and student interactions. Symbolic interaction also allows a researcher to collect and analyze data concerning teaching practices that promote collaborative learning in undergraduate mathematics classrooms (Yackel, 2001). Similarly, symbolic interaction allowed me to collect and analyze study data and answer each of the research questions.

According to Sandstrom, Martin, and Fine (2001), philosopher George Herbert Mead is the foundational figure of symbolic interaction theory, drawing his ideas of symbolic interaction theory from pragmatist founders and also from John Dewey, who was his colleague at the University of Chicago (Sandstrom, Martin, \& Fine, 2001). Crotty (2003) concurred that "it is from the thought of Mead that symbolic interactionism was born. Symbolic interactionism is pragmatism in sociological attire. In Mead's thought every person is a social constructionist. We come to be persons in and out of interaction with our society" (Crotty, 2003, p. 62). Crotty (2003) further stated that symbolic interaction involves the use of language and other communicative tools, as the use of dialogue creates opportunities for individuals to "become aware of the perceptions, feelings and attitudes of others, and interpret their meanings and intent" (pp. 75-76).

Many scholars have attempted to define symbolic interaction. For instance, Stryker (2000) defined symbolic interaction as a theoretical framework with the central idea that society is viewed "as a web of communication or symbolic interaction, conducted through meanings developed in persons' interdependent activity" (p. 527). Engaging in interaction makes humans active and creative. According to Schwandt (1994) (as cited in Jeon, 2004), symbolic interactions are the "theory and approach for the study of individuals' social and psychological action/interaction in search of portraying and understanding the process of meaning making" (p.250). The aim of symbolic interactionism is to comprehend "the complex world of lived experience from the point of view of those who live it" (as cited in Jeon, 2004, p. 250). It is through interaction that individuals construct meaning. People in a given situation (e.g., students in a particular class) often develop common definitions (or "shared perspectives" in the symbolic interactionist language) since they regularly interact and share experiences (Bogdan \& Biklen, 2007).

Herbert Blumer, one of Mead's students, was another scholar that significantly contributed to symbolic interaction theory. It is Blumer who compiled Mead's writings and developed the book Symbolic Interactionism (Blumer, 1969b). According to Blumer (as cited in Sandstrom, Martin, \& Fine, 2001):

The first premise is that human beings act toward things on the basis of the meanings those things have for them ... The second premise is that the meaning of such things is derived from, or arises out of, the social interaction that one has with one's fellows. The third premise is that these meanings are handled in, and
modified through, an interpretive process used by the person dealing with the things he [or she] encounters. (p. 218)

Sandstrom, Martin, and Fine (2001) also explored other assumptions of symbolic interactionism. One of these assumptions is that people are unique creatures because of their ability to use symbols. Symbolic interactionists believe that people's symbolic capabilities include use of language, which helps them to give meanings to things. Human beings learn the meanings of things through interaction among themselves, and the use of language and communicative processes makes this interaction possible. Secondly, symbolic interactionists assume that people become distinctively human through their interactions. That is, they believe that "people develop into distinctively human beings as they take part in social interaction" (Sandstrom, Martin, \& Fine, 2001, p. 218). These distinctively human qualities include but are not limited to the ability to think, make plans, use symbols, and participate in communication with others. Thirdly, interactionists emphasize that people are conscious and self-reflexive beings who actively shape their own behaviors. This means that humans develop their "minds" and "selves" through social interaction, which affects their actions. Human actions are influenced by social factors such as language, race, class, gender, people's interpretations, and behaviors. It is important for humans to engage in social interactions so that meaning can be shared. Fourthly, interactionists stress that people are purposive creatures who act in and toward targeted situations. The authors (Sandstrom, Martin, \& Fine, 2001) pointed out that humans do not pursue goals individually but rather through interacting with others and creating lines of action. Fifthly, symbolic interactionists assume that human society consists of people engaging in symbolic interactions. That is "interactionists stress that [humans] actively shape our identities and behaviors as we make plans, seek goals and interact with others in specific situations" (Sandstrom, Martin, \& Fine, 2001, p.
219). The sixth assumption is that to understand people's social acts, we need to use methods that enable us to discern the meanings they attribute to these acts. Sandstrom, Martin, and Fine (2001) further explained that to understand these acts, "the researchers must empathize with... the individuals or groups they are studying. They also must observe and interact with these individuals or groups they are studying. They also must observe and interact with these individuals or groups in an unobtrusive way" (p. 219).

Moreover, Yackel (2001) stated that one of the principles of symbolic interactionism is "centrality given to the process of interpretation in interaction." Symbolic interactionists maintain that social interaction allows individuals to interpret each other's actions. An individual's actions are affected by actions of the others. Yackel (2001) (citing Blumer, 1969) stated that "one has to fit one's own line of activity in some manner to the actions of others. The actions of others have to be taken into account and cannot be merely an arena for the expression of what one is disposed to do or sets out to do." Yackel (2001) further reported that symbolic interaction involves joint action and cited Blumer's (1969) idea that "the joint action of the collective is an interlinkage of the separate acts of the participants" (p. 5). Quoting Blumer, Yackel (2001) defined joint actions:

A joint action, while made up of diverse component acts that enter into its formation, is different from any one of them and from their mere aggression. The joint action has a distinctive character in its own right, a character that lies in the articulation or linkage as apart from what may be spoken of and handled without having to break it down into the separate acts that comprise it. (p. 5)

In addition, Yackel (2001) explained that the idea of "joint interaction supports the position that social rules, norms, and values are upheld by a process of social interaction and not the other way around" (p. 5).

Yackel (2001) mentioned that another principle of symbolic interaction is that meaning is regarded as a social product. Quoting Blumer's view of symbolic interaction as a social product, Yackel (2001) states:

It does not regard meaning as emanating from the intrinsic makeup of the thing that has meaning, nor does it see meaning as arising through a coalescence of psychological elements in the person. Instead, it sees meaning as arising in the process of interaction between people. The meaning of a thing for a person grows out of the way in which other persons act toward the person with regard to the thing. Their actions operate to define the thing for the person. Thus, symbolic interactionism sees meaning as social products, as creations that are formed in and through the defining activities of people as they interact. (p. 5)

Yackel (2001) also discussed the implications of symbolic interaction in teaching practices. He argued that meanings and understandings grow from instruction that involves dialogue and social interaction. As Freire (1970/2000) stated:

Through dialogue, the teacher-of-the-students and the students-of-the-teacher cease to exist and a new term emerges: teacher-student with students-teachers. The teacher is no longer merely the-one-who-teaches, but the one who is himself taught in dialogue with the students, who in turn while being taught also teach. (p. 80)

Therefore, it is important for teachers to increase student understanding through interactions among themselves and with teachers. The characteristics of symbolic interaction theory helped me collect data and answer the research questions guiding the current study by enabling me to focus on the ways the participant teacher related his own teaching style to characteristics of symbolic interaction theory.

Yackel (2001) investigated reform practices including explanation, justification, and argumentation in mathematics classrooms. Yackel's (2001) qualitative study embraced the theory of symbolic interaction as a theoretical framework, helping the researcher make sense of the social occurring in the classroom including social and sociomathematical norms. Data were collected through field notes, student work, student
interviews, and videotapes of each class session. Yackel (2001) indicated that the participant, both an educator and a researcher, used instructional strategies such as discussions that involved the entire class and small group discussions. These strategies allowed students to effectively interact with each other and the teacher. The students asked questions and explained and justified their solutions. Yackel (2001) reported that this type of "instructional approach seems to have considerable potential for in-depth conceptual development that grows out of students' discursive activity" (p. 12). The instructor in the study often asked students to provide reasons for their claims, which he did not evaluate on his own; rather, he asked other students what they thought about each other's claims. This approach led to further mathematics-based argument among students. Yackel (2001) mentioned that the nature of small group discussions was not to solve problems, but to allow students to collectively engage in critical thinking, which was referred to as "group thinking." This interactive mode of instruction had a positive impact on student learning. In Yackel (2001), one of the students commented:

A specific problem I liked was the predator-prey problem. Everyone had a different idea about it, which made everyone have to think. The group thinking helps me sort ideas out. Also, group thinking helps me put in words what I am trying to say. Group thinking in a math class is new to me, but I like it so far. (p. 15)

Yackel (2001) argued that incorporation of group thinking in teaching is a powerful instructional strategy because it promotes student learning and because most mathematicians and mathematics educators consider this the essence of mathematics. Therefore, it is vital for teachers to consider teaching practices that promote student interaction, which leads to group thinking.

Cobb, Yackel, and Wood (1992) also reported in their study (on interaction and learning in mathematics classrooms) that their interest in social interaction in the classroom was influenced by symbolic interactionists (e.g., Bauersfeld, 1980; Blumer, 1969; Mead, 1934; Voigt, 1985, 1989). Cobb, Yackel, and Wood (1992) confirmed that instructional practice (social interaction) helped students effectively learn mathematical concepts. This approach allowed students to validate and justify their solutions. The authors (Cobb, Yackel, \& Wood, 1992) stressed that students' mathematical activity was viewed as "shared" and that each student experienced intersubjectivity with one another, which implies that interactive methods promote the learning of mathematics.

## Experiential Instruction Theory

Experiential instruction theory was another theoretical framework that informed the current study. Advocates of standards-based instruction, including the NCTM and the AMATYC, stress the need for students to experience mathematics. Specifically, the AMATYC (1995) stated that in order for students to experience mathematics, instructors should:
...provide learning activities, including projects and apprenticeships, that promote independent thinking and require sustained effort and time so that students will have the confidence to access and use needed mathematics and other technical information independently, to form conjectures from an array of specific examples, and to draw conclusions from general principles. (p. 17)

Therefore, the definitions and characteristics of experiential instruction theory (discussed below) gave more insight into data collection in the current study. As I collected data, I focused my attention the ways the participant teacher incorporated elements of experiential teaching into his instruction.

According to Southcott (2004), Rousseau and Dewey are founders of experiential teaching theory. Referring to Dewey (1916) (as cited in Southcott, 2004), "an ounce of experience is better than a ton of theory simply because it is only in experience that theory has vital and verifiable significance" (p. 2). Dewey further stated (as cited in Bialeschki, 2007) that "all genuine education comes through experience" (p. 366). Dewey, however, emphasized that not all experiences are educative. Therefore, it is important for educators to consider the quality of the experience. Southcott (2004) (citing Drengson, 1995), argued that experiential education instruction is "the process of practical engagement with concepts and skills applied in the practical setting through physical and practical mental activity" (p. 3). This process is supported by reflection, critical analysis, and synthesis. Also, experiential education involves "the challenging action, proceeded by focusing, and followed by debriefing, which is surrounded by feedback and support" (Joplin, 1995, as cited in Southcott, 2004, p. 9). Southcott (2004) further emphasized that the major tenet of experiential education is that students are active participants in the learning process. Experiential instruction should be studentcentered instead of teacher-centered.

Further, experiential education instruction is "... a process through which a learner constructs knowledge, skill, and value from direct experience" (Association for Experiential Education, 1991, p. 1, as cited in Ives \& Obenchain, 2006). Peplau (2009) defined experiential teaching as the type of instruction that engages students in learning by experience. Peplau (2009) argued that experiential teaching helps students organize the meaning of experiences, which includes students' experiences, the things students have learned, and the interacting roles of the teacher and students in classroom
discussions. The experiential mode of instruction is "the instructional treatment in which students are provided concrete experiences" (Kolb, 1976, as cited in Fowler, 1991, p. 8). According to this theory, learning only occurs when instruction is related to student experiences. It is experiential teaching that leads to experiential learning, which includes four stages (i.e., concrete experience, observations and reflection, formation of abstract concepts and generalizations, and testing implications of concepts in new situations) (Kolb \& Lewis, 1986). Students should be active learners in experiential instruction. Newsome, Wardlow, and Johnson (2005) maintained that experiential teaching should be a hands-on, problem-based teaching method.

Newsome, Wardlow, and Johnson (2005) listed characteristics of experiential teaching, explaining that experiential instructions create environments for active learning and allow students to solve problems on their own. Experiential teaching methods increase students' information retention and cognitive achievement and develops students' critical thinking. The critical-thinking abilities help students retain, obtain, and retrieve knowledge. Gentry and McGinnis (2007) also reported that experiential teaching leads to student learning. Gentry and McGinnis (2007) mentioned the characteristics of experiential teaching: teacher's clarity, task-oriented behavior, ability to ask higher order questions, use student ideas, probe student comments, and enthusiasm. The authors (Gentry \& McGinnis, 2007) commented that "enthusiasm...is one of the leading factors in achieving experiential teaching effectiveness, due to the fact that it not only inspires students but inspires great teaching through preparation" (p. 2). Ives and Obenchain (2006) also mentioned three characteristics of experiential teaching, including opportunities for student-direction, connections to the real world, and critical reflection.

Estes (2004) noted that experiential instruction supports student-centered learning, which gives the students the opportunity to "actively engage in posing questions, investigating, experimenting, being curious, solving problems, assuming responsibility, being creative, and constructing meaning" (Association for Experiential Education, n.d., as cited in Estes, 2004, p. 142 ). The core of experiential teaching is to bring students' experience to the learning process. Dewey emphasized that "all genuine education comes about through experience" (Dewey, 1938/1988, as cited in Estes, 2004, p. 145). Dewey further added that the experience has to be educative, engaging to the students, and connecting to the students' further experiences (as cited in Estes, 2004). Estes (2004) further stated that for students to understand and use experience, they have to develop critical-thinking skills. Teachers play vital roles in implementing experiential instruction properly, which is further explained below.

A teacher in experiential instruction should be a facilitator and "listens intelligently to students' descriptions of experience and uses her own capacities and skills to encourage learning" (Peplau, 2009, pp. 885-886). Also, the teacher should ask for explanations, ask provocative questions, and call attention to extended exploration of important information. Estes (2004) noted that reflection is also an important aspect of experiential instruction. Therefore, teachers have to use facilitation strategies that promote reflectivity. Crosby (1995) (as cited in Estes, 2004) stressed that "after resolution comes reflection [by the student], on the movement [experience] so that what is learned may be generalized and used again" (p. 146). Estes (2004) also emphasized that the major tenet of experiential teaching is a student-centered method that involves changing teacher-centered teaching and places students at the center of the learning
process. Estes (2004) explained that students learn best through student-centered experiential instruction. According to Estes (2004), teachers can achieve student-centered experiential instruction "by embracing values similar to Paulo Freire's approach to education where teachers and students transform learning into a collaborative process" (p. 152). Freire (1970/2000) supported the use of dialogue in student learning rather than the banking method (traditional method). Dialogue creates room for both teachers and students to share ideas and listen to one another. As Freire (1970/2000) pointed out, "if [teachers] don't learn how to listen to these voices, in truth we don't really learn how to speak. Only those who listen, speak. Those who do not listen, end up merely yelling, barking out the language while imposing their ideas" (p. 306). Estes (2004) remarked that "Freire's approach fits well within experiential education...because it requires students to have prior experiential contact with the object of learning before dialogue" (pp.

153-154). Further, to achieve student-centered experiential instruction, teachers should use creative techniques to facilitate student reflection.

Ives and Obenchain (2006) also contributed to the literature on experiential instruction theory and its applications, using a mixed methods approach to compare experiential instruction and traditional teaching. Data were collected through interviews, classroom observations, and pre- and post-tests and were based on characteristics of experiential teaching, which included student directedness, real-world connections, and critical reflection. These characteristics are defined by Ives and Obenchain (2006) as follows:

1. Student directedness was student involvement in decision-making on course content, experiences, assessment, and classroom procedures.
2. Real-world connections were student actions on, or recognitions that they could act on, connections between content and applications outside the classroom.
3. Critical reflection was evidence of student thinking at the evaluation level of Bloom's Taxonomy applied to course content. (p. 68)

There were two groups of participants in the Ives and Obenchain (2006) study (i.e., experiential classes, traditional classes). The experiential class outperformed the traditional class in higher order thinking. The authors reported that higher order thinking included but was not limited to abstract thinking, integrating information into systems, following rules of logic and judgment, solving problems, and thinking critically. It also included subskills such as comparison, categorization, inference, prioritizing, analysis, question posing, argumentation, system thinking, discovery, organizing, analyzing, synthesizing, and evaluating. Ives and Obenchain (2006) reported that the experiential instruction class showed significant increase in student retention of concepts through real-life experiences.

McGlinn (1999), also contributing to the on experiential instruction literature, worked with colleagues to redesign their instruction and shift from a traditional method to experiential teaching. This change was based on Dewey's (1938) emphasis on learning through experience, Freire's (1970) criticism of the banking education, and Kolb's (1984) experiential learning cycle. McGlinn (1999) stressed that students are not empty vessels where teachers deposit their knowledge; rather, they are learners with prior experiences and knowledge. It is crucial for teachers to "draw on students' background information, build knowledge which is needed to develop new concepts by experiences, guide students in experiences in which they can create new knowledge, and provide a structure for
reflection" (McGlinn, 1999, p. 2). McGlinn (1999) also cited Dewey's (1938) views about experiential teaching, stating that experience "arouses curiosity, strengthens initiative, and sets up desires and purposes" which has positive effect on learning (p.3). Also, Dewey (1938/1988) reported... "every experience enacted and undergone modifies the one who acts undergoes, while this modification affects, whether we wish it or not, the quality of subsequent experiences" (as cited in McGlinn, 1999, p. 3). McGlinn and colleagues drew from Shor's (1996) ideas of experiential instruction when modifying their teaching. Shor (1996), following Freire's footsteps, believed that dialogue and questioning techniques help teachers listen to their students and allow students' voices to be heard. This method, which Shor (1996) calls "backloaded teacher commentary," gives students the opportunity to use their experiences in the learning process. Building on the ideas of these scholars, McGlinn (1999) developed experiential teaching strategies. These strategies were (a) getting students to draw on their background knowledge; (b) creating opportunities for students to engage in actual experiences, and (c) expanding student interactions with each other so that the interactions within the classroom themselves would become an experience. Moreover, they considered Shor's (1996) method of the backloading teaching technique, which involves asking students open-ended and thoughtprovoking questions based on their experiences. McGlinn noted (1999) that:

In an experiential model, the teacher is no longer considered the only expert in the classroom. Student experiences are significant. When this is acknowledged both teachers and students have to change their attitude towards the class. What happens in the class depends on the input and participation of both students and teachers. Students can begin to require as much from each other as they want from the teacher. This could lead to exciting possibilities where everyone in the class is engaged in learning. (pp. 6-7)

Fowler (1991) also supported the effectiveness of experiential instruction. Fowler (1991) indicated that the experiential mode of instruction helped students in the acquisition and retention of new knowledge. The students in the Fowler (1991) study receiving experiential instruction performed better than the control group. Fowler (1991) strongly recommended implementation of experiential activities in teaching.

As previously mentioned, experiential teaching strategies are (a) getting students to draw on their background knowledge, (b) creating opportunities for students to engage in actual experiences, and (c) expanding students' interactions with each other so that interaction in the classroom becomes an experience (McGlinn, 1999). Some researchers (Gutstein, 2006; McCoy, 2008; Moses \& Cobb, 2001) have explained ways to implement experiential instruction in mathematics classrooms. For instance, McCoy (2008) identified poverty as one of the cultural experiences of her students and purposefully assigned her students a project about poverty. First, McCoy asked her students to discuss the definition of poverty. Students were asked to view a documentary, "Tour Poverty USA," (produced by the U.S. Conference of Catholic Bishops) and then reflect on and discuss questions raised by the film. Next, students were encouraged to discuss who the poor are. In small groups, students examined poverty according to gender, age, race, and level of education. Students then used the information to construct bar graphs, and they discussed their findings. Next, students were required to discuss the ways in which poverty was related to academic achievement. In this activity, the students collected data from the U.S. Census Bureau's website, analyzed poverty within state school districts, graphed the data in scatter plot, calculated correlation coefficients and regression equations, and discussed their results. Students found that there was a negative
relationship between school achievement and poverty. The students also noticed that their school has been identified as having low test scores. In creating this kind of learning environment, McCoy believed, "once a fabric of relevance has been constructed, content learning naturally follows" (Kincheloe \& Steinburg, 1996, p. 189), as previously stated. With such a project, students were able to" read and write the world with mathematics" (Gutstein, 2006, p. 4). This experience allowed students to engage in practical aspects of mathematics, reflect, and think critically about mathematics concepts, hereby leading to a deeper understanding of the subject, supporting Dewey's (1916) philosophy that all true learning comes through experience.

The current study embraces three major theories, including culturally relevant pedagogy theory, symbolic interaction theory, and experiential instruction theory, all of which relate to experiences and actions within the classroom. These theories, all focused on teachers' actions, play a role in the reduction of student attrition in undergraduate mathematics classes through the improvement of students' understanding and achievement.

Although each of theory has a primary goal, there are similarities among these theories. The main focus of culturally relevant pedagogy is connecting teaching to the cultural knowledge of students. Symbolic interaction theory also considers cultural aspects of the student body. In addition, symbolic interactionists emphasize that meaningmaking evolves from social interactions. Humans, as parts of social interaction, assign meaning to objects based on their cultural and prior experiences, meaning that the symbolic interaction theory focuses on how people make meanings and interact with culture. Bogdan and Biklen (2007) mentioned that "the meaning people give to their
experience and their process of interpretation are essential and constitutive, not accidental or secondary to what the experience is" (p.27). It was pointed out that "the concept of culture as acquired knowledge has much in common with symbolic interaction" (Spradley, 1980, as cited in Bogdan \& Biklen, 2007, p. 32). Therefore, the primary goal of symbolic interactionism is to "understand the complex world of lived experience from the point of view of those who live it" (Bogdan \& Biklen, 2007, p. 250). Like the other two theories, experiential teaching theory, whose major tenet is to allow students to experience learning, also discusses the importance of students' cultural backgrounds. This theory dictates that for experiences to be useful to students, it has to relate to their life experiences. The implication of relating teaching to students' life and cultural experience is that it helps students become active learners.

The theories guiding the current study aim to place students at the center of the teaching and learning process to achieve academic success of the students, which is the central focus of the current study of standards-based instruction. These theories are not mutually exclusive; rather, they complement each other to support the study. However, the standards for instruction are not theories. It is vital to mention again that these theories, which are components of the standards for instruction proposed by NCTM, MAA, and AMATYC, supported the standards to serve as lens through which data was collected and analyzed and research questions were answered.

## Aligning the Theoretical Framework with My Ideological Paradigm

As mentioned earlier, college algebra is the gateway to upper-level mathematics courses. Unfortunately, it is also a barrier for many students who need the course to
continue in higher education. Teaching many years of college algebra has allowed me to witness the problem of college algebra serving as a barrier for many students, thus leading to their high attrition in the course. While teaching college algebra, although I (a) implemented strategies learned via rigorous teacher training, professional development sessions, and graduate courses and (b) also had some success in teaching my students, I often wondered what else could be done to make a more positive impact on my students. I realized the role of college algebra in students' academic plans; this is what sparked my curiosity for change and served as the impetus for this study.

My teaching experience and position in mathematics education affects my subjectivity towards the teaching and learning of mathematics. I am an international mathematics teacher, and I have many years of teaching experience in mathematics at both the secondary and university levels. My experience has enabled me to "think and rethink" mathematics education. Building on my personal experiences and professional development, I feel that the teaching and learning of mathematics should be studentcentered, which can be achieved through "cooperative learning, language experience, process writing, reciprocal teaching, and whole language activities" (Bartolome, 1996, p. 240). Students understand mathematical concepts when they are actively engaged in the learning process. Bartolome (1996) stated that education is:
a process in which teacher and students mutually participate in the intellectually existing undertaking we call learning. Students can become active subjects in their
own learning, instead of passive objects waiting to be filled with facts and figures by the teacher. (p. 240).

My beliefs and philosophy are based on Bartolome's (1996) argument that students should be active instead of passive learners. To improve student understanding in
mathematics, teachers should encourage students to use their previous knowledge and find their own solutions. Students are not empty vessels to which teachers transfer their knowledge; students have different backgrounds, and new information should build upon existing knowledge to make learning meaningful. Bartolome (1996) mentioned that teaching and learning is "the act of linking new information to prior knowledge" (p. 240). Students should be challenged to be active thinkers and to interact with one another. Students also learn better when learning is culturally and socially related to them. The traditional method of teaching where students just listen while the teacher talks does not promote effective learning. Freire (1970/2000) referred to the traditional method as "banking education," arguing that this method makes students memorize concepts without understanding them. Freire (1970/2000) stated that "education is suffering from narration sickness" (p. 71). He further explained that this problem of narration sickness in education is "an act of depositing, which the students are the depositories and the teacher is the depositor as stated earlier. Instead of communicating, the teacher issues communiqués and makes deposits which the students patiently receive, memorize, and repeat" (Freire, 1970/2000, p. 72). Gutstein and Peterson (2005) echoed the same idea, stating that "a text-driven, teacher-centered approach does not foster the kind of questioning and reflection that should take place in all classrooms, including those where math is studied" (p. 4).

Given that students should be active learners, I think it is important to connect teaching to culture of the students. I have been both an international student and teacher, which has aided my understanding of the need to relate teaching to culture. If teaching and learning incorporates students' cultural values, students will be more interested and
motivated, which will lead to better understanding of the material. Kincheloe and Steinburg (1996) stated that "once a fabric of relevance has been constructed, content learning naturally follows" (p. 189). Gutstein (2006) also emphasized that it is vital for teachers to "maintain [students' cultural] integrity while succeeding academically" (p. 148). Moreover, I believe that academic success in mathematics can be achieved in multiple ways, including setting high expectations, using technology properly, and helping students to build networks of knowledge.

In addition, teachers should encourage social interaction in mathematics classrooms. Interaction in classrooms can be achieved through student-to-student interaction, teacher-to-student interaction, rounded feedback from students and teachers, questioning techniques, small group discussions, discussions among the entire class, and students' explanations and justification of solutions. Freire (1970/2000) described that "only through communication can human life hold meaning. The teacher cannot think for her students, none can she impose her thought on them" (p. 77). Freire (1970/2000) also emphasized the need for dialogue in teaching, which allows the teacher to listen to the students and the teacher and students work together to share their ideas. Skovsmose (2005) stressed that "dialogic processes of learning and teaching can be a resource for reflections and for critical learning" (p. 179). Therefore, I believe it is crucial for teachers to allow their students to experience mathematics, which can be achieved through assigning projects and problem-posing education. Many scholars in education have endorsed the need for experiential education (e.g., Dewey, 1916; Gutstein, 2006). Teachers should be careful that the experience has to be meaningful to the students and of good quality.

There are similarities between my ideological paradigm (i.e., standards-based instruction) and the theoretical frameworks used in the current study (i.e., culturally relevant pedagogy theory, symbolic interaction theory, experiential teaching theory). The intersection of my beliefs and the theoretical frameworks is that teachers should consider student culture and experiences, encourage interaction within classrooms in different ways, support academic success, and promote active learning. These techniques allow opportunities for students to think critically and master mathematics concepts with understanding, which reduces student attrition in undergraduate mathematics when integrated in courses such as college algebra.

## Summary

Knowledge of mathematics creates future opportunities for college students. The use of standards-based instruction techniques in teaching of college algebra concepts increases student success and reduces student attrition in college algebra, thereby increasing their enrollment and success in upper-level undergraduate mathematics courses (Burmeisters, Kenney, \& Nice, 1996). However, there is a need for further study of standards-based instruction in undergraduate mathematics. The primary goal of the current study is to identify the teaching strategies of a college algebra teacher who applies a standards-based approach in his classroom. This study examines the ways these teaching practices are aligned with the characteristics of culturally relevant pedagogy, symbolic interaction, and experiential teaching theories, all of which guided the study. Using these theories, which emphasize the need for student-centered teaching, an interpretative case study was developed to provide insight into this standards-based approach.

## CHAPTER 2

## REVIEW OF THE LITERATURE

This chapter presents a review of the literature on standards-based instruction in undergraduate mathematics education, and this review was guided by the following research questions: (1) What teaching practices are used in the mathematics classroom of a college algebra teacher? (2) How are the teaching practices of the college algebra aligned with the characteristics of standards-based instruction? Since the primary goal of the current study was to reduce student attrition in undergraduate mathematics, I begin with a background of student attrition in undergraduate mathematics. Next, I provide a historical background and views of advocates of standards-based instruction, including the National Council of Teachers of Mathematics (NCTM), the American Mathematical Association of Two-Year Colleges (AMATYC), and the Mathematical Association of America (MAA). It is hoped that through describing how standards-based instruction is applied in different educational settings, a better understanding of how standards-based pedagogical practices impact student learning can be achieved.

## An International Look at Student Attrition in Undergraduate Mathematics

Student attrition in undergraduate mathematics and upper-level mathematics courses is an international concern (Fargasz \& Leder, 2000). Similarly, Engelbrecht and Harding (2009) pointed out that there is concern about the decline in student enrollment in college mathematics courses. Engelbrecht and Harding (2009), investigating the international trends of student enrollment in mathematics, reported that the International Commission on Mathematics Instruction, the American Mathematical Society, the Mathematical Sciences Joint Data Committee, and the National Science Foundation also
concur with the decline in the number of students taking college mathematics. Garfunkel and Young (1998) (as cited in Engelbrecht and Harding, 2009) stress the degree of this issue of student attrition in college mathematics as follows:

The reason for writing this piece is our belief that our profession is in desperate trouble-immediate and present danger. The absolute numbers and the trends are clear. If something is not done soon, we will see mathematics department faculties decimated and an already dismal job market completely collapse. Simply put, we are losing our students. (p. 256)

Engelbrecht and Harding (2009) mentioned that Australian universities have the same problem of low enrollment in college mathematics, specifically stating that: "nationally, the percentage of...students taking advanced and intermediate mathematics fell from $41 \%$ in 1995 to $34 \%$ in 2004" (p. 76). The authors (Engelbrecht \& Harding, 2009) referenced the National Committee for the Mathematical Sciences of the Australian Academy of Science by stating that: "Australia will be unable to produce their next generation of students with an understanding of fundamental mathematical concepts, problem-solving abilities and training in modern developments to meet projected needs and remain globally competitive" (p. 5). Engelbrecht and Harding (2003, as cited in Engelbrecth \& Harding, 2009) discussed South Africa's documentation of the decrease in student enrollment in college mathematics. Results of Engelbrecht and Harding (2009) indicated that the total number of students in college mathematics had decreased by $32 \%$ "...over the 10-year period 1990-2000 and raised alarm" (p. 78).

## A Local Look at Student Attrition in Undergraduate Mathematics

Discussing the problem of student attrition in the United States, Hutchison (2006) stated that few Americans are successful in mathematics and too few are earning advanced degrees in mathematics. Other researchers have also emphasized that students
are leaving the field of mathematics at the undergraduate level (Pierce, Turville, \& Giri, 2003; Rasmussen \& Kwon, 2007). Astin and Astin (1993, as cited in Seymour, 2001) reported that from the freshman to senior years, student enrollment declined about $40 \%$ in undergraduate mathematics and other mathematics-related subjects. Specifically, college algebra was identified as a high-attrition course that affects student success (Waller, 2006). Owens (2003) stressed that in:
...fall 2002, the overall attrition rate at [Austin Community College] for College Algebra was $41 \%$ ranging from $13 \%$ to $81 \%$ per section. [Zero] attrition cannot be reached, but I think we can do much better than we are doing now. (p. 1)

The aforementioned studies indicate that there is indeed a problem with student attrition in undergraduate mathematics.

## Reasons for Student Attrition in Undergraduate Mathematics

Seymour (2001) reported a reason for student attrition in undergraduate mathematics as lack of proper teaching practices on part of college mathematics faculty members. According to Bergsten (2007), undergraduate mathematics education has been dominated by the lecture method. Fargasz and Leder (2000) indicated that a predominant concern was the quality of teaching methods used by mathematics faculty at the undergraduate level. One of the interview questions in the Forgasz and Leder (2000) study concerned "what [the students] would like to change about the mathematics department if they could" (p. 40). A comment received was:

Ensure the lecturer is a good teacher, not just a good mathematician. Promote more group work between students. Lectures may be more interactive...with more examples and more levels of communication to ensure that everyone understands the material. Try to focus on students more, be more encouraging to those having problems, try somehow to make the lecture content more interesting and accessible. (Forgasz \& Leder, 2000, p. 40)

This comment suggested that students were not satisfied with the teaching practices of the mathematics faculty.

Using an ethnographic study method, Seymour (2001) examined changes in undergraduate mathematics and related disciplines in the United States. This study was based on a review of the last ten years of Seymour's work. In Seymour (2001), data was collected via interviews, and data were collected from student participants at seven universities. Participants included students at these 7 universities who scored 650 or above on their SAT (or equivalent). Seymour (2001) found that the quality of instruction was one of the issues affecting mathematics departments and other related disciplines. According to Seymour (2001): "...paramount among these [issues] were reports of poor teaching ...which was mentioned by $90 \%$ " (p. 82) of students leaving the field of mathematics and $74 \%$ of students remaining in mathematics.

Daempfle (2003-2004) reviewed student attrition at the undergraduate level across several disciplines, including mathematics. Seymour and Hewitt (1997, as cited in Daempfle, 2003-2004) reported that "the largest declines were seen in mathematics from 4.6 percent to 0.6 percent" (p.37). Daempfle identified teaching practices of mathematics teachers as one of the causes. Findings of the Seymour and Hewitt (1994) study showed that "pedagogical effectiveness...appeared in all issues raised [and] that students strongly believed that faculty did not like to teach, did not value teaching as a professional activity" (as cited in Daempfle, 2003-2004, p. 39). In addition, students' comments indicated that lecture was the teaching method used, which does not include discussion; rather, teachers read verbatim from textbooks. Teachers' encouragement of rote learning was also a factor contributing to student dissatisfaction with mathematics instruction.

Strenta et al. (1994) study also pointed out poor instruction by mathematics faculty (as cited in Daempfle, 2003-2004). Therefore, to reduce rates of student attrition, it is important to change the climate of college classrooms by introducing standards-based instructional methods, such as cooperative learning (Gavien, 1995, as cited in Daempfle, 2003-2004). Fenwick-Sehl, Fioroni, and Lovric (2009) discussed retention of students in undergraduate mathematics classes, citing Seymour and Hewitt (1997) who identified a common reason that students leave the field of mathematics as "poor teaching" (p. 32). Linn and Kessel (1996) investigated the progress of undergraduate mathematics students using a qualitative research method. Linn and Kessel (1996) found that dissatisfaction with the instruction received often made students leave the field of mathematics. Linn and Kessel also referenced student participants of Seymour and Hewitt (1991), who indicated that undergraduate mathematics teaching is designed to "weed-out" students instead of motivating them to succeed and finish as well as the idea that some mathematics professors "read directly from the textbook" (p. 116). Linn and Kessel (1996) also illustrated the poor teaching of undergraduate mathematics, concurring with the notion that most university professors use a lecture-based method: (a) They stand in front of the class without engaging their student; (b) Students were bored, and some arrived late and while others left early; (c) Some professors do not speak clearly; (d) The questions professors asked, which they quickly answered themselves, did not encourage student participation; and (e) Professors do not use eye contact to gauge the level of student understanding. Astin and Astin explained, "Although all of the professors we observed had Ph.D.s and were considered experts in their fields, few of them seemed able to present their knowledge in an interesting or provocative way" (as
cited in Linn \& Kessel, 1996, p. 117). Linn and Kessel (2006) also referenced a study in which a student mentioned taking an undergraduate mathematics class but "...was frustrated by the professor-disappointed by his teaching method. He mumbled. [Mathematics departments] hire a professor for a different purpose than teaching. They are here to do their masters or writing a book" (p. 117). This means that instruction of introductory courses in mathematics is mainly lecture-based, which does not capture the interest of the students.

## Advocates of Standards-Based Instruction

The National Council of Teachers of Mathematics (NCTM), the American Mathematical Association of Two-Year Colleges (AMATYC), and the Mathematical Association of America (MAA) all stress the importance and implementation of standards-based instruction in mathematics classrooms. These organizations share an agenda, which focuses on student-centered teaching. The historical backgrounds and viewpoints of these organizations advocating standards-based instruction will be presented in this section.

## The National Council of Teachers of Mathematics

The National Council of Teachers of Mathematics (NCTM) is an international organization whose aim is excellence in mathematics education for all students. To promote their ideas, the NCTM published Curriculum and Evaluation Standards for School Mathematics (1989), Professional Standards for Teaching Mathematics (1991), and Assessment Standards for School Mathematics (1995). Again, these standards were developed to improve mathematics education in classes ranging from pre-kindergarten to 12th grade. These standards were upgraded and revised in Principles and Standards for

School Mathematics (2000), which calls for learning of mathematics for all students who understands the concepts. The six standards presented in this document are equity, curriculum, teaching, learning, assessment, and technology.

The NCTM (2000) believes that improvement in mathematics depends on effective teaching. The organization makes it clear that "there is no one right way to teach" (NCTM, 2000, p. 18). Effective teaching requires teachers to know the content as well as multiple teaching and assessment strategies. It also involves reflection and professional development on part of the teachers. Effective teaching should lead to better understanding on part of students, which often means that they are able to connect new knowledge to prior knowledge. Teachers should be able to plan, organize, manage classrooms effectively, and engage students through questioning.

Further, the NCTM (2000) explains that effective teaching requires teachers to create challenging and conducive learning environments. A learning environment is more than the physical structure of the classroom-it involves creating an environment where students can discuss, collaborate, justify, and experiment with different methods. Careful planning and questioning from teachers enables a conducive learning environment. It is also important for teachers to expose students to challenging tasks that might involve real-world experiences. Also, to teach for understanding, teachers have to observe, listen, and interact with their students.

The NCTM (2000) mentions that assessment is also needed in standards-based instructional practices, stating that "assessment and instruction must be integrated so that assessment becomes a routine part of the ongoing classroom activity rather than an interruption" (NCTM, 2000, p. 23). Assessment should give all students the opportunity
to show what they have learned. Some of the assessment techniques recommended by the NCTM (2000) are "open-ended questions, constructed-response tasks, selected-response items, performance tasks, observations, conversations, journals, and portfolios" (p. 23). Most importantly, student assessment should not look for errors only, but it should seek ways to improve student learning. As noted by the NCTM (2000), an appropriate assessment technique is not the only part of the NCTM's standards-based instruction, but technology is also an important feature. "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances learning" (NCTM, 2000, p. 24). The NCTM emphasizes that using technology should enrich student understanding, further explaining that appropriate use of technology helps students focus on reflective practices, reasoning, and problem solving. It also helps students visualize mathematics concepts and verify their calculations. In addition, students can make conjectures, explore, graph, and investigate multiple approaches to mathematics problems. Technology can also stimulate discussion among students. The NCTM emphasizes that proper use of technology depends on the teachers, as teachers have to know when to use technology in their classrooms. To use technology effectively, teachers must consider the concept being covered and the ways technology can benefit students. The importance of technology is not limited to effective teaching and learning of mathematics, but technology also affects the mathematics that is being taught. Using technology, students can better understand topics that would have otherwise been difficult otherwise.

## The American Mathematics Association of Two-Year Colleges

Another organization that focuses on standards-based instruction is the American Mathematics Association of Two-Year Colleges (AMATYC). The AMATYC discusses prior examinations of undergraduate student development in mathematics: Curriculum in Flux (Davis, 1989), which provides guidelines for curriculum at two-year colleges; Reshaping College Mathematics (Steen, 1989), which also provides undergraduate curriculum; Moving Beyond Myth (National Research Council [NRC], 1991) which discusses the need to make changes in undergraduate education; and Everybody Counts (NRC, 1989), which makes specific suggestions for program change in mathematics from kindergarten through graduate school. The AMATYC further mentioned that no organization has discussed standards for mathematics programs for developmental mathematics courses.

To that effect, in 1995, the AMATYC (1995) published its first document about developmental mathematics courses, Crossroads in Mathematics: Standards for Introductory College Mathematics before Calculus. This publication was aligned with the organization's purpose of improving the teaching and learning of lower-level mathematics courses at two-year colleges. This first publication contains standards for intellectual development, content, and pedagogy.

The AMATYC then published a second standards-related document, Beyond Crossroads, revising and upgrading their standards and including their implementation standards. These implementation standards are student learning and learning environment, assessment of student learning, curriculum and program development,
instruction, and professionalism. The AMATYC's standards-based instructional model is formed from a constructivist perspective (AMATYC, 1995):

Standards for pedagogy... are compatible with the constructivist point of view. They recommend the use of instructional strategies that provide for student activity and student-constructed knowledge. Furthermore, the standards are in agreement with the instructional recommendations contained in Professional Standards for Teaching Mathematics (NCTM, 1991). (p. 15)

According to the AMATYC (2006), instruction standard requires that "mathematics faculty [should] use a variety of teaching strategies that reflect the results of research to enhance learning" (p. 51). This instruction standard states the importance for mathematics teachers to understand how students learn and use the different teaching strategies designed to promote active learning. It also states that teaching strategies should fully address students' different learning styles. The AMAYTC (2006) recommends several instructional strategies that promote active learning: cooperative learning, discovery-based learning, interactive lecturing and question-posing, and writing. They (AMATYC, 2006) also list ways mathematics faculty can achieve active learning in mathematics classrooms, which include:

- Design and implement multiple instructional approaches that promote active students participation in the learning process.
- Formulate activities that require students to memorize, comprehend, apply, analyze, and synthesize mathematical concepts.
- Allow discovery-based and thought-provoking questions and activities to guide classroom discussions.
- Provide opportunities for and encourage students to think, reflect, discuss, and write about mathematical ideas and concepts. (p. 55)

Other research also supports active and cooperative learning in teaching. Chickering and Gamson (1999) argue that effective teachers should allow their students to be active instead of passive learners. They also explain that students learn better when they work together and share their ideas. It is further stated that students do not learn just by listening to their teachers (Chickering \& Gamson, 1999). That is, learning should involve interaction, writing, and relating new materials to previous knowledge and life experiences. Mckeachie and Svinicki (2006) concur with the idea that cooperative learning is an effective teaching strategy at collegiate level. Peer learning (collaborative and cooperative learning) "is extremely effective for a wide range of goals, content, and students of different levels and personalities."(Mckeachie \& Svinicki, 2006, p. 214). Springer, Stanne, and Donovan (1999), discussing the effects of small-group learning, also demonstrated that cooperative learning is an effective way to accomplish academic improvement. Active learning can be achieved by assigning group projects and homework, allowing students to present their work, providing opportunities for class discussions, and encouraging students to justify their answers and pose questions.

In addition, the AMATYC (2006) recommends that proper use of technology, including "graphing calculators, student response systems, online laboratories, simulations and visualization, mathematical software, spreadsheets, multimedia, computers or the internet, and other innovations yet to be discovered" (p. 55) are effective standards-based instructional strategies. Effective implementation of technology in the teaching of mathematics can engage students in mathematics learning both inside and outside the classroom. Technology should be used for conceptual understanding and mastery of basic skills in mathematics. The AMATYC(2006) mentions that mathematics
teachers should consider both the content and the appropriate way students should learn the content before integrating technology into their teaching. However, technology should not replace teaching for understanding; rather, proper use of technology should help students explore and visualize new mathematics concepts, test and proof conjectures in mathematics, and communicate mathematical ideas. The AMATYC(2006) recommends the following ways for appropriate use of technology in classroom:

- Integrate technology into their teaching of mathematics.
- Use technology tools for assessments that align with instruction.
- Align technology platforms with those familiar to students, required for future courses, and/or necessary for their future careers. (p. 57)

Tabach, Hershkowitz, and Arcavi (2008) also reported that successful use of technology contributes to student learning in mathematics, indicating that appropriate use of technology should incorporate class discussion, writing, student presentations, openended activities, and student freedom to approach problems in different ways. The student participants in the study were allowed to use multiple ways to represent algebraic expressions, which the authors stressed as important for student learning in mathematics. Tabach, Hershkowitz, \& Arcavi (2008) also emphasized that "classroom should be envisioned as communities of learning in which different ability levels, the availability of new tools, collaboration and ongoing discussions are assets to be exploited" (p. 803).

The AMATYC (1995) discussed the importance of connecting mathematics learning to other experiences. These experiences include students' life experiences and their experiences gained in other mathematics courses and other disciplines. With these connections, students should see the value of mathematics. Students' view of
mathematics as an "isolated subject" should be overcome through making mathematics meaningful to them. The use of real-world problems has an impact on student learning and performance in mathematics (Austin, Berceli, \& Sarae, 1999; Leonard \& Guha, 2002; Pierce, Turville, \& Giri, 2003).

Multiple approaches and "experiencing" mathematics are also recommended by the AMATYC (2005) as part of standards-based instruction; it is clearly shown that mathematics faculty should create opportunities for students to use different approaches (e.g., numerical, graphical, symbolic, and verbal) to solve problems. Furthermore, experiencing mathematics requires mathematics faculty to incorporate learning activities such as projects and apprenticeships that allow students to experiment what they have learned in mathematics. Pierce, Turville, and Giri (2003) showed that assigning student projects improved student learning in mathematics. Specifically, the AMATYC (2006) stated that in order for students to experience mathematics, instructors should:
...provide learning activities, including projects and apprenticeships, that promote independent thinking and require sustained effort and time so that students will have the confidence to access and use needed mathematics and other technical information independently, to form conjectures from an array of specific examples, and to draw conclusions from general principles. (p. 17)

## The Mathematical Association of America

The Mathematical Association of America (MAA) is another organization that advocates the use of standards-based instruction. The MAA is the largest professional society that focuses on undergraduate mathematics education (www.maa.org). Members of the MAA include university, college, and high school teachers; graduate and undergraduate students; pure and applied mathematicians; computer scientists; statisticians; and many others in academia, government, business, and industry. The
organization emphasizes that it accepts everybody who is interested in the mathematical sciences. The mission of the MAA (www.maa.org) is to improve the college mathematics; mission guides are:

- Education: We support learning in the mathematical sciences by encouraging effective curriculum, teaching, and assessment at all levels.
- Research: We support research, scholarship, and its exposition at all appropriate levels and venues, including research by undergraduates.
- Professional development: We provide resources and activities that foster scholarship, professional growth, and cooperation among teachers, other professionals, and students.
- Public policy: We influence institutional and public policy through advocacy for the importance, uses, and needs of the mathematical sciences.
- Public appreciation: We promote the general understanding and appreciation of mathematics. We encourage students of all ages, particularly those from underrepresented groups, to pursue activities and careers in the mathematical sciences.

The MAA has a special committee to help mathematics departments design their undergraduate curriculum, the Committee for Undergraduate Programs in Mathematics (CUPM). In 1953, the CUPM published their first issue of undergraduate mathematics curriculum, which is updated approximately every 10 years. The content of CUPM Guide published in 2004 lists specific recommendations on improving undergraduate mathematics curriculum. The CUPM's suggestions for undergraduate mathematics instruction are aligned with ideas of the NCTM (2000) and the AMATYC (2006).

Specifically, the CUPM (2004) emphasizes that teachers should encourage students to "develop mathematical thinking (pattern recognition, generalization, abstraction, problem solving, careful analysis, and rigorous argument) and communication skills"; encourage the "use of computer technology to support problem solving and to promote undergraduate understanding" (p. 5); and involve students in applying and connecting mathematics knowledge to real life and other disciplines. The CUPM also has a subcommittee, the Curriculum Renewal Across the First Two Years (CRAFTY) that conducts workshops on undergraduate mathematics curriculum. The MAA also publishes a news magazine that is published six times in a year and is available to all MAA members. This publication is focused on MAA activities, news about mathematics and the mathematical community, and interesting (and sometimes) new ideas in mathematics, mathematics education, and other related areas.

## Standards-Based Instructional Strategies in Grade Schools

Standards-based instructional strategies have been implemented at both the grade school and collegiate levels. The impacts of these strategies at the different educational levels prior to undergraduate mathematics, therefore, were essential to understand for purposes of the current study.

Boaler (2006) conducted a four-year study on three high schools, including Railside High School. Railside High School was located in an urban area with mostly low-income families; the other two schools were in suburban areas. At Railside, students were taught mathematics using standards-based strategies while the students at the other two schools were taught in a traditional manner. The freshmen entering Railside scored significantly lower on standardized tests than their counterparts at the other two schools.

However, due to the teaching methods and practices of the mathematics department at Railside High School, their students improved significantly and finally outperformed the students at the other two schools.

According to Boaler (2006), Railside students were taught in groups, and teachers used what Boaler called "complex instruction" that recognizes students' individual differences in learning. Using complex instruction, teachers at Railside created "multidimensional classrooms." The activities within these classes included "asking good questions, helping others, using different representations, rephrasing problems, explaining ideas, being logical, justifying methods, or bringing a different perspective to a problem" (Boaler, 2006, p. 4). The students at the other two schools were taught in more traditional, one-dimensional methods. Students from traditional classes viewed mathematics as a set of rules and procedures. Students carefully listened to the teachers' presentation of the procedures and memorized the rules (Boaler, 2006). The result of the study showed that the heterogeneous instruction practiced Railside contributed to student success in mathematics.

Another factor that contributed to the progress of Railside High School students was "group-worthy problems." The students worked in groups, and the problems assigned to them were considered "group worthy." That is, the problems assigned to the students were designed in a way that the students could solve them using different approaches. The problems also engaged students in multitasking, such as "asking questions, drawing diagrams, and making conjectures" (Boaler, 2006, p. 5). Boaler (2006) pointed out that the main reason why students succeeded with group work was because of the nature of the questions.

Boaler (2006) argued that "shared responsibility among students" also had a positive effect on student performance at Railside High School. Students in the mathematics department at Railside worked in groups, where they were responsible for each other's learning. Group members ensured that everyone understood before they moved to another topic. Students acknowledged that explaining mathematics problems to each other students helped them achieve a deeper understanding of related concepts. Similarly, Boaler (2006) illustrated that standards-based instruction is effective.

Thomas and Santiago (2002) illustrated how to make mathematics meaningful to students in urban schools, implementing a Reflective Teaching Model (RTM). According to Thomas and Santiago (2002):

RTM is a teacher-change model developed in the Atlanta Math Project (AMP) at Georgia State University. It is an innovative process of reflective lesson planning, teaching, and debriefing designed to facilitate standards-based teaching as documented in the National Council of Teachers of Mathematics (NCTM). (p. 149)

Santiago (a teacher) taught seventh-grade geometry, and Thomas was a researcher. Thomas and Santiago (2002) collaborated in developing the RTM and implementing its use in Santiago's classroom. Project-based instruction was employed in teaching geometry where they implemented the teaching strategies (e.g., problem solving, connections, and communication). For the problem-solving strategy, Thomas and Santiago used opening questions to engage the students. They also encouraged follow-up questions, which promoted student thinking and improved their problem-solving techniques. Students were able to make connections within mathematics topics and other disciplines. Thomas and Santiago (2002) also established good communication skills in the classroom, and the environment was conducive to student communication and
learning. The students were comfortable working in groups and sharing ideas. Writing was also incorporated in the mathematics classes. In addition to the above strategies, the researchers incorporated culturally relevant teaching in the classroom. Student performance on standardized tests was reported as a direct outcome of the implemented teaching practices. It was mentioned that "on the state's standardized test given in the spring of the year, Carmelita's seventh-grade outperformed all other seventh-grade students in the school" (Thomas \& Santiago, 2002, p. 160). This supported the idea that standards-based teaching is effective in improving student achievement in mathematics on standardized tests.

Gutierrez (2000) also showed that standards-based instruction is successful in mathematics instruction. One of the study's purposes was to examine factors that had encouraged African American students to take advanced mathematics courses and make great achievements in learning mathematics. According to Guteirrez (2000), low performance of African American students and other marginalized groups in math was due to these groups not being taught well. Gutierrez (2000) reported that the success of African American students in mathematics was based on two factors: teacher effectiveness and reform movement. The teachers described their typical day of teaching, which began with warm-up exercises, a discussion of previous class and homework, teaching the new concept, and classwork. In addition to this formal organization, the teachers adjusted their teaching to student need. They adopted some standards-based techniques recommended by NCTM (1989), such as use of cooperative learning, technological tools, knowledge construction, real-world problems, and less emphasis on
drilling and memorization. Findings (Gutierrez, 2000) indicated that these practices helped improve student learning in mathematics.

Although the focus of the current study is to contribute to the literature on standards-based instruction in undergraduate mathematics, I feel that it is important not only be mindful of the impact of standards-based instruction at undergraduate mathematics education but also to be aware of the impact of standards-based instruction in grade schools.

## The Importance of Standards-Based Strategies in Undergraduate Mathematics

Advocates of standards-based instruction emphasize that standards-based strategies such as cooperative learning, student-teacher interaction, writing, proper use of technology, and real-life applications should be used in a classroom instruction where an instructor is implementing standards-based instruction. These strategies improve student success in mathematics.

## Interaction and Use of Concrete Method

Iannone and Nardi (2005) used qualitative method to investigate mathematicians' reflections on their effective teaching strategies at university level in United Kingdom (UK). The study involved 6 university mathematics departments in the UK and 20 mathematicians from these universities. Data were collected via focus group interviews. The result of the study indicated that the main teaching strategies were interaction among students, interaction among students and professors, and representation of abstract mathematical concepts in concrete form. Participants mentioned repeatedly that they learned mathematics through interactive methods. As described by the participants, activities such as "oral examination" and "oral interaction" were especially encouraged in
their classrooms because these activities often enable the students to see mathematics as an argumentative subject. One of the participants explained that the approach to students' interaction was by "small table arrangement in the department where students are invited to congregate and work together" (Iannone \& Nardi, 2005, p. 197). The same participant reported that the need for learning through interaction cannot be overlooked in mathematics. The learning of mathematics involves many languages which cannot be replaced with one another, and these languages include students' discussion with each other, textbook language, and teacher's language either to the whole class or to individual students who need extra help.

Seminars and tutorials are also reported to be effective interaction with the students. Mathematics problems were discussed during seminars and tutorials. According to the informants, the students' homework problems were similar to the ones discussed during seminar sessions. They added that the process helped the students become familiar with the homework questions and motivated them to participate in the seminars.

Iannone and Nardi (2005) argued that another interactive strategy in mathematics learning was homework. Study participants pointed out that they were careful in selecting homework problems that promoted writing ability in mathematics. In other words, they often assigned mathematics problems as homework, which allowed the students to use symbols and to translate them into verbal language. This method helped the students understand the meaning of mathematical concepts. Participants also emphasized that it is necessary to consider students' needs in selecting homework problems; a way to identify these needs is through graded assignments. Students' graded assignments indicated areas where the students needed further exploration. The informants believed that feedback on
student papers was an efficient way to communicate. They also explained that they provided a solution sheet to the students after grading the homework, which helped them provide feedback "to their collective errors" (Iannone \& Nardi, 2005, p. 205).

As indicated before, another strategy discussed by Iannone and Nardi (2005) was use of concrete methods to represent abstract concepts in mathematics. They reported that one method was the use of numbers in mathematics concepts (limit of a sequence) and watching how the sequence behaves. For instance, numbers could be used in the teaching students how to identify "the limit of a sequence" (Iannone \& Nardi, 2005, p. 205). Technology is another technique the participants used to represent abstract ideas in concrete form for their students. This technology includes graphing calculators and other mathematical computer software. The participants mentioned that technology helped the students with tedious calculations and also convinced them that explanations of mathematical concepts are true.

The participants of the Iannone and Nardi (2005) study reported that lecturing is not the best method to teach mathematics. Lecturing makes it difficult for instructors to know which students who are having problems understanding the materials presented. However, informants mentioned that use of "tactics" and "skills of synthesis" (Iannone \& Nardi, 2005, pp. 199-200) are effective strategies that work with the lecturing method. Therefore, interaction and representation of abstract topics in concrete forms are demonstrated to be effective teaching techniques.

Roth-McDuffie, McGinnis, and Graeber (2000) investigated the effectiveness of reform-based teaching and learning in college mathematics teaching. The study was conducted in an undergraduate mathematics classroom at a large state university. Study
participants were a mathematics professor and his students. Data collection included interviews, classroom observations, and surveys. Analytic deduction was used to analyze the data. Students' perspectives on traditional and reform-based mathematics education were compared. Findings (Roth-McDuffie, McGinnis, \& Graeber, 2000) showed that participants viewed a traditional mathematics classroom as a classroom where students sit and listen to their teachers and carry out step-by-step procedures in solving mathematics problems without understanding. Contrarily, reform-based teaching is student-centered. The participant (a reform teacher) focused on student understanding instead of memorization of mathematics facts. The reform teacher created a learning environment that allowed students to reason, explain, connect, and learn meaningful mathematics. One of the students commented that "[our class] has definitely been more of understanding of how to solve the problems as opposed to the memorization of facts and stuff" (RothMcDuffie, McGinnis, \& Graeber, p. 240). Similar to other studies, the reform teacher also provided the opportunity for students to work in groups. While the teacher assigned his students problems that "stimulated discussion," he moved around the classroom and asked questions that promoted critical thinking. Students commented (Roth-McDuffie, McGinnis, \& Graeber, 2000):
...[our teacher] would step in and kind of guide us the right way, maybe asking us questions in different ways so that we can see in a different way what he's trying to get across, and that way remember it because we understand. (p. 241).

Furthermore, in student' evaluations of the class, they were required to articulate the teaching strategies they preferred in learning. It was noted that "lecture, a typical feature of college mathematics course, was not selected by any student" (Roth-McDuffie,

McGinnis, \& Graeber, 2000, p. 245). Another comment made by the students in RothMcDuffie, McGinnis, and Graeber (2000) was:

I think [the teacher] did an excellent job teaching math concepts [and] relating [them] to life and other subjects. This is the first time I have enjoyed the content of my math class and felt like I was learning valuable information (p. 245).

The comments from the participants indicated that standards-base instruction had positive effects on their learning.

## Collective Vision Report on Standards-Based Instruction

A collective vision report originated from a series of workshops organized by the MAA. The purpose of these workshops was to create "suggestions to guide reform of the treatment of all topics in the first two years of the college mathematics" (Marcus, Fukawa-Connelly, Conklin, \& Fey, p. 355). Suggestions included a discussion teaching strategies (e.g., group work, class discussions, in-class and out-of-class activities) that promote problem solving and project work, since group work helps students work together and share ideas. The use of technology in mathematics classrooms was also mentioned in the report. The collective vision report stressed that the use of technology helps students understand difficult mathematics concepts at beginning of their mathematics development. Further, the report also recommended using various assessment techniques that promote conceptual understanding. Another recommendation made in the report was to help students develop communication skills such as "reading, writing, speaking, and listening skills" (Marcus, Fukawa-Connelly, Conklin, \& Fey, 2007/2008, p. 355). These skills can help students explain and justify their answers.

According to Marcus, Fukawa-Connelly, Conklin, and Fey (2007/2008), these aforementioned recommendations agree with the NCTM's (2000) process standards,
learning principles, and curriculum principles. The process standards emphasize problem solving, reasoning and proof, communication, connections, and representation while learning and curriculum principles, respectively, emphasize understanding of mathematics concepts and use of mathematics knowledge outside the classroom. Taken together, the teaching strategies suggested in the collective vision report were studentcentered instead of traditional lecture-based teaching methods. These strategies pertained to class discussions, in-class and out-of-class activities, and communication skills such as reading, speaking, listening, and writing.

## Implementing Writing to Assess Student Learning

Kagesten and Engelbrecht (2006) examined teaching strategies and student understanding in mathematics. The purpose of the study was to incorporate writing in the teaching and learning of mathematics to improve student understanding, as discussed by Pierce, Turville, and Giri (2003). Kagesten and Engelbrecht (2006) reported that mathematics students at Swedish universities viewed mathematics as a mechanical subject that involves calculations and manipulations without explanations. To promote student understanding in undergraduate mathematics, instructors involved in the study implemented writing to assess student learning. Students were required to explain and justify calculations they made during the examination. There were two parts of the assessment. In the first phase, after students received written comments on their papers, they addressed the comments in writing and resubmitted the papers for full credit.

Examples of such comments were "What formula do you use and what are its assumptions? Explain what you do. Can you test your answer? ..." (Kagesten \& Engelbrecht, 2006, p. 710). Findings indicated that the writing technique effectively
increased the students' understanding in mathematics. Kagesten and Engelbrecht (2006) summarized that "almost all students emphasized the fact that the additional time that they spent at home, attempting to address the comments from the teacher that marked the test, contributed largely to deeper understanding of the particular concept" (p. 712). They also emphasized that if the aim of mathematics instruction is to increase student understanding, then the writing approach is a good method to achieve the goal. Study implications were that encouraging students to explain, justify, and evaluate answers to mathematics problems in writing is an effective standards-based teaching strategy in mathematics education (Kagesten \& Engelbrecht, 2006). Similarly, Pierce, Turville, and Giri (2003) reported the use of writing as one of the teaching strategies implemented in their curriculum redesign.

## Multiple Standards-Based Instruction Strategies in Undergraduate Mathematics

Pierce, Turville, and Giri (2003) discussed a curriculum revision in undergraduate mathematics at an Australian university, which was a program dominated by preservice teachers. The purpose of the new mathematics curriculum was to address the issue of student retention, poor understanding, and negative attitudes toward mathematics. Pierce, Turville, and Giri (2003) noted that in the 5 years prior to the study, there was a decline in student enrollment in mathematics. The method of curriculum change was geared toward the use of standards-based teaching strategies, including writing, real problems in mathematics, use of technology, justification in mathematics, incorporating portfolios and projects in evaluating students' progress, and engaging students to think mathematically.

Pierce, Turville, and Giri (2003) also reported that writing and explaining mathematics solutions was helpful in improving student understanding. Students were required to present mathematics topics to their classmates. According to the study (Pierce, Turville, \& Giri, 2003), the use of real problems helped the students realize that mathematics is more than a set of rules and formulas. It also increased student interest in mathematics. The teacher participants reported that interesting topics such as "logic puzzles, gambling games, 2-D and 3-D geometric constructions, and streetscape studies and data collection for time-distance graphs" worked effectively in motivating students to learn (Pierce, Turville, \& Giri, 2003, p. 2). The use of real problems changed students' view of mathematics, as one student commented, "I thought this unit overall was interesting and enjoyable. Use of real-life context made it easier [for students] to understand concepts taught" (Pierce, Turville, \& Giri, 2003, p. 4). Another effective method was evaluation of students through individual and group assessment, presentations, and submission of portfolios of solutions of examples.

The curriculum redesign also affected the students' attitude toward mathematics. It reduced student anxiety about mathematics and improved their understanding and achievement. Consequently, student performance increased from " $40 \%$ in 2001 to $70 \%$ in 2002" (Pierce, Turville, \& Giri, 2003, p. 5). This better performance also led to an increase in student retention and understanding in mathematics. As previously mentioned, proper use of technology was part of the curriculum redesign.

## Standards-Based Teaching Strategies of College Algebra

The purpose of the current study was to investigate the teaching practices of a college algebra teacher who uses standards-based instruction in his classroom. This section examines the literature on standards-based instructional strategies in college algebra and their importance. The NCTM, MAA, and AMATYC all note that such strategies include proper use of technology, cooperative learning, real-life applications, and other strategies in providing standards-based instruction classrooms.

## Proper Use of Technology

Mayes (1995) investigated the effect of computer use in teaching and learning college algebra. This quantitative study had a control group of 76 students and an experimental group of 61 students. The control group was taught using the lecture method without computers while the experimental group was taught with the help of computers. Findings (Mayes, 1995) indicated that the experimental group performed better than the control group in areas of inductive reasoning, visualization, and problem solving. Also, results showed no significant difference between the two groups in computational ability. Mayes (1995) mentioned that the difference in attitudes between the experimental group and control group was not only because of the use of computers but also knowledge on how to implement the technology. He stated that use of computers "without a proper pedagogy will not have much of an effect" (Mayes, 1995, p. 66). This implies technology in mathematics curriculum is a good teaching strategy, but effective pedagogy must accompany this use.

Thiel, Peterman, and Brown (2008) reported that their college algebra curriculum redesign involved the proper use of technology. They focused on factors that led to low
performance of students in college algebra. Thiel, Peterman, and Brown (2008) mentioned that one factor that had contributed to student success in mathematics was effective instruction. They emphasized that student understanding of college algebra is important because it is a gateway to many majors and to more advanced mathematics courses. However, student achievement in college algebra was low at their teaching institutions, so the mathematics department decided to redesign college algebra, which aimed at "use of technology to reduce lecture time and encourage active learning, continuous assessment of student progress, and techniques to keep students focused on the work necessary to succeed" (Thiel, Peterman, \& Brown, 2008, p. 46). Details of the redesign include decreased lecture time and the introduction of computer lab sessions so that students could be actively engaged in problem solving. The software installed on the computers helped students understand mathematics concepts. Software features included tutorials, practice problems, guided solutions, and homework problems (assigned every week and allowing multiple attempts). The software also provided immediate feedback.

The course redesign made it clear that instructors and graduate teaching assistants should be present in the lab to assist students. Computers in the lab were arranged in a circular form to allow peer collaboration, and white boards were also available. In addition to homework problems, students were required to take quizzes, exams, and comprehensive final exam in the computer lab. Moreover, the software has a gradebook, which allowed students to monitor their progress and an e-mail feature that helped the instructors to contact students when needed. Thiel, Peterman, and Brown (2008) stressed that for this type of redesign to be effective, instructors must adhere to homework deadlines and that quizzes should be similar to homework problems to encourage
students to do their homework. A primary implication (Thiel, Peterman, \& Brown, 2008) was that proper use of computer technology is an effective teaching method in college algebra classes.

Hagerty and Smith (2005) examined the incorporation of online assignments in college algebra classes. They used software called "assessment and learning in knowledge spaces" (ALEKS). Study participants consisted of an experimental group and a control group of students taking college algebra; four sections were taught using the ALEKS (the experimental group), and four sections were taught the traditional way. To minimize possible biases, one instructor taught one section of the experimental class and one section of the control class. The same instructor was responsible for making sure that the pretests and posttests reflected the objectives of the college algebra course ( Hagerty \& Smith, 2005).

Results ( Hagerty \& Smith, 2005) showed that the experimental group performed better than the control group. The success of the experimental group was attributed to the proper use of ALEKS. The two groups in the Hagerty and Smith (2005) study were also tested on collegiate assessment of academic proficiency (CAAP). The results of the CAAP test indicated that the experimental group showed greater retention of the algebra skills 14 months after the class. Since the appropriate use of ALEKS software improved student learning, Hagerty and Smith (2005) recommended that universities consider webbased software in teaching.

Acelajado (2003) focused on the impact of the use of graphing calculators on student achievement in college algebra. The study consisted of 66 freshmen students enrolled in two sections of college algebra. The participating students were divided into
three groups, namely, a high ability group (HAG), an average ability group (AAG), and a low ability group (LAG); each group was allowed to use graphing calculators. Pretests and posttests were used assess student achievement. Data were analyzed using the results from $t$ tests. Acelajado (2003) reported that there was a high significant difference in pretests and posttests for the whole group ( $t$ value $=2.53, \alpha=.01$ ). The difference was attributed to the proper integration of the graphing calculators. It was also indicated that the LAG showed greater improvement when compared with the other two groups, which might be due to the confidence that the LAG students earned from the implementation of technology. Results (Acelajado, 2003) revealed that students improved significantly in all college algebra topics, with highest mean in functions and their graphs. Study participants reported that graphing calculators had a positive impact on them learning graphs of functions, as these calculators made it possible for students to see the connections among different representations (algebraic, graphic, and numeric) of mathematics topics. Acelajado (2003) stated, "the use of the technology made the students realize and appreciate the fact that mathematics is not a "burden" subject but rather a relevant learning area that is present in almost all aspects of their everyday living" (p.16).

Because of the positive effects of graphing calculators on student learning, Acelajado (2003) suggested that the course syllabus should include laboratory hours for use of graphing calculators. In addition, it was said that both teachers and students need training on the use of graphing calculators. Acelajado (2003) further advised that teachers should get away from traditional ways of teaching and consider creative methods that promote student achievement, learning, and interest.

## Cooperative Learning

Rogers, Davidson, Reynolds, Czarnocha, and Aliaga (2001) explored a different teaching strategy, cooperative learning, in undergraduate mathematics. Rogers et al. (2001) discussed the incorporation of cooperative learning in college algebra classes. According to the authors, cooperative learning strategy is based on Dubinsky's cycle of activities, class discussion, and exercises (ACE-cycle). Rogers et al. (2001) also mentioned that the ACE-cycle strategy helped "students [understand] mathematical concepts at a level higher than they had previously" (p. 85). The implementation of cooperative learning began with dividing students into small groups. The class started with a brief introduction to the mathematics topic, and problems that related to the topic to different groups were assigned. Students worked in groups but were required to present their solutions to the entire class. During class discussions, students were given a summary of the topic. Different techniques were employed to increase student understanding, which included that each group made sure that all members understood and were able to explain the problems they were assigned. Students were also assigned problems with multiple methods so that they could discuss different ways of solving the problems. In addition, as the students worked in small groups, the instructor moved around the classroom to guide and answer questions. The teacher stated that "working in groups seems to alleviate much of the anxiety related to algebra. Students express the sentiment that collaborative efforts of the group help them to understand the concepts..." (Rogers et al., 2001, p. 85).

Similarly, Hagedorn, Sagher, and Siadat (2000), evaluating a college algebra program, identified cooperative learning as an instructional-based teaching strategies.

The study (Hagedorn, Sagher, \& Siadat, 2000) consisted of an experimental group and a control group; he control group used textbooks, worksheets, and tests, and the experimental group was taught using standards-based strategies. Instructors teaching the experimental group were strict with the time allocated for quizzes and as well as the attendance policy. Additional strategies included immediate feedback for students, review of topics that most students failed on the test, and cooperative learning when the standard deviation between high-achieving and low-achieving students was more than $25 \%$. During group work, students were required to record the weaknesses found on quizzes. After group work, the materials were tested again to gauge the students' mastery of the concepts. Grading was based on absolute scale (no curving). The experimental group and the control group were compared based on their performance on college board descriptive pretests and posttests. These tests examined student retention, mathematics performance, and full concentration. Hagedorn, Sagher, and Siadat (2000) illustrated that the students in the experimental group showed a greater retention rate $(z=2.1, \mathrm{p}<.05)$ and concentration than the control group. There was, however, no significant difference between the two groups in mathematics performance. Hagedorn, Sagher, and Siadat (2000) argued that this could be due to the students in the control group being college seniors planning to teach mathematics at the high school level.

Depree (1998) conducted a quasi-experiment on small group instruction (cooperative learning) at an urban community college. The study compared two groups of students in terms of their confidence in mathematics, algebraic skills, and completion rates. The study (Depree, 1998) involved a racially diverse group of 386 algebra students between the ages of 17 and 58 years. The experimental group was taught using
cooperative methods while the control group was taught via lecture. Student achievement was evaluated based on pretests and posttests. In Depree (1998), the experimental group showed greater increase in confidence in mathematics ability as compared to the traditional group. Findings also demonstrated that students in the experimental group had higher completion rates than students in control group $(z=1.60, p=0.05)$. (Of interest is that all Native American students in control group dropped the course.) Achievement tests showed no significant difference between the two groups. Depree (1998) explained the reason for no significant difference is possibly because most students from the control group that were having problems dropped the course, but the same students continued in the experimental group. In addition, students' comments indicated that working in groups helped them to understand the mathematical concepts. Depree (1998) stated that "a cooperative than competitive learning environment may enable students for the first time to make sense of mathematical thinking" (p. 6). Interestingly, group work was also part of freshman learning support strategies.

## Freshman Learning Program Supporting Cooperative Learning

Burmeister, Kenney, and Nice (1996) examined a learning support program as a teaching strategy called supplemental instruction (SI), which was developed first in 1970s to help improve student retention. Supplemental instruction was designed for "high risk courses" (i.e., courses where students had lots of Ds and Fs, courses with high withdrawal rates (Burmeister, Kenney, \& Nice, 1996, p. 145). College algebra was one of these courses. It was noted that SI was not a tutoring program; rather, it was a supervised program managed by trained, competent students and faculty who evaluated progress through the program.

Supplemental instruction was developed to help students expand their reasoning beyond concrete to operational level. Activities included study skills such as "note taking, text-reading, and problem solving" (Burmeister, Kenney, \& Nice, 1996, p. 146). Another important component of SI was cooperative learning. Students who participated in SI worked in small groups where they shared ideas, justified their answers, and asked probing questions. Burmeister, Kenney, and Nice (1996) mentioned that cooperative learning was a powerful teaching tool used by well-known mathematician Uri Treisman. Where Treisman taught, cooperative learning improved the performance of "minority students enrolled in calculus from $60 \%$ failure rate before the [implementation of cooperative learning] to a $4 \%$ failure rate after the [implementation of cooperative learning] was available to students" (Burmeister, Kenney, \& Nice, 1996, p. 147).

Other institutions also participated in the SI program (Burmeister, Kenney, \& Nice, 1996); data were collected from course evaluations from 45 other institutions. Each institution had SI-participating students and non-SI-participating students. Data analysis showed that students who SI-participating performed better than non-SI-participating students in all three courses (college algebra, calculus, and statistics). Conversion of numeric averages to "letter-grade equivalent" indicated SI-participating students had grades above "C". The non-SI-participating students had grades below "C". The implication was that more SI-participating students would enroll in upper-level mathematics classes because most of these courses required a passing grade of "C". Further analysis showed that SI-participating students had low withdrawal rates than non-SI-participating students. This implies that the standards-based techniques used in SI are
effective in improving student understanding in undergraduate mathematics (Burmeister, Kenney, \& Nice, 1996).

## Relating Mathematics to Real Life

Austin, Berceli, and Sarae (1999) reported the ways they incorporated real-world activities in teaching and learning of college algebra through a service learning project at a community college. Comments from students, teachers, and mentors indicated that the service learning project, which involved relating mathematics to the real world, was effective. For example, one student mentioned that "I learned how [college] algebra is important in everyday life" (Austin, Berceli, \& Sarae, 1999, p. 798). A mentor commented "showing students the real needs, as in practical situations, not only clarifies but makes a dry learning process fun (good job!)" (Austin, Berceli, \& Sarae, 1999, p. 799). Making mathematics learning meaningful agrees with Choike's (2000) multiple strategies for teaching college algebra.

## Other Standards-Based Instructional Strategies for College Algebra

The AMATYC (2006) supported the use of multiple strategies in teaching mathematics. Similarly, Choike (2000) introduced many strategies for teaching algebra for all, which includes college algebra, based on his teaching experiences at the university level and working with teachers and students. One strategy is considering student interest in "word problems." Interesting wording captures student interest and thereby increases their participation in solving the word problems. Choike (2000) emphasized that focusing on "big ideas" is a good teaching strategy. He reported that many college algebra teachers follow the textbooks by topics and there are many topics in almost every chapter of college algebra books. Choike (2000) advised that the best
approach, however, is to look for "big ideas." According to Choike (2000), focusing on "big ideas" can help students and teachers connect present knowledge to previous knowledge. Also, "big ideas" can create an outline for teaching upper-level mathematics courses, which can avoid following the textbook section-by-section despite of the disjoint nature of most of the textbooks. Choike (2000) also emphasized that word problems should be phrased and introduced to students with clarity; teachers have to ensure that words used in mathematics problems are clear and not confusing. Choike (2000) also mentioned that use of multiple representations (use of words together with tables, graphs, or symbols) is a good teaching strategy. Multiple representations help students understand how these representations are related. Also, multiple representations are believed to be able reach more students than single representation because different students learn better with different materials.

Furthermore, Choike (2000) emphasized that reviewing concepts that students have trouble with at the beginning of a semester is a waste of time. He argued that student deficiency vary in many topics. Instead of wasting 2-3 weeks dealing with those deficiencies, which students might still have after the revision, it is better to start with the content of the course and deal with the "prerequisite deficiencies" as learning progresses. In learning these concepts, teachers should be able to identity and help students who need assistance and instruction in particular areas. Choike (2000) suggested that one way to deal with prerequisite deficiencies is teachers' use the first 5-10 minutes of each class to give students warm-up problems that incorporate review material in "an interesting and conceptual way (for more details, see Choike, 2000, pp. 6-7). Moreover, Choike (2000) emphasized the importance of discovery learning and guided exploration. Incorporating
discovery learning and guided exploration engages students and increases their algebraic thinking, which leads to conceptual understanding of mathematical concepts. College algebra teachers should also be good listeners. Teachers should concentrate not only on the correctness of students' answers but the ideas behind the answers as well. That way, more students will be willing to participate in class discussions. Finally, "establishing a safe classroom environment" is also mentioned as an important strategy for college algebra teaching (Choike, 2000, p. 10). This involves including every student in the learning process, encouraging them to ask and answer questions, using different approaches in answering mathematics problems, and encouraging collaboration among students.

According to Ellington (2005), due to high attrition rate and low passing rate in college algebra, Virginia Commonwealth University (VCU) introduced standards-based strategies in college algebra classes. The Ellington (2005) study consisted of a modelbased college algebra section (experimental group) and traditional sections (control group). The experimental group was taught using standards-based instruction (e.g., group work, real world applications, modeling, and communications of mathematics concepts orally and in writing). Traditional sections were taught using the lecture method. Study results (Ellington, 2005) showed that the experimental group had a $5.63 \%$ attribution rate while the traditional group had a $20.34 \%$ attrition rate. Also, $71.83 \%$ of students in experimental class passed and performed better in subsequent mathematics courses; only $49.80 \%$ of the students in traditional sections passed.

The focus of the current study is to reduce student attrition rates in undergraduate mathematics and improve instruction through standards-based instructional strategies.

Thus, existing literature must be examined. The aforementioned strategies will serve as lens to analyze data collected from a college algebra teacher who uses standards-based instructional techniques.

## Summary

This chapter began with a discussion on research on students' attrition in undergraduate mathematics and the reason for the attrition. Next, the historical background and views of advocates of standards-based instruction are presented. In addition, studies on how standards-based instruction has been carried out in grade schools, undergraduate mathematics, and college algebra are presented.

The studies under review present different standard -based teaching strategies, which align with ideas of NCTM, MAA, and AMAYC. The strategies help boost student learning in undergraduate mathematics and college algebra. The teaching strategies suggested in the studies were social interaction, group work and scaffolding, use of real word problems, and effective use of technology. Other teaching techniques recommended in the studies were writing and justification in mathematics, incorporating portfolios and projects in students' assessment, discovery learning by exploration, cooperative learning, setting high standards, and freshman learning program that support cooperative learning. This chapter also supports standards-based instruction. All the reported studies reinforce Pesonen and Malvela's (2000) point of view that "traditional teaching is inadequate as the only mode of instruction . . . The traditional teaching and new teaching methods should be combined . . ." (p. 114).

## CHAPTER 3

## METHODOLOGY

According to Bogdan and Biklen (2007), "design is used in research to refer to the researcher's plan of how to proceed" (p. 54), suggesting that research design is subject to change as the research progresses and that it continues until the end of the study. This chapter, which describes the research methods utilized in the current study, describes the rationale for a qualitative case study, the research setting and the role of the researcher, the study participant, and data collection and analysis.

## Rationale for a Qualitative Case Study

The primary research questions guiding the current study were: (1) What teaching practices are used in a mathematics classroom of a college algebra teacher? and (2) How are the teaching practices of the college algebra teacher aligned with the characteristics of standards-based instruction? A qualitative method was chosen because of the interpretative and exploratory nature of guiding questions. I examined the teacher's actions in a naturalistic setting and provided a thorough description of his teaching. Creswell (1998) mentions that a few of the reasons researchers choose qualitative research are that they want to study their informants in a natural setting and "write long passages, because the evidence must substantiate claims and the writer needs to show multiple perspectives" (p. 17). In addition, Denzin and Lincoln (1994) report that qualitative study provides the opportunity to interpret the situation of a study based on the meanings and interpretations that the subjects bring to the study. A case study is an example of qualitative research. Yin (2003) states "a case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when
the boundaries between phenomenon and context are not clearly evident" (p. 13). Using a case study for the current study was logical because it followed the model.

Yin (2003) mentions that the components of a case study are study questions, propositions, units of analysis, the link between data and propositions, and data interpretation. Yin (2003) claims that stating the propositions helps direct the researcher's attention, which in turn helps with data collection. Moreover, Yin describes that the same case study may involve more than one unit of analysis, a situation that occurs when attention is given to a subunit or subunits. Yin explains that "no matter how the units are selected, the resulting design would be called embedded case study design" (Yin, 2003, pp. 42-43). Yin also discusses that if a case study investigates a "case" as a whole, then it is called a "holistic" design; the current study followed a holistic design. Yin further explains that an investigator should define the case and boundaries of the case by introducing the time frame, the participants, and the location. Linking data to the propositions can be achieved by using "pattern-matching" to connect information gathered in a case study to the related literature (Yin, 2003).

Wilson (1979) (as cited in Merriam, 1998) defines case study as a process, "which tries to describe and analyze some entity in qualitative, complex and comprehensive terms not infrequently as it unfolds over a period of time" (p. 29). A case study is "an examination of a specific phenomenon such as a program, an event, a person, a process, an institution, or a social group" (Merriam, 1988, p. 9). Merriam summarizes characteristics of the case study as particularistic (meaning it should be focused) descriptive (meaning it should be detailed and replete with rich information), and heuristic (meaning it should help the reader to understand the phenomenon being studied)
(Merriam, 1998). In addition, Merriam explains that a case study should be intrinsically bounded. That is, the researcher should be fully aware of the time frame, the participants, and the location. In addition, data collection in a case study should involve triangulation (Merriam, 1998). Stake (1981) states that the knowledge obtained from a case study is more concrete, more contextual, and more developed by the understanding of the reader.

Based on these characteristics the case study, this was indeed an effective way to investigate standards-based instruction: a case study of a college algebra teacher to allow the participant's voice to be heard. The current study investigated practices of a teacher who utilized standards-based teaching strategies for a semester at a state university. Data were collected through interviews and classroom observations. The study participant also shared several documents during the interviews, including tests, homework, quizzes, lesson plans, and a statement of teaching philosophy.

Following Yin (2003) regarding case study design, the research questions were considered "case study questions," which are exploratory questions. The theoretical framework for the study and the standards-based instruction strategies were the propositions brought to the study. According to Yin (2003), propositions "direct attention to something that should be examined within the scope of study" (p. 22). Yin also discusses the unit of analysis, which defines the case. This unit follows the primary research question. For design of the current study, the case was a teacher who uses standards-based pedagogy in classroom teaching. The current study was a holistic, single case study. I obtained detailed, rich information from the study participant. Linking data to propositions is another component of case study suggested by Yin (2003), who stated that linking data to propositions can be achieved through "pattern matching," whereby
several pieces of information from the same case may be related to theoretical propositions. For purpose of this study, I used "pattern matching" to link the data from the study to the theoretical propositions. Data interpretation incorporated Merriam's (1998) ideas of "category construction" and use of Yin's "pattern matching" to connect each category of the data to the related theories.

The current study is an interpretive case study. Interpretative case studies require rich, thick, and descriptive data. These data "are used to develop conceptual categories or to illustrate, support, or challenge theoretical assumptions held prior to the data gathering" (Merriam, 1998, p. 28). The goal of interpretative case studies is to understand "the intricacies of a particular situation, setting, organizations, culture, or individual, but that local understanding may be related to prevailing theories or models" (Willis, Jost, \& Nilakanta, 2007, p. 243).

Taken together, elements of interpretative case study research design were effective in presenting detailed actions of a teacher who adopts standards-based teaching strategies. In this study, an interpretative case study approach was utilized to identify and understand the participating teacher's pedagogical skills, conceptions, and beliefs about his teaching.

## Research Context

The current study took place at a state university in southeastern region of the United States. The University is known for its diverse student population among comprehensive baccalaureate-level colleges and universities in the southern United States. In Spring 2011, the University's student body included 6,759 students, 56.1\% fulltime and $43.9 \%$ part-time students. Female students made up $69.5 \%$, and the average age
of the students was 29 years. The student body consisted of $63.2 \%$ of African American students, $24.5 \%$ Caucasian students, and $12.2 \%$ from other ethnic groups. The University has six subschools, including colleges for arts and science, information and mathematical sciences, health, business, and graduate studies. At the time of the study, the University offered 40 baccalaureate majors and 8 Master's degree programs. The graduation rate of the students is $24 \%$, and the retention rate is $61 \%$.

## The Role of the Researcher

Vissier (2000) discussed that the position of a researcher affects access to the subjects and the information the researcher provides. Vissier (2000) also mentions that an insider's position (referred to as emic) in a research project has an advantage over an outsider's position (referred to as etic) because the insider will be viewed as neutral, which makes it easier to collect information. However, Mullings (1999) discusses that outsiders may be are more likely to be objective in observing behaviors without distorting their meanings. I personally feel that both positions have their merits and demerits.

Relating to the ideas of both positions, I consider myself an outsider. I directly observed the participant and do not work at the same school with this participant. Yin (2003) mentions that field trips to the research site, which might include data collection through observations and interviews, are considered direct observations.

In a qualitative study, the researcher is the primary data collector (Merriam, 1998). As the main data collector of the current study, I collected data through interviews, classroom observations, and shared documents. It was my responsibility to analyze and report the result effectively; it was also crucial to identify my biases and beliefs. Merriam (1998) discusses the importance of knowing the researcher's biases
because it could affect the final product. My biases and beliefs are a direct result of my teaching experiences.

I am an international mathematics teacher with many years of teaching experience at both the secondary and university levels, which has required several years of professional development. I have found that some students view mathematics as a difficult subject that is unrelated to real life. I also believe that student attrition rates and mathematics achievement depend on the things teachers do in classrooms. Mathematics teachers, instructors, and professors should not see mathematics as a subject that requires the memorization of formulas and procedures without understanding. Instead, mathematics instructors should allow students to experience mathematics, engage in social interaction, and use their own cultural knowledge. Students should be given the opportunity to bring their previous experiences to current learning and be actively involved in the learning process. Active learning can be achieved through standardsbased teaching techniques. For instance, students should be engaged in group work and interactive questioning from the teachers, and they should also be allowed to explore new information. Additionally, teaching and learning should be culturally related because of the variation in students' backgrounds. The theoretical frameworks guiding the current study were culturally relevant pedagogy theory, symbolic interaction theory, experiential teaching theory, and standards-based instruction techniques. My beliefs are aligned with the frameworks supporting the study.

## The Study Participant

The participant of the current study was a college algebra instructor who utilizes a standards-based pedagogical approach in his classroom. The participating instructor was
selected based on purposeful sampling. Purposeful sampling occurs when a researcher selects a "sample from which the most can be learned" (Merriam, 1998, p. 61). The advantage of purposeful sampling is to collect detailed information. Purposeful sampling is aligned with the idea that (as seen in Merriam, 1998) "the logic and power of purposeful sampling lies in selecting information-rich cases for study in depth" (p. 61). Also, the study participant is representative of instructors who implement standardsbased teaching strategies in their classrooms; the data collected in this study represents this participant's experiences.

## Participant Selection

To locate an instructor participant, I first contacted the chair of the mathematics department at the university selected for the study took place to obtain a list of all college algebra instructors. After the chair gave me permission to speak with these instructors, I contacted each person on the list, informing them of the study and requesting a date and time to meet with him or her to provide further information about the study. I met with everyone who agreed to meet with me discussed the nature and expectations of the study. I also shared the requirements for participation (as outlined in the consent form) and the parameters for standards-based instruction. The parameters used to identify participants were based on the AMATYC's framework for standards-based instruction. A potential participant would apply all or some of the following strategies: (a) use multiple instructional strategies that encourage active student learning; (b) use cooperative learning, discovery-based learning, interactive lecturing and question-posing, and writing manages a learning environment that engages student interaction; (c) integrate technology as a tool to help students discover and understand key mathematical concepts during
instruction and assessment; (d) connect mathematics with other experiences; (e) use multiple approaches to solving mathematics problems; and/or (f) allow students to "experience" mathematics.

Each instructor was also informed that only one person would be randomly selected from all of those meeting the requirements and agreeing to participate in the study. Upon random selection, a college algebra instructor, referred throughout the study as Paul (a pseudonym), was chosen.

## Data Collection

The methods of data collection used in the current study were interviews, documents shared at interviews, and classroom observations (see Appendix A). Some of the artifacts shared during the interviews were tests, homework, quizzes, lesson plans, and Paul's teaching philosophy. According to Merriam (1998), different sources of data help the researcher better understand and describe the case of a study. Smith and Glass (1987) discussed that the main ways of collecting data in naturally situated research studies are through field observations, interviews, and the gathering of related documents. In addition, Chase (2005) states, "a narrative may be oral or written and may be elicited or heard during fieldwork and interview" (p. 652). Smith and Glass (1987) also specify that sources utilized during data collection depend on the researcher and the type of research.

## Interviews

According to Yin (2003), "the most important source of case study information is the interview" (p. 89). The aim of an interview is to discover the thought process of the participant (Merriam, 1998). Merriam (1998) further argues that interviews are necessary
when we cannot directly observe an event. Dexter (1970) (as cited in Merriam, 1998) mentions that "interviewing is the preferred tactic of data collection ... it will get better data or more data or data at less cost than other tactics!" (p. 72). Rubin and Rubin (2005) point out that:

Interviewing is about obtaining interviewees' interpretations of their experiences and their understanding of the world in which they live and work. Interviewers should not impose their views on interviewees. They should ask broad enough questions to avoid limiting what interviewees can answer, listen to what interviewees tell them, and modify their questions to explore what they are hearing, not what they thought before they began the interview. (p. 36)

Chase (2005) specifies that because interviewees usually speak in generalities instead of being specific, it is vital for the interviewer to be prepared to handle the interview. Yin (2003) discusses the strengths of interviews, stating that interviews are "focused directly on case study topic and provides perceived causal inferences," while the weaknesses of interviews are "bias due to poorly constructed questions, response bias, inaccuracies due to poor recall," and the ability of the interviewee to give responses that the interviewer wants to hear (p. 86). Yin (2003) discusses different types of interview (e.g., open-ended interviews, focused interviews, and structured interviews), and Smith and Glass (1987) further mention interview types to include story-telling interviews and semistructured interviews.

I used a semistructured interview for the current study to guide the interview for the participant. (A copy of the interview protocol is presented in Appendix B.) I provided the participant with the interview questions a few days before each interview date because I wanted insightful, reflective answers. This approach worked well during the pilot study. I noticed during the pilot study that one participant, when given interview
questions 3 days before the interview, provided more detailed and useful information than other participants given the questions 3 hours in advance of the interview.

On the day of the first interview, I began with conversational exchange to make us both feel comfortable. After introductions, I revisited the purpose of the study and told the participant that there were no "wrong" or "right" answers. There was a total of three interviews. During the interview process, as I sought data to answer my research questions, I noticed that information seemed somewhat cyclical at one point. That is, I started hearing the same things from the participant. In addition, semistructured interview questions were used because they allowed me to ask follow-up questions when necessary. Further, the interview questions were open-ended, which encouraged the participant to talk more. This interview method agrees with Yin (2003), who argued that "case-study interviews are of an open-ended nature, in which you can ask key respondents about the facts of a matter as well as their opinions about events" (p. 90).

In addition to written notes, I used a tape recorder to capture the complete discussion; the participant was aware of use of the recorder. Recording the interviews allowed me to listen several times afterward. Rubin and Rubin (2005) suggest that probing and follow-up questions are very useful during interviews. Following this method, I transcribed each interview immediately following the interview, which allowed me to retrieve as much as possible from the interview, including his gestures, laughters, discourse, and intonation. Then, I explored each interview in search of themes that pertained to the research questions (Rubin \& Rubin, 2005). These themes were used to summarize each interview. After I had gathered, transcribed, briefly analyzed, and compiled each interview, I created a file for single analysis of each interview.

Although artifacts (also called "documents") were not part of original data collection plans, during the interview process, Paul shared several documents relating to his teaching. These documents were useful in the current study. Merriam (1998) defines document as a "wide range of written, visual, and physical material relevant to the study at hand" (p. 112). Merriam warned about the accuracy of documents, meaning that investigators should be very careful in reading and accepting the content of these documents. Merriam (1998) also discussed the importance of documents, explaining that "many documents are easily accessible, free, and contain information that would take an investigator enormous time and effort to gather otherwise" (p. 125). Yin (2003) concurs that documents are important in case study data collection. Documents can provide detailed information to support information from other sources. According to Bogdan and Biklen (2007), the document collection includes diaries, memos, statements of teaching philosophy, and scrapbooks. Bogdan and Biklen (2007) further discuss that the documents the participants have written themselves should be used with interview and observation data in qualitative research.

Paul shared various documents, such as tests, homework, quizzes, lesson plans, and his teaching philosophy. In addition, he shared his students' reflections on his teaching. I analyzed the artifacts presented and identified themes that related to the research questions.

## Classroom Observations

As mentioned previously, classroom observation was another source of data collection. Patton (1990) mentions that use of observation as a means of data collection can lead to a deeper understanding of a research environment. It also provides
opportunities for the researcher to collect more data that the participant might not have discussed or be willing to discuss during interviews. Furthermore, Yin (2003) explained that there are two types of observations: direct observations and participant observations. I utilized direct observation because I did not work in the same school as the participant. Yin (2003) described strengths of direct observation, including that it covers events in real time and covers the context of event. The weaknesses are that is can be timeconsuming, there is selectivity and reflexivity, and the cost/hours associated with human observation.

Merriam (1998) explains that observation should take place in the natural setting and that data is first-hand information. Merriam (1998) states, "observation is the best technique to use when an activity, event, or situation can be observed firsthand, when a fresh perspective is desired or when participants are not able or willing to discuss the topic under study" (p. 96). Thus, observations help the researcher observe nonverbal communication, which is not always possible via interviews. Observations help the investigator notice phenomena that may lead to further interview development.

In the current study, I observed the participant while teaching to help answer my research questions. Building on my past experiences and the literature, when I observed the class, I arrived to the room before the start of class so that I could make note of certain things. I made note of the environment, including the teacher's desk, the arrangement of visuals, and the posters on the wall. I counted the number of students in the class. I used observation guides to record field notes during the observations (see Appendix C). According to Dewalt and Dewalt (2001), field notes are the main source of data from participant observation. They mention that "memory is unfortunately more
fleeting and less trustworthy than field notes..." (Dewalt \& Dewalt, 2001, p. 142). I made great effort to document everything the teacher said and wrote on the board. All teacherstudent and student-student conversations were written down. I also noted nonverbal communication and even the time that some events occurred. After classroom observations, I read the field notes many times. Then I expanded these notes when I was able to add details. Dewalt and Dewalt (2001) noted the importance of expanding field note immediately afterwards. This way, details would not be lost. According to Dewalt and Dewalt (2001), an expanded field note "is the detail and completeness of the record that provide the richness and texture of the written product" (p. 148). By taking expanded field notes, valuable details were added to my classroom observations and further helped me answer the research questions. As I searched for themes related to the research questions, I planned subsequent observations as needed.

In conclusion, I believe direct observations were effective methods of data collection. They gave me the opportunity to triangulate the information the participant shared about his teaching practices.

## Data Analysis

After data collection-all field notes, interview transcripts, and other related documents-I began data analysis. Bogdan and Biklen (2007) define qualitative data analysis as "working with the data, organizing them, breaking them into manageable units, coding them, synthesizing them, and searching for patterns" (p. 159). According to Bogdan and Biklen (2007), analysis of qualitative data is a tedious task that might involve examination of many pages of field notes and interview transcripts. Miles and Huberman (1994) mention that coding data is the process of identifying themes
embedded in the data. Strauss and Corbin (1990) refer to the method of sorting data into categories (also called "themes") as "open coding." After identifying themes, a researcher must re-examine the categories and make adjustments based on the data present. Creswell (1998) explains that spiral data analysis, which includes data management; reading and memoing; describing, classifying, and interpreting; and representing and visualizing data.

Merriam (1998) reports that "data analysis is the process of making sense out of the data" (p. 178). Merriam (1998) further warns that a researcher should not wait until after data collection to start data analysis, advising that analysis should start immediately after the first instance of data collection and continue in the same fashion until all data is collected. Merriam (1998) points out that a study's findings can be presented in descriptive forms or theories or categories. According to Yin (2003), analysis of data includes three strategies, which include "relying on theoretical propositions," "thinking about rival explanations," and "developing a case description" (pp. 111-114).

Merriam (1998) and Yin (2003) both gave valuable suggestions that directed data analysis in the current study. My design was a single case study; thus, within-case analysis was utilized to analyze study and aided the formation of study themes (Merriam, 1998). As previously mentioned, I began data analysis after the first instance of data collection and continued throughout the study. My conceptual framework guided data analysis. This implies that standards-based instruction strategies served as rubric for data analysis. File folders were created during data analysis, as recommended by Merriam (1998). I created separate folders for the interviews, the classroom observations, and the artifacts. I used different color markers (one color for each teaching practice, one to code interview data, one for classroom observations, and one for artifacts) to identify Paul's
teaching practices. After coding, I grouped similar terms to form themes or categories, as instructed by Merriam (1998). In examining the ways these teaching practices were aligned with the characteristics of standards-based instruction strategies, I color-coded each teaching practices according to the tenet of standards-based instruction that it belonged. The data analysis process showed that the categories reflected "the purpose of the research [and were] exhaustive, mutually exclusive, sensitizing, and conceptually congruent" (Merriam, 1998, pp. 183-184). Based on the categories created during data analysis, I generated meaningful conclusions for the study.

In addition to using themes for the data analysis, I used pattern matching to relate the content of each theme to the related literature, which helped improve the validity of the study. Yin (2003) mentions that "for case study analysis, one of the most desirable techniques is using a pattern-matching logic" (p. 116).

## Ensuring Study Quality

Ensuring quality is important in developing study design. Lincoln and Guba (1985) suggest four criteria to ensure quality of a qualitative study, specifically credibility, transferability, dependability, and confirmability. Credibility can be achieved through persistent observation, triangulation, peer debriefing, and member-checking (Guba \& Lincoln, 1994). Transferability is accomplished through providing enough information to help the reader determine whether results are transferable to other situations (Lincoln \& Guba, 1985). Dependability involves implementing "inquiry audit," where the output and process of the research go through the review process while confirmability includes providing "raw data, analysis notes, reconstruction and synthesis
products, process notes, personal notes, and preliminary developmental information" (Hoepfl, 1997, p. 14).

Yin (2003) mentions criteria for judging the quality of a case study. These include construct validity, internal validity, external validity, and reliability. According to Yin, construct validity implies "use of multiple sources of evidence" (triangulation) and establishing "chain of evidence" in a study (p. 34). Also, it involves asking participants to review the data collected. Further, Yin describes that "a case study involves an inference every time an event cannot be directly observed" (p.36). An investigator who checks for the correctness of the inference and thinks about rival explanations is dealing with internal validity. Yin (2003) discusses that interval validity can be implemented in a study by "pattern-matching," "explanation building," "rival explanations," and "logic models" (p. 34). Merriam (1998) explains that internal validity concerns the reality of the study. Merriam (1998) lists six ways to check for internal validity: triangulation, member checks, long-term observation, peer examination, participatory or collaborative modes of research, and examination of the researcher's biases.

Further, According to Yin (2003), external validity refers to the possibility of generalizing findings of one case study to another context. Yin mentions that tactics for external validity are the use of "theory in single-case studies" and the use of "replication logic in multiple-case studies."(p. 34). Merriam (1998) shares Yin's (2003) definition of external validity. In addition, Yin defines reliability as repeating the same procedure in the same context to see whether the findings are the same. If the findings and conclusions match, then the study is reliable. Moreover, Yin argues that the purpose of reliability "is to minimize errors and biases in a study" (p. 37). Yin (2003) also describes the means
through which reliability is achieved as "use of case study protocol" and "developing case study database." I used Yin's (2003) and Merriam's (1998) criteria to ensure the quality of the current case study.

For construct validity, I referred to Yin (2003), using multiple sources of data collection. I also asked the participant for a member-check. In terms of internal validity, I incorporated views from Merriam (1998). This study involved triangulation, memberchecking, semester-long classroom observation, and examination of the researcher's biases. Pattern-matching, part of internal validity, was used in this study as well.

External validity was also used to ensure quality. Since this was a single-case study, I referred to Yin (2003) recommendations to apply theory to ensure the study's external validity. This external validity was achieved by using findings from the current study to make meaningful conclusions. For reliability, this meant to repeat the same study in the same context to see if findings will match up (Merriam, 1998). Repeating this doctoral study was impossible due to time constraints, but the reliability of the study was achieved through thorough case study design and methodology.

## Ethical Considerations

The current study involved human subjects; therefore, it is important to discuss the implementation of basic ethical principles. According to the Collaborative Institutional Training Initiatives (CITI) (www.citiprogram.org), the three basic ethical principles are respect of persons, beneficence, and justice. The participant in the current study was referred to using a pseudonymous name throughout the current study. In addition, I did not interfere with the participant's decision-making process in determining participation in the study. Relevant study information was shared with the participant; the
participant also had the opportunity to sign the informed consent form. The consent form contained information such as research procedures, purpose, potential risks, and benefits. I also included a confidentiality plan in the informed consent form. This confidentiality plan includes the use of pseudonyms, keeping the data in a locked file, and a discussion about the people having access to the data.

The principle of beneficence, according to CITI, means securing the well-being of the informants and it falls under two rules: "do not harm" and "maximize possible benefits and minimize possible harms" (Tisdale, 2004 p. 21). The current study follows these rules. I made sure the subject was protected despite the benefits of the study. For instance, the students' learning environment was not disturbed during classroom observations. Also, the process of collecting data did not affect the duties of the teacher. This is accord with Tisdale (2004), who states that "the principle of beneficence actually reminds researchers of what it means to protect and to do good" (p. 22).

Another ethical principle considered in the current study was the principle of justice. Tisdale (2004) narrates that it is not good for researchers to consider already burdened populations as participants. The participant of the current study, a college instructor, is not from a "burdened" group of people.

## Summary

The current qualitative single case study was chosen because I wanted to provide a thick, rich description of the actions of the college algebra teacher using standardsbased instruction. I sought to explain what standards-based instruction looks like in a mathematics classroom. The participant in the current study was a college algebra instructor utilizing standards-based instruction strategies. Data were collected during a
single academic semester at a State college located in southeastern region of the United States. Sources of data collection included three participant-teacher interviews, three classroom observations, and shared artifacts collected during the interviews. Interviews were teacher transcribed using a word processor. Data analysis involved data coding and theme formation. Study quality was achieved through construct validity, internal validity, external validity, and reliability. The CITI initiative rules about confidentiality and ethics were implemented as well.

## CHAPTER 4

## THE STORY OF STANDARDS-BASED TEACHING

The current study tells the story of a college algebra instructor, referred to as Paul, who uses a standards-based teaching style. As a researcher, I was interested in finding evidence to support Paul's use of standards-based teaching practices. I observed Paul in his teaching environment to determine the things that allow him to consider his teaching style "standards-based."

Paul, the study participant, usually begins a class period by checking attendance; calling the students' names was followed by important classroom announcements. After announcements, Paul usually tried to build a connection between previous topics and the present topic. During my first classroom observation, Paul reviewed previous topics (i.e., horizontal and vertical shift, reflection across $x$-axis, reflection across $y$-axis) before discussing the new topic, which was vertical stretching and shrinking. Paul used the questioning method as well as a graph program to get students engaged when reviewing previous topics. The question-answer interaction helped students recall what they had covered in the last lesson. The graph program helped the students visualize the shapes of the functions.

After reviewing the previous lesson on horizontal and vertical shifts, the discussion of the new topic (vertical stretching and shrinking) was the focus of Paul's teaching. Paul usually started new topics by clarifying learning objectives and defining terms related to the current topics using PowerPoint. During the discussion of each objective, Paul employed many different teaching strategies to engage the students. In one of my observations, Paul explained several examples on how to sketch functions of
vertical stretching and shrinking using the graph program. He used a pointer to indicate what he was explaining on the overhead. That way, the students could easily stay on the same page with the instructor. Paul involved the students while explaining the concepts, and he called students by name for them to answer questions. Students were free to ask questions, and there was a constant dialogue between the teacher and students and among the students themselves. Through dialogue, students were able to understand the difference between shrinking and stretching. During practice time, Paul presented the following problems (via PowerPoint) for his students to work on, asking students to:
...describe what happens to the graph of $f(x)=x^{2}$ given the following equations:
A. $f(x)=4(x-3)^{2}+2$
B. $f(x)=-1 / 2(x-3)^{2}-5$
C. $f(x)=-(x+2)^{2}-5$

When the students were working on the problems, Paul moved around the classroom and interacted one-to-one with some students. Some of the students called for his attention spontaneously, some interacted with their neighbors, and some worked individually. The students were free to ask questions and share their solutions with Paul. When the practice time was over, Paul called students by name to answer the problems. As the students gave their answers, they were given words of encouragements such as "very good" and "excellent." Paul carefully examined each student's answer, and when the answer was not correct, Paul asked probing questions instead of directly giving students the correct answers. The students were active in the learning process and were encouraged to give reasons for each answer they provided. Further, Paul asked his students to change the following verbal representations to symbolic representations:

Write an equation for $\mathrm{y}=\sqrt{ } \mathrm{x}$ with the following transformation.
A. Shifted 2 units to the right, reflected across the x -axis and stretched vertically by a factor of 3
B. Shifted 4 units to the left, stretched vertically by a factor of 2
C. Reflected across the $y$-axis and shifted up 3 units.
D. Reflected across the $y$-axis, shifted 2 units to the right and stretched vertically by a factor of 3 .

Paul also drew the students' attention to important concepts such as basic functions. He emphasized that the basic function is $y=\sqrt{ } x$; therefore, it has to be part of each equation. Again, as the students were working on the problems, Paul moved around the classroom answering and checking students' answers and providing feedback. He gave the students enough time to think critically, and there was plenty of interaction between Paul and the students. Again, some of the students worked in pairs or small groups to solve the problems. When it was time to discuss the problems, Paul once again called students by name for answers. As Paul and his students were discussing the problems, there were questions from the teacher to the students, from the students to the teacher, and from students to other students. Additionally, Paul used the opportunity to discuss the questions the students asked him individually with the whole class.

Everybody in the class seemed to enjoy the environment, and there was plenty of interaction between the teacher and the students. All of the students tried their best to solve the practice problems, and each student was engaged and focused on his or her learning. Paul repeated important concepts several times. He used the graph program to explain the symbolic form of the functions and constantly used the questioning technique. The above practice problems allowed students to represent mathematics problems in different forms. As seen when observing Paul's classes, before he moved to the next
objective of a lesson, he summarized and evaluated student understanding of the prior objective by asking them questions.

In another observation, Paul engaged students by asking them to solve problems on the board. During this observation, Paul and the students discussed how to graph $f(x)$ $=\log x$. They first derived the table of values using the function. With the table of values, they graphed the function with the help of the graph program. While deriving the table of values and graphing the function, Paul was not the sole owner of knowledge; rather, he was a facilitator of the student learning process by asking them questions and giving them the opportunities to interact among themselves. After Paul discussed many examples with the students, he asked them to graph $f(x)=\log (x-4)$. He distributed graph sheets to every student. During the one-on-one interaction time, he provided feedback to each student and was very patient with them. Paul also used a questioning technique. He did not provide answers to the students who needed help; rather, he used probing questions to guide them. The students also interacted with their neighbors for help and consulted each other. From my observation, Paul's students were happy learners in the classroom. When it was time to discuss the questions, Paul asked one of the students to graph the functions on the board. The teacher put up the graph sheet on the overhead. The student, appreciating the opportunity, used the questioning method with his fellow students similar to the way Paul taught. The student asked questions related to how to graph the function, which promoted interaction in the class.

Paul did not discuss any new topics in a disjointed form; rather, he allowed the students to build mathematical connections. This element was present during all classroom observations. In the third observation, while discussing properties of
logarithm, Paul asked students questions about properties of exponents and related their prior knowledge to these properties. Paul emphasized the importance of knowing the properties of logarithm because their roles in trigonometry and calculus. Additionally, during the second observation, when the class was discussing oblique asymptotes, Paul emphasized the importance of long division. He asked students if they remembered long division, and he related the concept of oblique asymptotes to student prior knowledge of long division.

When Paul taught new topics, he often repeated key terms, emphasizing and repeating important concepts when appropriate. During the second observation, Paul and his students discussed how to find vertical asymptotes and the three conditions for finding horizontal asymptotes. As the lesson went on, Paul repeated "to find the vertical asymptotes, if there is no common factor, set the denominator equal to zero" several times. Similarly, he repeated the three conditions for finding horizontal asymptotes many times. When repeating the concepts, Paul either asked his students questions to repeat the important concepts or repeated the concepts himself. Through the repetition of key concepts, Paul reminded the students of what they had discussed.

Paul usually ended his class by evaluating their understanding of the daily lesson. He evaluated students by asking them to solve problems on their own or answer questions orally. The evaluation phase helped Paul bring closure to his lessons. After evaluations, Paul often took a few minutes to answer individual student questions, and he was always the last person to leave the classroom.

My classroom observations of Paul collectively revealed that he created a conducive environment for students to learn the topics. This conducive atmosphere made
students feel relaxed, engaged, and free to communicate with the teacher and among themselves. There was periodic laughter in the class, which made the learning environment even more relaxing. Paul moved from one place to another when he explained the material. He encouraged student participation by giving the pupils handshakes and high fives and using phrases like "very good." Paul was cheerful, excited, and full of energy, and the students were excited as well; they focused on their learning and quickly responded to Paul's questions. The students' expressions clearly indicated that they enjoyed learning in the class. There was no frustration on behalf of the student. While observing Pau's classes, I wondered where the time went because everybody in the class, including myself, was excited about the teaching and learning process.

Paul repeated key terms and provided reasons and explanations for each topic. He was polite and listened attentively to questions from every student. Whenever he explained materials, he faced the students and maintained eye contact to gauge their understanding. He was also patient each time an incorrect answer was given. He did not tell the student that he or she was wrong; rather, he used probing questions to involve the student and the rest of the class to arrive at the correct answer. Paul called on a different student each time he asked a new question. That approach gave many students the opportunity to participate. Paul also gave his students his attention each time they asked questions, and he respected their opinions.

Through asking questions, students were able to discover new concepts by building on their existing knowledge. Paul's strategy of allowing the students to discover new concepts aligns with the AMATYC's (2006) idea that teachers should incorporate
discovery learning while teaching. In addition, use of technology was a regular part of Paul's lessons; this helped captivate and reach all students, considerate of the different learning styles. By building mathematical connections, Paul was able to show his students that mathematics is not a disjointed topic. The teaching practices observed during classroom observations were discussed during the interviews and were evident in the artifacts, which served as a means for data triangulation.

## CHAPTER 5

## FINDINGS AND DISCUSSIONS

In the current study, I used a qualitative case study to investigate the teaching practices of a college algebra instructor, Paul, who used a standards-based teaching approach in his classroom. The research questions that guided the study were:

1. What teaching practices are used in the mathematics classroom of a college algebra instructor?
2. How are the teaching practices of this college algebra instructor aligned with the characteristics of standards-based instruction?

In identifying Paul's teaching practices, I used color-coding to categorize each teaching practice observed according to the tenet of standards-based instruction to which it belonged and to sort and analyze interview data, class observations, and artifacts. However, it is important to understand Paul's background before discussing his teaching practices. Paul's teaching practices are also presented in this chapter as themes, all of which were derived from within case analysis of Paul's teaching (Merriam, 1998). Finally, this chapter shows how Paul's teaching practices are aligned with standardsbased instruction.

## Background of the Study Participant

Paul is an African American college algebra instructor with 31 years of teaching experience at both the secondary and collegiate levels. He has successfully implemented standards-based instruction strategies into his teaching and has gotten outstanding results while teaching of college algebra. Paul was an instructor at a state university in southeastern region of the United States. At the time of the study, Paul was teaching an

Introduction to Statistics course and two college algebra courses. Paul was also
responsible for supervising student teachers.
Paul's decision to become a mathematics teacher was triggered by his experience
as a student and the prospect of seeing his own students understand mathematics
concepts. As a student, Paul experienced "bad teachers" and "good teachers." According
to Paul, his 9th-grade mathematics teacher, his music teacher, and a few others were
"good" teachers. These teachers motivated Paul to become an educator because of their
excitement toward teaching. Paul particularly wanted to become a mathematics teacher so
that he could motivate his students the way his 9th-grade mathematics teacher motivated
him. During first interview, Paul stated:
As a student, well I would probably say that I was turned onto math by Mrs. [Edwards]. She was my 9th-grade math teacher and she had such a passion for teaching that it rubbed off on me. She would hold my paper up at the end of a test and say that nobody could sling a math pencil like [Paul] and hold my paper up to the class because I had made a 100 on a test or something. And she turned us on, turned me on to math. Mrs. [Edwards] and Mrs. [Brown], I had a double major math and music in college and Mrs. Brown was my choral teacher and my voice teacher in college and both of them had that passion for teaching. They - you knew that they loved what they were doing because each day they came with that same kind of excitement, that same kind of energy. I never saw them with low energy, with a low energy level. They were always so excited about what they were doing. I am so glad that I had Mrs. [Edwards] because it was her excitement about math that turned me onto math and that's why I really became a math teacher. And so my experiences as a student evolved around those experiences that turned me onto math and I also took piano when I was in grade school so it was those two kinds of things that gave me my passion for what I wanted to do and what I wanted to do was become a teacher.

However, Paul's experience with his history teacher was not rewarding, as he recalled, which made him decided that he would not bore his students like his history teacher. Paul shared:

I think seeing and I think, and I think it's a double-it's two kinds of teachers that I experienced. I also experienced bad teachers and I knew that I did not want to be
like them. I experienced the history teacher who made the class so boring that I said I would never want to be a teacher that would bore a student like I was bored and that is probably why I hate history to today, I hate history in any form, history of math, history of music, history of anything because my history teacher did not convey a passion for teaching. It was more of a job So I think it was the experience of the wonderful teachers as well as the experiences of those teachers that were nonmotivating that helped me evolve into the teacher that I have become as far as trying to generate excitability within the students, trying to get them motivated to learn the material and trying to find ways that I could get them to understand the material. It is about those little nuances, those what I call my tool bag of strategies to try to get them to understand and I think my experiences as a student seeing those different types of methodologies used, that helped me to become the teacher that I am.

In addition to Paul's experience as a student, his desire to become a mathematics teacher was also attributed to the prospect of seeing his students comprehend mathematics concepts and helping the students use their inside knowledge to learn mathematics. Paul mentioned that he also wanted to become a mathematics teacher because:

I enjoy seeing the "ah ha" moment. I enjoy seeing the light bulb. I think for me as Hymie Escalante say it, math is a great equalizer and so standing in front or facilitating the learning process and seeing students saying oh I got it, that makes sense, now I understand, those moments were-I mean those are the moments that I cherish because it says to me that the students are comprehending what you are trying to convey whether or not it is with a lecture or with them being in groups discussing or whether it is my asking them a question about a particular topic. The "ah ha" moment, the light bulb moment, it comes on because they then get an understanding of what the math is all about and they have an intuitive understanding of what it's about and so for me it is that experience of helping them evolve, helping them realize what's within them and making sure that they can possibly get all out that's within as far as math, learning math is concerned.

Following his dream, Paul obtained a Bachelor of Science degree in mathematics and music education. After graduating from a university and obtaining his teaching certification, he started teaching mathematics at the high school level. He taught at three different high schools, serving as department chair at each of the schools. While Paul was teaching in high school, he obtained his Master of Science degree in mathematics education, his education specialist degree in mathematics education, and his Doctor of

Philosophy in teaching and learning with a concentration in mathematics education. Paul also earned numerous awards during his secondary teaching career. Paul was awarded "teacher of the year" and "star teacher" for several years at each of school he taught at. He was the system-wide "teacher of the year" for the county he taught in during 2006 and was a top-ten finalist for the State of Georgia in 2007. Regarding his rewards, Paul said, "I think it was by the grace of God that I received the awards; I think it was student's understanding my passion for the job." In addition to Paul's high school teaching career, he taught part-time at the university level for years. He was a full-time assistant professor at a state university during the time of this study. Altogether, Paul had taught four semesters of college algebra. Paul's students recognize his dedication to teaching and have complimented him on his enthusiasm, his commitment, and the excitement he demonstrated while teaching. Paul's students mention that all of these characteristics served as inspiration to them.

## Standards-Based Teaching Practices: Emergent Themes

Six major themes emerged during data analysis. These themes were (1) mathematics communication, (2) proper use of technology, (3) building mathematical connections, (4) multiple representations, (5) motivating students to learn mathematics, and (6) repetition of key mathematical concepts.

## Mathematics Communication

Mathematics (or mathematical) communication "is a way of sharing ideas and clarifying understanding. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing" (National Council of Teachers of

Mathematics [NCTM], 2000 p. 60). Mathematics communication was identified as a major theme in Paul's teaching, as consistent suggestions from culturally relevant theory (Ladson-Billings, 1994) and symbolic interaction theory Yackel (2001) on the ways to improve mathematics learning. These two theorists argued that mathematics education should include communication between teachers and students as well as among students. Also, the American Mathematical Association for Two-Year Colleges (1995) pointed out that in standards-based instruction, mathematics faculty should incorporate teaching strategies that help students "develop both oral and written communication skills" (p. 11). As presented in the literature review, other studies (Boaler, 2006; Ellington, 2005; Gutierrez, 2000; Iannone \& Nardi, 2005; Kagesten \& Engelbrecht, 2006; Thomas \& Santiago, 2002) also support the idea that mathematical communication has a positive impact on student learning in mathematics.

Based on participant interviews and direct observation in the current study, mathematics communication was one of Paul's major teaching strategies. During the first interview, mathematics communication was mentioned several times as a successful teaching strategy. When I asked what his expectations are on his college algebra students, he responded:

Well I expect them to first come to class then I expect them to be a participant in the learning process and I explain that to them at the beginning, that I expect them to be a part of the learning process meaning that they are not just going to come and sit and not be a part of the process. I expect them to ask questions if they don't understand. I expect them to support other's answers or disagree with other answers and then support their disagreements there. It's all about the dialogue. I expect my students to communicate with me and as I tell them over and over again the only way that I am going to know is if you communicate with me. If you don't communicate with me I don't know what is going on and so therefore I firmly believe in strong communication.

Paul shared that students learn better through interaction with their teachers and among themselves. Paul stated that his role as a teacher is to promote communication in his classes:

I'm trying to not just stand in front of students and impact knowledge but it is a communication between myself and the students and it's that communication, it is that verbal, that audible kind of things that go on in the classroom.

Data analysis indicated that promoting communication in the classroom allowed Paul's students to interact with him and among themselves, ask questions, and freely express themselves in the learning process. Also, according to Paul, interaction helps students retain mathematical knowledge and develop deep understanding of mathematics concepts and also to determine whether they are solving a particular mathematics problem correctly. Dialogue in Paul's mathematics classroom allowed the students to make sense of mathematics knowledge, think, react to comments, and reason. Similarly, communication allowed Paul to provide feedback to students through asking questions, which helped him determine whether or not his students understood the material. He indicated that "by asking questions, asking them to repeat what I have just said, or tell me what it is that you understand. That is the only way that I can determine whether or not they are understanding" or "What did you hear me say? Once they repeat it then I have an understanding of whether or not they really heard what I said or if what they heard me say was totally foreign to them." Paul stated that through communication, he was able to figure out his students' needs and weaknesses and also find holes in their mathematics knowledge so that he could fill the holes. This approach has been one of Paul's best teaching experiences. He stated in the first interview that one of his best teaching experiences has been "breaking the mold of you're just going to come to class, you're
going to listen to me lecture, and then you're going to leave and I think it has been fun for me in breaking that stereotype." To maintain mathematics communication in his classrooms, Paul used questioning methods, one-on-one communication during problemsolving, board participation, and nonverbal communication in his teaching of college algebra.

The questioning method. Questioning is one of Paul's major teaching strategies, which is consistent with AMATYC (1995/2006) suggestions for the improvement of teaching and learning mathematics. In one interview, Paul shared:

My methodology of teaching is that of questioning. Even though, as I said, from the collegiate level, there are times that I don't have as much time to ask as many questions as I want, but I still - I would not feel comfortable going through a full lesson without having asked questions, without having asked for feedback from students: do you think this is right? Do you think this is wrong? Tell me, what would I do here? You tell me, I'm going to write it down. That's another way of my getting feedback.

While directly observing Paul's classes, questioning was part of his teaching from the beginning of class until the end. It was impossible that one of Paul's students would not answer or ask questions. Paul used many strategies to make his questioning methods effective. For example, he called students by their names during question and answer interactions. He mentioned that:

Because my name is important to me, and I've discovered that when students know that you know their names, they are more prone to learn. They're more eager to come to class. I will never forget one student that I spoke to when we were not in class, I spoke to him and I said, hello, [Kevin]. He looked at me and he said, hi, you spoke to me. You know my name. I said, yes, [Kevin], I know your name. So it's important. I think it provides that personal touch, also. Because I think if students realize that you care, you are personable, then I think that they are going to be more motivated to come and partake in the learning environment. So my name is important to me, so I know that my students' names are important to them. And so that's why I try to learn their names at least by the second week of class.

I noticed that during class observations, Paul knew all his students' names. Most university professors do not know students' names. Haar, Hall, Schoepp, and Smith (2002) discussed that an effective teaching strategy is for teachers to know their students on a personal level. Paul's students were alert and engaged in the learning process because they never knew when Paul would call on them to answer a question. The following example, in which Paul passed out graph sheets to his students, illustrates how questioning was used in Paul's classroom (The question was to find the $y$-intercept, x intercept, vertical asymptotes, and horizontal asymptotes for the function, $\mathrm{F}(\mathrm{x})=\mathrm{x}+1 / \mathrm{x}-$ 3; use test points and graph the function):

Paul: Find the x-intercept of the rational function. Do I have to set the numerator equal to zero or the denominator equal to zero.
Class was silent.
Paul: (Rephrased the question and asked) if I want that function to be equal to zero, which part of the function will be zero.
Class: The numerator.
Paul: (Set the numerator equal to zero and solve for x.) Therefore, the x-intercept is $(-1,0)$ and for the vertical asymptote is there a common factor between $\mathrm{x}+1$ and $\mathrm{x}-3$.
Class: No.
Paul: What do I set equal to zero to find the vertical asymptote?
Class: The denominator.
Paul: Therefore I set $\mathrm{x}-3=0$, so I get x equal to what?
Class: 3
Paul: The equation for my vertical asymptote is $x=3$. For horizontal asymptote, what I am looking at?
Class: Numerator and denominator
Paul: Do they have the same degree?
Class: Yes
Paul: What is the degree?
Class: 1
Paul: So what is my horizontal asymptote?
Class: 1
Paul: Not just 1. It is $y=1$. What I want you to do now is to draw dotted line at $\mathrm{x}=$ 3 for vertical asymptote and $\mathrm{y}=1$ for horizontal asymptote on your graph sheets. Also, graph x-intercepts and y-intercepts on the graph sheets.

Paul moved around the room to interact, provide feedback, and answer the students' questions. Some students interacted with their neighbors. Some students worked alone, and some students rolled their seats so that they could talk to their neighbors in the back. At Paul's discretion, when it was time to discuss the problem, he put the graph sheet on the overhead board so that students could see. Paul and his students graphed the horizontal and vertical asymptotes on the graph sheet, including $x$ - and $y$-intercepts. The dialogue continued:

Paul: As far as region is concerned how many regions do I have?
Student 1: 4
Paul: (Counts sections/regions to show the students there were 4 regions.) Now, I have to figure out where the two branches of the graph are going to be.

He reminded the students that they need to find the x -values less than vertical asymptote and $x$-values greater than the vertical asymptote so that they could determine whether the corresponding y-values are above or below 1(horizontal asymptote). So, Paul asked the students to find the values of $f(4), f(8), f(-4), f(-8)$ using the function $f(x)=x+$ $1 / \mathrm{x}-3$. Paul was moving around again, supervising the students' work. There was plenty of classroom interaction among neighbors and between teachers and students.

Paul: What did you get for $f(4)$ ?
Class: 5.
Paul: What did you get for $f(8)$ ?
Class: 9/5.
Paul: $f(-4)$.
Class: $3 / 7$.
Paul: f (-8).
Class: 7/11.
Paul: Now, you have to determine the values that are less than or greater than 1.
He pointed to the graph sheet on the overhead and asked whether each value of y is below or greater than $\mathrm{y}=1$ :

Paul: What about 3/7?
Class: Below.
Paul: What about 7/11?
Class: Below.
Paul: What about 9/5?
Class: Above.
Paul: What about 5?
Class: Above.
Paul explained for x -values equal to -4 and -8 , the y -values are below the line $\mathrm{y}=1$ and while $x$-values equal to 4 and 8 , the $y$-values are above the line $y=1$. He then graphed the right positions on the graph sheet and reasons behind them. Dialogue ensued:

Student 1: How do we determine the values of x ? That is, the test points? Paul: If you have one vertical asymptote, you have to get values to the right and left of the vertical asymptote but for two vertical asymptotes get values to the left, in between, and to the right.
Student 2: Why do we use $x$-intercept and y-intercept?
Paul: We use $x$-intercept and $y$-intercept because they are easy to find and it provides direction and the curves must pass through the x -intercepts and y -intercepts.

Paul explained in details how the vertical and horizontal, $x$-intercepts and $y$-intercepts help graph rational functions. He emphasized that they do not have to plot the points.

Paul's questioning method gave his students the opportunity to think because he required them to justify their answers, and he asked probing questions instead of directly providing answers. When I asked Paul why probing questions were important he stated:

That's standard-based all over the place. That is not telling, that is probing, that is questioning, that is asking scaffolding questions so that you are getting students to think rather than just giving them an answer. If you just give them an answer, it is sort of like giving a man a fish for a day and that's all he's going to eat for the day but if you teach him to fish, then he will have fish for the rest of his life and so it is my way of getting them to think rather than simply just giving them the answer. But again, it's to get them to think so that they can really monitor themselves. When they can think...because I've had it happen, they are about to ask a question and then they start talking to themselves as far as the answer is concerned...oh never mind, I just answered my own question and so they are thinking and so that gives them a way to monitor themselves because as I said, to them, I'm not going to be there on the day of the test to tell you what to do. I'm
not going to be with you in the dorm room when you're doing your homework so it's a way of getting them to really monitor themselves.

The questioning process in Paul's classroom was not one-dimensional. Students were free to ask questions, and Paul gave them undivided attention each time they asked questions.

## Paul emphasized:

I want every student to feel equally important and so it's important for me that when a student asks a question, that I give that student my undivided attention and let them know that it is okay to ask your question and I am geared to you to answering your question now and making sure that I've answered the question because I don't know if you've heard me say this, but after answering the questions, then I will say to them, does that make sense because again, I want to make sure that I have answered their question or I might even ask the question did I answer your question, does that make sense now, did I answer what you were asking and even I will have students to repeat their question to make sure I'm understanding what they're asking. Well, what are you truly asking me? I'm trying to make sure I'm understanding that so it's important for me as a person, if I were to ask someone a question, I would want their undivided attention, I would want them to respond and so I would want to do the same things with my students in that I want them to feel that I have given them my undivided attention and that I've answered their question.

Classroom observation revealed that the use of questions enabled students to communicate among themselves and respond to each other's question. The following example (pertaining to solving logarithm functions) from class observations illustrates the way Paul created opportunities for his students to respond to each other's questions. Students were asked to "Express as a single logarithm: $7 \log _{a} x+1 / 4 \log 16-1 / 3 \log _{a} 8$ ":

Paul: (Asked the students to share their answers.)
Student 1: $\log _{\mathrm{a}} \mathrm{x}^{7}\left(16^{1 / 4}\right) / 8^{1 / 3}$.
Paul: What is $16^{1 / 4}$ ?
Class: 2.
Paul: What is $8^{1 / 3}$ ?
Class: 2.
Paul: (Substituted16 ${ }^{1 / 4}$ with 2 and $8^{1 / 3}$ with 2 and wrote $\log _{a}\left(x^{7} \times 2\right) / 2$ on the board.) So, what happens to the 2 s ?
Class: They cancel out.
Paul: We have $\log _{\mathrm{a}} \mathrm{x}^{7}$.
Student 2: How is $8^{1 / 3}$ equal to 2 ?

Paul: Student 3, how is $8^{1 / 3}$ equal to 2 ?
Student 3: Because $2^{3}=8$ exponentially.
(Laughter in the class.)
Paul: Watch out for that. That is a big word.
Student 4: Why can't we leave $8^{1 / 3}$ as $3 \sqrt{ } 8$ ?
Student 5: Because $8^{1 / 3}$ is a perfect square.
Paul: If it is a perfect square, you do not have to leave the radical in there.
Paul allowed students to respond to their own questions. In addition to other strategies Paul used along with the questioning method, making mistakes on purpose to ensure that the students were paying attention was also incorporated. For example, one classroom observations showed Paul making mistakes on purpose:

Graph: $f(x)=\ln (x-2)$.
Paul: Which direction will -2 push the graph of $f(x)=\ln (x-2)$ ?
Class: To the right.
Paul: (Drew the graph $\mathrm{f}(\mathrm{x})=\ln \mathrm{x}$ on the board indicating the x -intercept as $(1,0)$. Then he connected to the current learning.) What will be the x -intercept for the graph of $f(x)=\ln (x-2)$ ?
Class: $(3,0)$.
Paul: What is the argument in $f(x)=\ln (x-2)$ ?
Class: $\mathrm{x}-2$.
Paul: What makes the argument zero?
Class: $\mathrm{x}=2$
Paul: Therefore, the vertical asymptote is $x=2$. You know what the graph of logarithm functions looks like. It opens to the left.
Class: No, it opens to the right.
Paul: I want to know whether you listen; it opens to the right. (He drew the graph on the board showing the x -intercepts and vertical asymptote.)

Paul insisted:
If you remember my talking about the comments about students feeling free to make comments about students questions, that's another way that they are interacting because the other day in class a student asked a question and a student chimed in to answer and that student's response gave the student that asked the question the student said "oh, that's what you were saying.

I said yes, but I've also learned to through the years that a student peer to peer response answer will be a thousand times better than anything I would have said so I also realized that and so that's another way that they interact with each other. It is because they're peers so they have an understanding of the question that they are asking and if one has an understanding he or she can provide that
explanation which would be different probably from mine and they get a better understanding and that's what I want.

Other strategies Paul incorporated while using the questioning method included positively commenting on students' responses by saying things such as "beautiful insight," "very good," and giving students handshakes and high fives.

One-on-one communication during problem solving. Paul also used one-onone communication to achieve mathematics communication in his class, which agrees with Yackel (2001). One-on-one communication often occurred during practice time. One of the components of Paul's model-objective demonstration participation model (ODP) is practice. Practice sessions allowed Paul's students to solve problems on their own in class. Paul mentioned:

With the college algebra, I think it is really important that they get a lot of practice in and that practice not just the practice from the standpoint of you being outside of the classroom but again, that practice that's inside so that I can really get a feel of where you are with this material and as I am monitoring, as I am walking around and seeing what they're doing, it gives me some general idea as to if they get this or no they don't get this.

Paul emphasized that practice time allowed him to assess student understanding because he did not want them to have the mentality of "I'm going to come, I'm going to sit, you're going to talk, I'm going to write down these notes, and then I'm going to leave."

Paul repeatedly noted that allowing students to practice mathematics problems in class is effective assessing their progress. He commented:

It plays a role as far as my assessing is concerned. I want to assess, as I said. Prior to giving a test, I want to see if my students understand. So when I am going through-because I told them my mode of operation is ODP, and that is the order of the day. What are the objectives? Are there any terms that we need to know? Then it is demonstration. That's what the D stands for. And then it is practice. And so it is for me that time that I can use to assess their understanding, because, again, I don't want them coming in, copying down three and four pages of notes not really knowing what they're copying down and then leave and have no
understanding. And then the next time they come back, I'm going over a new set of notes, another set of three pages or four pages, and them still leaving without any understanding. And for me, I want to at least understand that they have some intuitive idea of what's going on in class. And so for me, that is my way of assessing what they're doing.

I observed Paul's students solve problems during my classroom observations. As the students solved mathematics problems, Paul would move around and interact with students one-on-one. During this time, Paul was patient with the students and provided thorough feedback when necessary. Paul felt that students "feel more comfortable in that setting to ask a question rather than asking the question from the whole class standpoint, because it is more one on one." For example, during one classroom observations, Paul asked the students to simplify the logarithm expression: Write as a single logarithm: $6 \log _{b} x-2 \log _{b} 4+1 / 3 \log _{b} z$. During the discussion time, Paul reported that many questions came up as he was moving around. He said that one of the students asked how one knows that the negative sign between the first two terms in $6 \log _{b} x-2 \log _{b} y+1 / 3 \log _{b} 2$ is not a negative exponent. Another student asked Paul how they should know that the same negative sign does not affect the last two terms.

With this type of one-on-one interaction, students were able to share their questions with Paul individually, and Paul also used the opportunity to discuss the questions with the whole class. Without this one-on-one interaction, students would not have had the opportunities to ask these questions. In addition, Paul informed me that moving around during in-class practice allowed him to monitor what students were doing with their laptops. Paul shared:

I can't stand to be stationary when I teach and so I move around to sort of let them understand that and also with us being at [World University], because they have access to their laptops, they have laptops in class all of the time and so it is also a way of saying I'm monitoring what you're doing with the laptop because if you're
not doing something that we are engaging in class, then that really means that you're not supposed to be doing it and so it's another way of monitoring what they are actually doing because many times some of them do take notes with their laptops. Some of them are actually, as we are going over material in class, their on my math lab doing the assignment for that particular section so it's just a way of monitoring what they're doing.

Paul believed that one-on-one communication among students was best achieved during practice time. He mentioned instances in which students understand peer explanations better than the teacher's. Paul explained that one-on-one communication allowed the students to provide feedback to each other, to question each other, and to debate each other as to whether or not the answer is correct. As described before, students interacted with their neighbors and shared ideas during practice time. Rolling chairs in the classroom made it possible for the students to face their neighbors and discuss things.

Board participation. Another form of mathematics communication used in Paul's classes was board participation. During one of the interviews, Paul shared that he actually allows his students to go to the board and solve mathematics problems. Board participation allowed Paul to see "what they are actually doing at their desks and to have them do those kinds of problems again. It gives them a feel of either I know how to do this or maybe I told them how to do this." Paul explained further that:

When students do problems on the board, it gives them more of a foundation, they get a better understanding of the material because they're explaining it to someone else and again, the research says if I have to explain something to someone else and even if they ask me a question, then that means I'm going to have to think about what I just did and see if I either explain in the same way or if I have to reach into what I call the "tool box" and grab another strategy because I always tell them if you explain something to Johnny one way, Johnny does not understand and you explain it that same way to Johnny, then I say whose the dummy? So, you have to think about that. I think it really, really helps them in their understanding because they're going to have to explain it to other students and I think it really builds that foundation of understanding.

During classroom observation, after giving the class time to graph $f(x)=\log _{3}(x-$ 4) and $f(x)=\log _{4}(x+2)$, Paul signaled to the class that it was the time to go over the problems together. He asked one of the students to graph the functions on the board using a graph sheet, which was already on the overhead.

The student (called Student Teacher in the example dialogue) was excited to play the role of Paul. He used the questioning method just as Paul did to guide the students to solve questions:

The first problem was to graph $f(x)=\log _{3}(x-4)$.
Student Teacher: What is the vertical asymptote?
Class: 4.
Student Teacher: Which way are we gonna move it?
Class: Right.
Student Teacher: (Indicated the vertical asymptote on the graph sheet.) 1 plus 4 is what?
Class: 5.
Student Teacher: The ordered pair is $(5,0)$. (He represented the ordered pair on the graph and graphed the logarithm function which passed through the ordered pair. There was plenty of interaction between the Student Teacher and class. There was periodic laughter in the class).

The second problem was to graph $f(x)=\log _{4}(x+2)$.
Student Teacher: What is the vertical asymptote?
Class: 2
Student Teacher: 1 plus -2 is what?
Class: - 1 .
Student Teacher: So our point is what?
Class: $(-1,0)$.
Student Teacher: The graph will move to the right.
Class: No, it will move to the left
Student Teacher: I want to know whether you are paying attention.
(Laughter in the class because that is exactly what Paul does.)
Student Teacher represented the vertical asymptote and the ordered pair $(-1,0)$ and graphed the logarithm function on the board. The students were engaged and happy, and they participated in the learning process. Paul stood at the back of the classroom provided feedback.

During the second interview, I asked Paul how he selects students for board participation. He shared:

It just depends, I again, you have to know the student to know that it's okay. Some students, again, they have that fear of math, they have that fear of being in front of people and standing in front of people so you have to sort of know the student to know they love the limelight so they want to do these problems whether they're right or wrong and also some students don't want to be in front if they know they are not correct so we have to now the personality of the student and also know if it's okay, if they are not right, to send them to the board anyway to give them that exposure so it's about knowing the student as to.. Like there are some students in the class that I would not pick to go to the board because I know that they have that fear of not being correct and that fear of being in front of the students so you just have to get to know the student.

According to Paul, board participation is very useful in improving student achievement in mathematics. Paul shared his opinion that "all of the students that volunteered to go to the board to do problems; they did better on their tests because they were gaining that actual experience of doing the work. It was hands-on and they were doing it."

Nonverbal communication. Verbal communication was not the only channel through which mathematics interaction occurred in Paul's classes. He also created written communication opportunities. According to Kagesten and Engelbrecht (2006), writing in mathematics promotes learning. Paul mentioned that mathematics writing helped students think through the process of solving problems. For example, according to Paul, he asked his students to write a problem that dealt with a linear function. He communicated with the students by writing comments on their tests. That way, the students received positive feedback for the progress they made in mathematics. To illustrate, Paul gave the following example:

I will make notes on the test to say, for example, on the last test we had, they had to provide the equations for vertical asymptote. Well, in some of the problems, they had to cancel out common factors before they set the denominator equal to 0 so if they $\operatorname{did} x=4$ and $x=-3$, whereas the $x=4$ was the only equation, I would
put a simple note saying remember to cancel your common factors before you set the denominator equal to zero so it depends on, and even in solving logarithmic equations, some of them didn't change it to exponential so I would simply say change to exponential if you want to solve it so it really depends on the problem, what they have missed in the problem, that gives me the direction as to what notes I'm going to write about them.

In analyzing the tests, homework, and quizzes, Paul encouraged students to write
mathematical statements. He purposely stated on their tests to show their work when answering test questions:

As far as showing their work, I think it gives them a systematic way of doing things because I think it requires them to go step by step rather than trying to do the computations in their heads and again, I've had students that could actually do that but then I think for many students it's about the step by step procedure and if I give them a sort of prescription, then it allows them to be able to analyze and see what they're doing as they use the step by step procedure because again, for those that have the fear of math, I think doing it in order, it helps them as they go along and I think it's going to help them in future math courses to be able to organize their work so they can see what they're doing which means they can back track and see where their errors are because for example, in writing equations of polynomials, I say you want to write it in this form so that you can always go back to see did I do the step correctly and so it's a way of them really checking themselves.

The nonverbal communication Paul showed in his classes included smiles, eye contact, high fives, handshakes, changes in pitch, moving around the room, and hand gestures while teaching. All of these forms of communication showed the students that Paul enjoyed what he was doing, that he cared about their learning, and that he wanted them to feel relaxed while learning mathematics.

Paul's teaching is characterized by mathematics communication. With this communication, Paul was able to improve academic performance of the students, which led to outstanding results. Paul believed that there would be no true mathematics teaching without dialogue and interaction in the classroom. Only through dialogue and interaction do students feel free to share their knowledge and have their voices heard in the learning
process, which empowers students to become active instead of passive learners. His students' comments, which Paul shared with me, confirmed that they also perceived mathematics communication as one of the major strengths of Paul's teaching strategy. Survey questions and student comments, characteristic of mathematics communication, are presented in Appendix D. Students' comments were consistent with both classroom observations and interviews with Paul.

## Proper Use of Technology

The second theme that emerged as one of Paul's teaching practices was the proper use of technology. Paul's use of technology matches the theory supporting this study. The experiential teaching theory emphasized that teachers should allow students to "experience" mathematics. One way students can experience mathematics is through effective use of technology. Also, the AMATYC (1995/2006) and the NCTM (2000) argued that proper use of technology improves student learning in mathematics classrooms. These organizations emphasized that the use of technology helps students explore new mathematics concepts, graph and visualize mathematics concepts, test and prove conjectures in mathematics, and communicate mathematical ideas. Other research has shown that appropriate use of technology can positively affect students' mathematics learning (Mayes, 1995; Thiel, Peterman, \& Brown, 2008; Hagerty \& Smith, 2005).

Paul described in the first interview how he incorporates standards-based instruction strategies in his weekly lesson plans:

Well, mine would not necessarily be weekly, mine might be for each class session but I incorporate all of the different things as far as I try to incorporate technology because I utilize the graph program that we had; I also utilize the Power Point presentation. I do Power Point presentations along with using the graph and calculator; I have TI Smart View which is a calculator that is on the screen that I use that to show them different things; I use geometry sketch pad, I use a lot of
these things to try to make it as plain as I possibly can as far as what the concept is all about even I'm planning, I think about the questions that I want to ask and the questions that might be asked of me... I try to incorporate those things in the information that I am imparting to them. So it's all about knowing what the concepts are.

Paul shared that effective use of technology enhances student performance in mathematics, which was observed in his classes as well. He emphasized that technology is a captivating tool for the students:

Technology plays a big role in my class because, again, it's the picture. I want them to see it as well as the analytical part. I want them to see that if a polynomial has four zeroes, what it looks like. I want them to see that if the multiplicity of a particular zero is odd, I want them to see that it crosses the X axis. I want them to see that if it has an even multiplicity, that it's going to be tangent to the axis. So the technology for me is so critical because, again, we're living in a technological age. My first task as a teacher, and this is personal, I think my first task is to captivate my students. What is the best way for me to captivate my students? And I think that technology plays a role in that. If the students that I presently have, because we have to understand there is the iPods, there's the phone, there's the laptop. There's all of these things. If I go in with the old school way of doing things, I don't think it's going to captivate them. As loud as I talk, as in their faces as I try to be, I still think that the technology serves as a method of, I'm used to technology as a student, so I want to see technology at my class. So I think that it provides that mechanism of at least capturing their attention. And then serving all of the other purposes that I've talked about as far as the visual nature of the class.

Paul mentioned that technology plays a big role in his class because "it's the picture. I want them to see it as well as the analytical part. So I think that it provides that mechanism of at least capturing their attention." During direct observations, Paul used different kinds of technology, including graph programs, PowerPoint, TI SmartView, and Geometer's Sketchpad.

Graph programs. During the first classroom observation, Paul used a graph program throughout the class period. With this program, students were able to see the difference between horizontal and vertical translations and the difference between stretching and shrinking. The abstract nature of the topics was eased because students
actually saw the changes that took place with the graph of each function on the overhead.
Paul discussed the importance of students seeing solutions to mathematics problems both analytically and graphically:

I think it's important because, especially with the college algebra, it's the visual, because I want them to see why it makes sense. Why is it that, given $\mathrm{F}(\mathrm{x})=\mathrm{x}^{2}$, and given $F(x)=2 x^{2}$, why does that stretch the graph vertically? Because to me, sitting in a class without that graphical representation, it's trying to get them to understand an abstract concept without a graphical approach. And so they can make the connection when I say, well, the closer the coefficient of $x^{2}$ gets to zero, what does that do to the parabola? And as we substitute in values and then they can see, oh, the smaller it gets, the closer it's getting to zero, meaning the closer it's getting to $\mathrm{y}=0$. What's that equation? What does that represent? That represents the x -axis, so they can make the connection. Because I think without the graphical, without the visual, they can't really intuitively understand what's going on. Yes, I'm saying it, given $\mathrm{Y}=\mathrm{x}^{2}-2$. Well, with my just telling them, this moves the parabola to the right, when they can actually see it, and that's where I go back to that standards-based. But that's how I use it because I want them to see what I'm talking about from a conceptual standpoint so that they can see it. They see it analytically when we work it out, but then I want them to see it graphically, also.

The graphs of the following functions were illustrated with the help of Graph Program during my first classroom observation:


Figure 1. Graph of function $\mathrm{f}(\mathrm{x})=\mathrm{x}^{2}+4$.


Figure 2. Graph of function $\mathrm{f}(\mathrm{x})=-3 \mathrm{x}^{2}$.
These graphical representations captivated the students and made it easier for them to understand the real meanings of the concepts. Students were interested in and focused on their understanding of the material instead of solely memorizing facts. While I viewed the graphical representations of each function from the back of the classroom, they appeared very neat, and the use of different colors to identify each function made them easy to read. The way Paul was used technology in his class was both efficient and effective; it almost appeared as if he had majored in technology as well! During our third interview, I asked him how he learned the technology skills. He responded:

Well, through professional development, through graduate school, through learning things for myself, many times I've done workshops with teachers on TI83 and 84 and they've asked what classes did you take. I didn't take classes, I just read the manual. Practice it with my students and because again, I wanted to make my students more savvy and so to make my students more savvy, then I had to read it for myself and I had to understand it so I just took the time and did it.

PowerPoint. Similarly, PowerPoint was another form of technology Paul utilized during each classroom observation. He often used PowerPoint and the board simultaneously, which means that if he wanted to explain what he put on the PowerPoint,
he would write on the board to add details. Sometimes, he wrote more examples on the board. It should also be mentioned that the use of PowerPoint slides was not overwhelming. The PowerPoint shows mainly provided definitions and objectives. I asked Paul during the second interview why he uses PowerPoint while teaching. He explained:

I use PowerPoint because I realized years ago when I did part-time at [Star University], I went in with my notes written on this, and as far as time was concerned, I would run out of time before I got through all of the material. And I thought, wow, from the point - and several reasons, too. One, from a standpoint of when I would just use my lecture notes, that I can't do what I do now, and that is: putting all the PowerPoints online, so that if a student missed class or if a student wants to, again, go back over the notes, because I cannot tell you the number of times they said, can you please put PowerPoint notes online again? Can you put them up on GeorgiaVIEW, because they want to review them. And so it was a matter of, it's the convenience of being able to move through the material a little bit faster and being also accessible, making it accessible to my students. Also, it gives me an opportunity to face them many times when, if I were writing on the board - and, again, keep in mind I try to make a mixture of it. Because even though I'm using the PowerPoint, there are still times that I'm actually working problems from the PowerPoint on the board, so I'm actually using the technology and a mixture of the old school as far as working the problems on the board, but it also allows me to make it more available for them.

Via classroom observation, I noticed that PowerPoint gave students access to the material after class. For example, during the question and answer session at the end of one class, I heard several students ask Paul to post the PowerPoint presentations on "GeorgiaView." In addition, the use of PowerPoint allowed Paul to face his students when explaining certain concepts because the information was already projected on the overhead. PowerPoint presentations also saved the class time. For example, during my second classroom observation, some of the materials displayed on PowerPoint would have taken lots of class time if they had to be written on the board during class. As I observed the class, I noticed that the students focused more on understanding the material
instead of trying to copy everything down because they knew it would be available to them after the class. Students also concentrated on asking and answering questions and interacting with Paul and among themselves. The focus was on learning rather than notetaking. Paul was able to move around the classroom and face the students while explaining the material projected via PowerPoint presentations, which would have been impossible if he had to write on the board at the same time.

TI SmartView. TI SmartView is an on-screen calculator that allows an instructor to clearly demonstrate topics to students. Paul explained that TI SmartView helped students see what he was explaining to them and allowed them to practice along with him because they saw the buttons he pressed. Paul explained:

With the TI SmartView, because they can actually see it, they can actually follow me because they see what I'm actually pressing. Before, with the overhead calculator, I would have the calculator in my hand and I would have to tell them, row one, column one, row three, column two, but not with the TI SmartView because the calculator is actually up on the screen, they can actually see what button I'm pressing so that makes it much, much easier for them because I don't have to tell them row one, column one; they actually see the calculator up on the screen and they can see what I'm doing there.

Paul described that they had used TI SmartView for different college algebra topics. For example, in graphing an equation of a line, Paul told his students to "Go to the TI graphing calculator, put the equation, put the values in list one and list two." Then he said "guess what this can do? It can actually graph the equation." As Paul graphed the linear equation on TI SmartView, students were graphing on their different graphing calculators, which made it easier for them to follow because they were seeing it on the screen as opposed to instruction without visual representation. Paul also mentioned that he used TI SmartView to discuss quadratic equations:

Also, especially what they love is with quadratic equations and we're trying to evaluate a function, I would simply tell them to put the equation in Y1-and this always gives them that aha moment-when they put it in Y1 and then I say, go to vars. They pull up Y1, and I say, in parentheses put the number two, and they put the number two, and I say, press Enter. That's the value of it? Oh, let me do another one. You don't have to do the entire thing again; just do second enter, pull up the Y1 again with the two; take out the two and put in a three. That's the bomb. You mean this could do this all this time and nobody told me about it? Yes. So it's just, it's those aha moments that is just so precious. As a matter of fact, this morning. I was telling a student how to use the list and to actually find the-we were using it to find something with the list. But it was that aha moment. It was, I have had this for two years and nobody told me that I could do all this. So those are the times that I just love doing that because in College Algebra, so many of them have had the graphing calculators, but they didn't know the power of the graphing calculator, so when I use TI SmartView, show them on the screen, this is what you can do, they're amazed at the power of it.

Paul emphasized that the use of TI SmartView in his class helped students learn how to use their own graphing calculators. Paul recommended the TI-83 or the TI-84 to the students because of the version of TI SmartView available at the institution.

Geometer's Sketchpad. Paul also incorporated Geometer's Sketchpad into his teaching of college algebra. According to Paul, use of this device depends on the topic being taught. Paul explained that Sketchpad can be used to find the maximum or the minimum value of a parabola, a point on a circle, the point of intersection of a system of two equations, the point of intersection of two lines, or a line in one or two circles. He described that Geometer's Sketchpad has many functions. Following is his comments on the technology:

It allows you to find all those things so that students can actually see it. I guess I like it because you're doing it at the same time that you're talking about, and it's not some distant thing that you're talking about or that they have to go back to their dorms and pull it up and then use it, but I'm actually showing it at the same time. To me, it brings those aha moments. Oh, that's what you're talking about. So that's why I use the different types of programs, the Geometry Sketchpad, the graph, because it's right there, they can see it, they can see that it makes sense at that very moment.

He shared with me a particular example where he used Sketchpad in his class when discussing the intersection of two lines $f(x)=3 x+6$ and $g(x)=-2 x+4$. With the help of Geometer's Sketchpad, students found that the point of intersection of the two lines was ( $-0.40,4.80$ ). The graph is presented as:


Figure 3. Graph of two lines $\mathrm{f}(\mathrm{x})=3 \mathrm{x}+6$ and $\mathrm{g}(\mathrm{x})=-2 \mathrm{x}+4$.
Proper use of technology in assessment. During interview 1, Paul identified one
of his expectations for his students is for them to "regularly do their assignments because their assignments are online." He expects "them to readily do those and their quizzes are online also." In the second interview, I asked about the software students used for the online assignments. He responded:

For the online quizzes, they use what's called MyMathLab. That is from Pearson. What we do is we go in and preselect problems from each of the sections, and I assign at the beginning of the semester-all those problems are put on their MyMathLab page. They go in and there's a due date for each of those homework assignments. There's a due date. They have to go in and do the problems based on the due date. They have to make sure that they have them done by the due date. But it is called MyMathLab.

According to Paul, MyMathLab incorporates practice problems, homework problems, and quiz problems. The course coordinator can create original homework and quizzes, but individual instructors can make changes. Paul noted that he creates his own questions from a database. He added that his typical homework assignments consisted of about 20 questions. When asked about the number of questions, he replied:

It's about the concepts, and I don't believe in overdoing a particular concept if a student has worked two problems on a particular concept, then I don't feel that they need to work five and so it's about the different concepts that are in that section that's the number of problems that I would assign.

Paul emphasized that although he was strict with deadlines, there were times when he allowed late assignments because he had quite a few nontraditional students with both families and jobs. He considered each student's situation. He also added that the purpose of online assignments was student practice because more practice was better for them. This is also the reason he allowed roughly three attempts for the homework and two attempts for quizzes. Paul explained:

It is homework, and we want them to gain practice in doing the problems. We don't want to make it so finative that, because when they get to the test and they do the test in class, you've only got one shot. So we try to make it such that the homework, we're giving them practice with that because if they were doing homework for me not online, I would grade the homework from the standpoint of their attempt; not from the standpoint of correctness. So it's just a matter of allowing them the time and the practice so that when they do come to the test with that only one shot, they have had enough practice so that they will be able to do much better on the test because the tests weighs much more heavily on their grade.

Paul mentioned that with MyMathLab, students can save unfinished homework and complete it later. MyMathLab also has a gradebook feature, allowing students to instantly view their grades after submitting an assignment, which according to Paul was
beneficial because it kept students well-informed. According to Paul, MyMathLab also gives students feedback:

I love the feedback that MyMathLab gives them. It really-there is no reason for a student not to do well, especially on the homework with MyMathLab because there are many, many tutorials that allow them to get feedback on how to do a problem. It gives them step-by-step-by-step examples of how to do the problem.

In addition to the feedback students received from MyMathLab, they got feedback from Paul based on questions they sent to him through electronic mail. According to Paul, some of the questions included: "Can you help me with number one on assignment number five? and this is what I've done."; "Can you help me out?"; and "Can you give me some kind of guidance?" Paul would e-mail students back in a timely manner, giving hints, explaining the problems directly, or asking them to come to his office during his hours. Paul summarized that MyMathLab is crucial to student success in mathematics because it allows them to work at their own pace. Students can access assignments from anywhere, go back and review, and print assignments. Paul stated, "It allows them more leeway and more time as far as getting help, even from the standpoint of going to the center for academic success. They can assist them in working the problems."

Paul also used other calculators for student assessment, and the type of calculator used would depend on the particular test. Paul said:

It depends on the type of test. For example, on Thursday's test, they can only have a scientific calculator because on Thursday's test, it's finding zeroes of polynomials, it's graphing polynomials, so I told them that, see, on the TI graphing calculator, all you have to do is just plug it in, tell it to find the zeroes and you're done. So it depends on the material as to the type of calculator. Now, I can't think of a single test such that they're not allowed to use a particular calculator. They can always use a calculator. It just depends on the form on which type of calculator based on the material that we're covering.

During the third classroom observation, Paul announced at the end of class that students would be allowed to use nongraphing calculators for the upcoming test.

Paul's knowledge and understanding of different technologies enabled him to use them efficiently in his class. His use of technology in instruction and assessment allowed students to visually see the explanations behind each concept. Students related mathematics to their own experiences with technology, which made learning lively and fun for them. As mentioned previously, Paul emphasized that "We have to understand there are the iPods, there's the phone, there's the laptop. There's all of these things. If I go in with the old school way of doing things, I don't think it's going to captivate them." Students' comments also indicated that Paul used technology effectively. Paul shared his students' comments on his use of technology. Their positive comments spoke volumes regarding his proper use of technology (see Appendix E).

## Building Mathematical Connections

According to culturally relevant pedagogy theory, teachers should connect their teaching to their students' cultural experiences (Ladson-Billings, 1994; Tate, 1995). The AMAYTC (2006) advocates connecting mathematics to student experiences, including life experiences and experiences from other mathematics courses and in other disciplines. With such connections, students should see the value of mathematics. Student views of mathematics as an isolated subject should be overcome through making mathematics meaningful to them. Other studies (Austin, Berceli, \& Sarae, 1999; Choike, 2000; Leonard \& Guha, 2002; Pierce, Turville, \& Giri, 2003) also supported that the use of real-world problems indeed impacts student learning in mathematics

In all three interviews with Paul, he emphasized that he wanted mathematics procedures to be connected with students' experiences. Mathematical connections were evident from viewing tests and homework and also during classroom observations. Paul did not want to see that the students were "just doing the, I call it math, doing procedures without connections. If we're doing procedures without connections, I don't want it to be procedures without connections. I want it to be procedures with connections." Paul wanted his students to apply their knowledge of mathematics to real-world experience, which agrees with culturally relevant theory (Ladson-Billings, 1995a). Consequently, building mathematical connections emerged as another major theme in Paul's teaching practices. Paul attributed his ability to allow his students to make mathematical connections to the policies of the NCTM and the AMATYC. According to Paul:
...with the NCTM, it goes back to the standard-based. It goes back to reasoning and making sense. I think that is the biggest thing that always resonates in the back of my head as far as wanting students to make sense of mathematics, wanting students to reason through mathematics and again, this is a new, you probably see that too, this is a new area of research as far as college is concerned because for so long, we have not really want students to make sense of things in college. I think it has been more of a here it is aspect in college and you go away from here. Now, you can make sense of it if you want to but if you don't, then you're just going to fail the course and so I think NCTM helps me from the standpoint of I want my students to reason. I want them to make sense of mathematics so that I can see that they are making sense of the mathematics, not just throwing some concepts out and now you go get it and so I think it is that that helps me with making those statements of does this make sense to you or asking the questions of do you understand? I understand. Okay, now, then explain it back to me and so I think those are the things that help me in my teaching, even at the college level as to I still want students to make sense. I want them to problem solve, I want them to rationalize their answers, I want them make logical sense of what they're doing mathematically and I want them to be able to do the math and not just have procedures but they have no connection to the mathematics and I want them to be able to connect with it.

Through data analysis, I found that Paul built mathematical connections in his classes by building connections between mathematics and real-life experiences, between
prior knowledge and current learning, between college algebra and higher mathematic courses, and between mathematical terminologies and procedures.

Building connections to real life experiences. Ladson-Billings (1994) and Gutstein (2006) emphasized the importance of connecting teaching and learning to students' life experiences. Paul related his teaching to his students' life experiences. During the first interview, I asked him about his experiences while teaching college algebra. He shared:

I try to incorporate the application kinds of problems into teaching the course so that they can see this is an application of a linear function. It is not just y is going to be x plus v but here's a real world problem. Now, how do you apply that to this problem? What does the y intercept mean in this problem? What does the slope mean in this problem? Here is a manufacturer and he is selling so many radios and the radios cost $\$ 5.00$ for every radio that he sells, how can you transform that into a linear function and so it's about taking the algebra and making it real world and making it more applicable to the things that they might encounter in the real world and try to make that more interesting for them so that they just don't think of it as just some variables and some numbers.

Paul repeatedly mentioned that it is important to relate mathematics teaching to the students' real-life experiences. He wanted his students to get an:
...understanding of the true application of what's going on, not just plugging and chugging, but can you give me a practical understanding or give me an example so that I know that you have a practical understanding of the concept?

There were different examples that Paul used in his class to connect his teaching to the students' life experiences. For example, in teaching linear functions, Paul shared:

We have several different kinds of applications. If we're talking about linear functions, then we're talking about possibly a company making profit, to determine whether a company is going to make profit. We talk about selling so many items and saying if they start out with a certain amount and your item is selling for $\$ 3$ or $\$ 4$ per item, develop that linear equation for me. Tell me what that linear equation looks like. Tell me what that slope looks like. Tell me that that slope - and so that you're trying to see if they will come out of that realizing that my slope is that 3 , and then what does that mean as comparing to selling so many items? If I sell 35, do they know that that means to multiply that 3 times
that 35 , because in many instances you will find that, coming from, say, an Algebra I class from high school, they dealt with slope all those years. They dealt with slope, finding the equation of the line, substituting those points in, but when it comes down to a practical application which they would call, quote, oh, those word problems, can you make the connection from the linear to the-from the equation that you did in Algebra I to what we're doing now. So that's one example.
Also, students were asked to solve application questions in linear functions in their
homework assignments and tests. Another example pertained to quadratic functions, in
which Paul shared:
Also, with quadratics, and I find this quite interesting, with the quadratic functions, if you just give a model for a quadratic function-let's suppose it's a trajectory problem for a rocket that's being shot and you want to find when the rocket reaches its maximum. Oh, God, what is that asking me? Well, they've learned how to find the vertex of the parabola. We've already done that, but now it's applying that to the problem, and in many instances it is a difficulty in the conceptual from taking this quadratic equation, knowing what I need to do, negative $v$ over 2a, to find the x-coordinate of my vertex. But taking that, transferring it to this rocket that's being shot up in the air with the model equation and then determining when is the rocket going to reach its maximum height, not understanding that that is also supposed to be trying to find the coordinate of the vertex when I find the coordinate of the vertex, that x-coordinate, that is giving me my time; but also the y coordinate, that gives me my maximum. We take it from the conceptual to the practical application and see, can they make the transfer. And that's what I'm talking about with actually talking about the conceptual, making sure that they understand what that means in finding the coordinate of that vertex. If the parabola is opening downward, what does that really mean? That means that vertex is going to be the highest point of the parabola, so now, can they make their transition from this vertex being the highest point of the parabola to, oh, the rocket, its maximum height; that's it right there. Why didn't you tell us that? No, I didn't want to tell you that because I wanted to see if you can make the transfer from just writing this equation, having this quadratic equation, finding the vertex, to making the transfer to this application kind of problem.

In addition, by analyzing some of the written documents (e.g., tests, homework
sheets) from class, Paul assigned application-based problems about quadratic functions as
homework and tested his students on these applications. For example, students were
asked to answer the following questions in test 2 :

1. A model rocket is launched with an initial velocity of $100 \mathrm{ft} / \mathrm{sec}$. from the top of a hill that is 20 ft . high. Its height " t " seconds after it has been launched is given by the function $s(t)=-16 t^{2}+100 t+20$. Determine the exact time at which the rocket reaches its maximum height.
2. Find the dimensions of a Persian rug whose perimeter is 28 ft and whose area is $48 \mathrm{ft}^{2}$.
Paul also shared during the second interview that other application problems he discussed
with the students involved exponential functions. For example, in Paul's class,
exponential functions can be used to predict the population a country at a certain time.
Paul explained:
Well, again, it's that from the historical standpoint, if we try to say-the prime example-I think one time we had an application problem where it was talking about a population. It was a population at a certain particular time, and if there was an increase in the population, if there was an increase by a percentage of the population per year, then in 2004, 2005, 2006, here's the function. That function value of $x$ is used from the number of years since 2000. Well, they have to again take that problem and realize from 2000 to 2003, how many years that is from 2000 to 2005, and then substitute that value in. Then they can get the population of the United States or the population of whatever at a particular time. So it's about showing how to take whatever concept that we're doing, how to apply it to the science or the history, depending upon the word problem, whatever it is, how to apply it in that particular area.

The tests also contained the population problems and other applications of exponential functions, for example:

1. Population Growth of the United States. In 1990 the population in the United States was about 249 million and the exponential growth rate was $8 \%$ per decade. (Source: U.S. Census Bureau)
a. Find the exponential growth function.
b. What will the population be in 2020 ?
c. After how long will the population be double what it was in 1990 ?
2. The decay rate of a certain chemical is $9.4 \%$ per year. What is its half-life? Use the exponential decay model $\mathrm{P}(\mathrm{t})=\mathrm{P}_{0} \mathrm{e}^{-\mathrm{kt}}$ where k is the decay rate, and $\mathrm{P}_{0}$ is the original amount of chemical. The half-life of the chemical is how many years?

Paul really wanted his students to be able to make connections between college algebra topics and real-life experiences.

Building connections to higher mathematics courses. In addition to connecting
mathematics concepts to real-life experiences, building connections to higher mathematics courses was also a teaching strategy Paul employed in his classes. The

AMATYC (1995/2006) stresses the need to build connections among mathematics courses. Paul provided detailed information about this strategy:

It's a matter of, especially with my college algebra course, because I ask them: How many of you will have to go on and take trig? How many of you will go on and have to take calculus? And I say, it is critically important. For example, one of the biggest ones that I emphasize is $x^{2}$ squared equal to four? And I say, when you have $\mathrm{x}^{2}$ is equal to four, you have to remember that x is equal to plus or minus 2, not just the positive two, because when you get to trig and you have sine squared X equal to one-fourth, you want to make sure that your sine of x is equal to plus or minus one-half, because if you don't, then that's going to carry over and you're going to forget. You're going to miss half of that problem because you're going to miss half of your solutions. Also, when they get to calculus and they are trying to do volume of revolution and we're doing top minus bottom curve, I said, if you don't put that plus or minus, you're going to be looking for that bottom curve and if you don't have that plus or minus there, if you're solving for $y$ in terms of $x$, if you don't put that plus or minus there, you're looking for that equation for that bottom curve and you're thinking, I don't have that curve. But it goes back to that fundamental process and that fundamental procedure of $x$ squared is equal to four; what is x ? x is equal to plus or minus two. So there is a carryover. I tell them that absolute value, absolute value is one of the most important concepts in calculus because if they don't remember that the absolute value of $x$, if I have the absolute value of $x$ is equal to five and that's equal to plus or minus, when you are looking at doing limit problems, then you're going to lose that because you have to know that I have to apply the definition of absolute value in finding the limit of a particular function that involves absolute value. I always tell them that absolute value is saying to them, do something special with me; do something special with me, because I'm the absolute value. And so it has to carry over into all of the other courses.

Another method Paul used to build connections to higher mathematics was asking students not to rely on calculators to solve every mathematics problem. For example, one test question was:

Solve by completing the square (no decimal answers): $3 x^{2}+5 x-2=0$.

In a follow-up interview, I asked Paul why he emphasized on "no decimal answers"; he replied:

Well, because again, they want to rely on a calculator and so in completing the square, I want to make sure that they have used the fractions, that they can actually do it by hand rather than just putting something in the calculator because again, what they want to do with, for example nine halves, they want to just put that in the calculator and get four point five and I want them to be able to work with completing the square in that manner because again when they get to the trig and they have to find the equation for the ellipse, we won't be using decimals there so its again, looking ahead to see what they are going to have to utilize in future math courses with this concept so that they will understand how to do the concept then.

With this strategy, students were able to develop a solid foundation for future study of mathematics. This method also allowed students to make sense of mathematics and understand that mathematics topics are not disjoint.

## Building connections between prior knowledge and current knowledge.

Hiebert and Carpenter (1992) mentioned that mathematical knowledge can only be fully understood when "it is linked to existing networks with stronger or more numerous connections" (p. 67). Paul also emphasized on building network of mathematics knowledge. During the first interview, Paul stated:

I do believe that students learn and again my whole paradigm for students learning is constructivism from the standpoint of their constructing knowledge from their prior knowledge, they are building on from their prior knowledge because it is all about what they have in their toolbox now. So they take those things in their toolbox and then they try to build onto those types of things but I think that-and they are more apt to retain.

Paul discussed that building connections between existing knowledge and current knowledge helped him identify holes in student learning, which then allowed him to fill the gaps. With building these types of connections, Paul was able to find out where his students were, what they already knew, and what they did not know. Also, the strategy
helped him to figure out how the depth of which he should cover new knowledge and the appropriate pace. For instance, when Paul and his students discussed the long division of polynomials, he used this as an opportunity to make connections to their prior knowledge, which was long division of numbers. Paul described how he connected long division of polynomials to long division of numbers:

For example, when we talk about today, last week I introduced synthetic division and long division. With long division, it is making that connection back to just long division that they did in elementary school when they divided five into 256. Well, they first divided the five into-let's say 246. They divided the 5 into the 24. The biggest thing is remembering when I divide five into 24 , yes, I get that 4 as a part of my quotient, but I have to multiply that five times the four to get the 20 and then I say, what did you do with that 20 ? Uh. You did something with that 20; what did you do with it? Oh, we subtract it. Yes, you subtracted that 20 . So if I'm dividing $x$ plus two into a polynomial, when I divide-let's suppose the first term of the polynomial is $x$ cubed, so $I$ divide the $x$ into $x^{3} I$ get $x^{2}$. I multiply that $x^{2}$ times each term in that binomial. What are you going to have to do with this polynomial that you now multiplied by, what are you going to have to do relative to the original polynomial? You want that $\mathrm{x}^{3}$ to cancel out, right? Yes. So what are you going to have to do? Oh, we subtract it. So that means you have to subtract that polynomial that you get once you multiply. And that is one of the biggest things getting them to realize: there is a connection back to the long division that you did with just numbers, and connecting it to the long division that you did with polynomials.

During each classroom observation session, Paul built connections between students' prior knowledge and current knowledge. This strategy was a key element of Paul's instruction. As previously mentioned, the topic during the first classroom observation was "shrinking and stretching." Paul did not start immediately with the topic. Instead, he started with questions and dialogue, through which he reviewed the previous topics (i.e., horizontal shift, vertical shift, and reflection) with the students. Specifically, he graphed and discussed these functions: $f(x)=x^{2}, f(x)=x^{2}+4, f(x)=(x-2)^{2}$. It was after the review that he introduced the new topic of shrinking and stretching. Paul further explained the differences between the prior topic and the current topic. With this type of
introduction, students were able to connect the current learning to their prior knowledge.
This approach helped students build a network of knowledge and understand that mathematics concepts are interrelated.

The second example comes from the second classroom observation. The topic that day was horizontal asymptote. Again, Paul started the lesson with a discussion of prior knowledge on vertical asymptotes. The following dialogue took place during the introductory phase of the lesson:

Paul: (The participant's question is leading such as) To find the vertical asymptotes, if there is no common factor we set the denominator equal to what?
Class: 0.
Paul: (Repeated and explained himself) If there is no common factor then you set the denominator equal to 0 in order to find the vertical asymptotes. What is the equation for vertical asymptote?
Class: $\mathrm{x}=\mathrm{a}$ number
Paul made the connection between vertical asymptotes and that day's lesson, telling students they were going to discuss horizontal asymptote and would need both the vertical asymptote and horizontal asymptote for graphing rational functions. He wrote the equation for vertical asymptote on the board as $\mathrm{x}=\mathrm{a}$ number. He explained and wrote the equation for horizontal asymptote as $\mathrm{y}=\mathrm{a}$ number. He continued his discussion.

Similarly, during the third observation, students were learning to graph logarithm functions. Instead of starting with the topic, Paul reviewed exponential functions. He asked the students about the characteristics of exponential functions. The students responded with their existing knowledge. Paul then posed a follow up question: Since exponential functions and logarithm functions are inverses of each other, which mean the logarithm functions grow how? The students responded "slowly." At this point, the
students were able to clearly see the relationship between exponential functions and logarithm functions. From here, Paul began the day's discussion.

Connecting current knowledge to prior knowledge did not occur only at the beginning of the lessons. This strategy was also present throughout the lesson and at the close of the lesson. For example, Paul connected current knowledge to prior knowledge when they discuss ways to simplify logarithm functions involving radicals. First, Paul reviewed radical exponents with the students. To review, he asked the students to change the radical expressions $(3 \sqrt{4}, 4 \sqrt{11}, \sqrt{5})$ to fractional exponents. Due to the connection to the students' prior knowledge, they were able to simplify $\log _{\mathrm{a}} 5 \sqrt{11}, \log _{2} 3 \sqrt{21}$. This strategy allowed students to understand that there is a connection between prior mathematics knowledge and the current topic.

## Building connections between mathematics terminologies and procedures. As

 observed, Paul always defined and explained mathematical terminologies before engaging in problem-solving. In the three observations, Paul described horizontal and vertical asymptotes, rational functions, exponential and logarithm functions, horizontal and vertical translations, and shrinking and stretching in detail before introducing the procedures of solving problems involving the terms. Also, Paul provided proofs of formulas to students instead of asking them to use the formulas without knowing how they were derived. For example, he showed his students how to derive product rule in logarithm as follows:Paul: Given $\mathrm{a}^{\mathrm{x}}=\mathrm{M}$ (equation 1) and $\mathrm{a}^{\mathrm{y}}=\mathrm{N}$ (equation 2). We already dealt with how to change from exponential function to logarithm function. What is the logarithm form of $\mathrm{a}^{\mathrm{x}}=\mathrm{M}$ ?
Class: $\log _{\mathrm{a}} \mathrm{M}=\mathrm{x}$.
Paul: $\mathrm{a}^{\mathrm{y}}=\mathrm{N}$ would be what?
Class: $\log _{\mathrm{a}} \mathrm{N}=\mathrm{y}$.

Paul: Now let's look at the two equations $\mathrm{a}^{\mathrm{x}}=\mathrm{M}$ and $\mathrm{a}^{\mathrm{y}}=\mathrm{N}$. What do the equations have in common?
Class: Common "base a."
Paul: If I multiply the two equations, what will I do with the exponents?
Class: Add them.
Paul: $a^{x} a^{y}=M N$ implies $a^{x+y}=M N$ (equation 3 ). What type of equation is this?
(Class was silent.)
Paul: E word.
Class: Exponential equation.
Paul: If I want to write equation (3) in logarithm form, I will have what?
Class: $\log _{\mathrm{a}} \mathrm{MN}=\mathrm{x}+\mathrm{y}$.
Paul: Very good. Now what is x ?
Class: $\log _{\mathrm{a}} \mathrm{M}$
Paul: What is y?
Class: $\log _{\mathrm{a}} \mathrm{N}$
Paul: Now I have $\log _{\mathrm{a}} \mathrm{MN}$ equal to what?
Class: $\log _{\mathrm{a}} \mathrm{MN}=\log _{\mathrm{a}} \mathrm{M}+\log _{\mathrm{a}} \mathrm{N}$.
Paul wrote the complete equation on the board as $\log _{a} M N=\log _{a} M+\log _{a} N$. He explained and repeated that the product rule can be used to write logarithmic expressions as single logarithms or vice versa. He emphasized that to use this rule, there must be a common "base."

Student 1: How did you get $\mathrm{a}^{\mathrm{x}+\mathrm{y}}=\mathrm{MN}$ ?
Paul: (Instead of providing direct answer to the student, he asked a question.) If we have common base what happens to the exponents.
Class: We add them.
This questioning method promoted interaction between Paul and the whole class, which helped the students to think and work out the correct answer by themselves.

Student 2: What about the final equation $\log _{\mathrm{a}} \mathrm{MN}=\log _{\mathrm{a}} \mathrm{M}+\log _{\mathrm{a}} \mathrm{N}$.
Paul: I substituted $\log \mathrm{aM}$ for x and $\log _{\mathrm{a}} \mathrm{N}$ for y .
Discussing the proofs of formulas helped the students to understand how they are derived before using them in solving problems.

Connecting mathematical terminologies and procedures was also evident in Paul's tests, homework assignments, and lesson plans. For example, Paul asked students to
comment on the following statements on their test before engaging in problem solving questions:

1. An exponential function grows quickly.
2. The inverse of an exponential function is a power function.
3. A logarithmic function has no asymptotes.
4. A logarithmic function is the inverse of an exponential function.
5. A rational function is never undefined.
6. The graph of every quadratic function must cross the $x$-axis.
7. A linear function can have only one zero.
8. The vertex of a parabola is always the highest point of the graph
9. A function is even if it is symmetric with respect to the $x$-axis.
10. The discriminant of a quadratic function can never be negative.

This types of questions required students to know the definitions of terms and characteristics of each term. Paul noted that he wanted his students to learn mathematical terminology with procedures. He said:

I want them to understand what we're about to do, what we're about to cover, and I think it's important and I guess I really go back to research in math as far as standardized test are concerned and terminology. Many students understand. They can do something, they can do procedure, but there are no connections with it, and so what I want them to understand is that it's a language. It's a math talk community. That's what I talk about, a math talk community, and so I want them to understand it's not just about being able to do the problem, but it's about the terminology that goes with the problems and so again, knowing from here they have to continue to go up the ladder in math courses and I want them to understand that terminology is going to be important.

Connecting mathematical terminologies and procedures promoted student understanding rather than rote learning in Paul's class.

In summary, one of the main focuses of Paul's instruction was making mathematics meaningful to students and building networks of mathematics knowledge. According to Paul, building mathematics connections helped students think critically, discover new materials, and better retain mathematics knowledge. Establishing networks of mathematics knowledge is not regurgitative mathematics. Paul stated:

It's what I don't call regurgitative math in that here's just an example that you did in class and I just want you to regurgitate that back to me, and that's what I'm trying to get them out of because again, as they move up the ladder with taking math courses, they are going to have to critically think through and not just regurgitate math.

## Multiple Representations

The next theme that emerged during data analysis was multiple representations. The term "multiple representations" means that mathematics faculty should incorporate different ways of solving mathematics problems, such as numerical, graphical, symbolic, and verbal methods (AMATYC, 1995). Multiple representations aid student understanding and help students interpret mathematics concepts (National Research Council, 1989). The use of multiple representations gives students opportunities to experience different ways of solving mathematics problems. Dewey, founder of experiential instruction theory, stated that "all genuine education comes through experience" (as cited in Bialeschki, 2007, p. 366). Furthermore, Choike (2000) mentioned that use of multiple representations is a good teaching strategy that can help students better understand how these representations are related. Choike (2000) further explained that multiple representations can reach more students than single representations because students learn from different methods. Findings of the current study are aligned with the literature.

It was obvious during the first classroom observation that multiple representations were incorporated into Paul's teaching practices. When asked about the role multiple representations play in his teaching, he stated:

And that depends on the student because I know in my classes I always have the visual learner; I have the kinesthetic, all of those different types of learners. So what I try to do is-and I don't know if it was in the class where you were, but I have the graph program, so the graph program is there. So it's making sure that I
have all of those things available because even one day in class, one of the students said, can you please write it out? I'm a visual learner. And of course, you have to take that into consideration because everybody-there's one student, [Jones], who's in my 8:25 class. He is strictly an auditory learner because he does not write a single note, but he sits and he listens. And I asked him one day, I said, you're an auditory learner? He said, yeah, all I have to do is just sit and listen at you explain it and I'm going to get it because it's a matter of my sitting and looking, and once I get it, it registers up here. Whereas I know [Kim], for example, is a visual learner. I must write it down for her because, again, it's not going to make sense unless I write it down. So, knowing all of the different learning styles that are there, I try to have the graph program ready, even from the standpoint of the calculator, trying to have that ready, so that they can get an understanding of it no matter what their learning style is.

For Paul, using multiple representations in his classes helped accommodate different types of learners. He explained that he made every effort to reach all of his students, regardless of their learning style. To accomplish this, he used different representations, ranging from symbolic to verbal, verbal to symbolic, numerical to graphical, symbolic to numerical to graphical, and symbolic to graphical.

Symbolic to verbal representations and vice versa. During the first classroom observation, multiple representations were evident by Paul's request to change the descriptions of graph of functions from symbolic form to verbal form. In a previous example, Paul and his students discussed the following questions:

Describe what happens to the graph of $f(x)=x^{2}$ given the following equations:
A. $f(x)=4(x-3)^{2}+2$
B. $f(x)=-1 / 2(x-3)^{2}-5$
C. $f(x)=-(x+2)^{2}-5$

The students came up with these translations:
A. Stretch by a factor of 4 , shifted up 2 units, and shifted right 3 units.
B. Shifts down 5 units, shifts right 3 units, reflected across the $x$-axis because of the "negative," shrunk by a factor of $1 / 2$.
C. Reflected across the $x$-axis, shifts left 2 units, shifts down 5 units.

Students were also tested on the ways to change functions from symbolic form to verbal form. For example, in the second test, the students were required to answer these questions:

1. Given $\mathrm{f}(\mathrm{x})=\mathrm{x}^{2}$ and $\mathrm{g}(\mathrm{x})=-2(\mathrm{x}-4)^{2}+5$. Describe the transformation on $\mathrm{f}(\mathrm{x})$ to produce $\mathrm{g}(\mathrm{x})$.
2. Given $f(x)=\sqrt{x}$ and $g(x)=2 \sqrt{-x+3}+7$. Describe the transformation on $f(x)$ to produce $\mathrm{g}(\mathrm{x})$.
Similarly, during the first observation, Paul asked students to change the description of graph of functions from verbal to symbolic in the following questions:

Write an equation for $\mathrm{y}=\sqrt{ } \mathrm{x}$ with the following transformation.
A. Shifted 2 units to the right, reflected across the $x$-axis and stretched vertically by a factor of 3
B. Shifted 4 units to the left, stretched vertically by a factor of 2
C. Reflected across the $y$-axis and shifted up 3 units.
D. Reflected across the $y$-axis, shifted 2 units to the right and stretched vertically by a factor of 3 .

The students' responses were:
A. $Y=-3 \sqrt{x-2}$
B. $Y=2 \sqrt{x+4}$
C. $\mathrm{f}(\mathrm{x})=\sqrt{-x}+3$
D. $\mathrm{f}(\mathrm{x})=3 \sqrt{-x+2}$

Students were also tested on the way to move from verbal to symbolic representations.
For example, the students were asked:
Given $f(x)=\sqrt{x}$. Write the equation such that $f(x)$ is shifted 2 units up, 3 units to the right and reflected across the x -axis.

These questions allowed students to switch from one form of representation to another.
This strategy helped students think critically and build chains of knowledge, which led to deeper understanding of the concepts.

Numerical to graphical. The numerical to graphical approach was another method Paul used to accomplish multiple representations in his instruction. During an interview, Paul used the following example to illustrate how he engaged his students when changing from numerical form to graphical form. He explained:

I have the graph program. Then I have, from the standpoint of showing them the polynomial function, showing them, here are the zeroes, here's the numerical values of the zeroes. Well, what does that mean? That means that at these particular values, at $x$ equal to three, $x$ equal to two, $x$ equal to one, what does that mean when I go to the graph? That means that on the graph, if it says that these are the zeroes, then that simply means that that's where the polynomial is going to cross the x -axis. Well, if we're graphing that polynomial, how are we going to determine what we need to do in between those 0s? Well, we go back to the numerical approach from the standpoint of, well, if a 0 is one and another 0 is five, somewhere in between you pick a value, you pick a number. Suppose that number is two. You substitute that number into the polynomial because we dealt with function notation. F of two means to substitute the number into the polynomial. Well, once I substitute that number to the polynomial, it's either going to give me a positive or a negative value. That positive or negative value, if it's a y-value, then that tells me if it's positive, that means it's going to where relative to the x -axis? It's going to be above. If it's negative, it's going to be below. And many of them ask how high does the graph have to go. Well, at this particular stage of college algebra, we're not trying to actually find that maximum. I said, as long as I know that you have that graph above the x -axis or below the x-axis, I said, we will find the maximum at a later point, but right now I want to know that you know, based on these 0 s, where that graph is going to go.

Paul also showed me the graph of a polynomial showing the 0 s:


Figure 4. Graph of a Polynomial Function

Moreover, one of the questions on the students' test required them to change from numerical to graphical form. The question was:

Graph the following showing the zeros: $f(x)=x(x+1)(x-2)(x+4)$


Figure 5. A graph sheet.
In addition, an analysis of Paul's lessons plans revealed that he represented the 0s of quadratic functions in graphical form. Seeing the 0 s of quadratic functions in graphical form provided the opportunity for the students to see that when 0s exist, the graph crosses the x -axis, and when the solutions are imaginary, the graph is above the x -axis. A few examples are:


Figure 6. Graph of a function showing real zeroes.


Figure 7. Graph of a function showing imaginary zeros.
Symbolic to numerical to graphical. Paul often showed students multiple representations that involved symbolic form to numerical form to graphical form. During the third classroom observation, Paul and his students discussed how to graph $\mathrm{f}(\mathrm{x})=$ $\log _{6} \mathrm{x}$, which is in symbolic form. With this function, they were able to create a table of values in numerical forms. Paul used his usual questioning method to engage the students on ways to derive the table of values:

Paul: $y=\log _{6} \mathrm{x}$ is equivalent to what?
Class: $6^{y}=x$.
Paul: When x is 1 , y is what?
Class: Zero.
Paul: (Rephrased the question) What value of y makes the equation $6^{y}=1$ true? Class: Zero.
Paul: It is zero because anything raise to power zero is 1 that is how $\mathrm{y}=0$ when $\mathrm{x}=1$.

With this approach, students were able to complete the table of values (presented below):

Table 1
Values of Function $y=\log _{6} x$

| X, or $6^{\mathrm{y}}$ | y |
| :--- | :--- |
| 1 | 0 |
| 6 | 1 |
| 36 | 2 |
| 216 | 3 |
| $1 / 6$ | -1 |
| $1 / 36$ | -2 |

To graph the logarithm function, they also needed vertical asymptotes. Therefore, Paul asked the students what the argument in $\mathrm{y}=\log _{6} \mathrm{x}$ is. The students responded with the answer "x." Paul posed another question about what makes the argument $x$ equal to 0 ? He added that the value would be the vertical asymptote for the logarithm function. The students answered that the value is 0 . Paul also asked the students what the value of x intercept is. The students again gave the right answer- 1 . With the help of the table of values, vertical asymptotes, and x-intercepts, Paul and the students graphed the logarithm function, which is presented as:


Figure 8. Graph of function $\mathrm{y}=\log _{6} \mathrm{x}$.
This example allowed students to understand that they can represent mathematics concepts in different forms. More specifically, with this example, Paul illustrated ways to change the presentation of a mathematics concept from symbolic to numerical form and then to graphical form.

The symbolic to numerical to graphical strategy was also present in the students' tests and homework. For example, in test four, students were asked to answer the questions (see Appendix F) without using graphing calculators. The questions showed how Paul used the symbolic to numerical and then to graphical strategy.

As shown here, the inclusion of multiple representations was a major theme in Paul's teaching. The symbolic to verbal, verbal to symbolic, numerical to graphical, and symbolic to numerical to graphical presentations were four strategies Paul used to present multiple representations. Paul felt that multiple representations was a successful strategy in reaching all the students. He shared:

I try to make sure I incorporate it. I try to incorporate it most every single time because, even though-and I've discovered this, too: There will be many that will not say it, but if you do it, you'll see a change in their face as far as the facial expressions of, oh, now I get it, because you have now taken it to the graphical
approach rather than just simply writing the equation on the board.

## Motivating Students to Learn Mathematics

One component of culturally relevant pedagogy is to promote academic success (Ladson-Billings, 1994). Students are more likely to achieve academic success if they are motivated to learn in conducive learning environment (AMATYC, 2006; Choike, 2000; Roth-McDuffie, McGinnis, \& Graeber, 2000).

The next theme that emerged from data collection was motivating students to learn mathematics, and Paul made great efforts to do this while teaching. According to Paul, "teaching is consistent, deliberate, thoughtful and unselfish commitment to the cause of captivating and motivating learners to maximize their potential." Paul used several methods to motivate his students to learn mathematics, which included showing enthusiasm towards teaching mathematics, using words of encouragement, and creating a conducive learning environment.

Enthusiasm for teaching. Paul shared that one way he tried to motivate his students was by showing them that he was enthusiastic about his teaching:

I try to motivate my students with my motivation because I think it is important that I am motivated, that they can see in what I'm doing that I'm excited about what I'm doing because if they don't see some excitement in I'm doing, then I don't think that they're going to be excited... I love teaching, even to the standpoint that sometimes, when I was in secondary, they would say, calm down, you're getting too excited about this and even some have remarked in the classes now, 'Wow, you really love this stuff.' Yes, I do, and so I want them to see that I'm excited about teaching so that it sort of gets them excited about the learning because I really, really think that it means something. It's sort of like a product and I always tell this to teachers when I do workshops, it's about a product that I'm selling and if I'm not excited about the product, then no \body will want to buy so I'm an actor on stage, I still stay that to all teachers. We are actors on stage and we are trying to get the audience engaged in our play and what we're doing and the part that we're playing and so I look at myself as that actor on stage trying to get my students involved in math and how do I do that is to be excited about what I'm doing because if I'm not, then I don't think that they're going to
be excited.
According to Paul, he wanted to excite his students just like his 9th-grade mathematics and music teachers did for him. Lesson plans, just like other artifacts, cannot exemplify enthusiasm about teaching. According Paul:

I spend an exorbant amount of time preparing for each of the lessons because again I want it exciting for me and I want it exciting for the students. So for me it has been those moments of the being excited about it, the students becoming involved in the lesson because as I said at the beginning of the semester it was like pulling teeth trying to get them out of that habit of just coming to sit and just observe rather than participating.

Paul emphasized that a teacher has to know that his or her purpose is to teach and that he or she must plan "because teaching is not something that you just wake up one morning and say, 'Okay, I'm going to teach,' but you have to really plan to do it and then the passion has to go with it." He explained that he plans adequately for each class session, which gives him opportunities to wear his "coat of many colors," which includes collaboration, agency, questioning, dialogue, deliberation, authority, and discourse. According to Paul, these coats are present in his classes because of his passion for teaching. These strategies also made teaching exciting for Paul, which in turn excited his students. By showing passion, which is achieved through adequate planning, Paul stated that his students knew that he was serious about the subject matter, that he cared about the subject matter, and that he cared about their learning. Due to his excitement, students became active participants in the learning process. Paul stated:

When I ask those questions of 'Do you understand? Does that make sense?' Yes, that makes, or no, that does not make sense. That at least says that they're paying attention. Even if I am working problems on the board and I purposely make a mistake to see if they're going to bring it to my attention. And then I say to them, I just wanted to see if you were paying attention. So that tells me whether or not they're engaged in the learning environment.

During my classroom observations, it was obvious that Paul was enthusiastic about his teaching. He was always full of energy and happy. His facial expressions made it clear that he loves what he does. His "coat of many colors" made his lessons lively and fun. The stimulus variations and flexibility in his teaching showed that he was passionate about teaching. Students were engaged and actively participated in the learning process. There was periodic laughter, and everybody was focused on learning and enjoyed it. From my own teaching experiences, I know that some students do not like mathematics for various reasons, but it was different in Paul's classes. His students were excited to be there and responded to all of his questions.

Paul mentioned numerous times that his success as a teacher and his students' success was attributed to his passion for teaching. Paul said, "Several of my students have said it was because of excitement that I have for math and because of the excitement that I had they were successful." Paul's excitement about mathematics made his students more excited about it, which led to their success. He explained:

You know not necessarily that it was easy but because they knew my excitement about it they were then excited and so it was nothing for them to spend two hours on an assignment because the time went by fast because they were excited about the material.

Paul attached great importance to being excited about teaching. Some students even told Paul that his excitement for teaching mathematics motivated them to become mathematics teachers one day. He stated:

I think that is so important that we show students that we believe in them, that we believe that they can do it and that I think comes through the passion of us showing that that's the passion of our job. That's the nature of that compassion and passion that comes with teaching. Students have to know that you have that passion for your job because I think once students know that you are passionate about it then they are more eager to learn. I cannot tell you the number of students that I've had that are now teachers because they-are math teachers because they
said that I motivated them to become math teachers. So it's about the passion for the job and it's about knowing that you're interested and showing that you're interested in what you're doing.

Specifically, one commented that because of Paul's excitement about mathematics, "I was successful at it, so now I want to become a math teacher." Paul also shared students' comments about his college algebra teaching (see Appendix G).

Paul is a firm believer that excitement toward teaching motivates students and contributes to their success in mathematics. Paul described this in his own words:

My definition of teaching is motivating and captivating students to maximize their potential and so I think it's important that we first have to captivate them, we have to motivate them to realize what they have within them and the only way that we can do that is to be excited about what we're doing and to show that excitement because they will know it. They know from day one whether or not you're there to earn a check or if you're there to really teach them and if you love what you're doing.

Conducive learning environment. Motivating students to learn mathematics in Paul's class was achieved through creating a conducive learning environment. Creating an environment for learning promotes academic success in teaching mathematics (AMATYC, 2006; Choike, 2000; NCTM, 1991). Paul's model of teaching objective demonstration participation (ODP) "leaves little time for any type of mismanagement to occur." The model created opportunities for a conducive learning environment because it allowed students to focus on their learning. According to Paul:

My structure is that I tell them that I follow a model of, that I'm going to talk about the material, I'm going to introduce the material, I'm going to do some illustrations, then there's going to be some practice going on that they are participating, then we're going to discuss their practice and see how well they're doing and how well they're understanding.

Paul's students were part of this model because he created a conducive environment for them to learn mathematics. In this environment, students were free to participate in the
learning process, which contributed to their increased motivation and success in college algebra. In the first interview, Paul shared that one factor that contributing to his students' success in college algebra was the environment. He said:

> I think the environment which is that such that they feel that they can ask questions because I say to them at the beginning of class that they can feel free to ask any questions they want; I tell them that the very questions that they possibly want to ask if they are afraid to ask it, somebody else wants to ask that same question and so I think that it has to do with them being free to ask questions, free to explain if they feel the need to; feel free to discuss the dialogue as to [Paul], that doesn't really make sense, can you explain that to me again. Also, I tell my students to feel free to challenge, to challenge me if they feel that what I'm explaining and that if I have made a mistake or something, feel free to challenge me on that because again it's about my feeling good that they're understanding the material that I'm explaining and if you are challenging the instructor, don't be afraid to do that because that says that you truly, truly understand the material. So I think it's about the environment such that I've set a climate such that they feel comfortable in carrying on that dialogue, in asking the questions, in talking in mathematical terminology, and in terms of being-feeling free to dialogue with their classmates also.

During classroom observations, the conducive environment Paul created made
mathematics communication two-dimensional. The students were actively involved in the learning process. Paul explained, "Now, if I ask for answers, there is going to be at least one or two that are going to throw out an answer for me because they know that I want them to respond." There were minimum distractions, and the students arrived on time for each class period, which made the learning environment more conducive. I asked Paul what motivated the students to come to class on time and stay throughout the class period. He replied:

Well, I tell you, with this class, I told them, don't be late for my class. And I told you, I think it goes back to the structure. They know that I'm going to be there. They know that I'm going to start on time. And they know that I have asked them to not be late for my class, which means that I think it goes with the level of respect, knowing that we're going to be doing something constructive; it's going to start on time and I don't want to be late. So I think it's about communication from the beginning. It's about laying down the rules. I think all students want
structure and I think if the structure is set in place from the beginning, then I think that they will get a better feel for what's going on in the class and they will have a better understanding of the guidelines and the mode of operation for the class.

Paul made sure that from the very beginning of a semester, this conductive learning environment was prepared for his students. Due to the relaxing environment, students were comfortable enough to ask questions, to respond to each other's questions, and to interact with Paul. According to Paul, "it's setting that conducive atmosphere that allows that dialogue to go on, that chemistry to evolve."

Words of encouragement. Ladson-Billings (1995a, 1995b) supports academic success for all students, which can be achieved through words of encouragement. Paul used words of encouragement to motivate his students. Paul shared that his greatest hurdle while teaching college algebra was students not reading their mathematics textbooks and not working up to their potential:

I think it is the students not understanding that they actually have to read a textbook, that they actually have to read a math text book, I think that is the biggest hurdle in the college algebra course because for many of them, it is their first college experience and I think for many of them, they feel that all I have to do is come to class, copy these notes and I'm going to be okay or because it is their first college experience, they feel they might are going to be babied like they possibly were in high school. ...And they think that all I have to do is come to class and take notes rather than actually studying and reading the book along with the notes and call and try to get help.

To overcome this hurdle, Paul used words of encouragement to motivate the students to read their mathematics textbooks and improve their performance. He told them:

You have to develop better studying habits, and that really means reading the book. I told them how my family talks about me being a book nerd, and then I tell them, before the class even began, I would go get the book and start reading the book. Why would you do that? Because I want to be ahead. I want to be ahead of where the instructor is, so it's those kinds of things that I try to do to sort of get
them on board and try to make those worst moments into some better moments.
In addition, Paul believed that college algebra is the gatekeeper of more advanced mathematics courses. Due to this belief, Paul expected his students to have strong backgrounds in college algebra so that they would be successful in higher level mathematics courses. He used words of encouragement to communicate to his students the importance of mastering college algebra concepts. He mentioned:

I again communicate to them that a mind is a terrible thing to waste. I tell them that all the time, a mind is a terrible thing to waste so that I want them to be the very, very best that they can be in the college algebra course. But I also tell them that this is a building block for other courses and so the foundation is being laid now for the other courses because many of them if you would ask how many have to take the trig, how many have to take calculus, many of them have to go on to take trig, they have to go on and take calculus so the expectation is that they do their best in this course so that they build a strong, strong foundation because I tell them about prerequisites and I say a prerequisite means that when you get to trig and a question is asked about a parameter that you already know how to find the vertex, you already know what is the axis of symmetry because those are prerequisites, those are things that you already know coming into this course so I expect great things from them as far as their academic success in the course because it's not just for this one course. It is the building block for other courses that they are going to have to take.

Paul encouraged his students to work hard and to use all the resources available to them, including the center for academic success. Students were informed that the center provides tutors for all mathematics courses. Paul also asked his students how much time they spent studying right after class, how much time they spent on MyMathLab, and how much time they had spent in his office to getting help. Paul encouraged his students to become active in the learning process:

I told them at the beginning of the semester, 'If you just sit and watch me do problems and sit and watch me do problems and sit and watch me do problems,' I said, 'the only thing that you become good at is sitting and watching me do problems.' And I said, 'that is not what you want to do.' I say, 'You learn math by doing math,' and I want to get them involved in the process.

According to Paul, words of encouragement played a major role in student success. For example, Paul encouraged one of his students by saying "You can do it, it just takes hard work. I'm going to make it as easy as possible for you, but [you] just have to dedicate yourself to it." These words motivated the student, and she ended with a high grade in the class.

In all classroom observations, Paul used words of encouragement as positive reinforcement to student responses, including words like "very good," "beautiful insight," "good job," and "excellent." Paul did not use negative comments in correcting students; rather, he used probing questions to redirect their thinking. During an interview, I asked Paul about the role of "words of encouragement. He responded:

I think it gives them a sense of-I always want to give them a sense of I can do it because Antonia, and I'm sure you probably remember this too, I remember having professors in college that said I only give one A. You're only going to make one A in this class and I don't feel that way. I tell them if you all make As, I will be thoroughly happy and I don't mind giving you that and so I want to give them that sense of accomplishment rather than, I also had professors that want to make you feel really, really dumb to show you what you don't...it really shows you what you don't know. It's not my intent to give them a sense of this is not attainable but to give them a sense of its attainable. You can do it. ... so when I provide those words of encouragement, it's to help them understand that even though this is college, you can still do it. As I tell them, the class is on Tuesdays and Thursdays, so that gives you Monday, Wednesday, Friday, Saturday, Sunday and Monday again to prepare for the class, and I said if you really, really work at it, you can do that. So I'm trying to provide them with encouragement to say you can do it. It's attainable. If you put your mind to it and work extremely hard at this, you can do it.

By showing excitement for teaching, creating a conducive learning environment, and using words of encouragement, Paul was able to motivate his students to realize that mathematics is not only for selected groups of students. Students were able to understand that mathematics skill is attainable if they work hard, especially in foundation course like
college algebra. Words of encouragement might be overlooked by some teachers, but they provide excellent motivation.

## Repetition of Key Mathematical Concepts

The key to academic success in teaching and learning of mathematics is use of different teaching strategies, which includes the repetition of key terms (AMATYC, 1995; MAA, 2004; NCTM, 2000). Haar, Hall, Schoepp, and Schoepp (2002) also discussed that repetition is an effective teaching strategy. In each classroom observation, I noticed the repetition of key terms throughout each class period, which was the final theme emerging from data analysis. This took place at the beginning of the lessons, throughout the lessons, and at the end of the lessons. Paul employed summary, questions, and problem-solving methods when repeating key terms. According to Paul, repetition of key terms was a very important strategy because it was an effective way to help students remember important concepts. He said:

That it's about repetition. I think again, the research says that as far as repetition is concerned, that's very important and again, knowing that I am covering a section per day. That's a lot, and so to try to emphasize any of those things in each of the sections, either saw a repeat kind of thing, to make them more aware of what I'm trying to cover and so its keeping in mind the constraints as far as the curriculum is concerned and as far as time but trying to make it more palatable for them and reminding them these are the terms, these are the terms, and so it's sort of like trying to help them remember.

I noted that the repetition of key terms throughout Paul's classes helped the teacher and students keep track of the lessons. It reminded students of what they had covered during the lesson and what was important. This also helped students better retain mathematics knowledge because as Paul repeated the key concepts, students were often able to provide the rest of the related information without Paul's help.

The following example shows how Paul repeated key terms. In one of the lessons, Paul was discussing how to graph rational functions. He began with vertical asymptotes and explained to the students how to find these vertical asymptotes. He said that if there is no common factor, you set the denominator equal to 0 . He gave the students several examples of ways to find the vertical asymptote, and for each example, he repeated several times "Remember to check for common factor." He mentioned that if there is a common factor, one should cancel out the common factor before setting the denominator equal to zero. As Paul was repeating "remember to cancel out the common factor," I recalled (from my own teaching experience) a common mistake that students make when finding vertical asymptotes. With this repetition strategy, it is more likely for students to remember to cancel out the common factor.

I noted that Paul repeated important concepts when he explained horizontal asymptotes, listing three conditions for finding horizontal asymptotes. First, he told students that if the degree of the numerator is the same as the degree of the denominator, the horizontal asymptote is the quotient of the leading coefficients of the numerator and denominator. To illustrate this, Paul provided the example $f(x)=x^{2}+2 x-1 / 2 x^{2}+3 x+4$. In this example, the numerator and denominator have the same degree. Paul, repeating the key terms, engaged the students in the following dialogue:

Paul: What is the degree of the numerator?
Class: 2.
Paul: What is the degree of the denominator?
Class: 2.
Paul: The degree of the numerator and denominator are the same. What is the leading coefficient of the numerator?
Class: 1.
Paul: What is the leading coefficient of the denominator?
Class: 2.

Paul explained that the equation for the horizontal asymptote is $y=1 / 2$, which means that as x increases, the graph of the rational function will get closer to the line $\mathrm{y}=1 / 2$. Paul showed the graph of the example on the overhead; he emphasized and repeated several times that the degrees of the numerator and denominator are the same. He gave many examples to make sure that students understood the concept:

Paul: If I change the rational function as $3 x^{2}+2 x-1 / x^{2}+3 x+4$ what is the horizontal asymptote?
Class: 3
Paul: If I change the rational function to $3 x^{2}+2 x-1 / 4 x^{2}+3 x+4$ what is the horizontal asymptote?
Class: $3 / 4$.
Paul explained and repeated that in order to find the horizontal asymptote, if the degrees are the same, we use the leading coefficients. That is, the quotient of the leading coefficients of the numerator and denominator is the horizontal asymptote.

For the second condition, when the degree of the numerator is less than that of the denominator, Paul told the students that the horizontal asymptote is equal to 0 , which is the $x$-axis. He gave them an example $f(x)=2 / x+4$. Paul repeated that they have dealt with functions where the degrees are the same, now they are dealing with those where the degrees are not the same.

Paul: What is the degree of the numerator?
Class: Zero.
Paul: What is the degree of the denominator?
Class: One.
Paul repeated that if the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y=0$. He explained it on the graph, saying that you have to represent it with a dotted line.

For the last condition, Paul gave the students another example, $f(x)=5 x^{4}-3 x^{2}+$ $2 x-5 / x+1$. He explained that the degree of the numerator is greater than that of the denominator, which means that there will be no horizontal asymptote. He emphasized that if the degree of the numerator is one greater than that of the denominator, one will have what is called oblique or slant asymptote.

The teacher then repeated the three conditions for horizontal asymptote:

1. If the degrees are the same we have the leading coefficients as the horizontal asymptote.
2. If the degree of the numerator is less than the degree of the denominator we have $y=0$ as the horizontal asymptote.
3. If the degree of the numerator is greater than the degree of the denominator we have no horizontal asymptote.

One student asked a question about the horizontal asymptote being equal to zero, that is, $y=0$. Paul answered the student's question with the example $f(x)=1 / x+4$. He explained that the vertical asymptote is $x=-4$ and the horizontal asymptote is $y=0$. He sketched the graph on the board and demonstrated how the graph is approaching $\mathrm{y}=0$ but will never intersect $y=0$, which is the horizontal asymptotes.

To further explain, Paul gave the students another example, $f(x)=x^{2}-9 / x-3=$ $(x-3)(x+3) /(x+3)$, explaining that they had to factor it and cancel out common factors. He graphed the function on the board to show students that in this particular example, there will be no asymptotes. With this example, students saw when asymptotes exist and when asymptotes do not exist. Again, Paul repeated the three conditions for horizontal asymptote. He asked the students again how to find the vertical asymptote:

Class: You set the denominator equal to 0 .

Paul emphasized that the graph of rational function will never intersect with the horizontal asymptote.

The discussion of graphing rational functions continued with the repetition of key mathematical terms. In the middle of the lesson, Paul said that so far, they had discussed the terms x-intercept, y-intercepts, vertical asymptotes, and horizontal asymptotes. He repeated that if you are asked to graph rational functions, he or she should find the values of the x - and y -intercepts, vertical asymptotes, and horizontal asymptotes. For the test points on horizontal asymptote, pick x -values to the right and left of the vertical asymptote (for one vertical asymptote). For two vertical asymptotes, pick x-values to the right and left of the vertical asymptote as well as x-value from the middle section. He reminded the students that for the horizontal asymptote, they had discussed different cases depending on the degrees of the numerator and denominator. He continued with the lesson and introduced a subtopic, oblique or slant asymptote. While teaching the subtopic, Paul repeated several times that you never have an oblique asymptote and horizontal asymptote on the same graph. He pointed out that it is possible to have horizontal and vertical asymptotes on the same graph and oblique and vertical on the same graph. After dialogue with the students, Paul mentioned again that they should remember there would be either oblique or horizontal asymptotes, but never both. Instead, one can have oblique mixed with vertical asymptote and horizontal mixed with vertical, but no horizontal and oblique. In addition, Paul summarized the important concepts they had discussed, starting at the beginning of the lesson and moving to the current.

By asking questions, Paul reviewed the key concepts that had been covered. He asked his students to give the three conditions for horizontal asymptote and to show him how to find vertical asymptote. He reminded the students that if there is no common factor, they should set the denominator equal to 0 to find the vertical asymptote. He said for the oblique asymptote, the degree of the numerator must be one less than that of the denominator. After explaining, Paul posed a question: "If the leading term of the numerator of a rational function is $2 \mathrm{x}^{4}$ and the leading term of the denominator is $\mathrm{x}^{2}$, do I have oblique asymptote?"

## Class: No

Paul: Why not?
Class: Because the degree numerator is not one less than the degree of the denominator.
Paul: Very good. You are listening. Again remember there will be either oblique or horizontal asymptote but never both. You can have oblique mixed with vertical asymptote. You can have horizontal mixed with vertical, but no horizontal and oblique.

To close the lesson, Paul once again repeated the important concepts to evaluate students' understanding of the material. He told the students to write down and graph the function down $f(x)=x+1 / x-6$. He moved around, checking students' work and providing feedback. As usual, Paul interacted with his students to get answers to questions and to give feedback. Students not only communicated with Paul, but they communicated with their peers as well in solving the problem. When the discussion time arrived, Paul started asking questions again:

Paul: What is the horizontal asymptote?
Class: $\mathrm{y}=1$.
Paul: Vertical asymptote is what?
Class: $\mathrm{x}=+6$.
Paul: What is x-intercept?
Class: $(-1,0)$.
Paul: y-intercept is what?

Class: (0, -1/6).
Paul represented each value on the graph sheet and finally graphed and explained the function on the overhead.

Paul's strategy of repeating key terms occurred at different stages of his lessons. Paul used different methods when repeating important material, including the summary method, questioning method, and problem-solving method. Paul used repetition to remind students of the important facts in a lesson, which led to better retention and deeper understanding of the information covered.

## Summary of Themes

Concluding three interviews and three observations during the Spring 2011 academic semester, six major themes were identified that characterized Paul's teaching practices: The first theme was mathematics communication, which includes the questioning method, one-on-one communication, board participation, and non-verbal communication. This theme created opportunities for students' voices to be heard during the learning process, which made them active learners instead of passive learners.

The second theme was proper use of technology in instruction and assessment. Proper use of technology encompassed the use of graph programs, PowerPoint, TI SmartView, and Geometer's Sketchpad while technology in assessment consisted of MyMathLab for online assignments and calculators for in-class tests. Paul's use of technology helped captivate student interest in mathematics.

The third theme was building mathematical connections between mathematics and real-life experiences, between college algebra and upper-level mathematics courses, between prior knowledge and current knowledge, and between mathematical
terminologies and procedures. Through these connections, Paul showed his students that learning mathematics can be meaningful.

Motivating students to learn mathematics emerged as the fourth theme. Through being enthusiastic about his teaching, using of words of encouragement, and creating a conducive learning environment, Paul was able to motivate his students. Motivation in turn helped promote student performance in mathematics.

The fifth theme identified was the use of multiple representations. Paul employed different methods to change mathematic representations from symbolic to verbal, verbal to symbolic, numerical to graphical, and symbolic to graphical. These representations provided opportunities for the instructor to accommodate different learning styles in the classroom. Presenting mathematics concepts in different forms also allowed students to achieve a deeper understanding of mathematic concepts.

The final theme was the repetition of key mathematics terms. Paul used different methods when repeating the important material, including summary, questioning, and problem solving. Through repetition at different stages of the lesson, Paul made sure students had time to retain and remember important mathematics facts.

## Paul as a Standards-Based Instructor

The second research question guiding the current study pertained to the ways Paul's teaching practices were aligned with the characteristics of standards-based instruction. After thorough data analysis, I decided that the best method to answer this question was to present the components of standards-based instruction strategies and explain how Paul's teaching was aligned with each component. Although this method created some redundancy, this was the best approach to address the question and still
allow Paul's voice to be heard. Table 1 lists the AMATYC's instructional strategies and corresponding strategies from classroom observations, interviews, and artifacts.

## Active Student Learning

One component of standards-based instructional strategies is active student learning (AMATYC, 2006). According to the AMATYC (2006), active student learning can be achieved through collaborative/cooperative learning, discovery-based learning, interactive lecturing and question-posing, and writing in mathematics learning.

These characteristics were present in Paul's teaching, as he was able to provide opportunities for students to work with their neighbors to solve and discuss problems. Board participation and one-on-one communication also created room for mathematics interaction. Using the questioning method, Paul created opportunities for students to ask and answer questions. In addition, Paul encouraged students to write during class, which included writing mathematics problems related to linear functions, solving mathematics problems during class periods, writing the steps to get their answers on class tests, and taking notes in class. The teacher also gave students feedback using written communication. In addition, Paul's focus on discovery-based learning allowed students to be active learners. Students were asked to solve application problems on their own, and in the process of solving them, they were able to discover new ideas and find connections between procedures and real-life experiences. For example, in the linear functions application problems, students were able to discover the equations for the linear functions, the slope, and the $y$-intercepts from real-life application problems. Further, students learned that there are connections between mathematics courses.

Table 2
Alignment of Paul's Teaching Strategies with American Mathematics Association for Two-Year Colleges Standards

| AMATYC Instructional Strategy | Strategy observed from data collection |
| :--- | :--- |
| Strategy 1 | Students interaction during practice time |
| Active student learning: collaborative/ | Questioning method |
| cooperative learning both inside and | Mathematics writing |
| outside the classroom, discovery-based | One-on-one communication <br> learning, interactive lecturing and <br> question-posing, and writing |
| Board participation <br> Discovery-based learning through <br> application questions |  |
| Strategy 2 | Symbolic to verbal representations |
| Use of multiple approaches | Verbal to symbolic representations |
| (numerical, graphical, symbolic, and | Numerical to graphical representations |
| verbal) | Symbolic to numerical to graphical |
|  | representations |
| Strategy 3 | Problem solving |
| Allow students to experience | Use of application questions |
| mathematics | Proof of mathematical formulas |
| Strategy 4 | Use of graph program, PowerPoint, |
| Proper use of technology both in | TI SmartView, Geometer's Sketchpad |
| instruction and assessment | Use of MyMathLab and calculators |
| Strategy 5 | Connecting teaching of mathematics to |
| Connect math to other experiences | students' experiences |
|  | Connecting teaching of mathematics to |
|  | other mathematics courses |
|  | Connecting teaching of mathematics to |
| other disciplines |  |
| Strategy 6 | Creating a conducive learning environment |
| Actively manage the learning | Enthusiastic about teaching |
| environment | Repetition of key terms |
|  | Using of words of encouragement |

These strategies helped Paul's students become active learners, and his students participated in the learning process. The traditional method of teaching mathematics, where students listen and the teacher talks, was not used in Paul's classes.

## Multiple Approaches

Another characteristic of standards-based instruction strategies is use of multiple approaches. This means that mathematics faculty should incorporate different ways of solving mathematics problems, such as numerical, graphical, symbolic, and verbal (AMATYC, 1995). It helps students better understand and interpret mathematics concepts (NRC, 1989). This characteristic was also present in Paul's teaching. Paul used different representations in teaching functions. For example, Paul asked the students to describe what happens to the graph of $f(x)=x^{2}$ when given the following equations:
A. $f(x)=4(x-3)^{2}+2$
B. $f(x)=-1 / 2(x-3)^{2}-5$
C. $f(x)=-(x+2)^{2}-5$

Also, he asked them to write an equation for $\mathrm{y}=\sqrt{\mathrm{x}}$ with the following transformation:
A. Shifted 2 units to the right, reflected across the $x$-axis and stretched vertically by a factor of 3 .
B. Shifted 4 units to the left, stretched vertically by a factor of 2 .
C. Reflected across the $y$-axis and shifted up 3 units.
D. Reflected across the $y$-axis, shifted 2 units to the right and stretched vertically by a factor of 3 .

The above examples were not the only times that Paul used multiple approaches. He also employed multiple approaches when teaching rational functions. For instance, in graphing $\mathrm{h}(\mathrm{x})=2 \mathrm{x} / \mathrm{x}+3$, he showed the students how to use the function to get tables of values and use the tables of values to graph the function. The same thing applied to the graph of logarithm functions. Paul discussed with his students how to use $f(x)=\log _{6} x$ to derive tables of values and use the values to graph the function. In some cases, Paul also engaged the students in using symbolic forms of the function to graph the function without the tables of values. In graphing functions like $f(x)=x^{2}+4, f(x)=(x-2)^{2}, f(x)=$ $1 / 2 x^{2} f(x)=2 x^{2}$, and $f(x)=-3 x^{2}$. Paul explained to his students how to graph the functions without tables of values. Paul helped his students understand that mathematics
topics can be represented in different forms. The students were able to interpret from one form to another, which promoted their understanding of the topics.

## Experiencing Mathematics

Allowing students to experience mathematics is another characteristic of standards-based instruction strategy. Experiencing mathematics refers to providing learning opportunities that allow students to be independent and critical thinkers (AMAYTC, 1995). It can be achieved through activities that make the learning of mathematics meaningful to the students. Paul allowed his students to" experience mathematics" rather than do mathematics without connecting it to student experiences. The students were involved in problem solving, and Paul's strategy of allowing students to experience mathematics helped them to understand the applications and meanings of as well as reasons for concepts. Application-based problems in Paul's classes gave students chances to think critically and create their own ideas about solving problems. In addition, Paul was committed to engaging his students in finding out the "whys" and meanings of the materials. For example, students were able to define and understand a logarithm. When asked the $\log _{2} 4$, students would not simply say " 2 " without knowing the reasons. According to Paul, his students knew that the inverse of a logarithmic equation is an exponential equation. To answer the question about $\log _{2} 4$, they knew they would change it to exponential form rather than just give the answer without knowing the reason for it. Paul also challenged his students to understand how formulas are derived.

Paul helped his students develop problem-solving skills, and through Paul's assignments, students were challenged to become independent thinkers. For example, as mentioned before, the students were asked to solve this problem: "The decay rate of a
certain chemical is $9.4 \%$ per year. What is its half-life? Use the exponential decay model $\mathrm{P}(\mathrm{t})=\mathrm{P}_{0} \mathrm{e}^{-\mathrm{kt}}$ where k is the decay rate, and $\mathrm{P}_{0}$ is the original amount of chemical. The half-life of the chemical is $\qquad$ years." This type of question allowed the students to think critically and make mathematical conclusions. Rote learning was not evident in Paul's classroom.

## Integration of Technology in Instruction and Assessment

According to the AMATYC (2006), mathematics faculty should incorporate technology in their instruction and assessment of the instruction. The use of technology promotes the learning of mathematics in the following ways:

- Explore new concepts and discover patterns;
- Examine, organize, analyze, and visualize real-world data;
- Develop understandings of mathematical ideas;
- Make connections among and between mathematical ideas;
- Provide a visualization of mathematical models; and
- Provide symbolic, graphical, and/or numerical evidence to support or dispel student-formulated conjectures (AMATYC, 2006).

The use of technology emerged as one of the major themes of Paul's teaching. Paul used different types of technology to accommodate the different learning styles. This included use of graph programs, PowerPoint, TI SmartView, and Geometer's Sketchpad. These technological tools helped the students visualize the concepts so that college algebra was less abstract. Some of these tools allowed students to see exactly what was meant by vertical and horizontal translations, shrinking and stretching, reflections, and differences between logarithmic and exponential functions. In addition, the students' use of

MyMathLab and calculators for online assignments provided them with opportunities to discover new concepts and develop better understand mathematics.

## Connecting with Other Experiences

According to the AMATYC (1995), connecting with other experiences means:
Mathematics faculty will actively involve students in meaningful mathematics problems that build upon their experiences, focus on broad mathematical themes, and build connections within branches of mathematics and between mathematics and other disciplines so that students will view mathematics as a connected whole relevant to their lives. (p.16)

While teaching, Paul made sure his students had opportunities to see and make connections between mathematics and their other experiences. Paul connected his teaching to students' experiences, other mathematics courses, and other disciplines. To do this, he used application-based problems that were related to student experiences in topics like linear functions, quadratics functions, exponential functions, and logarithm functions. These application-based problems helped students understand that mathematics is meaningful and useful. Paul also connected his teaching to students' prior knowledge, which made them understand the relationships between mathematics topics. In addition, Paul showed his students how college algebra concepts could be used in other mathematics courses such as calculus and trigonometry. He challenged students to master the concepts so that they would not have problems in their future mathematics courses. In addition to relating mathematics to other mathematics courses, he also connected it to other disciplines. For example, Paul discussed with students how to predict a country's population using their college algebra knowledge. They also discussed finding the half-life of bacteria. Paul's ability to connect his instruction helped students understand that mathematics is not an isolated set of rules and procedures.

## Actively Manage the Learning Environment

According to the AMATYC (2006), mathematics faculty should create a learning environment that maximizes the learning opportunities for all students. The learning environment should be comfortable for learning, and the faculty should encourage and support students while they learned mathematics. Paul indeed created such a learning environment. Through thorough lesson planning that incorporated a variety of teaching practices and the ODP model, Paul was able to engage his students in mathematics communication "that leaves little time for any type of mismanagement to occur." The focus on mathematics communication created room for all students to express themselves during the learning process. Even though the environment was relaxing, the focus remained on learning. Paul had high student expectations; students understood that each time they were in his class, they had to apply their best efforts and that distractions were unacceptable. Paul's enthusiasm for teaching motivated students to learn, improve in mathematics learning, and enjoy the learning process. Students arrived on time to class and stayed throughout the class period because of the conducive learning environment. Additionally, Paul repeated important concepts, which allowed him to reach all students regardless of their learning styles.

Words of encouragement also helped Paul create a conducive learning environment. This strategy assisted in student motivation, and it allowed Paul to communicate to his students that mathematics can be successfully learned if they work hard. Through words of encouragement, Paul was able to demonstrate study skills to the students that positively affected their learning. Paul was patient and gave students his undivided attention when they asked and answered questions. These actions showed the
students that they were valued and respected, which in turn earned Paul respect from the students.

According to the AMATYC $(1995,2006)$, a teacher can be considered a standards-based instructor if his or her teaching practices include the characteristics of standards-based instruction: active student learning, use of multiple approaches, allowing students to experience mathematics, integration of technology in instruction and assessment, connecting mathematics to other experiences, and actively managing the learning environment. Based on the data collected in the current study (i.e., direct observation, personal interview, artifacts), Paul's teaching practices were aligned with the characteristics of standards-based instruction. His teaching approach was studentcentered, not teacher-centered, and his students actively participated in the learning process.

## Summary

The six themes that emerged from analysis of the data collected in the current study were mathematical communication, proper use of technology in instruction and assessment, building mathematical connections, multiple representations, motivating students to learn mathematics, and repetition of key terms-all of which have been discussed in detail. I also aligned the characteristics of standards-based instruction with characteristics of Paul's teaching. Based on the data analyzed in the current study, Paul appears to be a successful college algebra instructor who applies standards-based instruction strategies while teaching mathematics courses.

## CHAPTER 6

## CONCLUSIONS AND RECOMMENDATIONS

This study concludes with a re-examination of the research problem, the purpose, the research questions, the methods used in the study, the theoretical frameworks, and a summary of study results. Implications of the results are teachers, university administrators, textbook writers, researchers, and policy makers are also identified in this chapter along with recommendations for future research and limitations of the study.

## Conclusions

The problem that motivated the current study is that the lecture method, which has dominated undergraduate mathematics education (Bergsten, 2007), leads to passive learning instead of active learning among students, thus contributing to student attrition in undergraduate mathematics. However, research indicates that if college algebra is taught using standards-based teaching strategies, student attrition is reduced, which leads to higher enrollment numbers in collegiate mathematics classes (Burmeister, Kenney, \& Nice, 1996). Despite efforts to improve undergraduate mathematics instruction, there are still limited empirical studies on standards-based instruction in college algebra. Furthermore, most of the literature on mathematics education has been with grade schools (Selden \& Selden, 1993); research on undergraduate mathematics education is a new field of study (Brown \& Murphy, 2000).

Waller (2006) reported that overall student attrition in college algebra could be as high as $41 \%$ in community colleges. This high attrition rate in college algebra adversely affects students' continuation in upper-level mathematics classes and their interest in the field of mathematics, which may lead to a higher attrition rate for college overall. The
purpose of the current study was to explore the teaching practices of a college algebra teacher who utilizes standards-based instruction in his classes and the ways these strategies contributed to student learning in college algebra. The research questions that guided the study were:

1. What teaching practices are used in the mathematics classroom of a college algebra teacher?
2. How are the teaching practices of the college algebra teacher aligned with the characteristics of standards-based instruction?

The study was an interpretive single case study, and the participating instructor was a college algebra teacher who has successfully adopted standards-based instruction strategies. Data, which included three personal interviews, three direct observation session, and artifacts/documents (e.g., lesson plans, homework assignments, quizzes and tests), were collected during the Spring 2011 academic term. The study was framed in culturally relevant pedagogy theory, symbolic interaction theory, and experiential teaching theory. I incorporated these theories because they each address components of standards-based instruction, which was the focus of the study. The conceptual framework of the study was standards-based instruction.

The research questions were focused on the teaching practices of the participating instructor and the ways his teaching practices were aligned with characteristics of standards-based instruction. Six themes emerged during data analysis: mathematics communication, proper use of technology, building mathematical connections, multiple representations, motivating students to learn mathematics, and repetition of key concepts.

As described in the case study, mathematics communication was a major observed in Paul's teaching practices. Interviews and classroom observations showed that Paul allowed his students many opportunities to communicate with him in writing as well. Paul stated (presented in Chapter 5) that mathematics writing helped students think through the mathematics process. Also, Paul encouraged his students to ask and answer questions in class, and he gave them all his undivided attention. Paul's "methodology of teaching is that of questioning," which Paul said created opportunities for him to provide feedback to and receive feedback from students. He used many strategies to make his questioning method effective, including calling students by their names, using probing questions, making students respond to each other's questions, making mistakes on purpose, and using positive reinforcement. In addition, Paul created many chances for students to interact with their peers. Practice time during Paul's classes gave students opportunities to share ideas with their neighbors. Students were able to exchange information about solving mathematics problems, provide feedback to each other, question each other, and debate with each other, which all contributed to their learning. Paul reported that there were times that students understood their peers' explanations better than his. Practice time also enabled one-on-one communication between Paul and the students. As mentioned earlier, Paul moved around in the classroom during practice time. With this one-on-one communication strategy, some students were more comfortable asking questions instead of asking them in front of the entire class. Paul also used the opportunity to provide individual feedback to the students.

Proper use of technology was another major component of Paul's instruction and assessment. He used different types of technology in his classroom, including graph
programs, PowerPoint, TI SmartView, and Geometer's Sketchpad. Also, students used MyMathLab and calculators for online assignments. Paul used technology to enhance student learning and to captivate student interest. The use of graph programs helped students visualize graphs of rational functions, exponential functions, logarithmic functions, and translations of functions. Students were able to see and identify the way each function looks and see the differences between functions. PowerPoint programs saved the class time and made the teacher's notes accessible to students after class. Students were able to focus on learning instead of trying to write everything down because they knew it would be available to them after class. In class, they were able to ask and answer questions and interact with the teacher. The students focused on learning instead of note-taking. Use of PowerPoint allowed Paul to face students and move around the classroom while teaching. TI SmartView, an on-screen calculator, allowed Paul to clearly explain mathematics topics to student and the students to see Paul calculate things on the overhead. Students learned how to use their calculators by watching on-screen practice partnered with Paul's explanation. With the use of Geometer's Sketchpad, Paul shared ways to prove and verify mathematics concepts with his students. Students drew and visualized each step of mathematics construction and thus were able to make meaningful conclusions.

Online homework and quizzes were also part of the class. Paul used MyMathLab to make practice, quiz, and homework problems available online, which allowed students to practice as much as needed. The thorough feedback students received from the online assignment software as well as feedback they received from Paul through the email system improved their mathematical performance. Online assignments made it possible
for students to complete assignments at their own pace and convenience. The fact that assignments were instantly graded by the software helped students realize their mistakes in a timely manner. That way, students were able to correct their mistakes based on this instant feedback. If Paul had to manually grade everything, instant feedback would have been impossible.

Building connections between mathematics concepts was another major theme found during data analysis. As discussed in Chapter 5, Paul related his lessons to realworld situations when teaching linear functions, quadratic functions, system of equations, exponential functions, and logarithmic functions. He gave students many applicationsbased problems, which helped them understand the value of college algebra concepts. Also, Paul was able to relate his instruction to higher mathematics courses including calculus and trigonometry. For example, when he asked his students to solve $\mathrm{x}^{2}=4$, he insisted that the students write the answer as $x=+/-2$ not just +2 so that when they take trigonometry, they would not have problems transferring the knowledge to solve $\sin ^{2} \mathrm{x}=$ $1 / 4$. Building mathematical connections allowed students to develop solid foundations for future mathematics courses. Paul was interested in helping his students build networks of knowledge by connecting current learning to their existing knowledge. Through the interviews and classroom observations, I confirmed that Paul did not present mathematics knowledge in "disjointed" form; rather, he presented it in an overlapping manner. Connecting prior knowledge and future knowledge led to students gaining solid mathematics knowledge that can be retained and retrieved later. In addition, deep understanding of mathematics topics was achieved in Paul's class through thorough explanations of mathematics terminology.

As explained in Chapter 5, use of multiple representations emerged as another major theme. Paul utilized multiple approaches in his instruction to improve the student understanding in college algebra. He wanted his students to understand the connections between different mathematics forms. This included symbolic to verbal, verbal to symbolic, numerical to graphical, and symbolic to graphical forms, as observed in Paul's class. The tests and assignments also embodied multiple representations. Paul used multiple approaches to make sure his students did not solely depend on calculators. He ensured that students were able to use rational and exponential functions to derive tables of values and then use these values to graph functions. It is easy to enter functions on a calculator and press the "graph" button, but Paul did not want his students to learn mathematics without understanding the concepts. He showed students how to graph without calculators first, which deepened their understanding. These representations provided opportunities for Paul to accommodate students with different learning styles in the classroom. Students had the opportunity to utilize different learning styles; some students understood the material when it was presented verbally while some preferred graphic representation. Some students preferred to see the concepts in tables while some wanted to see all the forms before attempting to make sense of the topic. Multiple representations involved all students in the learning process, thereby transforming them from passive to active learners.

Motivating students to learn mathematics was another major theme emerging from data analysis. Paul used different methods to motivate his students, which included showing enthusiasm toward teaching mathematics, using words of encouragement, and creating a conducive learning environment. Paul used different strategies to create this
conducive learning environment, making students feel free to communicate with him and among themselves. Students also felt free to ask and answer questions. Paul was patient and gave students his undivided attention when they asked and answered questions. Students arrived to the class on time and stayed throughout the class period because of the conducive learning environment. Paul never used words that discouraged students from participating in class; rather, he used probing questions to redirect incorrect responses. These strategies showed that Paul cared about his students learning. There was periodic laughter throughout the class, and Paul and the students were in a happy mood each time the classes were observed. The environment Paul created encouraged students to learn, which led to their success and to low attrition rates.

The sixth major teaching practice emerging from data analysis was repetition of key terms, which occurred at different stages of his instruction. Paul used different methods when repeating important material, including summary, questioning, and problem-solving methods. Paul used repetition to remind students of important facts in a lesson, which may potentially lead to lower retention rates and deeper understanding of the information covered.

Paul's teaching practices, aligned with the characteristics of standards-based instruction, indicated that he was a successful college algebra instructor. The current study provides evidence that standards-based instruction strategies can improve student outcomes and student learning as well as reduce student attrition rates in undergraduate mathematics courses. Results of the current study also have implications on college algebra instruction.

## Implications of the Results

The American Mathematics Association of Two-Year Colleges (AMATYC) (2006) stresses that mathematics faculty should use different teaching strategies to improve student learning. The participant instructor, Paul, used a variety of teaching strategies in his classroom as aligned with characteristics of standards-based instruction. This study has important implications for the teachers and university administrators as well as textbook writers, researchers, and policy makers.

## Teachers

Paul created many opportunities for communication in his mathematics classroom, which allowed students to express themselves in their learning environment. Students were active participants in their learning process, which led to deeper understanding of mathematics concepts. Considering this, mathematics faculty should include mathematics communication in their teaching. Inclusion of mathematics communication in teaching is also emphasized by the AMATYC (1995/2006) instruction standards and other researchers (Boaler, 2006; Ellington, 2005; Gutierrez, 2000; Kagesten \& Engelbrecht, 2006; Iannone \& Nardi, 2005; Thomas \& Santiago, 2002).

Making mathematics learning meaningful to students is the key to academic success and reduced student attrition rates. Paul made every effort to connect his teaching students' real-world experiences and knowledge from other disciplines. Building these connections in mathematics both motivated students and captivated their interest in mathematics. Students were able to understand that mathematics is not just a set of rules and formulas. Making mathematics meaningful to the students is also supported by AMATYC instruction standards (1995/2006) and previous studies on college algebra
instruction (Austin, Berceli, \& Sarae, 1999; Choike, 2000; Leonard \& Guha, 2002;
Pierce, Turville, \& Giri, 2003). Therefore, the main implication is that teachers should connect mathematics to students' life experiences and to their knowledge of other disciplines.

A third implication for teachers to incorporate technology in their instruction. Because students learn differently, it is important for teachers to include technology in their instruction to reach all students regardless of their learning styles. As Paul mentioned, technology involves many captivating tools. It helps sustain student interest in learning mathematics, thereby leading to student success and low attrition rates. The AMATYC (2006) stresses:
...the integration of appropriately used technology can enhance student understanding of mathematics through pattern recognition, connections, and dynamic visualizations. Electronic teaching activities can attract attention to the mathematics to be learned and promote use of multiple methods. (p. 56)

Therefore, effective use of technology can greatly promote student understanding of mathematics (Mayes, 1995; Hagerty \& Smith, 2005; Thiel, Peterman, \& Brown, 2008).

## University Administrators

Standards-based instruction strategies have the capability to improve student learning and reduce student attrition in undergraduate mathematics (Ellington, 2005). However, undergraduate mathematics education is a relatively new field of study (Brown \& Murphy, 2000); therefore, most mathematics faculty at the collegiate level are not aware of the strategies. A key implication of the current study is that university administrators should organize workshops and professional development sessions about standards-based instruction strategies. Paul emphasized that both workshops and professional development sessions helped him realize the importance of standards-based
instruction. Very few college faculty are aware of these instructional strategies and are faced with issues of time and curriculum constraint, which Paul mentioned as issues he once faced. Mathematics department chairs should make sure that the topics covered in each course are necessary, and they should remove unnecessary topics so that teachers have enough time to teach for understanding. Teaching for understanding encompasses standards-based instruction strategies (National Council of Teachers of Mathematics, 2000). This way, students can improve academically, which leads to low attrition rates.

## Textbook Writers

Paul has been a successful standards-based instructor at the grade school and college level and has seen outstanding student results. One resource Paul uses is a mathematics textbook that supports his standards-based teaching model. The positive impact of standards-based instruction should spread to all areas of student learning, including the development of mathematics textbooks. Skovsmose (2005) mentioned that "the mathematics textbooks dominate the class teaching which the teachers follow page-by-page" (p. 9). Skovsmose (2005) further explained that "most textbook authors do not make empirical studies in order to provide realistic exercises.... Many exercises refer to a non-mathematical situation that nevertheless appears artificial" (p. 48). Skovsmose stated that the exercises in mathematics textbooks lack activities that allow students to be creative. Skovsmose (2005) also emphasized that virtual reality in mathematics textbooks creates problems because it leads to absolutism in mathematics learning. Due to this, it is important for mathematics textbook writers to include activities and exercises that promote standards-based instruction strategies, which in turn promote academic success among students. But it is important to note that solely utilizing a mathematics textbook
with standards-based activities does not qualify an instructor as a standards-based instructor.

## Researchers and Policy Makers

The literature on mathematics education should include information on standardsbased instruction at the undergraduate mathematics level, more specifically in college algebra. Telling the stories of collegiate-level standards-based instructors has important implications and suggestions for researchers who aim to further explore standards-based teaching strategies in undergraduate mathematics. With regards to policy makers, I recommend that policy makers include standards-based instruction strategies as part of teaching strategies in undergraduate mathematics education.

## Recommendations for Future Research

The current study investigates the teaching practices of a successful college algebra instructor who utilizes standards-based instruction strategies in his classes. Application of these strategies was found to reduce student attrition in undergraduate mathematics. However, many other areas deserve further study. One recommendation is to replicate the current study to other schools with different demographic compositions.

Paul, the instructor participant, reported that he changed from a dominant lecturer to a facilitator in his classes. He explained that the reason for change was the advice he had been given and his experiences with "different workshops that talked about standards-based classroom, looking at NCTM and GCTM, and seeing what the literature was saying about the standards-based classroom, what the standards-based classroom should look like and in graduate school reading several texts." This knowledge helped Paul achieve outstanding instructional results. Practical implementation of the standards-
based instruction skills that teachers gain during graduate school should be examined. As such, further research is needed to study the teaching practices of those with training in standards-based instruction. Again, seeing these results from an institution with a different demographic composition would be useful.

Even though the focus of the current study was on college algebra instruction at the undergraduate level, other disciplines, including science and engineering, are also experiencing high student attrition at undergraduate level (Seymour, 1994). Therefore, future researchers could seek ways to reduce student attrition rates in those disciplines. Symbolic interaction theory, culturally relevant pedagogy theory, and experiential teaching theory can still serve as the theoretical frameworks for such studies.

In the current study, data collection included interviews and classroom observations along with class tests, homework, and quizzes. The multiple data sources allowed data triangulation. For practical reasons, there was no direct student contact. Therefore, future research could include student interviews as an additional method of data collection. Paul mentioned that some of his students told him that his excitement for teaching mathematics motivated them to become mathematics teachers one day. One student even commented that "I was successful at it, so now I want to become a math teacher." A follow-up study could involve Paul's students who wanted to become mathematics teachers; Paul could identify these subjects.

The current study is focused on college algebra, a foundational course for upperlevel mathematics courses. Reducing student attrition rates in college algebra courses will increase student enrollment in upper-level mathematics courses and reduce college attrition overall. However, fighting student attrition is a never-ending battle. Students
need upper-level mathematics courses. Since few scholars (Iannone \& Nardi, 2005; Kagesten \& Engelbrecht, 2006; Roth-McDuffie, McGinnis, \& Graeber, 2000) have investigated the impact of standards-based instruction in upper-level undergraduate mathematics, future research describing the teaching practices of successful college instructors who teach upper-level mathematics courses and adopt standards-based instruction strategies is needed.

## Limitations of the Study

To begin, caution should be taken when generalizing findings of the current study. One limitation of this study is that it examined only one participating instructor over one academic semester. Including more participants would allow for cross-case analysis (Merriam, 1998), and extending the time frame of the study would increase the volume of data collected and other themes that were not identified during the semester might arise.

Another limitation of the current study is that the participant was selected using purposeful sampling. Creswell (1994) noted that purposeful selection may affect the generalization of study results to other settings. The participant was identified as a successful teacher who has implemented standards-based instruction at both the secondary and collegiate levels. The participant has taken courses and participated in professional development sessions on about standards-based instruction. Therefore, it may be difficult to transfer the findings of this study to other educators without any knowledge of standards-based instruction.

Another limitation of the current study is the site of the study, which was the southeastern region of the United States. Additionally, Paul's students consisted of more of "nontraditional" and also African American students. African American students learn
better from certain teaching strategies, including interaction and relating teaching to their culture (Ladson-Billings, 1994). Nontraditional students have more life experience than traditional students, which may also have an effect on their academic decisions. These things may all have some effect on the generalizability of the study findings.

## Summary

The purpose of the current study was to investigate the teaching practices of a college algebra instructor who uses standards-based instruction strategies. The practices used by Paul, the instructor participant, were aligned with the characteristics of standardsbased instruction presented by the AMATYC (1995/2006).

Although (a) the findings from this study provided a rich description of standardsbased instruction that lowers student attrition rates in college algebra and (b) the results of this study can have an effect on college algebra instruction, more qualitative research on standard-base instruction strategies in undergraduate mathematics is needed. The AMATYC (1995/2006) states that pedagogical standards include teaching with technology, making connections, using multiple approaches, active and interactive learning, "experiencing" mathematics, and actively managing the learning environment. If instructors are going to implement these strategies in their classrooms to reduce student attrition, researchers should further investigate the actions of those following standardsbased instruction strategies in mathematics classrooms.

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## APPENDIXES

Appendix A
Interview/Observation Schedule

| Date | Time | Participant | Activity |
| :--- | :--- | :--- | :--- |
| $2 / 8 / 11$ | 2 p.m | Paul | Interview 1 |
| $2 / 15 / 11$ | $12: 45$ p.m | Paul | Observation 1 |
| $2 / 29 / 11$ | 2 p.m | Paul | Interview 2 |
| $4 / 7 / 11$ | $12: 45$ p.m | Paul | Observation 2 |
| $4 / 21 / 11$ | $12: 45$ p.m | Paul | Observation 3 |
| $5 / 2 / 11$ | 2 p.m | Paul | Interview 3 |

## Appendix B

## Questions for All Three Interviews

## Initial Interview Questions

1. Why did you become a mathematics teacher?
2. What is your role as a mathematics teacher? What is your philosophy concerning mathematics instruction?
3. How long have you been a mathematics teacher? How long have you taught college algebra?
4. Tell me about your experiences teaching college algebra.
5. When did you start implementing standards-based instruction in your teaching? What factors contributed to your change?
6. Tell me stories about your experience as a student
7. Did your experiences influence your role as a mathematics teacher?
8. Do you think that there is students' attrition in undergraduate mathematics? If so, what factors do you believe contribute to this attrition?
9. Do you view standards-based instruction strategies useful in reducing students' attrition in your class? If so, explain and provide specific examples.

Questions for Second and Third Interview
The following were questions for the second and third interviews. However, some of the questions were revised based on previous data and classroom observations.

1. How do you relate teaching of mathematics to students' life experiences and their experiences gained in other mathematics courses and in other disciplines?
2. How do you allow students to experience mathematics?
3. How do you use multiple approaches including numerical, graphical, symbolic, and verbal in your teaching? Provide specific examples.
4. Do you integrate technology in your teaching? If so, explain what role does it play in your classroom?
5. In what ways do you incorporate questioning methods in your teaching?
6. How do you encourage interaction in your class?
7. How do you implement cooperative learning in your class? What function does it play in your classroom?
8. How do you encourage critical thinking in your class?
9. How do you encourage writing in your class?
10. How do you encourage reflection in your class?
11. How do you connect teaching of mathematics to other disciplines?
12. What role does cultural background of your students play in your teaching?
13. How do you promote academic achievement in your classroom?
14. How do you implement NCTM, MAA, and AMATYC standards in your teaching practices?
15. How do you allow your students' voices to be heard in your class?
16. How do you consider students' interest when preparing your lesson plans?
17. How do you offer any support to your students outside of the classroom?
18. In which other ways do you encourage active learning in your classroom?
19. In which other ways do you encourage student-centered learning?
20. Have you read students' reflection from your class?
21. How do you view your students' experiences about standards-based instruction?
22. How does your teaching influence your students' performance?
23. How will I know that you are implementing standards-based instruction strategies? When I visit your classroom.

## Appendix C

## Classroom Observation Guide

| Teacher's Name: | Date of Observation: | Class Period: |
| :--- | :--- | :--- |
| Number of Students: | Class Location: | Topic: |
| Opening of Lesson: | Body of Lesson: | Closing of Lesson: |
|  |  |  |

## Appendix D

## Survey Questions

1. What aspects of my instructions helped you the most?
2. Did any technology use help you in the course? If so, how?
3. Did any student-teacher interaction help you in the course?
4. Was there anything different about this course and a previous math course you have taken? If so, explain.

## Students Comments Pertaining to Mathematical Communication

The student teacher interaction overall was very helpful. Math is not my favorite subject and I sometimes don't pay attention in math classes in general. However with your class the way you interacted with us as students always had me interested and focused on the class. I never knew when you would throw me a question but I felt that I always had to be ready to respond.

Whenever I had a question about a particular problem and asked you for assistance you helped me.

I was more comfortable in asking questions about any material I didn't understand.

The interaction in the class itself was very good, therefore I was interested in the class and that helped me.
[Paul] used somewhat of a Socratic teaching method which allowed for students to answer questions, it made the class interesting and allowed us to engage with the Professor rather than a completely lecture based class room setting.
[Paul] allowed us to go up to the board and solve problems which not only helped the student who is solving the problem, but it also helped the other students to navigate how to solve the problem for the student working it out on the board.

Yes, while completing practice examples on our own, walking around the class and checking each students work to make sure we all understood the material and/or method taught

This course was very interactive. [Paul] gets his students involved in answering problems as well as allowing students to solve problems on the board. If there are
any questions he is sure to answer thoroughly before moving on. When assigning in-class problems [Paul] walks around to aide any student needing help.

The student-teacher interaction that occurred during class helped as many questions seemed futile, but were encouraged to be asked regardless. The instructor's humor also lessened the tension and permitted participation.

I really like the interaction during class. It shows that you want us to not only pass the class, but also gain an understanding of the concepts, and not just memorize, but learn the material for the short and long run. You also emphasize the concepts are merely building blocks upon which more blocks will be added.

I asked questions when I didn't understand a certain subject and I utilized the available office hours. [Paul] would also reply to emails in a timely manner.

The student-teacher interaction was very helpful because it kept the students on track and engaged.

The fact that you took the time to thoroughly explain each topic. Many teachers move at a fast pace and when they are asked a question it sometimes seems as if that they are in convinced by the question. However you answered everyone's questions no matter how much time it took for the student to understand.
You had more of a relationship with the students than in other math classes I've had.

Your style of teaching was different than any math teacher I have ever had in my entire education career. It is very engaging and makes me want to learn math. I can't fully explain the exact difference in words clearly, however I was very engaged and wanting to learn more.

Taking the time to answer all questions from students, detailed notes and PowerPoints, completing numerous examples as a class and then practicing examples on our own.

You were very engaging with the students which made it easier to ask questions and to pay attention.

## Appendix E

## Students Comments Pertaining to Use of Technology

Your use of PowerPoints helped. Having something to visually look at while going to a problem was useful.

The dual use of the projector and the board was very helpful because it allowed for you to change the problem as well as give more examples related to the particular problem in question.

The PowerPoints helped me out a lot especially out of class when doing homework and studying. Sometimes my notes were not understandable and I was able to refer back to the PowerPoint in order to understand what I was doing.

MyMathLab was very helpful. It gives great hints when your answer is off by a small error and the examples it gives helped me make it through the course.

My calculator TI 30 and the internet. I went on Tube to help me solve some problems and my calculator did most of the problems except graphing.

Well the PowerPoints helped break the book down. Also my math lab broke the material down and worked it out step by step.

The availability of the PowerPoints in class as well as online was very helpful. As a visual learner it was beneficial to my understanding the material and being able to look over things that I missed in class.

It was very helpful to use the graphing software on our laptops to better understand how to evaluate graphs

Yes, the PowerPoints and calculators helped the most because it allowed me to see a visual while learning the curriculum, enabling me to remember more material.

The notes and PowerPoints used during class, course compass is the best way ever created for completing homework and quizzes, the graph application used to demonstrate examples.

Technology played a major role in the completion of quizzes and homework assignments. The program used has extra practice problems as well as step by step instructions for those problems that give students difficulty.

My MathLab, when use frequently and consistently, helps practice the material and increases understanding

I used technology such as my laptop for my online homework and I also used a calculator to help me with my homework, test, and some classroom work.

Yes. The homework was a large help, if that counts as technology. It was extra help to further understand the concepts taught in class...

The technology that helped me in this course was the course compass materials that helped me to understand the homework and quiz assignments. This online material gave me step by step tutorial on how to solve math problems that were beneficial for tests

The description of the PowerPoint, accompanied by many examples was helpful in learning content.

Everything from Dr. [Paul's] teaching method, to the PowerPoints, to the in-class questions were different. My personal skills in Math have always been slightly below average. Through the use of course compass and the available technology that is provided today, it allowed for the class to be less overwhelming than my previous experience in math.

The fact that you had a prepared PowerPoint presentation for each lecture really helped me because I am a visual learner. I also liked the fact that you would try to get all the students to participate during class instead of just standing up there talking.

You mixed visual teaching methods with hands on. It helped when you first took us through a problem then gave us another problem to do on our own.

## Appendix F

## Sample Questions from Test 4

Graph the following functions, showing the intercepts, if any.

1. $f(x)=e^{x}$

2. $\mathrm{f}(\mathrm{x})=\frac{5}{\mathrm{x}-3}$

3. $\mathrm{f}(\mathrm{x})=\log _{2} \mathrm{x}$

4. $\mathrm{f}(\mathrm{x})=\frac{2 \mathrm{x}-2}{\mathrm{x}+3}$

5. $f(x)=e^{-x}$


## Appendix G

## Students Comments Pertaining to Excitement about Teaching

I was successful because I felt that you believed in me and because of the excitement that you had for math.

It was because of your passion, your believing in me that I was able to do this.
Unlike many other professors, [Paul] actually showed us that he cared about our education. His caring really made me strive harder and care more about my own education.

Your excitement about the material helped me to become more excited to learn and helped me to focus. You asking questions and forcing us to answer before moving on to the next question made us become more involved and I made sure I understood each section just in case I was called on.

This is truly the best math course I've ever taken. [Paul] takes the time each class to ensure that everyone understands what he's just covered before moving on to the next problem. The excitement shown over the material is something that I've never seen before. It's really the little things that made college Algebra a great class for me and helped me to fully understand the material and I was actually happy to take the course.

It really helped me because you have passion for math and you love being a teacher and you want all your students to learn the material. This has been the only math class through my educational life that I have really enjoyed coming to class-class was never boring and it kept my interest in math.

The way you explained the material, and the way you worked the examples on the board helped the most. And the way you were always smiling and so glad to teach math kept my attention in class.

I have not had any other math courses in college, but compared to the ones in high school I would say that I only had one other math instructor that kept my attention during class and who had put energy into teaching the material.

Yes, the teacher was enthusiastic as he taught the curriculum instead of just going through the movements.

The instructor's thorough explanation of each concept as well as his enthusiasm for teaching. [Paul] teaches in a way that anyone can comprehend and apply. The material became something I was able to do with ease and not have anxieties about.

The main thing that was different and that helped me succeed in this course was the instructor, [Paul]. Because of his enthusiasm for math and his helpful methods of teaching math became fun again. [Paul] has an admirable dedication to seeing his students succeed.

